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Foreword

THIS volume of the TRANSACTIONS OF THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS is of the same form and size as volumes 49 and 50. It complies with the publication policy of the Society consummated in 1928 under which the TRANSACTIONS are issued in pamphlet form in order that they may become more quickly available to members.

The pamphlets, forming sections of the TRANSACTIONS, are issued periodically at the rate of about three a month, and each section is sponsored by one of the Society's professional divisions. A member of the Society may register in any three professional divisions and may receive, without cost, the sections of the Transactions which are sponsored by these divisions. The periodical publication of these sections in magazine form has a feature of timeliness which justifies the inclusion of papers of specific interest to a single group of members without the cost of distribution to all. This policy, combined with increased activity on the part of the professional divisions, has made available a much greater number of engineering papers.

COMPLETE TRANSACTIONS BOUND FOR PERMANENT RECORD

For permanent record and for reference use in depositories and libraries a number of sets comprising all of the sections of the Transactions have been bound, and are issued in two books which form parts I and II of volume 51. The division into two books is for convenience in binding and handling, and together these comprise volume 51 (1929).

NUMBERING AND ARRANGEMENT OF PAPERS IN COMPLETE TRANSACTIONS

In this volume the papers of each section are grouped together and are numbered serially. The different sections follow each other alphabetically. Each paper is designated by a symbol composed of key letters followed by the volume number and paper number. The letters refer to the section of the Transactions to which the paper is assigned. Thus AER refers to the Aeronautic section, IS is the Iron and Steel section, etc. Each of the two books forming parts I and II of volume 51 contains a complete index of subjects, authors, and discussers to both parts of the volume so that either may be consulted. The arrangement of the index and designations for locating material are explained at the beginning of the index.

CONCERNING BLANK PAGES

The papers in each section of the Transactions are printed separately so that they may be available as reprints. This makes it necessary to print each paper so that the number of pages is a multiple of four. Because of this blank pages will be found at the ends of papers which do not completely fill the printer's form. This is the reason for the blank pages which will be found scattered throughout the volume.

THE PUBLICATIONS COMMITTEE.

Progress in Aeronautics

Contributed by the Aeronautic Division

Executive Committee: Edwin E. Aldrin, *Chairman*, Alexander Klemin, *Secretary*, Archibald Black, Elmer A. Sperry, Wm. F. Durand, and Chas. H. Colvin

Power Plants[1]

THE outstanding feature in the development of aircraft engines during the past year is the great number of engines, both in quantity and in new types, which were produced for non-military activities. The various points of aircraft-engine design to give a light-weight, reliable engine are sufficiently well established that many of these engines give satisfactory performance beginning with the first article produced. However, for the protection of the public, the Department of Commerce conducts tests to determine the airworthiness of aircraft engines. (Department of Commerce Aeronautics Bulletin No. 14, "Requirements for Approved Type Certificates.") By the end of September of this year, airworthiness certificates had been issued for nine engines. Data on these engines are summarized in Table 1. Engines which have passed the standard endurance tests of the United States Army Air Corps or the United States Navy Bureau of Aeronautics will be approved by the Department of Commerce.

There are at least fifty different aircraft engines below 300 hp. at various stages of development. Most of these engines weigh below 4 lb. per b.hp. About one-third weigh 2½ lb. or less per b.hp. The air-cooled radial engine is the predominant type of engine of this class. The price per brake horsepower ranges from $25 for the small engines to as low as $15 for the larger ones.

The Allison Engineering Company has redesigned the accessory drives of the inverted Liberty engine so that both oil and water pumps are gravity fed and the ignition distributers are driven from the camshaft driveshafts instead of from the end of the camshaft. This engine has been thoroughly ground-tested and is now being flight-tested by the Army Air Corps in a Loening Amphibian plane. It is designated as the Allison V-1650.

An item of considerable interest to many is that the Army Air Corps has let the last contracts for reconditioning Liberty 12 engines. Approximately 2000 engines are being reworked, of which about two-thirds are being done by the Allison Engineering Company and the remainder by the Steel Products Company of Springfield, Ohio. Some of the principal changes being incorporated in these engines to improve their reliability are replacement of the old carbon-brush ignition distributers by the air-gap type distributers with coils tested at high temperature, new stub-tooth timing and accessory gears, replacement of bronze-backed by new steel-backed connecting-rod bearings, modifications to provide drives for a fuel pump and the inertia type of starter, selection of heavy-type crankcases, and modification of pistons and rings to reduce oil consumption. After this stock of engines is depleted, the Army Air Corps can equip all of its airplanes with the latest-type high-performance engines.

The Curtiss Aeroplane and Motor Company has produced a new six-cylinder, double-throw crankshaft, air-cooled radial engine of 600 cu. in. displacement, which they have named the "Challenger." They guarantee 170 b.hp. at 1800 r.p.m. The Army Air Corps is to test one of these engines for training-airplane service.

[1] Prepared by N. N. Tilley, Chief, Engine Development and Specification Unit, Power Plant Branch, U. S. Army Air Corps, Wright Field, Dayton, Ohio.

The Army Air Corps is flight-testing a Curtiss H-1640 ("Chieftain") engine and has adopted, as standard, Curtiss V-1570 and GV-1570 ("Conqueror") engines. The H-1460 is an air-cooled engine. The V-1570 and GV-1570 engines are water-cooled direct drive and geared engines, respectively. These engines develop 600 b.hp. The Curtiss bombing airplane with full military load, equipped with two GV-1570 engines, has a high speed and a rate of climb as good as many pursuit-type airplanes at the end of the War.

The Packard Motor Car Company is developing an air-cooled radial Diesel engine. It is reported to weigh approximately 3 lb. per hp. This marks a big step forward in the aircraft Diesel-engine project. While the fuel economy of this type of power plant may not be appreciably better than that of the aircraft gasoline engine, the use of a non-volatile fuel should considerably reduce the fire hazard.

The Pratt and Whitney Aircraft Company has brought out the series B Wasp engine, developed from the earlier model. This engine is rated at 450 hp. at 2100 r.p.m. With the power of this engine somewhat boosted at sea level by speeding up the centrifugal fan in its induction system, the new Boeing superfighter is reported to have a speed in level flight of 180 m.p.h. with full military load, and to have remarkable maneuverability.

A quantity of Pratt and Whitney Hornet engines and Wright Aeronautical Corporation Cyclone engines is being procured by the Army Air Corps for twin-engine bombing planes, and by the Navy for torpedo planes. Hornet engines are being used in the Boeing Air Mail Service. These engines are rated at 525 hp. at 1900 r.p.m., with a dry weight of not over 775 lb. Both are nine-cylinder air-cooled radial engines.

The Wright Aeronautical Corporation has developed a new J-6 series of engines, each of which will weigh less than the J-5 (Whirlwind) engine. They are 5-, 7-, and 9-cylinder, air-cooled radial engines of 160, 220, and 300 hp., respectively. They offer many new possibilities to the designers of both military and non-military airplanes.

TABLE 1 DATA ON AIRCRAFT ENGINES GRANTED AIRWORTHINESS CERTIFICATES BY THE U. S. DEPARTMENT OF COMMERCE

Certificate No.	Manufacturer	Engine name and designation	Rating B.hp.	Rating propeller r.p.m.	Max. dry weight, lb.
1	Fairchild-Caminez Engine Corp.	447-C	120	960	340
2	Warner Aircraft Corp.	Scarab (R-420)	110	1850	270
3	Kinner Airplane & Motor Corp.	Kinner K-5 (R-370)	75	1725	260
4	Velie Motors Corp.	Velie (R-250)	45	1750	210
5	Curtiss Aeroplane & Motor Co., Inc.	Challenger (R-600)	170	1800	420
6	Curtiss Aeroplane & Motor Co., Inc.	Conquerer (V-1570)	600	2400	760
7	Curtiss Aeroplane & Motor Co., Inc.	Conqueror (GV-1570)	600	1200	870
8	Curtiss Aeroplane & Motor Co., Inc.	Chieftain (H-1640)	600	2200	925
9	Aircraft Engine Co., Inc.	Comet (R-610)	115	1775	350

Large-engine development lost considerable impetus when twin engines in preference to a single engine for bombing airplanes and three-engined transports were selected by the Army Air Corps. However, larger disposable loads or maximum speed at any cost will demand larger engines, even in multi-engined planes. Large engines awaiting test by the Army Air Corps are the Allison X-



THE BOEING TRANSPORT

4520 engine and the Packard 4A-2500 engine. The X engine is air-cooled, has 24 cylinders arranged in four banks of six cylinders each, at 90 deg. to each other, is geared 2:1, and is rated at 1200 hp.

The Packard engine, one of their 800-hp. engines, modified to incorporate two gear-driven centrifugal compressors, one on each side, and has developed 950 hp. during a one-hour acceptance test. Over 1000 b.hp. should be obtained from this engine. It is expected to test these engines when the new power-plant laboratories of the Material Division are completed.

A number of refinements and new features in engine design have appeared during this past year. Among these are the finer pitch of cooling fins resulting from research at the Massachusetts Institute of Technology, radio-shielded spark plugs, flange-and-spigot-mounted aircraft magnetos, lighter propeller reduction gears, and the Chandler-Marvel mechanical fuel-metering device which may be expected to improve airplane maneuverability, to eliminate intake-manifold distribution difficulties, hot spots, and air heaters, and to facilitate two-cycle development.

Supercharger development has reached the position that all high-speed, high-rate-of-climb or altitude airplanes will employ them. The General Electric exhaust-gas-turbine type now has excellent durability. Some of these units have been in service for over a year, with hundreds of hours of operation without failure.

A standard Army pursuit airplane, equipped with the General Electric turbo supercharger, with full military load, has gone to 20,000 ft. altitude in less than eleven minutes several times, and has a rate of climb at that altitude in excess of 1500 ft. per min., and a service ceiling of 32,000 ft., which is reached in approximately 30 min. The gear-driven centrifugal supercharger is built into all the large American radial engines. The Army, Navy, National Advisory Committee for Aeronautics, and engine manufacturers are experimenting with the Roots-blower type.

During the past year the Army and Navy have agreed on many power-plant standards and are considering many more items for A-N standardization. Some of the engine parts standardized or being standardized are propeller-shaft ends and hubs, drives and mounting flanges for starters, magnetos, electric generators, fuel pumps, and tachometers. Type designations have been assigned new engines which consist of a number representing the displacement in cubic inches to the nearest 10 cu. in., and a letter prefix to designate the cylinder arrangement and whether direct drive or geared down to propeller, as R for direct-drive radial, GR for geared radial, V for direct-drive vee, GV for geared vee, and H for hexagon.

AIRPLANE DESIGN AND CONSTRUCTION[2]

In the field of design and construction the airplane is still evidencing a steady improvement in details and refinements, with no radical departures from the accepted standards. Particular attention is being paid by commercial manufacturers relative to refinements in appointments of their machines. In the matter of design details a very marked showing is manifest in the so-called "cleaning up" of the exposed portions of the machine, in order that the limit of speed may be secured. Generous fillets given to the junction of the wings and tail surfaces with the body, retraction of fittings within the wings and fuselage, and a general streamlining of the landing gear and other exposed assemblies have directly resulted in higher speeds, the power and pay load remaining the same. The recent transcontinental air races are evidence of this fact, as here such planes as the Laird and Waco, identical with former designs except for details, made speeds as much as 20 per cent in excess of former records. With the introduction of tail wheels to supplant the tail skid and the continued increase in the use of wheel brakes, the ground-handling problem of the larger as well as smaller planes is much simpler. Hydraulic or oleo shock-absorbing struts for landing gears have received numerous modifications with the use of air, rubber in compression or tension, and steel springs for the static-load or taxiing absorbing medium. Most popular of these is the Aerol strut produced by the Cleveland Pneumatic Tool Co. Triangular or elliptical tail surfaces with almost a universal application of front-spar adjusting gears for the stabilizer are to be noticed. Inset "Frise" ailerons with the remaining control surfaces carrying little or no balance appear as the trend in design regardless of the size of the machine.

More attention is being given to the comfort of the pilot and passengers. Upholstery comparable to that in motor buses and adequate ventilation and heating facilities are replacing the somewhat makeshift appointments that appeared in the earlier cabin-type airplanes. Outstanding among the transport machines is the Boeing, with an electric grill and cabin boy as standard fixtures. Color has not been lacking in the airplane, as various wing and fuselage combinations with suitable striping have brought out lines in a manner similar to that of the modern motor car. With the demand for colors has come a call for special dopes and lacquers that will give high-luster finishes. Berry Bros., du Pont, and the American Varnish Company have contributed greatly to this problem. Military aircraft are not without some vestige of color, as the olive drab is giving way to high-visibility yellow and suitable combinations.

[2] Prepared by Mac Short, Vice-President and Chief Engineer, Stearman Aircraft Co., Wichita, Kan.



THE VELIE MONOCOUPE

Types among military and commercial manufacturers are definitely appointed, and it is rare that a factory enters both fields. Among the military planes the biplane arrangement is still in the majority; however, a number of Fokker monoplanes are being used by the services for transport. In the commercial field the interest is fairly well divided between the monoplane and the biplane, and again between open and closed models. A recent summary of the licensed airplanes as approved by the Department of Commerce gives interesting facts. Of approximately 50 approved types 25 per cent are three-place open-cockpit biplanes, 16 per cent 5- to 8-place monoplanes, 6 per cent open two-place biplanes, 6 per cent open two-place monoplanes, 6 per cent closed 5-place biplanes, 4 per cent closed 3-place biplanes, and 12 per cent miscellaneous types, such as amphibians, flying boats, and large transports. The latter have received an added impetus due to the rapid establishment and growth of large transport lines. At present the majority of transport planes are monoplanes; that is, the Fokker and Ford versus the Boeing. Much interest will be centered around these machines in the next year.

That the trend is toward more power may be evidenced by the introduction of the Whirlwind engine in smaller planes and the replacement of this engine by the Pratt and Whitney Wasp in the medium-sized cabin models. A further extension in this direction is the replacement of the Hornet and Cyclone engines for the 400-hp. ones. In the military and commercial field alike the larger machines have already reached the maximum-powered units and must either introduce more units or await further development of higher-powered engines. In the multi-engined class the tri-motor of an aggregate of 1200–1500 hp. is gaining favor. With such power a pay load of 14 passengers may be transported at a speed of 140 m.p.h., and, as the air-transport operator points out, speeds of this order are becoming increasingly more necessary. The four-motored combination in which two motors are in tandem has not been without favor in the prospectus or on the drafting board, but as yet it has not appeared on American air-transport lines.

Metal construction either as a composite of steel or duralumin or entirely of the latter is making added inroads into the more common method of construction wherein wood wings and steel-tube fuselages and tail surfaces comprise the main structure. The Pruden, Thaden, Hamilton, and Flamingo are newcomers that are using steel or aluminum alloys for the main structural parts and duralumin sheets for the covering, either corrugated or flat sheets, with frequent seams turned up. Numerous manufacturers are using tail surfaces of duralumin where otherwise little or no use of this metal is made; the assumption being that the operation is easier and the moment that the weight of the assembly produces is materially reduced. Simultaneously with the use of steel and duralumin sheets and sections appear castings and forgings. Many planes show the application of this method of manufacture where in former years elaborate welded or riveted fittings were used. A fair reduction in weight and a lessening of manufacturing cost of as much as 200 per cent has been effected by cast or forged fittings. It is evident, however, that greater production, both in military and commercial craft, has permitted this improvement in methods.

Design characteristics have been improved and construction refined by advances in accessories. Metal propellers, which have substantially replaced wood for all engines of 200 hp. or more, have increased in efficiency, the so-called wide-blade type being very popular. The Curtiss Company has developed a new forged propeller with an integral hub. Hydraulic and mechanical brakes are more prevalent, the latest being the "Steel-Draulic" built by the Aircraft Products Co. Wheels and tires, both large and small, are in greater demand than before. The transport machines have exceeded the working load on 44-in. tires and wheels, and the lighter craft have demanded smaller tail wheels than the 14-in. size. At the upper end of the scale the problem is as acute as at the lower, except here it is possible to dual the tires, while with small tires the problem of mounting and of the presence of a hub for the wheel becomes difficult. Instruments have become more standard in regard to mounting dimensions, and sizes are being reduced. This latter improvement allows a more desirable grouping of important instruments on those craft that do night and "blind" flying. Electrically heated pitot tubes for the air-speed indicator have aided in flying during the icing period that planes experience in freezing weather.

Records have been broken in all manner of flying. Crossing the continent in approximately 18 hours, remaining aloft in an endurance flight for more than 55 hours, high speeds in excess of 300 m.p.h., outside loops, altitude tests, and many other records have been shattered during the past year. Records in production of all aircraft have reached values never anticipated before. Outstanding in quantity for the past year are the factories making the Monocoupe and Waco planes, where production reached the high mark of 5 to 6 machines per day. In military craft Vought showed excellent manufacturing methods by producing a "Corsair" a day.

Some foreign planes have been introduced, principally in the light field. Avro and De Havilland have both been represented by slotted and non-slotted wing biplanes using the Cirrus engine. Klemm light planes are being manufactured by Aeromarine.

Probably the most radical design evolved this year is the new Bellanca P with its odd bracing and retracting landing gear.

Aerodynamics[3]

The British Aeronautical Research Committee reports some interesting experiments with a "pilot plane." A pilot plane is an auxiliary airfoil pivoted ahead of a wing so as to provide automatically a slotted wing at high incidence without much increase of drag at fine angles with a 10 per cent pilot plane. The maximum lift coefficient of a R.A.F. 15 airfoil may be increased to 0.80 (British absolute units), and the increase of drag at top speed is less than the minimum drag of the pilot plane when tested alone. The increase in drag is greater on the climb. By making the stop which limits the pilot angle vary with the aileron angle, the pilot plane gives an efficient slot-and-aileron control, the pilot plane reversing the sign of the yawing moments at angles just above the stall.

The delivery of the lecture by F. Handley Page on slotted wings before the Society (Detroit Aeronautic Meeting, June,1928) has further increased American interest in the subject of the slot, and at least three full-scale experiments are being made with this device in the United States.

The recent flight of the Autogiro from London to Paris has increased interest in this type of aircraft. With the application of more power the Autogiro has shown increased speed, although the maximum speed attained thus far is still below 100 miles per hour. The British have been steadily continuing full-flight wind-tunnel dropping and theoretical investigations on this type of aircraft. A summary of its characteristics indicates that its performance is comparable with that which would be predicted by the application of theory. The performance of the Autogiro is inferior to that of a corresponding airplane of the same weight and horsepower as regards speed, rate of climb, and distance to take off. The Autogiro is able to land more slowly and to pull up more quickly after landing, and, in contrast with an airplane of conventional design, remains stable at large angles of incidence. The only point which remains uncertain is its ability to descend vertically at low speed. This question is not considered to be of much practical importance, however, since the actual gliding descent at an inclination of 30 deg. to the horizontal implies the possibility of vertical descent when the wind speed exceeds 20 m.p.h. The rate of rotation of the windmill is found to increase slowly with the forward speed.

The theory of performance testing is continually being discussed by British writers. In this regard it is important to determine the law of variation of engine power with height. Glauert concludes that it is more accurate to state that the power of an engine varies directly with the pressure, rather than directly as the density. A more exact statement is that the power is a function of $(\text{pressure})^{3/4} \times (\text{density})^{1/4}$.

A non-dimensional form of the stability equations of an airplane has also been devised by Glauert. The comparison of the stability criteria of different airplanes should be facilitated thereby.

The lateral stability of an airplane under control is a subject of some importance which has not hitherto received much importance. Lateral stability depends upon the values of the resistance derivatives of side force, rolling moment, and yawing moment due to side slip, rate of roll, and rate of yaw. In actual design most of these derivatives are fixed before the lateral stability is considered. Designers will be gratified to learn, therefore, that stability depends mainly on the ratio of L_v to N_v, and not on their absolute values, provided L_v is negative.

In the aerodynamic design of the airplane, both British and American designers are seeking the utmost refinement in streamline. The De Havilland Tiger Moth and the Lockheed Vega are examples of the tendency to give a commercial airplane the form of a racer. The subject of interference between parts, the cowling of air-cooled engines, and propeller interference are engaging much attention. Prof. Melville Jones arrives at two conclusions of practical importance: (1) In a perfectly streamlined airplane, interference, except of the most extreme kind, can have little influence upon overall performance, apart from a slight superiority in favor of the pusher airscrew over the tractor airscrew. (2) Serious losses due to interference must therefore be attributed to bad streamlining. An important investigation of the Deutsche Versuchsanstalt für Luftfahrt on the oscillation of the internally braced monoplane indicates that the period of vibration of such a wing varies between 500 and 900 per minute. Danger of resonance between the engines and the wing is only likely to occur when the engine is throttled down to a large extent.

The propeller research laboratory of the National Advisory Committee for Aeronautics is being used energetically for investigation of propeller and body interference, the cowling of air-cooled engines, and other problems of practical importance.

Researches at Langley Field, Wright Field, and by the Bureau of Aeronautics seem to indicate that spin, whether flat or normal, can be controlled largely by mass distribution.

The amount of aerodynamic research now being conducted by American institutions renders even a summary difficult. It may be safely said that in this respect American effort is now fully on a par with European work.

Scale effects in particular are being studied intensively in the compressed-air tunnel of the National Advisory Committee for Aeronautics.

The problem of wing flutter has been receiving much attention. Experiments by Zahm and Bear at the Washington Navy Yard have established that resonant flutter is not likely to ensue from turbulence of air flow alone past wings and tail planes in usual flying conditions. Danger of flutter is minimized by so proportioning the torsional resisting moment of the wings to the air pitching moment at high-speed angles that the torsional flexure is always small. Green's paper, "An Introduction to the Problem of Wing Flutter," recently read before the Society, is an important presentation of the present status of the problem.

The importance of the flat-top lift curve as a factor in control at the stall has long been recognized. One or two sections have been designed and tested at Langley Field. They have given the desired flat-top lift curve, combined, however, with low aerodynamic efficiency and high moment coefficients.

Air Transport and Other Civil Flying[4]

Since the preparation of the Progress Report for 1927, further and very important progress has been made in the development of air transport in the United States. A considerable number of new routes have been added, and operations on many former ones have been extended. Several air-transport companies are now earning more than the cost of operation, and some are able to show comfortable profits. Already a tendency toward consolidation has become evident, and one of the noticeable changes in this line was the taking over of Pacific Air Transport by Boeing Air Transport, giving the latter company a total operating route of 3017 miles.

Air-mail traffic has continued to increase rapidly, and the inauguration of the reduced air-mail rates on August 1 brought a further sudden increase in traffic, almost doubling it. With the new rate in effect many concerns are sending all first-class mail

[3] Prepared by Alexander Klemin, Professor of Aeronautical Engineering, Daniel Guggenheim School of Aeronautics, New York University, New York, N. Y. Mem. A.S.M.E.

[4] Prepared by Archibald Black, President and General Manager, Black & Bigelow, Inc., Air Transport Engineers, New York, N. Y. Mem. A.S.M.E.

by air. Some of the more enterprising ones have gone further and are sending out much of their advertising material under air-mail postage. The increased traffic due to this has necessitated an increase in facilities on the part of some operators, and this in turn has resulted in reducing the unit operating costs. An official of one operating company (the Boeing) has recently been quoted as estimating that its airplanes carry 15 per cent of the total first-class mail on one route, the remainder being sent by rail. One development of more than usual importance is the extension of air-mail routes from the United States to Canada and to Central America.

With the air-mail traffic now firmly established, many of the operators have begun to turn their attention to the further development of passenger traffic. It is in this class of activity that the United States has trailed behind Europe, although taking the lead in many other branches of aviation. A number of passenger routes are now in operation, and greater effort is being spent on the development of this type of traffic than heretofore. While always dangerous to make predictions in aviation, it is safe to expect that passenger air travel in this country will soon show a growth comparable with that of air mail in the past.

The recent changes in equipment used for air transport in this country have been mainly in the construction of multiple-engined passenger-carrying airplanes of larger capacity, ranging up to as high as 20 passengers. On some routes there has been a tendency to increase the scheduled speed of trips for the purpose of advancing mail connections, and this has already become reflected in the construction of mail-carrying designs of somewhat higher speed and capable of maintaining an average of about 110 m.p.h. under even adverse conditions. Miscellaneous commercial flying (including "aerial taxi" services) has increased in parallel with the increase in air-mail operations. Several large corporations have purchased airplanes for their own use and are applying them to the transport of high-salaried executives, carriage of rush orders, repair parts, and similar uses where speed is of primary importance. Private flying has similarly undergone a substantial increase, due in a large measure to the attention drawn to aviation by the numerous long-distance flights. Flying clubs are now making their appearance in various parts of the country. In some cases these are formed by wealthy men owning their own airplanes, and in others, by the less prosperous who form a club for the joint purchase of an airplane. The development of aerial express traffic has had no spectacular increase in the past year, but some progress has been made nevertheless.

One of the most significant developments of the past 12 months has been the entry of the railroads into the aviation field. Such an extension of the scope of the railway operators has been discussed frequently during the past few years, but approached reality only with the organization of Transcontinental Air Transport. This corporation was formed by some of the leading railroads, airplane operators, and builders for the purpose of creating joint air-rail services in which the passenger is carried part of the way by rail and the remainder of the trip by air. Initial operation of services of this character was started during the past summer.

As a consequence of all of this increased interest in aviation there has been a very large increase in the number of schools and of students taking flight training. Figures compiled by the Department of Commerce indicate that the number of students now taking up flying about equals the total number of pilots licensed. Interest in airport construction has been continued and has increased to the extent where most of the larger cities have at least some kind of flying field, and the city without one is coming to be regarded as behind the times.

Continued and greatly increased attention has been given to the selection, design, construction, and equipment of airports. There are now several companies scattered throughout the United States which specialize in this work. While general agreement seems to exist as regards most angles of airport work, there appears to be some variation of opinion as regards the actual layout of the airport itself. In general, this difference seems to resolve itself into a question of planning the layout with regard to the handling of heavy traffic in the future. Some rather novel plans have been advanced, both here and abroad, but it seems rather early to comment on them, and to include description herein would needlessly burden this report.

Such figures as are available on air-mail traffic and mileage in air transport are included in Tables 1 and 2, and are fully self-explanatory.

TABLE 2 MILEAGE AND TRAFFIC STATISTICS FOR PRIVATELY OPERATED AIR-MAIL ROUTES WITHIN THE U. S. A., FOR YEAR ENDING JUNE 30, 1928

(Compiled from Post Office Department figures)

Date	Pounds of mail carried	Mail revenue to operator	Airplane-miles flown with mail
July, 1927	99,589	$250,891	375,923
August	102,051	258,963	376,998
September	146,486	312,810	444,519
October	153,649	329,532	449,562
November	141,992	304,083	413,054
December	165,768	353,253	402,267
January, 1928	144,289	300,281	431,310
February	153,363	329,380	450,058
March	173,929	368,191	523,287
April	171,028	359,182	513,144
May	199,409	425,565	616,179
June	210,957	447,642	595,054
Totals for year	1,862,510	$4,039,773	5,591,355
Increase over previous years, per cent	265	205	115

TABLE 1 AIR-TRANSPORT MILEAGE OF THE WORLD[1]

(Operations of scheduled services in airplane-miles over 12-month periods as nearly possible comparable)

Country of registry	Year 1919	1920	1921	1922	1923	1924	1925	1926	1927
Australia	.....	.....	.....	.....	235,582	262,895	352,847	417,964	.....
Austria	.....	.....	.....	.....	.....	.....	.....	.....	245,043
Belgium	.....	.....	.....	.....	.....	160,000	144,500	.....	.....
Canada	.....	.....	.....	.....	.....	294,778	446,648[4]	631,715[4]	.....
Colombia	.....	2,687	53,650	125,678	134,375	169,500	183,206	.....	.....
Czechoslovakia	.....	.....	.....	.....	.....	126,400	48,300	170,895	257,888
Denmark	.....	.....	.....	.....	56,000	.....	114,500	126,730	116,798
France	.....	505,500	1,460,000	2,015,000	2,115,000	2,249,000	2,946,000	3,243,900	3,755,369
Germany	.....	.....	.....	842,986	885,881	1,860,000[2]	3,070,000	3,816,144[6]	5,921,593
Great Britain	168,000	599,000	259,000	778,000	1,004,000	890,000	865,000	840,000	873,000
Italy	.....	.....	.....	.....	.....	.....	.....	.....	824,474
Mexico	.....	.....	.....	.....	.....	.....	.....	96,800	146,400
Netherlands	.....	50,850	217,000	246,200	336,000	482,800	679,753	597,500	813,510
Poland	.....	.....	.....	66,293	142,057	233,923[3]	465,847	.....	654,873
Soviet Russia	.....	.....	.....	.....	.....	288,600	292,595[5]	311,000[5]	.....
Sweden	.....	.....	.....	.....	.....	69,280	248,610[6]	222,000	206,766
Switzerland	.....	.....	.....	.....	.....	268,400[2]	87,427	210,340	459,720
United States	393,066	880,028	1,828,354	2,329,296	1,743,030	2,220,761	2,910,611	4,407,263	6,009,226
Totals	561,066	2,038,065	3,818,004	5,903,453	6,651,925	9,576,337	12,855,844	15,092,251	20,138,260

[1] Compiled by Black & Bigelow Inc., New York, N. Y. [2] Figure uncertain or known to include duplication. [3] Revised figures. [4] Including some non-scheduled flying. [5] Incomplete. [6] All services suspended for 3 months.

Progress of Naval Aviation[5]

The past year has witnessed greater aviation activity in the Fleet. The general tendency in Fleet aviation is toward reducing the size of the type for each purpose, at the same time maintaining or improving its performance.

The commissioning of the aircraft carriers *Lexington* and *Saratoga* has aided materially in solving the problem of the role of planes at sea; naturally their operation brings about new problems to be worked out, and progress in this direction is also being accomplished. More extensive use of aircraft on the battleships and cruisers has given the same result.

Further advance in connection with metal construction is in full swing. The prevention of corrosion of light alloys is progressing, and these alloys as well as high-strength non-corrosive alloys are entering into all recent construction.

Of our service aircraft on hand June 30 of this year, 82 were four years old, 16 were three years old, and the remainder had been in service two years or less, hence the Navy is outfitted with modern planes and equipment, wooden propellers and wartime engines being obsolete.

The development of air-cooled engines is in a healthy state, with two permanent companies competing vigorously and several other companies engaged on experimental projects which are of considerable interest. The same competition exists in the field of metal propellers, and experiments on instruments and accessories are also pending which promise desirable results.

Now that the demands of Fleet aviation are approaching solution, the patrol types are again being placed in production. While these planes are too large for operation from ships, they operate to great advantage with tenders or from shore bases. Recently a plane of this type, systematically developed as an experimental project, broke several world's records for endurance and weight carrying. This type is now in production, and three other types which are at present in the experimental stage will be completed before the end of the year.

Thick weather and fog still present problems to which several possible solutions have been offered, but which require extended investigation and development. Progress has been made in navigation, but the safe landing of heavier-than-air craft in a fog is yet to be satisfactorily solved.

Military requirements have resulted in improvement in design staffs of contractors. The Five-Year Building Program with its steady demand has resulted in better organization and a larger force of skilled mechanics. To a degree the same is true of materials, but in this case the increasing demands have served to develop the interest of manufacturers. These good effects are bound to increase, and therefore the Navy Department finds a direct interest in the promotion of civil aeronautics through the development of resources in skilled personnel, materials, and organization.

The Marine Expeditionary Force, particularly in Nicaragua, has gained much valuable experience with up-to-date aircraft under actual operating conditions, clearly demonstrating the value of aircraft and bringing out the lines on which progress should be made. Particularly has the field of military-transport operation and requirements, observation and attack, been well covered.

In liaison with the Army Air Corps, a number of interesting projects are being investigated; duplication is avoided by each service keeping in close contact with the other on questions of mutual interest. When practicable, specific problems are allocated to Government department especially equipped for their solution, notably the National Advisory Committee for Aeronautics, the Bureau of Standards, the Forest Products Laboratory, and the Bureau of Mines.

In the lighter-than-air field, the *Los Angeles* and the semi-rigids at Lakehurst have been steadily engaged in training and development projects. Methods of mooring and handling have been improved, and much information has been gathered which will be of benefit in the construction and equipment of the large rigids about to be contracted for. The *Los Angeles* has demonstrated that it can land aboard a carrier and be refueled, regassed, and reprovisioned, making possible a greatly extended field of usefulness as well as demonstrating that emergencies can be met. It is expected that two of the largest rigid airships ever built will visit this country shortly and give a convincing demonstration of the commercial possibilities of this type.

To sum up, the Five-Year Building Program, a wise piece of legislation, is progressing in good order, and Naval progress has been advanced thereby. The establishment of the Aviation Branch of the Department of Commerce, and of the three Secretaries for Aviation, has aided not only military development but commercial development. As shown, this commercial development is an important aid in the progress of development of military resources.

[5] Prepared by Rear-Admiral W. A. Moffett, U. S. N., Chief of the Bureau of Aeronautics, Washington, D. C.

Progress in Military Aviation[6]

Progress in military aviation and its effect upon aviation in general during the past year has been as rapid as in any preceding year, but this effect has not been so apparent because the development was more in the nature of standard types of planes than in actual change of design. The Air Corps has concentrated upon the training of pilots, refinement in tactics, engine development, and on the planes and matériel. As now established Air Corps planes constitute six principal types: Attack, Bombardment, Cargo, Observation, Pursuit, and Training. The development has been limited largely to adapting existing planes to the newly developed aircraft engines in the 500- and 600-hp. classes.

In the line of equipment, suitable electrical and inertia starters have been made available for all service engines, and initiatory development of double-voltage electric generators gives promise of a big saving in the weight of radio installations.

Progress is being made along the line of metal construction. Initial cost, difficulty of repair, corrosion prevention, and increased weight have been potent factors in retarding the extensive use of metal covering for airfoils, it being felt that a definite and favorable answer to all problems should be obtained before metal construction is generally adopted for peacetime use.

All service airplanes, except primary training, now purchased by the Air Corps are provided with separately controlled landing-wheel brakes operated by pedals on the rudder bar. Investigation of the so-called "flat spin" which caused the crash of a standard service-type airplane led to extensive flight and wind-tunnel research to determine what condition characterized this spin.

Attack Types. The present attack airplane issued to the service is a modification of the Standard 0-1 (Curtiss) Observation type known as the A-3. This airplane meets requirements of the Attack group better than any previous type heretofore used.

Bombardment Type. Continuation of the bombardment program initiated last year resulted in comparative tests of six different airplanes to determine their suitability for procurement to replace the present types.

The most suitable airplanes were found to be the XB-2 and the XLB-6, procurement being made of both models which are biplanes, the former mounting two VG-1570 (Curtiss) engines developing 600 hp. each, and the latter mounting two R-1750 (Wright) engines developing 525 hp. each. An equal quantity

[6] Prepared by Major-General J. E. Fechet, Air Corps, U. S. A., Chief of the Air Corps.

of LB-7 airplanes which differ from the LB-5 model only in the substitution of Pratt & Whitney R-1690 engines is also being procured. The performance and flying qualities of these bombers will greatly exceed those of any type now in service.

Cargo Types. In the cargo class two airplanes were adopted for service use, the three-engine C-2 (Atlantic) series as standard, and the single engine C-1 (Douglas) series as substitute standard. Following its momentous flight to the Hawaiian Islands last year, the C-2 completed a very satisfactory service test which demonstrated its reliability and utility for cargo use. On the other hand, the single-engine C-1 model affords a less costly airplane suitable for use where reliability is not such an important factor.

Observation Types. There are four standard observation airplanes at present: namely, the O-1 (Curtiss) series, the OA-1 (Loening Amphibian) series, the 0-2 (Douglas) series, and the 0-11 (Curtiss) series. All of these are equipped with Liberty engines with the exception of the 0-1 series, which uses the Curtiss D-12 engine.

Experimental development was carried on with a new model Amphibian, incorporating a new vee-type air-cooled power plant and a single landing wheel in the center of the hull.

Pursuit Types. The P-1 (Curtiss) and PW-9 (Boeing) series of airplanes, using Curtiss D-12 engines, continue as standard pursuit types.

A few supercharged P-1 type airplanes were recently procured, under model designation P-5, for high-altitude work. These are equipped with Curtiss D-12-F engines with side-type superchargers.

Training Types. The PT-3 with Wright J-5 air-cooled engine was approved as the standard training type, in addition to the PT-1 which mounts the Wright E water-cooled engine.

Other Developments. Considerable work was done in aerodynamical research studying control characteristics, automatic leading-edge slots, methods of predicting performance of airplanes, and in wind-tunnel experiments. In general airplane development, make-ups have been constructed to determine ideal cockpit arrangements for corps observation, twin-engine bombardment and twin-engine airplanes for night observation, long-range day observation, and light bombing.

Aircraft Power Plants. The outstanding characteristic in power-plant development of the year was expansion of the air-cooled-engine field, with progress centering about the conventional radial type. Demands for increased power and flight performance, however, emphasized the need for experimentation on other types of air-cooled engines, and the more important experimental developments have been in this direction.

Progress on air-cooled engine design has, while satisfactory, not warranted abandonment of the water-cooled type, because there is yet no definite assurance that high-speed supercharged engines can be cooled successfully with air, nor has the development of the air-cooled engine reached the stage where it can supplant the water-cooled engine satisfactorily for high-speed work. These reasons, together with excellent prospects of improving performance of water-cooled power plants through higher crankshaft speeds, supercharging, and higher cooling temperatures, have warranted continuance of experimental water-cooled development.

Supercharger development has continued on the exhaust-turbine type, the Roots type, and the built-in geared centrifugal type. A new exhaust-driven supercharger, the Form F-2, was developed for 600-hp. engines and incorporates several general improvements over previous models.

The most interesting and possibly the most important step in carburetor development is the application of the mechanical metering system to aircraft engines. This work started with preliminary tests of two systems. The most promising system was selected for development, and in bench tests which were highly satisfactory the distribution variation between cylinders was less than 2 per cent at all speeds. The units have been fitted to a Curtiss D-12 engine for performance and endurance tests.

The principal activity in ignition work involved double-magneto development, although some work was done on single magnetos, battery ignition systems, and spark-plug requirements.

Evaporative cooling and high-temperature cooling liquids have major considerations in cooling-system development as a means of reducing weight of radiation surface required.

The evaporative cooling system developed for the Wright E engine is undergoing service test, and experimental work has been extended to produce a similar system for other service engines.

Equipment. Development of all classes of aerial photographic, electrical, instrument, radio, and miscellaneous equipment and accessories has been characterized by a consistent effort to improve operating efficiency of aircraft, individually and collectively, by means of new and improved devices and better installation arrangements.

In cooperation with the Fairchild Aerial Camera Company, the Air Corps has now in production or in preparation for production the most complete and perfected set of aerial-camera equipment in its history, and which it is believed far surpasses the photographic equipment of any other nation.

Other of the more important projects involve the following phases of activity:

1 Development of double-voltage generators; standardization of airdrome lighting equipment, and methods of flood lighting.

2 New liquid and gaseous oxygen equipment; improved instruments and methods for navigation and fog flying, including capacity altimeters and leader-cable methods for landing.

3 Experimentation with radio beacons and short-wave sets.

4 Development of detachable and form-fitting parachutes and improvement of seat and lap types; redesign of heavy-duty refueling units; photographic trucks, and balloon winches; redesign of electric-driven orientators and development of an air-blast-controlled orientator.

5 Development of tow targets, and miscellaneous aircraft accessories.

Lighter-Than-Air Craft. The trend of airship development for training purposes is toward enclosed cars having a welded tubular structure covered with doped cloth to afford complete protection for the crew. Air-cooled power plants are preferred to water-cooled types for the reason that no maintenance of the cooling system is required.

A number of other developments have been made in airship accessory equipment, including the helium purity indicator, mooring and handling equipment, winches, and other station equipment.

The rate per thousand flying hours of accidents due to engine trouble was reduced during the past year to one-third of the average rate prevailing in the two previous years. These accident rates indicate that 63 crashes from engine troubles were prevented by new methods and equipment. The lessening of aircraft accidents has, in a measure, been due to the installation at all Air Corps activities of the visual inspection system. By means of large charts posted on the wall of the hangar adjacent to each airplane, on which notations are entered from time to time in spaces provided therefor as to the operating conditions, flying time, performance, etc., of the airplane, an exact history of each airplane is available at first hand. Special sets of tools and equipment—part of the visual inspection system—have been furnished for each airplane in commission.

The equipment of a Transport type C-2 airplane as a laboratory and its use in connection with research work on aircraft radio

problems while in the air made it possible to investigate problems and conduct tests heretofore not possible, and further, has contributed more than any one thing toward overcoming difficulties encountered in this line of work. Important investigations have been carried out in connection with radio beacons, high-frequency transmitters, and receivers, transmission phenomena, double-voltage generators, and capacity altimeters.

The increase in radius of action of planes now used by the Air Corps has necessitated closer and more general study of aerial navigation by Air Corps personnel. The Flying Laboratory has also been equipped with special instruments for the purpose of carrying out research problems in aerial navigation.

During the fiscal year 1928 the Army Air Corps photographed areas in the United States approximating 35,000 square miles for the War Department and other Governmental bureaus.

Two methods of training were placed in operation at the Air Corps Primary Flying Schools, the "all through system" at Brooks Field, and the "stage system" at March Field. Both methods of flying instruction are being carefully studied, and it is possible that at some future date a training system will be evolved incorporating the relative merits of each.

If the succeeding years show a rate of improvement in personnel, matériel, and tactics commensurate with that of this year, there is no doubt but that the United States will continue its place as one of the foremost countries in military and commercial aeronautics, if not the greatest.

Airships[7]

The outstanding airship event of the year was the voyage of the *Graf Zeppelin* with passengers and mail from Friedrichshafen to Lakehurst. The successful completion of the voyage in spite of bad weather and an accident to the port stabilizing fin proved that the airship shares with the steamship the ability to reach its destination in spite of adverse circumstances. The general public compared the time of the *Graf Zeppelin*, 110 hours from the middle of the European Continent, 3425 nautical miles from Lakehurst by the great circle, with the *Mauretania's* record of 122 hours from Sandy Hook to Cherbourg, a distance of only 2925 nautical miles. The difference in the lengths of the voyages and the fact that steamships are frequently delayed by storms and mishaps were overlooked.

TABLE 3 COMPARATIVE CHARACTERISTICS OF THE AIRSHIPS "LOS ANGELES," "GRAF ZEPPELIN," AND "ZRS-4"

	Airship		
	Los Angeles	*Graf Zeppelin*[1]	*ZRS-4*
Nominal gas volume, cu. ft.	2,470,000	3,708,000	6,500,000
Length overall, ft.	658.3	776.2	785
Maximum diameter, ft.	90.7	100.1	132.9
Height overall, ft.	104.4	110.6	146.5
Gross lift, lb.	153,000	230,000	403,000
Useful lift, lb.	60,000	108,000	182,000
Number of engines.	5	5	8
Total horsepower.	2000	2550	4480
Maximum speed, knots.	63.5	69.0	72.8
Range at 50 knots, naut. miles.	3500	6850	9180
Slenderness ratio.	7.25	7.75	5.90
Ratio of useful to gross lift.	0.39	0.47	0.45

[1] NOTE: For purposes of comparison, the *Graf Zeppelin* is assumed to be inflated with helium. When inflated with hydrogen and fuel gas, her range at 50 knots is about 9000 nautical miles.

The U. S. S. *Los Angeles* made a notable voyage in February and March to the Panama Canal Zone. The outward voyage was a non-stop flight from Lakehurst to New York and thence to Panama. Returning the ship refueled at the mooring mast of the U. S. S. *Patoka* off the south coast of Cuba.

Construction of the two great British airships, *R 100* by the Airship Guarantee Co. and *R 101* by the Air Ministry, has continued, but hopes of their getting into the air in 1928 were disappointed.

In the United States, a contract has been placed by the Navy Department with the Goodyear-Zeppelin Corporation for the construction of two rigid airships of 6,500,000 cu. ft. volume each. The designs of these ships show a very great advance over any previous airships. They will have a full speed of 72.8 knots, and a range of 9180 nautical miles without refueling at 50 knots speed, as compared with 63.5 knots full speed and 3500 nautical miles range at 50 knots for the *Los Angeles*. Their hulls will contain hangar space for five small fighting or scouting airplanes. A new type of girder construction will be employed; and the strength of the ships will be relatively about double that of the *Los Angeles*.

The principal characteristics of the *Los Angeles*, *Graf Zeppelin*, and *ZRS-4* are compared in Table 3.

Construction of the small experimental metalclad airship *MC-2* for the Navy Department has been continued by the Aircraft Development Corp., of Detroit, Mich. A decision to change the hull plating from duralumin to the new metal, alclad, which is very much more resistant to corrosion, afforded an opportunity to test to destruction the tail cone which had already been constructed of duralumin. The test showed that the structure had great tenacity, even when badly crumpled by internal suction amounting to nearly half an inch of water. It was impossible to burst the plating by an internal pressure as high as 14 in. of water.

ALEXANDER KLEMIN, *Secretary.*

[7] Prepared by C. P. Burgess, Bureau of Aeronautics, Navy Department, Washington, D. C.

AER-51-2

Problems of Commercial-Plane Production

By WALTER H. BEECH,[1] WICHITA, KANSAS

The author compares conditions in the aircraft industry today with those which obtained in the automobile industry during its infancy; there are no precedents to guide, no sources of supply established, and no types determined. He then discusses six factors which enter the construction of a commercial plane—safety, airworthiness, performance, durability, comfort, and appearance. Considerable attention is paid by the author to the airplane-engine situation, which is unfavorable to airplane manufacturers. This is followed by a discussion of factory machinery and equipment, labor, and a brief description of the straight-line production system of the Travel Air factory. The paper closes with a short discussion of production methods of the future.

ESPECIALLY in these days of the infancy of our industry is a full and free exchange of experiences of great value, that no steps of progress need be retraced and that forward-looking airplane manufacturers may, by exchange of ideas, work together for the advancement of every legitimate phase of the business as a whole.

These problems are not unlike the problems that confronted the automobile manufacturer in the early days of that now great business. There were no precedents to guide, no sources of supply established, no type or types determined. It was a case of dig, search, and dig again, with thousands of dollars spent in finding out what to do and how to do it.

The airplane industry has been able to draw heavily upon the experience of that industry which, from the standpoint of individual transportation, is our nearest and most valuable ally.

Naturally the first airplanes, like the first automobiles, were made by hand one at a time. In no other manner could the structural ideas of the designer be proved or disproved. Only through trial and test could experience be gained, stresses and strains determined, airworthiness demonstrated, and performance proved. In the machine shop, at the forge and bench experienced pilots and mechanics worked side by side to bring through working models embodying the designer's theory of construction. The fundamental factors of airplane construction were recognized by all, but the adaptation of these factors to particular types of construction challenged and is still challenging the skill, patience, and inventive genius of the best engineers and designers in the aviation world.

Out of all this experimental work, out of all the months and years of patience and unceasing effort, certain types of airplanes have been developed that are no longer considered experimental in design. These types have the approval of the United States Government through a type certificate. The airplane designer or manufacturer who has reached that place in the development of his product is then ready to consider the problem of the production of his airplane in sufficient quantities to meet competition, take care of the expense of distribution, and yet hope to save a sufficient margin of profit to justify the faith of the stockholders who stake their money on the commercial success of the venture.

Approximately four years ago the Travel Air Manufacturing Company reached the place in its development and progress where it felt justified in aggressively entering the commercial-airplane field. With this decision made, the company was immediately confronted with the even more important problem of a factory arranged, manned, and equipped for volume production at competitive selling prices yet without sacrifice of quality. Even with the experience of the automobile manufacture as a general guide, there were no airplane precedents whatsoever to follow, for this was one of the first airplane manufacturers to undertake volume production.

Study and analysis led to the conclusion that six factors enter into the construction of a good commercial airplane. These are: safety, airworthiness, performance, durability, comfort, and appearance. While the relative importance of these qualities varies somewhat with the type of ship, in all cases safety is the paramount issue.

A safe airplane must leave the ground within a reasonably short distance, be able to climb rapidly under good control, have a reliable motor and controls, be structurally substantial, not spin dangerously if stalled, come out of action quickly, and land easily.

Airworthiness embodies stability, steadiness in rough air, and controllability with light or heavy loads.

Performance includes take-off distance and time, rate of climb, ceiling, high speed, land speed, landing distance, fuel economy, and cruising range.

The other factors of durability, comfort, and appearance need no explanation.

Planning for Production

Assuming the analysis just stated to be fundamentally correct, the important problem remained of planning for production on that basis and then putting the plan into practical operation.

First, consider for a moment the steps involved in producing a new model. They are: (1) design for simplicity of construction and assembly with as many identical fittings and parts as possible, (2) engineering, (3) detail, (4) fabrication, (5) testing. The general design is first worked up in the rough. Then a stress analysis is made of the structure to determine the size of the members. Changes in the original design are frequently made at this point to improve conditions which by analysis are shown to be unsatisfactory. The detail drawings are then made and sent to the shop. The production of the first two or three planes is always more or less experimental, but if the engineering is well done, the performance is accurately calculated and is found to vary only slightly from that found by actual tests. From then on the problems of commercial production include materials, factory equipment, labor, and a production schedule.

Materials

The materials for construction should be chosen, first, for strength, durability and lightness; second, for ease of manufacture; third, for reliability of supply; and fourth, for price.

Any well-organized airplane manufacturer has no especial difficulty in selecting materials for strength, durability, and lightness, nor for ease of manufacture or assembly. The difficulty comes in the reliability of the supply and the price that must be paid.

Consider as examples of the gravity of this problem the question of the proper sizes and types of steel tubing and the matter of engines.

Every manufacturer in the industry knows from personal and expensive experience the difficulties surrounding the purchase of the steel tubing he wants from a source of supply that is depend-

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[1] President, Travel Air Mfg. Co., Inc.

Presented at the National Meeting of the A.S.M.E. Aeronautic Division, Wichita, Kansas, September 21, 1928.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

able in both quality of product and the delivery of that product at the time it is needed. There seems to be either a feast or a famine—a long inventory or no inventory at all, with telegrams and long-distance calls to the parts manufacturer that the week's production of ships may not be clogged. All this effort adds to the cost of the finished product, and this cost must either be absorbed, with consequent loss of profit, or passed on to the purchaser in the higher price of the ship itself. The former is disastrous to the balance sheet of the company. The latter is not practical because of contract relations with dealers and distributors at fixed list and discount figures.

The motor situation is an even greater problem. The OX-5 situation following the war is probably a large part of the reason why the manufacturers of airplane motors were reluctant about making volume production plans on definite delivery schedules. A policy of watchful waiting with an ear to the ground seems to have been, and in many instances still is, their policy. What would be the condition of all airplane manufacturers today had they waited, like the motor folks, for a clearly defined market to be established before they started to make airplanes? We all would still be making airplanes one at a time by hand in the back room of some planing mill.

It is difficult for airplane manufacturers to see why engine manufacturers could not foresee the exhaustion of the supply of OX-5 motors within a comparatively short time as well as the ultimate demand of the airplane buyer for more power or more performance than could be expected from this war-time engine. Had they done so the situation of motor production in line with the present demand would be much more nearly in balance today.

There is another phase of the motor question that challenges the attention of any airplane manufacturer who is on a regular production basis—that is the practice of many engine companies in accepting orders of one or maybe a few engines from airplane companies who are as yet in the experimental stage and with whom commercial production is at best a remote possibility.

With a limited supply at best of motors that have proved themselves of unquestioned value in an airplane, the allotment by the engine builder, from that limited supply, to everybody who wants one or perhaps a few, works a decided hardship upon the airplane companies who have large sums of money invested in buildings, equipment, personnel, and selling organizations. It is not intended to suggest denying the airplane company just started the same right to all sources of supply as the older and more firmly established companies, but it is thought that a better understanding between engine builders and airplane manufacturers who are in volume production would give the former a steadier volume of business in the years to come, and would give the commercial-plane producer a far better chance to protect his investment, as well as more aggressively to seek and develop new markets for airplanes. Both would then be benefited, where at present the engine builder is the sole temporary gainer.

Factory Machinery and Equipment

Factory machinery and equipment must be commensurate with production. It requires perhaps less equipment to manufacture an airplane than any product selling at its price. This means that any one with a few tools, an acetylene tank and blow torch, some steel tubing, and an engine on order can start "manufacturing" airplanes. This condition is unfortunate because it permits many people to enter the business who are not adequately equipped in experience and financial backing, and accounts in part for the existence of some 140 or more airplane companies in the United States today.

This situation is somewhat parallel to that of the radio of a few years ago. Radios were easy to make by hand if the maker had even a slight knowledge of electricity and could wind an armature and set up an aerial. The electrical-supply houses naturally flooded this new market with coils, wire, controls, and all other accessories. Hand-made sets went into thousands of homes. The maker could operate them, but not the average user. Nothing was standard. When there was a failure of "reception" there was no service. Not until the radio was brought to scientific standards of construction and operation by a comparatively few aggressive and far-sighted manufacturers did radio come into its own and earn a permanent place in practically every home.

The so-called saturation point will never be reached in the sale of airplanes any more than it has been reached or ever will be reached in the sale of automobiles. This means that every airplane of proved type and construction that is made can be sold and that it will stay sold and bring new business through owner satisfaction.

The danger lies in placing on the market the untried and unproved product. As yet the public as a whole is not familiar enough with types and construction to discriminate. Herein lies the danger with the "experiment" that is offered for sale and the expense of repairs and upkeep of the "orphan" for which service is not available.

No undue discouragement should be felt over this situation. The loss will have to be borne by the investor who did not get down to fundamentals before purchasing his stock and by the purchaser who did not concern himself about the untried design or the question of service after sale. It is one of the burdens of a new industry which modern factories with modern machinery building ships of proved design will overcome in time.

Labor

At the start of any new manufacturing endeavor, labor usually consists of all-round mechanics upon whose ingenuity depends the success of the project. This applies with special force to the airplane. As the organization grows the labor involved in production becomes more specialized, for specialization is the basis of efficient production. The all-round man is replaced by a craftsman who devotes his time to one particular operation and gains skill and efficiency each day. Often the all-round man develops into the specialized job. Departmentalization of the factory rapidly follows, the old and tried men becoming foremen of the various departments. Personnel work comes into being. The welfare of the workmen is found to be for the good of the product as well as the contentment and satisfaction of all employees.

With us the process of specialization is still in its infancy, but is being studied constantly that the men may be legitimately spurred on to their best efforts.

We try, wherever practical, to give our men the "feel of the air" by taking them up. This gives them a feeling of grave responsibility in their work in knowing that the lives of the users of airplanes are largely in their hands in building safety into the airplane.

Travel Air Factory and Production Scheme

At present there are fifteen departments in the Travel Air factory, each in charge of a foreman and a few with sub-foreman. These departments are: welding, two fuselage assemblies, fuselage paint, wing paint, milling, wing assembly, final assembly, testing, maintenance, transportation, office, engineering, stock room, first aid, and safety.

Let me trace the production of a ship through our factory. At the west end of factory "A" the fuselage is begun. Tubing is drawn out of the stock room nearby, cut to the proper lengths on a tubing saw, welded on jigs, the engine mount built up and bolted on. The fuselage is then passed down the line through the fuselage-assembly department. The fitting department occupies the

side of the building at this point. Here the completed fittings are assembled on the fuselage. The wire department occupies a balcony on one side, and the upholstering department occupies a balcony on the other side. The purpose of these balconies is to concentrate the parts departments near the assembly department so that wires, fittings, etc., will not have to be moved more than a few feet before they can be assembled on the ship. Next the engine is installed in the plane and the cowling and tanks are added at a point opposite the sheet-metal department. The fuselate is then covered and passes into the dope room. Dope and paint are sprayed on and the fuselage passes out into the final fuselage-assembly department. Here the landing gear, tail surfaces, and center section are assembled.

Factory "B" abuts factory "A." Passing to the far end of factory "B" we find the mill room. Here lumber is milled to the proper size and shaped for wings, turtle backs, and cabin framework on the monoplane. The parts are assembled in the wing assembly department which joins the mill room. The fittings are added and the wings covered. The wing then passes into paint and dope room No. 2. The completed wing is then moved to the final assembly and rigging department where it is dropped on the fuselage which has been moved across the court from factory "A." After the ship has been rigged it is given a final inspection and then test-flown.

From this brief outline it will be obvious that we have adhered strictly to straight-line production and a policy of specialization. We have also facilitated production in our design. Fittings which formerly were built up of many pieces are now formed from one piece. Brazing has been cut to a minimum, and by slight changes in design nearly all fittings are now welded.

Production Methods of the Future

As to the production methods of the future only a guess can be made. We feel the use of machinery will increase, and as a result metal will enter into the construction more and more. Metal wings will be justified as soon as production mounts to a scale which permits the investment in the necessary equipment for their construction. Machined fittings will replace welded fittings, when the quantity of fittings produced is such that the machinery necessary to produce them will pay for itself.

Labor incentives will play a more important part in the future. The present status of the industry is such that little work can be done on time study and incentives. We are now at the beginning of a period when this phase of production will obtain considerable proportions. Bonus systems and piecework will come as soon as the necessary preliminary time studies can be made.

Present selling prices are to be maintained or lowered as time goes on only through volume production raised to the highest point of efficiency. This means sharp and clearly defined decisions on types of planes that are to be built and a sales program to the vast potential market that has not as yet been scratched.

This only requires aggressive selling on a constructive basis to insure an even flow of business to the sales manager's desk so that an even flow of ships sold at a fair price and yielding a fair profit may daily leave the factory door.

Historical Sketch of the Development of Aerodynamic Theory

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This paper is intended to give, in brief historical outline, a sketch of the development of aerodynamic theory from the time of Newton to the present day. Newton's theory of fluid motion and the resulting formula for the force reaction resulting between an inclined plane and a fluid in relative motion are briefly stated and the insufficiency of the formulas noted. Reference is then made to the various mathematical aids which were developed during the next century, and which made possible the investigation of fluid motion based on the concept of continuous change and the "perfect fluid" of the mathematician. The limitations of this concept, especially with reference to the explanation of force reactions between the body and the fluid, are then noted, together with the introduction of the idea of "circulation" and generally of the physical picture as developed by Lanchester and then in mathematical form by Prandtl and his school. These are then briefly sketched and the results of the introduction of these new ideas indicated.

Reference is then made to the problem of the drag, only in part taken care of by the Lanchester-Prandtl vortex theory, and to von Karman's work leading to his concept of the "Karman vortex street" and thence to the measurement of a further element of the drag; and finally to the third element (depending on skin friction) as at present dependent chiefly on direct physical measurement. In addition to the problem of lift and drag, the problems of stability and control in the air are briefly referred to, followed by mention of the dependence of theory on experimental research and to the vast amount of wind-tunnel investigation which has been made during the past two decades. Brief mention is then made of the two problems introduced by such work—the correction for "wall interference" and the "scale factor" due to the imperfect fulfilment of the conditions for kinematic similitude. This section closes with a mention of some of the problems which characterize the advancing fringe of progress in aerodynamic theory and which in large part still remain for future examination and study.

Then follows a brief section dealing with the screw propeller with reference to the chief stages of the development of propeller theory as suggested by the names Rankine, Froude, Drzewiecki, Prandtl, and closing with a suggestion of some of the problems which characterize the advancing fringe of this phase of general aerodynamic theory.

AVIATION in its broader aspects presents at least four well-marked divisions:

1 Aerodynamic theory
2 Aerodynamic experiment and research
3 Airplane design and construction
4 Airplane use, civil or military as the case may be.

Logically these should proceed in the order as stated.

Thus aerodynamic theory should lie at the foundation and when justified and supplemented by experiment and research should point the way to design and construction, after which come utilization in such manner as may be desired.

Historically, experiment and research, design and construction have often preceded theory. Especially was this true of the conditions a quarter century ago in their relations to the present body of aerodynamic theory.

The present paper is restricted to a sketch of the historical development of division (1), aerodynamic theory. No attempt will be made to bring in, except as incidental or auxiliary, any reference to the other three divisions.

The basic problem in aerodynamics involves the interaction between a plane or wing-shaped body and a fluid (the air) in relative motion one with the other.

We can hardly go as far back as Leonardo da Vinci for any beginnings of what properly may be called aerodynamic theory. Great as was the genius of Leonardo and interesting and suggestive as are his writings on aviation, they can hardly be said to have formed a start for aerodynamic theory.

For the first definite beginnings of such theory we must come down to Newton and his times. We find here three elements which served as foundation for the first approach to a theory of the interaction between a fluid and a plane in relative motion, one with the other. These were as follows:

1 The Newtonian concept of relativity in virtue of which the problem might be examined either by considering the fluid at rest and the plane moving through it, or the plane at rest and an infinite fluid streaming past.

2 The concept of a fluid as composed of discrete particles without interaction among themselves.

3 The Newtonian mechanics, through which it became possible with the physical picture provided by (1) and (2) to express in quantitative form the force and momentum interaction between, for example, a plane and a fluid in relative motion along a line oblique to the plane.

Based on Newton's second and third laws, it became simply necessary, in order to determine the force reaction along a line inclined at any angle to the line of relative motion, to determine the time rate of the change of momentum produced in the fluid medium estimated along this same direction; and in particular, to determine what we now call the lift, or force reaction at right angles to the line of motion, it was only necessary to determine the time rate of the change of momentum in such direction, and similarly for the drag or force reaction along the line of motion.

At this point, however, it became necessary to make some further assumption regarding the elastic properties of the particle assumed as the element of the fluid. The picture of the fluid assumed provides in effect a stream of moving particles or little bullets, as it were, moving independently and impacting with the plane. Three assumptions are then possible:

1 Perfect elasticity as between particle and surface at the point of contact
2 Complete inelasticity as between particle and surface
3 Some intermediate condition of partial elasticity.

In the first case the particle will rebound from the plane with velocity unchanged and at an angle with the plane equal to that of approach. In modern terminology with an angle of approach or of attack = Θ, this would produce a vector change of velocity represented by turning the vector V through the angle 2Θ.

In the second case there will be no rebound and we may assume the particles to slide along the plane and leave it in the line of the plane itself. This would produce a vector change represented by turning the vector V through the angle Θ.

In the third case there will be some intermediate condition and the vector change will be represented by turning the vector V

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through some angle intermediate between Θ and 2Θ. It is well known that Newton's assumptions give rise to the "sine square" law relating the total force to the angle of inclination—a law which persisted in textbooks and elsewhere long after experiment had shown it to be in error.

It will not be necessary to further discuss the results of these various possible assumptions. It is known that experimental research by Borda, d'Alembert, Dubuat, Hutton, Duchemin and others showed that the results of the application of this physical picture to the problem in hand indicated clearly the inaccuracy of the picture and the need of a complete revision of the physical concepts involved.

Reference in passing may, however, be made to another line of reasoning attributed to Euler which gives the first power of the sine instead of the second. While based on a questionable reasoning, this law was later shown, experimentally, to have, for small angles, a very satisfactory degree of accuracy. Still later Duchemin's empirical formula gave a still more satisfactory expression for the relation between the total force and the angle of inclination. In all this, however, there was nothing which gave any rational explanation of the development of these force reactions or of the particular physical conditions upon which they were dependent.

It thus became clear that this particular picture of the relations between a plane and a fluid in relative motion gave results obviously incomplete and imperfect. In fact, there is no reason for assuming that Newton himself considered this picture of a fluid and of its action on the plane as anything more than a trial hypothesis—the best he was able to make and use with the tools at his disposal in terms of mathematics. Furthermore, it is not easy to see what other pictures could have been treated mathematically with the means at his disposal. His own concepts of the calculus had not reached the point which made available the differential equation as an instrument permitting the study of the problem from the entirely different and more natural viewpoint furnished by the picture of a fluid as a continuous medium.

The development, during the century which followed Newton and Leibnitz, of the differential equation as a means for the study of the phenomena in nature which are assumed to involve continuous change in space and time, was an achievement the significance of which it would be hard to exaggerate; but no such means was available in Newton's day, though the germ of the entire development lay in the concept of his "fluxion" or rate.

However, as time passed it became increasingly clear that the trouble with the formulas for the force reactions between a plane and a fluid medium, the two in relative motion, was to be sought in the picture of the fluid and its relation to the plane around which it is considered to be flowing.

Finally, about a century later, with the development and provision by the differential equation of a tool fitted to the hand of the student of such natural phenomena, it became possible to approach this problem from the standpoint furnished by an entirely different concept regarding the physical constitution of a fluid.

The picture which thus developed, viewed from the standpoint of the possibility of mathematical treatment, was that of a continuous medium with infinite mobility—that is, a medium, the parts of which, however conceived of, may move past each other with no tangential or shearing resistance. This gives us the so-called "perfect" fluid of the mathematician—the fluid of viscosity zero. It is well known that in Nature there is no such fluid—that all fluids exhibit some degree of viscosity—some degree of tangential or shearing stress resisting the sliding of one portion or element of the fluid relative to another. However, the admission of this element into the picture increases the difficulties of the mathematical treatment to a degree which has thus far rendered impracticable any solution except in certain very special and restricted cases.

On the other hand, the omission of this feature of the problem, reducing the medium to the so-called perfect fluid condition, has rendered possible, through the application of various mathematical devices, a very considerable generality of treatment—such a development of the details of the physical picture, in fact, as should give full information regarding all dynamic relations involved in the problem—but only true, of course, for the "perfect" fluid.

Before speaking of this development of the picture, however, it will be well to go back for a moment to note the two forms which the general differential equations applying to this problem may take.

These two forms have been given names from Euler and Lagrange, though both trace back to Euler. In the Eulerian form, the results are calculated to give information regarding the field distribution of velocity and pressure while the Lagrangian relate to the history of a particular particle.

It is well known that the equations in the Eulerian form are much more manageable than in the Lagrangian form, and furthermore the former seem better adapted to give more immediately the information desired. It is clear that with a field knowledge of velocities, the entire field as regards paths of flow, distribution of pressure and of momentum becomes known, and most of the things with which we are concerned become immediately determinable.

For these reasons, the equations in the so-called Eulerian form have been usually employed, and have rendered chief service in building up this phase of the general treatment of the problem of fluid motion.

Granted then that we thus have means whereby, for bodies of certain forms at least, we may obtain a complete geometric and dynamic account of the field occupied by a fluid between which and the body there is a motion of translation measured by a velocity V, what further useful results may be drawn from the knowledge thus acquired? Here enters the tragedy attaching to this splendid development of mathematical science as applied to a concept of a fluid which seems, at least, to be closer to reality than the one employed by Newton.

The results, as well known, furnish no indication of any over-all force reaction between the body and the fluid, or rather they show that, with the "perfect" fluid as defined by the equations, there can be no such over-all force reaction. These results are therefore valueless so far as giving us any explanation of how a "lifting" force can be obtained upon a plane moving in a fluid medium and in an attitude oblique to its line of motion; or again as to how or why, in such case, there develops a force in the line of motion and opposed to it, or again, as to any measure of these forces.

It might well be questioned therefore whether, with this complete failure so far as any measurement of force reactions is concerned, any useful result had been accomplished through this new attack on the problem. The answer is that, though in itself this particular development was a failure as regards the furnishing of either an explanation of the origin of any such force reaction or its measure, it nevertheless did furnish a foundation and a general method, which, with the addition of certain further elements of detail in the picture, have carried us a long way toward the desired goal.

Before considering this further advance, however, some note should be taken of certain matters of detail regarding the development thus far.

In order to be able to trace stream-line paths and thus to determine the geometrical and dynamical details of the fluid field, use is found for a combination of five mathematical and physical agencies or concepts as follows:

1 The differential equation
2 The Newtonian mechanics
3 The principle of potential
4 The concept of sources and sinks
5 The method of conformal transformation.

The differential equation, as already noted, furnishes the mathematical means whereby we are able to trace the details of continuous change.

The Newtonian mechanics furnishes the tie between geometry and the dynamic details of the fluid field.

The principle of potential vastly simplifies the procedure in detail by furnishing a single expression, the vector differentiation of which gives immediately the velocity in that direction.

The concept of sources and sinks in two dimensions and in its simpler form furnishes the start for the determination of stream-line paths for certain resultant geometrical forms of cross-section of body. In its more generalized form, especially as developed by D. W. Taylor in his paper before the British Society of Naval Architects, it can be made to give the stream-line paths for flow either in two or in three dimensions and for a wide variety of forms symmetrical about an axis parallel to the direction of motion. In all such cases, however, the procedure is from an assumed distribution of sources and sinks, to the resultant stream-line form and the lines of stream flow about it. This procedure, unfortunately, is not reversible. That is, there is no direct way, having given a particular form, whereby we may determine the distribution of sources and sinks which would give such a form as a stream-line body with its accompanying field of stream-line flow. However, by a process of trial and approach, using a series of sources and sinks distributed along a line and with suitable variation of interval and strength, close approximations may be reached to any proposed form not too far removed from what are usually classed as "stream line" forms.

To Rankine in the last century must be given, perhaps, the chief credit for the application and development of this method to the production of stream line, or as he called them, "neoid" forms for the water lines of ships. The same general method was again taken up in the late years of the last century by D. W. Taylor, and generalized by the substitution of the concept of a continuous slot, so to speak, for a series of separate points. Such a slot, with continuously varying strength of flow, positive or negative, becomes, then, the final generalization of the source and sink concept, and as Taylor has shown, may be made to produce almost any form within the range of so-called stream-line forms, together with its accompanying field of stream-line flow.

The method of sources and sinks, is not, however, so readily applicable to the treatment of problems of flow about unsymmetrical forms, such for example, as the airplane wing. For the treatment of such cases, the Schwartz-Christoffel transformation (or as it is now commonly called, the method of conformal transformation) has provided us with an agency of marvelous and far reaching power. In the whole domain of the application of mathematical methods to the treatment of physical problems, there is perhaps nothing more impressive than the application of the wizardry which attaches to the imaginary $\sqrt{-1}$, to the transformation of one entire geometrical field into another of widely varying form, but with fundamental relations subsisting between the two such that we are justified in saying that if field number one represents the field of stream-line flow about a particular form of solid boundary, then will field number two represent likewise the field of flow about the transformed boundary form.

Now one of the earliest problems solved by the source and sink method was the determination of the flow about an infinite cylinder placed in a stream flowing in a direction at right angles to its axis. This means, in two-dimensional motion, the flow about a thin circular disk. Now comes in the method of conformal transformation and provides us with means for transforming the circle as a contour into airfoil shaped forms, and with it, the flow about the circle into what it would be about the foil.

Here again, however, we must accept the limitation of a "one-way street." Having given the circle and its field of flow and an assumed formula of transformation, we can find, by straightforward methods, the resulting form and field of flow. Having given the airfoil form, however, we cannot determine directly the formula of transformation which would produce such form from a circle and thus we cannot, in general, determine by direct procedure the field of flow for a given form moving in an indefinite fluid.

However, by indirect methods—by trial and approach, we may continuously so modify the formulas of transformation as to give a satisfactory approach to all ordinary forms of airfoil sections.

Joukowsky's pioneer work in this field during the second decade of the present century laid the foundation for the development of an entire series of airfoil sections named for him, and the field of flow about which could be immediately determined by these methods.

By such means, therefore, the mathematical agencies for which were developed largely during the latter half of the last century, we are now able to determine, at least with close approach, the field of flow for almost any proposed airfoil form.

It should also be noted at this point that by the use of the generalized source and sink method, as referred to above, the field of flow in three dimensions about spindle-shaped bodies may be also determined to any degree of detail necessary. In this manner the conditions of flow and the distribution of pressure about airship forms (aside from appendages) may be developed to a surprising degree of accuracy as determined by comparison with measurements on full-sized forms.

These various agencies serve, therefore, to give with reasonable accuracy the field of flow for a perfect fluid about any form of airfoil, considered as a two-dimensional flow, or about any spindle or fusiform body considered as a three-dimensional flow.

It thus results that for the case of two-dimensional flow, or what would correspond to the flow near the middle portion of an airplane of very long span, or again for any spindle-shaped body in three-dimensional flow, and assuming always a non-viscous fluid and rectilinear flow (not necessarily parallel to the axis), we are in a position to map out with a high degree of accuracy the lines of stream flow and to investigate the velocity and pressure conditions about the body. But as before indicated, while all of this resulted in a picture not far wrong, at least in many ways, it gave, nevertheless, no measure of the ultimate force reaction between the body and the stream, or rather, it indicated that such force reaction was always zero.

Confining our attention to the airplane, the problem then became, how to develop a theory or a picture which would explain, for an airfoil shaped body suitably exposed in a current of air, the lift or the force at right angles to the direction of relative motion, and the resistance or drag—the force along the line of relative motion.

The next step in the solution of the problem traces back to Helmholtz, who in the latter part of the last century, developed his theory of vortex motion and thus, quite unwittingly, laid the foundation for one of the most significant of the more recent advances in the field of theoretical aerodynamics.

The student of these problems thus became furnished with two basic modes of flow which he might use in his investigations—rectilinear flow and rotating or cyclic flow, and it was soon realized that these modes of flow might be combined in an infinite variety of ways.

In particular it was seen that assuming a combination of a

rectilinear two-dimensional flow with a simple rotating or cyclic flow, about a solid circular disk (two-dimensional body) the theory as it was already developed in the basic equations of fluid mechanics would furnish immediately the explanation and the measure of the force at right angles to the direction of relative motion.

Thus with a thin circular disk and a cyclic flow (the simplest possible case of the type of flow investigated by Helmholz) the explanation of one of the forces acting between the solid and the fluid came immediately to hand, and not only its explanation but its measure as well, and to a very satisfactory degree of accuracy when compared with the results of experiments.

Thus came into the picture the concept of "circulation"—the line integral of the velocity taken about any closed path—as a factor bearing a direct relation to the measure of the lift force. But while this was all simple enough, granting the cyclic motion, there seemed no possible way of explaining the development of any such motion in a perfect fluid. Or rather, the mathematical expression of a "perfect fluid" carried with it the impossibility of generating any such motion in a body of fluid not already endowed with it, or otherwise of destroying such motion once it be assumed to exist. But we are not, in fact, dealing with the perfect fluid of the mathematical equations; but with a medium (air) having a small though definite viscosity.

At this point Lanchester in 1907 came forward, with his remarkable physical insight, and gave a reasonable and convincing explanation of the generation of just the type of circulation about the wing which was needed to furnish the explanation of the lift force.

This was followed shortly by Kutta, who gave to the problem its expression in mathematical form and deduced the now classic formula relating the lift force to the circulation, the linear velocity and the density of the medium.

But this relation, important as it is, is not enough to deal with the problem of the actual airplane wing. The flow must here be considered in three dimensions and many secondary phenomena not present in the two-dimensional problem must be taken into account.

Here again came Lanchester and extended his vision regarding the case of circulation about an infinite wing to the vastly more complete case of the actual finite wing. Without attempting expression in mathematical terms, he developed in measurably full detail the physical picture of the complex system of vortex motion shed from the airplane wing, and which we have since come to accept as our point of departure in the detailed study of the wing, especially in combinations of two or more.

But the Lanchester picture lacked quantitative interpretation. In particular, there was needed some means for analyzing and evaluating the special distortion of the rectilinear field of flow produced by the presence of the wing—a distortion which we now express in terms of the downward component of velocity at the wing, and to which the name "downwash" is commonly given. It was seen that a given wing must be considered as acting in a field thus distorted due to its own presence, and further that in the case of two or more wings, the field in which each one acts will be subject to the distorting influence of each of the others.

At this point, Prandtl brought to the problem the resources of high mathematical skill, familiarity with the vortex theory of Helmholz and a physical picture, gained at first largely independently, but nevertheless quite in accord with that previously developed by Lanchester. In his later work, Prandtl had the advantage of familiarity with Lanchester's development of the physical picture and in public addresses has given to Lanchester generous recognition for his pioneer vision in picturing the physical conditions of this problem.

In the development of this splendid capstone to the modern aerodynamic theory of non-viscous fluids, we must give to Lanchester the credit for his pioneer work in the development of so complete and so satisfying a physical picture, while to Prandtl and his school must go the credit for translating this splendid picture into mathematical terms and for reducing the theory to the point of practical quantitative application.

One of the most interesting features of the theory of vortex motion is the perfect mathematical equivalence between a line vortex in an indefinite fluid and a conductor carrying an electric current, with reference to the so-called "induced velocity" in the fluid field on the one hand and the magnetic induction on the other. Aside from a constant factor, or in other words, aside from the arbitrary choice of the units of measure, both of these quantities have the same expression.

It follows that to the extent to which the action of a wing may be represented by a line vortex, to that extent may the action of such a wing or of any number of wings in combination (with reference to the production of the downwash velocity) be represented by an assemblage of electrical conductors with a specified current flowing in each, the magnetic induction at any point in the field representing the analogue of the downwash velocity at that point.

The development of this theory at once shed a flood of light on the general problem of the airplane wing, both with reference to the production of lift and the occurrence of one component of the drag.

As chief contributors to this result in its various details, mention should be made of the names of Prandtl, Joukowsky, Kutta, Betz, Munk, and Mises.

Thus far, then, we have a theory, satisfactory and practicable, and admitting of application under conditions as follows:

1 To the case of non-viscous fluids, or practically to fluids with viscosity small, or where, from the nature of the case, viscosity will play a relatively unimportant part.

2 To the extent to which it may be proper to replace the wing itself by a so-called "lifting line" and then the latter, mathematically, by a line vortex with its attendant system of tip and trailing edge vortices.

The theory may then be applied to:

(*a*) The explanation of lift, its measure and its relation to other conditions and phenomena involved in the problem.

(*b*) The explanation of downwash, its measure, and consequent influence on the field of flow.

(*c*) The explanation of the occurrence of a part of the drag force (induced drag) and its measure.

As noteworthy as were these achievements and as far as they seem to carry us toward a full and adequate treatment of the problem of the forces acting on the wing of an airplane, they yet fail to give any adequate treatment whatever of the drag, the force in the line of relative motion of the fluid and the wing.

It is true that the theory does give one component of such force—the induced drag. The lift force reckoned at right angles to the direction of relative motion at the wing is now no longer at right angles to the direction of movement relative to the great body of undisturbed air. The downwash u combining with the linear velocity V produces a resultant velocity at the plane inclined to the direction of V at an angle whose tangent is u/V. And, assuming horizontal flight, this has the effect of tipping the direction of flow at the plane up from the horizontal and hence of tipping the line along which we must compute the lift, back from the vertical by the same angle. This gives a component of the lift in a direction opposed to the motion and thus gives rise to a drag force to which has been given the name of "induced drag."

But this all develops out of a theory applied to a nonviscous fluid. Now we know that the induced drag does not account for

the actual drag as measured; and we know also that air is viscous and we naturally seek for the balance of the drag force in conditions which we trace back to viscosity and its influence on the phenomena of fluid motion.

Here again steps in the mathematician and obligingly furnishes general differential equations including the influence due to viscous forces. But while such equations can be written down in their general form, it is quite a different matter to effect their solution as applied to particular cases. In this matter the engineer is still waiting upon the mathematician. However, by making various special and simplifying assumptions, these equations may be made use of to give valuable indications regarding the phenomena attendant upon the movement of a body in a viscous fluid.

Historically, however, efforts had been made by Helmholz and Kirchhoff in the later years of the last century to devise some modification in the accepted ideas of fluid motion which should serve as a basis for the explanation of a force acting along the line of relative motion and opposed to it. Thus developed the concept of discontinuity which, with the aid of conformal transformation, was made to yield indications of a quantitative character in certain special cases. This concept of the conditions of fluid motion was developed, of course, without reference to the problems of present-day aeronautic engineering, and while suggestive and interesting and while supported by observation as to the broad fact of the existence of discontinuities, it was not, however, capable of giving results of any very great value in connection with these later problems.

At this point, von Karman, accepting the concept of discontinuity and at the same time including viscosity as a definite factor in the picture, together with the Helmholz theory of vortex motion, developed a picture of two series of vortices, the members of which are given off alternately from the two sides of a body past which a viscous fluid is moving, and which then pass to the rear forming between them the so-called "Karman vortex street."

Here we see, continuously passing to the rear, a series of vortices, each one of which embodies a certain amount of energy which can only be supplied from the source which is producing the movement of the body. This translated into terms of force means a resistance to the movement—a drag force, opposed to the line of motion.

To this component of the resistance the name of "form drag" has been given.

The existence of such a double set of vortices given off alternately on opposite sides of a body, especially if of bluff form (such as a cylinder), past which a fluid is flowing, had been already indicated by observation and more recent experimental research has served to show their existence quite in the manner as assumed in the theory.

The mathematical part of the problem involved the examination of the conditions which would secure the stability of such a system of vortices, as well as the development of a relation between the circumstances of the motion and the frequency and individual energy of the vortices as shed from the body.

This has all been carried out by von Karman in part by empirical formulas, thus furnishing a basis for the quantitative estimate of the rate at which energy must be supplied in order to secure the continued formation of the members of this series of vortices, and hence an estimate of the form drag. The results of comparison with experiments directed to test the formulas as thus deduced from the theory seem to give a good degree of confirmation. Further experience may indicate change in the empirical factors. In any event, it makes a definite step in advance as an attempt to supply a rational and definite means for the quantitative measure of this element of the total drag force.

Reference has thus far been made to two components of drag or resistance. There still remains a third, the existence of which is related to the formation of a blanket of eddying and turbulent fluid as the immediate envelope of a body in motion relative to the fluid. The existence of such an envelope had long been recognized as a matter of observation. Its existence, as well as that of the so-called "boundary layer," found ready explanation (as physical phenomena) in the theory of viscosity and its action as a factor in fluid motion. The continued generation of this blanket of eddying fluid which passes away downstream as formed, requires the continued expenditure of energy, and this again, as in the case of the eddies of the Karman vortex street, becomes translated into a resistance to the movement.

This gives us, finally, three components of resistance or drag: The induced drag due to the distortion of the field of flow by the presence of the wing, the profile drag due to the formation of the Karman vortices and the skin friction due to the effect of viscosity on a fluid undergoing rapid changes in velocity in the neighborhood of the boundary surface of the body.

It must be noted, however, that while the theory supplies a direct and simple measure of the induced drag, and while von Karman's analysis supplies a basis for estimating the measure of the form drag, there is as yet no adequate basis in theory for a direct measure of the frictional resistance. This item must still depend on empirical formulas for its measure.

It thus appears that existent theory furnishes a physical explanation of the development of the lift and drag forces, together with a basis for the measure of the lift and of two components of the drag. This does not imply that the theory is complete or that it is capable of taking care of all of the circumstances involved in actual engineering constructions, but rather that aside from secondary disturbing elements, it does furnish, as tested by actual experience, a reasonably satisfactory basis for the estimation of the operating characteristics of airplane wings of anything approaching conventional form and at small or moderate values of the angle of attack.

Stability and Control

While the basic problems of aerodynamics are concerned with the explanation and measure of the fundamental force reaction between a solid and a fluid in relative motion, yet the present sketch would be incomplete without some mention of the problem of stability and control. This problem, assuming the various air reaction forces on the plane as known or knowable, calls then for a complete investigation of the stability of the plane in rectilinear and curvilinear flight and for a study of the oscillatory or periodic movements which may develop under such conditions.

This problem has been investigated by Ferber, Bryan, Reissner, Bairstow and others and may be considered as in a very satisfactory state of progress. With a knowledge of the geometry of the plane, its aerodynamic characteristics, the moments of inertia about the principal axes, and the conditions of flight, the equations will serve to give a satisfactory account of matters coming under the head of stability and control under normal conditions of flight. Further extensions to conditions developing in aeronautics and especially in connection with the phenomena of spins are yet needed to make the theory satisfactorily complete in all respects.

Wind-Tunnel Research

Parallel with this development of theory, recent years have witnessed a most extended development of wind tunnel installations, wherein models of airfoil sections have been examined and measured for their aerodynamic characteristics. Literally hundreds upon hundreds of such forms have been tested and their characteristics measured with a view to their use for full scale estimate through the principles of kinematic similitude.

Such experimental examination is, in fact, a necessary element of the situation at the present time, since such measures have been needed as a check on the indications of theory, and since moreover, theory does not give a complete account of the drag force. We may, furthermore, expect that there always will be need for such laboratory research, since it can hardly be expected that theory alone will ever be able to give so complete an account of the circumstances and details of airfoil action as to eliminate the need of further experimental check and measure.

However, the use of reduced models calls for the application of the law of kinematic similitude and where the model experiments are carried on with air at atmospheric pressure, it is well known that the conditions for kinematic similitude (the same Reynolds' number) cannot be realized as between the model and the full scale body. The problem of the correcting factor thus enters—the factor for "scale effect" so-called. There is also a factor of correction for wall effect where the measures are made in a closed tunnel. Prandtl has given a seemingly satisfactory method for the estimate of the latter of these two corrections, but so far we have made little progress in the way of any development of a theory to account for the variation of results with change in Reynolds' number.

At present, the only assured method seems to be to eliminate the need for this correction by securing the conditions for kinematic similitude through the use of a wind tunnel in which the medium employed is air under high pressure, as in the installation at Langley Field, and by which means the full Reynolds' number may be realized for all ordinary operating conditions.

Along with the development of what may be called the main stem of aerodynamic theory as above briefly outlined, the last quarter century and in particular the last decade have witnessed a vast number of special studies and developments in detail; some lying outside the main line of development as here noted and others concerned with the examination of special points intended to make application of the theory in its simpler form either more broadly general, or more certain and specific in particular cases.

Much of this work constitutes what may be called the advancing fringe of aerodynamic theory and there are problems lying still further ahead which represent outlying portions of the field still awaiting investigation and study.

In any such account as the present brief sketch can attempt to give, it would be unwise to attempt to cite authors, names, or particular titles. The field is too rich and it would be well nigh impossible to know just where to stop. However, some indication of the general character and content of this field may well be attempted.

Thus we have:

The extension of airfoil theory to the case of large angles of attack.

More generally the development of some theory of accounting for the intimate phenomena attendant on the operation of airfoils at large angles of attack and including the separation of the boundary layer and the formation of reverse flow.

The explanation and quantitative estimate of the influence of the various means which have been proposed and employed for the removal of the boundary layer and including the phenomena attendant on the use of the Handley Page slot or other like devices.

The extension of theory to a more adequate account of the forces acting on control surfaces (rudder and elevator).

Further development in detail of the theory of the formation of the Karman vortex street including the periodicity, spacing, velocity and inherent energy of the individual members of the series, all as dependent on the circumstances of the body and of its motion.

Broad studies on the phenomena of turbulent flow and its relation to the other circumstances of specific cases.

The extension of studies on viscosity and its influence as a factor in the problems of aerodynamics.

Further study of the problem of skin friction with a view to the development of some form of rational relation between the body as to form and character of surface, the velocity of the relative motion and the amount of the skin resistance.

Studies on the mutual interference of airplane wings in various combinations and the development of optimum combinations in this respect.

Studies on the relations between aerodynamic conditions, laws and requirements, and the extension of airplane construction to continuously increasing dimensions.

The development of some theory adequate to account for the change in the value of the scale factor with change in Reynolds' number and to give an estimate of its value in specific cases.

The Screw Propeller

With the preceding sketch of the development of general aerodynamic theory, it will not be necessary to consider the parallel growth in the theory of the helicoidal propeller in other than broad outline. Four names may be used to mark the essential stages in the development of this theory. These are Rankine, Froude, Drzwiecki, and Prandtl.

Rankine, now some sixty years ago, took as the point of departure for his theory, consideration of the momentum change in the fluid acted on. That is, he studied the phenomena in the fluid and derived therefrom expressions for thrust (or pull) and effective work. Froude, a quarter century later, built upon the same theory and added many refinements in detail, especially the concept of inflow velocity, to which he gave the now commonly accepted value of one-half the ultimate increase in value.

Drzwiecki, early in the present century, took as his point of departure the blade of the propeller rather than the fluid medium. Considering the blade as made up of a radial series of airfoil sections, he took the aerodynamic characteristics of these sections and by a simple process of arithmetic integration was able to derive expressions for thrust, input work, and efficiency. This theory provides readily for taking account of the inflow velocity and this feature soon became incorporated as an essential part of the physical picture upon which the theory is developed.

Finally, Prandtl's vortex theory has been developed to apply to the propeller, and in such way as to round out the Drzwiecki method by supplying the resources of the vortex theory as an aid in estimating the aerodynamic characteristics of the series of elementary airfoil sections of which it is assumed that the blade is composed.

We have, therefore, at the present time, three methods of approach to the problem of the screw propeller:

The Drzwiecki method in which the aerodynamic characteristics of the sections are determined by direct model investigation in a wind tunnel.

The same general method, but using the further resources of the Prandtl's vortex theory of airfoil action.

Direct experimental research by the use of models and the law of kinematic similitude, in so far as it may seem proper to apply such law, where the conditions of similitude are not completely realized.

The literature on the subject of the screw propeller has enormously increased in recent years. A survey of aeronautic literature covering the last ten years shows some three hundred titles relating to the screw propeller, either theory or experimental research.

The advancing edge of propeller theory is concerned chiefly with problems relating to a more complete adaptation of

Prandtl's theory to the complicated phenomena presented by the operation of a screw propeller, especially as regards mutual blade interference and the periodic effect of the blade action on the inflow velocity. General advance in our knowledge of the screw propeller will require properly coordinated work along several lines as follows:

1 Extension of the general theory as above indicated.

2 Experimental investigation on model scale as a check on theory and as additions to our general store of observational material.

3 Experimental investigation on full scale as a check on model scale work and for the determination of ranges of "scale effect" factors.

4 Investigation, both theoretically and experimentally, of the mutual reaction between the propeller and the plane with which it is associated, and as affected by the circumstances of the case.

Aerodynamic theory is only one of the factors which make for advance in the general field of aeronautics, but it is one nevertheless of basic importance. It should point the way to useful and effective improvement, it should give indication of the goals toward which we may hope to carry forward our lines of progress, and it should warn where the way seems to be closed. Building upon the splendid body of aerodynamic theory which has been developed during the past two decades—a body of theory which, in its main features, has been so abundantly justified by experience, and guided always by resort to experimental test in the future as in the past, we may safely predict that the same spirit which has characterized the development thus far will carry on worthily in the future and that the aeronautic engineer and designer will not lack for adequate guidance in theory in his efforts to meet the demands of an ever-advancing use of aircraft as an agency of human progress.

Some Notes on Airplane Production

By JEROME LEDERER,[1] MOLINE, ILL.

RECENTLY a well-known automobile company decided to get into production on a two-place cabin monoplane. The company had had no previous experience in aircraft construction, but ample facilities and experience in producing good stampings, tanks, cowling, and machined parts. Plenty of floor space and a personnel trained in automobile production were available. The airplane company which they had taken over supplied one set of jigs and a small but well-trained aircraft personnel. The decision to build airplanes was made in early spring. A huge spring and summer market was immediately ahead. No time had to be lost in getting into production.

A production manager was supplied by the automobile company. His first step was to get a thoroughly complete material list. Every detail of the airplane down to the last cotter pin was listed, together with the number required, materials, etc. Orders were immediately placed for the required amounts of materials and parts. In the meantime dope rooms were being erected, dies made, jigs improved and built in multiple, an overhead conveyor system for carrying wings was installed, numerous welders were hired, tested, and the best retained, men and women were taken on and taught to do one job well.

Metal parts were stamped out by the automobile company's machine shop; wooden members for wings and fuselages were also made by the automobile company's wood shop. A few days' work in these shops produced sufficient parts to supply the airplane company weeks ahead. Difficulty was encountered with quantity production of welded parts such as fuselages and tail surfaces. Therefore, more welders were hired, more jigs built, and production increased to the desired quantity.

Welders were trained by putting them on scrap metal for a week under constant supervision of an experienced welder, then they were given odd jobs about the plant such as welding jigs and fixtures, after this they were allowed to weld on unimportant parts of the plane, and when they became good enough they were put on fuselages.

In six weeks the company was producing one plane a day; in two months, two a day; in three months, five a day. There has been no structural failure in any of these planes, although each plane is stunted to the limit before delivery.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

The difficulties in securing good welds and welders has turned this company's attention to the use of riveted joints, using structural members of duralumin. The advantages appear to be that a good riveter is cheaper than a good welder; a riveted joint can be inspected for strength and workmanship, whereas a good weld depends on the conscience of the welder; there is less chance of warpage; jigs are easier to build and handle; a lighter ship results. How these advantages work out in large production remains to be seen, but all indications point in their favor.

It may be reassuring to some manufacturers to know that the Germans have made numerous tests on the efficiency of welded joints made by good welders and by novices, and have found that the strength of the welds made by beginners was about 79 per cent of the strength of good welds. If this high ratio did not hold, there would be many more structural failures than now occur.

Women are generally employed for all sewing work; they are fast and efficient workers, but the invention of a special sewing machine or of some other good means of quickly attaching fabrics to airplane structures would meet with instantaneous approval by manufacturers.

Aside from production problems, manufacturers of approved types of ships have worries in connection with the requirements of the Department of Commerce. A small change in design may mean lower production cost, or increase in performance, or an improvement in the appearance of the plane, but before the change can be made, approval must be secured from the Department of Commerce. This may take several days or several weeks, depending on how busy the department is. It is suggested that a special division of the engineering branch be organized and devoted to the checking of minor changes in approved-type ships.

An example of possible damage due to delay in having a change approved is furnished by a manufacturer who was about to install a new type of landing gear on his ships, the installation to begin as soon as the department had given its approval. Rumors of the proposed change reached prospective buyers, who immediately canceled their orders and decided to wait for the improved ships.

It is apparent to many that the industry, especially the smaller companies, is in a state of great excitement. It would appear that older business heads are needed to stabilize things and provide a certain degree of soberness and experience now very much lacking. Otherwise unforeseen circumstances will force many companies to the wall.

THE ENGINEERING INDEX

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Items From Aeronautics Classification

AIRCRAFT ENGINES

Cylinders. Improved Cast Iron Cylinder for Air-Cooled Engines, R. A. Brewer. Inco, vol. 8, no. 3, 1928. pp. 15 and 21, 1 fig. Analyzing cost of major units, cylinder stands out predominant as possible source of cost reduction; problem was to obtain uniformity of structure, eliminate weak spots and also to apply support to cylinder so that stresses were not concentrated and complicated; most satisfactory metal, contents of iron.

Diesel. Advantages of Diesel-Type Aircraft Engine Pictured From Tests, L. M. Woolson. Automotive Industries, vol. 59, no. 25, Dec. 22, 1928, pp. 892–897, 6 figs. Details of high-speed compression-ignition solid-fuel injection Packard Diesel engine developed for airplanes; ignition and carburetor troubles practically eliminated; fire danger reduced; fuel cost lower; less noise from exhaust; engine will operate satisfactorily from any position and is not affected by rain or water; elimination of radio interference; possible effect of engine on airplane design. Abstract of paper presented before Soc. Automotive Engrs.

Testing. Aircraft Engine Testing, G. H. Walker. Aeroplane (Lond.), vol. 35, no. 26, Dec. 26, 1928, pp. 1007–1008 and 1010, 7 figs. See also Flight (Lond.), vol. 20, no. 52, Dec. 27, 1928 (Aircraft Engineer), pp., 100–104, 8 figs. Requirements of British Air Ministry in testing of airplane engines; details of Froude hydraulic dynamometer and of Heenan-Fell air-brake dynamometers; description of wind tunnels purchased by Bristol Aeroplane Co. from Heenan and Froude for use in testing airplane engines; testing Bristol Jupiter engine; Heenan and Froude equipment furnished Argentine Naval Aviation Station.

Dynamometer Testing of Airplane Engines, C. E. Platzer and R. F. Johnston. Aviation Eng., vol. 1, no. 3, Dec. 1928, pp. 10–11, 3 figs. Principles of operations of electric dynamometer and its advantages as means for testing airplane engines are discussed; test of water-cooled engines; altitude chambers used in tests; load and fuel consumption curves for Wright Whirlwind J-5C and J-5CA taken on dynamometer; problem of providing air for air-cooled engines while testing on dynamometer.

AIRCRAFT MANUFACTURE

Gluing. About Glue and Gluing Practice, C. L. Ofenstein. Aviation, vol. 24, no. 27, Dec. 29, 1928, pp. 2099, 2112, 2114, 2116 and 2118, 3 figs. Glue and gluing practice among commercial aircraft factories; casein especially described; testing for adhesiveness and water resistance after glue has been mixed according to directions, some sort of mechanical mixer having two speeds is recommended; glue receptacles should be cleaned out once a day; high pressures for thick glues.

AIRPLANE ENGINES

Berlin Exhibition. New German Aero Engines, E. P. A. Heinze. Aero Digest, vol. 13, no. 6, Dec. 1928, pp. 1168, 1170 and 1172, 8 figs. New German engines exhibited at International Aircraft Show in Berlin are described; B. M. W. leading with Junkers second, and Siemens and Halske third. Details of one new water-cooled 12-cylinder and two 6-cylinder B. M. W. engines, 6-cylinder and 12-cylinder V water-cooled Junkers engines, Siemens-Halske air-cooled radial engines, Daimler-Benz 12-cylinder V water-cooled and 3-cylinder water-cooled engines, and Argus two 12-cylinder inverted V engines.

Cowling. Drag and Cooling With Various Forms of Cowling for a "Whirlwind" Engine in a Cabin Fuselage, F. E. Weick. Nat. Advisory Committee for Aeronautics—Tech. Notes, no. 301, Nov. 1928, 25 pp., 35 figs. on supp. plates. Results of investigation in 20-ft. propeller tunnel on engines entirely exposed up to completely cowled types; cooling tests made and cowling modified; drag tests; great drag in uncowled engine; effect on airplane performance; cowling entirely covering engine reduces drag 2.6 times conventional cowling; careful design for proper cooling; with cabin fuselage its use should substantially increase high speed.

The New N.A.C.A. Low Drag Cowling, F. E. Weick. Aviation, vol. 25, no. 21, Nov. 17, 1928, pp. 1556–1557, 1586, 1588 and 1590, 6 figs. Wind-tunnel tests undertaken by National Advisory Committee for Aeronautics on different designs of cowling on air-cooled engines; enormous drag due to uncowled engine; conventional cowlings covering only central portion of engine only slightly reduced drag; new cowling of N.A.C.A. design completely enclosed engine but provided for cooling; great reduction in drag in wind-tunnel and flight tests; substantial increase in high speed and all-round performance.

Efficiency. Thrust Horsepower and Engine Efficiency, J. M. Shoemaker. Aviation (Aeronautical Eng. Supp.), vol. 26, no. 3, Jan. 19, 1929, pp. 6–7, 1 fig. Maximum effective thrust per horsepower, or effective thrust available in aircraft power plant is discussed; high-speed engine with propeller driven at crankshaft speed is less efficient than slower turning propeller in same aircraft; high-speed direct-drive propeller may also suffer loss of efficiency due to excessive propeller tip speed; latest practice in regard to reduction gear ratios.

Heavy-Oil. Heavy-Oil Engines for Aircraft, D. R. Pye. Aeronautics (A.S.M.E. Trans.), vol. 50, no. 26, Sept.-Dec. 1928, pp. 45–47 and (discussion) 47–50, 2 figs. Early experimental work with single-cylinder research unit to explore possibilities of compression-ignition operation; alternative designs; results in single-cylinder experiments showing effect of maximum cylinder pressure upon brake m.e.p. and fuel consumption; fuel-valve and nozzle design; air swirl; valve mechanism; possibilities of two-stroke cycle; performance at altitude; earlier use of heavy-oil engines in airships probable.

Problems. Problems of Aircraft Engines (Flygmotorproblemet), E. Hubendick. Ingenioers Vetenskaps Akademien (Stockholm), no. 84, 1928, 34 pp., 29 figs. Account of problems of aircraft engines; J. E. Cederblom's research work on ideal aircraft engine; climatic conditions of Scandinavia call for special measures for cooling and starting; adaptation of Diesel engine for aircraft; possibilities in design conditioned by new materials, etc.

AIRPLANE PROPELLERS

Characteristics in Flight. Characteristics of Five Propellers in Flight, J. W. Crowley, Jr. and R. E. Mixson. Nat. Advisory Committee for Aeronautics—Report No. 292, 1928, 23 pp., 22 figs. Investigation to determine characteristics in flight of four propellers differing uniformly in thickness and pitch, and of fifth which was identical with one of others except for change in airfoil section; efficiencies in flight were consistently lower than in model tests; conclusion that for accurate comparisons it is necessary to know propeller pitch angles under actual operating conditions.

Design. Aircraft Propellers, C. H. Havill. Soc. Automotive Engrs.—Jl., vol. 24, no. 1, Jan. 1929, pp. 17–23, 4 figs.; see also Aviation Eng., vol. 2, no. 1, Jan. 1929, pp. 7–12, 3 figs. Nearly all Army and Navy aircraft propellers of detachable-blade type; Navy has found it necessary to make its own designs; sources of designer's information; discrepancies between model tests and wind-tunnel tests; substitute propellers; strength of propellers; empirical formulas for maximum fiber stress; whirl test for strength; necessity for use of gearing; considerations applying to variable-pitch propellers.

Vortex Theory of Airplane Propellers (Zur Wirbeltheorie der Luftschrauben), T. Troller. Zeit. fuer Angewandte Methematik und Mechanik (Berlin), vol. 8, no. 6, Dec. 1928, pp. 426–430, 8 figs. Diagrams are presented showing results of calculation of induced speeds in axial direction, based on Prandtl's approximate method; there are cases in which more exact calculation is necessary, but in most cases it is believed that Prandtl's method is sufficiently accurate.

Design of Aircraft Propellers (Calcolo delle eliche), A. Zezi. Notiziario Tecnico di Aeronautics (Rome), vol. 7, no. 11, Nov. 1928, pp. 1–80, 36 figs. Review of theoretical researches; presentation of author's original method of analysis and design; author's method compared with study by Durand and Leslie.

Influence of Fuselage on Propeller Design, T. Troller. Nat. Advisory Committee for Aeronautics—Tech. Memorandums no. 492, Dec. 1928, 11 pp., 5 figs. on supp. page. Simple method for designing propeller for given arrangement of airplane parts; inflow to propeller and efficiency of propeller affected most by fuselage; flow conditions in plane of propeller, which are affected by shape of fuselage; determined mathematically; condition for best thrust distribution; induced efficiency of propeller calculated. From Zeit. fuer Flugtecnik und Motorluftschiffahrt, July 28, 1928, previously indexed.

Propeller Problems, A. Betz. Nat. Advisory Committee for Aeronautics—Tech. Memorandums, no. 491, Dec. 1928, 13 pp., 3 figs. on supp. plate. Comprehensive survey of most important propeller problems; detached propeller considered; propeller slipstream and propeller blade theories; conditions on best thrust distribution; special care must be exercised to avoid excessive angle of attack in starting which would cause air flow to separate from blades and thus interfere with their normal functioning; reactions between propeller and vehicle are discussed.

Manufacture. Making Micarta Airplane Propellers, R. L. Knight. Machy. (N. Y.), vol. 35, no. 6, Feb. 1929, pp. 409–411, 8 figs. Manufacture of micarta propellers is described; extreme care throughout manufacturing process to assure uniform and high-grade material, perfect balance of blades, and perfect fit of blades to hub; machining to fit blade to metal hub; final balancing and setting of pitch angle.

Metal. Technical Development of the Reed Metal Propeller, S. A. Reed. Aeronautics (A.S.M.E. Trans.), vol. 50, no. 26, Sept.-Dec. 1928, pp. 55–62, 14 figs. Details of aircraft propeller with solid, thin, and almost knife-like blades of forged or rolled duralumin or other aluminum alloys of similar type; development of metal propeller; propeller and its function; types of Reed metal propellers; materials of construction; methods of manufacture.

Metal, Failure of. Preliminary Study of Fatigue Failures of Metal Propellers Caused by Engine Impulses and Vibrations. Air Corps Information Cir., vol. 7, no. 618, Aug. 15, 1928, 17 pp., 7 figs. Study of cause and prevention of breakage of certain metal propellers in bending in plane perpendicular to propeller axis; causes investigated with particular reference to phenomena of resonance; theory presented in detail; cantilever beam, dynamic load, on propeller blade due to constant acceleration of hub; mathematical analysis of vibration of propeller in plane of rotation; frequencies of similar propellers.

AIRPLANES

Ailerons, Forces Produced by. Determination of Air Forces and Moments Produced by Ailerons of Airplane (Bestimmung der durch die Querruder eines Tragfluegels erzeugten Luftkraette und Momente), C. Wieselsberger and T. Asano. Zeit. fuer Flugtechnik und Motorluftschiffahrt (Munich), vol. 19, no. 13, July 14, 1928, pp. 289–293, 10 figs. Lift distribution produced by aileron deflections for rectangular wing on straight flight previously determined on theoretical principles for certain case, and air forces and moments produced by ailerons derived; approximate values deduced from former results for use in calculating acting forces and moments for all cases; results expressed by non-dimensional coefficients represented in diagrams.

Alloys for Use in. The Use of Light Alloys in Aircraft From the Point of View of Corrosion, H. Sutton. Royal Aeronautical Soc.—Jl. (Lond.) vol. 33, no. 217, Jan. 1929, pp. 38–54 and (discussion) 54–74, 19 figs. Value of various light alloys in aircraft construction and question of corrosion are discussed; aluminum and its alloys; restrictions of commercially pure aluminum; duralumin; aluminum-copper-silicon alloys; wrought aluminum-silicon alloys; wrought aluminum-copper-nickel alloys; aluminum casting alloys; protection of aluminum and aluminum alloys; magnesium alloys assuming increasing

importance; influence of corrosion on fatigue properties.

Anti-Friction Bearings in. Anti-Friction Bearings in Aircraft Construction, W. B. Moore Aviation Eng., vol. 1, no. 3, Dec. 1928, pp. 7–9, 5 figs. Application of anti-friction bearings to airplanes from engineering and practical view is discussed; suitability of bearings of any type for service; advantages; application of tapered roller bearings to main crankshaft and rocker arms of airplane engine to carry both radial and thrust loads; load characteristics and service requirements encountered in application of anti-friction bearings to landing and tail-wheels.

Autogiro. The Autogyro, V. E. Bertrandias. Engrs. and Eng., vol. 46, no. 1, Jan. 1929, pp. 26–27 and (discussion) 27–28. Interesting points of this new windmill plane are described; autogiro plane possesses valuable qualities of stabilty at large angles of incidence and ease of landing.

The Autogiros of Cierva (L'autogire de la Cierva), M. Ducout. Aérophile (Paris), vol. 36, no. 19–20, Oct. 1928, pp. 295–298, 7 figs. Cierva autogiro has carrying surface of 11 sq. m.; total weight 1120 kg.; motor 220 hp.; theory of design; rotation of lifting screw; speed of air relative to blade and forces applied; speed of autogiro.

The Cierva Autogiro, W. H. Sayers. Aviation, vol. 25, no. 18, Oct. 27, 1928, pp. 1320–1321, 1340–1342, 1344 and 1350, 8 figs. Aerodynamic principles involved in design and operation of this craft are discussed; all autogiros so far have standard airplane fuselage, landing gear and tail unit; rotating system of supporting surfaces in place of wings; ailerons are separate organs carried on each side of fuselage; blades in addition to rotating, move up and down; speed of vane system constant; auxiliary drive being developed will allow spinning up of rotating wings directly by main engine.

Biplane, Freight. Boeing Building Mail Plane Fleet. Aviation, vol. 25, no. 27, Dec. 29, p. 2106. Few details of new biplane Model 95 for handling mail and express only; 525-hp. Hornet engine; four mail and express compartments have cargo capacity of 1600 lb.; wing span 44 ft. 3 in.; body of steel and dural construction, durat and fabric covered; wood wings.

Commercial, Design of. Commercial-Airplane Design (Der Verkehrsflugzeugbau), A. Gymnich. Luftfahrt (Berlin), vol. 32, no. 23, Dec. 7, 1928, pp. 364–368, 7 figs. Problems of design are discussed from standpoint of pilot; monoplane is favored in Germany, Holland, and United States, whereas England favors biplane and in France both types are employed to about equal extent; one cannot be called superior to other; aileron construction; steering gear.

Control. Research on the Control of Airplanes, B. M. Jones. Nat. Advisory Committee for Aeronautics—Tech. Memorandum, no. 485, Oct. 1928, 23 pp., 9 figs. on supp. plates. Controversy between theory of airplane with control organs and uncontrolled stable airplane; provision of good control qualities is more a question of proportioning airplane and adjusting its load than of new control organs; motions of stalled airplanes studied by Cambridge University Air Squadron; great influence of yawing motions on rolling couples; ailerons positive source of danger; rudder provides control difficult to use; solutions offered. From Nature (Lond.), May 12, 1928. Lecture delivered at Royal Instn.

Control, Automatic. Device for the Automatic Control of Airplanes, A. Gradenwitz. Nat. Advisory Committee for Aeronautics—Tech. Memorandums, no. 495, Jan. 1929, 6 pp., 6 figs. partly on supp. plates. Control device of Franz Drexler has three switch buttons, with which three-part automatic control can be switched on or off in whole or part, leaving controls to be operated by hand or foot; for intermediate control, purely electric transmission used; each mechanism has special contact-making device by which reversing motor is set rotating; direct current produced by windmill-driven generator; provisions for correction. From Motorwagen, March 31, 1928, pp. 204–206.

Design. The Modern Aerodynamic Theory in Design, I. H. Driggs. Aviation (Aeronautical Eng. Supp.), vol. 26, no. 3, Jan. 19, 1929, pp. 2–5, 2 figs. Discussion of Prandtl theory with formulas developed from study of subject; rational formula for drag curve of airplane over useful range developed and from that minimum drag and consequently maximum L/D and velocity at which they occur; from resulting power equation value of minimum power and speed at which it occurs is found; wing are a should be determined from speed of minimum power rather than landing speed desired; case of glider and soaring speed; minimum sinking speed for soarer.

Design Stresses. Design Stresses, C. D. Holland. Flight (Lond.), vol. 20, no. 43, Oct. 25, 1928; (Aircraft Engr.) pp. 936f-936g, 3 figs. Design stress defined as maximum stress material will be subjected to when structure is loaded to amount of load factor times normal load; determining tension, pure compression, compression for long struts, shear, bearing, Young's modulus, modulus of rigidity, and Poissons' ratio; effect of workshop processes on normal design stresses of materials; effect on design-stress value due to different types of loading.

Dope. Aircraft Dope and Finish Problems, E. G. Davis. Aviation Eng., vol. 2, no. 2, Feb. 1929, p. 10. Discussion of dope failures, and complete peeling of dope film away from fabric; this trouble was noticeable some 12 years ago, but disappeared with development of dopes after war, only to appear again now; military and naval aircraft free from these troubles; tendency of aircraft industry toward heavy, thick, smooth coatings which do not show any fabric is mechanically unsound.

Landing Gear, Caterpillar. A Caterpillar Landing Gear for Airplane, P. E. Lamarche, Jr. Aero Digest, vol. 14, no. 1, Jan. 1929, p. 88, 1 fig. Details of caterpillar tread device called "petin atterrisseur" developed by Louis Vinay, Paris; caterpillar tread is attached to landing-gear axle in such a way that it allows plane to pivot on its axis when it meets an obstacle; it can overcome obstacles on ground up to height of about 16 in. with ease; Westinghouse air brakes mounted on each caterpillar tread inside plates that form sides.

Metal Construction. Armstrong-Whitworth Metal Construction. Flight (Lond.), vol. 20, no. 52, Dec. 27, 1928, pp. 1083–1087, 28 figs. Typical Armstrong-Whitworth forms of metal construction and methods employed are described; both circular-section tubes and rolled or drawn strip used; tubular construction confined to fuselage; two types of mechanical joints; each side of fuselage built up on jigs; steel strip wing spar construction; heat treatment and rust protection.

Metal, Manufacture of. New Construction in Metal Airplanes, F. H. Colvin. Am. Mach., vol. 69, no. 26, Dec. 27, 1928, pp. 983–986, 7 figs. Advanced method of constructing airplane fuselage and way in which aluminum alloy sections are built up, drilled, and riveted for Navy planes in plant of Glenn L. Martin Co. are described; alignment of fuselage claimed difficult with construction of welded tubing; Eureka built-up section developed by engineers of Martin Co. is described; fuselages of high-strength aluminum alloy with chrome molyldenum steel fittings.

Structures of Thin Sheet Metal, Their Design and Construction. H. Wagner. Nat Advisory Committee for Aeronautics—Tech. Memorandums, no. 490, Dec. 1928, 21 pp., 32 figs. on supp. plates. Uses of sheet-metal coverings in conjunction with inner structure; special methods of construction on which author worked with Rohrbach firm, and which enables simple and cheap shop work combined with great simplicity and lightness; Junkers, Brequet, and Dornier construction; flat sheet-metal girders with very thin webs; buckling strength of flat metal sheets; curved sheet-metal girders.

Performance Testing. The Reduction of Performance Tests, H. L. Stevens. Roy Aeronautical Soc.—Jl. (Lond.), vol. 32, no. 215, Nov. 1928, pp. 934–957, 11 figs. and (discussion) 957–958. Computing work necessary between receipt of performance figures from pilots and issue of complete report; method adopted for all but supercharged engines assumes power depends on pressure only; effects of atmospheric temperature on altimeters and air-speed indicators; correction of air-speed readings for propeller-efficiency correction; density method for correcting performace tests of supercharged engines; pressure methods explained.

Speed Control. Increasing Airplane Speed-Range, H. Alfaro. Soc. Automotive Engrs.—Jl., vol. 24, no. 1, Jan. 1929, pp. 99–100. Devices for improving control at slow speed are discussed; recommendations made by Daniel Guggenheim Foundation for safety and efficiency of aircraft.

Spinning Characteristics. The Spinning of Airplane, M. Watter. Aero Digest, vol. 13 no. 5, Nov. 1928, pp. 912, 914 and 916, 2 figs. Spinning and efforts directed toward explaining it; spinning sometimes considered desirable maneuver in combat; aerodynamical and dynamical causes of spinning; mastery lies in reliable means of recovery and not in prevention; increase in size of stabilizing and controlling surfaces only temporary remedy; possibility of using automatic leading edge slots on horizontal tail surfaces; rudder action is of utmost importance in spinning and efforts should be directed to increase its efficiency by preventing its shielding; inclination of rudder axis suggested.

Wing Design. A New Method of Stress Calculation of Monoplane Wings, S. Kaneko. Tokyo Imperial Univ.—Jl. (Tokyo), vol. 17, no. 12, Oct. 1928, pp. 213–244, 22 figs. General equations for deflection and torsion of wing and expressions for stresses in its structure; actual strength of monoplane wing is widely different from results obtained by conventional method of stress calculation; it must be calculated from combined considerations of deflection and torsion of wing as whole; problems solved for special cases; comparison of numerical results obtained by present method and by conventional one. (In English.)

AIRSHIPS

Metal, Construction of. Metal Airship Construction. Engineer (Lond.), vol. 146, nos. 3803 and 3804, Nov. 30 and Dec. 7, 1928, pp. 598–599 and 622–625, 18 figs. Nov. 30: Description of Boulton and Paul's most recent practice with particular reference to work which it has carried out for Government airship R 101; materials entering into airship structure. Dec. 7: In addition to rolling and drawing of various sections in steel and in duralumin, contract involves manufacture of large number of joints, sockets, shackles, and other details.

Rigid. Airship R 101. Flight (Lond.), vol. 20, no. 47, Nov. 22, 1928, pp. 997–998. Single-line diagrams stressed by Royal Airship Works and geometry and load used by Boulton and Paul to convert line diagram into actual structure; airship is compromise between perfect aerodynamic shape and practical manufacturing possibilities; very fundamental differences between R 101 form of construction and that of Zepplin; accuracies to which manufacturers had to work; some dimensions show magnitude of task.

AER-51-5

Production Problems of the Commercial-Aircraft Manufacturer

By E. E. PORTERFIELD, JR.,[1] KANSAS CITY, MO.

The author divides the problems confronting the manufacturer of commercial airplanes into those of production and sales, and confines himself to the first of these. The problems of production include inadequate supply of motors and lack of an outstanding standard type, unreliability of supply of materials of construction, uncertainty of schedule to follow in large-scale production, difficulties arising out of the certification required by the Department of Commerce, standardization of product, methods, and materials, and the coordination of production and distribution.

THE producer of commercial aircraft has enjoyed a period of tremendous growth within the past two years. The industry has expanded amazingly during this time, thanks to the startling achievements of a score of heroic figures whose feats have crystallized general interest and created a public air-consciousness. The airplane manufacturer has been riding the wave of the industry's first great boom, so to speak. However, his initial success has not been achieved without a number of attending difficulties.

Any industry, of course, may expect to encounter circumstances in its early stages of development which only time and intelligent effort at organization can overcome. Every business goes through a formative period in which it must experience its growing pains. Of these, commercial aviation has had its share and, in fact, is still having its share. Such problems as may be considered common in the early years of every industry need not be studied at any great length here. However, the manufacturer of commercial aircraft is confronted with certain problems which are peculiar to the aircraft industry. These fall naturally into two classes: namely, the problems of distribution or sales, and the problems of production.

In our own industry, the difficulties arising in production have assumed an importance to the manufacturer which has, to date, practically excluded serious consideration of sales development. Although it is my task to consider the problems of production, I want to call particular attention to the increasing importance of sales effort in the industry. For, as we apply ourselves to the problems of building more airplanes and building them better and more economically, we shall find ourselves reaching production capacities for which sales outlets must be created. In other words, in the solution of production problems we shall be creating problems of selling.

The groundwork in our selling organizations cannot be established too soon. Probably every prominent builder has seen the necessity of devoting more and more effort to sales organization and is acting accordingly. He realizes that large-scale production will drive him to a higher pitch of selling activity.

The point I would make regards the vital importance of solidarity and reliability of the selling organization as it affects production. There can be no such thing as stable production until there exists a distributing system in which we can place dependence. How shall we estimate production for a given period unless we can gage accurately the capacity of distribution over that period? The manufacturer who creates a sales organization from which he can accurately predetermine sales potentialities has gone a long way in the matter of simplifying production problems.

Motor Problems

But perhaps of more immediate interest to every producer is a discussion of problems now affecting production. For after all, we are still in the formative period, in which we find our most serious problems in the factory. Of these, one of the most important—and by the same token, one of the most vexing—is the difficulty of powering planes.

Although there is promise of new production motors in adequate quantity, and at prices which will permit their use in the medium and lower priced plane, we are still struggling with the problems arising with present motor supply. In the first place, no single new production power plant has proved itself to be outstanding. In fact, no two or three engines have developed what might be called public preference. It has therefore been necessary for the manufacturer to diversify production to satisfy a wide range of public demand. The result is reduced production and increased production costs.

The war-surplus motor has been both an invaluable aid and a decided handicap to commercial-aircraft builders. These power plants have made it possible for aircraft companies to meet the growing demand. They have made it possible to put ships on the market at prices which make possible something approaching large-scale production and distribution. But on the other hand, they have greatly retarded the development of new motors for the simple reason that motor builders have not been able to compete with this cheaper type of power. Of course, with a very limited market for new engines, the builders could not possibly hope to reach a point in production where their motors would receive general acceptance. The day of the war-surplus motor is about over, and manufacturers are awaiting the end of the OX-5's and "Hisso's" with mixed feelings of joy and misgivings. Will it be possible to continue large production with ships powered with expensive new motors? I think the let-down will be scarcely noticeable—and then only for the few months it takes engine builders to reach large-scale production. When that point is reached we shall all look back upon the war-surplus power plant as a necessary evil which both helped and harmed the industry. In the meantime, we can only fight for standardization of the power equipment and foster the efforts of motor builders who are striving to give us the type of power we must have before commercial aviation can make substantial progress.

Problems of Materials

The motor situation is largely duplicated in much of the materials used in plane construction. The industry has actually grown so fast within the past two years that the sources of material have proved entirely inadequate. The manufacturer, unfortunately, has been entirely dependent upon independent sources for practically the entire list of materials used in plane construction. The rapid growth of the industry calls not alone, therefore, on the aircraft builders, but upon a dozen other industries as well. And in the case of most materials, the companies that are called upon to supply the aircraft builder find themselves faced with the necessity of producing this material to

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

special specifications. As a result, it has taken time and a great deal of effort to secure anything approaching an adequate, reliable supply of aircraft production materials.

In view of the fact that there are so many different kinds and types of material involved in the production of the airplane, we must congratulate ourselves upon being able to accomplish as much as we have. Much has been done toward the solution of this problem. Whereas a year or two years ago the business of supplying airplane materials was incidental to other lines of business, we have today a group of organizations handling airplane supplies exclusively, or specializing in the materials the industry demands. Still there is much to be done in this connection. There must be greater standardization of parts and materials; there must be more definite and more rigid specifications imposed on the producers, and more thorough and painstaking inspection of supplies in instances where we are dependent upon outside sources. We have fought, and are still fighting, for better motors. We have accomplished a great deal in the matter of supplies coming from outside sources, but still find ourselves handicapped at every turn. We still, for instance, are forced to tie up a practically completed ship while it awaits the delayed delivery of a propeller, or some other part over which we have no control. But greater than either of these difficulties, perhaps, is the one arising within our own organizations.

Problems of Organization

There is the vital question, "What shall our program be this year, or this month, or a year from now?" We have found it almost impossible to gage public demand. Fortunately, so far we have underestimated it. Fortunately, I say, for such a condition leaves us financially safe for the moment. But it adds to our problem. Public demand is calling upon us early for large-scale production. Still, we find ourselves with many of our initial problems unsolved and with our policies consequently undetermined. The industry is still in its formative period, even though we may say that we are approaching the close of that stage.

As a result, we are facing the problem of constantly increasing production without being sure of the ground over which we must travel. None of us knows what the trend of public demand may take a year from now or even a month hence. We have by no means completed our engineering and research work. We are by no means satisfied with our present product and present methods, gratifying as they may be. Therefore, no manufacturer cares to invest heavily in machinery and equipment necessary to make large-scale production possible. As a result, we are faced with the alternative of forcing production through the continued use of present methods, namely, through hand work or small machine-tool work, upon materials provided through outside sources. Obviously, such a program means a high cost of production. Equally obviously, the continued success of the industry demands the use of efficient machi nery for large-scale production—the only course to reduce production costs. We are agreed, I think, that we want to move into the era of greater production, with its many obvious advantages for the manufacturer. We must reach the large-production stage soon. In fact, we are being forced toward it daily. But how soon shall that time arrive? How soon shall we be fully prepared to enter it?

Three Essential Conditions

I would say that three things are necessary in the meantime. One is beyond our control, except as we can cooperate as a group to remedy the situation now existing. Due to the fact that the Engineering Department at Washington has been so overloaded with the mass of applications piled up on it by manufacturers, we are all vitally interested in seeing some steps taken to alleviate that situation, and to speed up the part which the Department of Commerce must play in our progress. The approved type certificate is vitally necessary to the commercial builder. The mass of work thrown upon the department is seriously delaying the issuance of important approvals, and consequently seriously hampering our efforts at determining upon a definite program. We must determine among ourselves some method of reducing the demands being made upon this department; and recommend, if possible, some method of simplifying the work of the department.

Within our own organizations there are two courses of action which suggest themselves. First of these is standardization as far as possible in every phase of factory operation. It means standardization of product, of methods, and of materials. The manufacturer obviously must cease to experiment with a dozen different types of planes and motors. He cannot hope to continue, as he has in the past, a system which makes practically every ship an individual production problem. Neither can he allow such wide latitude in the installation of special equipment. He must standardize upon a given line of planes, powered with certain motors and equipped in a given manner. Such a program leaves the manufacturer with a number of engineering problems and only a limited time in which to solve them. It is simply his own responsibility to determine his program and to institute it without delay. And if he should waver too long in his decision, he will find himself trailing the leaders, who have had the foresight and initiative to fix their programs early.

Further standardization is vitally necessary to materials. I have already mentioned the necessity of demanding more reliable sources of supply, and of imposing more rigid standards of specifications and inspection. In this phase, the initiative rests with the manufacturer. When he has fixed his program of production more carefully, a certain degree of the confusion now existing regarding supplies will automatically be eliminated.

Finally with our policy of action definitely settled, we shall have a basis from which to proceed. Our only problem from that point is the coordination of production and distribution. The responsibility then passes largely to the sales department, which must establish a definite method of arriving at potential sales over a known period. Knowing sales capacity, the question of accomplishing stable production is comparatively simple.

The industry is facing an important year. It must prepare to move from its formative period to the stage of permanent development and expansion. If it would lose none of the impetus it has gained within the past two years, it must move quickly, but carefully. There are any number of problems confronting the aircraft builder, each one of them vitally important. There can be no set of rules offered for the solution of any one of these difficulties, for after all the problem is largely individual and peculiar to each manufacturer. However, there is much to be gained through cooperation within the industry. If there is any one thing I hope we shall all learn early in the game, it is that we shall learn to get together with our problems; to cooperate—for there will be much that we can accomplish together that will be impossible in any other manner.

The industry has prospered in its early years. It will prosper even more if we grasp the opportunities before us—if we devote the thought and energy necessary to carry us into the more permanent and important phases of development and expansion still ahead of us.

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FIG. 1 T. C. AIRSHIP WITH AIRPLANE ATTACHED

Design, Construction, and Handling of Non-Rigid Airships

BY THOMAS L. BLAKEMORE,[1] LAKEHURST, N. J.; J. F. BOYLE,[2] NEW YORK, N. Y.; AND NORMAN MEADOWCROFT,[3] HAMMONDSPORT, N. Y.

FOR AIRSHIP construction the fabric must be light in weight, strong, and gas tight. The cloth used should be high-grade cotton sheeting, closely woven, and as free from imperfections as it is possible to get it in order that it will rubberize perfectly. Airship cloths were formerly made of Egyptian cotton. Now the best grade of long staple American cotton is used. Silk is unsatisfactory since it does not rubberize. There is a standard grade of balloon cloth which weighs two ounces per square yard, and has a strength of approximately 40 lb. per inch of width. All cloth received from the manufacturer is inspected carefully for dropped threads, slubs, and blemishes. All of these are combed out or the cloth is discarded. This work is preparatory to spreading rubber on the fabric.

Spreading the fabric is the process of making it gas tight. A spreader machine is about 20 ft. long and five feet wide. It has a series of steam-heated pipes over which the spread fabric travels, driving off the solvents of the rubber. At the feeding end of the spreader is a knife extending all the way across it. This knife is adjustable, up and down, and is brought down very close to the fabric which runs under it. The rubber compound, which is spread on the fabric, is a plastic mass. It is placed in front of the spreader knife on the cloth as the cloth runs through. The knife scrapes off the compound leaving a very thin film on it. Seven passes through the spreader deposit approximately one ounce of gum per square yard on the surface of the fabric.

After the various plies of cloth have been rubber coated on both sides, they are plied together into the finished fabric. This finished fabric is then vulcanized. Airships are generally made of three-ply fabric of the following construction:

Outside aluminum	0.7 oz. per sq. yd.
Outside gum	1.0 oz. per sq. yd.
Outside cloth (straight)	2.0 oz. per sq. yd.
Film (gum)	2.0 oz. per sq. yd.
Middle cloth (bias)	2.0 oz. per sq. yd.
Film (gum)	3.5 oz. per sq. yd.
Inside cloth (straight)	2.0 oz. per sq. yd.
Inside gum	0.5 oz. per sq. yd.
Total	13.7 oz. per sq. yd.
Strength warp	80 lb. per sq. yd.
Strength filler	80 lb. per sq. yd.

Maximum permeability, 15 liters per sq. m. in 24 hr.

Other fabrics used for various purposes are two-ply bias and one-ply straight. The object of a bias ply in a fabric is to prevent spreading of tears. The vulcanizing or curing of the fabric is accomplished by winding the completed fabric on a roll very tightly. The roll is wrapped in cloth and the whole placed in a steam cabinet. Steam pressure is increased to a certain point, maintained a certain time, and then gradually reduced. About an hour is required to cure a roll of fabric. This vulcanization process changes the gum from a plastic to an elastic mass. The vulcanization is not carried to completion.

The life of airship fabric depends greatly upon the cure and condition of service. Airships operated at Scott Field, Ill.,

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Presented at the Third National Meeting of the A.S.M.E. Aeronautic Division, St. Louis, Mo., May 27 to 30, 1929.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

have been in constant service, under pressure, for over two years. One "TC" ship has a record of 1300 flying hours. The pressure of the gas, combined with the loads on the fabric due to the suspended weights, gradually weaken the fabric, which in turn begins leaking. Gas bags are usually replaced after two years of service.

The general practice now is to paraffin the entire inner surface of airship envelopes. Paraffining increases the life of the fabric, and decreases the permeability to gas. The process of paraffining an envelope is purely a mechanical one. The completed envelope is turned inside out, and melted paraffin mixed with hot benzol is spread on the surface with common garden sprinkling cans, and is brushed quickly over the surface with corn brooms.

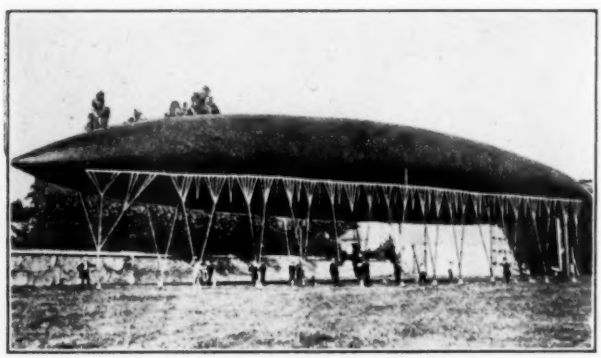


FIG. 2 EARLY NON-RIGID AIRSHIP WITH SUSPENSION GIRDLE OR BAND

The hot benzol carries the melted-paraffin solution into the gum and leaves a film of paraffin on the surface. This benzol penetrates so rapidly that it carries the paraffin clear through the entire fabric, and in some cases blooms out on the opposite side. The envelope is allowed to stand twenty-four hours until all the benzol evaporates, leaving the paraffin alone on the surface.

The red rays of the sun are the worst enemy of rubber, consequently the outer surface of the fabric is coated with aluminum. The aluminum is put on the fabric during the spreading process. It is in the form of powder, and is mixed with gum compounds in some cases, and spread on the surface. In other cases the aluminum powder is printed on to the surface by a printing machine, which consists of several rolls through which the fabric is passed, the powder being pressed into the outer surface of gum. This metallic-aluminum surface reflects the red rays of the sun.

All airship fabric is tested for diffusion and tensile strength. The diffusion must meet the specification requirements, which are very rigid. Diffusion varies considerably with the cure. Some fabrics when new have a relatively high diffusion, which drops shortly after being put into service. Other fabrics have a low diffusion when new, which rises in service. It has been found that practically all fabrics reach a common level of diffusion.

Diffusion of gas through fabric is a process of leakage or osmosis. There is a leakage of air from the outside in through the fabric. The final result is that the filling gas becomes mixed with a certain amount of air and becomes impure. In the case of hydrogen-filled balloons it is necessary to test the purity constantly, since hydrogen mixed with gas becomes an explosive mixture, and when the purity of hydrogen gas comes to a certain low point, it is necessary to purge the ship. This purging process consists simply in adding more pure hydrogen to the filling, thereby raising the purity of the entire contents. When, however, the hydrogen purity drops to a certain point it is necessary to deflate and refill. The old hydrogen is discarded.

In the case of helium-filled ships purity is checked constantly. However, when the purity of helium reaches a certain low point, the helium is removed, purified, and returned to the ship. Although the initial cost of helium filling is higher than that of hydrogen, when it is considered that helium filling can be purified and returned to the ship, the initial cost is not such an important factor.

There are no hydrogen-filled military airships operated in this country, since there is an act of congress forbidding the use of hydrogen in airships. Helium, of course, is non-inflammable and although the efficiency of a helium-filled ship is below that of a hydrogen-filled one, the increased advantage of non-inflammable helium compensates for the loss of efficiency.

FABRICATION

The process of fabricating an envelope of an airship is as follows:

The rubberized fabric is cut into panels. The largest panels in a standard training ship of the "TC" type are eight feet long

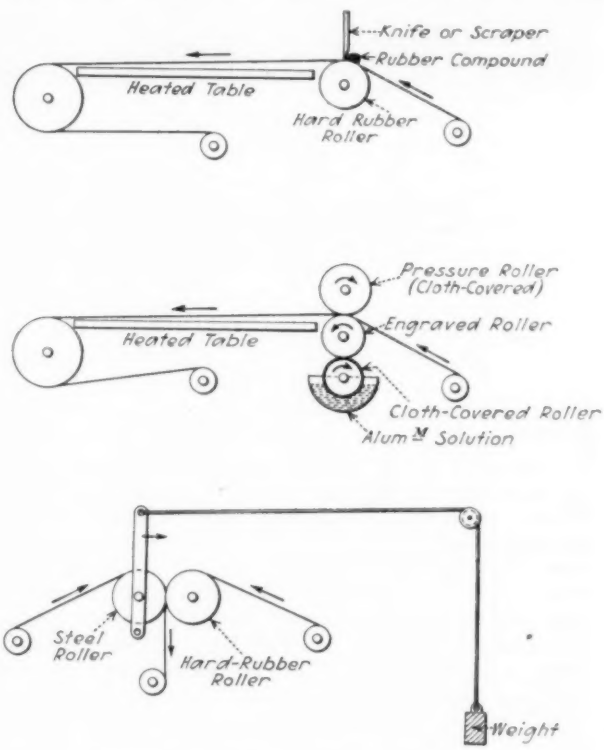


FIG. 3 OPERATIONS IN AIRSHIP FABRIC MANUFACTURE. AT TOP, SPREADING; IN CENTER, PRINTING; AT BOTTOM, DOUBLING

and the width of the fabric, which is 40 in. The panels are developed on the conic-frustrum method. The envelope being a surface of revolution, it is necessary that the panels be developed on this conic-frustrum method. The cutting of the fabric in the panels is accomplished in much the same way as the cutting of clothes is done in the tailoring industry. Cardboard patterns of individual panels are laid upon the fabric, which in turn is placed on a cutting table in several layers. Cutting is done with a cloth cutter. It is possible to cut as many as fifty layers at one cutting.

The panels of fabric are seamed together with an ordinary lap seam. The lap is three-quarters of an inch wide. The joints are made with rubber cement. It is necessary to buff the aluminum surface where a seam is made. After the cemented

seam is completed it is sewed with a double row of stitches. The stitches are covered inside and outside with rubberized-fabric tape to prevent leakage.

The rubberized cement used is a non-curing cement, although the latest cements are air curing. A three-quarter inch lap is sufficiently strong to break the fabric in a test. These tests are made before the seam is sewed. The sewing is simply an extra precaution. The seams are tested at a temperature of 115 deg. fahr., which has been found to be the maximum temperature obtained on airships in hot climates. This temperature is a superheat due to the sun's rays and to the heating of the filling gas. At this temperature the rubber cement (three-quarters of an inch seam) is capable of holding 40 lb. per in. of width.

Balloon fabric is made right-and-left bias. The bias is alternated in the balloon in gores. A "TC" ship, which has a diameter of about 45 ft., is made up of twelve gores running lengthwise of the ship. The gores are made up of individual panels. All the panels in one gore are of the same bias, and the bias in the adjacent gores are alternated. This alternation of the bias keeps the ship from twisting out of shape.

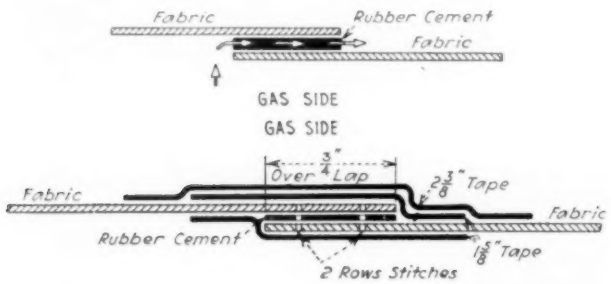


Fig. 4 At Top, Leakage of Gas Through Plain Seam. At Bottom, Typical Envelope Seam

Due to the weight of the tail surfaces the ship droops at the tail after being in service, consequently the envelopes are tailored, that is, pie-shaped pieces of fabric are cut out of the envelopes at various sections so that the envelope when new actually has a rising tail. When it has been in service awhile and has stretched, the tail pulls down level.

The internal ballonets and air lines are assembled in a similar manner to the envelope, and are attached inside the envelope. The internal ballonets are filled with air, and are used for maintaining constant pressure in the envelope.

When an airship rises the gas expands due to the decrease of atmospheric pressure, consequently when a ship leaves the ground it is inflated only partly full of gas. The remainder of the inflation is air. As the ship rises and the gas expands, it forces the air out of the ballonets; as the ship descends again air is forced back into the ballonet through air scoops which are raised and lowered in the slip stream of the propeller. In this manner a constant pressure is maintained in the bag. This pressure is recorded on pressure gages which are visible in the car of the ship.

The internal pressure of an airship is about one inch of water pressure at the center of the bag. This means about two inches of pressure at the top of the bag and one-half inch of pressure at the bottom, the pressure varying with the height of the bag. This pressure of one inch above atmospheric is sufficient to maintain a tight bag.

The airship car is suspended by cables attached to finger patches which are cemented only to the outside surface of the bag. The finger patches are built of combed-out rope cemented and attached to fabric fingers. The standard four-finger patch is capable of holding a load of 2000 lb. These patches are assembled on the ship in groups according to the load which they are to carry. The suspension lines themselves are steel cables spliced into the patches and attached to the suspension points on the car by means of cable clamps. It is necessary to adjust the suspension cables constantly at the car end.

An airship is designed usually so that it trims down three degrees at the nose. This trim down at the nose compensates for the force couple due to the line of thrust of the propeller being below the center resistance of the ship. The fins or control surfaces are suspended from the bag by patches. The various handling lines and mooring lines are attached to the bag also by suspension patches.

It is necessary to stiffen the nose of an airship to prevent its caving in at high speeds. Light-weight box-type nose battens are installed in much the same manner as umbrella ribs. The battens are lashed to small patches, which are cemented to the ship. The effect of the bow stiffeners is to reduce considerably the pressure which must be maintained inside of the envelope. It is evident that the internal pressure must be greater than the dynamic pressure against the outside of the envelope. The maintenance of this greater pressure at the nose when diving at an angle means that the fabric of the ship would be unduly stressed, and the life would be reduced greatly. The effect of bow stiffeners in non-rigid airships is completely described in Technical Memo. No. 221 of the National Advisory Committee for Aeronautics.

Airship envelopes are equipped with gas and air valves. These valves have opening and closing devices operated from the control car. The airship valves are very delicate instruments. The latest type are the result of a great deal of experimenting.

The standard form of car used on the training ship of the "TC" type is a wooden structure about 40 ft. long. The cars carry two motors on outriggers. Water-cooled motors of about 180 hp. have been used. The latest types have air-cooled motors of 200 hp. The cars usually have open cock pits.

The cars accommodate a crew of eight. The latest types of cars have enclosed cabins for the crew. Airship cars are equipped with landing pontoons which are rubber pontoons, air inflated. These take the landing shocks, and support the weight of the car at rest. An airship lands in the air, that is, it comes in at a state of equilibrium at about 100 ft. from the ground, and is hauled down to the ground by the handling ropes. There are usually no landing shocks similar to those experienced in airplane landing. Landing wheels have been experimented with, and work very satisfactorily, especially on two-motor ships. A caster-type wheel has been found to assist greatly in maneuvering the ship.

Water recovery has been installed on non-rigid airships, and has been found to increase the range greatly. The theory of water recovery is simply the condensation of the exhaust gases from the engine. This condensation is accomplished by running exhaust gases through aluminum tubing which is placed in the slip stream of the propeller. The amount of water is equal to the amount of fuel burned, and is recovered in this manner, thereby maintaining the trim of the ship. It is seldom necessary to valve gas in modern airships.

Control surfaces of airships are very large as compared with the size of airplane wings. They are built up in much the same manner as an airplane wing using wooden latticed girders, although some of the later types are made of metal. All surfaces to date are cloth covered. The control surfaces are very light, weighing but one-half pound to a square foot of area. This light surface will sustain loads of 10 lb. per sq. ft. The rudders and elevators are of the balanced type. Most airships carry, in addition to the bottom fin, rudder, and the side fins, an elevator or a top fin. It has been found that a top rudder

is very advantageous, but it is difficult to operate, in that the long control line must be brought down over the top of the ship to the car.

Rip Panels. All non-rigid airships are fitted with a device for releasing the lifting gas almost instantly in an emergency. If, for instance, a ship runs out of fuel or has complete motor breakdown, and a landing must be made in a strong wind, the rip panels are pulled, when reaching the ground, and the envelope is deflated, allowing it to fall over the ground without material damage. It may then be rolled up and removed for new inflation. The rip panel is a specially strong panel of fabric fitted over a specially reinforced hole in the top of the envelope. To one end is attached a heavy rope which passes down to the

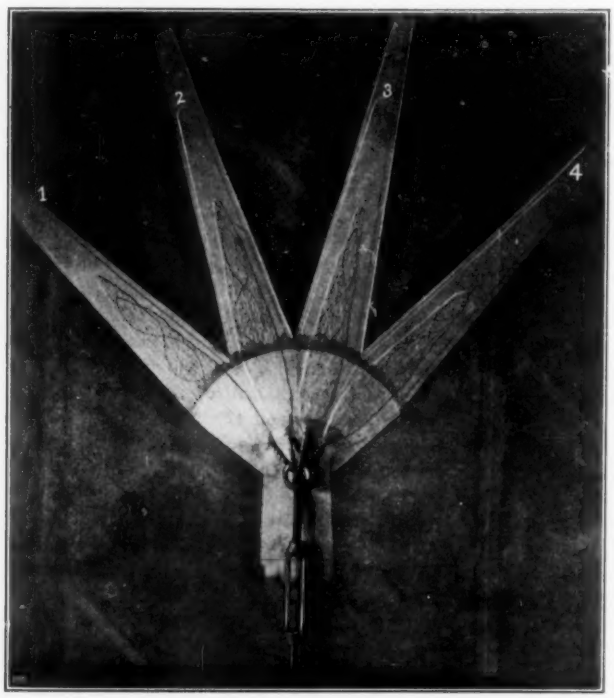


FIG. 5 FOUR-FINGER SUSPENSION PATCH

control car, and is pulled by the pilot when required. This pulling tears out the panel but does not harm the envelope. The panel may be refitted before the next inflation. For ships up to about 200,000 cu. ft., one panel is sufficient, larger ships usually having two rip panels.

Suspensions. The method of suspending the relatively short car to the long envelope of an airship is a whole problem in itself. The early pioneers used the simple net which completely surrounded the envelope, and was brought down to the car in a smaller number of larger ropes. This system had numerous disadvantages, notably excessive weight, resistance, tendency to slip over the envelope, and the difficulty of properly apportioning the loads. The next step in suspension was that of a girdle or band arranged on each side of the envelope on such a line as to allow of a tangential pull of the car ropes. The simplest form of band is that used in kite balloons where the total load to be carried by the envelope is small. This girdle consists of a doubled strip of fabric into which are sewed small wooden toggles, the complete band being then stuck or sewed to the envelope. A slot is cut in the center of the band at each toggle, and the suspension cords are passed around the toggle, and either tied or spliced securely.

Suspension girdles for larger ships are made by developing strips of heavy fabric in trajectories for each group of suspension ropes. The edge of the trajectory is reinforced by rope, formed into loops at each point, into which the actual-suspension lines are fastened. In the German "Parseval" ships suspension bands are fitted, which consist of lengths of strong webbing fastened in at each point of suspension, and passing through fabric pockets completely over the envelope to the suspension band on the other side. These bands distribute the loads over the envelope, and also add enormously to its strength.

Another form of suspension is that of individual patches for each suspension rope or each group. This type was first developed by the British Airship Service and first used on the

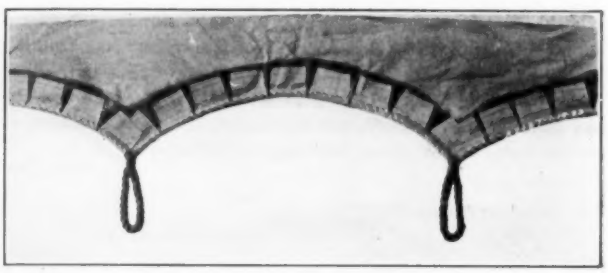


FIG. 7 SIMPLE LOOP SUSPENSION BAND FOR LIGHT LOADS

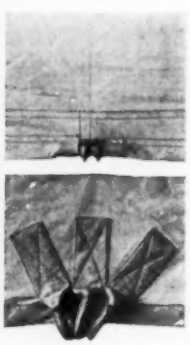


FIG. 6 AT TOP, TOGGLE SUSPENSION. AT BOTTOM, CROWS-FOOT SUSPENSION

British army airship *Eta*. The patches were afterward known as Eta patches. In this type of patch, lengths of strong webbing are passed around a tubular-steel "D" ring forming the bottom of the patch, and spread out fan-wise on to or between heavy rubberized fabric. The complete patch is about three feet overall each way, and will hold a load of about 1800 lb. when stuck with rubber cement on to the fabric envelope.

About ten years ago developments were started in this country with a modified suspension having the same features as the Eta patch, but formed by using manilla rope frayed out instead of webbing. Also, instead of using a fan-shaped patch, each strand of rope was stuck between fingers of fabric. This type is known as the Finger patch, and is now the standard type used for suspensions in U. S. airships. The regular four-finger patch will hold 2000 lb. A patch with two fingers is used for light loads, such as guy wires for stabilizers, etc., and holds about 1000 lb.

INFLATION, ASSEMBLY, AND RIGGING OF NON-RIGID SHIPS

To inflate and erect a non-rigid airship, the floor of the hangar must first be swept clear of dirt and all obstructions which would be apt to cut the envelope fabric, and the envelope is laid out flat on a clean ground cloth of duck. All air must be rolled out of the envelope, and the rip lines properly connected inside. The gas and air valves should be fitted, and the envelope left topside up, in order that it will rise from the ground straight, when the gas is allowed to flow in. A net made of soft-cotton cord is then placed over the whole envelope, and sufficient sandbags arranged along each side to take care of the total lift of the full envelope. The gas hose of heavy rubberized fabric and about 11 in. diameter is next connected to the filling sleeve near the nose of the envelope on the underside. A row of sandbags is placed across the bag just forward of the place

where the stabilizers will be fitted. This is to hold the gas in the front end until sufficient lift is obtained to carry the stabilizers. Gas is then allowed to flow into the bag, and as the lift increases, sandbags are hung on the net along each side, and more added as may be necessary. During this time the top stabilizer (if fitted) should be attached, and its guy wires fitted slackly. The side stabilizers should also be attached with their wires. Tackles should be attached to each stabilizer with ropes to the roof of the hangar in order to relieve the weight as they are lifted by the gas. Inflation is carried on with the row of sandbags in place until it is judged that there is sufficient lift to support the stabilizers. Then the bags are carefully removed, and the bag allowed to lift the surfaces, taking up on the tackles accordingly.

When the bag is full, plumb-bob lines are attached at nose, tail, and on each side at the maximum width. These are to check the trim of the envelope, and it is leveled up in both directions by moving sandbags along the inflation net as may be necessary.

When the correct conditions are obtained the car is moved into position under the bag, and the various suspension ropes or cables attached. After the position of car is checked, and the ropes at a proper tension, the inflation net may be removed by pulling it carefully over the top of the envelope to the ground. The car is held down during this process by changing the sandbags to the inside of the car or hanging them on the places provided on the outside. The stabilizer guy wires may now be adjusted, and the various control lines fitted. A blower is hooked up to one or both of the ballonets to maintain pressure by these air balloons during the changes of atmosphere. Valves and controls are tested, and the engines run, and after satisfactory trials of all parts have been made the ship is ready for flight.

Commercial Possibilities of Non-Rigid Airships

The non-rigid airship, particularly the smaller sizes, has many possible uses. Some of these are suitable for airplane operations, and others are distinctly adaptable for the airship. Among the uses of non-rigid ships are the following:

1 Survey and photographing of large areas of difficult, wooded country. This makes it suitable for use in the Forestry and Geological departments.

2 Dusting of crops or trees where large areas have to be covered. In the case of hilly country it is hazardous for a plane to operate close to the ground where the most beneficial results are obtained. The airships' ease of operation close to the ground and at slow speeds makes it ideal for this work.

3 Examination of territory by mining interests where the terrain may be examined close up and at slow speeds, and further, in difficult country where a plane could not be used safely.

4 Advertising in the air.

5 Transportation of passengers and freight.

For all of the above named fields excepting (5), a fairly small ship would be used. The design would be such as to offer the greatest ease of inflating and assembly in the open without the necessity of a hangar. Some form of small, portable mooring mast would be used, and the base of operations moved after each set area was covered. A truck or trucks would carry the portable mast, hydrogen cylinders, and working crew, and if the project was large enough, a portable hydrogen-making apparatus would be carried. Some six or seven years ago a medium-sized non-rigid ship built for the U. S. Army (the M. B.) was loaned to the Department of Agriculture for tests in dusting areas in New England infested by the Gypsy moth. Though this ship was not designed primarily for dusting, and much trouble was encountered due to lack of knowledge, this ship demonstrated the practicability of its use for dusting. In the light of present-day knowledge it is known that a small ship especially designed for the work would be of tremendous assistance in the dusting of large areas of hilly country where the Gypsy moth is a pest.

Similarly, in 1928, a small non-rigid ship was built for advertising automobiles in Detroit, and sufficient experience obtained to prove the uses of such a ship for the purpose. In this case, the ship was the smallest practical design ever used, having a gas capacity of only 22,000 cu. ft., compared with the 57,000 cu. ft. of the smallest passenger ship, the Goodyear *Pilgrim*. One of the chief features of this ship was its extreme portability, the whole ship consisting of envelope, car with engine, controls,

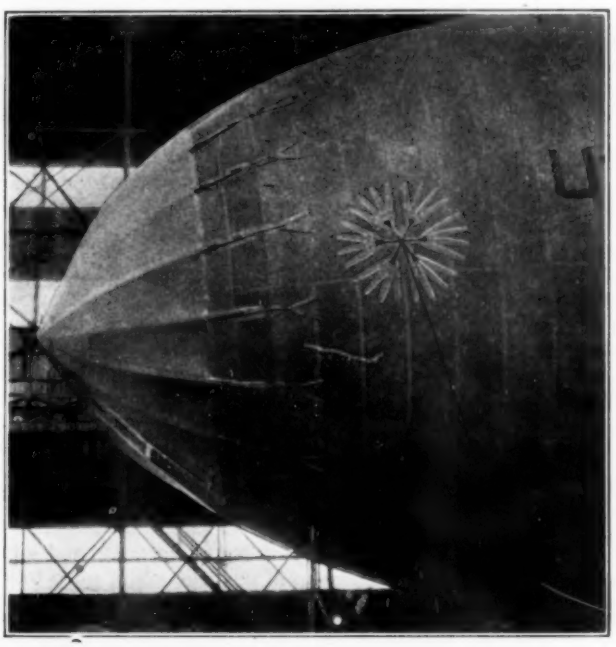


Fig. 8 Nose Stiffening of U. S. Army Semi-Rigid Ship R.S.-1

rigging, etc., being easily packed on to a small trailer, and the ship was actually transported several thousand miles in this way behind a light car. This ship was also inflated in the open several times with the same ease as the inflation of a spherical balloon, and was handled by one man when in the air. Further a ground crew of five men handled the ship for several weeks, compared with the sometimes two or three hundred used for larger ships in military use. The U. S. Army Air Corps made a technical report on this ship, and great possibilities were foreseen for the type. The small cost of a ship of this type is a notable feature. The Goodyear non-rigid ships, the *Pilgrim* and the *Puritan* of 57,000 and 85,000 cu. ft. capacity, respectively, have shown clearly the safety and reliability of airships for passenger carrying. These two ships have flown between them several thousand hours, and have carried a very large number of passengers, in addition to making many long-distance cross-country flights. The larger ship has a maximum speed of 55 m.p.h., and is, therefore, able to operate in winds of considerable velocity. Ships of this type may be used with advantage to create interest in lighter-than-air activities, and offer transport to people who cannot be appealed to by airplane passage. The satisfaction gained by the knowledge that if the motors stop functioning, the ship will still remain in the air and safe, gives it a decided advantage over the airplane, par-

ticularly a single-motor plane. Also, the passengers may be comfortably seated in a closed cabin with little trouble due to vibration or engine exhaust, and the ships ride more easily than a plane. A small airship of this type is also easily throttled down to any speed required, giving the opportunity of seeing the country at any particular time without danger.

It is foreseen that at a date not very far into the future, the non-rigid ship will be accepted as an ideal unit for passenger transportation within limited range and under limited weather conditions. It should be mentioned that a hangar will be required for the assembly and overhaul of these larger ships, but the regular operations may easily be carried on from mooring masts.

Ground Handling

There is one factor in the operation of airships that prevents the profitable operation of non-rigid as well as other types of airships. This is the handling element which includes the following operations:

(*a*) Maneuvering in and out of the hangar
(*b*) Take off and landing
(*c*) Mooring out where hangars are not available or when winds are too high for maneuvering the ship into a hangar.

The problem of handling rigid airships is receiving serious and active attention at the U. S. Naval Air Station, Lakehurst, N. J. The problem is a paramount issue, in view of the recent awarding of the contract for the construction of two, six and one-half million cubic-foot airships, and also the necessity of handling visiting-foreign airships, namely, the *Graf Zeppelin* and the British airships now nearing completion.

In consequence of this the non-rigid airship problem is practically eclipsed and will, therefore, receive little support from the government. However, the recent accident to the Navy's J-3 and J-4 ships, requiring their forced deflation at Bolling Field immediately following the presidential inauguration, has given impetus to the need for providing an adequate and safe method of mooring for an airship in the field, where entry to a hangar is impracticable or impossible.

Maneuvering In and Out of Hangar. The maneuvering crew required for non-rigid airships varies directly as the square of the wind velocity, and as the cross-sectional area of the ship. This may be expressed algebraically as follows:

$$N = K\ Vol.^{2/3}\ v^2$$

where,

N = Number of men
$Vol.$ = Volume in cubic feet
V = Velocity of wind in miles per hour
K = Factor of experience which is determined as 0.00014

Therefore,

$$N = 0.00014\ Vol.^{2/3}\ v^2$$

The various existing types of non-rigid airships would require the following sizes of maneuvering crew on the basis of the above equation when considering a ten mile an hour wind:

Type	Volume cu. ft.	No. Men
Pilgrim	57,000	15
TC and J	210,000	50
TA	130,000	36
TE	84,000	27
Puritan	85,000	27
Proposed Navy	320,000	65

This crew is split into five sections as follows:

Starboard Forward	25 per cent
Port Forward	25 per cent
Starboard Aft	15 per cent
Port Aft	15 per cent
Rudder and Car	20 per cent

The following is a brief description of the present method of maneuvering prior to a take off, and immediately on landing an airship. This is extracted from the Scott Field Airship School Manual on Airship Operation:

(*a*) The motion of the ship when in the hands of the maneuvering crew is controlled entirely by the handling guys. The function of the car party is not to move the ship but to keep the car off the ground. The car party lifts the car clear of the ground, and then the command to march forward is given.

(*b*) As soon as the handling-guy parties are clear of the hangar doors they move out at double time or walk out to the full extent of the handling guys, so that their pull is as horizontal as possible on the ship. When the ship is, at least, its own length from the hangar and nose into the wind, all the men on the after guys, except two to each guy, drop their lines and double time up to the forward guys.

(*c*) The above manner of handling the ship from the forward guys only is satisfactory for small and large ships in a mild wind, say ten miles per hour. In rough weather it is always necessary to retain enough additional men on the after guys to assist the ship in swinging into the wind. Should the wind change direction so as to strike the ship on the side, the car would tend to strike the ground due to the higher resistance of the ship when in a cross wind which produces a higher vertical component on the handling guys. Should the wind be of sufficient strength to cause the car to strike the ground, the ship would be turned broad side, in which position the ship is liable to be wrecked. But if the forward guys are held firmly and the car kept clear of the ground, the ship will swing around into the wind.

(*d*) When the ship has arrived at the point designated for taking off, it is brought to a standstill easily, the car being held clear of the ground so as to facilitate holding the ship into the wind. Assistance in this effort is rendered by the rudder man whereas the elevator man also aids in holding the ship clear of the ground. This assistance is very imperative in the case of rough gusty weather.

(*e*) In taking off, the car must be lifted waist high, then the guys are eased off and the car released.

(*f*) In landing the ship, the car party takes the position directly under the car as soon as the long forward-handling lines are in the hands of parties designated for them. The car party, obviously, prevents the car from bumping or scraping the ground. Maneuvering the ship into the hangar is similar to maneuvering it out from the hangar as far as the requirements of the maneuvering crew are concerned.

Requirements for Mechanical-Handling Equipment

From the foregoing it is apparent that the requirements for mechanical-handling equipment for non-rigid airships are fundamentally as follows:

(1) *Functional.*

(*a*) It must protect the car and surfaces from crashing into the ground.

(*b*) It should be able to keep the ship swung into the wind, as the resistance for zero inclination of the ship's axis to the wind is considerably less than when the ship's axis is at an angle to the wind. At zero degree incidence the drag is $^1/_3$ that for

20 deg. and about $^1/_6$ of the force normal to the wind (cross-wind force). These forces may be calculated on the basis of the convenient formula for resistance at zero degree angle of incidence:

$$\text{Resistance} = 0.03\ \rho\ V^2\ (Vol.)^{\,2/3}$$

Where ρ = weight in pounds of a cubic foot of air divided by g in ft. per sec. per sec., and has the numerical value of 0.00237, V is the velocity of the airship in ft. per sec., and *Vol.* is the volume of the airship in cubic feet.

(*c*) It should be provided with a weighing-off device, that is, a means for determining how much the ship is heavy or light.

(*d*) It should be quickly attachable and be provided with a master control by which it can release the ship.

(*e*) Provision for servicing the ship is desirable, such as refueling, ballasting, and gassing.

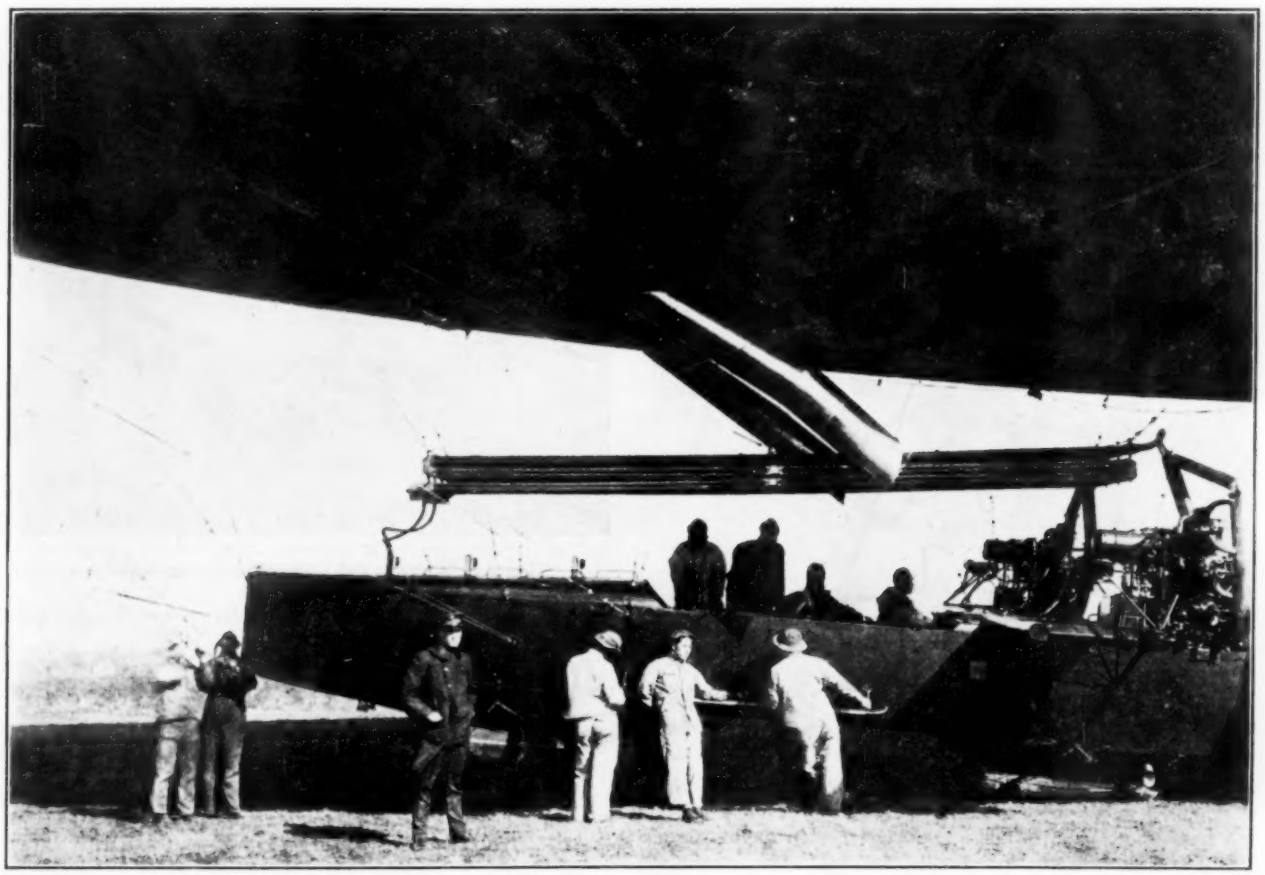


FIG. 9 WATER RECOVERY SYSTEM ON CAR OF NAVY "J" SHIP

(2) *Cost.*

(*a*) Crew required to operate should be a minimum.

(*b*) Low structural weight to reduce material and operating costs.

STATUS OF DEVELOPMENT OF MECHANICAL-HANDLING EQUIPMENT FOR NON-RIGID AIRSHIPS

Very little progress has been made in the past decade in the mooring and handling of airships of this type owing to the two retarding factors, that is, lack of funds for development, and the availability of ample manpower at a military post.

A very interesting and instructive paper prepared by Flight Lieutenant F. L. C. Butcher was read before the Royal Aeronautical Society on December 2, 1920, explaining the various methods which had been tried for both rigid and non-rigid airships in Great Britain with their advantages and disadvantages. Very little has been done in this country on non-rigids outside of verifying some of the British experience, and making some advancement in ground-mooring methods. An endeavor will be made to describe this experience.

Windscreens diverging from the sides of a hangar are a great aid to maneuvering a ship out of or into a hangar. The ship can be pivoted between the screens so as to head into the direction of the wind prevailing beyond the hangar doors.

(*a*) *Research.* Free-air tests conducted at the Bureau of Standards on models of $^1/_{20}$ scale in conjunction with wind-tunnel tests showed that a screen of the picket-fence type with the same spacing of pickets all over the screen was as good as any complicated method of perforation proposed or tried out in France or Great Britain. The most favorable spacing of pickets was found to be with $^2/_3$ of the area closed. The forces, when the ship is thus screened, are about 5 per cent of those measured without screens.

(*b*) *The method of maneuvering* an airship into a hangar equipped with windscreens is indicated diagrammatically in Fig. 10. Obviously, the maximum utility of windscreens is realized in cross-hangar winds as they minimize the danger of wrecking the airship against the doors.

For up-and-down hangar winds the danger in airship handling is caused by the eddy currents produced by the obstruction of the doors when open, and the downward pouring of the air at

the leeward end. These destructive air currents may be broken up or straightened out by opening the windward doors a distance, depending on the direction and magnitude of the wind.

(*c*) *Service Tests and Costs.* Screens have been used extensively in France and England. They have not, however, received very great support in England due partly to the fact that with large rigid airships the screens would have to be placed a great distance apart to be of any use and this would increase their cost. Furthermore, it is believed that the omission of docking-rail facilities in connection with windscreens had a great deal to do with England's decision to discontinue their use.

In this country windscreens were installed at the Fairfield,

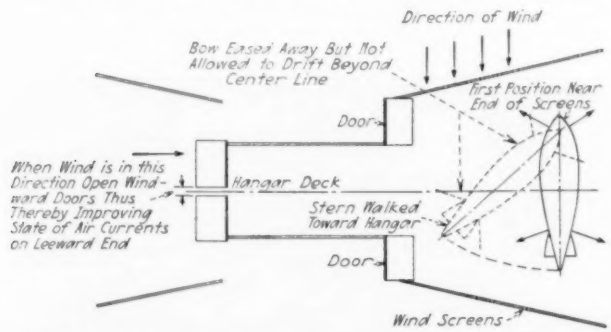


FIG. 10 METHOD OF MANEUVERING SHIP INTO HANGAR WITH WINDSCREENS

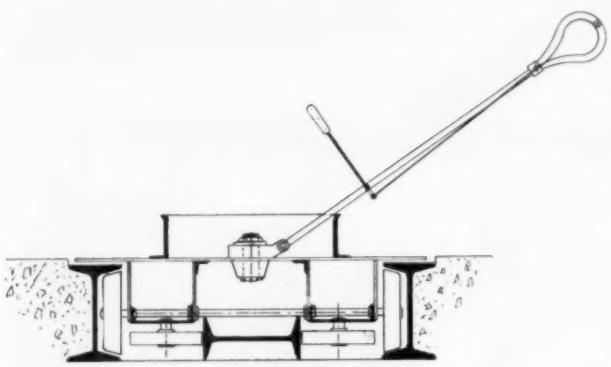


FIG. 11 OPEN-TRENCH TYPE OF DOCKING RAIL AND TROLLEY DEVELOPED IN GERMANY

Ohio, Air Intermediate Depot where a 43,500-cu. ft. airship was housed in a correspondingly small shed. As this hangar was just wide enough to house the single airship the maximum utility of the screens was hardly realized. The maximum utility of windscreens can be had only when the screens do not have to extend beyond the length of a ship, to be far enough apart to include the full length of the ship when placed broadside to the center line of the hangar.

Applying this generalization to an airship of 350,000-cu. ft. size, which is generally conceded to be the size for maximum utility, the following dimensions for hangar and screens are obtained:

Ship's length	250 feet
Ship's diameter	50 feet
Hangar height	75 feet in the clear
Hangar width	100 feet in the clear
Hangar length	275 feet in the clear

Screen diverging from 200 feet at the doors to 275 feet at the outer ends.

Here it is seen that the screens need not be longer than 250 ft., the length of the ship.

In 1926, the U. S. Army Air Corps proposed to install at Scott Field a set of moveable windscreens, as several ships had been damaged considerably in maneuvering in and out of the hangar. In order to accommodate the semi-rigid airship RS-1, then being service tested, it was decided that the dimension of each screen would be 300 ft. long and 80 ft. high. An estimate was obtained from the Wilkins and Phillipi Construction Company of St. Louis. This included figures for one screen to be mounted on 2000 ft. of trackage on one side of the hangar so that the screen could be used to protect a ship entering the hangar at either end. An estimate was also made for two screens and trackage on both sides of the hangar. In either case the cost was too high for funds then available. These figures and specifications are as follows:

COSTS AND CONSTRUCTION OF WINDSCREENS

Cost for labor and material for complete installation:

One Screen and 2000 ft. of double track $250,000
Four Screens and 4000 ft. of double track $800,000.

From this it is seen that the screens were estimated at $150,000 each and 2000 ft. of double track at $100,000.

Track to be of 100-lb. A. R. A. type-B rail bolted down to

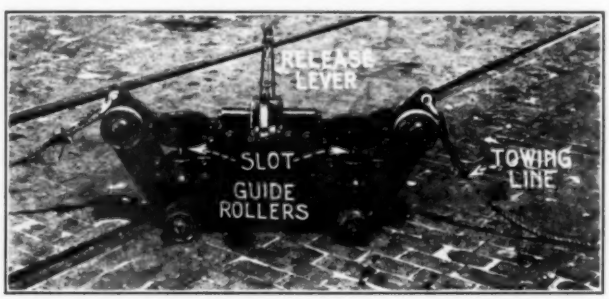


FIG. 12 TROLLEY FOR COMPOUND DOCKING RAILS

reinforced-concrete foundation with reinforced-concrete bumpers at the ends.

Screens to be constructed with structural-steel frame which was to be covered with No. 24-gage galvanized-rib metal lath wired on structural supports with copper wire. Specially designed trucks were to support these frames and air brakes and pneumatic bumpers were to be furnished for the trucks.

Motive Power. To consist of two 240-hp., two-cycle, direct-reversible, marine-type diesel engines approved by the American Board of Shipping. The engines to be equipped with clutch and to be geared to trucks to give the desired speed to the screen.

DOCKING RAILS AND TROLLEYS

An attempt has been made to reduce maneuvering personnel by the use of docking rails and trolleys. This has met with partial success. From the open-trench type developed in Germany as shown in Fig. 11, was evolved the compound-docking rail with trolley protruding through a slot (see Fig. 12). Various types of trolleys with varying degrees of success were developed to overcome difficulties experienced in operation. The principal difficulties, however, concerned the rails which being of the subsurface type required draining. During freezing weather these rails are liable to become jammed with ice and snow. In hot weather the slots will become too narrow for the passage of the trolley if the rails are set in concrete without adequate expansion joints to allow for the expansion

of the concrete. Where the trolley projects through the slots, rollers have been provided to prevent the trolley plate from scraping the sides of the slot. The size of these rollers being limited by the width of the slot, have proven too small in diameter to function as rollers. They fail to revolve under light loads, thereby producing a flat side which further prevents their action as a roller. At this stage, the roller requires replacement. It is admitted that a trolley can be developed to overcome this or any other difficulties if adequate study could be allowed for the design of a successful type.

After a study of the available data which at that time were meagre, it was decided to install the single rail and trolley as indicated in Fig. 13 for non-rigid airships. This system was installed at the Aberdeen Proving Grounds. Two parallel rails, 62 ft. apart, started inside the hangar 200 ft. from the doors, and extended straight out 400 ft. beyond the doors. This layout is similar to the German method, but the Italian method is to curve the outer ends, diverging the rails until they are at right angles to the centerline of the hangar. This latter method requires the use of one rail, obviously the one to the windward, and is based on the theory that anchorage to the leeward rail is unnecessary.

A description of the method of handling a ship with docking rails and trolleys is unnecessary as it does not differ from that of handling a ship without them, except that the guys are secured to the trolleys instead of being manned by parties. The trolleys are provided with quick-releasing gear for securing the guy lines.

Very little experience has been gained by the use of this system of handling, first because of the availability of ample manpower and second, because of the uncertainty of its success, the operating personnel being very skeptical concerning methods or devices differing from usual practice which they have found dependable and safe based on ample manpower.

Had experiments been conducted to a conclusion it is very probable that this gear would have been augmented by a cradle to receive the airship car, this cradle to be mounted on a car to run on a standard-gage railroad or to be equipped with a pneumatic-tired castor wheel such as is used with the *Los Angeles* airship at Lakehust.

Ground Cable and Snatch Blocks. Under the caption, "Emergency handling gear for small non-rigid airships" in the "Pressure-Airships" volume of the Ronald Aeronautic Library is described a method of launching an airship which is no doubt an evolution of the more-permanent docking rail and trolley system. It consists of a cable anchored a few inches above the ground on which are mounted two snatch blocks, one for each end of the ship. This gear was tested in 1918 and 1919 at the Hampton Roads Naval Air Station and at Camp Bierne, Texas and was proven practical. Later, about. 1923, it was installed at the Fairfield Air Intermediate Depot for handling the small towing airship turned over to the Army Air Corps by the Navy.

Maneuvering winches have been experimented with at Scott Field. A type designed by the McCook Field Engineering Division was originally equipped with a rope-grab mechanism located on the front end for automatically grabbing the handling rope released from the ship. This, however, proved impracticable and was replaced by the capstan and tested in the fall of 1926. The tracks of the caterpillar wheels were powered by a 60-hp. Waukesha engine. There was also a power take-off from the transmission for the capstan.

The following is quoted from a report of the test engineer W. E. Huffman for the McCook Field design of maneuvering winch. The TA airship noted therein had a volume of 135,000 cu. ft. and a length of 162 ft.

"The tests were made with a TA-type airship, using the winch on one of the forward-handling guys and the crew on the other. The first few landings were simulated ones, in which the ship came in over the winch but no attempt was made to snub it, the operator of the capstan merely taking hold of the handling guy and then releasing it after the winch had followed the ship's course some distance across the field. Due to the fact that there was little or no wind during these tests, most of the landings were quite fast, and some difficulty was experienced in keeping up with the ship.

"The first few landings proved that the best method of landing a ship is to station a man about 200 ft. ahead of the winch to grab the handling guy and pass it to the operator of the capstan. It was also found that in order to prevent running over and fouling the rope, and to enable the driver to see what is going

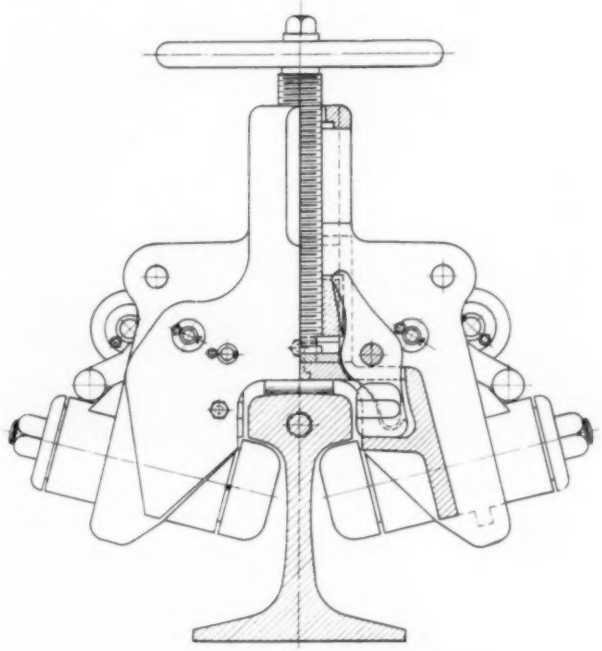


Fig. 13 Single Docking-Rail Trolley

on, it is best to head the winch toward the ship and run backward while a landing is being effected. As soon as the capstan operator is in possession of the rope, the driver reverses the winch and runs backward at the approximate speed of the ship while the capstan operator passes the rope through the hook and around the capstan. As soon as a couple of turns of the rope have been made and the slack taken up, the ship is gradually brought to a stop and then hauled in with the capstan. During these tests, handling-guy landings only were made, and of course it was necessary to use the regular crew on one guy.

"After an hour's practice a number of successful landings were made and it was found to be a very simple matter in spite of the fact that this winch is underpowered. It was proven, however, that an airship can be landed with machines of this kind without a large crew of men. The only men necessary outside of the three for each winch would be a sufficient number to handle the car upon landing. Such equipment would be especially valuable in landing ships that are very light, or the larger ships under all conditions where a great number of men are required. Using the winch to maneuver a ship across the field was successfully carried out at McCook Field.

"From the information obtained during the tests it is now possible to set forth the characteristics that a winch of this

type should possess. The first requirements are speed and maneuverability. In maneuverability the present winch is equaled by none, but its speed is not sufficient on soft ground due to lack of power. A speed of 20 m.p.h. should be possible over an ordinary flying field which is in fair condition.

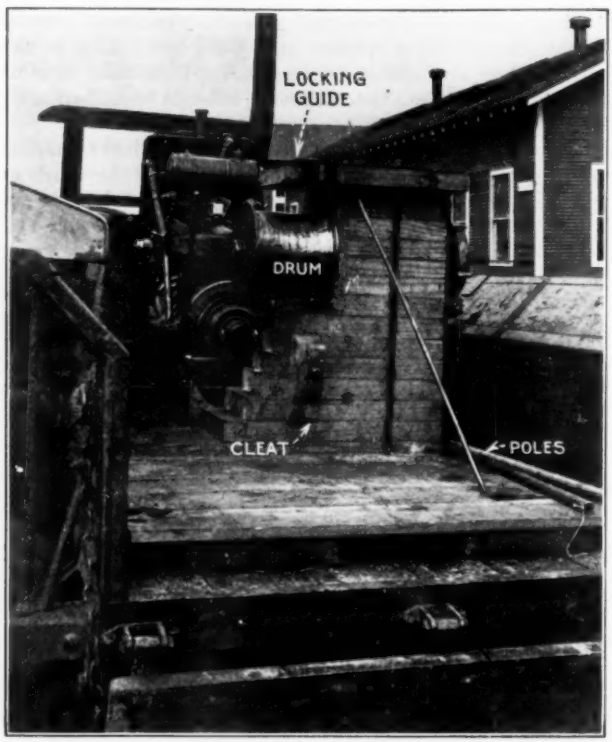


FIG. 14 MANEUVERING WINCH MOUNTED ON ARMY TRUCK

"The hook or other device through which the rope is passed before it reaches the capstan should be as near as possible to the pivot point of the winch in order to prevent jerking the ship in changing direction or in passing over rough ground.

"The capstan should be two and one-half or three feet above the floor. As it is, a man is required to stoop over while operating it, which makes it hard for him to maintain his balance when the winch is in motion. Operation of the capstan clutch should be by means of a pedal which can be reached conveniently by the driver's left foot."

In 1927 Captain H. C. Gray mounted a windlass on an army truck using miscellaneous parts from reclamation and conducted tests at Scott Field in landing both TA and TC ships. His setup is shown in Fig. 14 and was accomplished with very little funds and some ingenuity. However, definite progress along these lines can be had only by placing such equipment in service in order to obtain constructive service criticism to the end of perfecting the devices proposed.

The engine on this windlass is a small Ballot motor reclaimed from the French Zodiac non-rigid airship, of 325,000 cu. ft. capacity. The winch-driven mechanism is taken from the OB-1 airship (43,500 cu. ft.) and the radiator from the S. S. T. an English submarine-scout twin-engined airship (100,000 cu. ft.). The drum is made from a hydrogen cylinder.

The truck speed is 25 to 30 m.p.h., ample for the operator to pass the handling rope through the locking guide and wind it around the cleat thereby snubbing the ship. The poles on the floor of the car had wire hooks bent on the ends and were about eight feet long, enabling the operator to snare the rope as the truck passed by it. It was reported that a crew of two men easily landed both TA and TC ships with this device. Landings with ships light and heavy were made satisfactorily.

Captain Gray's recommendation for rigging the ship was as follows:

The ship should be rigged with the "Fixed Three-Wire Mooring System," a $^5/_8$-in. rope about 75 ft. long, being suspended from the quick release, and a $^3/_4$-in. rope 25 ft. long being attached to one front-handling rail on the car for the purpose of hauling the car down to change crews.



FIG. 15 NOSE CONE-TYPE MOORING MAST WITH TC-1 AIRSHIP

MOORING OUT EQUIPMENT

Masts. In addition to Flight Lieutenant Butcher's paper of 1921 quoted above, in which he gives a very complete history of mooring there have been rendered many papers on mast mooring, concerning tests and analyses of stresses of a ship thus moored. In the appendix of his paper on the "Development of Airship Mooring" presented to the Royal Aeronautical

Society, on March 4, 1926, and published in the Society's Journal in August of that year, Major G. H. Scott gives a brief history of mast mooring.

It appears that an Italian took out first patents on mast mooring which were allowed to lapse. Experiments with this type of mooring were started in England in 1911, continued in 1912 with non-rigids, then suspended until 1917. From 1921 to 1925 no flying was carried on in England while, however, a considerable amount of experimental work on mast mooring was carried out in this country.

In connection with rigid airships some success has been realized in this type of mooring, the initial difficulties of bringing a ship up to the mast and securing thereto having been surmounted, and the stresses on an airship thus moored determined, and structural strength being accordingly provided so as to minimize recurrence of breakaways experienced with the R-33 and the *Shenandoah*. Furthermore, servicing facilities such as waterballasting, refueling, gassing, and battery charging have been perfected. Therefore mast mooring bids well to become an established institution for large commercial airships since facilities can easily be provided for handling passengers and freight, such having been developed to meet military requirements.

Mast mooring has not however proven successful to such a degree for non-rigid airships as is evident from the following outline of the development:

Stationary types of mast were developed in the United States, patents having been taken out by V. H. Dill of the U. S. Army Air Service, the design however, having been originally conceived in England, where tests were conducted on it by the Army

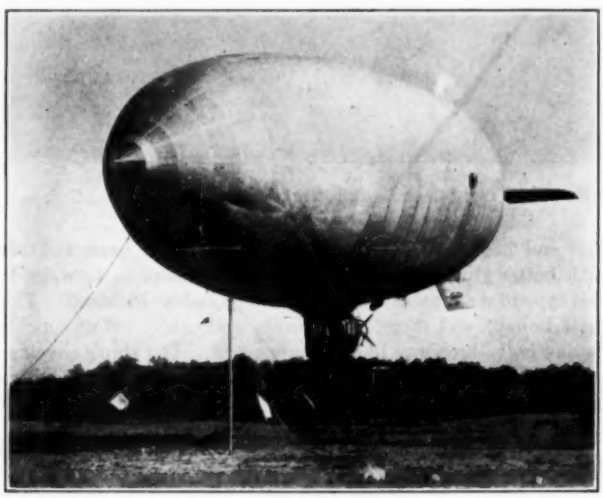


Fig. 16 Airship "Pilgrim" on Mooring Mast

in 1912 at Laffans Plain, South Farnborough with the *Beta*, an airship of about 65,000 cu. ft. capacity. Quoting Lieut. Butcher:

"This method proved satisfactory in light or steady winds, but in gusty weather when the movement of the ship was rapid a bending stress was set up in the fabric just behind the ring; this wrinkled one side of the nose and put a corresponding tension on the other, which tore the fabric after a short while.

"The reason for this was the fact that a few feet of the nose were held inside the cone and that the point of attachment of the ship was not directly above the pivot of the top of the mast.

"The cone was done away with, and a ship was moored directly to the top of the mast, but it was found that the fabric was not strong enough to stand the force of the wind on the ship. Consequently an axial spar, stayed to the after ends of the nose stiffeners, and projecting through the nose, was fitted to a ship and the spar secured to the top of the mast. This system was tried out with satisfactory results until the spar broke, and the experiments were discontinued."

Despite the British conclusions attempts were made to prove this type of mast, a number having been contracted for in 1921 for the purpose of trying out various types of cones and mast gear, and the results are rather optimistically set forth in McCook Field Serial No. 2063 prepared by V. H. Dill, July, 1922, in that the prediction is made that the deficiencies will be overcome by a suitably designed buffer.

Assuming that a buffer properly designed reduces the intensity of pressure caused by the ring type of buffer which gives

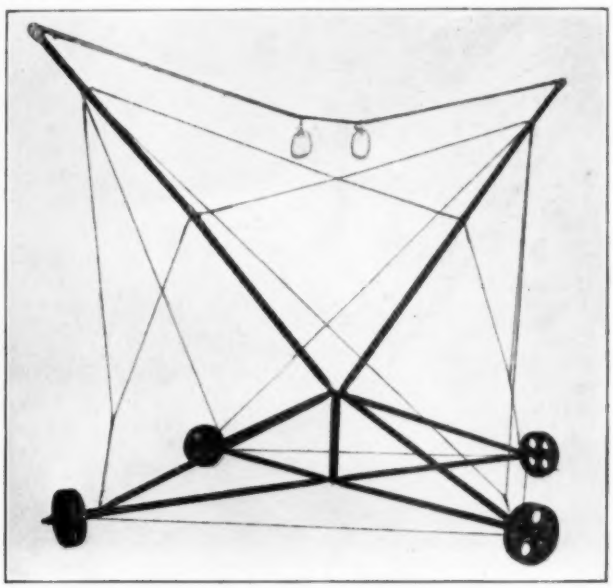


Fig. 17 Suspension-Type Mooring Mast

a line contact to the nose of the envelope, there still remains the bending stresses noted above, "the point of attachment of the ship being not directly above the pivot of the top of the mast."

Although there is no very definite data concerning the actual forces on an airship at a mooring mast, the approximation to the maximum transverse force at the bow as given in the Airship-Design Volume of the Ronald Aeronautic Library by C. P. Burgess will serve as a conservative basis for the determination of the reaction of the airship nose to the buffer, which is attempted as follows:

Given: $Vol.$ of TC airship 200,000 cu. ft. (approx.)
L of TC airship 200 ft.
Wind velocity 40 m.p.h. = 58.7 f.p.s.
$F = C\,q\,Vol^{2}/_{3}$

where F = transverse force
C = coefficient = 0.12
q = aerodynamic pressure $= \rho \dfrac{V^2}{2}$
$Vol.$ = volume of airship

Therefore:

$$F = 0.12 \times \frac{0.00236 \times 58.7^2}{2} \times 3400 = 1660 \text{ lb.}$$



FIG. 18 FIXED THREE-WIRE MOORING SYSTEM WITH J-3 AT LAKEHURST

With the nose of the airship six feet behind the pivot, and the buffer ring four feet aft of this point the vertical reaction at the buffer would be

$$\text{Reaction} = \frac{6}{10} \times 1660 = 1000 \text{ lb. approx.}$$

The component of this reaction normal to the envelope surface, assuming this surface to be 45 deg. with the reaction, would then be equal to

$$1000 \times \cos 45 \text{ deg.} = 707 \text{ lb.}$$

From this result it is difficult to conceive a buffer that would satisfactorily distribute the pressure over the nose of a non-rigid airship without stiffening the nose considerably thus introducing a prohibitive weight. This calculation further accounts for the failure of nose battens resulting from a mooring at this type of mast.

Mooring from a single point aft of the nose where the forward-handling guys are usually fitted was next attempted in England in 1918. A similar type of mast has been used by the Goodyear Tire and Rubber Company with the *Pilgrim* airship (Fig. 16) and by the Goodyear-Zeppelin Corporation with the *Puritan* airship. It is reported that the *Pilgrim* airship has moored to this mast in a 17-m.p.h. wind and the *Puritan* in a 28-m.p.h. wind. However in the latter instance one of the anchor bolts pulled loose from the ground, necessitating unhooking the ship and holding it by a ground crew. It was predicted that with better ground fastenings it would be possible to moor to this type of mast at a wind speed of probably 35 m.p.h. This mast is light and therefore portable, and can be set up quickly having only three guy lines to support it. The ship's envelope is fitted with a metal disk, and the mast is secured thereto with a ball-and-socket fitting. The British report this type of mast unsatisfactory asserting that the ship rolls badly.

From this type the Vickers Company of England evolved the crutch type shown in Fig. 19. The ends of the crutch arms are padded for fitting against ring patches attached to the envelope on opposite sides of the nose in the horizontal plane. The crutch is integral with the mast, the top part of which rides in a bearing, the housing being secured to cable guys anchored to the ground.

Cables are led through the crutch pads and secured to the envelope through ring bolts. Hand windlasses are located in the crutch arms for drawing the cables taut thereby securing the envelope rigidly to the mast crutch. Quoting Lieut. Butcher:

"Before bringing a ship to the mast it is turned so that a line drawn through the pads would be at right angles to the direction of the wind, then the ship is walked as close as possible to the mast by the handling party.

"A wire led from another small winch at the foot of the mast through a block in the bottom of the crutch is secured to the mooring point of the ship, and the ship hauled down until two

men, one stationed on each bow, are able to secure their winch wires to the ring bolts on the envelope.

"By hauling in on these winches the ring patches are brought hard against the pads on the bows, and the ship is secure. To keep the ship as nearly as possible in equilibrium chains are made fast to the car and allowed to trail on the ground. Thus when the ship becomes light extra chain is lifted, and if the reverse happens and she commences to fall, more weight of chain is supported by the ground."

In tests at Barrow-in-Furness a 70,000-cu. ft. ship was moored for eight days and rode without sustaining damage in winds of over 40 m.p.h.

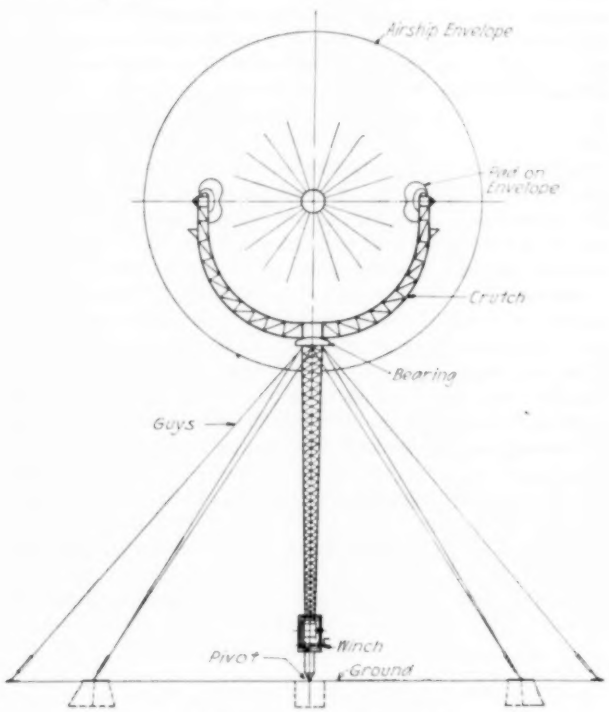


Fig. 19 Crutch-Type Mooring Mast

The disadvantages of this type of mast are:

(*a*) It is considered hazardous to have men stationed in the arms of the crutch as is required to operate the winches.

(*b*) If both sides of the ship are not attached to the mooring cables of the mast simultaneously, it is possible for the ship to swing away from the side not secured, thereby placing all the load on one patch, and subject the envelope to a tearing action.

Mobile Types

In 1922 the Air Corps initiated the development of a collapsible mooring mast with a cone top. This was designed to be mounted on a gun chassis but was found to be too heavy for transporting over flying-field terrain owing to the weight of the hoisting gear required. In fact, the mechanism was too heavy for the gun carriages then available. One of this type was constructed but owing to its excessive weight was not put through trial tests.

In Fig. 17 is shown a model of a mooring mast invented by Matt Q. Corbett, General Manager of the Aircraft Specialties, Inc., Wyandotte, Mich., which was developed and tested by the Air Corps at McCook Field in 1923. This was known as the suspension type, as the ship was attached at its bow mooring point to the cable suspended between the outer points of the upright tubular arms. The experimental units of this mast differed from the model only in that a pivot bearing was provided for anchoring the mast's position.

As the ship pivoted about the mooring point the mast would be rotated manually about its pivot point, the weight being carried by the four wheels fixed to the mast so as to travel in a circular path. It is also possible to rotate the mast automatically by securing the ship thereto with appropriately arranged yaw guys.

The advantages claimed for this type of mast were:

(*a*) Elimination of the excessive tearing strains found to exist with the nose-cone type previously described.

(*b*) Reduction of oscillations in thus mooring a ship at or near its center of resistance.

(*c*) Lessening of aerial obstruction in that it could be built closer to the ground than other types herein discussed.

During experimental tests of this mast the U.S.M.B. airship was satisfactorily moored thereto while executing its Gypsy-moth mission in the New England states in 1923. However, conclusive tests did not culminate as the urgency of mast mooring was not then realized. Consequently no further development was carried on.

Cable Mooring

From the beginning of the Air Corps' operations with non-rigid airships, the need was felt for establishing suitable anchorages at the then-existing air stations for the purpose of carrying on extensive cross-country flying. The first step in meeting this need was realized by installing anchors set below the ground surface and covered with a manhole cover.

There was also developed a snatch-block anchorage of the disappearing type, the block being mounted on a hinged plate designed to swing downward into a pit which could then be closed with a cover plate, thereby removing the obstruction from the landing field. This was known as the Terry maneuvering block, and was intended to aid the maneuvering crew in pulling the ship to the ground. The drag rope or trailing line would be reeved through the block, and manned by a party of men who would continue to haul on the line until the car was received by the car party and the handling lines by the rest of the maneuvering crew.

The next step in this development was in trying out various cable-mooring rigs such as described in Flight Lieutenant Butcher's lecture which has been previously referred to. The disadvantages of these types of moorings were early realized, being in general as follows:

(*a*) Considerable amount of cable rigging is required to be kept in working order by a ground crew.

(*b*) Ample notification would have to be given prior to making a landing in order to set up this rigging for the reception of the ship.

(*c*) The weight of such rigging made them cumbersome to handle. Ease in handling is a prime requirement for any airship-mooring rig.

(*d*) The ship was required to be moored at some distance above the ground in order to give the required kiting angle, and at the same time keep the tail clear of the ground. This led to the impracticability of servicing the ship, changing crews, etc.

In order to overcome the difficulties inherent in the existing types of cable mooring, a mooring device was conceived by the late Captain H. C. Gray of Scott Field, which was termed the "Fixed 3-Wire Mooring System." Its installation is shown in Fig. 18. The purpose of this design obviously was to reduce the weight of rigging, to facilitate installation, and to moor the ship close to the ground thereby making it accessible for servicing.

The initial service test of this device was conducted by Captain

Charles P. Clark, formerly of Langley Field, with a TC-airship. Previous experimental tests had been conducted by Captain Gray at Scott Field. In the service test which was conducted during the fall of 1926, the ship was moored at the Boston airport. Three hundred pounds of sand ballast was secured to a quick-release hook at the nose of the car. With this weight and an elevator setting of 10 deg. up, the nose of the car was kept just clear of the ground. The angle of trim of the ship averaged about 5 deg. nose down.

The wind was very gusty, ranging from 10 to 40 m.p.h. Captain Clark observed that the ship would ride the most extreme gust without the slightest danger provided elevators were used to right the ship when it surged forward or oscillated heavily after a gust. The rudder was used just sufficiently to overcome extreme yawing, but for the most part was left in neutral position.

The ground anchorage is provided with a quick-release hook, a torpedo-release hook being in the J-3-airship installation at the Naval Air Station, Lakehurst, N. J. In releasing from this anchorage the sand bags are dropped first, and the ground anchorage released immediately thereafter. This permits a quick get away in case the wind becomes too heavy for safe mooring.

The above cable-mooring gear has been used quite successfully by the U. S. Army Air Corps pilots operating out of Scott Field and Langley Field.

Summary

The salient features of the research thus far conducted may be summarized as follows:

(*a*) The present practice of handling an airship by a ground or maneuvering crew has proved inadequate, hazardous, and prohibitively costly.

(*b*) Docking rails and trolleys in conjunction with windscreens will no doubt reduce maneuvering crews, but research and experimentation conducted with this method of handling to date are not conclusive, having resulted only in the following generalizations:

Docking trolleys are considered too slow and unwieldy, rapid maneuvering being required in swinging an airship into the wind when leaving the hangar with a cross-hangar wind. A releasing and braking mechanism should be developed so that the guy lines may be reeved through at any required speed.

Wind screens have a high initial cost. Revolving hangars without screens are a logical substitute.

(*c*) Maneuvering winches have proved their utility on several occasions. It is reported that a tractor saved the R-9 from destruction when it was necessary to take it into the shed at Howden in a thirty mile cross wind. High maneuverability and a speed of 25 to 30 m.p.h. is a prime requirement of these tractors.

(*d*) Masts or towers which are proving a commercial possibility by Great Britain and the United States for rigid airships are far from being an unqualified success for non-rigid airships. This is due to the requirement, in case of the nose-cone type of a rigid nose on the airship for engaging the mast which would introduce a prohibitive weight. Masts for mooring the airship at the bow-mooring point aft of the nose have not proved entirely successful because of the excessive oscillations of the ship about this point.

(*e*) Cable Mooring. As in the case of the mast this means of mooring a ship in the field was developed to obviate the necessity of taking the ship into the hangar except for repairs or major overhauls requiring hangar equipment. The "fixed three-wire mooring system" is the latest and most approved type of such mooring, as it has saved ships from destruction on many occasions.

Conclusion

Non-rigid airship transportation is technically possible, and has certain inherent advantages over terrestrial modes of transportation. Exploitation of its commercial possibilities cannot, however, be accomplished without a satisfactory solution of the handling and mooring problems involved.

Obviously, the solution will be the one that affords the greatest number of flying hours to an airship. In this connection it may be observed that naval operations leading up to the battle of Jutland were held up by the German command because the airships were weather-bound in their sheds, otherwise the possibility of a German victory would have been greatly enhanced.

The prime requirements for a successful handling and mooring system are:

(*a*) Complete protection of the ship from the destructive elements of weather.

(*b*) Provision for determining the static lift of the ship, i.e., a weigh-off device.

(*c*) Provision for completely servicing the ship with fuel, oil, water, etc.

(*d*) Passenger facilities for embarking and disembarking.

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AER-51-7

Airports and Airport Engineering

By JOHN BONFORTE,[1] NEW YORK, N. Y.

DEVELOPMENT of efficient airports is one of the most vital problems confronting the aeronautical world today. Looking back to the war period and the years immediately following, aeronautical progress was marked by alternate spurts in the development of airplanes and airplane engines. After airplanes and airplane engines had arrived at a fair degree of efficiency, the thought of aeronautically minded men was turned to commercial air transportation. Civil air transportation was begun and carried on by the Government air services at a time when its future was uncertain. When the practicability, usefulness, and profit of air transportation had been fully demonstrated, the Government air services were replaced by private contractors.

While the Government stepped out of commercial air transportation, it continued to manifest its vital interest in American aeronautical activities by the passage of the Air Commerce Act of 1926. Since the inauguration of this Act, all commercial air activities have expanded steadily.

But, one important item is lacking in the complete air-transportation program, that is, airports which are efficient from an engineering viewpoint. Apparently, what is needed to develop good airports is specialized airport engineering. But the term "airport engineering" is so new that its meaning is apt to be somewhat loose; its application general; and its abuse by self-seekers very wide. Airport engineering is not a brand-new science or business but only a profession combining portions of the older engineering sciences in a new application.

In the absence of any accepted definition of airport engineering, the following is submitted:

Airport engineering is the profession that combines the existing knowledge of civil, mechanical, and electrical engineering, and applies them efficiently in the preparation of an area for landing field use.

The broad scope of this definition of airport engineering necessitates that the airport engineer possess the qualities of a chief engineer with the flier's knowledge of a desirable landing-field area; the city-planning expert's knowledge of city growth; the civil engineer's knowledge of surveying and topography; the soil specialist's knowledge of drainage and of sod growth; the highway engineer's knowledge of road materials and of road building; the construction engineer's knowledge of building; the electrical engineer's knowledge of the proper distribution of lighting; and the architect's knowledge of planning for present needs and future growth.

The problems confronting the airport engineer can be roughly divided into four classifications—selection, planning, construction, and lighting.

The selection of an airport site should include consideration of (1) probable users of the field, (2) geographical location of the city, (3) topography of the site and of the airways leading to the airport, (4) local weather conditions, (5) area and the shape of the airport, (6) surrounding obstructions, (7) accessibility to the site by highways and railroads, (8) availability of electricity, telephone service and water, and (9) drainage and nature of soil.

The probable users of an airport site determine largely and sometimes entirely the proper selection of an airport site. The probable users of an airport can be classified into air mail, express, and freight carriers; passenger carriers; flying-school operators; service operators; and "joy riders."

At one time joy riders constituted the bulk of the airplane passengers, and so the control lever of the airplane was called the "joy stick." Each of these users of a landing field has a different main requirement.

An airport suitable to an air-mail, express, and freight carrier should consider primarily the airway of which it is a terminal or depot. The inter-city passenger-carrier's airport should consider primarily its connection with the highway or railroad, transporting the passengers to the airport. The flying-school field should be located where ample level area is available with few surrounding obstructions and available to some nearby community by a good road. The service operators' field should be located primarily with a view to its availability to the business section of the community which it is serving. The joy-riders' field can be located anywhere that a crowd can be attracted and conveniently accommodated with parking space. And if a dance floor is provided all conveniences may be forgotten.

Local weather conditions should be given careful consideration in the location of an airport. There are sometimes small local areas in parts of the country which are frequently enveloped in fog due to the proximity of a lake, a marsh, a river, or the ocean. There are also areas in the country which are subject to peculiar local wind conditions. There may be winds that are almost blowing from one or two directions; or winds that blow incessantly and strong, or winds that fluctuate widely, causing the formation of eddy currents.

It seems superfluous to say that no airport should be selected and no airport layout should be planned without a careful study being made of the wind conditions; yet one of America's famous fliers laid out the runways for a proposed airport north and south, and east and west; when seventy-five per cent of the prevailing winds for that locality were from the southwest to northeast and from southeast to northwest, or at angles of 45 deg. to the proposed runways.

The area and shape of airports may be widely different and yet each be efficient; or the area and shape of airports may be alike and some be very good, some fair, and some very bad. The area and shape of an airport is only one of the factors entering into an airport selection, and for this reason it alone is not a criterion of the worth of an airport.

Airport drainage and the nature of the soil are two factors in airport selection which should also be given very careful attention. An airport that is excellent in summer and a marsh in spring cannot be expected to serve as an industrial center.

Airport Planning

One important consideration in airport planning is the possibility of its use for night flying. Night flying increases the safety requirements of an airport as night automobile driving increases the safety requirements of a highway. Obstructions which seem minor in the daytime may become hazardous at nighttime. The airport layout should consider in turn, the prime use of the field, the runways and landing area, the obstructions about the field, the location of the structures to be erected immediately and in the future, and the location of roads and parking space.

The location of the buildings at an airport depends upon so many other considerations that their position is frequently de-

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

termined by these considerations if sufficient careful forethought is given to them. A few of these predetermining factors are: (1) The approaches to the field; (2) their availability and proximity to the highway; (3) the location of existing obstructions; and (4) the direction of the electric feeder lines. This last is an important point which is sometimes overlooked, necessitating the stringing of overhead electric lines around a border of the field, or burying them underground at considerable cost.

Construction Work and Lighting

The value of the construction work done at an airport depends largely upon the care and ability exercised in making the airport plans. Airport lighting is a new application of industrial lighting; and like it, airport lighting may enhance or impair the utility of the airport. An airport should have the usual night light equipment described by the Government, that is, a strong, distinctive beacon, field and obstruction lights, a ceiling light, and flood-lights. Flood lights are required for field and hangar illumination.

A single source of light for flood lighting is preferable to scattered sources of lights, because it is easier for the eye to accommodate itself to the uniformly graduated light from a single source than it is to accommodate itself to the uneven distribution of light from scattered sources.

At least one hangar should be illuminated by flood lighting. The same thought regarding having a single source of light, rather than a group of lights, applies to hangar illumination as well as to field illumination. Scattered goosenecks along the eaves and ridge of a building are an ineffective and insufficient means of lighting a hangar. A more efficient method is to flood light the roof by two or four flood lights mounted at short distances from opposite corners of the roof.

The term "airports" should be limited to those flying fields properly located, having a prepared surface of sufficient area to permit the safe landing and take-off of the largest airplanes, carrying their maximum loads, by day or night, in all directions of the prevailing wind currents for that locality; and having the following accommodations: (1) A maximum 30-minute traction or automobile service to the heart of the community being served, (2) a good telephone and telegraph service, (3) an ample water supply for drinking, sanitary field service, and fire-prevention use, (4) ample hangar space for the largest airplanes, containing lighting, heating, and repair equipment, (5) electric fueling service, (6) an efficient field and flood-lighting system, (7) day and night field service, (8) a weather communication office, and (9) a radio receiving and sending station.

Inefficient airports result from (1) general lack of knowledge of airport engineering, (2) widespread belief that well-known aviators are experts of everything aeronautical, including airport engineering, (3) diversified control of airports by municipal and civic bodies, who cannot profit by their early experiences, because they only control one airport, and (4) lack of experienced personnel in airport engineering.

The future remedy for the proper development of airports perhaps lies in governmental supervision. This supervision is coming very rapidly. But, instead of airport development and maintenance supervision being conducted by the Federal Government, as is the case with practically every other phase of aeronautical work, to all present indications governmental supervision of airports will apparently be conducted by state, county, and possibly municipal governments.

In an industry so widespread as aviation, such diversified control is unfortunate. However, since the Federal Government will not interest itself at the present time in airport legislation; and the states, counties, and local governments are doing so very rapidly, it is to the benefit of the entire aeronautical industry that those interested in airport legislation endeavor to make the regulatory measures as nearly uniform and helpful to the industry as possible.

Proper governmental regulation of airports should result in (1) standardization of airport planning, development, and lighting, (2) selection of air-mail airports relatively primarily to the airways of which they are a part, (3) maintenance of airports at a minimum standard, (4) increased public confidence in the security of air transportation, with a consequent increase of business for the reliable operators, and (5) an increase in the number of airports as State aid is obtained for the development of airports at strategic points in communities unable to bear the financial burden of the airport.

However, legislation at its best is only regulatory. The final solution of airport problems must lie in the hands of the airport planners and builders. Colonel Blee, Chief of the Airport Section of the Department of Commerce, summarizes the whole airport problem fully and concisely by saying: "The selection of airport sites cannot be reduced to a matter of mechanical routine; it necessitates the exercise of sound engineering and business judgment, coupled with a thorough understanding of the aeronautical problems involved."

Type Testing of Commercial Airplane Engines of Medium Power[1]

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ONE OF THE major requirements for an airworthy airplane is a reliable power plant of adequate power, suitably installed and fitted with an appropriate propeller. When a new airplane engine is built for military use, it is submitted to a type test for the purpose of determining its performance characteristics and demonstrating its reliability. The type test made by the Navy Department (aeronautical specifications E-4E) comprises (1) determination of the sea-level performance characteristics of the engine, (2) a 50-hour endurance run, (3) redetermination of the sea-level performance characteristics, and (4) a complete teardown inspection of the engine.

Under the first heading, tests are made at full throttle every 100 r.p.m. from 75 per cent to 115 per cent of rated speed. The quantities measured at each speed include brake horsepower, specific fuel consumption, friction horsepower, oil-pump capacity, oil pressure, and manifold vacuum. A series of propeller-load runs are made also, based on maximum power at the rated speed. At full throttle and rated speed, the effects on power of oil-outlet and air-inlet temperature are independently studied. The endurance test is run in five-hour periods. There are nine periods at rated power and speed followed by one period at full throttle and rated speed. Rated power is defined as 95 per cent of the maximum power developed at rated speed with the throttle wide open. The fiftieth hour of the endurance test is sometimes run at rated power but at 5 per cent above rated speed and the entire 50-hour test may be run without stops between periods. The failure of any vital part of the engine prior to the endurance test automatically rejects the engine, and any stoppage of the engine due to failure of its own mechanism terminates the test, unless such defect could be repaired while in flight. No engine is accepted for the standard type until it has passed an acceptance test (aeronautical specifications E-3E) in which it is run-in for at least five hours and then given two tests at rated speed with an inspection between. The first of these tests consists of $1^1/_2$ hours at 90 per cent rated power at rated speed followed by $^1/_2$ hour at rated speed and power. During this test the maker may make minor adjustments. The final acceptance test consists of $^1/_2$ hour at 90 per cent rated power and $^1/_2$ hour at rated power. At the conclusion of this test, the maximum power at rated speed is determined. Defective parts found on inspection and leaks developing during tests require one hour penalty runs. Not more than two such penalty runs are permitted.

The British Air Ministry prescribes a type test (Air Publication 840) as the basis for approving engines to be installed in licensed aircraft. The maker of a new type of commercial airplane engine must specify: (1) The normal speed of the engine (i.e., the maximum crankshaft speed at which it may be operated for long periods); (2) the maximum permissible speed (i.e., the maximum safe crankshaft speed for short periods of operation), which must be at least 10 per cent greater than normal speed; (3) the rated full power (i.e., the maximum power at normal speed), which must be at least $97^1/_2$ per cent of the power obtained at full throttle at normal speed; (4) the specific fuel consumption at normal speed; and (5) the specific oil consumption (maximum and minimum) at normal speed.

Prior to the type test, the maker must run a new engine in for at least two hours, tune it up on gasoline, and submit it to a standard two-hour endurance test at normal speed and 90 per cent full power. After a complete teardown inspection has been made, the engine is reassembled and given a final acceptance test for at least $^1/_2$ hour at normal speed and 90 per cent full power. (Unless otherwise specified, all test runs at 90 per cent full power end with a five-minute run at full throttle.) The type test comprises the following: (1) A power curve test, (2) a 50-hour endurance test, (3) tests of slow running and acceleration, (4) a high-speed test, (5) a high-power test, (6) a second power-curve test, and (7) a complete teardown inspection followed by a final $^1/_2$-hour test at normal speed and 90 per cent full power. The power curve is obtained by adjusting the load at full throttle to obtain successively maximum power at maximum permissible speed and at lower speeds (by 100 r.p.m. steps) down to at least 400 r.p.m. below normal speed. The engine must be run at least five minutes at each speed before the power reading is taken. The endurance test consists of five 10-hour non-stop periods at normal speed and 90 per cent full power. At least two periods are run on a torque stand and at least the fifth period is run on a dynamometer. For the final hour of this test, the load is increased to develop full rated power at normal speed and the test concludes with a five-minute run at normal speed with the throttle fully open. Fuel and oil consumptions on any 10-hour non-stop run during the endurance test must be within 20 per cent of the maker's rating. After the endurance test, while the engine is still warm, tests are made to see that the engine runs reasonably well at 90 per cent of full load when throttled down to 20 per cent of normal speed and that it accelerates to normal speed in five seconds without excessive vibration or carburetor trouble. The high-speed test consists of a one-hour continuous run at 10 per cent above the specified maximum permissible speed. For this test the load will be that recommended by the maker. The high-power test is a one-hour continuous full-throttle run at the maximum permissible speed specified by the maker. The failure of any vital part of the engine during test automatically rejects the engine, and after the replacement or rejection of any parts the government representative may require the engine to be resubmitted exactly as a new engine. The British requirements for the type testing of military engines differ only in setting maximum limits on the fuel and oil consumption and in calling for a 100-hour endurance test.

The air commerce regulations of the Department of Commerce provide that engines which have passed the regular endurance tests of the Army Air Corps or of the Navy Department will be approved for use in licensed aircraft and that all other engines submitted for approval will be tested at the Bureau of Standards. The present test requirements (Aeronautics Bulletin No. 14, pages 40–42) call for (1) a 50-hour endurance test, (2) a power-curve test, and (3) a flight test. However, the flight test is being

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[1] Publication approved by the Director of the Bureau of Standards of the U. S. Department of Commerce.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

waived in the case of engines which have a record of successful performance in the air.

The endurance test is run in ten five-hour periods. During the first period the engine is run at full throttle, the speed being at least equal to the rated speed and the power at least 10 per cent in excess of the rated power. The balance of the test is run at rated speed and power. This gives the manufacturer three options: (1) He may select as his rated power 90 per cent of full throttle power at the rated speed, in which case the first period will be run at rated speed; (2) he may select full throttle power at rated speed as the rated power, in which case the first period may have to be run at a speed 10 or 15 per cent above rated speed to obtain the 10 per cent excess power; (3) he may select as rated power 95 per cent of full throttle power at the rated speed and obtain the increased power at a speed about 3 per cent above rated speed. It will be seen that the first case corresponds approximately to the British endurance test but yields a lower power rating. The second case gives a rating equivalent to the British rating but makes the test rather more severe than the British endurance and high-power tests combined. The third case corresponds closely with the Navy endurance test but the first period is a trifle more severe than the final period required by the Navy department. This is the option usually chosen and is the most convenient one for torque stand testing since the same propeller which absorbs 110 per cent rated power at 103 per cent rated speed with the throttle wide open will also provide the proper load when the engine is throttled down to rated speed. This test is less severe than it sounds, for, contrary to both British and Navy practice, up to three forced stops are permitted. Forced stops may be required to correct fuel or oil leaks at the engine, to replace broken rocker arms or valve springs, and to clean spark plugs or repair accessories. Each forced stop becomes part of the test record and requires a two-hour penalty run at the end of the test period in which it occurred. The manufacturer's representative is also permitted to make minor repairs and replacements between test periods without penalty, provided the test schedule is not delayed thereby. The failure of a major part of the engine, such as a piston, cylinder, connecting rod, or crankshaft, automatically terminates the test, and inability to maintain rated power may necessitate a revision of the manufacturer's power rating.

The Bureau of Standards has determined with an electric dynamometer the performance characteristics of numerous water-cooled engines both under sea-level conditions and under conditions corresponding to various altitudes in its altitude chamber where temperature and pressure conditions can be regulated as desired. A 400-hp. dynamometer is available for the testing of air-cooled engines, but the air-blast required for suitably cooling radial engines of large frontal area is not yet available in the laboratory. For this reason, all tests thus far completed have been made on a torque stand located at College Park, Maryland, where the bureau's radio beacon is also located. An oversize four-blade wooden propeller is built for each engine and is adjusted until it absorbs the desired amount of power and also gives an air blast of at least 90 m.p.h. across the engine cylinders.

The torque stand is equipped with platform scales which are read to the nearest half pound, the average of three readings being taken for each $^1/_2$-hour interval. The horsepower is figured as equal to the product of the torque multiplied by the speed and divided by a constant for the torque arm. This result is increased by 5 per cent to allow for the reaction of the slipstream on the torque stand. The barometric pressure is read on a mercury barometer. Air temperatures are read on a mercury-in-glass thermometer. A stop watch is used to obtain the time required for the consumption of five pounds of fuel, weighed on a sensitive torsion balance. The oil consumption is determined by weighing at the end of each $^1/_2$-hour interval. Engine revolutions per minute are adjusted approximately by using a Navy type tachometer and are determined accurately by using a revolution counter and stop watch. The oil pressure is measured on an accurate test gage and two distance thermometers are used to indicate the oil inlet and outlet temperatures. The pressures through the fuel induction system are measured by mercury manometers. All the instruments are calibrated by the Bureau of Standards.

The curve of power versus speed is obtained by varying the load with the throttle wide open. The load is varied by adjusting a variable-pitch propeller or by cutting down a wooden propeller. At each load, the speed must be maintained approximately constant for five minutes before the torque is recorded. The test speeds range from 75 per cent to at least 110 per cent of rated speed. These full-throttle runs are normally made at the conclusion of the endurance tests. However, they may be made first for the information of the manufacturer in case his proposed power rating seems excessive.

As implied above, under existing regulations, the approval of an engine means only that it has gone through a 50-hour endurance test without a major failure and it may have escaped failure by a narrow margin. After the torque-stand tests and the final teardown inspection of the engine are completed, a report is made to the Director of Aeronautics which contains all the test data together with corrected horsepower, fuel consumption, oil consumption, cylinder temperatures, and air velocities. A detailed history of the test is given, illustrated by photographs of all parts which failed, were replaced, or showed undue wear. The design and performance of the engine are discussed and suggestions are offered as to the probable causes of any failures. Definite recommendations are made as to the approval of the engine. The fuel used during the test is stated and the engine is only approved for use with fuels of equal or higher anti-knock value. In case an engine cannot be recommended for approval, the conditions to be met by the manufacturer before a retest can be undertaken are specified. When an engine fails the test report is confidential, but the prospective user of an approved engine can and should obtain from the department information as to the actual performance of that engine under test.

Although all the engines tested thus far have been flown more or less extensively, about half of them have suffered major failures on the test stand. These failures were due in most cases to poor design, but the faults should have been discovered by the manufacturer on his own test stand. To be sure, the department requires merely that the engine submitted for test shall have been given a 25-hour bench test and that the average horsepower developed during this test shall not average less than one-half rated power, but this does not prohibit preliminary tests at rated power and speed. Difficulties with valves, valve stems, valve springs, rocker arms, rocker-arm pins, and piston rings indicate that more attention should be given these parts with respect to design and materials. In one case a poorly designed valve stem alone was responsible for a major failure. Induction, ignition, and lubrication systems have all given more or less trouble. The manufacturer furnishes whatever grade of fuel and lubricant is to be used and is permitted to have his carburetor and ignition system checked up by experts.

In conclusion, the use of a propeller which permits an approved airplane engine to run at full throttle in level flight greatly in excess of its rated speed is generally unwise and should never be permitted in licensed aircraft. An airplane engine approved for use on California aviation gasoline should never be permitted to operate at full throttle on straight Eastern domestic

aviation gasoline. The following ratings have been recommended by the Bureau of Standards:

Name	Engine description	Rating Hp.	Rating Speed	Fuel
Warner	7-cylinder (R-420)	110	1850	Domestic aviation gasoline (S. O. Indiana)
Kinner	5-cylinder (R-370)	75	1725	Red Crown aviation gasoline (S. O. Cal.)
Velie	5-cylinder (R-250)	45	1750	Esso or California aviation gasoline
Comet	7-cylinder (R-610)	115	1775	California aviation gasoline

Author's Closure

The printed airplane-engine test requirements were modified two months after the Wichita meeting in the following respects:

The paragraph specifying the endurance test was changed to read as follows:

The 50-hour endurance test shall be run in ten five-hour periods. During the first five hours the engine shall be run with the throttle wide open, the speed being at least equal to the rated speed specified by the manufacturer. A provisional power rating of the engine will be based on the average horsepower developed during the first period. During the remaining nine periods the engine shall be run at approximately 97 per cent of rated speed, and the horsepower developed should at no time be less than nine-tenths of the rated horsepower. The provisional power ratings shall remain in force for one year or until design changes or further tests justify a change.

The following new paragraph was added:

(9) No approved engine installed in a licensed airplane shall be equipped with a propeller so designed or adjusted that at full throttle the engine speed will exceed by more than 3 per cent the official rated speed in level flight at approximately sea level altitude.

The tolerance has since been increased from 3 to 5 per cent.

The effect of the first change is to eliminate the three options mentioned in the paper and to require of the manufacturer only a speed rating. The second change assures the engine manufacturer that his engine will not be permitted to operate at speeds greatly in excess of its speed rating and thus meets the point raised in the final paragraph of the paper. Under the revised regulations the power rating is determined by the horsepower developed during the first five hours instead of by the lowest horsepower developed in any five-hour period. It was possible, therefore, to raise the power ratings of the Kinner, Velie, and Comet engines by more than 10 per cent.

The present ratings of all engines which had received approved type certificates by May 15, 1929, are given herewith. In each case, the A.T.C. number is followed by the name and description of the engine and the last two figures specify the power and speed rating.

Engines Having Approved Type Certificates

1 Fairchild Caminez 447-C, 4 cyl., radial, air-cooled, 120/960
2 Warner Scarab, 7 cyl., radial, air-cooled, 110/1850
3 Kinner K-5, 5 cyl., radial, air-cooled, 90/1810
4 Velie M-5, 5 cyl., radial, air-cooled, 55/1815
5 Curtiss Challenger R-600, 6 cyl., radial, air-cooled, 170/1800
6 Curtiss Conqueror V-1550, 12 cyl., vee, water-cooled, 600/2400
7 Curtiss Conqueror GV-1570, 12 cyl., vee, water-cooled, 600/2400
8 Curtiss Chieftain H-1640, 12 cyl., radial, air-cooled, 600/2200
9 Aircraft Comet, 7 cyl., radial, air-cooled, 130/1825
10 Curtiss D-12, 12 cyl., vee, water-cooled, 435/2300
11 Dayton Bear, 4 cyl., line, air-cooled, 100/1500
12 LeBlond 60, 5 cyl., radial, air-cooled, 65/1950
13 Wright Whirlwind J-5, 9-cyl. radial, air-cooled, 220/2000
14 Pratt & Whitney Wasp, 9 cyl., radial, air-cooled, 450/2100
15 Pratt & Whitney Hornet, 9 cyl., radial, air-cooled, 525/1900
16 Axelson R-610, 7 cyl., radial, air-cooled, 115/1800
17 Wright Cyclone R-1750A, 9 cyl., radial, air-cooled, 525/1900
18 Packard 3A-1500 direct, 12 cyl., vee, water-cooled, 525/2100
19 Packard 3A-2500 direct, 12 cyl., vee, water-cooled, 800/2000
20 LeBlond 90, 7 cyl., radial, air-cooled, 90/1975
21 Wright R-975, 9 cyl., radial, air-cooled, 300/2000
22 Harris B-1, 8 cyl., vee, water-cooled, 90/1400
23 Wright R-540, 5 cyl., radial, air-cooled, 165/2000

Further work has shown that the windage correction on torque-stand measurements with a four-blade propeller may be quite large in case there is not room to use a straightening grid between propeller and engine. In such cases the corrected factor must be determined experimentally for each engine.

Development of Welded Aircraft Construction

By S. C. CLARK,[1] KANSAS CITY, MO., and W. I. GASTON,[2] NEW YORK, N. Y.

In this paper the authors point out that steel tubing is coming into common use for fuselage construction and with this there is a trend toward general use of welding. It is desirable to establish standard designs for welded joints and to this end further research is needed. Suggestions are made for improvements in welding practice that will add safety and insure uniformly well-made welds.

IN THE EARLY days of aviation it was natural that the pioneer experimenters should turn to wood as a structural material for airplanes. It was about the only suitable material available. Seamless-steel tubing was not unknown, but it is doubtful if it could be had in the extremely thin-walled variety, having the physical properties necessary to produce a very strong, light structure. High-strength alloys of aluminum were not developed until after the World War. Even if suitable steel had been available commercially, a convenient method of fabrication would have been lacking, for, at the time of the first memorable experiments of the Wright Brothers, gas welding was in about the same stage of commercial development as aviation. It is interesting to note that the development of the process is somewhat coincidental with that of aviation, having had its inception at about the same time and having received its greatest impetus during the World War.

Change From Wood to Metal

Airplane structures were, therefore, first constructed of wood. But, although wood still has its adherents, notably in foreign countries, many engineers soon began to see its shortcomings as a material for fuselage construction. In the first place wood, being a natural material, does not have the same uniformity of physical properties as metals, which are manufactured. Also, it must be remembered that a wooden fuselage is necessarily a composite structure requiring metal fittings at the joints and piercing of the wooden members, with consequent weakening of the section. It is very difficult to make such a structure function as a unit when stressed, rather than have individual members bear the brunt of the load at the point of application. Considerations of safety also favor metal construction since metal will bend while wood has a tendency to splinter leaving sharp ends. Small cracks are much more likely to exist undetected in a wooden structure than in one of metal.

However, the choice of a material for a given airplane part is largely a matter of proportion. For a part as deep as a fuselage, most of whose members function either as long columns or as tension members, according to the conditions of flight, metal is thought by the majority of engineers to produce the most economical section, considering the strength-weight ratio. On the other hand, for wings, which are of less depth and under flexural stress at all times, many feel that wood, even allowing an extra factor of safety for its lack of uniformity, produces the most economical section for wings of small or moderate size. As wing sections become deeper, metal construction becomes relatively more economical and therefore metal is being adopted rapidly for wing construction in the case of large planes. The present trend seems to be toward composite metal construction for wing structures, although some large wing structures have been fabricated entirely of welded chrome-molybdenum steel tubing. Welded steel wing beams, or spars, are being used with ribs of aluminum alloy.

Welded Steel Fuselages

For fuselage construction, mild-steel or alloy-steel tubing with welded joints is general practice. The most complicated joints or fittings can be fabricated easily from light-weight steel sheet and tubing by welding and finished fittings can be heat-treated if desired. Dies and patterns are not required, as would be the case if forged or cast fittings were used. This is especially important because of the rapidity with which airplane models change.

All forms of riveted or bolted construction will, sooner or later, loosen up to some extent at the joints. A welded fuselage is rigid and cannot develop play at the joints. Mechanical deterioration in joints is eliminated by this type of construction. A welded fuselage, if damaged, can be repaired easily, in any location, to be just as strong as it was originally.

A welded-steel fuselage, can be built, maintained and repaired economically. These considerations, in addition to the fact, that the majority of American aeronautical designers believe that the most economical strength-weight ratio is obtained through this construction, have made welded steel tube fuselages common in this country. About 90 per cent of American-made ships are being so constructed at the present time. Even when fuselages are constructed from duralumin, the highly-stressed landing gear, tail skid and motor mount are fabricated from chrome-molybdenum steel by welding and are then heat-treated.

Present Status of Aircraft Welding

The ability of welded-steel fuselages to stand the severe stresses of sustained flight with heavy loads has been amply demonstrated. All of the American planes that successfully completed trans-Atlantic and Hawaiian flights were equipped with welded-steel tubing fuselages.

Many pilots owe their lives to these crash-proof fuselages. When "crack-ups" occur the tubes bend and buckle, but the welds hold, keeping the structure largely intact. Welded aluminum gasoline tanks, which will often stand enormous impacts without even springing a leak, have also been a valuable contribution to the cause of safety. These are common equipment on the airplane of today. Fig. 1 shows the condition of a welded joint after a bad crack-up. The tubing is bent but the welds all held. Fig. 2 shows two 150-gal. gasoline tanks that survived the same crash without springing a leak.

F. F. Sisco, Chief of the Metallurgical Laboratory, Air Corps, Wright Field, Dayton, Ohio, said in a paper presented at the Tenth Annual Convention of the American Society for Steel Treating at Philadelphia:

"The art of welding, especially of thin tubular products, has progressed to the point where it is practically 100 per cent perfect.

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Presented at the Third National Meeting of the A.S.M.E. Aeronautic Division, St. Louis, Mo., May 27 to 30, 1929.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

The perfection attained is so high that at present the welded fuselage is standard in airplane construction. Other applications of welding in airplane and engine construction are too numerous and too well known to demand further consideration. Failures due to improper welding in the construction of the airplane are exceedingly rare; when they do occur they are almost always due to the welder and seldom if ever have their origin in the materials or the process."

Some of the best features of gas-welding practice have been successfully adapted to aircraft manufacture from other industries. For example, joints made by inserting a reinforcing plate in the plane of the axes of two or more tubes (in saw kerfs in the tubes) and then welding all seams are an adaptation of structural welding, this design having first been used in tubular roof trusses.

Thus it is seen that welding is at present an entirely practical and recognized process for the fabrication of aircraft fuselages and other parts. Future developments must, therefore, be in the nature of refinements.

Desirability of Further Research

J. B. Johnson, Chief, Material Division, Army Air Corps, Wright Field, Dayton, Ohio, said in a paper presented at the annual meeting of the American Welding Society in September, 1927, "There are no established rules for the design of welded joints which are directly applicable to aircraft. The Air Corps has published a few for the assistance of designers which are based on the results of tests and service requirements. A reduction of 15,000 lb. per sq. in. in the tensile strength of a chrome-



Fig. 1 Condition of Welded Joints After a Bad Crash

molybdenum steel having an ultimate strength of 95,000 lb. per sq. in. is recommended as a conservative figure for a straight butt weld. If the joint has a fish-tail form or is reinforced by gusset plates, the value may be increased and designs have been approved in which an allowable stress of 95,000 lb. per sq. in. has been used in calculation."

A new design for an airplane is checked by experts of the Aeronautics Branch of the Department of Commerce to determine whether or not it is air-worthy. Since available information on the strength and other properties of welded joints was not considered adequate by the Aeronautics Branch for checking airplane designs, the Bureau of Standards was requested to conduct an investigation of the subject, which is now nearing completion.

From these commentaries it is seen that aircraft welding in its present state may be considered a product of natural growth, which has not yet received its share of scientific research, although certain individuals have been very active in the latter field. Having achieved its remarkable success under these conditions, it is reasonable to assume that considerable further improvement can be made.

Work of the Bureau of Standards

The investigation now in progress at the Bureau of Standards includes 134 different joints of the "T" and "lattice" types. Materials used are mild steel under Army specifications No. 57–180–1, which is the same as S.A.E. No. 1025, and chrome-molybdenum steel under Army specification No. 57–180–2, which is the same as S.A.E. No. 4130 X. By systematically varying

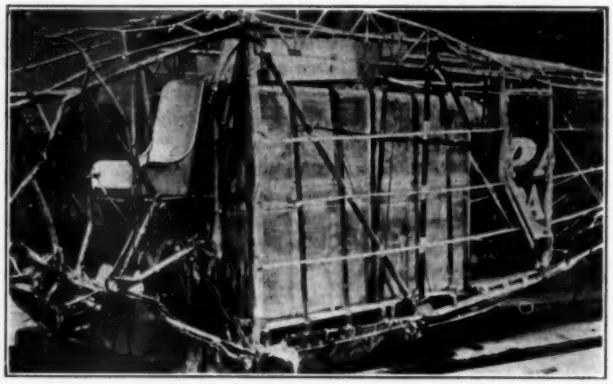


Fig. 2 Tangled Mass in Pilot Seat. There Were no Failures at Joints

the designs of the joints, including dimensions of inserted gusset plates, where used, and noting the effect on the strength, weight and cost, the most efficient types of joints will be determined. Handbook information concerning these joints will be available to all aircraft manufacturers at the conclusion of the investigation. Types of joints being tested at the Bureau of Standards are shown in Fig. 3. The ones on the right are the inserted gusset-plate type, previously mentioned.

Welding at the Bureau of Standards is being done in accordance with a "Procedure Control" drawn up by the representatives of the gas-welding industry on the sub-committee on Procedure of the American Bureau of Welding, which is the research department of the American Welding Society. A "Procedure Control" is a detailed production specification covering: (1) Check of welders' ability, (2) selection and inspection of materials, (3) design and layout of welded joints, (4) preparation for welding, (5) welding technique and (6) inspection and tests. Experience with procedure control in other fields where strength and reliability are paramount has shown that it does insure work of uniform high quality. A carefully prepared procedure control removes the human element and places the responsibility for good welding upon the designer and inspector rather than upon the welder.

Suggestions for Improvement of Present Practice

Failures of welded joints in service have been practically unknown for the following reasons: (1) Properly made welds are inherently strong. (2) In the application of Euler's formula for determining tube sizes for fuselage members, a coefficient of restraint of only 1 has been used; thus it has been assumed that the structure is pin-ended at every joint, although this is not the case. (3) The testing of new types of airplane construction has been very thorough. Nevertheless, procedure control as applied to aircraft welding has not yet been developed to the same extent as it has in other industrial fields. From the results obtained in other important fields, it is certain that by the adoption of standardized procedure control methods much can be done toward better design and execution of the weld, increase of the strength-weight ratio, and a closer approach to the ideal of a

plane that cannot be broken in the air. The procedure control prepared by The American Bureau of Welding and now being followed by the Bureau of Standards will, when finally revised, constitute an authoritative guide, available to all aircraft manufacturers, on this important subject.

Check of Welder's Ability

A standard specification that will insure the hiring of good welders has been drawn up. This includes, in detail, previous

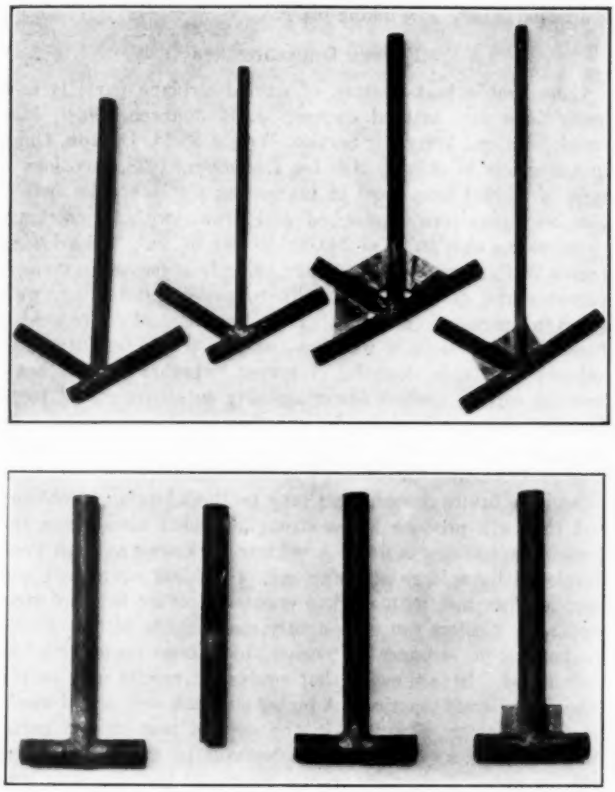


Fig. 3 Types of Welded Joints Being Tested by the Bureau of Standards

experience or training required, specimen welds to be made, observation of the manner in which the prospective welder handles the apparatus and the factors of manual skill to be looked for by the welding inspector in a preliminary qualification test. Conformity to dimension, contour of the weld, penetration and fusion are taken into consideration. If the welder satisfactorily passes this preliminary test, he is required to pass a final qualification test consisting of certain rotative and position welds, which are then examined visually and subjected to prescribed strength and ductility tests. A welder having passed these tests is certain to be capable of making satisfactory aircraft welds.

Selection and Inspection of Material

Certain grades of mild-carbon and chrome-molybdenum steel, mentioned previously, are the materials used at present. The latter has largely replaced mild-carbon steel because of its superior strength, and due to its air-hardening properties, the strength of the welded tubing approaches that of the original material. Mild steel is still used to some extent. Its modulus of elasticity is equal to that of chrome-molybdenum steel, and it is cheaper. Therefore, it is considered more economical for certain members by some manufacturers. Others have standardized on the alloy steel for all members.

The procedure control contains detailed specifications for check testing this material against mill specification for physical properties. Weldability tests are also prescribed for base metal and welding rod.

Design and Layout of Welded Joints

Design of welded joints is an extremely important consideration. While the joints used at present are very efficient, a standard design for each type of joint, which is strongest, easiest to weld, and most economical of base metal, welding rod, and gases, and consequently lightest for the required strength, is highly desirable.

Position of tack welds and sequence of welding to avoid cracking and distortion of the members joined, from the contraction of

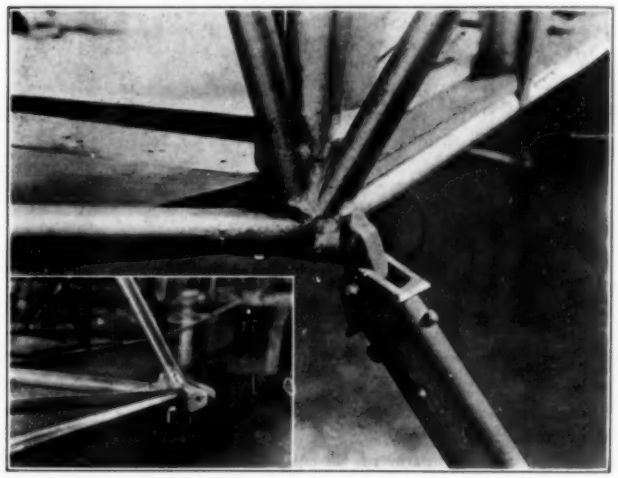


Fig. 4 Some Types of Joints With Many Members Coming Together

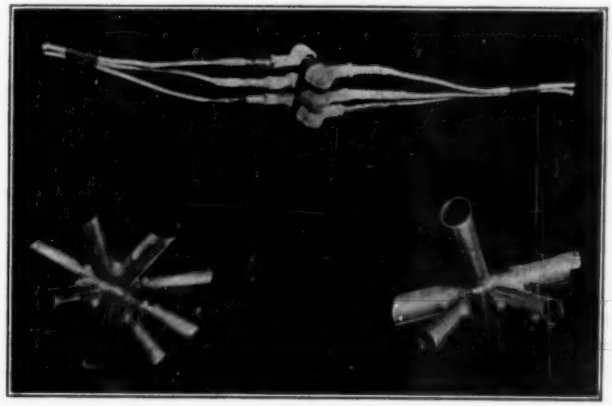


Fig. 5 Some Complicated Types of Welded Joints

the cooling weld metal, are problems yet to be thoroughly worked out. The principles of heat control are already well understood, especially for the simpler joints, but there is undoubtedly much room for improved procedure in the case of the more complicated joints containing gusset plates and fittings and where many members come together. Figs. 4 and 5 show some joints of this general type.

Preparation for Welding

The actual layout or assembly of members is usually accom-

plished by the aid of jigs, although special or experimental work may be set up largely by means of clamps. Jig design is well understood at present although this too is susceptible of improvement and standardization. A jig for interchangeable-fuselage sections for large Army bombers is shown in Fig. 6. Probably the most important problem to be worked out in preparation for welding is the arrangement of work for line production. For instance, is it more economical to have six welders working

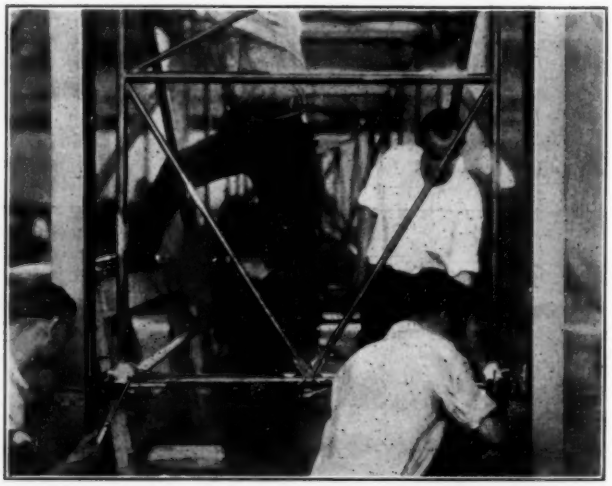


FIG. 6 TYPE OF JIG USED IN FUSELAGE CONSTRUCTION

simultaneously on one assembly or to have two welders working on each of three assemblies? Such details as this have not yet been satisfactorily determined, in most cases, due to the newness of the industry.

Actual manipulation of blowpipe and welding rod to secure a good weld needs little comment. A qualified welder, given detailed instructions, will experience no difficulty in properly executing the weld. Welding technique is, however, thoroughly covered in the procedure control.

INSPECTION AND TEST

An authoritative scheme for inspecting the work of the welder, which will insure against defective welds in the finished product, is also being worked out. Individual manufacturers have their own ideas on this subject and some of them are undoubtedly better than others. What is needed is a standard method that can be used successfully by any manufacturer. Methods of periodic testing of welders' work and test of the finished product that will apply in all cases are included in the procedure control.

Individual welders are, as has been mentioned, required to pass qualification tests before undertaking production welding. Welding inspectors should be qualified by a period of training in a good welding laboratory. By this means they will obtain a laboratory point of view and a grasp of fundamental principles, obtainable in no other way, through contact with men extremely well versed in the theory and practice of welding as well as testing methods.

Additional items covered by the procedure control are welding apparatus and gases, characteristics of flame to be used, gaging welds and weld tightness as a possible means of preventing interior corrosion.

In the foregoing an effort has been made to set forth the lines along which present aircraft-welding practice may be improved. It is not possible to offer conclusions since the process of standardization is only now taking place.

FUTURE DEVELOPMENTS

Considerable heat-treating of welded airplane parts is now being done with marked success. J. B. Johnson, Chief, Materials Section, Army Air Service, Wright Field, Dayton, Ohio, in an article in *Airway Age* for December, 1928, mentions a type of welded joint used in fabricating airplane axles, which can be satisfactorily quenched and drawn without cracking, to develop a unit stress of 200,000 lb. per sq. in. Welded wing beams 20 ft. or more in length are being heat-treated in vertical furnaces and quenched vertically to prevent distortion, with resultant increase in strength. Heat treatment of entire welded fuselages, or sections of fuselages, may be a field for future development. It is doubtful, however, whether or not heat-treating can be applied advantageously to all structural parts because, although it materially improves tensile and compressive strength, it does not increase the modulus of elasticity of the steel and therefore the efficiency of the members.

Another future development may be the adoption of welding rod that will produce higher-strength welded metal than the low-carbon rod now in use. A welding rod known as High Test, developed by a large manufacturer of welding equipment and supplies, has met with marked success in other fields of steel welding. Welders can consistently make welds with it 11,000 lb. per sq. in. stronger in tension than those made with low carbon rod. It is thought that equivalent results may be obtained in aircraft practice. A higher strength weld metal would be of particular advantage in the case of heat-treated parts. Manufacturers so far have been hesitant to adopt alloy steel welding rods because they are not convinced that the same uniformity of results can be obtained as with the type of rod now used. This matter is one that will be answered by future developments and experience. Another future possibility is the development of a new alloy steel, but the present chrome-molybdenum seems to be highly satisfactory.

The welding industry is keenly alert to the possibilities for developing further improvements in welding as applied in aircraft construction. Its engineers, metallurgists and physicists are giving serious study to all phases of the subject. They invite cooperation of aviation executives by submitting their problems, in order that the same facilities and technical guidance may be applied to aircraft welding as have been adopted with such splendid results in other industries where procedure control has proved to be the key to successful production welding.

Relative Merits of Various Types of Wing Structure in Monoplane Design

By GEORGE W. DeBELL,[1] BUFFALO, N. Y.

WITH reference to wing design, airplanes may be divided into four classes: the sport type under 2500 lb.; the small transport of about 4000 lb. for five to eight passengers; the medium-sized transport around 15,000 lb. for ten to twenty passengers; and the large transport over 20,000 lb. for more than twenty passengers. The size of the airplane has considerable influence on the type of wing structure used. For instance, in the small sport-type plane, two-spar cantilever construction may be used to advantage as the wings are so small and relatively short that torsional deflection is not a problem, and the loads are so light that two spars are sufficient to carry them to advantage. If more than two spars were used on this small airplane, they would be composed almost completely of minimum sizes and, therefore, the weights would be higher than the two-spar construction. But, if two-spar construction were applied to the large cantilever airplane weighing in the neighborhood of 20,000 lb. it would make the construction quite difficult and probably unsafe, due to the lack of torsional rigidity. Due to the tremendous sizes which would be required, when the applied loads were concentrated on only two spars, the material would be of such size as to be difficult of fabrication. Considering only cantilever construction, the following is an analysis of the types of spar structure adaptable to the different size airplanes.

In the sport type either single or two-spar construction is most efficient as the loads are concentrated and there are fewer members to fabricate. In the small transport (4000 lb.) the single-spar construction would not be adaptable as it has not sufficient torsional rigidity. Two-spar construction would probably be the most economical, particularly if the airfoil section were deep enough to give a good drag truss depth. Three-spar construction on this size airplane would again become uneconomical due to the excessive use of minimum sizes. In the medium size transport, around 15,000 lb., the two-spar construction would not be adaptable unless it was aided by a plywood or metal-wing covering as the members would become excessively heavy and the torsional rigidity would be questionable. Three-spar construction would seem the most efficient for this size airplane as any greater number than three would require minimum sizes. Two notable cases in this are the Fokker tri-motor using two-spar construction but relying for torsional stiffness on the plywood wing covering, and the Ford tri-motor using three spars of built-up channel sections and using a corrugated aluminum-alloy wing covering. In the Ford both the three-spar construction and the wing covering tend to make for torsional stiffness, while in the Fokker the plywood wing covering gives the needed torsional stiffness and also may aid in the beam forces, thus reducing the theoretical size of the members. In the large transport over 20,000 lb., multi-spar construction is practically a necessity since any fewer number of parts would require excessively large size and so would be difficult to fabricate.

The above statements apply to pure cantilever construction only. When external wing bracing of any sort is used, the number of spars required for any particular size of airplane may be diminished over the number required for the same airplane in cantilever construction. It is possible, when using deep-external bracings, to design a two-spar wing for the largest type transport and feel assured that the wing will be torsionally stiff enough to prevent flutter. There are two methods in which the wing may be externally braced. One in which the external bracing takes load in either direction, and the second in which the external bracing acts under normal flight loads and the wings act as a pure cantilever under inverted flight loads. In the second type, the front spar should be checked for rigidity as in nose dive the front spar would have inverted flight loads, while the rear spar had normal flight loads. This makes the external bracing in the plane of the rear spar act while the front spar is required to take the loads as a cantilever beam. For this reason the front spar should be rigid enough to prevent excessive washout of the wing tips.

There are two main types of drag trussing used in wing construction, the single-bay drag truss and the double-bay drag truss. The single-bay drag truss has gone more or less out of favor since the advent of the thick airfoil, although it is still used on small planes such as training planes, where cheapness of construction is one of the primary requirements. It has two disadvantages, that it does not prevent rotation of the spars, and that it does not aid in torsion. In the two-bay, or double-drag truss, where one truss connects the tops of the spars and the other truss the bottoms, the spars are held in relation to each other and rotation is prevented by the use of a trussed drag strut. It has the additional advantage of adding greatly to the torsional stiffness of the wing. In fact, the most efficient place to add weight to resist torsional flexibility is in the drag truss. This follows from the formulas of mechanics which show that in the beam of rectangular cross section the greatest torsional stress occurs in the wide face, which in the wing is represented by the drag truss, the narrow faces being the spars. In the single-bay drag system the truss is usually a pure Pratt truss with cross wires and compression struts, but in the double drag-truss system there are several types of construction used, each one being particularly adaptable to a single type of spar structure. The strut and wire, or pure Pratt type is best adaptable to two-spar construction. The Warren truss, or modified Pratt, in which the wires of the Pratt truss are replaced by tubes, is best adaptable to three-spar construction. This is particularly true if the middle spar is exactly half way between the front and rear spars, as the crossing points then occur on the middle spar and so reduce its lateral column action to half the drag bay length. In multi-spar construction where fabric wing covering is used, a drag truss must be employed to hold the spars in their relative positions. For this type of construction the drag truss is usually of the lattice type interlocking all the spars. (See Fig. 1.)

In any type of double-drag system, the drag struts should be shear braced in order to derive maximum benefit from the system in resisting torsional deflection. When a metal or plywood covering is used on the wing, the drag truss is usually omitted. However, compression ribs must still be used at intervals to replace the drag struts and hold the skin to the true contour. In

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

these cases the skin is assumed to take all the drag load and in the case of plywood covering the skin is, in addition, assumed to take a small amount of the beam load.

In the detail design of the spars there are several types of construction which may be used. In metal construction there are two major classes with spars made of steel and of strong-aluminum alloys. There are two main sub-divisions of the

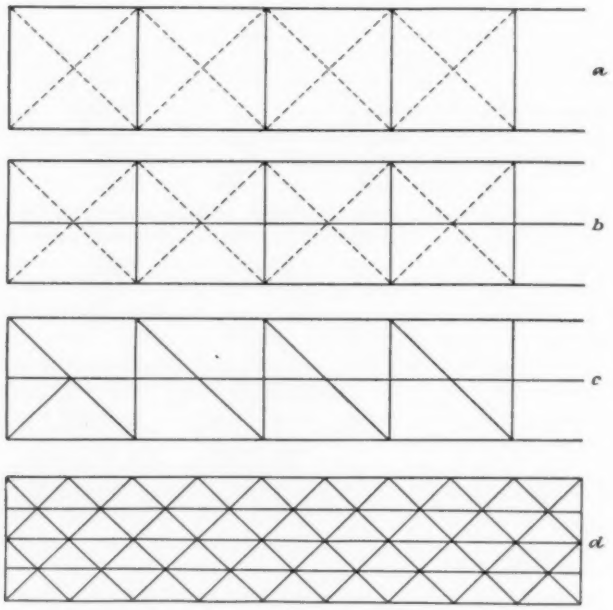


FIG. 1 TYPES OF MULTI-SPAR CONSTRUCTION: *a*, TWO-SPAR; *b*, THREE-SPAR WITH WIRES; *c*, THREE-SPAR WITH STRUTS; *d*, MULTI-SPAR LATTICE TYPE

types of metal spars; one, the continuous-web type, and the other the trussed type. There are many ramifications of the continuous-web type, several of which are used in this country.

In the trussed type there are two major methods of fabrication; one in which the parts are welded together, and the other in which they are riveted. At present the welded type is limited entirely to steel, while the riveted type may be used on either steel or aluminum alloys. The welded type does not lend itself readily to production as the welding shrinkage seriously distorts the spar and requires either considerable rework or a large percentage of rejection, with the obvious disadvantages to interchangeability. Slight distortions have even been noticed in riveted structures due to variations of the order in which the rivets are driven and of the forces used in setting them. Spar designers incorporating tubing are limited to the use of duralumin or steel. Structural, drawn, or rolled, shapes may be used in either steel, duralumin, or Alclad construction, and in many cases it is more advantageous to use some of these shapes since riveting of tubing requires long mandrels to hold, or head up the rivets on the inside of the tubing. By proper design of shapes made of flat stock, the riveting may be made external and high speed power riveters may be used to advantage. In the design of special-shaped spars, care should be taken in selecting the allowable design stresses, as very little is at present known of the form factors for various drawn or rolled shapes. The best way to determine the maximum allowable stress, or form factor, is by actual tests of samples built to the identical dimensions of the spars. In the design of spars, manufacturers differ considerably as to the amount of fixity they assume to be derived from the ribs. Most manufacturers either ignore the fixity of the ribs entirely or run actual tests to determine exactly how much fixity they derive. Designers should be warned against using rib fixity figures procured from other manufacturers as differences in detail rib construction will have considerable bearing on these fixity factors. Aside from this, rib construction has very little effect on the structural strength of the wing.

In the selection of the type of rib to be used, the main considerations should be the rib weight versus the cost of manufacturing. In large production of untapered wings, pressed- or stamped-metal ribs may be used to advantage with considerable saving in manufacturing cost. On tapered wings, unless the production becomes equivalent to that in the automobile industry, pressed or stamped ribs are out of the question as the die costs are entirely too heavy for the output. Therefore, for this type of work the most efficient type of rib construction is the trussed or built-up rib. These ribs may be fabricated either from tubing or from drawn or rolled shapes made from flat wire. The latter type usually proves to be the most economical as the material cost is lower and the riveting cost, in some cases, is less. Pressed or stamped ribs have not come into extensive use as many of the manufacturers of wood wings claim that they can manufacture wood ribs cheaper than they can purchase or make the stamped ribs. However, the moment that metal construction becomes predominant, the stamped or pressed rib will come into its own in straight wings.

In fabric covered multi-spar construction, the ribs consist merely of cap-strips with special nose formers and one or two truss members in the tail. If the multi-spar wing were untapered these nose formers could advantageously be made as stampings, but since the majority of multi-spar wings are tapered the nose formers are usually built up from structural sections. Between the nose and the tail the staggered spars of the multi-spar construction act as the rib trussing, therefore with this arrangement the ribs require no trusses of their own. In well-designed, built-up ribs of aluminum-alloy construction, strength weight ratios as high as 800 have been obtained in high incidence loading conditions, and ratios as high as 700 have been obtained in low incidence conditions.

Since torsional flexibility is so detrimental to monoplane design, the drag truss should be designed so as to be strong enough to insure torsional stiffness. In order to obtain a rigid drag truss a load factor of 2 should be used for nose-dive conditions. In cases where the aspect ratio of the wing is exceptionally high, or where the airfoil used is exceptionally thin, design requirements for the drag truss should be even more severe.

In designing the drag truss of the three-spar wing, a simple expedient may be used to prevent redundancy of the drag truss connecting all three spars. This is to assume the two outside spars to be the chords of the drag truss and the middle spar to be placed midway between the front and rear. By this means the lines of action of the drag truss members may pass directly through the center spar without putting any loads on it. At the same time, if properly designed, this method reduces the lateral column action of the middle spar to one-half the drag bay length and so lightens the most heavily loaded spar.

In three-spar construction, the drag truss can usually be built entirely of compression members more efficiently than it may be built if the standard strut and wire type is used. This, in addition to being more efficient because of fewer fittings, is, at the same time, more rigid since the allowable stresses are much lower in compression tubing than in tension wires, thereby requiring a greater area and so reducing deflection of the members. In designing the drag truss, the diagonal drag struts in the bottom truss should run from inner front to outer rear while those in the top truss should run from inner rear to outer front. By diagonal drag struts are meant those struts which replace the cross wires

in the standard type drag truss. If the diagonal struts are placed as mentioned, both tubes will be stressed in tension for torsional washout of the wing tip. In nose dive conditions, therefore, torsional deflection will not be increased due to column action bending.

In designing the spars of a three-spar wing, the best method of proportioning the load between the spars is the Burgess method of distribution. This method is explained in detail in Air Service Information Circular No. 394. This circular takes up several different methods and compares the computed results with the actual test results obtained on a three-spar test wing.

In multi-spar construction, the structure must be analyzed in three component parts instead of two (spars and drag truss), as in the two- or three-spar construction. These component parts are the spar chords, the spar trusses, and the drag truss. Each chord member is the chord of two trusses and it is for this reason that the chord must be analyzed separately from the trusses. There are two methods which may be used in computing the loads carried in the chord members. The first method neglects the center of pressure movement and assumes the resultant of the load to pass through the elastic centrum of the wing. The second method takes account of the center of pressure travel and is the one preferred in this country. The Germans in designing their original multi-spar wings used the first method. Both methods are described in an article on "Multi-Spar Wing-Design and Analysis" published in the February sixteenth issue of Aviation.

In strut-braced monoplane design, the wing, as far as analysis is concerned, is similar to the biplane wing in that the spars in the inner portion or bay are subjected to end loads. This requires the spars to be analyzed as eccentrically loaded columns or columns under initial side load. In biplane design, particularly in the smaller types of airplanes, the spars are constant in section and the "precise equation" may be used in computing the ultimate stresses, but in strut-braced monoplanes, or very large biplanes, the spars must be of varying section in order to save weight. This prevents the use of the precise equation and requires some other method of analysis to be used. There are several approximate methods mentioned in various text books but the majority of them deal only with constant-section spars. The following is a method which the author developed for analyzing beams and trusses of varying section. It is what might be termed a graphical solution of the precise equation. Its application to beams with a continuous web is quite simple. Its application to trusses is simple but rather tedious. The following is an outline of its application to the continuous-web beam. The primary moments (moments due to side load only) are computed for the beam. To these are added, the corresponding eccentric moments due to end load, if such eccentric moments exist. At any section, the eccentric moment of the end load is equal to the distance between the point of application of the end load and the neutral axis of the section multiplied by the end load itself. (See Fig. 2.)

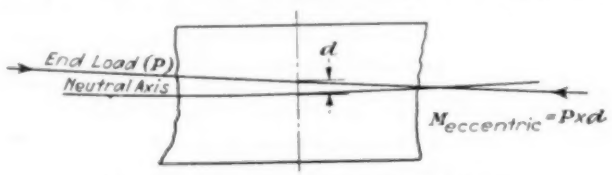


Fig. 2 Eccentric Moment of End Load

The resultant moment (primary plus eccentric) is applied to the beam and the deflection of the beam under this moment is found by the method of $\frac{M}{I}$. This deflection is the first approximate deflection and the first term of a series of which the ultimate deflection is the limit. Multiplying these deflections by the end load gives the first approximate secondary bending moment. If the deflection under this bending moment alone is computed by the same method the resulting curve will be the second term of the series, and if the ordinates of this deflection curve are multiplied by the end load, the result will be the second approximate secondary bending moment. Generally, the shape of the primary-moment curve will be entirely different from the shape of the first approximate secondary bending-moment curve.

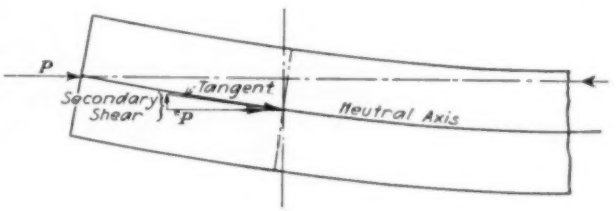


Fig. 3 Illustration of Secondary Shear

Due to beam continuity, the shape of the deflection curves will more and more closely approximate each other as each successive increment is solved. Usually it will be found that the first and second approximate deflection curves have practically the same equation, or are similar, therefore the ultimate deflection may be computed by calculating the limit of the geometric series of which these curves are the first two terms. The use of the geometric limit is reasonable, since the end load is a constant. As can readily be seen, this method is the solution of the precise equation by successive increments. In solving a truss, the same method can be applied except that the Williot diagram is used, in place of the $\frac{M}{I}$ curve, for solving the deflections.

In both types of beams, and particularly the truss type, some account should be taken of the secondary shear as this shear may in many cases be practically as great as the normal shear. By secondary shear is meant the shear component of the end load on the deflected beam. When the beam deflects, the neutral axis is given a slope and as the end load does not change in direction it must be divided into two components, parallel and perpendicular to the deflected neutral axis. The perpendicular component is the secondary shear. (See Fig. 3.) To compute the secondary shear at any section, the sine of the angle whose tangent is the slope of the deflection curve at the section should be multiplied by the end load. In designing a trussed beam, allowances for the secondary shear should be made in selecting the original tube sizes. Then, after the deflection curves have been computed, the shear truss should be checked to determine if any of the shear parts are unsafe.

From the standpoint of manufacture, the cheapest type of construction would be one in which the structure was composed entirely of flat sheets and structural shapes such as channels and angles. There is a limit to which this type of construction may be used since the flat, unsupported surfaces and free edges which form an appreciable part of the structure have a tendency to buckle at low stress values. This type of construction would require the use of very low form factors and so would produce a low strength-weight ratio. The use of curved sheet prevents the tendency to buckle and allows much higher form factors or stress values to be used. However, this type of construction requires the use of dies or rolls in the fabrication of the material, and the extent to which this die cost is warranted, depends entirely upon the anticipated production. With either flat or curved sheet, free edges are especially detrimental since they

have a tendency to buckle and so reduce the form factor of the section. There are two methods by which this effect may be overcome. The first is to place the free edges on or near the neutral axis of the piece and by this means reduce the stress to a safe value. The second method is to make the ratio of overhang length to thickness of all free edges as low as possible. By this means the material furnishes its own support and allows the use of higher-stress values. No direct formula can be given by which the manufacturer may determine the most efficient design for his particular case, as it would depend on the shop facilities, the anticipated production, and the location of the sources of supply of material.

AER-51-11

Radio Developments Applied to Aircraft

By J. H. DELLINGER[1] and H. DIAMOND,[1] WASHINGTON, D. C.

In this paper, the authors point out the need for equipment and methods, that will reduce the weather hazards of air transportation. Radio offers great possibilities as an aid to flight. Research work carried on by the Bureau of Standards has lead to developments, through which weather and landing conditions can be communicated to pilots while in flight and through course navigation, flying can be done satisfactorily regardless of fog. Many interesting applications of radio to aircraft are described.

WITH the transportation of air mail, express and passengers constituting its basic source of revenue, the success of air transportation in the United States depends in large measure upon rigorous maintenance of flying schedules by day and night. Means are at hand to cope with every limitation on flight except hazards of weather. Multiple engines assure safety, landing fields are being provided in abundance, aircraft of adequate strength and stability are available, every comfort and convenience are offered the air traveler. And yet air traffic is still halted when meteorological conditions make the pilot uncertain that he can see landmarks or landing field.

Limitations on regular commercial flight imposed by weather are, however, being overcome by the aids to air navigation being provided on the civil airways by the Department of Commerce. These aids include the lighting of the airways for night flying, special weather reporting service, and radio aids.

Possibilities of radio as an aid to flight include: (1) Communication, (2) course navigation and (3) fog landing. The most essential of these is considered by pilots to be the first, communication. The pilot is interested in receiving information while in the air on weather and landing conditions at possible destinations. A successful system of course navigation, which will permit the pilot to fly his route regardless of fog, is the second element in the conquest of weather hazards. The use of radio to enable a pilot to land in fog is not yet practicable, but promising developments are under way.

Communication

Communication between aircraft and ground is the simplest and most obvious application of radio as a navigational aid. In times of storm and low visibility the pilot can, by radio communication, be in touch with conditions along his route and learn where a landing can be made most safely. A number of problems at once arise. Shall the communication be by telegraphy or telephony? For the conditions of the U. S. Airways the answer is telephony. In some services, military, for instance, radio telegraphy is more suitable, since communication over great distances must be provided for and suitable radio-telephone equipment would be too bulky and heavy. On the U. S. Airways, however, the greater simplicity of radio telephone operation, the short distances, and the impracticability of carrying on the communications both on the aircraft and on the ground through code operators, leave no choice.

Another question is whether one-way or two-way telephony is required. Both have their place and will be used. Two-way telephony between plane and ground, and between plane and plane, permits individual handling of the problems of each pilot; this is secured at the cost of much heavier equipment and more cumbersome antenna than one-way telephony from ground to plane. The radio telephone service provided by the Government is one-way telephony from ground stations. It may be utilized by all airplanes on the airway by use of only a very simple receiving set weighing a few pounds.

Another problem of a technical-radio character is the choice of frequencies for these services. This is being gradually worked out. The beacon and telephone broadcast service are on frequencies between 285 and 350 kilocycles, and the tendency is for the two-way telephone service to be handled on frequencies between 1600 and 6500 kilocycles.

Low-Frequency Telephone Broadcast Service—A definite system of weather reports and radio-telephone communications to aircraft has been inaugurated by the Department of Commerce. Before departing, the pilot is advised as to weather conditions at his destination. After departure, the pilot is advised by radio telephone broadcasts, at least once an hour, regarding landing and weather conditions along his route, and, if landing at the chosen destination appears hazardous, he is told of alternate landing fields where a safe landing may be made. Pertinent information, such as ceiling heights, barometric pressure at the ground, elevation of the landing field, arrangements made for transportation of mail, express and passengers, and orders from the transport company, are given the pilot enroute.

With a view to making this information most effective, the Department has begun the use of an automatic telegraph typewriter system for instantaneous communication between the radio stations at landing fields along the route, weather stations, and operations offices. Timely and accurate information is thus made available to be broadcast to the pilots.

Three radio-telephone weather-broadcast stations have been installed and twelve more are under construction on the transcontinental air route. By June 30 it was hoped to have thirty of these stations in operation on the airways. The frequency band, 315 to 350 kilocycles has been allocated to this service.

High-Frequency, Two-Way Telephony—While the above system provides for the broadcasting of such information as is necessary to the safe operation of aircraft, it does not provide the complete facilities for the dispatching of aircraft, essential to the most efficient operation of a transportation service. The air-transport operators are themselves planning to satisfy this need by the establishment of high-frequency radio-telephone two-way communication between aircraft and ground. Sixty-four channels have been reserved for this purpose by the Federal Radio Commission in the 1600–6500 kilocycle band. Radio-telephone transmitting stations on the assigned frequencies will be constructed at the principal airports of certain airways, and the airplanes will be equipped with a high-frequency receiving and transmitting set in addition to the low-frequency receiving set required in connection with the government-operated radio aids. The operations office of each route will maintain a watch on the frequency or frequencies assigned to its aircraft, while the Commerce Department will also maintain a watch on a standard calling frequency. A frequency in the neighborhood of 4000 kilocycles will be used for this purpose. This constitutes a service which admirably supplements the telephone broadcast service provided by the Government on 315–350 kilocycles, and should measurably increase the safety of operations.

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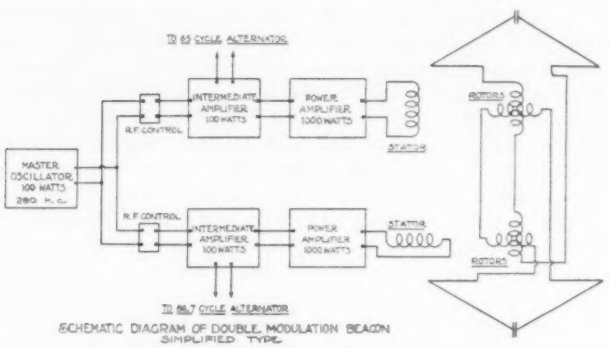
[1] Bureau of Standards, Department of Commerce, Washington, D. C.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

Suppose, for example, that the weather information is being broadcast from a low-frequency station, and an airplane gets into difficulties and communicates on its assigned high frequency requesting specific information. The air-transport station will receive that message and will reply on its assigned high frequency, without interrupting the weather broadcast which may be absolutely essential to other airplanes in the air. In the event that the air-transport station does not receive the message, the pilot may communicate on the standard high-frequency calling wave, on which the Department of Commerce maintains a watch. This message will then be communicated over the telegraph-typewriter system to the proper operations office for reply; or, if particularly urgent, the weather broadcast on the low frequency may be interrupted or extended in order to transmit the desired information.

Furthermore, in the case of a breakdown of the low-frequency station, the high-frequency station may be employed for disseminating the weather information. Finally, the two-way communication between aircraft and ground may well be used as an auxiliary to landing operations under adverse weather conditions.



SCHEMATIC DIAGRAM OF DOUBLE MODULATION BEACON SIMPLIFIED TYPE

FIG. 1 SCHEMATIC DIAGRAM OF DOUBLE MODULATION BEACON—SIMPLIFIED TYPE

COURSE NAVIGATION ON FIXED AIRWAYS

Requirements.—Consideration of the relative usefulness of possible radio systems for guiding aircraft divides naturally into two parts: (1) aids for aircraft flying the established airways; (2) aids for aircraft flying over independent routes. The first is the more important in the United States. All commercial-transport airplanes use fixed airways. The Government's aids to air navigation are being provided with the primary view of serving aircraft flying these airways.

The fundamental requirement of a radio system for guiding airplanes traveling the fixed airways is, that it shall give the pilot information to enable him to continue along a given airway, when no landmarks or sky are visible. If he leaves the course, it should tell him how far off he is, and to which side, should show him the way back to the course, and should inform him when he arrives at his destination. Guidance along the airway means guidance along the route regularly flown, with its emergency fields, lights, and other facilities. Thus the airways guidance system is sharply marked off from direction finding and other systems which are mere homing devices, and with which the pilot can unwittingly be driven off the desired course by side winds. The guidance system must be entirely free from errors of wind drift, since one of its purposes is to prevent collisions, both between airplanes and with the sides of mountain passes, etc. The system must provide service to all airplanes flying the course, and must be adaptable to the complex conditions on the busiest airways.

In addition to these requirements, there are a number of additional desiderata which are not strict necessities. The identity of the portion of the airway traversed should be indicated by the radio signals utilized. The service should be continuous rather than intermittent. The pilot should receive the service by a mere glance at an instrument, being free from any necessity of using ear phones, or adjusting anything, or correlating with other instruments, or changing the course of the airplane. The radio frequencies, power, type of emission, and location of transmitting stations should be so chosen as to serve the needs with maximum efficiency and conservation of the limited radio channels. The radio equipment on the airplane should be simple and inexpensive.

All of these requirements and desiderata are fulfilled by the radio-beacon system adopted. This system has been worked out to be applicable under the actual flying conditions encountered on the U. S. airways.

Simplified Visual Radio Beacon—The radio-beacon system employs a special transmitting station on the ground, known as a radio beacon. This is usually located at an airport, just off the landing field. Instead of having a single antenna like an ordinary radio station, it has two loop antennas at right angles with each other. Each of these emits a set of waves which is directive, i.e., it is stronger in one direction than others. When an airplane flies along the line, exactly equidistant from the two beams of radio waves, it receives signals of equal intensity from the two. If the airplane gets off this line, it receives a stronger signal from one than the other.

The current in the two antennas is of exactly the same radio frequency, but is modulated at a different low frequency in each, i.e., the current in one antenna has a tone of 65 cycles impressed on it, and the current in the other antenna has a tone of 86.7 cycles impressed on it.

On the airplane, a visual-indicating instrument is used, consisting of two vibrating steel reeds, mechanically tuned to the two beacon modulation frequencies (65 and 86.7 cycles). This instrument is mounted on the instrument board of the airplane and is connected to the output of the receiving set. When the beacon signals are received, the two reeds vibrate, and, since they are tuned to the two modulation frequencies used at the beacon, serve as a device for indicating equality of received signals from the two antennas. The tips of the reeds are white against a dark background, so that when vibrating they appear as vertical white lines. When the two lines are equal in length the airplane is on its course. A deviation from the course to the left serves to increase the relative deflection of the left reed, and the reverse is true if the airplane deviates to the right. To return to the course, the pilot turns in the direction of the shorter reed deflection.

The radio beacon used in this system is known as the double-modulation beacon. A detailed description was published a year ago.[2] The transmitting apparatus has since been considerably simplified. Fig. 1 is a schematic circuit diagram of the beacon-transmitting station in its simplified form. A 100-watt master oscillator supplies power at 290 kilocycles to two 100-watt intermediate amplifiers which in turn supply power to two 1000-watt power amplifiers. These feed separately into two loop antennas crossed at an angle of 90 deg. to each other. Each intermediate amplifier is modulated to a selected low frequency, 65 cycles being chosen as one modulation frequency and 86.7 cycles as the second. One loop antenna therefore radiates a 290-kilocycle wave modulated to 65 cycles while the other emits a 290-kilocycle wave modulated to 86.7 cycles. Owing to the directive properties of the loop antennas, the intensity of the waves is a maximum in the directions of the planes of the antennas, transmitting them, and zero in the directions perpendicular

[2] Development of radio aids to air navigation. J. H. Dellinger and H. Pratt. *Proc. I.R.E.*, 16, p. 890; July, 1928.

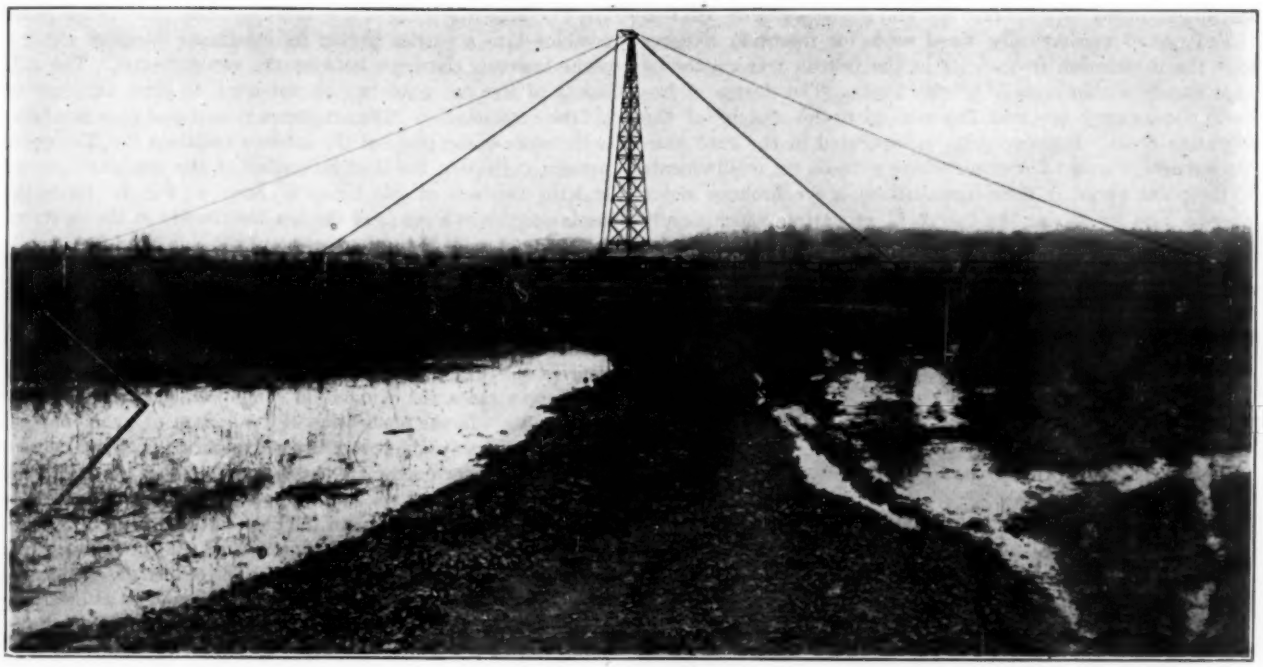


FIG. 2 CROSSED LOOP ANTENNA SYSTEM USED WITH DIRECTIVE RADIO BEACON. EXPERIMENTAL STATION OF BUREAU OF STANDARDS AT COLLEGE PARK, MD.

to these planes. Equality of received signal thus indicates a fixed line or "course" in space, and provides a means for guiding aircraft along that course. For the purpose of orienting the course in any given direction, a four-coil goniometer, consisting of two stator coils crossed at an angle of 90 deg. with each other and two rotor coils also crossed at 90 deg., are employed. The stator coils are connected to the plates of the power amplifiers, while the rotor coils are connected, each in series with one antenna. The currents in the antennas due to the driving voltage of each stator coil, create a resultant field in a direction such as would be produced by an imaginary or phantom antenna, rotatable with the goniometer rotor. Since there are two stator coils, there are two such phantom antennas rotatable as the two rotor coils rotate together, allowing the equisignal zone or "course" in space to be oriented in any direction.

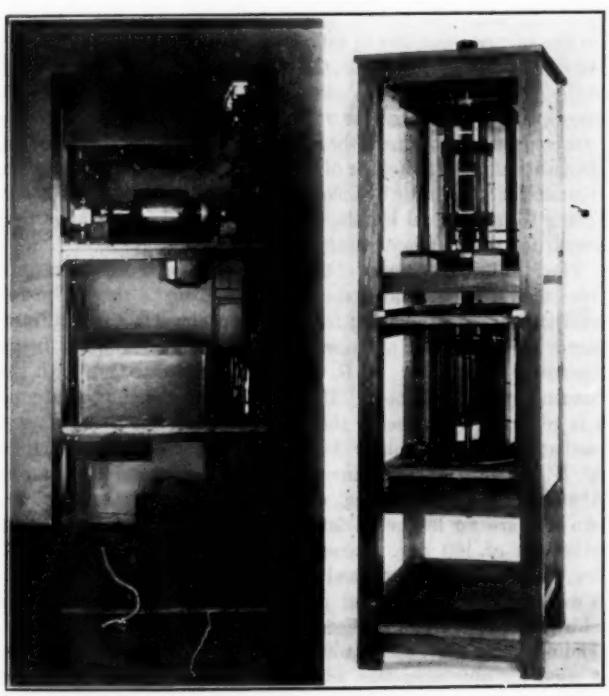


FIG. 3 TRANSMITTING SET FOR DOUBLE-MODULATION BEACON AS USED WITH THE SIMPLIFIED SYSTEM

FIG. 4 GONIOMETER USED IN BEACON STATION TO ORIENT BEACON COURSE IN ANY DESIRED DIRECTION

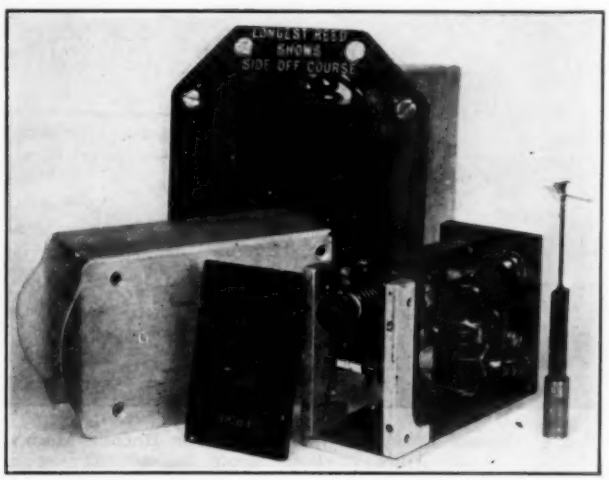


FIG. 5 VISUAL BEACON COURSE INDICATOR

A photograph of the crossed-coil antenna system is shown in Fig. 2. The beacon-transmitting equipment is shown in Fig. 3 and the goniometer used for orienting the course in Fig. 4. A photograph of the vibrating-reed course indicator showing the working parts and also the shockproof holder for mounting on the airplane is given in Fig. 5. Fig. 6 shows an airplane

instrument board with the reed box and holder suitably located.

The use of mechanically tuned reeds for reception requires that the modulation frequencies in the beacon transmitter be kept steady within certain definite limits. The degree of frequency constancy required depends upon the design[3] of the vibrating reeds. Improvements incorporated in the reed indicator during the past few months have reduced the requirements to the point where, in many installations, a synchronous motor may be used for driving the two A. C. generators which supply the modulation frequencies. At locations where the variation of

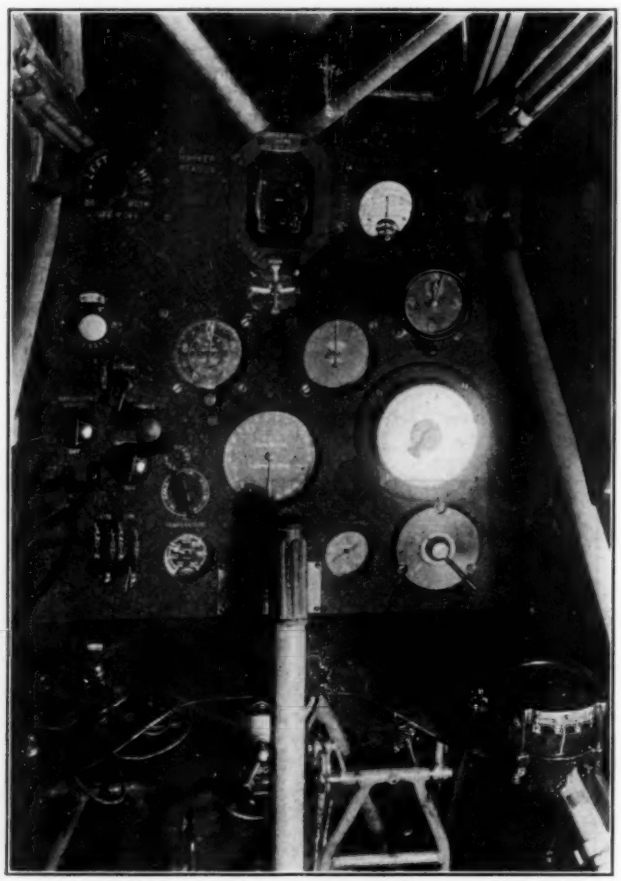


FIG. 6 COURSE INDICATOR AND SHOCKPROOF HOLDER MOUNTED ON AIRPLANE INSTRUMENT BOARD

the power-supply frequency exceeds 0.2 cycle, it is necessary to employ a motor other than of the synchronous type with provisions for controlling its speed within the desired limit of accuracy. A constant-speed unit having the requisite characteristics has been in satisfactory operation at College Park for the past few months. The speed of this unit is controlled by means of a vacuum tube driven tuning fork.

Adaptation of Beacon to Four Independent Courses—It is desirable, at this point, to study the polar pattern of the field radiated by the double-modulation beacon described. One loop antenna radiates a 290-kilocycle wave modulated to 65 cycles, while the other loop antenna radiates a 290-kilocycle wave modulated to 86.7 cycles. The wave due to each antenna, may be resolved into a carrier and two side bands. The carriers in the two loop antennas being of the same frequency and in time phase combine into a carrier having its maximum intensity along a plane bisecting the angle between the two antennas. The side bands of the two antennas do not combine since they are of different frequencies. The maximum intensity of each side band is therefore in the plane of the antenna radiating it. The space pattern indicating the field intensities of the combined carrier and the two sets of side bands is shown in Fig. 7. Since the reeds operate as a result of the low frequencies in the receiving set output produced by the beating of the side band frequencies with the carrier, they respond to a space pattern characteristic as indicated in Fig. 8. This assumes a square law detector. It will be noted that but two courses are produced, practically no signal being radiated in the directions at right angles to these courses.

In many cases, the elimination of the two right-angle courses is desirable. In the application of the beacon to course navigation on the airways, however, airports requiring only two courses 180 deg. from each other are the exception rather than the rule. At those airports where two courses are sufficient, these are usually at an angle other than 180 deg. At many airports, more than two radiating courses are necessary, the number varying anywhere from 3 to 8 or even more. The beacon described may be easily modified to permit the use of four courses at arbitrary angles. By a suitable coupling arrangement to the master oscillator, one of the carrier-frequency currents can be advanced in phase 90 deg. ahead of the other. Since the two loop antennas are in 90 deg. space phase and their carrier currents in 90 deg. time phase relationship, a revolving field is set up in space, so that the carrier space pattern becomes circular. See Fig. 9. The polar pattern as received by the reeds is that indicated in Fig. 11, four courses being obtained.

A second method, perhaps preferable from a practical standpoint, is to excite one antenna at a time. The carrier waves radiated by the two loop antennas, therefore, cannot combine since they are never sent out simultaneously. The space pattern for the carrier wave due to each antenna is then the same as for the corresponding side bands, the maximum power being radiated in the plane of the antenna considered. See Fig. 10. The polar diagram as received by the reeds is shown in Fig. 12. Again four courses are obtained, these courses being somewhat sharper than above. The switching of the power so that but one antenna is excited at a time may be accomplished electrically at high speed. Owing to mechanical inertia, the reeds maintain their vibration amplitude without interruption or flutter.

The four beacon courses obtained may be shifted considerably from their 90-deg. relationship in order to make them coincide with the airways radiating from a given airport. Two adjustments at the beacon are possible, the first and simplest being illustrated in Fig. 13. A, B, C, and D are the courses with the beacon normally adjusted. If the figure-of-eight due to antenna 1 is reduced, by reducing the voltage supply from the master oscillator to the amplifier branch feeding antenna 1, courses A′, B′, C′ and D′ are obtained. It will be observed that courses A′ and C′ are still at 180 deg., also courses B′ and D′, but that the two sets are no longer at 90 deg. with each other. To shift either set of 180 deg. courses (say B′ and D′) from their 180 deg. relationship, the method illustrated by Fig. 15 is adopted. A non-directional carrier, of proper phase value, is radiated in addition to the ordinary beacon radiation. A vertical wire, extending along the beacon tower and coupled to the beacon set, is used for obtaining this additional radiation. The resultant space pattern is indicated by the heavy lines in Fig. 15. It will be observed that courses B′ and D′ are no longer at 180 deg.

Applying these principles, the Airways Division, in cooperation with the Bureau of Standards, has succeeded in fitting the aural

[3] Design of tuned-reed course indicators for aircraft radio beacon. F. W. Dunmore. *Bureau of Standards Journal of Research*, p. 751; Nov., 1928. Research Paper No. 28.

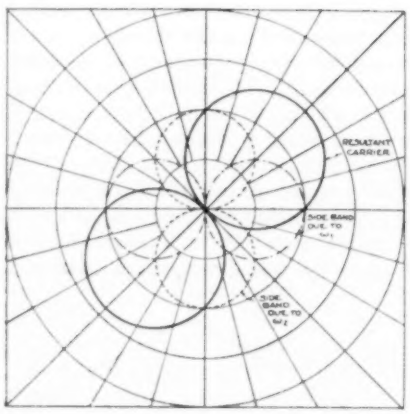


FIG. 7 THEORETICAL SPACE PATTERN RADIATED BY DOUBLE MODULATION BEACON

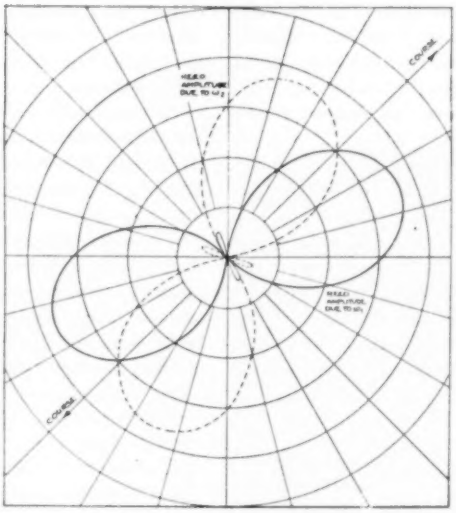


FIG. 8 THEORETICAL RECEIVED POLAR DIAGRAM

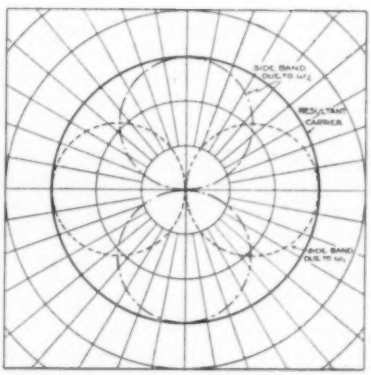


FIG. 9 THEORETICAL SPACE PATTERN WHEN ANTENNA CURRENTS ARE 90° OUT OF PHASE

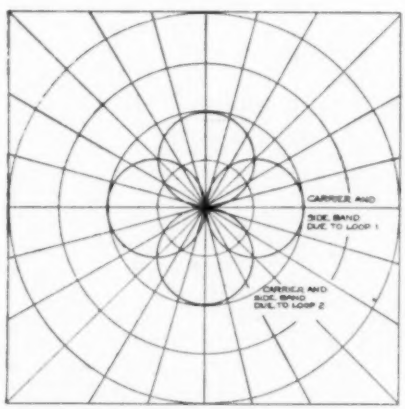


FIG. 10 ALTERNATE SPACE PATTERN FOR OBTAINING FOUR COURSES

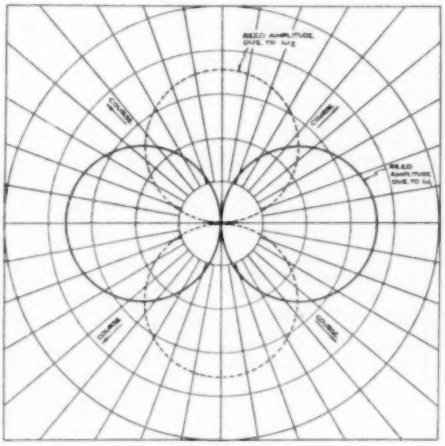


FIG. 11 THEORETICAL RECEIVED POLAR DIAGRAM SHOWING FOUR COURSES

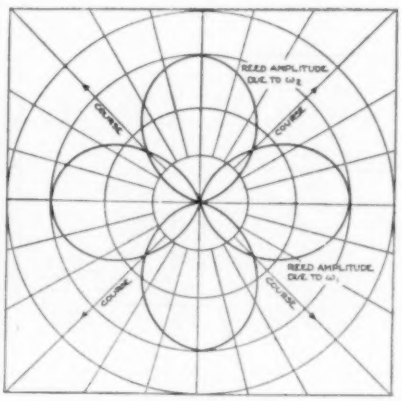


FIG. 12 THEORETICAL RECEIVED POLAR DIAGRAM

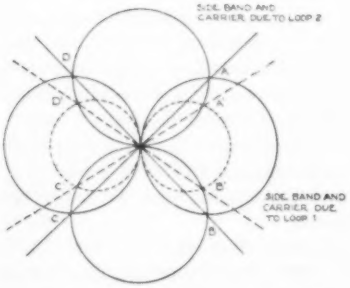


FIG. 13 THEORETICAL SPACE PATTERN FOR SHIFTING TWO OF THE FOUR BEACON COURSES

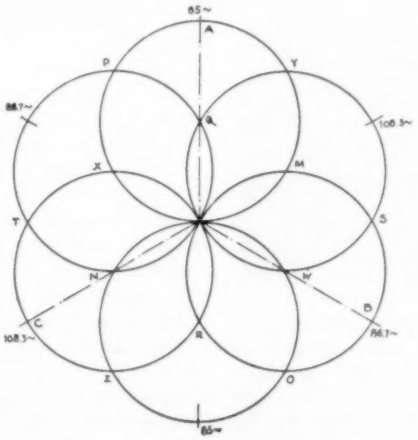


FIG. 14 THEORETICAL SPACE PATTERN RADIATED BY MULTIDIRECTIONAL BEACON

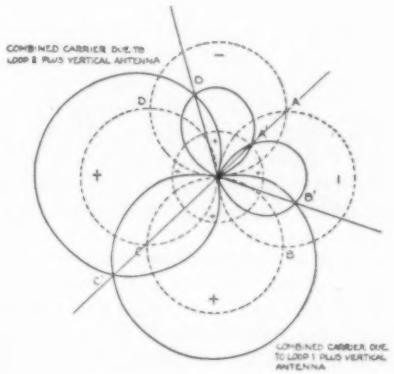


FIG. 15 ALTERNATVE SPACE PATTERN FOR SHIFTING TWO OF THE FOUR BEACON COURSES

beacons at Bellefonte, Pa., and Hadley Field, N. J., to the airways. At Bellefonte, two courses at an angle of 165 deg. are used, while at Hadley the three courses leading to Hartford, Conn., Bellefonte, Pa., and Washington, D. C., are served simultaneously.

With the visual beacon system, shifting the courses at the beacon is not entirely necessary. A slight change in the receiving equipment aboard the airplane makes it possible for the pilot to choose his own course, so long as he is within about 20 deg. on either side of an equisignal zone. This is accomplished by shunting a suitable resistance across the coil actuating one of the two reeds, thereby reducing the sensitivity of that reed. The course, as determined by equality of reed deflections, is therefore shifted from the true equisignal zone in the direction of the shunted reed. The same effect is thus accomplished as by shifting the courses at the beacon. The shunt resistance may be made variable and calibrated in terms of degrees off the true equisignal zone, to give additional flexibility to the system.

Multidirectional Beacon (12 Courses)—To render the beacon system still more flexible and thus make it suitable for use at cities located at the junction of a large number of airways, a new beacon has been developed capable of serving twelve courses simultaneously. The increase in apparatus over the beacon described above is not great. The same crossed-coil antenna system and the same circuit arrangements are employed (see Fig. 16) except that three amplifier branches, modulated to three different frequencies, are necessary. The modulation frequencies used are 65, 86.7 and 108.3 cycles, respectively. A special goniometer is also required. The rotor system of this goniometer is the same as before; three stator coils are, however, required, one stator coil being connected to each power-amplifier tube. The stator coils are disposed at 120 deg. to each other; these angles may be deviated from in any manner to obtain certain desired conditions.

Since the stator coils are not at right angles to each other, it is essential that but one stator be excited at any given time in order that coupling between stators be avoided. A high-speed electrical switching arrangement has been provided in the grid circuits of the intermediate amplifier tubes for accomplishing this purpose.

Referring to Fig. 16, it will be observed that one stator carries a 290-kilocycle current modulated to 65 cycles, the second stator a 290-kilocycle current modulated to 86.7 cycles, and the third stator a 290-kilocycle current modulated to 108.3 cycles. Each stator, acting in conjunction with the two crossed rotor coils and the two crossed loop antennas, sets up a system which is electrically equivalent to a single loop antenna. The plane of this phantom antenna coincides with the plane of the stator coil considered. The space pattern is therefore a figure-of-eight with its axis in the plane of the stator coil. Since but one stator is excited at a given time, the combined space pattern consists of three figures-of-eight with their axes crossed at the same angles as the stator coils. See Fig. 14.

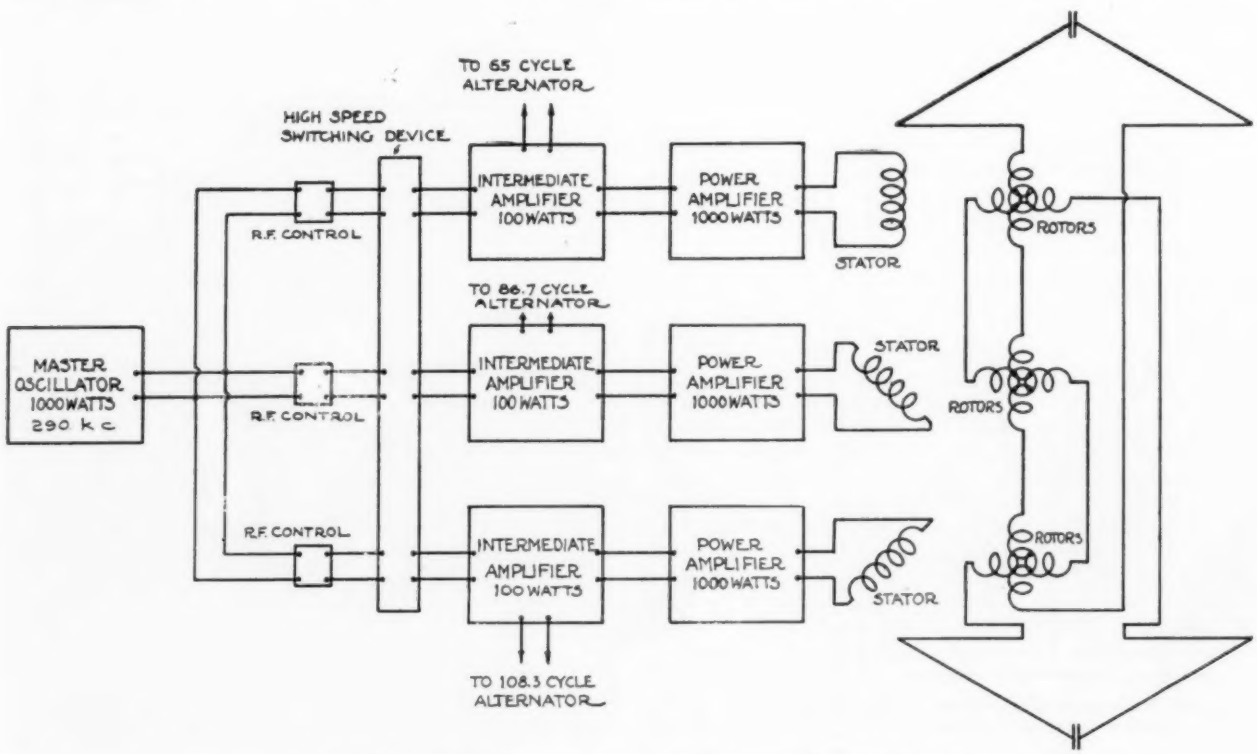


FIG. 16 SCHEMATIC CIRCUIT DIAGRAM OF MULTIDIRECTIONAL BEACON

Now assume that a pilot is equipped with three reed boxes, the first tuned to 65 and 86.7 cycles, the second to 86.7 and 108.3 cycles, and the third to 108.3 and 65 cycles, respectively. Using the first reed box, he will find two sharp courses at M and N and two broad courses at O and P. With the second reed box, two sharp courses will be obtained at Q and R and two broad courses at S and T. Similarly with the third reed box there will be two sharp courses at W and X and two broad courses at Y and Z. The sharp courses have an equisignal zone of 1 to 1.5 deg. while the equisignal zone for the broad courses is from 3 to 4 deg. wide. There are thus twelve useful courses provided, any two adjacent courses being separated by 30 deg.

Some routes, radiating from an airport may, however, be at angles differing from 30 deg. This difficulty is not insurmountable, since the angles between stators may be so adjusted as to fit the courses to those airways.

A 12-course beacon of the type described has been installed at

College Park, Md., and is giving satisfactory results. A photograph of the transmitting set used is shown in Fig. 17 and of the special three-stator goniometer required in Fig. 18. A special course indicator containing three reeds has been designed which makes it possible to receive any of the twelve courses radiated by a beacon of this type without confusion. The dimensions of this indicator are the same as of the two-reed indicator, the two types being therefore interchangeable on the airplane, which is of great convenience.

Marker Beacons—The directive radio beacon will successfully guide a pilot along a given course but does not directly indicate his exact location along that course. Where it is desirable that this indication be given, for example at the intersection of two beacon courses or at a dangerous portion of the airway, it can be obtained by the installation of a small non-directive marker beacon, which is of very low power (a few watts) and emits a characteristic coded signal. This signal operates a 120-cycle reed indicator mounted at a suitable location on the airplane instrument board, and is received for two or three minutes as the airplane passes over the marker. The pilot can thus locate his exact

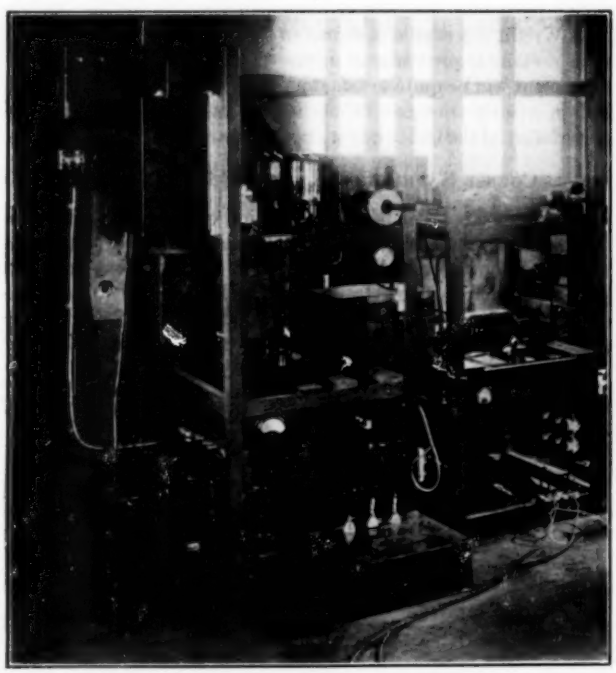


FIG. 17 EXPERIMENTAL TRANSMITTING APPARATUS FOR MULTIDIRECTIONAL BEACON INSTALLED AT COLLEGE PARK, MD.

position on the airway. The coded signal of the marker beacon can have the same characteristics as the light beacon if there is one there.

A photograph of the marker beacon transmitting equipment is given in Fig. 19. When located at the intersection of two main beacon courses, an arrangement must be provided on the marker beacon to permit the emission of its characteristic signal on the two radio frequencies used by the main beacons. A satisfactory switching device for accomplishing this purpose has been developed.

Airplane Receiving Sets—The beacon system can be used with any receiving set which operates at the frequencies employed, merely replacing the telephone receivers by the simple reed indicator unit. There are, however, a number of special conditions involved in receiving on an airplane, and the Bureau has developed special receiving sets[4] in order to use the beacon system under the most advantageous conditions.

The receiving set designed weighs less than 15 lb., the auxiliary batteries weighing an additional 10 lb. The receiving set operates in the frequency range from 285 to 350 kilocycles and is used

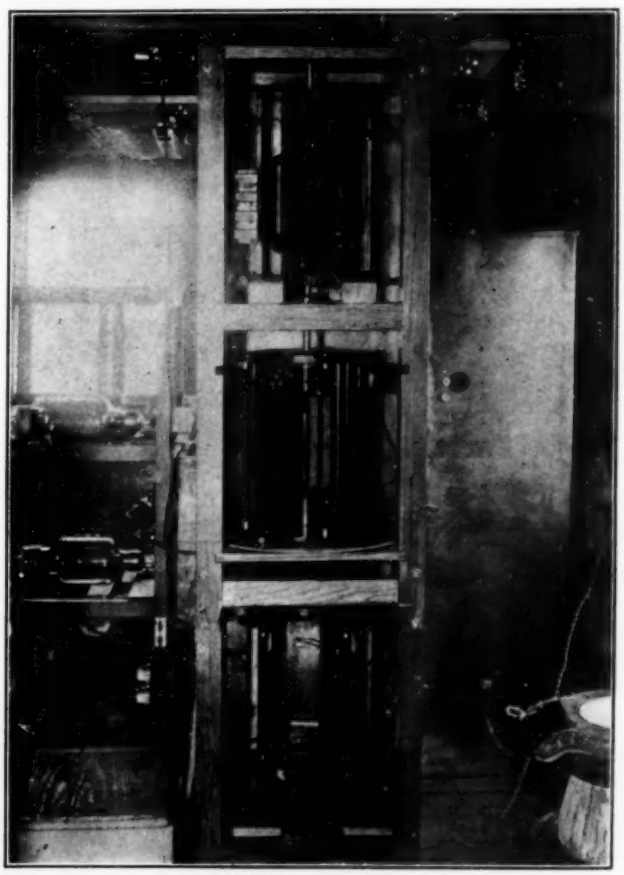


FIG. 18 SPECIAL GONIOMETER USED WITH MULTIDIRECTIONAL BEACON



FIG. 19 MARKER BEACON TRANSMITTING APPARATUS

to receive either the beacon signals or radio-telephone weather messages at will. It is highly selective as well as sensitive and is provided with interstage shielding as well as shielding against

[4] Receiving sets for aircraft beacon and telephony. H. Pratt and H. Diamond. *Bureau of Standards Journal of Research*, 1, p. 543, Oct., 1928. Research Paper No. 19.

direct extraneous interference. The audio-frequency amplifier is specially designed to be efficient at the low modulation frequencies used in the beacon. The selectivity of the set is supplemented by the selectivity of the vibrating reeds, which help greatly in reducing interference. Nevertheless, it is important that a good degree of set selectivity be provided in order to make the most efficient use of the frequency channels provided for this service and also to make the receiving set suitable for the reception of

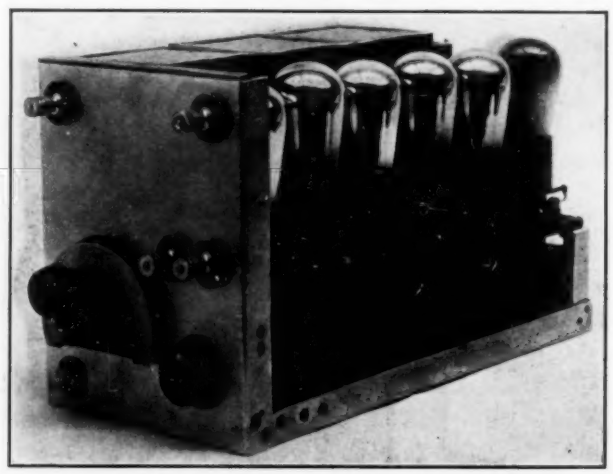


FIG. 20 UNICONTROL AIRCRAFT RECEIVING SET

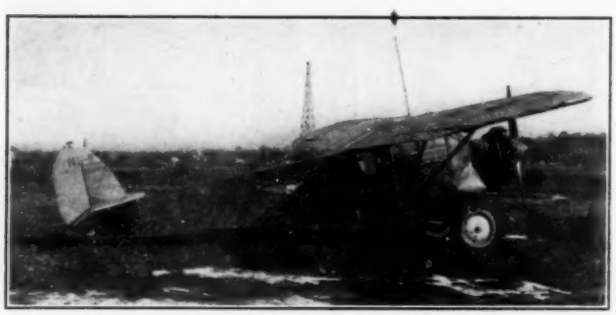


FIG. 21 AIRPLANE SHOWING VERTICAL POLE ANTENNA

weather telephone messages. The set has remote control arrangements for tuning and volume, so that it may be mounted in any location on the airplane. Fig. 20 is a photograph of this receiving set with the outside casing removed.

The Bureau has interested several manufacturing companies in building receiving sets of suitable specifications. One model, already available commercially, has proved very satisfactory; this model, with all auxiliary batteries necessary, weighs approximately 20 lb. Two more sets will become available within a short time. These will incorporate several features not included in the present receivers.

Airplane Antenna—The development of receiving sets having the necessary sensitivity made possible the use of an antenna system on the airplane, consisting of a metal pole extending vertically from the fuselage, the total length of this pole being but 8 ft.

The use of a trailing-wire antenna with its attendant inconvenience and possible danger was thus eliminated. A great advantage of the vertical pole antenna is that it is entirely nondirective. As a consequence, spurious direction variations in the beacon course, which have previously been observed at night, are practically eliminated. In addition, since this type of antenna is not affected by horizontal components of the electric field radiated by the beacon, a region of zero signal strength is met with directly above the beacon tower (where no vertical field exists). This serves quite effectively as a field localizer locating the landing field within 1000 to 2000 ft. A view of the Bureau's experimental airplane showing the vertical pole antenna is given in Fig. 21.

Ignition Shielding—The use of a sensitive receiving set together with the short vertical antenna, located near the airplane engine ignition system, makes necessary a considerably more rigorous shielding of the ignition system than is ordinarily required to prevent ignition interference. Merely shielding the high- and-low tension wires is not sufficient. The spark plugs and the magneto-distributor blocks must also be shielded.

The Bureau has been in active cooperation with airplane engine, magneto, spark plug and cable manufacturers in an effort to develop a safe method for effecting this shielding and to make the necessary equipment available commercially. The problem in ignition shielding is chiefly a mechanical one, it being much more difficult to arrive at an arrangement which will not disturb the airplane-engine ignition system than to obtain complete freedom from ignition interference for the radio equipment on board the airplane.

One particular shielding arrangement suitable for installation on a Wright J-5 engine is shown in Fig. 22. It is believed that with certain improvements in the mechanical design, this arrangement will be quite satisfactory from an operating viewpoint. Full provisions are made for mechanical sturdiness, freedom from disturbance of the ignition system, and water-proofing of all parts.

COURSE NAVIGATION ON INDEPENDENT ROUTES

As may be apparent in the description of the multidirectional beacon, an airplane receives beacon indications at all points and not merely on the established airways. It thus gives guidance service to any one flying to or from the beacon, even though he fly an independent route. There are, however, a variety of other possible radio-navigational systems for the independent flyer.

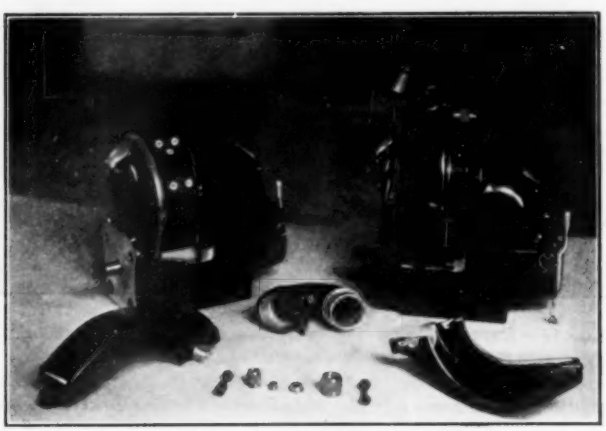


FIG. 22 IGNITION SHIELDING ASSEMBLY FOR WRIGHT J-5 AIRPLANE ENGINE

Direction Finder on Ground—This is the system used on the airways of Europe. Radio direction-finding stations are maintained by the Governments at various airports, and each airplane carries both a transmitting and a receiving set. Upon request by radio from an airplane, two or more of the direction-finding stations determine the direction of travel of radio waves from the airplane; combining their determinations, they calculate the airplane's position and send this information by radio to the airplane. This system has had a little application in the United States during the past year. On the airways, it may receive

some development, in conjunction with the system of high-frequency telephony between airplane and ground, as an alternative or auxiliary to the radio-beacon system.

Direction Finder on Airplane—A second means of radio navigation for the independent flyer is the use of a radio direction finder on the airplane. By steering a course in the indicated direction of a radio station on the ground, the airplane can be certain of reaching that point, the accuracy of the indicated direction increasing as the objective is approached. Direction finders are used extensively as a navigational instrument on marine vessels and on lighter-than-air craft. On airplanes it is considerably more difficult to protect them from error and disturbances caused by the engine ignition and other sources. They have been used successfully to some extent, and their use will doubtless increase; they do, however, require expert handling. This method of navigation has the inherent limitation that it does not prevent wind-drift from shifting the airplane off its course; the method does, eventually, bring the airplane to its destination, although by a circuitous route if there is a side wind.

Rotating Directive Beacon—A third method of furnishing navigational aid to the independent flyer is the rotating radio beacon. This is a radio transmitting station, located at an airport, which has a directive antenna rotating at a constant speed of one revolution per minute. A figure-of-eight pattern is thus rotated in space. A special signal indicates when the figure-of-eight minimum passes through north and also when it passes through east. A pilot listening for the beacon signal with his receiving set can start a stop watch when the north signal is received and stop it when the figure-of-eight minimum reaches him. The number of seconds multiplied by 6 gives him his true direction in degrees from north. The stop watch may be calibrated directly in degrees, so that the position of the second hand when the minimum signal is received gives the bearing directly. This system thus serves any course within its range. It has, however, the disadvantages of slowness of operation and also of being difficult to use during conditions of severe atmospheric disturbances or interference from outside sources, besides the common disadvantage of all these methods, the use of headphones.

Fog Landing

By the use of the radio aids being installed on the U. S. airways, it will become entirely feasible to fly between any two points along a given airway and to arrive within a few thousand feet of the desired destination regardless of weather conditions. The problem of effecting a safe landing during fog and extremely low visibility is not yet, however, an assured accomplishment.

Recent tests indicate that fog-penetrating lights are not yet available. The dissipation of fog by mechanical means has been the subject of considerable laboratory experimentation, the general result being that it is not economically possible on a useful scale.

The most feasible method of attack at the present time appears to be in the application of radio. When attacked from this angle, the problem of fog landing resolves itself into two separate problems; namely, field localizing, and the development of a suitable altimeter.

Field Localizers—Practically all previous experimentation in this field has been in the use of "leader cables." The principal installations are those of the British Government at Farnborough and of the French Government at Chartres. The British installation uses a complete circuit around the landing field with a visual indicator in the airplane instrument board. The French installation uses straight cables. The Loth Company of Paris has also conducted experimental work using straight cables.

The ground equipment at Farnborough consists of an oval loop of cable about 5 × 2 miles, buried 2 ft. in the ground. One of the straight sides of this oval loop passes through the airdrome. Near this straight side and surrounding the portion of the airdrome free from obstructions is a second smaller oval loop. A pilot circles around the major loop coming lower and lower.

Once each revolution, he receives a signal from the smaller loop indicating that he is directly over the airdrome. When he is at a sufficiently low altitude and over the airdrome a safe landing may be effected. An obvious disadvantage of the leader cable method of field localizing is its great cost.

The success obtained with the directive radio beacon in its application to point-to-point flying, suggests its possibilities as a field localizer. With the main beacon used for guiding an airplane to a given landing field, a low-power beacon with small loop antennas may be employed for marking out the major, or most desirable axis, or runway of the landing field. This can be done

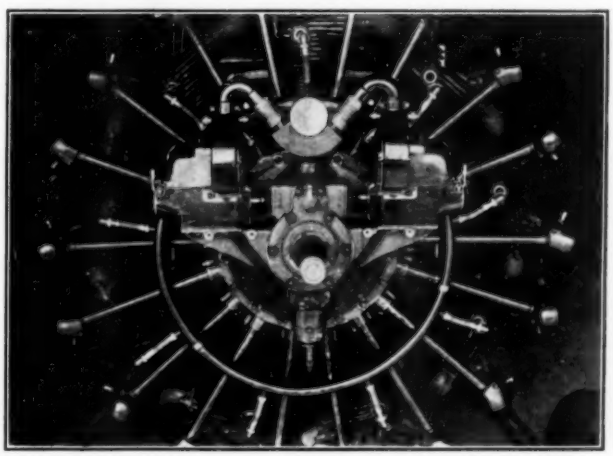


Fig. 23 Engine With Shielding Equipment

quite as effectively as with leader cables. For example, consider a class A field having the required dimensions of 2500 × 2500 ft. with two perpendicular runways at least 300 ft. wide. The equisignal zone of the two-course radio beacon is at a maximum 5 deg. wide. (The use of the two-course beacon is here assumed because of the obvious advantages of eliminating the 90 deg. courses.) If, then, a course is oriented along a particular runway, with the beacon at one end of the field, a pilot can locate the axis of this runway at the other end of the field within 100 ft. He is thus certain to land on the runway.

The complete details for effecting a safe landing have not yet been worked out, but several arrangements appear practicable. An installation is now being made at College Park, Md., to test their practicability.

Altimeters—A desirable adjunct to any method of field localizing is the development of an instrument to show accurately the height of an airplane above the ground, allowing an error of but a few feet. The barometric instruments now in general use are not accurate enough for the purpose, particularly since they measure height above sea level and not height above the earth.

Experiments are being undertaken by a number of investigators looking toward the development by several means of suitable sensitive altimeters. One is the development of a sonic altimeter. This is an instrument on the airplane, which measures the time taken by a sound wave emitted from the airplane to reach the ground and return to the airplane by reflection. Knowing the velocity of sound, the height of the airplane above the ground is determined.

Another device is the capacity altimeter which measures the distance from the ground by detecting the change in the elec-

trical capacity between two plates on the airplane, as the airplane approaches the ground. A definite limit to the possibilities of this type of instrument arises from the fact that the change in capacity becomes infinitesimal at altitudes exceeding 100 ft.

A third method is by the use of direct reflection with radio waves. The most recent work on this type of altimeter is reported by Dr. E. F. W. Alexanderson.[5] With the development of an accurate altimeter and field localizing system the last obstacle in the battle with fog will be overcome.

Recent Demonstrations

The practicability of the visual beacon system described may be illustrated by several typical tests.

1 On a day of low visibility, a pilot, unfamiliar with the route, took the air in Philadelphia for Washington with no maps or instructions as to landmarks; he was told to proceed to Washington (a distance of 120 miles) and land at College Park Field solely in accordance with the guidance given by the beacon indicator on his instrument board. He not only flew in a straight line to Washington, but when over College Park field, the location of which he did not know, the sudden drop in the reed deflections told him he was at his journey's end, whereupon he landed.

2 The remarkable selectivity of the reed indicator has been demonstrated in several test flights from College Park, Md. to Hadley Field, N. J., with the double-modulation type College Park beacon and the aural-type Hadley beacon, both operating on the same radio frequency and with approximately the same power. With the particular type of receiving set employed, the interference of the Hadley-Field beacon does not interfere with the operation of the reeds until the airplane is approximately 140 miles from College Park and only 50 miles from Hadley. This is an interference-to-signal ratio of about 8 to 1.

3 An additional test also demonstrating the degree of freedom from interference of the reed indicator was made on an unshielded airplane at Hadley Field, using a different type receiving set. The College-Park beacon signals were used, the reeds functioning perfectly at all engine speeds, even though the ignition interference present was sufficient to render a signal from a nearby radiotelephone set unintelligible.

[5] *Radio Engineering*, pp. 34–35, Feb., 1929.

AER-51-12

Airplane Fuel and Lubricants

By C. K. FRANCIS,[1] TULSA, OKLAHOMA

GASOLINE is the product which distils from petroleum at the lower temperatures. The desirable product of the refiner in the early days of the industry was kerosene, separated from the crude after the gasoline fraction at a higher temperature. It was necessary to set the gasoline produced aside; a very inflammable, troublesome product of no commercial value. Its combustible nature and the fact that it would ignite at normal temperatures, furnished characteristics which enabled the operator to identify it. The inflammable property was the characteristic which formed the basis for the first specification for gasoline.

The methods which were devised for the determination of the temperatures at which the illuminating oil would flash and fire were employed for the purpose of showing when a separation of the gasoline from the kerosene had been made. Some refiners did not succeed in getting all of the gasoline out of the kerosene, and the volatile nature of the gasoline caused many explosions of lamps. Numerous laws were enacted stipulating the minimum flashing temperature of commercial illuminating oils.

For many years, the petroleum refiner continued to find gasoline a source of annoyance. It was the readily combustible nature of gasoline which encouraged the original students of the internal-combustion type. With the perfection of the automobile engine, which functioned because of the explosive mixture of gasoline and air, the foundation was laid for a market which was to consume more of this troublesome product than any refiner could have imagined would be possible.

Then it was found that gravity could be used to classify gasolines, and it was thought that the higher the gravity, the better the gasoline. This explains the origin of the term "high test." Since that day, progress has been rapid. Distillation has served to indicate volatility. Sulphur, gum, and corrosive specifications have appeared. A recent development has shown that many gasolines cause knocking in the motor, consequently, there is now an endeavor to classify gasolines for this property. It appears that a condition is rapidly approaching where the chemical composition of gasolines must be known to understand their real worth.

The value of gasoline consists in its performance as a motor fuel, but our physical and chemical methods of investigation have not advanced far enough to enable us to predetermine with certainty just how a specific gasoline will function in a motor. If we are to have complete appreciation of the characteristics of a gasoline, more definite knowledge concerning the influence of molecular structure will have to be known together with the permissible limits, which may be established by means of physical tests.

There are numerous physical tests which are used in determining the characteristics of gasoline. A few chemical tests are required. The gravity, color, and odor are determined; the quantity of gum and sulphur, antiknock rating, and volatility range comply with standard specifications. Water must be absent from aeroplane gasoline.

There appears to be considerable difference in the consumption of gasoline by aeroplanes. A good performance is reported to be 0.5 lb. per hp-hr. A single motor developing 140 hp. at maximum speed would ordinarily require about 60 lb. per hr. Single motor planes use around one gallon to each 10 to 12 miles. The large ships, with three motors which carry up to 16 persons, and travel at the rate of 100 m.p.h., consume approximately 60 gal. an hour.

The problems surrounding the development of the aero engine have included similar ones to those encountered in the perfection of the automobile engine. These have been faulty lubrication and poor lubricants. The short life of many of the first ships was due to poor mechanical conditions for handling the lubricating oil. In the early types of rotary engines, the inlet valve for the petroleum-air mixture was in the center of the piston, and this valve soon became choked with deposits. In one engine, there was only one piston ring which, aside from seldom fitting, furnished a straight opening where the two ends were supposed to meet. This part of the ring was soon decomposed by the action of the slightest trace of acid in the oil used for lubrication. As soon as the corners of the ring became rounded, the ring no longer prevented the explosion gases from passing the piston, and the cylinder became overheated.

When the lubricating oil is fed to the oil pump through an elevated tank, it is necessary to use oil with a very low cold test, and to preheat the oil before a flight; this is especially advisable during the cold months.

The great majority of aero engines are now of the radial type. The oil is kept warm in the engine and circulates under pressure, consequently pure mineral oils may usually function with entire satisfaction. Some authorities advise the addition of about 10 per cent of fixed oil, such as neatsfoot oil, to reduce the danger of seizure. Castor oil can also be used in such mixtures, but only in the presence of some animal oil. An admixture of 5 per cent castor oil and 5 per cent lard or neatsfoot makes an excellent mixture. If more castor oil is desired, as much as 20 per cent can be mixed with 74 per cent paraffin free mineral oil, and 6 per cent lard oil.

The early hours, that is the breaking in period, of a new aero motor constitute the period when such oils as castor oil and castor blends are important because of the tendency of the aluminum pistons to cut and freeze owing to the initial growth of the aluminum. When the engine is running at maximum power, the piston clearance is reduced to proper working value, but when the engines are running at reduced speed, the lower temperature of the piston brings about larger piston clearances, which explains why under these conditions aero engines may have a high oil consumption.

Excessive oil consumption follows excessive oil pressure as illustrated by the following:

Oil pressure lb. per sq. in.	Oil consumption, gals. per hr.
70	1.5
50	0.9
30	0.5

Airplane motors must have superior quality oil which has been thoroughly refined. An oil that is free-flowing, at the same time maintains its body or viscosity under high pressure, high temperature, changes in atmospheric conditions, etc. Airplane motors are operated much nearer their maximum speed than automobile motors. Paraffin-base oils have proved to be much better lubricants for airplane motors than asphalt-base oils on account of the ability to resist heat.

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[1] Chief Technologist, Skelly Oil Co., Tulsa, Oklahoma.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and are not those of the Society.

Following are representative specifications:

Oil	Grav. A.P.I.	Flash F°	Fire F°	Pour point F°	Viscosity at 100° F	Viscosity at 210° F	Carbon
1	23–24	460–470	535–545	20–25		115–125	0.968
2	23.6	520	600	24	1408	114	1.67
3	23.7	490	570	0		125	1.92

Castor oil is insoluble in gasoline, and the fact that it does not decompose under high pressure and temperature makes it a good lubricant for certain types of motors. The fact that it is soluble when compounded with some other substances has caused large quantities to be used in the preparation of artificial leather, in the manufacture of linoleum, in the dye industry, and for the fabrication of rubber goods. Some high-grade typewriter inks contain castor oil. It is used in the varnish industry, and for making flypaper.

Prior to the war, India furnished 95 per cent of the world's castor-bean supply; with the rise in the economic position of the bean, however, this figure is gradually declining. While the eastern hemisphere looks to India and China to support its laurels, Brazil cultivates the largest acreage in the western hemisphere. It is ready to increase its production as needed. The crop grown in the Argentine is estimated at between 7000 and 10,000 acres, and in addition to these the Central American countries, including Mexico, are well adapted for cultivation of the bean. So well are they adapted, in fact, that it is said they could supply the United States' need without difficulty, should anything happen to the Indian crop.

The very high rating of castor oil as a lubricant is quite widely recognized. It is used for the lubrication of bearings, especially of the heavy-duty type, and is extensively used for lubricating the rotary-airplane engines to which service it is especially adapted. The chief objection to the use of castor oil as a lubricant appears to be its tendency to form sticky and gummy substances through oxidation. When decomposition sets in acids are produced. These facts are the cause of some corrosion and the sticking of connecting rods, etc.

Castor oil has a Saybolt viscosity over 1000 at 100 and above 90 at 210 deg. fahr. A sample tested as follows: Gravity, 15.5 A.P.I.; flash, closed cup 485 deg. fahr.; flash, open cup, 505 deg. fahr.; Saybolt viscosity at 100 deg. fahr., 1485; Saybolt viscosity at 210 deg. fahr., 110; pour, 14 deg. fahr.

This oil does not blend readily with mineral oil, but is freely soluble in rosin oil; it becomes soluble in mineral lubricating oils in the presence of rape-seed oil, lard oil, and tallow. When heated under pressure for several hours it becomes more soluble in the mineral oils. There is one peculiar difficulty that may develop when castor oil is used to cool bearings, and its use is not recommended in a circulation system; that is the formation of deposits.

After a bearing has become accustomed to castor oil it is not always easy to return to the original conditions. This appears to be due to rapid loosening of the deposits which cause trouble. When a change is to be made from castor oil to a mineral oil there should be a gradual dilution of the castor oil with a good grade of lubricating oil.

The Society of Automotive Engineers has classified lubricating oils by numbers, known as S.A.E. numbers. The table indicates how these numbers correspond with viscosities.

S.A.E. Visc. number	Viscosity range At 100 deg. fahr.	At 130 deg. fahr.	At 210 deg. fahr.
10	135–200	90–115	
20	220–330	120–150	
30	410–500	185–220	
40	600–630	255–	–70
50	635–1000		75–95
60	1300–1800		105–120

On the tag attached to each new Wright aeroplane engine is the following warning: "This engine is designed to use only high-test aviation gasoline as fuel. The use of other fuels is apt to lead to unsatisfactory operation and serious damage to the engine. The manufacturer will assume no responsibility for the engine's performance when other fuels are used. In the event, high-test gasoline is not available, always operate at reduced throttle with the mixture control in the full-rich position. Use benzol gas, ethyl gas, or high-test automobile gasoline if available. Gasoline from California base crude is much superior to gas from Mid-Continent and eastern crudes."

The weight of gasoline for airplane use is important. A plane was fueled for the Pacific flight with 500 gallons of selected Mid-Continent, 76.9 gravity gasoline, which weighed 2826 lb. or 5.652 lb. to the gallon as compared with California, 58.5 gravity gasoline weighing 6.2 lb. per gallon. This is a saving of 0.548 lb. for each gallon, or a total saving in weight of 274 lb. The consumption of the lighter gasoline has been reported to be from one gallon to a gallon and half per hour less than the heavier product in the Wright radial motor when the normal consumption was 12 gal. per hr.

The light gasoline has been tested in numerous types of airplane motors at all altitudes from sea level to above 16,000 ft., and at temperatures from below freezing to 105 deg. fahr. These tests demonstrated that the light gasoline produced an increase of 125 r.p.m. above the speed obtained with ordinary gasoline. The travel speed was increased from 5 to 11 m.p.h. With this greater speed there was a lower motor temperature of 5 deg. Flight tests with five planes showed a saving in the fuel of 1.4 to 3.25 gal. per hr.

The Skelly Oil Company has made a test with this special gasoline in a Spartan plane from Tulsa to El Dorado, Kansas, and return, a distance of 260 miles. The plane with two persons flew the distance each way in an hour and 45 minutes. Eighteen gallons of the gasoline were used, which means that a little over 14 miles to the gallon were made.

Preliminary tests had indicated that best results were obtained in the airplane motor with straight natural gasoline when jet 110 was used in the carburetor. With U. S. motor gasoline the best setting was 120. In the tests jet 115 was used as an average of the two.

Natural gasoline Gr. 74.2			Domestic aviation Gr. 64.6		
	5%	85			
	10	94	10%	136	
	20	112	20	168	
	30	130	30	192	
	40	152	40	216	
I.B.P.	50	170	50	236	I.B.P.
68	60	188	60	254	86
	70	204	70	274	
	80	224	80	298	
	90	256	90	326	
	96	316	98	368	

Dr. Brown of the University of Michigan has investigated gasoline of this type and says: "Although it seems unnecessary to use more than 50 per cent of natural gasoline in a blend with motor fuel to obtain the best engine performance for motor cars, the exacting conditions demanded from airplane engines, particularly in obtaining the maximum number of revolutions per minute possible, give natural gasoline a peculiar advantage in aviation work. Natural gasoline not only gives the maximum speed and power, but also, being of higher A.P.I. gravity, gives the plane a wider cruising for the same weight of fuel carried. It is interesting to compare the European specifications for aviation gasoline with the distillation curves of grade A natural. The specifications of aviation gasoline used in Europe are 50 per cent over at 212 deg. fahr. and an end point of 302 deg. fahr. Ap-

parently these rigid specifications for aviation gasoline for use on the Continent can best be met by natural gasoline, which appears to be an ideal airplane fuel."

Records of actual flight mileage of the various component phases of aviation in the United States are not available except in the form of estimates, mostly of an unofficial nature. The only available government statistics over any period of time are those for the air-mail service, which show a decrease over the period of 1920–23 inclusive, and from thence forward a decidedly progressive increase until it is estimated that there will be 8,250,000 miles flown with the air mail in 1928. Since all of the mail is now carried by contract operators, air transport will also show a commensurate increase for 1928. Aerial-transport service is the phase of civil aviation by which the greatest amount of flying is done, and the Department of Commerce estimates that this branch flew 18,746,640 miles in 1926. While government figures are not yet available for 1927, it is estimated that aerial-service operators flew about 20,000,000 miles in that year and will fly more than 80,000,000 miles during 1928. The latter estimate includes private, itinerant, and all other classes of flying.

Including both civil and Government aviation, probably 80,000,000 miles will be flown in the United States during 1928, or a daily average of approximately 219,000 miles. This represents an increase over 1927 of over 50 per cent.

The gasoline consumed by aviation flying these 80,000,000 air miles will approximate 530,000 barrels, while about 27,210 barrels of motor oils will be used. The process of arriving at the average number of miles flown per gallon of gasoline and oil resulted in an approximation of 4 miles per gallon of gasoline and 70 miles per gallon of oil consumed. Flight figures over the past ten years have shown that the actual operating consumption of fuel ranged from 2 flight miles per gallon of gasoline to 12 flight miles per gallon.

In October, 1918, the Post Office Department flew 12,671 miles and in doing so used 2293 gallons of gasoline, or an average of 5.52 miles per gallon at an average speed of 69 miles per hour. From May 15, 1918, to June 30, 1919, the Post Office Department flew 184,034 miles on the New York to Washington route at an average speed of 73 m.p.h., and at the rate of 4.08 miles per gallon of gasoline consumed. The official figures of the operation of the U. S. Army Model Airway for the years 1923–26 inclusive, covering the operations of all types of planes, gives a total of 1,201,370 flight miles at an average speed of 81 miles per hour. A total of 332,011 gallons of gasoline and 20,524 gallons of oil were used over the four-year period, or an average of 3.62 flight miles per gallon of gasoline and 58.52 miles per gallon of oil.

Figures on notable long distance and other record flights do not give a correct impression as to the average quantity of gasoline consumed under commercial conditions. This is due to the fact that huge quantities of gasoline are consumed in cleaning motors and testing same, which must be included in total consumption figures when cost is to be accounted for. On Lindbergh's flight to Paris of 3610 miles, 381 gallons of gasoline were consumed, which gives an average of 9.48 miles per gallon. On Art Goebel's recent West-East flight, he was in the air 18 hours and 51 minutes and traveled at an approximate average speed of 150 m.p.h.; 360 gallons of gasoline and 20 gallons of oil were consumed, which approximates an average of 8 miles per gallon of gasoline and 130 miles per gallon of oil.

The above comparisons show that the actual gasoline consumption of the average plane under all conditions, and over a long period of time will approximate one gallon for every four flight miles. This is further substantiated by operating figures furnished by two large air-transport companies who utilize various types of planes in their service of several hundred thousand miles per year on regular schedule. Both companies report an average mileage of 4.0 to 4.5 miles per gallon of gasoline consumed. One company reports oil consumed at the rate of 70.5 flight miles per gallon with one type of motor, and 108.5 miles per gallon with another type of motor, but estimates an average consumption of 70 miles per gallon of oil for all types of motors used in air transport.

Discussion

HERBERT K. CUMMINGS.[2] The Bureau of Standards receives many letters from filling-station and air-port operators, inquiring what they should stock in the way of gasolines. For obvious reasons, particular brands of gasoline cannot be recommended by a government bureau and the reply that "the gasoline should have suitable volatility, and be free from excessive tendency to knock" usually requires some further explanation. However, assuming that the gasolines under consideration do not contain sufficient unstable compounds to cause valve sticking, nor sufficient sulphur to cause corrosion, volatility and anti-knock value are the two points to consider in selecting an automobile- or aircraft-engine fuel.

High-compression engines require fuels of higher anti-knock value than low-compression engines. The tag used on the Wright engines recognizes this, and every aircraft engine might well wear a tag to indicate its fuel requirements. However, the writer objects to the term "high-test gasoline" because there is an unfortunate tendency in this country to use it as a synonym for premium gasoline. Gasolines may command a premium either because they are extra volatile, or because they are unlikely to knock, and there should be a distinction, keeping the term "high test" for the volatility factor, and using the term "high compression" for gasolines that have a high anti-knock value.

The federal specifications for aviation gasoline tie down the distillation curve which the author has been talking about by requiring that certain temperatures shall not be exceeded at 5 per cent, 50 per cent, 90 per cent, and 96 per cent distilled, and at the dry point. The Bureau's work on fuel volatility has shown that the portion of the distillation curve from 5 to 25 per cent distilled defines the relative starting performance of different gasolines. To ensure good starting performance it is specified that the temperature at 5 per cent distilled shall not exceed 167 deg. fahr. for domestic aviation gasoline and 149 deg. fahr. for the fighting grade. The specifications also provide that the temperature at 5 per cent distilled shall not be less than 122 deg. fahr. for either grade. This requirement was intended to prevent the excessive gas formation mentioned by the author. When too much air and natural gasoline are dissolved in a fuel, "gassing" is likely to occur in the fuel lines or in the carburetor at high temperatures or low pressures. Vapor lock due to this rapid evolution of gas is probably a not infrequent cause of engine trouble. It is not often reported as such because the same effect could be caused by dirt in the gasoline clogging the line. Since gassing is not by any means confined to gasolines having 5 per cent points below 122 deg. fahr., a further investigation of vapor lock in aircraft fuel systems is now in progress.

The upper end of the distillation curve is of interest in connection with the amount of heating necessary to vaporize the gasoline completely. Recent work at the Bureau of Standards on the dew points of gasolines has shown that the minimum temperature for the complete vaporization of gasoline mixed with air can be computed from the 90 per cent point. There appears to be no relation between the dry point in the A.S.T.M. distillation test, and the dew point, although the former is the sole distillation

[2] Bureau of Standards, Washington, D. C.

temperature mentioned in many commercial-gasoline quotations. The maximum temperatures specified at 90 per cent distilled are 311 deg. fahr. for domestic aviation gasoline and 257 deg. fahr. for the fighting grade. These correspond to dew points of 91 deg. fahr. and 52 deg. fahr. for air-gasoline mixtures containing twelve parts of air to one of gasoline by weight. For motor gasoline having a 90 per cent point of 392 deg. fahr. the dew point of a 12:1 air-gasoline mixture is 149 deg. fahr.

Unfortunately, there is as yet no entirely satisfactory way of determining or specifying just how good a gasoline is with respect to detonation. Nothing but a trial in the engine itself will determine with certainty how readily a given gasoline will knock in that particular engine. All present knock ratings are based on the assumption (plausible but unproved) that, if fuel A knocks more violently than fuel B in a given engine under given operating conditions, fuel A will have more tendency to knock than fuel B in other engines, and under other operating conditions.

Experience indicates in general that straight-run California gasolines are less likely to knock than straight-run eastern gasolines, and that some mid-continent straight-run gasolines are almost as good as the best California gasolines in this respect while others are nearly as bad as the worst eastern gasolines. There are some cracked gasolines, particularly from mid-continent crudes, which equal or exceed straight-run California gasolines in anti-knock value, but it is impossible to predict how much better one gasoline will be than another without actual engine tests. A large-scale attempt to market a fuel of constant anti-knock value is found in the case of gasolines containing lead, since by contract those licensed to sell ethyl gasoline in bulk are required to use sufficient lead to match under specified conditions an arbitrary standard fuel which approaches straight-run California gasoline in anti-knock value.

The domestic aviation gasoline (D.A.G.) that the government buys on the east coast under specifications knocks about as readily as any gasoline which the Bureau of Standards has occasion to test, although some of the straight-run eastern motor gasolines are even lower in anti-knock value. The navy seaplanes along the Atlantic coast use this eastern D.A.G. treated with lead, while the army uses a benzol blend of the same eastern D.A.G. The fuels used in type-testing commercial-aircraft engines are supplied by the engine manufacturers, but these fuels are compared with the Bureau's current supply of eastern D.A.G., and each engine which passes is approved for use with a certain grade of fuel, or better. No engine has yet been approved for use with straight-eastern D.A.G.

At the present time the average aircraft engine will find many volatile gasolines which are unsatisfactory for use at full throttle at the ground without treatment with benzol or lead. There are, however, a great many gasolines which are satisfactory, and the difficulty is to pick out those that can be used safely in a given engine. Knocking is objectionable in a motor-car engine, but in the aircraft engine where weight is cut down to a minimum it may actually be hazardous, resulting in mechanical damage as well as lower power, and increased heat dissipation.

Cooperative work by representative laboratories on the problem of developing a standard method for determining and expressing anti-knock value is in progress, and it is hoped that it will not be long before a definite knock test can be included in specifications for aviation gasoline.

In concluding, there is just one point that the writer wishes to make regarding lubrication. The aviation engine has a great advantage over the automobile engine, because with aviation gasoline there is practically no loss in viscosity due to progressive dilution of the crankcase oil with unburned fuel.

AER-51-13

Air-Mail and Express Traffic

By R. W. IRELAND,[1] CHICAGO, ILL.

The organization of most of the air-transport lines of the country is patterned somewhat after the organization of the railroads, as it is only a newer and faster method of moving persons or things. The number of air-transport lines is increasing with each year. The twenty-five existing lines are supported by private capital. An average of 42,000 miles is flown over these lines every 24 hours. The Department of Commerce has completed the lighting on over 9000 miles of the 20,000 miles of airways now in operation.

SELLING service is the real function of the traffic department of an air-transport company. It bears the same relation to the organization as does the traffic department of a railroad. While the actual selling does not begin until the line is operating, the work of the traffic department, or at least the traffic work of the company, should come into action as one of the first activities in connection with the organization of a line. The probable amount of traffic should be given primary consideration in the selection of the route to be flown, because if the route is not laid out properly from a traffic point of view, it will never mean anything financially. Planes may fly on time, but if they have nothing to carry, that is, if they have no traffic, the line will not continue long in existence. Therefore, before the line is actually laid out, the proposed route should be surveyed carefully from a traffic point of view.

Consideration must be given to the distance between points to be served by the proposed route and its connections. In order to offer a salable advantage to the public, an air-transport line, either in itself or in combination with connecting routes, must save time in comparison with other existing means of transportation. This requires that the distance between points to be served be great enough so that the airplane will be in the air a sufficient number of hours to effect a real saving in delivery to destination. What this distance must be depends upon the other forms of transportation with which the airplane competes. Obviously if this competitor is water transportation with a speed of only 15 or 20 miles an hour, the length of the route will not have to be as great, in order to gain time, as in a case where the air route is competing with fast trains averaging 40 or 50 miles an hour. In general, the distance between points to be served by an air route, even if flown at night, must be greater than can be traveled overnight by other means of transportation, because, ordinarily, overnight delivery is all that is needed from any form of mail or express transportation.

In some unusual cases, the airplane will save time in delivery between cities a short distanee apart, 100 or 150 miles or even less. This is true where mail or express matter can be picked up late in the forenoon at point of origin and carried to its destination in time for delivery to the consignee the same day and where such delivery is not possible with other means of transportation.

The desirability of night operation of an air line is obvious. When this is possible, the movement takes place without any loss of business time. An outstanding example of this is the overnight line between Chicago and New York. Mail or express leaves either end of this route after the close of the business day and is delivered at the other end prior to the opening of business on the next day.

Night flying, of course, depends upon the existence of a lighted airway. The Department of Commerce is proceeding with the lighting of these airways as rapidly as possible, and has completed this work on between nine and ten thousand of the 20,000 miles of airways now in operation. In many instances the installation of these lights and the inauguration of night flying has been the means of greatly increasing the cargoes carried on air-mail and express lines. A case in point is the Southwestern route operated by National Air Transport between Chicago and Dallas, Texas. Originally this line could be flown only during daylight hours on account of the absence of lights. While the plane made the thousand-mile trip from Chicago to Dallas in 12 hours elapsed time, including stops, as compared with a minimum of 26 or 27 by rail, the fact that the flight was made in the daytime during business hours destroyed its advantage so far as service between Chicago and Dallas was concerned, and while it did offer a saving between the Southwest and New York through connection made at Chicago with the Chicago–New York overnight route, it offered the man who wanted to send mail and express between Chicago and Dallas next to nothing so far as time saving was concerned. Mail posted Monday evening in Chicago was not started on its flight until Tuesday morning and did not reach Dallas until Tuesday evening, too late for delivery to the addressee before Wednesday morning. Practically this same service was available by rail. Hence very little Chicago–Dallas mail was carried. This situation was entirely changed by the installation of lights. Monday's air mail and express now leaves Chicago at 8 p.m. Monday and arrives at Dallas the next morning, saving an additional day as compared with the day flight. The improved service has been reflected by a substantial increase in cargo.

The question that must be answered in determining if a proposed air line will offer a service that the public will buy is whether the line will deliver air mail or express far enough in advance of other methods to make the saving of time valuable to those who use the service. Sound business considerations must govern the establishment of an air line, as well as any other form of business, and unless the people to be served can be offered something better than they already have, the results will be disappointing.

Obviously, careful thought must be given to the potential air-mail and express cargo in the territory to be served. A city or group of cities whose business is entirely local or is confined to points only a few miles beyond its limits will mean little to the prospective air-transport operator.

The character of population has considerable bearing on the use that will be made of a proposed air line. This is particularly true with reference to use of air mail for social or personal correspondence as distinguished from business uses. Certain classes of population will adopt new methods more readily than will others. Comparatively new communities, the population of which comes largely from other distant parts of the country, may be expected to make liberal use of air mail, as well as of all other classes of mail, on account of connections back home.

A basis for estimating the potential air-mail or air-express traffic that may be expected from a given locality may be obtained by a count of the ordinary mail and express moving by rail from that locality to points far enough distant so that a substantial saving of time may be offered by the use of air transportation. Such

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[1] General Traffic Manager, National Air Transport, Inc.

Presented at the Third National Meeting of the A.S.M.E. Aeronautic Division, St. Louis, Mo., May 27 to 30, 1929.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

figures, while not giving accurate information as to the amount of air cargo that may be expected, at least give some basis for a more or less intelligent estimate. The percentage of air mail that may be expected from a given quantity of ordinary mail varies considerably, in some cases running as high as 15 or 20 per cent. Generally, however, the percentage is smaller.

Because of the great public interest in aviation and air transportation, and because of the desire of most cities to secure for themselves places on air-transport lines, chambers of commerce are quite often willing to institute surveys to determine the potential business which may be expected if a line is placed in operation. The common method of securing this is for the chamber of commerce to send questionnaires to the leading business firms telling them what the service will offer and asking them to indicate approximately the extent to which they would use it.

In some localities, chambers of commerce have even gone to the extent of securing guarantees of traffic, that is, to ask the leading business men to guarantee to send a certain quantity of mail or express by air for a period of a year or so, in order to help the line in its early stages. Such a guarantee as a general rule, however, amounts to little more than a good-will gesture, because it cannot be expected that the public will make permanent use of the service unless it does offer an advantage sufficient to compensate for the increased cost as compared with the cost of other means of transportation or communication. Such guarantees do have considerable value in getting a given community into the habit of using air transport and thus finding out what such a service accomplishes. These guarantees, however, contain an element of danger unless the company recognizes the traffic which comes from such a source. If this is not done, a false impression will be gained of the business to be expected and after the period of the guarantees expire, the traffic is likely to take a disconcerting drop. Taking them as a whole, I believe they are undesirable but they do offer a tempting source of revenue to a new company, particularly to one with limited financial backing and which cannot operate for a very long period of time at a loss.

In estimating the potential business which may come to a proposed line, there must be considered not only the business originating in the cities touched by the line but also business which may come into the route cities from the surrounding territory, from other air lines or from rail lines. Many times, by a combination of air and rail, as much time can be saved as in the case of an all-air route, and sometimes even more.

One matter of importance in estimating probable air-mail and express cargoes is the scheduled departure time of the plane. If this can be made late in the afternoon or in the evening, it will be much easier to induce business houses to use the service than if the plane leaves so early that a special effort must be made in order to get mail out before the normal mailing time at the close of the business day.

To function properly, a proposed air line must tie into the air-transport system of the country. The transcontinental line from New York to San Francisco forms the backbone of the system as it exists today and practically all of the other lines function as feeders to this transcontinental route.

Obviously it is an advantage to have an air-transport line follow a straight line as nearly as possible, in order that the time gained by the speed of the airplane may not be lost by following a circuitous route. However, in some cases, it may be wise to deviate slightly from a direct route when by doing so the plane may follow the route of a railroad going in the same direction as the air route. The reason for this is that, in the present stage of aviation, bad weather offers an obstacle to airplane operation, which sometimes makes it necessary to land and forward the cargo by ground transportation. When this is necessary, if the landing can be made along the route of fast rail service, delivery of the cargo to its destination will be greatly expedited as compared with the situation where a considerable period of time is required to transfer the cargo from the plane to the railroad after a forced landing.

The same consideration leads to a limitation of the number of stops to be made by the plane, because, with too many stops, a substantial portion of the time gained by the speed of the airplane will be lost on account of the time required to land and take off.

The probable amount of traffic obviously must be given careful thought in determining bids made on air-mail contracts. With the exception of some foreign routes, payment to an air-mail contractor is based directly upon the amount of mail that he carries, which in turn depends upon the amount of mail on which the people served by him are willing to pay air-mail postage. It is therefore of first importance that the bidder have some idea of the cargo that he may expect to secure before he makes his proposal.

Under the law, air-mail contracts are awarded on a competitive basis, with the limitation that the law does not permit the Postmaster General to consider a bid in excess of $3 per pound. Contracts now in existence vary considerably as regards rates, ranging from the maximum of $3 to a minimum of around 75 cents. In the earlier contracts, competition was not especially severe, but within the last year or year and a half this situation has changed, and any contracts now thrown open to bids may be expected to go at a rather low rate due to the large number of people interested in engaging in such service. This condition may lead to a bad situation, inasmuch as many contractors in order to secure a line may bid much lower than the prospective business warrants, with the result that on account of the small amount of business secured and the low rate received, the contractor will not be able to purchase the most efficient equipment nor to develop his line in the way he should.

Eventually it is felt that the method of payment to air-mail contractors will be changed to something similar to the system in force with reference to rail-mail contracts where the operator is paid on a space basis regardless of day-to-day variation in cargo.

The second and the permanent phase of the traffic work of an air-transport operator is the sale of his service to the public after the line has been laid out and operations started.

Selling air transportation, particularly air-mail and express service, is one of the most agreeable and at the same time one of the most unsatisfactory forms of selling of which I know. It is agreeable because of the general attitude of the public toward aviation. Probably 999 out of every 1000 people in the United States are interested in aviation and like to hear about it. This makes it easy for the air-traffic solicitor to gain an audience. Because of this great public interest, chambers of commerce and other civic bodies will go to great lengths to assist the air-transport company in obtaining business. I have never yet visited a city not located on an air line that was not anxious to have a place on one. In many cases, municipalities will spend large sums of money in buying and improving landing fields and constructing hangars and other facilities to secure a place on an air line, when by no stretch of the imagination can they be expected to secure any financial return proportionate to the investment required. This varies in inverse proportion to the size of the city involved. The larger cities of the country very seldom get excited about an air line, while the smaller communities (and by smaller, I mean cities of a half-million or under) will go to great lengths to become stations on such lines.

The unsatisfactory phase of air-transport traffic promotion, and this again is particularly true in the case of air-mail and express traffic, lies in the fact that the business solicitor very seldom knows definitely that he has made a sale. Air mail and air express do not come direct to the carrier, but are received through the post

offices and express agents. The mail comes to the plane already pouched, so the carrier does not know definitely who is using his service. An automobile salesman or a real estate salesman knows when he has made a sale, but the air-mail and express salesman does not except as may be reflected in the aggregate by the increased loads which his company carries.

The successful and permanent sale of air-transport service depends upon three things. First, the service must be made easy to use. Second, the public must be told of its existence and how to use it. Third, a satisfactory service must be delivered. The most important of the things that have been done by the Post Office Department to make the air-mail service easy to use is the establishment of a uniform postage rate. Originally the air-mail rate was very complicated, the rate depending upon distance flown and zones and routes traversed. Since August 1, 1928, however, a uniform rate of 5 cents for the first ounce and 10 cents for each additional ounce has been in force, which carries air mail between any points in the United States, Canada, and Mexico regardless of distance.

When the service was first inaugurated, it was necessary for air mail to be deposited in special air-mail boxes. Now it may be deposited in any mail box regardless of location, and most all boxes and chutes have been marked with the air-mail pick-up time. In addition the department permits the use of any ordinary postage stamps and any envelopes so long as the letter or package is marked plainly to indicate that it is to go by air. These things, all of which have been done to eliminate complications in the use of air mail, undoubtedly have contributed much to the increased use of this method of communication.

Air express has at its command all of the facilities of the Railway Express Agency, including pick-up, delivery, insurance, etc. The express company in many instances makes special delivery of air-express packages.

The second phase of the air-transport sales effort is largely an educational one. The public must be told that the service is available and must be instructed in the method of using it. Changing habits is not an easy task, but it is one that must be accomplished before the use of air mail and air express will become general in a community. The American business man has been accustomed to putting a two-cent stamp on a letter and to sending packages by ordinary express at the cost of only a few cents a pound. He must be given a good reason for doing something that costs him more money. He must be told about this, not once, but many times in order that air transport may become a routine business tool that is used as a matter of course. This is the principal and continuing work of the traffic department. Such a department consists ordinarily of a general traffic manager having headquarters at the home offices of the company, with division traffic managers distributed through the territory served by the line.

Publicity forms one of the most important traffic activities and is ordinarily in charge of an experienced newspaper man operating as a part of the traffic department of the company. Public interest in aviation has been and still is of such a high degree that the press welcomes legitimate items of news regarding air-transport activities. Most of the larger newspapers of the country have even gone to the extent of inaugurating aviation sections or columns, in some cases designating aviation editors who devote all of their time to this class of news. Many papers have adopted the development of aviation as a definite part of their editorial programs. There is no doubt that this liberal attitude of the press has been one of the most potent factors in building up air-mindedness in America. More columns of newspaper space were devoted to Colonel Lindbergh's flight than to any other event in history. This flight, with its attendant publicity, was of immeasurable value to the development of all branches of aviation including air transportation. Trade journals and national publications have generally adopted a progressive attitude toward aviation.

Supplementing the publicity departments of the various air-transport companies, the American Air Transport Association, made up of the principal operators of the country, maintains an office in Chicago for the dissemination of information regarding air transportation in general. The direct sales work of the air-transport company is carried on by the division traffic managers previously mentioned. One of these men is ordinarily stationed in each of the subdivisions of the territory served by the company, and is responsible for the company's sales efforts in his community. His sales assignment includes not only cities on the air line but all surrounding territory. This man is the company's contact with the public and to be successful he must combine the qualifications of a salesman with those of a diplomat. He must have a good general knowledge of business, so that he may be able to determine how any type of business may use air transport to advantage. This knowledge is ordinarily acquired by visiting one or more representative members of a given class of business and determining all the specific applications of air mail and air express for that group. The knowledge thus gained is used in solicitation of other members of the same industry, either by personal solicitation, advertising, publicity through trade journals and the press, or by other methods.

The cooperation of the Post Office Department has been invaluable in publicizing and advertising the air-mail service. The sales efforts of the Railway Express Agency (formerly American Railway Express Company) have included advertising of air express. The routine of air-transport sales activities includes many other forms of advertising and publicity, such as air-mail and express window displays, talks before civic bodies and clubs and over the radio, direct mail advertising, distribution of printed matter, educational motion pictures, etc.

Many ways have been found by which American business can use to advantage the speed of air transport. Saving, as it does in most cases, a business day, in some cases two days, air transport performs a valuable economic function. Time is our most valuable commodity. The airplane, by cutting to a half or a third the number of hours required for transportation of mail and express, enables the saving of this commodity. In effect, it enables the completion of a given task in one-half or one-third of the time ordinarily required, so far as transportation or communication is involved.

The records of the traffic department of any air-transport company will show many interesting applications of air mail and express to business. Banks derive much profit from the use of air mail and express for clearings. Clearing a day earlier than is possible by other methods means the saving of a day's interest on the amount involved. Many large banks save several thousand dollars a month by using this service. This service is used extensively for shipment of stocks, bonds, and other financial papers where an earlier delivery will mean the saving of interest.

Where schedules permit, air mail is quite often used as a substitute for night telegrams. Merchants often use this service for rush shipments of goods. Many cases are on record where air mail or express has saved the closing down of manufacturing plants by quick delivery of parts needed to keep disabled machines in operation. Air mail and express are used daily for shipment of motion picture negatives and films, especially news reels. Pictures filmed in Chicago on one afternoon may be shown on the screen in Houston, 1300 miles away, on the next day by using this method of fast transportation.

Insurance companies use the service for re-insurance, securing home-office approval of loans and for all classes of correspondence where speed is an object. Social correspondence forms a sub-

stantial portion of the mail carried in the closely packed pouches traveling by air through all hours of the day and night. The time is not far off when good form will call for the use of air mail in all personal correspondence going to distant points.

The value of this service to the American people is best evidenced by its increasing use. In 1926, 810,000 lb. of air mail were carried on all lines of the country; in 1927, this had increased to 1,650,000 lb.; and in 1928, to 4,060,000 lb. At the present rate 1929 will show a total of over 6,000,000 lb.

The final phase in the sale of air transport is not a sales problem at all but an operating one and involves delivery of service after the customer has decided to use it. This is hardly within the scope of this discussion so I shall touch on it only very briefly. Maintenance of service involves a proper flying and ground organization. Accurate weather reports must be available. Satisfactory radio or other communication service must be had.

Planes and motors must be checked and overhauled carefully at periodic intervals. Probably 99 per cent of delays in air-transport flying result from bad weather. Continuous progress is being made in overcoming this handicap. The radio directional beacon, radio ground to plane communication, and improved navigating instruments are expected to do much in this direction.

From the economic viewpoint I think it may be fairly said that air transport is making satisfactory progress. The number of air-transport lines is showing a healthy increase each year. Three and a half years ago there were but two air-mail lines in operation, the transcontinental and New York–Chicago overnight routes, both operated by the Post Office Department. Now both of these routes, as well as twenty-three others, are operated by private capital. An average of 42,000 miles is flown over these lines each twenty-four hours. Practically all of these lines are carrying substantial cargoes and are earning a fair profit. The airplane is proving itself to be a useful peace-time economic tool and one that can sustain itself financially as well as aerodynamically.

Discussion

AMELIA EARHART.[2] Women should be included in a traffic selling program. Women's organizations offer an excellent opportunity for air-mail campaigns, and one which is often neglected.

Social mail will go by air, and a very good selling point is the little added distinction which the air-mail stamp still bears. Guarantees of use similar to those from chambers of commerce can be wrung cheerfully from women's clubs with the same benefit to the operating company as from the other source.

Not only must women be sold on aviation for themselves, but for the influence they have on men. So I urge that important feminine names be added to the list of prospects to be visited by (the handsomest) traffic representatives.

[2] Aviation Editor, Hearst's International, New York, N. Y.

AUTHOR'S CLOSURE

The author agrees heartily with Miss Earhart's suggestion that air-mail traffic sales should include women as well as men. While no special point of that is made in the paper, nevertheless the work of all operators has included efforts along this line. For example, in National Air Transport, some of the things done to encourage the use of air mail for social correspondence have included talks before women's clubs, and in cooperation with Post Office department, posters have been placed on mail trucks and printed matter is distributed through mail carriers. Moving pictures are given before women's clubs and talks over the radio. Essay contests have been inagurated in schools. Air-mail stickers have been placed on mail boxes and mail chutes, including those in hotels and some apartment buildings. General publicity is featured in newspapers and magazines, and a limited amount of general advertising is done in newspapers. Air-mail literature is distributed with gas bills, telephone bills, etc., a large percentage of which reach women.

In the early days of the air-mail service perhaps the sales efforts of many of the air-mail contractors were pretty generally confined to business houses, which was quite natural in view of the fact that business houses are much smaller in number than prospective social users, and are much more easily reached by the air-mail solicitor. But as the service has progressed, everybody in the business has recognized the importance of urging the use of air mail for social correspondence, and considerable effort and money have been expended in reaching this class of mail. In an effort to determine the relative percentage of business and social correspondence which is being carried by air mail, a survey has been made recently of the air mail in certain cities served by National Air Transport lines and it was found that of the total air mail originating in each of these cities, the percentages of social correspondence were as follows:

Chicago, Ill.	19.4	Brooklyn, N. Y.	65.6
New York City	24	Newark, N. J.	35
St. Joseph, Mo.	40	Kansas City, Mo.	30
Wichita, Kansas	40	Ponca City	59
Tulsa, Okla.	43.4	Okla. City	30.8
Fort Worth, Texas	37.8	Dallas, Texas	22.3

The use of air mail as a medium for social correspondence, and the attention getting value of this class of mail was recently recognized by Miss Emily Post who is perhaps the world's foremost authority on etiquette and social usage. In an article in Collier's, Miss Post stated, in part: "As letters arrive, they are opened and laid flat on a special desk and held in place by a heavy book. Air mail and special delivery letters are put on the blotter of my writing table; these are answered the day they are received, the others are answered as I find time, in turn."

Landing Speed of Airplanes

By JOHN G. LEE,[1] FARMINGDALE, N. Y.

Results of a large number of wind-tunnel and flight tests on the maximum lift of wings are presented, analyzed, and compared. Curves are given showing the relationship between the shape and lift of wings and for the landing speeds obtained with different wing loadings and lift coefficients. The subjects of sinking speeds, landing impacts, and the effect of flaps and slots are discussed in relation to landing speed and the general problem of easy landing.

THERE is no item of airplane performance as universally misrepresented as the landing speed. It has become common practice to advertise landing speeds much lower than they really are. Reliable flight tests are scarce. Wind-tunnel tests are plentiful, but differ greatly among themselves, even for the same wing. In addition, many pilots feel that a low landing speed is only one of several qualities required by an easy-landing airplane.

The first problem is to get some sort of correlation between the wind-tunnel tests and flight tests. The maximum lift obtained in a wind tunnel is influenced greatly by the turbulence of the tunnel, so that tests in different tunnels which agree in other respects usually do not agree in maximum lift, particularly with the high-lift wings. In addition, accurate flight tests are quite rare, and are available on only a few wings.

It is equally desirable to establish a relationship between the maximum lift and the shape of the wing. By this means the lift of a wing can be predicted from flight tests on similar wings. It has been established rather generally that the lift of a wing increases with the mean camber and that of two wings having the same mean camber the thicker will have the higher lift.

Attempts to plot the maximum lift of several wings against the mean camber or the thickness gave fairly uniform results. Much better results were obtained, however, by plotting the lift against the maximum camber of the upper surface. This latter really takes into consideration both of the other variables, since increasing either the mean camber or the thickness automatically increases the upper camber. Moreover, the maximum lift occurs when the flow over the upper surface breaks down, which lends a physical significance to the upper surface of the wing.

The maximum upper camber is measured perpendicular to the common chord line for all wing sections as shown in Fig. 1, and is expressed in per cent of that chord. It will be noted that the common chord line is taken as the line joining the extreme leading and trailing edges, and in general does not coincide with the usual chord or baseline from which the wing profile is layed out, since that is located in different places on different wings.

Fig. 2 shows the maximum-lift coefficient plotted against upper camber for wings tested on the 7½-ft. tunnel of the Massachusetts Institute of Technology. This represents a considerable body of data, 28 wings, all tested under identical conditions and covering the entire range of types. The variations from the straight line are generally small, and the three or four exceptions may be explained easily as turbulence effects. The maximum-lift coefficients are in engineering units.

In Fig. 3, similar data are plotted for wings tested in the N.A.C.A. high-density wind tunnel, at 20 atmospheres pressure. A total of 28 different wings are again shown, of which about one-fourth are duplicates of those tested at M.I.T. The lift coefficients are in absolute units. It is apparent at once that while the agreement is excellent for low-lift wings, the high-lift wings diverge largely, and the question of whether this is a turbulence effect dependent upon the tunnel or whether it is actually a characteristic of the wings themselves still remains.

In Fig. 4, the maximum lift is plotted against upper camber for wings tested in flight. In this case, only 15 different wing sections are available, but the plotted points represent tests on 47 different airplanes and certain of the points are the average of tests on many planes of different types, all having the same wing section. It will be noted that the agreement of the points with the curve is quite as good as with the wind-tunnel tests, which is remarkable when it is realized that a flight test includes not only the lift on the wings in landing, but the lift on the fuselage as well, and the down load on the tail—both of which are indeterminate variables.

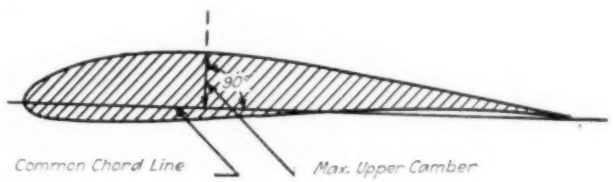


FIG. 1 THE MAXIMUM UPPER CAMBER IS MEASURED PERPENDICULAR TO THE COMMON CHORD LINE

The original data from which this plot was obtained are summarized in Table 1, together with the maximum lifts obtained in the M.I.T. and N.A.C.A. wind tunnels. The maximum thickness, mean camber, and upper camber are also given by way of general information. In this table will be found the maximum lifts of most of the generally used wings. The maximum lift may be found for other wings by determining the upper camber graphically and referring to Fig. 4.

TABLE 1 MAXIMUM LIFT OF WINGS FROM FLIGHT TESTS

Aerofoil	Flight test max. lift coeff.	Number of airplanes tested	Wind tunnel max. lift coeff. M.I.T.	Wind tunnel max. lift coeff. N.A.C.A.	Maximum upper ordinate, per cent	Maximum thickness, per cent	Mean camber, per cent
Clark Y	17.78	12	17.8	16.9	9.25	11.68	3.67
RAF-15	18.87	7	19.6	17.9	5.70	6.38	2.70
Göttingen 387	15.30	3	16.5	17.2	12.85	15.21	5.40
Göttingen 398	16.12	5	..	..	11.15	13.85	4.45
Göttingen 436	16.33	3	18.0	..	9.10	11.08	3.70
"Albatross"	17.95	3	..	..	9.15	7.10	5.55
Boeing 103	17.93	2	..	..	9.80	12.70	3.45
Curtiss C-72	17.15	2	..	..	9.58	...	..
Loening 10-A	17.95	2	..	..	7.20	8.20	3.35
USA-5	17.25	2	..	18.1	8.38	6.42	4.75
USA-27	17.15	2	17.0	16.8	10.50	11.06	5.05
USA-35b	16.7	1	17.3	18.8	9.80	11.63	4.35
Eifel 36	18.2	1	..	..	8.40	6.95	5.15
Aeromarine 2-A	18.1	1	..	..	9.20	8.59	5.25
Curtiss C-62	18.4	1	20.7	..	6.00	8.04	2.05

Several of these flight tests have not yet been published, which, together with the scarcity of reliable data on the subject, makes them of sufficient interest to give in detail in Table 2. The author wishes to thank ex-Secretary Warner for the Navy data and Major C. W. Howard for most of the Army data, as well as the Curtiss, Keystone, and Fairchild companies for their contribution. All of the wind-tunnel tests, together with the

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[1] Fairchild Airplane Manufacturing Co.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

N.A.C.A. and several of the Army flight tests, have been published as N.A.C.A. reports or Air Service Information circulars, and are generally available.

Two different methods have been employed to obtain data. The first consists of calibrating the air-speed meter by flying over a course at various speeds and then reading the air speed at the moment of landing. Generally the average of several landings is taken. This is the most usual method. The second method consists of attaching an electric recording instrument to the landing wheels which records their rate of revolution graphically. Knowing the circumference of the wheel, the virtual landing speed is obtained which must be corrected by the velocity of the wind taken simultaneously by an observer on the ground. Both of these methods are open to certain inaccuracies, but are probably accurate within 5 per cent. In any case, it is necessary to make a careful three-point landing without stalling or dropping the machine. A test on an uncalibrated air-speed meter is very unreliable, since the position of the air-speed head on a given airplane often introduces an error of 20 per cent at slow speeds (usually too low). This explains many of the advertised slow landing speeds.

The curves from Figs. 2, 3, and 4 have been plotted in Fig. 5 using the same coefficients. The N.A.C.A. curve gives landing speeds approximately 1 per cent too low, and the M.I.T. curve 6 per cent too high relative to the flight tests, which is an excellent agreement. Unfortunately, this agreement is of academic interest only since it deals with averages. Comparisons of individual wings show great variations and inconsistencies and have led to great confusion in calculating landing speeds.

TABLE 2 LIFT COEFFICIENTS FROM FLIGHT TEST

Aerofoil	Flight test lift coeff.	Airplane used in test	Description	Landing speed, m.p.h.	Wing loading lb. per sq. ft.	Source of data
RAF-15	18.6	Vought, VE-7	Observation	51	7.57	N.A.C.A.
	18.6	Vought, UO-1	Observation	55	8.76	Army
	18.4	SE-5a	Pursuit Plane	54	8.67	N.A.C.A.
	18.4	Thomas, MB-3	Pursuit Plane	57	9.63	N.A.C.A.
	19.7	DH-4	Observation	61.5	9.77	Army
	18.8	DH-4b	Observation	56.5	9.10	N.A.C.A.
	19.6	Mail DH-M2	Mail Conversion	60.0	9.40	Army
	18.87	Average				
Clark-Y	17.4	Curtiss, OC-1	Observation	58	11.11	Navy
	17.2	Curtiss, O-1B	Observation	60	12.20	Curtiss
	17.2	Curtiss, P3-A	Pursuit	58	11.45	Army
	17.5	Curtiss, XP-6	Pursuit	61	12.14	Army
	16.9	Consolidated XO-17	Training	52	9.49	Army
	16.9	Thomas, XO-19	Observation	58	11.78	Army
	17.8	Curtiss, O-11	Observation	64	12.92	Army
	19.0	Douglas, O-2	Observation	65	11.66	Army
	18.8	Curtiss, AT-5A	Adv. Training	59	9.83	Army
	16.9	Consolidated, PT-3A	Pri. Training	48	8.10	Army
	19.7	Douglas, C-1	Transport	57	8.37	Army
	18.0	Curtiss, P-1	Pursuit	63	12.22	Army
	17.78	Average				
Göttingen 398	16.7	Douglas XT-2D-1	Observation	55	10.82	Navy
	16.5	Loening OL-7	Amphibian	55	11.17	Navy
	15.8	Douglas O-2H	Observation	56	12.65	Army
	16.4	Keystone XLB-6	Light Bomber	58	11.48	Army
	15.2	Keystone Patrician	Tri-Motor Monoplane	59	15.00	Keystone
	16.12	Average				
Göttingen 436	14.8	Boeing Pursuit I	Pursuit Plane	64	18.68	Army
	17.2	Boeing PW-9D	Pursuit Plane	63	13.42	Army
	17.0	Boeing XP-7	Pursuit Plane	62	13.12	Army
	16.33	Average				
Loening 10-A	19.5	Atlantic XCO-8	Observation	60	9.45	Army
	16.4	Loening XOL-8	Amphibian	55	9.34	Navy
	17.95	Average				
"Albatross"	18.7	Martin, MB-2	Bomber	58	9.70	N.A.C.A.
	18.0	Martin, NBS-1	Bomber	59	10.7	Army
	17.1	Martin, NBS-1	Bomber	52	9.25	Army
	17.93	Average				
USA-27	17.7	Consolidated PT-1	Training Biplane	53	9.02	Army
	16.6	Consolidated PT-1	Training Biplane	50	9.11	Army
	17.15	Average				
USA-5	17.2	Sperry Messenger	Small Biplane	41	5.70	Army
	17.3	Sperry Messenger	Small Biplane	44	6.50	N.A.C.A.
	17.25	Average				
Curtiss C-72	16.5	Curtiss XN2C1	Training Biplane	45	7.45	Navy
	16.1	Curtiss XB-2	Bomber	53	10.88	Army
	16.30	Average				
Boeing 103	16.7	Boeing F2B1	Pursuit Plane	57	11.60	Navy
	16.4	Boeing XF3B1	Pursuit Plane	55	11.32	Navy
	16.55	Average				
Göttingen 387	16.7	Fairchild FB-2	Monoplane Boat	70	17.5	Fairchild
	14.9	Fairchild FC-2	Cabin Monoplane	46.9	9.94	Fairchild
	14.9	Fairchild FC-1	Cabin Monoplane	42.5	9.03	Fairchild
	15.30	Average				
USA-35b	16.7	Keystone XNK-1	Training Biplane	47	7.97	Navy
Eifel-36	18.2	Curtiss JN-6h	Training Biplane	51	7.85	N.A.C.A.
Aeromarine 2A	18.1	Waco 9	3-Place Biplane	45	6.20	Army
Curtiss C-62	18.4	Curtiss PW-8A	Pursuit Plane	61	11.05	Curtiss

The formula for the landing speed of an airplane is given in any aeronautical textbook in one of the three following forms:

$$V = \sqrt{1/L_e}\,\sqrt{W/A} \text{ engineering units (M.I.T.)} \quad [1]$$

$$V = 19.75\,\sqrt{1/C_L}\,\sqrt{W/A} \text{ absolute units (N.A.C.A.)} \quad [2]$$

$$V = K\,\sqrt{W/A} \text{ as in Fig. 4} \quad [3]$$

where V = landing speed in m.p.h.
W/A = wing loading in lb. per sq. ft.
L_e = max. lift coefficient (engineering units)
C_L = max. lift coefficient (absolute units)
$K = \sqrt{1/L_e}$

The formula for converting absolute units into engineering units is $L_e = 0.002558\,C_L$.

Of the three forms, the third is the simplest, and has given rise to a series of handy formulas for estimating landing speed roughly, wherein a definite value is assigned to K for all wings. The coefficient K may well be called a "landing-speed coefficient."

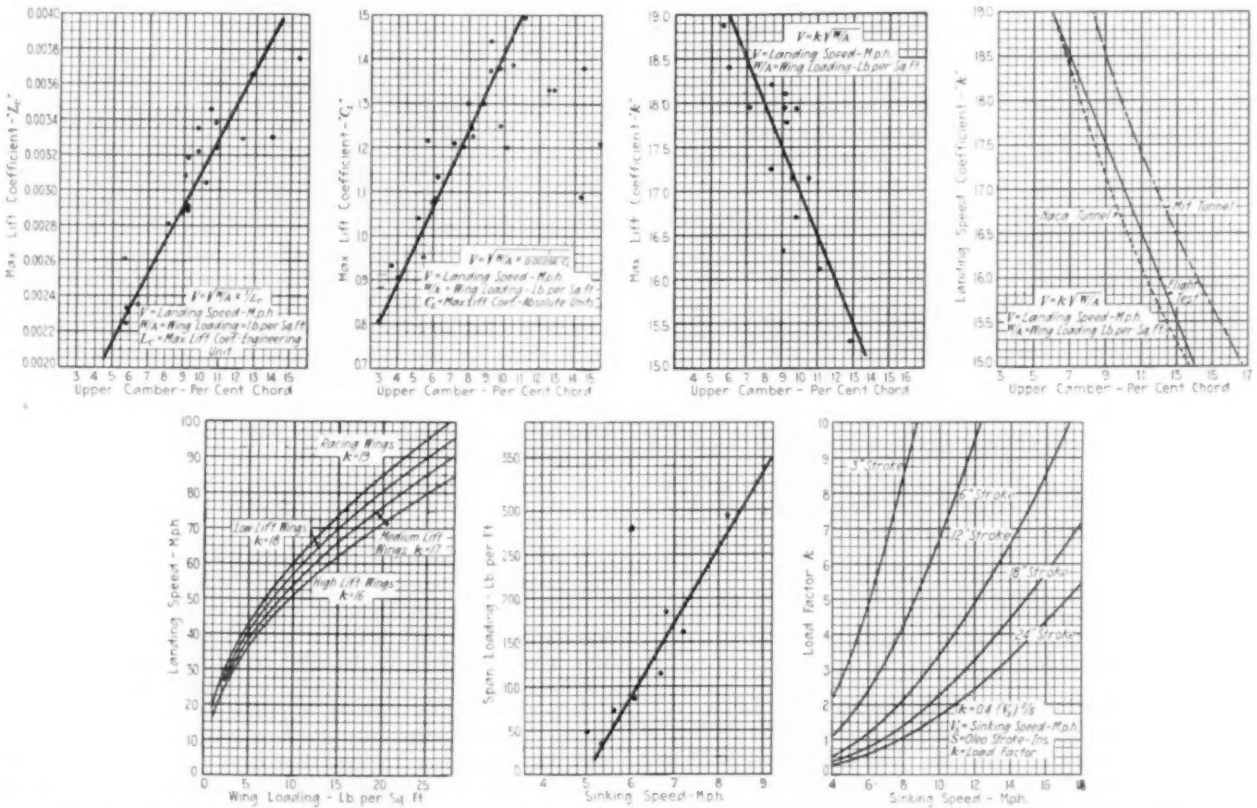
Returning to Fig. 4, the "landing-speed coefficient," K varies between the values 16 for high-lift wings and 19 for very low-lift or racing wings. This makes it possible to plot landing speed against wing loading, obtaining a family of curves for different values of K, as has been done in Fig. 6. Given the wing loading of any airplane, the landing speed can be determined from Fig. 6, the only judgment required being to decide whether the wings are of the high-, low-, or medium-lift type. To take some examples, our heavily-loaded transport monoplanes with a wing loading of 17 to 18 lb. per sq. ft. land at upwards of 70 m.p.h. The usual run of cabin planes with wing loadings of 12 lb. per sq. ft. land at speeds between 55 and 60 m.p.h. For a plane to land at 40 m.p.h. the wing loading would have to be less than 6 lb. per sq. ft., and to land at 30 m.p.h. it could not be more than 3.

While a landing speed of between 35 and 40 m.p.h. is extremely desirable (although practically never obtained) for instruction and for the inexpert private owner, our present landing speeds are not excessive for competent commercial pilots. The point is to recognize our landing speeds for what they are. There is no use fooling ourselves or falsifying our advertising. It simply reflects back upon the manufacturers and throws the whole subject of airplane performance into disrepute.

The most important characteristics which make an easy landing airplane may be placed in this order: (1) adequate control at slow speeds; (2) low vertical velocity or sinking speed; (3) easy-acting and non-rebounding landing gear.

An excessive sinking speed will make a hard landing no matter how slow the horizontal speed may be. Also, the faster the sinking speed, the quicker the pilot must be. On the other hand, a plane having unusually low sinking speed will take the whole flying field after levelling out before it touches the ground and is usually known as a "floater." There is practically no test data available on this subject. A theoretical investigation is therefore of interest.

determine the total drag at any air speed. Substituting this drag back into Equation [4] and writing gross weight in place of lift, we have the sinking speed corresponding to any horizontal speed of flight. The minimum sinking speed generally corresponds to a speed of flight near the landing speed of most airplanes and no great error is introduced by taking the sinking speed which corresponds to the landing speed as the actual minimum.



Top Row Left to Right: Fig. 2 Maximum-Lift Coefficients From M.I.T. Wind-Tunnel Tests. 36 In. × 6 In. Models Used. 40 M.p.h. 7½ Ft. Tunnel. Fig. 3 Maximum-Lift Coefficients From N.A.C.A. Wind-Tunnel Tests. Fig. 4 Maximum-Lift Coefficients From Flight Tests. Fig. 5 Comparison of Landing-Speed Coefficients From Wind-Tunnel and Flight Tests

Bottom Row Left to Right; Fig. 6 Landing Speed Vs. Wing Loading for Various Lift Coefficients. Fig. 7 Theoretical Sinking Speed for Typical Airplane. Fig. 8 Load Factors in Landing

The sinking speed is easily calculated by the formula:

$$V_s = V/(L/D) \quad \ldots\ldots\ldots\ldots\ldots\ldots [4]$$

where V_s = sinking speed in m.p.h.
V = speed of horizontal flight in m.p.h.
L/D = lift/drag ratio of the airplane.

The derivation is quite obvious when it is remembered that $1/(L/D)$ is the tangent of the angle of descent in a glide.

Generally the L/D of an airplane under flight conditions is not known, but may be estimated using Driggs method of simplified performance calculation, where the drag is taken as the sum of the induced drag and the parasite drag. The parasite-drag coefficient is assumed constant for all air speeds.

Knowing the power and high speed of the airplane, the total drag at high speed can be determined by assuming a value for the propeller efficiency, usually 75 per cent. If the induced drag is calculated and subtracted from the total, the remainder is the parasite from which the parasite-drag coefficient can be computed. Since this coefficient is assumed constant, we can

In Table 3, the minimum theoretical sinking speed has been calculated in this way for a number of typical airplanes, using the published high speed and rated power. Since the induced drag is the most important resistance factor in slow-speed flight, and since it depends directly on the span loading, the theoretical sinking speed has been plotted against span loading in Fig. 7. The sinking speeds vary between 5 and 8 m.p.h. This is obviously too slow. Any one who has landed without levelling out will bear witness to the fact that he hit harder than 8 m.p.h.

There are two probable reasons for these low theoretical values. First, the parasite drag has been assumed constant, which is correct over the usual flying range. However, the profile drag of the wing increases very rapidly at maximum lift, and hence the parasite drag is increased. Second, the drag of an idling propeller is a very large factor. The result of both of these drags is to reduce the L/D and increase the sinking speed. Taking one of the planes from Table 3 with a theoretical sinking speed of 6 m.p.h. and making approximate allowance for the above factors, the sinking speed would be increased to roughly 17 m.p.h., a much more reasonable figure.

It therefore appears that this theory is scarcely adequate, and in the absence of flight test data, we are largely ignorant of one of the important phases of landing, the value of the sinking speed.

When an airplane strikes the ground with a vertical velocity or sinking speed, the shock absorbers resist with a nearly constant force (the modern oleo type) and the load factor K imposed upon the airplane is given by the equation:

$$K = v^2/2gs \quad \ldots\ldots\ldots\ldots\ldots\ldots [5]$$

In other words, the load factor in landing is equal to the square of the vertical velocity divided by $2g$ and divided by the stroke of the oleo. The actual weight of the airplane does not enter in. The larger the stroke, the smaller the load factor and the easier the landing. We are apt to forget the fact that a long stroke is just as important on a small airplane as on a big one.

TABLE 3 THEORETICAL SINKING SPEEDS OF TYPICAL AIRPLANES

Airplane	Type	Sinking speed, m.p.h.	Span loading, lb. per ft.
Dornier "Superwal"	2000 hp. Flying Boat	7.98	295
Fokker F-10	1275 hp. Tri-Motor	7.20	162
Boeing 80 Transport	Biplane Tri-Motor	6.82	185*
Fairchild 71 (loaded)	425 hp. Cabin Type	6.69	115
Fairchild 71 (light)		5.43	73
Fairchild FC-2	200 hp. Cabin Type	6.12	86
Monocoupe	60 hp. Cabin Type	5.35	39
D.H. "Gypsy Moth"	90 hp. Open Biplane	5.07	48.5*

* Biplanes are figured with equivalent monoplane span.

The above equation is in the usual foot-pound-second units of theoretical mechanics. Substituting for g its value 32.17, and converting v into m.p.h. and s into inches, we obtain:

$$K = 0.4(V_s)^2/s$$

where V_s = sinking speed, m.p.h.
K = load factor obtained in landing
s = travel of oleo, in.

The value 0.4 is in reality 0.4012, the conversion factor.

In Fig. 8, the load factor is plotted against sinking speed for oleos of various lengths of stroke. The importance of the long stroke is at once obvious. Taking a sinking speed of 8 m.p.h., a 3 in. oleo travel will give a load factor of 8.5, while the factor is only 4.3 for a 6-in. travel and 2.2 for a 12-in. travel. The travel of our present-day oleos is between 4 and 8 in. If we could design our plane to have an actual sinking speed of 12 m.p.h. with an oleo travel of 12 in., the pilot could cut the switch and settle into the ground without leveling out and the load factor would be only 5, which is not prohibitive.

If the engine is running at part throttle, the drag of the propeller is eliminated, and if the landing is made at somewhat above the minimum landing speed (that is, a two-point landing at part throttle), the rapid increase in wing drag is also eliminated and we approach the theoretical sinking speeds shown in Fig. 7. Thus it is perfectly possible with present-day aircraft to land without levelling out, as has been repeatedly demonstrated by students, without damage to themselves or to the plane. The load factor in these cases must be below 4, and the sinking speed between 6 and 8 m.p.h.

No discussion of landing speed is complete without mention of the usual auxiliary devices for reducing the landing speed, namely, flaps and slots. This is a very controversial subject and we shall only touch upon it lightly. We must again recognize that the flight test is the only reliable test, and that the wind tunnel is open to error in high-lift tests because of scale and turbulence effects.

The original Fairchild monoplane of 1926 was equipped with flaps. This machine is still in commission. A reduction in landing speed of 3 to 4 m.p.h. was actually obtained. In spite of this, it is our constant experience that the pilots prefer to land at the higher speed rather than operate the very simple flap control, even after considerable experience in using it. More recently, a standard Fairchild FC-2 was fitted with an auxiliary leading edge along the entire span, closely resembling the well-known Handley Page slot. The landing speed was reduced about 5 m.p.h., corresponding to an increase in area or in lift coefficient of 17.5 per cent. In this installation, difficulty was experienced in adjusting the leading edge so that the ship was neither wing-heavy nor inclined to yaw.

The most successful slot installations are not intended to decrease the landing speed, but to increase the lateral stability in a stall, in which they are undeniably effective. Such installations are found in the DH "Moth" and the Avro "Avian." We must not expect these machines to land appreciably slower. No amount of wing slots or flaps will decrease the vertical velocity or sinking speed. On the contrary, they are apt to increase it, since they increase the parasite drag and decrease the L/D. The best way to get low landing speed is to have low wing loading. No matter how effective the auxiliary devices may be, they are open to the distrust of most pilots and all maintenance men who see in them just one more "gadget" to get out of order. To solve the problem of the easy landing airplane, as with every other problem, we must not only recognize the facts which are known, but we must form a very clear idea as to the extent of our ignorance.

Discussion

HARLAN D. FOWLER.[2] The desire for low landing speed is always with us because of its safety element. Its limitation is gradually being deliberately evaded in the necessity of obtaining increased high speed due to commercial demands. That this is not conducive to the ideal condition is of course recognized by those who are bringing about higher landing speed.

It is an economic necessity to provide some reasonable wing device that is mechanically simple enough in principle to warrant its adaptation to everyday service requirements. We are advocates of the variable area wing, demonstrations of which have given very promising results. It is quite obvious that high landing speed brings with it higher sinking speed. It can be shown that with the variable area wing the sinking speed may be appreciably reduced from that of a high landing speed plane.

We will say, however, that for the same landing speed the extended wing will have a little higher sinking speed. The following table may be of interest in this respect. The actual sinking speed is taken as three times the calculated V_s.

Type	Actual V_s	Landing speed	High speed	Wing area	Span load, lb.
Conventional biplane	15.5	46.2	89.3	338	55
Normal monoplane..	25.2	58.2	98.8	136	66
Extended area.....	20.0	44.5	78.8	166	66

It is noted that for one-half the wing area a lower landing speed for the extended wing is obtained than for the biplane, with an increase in sinking speed of 4.5 m.p.h. Also that the normal plane had its landing speed decreased about 14 m.p.h. by using the extended area and also a decrease in sinking speed of 5 m.p.h. These figures are based on full flight data. An OX-6 engine was used.

It should be noted that the Fowler variable area wing has the decided advantage in that it can be set for any desired position, whether for best climb, high speed, or landing, and it will stay there without distracting the pilot's attention. Also

[2] Chief Engineer, Miller Corporation, New Brunswick, N. J.

that it cannot become misaligned or cause unequal controlling forces because of its positive means of sliding location.

The value of the variable area wing should be considered in the light of preference, whether to continue to go to still higher and more dangerous landing speeds or to accept the comparatively simple mechanical wing device and go back to where we do know what is best for us and still maintain the higher performances.



FIG. 9 FOWLER VARIABLE AERO WING

AUTHOR'S CLOSURE

Up to the present not one of the more experienced airplane manufacturers has been able to make use of a variable area wing, because of the complications in structure which this involves. It is extremely doubtful whether an airplane with a variable area wing could be sold to any operating company for the same reason. A variable area has the same drawbacks that are inherent in flaps and slots; namely an increase in the complication and maintenance features of the airplane, only with the variable area the case is somewhat more severe. At the present time the only means of obtaining low landing speed, which has proved successful in commercial operation, is a light wing loading, and it is yet to be shown that a light wing loading necessarily brings about a slow speed airplane.

Air Transport in New England

By SUMNER SEWALL,[1] BOSTON, MASS.

THE measure of usefulness of any vehicle of transport is its ability to deliver its load safely and speedily at a predetermined point. To perform this function today in New England with the conventional aeroplanes equipped with wheel landing gear is, with few exceptions, little short of complete folly. Few towns have seen their way clear to finance the construction of suitable airports, and there are still fewer natural landing fields in this beautiful scenic region.

There is another solution of the problem. If enough suitable landing fields for our present type aeroplanes cannot be constructed soon, airplanes should be built which will be suitable for landing on the surfaces which are already available. Efforts should be made immediately to develop and produce a satisfactory amphibian plane. This will extend in a large measure the advantages of air transport to every city of any size in New England, for almost without exception, these will be found to lie on bodies of water suitable in size and character to handle flying-boat operations safely. Not the boats of today, it is true, but those of tomorrow, for this type has demonstrated its capacity for greatly improved performance, under further scientific development.

Successful prosecution of this will open up the bays and harbors of the coastline, and the lakes and rivers of the interior. The tourist and sportsman can be transported to his favorite resorts, and the business man to his trade centers. The ski-pontoon combination if developed properly will land on the snow or ice if the surface of a body of water be frozen, and on the open water if it be thawed. A ski-wheel combination will land on either snow or bare ground as conditions may present themselves.

Such amphibian types have far greater value than is perhaps first realized. In addition to enlarging immediately the useful scope of the airplane many fold by increasing its landing availability, the amphibian also offers a far greater factor of flying safety than is possible in most types of conventional planes equipped with landing wheels. In this respect, it compares most favorably with that costly luxury know as the multi-motor plane in which machine at a tremendous cost in both capital investment and pay load, one is given reasonable assurance of staying up long enough to reach a suitable landing place, in case mechanical trouble develops. Isn't it better to have a plane constructed so that there is reasonable assurance of being able to effect a safe landing at almost any point along the route without need of elaborate airport facilities? At the present time, weather will force down a multi-motored ship as quickly as one powered with a single unit.

Another point, strongly in favor of the amphibian type, yet apparently entirely overlooked by the majority of operators, is its ability to cruise with the utmost safety at an extremely low altitude along rivers and waterways. It has often been strikingly demonstrated that passengers enjoy flying infinitely more at low altitudes than at high, as it is only when comparatively near the ground that one can appreciate the speed of flight, and view with interest the details of the country over which one is passing.

Furthermore, on days of low ceilings, it is not infrequent that a flying boat can, by flying close to the water, safely carry out a flight which would be extremely hazardous, if not impossible, to operate with a land plane. Thus, in this safe low-flying characteristic of the amphibian, there is both increased salability and improved regularity of service. Upon this latter point stands the basis of traffic.

During the last two years United States air-transport operators have been shown beyond shadow of doubt that Americans will fly. There is no country in the world where speed is at a greater premium than in the United States. Demand for aviation certainly exists. Great potential traffic awaits only suitable air-transport service. In New England at least, landing surfaces exist in generous abundance, but planes must be adapted to take advantage of them. This is a problem in design that is up to the engineer.

Discussion

B. A. POLLETT.[2] The basic reason for delay in the transporting of air mail is the weather. The figures of a year's operation of the Colonial show that 97 per cent of the uncompleted trips were uncompleted because of weather. Facilities for handling mail once it gets in the airport is very important, and perhaps for that reason it has not actually to do with the operation of the airplane itself. The airplane system is set up to operate in some 72 or 75 places, and it has attempted to coordinate with half a dozen different agencies which have been in existence for a period of time, and which find it difficult to change their methods. This refers primarily to railroad schedules, postoffice service, and the railway-mail service, which is another part of the postoffice department. The organization of the flying part of the air mail is gradually being coordinated with that of the various agencies, but when it is recalled that there are many places where adjustments have to be made in order to interlock with another, one can see where single letters or single pouches will be delayed, and will be transferred to the wrong train or to the wrong point.

An organization of that sort is allocated to different individuals who often are not very well informed. Things come up that they are not prepared for. A good example, and one which will more or less answer the question, is one that happens on the line with which the writer is connected. It takes mail from Boston to New York. Should the pilot fly as far as Stafford Springs, which is 15 miles this side of Hartford, and find a fog on top of the landing field, which happens time and again, the mail is taken back and discharged at an emergency field at Webster, ten miles from Worcester. From Webster it is taken to Worcester and supposedly transferred to the next train to go out.

The organization has not been formulated so as to take care of exigencies of that sort, and mail will arrive there and a number of men will look at the mail bag and wonder what is going to happen to it. They have little idea of the organization of the air-mail service, and they have only a general idea of what direction that mail has to go. That is gradually being eliminated, and one of the primary moves to eliminate that particular source of trouble is the reorganization of the postoffice department which makes the railway-mail service entirely responsible for the despatch of air mail between points.

REGINALD W. THOMAS.[3] The writer has flown seaplanes and

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] Traffic Mgr., Colonial Air Transport Co., Boston, Mass.

[3] Lieutenant, U. S. N. Air Station, Quincy, Mass.

landplanes, and agrees with everything that Mr. Pollett has said about the feasibility of using amphibians. He states it is absolutely a job for the designer of new craft. After analyzing the best types of flying boats both in this country and in France, England, Germany, and Italy, it appears that there are only a few engineers in the world today who put all their time into developing flying boats or amphibian types of planes.

In this country Curtiss brought out the original flying boat, and every other design in this country has been influenced by Curtiss design. It happens that Curtiss' first boats were not of a graceful type to fly. They did not have especially good performance and did not give the pilots confidence. Practically every other boat in the United States, that has been built along that design, has had the same characteristics, which did not render them conducive to further development or use.

During the War Curtiss sold England many boats, and the English aeronautic press stated that if England had not seen a Curtiss boat, it would be five years ahead of its wartime flying-boat program. By the time it was found that the Curtiss boat was not what was wanted, they were actually five years behind, in developing a satisfactory machine.

There are four or five companies in France that are building flying boats. Germany has one or two, the Dornier and the Rohrbach. In Italy there are the Savoia and the Macchi boats. There is no question that flying boats and amphibian-type planes are very desirable, and are going to eliminate many of the conditions which now hamper transportation. As Mr. Pollet says, it is a job for new engineering by naval architects instead of aeronautic engineers to start thinking of boats. They have built very satisfactory land planes. The designers are concentrated on the land-plane type, and as soon as somebody says they want a plane for water transportation they stick on a couple of pontoons. The pontoon flying machine works out fairly well but it will not do the job as well as a well-designed boat. It is a job for engineers to get busy and build them from the bottom. French flying boats fly gracefully and easily, and they are just as easy to handle as any good-flying land plane. If flying boats are mentioned to nine-tenths of the pilots, they will say they do not fly well, that they do not like them. That is too bad, because there are a lot of boys who think a boat will not fly well and gracefully, and naturally that has been one reason boats have not been developed further.

Author's Closure

One discusser wished to know if it is practicable to use amphibian planes to reach inland cities on bodies of water. It is entirely practicable and has been done and is being done. Probably the only reason that amphibian planes have not been developed more is because their design at the present time has not been brought to the point where certain basic defects have been overcome.

The amphibian plane in order to be practicable for landing on water has to combine the quality of a boat and that of an airplane. They are heavy, they have considerable air resistance, and they require more power to get certain speeds. All of those things will have to be overcome. The design of them will have to be refined before the amphibian can be used practically by commercial operators. I think a good example of that is the Loening amphibian which is not commercially a good ship. It is reliable and well built, but it uses too much of its potential speed, its potential maneuverability, and of its potential dead load capacity in its design. That is the one impracticable feature of the amphibian, and is probably the reason it is not being used more generally by commercial operators.

The amphibian, combines the quality of the boat with an arrangement of wheels, retractable, or whatever arrangement there may be, which makes it possible to land at an airport or any place where an airplane can land. It can with practicability fly to any inland city and land provided the inland city is alongside a body of water.

AER-51-16

Ball and Roller Bearings in Aircraft

By FREDERICK W. MESINGER,[1] NEW YORK, N. Y.

MECHANICAL equipment of modern aircraft can be listed in a general way as engines, starters, magnetos, superchargers, instruments, generators, radio equipment, gearing and controls. This list does not, of course, include special equipment for aircraft designed for the Army and Navy. Bearings finding favor in aircraft construction include the straight cylindrical roller bearing, the orthodox rigid radial ball bearing, and the open-type ball bearing. These three types practically cover all the requirements of the makers of the mechanical equipment. Each of these bearings has a definite part to play.

The straight cylindrical roller bearing is selected where heavy radial loads are imposed and where definite space limitations control the size of the bearings. Radial ball bearings of the closed-race type are used where ordinary radial loads and thrust loads are encountered, while the open type is used in applications where advantage can be taken of the easy assembly feature of this type of bearings and its high degree of precision.

It is not possible to discuss every anti-friction bearing application that exists in the completely equipped airplane. It is, however, possible to examine the more interesting mountings that have been definitely adopted.

Engines

The Wright "Whirlwind" J-5 engine is the result of many years of development and research and is designed with equal thought to reliability and efficiency. The crankshaft is suspended on three bearings, two of which are of the straight roller type and the other a single-row radial ball bearing. The function of these bearings is very definite. The two roller bearings carry the main crankshaft radial loads. These loads are necessarily heavy and the bearings are subjected to considerable shock, since these two bearings must withstand the loads created by the power impulses in nine cylinders firing in rapid succession. At 1800 r.p.m., these bearings are subjected to 8100 load impulses per minute. All thrust load in the engine and created by the propeller is taken by the ball bearing. The propeller thrust, obviously, is considerable, but well within the capacity of the bearing selected. It has rarely been found necessary to replace these bearings during the life of the engine. Apart from their ability to handle the loads encountered, these bearings, because of their low weight per pound of load-carrying capacity and their small proportions, are highly desirable as keeping the weight per horsepower of this type of engine at such a low figure. The narrowness of these bearings has a direct effect on the overall dimensions of the engine and in reducing the amount of metal in the housing.

The Pratt & Whitney "Wasp" engine develops 425 hp. and together with its larger model the "Hornet" is finding much favor with the services as well as commercial organizations having use for such a large engine. The crankshaft bearing arrangement is virtually the same as that of the "Whirlwind" engine. There are some eight or ten distinct makes of radial engines available here and in Europe, all of which make use of this construction. The most interesting feature of this engine, from the viewpoint of the study of anti-friction bearing applications, is that of the built-in supercharger. This gear-driven unit, employing the General Electric type of impeller, operates at 22,000 r.p.m. at full throttle. Both the gearing and the impeller shaft are mounted on anti-friction bearings with the roller bearing receiving the assignment at the point of heaviest radial load, and the ball bearing doing the anchoring and absorbing of the thrust. A well-made and well-designed roller bearing, of the straight cylindrical type, can be operated at very high speeds equally as well as high-grade ball bearings. One of these engines has a log of over 800 hours. On the basis of average operation at three-quarter throttle, the supercharger impeller bearings have revolved over 400,000,000 times. Owing to the difficulty of lubrication and the angular application of the load, it has been found practical to use ball bearings in the rocker arms. Ball bearings are also used on the starter shaft and gun-fire control drive.

The Fairchild-Caminez four-cylindered unit has the camshaft assembly mounted on ball and roller bearings. The cam actuates the pistons by means of a cam roller built into the piston. This cam roller rotates on roller bearings built up to special dimensions to conform to the space limitations. The side links that control the proper relationship of the piston in the cylinders that are not transmitting power are also fitted with roller bearings. These again are special, due to the space limitations.

On the crankshaft assembly of the D-12 Curtiss V-type water-cooled engine, only one ball bearing is used. This bearing is incorporated to take care of the propeller thrust. The main bearings in this engine are still of the sleeve type, as the application of anti-friction bearings to multiple crankshafts presents a nice problem. It can, however, be accomplished either by means of a built-up crankshaft or by using a bearing with a large bore and introducing a split sleeve to make up this extra diameter. Ball bearings support the drives of the magnetos, distributors, cam, gun-fire control, generator, and water pump. Fifteen or sixteen radial ball bearings are used in this complete assembly. The simplified oiling system and the ability of these bearings to absorb the thrust created by the bevel gears have been the designer's justification for their use.

One engine that offers a particularly fitting application is the British Napier "Lion." This 12-cylindered engine is composed of three banks of four cylinders. Roller bearings have been applied to the main crankshaft. The means of accomplishing this is by supplying split sleeves to take care of the large bore necessary in threading over the crankshaft. It is often found in European practice that these bearings have been made somewhat special in order to simplify their housing construction. Contrary to American practice, the propeller thrust is taken on an orthodox thrust bearing, while roller bearings and ball bearings will be observed in practically all places where a bearing of any description is required. European aviation engines use anti-friction bearings extensively, and there are several instances where connecting rods are also equipped with ball bearings on the big ends.

Instruments

Under this heading, from an anti-friction point of view, can appear any instruments that have rotating elements, and possibly the most interesting is the Pioneer earth inducter compass.

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Presented at the Boston Regional Meeting of The American Society of Mechanical Engineers, Oct. 1 to 4, 1928.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

This instrument employs an anti-friction bearing at every point of rotation. The main shaft, which carries the armature, is suspended on three rigid-type ball bearings. In addition to this the governor uses four small ball bearings on the gimbles. Dealing with such minute currents, it was found necessary that

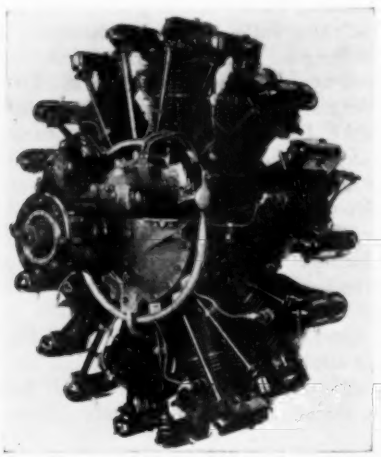


FIG. 1 WRIGHT "WHIRLWIND" J 5 AERO ENGINE

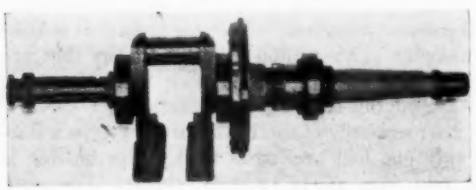


FIG. 2 CRANKSHAFT ASSEMBLY OF WRIGHT "WHIRLWIND" J 5 ENGINE

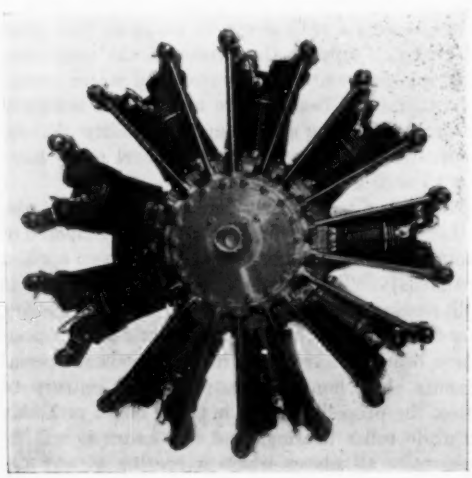


FIG. 3 WRIGHT "CYCLONE" AERO ENGINE

constant friction should be maintained in varying temperatures and with varying speeds. With this and the simplified lubrication required by these bearings, the use of anti-friction bearings was found to be a necessity.

Another instrument equipped with anti-friction bearings is the air-distance recorder. This instrument records the air distance by registering the number of rotations of the small propeller in the head. A fixed constant must be designed into the recording mechanism based on a uniform coefficient of friction for the bearings. Should this friction vary, the speed of rotation of the propeller for given velocity through the air would vary and cause the reading to be inaccurate. With varying temperatures, due to low and high altitude flying, any bearing depending upon an oil film, the consistency of which would undoubtedly change with the temperature variation, would not be able to maintain a coefficient of friction sufficiently constant to permit this instrument to return an accurate reading. Modern tachometers, to conform with Navy specifications, must now be fitted throughout with ball bearings. There is little doubt that commercial units will follow this specification also.

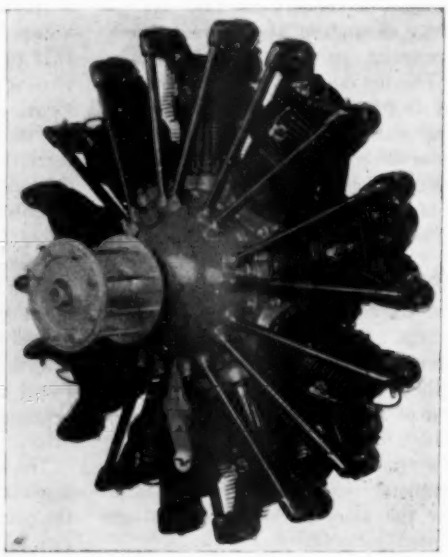


FIG. 4 PRATT & WHITNEY "WASP" AERO ENGINE

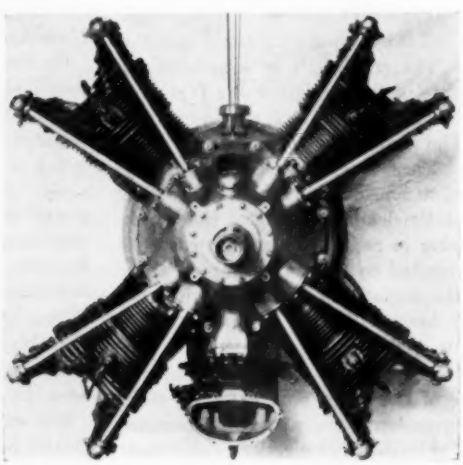


FIG. 5 FAIRCHILD-CAMINEZ AERO ENGINE

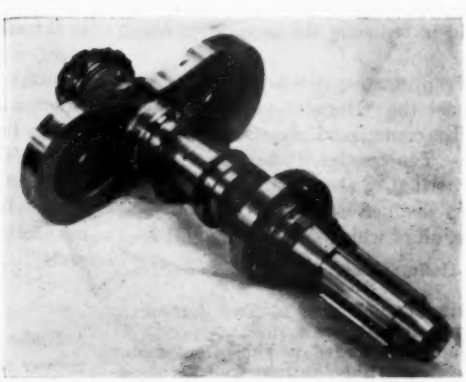


FIG. 6 CAMSHAFT OF FAIRCHILD-CAMINEZ ENGINE

STARTERS

There are not many makes of airplane engine starters on the market, but of those that are, undoubtedly the Eclipse starter is best known and more generally used. This starter is practically completely equipped with anti-friction bearings. The compactness of this unit is the direct result of use of anti-friction bearings. Due to the impracticability of fitting an airplane engine with a ring gear such as is used in automobile engines, the airplane starter is faced with the difficult problem of transmitting sufficient torque directly to the crankshaft. To do this a small flywheel is rotated at high speed to store energy later transmitted to the crankshaft through a clutch. The gear ratio from hand crank to fly wheel is 150 to 1, giving a flywheel speed of from 12,000 to 16,000 r.p.m. depending on the operator. Enormous pressures are exerted on the gears

and bearings, and with the simplified lubrication arrangement it is found that ball and roller bearings meet all requirements. This is a really severe service application.

Magnetos

While the magnetos on aircraft engines differ only very slightly from the familiar designs of the automobile type, they are nevertheless a very interesting application, since absolute reliability must be obtained. The rotating element must maintain a minute and uniform air gap between itself and the magnets, consequently every effort is put forward to obtain rotation with a minimum of eccentricity. It is for this reason, together with simplified lubrication at high speed, that the open-type ball bearing is selected for this service. The open-type bearing also lends itself admirably to ease of assembly. The Scintilla magneto uses this type of bearing. The distributor is also mounted on open-type bearings. A distinct development directed at the aircraft engine is the dual magneto. The saving of weight in this case, both in the magneto itself and the mount, will undoubtedly make this type very popular with makers of multicylindered aircraft engines. Ball bearings are generously used in this design.

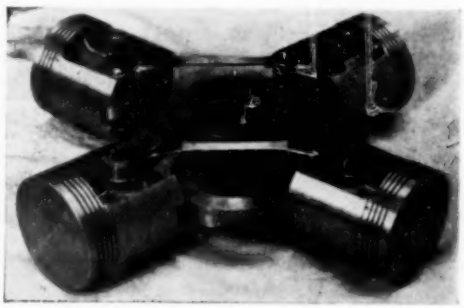


Fig. 7 Fairchild-Caminez Piston and Link Assembly

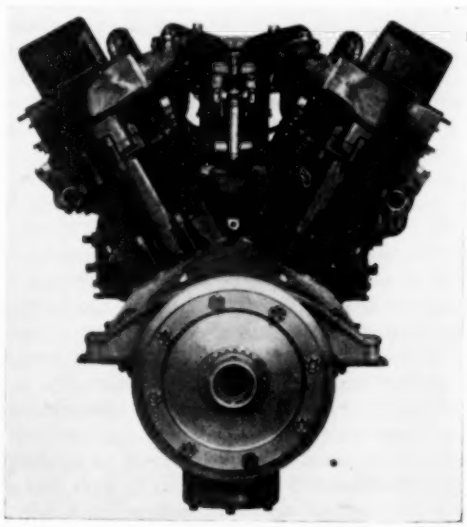


Fig. 8 End View of Curtiss D 12 Engine

Superchargers

Superchargers are becoming essential elements to the well-designed aircraft engine installation to such an extent that manufacturers are building them directly into the engine. This has already been mentioned in the cases of Wright "Cyclone" and the Pratt & Whitney "Wasp" and "Hornet," to which might be added the Bristol "Jupiter," the Curtiss "Chieftain," and many others. There are still a large number of engines that employ a supercharger that is attached. Of these the high-speed impeller type such as is developed by the General Electric Company, the Wright field engineers and the Roots blower modifications as built by the Allison Engineering Company predominate. In all cases anti-friction bearings are used, while in some instances bearing speeds of 50,000 r.p.m. are maintained.

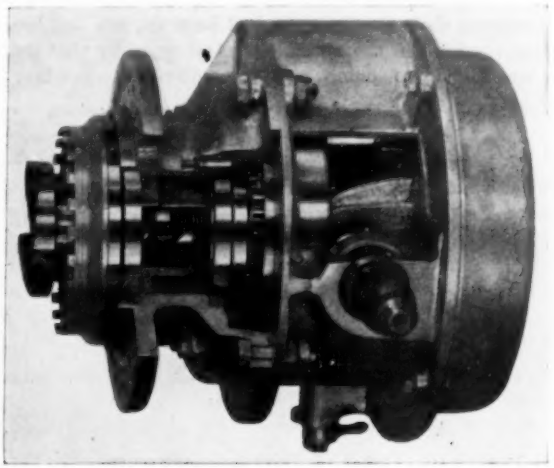


Fig. 10 Eclipse Inertia Starter

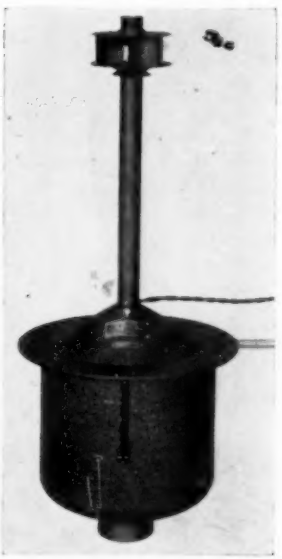


Fig. 9 Pioneer Earth Inductor Compass

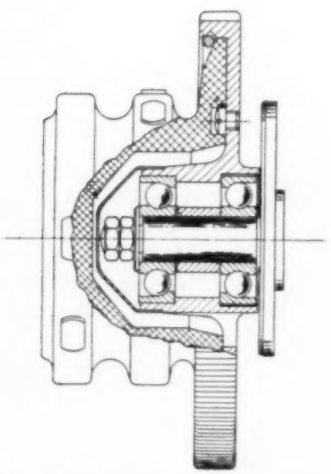


Fig. 11 Distributor Gear Assembly on Scintilla Aero Magneto

Generators

There is not much to be said under this heading, except that like the generators on automobiles, ball bearings are generally used. The service is more severe in the aircraft application, and therefore the design of the mechanical part of the electric generator must be somewhat more substantial.

Gearing

The geared engine in conjunction with three- and four-blade propellers has a definite position in the industry. These gear

reductions are sometimes built in and sometimes attached. In both forms ball and roller bearings are extensively used. The built in gearing of the Napier "Lion" engine has already been referred to. This application, as in most gear reductions for aircraft, is of the orthodox type employing spur gears.

There are still a number of applications that merit mention but which are more or less isolated. Wind-driven radio generators suspend their armatures on ball bearings, and ball bearings are incorporated in the variable-pitched propeller that governs the speed. Gyro bomb sights, bomb-release handles, fuel pumps, etc., depend upon ball bearings for proper functioning.

Ball bearings are being used extensively on the Ford ships. They are used in the hinges of the ailerons, the elevator fins, and the rudder. Throttle control for the outboard engines on these multiengined ships is obtained through a series of levers and cranks. In order to preserve the essential sensitiveness, ball bearings are used in all cranks. Other plane builders have designed ball bearings into vital control positions, and there seems little doubt that as airplanes grow larger this practice will increase.

Commercial Aircraft Engines of Medium Power

By RICHARD M. MOCK,[1] NEW CASTLE, DEL.

AMERICAN commercial airplanes were not produced in any considerable quantity until approximately 1925, when the OX-5 engines were selling at about $300 each. Naturally a power plant on the market at this price discouraged the development and manufacture of any new engines solely for commercial purposes. Commercial airplanes, rather than being built for a definite purpose, were being designed as a general type around this engine; hence, the great similarity in designs. This condition continued for some time, with the price of the OX-5 slowly rising and the supply diminishing. Toward the end of 1926 or early in 1927, manufacturers of airplanes began to sense that soon the supply of these engines would be exhausted; the price was then about $500. At this time a few modern engines were under development to replace the OX-5, but all were still far from the production stage. By the end of 1927, the OX-5 was no longer so cheap an engine that it could be discarded rather than go to the expense of overhaul, as had been the case in the past. In fact, aircraft engines of this size were so scarce that many airplane manufacturers found themselves with a number of completed planes in their factories but without engines. Desperately they began to look about for available engines, and many used and overhauled OX-5's were installed in new production airplanes. Other types that had been produced in smaller quantities during the war were used more and more to overcome the scarcity.

It was not long before airplane manufacturers began to raise their prices or sell their planes without engines, leaving it to the purchaser to supply his own power plant; of course, to be installed at the airplane factory. This situation brought a number of European engines to this country. In Germany, after the war, military machines were destroyed and it was some time before she was allowed to build powered airplanes. However, after the ban was lifted, Germany began production of airplane engines, and it was not long before they were being exported to this country. When the shortage of Curtiss OX-5 engines was first felt, a number of companies were formed or expanded for the importation of aircraft engines.

During this shortage of medium-powered engines, American manufacturers were not idle and many new designs were nearing the production stage. In the past year a number of new engines have appeared on the market, approximately a dozen. Many of these new engines are the result of considerable experiment, while a few were produced by organizations which seemingly had little use for or no intention of producing aircraft engines a year or a year and a half ago. In spite of this fact, it seems that most of the new engines are performing creditably.

It is quite generally accepted that the proper engine for a medium-powered plane shall be air cooled. As to the horsepower desirable, much depends upon the skill of the airplane designer. It appears that an engine of from 60 to 200 hp. should be sufficient for a plane carrying two to four people with a reasonable amount of luggage and gasoline. In order not to have the power loading too high and to have a reasonable amount of reserve power, the rated horsepower might be anywhere between these figures, depending upon the gross weight and the efficiency of the airplane. Because of the lack of power plants of intermediate power, there has been a tendency in the past to design airplanes that are underpowered.

On the other hand, some more conservative designers, also lacking a power plant of intermediate power, have installed engines in their planes developing power greatly in excess of that required. The idea is to use the engine at part throttle and thus prolong its life as well as to have a reserve of power for use in an emergency such as taking off from a small field.

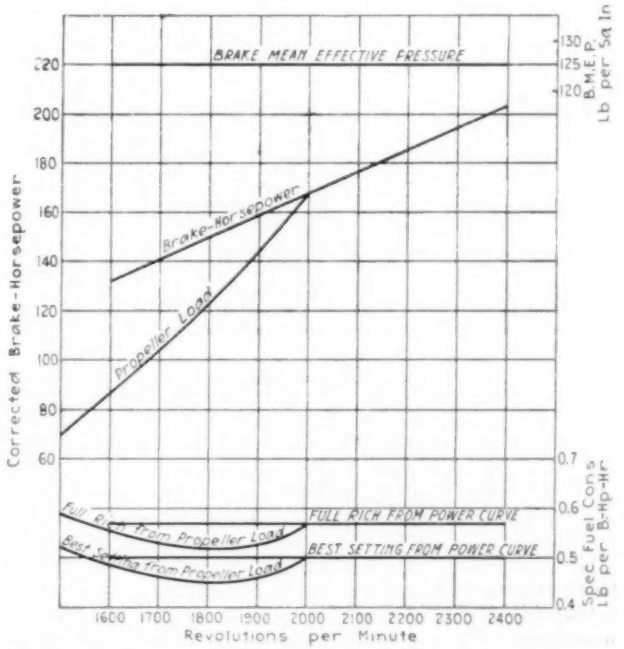


FIG. 1 ESTIMATED POWER AND FUEL CONSUMPTION OF WRIGHT MODEL R-540, FIVE-CYLINDER AIRCRAFT ENGINE. BORE 5 IN., STROKE 5½ IN., COMPRESSION RATIO 5.2 TO 1, ROTARY INDUCTION, AVIATION GASOLINE

It seems that the solution to the problem lies in having a variety of engine sizes available. These engines should have interchangeable mounting to facilitate installation in various types of planes without any changes in the primary structure of the airplane. Of course, this pertains only to engines with the same cylinder arrangement. Today, though many of the commercially available engines are similar in design, there are hardly two that have the same mounting-bolt dimensions.

There has been much discussion as to the best disposition of cylinders for a medium-powered, air-cooled engine. Of the engines now commercially available, there is a remarkable similarity in their design. The majority are air-cooled radials of the general type, construction, and arrangement of accessories of the Wright Whirlwind J5 except that, unlike the Whirlwind, the magnetos are behind the cylinders. Most of these engines, though using the best quality materials, are of somewhat cheaper construction, employing cast-iron cylinders, etc.

[1] Aeronautical Engineer, Bellanca Aircraft Corp. Jun. A.S.M.E. Presented at the Meeting of the A.S.N.E. Aeronautic Division, Wichita, Kan., September 21, 1928.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

TABLE 1 MANUFACTURERS' SPECIFICATIONS ON
(The table is believed to be accurate but no

Manufacturer or agent	Engine	Cooling system	Cylinder arrangement	No. of cylinders	No. of cycles	Propeller drive	Rated hp. and r.p.m. at sea level	Maximum hp. and r.p.m. at sea level	Weight dry without hub or starter	Weight per rated hp.	Bore and stroke (in.)	Compression ratio	B.m.e.p.	Displacement (cu. in.)
Aircraft Engine Co.	Comet	A	Rad	7	4	D	150–1800	.....	...	...	$4^{1}/_{2} \times 5^{1}/_{2}$	...	...	...
Allison Engineering Co.	V1410	A	IV-45°	12	4	D	410–1800	430–1900	1000	2.44	$4^{5}/_{8} \times 7$	5.3	128	1410
Allison Engineering Co.	VG1410	A	IV-45°	12	4	G^{1}	410–1800	430–1900	1125	2.62	$4^{5}/_{8} \times 7$	5.3	128	1410
Allison Engineering Co.	V1650	W	IV-45°	12	4	D	420–1700	450–1800	885	2.10	5×7	5.4	122	1650
Brownback Motor Laboratories*	Anzani 3-35	A	Rad	3	4	D	35–1700	45–1800	115	3.29	$3^{23}/_{64} \times 4^{23}/_{32}$	5.1	...	119
Brownback Motor Laboratories*	Anzani 6-80	A	Rad	6	4	D	80–1500	89–1600	215	2.69	$4^{1}/_{8} \times 4^{15}/_{16}$	5.1	...	397
Brownback Motor Laboratories*	Anzani 10-120	A	Rad	10	4	D	120–1500	131–1603	320	2.67	$4^{1}/_{8} \times 5^{1}/_{2}$	5.5	...	736
Curtiss Aeroplane & Motor Co.	OX-5	W	V-90°	8	4	D	90–1400	.....	375	4.17	4×5	4.9	...	502
Curtiss Aeroplane & Motor Co.	OXX6	W	V-90°	8	4	D	110–1900	.....	390	3.55	$4^{1}/_{4} \times 5$	4.9	...	568
Curtiss Aeroplane & Motor Co.	C-6A	W	L	6	4	D	160–1700	165–1750	420	2.62	$4^{1}/_{2} \times 6$	5.2	133	573
Curtiss Aeroplane & Motor Co.	D12	W	V-60°	12	4	D	435–2300	445–2300	680	1.47	$4^{1}/_{2} \times 6$	5.3	135	1145
Curtiss Aeroplane & Motor Co.	Conqueror V-1570	W	V-60°	12	4	D	600–2400	615–2400	760	1.26	$5^{1}/_{8} \times 6^{1}/_{4}$	5.8	130	1569
Curtiss Aeroplane & Motor Co.	Conqueror GV-1570	W	V-60°	12	4	G^{2}	600–2400	615–2400	840	1.40	$5^{1}/_{8} \times 6^{1}/_{4}$	5.8	128	1569
Dayton Airplane Engine Co.	Bear	A	L	4	4	D	110–1550	125–1800	375	3.41	$4^{1}/_{2} \times 7$	5.3	118	441
Fairchild-Caminez Eng. Corp.	Caminez 447-C	A	Rad	4	4	D	120–960	140–1050	350	2.50	$5^{3}/_{8} \times 4^{1}/_{2}$	5.0	120	447
Hallet Mfg. Co.	H-526	A	Rad	7	4	D	130–1800	.....	425	3.26	$4^{3}/_{8} \times 5$	5.2	...	526
Irwin Aircraft Co.	Irwin 79	A	Rad	4	2	D	20–1700	25–2200	60	3.00	$2^{7}/_{8} \times 2^{3}/_{4}$	5.0	110	79
Kinner Airplane & Motor Corp.	K-5	A	Rad	5	4	D	75–1725	112–1880	231	2.30	4.25×5.25	5.0	...	372
Menasco Motors Co.	B-2 Salmson	A	Rad	9	4	D	260–1500	278–1760	542	2.08	4.92×6.69	5.2	...	...
Axelson Machine Co.	Axelson	A	Rad	7	4	D	150–1800	.....	430	2.87	$4^{1}/_{2} \times 5^{1}/_{2}$	...	...	612
O. E. Szekely Corp.	SR-3	A	Rad	3	4	D	40–1800	45.6–1900	144	3.8	$4^{1}/_{8} \times 4^{3}/_{4}$	4.8	100	191
Packard Motor Car Co.	1A-2775	W	X-60°	24	4	D	1200–2600	1300–2700	1500	1.15	$5^{3}/_{8} \times 5$	7	150	2775
Packard Motor Car Co.	3A-1500	W	V-60°	12	4	D	600–2500	650–2700	760	1.26	$5^{3}/_{8} \times 5^{1}/_{2}$	5.5	140	1498
Packard Motor Car Co.	3A-1500	W	V-60°	12	4	D	600–2500	650–2700	880	1.46	$5^{3}/_{8} \times 5^{1}/_{2}$	5.5	140	1498
Packard Motor Car Co.	3A-1500	W	V-60°	12	4	D	600–2500	650–2700	780	1.30	$5^{3}/_{8} \times 5^{1}/_{2}$	5.5	140	1498
Packard Motor Car Co.	3A-2500	W	V-60°	12	4	D	800–2000	835–2100	1160	1.45	$6^{3}/_{8} \times 6^{1}/_{2}$	5.7	135	2490
Packard Motor Car Co.	3A-2500	W	V-60°	12	4	G	800–2000	835–2100	1380	1.72	$6^{3}/_{8} \times 6^{1}/_{2}$	5.7	135	2490
Pratt & Whitney Aircraft Co.	Wasp	A	Rad	9	4	D	400–1900	.....	670	1.67	$5 \times 5^{3}/_{4}$	5.2	124	1344
Pratt & Whitney Aircraft Co.	Hornet	A	Rad	9	4	D	500–1900	.....	760	1.52	$6^{1}/_{8} \times 6^{3}/_{8}$	5.0	123.5	1690
Quick Motors Co.	Quick-Radial	A	Rad	9	4	D	125–1450	.....	325	2.6	4.13×5.51	5	...	667
Ryan Aeronautical Corp.*	Ryan-Siemens 5	A	Rad	5	4	D	70–1750	75–1850	258	3.68	3.94×4.72	5.6	110.5	287
Ryan Aeronautical Corp.*	Ryan-Siemens 7	A	Rad	7	4	D	96–1750	102–1850	328	3.40	3.94×4.72	5.6	108.0	402
Ryan Aeronautical Corp.*	Ryan-Siemens 9	A	Rad	9	4	D	125–1850	132–1850	382	3.05	3.94×4.72	5.6	109.5	517
Tips & Smith, Inc.*	Super-Rhone	A	Rad	9	4	D	125–1450	133–1650	15	2.52	$4^{1}/_{8} \times 5^{1}/_{2}$	5.2	...	667
Velie Motors Corp.	Velie M5	A	Rad	5	4	D	45–1750	80–2000	210	2.63	$4^{1}/_{8} \times 3^{3}/_{4}$	5.2	...	2506
Warner Aircraft Corp.	Scarab	A	Rad	7	4	D	110–1850	120–1925	270	2.45	$4^{1}/_{4} \times 4^{1}/_{4}$	5.2	112	422
Wright Aeronautical Corp.	Whirlwind J5	A	Rad	9	4	D	200–1800	225–2000	500	2.50	$4^{1}/_{2} \times 5^{1}/_{2}$	5.2	123	788
Wright Aeronautical Corp.	Cyclone R1750	A	Rad	9	4	D	525–1900	560–2000	760	1.52	$6 \times 6^{7}/_{8}$	5.0	130	1753
Wright Aeronautical Corp.	Hispano Suiza E-4	W	V-90°	8	4	D	190–1800	204–1800	480	2.52	$4^{3}/_{4} \times 5^{1}/_{8}$	5.3	125	718

*—Engines of foreign manufacture
†—Rebuilt
a—Applied for
A—Air cooled
Aa—Aluminum alloy
Ae—Aeromarine
Al—Aluminum
AS—Anzani or Scintilla
Au—Automatic
B—Battery ignition
Bs—Scintilla booster magneto
C—Cast iron
Ch—Champion
Cylinder Construction
first—head
second—walls or waterjacket
third—sleeve, if any
D—Direct
De—Delco
Dx—Dixie
E—Eclipse
G—Geared
G^{1}—5:3 epicyclic gear
G^{2}—2:1 reduction gear
G.E.—General Electric
I—Inverted
L—Cylinders in line
Mo—Model
M—Magneto
NI—Nickel Iron

The question of cylinder arrangement is a subject in itself and becomes too involved for a complete discussion here. However, a few of the salient points of each of the various types will be mentioned. The single-bank radial engine has the advantage of even cylinder cooling and light weight, versus the disadvantage of a large frontal area, resulting in increased air resistance, poor vision, and valves actuated by push rods limiting the crankshaft speed. The two-bank, air-cooled radial with every odd cylinder slightly to the rear and having a two-throw crankshaft is very similar to the single-bank radial except that its overall length is slightly greater and that it is slightly heavier. It has an even number of cylinders, while the single-row radial must have an odd number. In a medium-powered radial engine an odd number of cylinders might not give proper mixture distribution and high enough mean effective pressure without a rotary induction system and it might therefore be necessary to reduce the number of cylinders. To develop the desired power, this might not be possible without having too few cylinders, which would result in a large bore and not allow sufficient cooling or result in an uneven-running engine. Hence it is often necessary in a radial to shift from a single bank to a double bank to obtain the required number of cylinders.

The "air cooled in line," which is usually of four cylinders, has the advantage of allowing an overhead camshaft and reduction of frontal area. There are now on the market a few

TABLE 2 ENGINES AVAILABLE FOR COMMERCIAL USE

Engine	Rated power and r.p.m.	Weight dry, lb.	Compression ratio	Displacement	Wt. per hp.	Displac. wt.	Displac. hp.
Anzani 80	80–1500	215	5.1	397	2.69	1.85	4.97
Anzani 120	120–1500	320	5.5	736	2.67	2.3	6.13
Axelson	150–1800	430	...	612	2.81	1.43	4.08
Cornet	150–1800	...	...	...	...	...	...
Curtiss OX-5	90–1400	375	4.9	502	4.17	1.34	5.57
Curtiss Challenger	170–1800	420	...	603	2.47	1.44	3.55
Dayton Bear	110–1550	375	5.3	441	3.41	1.18	4.0
Fairchild-Caminez	120–960	350	5.0	447	2.6	1.28	3.73
Fisher & Jacobs	120–1700	...	...	521	...	...	...
Hallett	130–1800	425	5.2	...	3.26	...	...
Kimball Beetle	135–	400	5.4	585	2.95	1.46	4.33
Kinner K5	75–1725	231	5.0	372	2.3	1.61	4.97
Le Blond 60	60–1800	210	...	250	3.5	1.19	4.13
Le Blond 90	90–1850	250	...	350	2.75	1.4	3.90
Quick	125–1450	325	5.0	667	2.6	2.05	5.34
Ryan 5	70–1750	258	5.6	287	3.68	1.11	4.10
Ryan 7	96–1750	328	5.6	402	3.40	1.23	4.19
Ryan 9	125–1750	382	5.6	517	3.05	1.78	4.13
Scorpion	100–1800	290	...	...	2.90	...	...
Tips & Smith	125–1450	315	5.2	667	2.52	2.12	5.33
Velie M5	45–1750	210	5.2	506	2.63	1.19	5.58
Warner Scarab	110–1850	270	5.2	422	2.45	1.56	3.83
Wright J5	200–1800	500	5.2	788	2.5	1.58	3.94
Wright R-540	150–1800	370	5.2	540	2.46	1.46	3.6
Wright R-760	200–1800	417	5.2	760	2.09	1.82	3.8
Wright R-975	300–2000	485	5.2	975	1.62	2.01	3.25

air-cooled-in-line engines, namely, the Dayton Bear, a war surplus, water-cooled type rebuilt into an air-cooled engine, and two new production engines, the Cameron and the Scorpion. It appears that neither of the first two is in production at this moment. These engines resemble very closely the British

ENGINES AVAILABLE FOR COMMERCIAL USE[1]
responsibility is assumed for the figures)

Frontal area (sq. ft.)	Diameter of mounting ring or distance between bearers (in.)	Height or overall diameter (in.)	Length without starter (in.)	Guaranteed fuel consumption at rated hp., lb. per hp. per hr.	Oil consumption, lb. per hp. per hr.	Valve mechanism	Cylinder construction (see key)	No. intake valves per cylinder	No. exhaust valves per cylinder	Make spark plugs supplied with engine	Type of ignition	Make ignition	No. carburetors	Make carburetors	Make starter	Are exhaust manifolds provided	Shipping weight crated	Dept. of Commerce license	Cost at factory
..	...	...	...	...	....	prr	Al, S	1	1	...	2M	Sc	..	Sy, E	Bs	..	..	N	.....
..	17	46 3/4	78 1/2	0.55	0.03	ohc	Al, S	1	1	BG	2B	De	1	Sg[2]	E	..	..	Y	.....
..	17	...	85 1/8	0.55	0.03	ohc	Al, S	1	1	BG	2B	De	1	Sg[2]	E	..	..	Y	.....
..	17	46 7/16	74 1/4	0.53	0.025	ohc	Al, S	1	1	BG	2B	De	1	Sg[2]	E	..	..	Y	.....
..	9.5	24 13/16	21 13/32	0.53	0.025	prr	S	1	1	...	2M	AS	..	Z	..	Y	..	..	$ 735
..	10.6	35 13/16	29	0.53	0.027	prr	S	1	1	...	2M	AS	..	Z	..	Y	..	..	$ 1,650
..	11.0	43 3/16	34 15/32	0.60	0.031	prr	S	1	1	...	2M	AS	..	Z	..	Y	..	..	$ 2,170
..	12 7/8	...	...	0.55	0.030	prr	Mo, C	1	1	...	1M	...	..	Z	..	N	..	Y	.....
..	12 7/8	...	...	0.55	0.030	...	Mo, C	1	1	...	2M	...	..	Z	..	N	..	Y	.....
..	15 3/4	...	57 11/32	0.52	0.015	ohc	Al, Al, S	2	2	AC	2M	Sp	1	Z	U	N	865	Y	.....
..	15 3/4	...	56 3/4	0.52	0.015	ohc	Al, Al, S	2	2	BG	2M	Sc	2	Sg	E	N	1080	Y	.....
..	15 3/4	...	56 3/4	0.52	0.015	ohc	Al, Al, S	2	2	BG	1M	Sp	2	Sg	E	N	1040	Y	.....
..	15 3/4	...	56 1/32	0.52	0.015	ohc	Al, Al, S	2	2	BG	1M	Sp	2	Sg	E	N	1120	Y	.....
..	16	39	47 5/8	0.505	0.0175	ohc	Al, Ni	1	1	...	2M	Sp	1	Z	N	N	550	..	$ 1,600
3.9	21 1/2	36	35	0.55	0.035	prr	Al, S	1	1	AC	2M	Sc	1	Sg[1]	E	X	775	Y	.....
..	18 1/2	46	37	...	....	prr	C	1	1	BG	2M	Sp	2	Z	..	N	..	N	.....
2.0	4 1/8	23	11	0.45	0.031	Au	Aa, Aa, S	1	1	RB	1M	RB	1	Wf	..	N	100	N	$625
..	14	42	...	...	....	prr	Al, Al, S	1	1	CH	2M	Sc	1	Sg	..	N	400	Y	$ 1,875
..	...	49 1/8	38 1/2	0.42	0.025	prr	Al, S	1	1	...	2M	O	1	Z	..	..	..	..	$ 3,250
..	...	45 1/2	35	...	....	..	.....	...	...	...	M	Sc	1	Sg	..	..	750	N	.....
2.3	9 7/8	...	23 3/4	0.57	....	p	C	1	1	...	2M	Sc	1	Z	N	N	205	..	.....
6.22	14 1/4	45 1/32	78 1/16	0.52	0.03	ohc	S	2	2	BG	4B	Sc	4	Sg	O	N	..	Y	$25,000
4.03	15 3/4	38 15/64	62 11/32	0.52	0.015	ohc	S	2	2	BG	2M	Sc	2	Sg	Ae	N	1275	Y	$12,000
4.03	15 3/4	38 15/64	65	0.52	0.025	ohc	S	2	2	BG	2B	De	2	Sg	Ae	N	1390	Y	$17,375
4.03	16 1/4	38 5/8	62 11/32	0.52	0.025	ohc	S	2	2	BG	2M	Se	2	Sg	Ae	N	1275	Y	$12,000
4.8	18 1/4	36 1/8	69 3/4	0.52	0.015	ohc	S	2	2	BG	2M	Sc	2	Sg	Ae	N	2150	Y	$15,625
5.1	18 1/4	36 1/8	76 1/2	0.52	0.015	ohc	S	2	2	BG	2B	De	2	Sg	Ae	N	2340	Y	$19,375
..	23 3/8	....	43 3/8	0.55	0.035	prr	Al, S	1	1	BG	2M	Se	1	Sg	..	N	1150	Y	.....
..	23 3/8	....	44 1/8	0.55	0.035	prr	Al, S	1	1	BG	2M	Sc	1	Sg	..	N	1325	Y	.....
..	19 5/16	36	36	...	....	prr	Al, S	1	1	CH	1M	Sc	1	Z	..	..	..	..	$ 1,750
..	14	40 1/2	34	0.50	0.022	prr	Al, S	1	1	O	2M	O	1	Sm	RB	Y	630	Y	$ 1,800
..	14	40 1/2	32	0.50	0.022	prr	Al, S	1	1	O	2M	O	2	Sm	RB	Y	675	Y	$ 2,340
..	14.6	40 1/2	32	0.50	0.022	prr	Al, S	1	1	O	2M	O	2	Sm	RB	Y	720	Y	$ 2,970
4.75	11	...	39	0.31	0.036	prr	-, S, C	1	1	Sp	1M	Dx	1	Z	..	..	400	N	.....
..	12 3/4	32	27	0.55	0.025	prr	Al, S	1	1	AC	2M	Sc	1	Z	X, E	Y	..	Y	.....
..	17	35 1/2	27 1/2	0.50	0.01	prr	Al, S	1	1	AC	2M	Sc	1	Sg	..	N	..	..	$ 2,485
..	19 1/4	45	33 3/4	0.53	0.035	prr	Al, S	1	1	AC	2M	Sc	1	Sg	..	N	1000	Y	$ 4,980
..	23 3/8	53 13/16	40 5/8	0.55	0.035	prr	Al, S	1	1	BG	2M	Sc	1	Sg	..	N	1530	Y	.....
..	11 1/2	...	49	0.52	0.025	ohc	S	1	1	...	2M	...	1	Sg	..	N	..	Y	.....

[1] Reprinted from table prepared by the author for *Aviation*.

N—No
O—Own
ohc—Overhead cam
prr—Push rods and rocker arms
Rad—Radial
RB—Robert Bosch
Sc—Scintilla
Sg—Stromberg
Sg[1]—Stromberg NA-U5 duplex
Sg[2]—Stromberg NAS-8E
Sm—Sum
Sp—Spitdorf
S—Steel
U—U.S.L. Electric
V—Vee
W—Water cooled
Wf—Winfield
X—Extra
Y—Yes
Z—Zenith

60–80 hp. Cirrus, four-cylinder-in-line air-cooled type. The cylinder arrangement, though causing an engine of slightly heavier weight than that of the radial type, has comparatively little head resistance and should therefore prove quite successful. It is understood that Louis Chevrolet also has a six-cylinder engine of this type under development in Baltimore, Md.

Another type that has been neglected, though it is believed to have possibilities as a commercial type, is the inverted one, air cooled in line with the crankshaft and with the crankcase above the cylinders. By inverting the engine, the propeller-thrust line is raised, allowing one to design the airplane structure close to the ground, avoiding a high landing gear. It also permits the thrust line to be closer to the wing on a monoplane. This is a subject that has brought much comment of late. In addition the cowling, from the point of view of appearance and cost, is cheaper and better as the crankcase itself can be of streamline shape with no covering. Other cylinder arrangements such as the air-cooled Vee and the "Hex" type are better adapted to large engines and have not yet been used for engines of medium power.

Though there is some tendency to use reduction gears on large engines, it is doubtful if the increased efficiency due to the slow propeller speed is worth the increased weight on an engine of medium size. However, the question of propeller and crankshaft speed is a vital one, and though more power may be developed for a given weight by increasing the crankshaft speed it has been estimated that an increase above 1800 r.p.m. for an airplane with a top speed of 120 m.p.h. is inefficient. In the design of these engines, a subject which until recently has been neglected is the arrangement for the exhaust manifolding. It appears that the tendency on many new radial engines is to have the exhaust port in front of the cylinder and to employ an exhaust collector ring which conforms with the streamline of the airplane. Another feature which deserves attention is the silencing of the exhaust. The British Cirrus engine is usually provided with an exceptionally long exhaust manifold over half the length of the airplane; this reduces the noise considerably. A muffled engine is a great factor in selling an engine or plane, as a quiet, smooth-running engine always creates a better impression, especially to the layman or the partly initiated buyer who is very apt to be the purchaser of a plane of this size.

There are now in this country in production about ten makes of medium-powered engines with another five in the experimental stage, but so far along that orders are being placed and promises made for delivery. In addition, there are between five and ten new engines which are undergoing tests either on the ground or in the air prior to being prepared for production. Though there are a number of commercial engines available, only five new production engines of this size have been approved by the Aeronautics Branch of the U. S. Department of Commerce at this time. These five are: (1) Fairchild-Caminez, 120 hp. at 960 r.p.m.; (2) Warner, 110 hp. at 1850 r.p.m.; (3) Kinner, 75 hp. at 1725 r.p.m.; (4) Velie, 45 hp. at 1750 r.p.m., and (5) Curtiss Challenger, 170 hp. at 1800 r.p.m. A sixth engine, the Curtiss Conqueror, is the only other engine to be approved by the Department of Commerce for use in licensed airplanes. In addition, there are the engines which have been approved

by either the Army or Navy and which are then automatically accepted by the Department of Commerce. Practically all of the war surplus engines come under this heading. It is understood that a number of new production engines are now undergoing tests and will be approved shortly.

The problem of choosing a power plant, with so many new and only partially proved engines on the market, is an exceedingly difficult one. The aircraft manufacturer does not want to do the test work for the engine he is buying nor does he want to sell

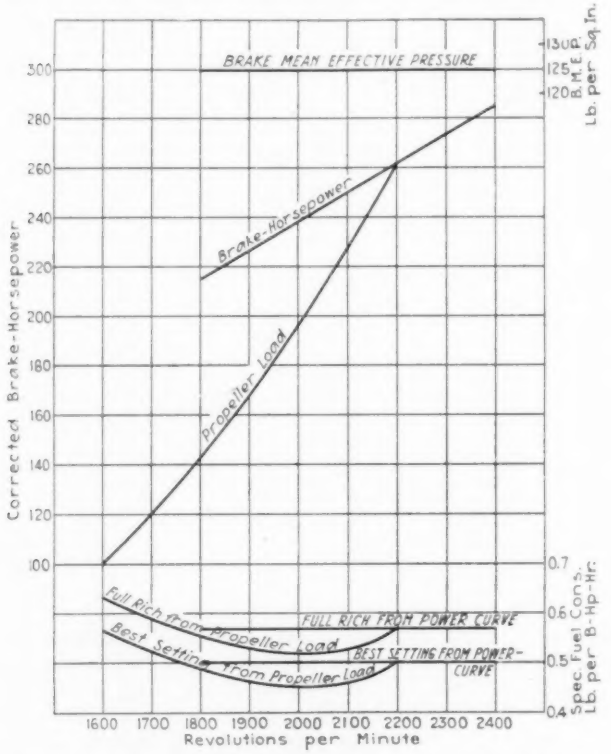


FIG. 2 ESTIMATED POWER AND FUEL CONSUMPTION OF WRIGHT WHIRLWIND MODEL R-760, SEVEN-CYLINDER AIRCRAFT ENGINE. BORE 5 IN., STROKE 5½ IN., COMPRESSION RATIO 5.2 TO 1, AVIATION GASOLINE

an airplane with an unknown engine. If he is assured that the engine functions properly, there is the additional problem of maintenance and cost of installation and assembly. In an effort to gain simplicity and low price, it is feared that a number of engine manufacturers will reduce their standards; that is apt to prove disastrous. Another important point is the availability of the product. Is the engine manufacturer properly financed and has he facilities to meet the production requirements of the airplane manufacturer without sacrificing the quality of his product? If the engine is available and is satisfactory mechanically and the airplane manufacturer decides to use it as standard equipment, he

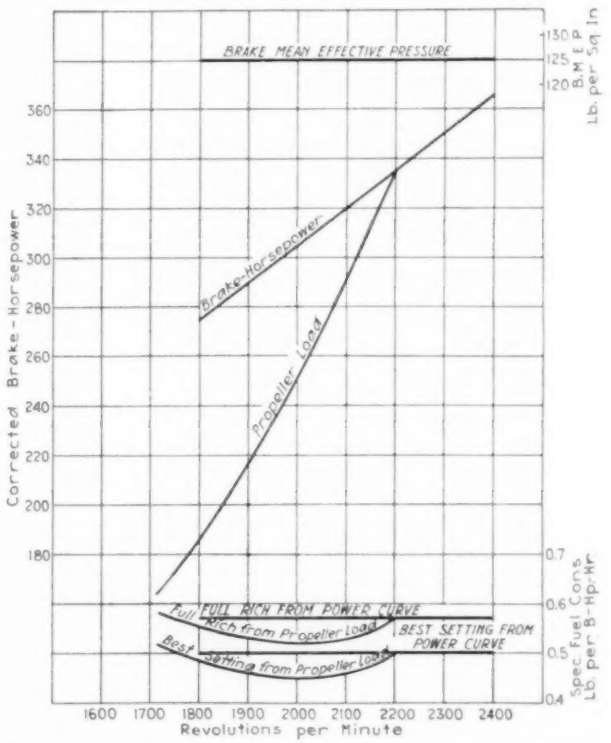


FIG. 3 ESTIMATED POWER AND FUEL CONSUMPTION OF WRIGHT WHIRLWIND MODEL R-975, NINE-CYLINDER AIRCRAFT ENGINE. BORE 5 IN., STROKE 5½ IN., COMPRESSION RATIO 5.2 TO 1, AVIATION GASOLINE

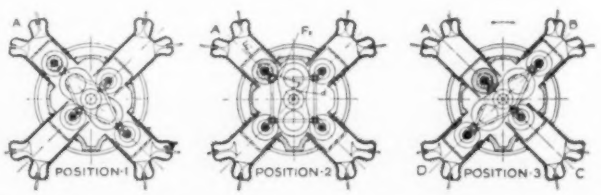


FIG. 4 DIAGRAM SHOWING CAM MECHANISM IN THREE POSITIONS DURING ONE PISTON STROKE OF FAIRCHILD-CAMINEZ ENGINE

must sell the engines as well as the airplane. For this reason, the question of interchangeable power plants is again stressed. A purchaser may then choose his own power plant and with little or no extra cost have it installed in a new plane.

AER-51-18

Aerial Photography Engineering

By E. R. POLLEY,[1] NEW YORK, N. Y.

Aerial photographic surveying is described, and its advantages over ground work are given. Maps may be made with almost perfect detail and at far less cost of time and money than required by survey work in the usual manner. The methods by which the many overlapping photos are taken and then assembled in the scale map are outlined, together with the supplying of stereoscopic photos for studying the contour and physical features of the terrain. Maps made from the air have a value to municipalities in zoning and taxation problems and to counties and states for highway and park and reforestation and waste-land activities.

THE subject of aeronautical engineering is of prime importance to every branch of engineering because of the influence which it exerts in the entire engineering world. Aerial engineering, aside from its extreme practicability, is injecting the factors of glamor and adventure into this field of endeavor. Aerial photographic surveying is comparatively new, and people are unfamiliar with this work and with its engineering position in this inventive age.

Of all the great steps in engineering one of the greatest was the development of the lens, with its resultant precision instruments, and in this field the last and greatest stride of all was the coupling of the airplane and the aerial camera. No longer does the engineer have to hew his way through canebrake and scrub nor wade through swamp and morass, but the modern surveyor takes unto himself wings and from a vantage point two to three miles in the air he maps portions of the earth's terrain accurately to scale, with almost perfect detail and at a great saving in both cost and time.

Method of Making the Aerial Photographic Map

The camera is installed in the plane with the lens pointing down through a hole in the fuselage. The elevation at which the plane must be flown is determined; for instance, if the area in question is to be mapped at a scale of 800 ft. to the inch and a lens of 20 in. focal length is used, multiplication of the two factors gives the altitude of 16,000 ft. which the plane must maintain.

The pilot outlines upon United States Geological Survey sheets the area to be covered and also denotes the guide or flight lines. From this marked map the "job is flown." The pilot flies along the guide or flight lines, back and forth, forming "strips," until the required area has been covered. The photographic film covered by each exposure is $7^1/_2 \times 9^1/_2$ in., the larger dimension forming the width of each strip. However, the net effective area of one exposure is about one-fifth of the total area, because of the overlap. The photographer times the consecutive exposures so that they overlap each other like shingles by about 60 per cent, and the pilot plots his flight lines so that the strips overlap each other by about 50 per cent.

Thus each object on the terrain is photographed five times. The overlap makes it possible to use only the center portion of each photograph, which alone is in true vertical projection. Discarding the edges of the photographs in making the map also avoids a certain amount of displacement of elevated ground objects due to conical projection caused by the diverging lens' rays. The overlap, moreover, allows stereoscopic study of the terrain, which adds the third dimension to any portion of the ground covered. With the completion of the photography the air work is done, and the films after development go to the engineering and mapping departments.

The layman usually is astonished when told that aerial maps are photographed from such altitudes as 16,000 and 20,000 feet, yet in a recent test an Army pilot and photographer took on one exposure 19 square miles of the city of Dayton, Ohio, from an altitude of 32,000 ft. This six-mile airplane climb put the photographer 3000 ft. higher than the top of Mt. Everest and brought him into a temperature of 60 deg. below zero.

The aerial photographic map is assembled as a mosaic by first plotting as a control map the existing survey data upon a suitable composition board and then superimposing upon this control the selected portions of the aerial photographs after they have been corrected to scale, now called ratio prints. This master map is then rephotographed in large sections. For study of an area enlargements are made, and a negative at, say, 800 ft. to the inch, can be enlarged to 200 ft. to the inch without appreciable loss of definition.

The practical application of this type of survey is typified by a recent instance. A few months ago one of the executives of the New England Telephone and Telegraph Company was impressed by a phrase in a scientific magazine which read: "In this modern period every executive is in duty bound to ask himself what is obsolete and what is more efficient." At the time he was engrossed in solving the problem of a potential telephone line. Knowing that in surveying a right-of-way it is impossible to secure too much detailed information, and with the quoted phrase in mind, a contract was given through his engineering department to have this right-of-way surveyed photographically from the air.

Briefly, the record is as follows: On November 17, 1928, the New England Telephone and Telegraph Company placed the order with Fairchild Aerial Surveys to map their telephone right-of-way area of about 26 square miles. On December 7 the Fairchild monoplane equipped with aerial camera climbed to an elevation of 9600 ft. and began the survey. As soon as the plane arrived over the eastern boundary of the area the photographer turned a switch which set the automatically controlled camera in action.

As soon as the aerial survey was completed the pilot and photographer descended from their chilly elevation, with a temperature of 17 deg. below zero, and rushed the film to the Fairchild photographic laboratory, where it was developed and printed. The photographs were delivered on December 17.

Closely following the delivery of the contact prints there was made a mosaic map which shows the entire area as a continuous photograph to the scale of 400 ft. to the inch, also a set of enlargements for field work, and a stereoscopic study to the control scale.

Thus from a height of nearly two miles in the air, this winged surveyor, invisible from the ground, used the modern instruments of his craft to secure an accurately scaled map of this right-of-way which enabled the company engineers to view each house, tree, road, and stream exactly as it is and with complete photographic detail.

There were no weary miles of walking and of carrying weighty ground instruments, no tedious sighting and compilation of field notes, no entrance of the human equation with its possibilities of forgotten chain lengths and mistaken bench marks and

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[1] Fairchild Aerial Surveys, Inc.

Presented at the Third National Meeting of the A.S.M.E. Aeronautic Division, St. Louis, Mo., May 27 to 30, 1929.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

errors of computation, no protracted drafting of maps over a long period of time. The following points are so obvious that they lend their own emphasis:

First, instead of a period of months, only ten days elapsed from the time the first datum was secured to the delivery of the contact prints.

Second, the cost was less than one-fifth that which would have been incurred by an equivalent ground survey.

Third, instead of a line-drawn map embracing a maximum detail of, say, 20 per cent, as would have been the case in ground survey, the aerial map gives 100 per cent detail showing every object on the terrain exactly as it exists.

Fourth, it is a well-known fact that a line-drawn map is not always easy of interpretation even by the engineers, and to the layman it is a sealed book, whereas the aerial photographic map is readily interpreted by the non-technical executive.

Fifth, aside from the engineering value of the map itself, any two overlapping prints can be placed under the Fairchild mirror stereoscope and the relief of terrain be observed. Thus the company engineers can view the area with the hills and valleys showing in the third dimension, without the necessity of visiting the locality.

This new branch of engineering calls for the highest type of trained personnel. For instance, the Fairchild Company, which has mapped over 80,000 square miles by this method, has found that out of every one hundred trained commercial pilots only four are good mapping pilots, possessing that coordination of mind and muscle which enables them to maintain altitude, to fly the ship steady under all conditions, to orient themselves as to locality, and to follow imaginary flight lines on the ground below from an elevation of some two to three miles in the air.

It is believed that from this simply told story there may be envisioned the romance and adventure which are concealed within the telling—the cold of the upper-air strata; the carefully trained men, who are able to orient themselves as do the eagles from their station in the sky; the age-old battle of human progress in devising these perfected instruments of airplane and camera; the working of modern chemistry in the photographic darkroom; and the spirit of adventure which prompts both pilot and photographers—young men who no longer fight the hazards of new frontiers and boundaries, but who eliminate frontiers by facing and defeating the hazards of the air.

Briefly there may be outlined the practical application of aerial survey to the problems of municipal, county, and state engineers.

Tax Equalization

The president of a prominent appraisal company recently published the following statement:

"The first municipality ever reappraised in which aerial photographic maps were used is, to the best of our knowledge, that of Middletown, Conn. Let us compare the old and the new method in the two cities of New Britain and Middletown in regard to time and cost of mapping: Old method in New Britain—Area, 13 square miles; time required to secure maps, 4 years; cost, $48,600. Aerial method in Middletown—Area, 42 square miles; time required to secure maps, 60 days; cost, $4000.

"At the end of fourteen months it was made public that 1896 pieces of city property had been discovered which were not previously on the tax list and which had never before paid taxes. Aerial mapping has proved its case, and at the end of the first year the city or township finds itself firmly inventoried, soundly financed, well equipped with maps, and in possession of complete data with which to enter the next logical step of zoning."

Incidentally, since the Middletown maps were made, the Fairchild Aerial Surveys have mapped 12⅓ per cent of the area of the State of Connecticut.

City Planning and Zoning

The aerial map is of most practical use in city planning and zoning. Owing to the full knowledge of local atmosphere and conditions held by the municipal executives, the 100 per cent photographic detail of the aerial map lends itself to perfect zoning by the local executives. By the use of transparent colors on maps, the zones are not only established, but even the non-technical citizen can read at a glance what lies within those zones and why changes are advocated.

Aerial mosaic maps greatly reduce zoning costs and also eliminate the major part of the battles which are the almost inevitable aftermath of city planning and zoning.

Public Utilities

Men in public service companies find aerial maps of great value. F. A. Allner, general superintendent of the Pennsylvania Water and Power Company, referring to a 140-mile power-transmission-line route, writes as follows:

"We estimate that about one-half the cost of survey has been saved by aerial mapping. No preliminary surveys were run in the field. The alignment was practically fixed in the office on the map. We believe the saving of the preliminary survey would not be possible were it not for the accurate alignment of the photographs."

Timber Survey

Enthusiastic reception of this type of map has been accorded in timber survey. It eliminates the guesswork of the old-time timber cruising. All of the timber area is covered. It shows burnt-over areas, rock outcroppings, blowdowns, swamps, and stranded logs above river-flood lines. Distinctly separated are scrub brush, second growth, and primitive timber. A sharp line of demarcation separates the coniferous and the deciduous trees, and groupings of kinds are easily recognized.

Hydraulics

Recently aerial survey has given material aid in the study of watersheds and reservoir sites. Studied with the aid of the stereoscope, the terrain shows in relief, and the aerial map gives immediate information as to slopes, feeders, and approximate flood lines. To secure this data by old methods would take months of effort.

State Uses

If entire states were mapped photographically from the air, from these maps the agricultural department could note the percentage of land under cultivation, could mark down the waste lands for reclamation, could check off the sour marsh lands for chemical reclamation, and could plan for maximum agricultural efficiency.

Transportation lines and spurs could be located from these maps for commercially opening up present dead areas. The state highway department would be able to more intelligently plan main arteries of travel.

State park departments would be able to locate unerringly the proper areas for parks and reservations. The department of forestry could read these aerial maps and locate fire-observation towers, mark off fire lanes, and intelligently apportion ranger patrols. The forester would be able to recognize denuded areas where acres of stumps and dry brook beds might be benefited by reforestation.

The hydraulic engineer could locate potential power sites and figure to a nicety the undeveloped power of his state.

Regional Planning

Where original suburban settlements have themselves grown into cities, crowding and hemming in the parent municipality, or where a number of cities are separated by only a few miles, under these conditions the aerial map reaches the apex of its efficiency.

Not one of these cities has a problem confined to itself. The problems are regional. If traffic lanes are developed or widened in one city, they must connect with similar thoroughfares in adjoining municipalities, or otherwise the traffic load will "bottleneck" and results will be negative.

Regional service must be established in the form of communal sewer and water systems, power and light distribution, major zoning ordinances, and traffic regulation. Broad actual view of the entire area must be secured before real data can be accumulated and efficacious methods advocated. Obviously the aerial mosaic map is the only solution, and it has nobly proved its worth. Numerous other possible uses of aerial maps are cropping out.



FIG. 1 VOUGHT O2U-2 AIRPLANE

Application of Advanced Methods to Airplane Structural Analyses

BY DR. MICHAEL WATTER,[1] LONG ISLAND CITY, N. Y.

This paper urges the necessity of wider use of finer methods of structural analysis. It illustrates the importance of use of advanced methods not only from standpoint of structural safety, but also as a means of evolving new forms.

It outlines briefly some results from the author's experience, and in connection with the desirability of finer methods urges closer coordination between aerodynamical facts and structural requirements.

Certain present requirements are criticized and a suggestion is made to revise the conception of load factors in their relation to individual members.

IF ECONOMY is a purpose in other branches of engineering, it is a necessity in aeronautics. The performance of any airplane is a function of its weight, and military and commercial airplanes alike must be built as light as possible in order to allow greater military load in the former and greater pay load in the latter. The efficiency of any structure hinges on its uniformity of strength, and because of the comparatively small factors of safety allowed in airplane structures it is imperative that the designer should know, with the precision afforded to him through the finest methods available at present, the nature and distribution of the working loads. This necessity is not based solely on the requirements of structural safety or economy, for it also aids powerfully in the evolution of new and more perfect forms which may be desired because of performance or service requirements. The complexity of airplane structures dictates the use of advanced methods of structural analyses in order that the nature and distribution of loads may be properly determined.

While in other branches of engineering the method of calculation would more strictly apply to the determination of stresses, in aeronautical engineering the problem is further complicated by the necessity for evaluating acting loads.

Structural analyses of the airplane are intimately related to an accurate knowledge of the aerodynamic and dynamic loads acting on the structure, and the title of this paper refers not only to advanced methods of purely structural analyses but also to more rigorous requirements coordinated with available aerodynamic data. Application of the Berry method, the least-work theorem, the Williot diagram, method of elastic weights, method of work, etc., may well be wasted if the engineer does not take into account the actual condition of loading or if, in dimensioning the sizes of members, he neglects the real meaning of the factor of safety.

The author will endeavor to show briefly and by means of illustrations of actual cases met in practice the importance of finer methods of calculation, the better coordination of aerodynamical facts and structural requirements, and also the problems which must be taken care of in assigning the proper factor of safety to the main structure, and details in actual design.

Referring only to the method of determining the loads in various members of the structure, it may be well to point out that even conventional structural arrangements may in the case of simplified analysis lead to results which are sometimes quite far from the truth, as finer analysis would readily show.

STATICALLY INDETERMINATE STRUCTURES

It is beyond the scope of the paper to discuss the relative merits of statically determinate or indeterminate structures.

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[1] Design Engineer, Chance Vought Corporation.

Presented at the Third National Meeting of the A.S.M.E. Aeronautic, Division of the A.S.M.E., St. Louis, Mo., May 27 to 30, 1929.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

It is true that in statically determinate structures the loads in the different members can be determined readily with the aid of definite rules, and often without the necessity of simplifying assumptions. On the other hand, with our improved control and knowledge of properties of materials and fine workmanship in airplane construction, the behavior of a statically indeterminate structure can be very closely analyzed by available methods handled by an experienced engineer. It is also well to point out that ideal statically determinate structures are never met in practice, and that in a number of cases the service requirements are such that an engineer is forced to adopt more complicated arrangements.

It is always tempting to make a simplified assumption, but it will be found that finer analyses will give a much more accurate picture of the load distribution, and thus lead to a more economical design of structure.

The Single-Bay Cell

By way of illustration the author will cite the case of a normal-type single-bay biplane cell designed for use on the latest Vought two-seater fighter. In this machine both upper and lower wings are parallel, have no sweepback, and are arranged with a stagger of about 12.9 deg. The flying wires are not in the plane of the beams, but are slightly brought forward. The outboard strut is a conventional N-strut arrangement.

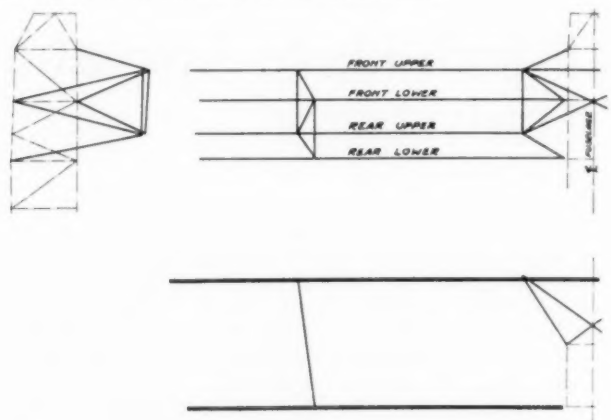


Fig. 2 Cell Arrangement

Fig. 2 gives the cell diagram. If an engineer had never made a least-work analysis of a similar cell, he would be tempted to make a few assumptions, with the intention of keeping reasonably on the safe side. Since the redundancy is caused by the N-strut, in view of the comparatively small stagger it would appear that in case of the high incidence condition the anti-drag components on the lower wing could be assumed as taken by the internal trussing of the wing, while the lift reactions could be resolved in the respective members of the N-brace. It would be safer to assume that the total air reaction (which, it should be remembered, includes the anti-drag) at the lower-wing-strut points is carried by the N-brace to the upper wing, but experience will prove that this assumption is fallacious.

The truss was solved on the basis of the first assumption, and the following loads were obtained:

Compression in the front beam = —17,620 lb.
Load in the front flying wires = +13,500 lb.

Since for design purposes it was necessary to analyze the structure as closely as possible, a least-work analysis was made, assuming as the redundant member the front strut of the N-brace, and the loads obtained were as follows:

Compression in the front beam = —12,516 lb.
Load in the front flying wires = +10,105 lb.

Similarly the loads in the upper anti-drag bracing proved to be considerably less than a simple analysis would seem to indicate. On the other hand, the loads in the rear flying wires and rear beam in case of a simplified analysis would be considerably underestimated. This is a very good illustration of how much it is possible to be in error as regards the magnitude of the loads in the structure if a finer method of analysis is not resorted to.

In connection with this the author would like to bring up a point in regard to present requirements in the case of statically indeterminate structures. These requirements specify that when allowance must be made for redundant members the loads shall first be computed for the statically determinate system obtained by neglecting the redundant members. The effect of the redundant members shall then be considered by the method of least work or by some other method, and the net loads obtained. In computing these net loads the effect of the redundant members shall be increased or decreased 25 per cent, whichever is the more conservative, in order to allow for play in joints, variation in cross-section, etc. It must be pointed out, however, that the loads obtained by means of a least-work analysis or other precise method will be the same irrespective of which members were assumed to be redundant, but widely different in the case of a simple analysis when different redundant members are assumed to be inactive. This leads to an uncertainty in deciding on the above-mentioned 25 per cent correction because, for instance, in the case illustrated above, the assumption that diagonal brace of the N-strut is redundant would lead to somewhat higher loads in the front beam and flying wires, and the design loads in consequence should be increased. It would be desirable to have this rule made more specific because in its present form it is somewhat ambiguous.

Beams

Since the purpose of the present paper is mainly to illustrate special cases and problems actually met in the author's experience, the design of beams and shallow trusses will not be discussed in detail, because besides classical treatises there are excellent papers available on the subject of combined axial and transverse loading and on the use of the methods of work, Williot diagrams, and of elastic weights in the design of trussed beams.

In practice it will be found that because of the comparative complexity of the latter problem there is a great deal more information available on stresses in individual parts of a trussed beam than on an ordinary webbed beam. This is of course partly due to the method of analysis necessary in case of trussed beams and partly because the webbed beam is usually proportioned by merely obtaining the maximum bending moment and the distance between the points of inflection. A word of warning may not be out of place that there may be a case in design of a webbed beam where the design section may not be at the point of maximum bending, but at the point of the next largest drag-load component.

If the engineer has available the true bending-moment diagram with a diagram of the axial loads drawn in their proper relation to the sections of the beam, he will be able to judge the condition and verify by calculation sections which appear to be more stressed than the section at the maximum bending moment.

In practice it will also be found that many times the shop has made small errors here and there which are not serious enough to warrant rejection of the beam and yet are such that the engineer may be in doubt whether the section in question is sufficiently strong. A diagram of the nature suggested will be of great help to him in such a case.

The author has evolved a simple and rapid graphical method of constructing the bending-moment diagram for a beam under combined axial and transverse loading which he has found of great help in his routine experience. The method was described in detail in *Aviation* of February 22, 1926 (vol. 20, no. 8) and only a brief outline of it will be given here.

Graphical Construction of Bending-Moment Curve for a Beam of Uniform Section Under Combined Transverse and Axial Loading

Referring to Fig. 3, draw the beam AB to any convenient scale and, on the vertical lines passing through the points of support, lay off downward the values of M_A and M_B. (The same notation and method are used here as in Pippard and Pritchard's book "Aeroplane Structures.") Join the two points thereby obtained and project the point of intersection of this line with the vertical line passing through the middle of the beam

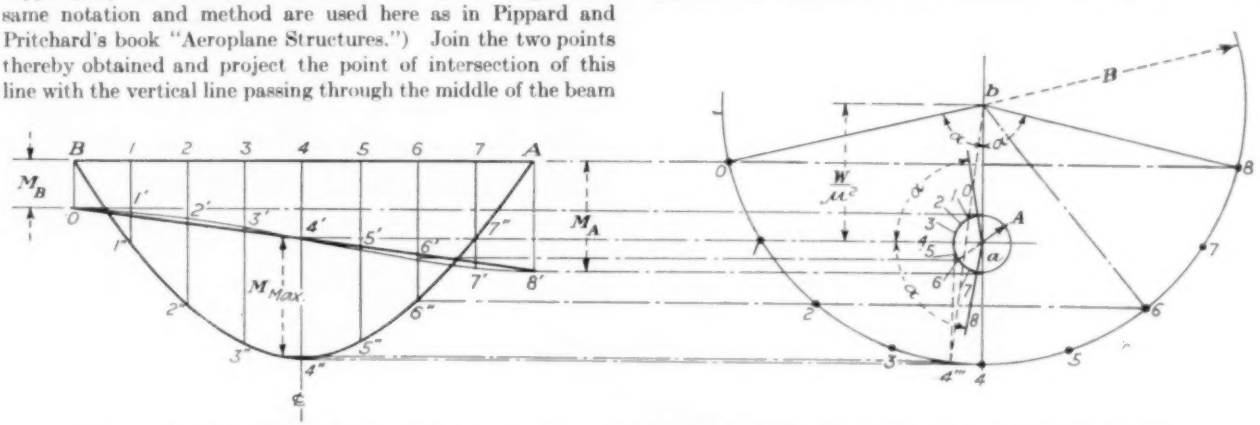


Fig. 3 Construction of Bending-Moment Curve for a Beam Under Combined Axial and Transverse Loading

on any arbitrary vertical line drawn on either side of the bay. In Fig. 3 this would be the line ab. Lay off on this line vertically upward, the value of W/μ^2 to a definite scale. At the points a and b draw the angles 2α where $\alpha = 57.3° \frac{l}{2}\sqrt{\frac{P}{EI}}$. Extend the line AB until it intersects the lines $b0$ and $b8$. The segments thus obtained give the magnitude of B. By drawing two horizontal lines through the points $0'$ and $8'$ until they intersect the lines $a0$ and $a8$, the magnitude of A may be obtained. It must be observed that $b0$ and $b8$ are drawn by measuring α from the vertical line, while $a0$ and $a8$ are drawn by measuring it from the horizontal line. With the radii thus obtained draw two circles B and A and divide the length of the beam and both segments into the same number of equal parts. Through point $1, 2, 3 \ldots 2m$ of the segments draw lines parallel to the axis of the beam, and their intersection with corresponding vertical lines will determine points $1', 2', 3' \ldots 2'm$, and $1'', 2'', 3'' \ldots 2m''$. It can be proved that the distances $1'\ 1'', 2', 2''$, etc., give the bending moment at the corresponding points in the scale in which W/μ^2 was originally drawn.

To locate the maximum bending moment lay off on the line $44''$ point $4'''$ such that $44'''$ is equal to A, and join the point $4'''$ with the center of circle B. Project the point of intersection of this line with the circle B on the curve $0''$, $1''$, $2'' \ldots 2m''$. The corresponding ordinate of the bending moment diagram gives M_{max}. Since there are two such points of intersection, it must be understood that the point in question is on the side of the diagram where curve $0'$, $1'$, $2' \ldots 2'm$ mounts upward.

New Structural Forms

In connection with the mention of the cell arrangement, it is interesting to point out one novel feature which was incorporated in the center section. While it was believed that the wing arrangement was the most advantageous from a combat viewpoint, it was nevertheless felt that for gunnery and general use a large center section was undesirable because of the blind angle resulting from it.

In order to retain the general cell arrangement and yet to improve the upward and forward vision of the pilot, attention was concentrated on the center section. Since ordinarily the location of the rear spar limits the depth of cut-out, it was decided to change the standard structure of the center section by employing two rear beams crossed in the form of the letter X, which made it possible to double the depth of the cut-out obtainable. This resulted in the Vought "vision" center section, which was first incorporated in the Vought special observation airplane submitted to the U. S. Army and made its first actual appearance on the Navy 02U–2 airplanes. Figs. 4 and 5 show respectively the standard and "vision" type sections.

The first center section was metal with latticed beams and channel-type ribs. Because of the type of the structure and in order to improve vision, the center section was built in such a way that the original airfoil was approximately preserved across

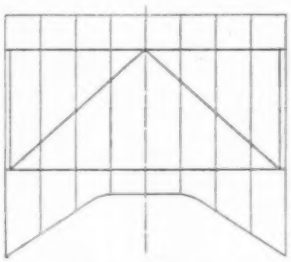


Fig. 4 Standard Center Section

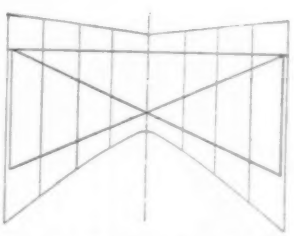


Fig. 5 "Vision"-Type Center Section

the center section, and therefore the beams were tapered with the lowest depth at the center. Further, the diagonal beams were not symmetrical about the mid-point. This presented almost a general case of a non-uniform beam under combined lateral and axial loads in which the lateral load varied throughout the length, the axial load being applied eccentrically and the taper reversed in the sense that the depth of the beam decreased toward the center.

In analyzing this structure the following method was adopted. The axial loads were obtained from the least-work analysis of the cell. In view of the uncertainties of the lift distribution over the span, it was felt that it was sufficiently accurate to employ the following assumptions. The load was considered to be a distributed load along the beam. The load per inch run was taken as proportional to the respective chord length.

The proportion of this load on each beam was then calculated by means of rib loading curves, the front beam taking the load from the leading edge to a point half-way to the middle beam, the middle one taking it from there to a point half-way to the rear beam, and the rear carrying the load from this mid-point to the trailing edge. In addition to the non-uniform loading and a tapered beam, there were also to be considered the eccentricities at both ends where the axial load was applied. The beams were analyzed graphically by drawing primary bending moments due to the axial and distributed loads. Then the primary M/I curve was drawn and deflection determined from it in order to obtain the secondary bending moment. In calculating the deflections in order to account for the fact that the beam in reality was a shallow truss, the value of $E = 10{,}400{,}000$ lb. per sq. in. for duralumin was taken as only 8,000,000 lb. per sq. in. After the secondary bending was obtained the new deflections were determined and the process repeated until the beam appeared to approach the condition of equilibrium, at

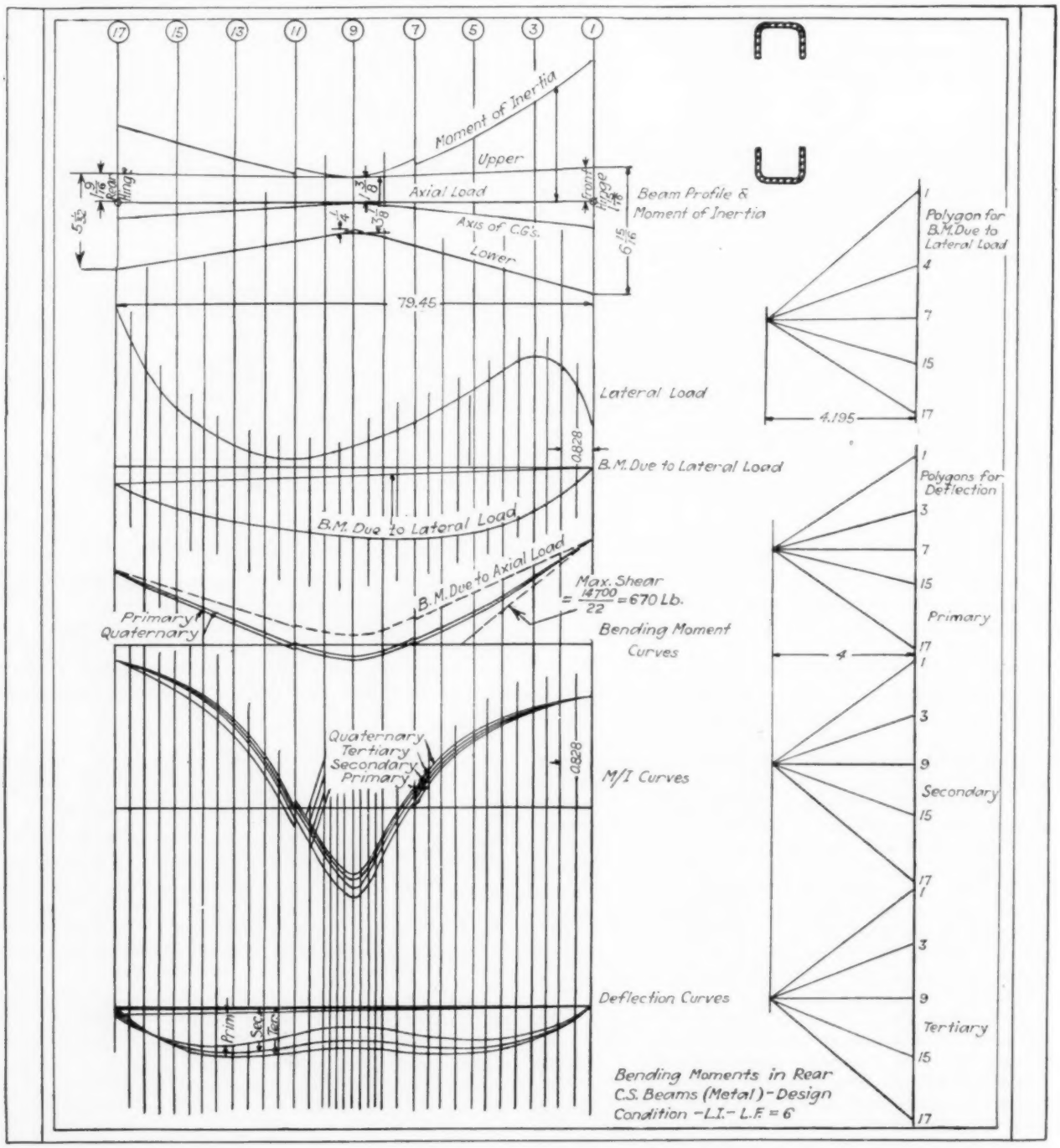


FIG. 6 GRAPHICAL DETERMINATION OF THE BENDING MOMENT DUE TO TRANSVERSE AND ECCENTRIC AXIAL LOADS ON THE REAR BEAM OF THE "VISION" CENTER SECTION

which point a final bending-moment curve was determined which was used in calculating the stresses in the beam. In order to design the diagonals, the shear was obtained from the slope of the final bending-moment curve.

In order to verify the assumptions used in the design of the beams, the front beam was tested under simple bending and the value of E obtained from the observed deflection. The figure obtained was 8,400,000, which was quite close to the assumed value.

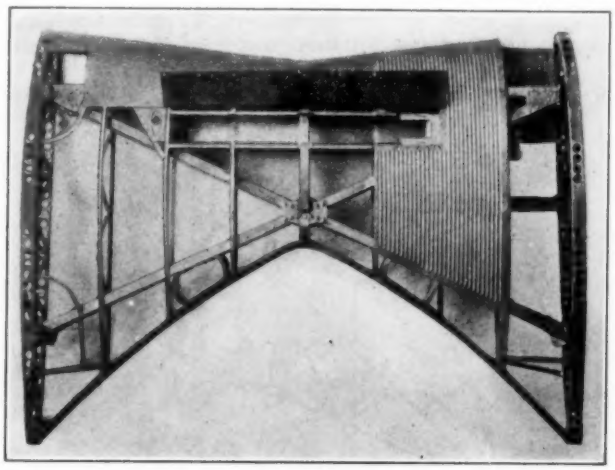


FIG. 7 METAL "VISION" CENTER SECTION

Split-Axle Chassis. Another example of a new structural form which necessitated the use of rather elaborate analysis is the split-axle-type chassis which was designed for the Vought "Corsair." Because of certain service requirements it was felt desirable that the wheels should move only in the vertical plane and that the chassis should be independent of the wing structure. The arrangement which met these requirements, despite its simplicity, was arrived at only after several tentative designs.

The present chassis structure consists of a total of six members. Each wheel is supported by a front oleo leg of the spring-oleo type, a diagonal axle and a rear strut, these two being in the same plane. Since the axles cross in the same plane, one is provided with a ring structure to allow for independent movment of either of the two wheels. When the front oleo leg shortens under the load, the rotation occurs about the transverse axis passing through the upper attachment of the axle and rear strut. The lower rear-strut attachment is such that theoretically it does not take any bending caused by the part of the component of the load in the plane of the rear struts.

In view of the complexity of the problem the following assumptions were made:

1 Referring to Fig. 9, the loads applied to the frame by the wheels were resolved at point A into axial loads in the front strut, rear strut, and axle; a bending moment in the axle from the cantilever load in the plane of rear strut and axle; and a torsional couple in the plane perpendicular to the plane of rear struts and axle.

2 The fact that the axle and rear strut are not perpendicular to the plane of the torsional couple was neglected, and sections were taken perpendicular to the longitudinal axis of the member.

3 In calculating the angle of twist of the rear strut an elliptical section of the same major and minor axes was substituted for the streamline section actually used.

For convenience the bending moment caused by the component of the load in the plane of the rear struts and axle was called the primary bending moment, and that resulting from the method of attachment and the torsional couple at the lower point A was called the secondary bending.

In endeavoring to solve for the secondary bending and torsion in the rear strut and axles, it was found that the standard equations of statics and least work were inadequate and the question arose as to what equation or equations might be lacking. By studying the structure, however, it became evident that what was lacking was an expression of the equality of the torsional deflections of the rear strut and axle.

In connection with this stress analysis it became necessary to find an expression for the angle of twist of a streamline tube, and a study was made of the problem. The difficulty of this problem lay not only in finding a suitable formula for the angle of twist, but also in the lack of information on the modulus of rigidity of chrome-molybdenum tubing; however, it was felt that for the latter a value of 12,000,000 lb. per sq. in. should be sufficiently close for all practical purposes. The problem of finding a suitable formula for the angle of twist and the shearing stress of a streamline tube was approached in several ways, and since the theoretical solution presented insurmountable difficulties, several approximate methods were resorted to.

The first method, the one actually used in the analysis, was to assume instead of a streamline tube an elliptical tube whose major and minor axes were equal to the axes of the streamline tube. Since, to the author's knowledge, there is no expression of the angle of twist of a hollow ellipse available, a formula was

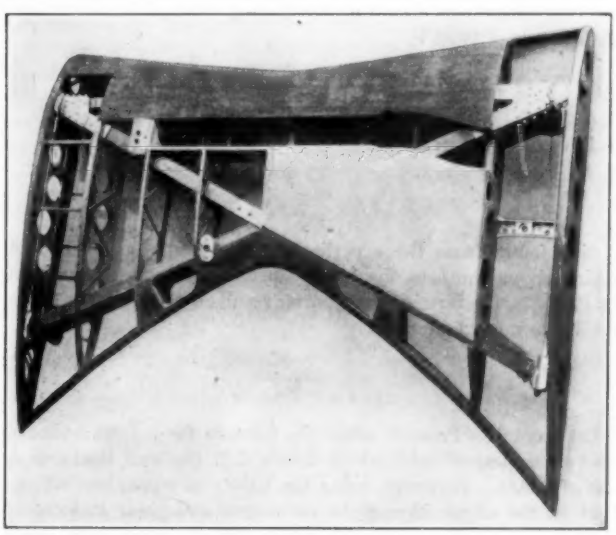


FIG. 8 WOODEN "VISION" CENTER SECTION

derived by analogy with a circular tube, the following nomenclature being employed:

θ = angle of twist in radians
f_s = shearing stress
a = major axis
b = minor axis
subscript 1 indicates outer dimensions
subscript 2 indicates inner dimensions
T = torsional moment
l = length
G = modulus of rigidity
A_1 = area enclosed by outer boundary of section
L_0 = length of the median line.

The angle of twist of a solid elliptical section as given in Morley's "Strength of Materials," p. 295, is:

$$\theta = \frac{16}{\pi} \frac{(a^2 + b^2)}{a^3 b^3} \frac{Tl}{G}$$

For a hollow circular section (D_1 and D_2, respectively, outside and inside diameters),

$$\theta = \frac{32}{\pi} \frac{1}{(D_1^4 - D_2^4)} \frac{Tl}{G}$$

so that the twist of the hollow ellipse may be written by analogy as

$$\theta = \frac{16}{\pi} \frac{Tl}{G} \left[\frac{1}{\frac{a_1^3 b_1^3}{a_1^2 + b_1^2} - \frac{a_2^3 b_2^3}{a_2^2 + b_2^2}} \right] \quad \ldots\ldots [1]$$

One of Bach's works gives the following formula for the strength of a hollow ellipse which was used to determine f_s:

$$f_s = \frac{16T}{\pi} \left[\frac{b_1}{a_1 b_1^3 - a_2 b_2^3} \right] \quad \ldots\ldots [2]$$

In order to check [1] several other methods were used, and it is of interest to review them here.

In Prescott's "Applied Elasticity," on pp. 165 and 166, are given formulas which hold true for thin tubes, that is, those where the wall thickness is small compared to the curvature of the inner boundary of the section. This is the case for streamline tubular sections except at the trailing edge. The expression of the angle of twist is

$$\theta = \frac{TlL_0}{4GtA_0^2} \quad \ldots\ldots [3]$$

and for the shearing stress:

$$f_s = \frac{T}{2tA_0}$$

It was found from the experiments, however, that instead of taking A_0 as the area inside the median line as recommended by Prescott, the formula gave better results if the area was taken within the outer boundary.

$$f_s = \frac{T}{2tA_1} \quad \ldots\ldots [4]$$

On page 168 Prescott gives the formula for a tube bounded by two similar ellipses, which means that the wall thickness is not constant. However, using the values of equivalent ellipse, that is, the ellipse having the same area and polar moment of inertia as the streamline tube, this formula gave fairly good results.

$$\theta = \frac{2}{\pi} \frac{Tl}{G} \frac{(a_2^2 + b_2^2) b_1}{a_2^3 b_2^3 t} \quad \ldots\ldots [5]$$

Another formula for the angle of twist was obtained as follows: For a solid ellipse the angle of twist in terms of fiber stress, as obtained by solving the equations for T and θ in Morley's work, p. 295, is

$$\theta = \frac{f_s l}{G} \frac{(a^2 + b^2)}{a^2 b}$$

Assuming that it holds true for a hollow ellipse also and substituting for f_s the value in Equation [2],

$$\theta = \frac{16}{\pi} \frac{Tl}{G} \frac{(a_1^2 + b_1^2)}{a_1^2 (a_1 b_1^3 - a_2 b_2^3)} \quad \ldots\ldots [6]$$

Tests. In order to obtain data on streamline tubing in torsion, for the purpose of checking the foregoing formulas as well as to obtain some idea in regard to the allowable shearing stress, two pieces of 1.25 × 3 × 0.064-in. chrome-molybdenum streamline tubing were tested in torsion. Tensile tests of two specimens cut from the same tube gave the following properties: Ultimate strength, 102,000 lb. per sq. in.; elongation in 2 in., 14.2 per cent. The average angle of twist of the tubes in radians under a moment of 4000 in-lb. was 0.0172. The angle as calculated by the formulas given above is as follows:

Equation	[1]	[3]	[5]	[6]
Angle of twist	0.0191	0.0191	0.0181	0.0171

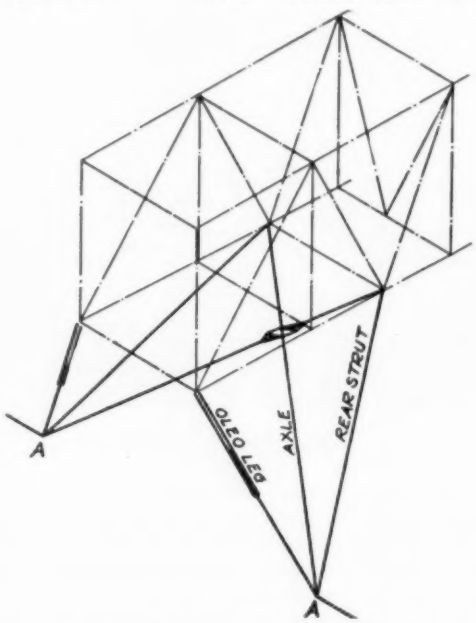


FIG. 9 LANDING-GEAR STRUCTURE

Using in Equation [3] the dimensions of an equivalent ellipse, the calculated value of θ is 0.0176.

Based on these two tests all formulas seem to give fairly accurate results. The torsional modulus of rupture as determined from the test for this tubing was 51,000 lb. per sq. in. (Eq. [2]) and 41,700 lb. per sq. in. (Eq. [4]). Since the torsional modulus is a function of the D/t ratio, it is rather hard to say which formula gives closer results. It is hoped that more experiments will be made on the torsion of streamline and oval tubing, when it will be possible to obtain accurately the variation of the modulus of torsion with the D/t ratio.

Because of the wide use of streamline tubing, the main properties of any standard streamline tubing of any practical fineness ratio are given below

$$I_{major} = 0.0838\, D^2 t(3L + D)$$
$$I_{minor} = 0.0960\, L^2 t(L + 3D)$$
$$A = 1.635t(D + L - 2t)$$
$$y = \text{distance of the minor axis from L.E.} = 0.485L$$

DESIGN OF DETAILS

The use of finer methods of analysis should not be limited to the major structures since analyses of details have still greater bearing on the strength and economy of the structure. Details are points of concentrated stresses and are often more susceptible to wear and deterioration than the main members of the structure.

Greater economy and better uniformity can be attained if every detail is carefully analyzed and designed.

A wing rib which is so often proportioned by simple experience can be much better designed if a truss analysis is made from the rib loading diagram and the member dimensioned by assuming a fixity of 2. In case of wooden ribs the strength of glue joints can be taken between 250 to 300 lb. per sq. in.

Requirements

Careful design and analysis of structure, however, can be economically justified only if the requirements are coordinated with actual loading conditions encountered in the airplane service. This hinges on our knowledge of aerodynamical facts and service conditions, close control of properties



Fig. 10 Landing Gear of the Vought "Corsair"

of material, and good workmanship. The problem of materials and workmanship will not be discussed here, but the coordination of aerodynamical and service facts and design conditions is a vital matter in structural analysis and its importance should be stressed. A number of failures of different parts of the airplane occur on the ground, and very seldom is an adequate explanation found. A case often met in the practice is the failure of cabane struts in bad landing. These struts may stand up successfully under the most severe conditions of aerial acrobatics and may show ample margins of safety for design conditions required by the present specifications, so it would seem unexplainable how they could ever fail in bad landing. A plausible reason would seem to lie in the inadequacy of the present side-load condition. The present specifications make no mention of how the structure should be assumed to be balanced to react against the external loads nor of the members which should be considered in this condition. It would appear logical to assume that the side loads are balanced by inertia forces of equal total magnitude acting in the opposite direction, and by rotative inertia forces opposing the rotation of the airplane about its own center of gravity. The latter forces are proportional to the weights and also to the square of their distance from the center of gravity of the airplane. Thus knowing the loads and reactions, it is possible to make a careful analysis of the structure. A similar analysis was made by the author and, while somewhat elaborate, it led to interesting results. It indicated that it is quite possible that cabane members and certain members in the fuselage truss may be designed by this condition, and because of that it is suggested that the side-load landing condition be revised and made more accurate and specific. It is believed that the requirements should not be made too elaborate, nevertheless they must more nearly approach the actual condition and should specify, if possible, definite means of analysis. In connection with the requirements for the landing condition a new condition should be added to protect the crew and passengers in case the plane should completely turn over in a bad landing. It is the author's practice always to analyze the cabane and center-section structure for this "tipping over" condition, and the load factor which he uses is 4. It may be safer to increase this factor to 6, but whatever it may be it must be ample to protect the occupants of the plane from possible failure of the center-section structure or the cabin superstructure.

As regards flying conditions, the nose-dive condition should be changed to more closely represent the aerodynamical facts, inasmuch as this condition is very important for military combat airplanes. We all are acquainted with the present requirements and specifications for analyzing the nose-dive condition and with the aerodynamical facts of the real nature of the loads and couples acting in nose dive.

A more accurate analysis can be easily made, and while it entails slight complication, it is, nevertheless, worth while to determine the structural strength of the craft by making full use of available aerodynamical information. The wind-tunnel test of the airfoil used gives the necessary moment coefficient to obtain the couples on the wings:

$$M = \rho C_m S V^2 c$$

where ρ = density
C_m = moment coefficient
s = wing area
c = chord
V = terminal velocity.

The drag loads must be obtained by writing the equation

$$D = \frac{C_{Dw}}{C_{Dp} + C_{Ds} + C_{Dw}}$$

where D = drag of wings
C_{Dw} = drag coefficient of the wings
C_{Dp} = parasite drag coefficient
C_{Ds} = drag coefficient of the propeller
W = gross flying weight.

To obtain the magnitude of the terminal velocity V in the dive, it is necessary to obtain the magnitude of C_{Ds} from available information on the propeller used (see Diehl, "Engineering Aerodynamics," pp. 134 and 221) and write the standard equation

$$\rho(C_{Dw} + C_{Dp} + C_{Ds})SV^2 = W$$

For planes not to be used in stunt flying the factor of safety must be sufficiently high to preclude any possibility of failure in the air, but it does not have to prevent the members of the structure from taking permanent deformation. On the other hand, planes which because of their service requirements have

to dive repeatedly must have a factor of safety sufficiently high to keep the stresses in all the members of the structure within the elastic limit of the material.

Factor of Safety

In aeronautical structural engineering we deal with load factors which are actually overload factors multiplied by a factor of safety. While this is a logical procedure, it is well to keep in mind that it provides only a general factor to be used in calculations of design loads, and that in the actual proportioning of the members one must take into consideration the nature of the stresses and the service conditions. This is particularly true in the design of details, but its importance should not be lost sight of in proportioning the major members of the structure. The actual factor of safety is equal to the breaking load of the structure divided by the maximum possible load on the structure in flight.

Since present design requirements are based on the ultimate strength of the structure, one must be particularly cautious that at no time shall any part of the structure be stressed beyond its elastic limit.

The factor of safety, generally speaking, must take care of—

1 Inaccuracies of calculations and overload assumptions
2 Possible increase in weight in future
3 Deterioration of the material with time
4 The condition that at no time of normal service conditions shall actual working stresses exceed the elastic limit of the material.

In the design of details there are three other conditions which should not be lost sight of by the design engineer, namely,

5 Variation in the material and inaccuracies in workmanship
6 Defects in the material
7 The effect of vibrations, reversal of loads, etc.

Conclusions

It is hoped that the examples of the structural problems given in this paper and met in a routine practice will emphasize the importance of using finer methods of calculation. A closer coordination of aerodynamical data and structural requirements will still further perfect our methods of structural analysis and thereby lead to mechanical and aerodynamical perfection of airplanes. Structural failures of well-designed planes in the air are almost unknown, and if such failures occur their origin can almost invariably be traced to causes other than the imperfection of present methods and requirements. But future development hinges not only on safety, which can always be obtained at the sacrifice of performance, but on economy as well.

Economy and efficiency of structure are the paramount problems in the design of the airplane of today and tomorrow.

The author is indebted to the Chance Vought Corporation for permission to use the examples of structures described in this article, to Mr. Charles J. McCarthy for his criticism and comments and to his assistants, Messrs. George W. Brady and Charles R. Strang, who made most of the calculations which are mentioned in the paper.

Discussion

Alfred S. Niles.[2] The author has criticized the rule that the effect of the redundant members must be modified by 25 per cent on the grounds that the resultant design load in any given member will depend on which parts of the structure were assumed to be redundant. This effect of the rule was thoroughly recognized when it was first adopted by the Air Corps, but no attempt was made to be more specific as to which members should be considered redundant for two chief reasons. The first was the practical impossibility of writing a generally inclusive rule unless it was provided that the members chosen as redundant should be those which would have the least influence on the stresses in the other members. This would often involve many computations to determine which members should be chosen, and the checking authorities would seldom be inclined to reject a stress analysis because it erred somewhat on the safe side. Furthermore, it would be difficult in some cases to decide which of several possible groups of redundant members had the least influence on the stresses in the others. The second reason was that the rule as worded gave a slight advantage to the clever designer who was able to pick out the members which would have the least influence on the others. The author shows that he obtained lower net loads by using the front member of the "N" instead of the center member, where another engineer might easily overlook this fact. This would permit him to use a lighter structure, a reward for intelligent design that the Air Corps would have been glad to allow him.

Though not mentioned by the author, the rule in question is ambiguous in another direction. It is not clear as to how the 25 per cent allowance should be computed when there are two or more redundant members that have opposite influences on a member of the included statically determinate structure. Suppose for example a certain member is subjected to a load of —4000 lb. if the redundant members are out of action; the effect of one of the redundant members is to subject it to + 2000 lb., and that of the other to subject it to —1000 lb. What should be the design load according to the rule? The most conservative interpretation is to compute the design load as follows: $P = -4000 - 1250 + 1500 = -3750$ lb., the 25 per cent being applied separately to the effect of each redundancy. An alternative is to apply the 25 per cent to the net effect of the redundant members, thus: $P = -4000 + 0.75\ (2000 - 1000) = -3250$. The writer believes the latter method to be sufficiently conservative, and hopes that the Department of Commerce will reword the rule so this interpretation will be specified.

The writer agrees that the present rules for the conditions of diving should be modified in such a direction that they will depend on the moment coefficients of the airfoil used. He does not believe, however, that the author's method is practical. Its chief defects are that it assumes that the airplane is diving at terminal velocity, and that it requires a knowledge of the drag coefficient of the propeller. With the exception of a few types of military designs, few if any airplanes ever dive at terminal velocity, and it would be as unfair to require that a three-engined airliner be designed for a dive at terminal velocity as to require that it be designed for a high-angle-of-attack load factor equal to the maximum that could be theoretically imposed in a sudden pull-out from a dive multiplied by a factor of safety. It would be much more logical and also more practical to design for a condition of a dive at a specified number of times the stalling speed. This number could depend on the size and type of airplane, if desired. The speed of dive would not be the only quantity specified, but it would be the chief one, and the others could be selected by the Department of Commerce when the revised rule is formulated. For pursuit airplanes the dive at terminal velocity is a reasonable design condition, but this could be covered by an empirically determined ratio of diving speed to stalling speed as well as by a study of drag coefficients.

The second weakness of the proposed method of determining the forces in the dive is lack of data on the drag coefficients of propellers at diving speeds. As long as there is a lack of reliable

[2] Guggenheim Aeronautic Laboratory, Stanford University, Stanford University, Cal. Mem. A.S.M.E.

data on this quantity one would be forced to make assumptions, and the writer believes that at present it is possible to assume diving speeds with more precision than propeller-drag coefficients. In fact, it would be necessary to compute the resulting diving speed from an assumed propeller-drag coefficient to learn whether or not the latter was reasonable.

JOSEPH S. NEWELL.[3] To the statement that "structural analyses of the airplane are intimately related to an accurate knowledge of the aerodynamic and dynamic loads acting on the structure" one might add "and to the allowable strength properties of the materials employed in the structure." With the conventional single-bay biplane this added phrase may be construed as applying to fittings and connection details, since it seems to be these items that give trouble during static tests. So far as the writer knows no airplane has yet passed the complete schedule of static tests established by the Army Air Corps without failures occurring in one of the test conditions at loads less than those specified for design.

A majority of these failures occur in or near connections even though the analyses are made on the conventional and generally conservative assumption that the diagonal members in the outer interplane-strut assembly are out of action. Such an assumption results in the design of a front truss having sufficient strength to carry a majority of the load on the wings when the center of pressure is forward, and a rear truss of sufficient strength to carry practically all of the load when the center of pressure is in its rearward position. In either of these conditions the heavily loaded truss would tend to deflect to a greater extent than the more lightly loaded one, and it is the function of the diagonal members in the interplane-strut group to equalize the deflection of the two trusses, or to transfer some of the load from the more heavily to the more lightly loaded truss.

The actual result of the incidence wires, or the diagonal of an N-strut, is therefore, a tendency to relieve one truss and add load to the other so one would naturally assume that conventional biplane cellules would withstand test loads greater than those used in making the structural analyses. In the case cited by Dr. Watter one would expect the ratio of actual load carried in static test to design load to be in the neighborhood of 17,620/12,516 if limited by the sizes of the front beam, or 13,500/10,105 if limited by the flying wires or their fittings, as the method of analysis takes into consideration the effect of the incidence wires. Or if, as static tests seem to show on other single-bay biplanes, connection details proportioned by conventional methods will not stand up to the anticipated load factors because of lack of knowledge concerning the allowable loads on thin steel plates, or concerning the actual loads imposted upon the members themselves, one would expect details designed for the loadings resulting from a least-work analysis to have strengths about in the ratio of 12,500/17,600. That is, roughly two-thirds of those obtained by conventional methods. In other words, the weakest details in a conventional cellule might well be further weakened if proportioned by more advanced methods of analysis.

That such a condition might logically be expected is obvious when one considers the simplifying assumptions ordinarily made when applying the method of least work to a complete cellule. In the first place, the work involved in the bending of the spars is entirely neglected although it may very well represent two-thirds or three-quarters of the total work done by the members of the cellule. The work done by the counter-wires in the drag trusses before they go out of action is also neglected, and it is seldom that any provision is made for effective changes in length of members due to play in pin joints and connections. Each of these simplifying assumptions results, therefore, in a modification of the relative deformations of the various panel points in the structure, and so renders the method of least work, which in the final analysis depends on the relative deformations of the structure, of doubtful value when applied to a wing cellule.

The 25 per cent increase or decrease in stress provided by the present rules of the Army, Navy, and Commerce departments is ambiguous and unsatisfactory as Dr. Watter points out. That some allowance must be made to provide for the "work" neglected in the ordinary least-work analysis is obvious, and the Department of Commerce is modifying its rule in this regard to differentiate between structures involving pinned connections, and those having rigid connections such as a welded fuselage. It is the writer's opinion that any of the methods for analyzing redundant structures should be applied with extreme caution to a structure involving a multiplicity of members and joints unless a final proof of the strength of the resulting "light" structure is obtained by sand testing a complete assembly to destruction.

The sole justification for present-day load factors lies in the fact that with conventional methods of analysis, they work out reasonably well in practice, and any method of reducing the effective strength of any part of an airplane structure, although having the virtue of lightening it, should be used with caution and with full knowledge of the uncertainties included in the simplifying assumptions which it involves. With such a knowledge, and with a judicious replacement of a portion of the material "saved" by the more precise method of analysis at points shown by static tests to be critical, it is probable that a somewhat better proportioned structure may be obtained with the more advanced methods of analysis than with the conventional. But such methods cannot be used as a cure-all, and a designer must constantly bear in mind that they are highly approximate as ordinarily applied to wing structures.

Moreover, when full-flight or model pressure data for complete cellules are investigated, and it is found that the front upper spar of a biplane may be subjected to loads approximately 25 per cent greater than those obtained by conventional assumptions as to distribution, a designer hesitates to employ advanced methods of computation which may reduce the strength of the structure 20 to 30 per cent below the "conventional."

From all of which one may conclude that the writer does not advocate the general application of advanced methods of structural analysis until research in the two fields of "actual load, or pressure, distribution" and "allowable strength of materials" has justified them. The advanced methods of analysis already exist, the knowledge of actual loads on the structure or of actual allowable stresses in the materials used does not, and it appears irrational to attempt extreme precision in one phase of design when such gross inaccuracies still exist in the other two types of construction.

It is imperative that these inaccuracies, or approximations, be eliminated as soon as possible and, as Dr. Watter has indicated, the landing chassis offers great opportunities for research. That we know little or nothing of the loads coming on landing gears or of the allowable stresses on the tubes used in chassis structures is indicated by the success of a split-axle landing gear, each member of which, by conventional systems of analysis and with conventional, conservative allowable stress data, showed a negative margin of safety in excess of 15 per cent. This particular type of chassis, to the writer's knowledge, passed a drop test satisfactorily, and has given no trouble in service during the past year although used on several airplanes in different parts of the country. Efforts should be concentrated on the determination of actual loads to be carried and actual allowable strengths of various materials, and use such methods of analysis as the precision of the "load" and "allowable load" data justify.

[3] Assistant Professor of Structural Engineering, Dept. of Civil Eng., Mass. Inst. of Tech., Cambridge, Mass.

Author's Closure

Since the purpose of specifications is to standardize procedure and obtain uniformity, it is hardly fair to attempt justification of any rule because of the advantages which it may give to a clever designer. Mr. Niles' defense of the 25 per cent rule still leaves the question of ambiguity unanswered. As long as the rule remains in effect, however, it is very helpful to find Mr. Niles' example for the case of several redundancies.

Mr. Niles' criticism of the proposed diving condition may be justified in regard to diving velocity to be chosen for the design conditions. The author was concerned mainly with methods of design, and the choice of factors would be preferably left to a committee with sufficient experimental and theoretical data to justify their findings. Regarding the impracticability of calculating the terminal velocity of an airplane, the magnitude of the negative drag of the propeller is readily determined, and the necessary data could be gathered easily if required.

It seems that the warning sounded by Mr. Newell is based mainly on the fact that although the primary structure of an airplane usually withstands the loads imposed on it, the details of construction such as fittings usually fail before the design load is reached.

Although the author knows a number of cases where carefully designed structures withstood the loads imposed on them, the fact that details often fail is only another excellent justification for the necessity of finer analysis. In many cases the details present problems much more complicated than some primary structures, and in the design of a fitting, in a number of cases, the engineer has to resort to very fine analysis which must be further supplemented with knowledge of shop practice and working of material.

To attempt to justify simplified analysis, only because of slight inexactness in known data, may lead one to serious errors, and the dangers involved in approximations fully warrant the extra work which finer analysis usually entails. In the author's opinion, present knowledge of properties of material combined with the fine inspection method employed fully justify the use of advanced methods of analysis.

The Production and Uses of Helium Gas

By R. R. BOTTOMS,[1] LOUISVILLE, KY.

The author, after discussing the origin and occurrence of helium in natural-gas deposits, presents an estimate of the quantity available. He then describes in detail processes for separating the gas from the other gases with which it is associated, and for purifying and liquefying it. Following this he deals briefly with the questions of storage and transportation. In conclusion he enumerates the remarkable physical properties of helium, discusses its use for the inflation of airships, both pure and in a non-inflammable mixture with 20 per cent of hydrogen, and then gives estimates of the cost of helium storage, transportation, and purification facilities at an airship terminal designed to care for four ships.

HELIUM was first discovered as a terrestrial element in 1895 by Sir William Ramsay while investigating an inert gas given off from the mineral cleveite on boiling it with sulphuric acid. Ramsay thought this gas was argon, but upon examining the spectrum of the gas in a Crookes tube it was found to give a brilliant yellow line, designated as the D_3 line of the spectrum. This D_3 line had already been discovered in 1868 to exist in the spectrum of the sun and had been ascribed to a new solar element. Thus the solar element had been proved to exist on the earth. It was therefore called helium.

Helium belongs to the group of elements known as the rare gases of the atmosphere. The other gases of this group are neon, argon, krypton, and xenon. All are characterized by their comparative rarity, chemical inertness, and the fact that they are monatomic elements.

For about 15 years succeeding Ramsay's discovery of helium in 1895, a great deal of research was carried out on the occurrence and properties of helium. It was found to be a constituent, in small amounts, of the atmosphere, of gases from mineral springs, of volcanic gases, and occurred in minute quantities in sea water and was found to be present in the atmosphere to the extent of one part in 185,000, and to occur in minute traces in nearly all rocks and minerals.

Occurrence

In 1905, Cady and MacFarlane, of the University of Kansas, investigating a peculiar noncombustible gas from Dexter, Kansas, found 1.84 per cent helium in the gas. This discovery led them to examine other gases from the same locality and also gases from other points in Kansas, and in all of them helium was found. These investigations have been greatly extended by Cady and also by others since these discoveries, and today almost all known deposits of natural gas in the world have been tested for helium.

Only in the United States and Canada are helium-bearing gas deposits found with a sufficient helium content to make it economical to process them. In Italy, however, there are certain gases issuing with steam from boric acid fumaroles which might possibly be treated for the recovery of helium. These gases, after the condensation of steam, consist principally of carbon dioxide, but since the latter gas is very easily liquefied there would be no great technical difficulty in treating the gas. It has been estimated that from 200,000 to 250,000 cubic feet of helium per day could be obtained from this source.

In Canada there are two gas fields carrying a small percentage of helium—the Bow Island field in Alberta, a few miles north of the Montana line, and a field in Ontario north of Lake Erie. Neither of these fields carries a helium content of over 0.5 per cent. In this country, under present conditions, it would not be considered a commercial grade of helium-bearing gas.

In the United States helium occurs to some extent in most the natural-gas deposits. However, there are some gas deposits occurring on the Pacific Coast, on the Gulf Coast in Louisiana, and in the Rocky Mountain region which show no helium that can be detected by the usual analytical method.

TABLE 1 TYPICAL ANALYSES OF NATURAL GAS

State	County	Location	Depth, ft.	He	O_2	CO_2	CH_4+	Nitrogen	Type
Kan.	Cowley	29–33–5	600	0.34	0.3	0.5	87.5	11.36	Hydrocarbon
Kan.	Cowley	24–33–6	1080	2.23	0.23	0.06	19.8	77.68	Nitrogen
Kan.	Cowley	4–33–7	1015	2.87	0.31	0.12	1.7	96.2	Nitrogen
Kan.	Cowley	9–32–5	87	0.00	0.00	0.016	0.0	99.8+	Nitrogen
Kan.	Russell	17–12–15	2500	1.64	0.20	0.04	45.0	53.12	Nitrogen
Kan.	McPherson	32–2–18	1450	2.45	0.00	0.00	41.0	56.65	Nitrogen
Kan.	Wabansee	9–12–10	276	3.6	0.15	0.02	11.0	85.23	Nitrogen
Kan.	Elk	26–31–10	1995	3.4	0.20	0.25	26.0	70.15	Nitrogen
Kan.	Greenwood	22–26–11	675	1.38	0.15	0.00	64.5	33.97	Hydrocarbon
Kan.	Rush	27–17–17	3578	1.78	0.00	0.31	86.7	11.21	Hydrocarbon
Okla.	Jefferson	19–7–5	739	0.97	0.30	0.33	62.2	36.2	Hydrocarbon
Okla.	Jefferson	19–7–5	602	1.00	..	...	...	....	.....
Okla.	Osage	28–28–7	1828	0.56	..	...	...	....	.....
Okla.	Texas	(Pipe-line sample)		0.36	..	...	...	....	.....
Okla.	Grady	25–4–8	2307	0.17	0.34	0.14	85.17	14.18	Hydrocarbon
Tex.	Potter	Tuck Trigg	3180	1.70	..	...	...	....	.....
Tex.	Wheeler	.....	..	0.16	..	...	...	....	.....
Tex.	Moore	Skelly	..	0.46	..	...	...	....	.....
Tex.	Hartley	A-7	..	0.96	..	...	...	....	.....
Ark.	Eldorado	(Pipe-line sample)	..	0.04	0.20	4.79	74.97	20	Hydrocarbon
Calif.	Kern	30–31–24	..	0.00	0.89	6.56	88.56	4.09	Hydrocarbon
Colo.	Jackson	Walden	5200	0.16	0.00	92	4	3.84	CO_2 type
Colo.	Las Animas	(Pipe line sample)	..	0.07	0.00	0.3	98.0	1.63	Hydrocarbon
Utah	Grand	Harley Dome	850	7.07	0.5	3.5	17	71.93	Nitrogen
Utah	Grand	Harley Dome	450	2.23	0.3	...	...	....	.....
Utah	Emery	Woodside Dome	450	1.3	..	82.0	...	....	Carbon dioxide
Utah	San Juan	22–43–22	4525	0.76	0.00	0.26	76.20	22.78	Hydrocarbon
N. M.	Eddy	4–19–28	690	0.22	0.00	0.11	62.45	37.22	Hydrocarbon
N. Y.	Allegheny	Andover	..	0.3	0.00	0.00	99+	Trace	Hydrocarbon
Ohio	Fairfield	Rush Creek	2721	0.5	0.00	0.00	69.26	30+	Hydrocarbon
Ky.	Princeton	Princeton	..	0.1	0.00	0.40	99+	0.2	Hydrocarbon
Mich.	Muskegon	Laketon Twp.	2050	0.24	0.2	0.4	85.96	14	Hydrocarbon
Tenn.	Davidson	Dean No. 1	125	0.24	0.53	0.22	94.4	4.61	Hydrocarbon
Ill.	Lawrence	(Pipe-line sample)	..	0.04	0.4	0.10	96.63	2.83	Hydrocarbon
La.	Richland Parish	.....	2300	0.09	0.15	0.00	82.0	17.76	Hydrocarbon
Pa.	Mayport	.....	..	0.08	0.00	0.12	98.7	1.2	Hydrocarbon

Natural gases from different fields and even from different geological strata in the same field may differ widely in chemical composition and helium content. They fall into four main types: hydrocarbon, nitrogen, carbon dioxide, and sulphur, according as hydrocarbons, nitrogen, CO_2, and H_2S, are the most prominent constituents. A few typical analyses are given in Table 1.

The Helium Company has, during the last two years, been conducting an active survey of all the natural-gas resources of the United States in order to determine sources of helium-bearing gas.

[1] Kentucky Oxygen-Hydrogen Co., Inc.

Presented at the Third National Meeting of the A.S.M.E. Aeronautic Division, St. Louis, Mo., May 27 to 30, 1929.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

Table 1 deals only with a few of the many hundreds of samples tested, but it gives an idea of the remarkable results obtained.

Origin of Helium

The facts relating to the origin of helium and the reason for its accumulation in certain natural-gas deposits have not been entirely cleared up. There are several theories relating to the origin of helium:

1 The radioactive theory, which assumes that the helium in natural-gas deposits has been accumulated by radioactive disintegration of radium-bearing minerals, distributed through the sedimentary rocks.

2 The theory propounded by Rogers that the helium occurring in natural-gas deposits is primordial in origin and has nothing to do with radioactivity. The evidence in either case is so meager that it is practically impossible to determine which of these theories is correct. Rogers objects to the radioactive theory on the basis that the quantity of radioactive elements necessary to produce the enormous volume of deposits would be greatly in excess of what is actually found in the crust of the earth.

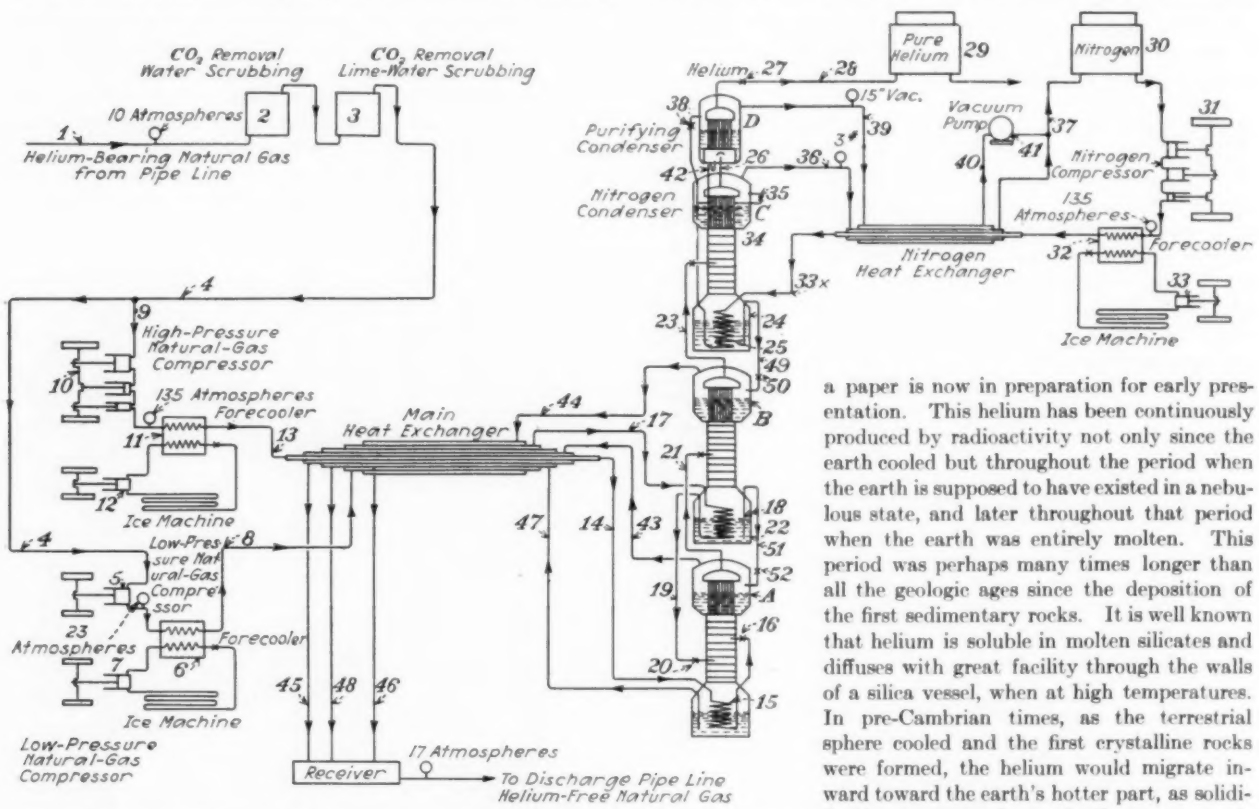


Fig. 1 Apparatus Used Until Recently at the Government Helium Plant at Fort Worth, Texas

The facts relating to the accumulation of helium of course are simple. A great deal of investigation has shown that it occurs in various sedimentary rocks overlying, or at least in the vicinity of, granite uplifts which have been faulted, and we find that along these granite folds the natural gas will usually contain anywhere from 0.1 per cent helium up to as high as 8 per cent, while the gas occurring outside of the faulted area is nearly always low in helium content.

A theory has been evolved by the author regarding sources of helium in natural gases which, if correct, will aid in the location of helium-bearing gas. Briefly stated, this theory is that the original source of helium is the basal crystalline rocks, underlying the sedimentary deposits, and that the helium with large amounts of nitrogen and carbon dioxide in the granite seeps into the sedimentary deposits through fissures, crevices, and fractures in the basal rocks and diffuses and migrates through the structural rocks until it is trapped underneath an impervious cap rock in anticlinal folds or domes. Whether the helium in the granite rocks is primordial helium or results from radioactive disintegration of uranium, radium, and thorium makes little difference.

In this connection it seems most probable that the helium now found in natural-gas deposits was produced entirely by the radioactive disintegration of elements with high atomic weights. A revolutionary theory has been evolved concerning this point and a paper is now in preparation for early presentation. This helium has been continuously produced by radioactivity not only since the earth cooled but throughout the period when the earth is supposed to have existed in a nebulous state, and later throughout that period when the earth was entirely molten. This period was perhaps many times longer than all the geologic ages since the deposition of the first sedimentary rocks. It is well known that helium is soluble in molten silicates and diffuses with great facility through the walls of a silica vessel, when at high temperatures. In pre-Cambrian times, as the terrestrial sphere cooled and the first crystalline rocks were formed, the helium would migrate inward toward the earth's hotter part, as solidification progressed, much the same as air migrates to the interior of a cake of ice as it freezes. Thus, whenever in later times there has been any volcanic activity tending to form fractures in the crystalline rocks, this concentrated helium tends to escape. These fissures, cracks, or fractures may occur whenever there is an area uplifted by volcanic activity, especially if accompanied by faulting.

If this theory is correct then we should feel more certain of finding helium in the vicinity of major granite uplifts or folds, especially if faulted, and in the vicinity of other volcanic activity which would tend to open up passageways for helium from the interior of the earth.

The helium-bearing gas, accumulating as just described, will probably consist entirely of non-combustible gases, principally nitrogen and carbon dioxide. Unless hydrocarbon gas forms in surrounding source beds and migrates into the same reservoir and dilutes the accumulating volcanic gas, the helium-bearing gas as described would consist almost solely of nitrogen, carbon dioxide, and helium. If dilution has taken place the helium content of the gas would depend upon the extent of that dilution.

The Helium Company, the latter part of 1928, using this theory as a guide began some investigations looking toward the discovery of helium in those parts of the country where the necessary geological conditions seemed to be favorable toward the accumulation of helium-bearing gas. As shown in Table 1, data are being accumulated which will form a favorable guide for intelligent exploration and will ultimately give the airship industry the necessary assurance as to the location and extent of its helium supply.

Quantity of Helium Available

There seem to be some misgivings in the minds of those interested in airship development regarding the amount of helium available for airship use. These misgivings are entirely unfounded. A recent estimate of the total available helium already in sight is about as follows:

Kansas	2,000,000,000 cu. ft.
Texas	3,000,000,000 cu. ft.
Colorado	2,000,000,000 cu. ft.
Other states	3,000,000,000 cu. ft.

This gives us a grand total of 10 billion cubic feet of helium, which, if used at the rate of 10 million feet a year, would last for 1000 years, and if used at 10 times that rate would last for 100 years.

This estimate merely gives the present status of the helium supply. About a year ago the estimate of the available supply

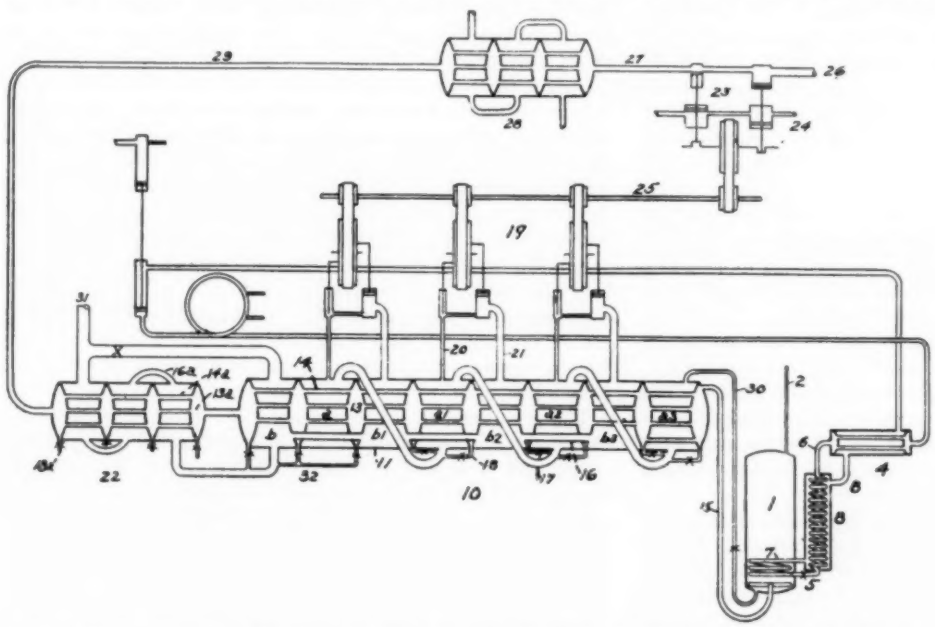


Fig. 2 Modification of Jeffries-Norton Process Used in 1918–1919 at Bureau of Mines Helium Plant, Petrolia, Tex.

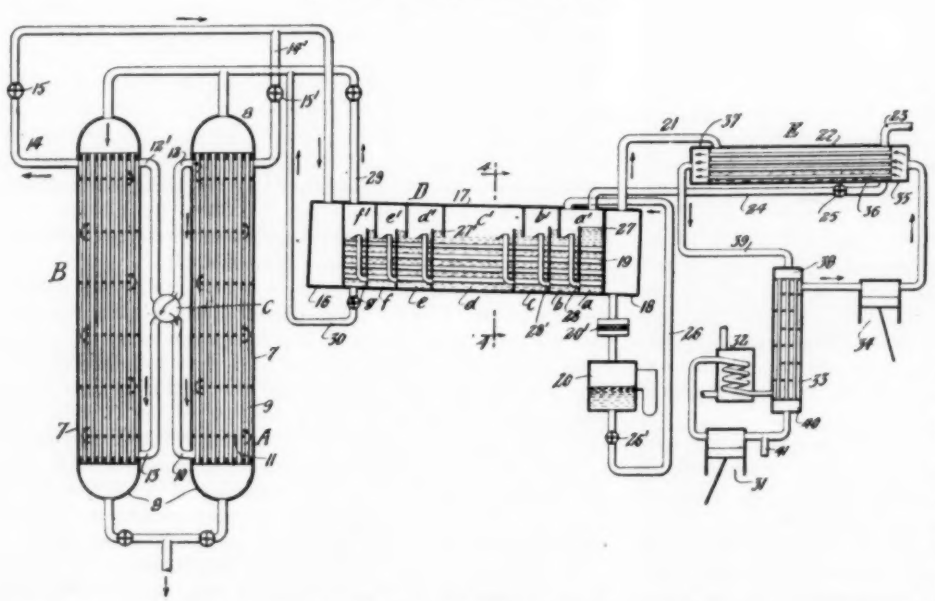


Fig. 3 Schematic Drawing of Apparatus Now Being Used in the Government's Plant at Amarillo, Tex.

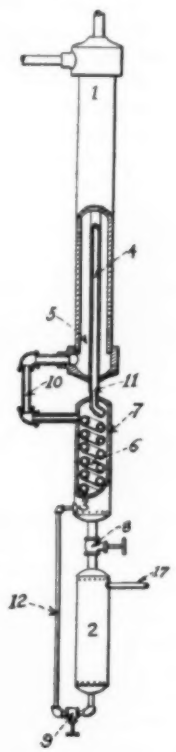


Fig. 4 Author's Process as Installed in the Helium Company's Plant at Dexter, Kan.

in sight amounted to about 7 billion cubic feet, but two of the largest helium-bearing areas in the world have been discovered since that time.

Methods of Production

The technical methods of producing helium are based on the fact that all gases, with the exception of helium, can be reduced to the liquid phase at readily acquired temperatures. Helium being a most difficult gas to liquefy and so refractory in this connection that it has been liquefied at only two places in the world, makes it very easy, therefore, to separate helium from

the other constituents with which it is associated by simple liquefaction of the other constituents and the separation of the gaseous helium from the liquids. Every process for the production of helium is based on these facts.

The accompanying illustrations, Figs. 1 to 4, are given in schematic form and show processes for the production of helium.

Government Plant, FORT WORTH, TEX. Fig. 1 shows the apparatus which was used until recently at the Government plant at Fort Worth, Texas. This is a highly complicated apparatus. Its action is about as follows:

The helium-bearing gas, previously deprived of its gasoline constituents, and arriving at the plant through pipe line 1, at or above 10 atmos. pressure, is scrubbed at that pressure in scrubber 2 for the removal of carbon dioxide. Owing to the increased solubility of carbon dioxide at such pressures, water alone acts as a very effective agent for removing carbon dioxide. If a more complete removal of carbon dioxide than can be obtained with water alone is desired, the gas may be passed to a second scrubber 3, where it is treated with a solution of an alkaline substance such as lime water. The absorbent action of lime water is also increased by the pressure.

From scrubber 3 a portion of the gas passes through line 4 to low-pressure compressor 5, where its pressure is brought to about 23 atmos. The compressed gas passes through a forecooler 6 refrigerated by an ice machine 7, and thence through line 8 to the main heat exchanger.

Another portion of the helium-bearing gas, after removal of the carbon dioxide, is drawn through line 9 to high-pressure multistage compressor 10, and is brought to about 135 atmos. The high-pressure gas passes through a forecooler 11 refrigerated by an ice machine 12, and is carried through line 13 to the main heat exchanger, but is kept separate therein from the low-pressure gas entering through 8.

The high-pressure gas emerges from the heat exchanger through line 14, whence it passes to coil 15 immersed in a bath of boiling liquid at low temperature. After its final cooling in the coil, the high-pressure gas expands through valve 16 into the column at a point below condenser *A*. The low-pressure gas leaves the heat exchanger by line 17, passes through a coil 18 likewise immersed in a low-temperature bath of boiling liquid, and then flows through line 19 and valve 20 to the same column into which the high-pressure gas is expanded.

The expansion of the cooled high-pressure gas to the column pressure, about 22 atmos., absorbs a large quantity of heat. The less volatile constituents of both high- and low-pressure gas, for example the propane, most of the ethane, and some methane, are condensed, flow down through the column where they are subjected to a rectification to remove any dissolved helium, and collect in the kettle containing the coil 15.

Those constituents of the gas which are not liquefied on expansion, and including all of the helium of the gas, pass upward through the column to condenser *A*, where they are subjected to the temperature of the liquid boiling about the condenser, this liquid being held under a pressure of about 17 atmos. A further quantity of relatively non-volatile material is liquefied in condenser *A* and returns to the column below, in which it also is rectified.

The helium and other gases not liquefied in condenser *A* pass through line 21 to a column which is surmounted by condenser *B*. Condenser *B* is surrounded by a boiling liquid which is held at the same pressure as that which obtains about the condenser *A*, but the liquid surrounding condenser *B* contains most of the volatile constituents of the natural gas, such as nitrogen and methane, and hence is at a lower temperature than the liquid about the condenser *A*. As a result, an additional quantity of material is liquefied in condenser *B* and descends through and is rectified in the column immediately below, collecting finally in the kettle 22 in which is contained the coil 18 previously referred to.

The helium, still admixed with many times its volume of gaseous impurities, leaves condenser *B* through line 23 and enters the column which is immediately below condenser *C*, the helium-containing mixture being still at the same pressure of 22 atmos. Condenser *C* is cooled by substantially pure liquid nitrogen boiling at about atmospheric pressure, and hence the temperature of *C* approaches closely the boiling point of nitrogen, —193 deg. cent. As this temperature is considerably below the temperatures of condensers *A* and *B*, a further quantity of gas associated with the helium, now chiefly nitrogen, is condensed, and passes down through the column immediately below, where it is rectified to remove any dissolved helium. The liquid collects in kettle 24 at the bottom of the column and there surrounds and cools coil 25, which performs the function presently to be described.

Due to the temperature and pressure prevailing in condenser *C*, a helium concentrate containing about 90 per cent of helium will be produced. This concentrate passes through line 26 to the final condenser *D* where it is subjected, still at a pressure of about 22 atmos., to the temperature of nitrogen boiling under reduced pressure, say, at about 0.5 atmos. abs. This will give a temperature below —195 deg. cent. in condenser *D*, and as a result some of the small amount of residual nitrogen will be liquefied and a concentrate containing about 98 per cent helium, and usually considered pure enough for aeronautical purposes, will pass the condenser, expand through valve 27, and pass through line 28 to the helium holder 29.

The arrangement for producing the nitrogen baths which surround condensers *C* and *D* will now be described.

Nitrogen is drawn from holder 30 and compressed in multistage compressor 31 to a pressure of about 135 atmos. The compressed gas then passes through forecooler 32 refrigerated by the ice machine 33, and thence through the nitrogen heat exchanger. The cooled high-pressure nitrogen emerges from the heat exchanger through line 33 and passes through coil 25 previously referred to, where it is brought into thermal contact with the very cold boiling liquid in kettle 24. The liquid in kettle 24, it will be noted, is under a pressure of about 22 atmos. The nitrogen now passes through coil 34 disposed in the bath around condenser *C* and hence immersed in nitrogen boiling at a little above atmospheric pressure.

After the final cooling the nitrogen expands through valve 35, which results in its partial liquefaction. The liquid nitrogen collects about coil 34 in condenser *C*, where it boils away continuously and produces the refrigerating effect already referred to. The gaseous nitrogen produced by the boiling of this bath passes at slight pressure through line 36 to the nitrogen heat exchanger, where it cools the high-pressure nitrogen and then flows through line 37 to the nitrogen holder 30.

The space around condenser *D* is held, as already stated, under a partial vacuum. Liquid nitrogen will rise at a regulated rate through valve 38 and collect about condenser *D*, boiling there under the reduced pressure at a very low temperature. The gaseous nitrogen produced by the boiling of this liquid passes through line 39 to another section of the nitrogen heat exchanger, where it assists in the cooling of the high-pressure nitrogen, and then passes through line 40 and vacuum pump 41 to the nitrogen holder. The small quantity of liquid formed in condenser *D*, and consisting of pure nitrogen, returns through the trap 42 to the top of the column surmounted by condenser *C*.

The gases produced by the boiling liquids around condenser *A* and *B* pass through lines 43 and 44, respectively, to the main heat exchanger, where they cool the high- and low-pressure helium-bearing gas, and then pass through lines 45 and 46, re-

spectively, to a receiver which discharges helium-free gas into a pipe line to be transported to the place of its consumption. Since the baths about condensers *A* and *B* are maintained at 17 atmos. pressure, and the reverse-flow passages of the main heat exchanger are at the same pressure, the exit gases may enter the line without being further compressed. A portion of the liquid contained in the kettle around coil 15 is also continuously withdrawn through line 47 to the main heat exchanger, in which it evaporates and assists in cooling the high- and low-pressure helium-bearing gas, passing from the heat exchanger through line 48 to the receiver and pipe line already referred to. The exit gases passing through the main heat exchanger and lines 45, 46, and 48 constitute practically all the natural gas entering the plant with the exception of the helium.



FIG. 5 VIEW OF HELIUM COMPANY'S PLANT AT DEXTER, KAN.

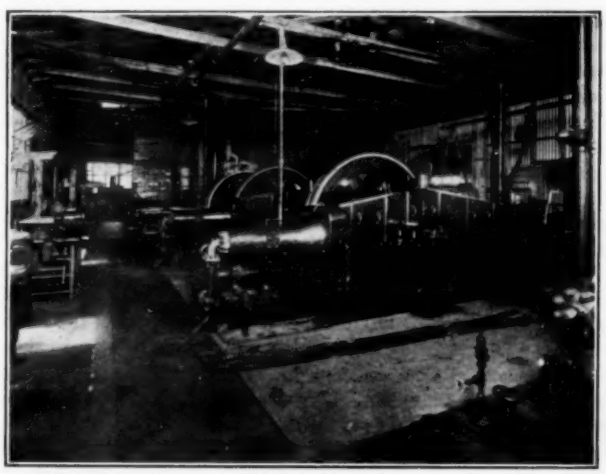


FIG. 6 INTERIOR VIEW OF HELIUM COMPANY'S PLANT AT DEXTER, KAN.

The condenser *B* is cooled by liquid which collects in kettle 24, and flows through line 49 and valve 50 into the space surrounding condenser *B*. Condenser *A* is similarly cooled by liquid which collects in kettle 22 and flows through line 51 and valve 52 to the space surrounding condenser *A*. At valves 50 and 52, the pressure on the liquids just referred to is reduced from about 22 atmos. to about 17 atmos.

Should it be desired to remove from the helium leaving condenser *D* the very small quantity of impurities contained in it, the apparatus shown in Fig. 4 may be used, being introduced into the system at an appropriate point—for example, between condenser *D* and the helium holder.

This apparatus consists of a series of identical absorbers comprising an inner chamber packed with activated carbon 53 and held between foraminous (porous) partitions 54. To facilitate the absorption of the impurity (nitrogen) in the carbon, the latter should be cooled to a very low temperature, and the author has shown the absorbers provided with jackets 55, through which may be passed a very cold medium, such as nitrogen from the space around condenser *C*. The absorbers may be small, as they are designed to take up an impurity which consititutes only about 2 per cent of the helium, the helium being less than 1 per cent of the gas treated.

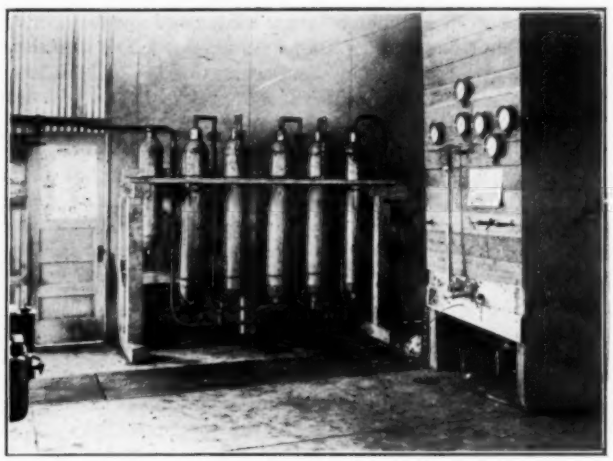


FIG. 7 GAS-DRYING BATTERY AND COLUMN IN DEXTER PLANT



FIG. 8 END VIEW OF NAVY DEPARTMENT'S HELIUM TANK CAR

When cooled to a temperature such as that indicated, activated carbon selectively absorbs the impurity with great efficiency, and the purity of the helium can be raised to any desired degree. When one absorber is saturated to the desired degree with impurity, it is cut out of both the helium and the refrigeration circuits and allowed to warm up to room temperature, whereupon the absorbed impurity will pass off through 56. The absorber is

then ready for use again as soon as it has been cooled to the required low temperature.

Fig. 2 shows a modification of the old Jeffries-Norton process which was used by the Bureau of Mines at Petrolia, Texas, in 1918 and 1919, but which proved to be a failure. No helium has ever been produced by this apparatus and a description of the process does not warrant the use of the space.

Government Plant, AMARILLO, TEX. Fig. 3 shows a schematic drawing of the plant which is now being used in the Government plant at Amarillo, Texas. This plant operates as follows:

The drawings show an interchanger having two units *A* and *B*, each unit comprising a casing 7, a pair of headers 8, and the tubes 9 extending within the casing 7 and connecting the two headers. The gas is admitted through the valve *C*, preferably at a pressure of approximately 30 atmos. and at a temperature of approximately 35 deg. cent., and passes via the pipe 10 into the casing 7 of the unit *A*, flowing from one end to the other in a tortuous path about the tubes 9, as indicated by the arrows, the interior of the casing being provided with several baffles 11 for this purpose. In passing through the unit *A*, the gases are somewhat cooled by thawing out the frozen water vapor in the unit, and they leave the unit *A* by way of the pipe 12, crossing over at the valve to the pipe 13 leading to the bottom of the unit *B*, where the gas is cooled by the return substances going through the tubes 9. A pipe 12′, corresponding to the pipe 12, is connected with the casing of the unit *B* so that the interchanger units may be reversed. The cooled gases pass out of the upper end of the unit *B* through the pipe 14, which is provided with a suitable shut-off valve 15 and which leads to the header 16 of the liquefier, indicated as a whole by the reference letter *D*. The gas enters the header 16 approximately at a pressure of 30 atmos. and a temperature of approximately —100 deg. cent., which is practically the dewpoint. An outlet pipe 14′ leads from the upper end of the casing 7 of the unit *A* and is connected to the pipe 14 and is controlled by a shut-off valve 15′, such valve being closed when the flow is first through the unit *A* and then through the unit *B*, as described, the valve 15 being closed and valve 15′ being open when the flow takes place through the unit *B* and then through the unit *A*.

The liquefier *D* comprises the casing 17, and the header 16 previously mentioned, the header 18 and the tubes 19 connecting the headers 16 and 18. This liquefier is preferably substantially horizontally disposed, with a slight inclination from the horizontal. The reason for inclining the liquefier is to cause the liquid condensing in the tubes 19 to flow as rapidly through such tubes as the gas or vapor above the liquid is being conducted through the tubes by the pressure drop. Thus the gases entering the header 16 at approximately the dewpoint are subjected to the action of the returning fluid as they pass through the pipes 19, and practically all of the readily liquefiable contents of the gas are liquefied, the liquid and gas passing along together, in intimate contact with each other and in substantial phase equilibrium, the composition of the mixture being approximately uniform through the length of the tubes 19, with a minimum variation in the composition of the vapor above the liquid. The condensate leaving the pipes 19 passes down into the chamber 20 connected with the header 18 through a filter 20, and the uncondensed vapors pass out of the header 18 approximately at a pressure of 30 atmos. and a temperature somewhat lower than the dewpoint mentioned, say, for example, —117 deg. cent., by way of the pipe 21 to the purifier, indicated as a whole by the letter *E*.

The gas entering the purifier *E* is an impure helium, the impurities being a small quantity of nitrogen and methane and traces of other constituents. The gas is subjected to the cooling action of an external cycle of refrigeration and the impurities referred to above are liquefied, collecting in the bottom of the horizontal casing 22 of the purifier, practically pure helium being drawn off by pipe 23 at a pressure of approximately 30 atmos. and a temperature of approximately —170 deg. cent. The liquid condensed in the purifier is returned to the liquefier by means of the pipe 24 provided with a throttling valve 25 which throttles the pressure down to preferably 20 atmos.

The liquid condensed in the tubes of the liquefier *D* is returned to the shell of the liquefier through the pipe 26 at a pressure of approximately 20 atmos. obtained by means of a throttling valve 26 and at a temperature of approximately —121 deg. cent.

The external cycle of refrigeration referred to will be described later.

The lower portion of the casing 17 of the liquefier is divided into compartments by means of division walls 27, forming the compartments *a*, *b*, *c*, etc., in the lower portion of the liquefier casing. The space above the liquid in the casing is also divided into compartments *a′*, *b′*, *c′*, etc., by hanging baffles or deflection plates 27. From the bottom of each lower compartment a pipe 28, 28′, etc., leads to the upper region of the next lower compartment. The return liquid is delivered by pipe 24 to the compartment *a* and by pipe 26 to the compartment *b*. From compartment *a* the liquid not evaporated flows through pipe 28 to compartment *b;* and from such compartment, the unevaporated liquid flows into compartment *c* by means of pipe 28′; and so on from compartment to compartment. Since the pressure of the vapor above compartment *a* is the same as the vapor pressure above the mouth of pipe 28, liquid accumulates in compartment *a* until there is a sufficient head to cause liquid to flow out of pipe 28 into compartment *b*, the liquid being discharged into the vapor in the upper compartment *a″*. This is true of the other compartments. The arrangement of baffling is therefore one tending to keep the liquid and vapor in intimate contact and to maintain phase equilibrium, this purpose being furthered by providing a broad lip or plate 28*a* at the orifice of the pipes 28, 28′, etc., over which the discharging liquid flows in a thin, wide, sheet-like stream insuring close contact between the liquid and the vapor. Also the vapor evaporated cannot take a free course directly to the outlet 29 of the liquefier but must bubble through the liquid in passing progressively under the baffles 27′ from compartment to compartment.

The gases returning from the casing 17 to the outlet pipe 29 are delivered to the headers 8 and pass through the tubes 9 of the interchanger securing the necessary heat transfer in the interchanger, the gas leaving the casing 17 at approximately a pressure of 20 atmos. and a temperature of —111 deg. cent., and leaving the interchanger at substantially 20 atmos. pressure and a temperature of +17 deg. cent. It will of course be understood that when the flow of the incoming gas is first through the unit *A* of the interchanger the gas from the liquefier is conducted through the tubes of the unit *B*, cooling the gases coming into such unit from the unit *A*, as before described; and when the flow of incoming gas is first through unit *B* and then through unit *A*, the gases from the liquefier are led through unit *A* to cool the gases coming thereinto from unit *B*. To this end the outlet pipes leading from the bottom headers of the unit are provided with valves.

Depending upon how closely equilibrium is attained, the last compartment of the liquefier, as it is arranged, may contain a liquid (such, for example, as methane) which has a relatively high boiling point, that is to say, is of low volatility; and if provision were not made for the abstraction of such liquid from this compartment, the liquefier would eventually fill with high-boiling-point liquid and its operation would be greatly impaired, if not destroyed. To meet this condition a lead 30 is taken from the bottom of the last compartment and connected to the return pipe 29 so that any relatively high-boiling-point liquid which may

be present in the liquefier shall be returned along with the gases, leaving the upper region of the liquefier casing to the interchanger where, of course, such liquid will be evaporated and effective heat transfer secured in the interchanger.

Referring now to the cycle of refrigeration by means of which the helium is purified in the purifier *E* by the liquefaction of the impurities, this is, as stated, an external cycle, whereas the refrigeration in the process up to this point is accomplished primarily by means of an internal cycle. Any suitable form of refrigeration may be employed for the purifier, but apparatus such as diagrammatically indicated is preferred, using an inert gas which does not condense easily. Nitrogen or helium is mentioned as a suitable medium for this purpose. This is drawn into the system and its pressure is raised to approximately 30 atmos. by the compressor 31, the gas being cooled to a temperature of about +35 deg. cent. in the aftercooler 32, from whence it is conducted to the interchanger 33 in which the gases are cooled to a temperature of approximately —104 deg. cent. The gases leaving the interchanger are then expanded by means of the expansion engine 34 to approximately one atmosphere of pressure and thereby cooled to a temperature of approximately —180 deg. cent., the expanded gases being led to the purifier at the header 35, passing therefrom through the tubes 36 to the header 37, and from thence to the header 38 of the interchanger by means of the pipe 39. The gases enter the header 38 at a temperature of approximately —123 deg. cent. and serve to cool the incoming gas. The gases leave the outlet header 40 of the interchanger at approximately +30 deg. cent. and are returned to the inlet side of the compressor. The cycle is thus in effect a substantially closed one, and any make-up necessary is supplied to the low-pressure side of the compressor at the point marked 41.

The reason for placing the helium purifier in combination with the auxiliary refrigerating system in a substantially horizontal position is the desire to keep the liquid and gas in contact so far as possible in order to approximate equilibrium between all of the gas and the liquid, rather than between the gas and the last portion to be condensed out, thus obtaining a higher purity of helium.

The progressive evaporation of the returning liquid provides for the elimination of high-boiling-point substances such as ethane and the higher hydrocarbons, as it insures their collection in the warm end of the liquefier where they can be returned to the interchanger without disorganizing or impairing the functioning of the system, particularly the liquefaction step. This also obtains maximum efficiency in the interchangers. The evaporation of the liquid in steps and the return of the liquid and gas together also make possible the condensation of the gas by evaporation of mixed liquids with a minimum pressure drop in the system.

Helium Company's Plant, DEXTER, KAN. Helium is produced at the Helium Company's plant at Dexter, Kansas, by means of the Bottoms process, one phase of which is illustrated in Fig. 4 and which is probably the least complicated of any process yet developed. This apparatus, which of course is shown schematically, operates about as follows:

Helium-bearing gas, previously compressed to a pressure of from 40 to 200 atmos., and freed of the greater part of its carbon dioxide and water vapor, is led into the apparatus through passages 4 of the heat interchanger 1, where it is cooled to a low temperature by a means that will shortly be more fully explained. The gaseous mixture passes from interchanger 1 through pipe 11 and enters tubes or coils 6 in condenser 3. Condenser tank 7 of condenser 3 is filled with the liquid constituents of the natural gas boiling at approximately atmospheric pressure. It is this liquefied gas that forms the cooling or refrigerating medium necessary to liquefy the before-mentioned easily liquefied constituents of the natural gas contained within tubes 6, the said liquefaction being facilitated by the natural gas contained in tubes 6 being at high pressure. The easily liquefiable constituents of the natural gas contained at high pressure within tubes 6 condense to a liquid, holding the greater part of the helium in solution and in the liquid phase. From tubes 6 the mixture of liquefied gases is passed to the liquid-collecting pot 2 through valve 8. In pot 2 the helium held in solution in the liquids is given off as a gas due to the reduction in pressure much in the same way that carbon dioxide is



FIG. 9 HIGH-PRESSURE HELIUM STORAGE AT LAKEHURST, N. J., BEFORE COVERING



FIG. 10 DETAIL OF STORAGE PLANT SHOWN IN FIG. 9

released from carbonated water upon release of pressure, the liquid gases and the gaseous helium separate.

The liquid settles to the bottom of the pot 2 while the helium, still at a pressure much above atmospheric, passes out of the pot through pipe 17 to the place where it is to be stored or used. The liquid separating out in pot 2 is drawn off through valve 9 and transferred to condenser 3 through pipe 12 where it is allowed to evaporate at approximately atmospheric pressure. The vapors from the liquid in condenser 3 pass out through pipe 10 to passage 5 of interchanger 1, and thence through passages 5 countercurrent to the ingoing gases in passages 4, where they are cooled to a low

temperature as previously stated. The said vapors in their passage from end to end of the interchanger 1 are warmed at the same time to a temperature that approaches the temperature of the ingoing gases.

At this time of writing (April 17, 1929) there is only one plant operating at all, and this is the plant of the Helium Company at Dexter, Kansas. The new plant at Amarillo recently built by the U. S. Bureau of Mines is not yet in operation, and it is not known when it will be placed in operation.

Storage of Helium

Since it is necessary to have large volumes of helium on hand at any major airship-operating station, some kind of storage

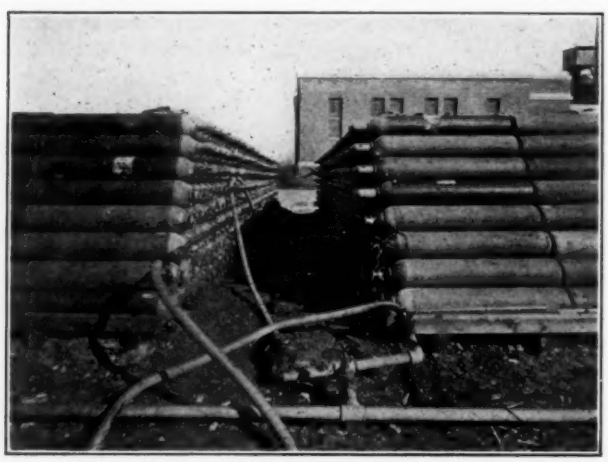


Fig. 11 Method of Discharging Helium From Small Cylinders at Lakehurst



Fig. 12 Interior View of Helium Purification Plant at Lakehurst

facilities will have to be provided. There are three general methods of storage:

1 Atmospheric holders or gasometers. A 6,000,000 cu. ft. holder of this type will cost about $1,200,000.
2 Storage in medium-pressure tanks, that is, about 60 lb. per sq. in. Tanks of this type having sufficient capacity to hold 6,000,000 cu. ft. would cost erected something like $560,000.
3 Storage in high-pressure containers, that is, in containers that will withstand an internal pressure of about 2000 lb. per sq. in. Such storage as this has been provided at the Naval Air station at Lakehurst, N. J. The present cost of storage facilities of this kind for 6,000,000 cu. ft. of helium would be, installed, about $400,000.

It therefore appears that the most economical method of storing helium at an airship depot would be in relatively large-size high-pressure storage tanks. There are three general types of these cylinders:

1 Relatively small cylinders capable of holding about 400 cu. ft. of helium each. These cylinders would be connected up in banks and manifolded with a common valve.
2 Medium-size tanks, approximately 4 ft. in diameter by 45 ft. long which are usually buried under the ground. Both of the above types of cylinders are seamless, forged, or drawn cylinders.
3 The third type is of welded steel, about 10 ft. in diameter by 100 ft. long.

Transportation of Helium

Due to the fact that helium deposits are usually located long distances from operating bases, it is necessary to provide transportation facilities. This transportation is one of the greatest problems in the use of helium in airships. The usual method of transporting has been in steel cylinders of approximately $1^1/_2$ cu. ft. dead volume. These cylinders are filled at about 2000 lb. pressure, and at this pressure hold something like 190 cu. ft. of helium. Each cylinder weighs from 125 to 135 lb., and approximately 600 can be loaded into an ordinary box car. The helium content of these cylinders will amount to approximately 100,000 cu. ft. The transportation charges on this amount of helium from the Mid-Continent field to the Eastern Seaboard will amount to about $1200, or approximately $12 per 1000 cu. ft. The freight on the empty cylinders returned will be about $600, making a total of $1800 or $18 per 1000 cu. ft. of helium.

In their efforts to reduce transportation costs both in freight charges and in handling charges at their terminal, the Navy Department, several years ago, developed a tank car which would transport approximately 200,000 cu. ft. of helium at 2000 lb. pressure. Due to a favorable freight rate obtained for this car a very material reduction in the total transportation charges has been obtained. Using such transportation the cost of transporting helium from Dexter to Lakehurst has been reduced to less than $4 per 1000 cu. ft. This tank car, which consists of three cylinders about $4^1/_2$ ft. in diameter by 39 ft. long, cost originally in the neighborhood of $98,000, but later developments have made it possible to reduce this cost to approximately $60,000 per car. These large helium tank cars can also be considered as mobile helium storage. Cars of this character can be purchased almost as cheaply as fixed storage of the same kind.

Helium transportation cars of this character were used to some extent in Germany during the war. In this country, however, the first tank car of this kind was built in 1923. The Navy Department, as early as 1920, had been looking into the matter of having tank cars of this type built, but it was not until 1923 that manufacturers were willing to undertake the work.

Properties and Uses

In spite of the extremely small quantities of helium available prior to 1917, its properties and physical constants had already been most thoroughly investigated.

Helium has several remarkable properties which make it of great interest and which will make it one of the most important of the industrial gases. It is colorless and odorless. It has a molecular weight of 4 and is only slightly heavier than hydrogen, which is the lightest substance known.

Helium is almost completely inert. It cannot be made to enter

into chemical association with any other substance, with the possible exception of mercury and platinum under special conditions.

It has been shown that helium is not soluble in metals, either liquid or solid. This property may indicate that helium may find extensive use in metallurgical operations.

The solubility of helium in water and in aqueous solutions is only about half that of nitrogen and its rate of effusion is more than three times that of nitrogen. For this reason Dr. Elihu Thompson suggested that helium might be used in admixture with oxygen as an atmosphere for deep-sea divers and caisson workers. Since that time the Bureau of Mines at Pittsburgh has carried out a series of investigations to establish the feasibility of using a helium-oxygen mixture for this work. It has been proved that divers can work at a greater depth with greater comfort and with a much shorter decompression period when a helium-oxygen mixture is used instead of air. In other words, helium prevents the development of caisson disease or "bends." "Bends" is supposed to result from the liberation of small bubbles of nitrogen which have been dissolved in the body fluids at high pressure, and which later is released, some of it perhaps in the brain tissue, which renders the subject unconscious. Helium mixed with oxygen as a breathing medium prevents this, due to the fact that it is less soluble and upon release of pressure escapes into the lungs at a more rapid rate than nitrogen.

The weight of 1000 cu. ft. of helium, under normal atmospheric conditions, is 11 lb. That of air under the same conditions is about 75 lb. Therefore the buoyant effect or "lift" of helium is 65 lb. per 1000 cu. ft. The two new ships recently contracted for by the Navy Department will have a capacity of 6,500,000 cu. ft. The total lift or buoyant effect of the helium in these ships will be over 422,000 lb. The actual weight of the helium used would be 71,500 lb., or over 35 tons.

Outside of the aeronautical field there are few uses to which helium is now put. The largest use is perhaps for the inflation of toy balloons, and the next largest volume goes for research work. As this latter progresses it is quite certain that increasing uses will be found for the gas.

The first serious suggestion to use helium for the inflation of airships was made in England in 1914, and the matter was taken up seriously and studied by Sir Richard Threllfall. Early in January, 1915, Threllfall sent a chemist to Fredonia, Kansas, who obtained samples of the natural gas there, which were sent to England for analysis. He then presented complete plans and specifications for an extraction plant to be built in Kansas by methods which are now employed. The cost, according to his estimate, was about £3,000,000, or approximately $15,000,000. Therefore the British had already gone far in their investigations relative to the use of helium in airships before the United States entered the war. As soon as the United States entered the war the matter of using helium in airships was brought to the attention of the Army and Navy officials and immediately met with a very favorable reception. Funds were allotted, and three experimental plants were built immediately. The results of these experiments pointed to the fact that the production of helium was feasible and that it could be made at a reasonable price. For that reason the Government built a large helium plant at Fort Worth, Texas, which was under the direction of the Naval Bureau of Steam Engineering. This plant has been operating and has produced, during its life, about 50,000,000 cu. ft. of helium.

This plant, which was later transferred to the jurisdiction of the Bureau of Mines of the Department of Commerce, has now been abandoned. In its stead the Bureau has built a new plant of large capacity near Amarillo, Texas. The manufacture of helium on a large scale by the Government tends to make it difficult for private individuals to establish themselves in the business.

The use of helium in airships presents two or three special problems, chief of which is the saving of the gas. It is necessary that airship fabrics be tight so as to avoid loss of helium, and that means be provided to prevent valving the gas in the maneuvering of the ship. The first of these problems has been taken care of through the use of goldbeaters' skin as a lining for the gas cells, and the second through the use of condensers for recovering water from the exhaust of the engines.

Assuming a 6,500,000-cu. ft. ship, the total lift will be 422,000 lb. The lift of the same ship, using hydrogen, will be 455,000 lb. Therefore the gain in lift by using hydrogen is 33,000 lb. Recent tests have shown that hydrogen can be used mixed with helium to the extent of 20 per cent by volume without danger of the mixture being inflammable. Methods of purification have been devised for purifying this hydrogen-helium mixture without any danger of explosion. Therefore with an 80–20 mixture of helium and hydrogen the total lift of a 6,500,000 cu. ft. ship would be 429,000 lb., or a gain by using 20 per cent hydrogen of 6500 lb. or a little better than three tons.

The helium requirements for the operation of an airship of 6,500,000 cu. ft. capacity, are about as follows:

For initial inflation, assuming 90 per cent of capacity	5,850,000 cu. ft.
Operating reserve, 50 per cent of capacity	3,250,000 cu. ft.
Make-up helium, to replace loss, about 60 per cent per year	3,900,000 cu. ft.

Therefore the first year's operation of one ship would require about 13,000,000 cu. ft. of helium, of which 9,100,000 cu. ft. will remain at the end of the year.

Of the above three items the first must be on hand at the initial inflation of the ship. As to the second item, steps should be taken immediately after the initial inflation to start building up this reserve to take care of contingencies and insure continuous operation. The third item should be delivered continuously throughout the year in approximately equal monthly quantities. Should more than one ship be operated, the first and third items would have to be increased by the number of ships operated. The second item would not necessarily be any greater for several ships than for one.

In major airship operations a helium-purification plant is necessary. A complete purification plant, including building, consisting of two units with all necessary piping and auxiliaries, will cost around $165,000. Experience indicates that the helium in a ship under usual operating conditions should be purified about four times a year. This does not mean that every three months the entire volume in the ship should be removed and purified, but that the helium in the various cells, as it becomes contaminated by the air, should be removed and purified. In this way there would be a continuous cycle of the purification of the gas throughout the year. The cost of purifying helium is approximately $1 per 1000 cu. ft.

It has been estimated that the investment in helium storage, transportation, and purification facilities at an airship terminal, for four ships, would cost about as follows:

Tank cars (8) for 1,600,000 cu. ft.	$ 480,000
Storage for 6,000,000 cu. ft.	450,000
Purification plant	165,000
	$1,095,000

The investment in helium inventory would be as follows:

Helium in 4 ships (5,825,000 cu. ft. in each ship or 23,300,000 cu. ft. for four ships at $35 per M, plus $5 freight	$ 928,000
3,500,000 cu. ft. reserve at $35 per M, plus $5 freight	140,000
	$1,068,000

Total investment in helium and facilities, $2,163,000.
The total annual cost of helium for 4 ships would be as follows:

10 per cent depreciation on investment	$ 111,500
Insurance, taxes, maintenance, and repair, 10 per cent	111,500
Loss of helium, 15,600,000 cu. ft. at $35 plus $5 freight	624,000
Carrying charges on inventory, 6 per cent	55,000
Labor (not including labor of purification)	10,000
Administrative overhead	101,000
Purification 92,800 cu. ft. per year at $1	92,800
Total	$1,105,800

This cost represents all of the costs of helium at the home port for four ships operating continuously. There will be required perhaps 500,000 cu. ft. of helium storage in each foreign port, which will increase the investment in storage facilities by about $40,000 and the helium inventory by about $25,000.

By using 20 per cent of hydrogen in the helium, the inventory for helium would be reduced about 20 per cent; instead of an annual cost for four ships of $624,000 as given, the cost would be $499,200, plus cost of hydrogen at $2.50, or $8875, making a total of $508,075, or a saving of $116,000 per year for four ships.

If hydrogen were used alone, and estimating the cost at $2.50 per 1000 cu. ft., the loss through valving to maintain purity, especially if that purity is maintained at a point to give any advantage over the use of helium, would amount to 40 per cent of the

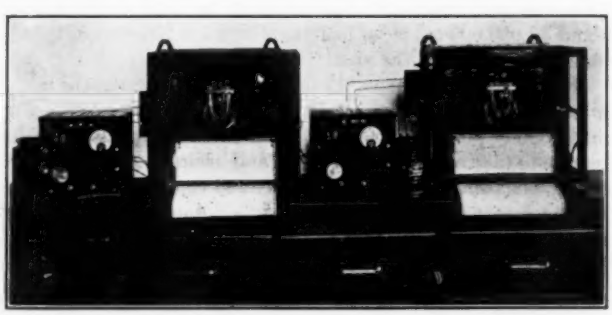


FIG. 13 HELIUM PURITY RECORDERS

(These instruments automatically record the purity of helium at the purification plant at Lakehurst. Instruments of the same type are used at Dexter, Kan.)

total volume per month, or 500 per cent per year. Therefore the annual cost of hydrogen would be $325,000 as against $624,000 for helium. This is less by $300,000 for the four ships, or $80,000 per ship. The saving in insurance on a helium-filled ship over a hydrogen ship would be very much greater than this, which would result in helium being the more economical of the two gases.

It is hoped that the presentation of these facts will serve to increase the public's faith in helium and strengthen its confidence in the continued supply of this commodity.

Discussion

R. A. CATTELL.[2] We wish to call attention to the following facts: The Amarillo helium plant of the U. S. Bureau of Mines has been in practically continuous productive operation since April 9, 1929. The Fort Worth helium plant was abandoned because the rapidly diminishing supply of helium-bearing gas there available did not warrant its further operation. The first helium tank car was supplied by the Army rather than the Navy.

[2] Chief Engineer, Helium Division, Washington, D. C.

R. H. UPSON.[3] The story of helium is one of the most fascinating in modern science, and it is natural to suppose that many practical uses will be found for a material of such unusual properties. But, although Mr. Bottoms does not say so, some one might get the impression that helium inflation is commercially a great boon to lighter-than-air craft and ignore the serious engineering objections to its use.

The direct cost of helium is not the big difficulty. The operating cost chargeable to the lifting gas in any case may be made comparatively small, especially with the improved gas-holding materials that may now be confidently predicted. But by the same token there should be no great difficulty about keeping up the purity of either hydrogen or helium. Helium at 97 per cent purity has a lift equivalent to hydrogen at about 90 per cent purity, or for equal purities there is about 7 per cent difference in lift. Then there is the important fact that a helium ship must take off with a considerable reserve space for expansion. Mr. Bottoms has mentioned 10 per cent, which seems a reasonable minimum, as a further deduction from the gross lift.

The next objection to helium lies in the very quality which is thought to be its chief virtue, its non-inflammability. If hydrogen, equivalent in lift to the gasoline consumed, is also burned in the engines, an additional distance range of about 20 per cent is obtained. This was demonstrated in England. The Germans accomplish the same thing more uniformly with their "fuel gas," filling the entire lower part of the hull.

There are of course redeeming features for helium. Its non-inflammability, as such, must not be passed lightly aside; though any ship in which gasoline and fabric play major parts can hardly be called fireproof. In a ship designed exclusively for helium, a slight saving could be made in structural strength and weight. Putting the engines within the hull is a substantial improvement, though not solely attributable to helium. On the whole the weights are probably about even, due to the extra water-recovery apparatus and other special equipment required by the helium ships.

Coming back to the more serious questions of lift and range, it is obvious that a pay-load of 20 per cent to 30 per cent of the gross, which is a good figure for hydrogen, will be easily cut in half if helium is substituted. With such a handicap in face of the rising efficiency of large planes, the future of commercial airships would be indeed dark.

But fortunately there are other possibilities besides the 100 per cent use of helium to which the foregoing remarks apply. A metal hull, Diesel engines, helium around the engine rooms and for emergency valving, and a substantial increase in strength and speed would seem to give to the airship its recognized advantages plus safety from fire and storm. Then will helium come into its own, and the fine work of such pioneers as Mr. Bottoms will have its due reward—but the way to get this golden egg is not by killing the goose!

R. B. MOORE.[4] The author's report on a natural gas containing as much as 7 per cent helium is extremely unusual, and if the volume of gas available is large, such a gas field should be of great use. I have been asked frequently whether there is enough helium in existence to take care of a reasonable demand for airship use when the number of airships is increased due to commercial demand for this kind of travel. The Amarillo gas field in Texas is probably the largest gas field in the world, and the Government has a lease on a portion of the field, containing a large volume of gas with a content of 1.7 per cent helium. For many years I tried to get Congress to appropriate money to acquire the whole of this field, but without success. If this

[3] Aeromarine Klemm Corporation, Keyport, N. J.

[4] Dean of Science, Purdue University, Lafayette, Ind.

could have been done, enough helium would have been available, and stored in the ground until needed, to take care of all the needs of the Army and the Navy for the next twenty-five or thirty years, besides having a surplus for commercial purposes. The United States is extremely fortunate in being the only country in the world having large supplies of helium-bearing natural gas.

The origin of helium in natural gas is very uncertain, as Mr. Bottoms has pointed out. I am not inclined to agree with any theory that depends on radioactivity for the origin of helium in natural gas. Of course, some of the helium in the earth's crust has a radioactive origin, as all rocks and minerals contain some radioactive material, but the helium that is concentrated in natural gas probably has another origin. The determination of the diffusion of helium through various types of rocks should be of great interest.

I trust that the company which Mr. Bottoms represents will be able to develop a satisfactory market for its helium. The Bureau of Mines has shown that helium can be substituted for nitrogen and an artificial atmosphere produced which can be used by those who are working in caissons under pressure. As helium is not as soluble in the blood lymph as is nitrogen, the use of such an artificial atmosphere instead of air largely eliminates trouble from the "bends." Further experimental work along this line should be carried out.

Author's Closure

Mr. Upson states that, although I do not say so, some one might get the impression that helium inflation is a great boon to lighter-than-air craft. I am sorry that I did not make myself clear on this point. If we remember correctly, the burning of the Goodyear blimp in Chicago, the destruction by fire of the *Roma*, the explosion of the *R-38* in England, and the burning of the French airship *Dixmude* were all milestones in the death march of lighter-than-air craft until helium appeared and gave renewed life to this mode of travel. It is my firm belief that without helium there would be no lighter-than-air activity in the United States today, and it seems a conclusion that helium is a great boon to lighter-than-air craft, regardless of any engineering objection to its use.

In regard to the difference in lift between helium and hydrogen, that difference is about 7 per cent, but Mr. Upson gives the impression that to this difference must be added the 10 per cent reserve space for expansion. Now this 10 per cent reserve space for expansion is necessary or desirable even though hydrogen is used; consequently, the difference is not 17 per cent but the original 7 per cent.

This difference in lift seems to the author to be the only point in which helium does not compare so favorably with hydrogen, but, on the other hand, in the ships now being built by the Navy Department the useful lift will probably be in excess of 40 per cent of the gross lift. Allowing for fuel for a 5000-mile cruise will still leave 25 per cent of the gross lift for pay-load even with the use of helium, and inasmuch as fuel consumed in a ship of this size is not over 3 gallons per mile it would seem that there would be no reason for the use of hydrogen even though it did not possess the dangerous qualities of explosion and inflammability.

Mr. Upson seems to assume that the non-inflammability of helium is a hindrance in view of the fact that hydrogen can be used as additional fuel. Hydrogen's B.t.u. value is only about 300 per cubic foot and the fuel gas used by the Graf Zeppelin about 1800 B.t.u. per cubic foot. The fuel gas floats in the air without giving either weight or lift, and these facts would seem to answer his question as to the inability of using helium as additional fuel in the engines.

He states further that a ship in which gasoline and fabric play major parts can hardly be called fireproof, but that the use of Diesel-engine and metal-clad ships will be one of the future developments. There seems to be no reason why helium cannot be used in metal-clad ships, together with Diesel engines, and make the whole thing 100 per cent fireproof. When the metal-clad ship is fully developed and the Diesel engine is perfected, there is no reason why helium will not add its further advantage of non-inflammability and finally bring lighter-than-air craft to a point of development where they will compete not with airplanes but with ocean-going surface craft. The day when the airplane can compete with airships in transoceanic service is indeed remote.

Another thing that would seem to make the use of hydrogen practically out of the question in this country is that hydrogen filled ships cannot be insured, while helium-filled ships can be.

Replying to Dr. Moore, I would say, first, on the question of new uses for helium, a research program is now being carried out for the purpose of discovering new uses and extending present uses for helium. One of the uses is of course in diving and caisson work, and it is planned, if possible, to establish a fellowship at one of the large universities for carrying on further study of the matter.

Dr. Moore does not agree with me thoroughly on the origin and occurrence of helium, but since the whole question as to the occurrence of helium has been unsatisfactorily explained, it is admitted that one guess is probably as good as another and probably will be until sufficient data can be collected to clear up the points not very well understood.

AER-51-21

Recent Developments in Aircraft Instruments[1]

By W. G. BROMBACHER,[2] WASHINGTON, D. C.

Aircraft instruments are separated into four groups by the author, (1) for the power plant, (2) for speed, (3) for altitude, and (4) for navigation. General performance requirements are outlined, and testing equipment and procedure used at the Bureau of Standards are described. Among operating problems the author considers instrument-board vibration as most serious. The variation in temperature to which aircraft instruments are subjected causes a severe lubrication problem.

Standard dimensions for some of the instruments have been adopted. This makes these instruments interchangeable on the instrument board, and this is considered an outstanding achievement.

AERONAUTIC instruments may be divided into two classes, flight instruments and ground instruments. Flight instruments include those installed in the aircraft and ground instruments those required on the ground in connection with aircraft operation. The dividing line between flight and ground instruments is very sharp. Flight instruments must be simple in construction and light in weight, must operate under difficult conditions, and must require absolutely the minimum amount of adjustment, etc.; while in contrast, ground instruments may be as complicated and heavy as is necessary.

Flight instruments may be divided into service instruments and experimental instruments, although the dividing line between the two classes is not sharp. Service instruments include, primarily, those which it is customary to install to aid the pilot in operating and navigating the aircraft. Experimental instruments are those used for performance testing in flight, for competitive flights, for free-flight investigations, new types of instruments not yet fully developed to the service stage, and instruments which are not generally used. Experimental instruments include many types which will be in common use tomorrow. An important distinction exists between service and experimental instruments from the viewpoint of the instrument manufacturer, since service instruments in general are needed for every aircraft, while experimental instruments are required only to a limited extent by aircraft manufacturers.

Service instruments as defined also include those needed especially for high-altitude flying, and in aerial photography. At the present time these instruments may be designated as experimental, since relatively few are required at the present time.

Aircraft instruments are particularly interdependent upon each other. Thus, a change from level flight to climbing will cause a decrease in air speed provided the engine speed remains constant, an increase in altitude, and a change in the indication of the fore-and-aft inclinometer. Further, a rate-of-climb indicator is used now as an inclinometer rather than to measure rate of climb. It is, therefore, with some reservation that the customary classification of service instruments is given. A number of experimental instruments under each class is included. Astronomical-navigation instruments are classed as experimental because their use is not common. They are needed only in experimental flying and in special long-distance flights, such as that of Hegenberger and Maitland from California to Hawaii and that of the *Southern Cross* to Australia.

CLASSIFICATION OF AIRCRAFT INSTRUMENTS

1	Power Plant Instruments	
	Service	Experimental
	Tachometers	Supercharger instruments
	Oil or water thermometer	Recording tachometer
	Oil- and fuel-pressure gages	
	Fuel-quantity gage	
	Fuel-flow meter	
2	Speed Instruments	
	Service	Experimental
	Air-speed meter	Air-logs
	True air-speed meter	Air-speed recorders
		Ground-speed meter
3	Altitude Instruments	
	Service	Experimental
	Aneroid altimeter	Absolute altimeter
		Barograph
		Free-air thermometers
4	Navigation Instruments	
	Service	Experimental
	Magnetic compass	Sextant
	Turn indicator	Chronometer or watch
	Bank indicator	Induction compass
	Drift indicator	
	Fore-and-aft inclinometers	
	Radio beacon	
	Clock	

The discussion which follows is almost entirely restricted to the round-dial instruments, however, some mention should be made here of vertical-scale instruments. Tachometers, thermometers, pressure gages, and air-speed indicators are available both in the vertical-scale and round-dial types. A combination vertical-scale instrument, known as an engine-gage unit, consists of the two pressure gages and the oil or water thermometer. Although the vertical-scale instruments have some advantages, notably that of conserving space on the instrument board, they compare unfavorably with the round-dial type in such essential factors as performance and cost. For single-engined ships the present trend seems to be toward the general use of the round-dial instrument. In multi-engined ships there is considerable advantage in grouping the power-plant instruments together, so that the use of the engine-gage unit mentioned is finding considerable favor. The use of the vertical-scale tachometer is also advantageous when it is possible to have all of the indicators side by side on the instrument board, to show at a glance comparative performance in case of multiple-engine installations.

General-Performance Requirements. The performance of aircraft instruments must be satisfactory under conditions of use which are comparatively very severe. Many of these conditions either do not exist or do not occur with the same severity in ordinary engineering practice, and therefore have not required special consideration from the manufacturer of engineering instruments. The severe conditions of use have been reflected in the acceptance tests required in the current specifications of the Army Air Corps and Bureau of Aeronautics. The tests have been divided into two classes, one including tests to which all instruments are subjected, the other including what are known as type tests, to which a limited number of samples are subjected. The individual tests under the first class include a scale-error test at room temperature, if the instrument is calibrated quantitatively, and such other tests as bring out individual differences in performance due to workmanship rather than to design. Low cost and ease of making the individual test are important factors. In type tests the instruments, in general, are tested for the effect of temperature in the range —35 deg. cent. to +45 deg.

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[1] Publication approved by the Director of the Bureau of Standards of the U. S. Department of Commerce.

[2] Chief, Aeronautic Instruments Section, Bureau of Standards, Washington, D. C.

Presented at the Third National Meeting of the A.S.M.E. Aeronautic Division, St. Louis, Mo., May 27 to 30, 1929.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

cent., the effect of pressure change, vibration, elastic defects such as drift, lag, hysteresis, and after-effect, seasoning, excessive friction, position error, time lag in the indication, proper damping, strength of parts subject to external stress, durability, and other properties which may have a bearing on the desired performance of the instrument. In addition to the above characteristics, all of which can be determined quantitatively in the laboratory, the weights of the instruments must be a minimum, they must have luminous dials, must be made of non-corrosive parts as far as possible, must have the mechanism balanced to minimize the effects on the indication of large accelerations, and must in general operate satisfactorily under all conditions of severe usage which occur in practice. In the current military specifications

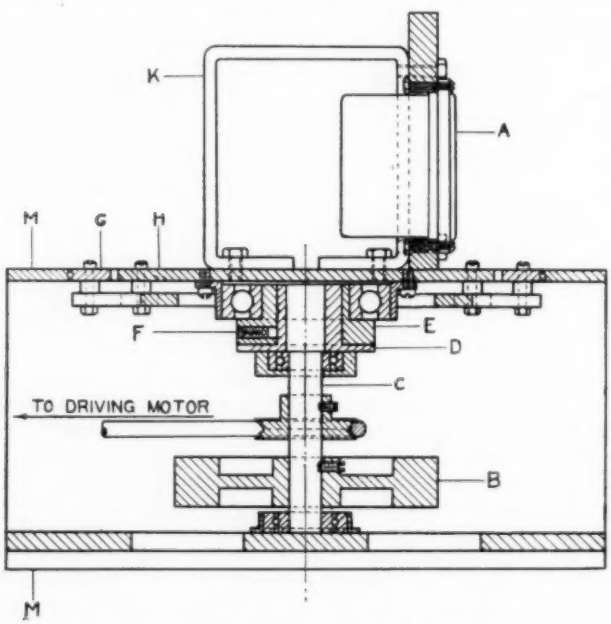


FIG. 1 BUREAU OF STANDARDS VIBRATION RACK

(E and D form an adjustable cam, which imparts a simple harmonic motion to frame G at right angles to the plane of the figure and to frame H in the plane of the figure. The amplitude is adjusted by rotating E with reference to D after loosening set screw F. An instrument under test is shown at A.)

the tolerances are based on the accuracy needed by the pilot when the instruments can be made to meet it. In the cases where the art has not progressed to the point of furnishing instruments possessing the needed accuracy, the tolerances express approximately the best performance which can reasonably be obtained. The performance of a number of service instruments will be given in more detail in following sections.

TESTING EQUIPMENT AT THE BUREAU OF STANDARDS

It may be of value to describe briefly the test procedure, and the quite complete testing equipment of the Bureau of Standards, since relatively few laboratories are equipped to test aircraft instruments completely. The equipment is described in a series of technical reports issued in 1921 (references 2 to 7) of the National Advisory Committee for Aeronautics, but in many instances improvements in equipment have made these descriptions out of date.

Practically all types of instruments must be tested for the extraneous effects of temperature, pressure, and vibration. Temperature and pressure control are required also for testing thermometers and pressure-measuring instruments. The equipment for these tests is, (*a*) temperature-control chambers, (*b*) pressure-control apparatus, and (*c*) vibration board.

Temperature-Control Chambers. At the present time there are available in the aeronautic-instruments laboratory three cork-insulated temperature chambers which can be cooled below room temperature by means of a common ammonia-refrigeration system, and can be heated by individual electric heaters. One of the chambers (inside dimensions 48 in. × 36 in. × 30 in. high) has been found sufficiently large to contain such instruments and testing equipment as it has been desired to test. As has been stated, specifications require tests at −35 deg. cent., +20 deg. cent. and +45 deg. cent. The ammonia-refrigeration system is inadequate in that −35 deg. cent. is practically the lowest temperature which can be attained with it, while a temperature of −50 deg. cent. is required often for special tests. Temperatures as low as −45 deg. cent. are now obtained in a bell jar of fair size using liquid CO_2 and it is planned eventually to change from the ammonia to a CO_2 refrigeration system. It may be interesting to know that a suitable apparatus for experimental-instrument work has been designed and constructed at the Langley Memorial Laboratory of the National Advisory Committee for Aeronautics (reference 15) in which a bell jar is cooled by liquid CO_2 from commercial cylinders.

The glass windows of the temperature chambers are prevented from frosting by using four to six individual glass windows, each separated by an air gap. A source of trouble is the difficulty in securing good service from electric ventilating fans mounted in the chambers. A non-freezing lubricant must be used if they are mounted inside of the chambers, but better service may be secured by mounting the motor outside the chamber, and passing the fan shaft through the wall.

Pressure-Control Apparatus. For testing altimeters, barographs, supercharger gages, compasses, and most other instruments the pressure-control apparatus consists usually of a bell jar and metal base, together with a suitable vacuum pump. For pressures above atmospheric, which are required for some of the above instruments, a bell jar is unsuitable for constant use, although with proper clamps it can be made to "hold." Shallow metal containers with glass covers are also available for pressure control, especially for values about atmospheric.

The size of glass bell jars is limited, and is often inadequate, notably in testing aircraft thermometers. For the above, and for other reasons a steel altitude chamber has been constructed. This has a diameter of 54 in. and a useful height of 96 in. with a door opening of 16 in. × 53 in. Pressures ranging from atmospheric to that equivalent to 30,000 ft. are obtained, and provision has been made for temperature control. A port hole 5.5 in. in diameter was found to permit ample vision into the chamber.

Pressure control for instruments with external pressure connections such as air-speed indicators is an individual and relatively simple problem.

Vibration Board. Within the past two years there has been designed and constructed a new vibration board which is now in constant use. Credit for the design should be given to Dr. V. E. Whitman. The chief improvement over the older type lies in the independent control of frequency and amplitude. In the older models the centrifugal force developed by a revolving eccentric weight was used to vibrate a panel containing the instruments. The panel was connected to its support by means of helical springs. The amplitude of the vibration depended on both the frequency, and the combined weight of the panel and instruments.

A cross section of the new model is shown in Fig. 1. The construction is essentially as follows: A motor-driven shaft carries an adjustable eccentric by means of which each of two plates properly restrained move in simple harmonic motion in planes at right angles to each other. The instrument under test

is mounted on one of the plates and is thus subjected to a uniform circular vibration.

Standards and Standard Instruments

Essentially the performance of instruments is determined by comparison of their reading with that of a master instrument. The latter may be a primary standard, but more often is a secondary standard which is used because of its greater convenience, its accuracy being determined from time to time by comparison with a primary standard. A brief discussion of the standard instruments, and of certain standard relations which are used in testing aircraft instruments follows.

Air-speed Standards. The air speed of airplanes is now usually determined by means of the differential pressure developed by a pitot-static tube headed into the free airstream. The relation between air speed and differential pressure which has been adopted for calibrating air-speed indicators contains the assumption that the air is compressible, and is as follows:

$$\frac{p + p_0}{p_0} = (1 + 0.34526 \times 10^{-6} V^2)^{3.5} \quad \ldots\ldots\ldots [1]$$

in which p is the differential pressure, p_0 the standard atmospheric pressure (760 mm. of mercury) and V the air speed in miles per hour.

The pressures determined by formula [1] are higher than those given by the familiar relation $p = {}^1/_2 \rho V^2$, in which the air is assumed incompressible, by 1 per cent at 150 m.p.h. and by 1.7 per cent at 200 m.p.h. Here ρ is the standard density (at 15 deg. cent. and 760 mm. of mercury). For further details see references 10 and 14.

Table 1 gives convenient values of the standard air speed and pressures for the calibration of indicators.

TABLE 1 AIRSPEED CALIBRATION VALUES

Airspeed, m.p.h.	Differential pressure, inches of water	Airspeed, knots	Differential pressure, inches of water
0	0	0	0
10	0.049	10	0.065
20	0.197	20	0.261
30	0.443	30	0.588
40	0.788	40	1.045
50	1.231	50	1.634
60	1.774	60	2.354
70	2.416	70	3.207
80	3.158	80	4.192
90	4.000	90	5.310
100	4.942	100	6.563
110	5.985	110	7.951
120	7.130	120	9.475
130	8.377	130	11.14
140	9.726	140	12.94
150	11.18	150	14.87
160	12.74	160	16.95
170	14.40	170	19.17
180	16.17	180	21.54
190	18.04	190	24.05
200	20.02	200	26.71
210	22.12	210	29.51
220	24.32	220	32.47
230	26.63	230	35.58
240	29.06	240	38.84
250	31.59	250	42.27
260	34.25	260	45.85
270	37.01	270	49.59
280	39.90	280	53.50
290	42.91	290	57.57
300	46.03	300	61.82

A suitable cistern-type water manometer is used as the standard in the calibration of the indicators.

Altitude Standards. The altitude of aircraft is still determined by the barometric method in view of the lack of an absolute type of instrument. A standard atmosphere was adopted in 1925 for use in the United States based on an arbitrary altitude-temperature assumption. The altitude-pressure relation of this atmosphere is used in calibrating altimeters. (See references 16 and 17.) Table 2 gives the pressure and temperature at standard altitude in the range 0 to 40,000 ft.

In laboratory tests the indication of the altimeter is compared with that of a standard mercurial barometer which differs from the Fortin type in that the cistern is made air-tight except for a pressure connection through which the pressure above the mercury in the cistern can be controlled. (See reference 2.)

Improvement is desirable in the standard for testing altimeters in field laboratories, since the latter are usually without facilities to check the mercurial barometers. One arrangement which has some advantages, and has given quite satisfactory service is that of a reservoir-mercurial manometer used with the altimeter testing equipment together with a Fortin-type mercurial barometer, the latter measuring the atmospheric pressure so that the absolute pressure can be determined. Another suggestion would substitute the use of an aneroid barometer as the standard. The latter suggestion has found little favor thus far because of the lack of sufficiently good instruments. The recent improvement in altimeters has gone far toward meeting this objection.

TABLE 2 U. S. STANDARD ATMOSPHERE PRESSURES

Standard Altitude thousand feet	Pressure Inches of mercury	Mm. of mercury	Temperature, deg. cent.
0	29.921	760.0	+15.0
1000	28.86	732.9	+13.0
2000	27.82	706.6	+11.0
3000	26.81	681.1	+ 9.1
4000	25.84	656.3	+ 7.1
5000	24.89	632.3	+ 5.1
6000	23.98	609.0	+ 3.1
7000	23.09	586.4	+ 1.1
8000	22.22	564.4	− 0.8
9000	21.38	543.2	− 2.8
10000	20.58	522.6	− 4.8
11000	19.79	502.6	− 6.8
12000	19.03	483.3	− 8.8
13000	18.29	464.5	−10.8
14000	17.57	446.4	−12.7
15000	16.88	428.8	−14.7
16000	16.21	411.8	−16.7
17000	15.56	395.3	−18.7
18000	14.94	379.4	−20.7
19000	14.33	364.0	−22.6
20000	13.75	349.1	−24.6
21000	13.18	334.7	−26.6
22000	12.63	320.8	−28.6
23000	12.10	307.4	−30.6
24000	11.59	294.4	−32.5
25000	11.10	281.9	−34.5
26000	10.62	269.8	−36.5
27000	10.16	258.1	−38.5
28000	9.72	246.9	−40.5
29000	9.29	236.0	−42.5
30000	8.88	225.6	−44.4
31000	8.48	215.5	−46.4
32000	8.10	205.8	−48.4
33000	7.73	196.4	−50.4
34000	7.38	187.4	−52.4
35000	7.04	178.7	−54.3
36000	6.71	170.4	−55.0
37000	6.39	162.4	−55.0
38000	6.10	154.9	−55.0
39000	5.81	147.6	−55.0
40000	5.54	140.7	−55.0

A very convenient standard instrument for field use is the mercurial barometer suggested by Barnes (reference 18) which can be transported empty of mercury, and the proper vacuum above the mercury column obtained when desired by the use of a fairly good vacuum pump. Essentially the instrument consists of a U-tube manometer inverted into two cisterns. By controlling the pressure in both cisterns the mercury can be made to fill completely the manometer legs. If the mercury column is then broken at the bend of the U-tube, a vacuum will be formed above the mercury column, and one leg with its cistern then constitutes a mercurial barometer.

Engine-Speed Standards. Essentially the equipment already described (reference 5) is still in use and will not be discussed.

Miscellaneous Standards. A dead-weight oil-gage tester is used to test oil-pressure gages. In testing the gages at low temperatures (—35 deg. cent.) it was found necessary to avoid freezing the liquid in the cold part of the apparatus. The difficulty was finally met in a satisfactory manner by using an oil in the system which had a pour point of —40 deg. cent.

Standards and associated equipment are also available to test flow meters and accelerometers. This equipment is now undergoing further development, and a description will be omitted at present. Most other service instruments require standards of such relatively low accuracy, and are so well known that a description is not essential here.

Power-Plant Instruments

At present supercharger instruments are classed as experimental, due to the fact that superchargers themselves are experimental. No serious problems are involved in furnishing the pressure gages and thermometers which may be needed to control the operation of the supercharger. Recording tachometers are useful in flight-test work, and as an aid in keeping an engine

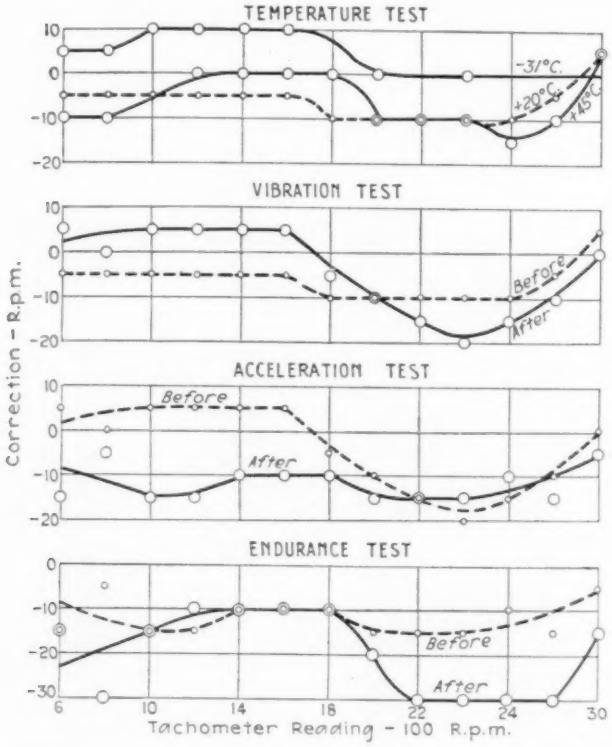


Fig. 2 Performance of a Recently Developed Centrifugal Tachometer

log. No recording tachometer of American make is commercially available at the present time. A detailed consideration of the service power-plant instruments follows.

Tachometers. At present the choice of a tachometer for single-engined airplanes is usually made between the chronometric and centrifugal-type instrument. These instruments are practically restricted to the case when a short drive shaft with few bends is possible. The essential features of these instruments have not changed in the last few years. For descriptions see reference 5. The performance of the chronometric instrument manufactured in this country has remained unchanged for many years. Except possibly for endurance, the performance is adequate. See reference 5.

Centrifugal tachometers are now being made by a number of manufacturers in this country. A recent development in these instruments has been the reduction from a $3^3/_4$-in. to a $2^3/_4$-in. dial by the manufacturers at the initiation of the Navy Bureau of Aeronautics. Improvement of this type of instrument is still believed desirable, and no doubt will be effected in the near future.

The performance required of centrifugal tachometers by the military services is based on securing an accuracy of approximately one per cent under all conditions. The instruments are subjected to the following tests:

a Test at room temperature for accuracy of calibration.
b Scale-error test at —35 deg. cent. and +45 deg. cent.
c Vibration for 3 hours and test to determine change in scale errors.
d Effect of over speed on scale errors.
e Effect on scale errors of accelerating to a given speed in one second for 500 times.
f Position error at various speeds.
g Change-in-scale errors after an endurance run of 300 hr.

The performance tolerances are most conveniently expressed for most of the type tests in terms of the average change in the error. A small number from a production lot of a recently developed centrifugal tachometer have been tested. These instruments have an equally-divided scale from 500 to 3000 r.p.m. indicated speed with 20 r.p.m. for the smallest scale division. Some of the test results on the best instrument are given in Fig. 2.

The direct-drive feature of centrifugal and chronometric tachometers is unsatisfactory in multi-engined ships where the distance from the cockpit to the engine is great. Current practice has been to mount the tachometer on a strut close to the outboard engine, thus forcing the pilot to turn from the instrument board in order to read the speed. Two of the possible types of instruments which permit an indication on the instrument board are now being developed by commercial organizations. One of these instruments consists of a commutator attached to the cam shaft of the engine, and an indicator consisting of an electromagnet and chronometric tachometer combined. A battery furnishes the power. The commutator controls the rate of operation of the electromagnet, which in turn operates a pawl forming the connecting link with the chronometric tachometer.

The other type of distant-indicating tachometer consists of the well-known direct-current magneto and voltmeter. Modifications are necessary however in order to make the instruments now available satisfactory for aircraft use. The bulk and weight of the magneto must be as small as possible, and the indicator should have a pointer motion greatly in excess of the customary 120 deg. in order to permit the use of an instrument of standard-case dimensions. A recent investigation in the Aeronautic Instruments Section of the Bureau of Standards conducted by J. B. Peterson has indicated the method by which complete compensation for the effect of temperature changes can be effected, and the extent to which this method can be used. It is essential to compensate the indicator and magneto separately since the temperature of each varies independently.

The importance of proper lubrication for tachometers should not be overlooked. If the instrument is to meet laboratory tests the lubricant should not freeze at —35 deg. cent., evaporate to any great extent at +45 deg. cent., nor require replacement during an endurance run of 300 hr. These conditions are not believed to be more severe than those obtained in service. Further experimentation on this point is desirable.

Aircraft Thermometer. These instruments are furnished in the vapor-pressure and the liquid-filled types. For descriptions see reference 5. An accuracy of about 4 deg. cent. is required of oil or water thermometers purchased under military specifications. This requirement causes no special difficulty to the manufacturer except for the effect of temperature on the line and indicator of liquid-filled instruments. Vapor-pressure instruments on the other hand have the disadvantage of being affected by changes in altitude (pressure effect), and also of having roughly only one-sixth the operating pressure per degree centigrade at 0 deg.

cent. as at 100 deg. cent. The vapor-pressure instrument appears on the whole to be the preferable type.

Oil- and Fuel-Pressure Gages. Since only a low degree of accuracy is required in measuring fuel and oil pressure (about 3 per cent of the maximum indicated pressure) no development work has been necessary on the gages. There have been changes, such as the reduction of the dial size to $1^7/_8$ in., and improvements in the method of connecting the tubing to the indicator. A further change may be desirable in the oil-pressure gage design in order to obviate the effect on the indication, of the oil freezing in the indicator and tubing at low temperatures. Fig. 3 is included to show the effect of temperature on Bourdon tube pressure gages.

Fuel-Quantity Gage. None of the variety of gages now available appears to give universal satisfaction. The instrument may be divided into the hydrostatic and float types. The indication of the hydrostatic-type instrument depends upon the density of the fuel as well as the head. The chief difficulty appears in securing a satisfactory instrument which indicates on the instrument board. In addition to the familiar aircraft and various automobile fuel gages available, two types may be classed as new developments, although not yet out of the experimental stage. In the first of these a pressure indicator at the instrument board is connected to the bottom and top of the tank by means of air lines. The line to the bottom of the tank is connected to a cellophane capsule. A quantity of air remains in the airtight system consisting of the pressure element of the indicator, the air line, and the cellophane capsule. The volume of the system is so proportioned, and the cellophane capsule so flexible that neither changes in air pressure nor temperature cause any change in the indication. The cellophane capsule is surrounded by the fuel so that its internal pressure is controlled by the hydrostatic head of the fuel thus affording a measure of the fuel quantity.

The laboratory of a large electrical company has recently announced the successful development of a distant indicating instrument which depends on the effect of the pressure head of the liquid on a carbon pile. No further details have come to the attention of the writer. Attention is directed to two gages of the float type (described in reference 9), which have met with some favor abroad.

Fuel-Flow Meters. A fuel-flow meter recently brought out commercially consists of a Venturi tube in the gas line, and an indicator on the instrument board. Two fuel lines lead from the Venturi tube to the indicator, one to a diaphragm capsule, and the other to the case of the instrument. This requires that the case be made airtight against a differential pressure of about five lb. per sq. in. For one type of Venturi tube now furnished, the differential pressure operating the indicator mechanism is about 16 in. of water for a rate of flow of 60 gal. per hour. Maintaining a leak-tight case under all conditions is problematical, so that it appears desirable to have a by-pass for the fuel around the case which can be controlled by the pilot. An indicator with two opposed diaphragm capsules would be a desirable development since it would make unnecessary the filling of the instrument case with the fuel. When calibrated in pounds per hour the following equation holds:

$$W = K_1\sqrt{pd} \quad \ldots\ldots [4]$$

When in gallons per hour

$$G = K_2\sqrt{\frac{p}{d}} \quad \ldots\ldots [5]$$

in which W and G are the rates of flow in pounds and gallons per hour respectively, p the differential pressure developed by the Venturi tube, d the density of the fuel, and K_1 and K_2 constants depending on the design of the Venturi, and on units of measurement.

Attention should be called to the recent announcement of the production by the British Royal Air Force of a fuel-flow meter of the sink-and-tube type (reference 9).

Speed Instruments

Of the instruments classified as experimental, the air log is useful in the dead-reckoning method of position finding, and the air-speed recorder in flight testing and flight research. The ground-speed meter is an example of an instrument which is desired for regular equipment, but none of the types now available are adequate since all depend upon the visibility of the ground for their operation.

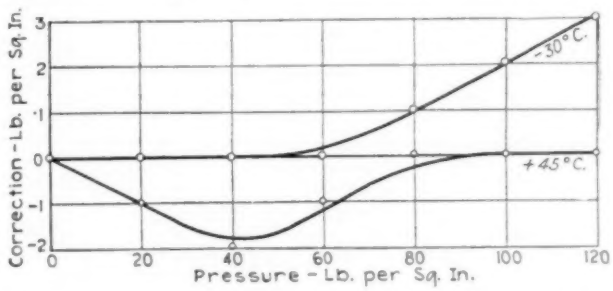


Fig. 3 Effect of Temperature on Scale Errors of an Oil-Pressure Gage

Air-Speed Meter. The air speed of airplanes is usually determined from the differential pressure developed by tubes such as the pitot-static and pitot-venturi types. The indications of the differential-pressure type instruments depend on the air density as well as the air speed. The instruments are calibrated to indicate correctly at one air density. A correction must be applied to the indications for deviations from this density. The reading of the indicator, corrected for instrumental errors but uncorrected for deviations in air density, is called the indicated air speed.

Since 1920 steady progress has been made in changing from the pitot-venturi type to the pitot-static tube-type instrument. The adoption of the pitot-static type instrument has required the development of the technique of making diaphragm-pressure capsules for the measurement of low pressures. Today there is in use an indicator with a $2^3/_4$-in. dial weighing 11 oz. in which the pointer has an angular movement of about 300 deg. for a pressure change as low as 10 in. of water (equivalent to 140 m.p.h.). The performance of the indicator is determined by the following laboratory tests:

a Test at room temperature for scale errors.
b Scale-error tests at —35 deg. cent. and +45 deg. cent.
c Vibration for three hours and test to determine change in scale errors.
d Drift test or change in reading in five hours at a given (high) speed.
e Seasoning or effect on scale errors of subjecting instrument to 100 full-scale deflections.
f Position error at various readings.
g Effect on scale errors of subjecting instrument to specified over pressures.

Usually the smallest scale division is five miles per hour or five knots which makes the least reading at best 0.5 m.p.h. or knots. Scale errors up to 1.5 per cent of the range are allowed. Average changes in calibration due to vibration, seasoning, and over-pressure have 0.5 per cent tolerances, and a permitted drift 0.6 per cent of the speed range. The change in scale value with temperature is somewhat larger, amounting in speed units to

about one to 1.5 per cent in the temperature interval —35 deg. cent. to +45 deg. cent. This is, as predicted by theory, about one-half of that caused by the change in the elastic modulus, but it can, of course, be much greater especially if the design is such that differential expansion is a factor. Fig. 4 shows the effect of temperature, vibration, and seasoning on a typical good indicator. The errors are larger at very low speeds owing to the small differential pressures available at lower speeds and the pointer position is also more uncertain.

The size of the pitot-static tube of the service-type air-speed meter is restricted because of the conditions involved in handling an airplane so that the openings are located in a position very close to the tube support. It is well known that the openings, especially the static openings, must be placed in the undisturbed air stream in order to develop the theoretically correct pressures. Finding the best location for a pitot-static tube with openings

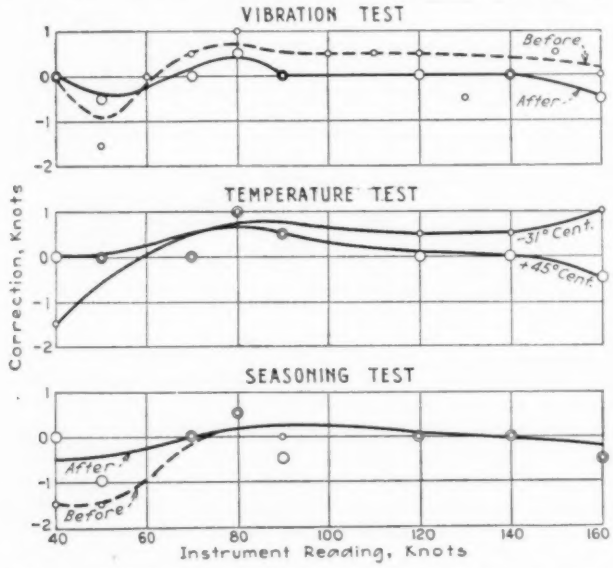


FIG. 4 PERFORMANCE OF AN AIR-SPEED INDICATOR OF THE PITOT-STATIC TYPE

four or six inches from a strut or 12 in. in front of the leading edge of a wing is an individual problem for each type of aircraft. In nearly all cases it is necessary to calibrate the complete installation over a speed course in order to secure the accuracy in the air-speed measurement consistent with the present high quality of the indicating instrument. The errors in air-speed measurement in flight with the equipment usually installed are greatest at the stalling speeds. The simplest solution of the problem appears to be that of having available for the pilot's use, the results of a speed-course calibration of the air-speed meter installation, at least for the cruising and stalling speeds.

In the operation of the pitot-static air-speed meter consideration must be given to preventing stoppage of the line by rain, snow, and the freezing of rain. Trouble with leakage in the line is largely avoided by the present practice of using an all-metal tube, avoiding even the use of rubber connecting tubes.

True Air-Speed Indicators. By definition these are instruments which indicate the air speed directly, without the necessity of correcting for deviations of the air density from the standard value. In terms of true air speed the stalling speed of an airplane varies with air density, but in terms of indicated air speed it is independent of density. A knowledge of indicated air speed is therefore of primary importance in airplane operation. However, for the purposes of navigation it is also desirable to know the true air speed.

In airship operation a knowledge of the true air speed is desired since the operation of the ship does not, as in the case of airplanes, depend upon a knowledge of the indicated air speed. This is made necessary further by the difficulty of measuring small values of the indicated air speed. In order to mount the air-speed element in the undisturbed air stream it must be mounted well away from the airship which almost necessitates a distant indicating instrument of the electrical type. For use on Navy airships the Bureau of Standards has developed the commutator-condenser type of air-speed indicator. A propeller in the air stream operates a commutator through which an electric capacity is alternately charged from a battery and then discharged through an ammeter. The ammeter can be calibrated in terms of true air speed since the number of discharges per second is proportional to the true air speed. (Reference 19.)

ALTITUDE INSTRUMENTS

Of the altitude instruments called experimental in this paper the absolute altimeter will undoubtedly be a future service instrument. Three possible methods of measuring the tape-line altitude of the aircraft, as contrasted with the pressure altitude, have been under intensive development in a number of laboratories both here and abroad. These methods have given rise to the following instruments: the sonic altimeter, the capacity altimeter, and the radio altimeter. No detailed description of the individual instruments thus far developed is available, but it appears at the present writing that absolute altimeters of one or more of these types will soon be available.

The barograph and free-air thermometer included under experimental instruments are largely used in the flight testing of aircraft and in special investigations and flights. Their consideration is beyond the scope of this paper.

Aneroid Altimeter. The service altimeter of today varies greatly in external appearance and performance from its predecessor of 10 years ago. The mechanism is now enclosed in a bakelite case, the dial size has been reduced from $3^1/_2$ to $2^3/_4$ in. and the weight from 14 to 11 oz. The ranges of the instruments commonly used by the military services are now 20,000 and 35,000 ft. with a pointer movement of two revolutions for the 20,000-ft. instrument, and $1^3/_4$ revolutions for the higher-range instrument. The older instruments are based on a pointer movement of one revolution for 20,000 ft. The scales of the instruments are equally divided, as has always been customary in this country. The most striking feature is the improved performance of the best present-day instrument as shown by the accompanying table. See also Figs. 5 and 6.

PERFORMANCE OF ALTIMETERS

Error	Instrument 1919	1929 Range in Feet	1929
	20000	20000	35000
Scale errors at +20 deg. cent. in feet	0 to 400	0 to 100	0 to 400
Maximum elastic hysteresis in per cent range	1.5	0.1 to 0.2	0.1 to 0.2
After effect in per cent of range	1.0	0.1 to 0.2	0.1 to 0.2
Drift in per cent of range	0.8	0.4	0.4
Change in reading at 760 mm. Hg in temperature interval +40 to —10 deg. cent., in feet	100 to 150	10 to 50	100 to 150
Change in scale value in per cent			
+40 deg. cent. to —10 deg. cent.	3 to 5	.....	.....
+20 deg. cent. to —35 deg. cent.	.....	1.0	1.4

The instruments contrasted are those of the best quality made in production, and not special hand-made instruments. The definitions of the above errors as used in aircraft instrument work are given below.

The *elastic hysteresis* is the difference in the scale error of the altimeter at a given reading, determined by subjecting the instrument first to pressures decreasing to the highest indicated

altitude and then to pressures increasing back to the atmospheric pressure.

The *after effect* is the hysteresis at full atmospheric pressure (0 altitude).

The *scale value* is defined as the ratio of the actual altitude change to the corresponding change in the indication of the instrument.

The *drift*, often spoken of as *creep*, is defined as the change in reading with time at some altitude other than zero, while the pressure is held constant.

The improvement of the elastic qualities of the altimeter has opened up a number of possibilities, among which may be mentioned its increased utility for landing purposes. If an instrument of good quality and with known errors is used, it is believed that the over-all instrumental performance can be depended upon to within 50 ft., even under severe conditions of usage. In order to avoid the inherent defects of the barometric method of measuring altitude, the effective use of the instrument is linked up with obtaining information from the proposed landing field as to the pressure of the ground level so that the pointer of the altimeter may be reset in such a way that it will indicate zero upon landing.

One of the problems of aerial photography is that of flying level. Methods have been proposed involving the use of a statoscope, a rate-of-climb indicator, or a sensitive altimeter, as well as methods of a more indirect nature. The present trend of development in altimeters indicates that the use of a sensitive altimeter may be the ultimate solution. Instrument makers are making effective progress at the present time in the development of an altimeter of 20,000-ft. range with the pointer making 20 revolutions for this range. One manufacturer now has available an instrument of this type with a six-inch dial. Further developments are under way and will be looked forward to with interest.

Navigation Instruments

The navigation of an airplane consists principally in maintaining the normal flying attitude and the proper direction of flight toward the destination, and in being able to determine the position of the airplane and to clear obstacles. In scheduled flights over airways, and in a large amount of other flying when the ground is visible, the navigation is of the simplest sort. The navigator determines his position and flying attitude almost entirely by observing landmarks. These are observed to give the position, and are used also to determine the direction of flight. The present movement to have the names of towns, together with the direction of the meridian and location of nearest landing field, marked on roofs and other suitable spots is a great aid to this type of navigation. The service type of navigation instruments listed in this paper are useful but not essential under these conditions.

An entirely different situation exists when the ground is not visible. Three elements must be considered, maintaining the proper flying attitude, the proper direction of flight, and making the landing. The proper flying attitude can be maintained only by the aid of instruments, of which all of those needed are now available.

The magnetic compass and the turn indicator are inadequate to maintain the proper course in flight under these conditions, since the drift cannot be measured when the ground is invisible. The radio beacon has been developed to meet this problem, and has been found to be indispensable in flights which are made during poor visibility.

The most recent work on the improvement of navigation during poor visibility has been in devising methods for making a successful landing. To do this the navigator needs to locate the landing field, and know his absolute altitude above it. The progress in developing absolute altimeters has been discussed. Progress has been made in the method of locating the landing field based on a property of the radio beacon. Although a considerable amount of development work is still essential in order to make landings during fog a common operation, it appears that promising methods for doing so have been found.

Aerial navigation by astronomical methods is commonly employed only in flights over oceans, and in long flights over undeveloped or unfamiliar country. The necessity for their use is becoming less and less as the operation of the radio beacon is extended to cover more and more air lines. The astronomical method of position finding has an inherent advantage over other

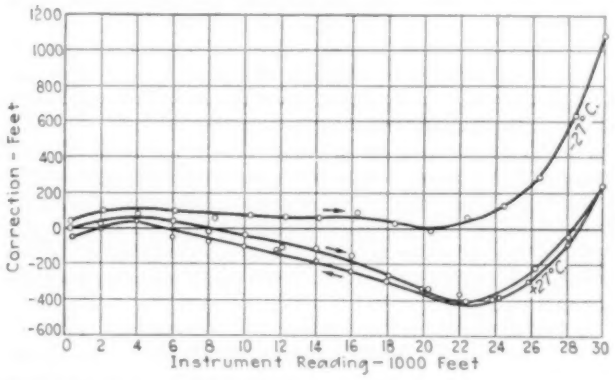


Fig. 5 Scale Errors and Elastic Hysteresis of, and Effect of Temperature on a 30,000-Ft. Altimeter. Pointer Motion for the Range Is 1½ Revolutions

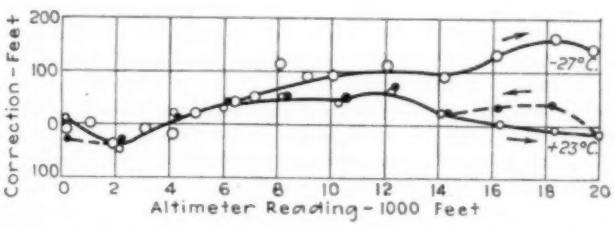


Fig. 6 Scale Errors at Two Temperatures and Elastic Hysteresis of a 20,000-Ft. Altimeter

methods in that it is independent of all ground equipment, for which reason its use will always continue in many cases. Important improvements are being made in the method of position finding by astronomical methods. The sextant and time pieces have been improved, as well as the methods of computing. For the latter see references 20 and 21. A discussion in detail is beyond the scope of this paper.

Magnetic Compass. There has been a slow and steady improvement in the performance of magnetic compasses. The important features of design are:

(*a*) Adequate provision for the expansion of the damping liquid due to temperature, and at the same time providing for the effect of low pressure.

(*b*) The reduction of the weight of the compass card consistent with securing adequate damping.

(*c*) Use of better magnet steels as these become available. Cobalt magnet steels are now being used, from which relatively stronger magnets per unit volume can be produced.

(*d*) The selection of the proper damping liquid. For some years mineral spirits or varnolene has been used as the damping liquid. A recent study by Keulegan (reference 13) of damping liquids from the viewpoint of their use in aircraft instru-

ments may make possible greater freedom in the selection of the liquid.

The chief difficulty with compasses at the present time is that associated with their installation in the aircraft. The effect of vibration and the effect of extraneous magnetic fields produced in the aircraft combine to make proper installation a comparatively difficult matter in many cases. The effect of vibration is being minimized by the use of vibration-absorbing mountings. No general solution for the effect of the extraneous magnetic fields has been found save that of changing the location of the compass. The compass must be compensated after installation against the effect of the magnetized parts of the ship, and for the deviation. For details see reference 12.

Compasses of new design are subjected to the following laboratory tests:

(*a*) The instrument is subjected to a pressure corresponding to the lowest altitude at which it will be used in order to determine the effect on the expansion chamber, and to note generally possible injurious effects of low pressure.

(*b*) The instrument is subjected to a temperature of —35 deg. cent. The liquid should not freeze at this temperature.

(*c*) The critical test for the adequacy of the expansion chamber is to subject the instrument to a temperature of +45 deg. cent. There should be no leaks at this temperature, nor should a bubble form in the compass bowl upon reducing the temperature.

(*d*) The period of the compass is determined at +20 deg. cent. The procedure usually consists in deflecting the compass card 45 deg. then allowing it to swing freely. While swinging the time interval is obtained for the card to swing through the zero position for the first and second time. Twice this time is the period. In order to be of any value these data must be obtained when the compass is in a horizontal magnetic field of standard strength (0.180 gauss). This magnetic field is practically never found inside of a building, so that recourse must be had to a suitable location out of doors. The period should be obtained at low temperature, but it is difficult to control both the external magnetic field and the temperature simultaneously. Qualitative data of some value may be obtained on this point in the low-temperature test above mentioned.

(*e*) Current specifications also give tolerances for the card error, balance of card, friction error, heeling, and levelling.

Although an experimental instrument, the induction compass should be mentioned. It may be also called a distant indicating compass since the chief advantage of this instrument is that its magnetic element may be placed in the most favorable part of the aircraft as regards freedom from magnetic disturbances. Since the announcement by Heyl and Briggs of the first working model in January, 1922, development work has been continuously in progress. Only one type, the Pioneer compass, is at present available commercially. Other types are being worked on experimentally which may lead to improved instruments. The improvements will consist of reduction in weight, improved reliability and perhaps reduction in cost. The induction compass is at present classed as an experimental instrument because it is not generally used, which is largely due to its relatively higher cost and its present lack of reliability as compared with the magnetic compass. Its ultimate adoption as a service instrument is sure.

Turn Indicator. New developments in the gyroscopic turn indicator have been largely along the lines of a general expansion of its use rather than along its further mechanical development. The wind-driven instrument giving qualitative indications has been available in substantially the same design for a number of years, and has usually given satisfactory service (reference 11).

Tests show that when properly lubricated with a low pour-point oil, the starting pressure and friction characteristics of the instrument are substantially the same in the temperature range —35 deg. to +45 deg. cent.

There is some difference of opinion on the part of pilots and navigators as to the proper sensitivity of the turn indicator. Military specifications require a deflection of $^1/_{16}$ in. in turning at a rate of 36 deg. per min., $^3/_8$ in. for 360 deg. per min., and $^3/_4$ in. for 1080 deg. per min. when the instrument is operating under a suction of 26 in. of water. This suction is developed only at one speed of the aircraft, so that the sensitivity of the instrument will in general be different from the above. The instruments, however, are provided with an adjustment for the sensitivity, and also for damping so that these can be adjusted as desired.

Bank Indicator. A curved-glass tube containing a steel ball, and filled with a suitable liquid for damping purposes is used as the bank indicator, and in the absence of turns, to indicate rolling. Until quite recently a $^1/_4$ in. ball has been used, but in the opinion of pilots this size of ball is too small. At the present time standardization appears to be taking place around the use of a steel ball $^7/_{16}$ in. in diameter. Proper damping is the controlling factor in a proper design. A satisfactory performance is determined by the results of two tests, vibration and time of roll at low temperature. If the viscosity of the damping liquid is inadequate at room temperature the ball spins when subjected to vibration, and in general will climb up the tube. Liquids which give entirely adequate damping at room temperatures will either be found to be frozen or so viscous as to be unsuitable when the inclinometer is cooled to —35 deg. cent. Preliminary experiments indicate that satisfactory low-temperature performance can be obtained with certain solutions having a viscosity as high as 0.04 poise at 25 deg. cent., which value is sufficient to damp out the effects of a vibration with an amplitude of approximately $^1/_{32}$ in. It is obvious, of course, that the performance of the inclinometer also depends upon the clearance between the ball and tube. If the inclinometer is designed to have a small clearance between the ball and tube, the cost of the instruments will be relatively great owing to the difficulty of obtaining glass tubing with a uniform bore. Tests show unexpectedly that better damping is secured by having relatively large clearances between the ball and tube. For data on damping liquids see reference 13.

Drift Indicators. The measurement of drift can at present be made only when the ground is visible. The methods used have not varied essentially in the past few years. There has been, however, a tendency toward more simple apparatus.

As is well known (see reference 7) the wind and ground speed of the aircraft can be determined from drift measurements if the true air speed be known. Further, the drift indicators commonly available contain suitable sights which with a stop watch, and a knowledge of the altitude also determine the ground speed. However, the common practice is to dispense with the possibility of determining wind and ground speed.

The proper heading only is determined. This is done by measuring the drift when on a compass course, adjusting the heading by the amount of the drift, and then repeating the process until on the proper course. When over water or flying at night smoke bombs or flares are used to furnish the object upon which to sight.

Fore-and-Aft Inclinometers. The instruments which may be termed fore-and-aft inclinometers are not strictly service instruments since their use is not common. Two types of instruments are available, one a liquid-in-glass type and the other a combined pendulum and gyro instrument. The first-named instrument is affected alike by inclination, and fore-and-aft accelerations. The pendulum-gyro type instrument gives indications which are

practically independent of the fore-and-aft acceleration. The indication of both instruments is affected by the angle of attack.

Experience with the capillary-leak tube-type indicator has shown the instrument to be of little use in indicating the rate of climb accurately. It has been further shown, however, that the instrument indicates qualitatively the fore-and-aft inclination. The instrument is free from the acceleration and angle of attack errors above mentioned. The indication depends on the rate of change of pressure.

A change in indication of the air-speed indicator associated with a constant speed of the propeller is also often used to indicate fore-and-aft inclination.

Radio Beacon. A discussion of the radio beacon is out of place in this paper except as regards its function as an aid to navigation. The chief use of the radio beacon is in flying during poor visibility and fog. It enables the proper direction toward the destination to be maintained. Without the radio beacon the direction of flight would have to be determined by means of the magnetic compass, a method which is unsatisfactory because of the impossibility of determining the drift, and correcting the heading for it. A further property of the radio beacon is now in the process of being utilized. When the airplane is equipped with a vertical-rod antenna, a drop in the strength of the signal received on the airplane occurs when directly over the beacon. This phenomenon gives a method of locating the landing field. The radio beacon is essential in fog flying since no other successful methods have been developed to maintain proper direction and to locate the landing field.

Operating Problems

A few general problems will be mentioned relating to the installation and satisfactory operation of aircraft instruments. The first of these problems is the effect of instrument-board vibration. This is most serious on the magnetic compass, and on other instruments of which the operation depends on very small forces. In some cases the vibration encountered is too severe for these instruments. There is apparently need to specify the vibration which instruments should be expected to withstand. All new designs should be subjected to this vibration, with suitable damping devices attached to the instrument when necessary. The damping device should be successful before the instrument is considered suitable for aircraft use. As has been stated a vibration test is now required in current military specifications, but it is believed that further experience will probably require some changes in the test.

It is well known that the static pressure in a cabin of an airplane differs from that of the free air. This difference affects the indication of altimeters and barographs particularly, and can be eliminated if found necessary by the use of airtight cases connected to the free air.

Trouble is often experienced with the breaking of the tubing of pressure gages, and oil or water thermometers. This is very often due to vibration, and occasionally to poor design of the connection or improper construction. It can be prevented in most cases.

A problem exists in securing proper lubrication on instruments with rotating elements such as tachometers, turn indicators, and induction compasses. The requirements are that the lubrication be satisfactory at least in the temperature range —35 to +45 deg. cent., and to lower temperatures occasionally. Further, the lubrication must be effective over a long interval of time as in some cases no provision is made for lubrication after installation, and in others the neglect of this necessity in operation must be anticipated. At the present time oils having a pour point of —40 deg. cent. can be obtained from several sources. The performance is satisfactory in the above temperature range, but it requires further experience to obtain data on its useful life.

Standardization of Instrument Dimensions

At the recent Army-Navy (AN) Conference at the Naval Aircraft Factory in Philadelphia attended by manufacturers, representatives of the Army Air Corps and Navy Bureau of Aeronautics, and others interested, new case dimensions were adopted for altimeters, tachometers, and air-speed indicators which are based on a $2^3/_4$-in. dial. This adoption makes these instruments interchangeable on the instrument board with minor modifications. Changed dimensions were also adopted for fuel- and oil-pressure gages, and oil or water thermometers, in which the dial size was reduced from two inches to $1^7/_8$ in. In each case the adoption of these new dimensions was preceded by a large amount of preliminary work which included the preparation of drawings and specifications by the Bureau of Aeronautics assisted by the Bureau of Standards on the tests and tolerances. This work was followed by the production of sample instruments by a number of manufacturers. This was essential since in some cases a redesign of the instrument mechanism had to be made in order to make it possible to insert it in the new size cases. In making the change other factors also had to be considered, such as the maximum pointer movement in its relation to sensitivity and the desired accuracy, the performance, and other design factors. The new sizes of instruments are being quite generally used in commercial aircraft, and it appears that the new sizes are in a fair way to be considered as the American standard, except possibly in the case in which vertical-scale instruments are used. This work is an outstanding achievement in the aircraft-instrument field and was made possible only by the close cooperation of a number of instrument manufacturers, and the Bureau of Aeronautics.

In order to make instruments generally interchangeable on the instrument board, and of course to simplify the board itself the manufacturers have produced instruments of other types which are interchangeable with the altimeter, tachometer, and air-speed indicator. These instruments with new case sizes include the bank and turn indicator, the clock, and magnetic compass, the rate-of-climb indicator, fuel-flow meter, fuel-quantity gage, and induction-compass indicators.

The assistance and cooperation of Messrs. H. C. Sontag, H. B. Henrickson, R. B. Block, and M. J. Evans, all of the Aeronautic Instruments Section of the Bureau of Standards, is acknowledged in the preparation of this paper, especially in securing the performance data on the various instruments. These data have been obtained largely on instruments submitted by the Bureau of Aeronautics for test in accordance with its specifications. The close cooperation of Lieut. T. C. Lonnquest, U. S. N., and C. L. Seward of the Bureau of Aeronautics of the Navy Department is hereby acknowledged. The credit for the initiation of many improvements in service instruments, and the carrying through of the recent standardization of instrument dials and case dimensions is largely due to them.

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Discussion

A. H. Mears.[3] The paper by Dr. Brombacher adequately summarizes the development of aeronautic instruments during the last ten years. It points out that the innovations in service instruments have been largely in improvements and developments of materials of which the instruments are constructed, and of production methods rather than the development of new designs or principles of operation. The testing methods are practically the same as they were ten years ago.

There are two interesting points contained in the paper that appear to be worth calling to one's attention. The first is the change in the pressure heads of air-speed indicators from pitot-venturi to pitot-static heads. Ten years ago the speed of heavier-than-air machines was relatively slow so that pitot-static heads did not develop sufficient pressure differences, therefore, much expense and effort was spent in developing pitot-venturi heads that would produce large pressure differences. These were hard to construct, complicated, and often unsatisfactory. While this development was in progress, the speed of the airplanes was increasing rapidly, hence much greater pressure differences could be developed by the pitot-static heads. By slightly improving the air-speed indicators the pitot-static heads became entirely satisfactory. Such situations as these must be watched carefully by the aeronautic-instrument manufacturer to avoid losses and unnecessary developments. The second point is the change in the use of the rate-of-climb indicator to that of an inclinometer thereby greatly increasing the use of the instrument as a service instrument in the heavier-than-air field.

With the improvement in the performance of the altimeter, and the necessity of connecting to a static head for accurate readings, it seems that there might be many advantages of installing all altimeters in airtight cases provided with pressure connections. This would enable them to be checked when on the instrument board of the plane, and simplify the testing apparatus of altimeters.

In the temperature tests at sea-level pressure, the 35,000-ft. altimeter seems to have much more tolerance than is necessary, when compared with the 20,000-ft. instrument.

Victor E. Showalter.[4] Dr. Brombacher's paper makes available to the aeronautical engineer information heretofore obtainable only through tremendous effort on the searcher's part. Speaking authoritatively as Dr. Brombacher does, having been in touch constantly with the development of aircraft instruments for several years, his effort must be received appreciatively. It may be read in full confidence that it does not contain any sales argument, and that the data imparted are accurate.

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The Application of the Principle of Least Work to the Primary Stress Calculations of Space Frameworks

By C. P. BURGESS,[1] WASHINGTON, D. C.

A structure similar to the framework of a rigid airship is analyzed by three approximate methods commonly applied to such structures, and also by the mathematically exact method of least work. It is shown that the approximate methods may be very considerably in error. A short-cut method of least work, giving very satisfactory results, is described and applied to the structure.

Introduction

THE hull structures of rigid airships are typical of a large class of space frameworks, including gasometers and the wind bracing of steel-framed towers and buildings, in which the shearing and bending forces are distributed between such highly complex and redundant systems of members that mathematically exact methods of primary stress calculations are so difficult as to be rarely or never attempted. Several approximate methods have been developed and used by engineers and designers, but the known methods are not very satisfactory.

Even a mathematically exact analysis may yield results quite different from reality, because the mathematical theories of stress are based upon the assumptions that there are no initial stresses in the unloaded structure and the material behaves strictly according to Hooke's law, i.e., the ratio of stress to strain is constant. In all indeterminate structures, particularly wire-braced ones, there may be very considerable departures from both of these assumptions.

It is fortunate for the success of redundant structures that when a member is stressed to near or beyond its yield point, the stress/strain ratio diminishes, with the result that the overloaded member takes a decreasing share of the load on the structure and the less-stressed members take an increasing share. A mathematically exact strength calculation takes no account of this transfer of load from the more heavily stressed members in extreme conditions and therefore errs on the safe side.

In this paper a simplified method is developed for applying the principle of least work or minimum-strain energy to the stress calculations of highly redundant framed structures. Some simplifying assumptions are introduced, involving slight departures from reality, but in most cases the errors will be much less than when the customary approximate methods of stress calculation are used.

By way of illustration the new method is applied to the stress calculation of a framework resembling the hull structure of a rigid airship; and for comparison, the stresses are also calculated by the exact method of least work and three different approximate methods commonly used for the stress analysis of airships and similar types of frameworks.

Approximate Methods of Stress Calculations

The bending and the shear theories are the bases of two well-known methods of approximating to the stress analysis in framed structures of the type under consideration. The bending method is an application of the familiar beam-bending theory to the distribution of the longitudinal forces. The sizes of the shear wires are not considered to influence the distribution of the forces, except to a slight extent in the modified bending theory proposed by Professor Hovgaard, in which fictitious or imaginary bars are introduced to represent the contribution of the shear wires to the cross-sectional area of the structure. The shear wires are assumed to take the longitudinal shear between the longitudinal members without regard to the sizes or elastic properties of the wires.

The shear theory contrasts with the bending theory in that the stress distribution is assumed to depend only upon the elastic properties of the shear members, ignoring these properties in the longitudinals. The stresses in the longitudinals are calculated from the forces applied to them by the shear wires at the joints.

Attempts have been made to justify the use of the bending method by proving that it is consistent with the principle of least work. The line of reasoning has been to show (see Appendix I) that if the forces in the longitudinals are distributed according to the bending theory, the strain energy in the longitudinals has its minimum value, and therefore the bending theory must be correct. The limitation to this argument is that it may be shown equally well (see Appendix II) that, if the forces are distributed according to the shear theory, the strain energy in the shear members is a minimum. The conditions of any particular problem determine which distribution of force will produce less total work in the structure. Neither may give the least possible. In the event of a discrepancy between the results of the shear and bending theories, it is to be expected that the least total work will result from a force distribution lying somewhere between the indications of the two theories; nearer to the bending theory when the greater part of the work is in the longitudinal members, and nearer to the shear theory when the internal work of the shearing forces predominates. It might be thought that the work in the longitudinal members would constitute the greater part of the total strain energy in most practical problems; but in structures such as airship hulls, in which the length/diameter ratio is not large, and the longitudinal girders are of much greater cross-sectional area than the diagonal wires, it usually happens that there is more internal work in the wires than in the longitudinals.

When the bending and shear theories give the same results, both must be correct, because in that case there is the least possible strain energy in both the longitudinal and diagonal members. The only qualification to be placed upon this statement is that it often happens that the stresses in the longitudinals are not entirely the result of the primary bending of the structure. The longitudinal components of the tensions in the shear wires produce some compression in the longitudinals, over and above the direct effect of the bending moment, leading to small departures of the stress distribution from the bending theory.

It has been argued that the forces or stresses in the longitudinal and shear members can be considered separately, on the

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

assumption that the stresses in the longitudinals are in accordance with the bending theory and the stresses in the diagonals conform to the shear theory. A stress calculation on this assumption will not show equilibrium of forces at the joints, except in the special case when the longitudinal and diagonal members are so proportioned to each other that the bending and shear theories are both correct. But it may be said in favor of this method of stress calculation that the largest bending stresses occur where shear is least, so that the internal work in the longitudinals predominates, and the bending theory should therefore have a good order of accuracy; and conversely, where the shear is greatest, the bending is small, with the result that the work is mainly in the shear members, favoring a distribution of stress according to the shear theory.

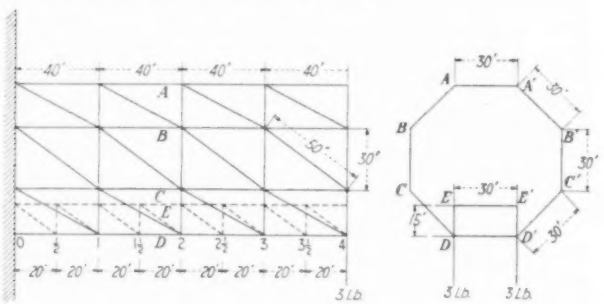


FIG. 1 SHOWING DIMENSIONS OF STRUCTURE

Attempts have been made to strike a weighted mean between the shear and bending theories, according to the ratio between the internal work of shear and bending, but the results have not been satisfactory.

A third approximate method of stress calculation is the inverse-ratio theory, which takes account of the elastic properties of all members. It is based upon the mathematically demonstrable theorem that, when a load is divided between two or more independent statically determinate systems, these systems share the load between them in inverse ratio to their internal work when each alone takes the entire load. This theorem ceases to be exact when the determinate systems react upon each other. It is impossible to divide a structure of the type under consideration into statically determinate systems which do not react upon each other to some extent. Consequently, the inverse-ratio theory must be classed among the approximate methods; but since it takes account of the elasticity of all members, it may be expected to give more nearly correct values than either the shear or bending theory when these differ from each other.

The proposed new method of stress analysis by a modification of the conventional least-work procedure is most conveniently explained after following the analysis in the illustrative problem by the principle of least work applied in the conventional manner.

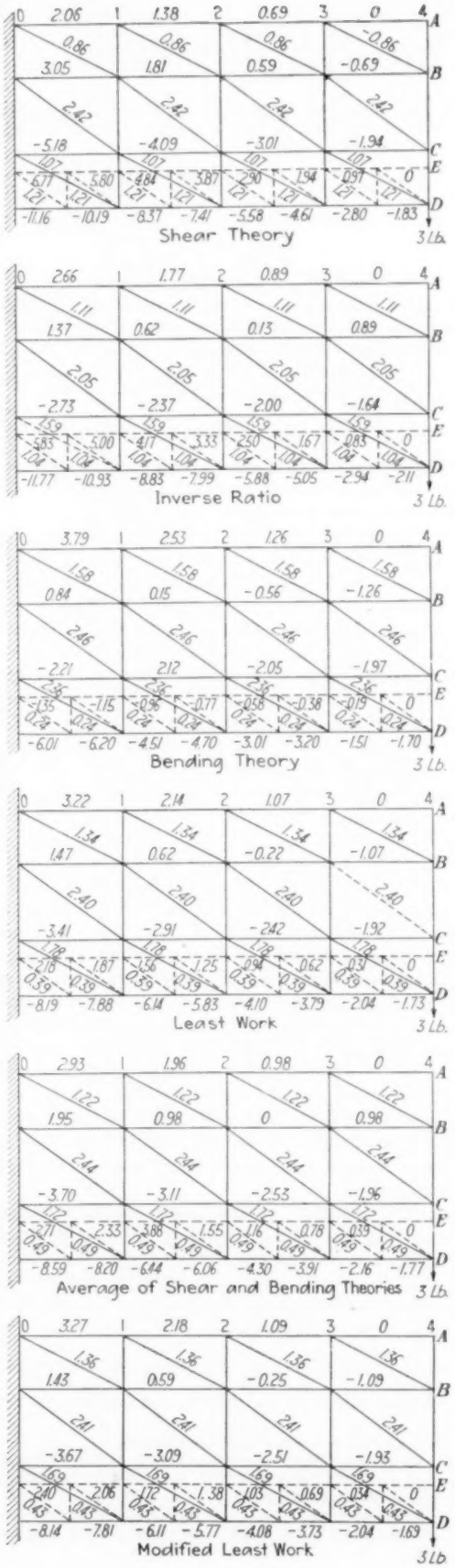
Before proceeding to the least-work analysis, approximate calculations by the three methods already discussed are set forth briefly, and it is shown that the approximate methods give widely divergent results.

The derivations of the formulas of the approximate methods are not given here. They may be found in the chapter on the Longitudinal Strength of Rigid Airships in the author's book, "Airship Design."

ILLUSTRATIVE PROBLEM

The framework shown in Fig. 1 is to be analyzed by the three approximate methods of shearing, bending, and inverse ratio,

FIG. 2 (IN THE RIGHT-HAND COLUMN) SUMMARY OF STRESSES BY VARIOUS METHODS OF CALCULATIONS



and also by the mathematically exact method of least work and by the proposed simplified adaptation of least work.

The following assumptions are made in regard to this structure:

(*a*) All joints are pinned
(*b*) At the left-hand end of the structure, all longitudinals and diagonals are pin-connected to an absolutely rigid wall
(*c*) The frame at the right-hand end is rigidly cross-braced
(*d*) Intermediate frames are without cross-bracing
(*e*) The internal work in all transverse members, including the cross-bracing of the right-hand end frame, is negligible. This is equivalent to assuming that EA is infinite for these members in the least-work and inverse ratio calculations
(*f*) A single load acting vertically downward is applied to the right-hand frame.

Owing to the absence of cross-bracing in the intermediate frames, there can be no transference of the load at these frames from one row of panels to another. Consequently, the tensions are constant throughout each horizontal row of diagonal members.

The Calculations

(1) *The Shear Theory.* The expression for the tensions in the shear wires according to the shear theory is:

$$S = \frac{FA \sin\theta \sin\phi}{L\Sigma\,(A/L)\sin^2\theta \sin^2\phi}$$

where S = tension in any shear wire
F = total shear in the frame bay
θ = inclination of the panel to the plane of shear and bending of the hull
ϕ = inclination of the shear wire to the longitudinals
A = cross-sectional area of the shear wire
L = length of the shear wire.

The summation includes all the shear wires in the frame bay.

The following table shows the application of this method to the structure under discussion:

TABLE 1 CALCULATION BY SHEAR THEORY

Member	A	L	$\sin\theta$	$\sin^2\theta$	$\sin\phi$	$\frac{A}{L}\sin^2\theta \sin^2\phi$	S
A0–B1	0.0167	5.0	0.7071	0.5	0.6	0.00060	0.86
B0–C1	0.0333	5.0	1.0	1.0	0.6	0.00240	2.42
C0–D1	0.0208	5.0	0.7071	0.5	0.6	0.00075	1.07
E0–D½ } E½–D1 }	0.0167	5.0	1.0	1.0	0.6	0.00120	1.21
Total						0.00495	

$$S = \frac{FA \sin\theta \sin\phi}{L\Sigma\,(A/L)\sin^2\theta \sin^2\phi} = \frac{3 \times 0.6 \times A \sin\theta}{5 \times 0.00495}$$
$$= 72.8\,A \sin\theta.$$

The stresses in all members according to the shear theory are summarized in Fig. 2.

(2) *The Bending Theory.* The expression for the tensions in the shear wires according to the bending theory is:

$$S = \frac{FL\,\Sigma\,Ay}{I}$$

where S = tension in any shear wire
F = total shear in the frame bay
L = length of the shear wire
I = moment of inertia of the cross-section of the structure
A = cross-sectional area of any longitudinal girder (not of a shear wire as in the shear theory)
y = the distance of any longitudinal from the neutral axis of the structure.

The summation $\Sigma\,Ay$ is taken over all longitudinals beyond the shear wire under consideration in the direction away from the neutral axis.

Tables 2 and 3 show the application of the bending theory to the problem in hand.

The stresses in the longitudinals are shown in Fig. 2, calculated from the forces applied to the joints by the shear wires, instead of by direct application of the bending theory to the longitudinals. By this procedure account is taken of the contribution of the shear wires to the longitudinal forces. In Table 2 the moment of the cross-sectional areas of the longitudinals are calculated about the geometrical axis (h = distance of a longitudinal member from the geometrical axis). The position of the neutral axis and the moment of inertia about that axis are then computed. From the data derived in Table 2, the shear-wire tensions are calculated in Table 3.

It may be noted that the keel shear wires are in compression. This is impossible in reality and would result in the counter-shear wires taking the load, but for the present purpose of comparison with other methods of stress calculation it is permissible to assume that the shear wires are able to sustain compression.

TABLE 2 MOMENT OF INERTIA CALCULATION

Member	A in.²	h ft.	Ah ft. in.²	y ft.	Ay ft. in.²	Ay^2 ft.² in.²
A	0.25	36.2	9.05	48.3	12.07	583.0
B	0.25	15.0	3.75	27.1	6.77	183.5
C	0.25	—15.0	— 3.75	— 2.9	— 0.73	2.1
D	0.60	—36.2	—21.72	—24.1	—14.46	348.5
E	0.40	—21.2	— 8.48	— 9.1	— 3.64	31.1
Total	1.75		—21.15			I = 1148.2

Neutral axis is at 21.15/1.75 = 12.1′ below the geometrical axis.

TABLE 3 STRESSES IN SHEAR WIRES BY BENDING THEORY

Member	L	$\Sigma\,Ay$	$L\,\Sigma\,Ay$	S
A0–B1	50	12.07	603.5	1.58
B0–C1	50	18.84	942.0	2.46
C0–D1	50	18.10	905.0	2.36
E0–D½	25	—3.64	—91.0	—0.24

$$S = \frac{FL\,\Sigma\,Ay}{I} = \frac{L\,\Sigma\,Ay}{383}$$

(3) *The Inverse-Ratio Theory.* In the application of the inverse-ratio theory, the structure is divided into four statically determinate systems, one for each row of shear wires. The internal work in each system when alone carrying the external load is calculated in Table 4.

Let Q_1, Q_2, Q_3, and Q_4 be the internal work in the top, middle, bottom, and keel panels, respectively, when each alone sustains the external load. Let q_1 to q_4 be the fractions of the total load which each panel takes when all are acting.

Then:

$$q_1 + q_2 + q_3 + q_4 = 1$$

And by the inverse ratio theory

$$q_1Q_1 = q_2Q_2 = q_3Q_3 = q_4Q_4$$

Whence

$$\frac{1}{q_1} = 1 + \frac{Q_1}{Q_2} + \frac{Q_1}{Q_3} + \frac{Q_1}{Q_4}$$
$$= 1 + \frac{425{,}280}{162{,}640} + \frac{425{,}280}{295{,}578} + \frac{425{,}280}{320{,}691}$$
$$= 1 + 2.615 + 1.439 + 1.326 = 6.380$$

TABLE 4 STRESS CALCULATION BY INVERSE RATIO METHOD

Member	L	A	E	$U = \frac{L}{AE}$	S_1	US_1^2	S_2	US_2^2	S_3	US_3^2	S_4	US_4^2	q_1S_1	q_2S_2	q_3S_3	q_4S_4	S
A0–A1	40	0.25	1	160	16.971	46,080	....	....	....	....	....	....	2.659	....	....	....	2.659
A1–A2	40	0.25	1	160	11.314	20,480	....	....	....	....	....	....	1.773	....	....	....	1.773
A2–A3	40	0.25	1	160	5.657	5,120	....	....	....	....	....	....	0.886	....	....	....	0.886
A3–A4	40	0.25	1	160	0	0	....	....	....	....	....	....	0	....	....	....	0
B0–B1	40	0.25	1	160	−22.628	81,920	12.0	23,040	....	....	....	....	−3.546	4.916	....	....	1.370
B1–B2	40	0.25	1	160	−16.971	46,080	8.0	10,240	....	....	....	....	−2.659	3.278	....	....	0.619
B2–B3	40	0.25	1	160	−11.314	20,480	4.0	2,560	....	....	....	....	−1.773	1.639	....	....	−0.134
B3–B4	40	0.25	1	160	−5.657	5,120	0	0	....	....	....	....	−0.886	0	....	....	−0.886
C0–C1	40	0.25	1	160	....	....	−16.0	40,960	16.971	46,080	....	....	....	−6.555	3.827	....	−2.728
C1–C2	40	0.25	1	160	....	....	−12.0	23,040	11.314	20,480	....	....	....	−4.916	2.551	....	−2.365
C2–C3	40	0.25	1	160	....	....	−8.0	10,240	5.657	5,120	....	....	....	−3.278	1.276	....	−2.002
C3–C4	40	0.25	1	160	....	....	−4.0	2,560	0	0	....	....	....	−1.639	0	....	−1.639
D0–D½	20	0.60	1	33.3	....	....	....	....	−22.628	17,039	−32.0	34,099	....	....	−5.103	−6.666	−11.769
D½–D1	20	0.60	1	33.3	....	....	....	....	−22.628	17,039	−28.0	26,107	....	....	−5.103	−5.832	−10.935
D1–D1½	20	0.60	1	33.3	....	....	....	....	−16.971	9,585	−24.0	19,181	....	....	−3.827	−4.999	−8.826
D1½–D2	20	0.60	1	33.3	....	....	....	....	−16.971	9,585	−20.0	13,320	....	....	−3.827	−4.166	−7.993
D2–D2½	20	0.60	1	33.3	....	....	....	....	−11.314	4,260	−16.0	8,525	....	....	−2.551	−3.333	−5.884
D2½–D3	20	0.60	1	33.3	....	....	....	....	−11.314	4,260	−12.0	4,795	....	....	−2.551	−2.500	−5.051
D3–D3½	20	0.60	1	33.3	....	....	....	....	−5.657	1,065	−8.0	2,131	....	....	−1.276	−1.666	−2.942
D3½–D4	20	0.60	1	33.3	....	....	....	....	−5.657	1,065	−4.0	533	....	....	−1.276	−0.833	−2.109
E0–E½	20	0.40	1	50	....	....	....	....	....	....	28.0	39,200	....	....	....	5.832	5.832
E½–E1	20	0.40	1	50	....	....	....	....	....	....	24.0	28,800	....	....	....	4.999	4.999
E1–E1½	20	0.40	1	50	....	....	....	....	....	....	20.0	20,000	....	....	....	4.166	4.166
E1½–E2	20	0.40	1	50	....	....	....	....	....	....	16.0	12,800	....	....	....	3.333	3.333
E2–E2½	20	0.40	1	50	....	....	....	....	....	....	12.0	7,200	....	....	....	2.500	2.500
E2½–E3	20	0.40	1	50	....	....	....	....	....	....	8.0	3,200	....	....	....	1.666	1.666
E3–E3½	20	0.40	1	50	....	....	....	....	....	....	4.0	800	....	....	....	0.833	0.833
E3½–E4	20	0.40	1	50	....	....	....	....	....	....	0	0	....	....	....	0	0
A0–B1	50	0.0167	3	1000	7.071	50,000	....	....	....	....	....	....	1.108	....	....	....	1.108
A1–B2	50	0.0167	3	1000	7.071	50,000	....	....	....	....	....	....	1.108	....	....	....	1.108
A2–B3	50	0.0167	3	1000	7.071	50,000	....	....	....	....	....	....	1.108	....	....	....	1.108
A3–B4	50	0.0167	3	1000	7.071	50,000	....	....	....	....	....	....	1.108	....	....	....	1.108
B0–C1	50	0.0333	3	500	....	....	5.0	12,500	....	....	....	....	....	2.048	....	....	2.048
B1–C2	50	0.0333	3	500	....	....	5.0	12,500	....	....	....	....	....	2.048	....	....	2.048
B2–C3	50	0.0333	3	500	....	....	5.0	12,500	....	....	....	....	....	2.048	....	....	2.048
B3–C4	50	0.0333	3	500	....	....	5.0	12,500	....	....	....	....	....	2.048	....	....	2.048
C0–D1	50	0.0208	3	800	....	....	....	....	7.071	40,000	....	....	....	....	1.595	....	1.595
C1–D2	50	0.0208	3	800	....	....	....	....	7.071	40,000	....	....	....	....	1.595	....	1.595
C2–D3	50	0.0208	3	800	....	....	....	....	7.071	40,000	....	....	....	....	1.595	....	1.595
C3–D4	50	0.0208	3	800	....	....	....	....	7.071	40,000	....	....	....	....	1.595	....	1.595
E0–D½	25	0.0167	3	500	....	....	....	....	....	....	5.0	12,500	....	....	....	1.041	1.041
E½–D1	25	0.0167	3	500	....	....	....	....	....	....	5.0	12,500	....	....	....	1.041	1.041
E1–D1½	25	0.0167	3	500	....	....	....	....	....	....	5.0	12,500	....	....	....	1.041	1.041
E1½–D2	25	0.0167	3	500	....	....	....	....	....	....	5.0	12,500	....	....	....	1.041	1.041
E2–D2½	25	0.0167	3	500	....	....	....	....	....	....	5.0	12,500	....	....	....	1.041	1.041
E2½–D3	25	0.0167	3	500	....	....	....	....	....	....	5.0	12,500	....	....	....	1.041	1.041
E3–D3½	25	0.0167	3	500	....	....	....	....	....	....	5.0	12,500	....	....	....	1.041	1.041
E3½–D4	25	0.0167	3	500	....	....	....	....	....	....	5.0	12,500	....	....	....	1.041	1.041
Totals						425,280		162,640		295,578		320,691					

$q_1Q_1 = 66{,}641; \quad q_1 = 0.1567$

$q_2 = 66{,}641/162{,}640 = 0.4097$

$q_3 = 66{,}641/295{,}578 = 0.2255$

$q_4 = 66{,}641/320{,}691 = 0.2081$

1.0000

The stresses in the members of the four determinate systems are designated by S_1 to S_4, respectively. The total stress in any member is $S = q_1S_1 + q_2S_2 + q_3S_3 + q_4S_4$. These stresses are calculated in Table 4 and summarized in Fig. 2.

(4) *Least-Work Calculations.* In the stress analysis by the method of least work, the re-entrant keel at the bottom of the framework is assumed to be the basic statically determinate structure. The stresses in the keel members, when the side panels are inactive, are listed in the column headed S_0 in Table 5. The stresses due to 1-lb. tension in the diagonals of each row of side panels taken consecutively are listed in the columns headed S_1, S_2, and S_3 in the same table. As already noted, the absence of cross-bracing in the intermediate transverse frames leads to the wire tensions being constant throughout each row of side panels. These tensions are the variables to be determined, designated by X_1, X_2, and X_3, proceeding in order from the top to the bottom row. The total stress S in any member is given by

$$S = S_0 + X_1S_1 + X_2S_2 + X_3S_3$$

The total internal work W is given by

$$W = \Sigma \frac{(S_0 + X_1S_1 + X_2S_2 + X_3S_3)^2 L}{2AE}$$

where the summation is taken over all members in the structure.

When W has its minimum value,

$$\frac{\partial W}{\partial X_1} = 0, \quad \frac{\partial W}{\partial X_2} = 0, \quad \text{etc.}$$

By performing these differentiations and equating to zero, the following equations are obtained:

$$X_1\Sigma S_1^2L/AE + X_2\Sigma S_1S_2L/AE + X_3\Sigma S_1S_3L/AE + \Sigma S_0S_1L/AE = 0$$

$$X_1\Sigma S_1S_2L/AE + X_2\Sigma S_2^2L/AE + X_3\Sigma S_2S_3L/AE + \Sigma S_0S_2L/AE = 0$$

$$X_1\Sigma S_1S_3L/AE + X_2\Sigma S_2S_3L/AE + X_3\Sigma S_3^2L/AE + \Sigma S_0S_3L/AE = 0$$

For convenience L/AE is usually represented by the symbol U.

In practice, equations of the foregoing form are written directly without carrying out the differentiation by which they are derived.

The coefficients of the three variables and the constant terms are calculated in Table 5, giving the following equations:

$$14{,}919X_1 + 7{,}023X_2 + 4{,}755X_3 = 45{,}352$$

$$7{,}023X_1 + 19{,}333X_2 + 4{,}678X_3 = 64{,}138$$

$$4{,}755X_1 + 4{,}678X_2 + 9{,}010X_3 = 33{,}631$$

The solution of these equations is:

$$X_1 = 1.34$$
$$X_2 = 2.40$$
$$X_3 = 1.78$$

From these values of the tensions in the diagonals, the stresses in the other members are obtained statically, as shown in Fig. 2.

Modified Least-Work Method

Least-work calculations are long and laborious because of the large number of members which have to be taken into account in complicated framed structures. It is proposed to effect a great economy of time and labor by assuming that the transfer of longitudinal force from the diagonal to the longitudinal members is continuous, instead of being concentrated at the joints of the structure. By this device, the stresses in the members may be expressed as continuous functions of easily integrated form, and the number of members to be considered is greatly reduced. Moreover, an old but little-known or used method of solving least-work equations and checking the calculations as the work proceeds is explained in Appendix III.

Let P be the transverse load applied to the structure, and let h be the depth of the truss formed by any pair of longitudinals. Then at distance x along the longitudinals from the point of application of P the end forces in the two longitudinals are given by

$$S = \pm P\,x/h.$$

In the illustrative problem, P = 3 lb., h = 30 ft. in each of the side panels and 15 ft. in the keel. In longitudinal D,

$$S_0 = -3x/15 = -x/5.$$

TABLE 5 STRESS CALCULATION BY METHOD OF LEAST WORK

Member	L	A	E	$U = \frac{L}{AE}$	S_0	S_1	S_2	S_3	US_0S_1	US_0S_2	US_0S_3	US_1^2	US_1S_2	US_1S_3	US_2^2	US_2S_3	US_3^2
A0–A1	40	0.25	1	160	...	2.4	...	...	...	...	...	921.6	...	...	...	...	...
A1–A2	40	0.25	1	160	...	1.6	...	...	...	...	...	409.6	...	...	...	...	...
A2–A3	40	0.25	1	160	...	0.8	...	...	...	...	...	102.4	...	...	...	...	...
A3–A4	40	0.25	1	160	...	0	...	...	...	...	...	0	...	...	...	...	...
B0–B1	40	0.25	1	160	...	−3.2	2.4	...	...	...	...	1638.4	−1228.8	...	921.6	...	...
B1–B2	40	0.25	1	160	...	−2.4	1.6	...	...	...	...	921.6	−614.4	...	409.6	...	...
B2–B3	40	0.25	1	160	...	−1.6	0.8	...	...	...	...	409.6	−204.8	...	102.4	...	...
B3–B4	40	0.25	1	160	...	−0.8	0	...	...	...	...	102.4	0	...	0	...	...
C0–C1	40	0.25	1	160	...	...	−3.2	2.4	...	...	...	...	...	...	1638.4	...	921.6
C1–C2	40	0.25	1	160	...	...	−2.4	1.6	...	...	...	...	...	...	921.6	...	409.6
C2–C3	40	0.25	1	160	...	...	−1.6	0.8	...	...	...	...	...	...	409.6	...	102.4
C3–C4	40	0.25	1	160	...	...	−0.8	0	...	...	...	...	...	...	102.4	...	0
D0–D½	20	0.60	1	33.3	−32	4.53	6.4	1.33	−4827.2	−6819.8	−1417.2	683.1	965.1	200.6	1364.0	283.4	58.9
D½–D1	20	0.60	1	33.3	−28	3.96	5.6	0.76	−3892.3	−5221.4	−708.6	522.3	738.6	100.2	1044.3	141.7	19.2
D1–D1½	20	0.60	1	33.3	−24	3.39	4.8	0.99	−2709.3	−3836.2	−791.2	382.7	541.9	111.8	767.2	158.2	32.6
D1½–D2	20	0.60	1	33.3	−20	2.83	4.0	0.43	−1884.8	−2664.0	−286.4	266.6	376.8	40.5	532.8	57.3	6.2
D2–D2½	20	0.60	1	33.3	−16	2.26	3.2	0.66	−1204.1	−1705.0	−351.6	170.1	240.8	49.7	341.0	70.3	14.5
D2½–D3	20	0.60	1	33.3	−12	1.70	2.4	0.10	−679.3	−959.0	−40.0	96.2	135.9	5.6	191.8	8.0	0.3
D3–D3½	20	0.60	1	33.3	−8	1.13	1.6	0.33	−301.0	−426.2	−87.9	42.5	60.2	12.4	85.2	17.6	3.6
D3½–D4	20	0.60	1	33.3	−4	0.57	0.8	−0.23	−75.9	−106.6	+30.6	10.8	15.2	−4.4	21.3	−6.1	1.1
E0–E½	20	0.40	1	50	28	−3.96	−5.6	−3.96	−5544	−7840	...	784.1	1108.8	784.1	1568.0	1108.8	784.1
E½–E1	20	0.40	1	50	24	−3.39	−4.8	−3.39	−4068	−5760	−4068	574.6	813.6	574.6	1152.0	813.6	574.6
E1–E1½	20	0.40	1	50	20	−2.83	−4.0	−2.83	−2830	−4000	−2830	400.4	566.0	400.4	800.0	566.0	400.4
E1½–E2	20	0.40	1	50	16	−2.26	−3.2	−2.26	−1808	−2560	−1808	255.4	361.6	255.4	512.0	361.6	255.4
E2–E2½	20	0.40	1	50	12	−1.70	−2.4	−1.70	−1020	−1440	−1020	144.5	204.0	144.5	288.0	204.0	144.5
E2½–E3	20	0.40	1	50	8	−1.13	−1.6	−1.13	−452	−640	−452	63.8	90.4	63.8	128.0	90.4	63.8
E3–E3½	20	0.40	1	50	4	−0.57	−0.8	−0.57	−114	−160	−114	16.2	22.8	16.2	32.0	22.8	16.2
E3½–E4	20	0.40	1	50	0	0	0	0	0	0	0	0	0	0	0	0	0
A0–B1	50	0.0167	3	1000	...	1	...	...	...	...	...	1000.0	...	...	...	...	...
A1–B2	50	0.0167	3	1000	...	1	...	...	...	...	...	1000.0	...	...	...	...	...
A2–B3	50	0.0167	3	1000	...	1	...	...	...	...	...	1000.0	...	...	...	...	...
A3–B4	50	0.0167	3	1000	...	1	...	...	...	...	...	1000.0	...	...	...	...	...
B0–C1	50	0.0333	3	500	...	...	1	...	...	...	...	...	...	...	500.0	...	...
B1–C2	50	0.0333	3	500	...	...	1	...	...	...	...	...	...	...	500.0	...	...
B2–C3	50	0.0333	3	500	...	...	1	...	...	...	...	...	...	...	500.0	...	...
B3–C4	50	0.0333	3	500	...	...	1	...	...	...	...	...	...	...	500.0	...	...
C0–D1	50	0.0208	3	800	...	...	...	1	...	...	...	...	...	...	...	...	800.0
C1–D2	50	0.0208	3	800	...	...	...	1	...	...	...	...	...	...	...	...	800.0
C2–D3	50	0.0208	3	800	...	...	...	1	...	...	...	...	...	...	...	...	800.0
C3–D4	50	0.0208	3	800	...	...	...	1	...	...	...	...	...	...	...	...	800.0
E0–D½	25	0.0167	3	500	5	−0.7071	−1	−0.7071	−1767.8	−2500.0	−1767.8	250.0	353.6	250.0	500.0	353.6	250.0
E½–D1	25	0.0167	3	500	5	−0.7071	−1	−0.7071	−1767.8	−2500.0	−1767.8	250.0	353.6	250.0	500.0	353.6	250.0
E1–D1½	25	0.0167	3	500	5	−0.7071	−1	−0.7071	−1767.8	−2500.0	−1767.8	250.0	353.6	250.0	500.0	353.6	250.0
E1½–D2	25	0.0167	3	500	5	−0.7071	−1	−0.7071	−1767.8	−2500.0	−1767.8	250.0	353.6	250.0	500.0	353.6	250.0
E2–D2½	25	0.0167	3	500	5	−0.7071	−1	−0.7071	−1767.8	−2500.0	−1767.8	250.0	353.6	250.0	500.0	353.6	250.0
E2½–D3	25	0.0167	3	500	5	−0.7071	−1	−0.7071	−1767.8	−2500.0	−1767.8	250.0	353.6	250.0	500.0	353.6	250.0
E3–D3½	25	0.0167	3	500	5	−0.7071	−1	−0.7071	−1767.8	−2500.0	−1767.8	250.0	353.6	250.0	500.0	353.6	250.0
E3½–D4	25	0.0167	3	500	5	−0.7071	−1	−0.7071	−1767.8	−2500.0	−1767.8	250.0	353.6	250.0	500.0	353.6	250.0
Totals									−45,352	−64,138	−33,631	14,919	7,023	4755	19,333	4,678	9010

Similarly, in longitudinal E,

$$S_0 = x/5.$$

The forces of 1 lb. in the diagonals of the side panels produce transverse forces of 0.6 lb. Hence in longitudinals A and B, the S_1 forces are, respectively:

$$\pm 0.6x/30 = \pm x/50.$$

The vertical component of the 1-lb. tension in the diagonals between A and B is 0.424 lb., producing S_1 forces of $\pm 0.424x/15 = \pm x/35.4$ in the keel members D and E, respectively. By similar procedure, simple expressions for the S_2 and S_3 forces in the redundant members and the keel are derived. The only catch to look out for is the S_3 force in longitudinal D; as a member of the truss between longitudinals C and D, its S_3 force is $-x/50$, but it also carries a force equal to $x/35.4$ as a member of the keel, so that $S_3 = -x/50 + x/35.4 = x/121$.

Since all four diagonals in any one row have the same tensions, they may be considered as one continuous member having a constant tension instead of four separate members.

The expressions for all the terms from S_0 to S_3 are tabulated in Table 6.

The calculation of terms of the form S_0S_1L/AE for the longitudinals is by a simple integration of the form

$$\int^{L} \frac{S_0S_1\,dx}{AE}$$

For example, in longitudinal D, taking the values of AE, S_0, and S_1 from Table 6,

$$\int_0^{L} \frac{S_0S_1\,dx}{AE} = 1.67\int_0^{L}\left(-\frac{x}{5}\right)\left(\frac{x}{35.4}\right)dx$$

$$= 1.67\int^{160}\frac{-x^2\,dx}{177}$$

$$= \frac{-1.67\times 160^3}{3\times 177} = -12{,}900.$$

The results of the integrations are listed in Table 6, and from the summations the following equations are obtained:

$$14{,}912X_1 + 7{,}076X_2 + 5{,}255X_3 = 46{,}308$$

$$7{,}076X_1 + 19{,}479X_2 + 5{,}254X_3 = 65{,}548$$

$$5{,}255X_1 + 5{,}254X_2 + 10{,}264X_3 = 37{,}195$$

TABLE 6 MODIFIED LEAST-WORK CALCULATION

Member	L	A	E	$U=\frac{1}{AE}$	S_0	S_1	S_2	S_3	$U\int S_0S_1dx$	$U\int S_0S_2dx$	$U\int S_0^2dx$	$U\int S_1S_2dx$	$U\int S_1^2dx$	$U\int S_2^2dx$	$U\int S_3^2dx$
A0–A4	160	0.25	1	4.00	...	$x/50$	...	...	...	...	...	...	2,185	...	...
B0–B4	160	0.25	1	4.00	...	$-x/50$	...	...	...	...	...	...	2,185	...	...
C0–C4	160	0.25	1	4.00	...	...	$-x/50$	$x/50$	...	...	...	...	...	...	...
D0–D4	160	0.60	1	1.67	$-x/5$	$x/35.4$	$x/25$	$x/121$	$-12,882$	$-18,241$	$-3,769$	532	2,576	3,648	754
E0–E4	160	0.40	1	2.50	$+x/5$	$-x/35.4$	$-x/25$	$-x/35.4$	$-19,284$	$-27,307$	$-19,284$	2,723	3,857	3,461	3,857
AB	200	0.0167	3	20.0	...	1	...	...	...	...	...	...	4,000	...	...
BC	200	0.0333	3	10.0	...	...	1	...	...	...	...	...	...	2,000	...
CD	200	0.0208	3	16.0	...	...	...	1	...	...	...	...	...	...	3,200
ED	200	0.0167	3	20.0	5	-0.7071	-1	-0.7071	$-14,142$	$-20,000$	$-14,142$	2,000	2,828	4,000	2,000
Totals									$-46,308$	$-65,548$	$-37,195$	5,255	7,076	19,479	10,264

The solution is:

$$X_1 = 1.364$$
$$X_2 = 2.414$$
$$X_3 = 1.690$$

The method recommended for solving least-work equations is explained in Appendix III and illustrated by solving these particular equations.

Discussion of the Results

Fig. 2 shows the results of six different methods of calculating the stresses in the given problem. These methods are:

(1) Shear theory
(2) Bending theory
(3) Average of shear and bending theories
(4) Inverse ratio
(5) True least work
(6) Modified least work.

The results of method (5) may be taken as the standard by which to judge the precision of the others.

There is a great difference between the results of the bending and shear methods; and as would be expected from the discussion of their theories, the truth of the matter lies between them. The simple average of the bending and shear methods gives quite good results, more nearly correct in fact, than the more laborious inverse-ratio method. This is, however, largely fortuitous and a result of the circumstance that the internal work in the structure happens to be about equally divided between the longitudinal and diagonal members.

The modified least-work method shows a very good order of precision indeed, in spite of the simplifying assumption made in regard to the transfer of force from the diagonal to the longitudinal members.

In the illustrative problem, the sizes of the members were selected with a view to bringing out the possible large discrepancy between the bending and shear theories. The least total internal work and probably the most efficient structure in regard to the ratio of strength to weight are obtained when the sizes of the members are so proportioned to each other that the bending and shear theories give concordant results; and also the stress calculations are very easily made in that case, since both of these simple methods are then correct. If it were not for the re-entrant keel, there would be no difficulty in proportioning the longitudinal and diagonal members to give the desired result. The keel makes this impossible, because, as may readily be seen, no matter what sizes of members are chosen, the bending theory will always show the re-entrant longitudinals to have stresses of the same sign as their neighbors, while the shear theory shows them having stresses of opposite signs. The inverse-ratio theory resembles the shear theory in this respect. A least-work calculation seems essential to determine the truth.

Stresses in the Second Main Frame Bay

At the cross-braced main frames, the stresses in the shear wires may be redistributed, even if no additional external load is imposed at the frame. The modified least-work method may readily be extended to the computation of the forces in the main-frame bays at successive intervals from the point of application of an external load. The second bay is assumed to be loaded by the end forces calculated at the fixed end of the first bay; although actually some discrepancy will result from the fact that when the second bay is introduced, the first bay no longer has a perfectly fixed end. The expressions to be integrated for the stresses in the longitudinals are the same functions as in the first bay, except that a constant term equal to the end load re-

ceived from the first bay is to be added. The modified least-work calculations for the second main frame bay are given in Table 7, from which the following equations are obtained:

$$\begin{aligned} 14{,}912X_1 + 7{,}076X_2 + 5{,}255X_3 &= 52{,}670 \\ 7{,}076X_1 + 19{,}479X_2 + 5{,}254X_3 &= 70{,}198 \\ 5{,}255X_1 + 5{,}254X_2 + 10{,}264X_3 &= 43{,}480 \end{aligned}$$

It may be noted that only the constant terms differ from the equations derived for the first main bay, so that in practice it is only necessary to compute the constant terms for successive bays, unless the sizes of the members are changed from bay to bay.

The solution of the new equations is:

$$\begin{aligned} X_1 &= 1.61 \\ X_2 &= 2.43 \\ X_3 &= 2.16 \end{aligned}$$

The stresses in all members of the second main bay are shown in Fig. 3. It is interesting to note that in the second bay the

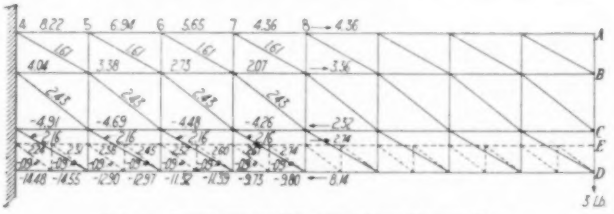


FIG. 3 STRESSES IN SECOND MAIN FRAME BAY

stresses are much closer to the bending than to the shear theory; and the stress in the keel diagonal is negative.

Appendix I

PROOF THAT DISTRIBUTION OF THE BENDING FORCES ACCORDING TO THE BENDING THEORY PRODUCES THE LEAST INTERNAL STRAIN ENERGY IN THE LONGITUDINAL MEMBERS

The fundamental assumptions of the ordinary theory of bending are that the stresses in the longitudinal elements of a body or structure subject to pure bending are proportional to their distances from the neutral axis, their cross-sectional areas, and moduli of elasticity. The validity of these assumptions will now be proved by the theorem of minimum strain energy; that is, distribution of the stresses according to the assumptions of the bending theory results in minimum internal work in the longitudinal members.

Let B and C be any two longitudinal members of a structure, or small elements of the cross-section of a body of length L subjected to a pure bending moment. Let M be the portion of the total bending moment resisted by B and C. Let A_B and A_c be the cross-sectional areas of B and C; E_B and E_c the moduli of elasticity of their materials; H_B and H_c their distances from the neutral axis; S_B and S_c their longitudinal stresses.

According to the bending theory,

$$\frac{S_B}{S_c} = \frac{A_B E_B H_B}{A_c E_c H_c}$$

It is required to prove this relation by the principle of least work or minimum strain energy.

Proof:

By moments

$$S_B H_B + S_C H_C = M$$

whence

$$S_B = \frac{M - S_c H_c}{H_B}$$

$$S_B^2 = \frac{M^2 - 2MS_c H_c + S_c^2 H_c^2}{H_B^2}$$

The internal work in the two members is given by

$$W = \frac{L}{2}\left(\frac{S_B^2}{E_B A_B} + \frac{S_c^2}{E_c A_c}\right)$$

$$= \frac{L}{2}\left(\frac{M^2 - 2MS_c H_c + S_c^2 H_c^2}{E_B A_B H_B^2} + \frac{S_c^2}{E_c A_c}\right)$$

$$\frac{dW}{dS_c} = L\left(\frac{-MH_c + H_c^2 S_c}{E_B A_B H_B^2} + \frac{S_c}{E_c A_c}\right)$$

When the strain energy is a minimum, $dW/dS_c = 0$, whence

$$\frac{-MH_c + H_c^2 S_c}{E_B A_B H_B^2} + \frac{S_c}{E_c A_c} = 0$$

$$S_c = \frac{ME_c A_c H_c}{E_c A_c H_c^2 + E_B A_B H_B^2}$$

Similarly,

$$S_B = \frac{ME_B A_B H_B}{E_B A_B H_B^2 + E_c A_c H_c^2}$$

and

$$\frac{S_B}{S_c} = \frac{E_B A_B H_B}{E_c A_c H_c}$$

which proves the assumptions of the bending theory.

Appendix II

PROOF THAT DISTRIBUTION OF THE SHEARING FORCES ACCORDING TO THE SHEAR THEORY PRODUCES THE LEAST INTERNAL ENERGY IN THE DIAGONAL SHEAR MEMBERS

The shear theory for the analysis of stresses in a redundant framework is based on the assumptions that when the shear between any two transverse sections of the structure is resisted by tension or compression in two or more diagonal members, the stress in these members is proportional to the sines of their angles of inclination to the direction of shear, to their cross-sectional areas and moduli of elasticity, and inversely proportional to their lengths. It will be proved that distribution of the stresses according to these assumptions results in the minimum internal work in the shear members.

Let B and C be any two diagonal shear members between two consecutive transverse frames. A transverse shearing force F is divided between these members. Let L_B and L_c be the lengths of B and C; A_B and A_c their cross-sectional areas; E_B and E_c their moduli of elasticity; θ_B and θ_c the transverse inclination of the panels containing B and C to the direction of shear; ϕ_B and ϕ_c the inclination of the wires to the longitudinals. The components of the wire tensions in the direction of the transverse shear are $S_B \sin\theta_B \sin\phi_B$ and $S_c \sin\theta_c \sin\phi_c$.

According to the shear theory:

$$\frac{S_B}{S_c} = \frac{L_c E_B A_B \sin\theta_B \sin\phi_B}{L_B E_c A_c \sin\theta_c \sin\phi_c}$$

It is required to prove this relation by the principle of least work.

Proof:

$$S_B \sin\theta_B \sin\phi_B + S_c \sin\theta_c \sin\phi_c = F$$

TABLE 7 SECOND MAIN-FRAME BAY BY MODIFIED LEAST-WORK METHOD

Member	L	A	E	$U = \frac{1}{AE}$	S_0	S_1	S_2	S_3	$U\int S_0S_1dx$	$U\int S_0S_2dx$	$U\int S_0S_3dx$
A4–A8.....	160	0.25	1	4.00	4.365	$x/50$	.....	.....	4,470	.....	.....
B4–B8.....	160	0.25	1	4.00	3.360	$-x/50$	$x/50$	.....	− 3,441	3,441	.....
C4–C8.....	160	0.25	1	4.00	−2.317	.....	$-x/50$	$x/50$	.....	2,373	− 2,373
D4–D8.....	160	0.60	1	1.67	$-8.141 - x/5$	$x/35.4$	$x/25$	$x/121$	−17,797	−25,200	− 5,207
E4–E8.....	160	0.40	1	2.50	$2.733 + x/5$	$-x/35.4$	$-x/25$	$-x/35.4$	−21,760	−30,812	−21,760
AB.....	200	0.0167	3	20.0	.....	1	.....	.....	.....	.....	.....
BC.....	200	0.0333	3	10.0	.....	.....	1	.....	.....	.....	.....
CD.....	200	0.0208	3	16.0	.....	.....	.....	1	.....	.....	.....
ED.....	200	0.0167	3	20.0	5	−0.7071	−1	−0.7071	−14,142	−20,000	−14,140
Totals.....									−52,670	−70,198	−43,480

$$S_B = \frac{F - S_c \sin \theta_c \sin \phi_c}{\sin \theta_B \sin \phi_B}$$

$$W = \frac{S_B^2 L_B}{2E_B A_B} + \frac{S_c^2 L_c}{2E_c A_c}$$

Substituting the value of S_B, differentiating, and equating to zero:

$$\frac{dW}{dS_c} = \frac{-FL_B \sin \theta_c \sin \phi_c + L_B S_c \sin^2 \theta_c \sin^2 \phi_c}{E_B A_B \sin^2 B \sin^2 B} + \frac{S_c L_c}{E_c A_c} = 0$$

$$S_c = \frac{FL_B E_c A_c \sin \theta_c \sin \phi_c}{E_c A_c L_B \sin^2 \theta_c \sin^2 \phi_c + E_B A_B L_c \sin^2 \theta_B \sin^2 \phi_B}$$

Similarly:

$$S_B = \frac{FL_c E_B A_B \sin \theta_B \sin \phi_B}{E_B A_B L_c \sin^2 \theta_B \sin^2 \phi_B + E_c A_c L_B \sin^2 \theta_c \sin^2 \phi_c}$$

$$\frac{S_B}{S_c} = \frac{L_c E_B A_B \sin \theta_B \sin \phi_B}{L_B E_c A_c \sin \theta_c \sin \phi_c}$$

which proves the assumptions of the shearing theory.

Appendix III

Method of Solving and Checking Least-Work Equations

The following method of solving simultaneous equations of the first degree was invented by the mathematician, Gauss, about a century ago. It was used in the stress analysis of the lock gates of the Panama Canal for solving least-work equations in which there were fourteen variables. It is described in a paper on "The Distribution of Stresses in Mitering Lock Gates, with Special Reference to the Gates on the Panama Canal," by Henry Goldmark, Mem. A.S.C.E., published in the Proceedings A.S.C.E., August, 1917. In spite of its ancient lineage, the method is not to be found in any structural textbook known to the author, and it appears to be practically unknown to aeronautic engineers. The method is so good and yet so little known and used that a repetition of Goldmark's account of it seems appropriate in this paper addressed to aeronautic engineers. By way of illustration, the solution of the three equations is given in detail.

The typical form for, say, three simultaneous least-work equations is:

$$ax + by + cz + m = 0 \quad [1]$$
$$bx + dy + ez + n = 0 \quad [2]$$
$$cx + ey + fz + p = 0 \quad [3]$$

It is required to find the values of x, y, and z.
From Equation [1],

$$x = -\frac{by + cz + m}{a}$$

and substituting in Equations [2] and [3],

$$\left(d - \frac{b}{a}b\right)y + \left(e - \frac{b}{a}c\right)z + \left(n - \frac{b}{a}m\right) = 0 \quad [4]$$

$$\left(e - \frac{c}{a}b\right)y + \left(f - \frac{c}{a}c\right)z + \left(p - \frac{c}{a}m\right) = 0 \quad [5]$$

which may be written:

$$d'y + e'z + n' = 0 \quad [4']$$
$$e'y + f'z + p' = 0 \quad [5']$$

These two equations are of the same form as the original three, but with x eliminated. By similar procedure y is eliminated from [4] and [5], and so on for any number of variables.

A check on the arithmetical work is obtained at each step by adding a summation term to each equation; for example, in Equations [1], [2], and [3], let

$$q = a + b + c + m$$
$$r = b + d + e + n$$
$$s = c + e + f + p$$

Similarly in Equations [4] and [5],

$$r' = d' + e' + n$$
$$s' = e' + f' + p'$$

But r' and s' may also be determined from r and s in the same way as the other terms of Equations [4] and [5] are found from the corresponding terms of the preceding set of equations. The check consists in computing the summation term for each new equation by the two methods and observing whether or not the results are the same. If there is a discrepancy, the work contains an arithmetical error.

In numerical examples the procedure is much less cumbersome than in the foregoing explanation where symbols are used throughout.

The coefficients of the variables, the constant terms, and the summation or check terms of the three equations are written out in tabular form in Table 8.

The first step in computing the terms of Equation [4] and [5] is to find b/a, that is 7076/14,912 = 0.47452.

All terms in Equation [1], except the coefficient of X_1, are then multiplied by 0.47452, giving the numbers 3358, 2494, 21,974, and 34,902, which are subtracted from the corresponding terms of Equation [2], giving the terms of Equation [4] in Table 8.

TABLE 8 SOLUTION OF LEAST-WORK EQUATIONS

No.	X_1	X_2	X_3	Constant	Check
1.....	14,912	7,076	5,255	46,308	73,551
2.....	7,076	19,479	5,254	65,548	97,357
3.....	5,255	5,254	10,264	37,195	57,968
4.....		16,121	2,760	43,574	62,455
5.....		2,760	8,412	20,876	32,048
6.....			7,939	13,416	21,355

$X_3 = 13,416/7,939 = 1.690$
$16,121X_2 = 43,574 - (1.690 \times 2,760) = 38,910$
$X_2 = 2.414$
$14,912X_1 = 46,308 - (2.414 \times 7,076) - (1.690 \times 5,255)$
$= 20,346$
$X_1 = 1.364$

Similarly, c/a = 5255/14,912 = 0.35240, by which the terms of Equation [1], except the coefficient of X_1, are multiplied, giving the numbers 2494, 1852, 16,319, and 25,919, which subtracted from the corresponding terms of Equation [3] give the terms of Equation [5].

The check is that the term calculated for the column headed "Check" in the new equations equals the sum of the other terms, and also the coefficient of X_2 in Equation [5] must be the same as the coefficient of X_3 in Equation [4].

Equation [6] is derived from Equations [4] and [5] in similar manner.

When there are only three variables, as in the present problem, the Gaussian method of solution has no very marked advantage over the more conventional tabular method in which the equations are successively divided by their first terms in order to obtain a new set of equations, all having the number "one" for the coefficient of the first term, so that the corresponding variable may be readily eliminated by adding or subtracting the equations in appropriate groups. The advantages of the Gaussian method become more pronounced as the number of variables increases.

Discussion

Ralph H. Upson.[2] There is not much to be said on Mr. Burgess' fine paper, as far as it goes. One can hardly argue with correct mathematics. The application of the method still leaves some room for discussion, however. Mr. Burgess has very well stated the principal physical limitations involved, but these in themselves should not be necessarily a bar to the more extended use of the least-work principle; for whenever design work can be founded upon fundamentally sound theory, the practical departure from the initially assumed ideal may usually be allowed for separately, provided the method itself is not too much involved in mathematical complications. The modified method proposed by Mr. Burgess is apparently well suited to structures with parallel longitudinals and as such is a step in the right direction; but unfortunately a modern airship hull or airplane fuselage is not only tapered but of continuously varying taper, for which Mr. Burgess' theory as it stands would still seem to be quite impractical. Is it too much to hope that Mr. Burgess himself will continue the good work which he has so ably begun toward making more general his method of taking the "work" out of least work?

[2] Aeromarine Klemin Corp., Keyport, N. J.

ALFRED S. NILES.[3] The problem discussed by the author is of much less direct interest to the airplane designer than to the airship designer. The former seldom has to design a structure of the degree of redundancy of the rigid airship framework, and so has less incentive to develop methods of handling a large number of redundancies. As a result he is likely to overlook methods of treating some of his design problems that would be very valuable to him. The author's modified method of least work appeals to the writer as pointing out the way to solve some design problems of considerable importance to the airplane designer.

The only application of his method illustrated by the author is the analysis of a cantilever space framework subjected to a single load at the free end. There appears to be no reason why it could not be applied to a similar framework subjected to a couple at the free end, or a combination of a single transverse load and a couple. This would make it apply to the analysis of a conventional fuselage in the maximum fin and rudder load condition.

By developing the method to provide for a load distributed along the span, it could be used for the analysis of a fabric-covered monoplane wing with metal truss spars and conventional drag trussing. The essential feature of the method is the treatment of the shear members as if they were continuous. In a monoplane with metal or plywood covering, or beam types of spars, the shear members are continuous, and it should be possible to extend the method to cover this type of construction also. It is probable that some difficulty might be encountered in extending the method to cover these additional cases, but the advantages to be gained by so doing would be very valuable to the designer.

As the author states, the use of the Gaussian method of solving simultaneous equations is of no particular advantage in solving only three equations, and he proposes it more for use when there is a large number of equations to be dealt with. The writer does not believe that the Gaussian method would ever result in less labor of correct computation than the ordinary system, no matter how many were the variables. Its real merit is in the fact that the steps can be checked as the computations proceed, errors are found almost as soon as they are made, and usually the engineer is saved from making calculations that are useless as they have been vitiated by a preceding error. When numerous simultaneous equations are involved this advantage in the checking becomes a most valuable feature, and the author is to be thanked for bringing the method to the notice of airplane designers.

[3] Professor of Aeronautic Engineering, Stanford University, Stanford University P.O., Cal. Mem. A.S.M.E.

AUTHOR'S CLOSURE

The author agrees with Mr. Upson and Mr. Niles that the particular problem taken as an example in this paper is much more simple than most of the space-framework problems likely to occur in practice. It does not seem possible to do nearly as much as one would wish toward taking the "work" out of least work. Nevertheless, the assumption of continuous, instead of discontinuous, transfer of the shear between the booms or longitudinals of a framed structure does seem to simplify the problem to a considerable extent. As Mr. Niles suggests, the assumption is particularly applicable to multi-spar airplane wings with metal or plywood covering.

The author does not agree with Mr. Upson that the proposed method is "quite impractical" with modern rigid airships because of the varying taper of the hull. For purposes of stress analysis, the hull is divided into bays or sections between consecutive main transverse frames; and the taper varies so little that no appreciable error results from considering the taper in these sections as continuous, or even as zero in that portion of the ship where the bending moment is large.

AER-51-23

Comparison of Reinforced-Shell and Steel-Tube Fuselage Construction

By W. C. CUMMING,[1] WICHITA, KANSAS

The author of this paper analyzes stresses in the reinforced-shell and steel-type fuselage of an airplane, and tells how various parts were constructed. Four types of construction were considered, and the reinforced-shell type resulted in a weight saving of 2 per cent.

In designing a high-performance sport and training airplane powered with from forty to sixty horsepower, it became necessary to refine the design to the utmost, both from the aerodynamic and structural standpoint. The aerodynamic design indicated an airplane of about the following characteristics: Wing area 175 sq. ft. with a tapered wing of 37-ft. span, root cord 72 in., and tip cord 43 in. The weight light was not to exceed 725 lb. Due to the high-aspect ratio, it was decided to make a wing in three sections, the two wings being of 15-ft. span with a 7-ft. center section. A normal two-spar construction was decided upon in the wing structure with box spars and built-up truss-type ribs. That gave a wing structure which was very efficient aerodynamically, but relatively heavy, and made it necessary to economize to the utmost on the weight of the rest of the airplane.

When a preliminary stress analysis was run, it was found that everyone of the members from the rear cockpit to the tail post, as designed according to the usual methods, was smaller than the minimum-design size recommended by the Department of Commerce. For example, the lower longeron which was critical in a maximum-stabilizer and elevator-load condition, had an indicated size of $^1/_2$ in. $\times$ 0.035 in., to the last two bays where the indicated size dropped to $^3/_8$ in. $\times$ 0.035 in. As these sizes were entirely too small to withstand the loads imposed by the shrinking of the fabric covering, and also those occuring in handling, it seemed advisable to make a survey to determine if other methods of construction might not be more suitable in this particular instance.

In view of these facts, four methods of construction were considered. (1) Steel tubing, (2) reinforced shell in metal, (3) reinforced shell using plywood, and (4) monocoque. Of course the tubing construction offers some disadvantages as mentioned. The all-metal construction was eliminated, due to difficulties experienced in experimental construction, and it was decided that a monocoque would be too difficult for field repairs in case a design was carried into production. The fact that two very successful English light planes, the Avro Avian and the D. H. Moth, used reinforced shell-type construction, and also that it had been more or less successfully used for some years in the DeHavilland mail planes operated by the government, seemed to point in favor of this type. It seemed advisable to carry out the construction from the rear cockpit forward in the usual steel tubing, not only because of the loads in this portion were of such a nature as to be carried economically by steel tubing, but also on account of added protection afforded the occupants in case of a crash. This would also allow the construction of the seven-foot center section of alloy-steel tubing as an integral portion of the front-section fuselage.

It was decided to design and construct an experimental fuselage section with the reinforced-shell type of plywood and spruce, and two methods of design were considered. These were as follows:

1. The plywood could be assumed to carry the entire load, the area of the truss members cut by any section being included with that of the plywood in calculating the properties of the section.

2. The truss framework could be assumed to carry the entire load, the plywood skin providing complete lateral support in its own plane only, to the truss members.

A preliminary estimate indicated that the second method was more conservative, and due to the fact that this type of construction was an experiment, this method was used.

The same preliminary stress analysis, which was used in the design of the steel-tubing fuselage, was used in the design of this new section, and no allowance was made for the change in the weight of the members or of their weight distribution. Since the change in the weight distribution was very small, and most of the members were critical in the maximum-stabilizer and elevator-load condition, this assumption effects the results very slightly. The design of the individual members of the truss was therefore reduced to taking the design load from the stress analysis, and in the case of compressive stress, designing them from a standard-column formula using the radius of gyration in a plane parallel to the plywood sidewall. In the case of the lower longerons which were supported in two directions, this was reduced to a determination of area required from the allowable compressive stress in spruce.

This portion of fuselage was designed and constructed, and was identical in its external dimensions to the steel-tubing design with the following exceptions. It was thought that due to the difficulty in attaching the fin, as designed for the tubing job, to the plywood construction, it would be advantageous to construct a cantilever fin as an integral part of the fuselage. For the same reason, an integral head-rest fairing was added.

Since this fuselage was not a production job, no attempt will be made to compare it with the steel-tubing production as to the amount of labor required. However, it was constructed from shop drawings by two men in approximately four days, which included application of plywood covering to the sides, turtledeck, and fin. The procedure in construction used was about as follows: All the members were cut to the size indicated in the drawing. The sides were built separately, fastening the web members to the longeron with small plywood gusset plates which were applied on the side, which afterward would be the inside of the fuselage. These two sides were then fastened together by means of the top- and bottom-web members using the same size gussets. The internal-fin structure was then added as were also the turtle-deck members. All of this, of course, was done in a jig similar to that used in the construction of a tubing fuselage with the exception that the jig was made very light with no provision to keep the members from warping as is necessary in a welding jig. After the framework was completed, it was checked for trueness and accuracy before the plywood was applied. The turtledeck, having been designed to a true conic section throughout its length, with the exception of a slight compound curve to the side, presented no especial difficulty in molding the plywood on it. The sides and bottom were covered with $^1/_{16}$-in. three-ply birch attached with water-proof glue and small nails. The turtledeck and fin were covered with $^1/_{25}$-in. three-ply birch using

[1] The Watkins Aircraft Co., Wichita, Kan.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

the same method of attachment. After the plywood was applied it was sprayed with three coats inside and two coats outside with the best spar varnish, and the outside was finished in colored lacquer.

This portion of the fuselage was attached to the steel forward portion by means of five $^{3}/_{16}$-in. nickel-steel bolts through the longerons which were blocked up to compensate for the drilling of these holes. These bolts passed through a $^{1}/_{16}$-in. alloy gusset plate which was slotted into the longeron, and welded to the steel portion. The design load of this section was 1810 lb., and the section in static tests resisted an average of 2900 lb. Two blocks were attached to the truss members in the tail end by means of $^{1}/_{8}$-in. birch plywood gussets for the attachment of a spring-leaf tail skid. The stabilizer was bolted onto two specially built turn buckles which were passed clear through the fuselage and screwed up tight. The compression of these turn buckles was carried by a small steel "U" section which enclosed them.

There follows an analysis of the comparative weights of the steel-tubing fuselage section and reinforced-shell fuselage section. These are especially interesting in view of the fact that both are designed from the same stress analysis. The members of the steel fuselage are all of the minimum size recommended by the Department of Commerce where the size indicated by the design loads was below the minimum. The four longerons are $^{3}/_{4}$ in. $\times$ 0.035 in. and the web members are $^{1}/_{2}$ in. $\times$ 0.035 in. The weight of the tubing is calculated accurately from the weight per 100 in. of tubing as given in "Niles Airplane Design."

Weights in Pounds of Steel Tubing Fuselage

Side trusses	19.00
Top and bottom trusses	7.70
Transverse members	2.60
Turtle-deck members	11.94
Welding material	2.00
Covering and finish	6.76
Total	50.00

While these weights are all calculated, they are conservative. The tubing fuselage section has not been built to date as construction is put off to find out how the plywood job will act in actual service. The fuselage section, actually built as described, weighed 41 lb. with finish, and subtracting three pounds (estimated) for the cantilever-fin construction in order to compare it with the steel tubing, would make the weight of the structure 38 lb. or a saving of 12 lb. over the steel tubing construction. As this is approximately 2 per cent of the weight of the airplane, it is considered well worth saving.

Problems in Flying

By J. H. DOOLITTLE,[1] NEW YORK, N. Y.

In this paper the author discusses many things that are now receiving attention to make flying safer. More and better airports are needed, and a low landing speed is essential. More attention is being given to airplane construction. The cockpit should be strong to protect the pilot in case of a crash, and there should be sufficient material between the pilot and ground to absorb the shock.

Work being done by the Weather Bureau in notifying pilots of weather conditions along the route is of great value. More frequent points for giving information will add to the service. Recent developments provide a satisfactory instrument for measuring altitude. More improvements are needed in the compass for blind flying. Flying in fog makes it necessary to direct pilots to landing ports, and to make it possible to land without seeing the ground. Much work is being done to solve these problems.

SAFETY in flight is the principal problem in aviation. Much has been done in recent years to this end, but there remains much yet to be done. More and better airports are needed. Many of our terminal airports are not as safe as it is possible to make them. They can be improved by increasing their size and by removing obstacles in the vicinity of the field. The ideal airway is one so studded with emergency landing fields that a pilot may land safely at any moment, should his motor fail or inclement weather make it advisable. Present-day motors are far superior to those of even a few years back, but power-plant reliability must be brought to such a point that motor failure will never occur where proper care is exercised in maintenance.

Sufficient importance is not given to the value of low landing speed even by pilots. If a pilot flies the same type of plane every day, he soon becomes accustomed to its landing characteristics, and is satisfied with them. He learns how large a landing field is required, and does not attempt to go into a smaller one. Still, when a crisis comes, one cannot get away from the fact that impact is a function of V^2 (velocity squared), and an airplane landing at 60 m.p.h. hits any obstacle in its path just four times as hard as an airplane landing at 30 m.p.h. The damage to plane and occupants would probably be severe or fatal in the first case and unimportant in the latter.

A feature to which insufficient attention is given is designing and constructing an airplane so that it crashes well. This sounds odd, but many lives have been saved because the cockpit was strong and there was sufficient material between the pilot and the ground to absorb the shock of crashing. This permits gradual, uniform deceleration between the instant of impact and the pilot's eventual coming to rest.

It is obviously desirable that airplanes be made as safe and as easy to fly as possible, but it is equally obvious that when in the near future the air becomes literally full of airplanes which will fly themselves, for the safety of one another the pilots should be able to manage them properly under any and all conditions. This will necessitate a very careful physical examination and adequate training before unrestricted solo flight.

A large percentage of the automobilists driving along the road should not be allowed to operate a car under present-day congested-traffic conditions. Take, for example, the one who always drives down the center of the road. He is not necessarily a "road hog," but stays in the center because his judgment of speed and distance is so poor that he is afraid to drive on his own side for fear of running off. He would rather depend on another's judgment than his own. How much more important that such a person should not be allowed to fly when the primary prerequisites for a pilot are accurate judgment of speed and distance, and quick, correct reactions, and that even with the present careful supervision over 80 per cent of the fatal accidents in aviation are due directly or indirectly to piloting errors.

From the point of view of congested traffic, aviation has one great advantage and one great disadvantage over any other means of transportation. The advantage is another dimension in which to dodge and the disadvantage is that a flier, to remain in the air, must "carry on." He cannot "lay to" when conditions become unfavorable. To stop is to fall. Thus, to fly requires more and quicker judgment than to drive a car.

The work that is being done by the Weather Bureau in notifying pilots of the weather along their routes is of inestimable value, but there is still room for considerable improvement. The two principal faults are:

1 The points from which weather information is available are too widely separated, so it is impossible to get more than a rough average estimate of the weather conditions along the route.

2 The weather information is now available for the past and present, and weather conditions for any time in the future cannot be predicted accurately. The pilot is not primarily interested in the present weather at his objective, but he wants to know what the weather will be several hours later upon his arrival. The hourly radio reports given out by the airways division of the Department of Commerce are a great step forward.

Our training in the past has been such that a pilot who used his flight instruments was termed "mechanical" and often thrown out of school. This condition should no longer exist. It has been proved that unless he can see some reference point a human being is not equipped with the senses necessary to permit him to maintain a level, directed course. He can do this only with the aid of suitable instruments.

For blind flying three primary instruments are required: first, something to give the attitude of the plane with respect to the horizon; second, something to give the direction of flight; and third, something to indicate whether or not the attitude and direction are coordinated properly. In the past it has been found that flight-attitude instruments operate satisfactorily only when weather conditions are favorable. For example, a pilot can fly blind for a considerable period of time if the air is smooth, but in bumpy air it is practically impossible to maintain a desired course at any given altitude. Furthermore, the pilot requires mental effort to utilize these instruments, and after flying blind for some time experiences fatigue.

There has recently been developed in this country the first satisfactory instrument for indicating the attitude of the airplane. This instrument consists of a single stabilized, nonpendulous gyroscope mounted with the axis vertical and operating, through a simple mechanism, a line which always remains parallel to the earth's surface. This line represents a "piece

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

of the horizon." Bank is shown by the apparent rotation of the line, and pitch by an up-and-down motion.

As the gyro in this instrument is not pendulous it is not affected by rough air, and as the gyro is stabilized it does not precess but remains always with its axis normal to the earth's surface. It is unaffected by a steep, continued bank to right or left, and after leveling out, the hand again immediately indicates level flight.

The magnetic compass is satisfactory for navigation in smooth air, but in very rough air, especially when flying blind, it becomes practically useless. This is true of any compass which operates through the effect of the earth's magnetic field. One confusing thing with the magnetic compass is that, due to the dip of the earth's magnetic field, when flying north if a turn is started to right or left the compass indicates that the pilot has started turning in the opposite direction. This is particularly annoying when flying blind. The tendency, inherent in all magnetic compasses except the aperiodic, is to swing past the course being flown, and require a considerable time to settle. This is bad.

An electric compass employing the earth's field and a heavy, pendulous, rotating armature will indicate a turn when the airplane pitches due to the precession of the gyroscope formed by the rotating part. The ultimate solution lies in a light, compact gyroscopic compass, and the immediate solution for blind flying lies in a turn indicator which mounts a compass card and which can be "indexed" or set practically on "north" at will by checking back from the magnetic compass. This will hold for some hours, depending upon the power and orientation of the gyroscopic element, and the pilot flying blind can immediately read his course, keep himself from deviating from it, and can at will change his course any desired amount by adding or subtracting the proper number of degrees, and turning until his lubber's line rests on the reading for the new course.

This latter maneuver is very difficult to execute at present, and can be done only by trial and error, and in very bumpy air only with the greatest skill and patience. This maneuver is of especial importance in landing blind as the beacon is usually some distance from the field and the pilot wishes, usually, to change his course an exact amount when coming in on the beam and getting the indication that he is exactly over the beacon.

The last fundamental instrument is merely the old level bubble or ball which tells the pilot if he is employing the proper bank in any turn, or, from the pilot's viewpoint, which rudder he should use to correct it if he is not.

The author has purposely avoided the problem of ice formation on the airplane as he feels that the excellent research work now being done by Dr. MacAdie at Blue Hill Observatory and the Weather Bureau with the cooperation of the Air Mail Services should soon give the contributory causes. It is then but a step to develop instruments which will indicate the approach of these conditions, and tell the pilot, provided he is equipped for indefinite blind flying, at what altitude or what course to fly in order to avoid them.

Another enemy to be overcome is fog. The problem of fog flying resolves itself into the ability to carry on flying operations under any and all weather conditions. That is, to take off from any point, arrive safely, and land at any other desired point regardless of the weather. One needs nothing more than the instruments at present developed in order to take off and fly indefinitely in fog no matter how thick it may be, but landing without being able to see the ground is a more difficult problem.

After taking off it is necessary that the pilot be directed to the general vicinity of the desired port. This can, at present, be done with either the visual or aural-type radio beacon. The aural type has the advantage of a somewhat greater range. The visual type has the advantages of greater ease in reading, less fatigue to the pilot, and of being affected less from interference with other stations operating on approximately the same wave length. This latter point is especially important near the seacoast where great difficulty is experienced in following the aural beacon due to interference from marine beacons.

It is possible, with either type of beacon, for the airplane to be led to a point exactly over the beacon tower, and for the pilot to know when he has arrived at that point. From here or from some other point along the beam it is necessary to direct the pilot to the exact point where he desires to land. For this a small localizer beacon or lead-in cable may be employed.

If we assume the worst conceivable case—when it is impossible to see the ground until the plane strikes it—a knowledge of the approximate altitude above the ground is required in order to avoid over- or under-shooting the field.

Several instruments to give true altitude are now in process of development, but no completely satisfactory one is at present available for general use. The simplest means of getting true altitude above the earth at any point now is by a combination of the aneroid or barometric altimeter and radio. If the ground station at the point of landing radios the ground pressure up to the pilot, he can set this off, and then he has true altitude. A rate of change of ground pressure corresponding to five feet per minute on that method is not uncommon, especially near storm centers, so the pilot must be acquainted with the exact ground pressure again just before landing. He can thus square off at a given altitude, at a predetermined point, and start his glide into the field following the localizer beam or lead-in cable. If a correction is made for the wind and he comes in at the proper air speed and rate of descent, he will arrive at exactly the desired point on the ground.

Recent tests have shown that in employing the method outlined, the pilot does not have to see the ground in order to land, but can make a good landing absolutely blind, provided the plane is held level in a steady glide at an airspeed of not over 20 per cent in excess of the stalling speed, and at a rate of descent of not over 500 ft. per min. In order that the angle of glide be sufficiently flat it is necessary to come in with the throttle cracked and the motor "reeving up" slightly. A little practice will give the best air speed and engine revolutions at which to come in. Care must be exercised to see that the glide into the field is straight, and started at an altitude sufficient to let conditions become steady before the airplane lands. The primary requisite is a good oleo landing gear. This maneuver cannot be executed with rubber shock absorbers as the plane will either bounce back into the air or the landing gear will carry away.

We have now, either in our hands or in process of construction, the instruments and equipment required to fly in fog and to land blind on a prepared airdrome. It is only necessary, before actual fog operation can be carried on, to coordinate these facilities and render them infallible.

Discussion

P. V. H. Weems.[2] In addition to the principal problem of safety, we have the important problem of flying on schedule. Our air-mail pilots have done wonders in going their appointed rounds regardless of adverse flying conditions. Much remains to be done to permit dependable scheduled flying over many parts of the country.

The importance of an instrument to indicate the attitude of the plane cannot be overestimated for blind flying. As Lieu-

[2] Lieutenant Commander, U. S. Navy.

tenant Doolittle brings out, it is when we need the aid of instruments for blind flying most that these instruments "act up" and are likely not to function properly. Hence the need is for exceptionally rugged instruments. Assuming that the instruments will withstand the conditions imposed in blind flying, an important point is to have the pilots so trained in good weather that they may react properly when the emergency occurs. I am of the opinion that the relief pilot in large passenger planes does not do as much training work as he should. He has an exceptionally fine opportunity to test instruments and to do as much navigation as facilities permit. Intensive training on the part of relief pilots would be a definite step toward greater safety and efficiency in air navigation.

Referring to the stabilized, non-pendulous gyroscope for indicating the attitude of the plane, there is a new style gyro mounted as described, but showing directly the attitude of the plane and the angle of bank, or angle of attack. The instrument was invented by Lieutenant Burgess of the Army, working with Mr. Schenck, a gyroscope expert who gained his experience in German Zeppelins and in German submarines. The instrument is called a turn-and-bank indicator, but it really shows the exact attitude of the plane to the earth. It is driven by air in the same way a windshield wiper is operated, but through a small turbine. It may be stopped and started in the air, and reset by leveling off by compass and an ordinary turn-and-bank indicator. Getting away from the deceptive bubble and the vagaries of the magnetic compass in blind flying is a big step in making blind flying practicable.

Another means for increasing the safety and efficiency flight lies in the use of practical celestial navigation. A skilled navigator, using present methods of celestial navigation, can take off through the fog, fly around the earth, and return within ten miles of the take-off point without once seeing the earth. For long hops over land or sea, celestial navigation affords a ready means of navigation independent of objects on the earth, and solves the problem of blind flying by keeping above the fog or clouds where the aircraft has sufficient ceiling.

With present equipment and methods (the latest), a position may be worked in a minute when stars are available, one subtraction being the only figuring necessary. During the day, when flying on schedule, a "line of position," at some point on which the observer's position lies, may be determined continuously, i.e., as quickly as the altitude of the sun or moon may be observed and plotted on a curve drawn on cross-section paper, say every 40 seconds, and this without so much as writing a figure! This method is possible by the precomputation of the sextant altitude which is suitably graphed on cross-section paper against time. All the computations for scheduled flying may be done before taking off. If the schedule is not followed in the air, considerable work over the older methods may be saved by computing the curve while in flight. Once the curve of altitudes is plotted, the observed altitudes and corresponding times give at once practically all the information which may be obtained from careful solutions by logarithms.

The promising feature about the latest methods and technique of celestial navigation is that a position may be determined easier than the drift is determined by the usual methods, and once the position is determined, the drift, course, and speed over the ground are given at once. It appears that the centuries of development of marine celestial navigation have in some respects been a disadvantage to the airman due to the fact that the extreme accuracy demanded in marine navigation has imposed somewhat difficult solutions which have always appeared a mystery to all but marine navigators. It might be added, however, that methods of marine navigation have already profited greatly by the methods developed to speed up air navigation.

With a suitable gyroscope for showing the attitude of the plane, with a dependable gyro compass, with efficient radio, and finally with a good sextant and other equipment needed for celestial navigation, a plane should be as dependable for following its schedule as an ocean liner so far as the navigation problem is concerned.

J. E. Gingrich.[3] Lieutenant Doolittle's excellent paper treats the subject from an experienced and understanding viewpoint, that of the flier. His are the troubles of the operator of the plane. How many airports are there along the route to be traveled? Does the plane have a sufficiently low landing speed to use a small landing field? If the plane crashes, is it so designed and constructed as to offer maximum chance of safety? In other words, he sees the problem from the viewpoint of immediate safety to the plane and pilot; and this, no doubt, covers the major part of the problems in aviation.

However, there is possibly another problem which the flier, when flying blind, and in making long distance flights, must face; that of fixing his position in the air with reference to the earth.

In the past, with few exceptions, long-distance flights have been attempted, flying by dead reckoning. Due to the fluidity and the variability of the medium in which the plane travels, and due to the unreliability of the magnetic compass, the plane may be swept hundreds of miles from its proper track; and the flight passes into the long list of unsolved mysteries.

Definite steps have recently been made toward facilitating the proper avigation of the plane by means of celo-avigation. The term avigation is derived from avis, a bird, and agere, to lead. Celo-avigation deals with the fixing of the plane's position in the air by means of astronomical observations.

The second-setting watch allows the second dial to be turned so that the watch may be set to keep exact Greenwich civil time. Sidereal watches with the movable second dial allow the keeping of exact Greenwich sidereal time. This solves about half the problem of working a sight.

The new Hydrographic Office navigation tables HO-208 conceived by Lieutenant Commander Dreisonstok, U. S. Navy, and the new line of position tables by Lieutenant Commander P. V. H. Weems, U. S. Navy, reduce to a minimum the labor of fixing the plane's position.

The problem of teaching the aviator sufficient spherical trigonometry to enable him properly to use the means of celo-avigation which he has at hand must be solved by proper instruction at the aviation school.

Sextants of the bubble type are in the process of development. This type of sextant was partially perfected, and was used by Commander Byrd on his North Pole flight. The present type of bubble sextant should be accurate to within 15 minutes of arc, or 15 miles. The further increase in accuracy of the sextant is still a problem.

The new sextant camera invented by Lieutenant Commander Pierce, U. S. Navy, is designed to photograph the time and the position of the sun or star, allowing the aviator to calculate his position from the picture. The further adaptation and use of this new instrument opens up vast possibilities in celestial avigation.

There are mechanical features of the celo-avigation problem which must be perfected, such as proper charts which will not tear or be ruined by the weather, proper map holders, and proper avigational instruments for rapid work while in the air. Above all, however, on long flights, the aviator must learn to understand and use the facilities for celo-avigation which are at present available.

[3] Lieutenant, U. S. Navy.

V. I. ZELOV.[4] Lieutenant Doolittle's paper is a valuable contribution to the rather scarce information available on the present status of flight instruments. The paper contains a general description of a number of new devices the development of which is apparently successfully approaching the desired results. But, as in most cases, the value is gaged upon the final shape of the instruments, and the discussion should be postponed until the new devices are completed. The principles upon which engineers are working are correct. It is hoped that instruments which are dependable, not prohibitive in price, and not too complicated will be available in the very near future.

It is extremely gratifying to the engineer in general and to the instrument designer and manufacturer in particular that a pilot rapidly and decidedly begins to appreciate the value of aircraft instruments. We do not have to remember far back when a pilot believing in his instruments was considered "scientific" or queer by his fellow fliers. The recent past has taught them differently; they now want better and more instruments. It is natural, since humans do not possess air-feel and since the same air either carries them on or brings them down, that one has to know when it is with or against him.

The accurately installed and maintained airspeed indicator shows not only the speed of flight but foretells stall and indicates proper landing speed. The accurate barometric altimeter should receive its share of attention and be properly read so as to warn of the approach to the ground when it is not visible. The magnetic compass is fully reliable if properly compensated and designed, so as to be free from spins and deviations. Rate-of-climb and turn indicators supplant the lacking "feels" and supply an ease in handling aircraft when erratic sensations begin to confuse usually cool-headed and experienced fliers.

It is apparent that a number of new instruments have to be developed to take care of the requisites of successful flight which do not exist or have not been experienced in our earlier modes of transportation. At the same time, the instrument board should not be studded with a multitude of indicators, which would certainly discourage the pilot. A number of instruments should be conveniently combined, their dials properly limited to operating zones and having unnecessary ranges of indication so placed as not to confuse the observer and detract his attention from more important observations.

Upon careful study of particular aircraft, the instruments should be set and their location adjusted so as to render them as easy as possible to read, with danger points well indicated.

Instrument knowledge can be easily imparted to the future pilot, prior to his exposure to the dangers of an enthusiastic beginner, and schools of blind ground "flying" will teach him to follow his indicators, which have to be foolproof and accurate. Movement of controls can be arranged with properly set limiting signals or stops interrelated to aerodynamic properties of definite aircraft. Wing-pressure indicators, center of pressure locators, stress recorders and other devices will prevent the abuse of a plane from a structural or aerodynamic point of view, thus eliminating a great number of so-called accident failures. With the elimination of fire hazards, proper knowledge of gliding flight, supplemented by a suitable system of airports and emergency flying fields, aviation will be made a sound undertaking and be assured as a successful method of transportation.

[4] Aircraft Control Corp., Philadelphia, Pa.

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Air Navigation

An Estimate of Progress Made, and a Discussion of the Minimum of Theoretical Knowledge Required by Pilots

By GEORGE R. FAIRLAMB, JR.,[1] RICHMOND, VIRGINIA

THE term "navigation" embraces a variety of meanings. Employed not only in the scientific sense but also in the vernacular, it appears reasonable at the outset to define it and then to confine it. Used scientifically, navigation denotes the science which enables one to conduct a craft from point to point such that its position at any time during the journey is known. In a wider sense it includes the ability to go with reasonable assurance of safety under a variety of conditions. The problems of safety in flight under conditions of low visibility and with all auxiliary aids to navigation might well be included in a discussion of aerial navigation, but they constitute special problems in themselves and are so being treated. As here discussed, navigation will be limited for the most part to its meaning of determination of position in flight. Development of some of the means and methods to this end will be pointed out, and an endeavor made to classify the nature and amount of navigation knowledge needed by airplane pilots.

Air Navigation Abreast of Other Branches of Aeronautics

There is no doubt that air navigation has kept pace with progress made in other branches of aeronautics. The marking of the airways with beacons, the collection and dissemination of hourly weather reports direct to terminals and to radio-equipped planes in flight, and the successful application of the radio beacon to mark the course on airways are known to every one who is familiar with the aeronautical industry. Assuming that instrument flying will eliminate the danger of fog and low visibility, the radio beacon makes navigation on the fixed routes a foolproof and automatic affair, while development of further aids in the nature of high-sensitivity altimeters and leader cables are pointing the way to landing on fog-covered airdromes. The capacity altimeter, designed for high sensitivity, uses the principle of varying air dielectric of a condenser with altitude, the earth and a portion of the airplane being used as the plates. It is possible also that a radio altimeter, similar in principle to the marine sonic depth finder, may prove successful in this field. This instrument would measure the altitude in terms of time taken to send a radio signal and receive the echo signal reflected back by the earth. The leader cable is an underground wire extending from the airdrome. It transmits signals to guide the plane to the landing field, after which landing must be effected by use of the sensitive altimeter.

Publication of new inland-airway maps proceeds by the Department of Commerce, while the Navy Department is publishing airway coastal maps. These, when completed, will cover not only our own coasts but also western Canada, Mexico, and the Central American coasts to the Canal Zone, not to mention the West Indian Islands from Cuba to Trinidad, and the South American coast westward from thence to Colon, thus completely girdling the Caribbean Sea. Each month it publishes the "Upper Air Pilot Chart," giving meteorological conditions that may be expected over the Pacific and Atlantic oceans, and showing the best routes for pilots who have transoceanic flight ambitions. On the back of these charts is printed valuable information of a general navigational nature—varying all the way from curves showing the shift of winds at altitude to sample solutions of problems in aerial dead reckoning and celestial navigation. It also publishes "Notices to Aviators" which like the charts are identical in nature with its similar publications for mariners. It has completed the drafting of special mercator navigational charts from the equator to 48 deg. latitude and has developed the "air plotter" to facilitate the use of these charts. The air plotter is very simple, being a clear piece of square celluloid with compass bearings and distances etched upon it. Each 10-deg. bearing is marked by a line radiating from the center, while intermediate degrees are indicated on the edge. Concentric circles mark distances in increments of 10 miles. It is designed to do the work of parallel rulers and dividers, thus facilitating plotting and laying down or picking off courses from a chart.

Developments in Air Sextants and in Celo-Navigation Methods

The most interesting developments, however, have been in air sextants and in methods used in celo-navigation. The first air sextants were converted marine sextants, adapted for air use by attaching just below the horizon mirror a spirit level and a mirror to reflect the bubble. The image of the spirit level provided the aviator with a much needed artificial horizon.

Practically all later sextants have been refinements of this principle, in which a spirit level informs the navigator when the sextant is held in the horizontal plane, the altitude of the sun or star then being measured by bringing its image down to the image of the bubble in the level. Vibration, rolling, and pitching of an aircraft make it extremely difficult to hold a sextant steady. The result is that the bubble is rarely ever quiescent in the center of the level, as it must be to obtain an accurate sight. New designs have endeavored to dampen the oscillations of the bubble; to provide a more compact and easily handled instrument; to give the navigator a mechanical advantage of position in holding the sextant so that both sun and bubble may be sighted through an optical system by looking down, instead of the customary method of looking out directly toward the sun. It has always been customary to hold the marine sextant with the right hand. The newest air sextants are being made to hold in the left hand, so that the right hand may be free to note down the time and altitude. Here, as elsewhere, necessities of convenience and speed for the air navigator have greatly accelerated the development of equipment and methods which are proving a boon also to their brother navigators of the sea. One of the most interesting sextants is a photographic instrument developed by Commander Pierce of the Airship *Los Angeles*. This sextant avoids the oscillating movement of the bubble as follows: A photograph of the sun and the bubble is made simultaneously after the image of the sun has been brought down to the *vicinity* of the bubble. The photograph is taken at a moment when the bubble is centered. The film is self-developing in about a minute's time, and a small scale is used to measure the distance between the sun and the bubble. This distance in arc measure is applied to the observed altitude to get the corrected

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

altitude. Commander Byrd took the first of these instruments on his polar expedition, and the second one is now being tested at Lakehurst. Taking celestial observations in an airplane is a difficult thing at best and requires much practice. When the air is rough, an error of forty or fifty miles in the observation is not uncommon. When the air conditions are good, the error of a skilful observer using a good instrument may be expected to average from 10 to 15 miles. But the air sextants of today are vastly superior to those of yesterday, and improvement continues.

Time is a vital element in celestial navigation, and there are three kinds of it to worry the navigator, namely, apparent solar, mean solar, and sidereal or star time. The declination of the sun, the equation of time, and the declinations and right ascensions of the stars are tabulated against Greenwich mean time, while the navigator solves the astronomical triangle for the moment of the apparent sun time or sidereal star time of his observation. To convert one time to another lengthens the solution and introduces additional probability of error. Hence it always has appealed to air navigators to have several chronometers—one carrying Greenwich mean time, one carrying Greenwich apparent time, and one carrying Greenwich sidereal time. No one has as yet built a watch, except on paper, to keep apparent time, yet a second-setting watch is now being made at the Naval Observatory which does almost as well. This watch, invented by Lieutenant-Commander Weems, has a ratchet which permits it to be set by the daily radio time signal to the exact second of time, only instead of setting it to the mean time a correction (the equation of time) is applied, and it then shows apparent time for the succeeding twenty-four hours. Incidentally, by resetting it each day the usual chronometer error is avoided. The time of an observation is noted from this watch or from the sidereal watch in case of a star, and several tedious arithmetical steps are saved in the solution for the position line.

Celo-Navigation and How It Is Done

Many have doubtless desired to know just how navigating by the heavenly bodies is done. Reduced to simplest terms and principles, celo-navigation enables the observer to obtain a locus of points—known as the Sumner line of position—upon which he is at the moment of observation. Intersection of two (or more) lines in a point, obtained by observations of two (or more) bodies, establishes a definite position called a "fix." The principle on which this is based is a pretty one, yet it has been known and practiced for less than a hundred years, having been accidentally discovered in 1837 by Capt. Thomas Sumner, an American Shipmaster. It depends upon the fact that for any given instant of time there is a series of positions on the earth at which a celestial body appears above the horizon at the same altitude. These positions lie on a circle described on the earth, the center being immediately below the body where the altitude is 90 deg., and the radius of the circle being in effect the given altitude. Such circles are called circles of equal altitude, and are of course nearly infinite in number. Looking at it another way, we may place the observer on one of these circles at an assumed position, denoted by latitude and longitude, and then calculate that at a given instant of time a particular body must have a certain definite altitude when viewed from that point. If at this instant the observer measures the actual altitude of the body by a sextant and it is different from the computed altitude, he is either inside the circle or outside according as the observed altitude is greater or less than the one computed. Solution of the astronomical triangle determines in effect, for the instant of observation, the direction of a very small portion of one of these circles and its distance from the center in terms of altitude of the observed body above the true horizon. The observer compares the actual altitude of the body as measured by a sextant with this computed altitude, based on a definite *assumed* position, and thus establishes a line of position coincident with or parallel to the computed portion of the circle of equal altitude that passes through the assumed position.

There has been continuous progress in the methods of solving for the line of position. The old methods required from fifteen to twenty minutes for solution, and involved many chances of faulty arithmetic, or mistakes in picking out the numerous arguments from the various tables. To obviate this labor and to obtain speed, air navigators resorted to many ingenious devices, but all involved considerable pre-flight labor in preparing specially computed tables covering various heavenly bodies for the expected period of flight, and in some cases special charts. This is no longer necessary. The navigator may now embark with a nautical almanac, one small book, and no pre-flight worries. Two new methods, invented and developed by naval officers especially for air work, permit the solution of the astronomical triangle in from two to three minutes' time. The latest of these books known as "Hydrographic Office Publication No. 208," is highly accurate and can be used not only to solve for the line of position but also to find direct the latitude at local noon, to identify unknown stars, and to compute great-circle courses and distances.

It should be understood that a single celestial observation does not determine the observer's position, but merely a line. Thus it is only at morning and evening twilight or at night that a "fix" can be obtained from the stars by celo-navigation, unless during the daytime the moon is visible and in favorable position for crossing its "line" with the "sun line." In the morning and late afternoon the sun line of position runs nearly north and south, and thus gives a good indication of longitude at the time of sight. At local noon, the sun being on the meridian, its position line runs due east and west and definitely establishes the latitude. If the morning sun line be moved parallel to itself in the direction of and for a distance equal to the track of the aircraft between the time of the morning sight and the local noon sight, its intersection with the moon line gives a noon position. It is obvious, however, that the actual track of an aircraft between these two sights may be considerably at variance with the track as computed or plotted by dead reckoning, and hence such a position found by "running forward," as it is called, an old line of position is of doubtful value. This, however, is the best that can be done by day at the present time, and it is evident that the radio beacon must be combined with celo-navigation to give the maximum results. The beacon not only solves the very important problem of drift (difficult enough over water by day and even more so by night), but it also establishes a definite line of position. The beacon line is a somewhat thick one, to be sure, but it will serve very well to obtain frequent and rapid "fixes" by the simple process of intersecting it with lines of position obtained by celestial navigation. In case of a forced landing the navigator can report a very accurate position. Thus the beacon and celo-navigation both have an important place in the scheme of future transoceanic air travel, although neither one can take the place of the other, both being essentially complementary to the general scheme of navigation.

Navigation may be said to fall naturally into two main branches, viz.: geo-navigation and celo-navigation. The former embraces the methods and means of determining position with reference solely to the earth and objects on the earth. It comprises what are known as piloting, dead reckoning, and directional radio. Piloting, perhaps better known as "map flying" in aviation, employs visible and plotted landmarks; dead reckoning employs the knowledge of ground speed and direction taken; directional radio may either delimit and mark a course or de-

termine the position, or by combination do both. Celo-navigation, as has been noted, is the branch which, based upon astronomy, enables position to be determined solely from the sun, moon, planets, and stars. We have a variety of navigation methods to choose from, but it is obvious that no single method can cover all classes of flying. Moreover, when we consider how to find the course, which is as much a part of navigation as the determination of position, this requires certain other knowledge depending also on the type of flight. In general, it is usually classed as being in the realm of dead reckoning. Finally, all methods and both general branches of navigation are very closely correlated.

It is often said that an airplane pilot must learn navigation. But we do not demand that Chesapeake Bay fishermen and tugboat captains be finished deep-sea navigators, nor are we surprised if inland-waterway navigators prove incompetent out of sight of land. Undue emphasis on navigation for aviators can therefore only lead to contempt for it on their part, for it is not very difficult to learn map flying and to get about cross-country successfully with only the most sketchy ideas as to the fundamental principles of navigation. It does not matter that the pilot who is thoroughly acquainted and familiar with the art of navigation is *potentially* a more valuable and perhaps safer pilot, for the other chap if he uses good judgment will arrive too.

The Kind and Amount of Navigational Knowledge Required by Aviators

What, then, is the kind and amount of navigating knowledge required in aviation? This depends, obviously, on the type of flying. It is well to remember also that the amount of navigation possible depends to a large extent not only on the type of airplane and the equipment, but also on whether the pilot must fly and navigate alone or whether there is a relief at the controls. A lone pilot frequently has a bad time trying to do both, and easily becomes confused as a result.

Heavier-than-air flying may be classified under three general types: first, transoceanic flights; secondly, flights of shorter length, but involving stretches of water that take the plane out of sight of landmarks ashore; and thirdly, flights over land. Such flying may be either commercial or military in nature, but the principles governing navigational requirements are nearly identical.

Transoceanic flights obviously demand a *navigator* skilled both in geo- and celo-navigation. It is true that some flights have been successfully made without the aid either of directional radio or celestial navigation, but this does not prove anything from a navigational standpoint. Transoceanic flying is still a stunt, and is likely to remain so for some years, but coastwise flights, involving stretches of overwater flying, best illustrated at present by Pan-American Airways, is no stunt. Such flying is a sound and logical forerunner of future commercial overseas flights. It is highly desirable that this type of flight should also be provided with a very skilled navigator aided by directional radio, and for the same reason that boats or amphibians are used, namely, as a factor of safety. Take for example the 110-mile stretch between Key West and Havana. Here there is no question of ability to make a satisfactory landfall, but in case of a forced landing there is a question of reporting the exact position. Since, as previously noted, it is not always possible to obtain a fix at sea by celo-navigation during the day, the radio beacon should be used either in connection with lines of position obtained by celo-navigation or with one or two well-located radio-compass stations to give cross-bearings. Whether we use just the beacon and celo-navigation or a system of directional radio (and this seems the better in these cases), the navigator must be of deep-sea caliber to get best results. It can only be a short time now before planes will run on schedules between the mainland and Bermuda, throughout the ring of West Indian Islands, and from the Canal Zone direct to the States via Cuba. The company that undertakes this without providing proper directional-radio equipment and competent navigators skilled in all forms of navigation, will be headed for an eventual disaster. So much for transoceanic and shorter trans-water flights. There *is* no minimum of navigational knowledge or ability for such types of flights, and this is true, be the weather fair or foul.

When we consider flights made exclusively over land, it is natural to think in terms of the flights that are made daily over our own well-mapped country. Here there is no need for celo-navigation at all. When the weather is fine and visibility is good, we need not even bother a great deal about any kind of navigation so long as we can read a map, while if the visibility is low one can very sensibly take a railroad or perhaps a highway route. Not a year since a lad of eighteen years made two passes at the large Air Corps field near Dayton and finally screwed his courage up and landed, one wing low. The day was fine and clear. He wanted gas and he wanted to know how to get to Troy, Ohio. The day before he had completed his first ten hours' instruction at a Long Island school and then on his first solo flight flown blithely on to Buffalo. He was en route to points west via Troy. He had three gallons of gas left, but the hard-boiled sergeant finally relented and instead of sending him on to the commercial field, gave him some gas and directions to follow the river. We scanned the papers for the next few days, but apparently there were no obsequies held. He got through. His navigation, one might say, was of the rough-and-ready kind, but it seemed to suffice, and apparently there is a great deal of that kind of navigating being done these days. As long as it works, it is all right, and it further appears that that kind of navigation is quite suitable for what might be classed as private cross-country flights from point to point for pleasure or profit. There is a minimum of navigational knowledge, however, for these pilots. They should be good map readers, have an elementary knowledge of the compass, and should either fly in fair weather or faithfully follow the railroads. There is no use in saying that this cannot be done or is unsafe, for the fact is that it is done daily, and, combined with good piloting ability and good judgment, it is not unsafe.

Minimum Navigational Knowledge for Overland Commercial Pilots

The most important flights, though, are scheduled commercial flights over regular airways, and special commercial flights from point to point not over regular airways but in the nature of air taxi service. It is possible for pilots to do such work with only a rudimentary knowledge of geo-navigation and especially if flying only along a regular airway, where daily trips familiarize them to such an extent with the terrain that they can find their way about within the narrow belt of their route as easily as one can navigate around one's bedroom in the dark. Such airplane pilots are comparable exactly to our river- and harbor-boat pilots. The demand of navigational knowledge on the taxi pilot is more exacting, but since this pilot is usually chosen from the staff operating on a regular run, it is proper to set a minimum for all overland commercial pilots.

This minimum should be set, because the pilot who has a comprehensive grasp of navigation is potentially the safest one to fly with, other things being equal; he will average better time and consequently less gas consumption; he will combine judiciously his general navigation knowledge with so-called railroad flying, and will not confine himself to railroads unnecessarily. The minimum for these pilots is everything short of celestial

navigation and the "sailings" of marine dead reckoning. This includes a knowledge of maps, the projections on which they are made, what the courses laid down on them represent, and how distances are measured on them; the true significance and meaning of direction on the earth's surface, of bearings, and how they may be expressed; the theory of the compass, the errors of variation and deviation and how they are found and used; the errors peculiar to the aircraft compass, and how to mount, compensate, and take care of the compass; the problems peculiar to aerial dead reckoning and their graphic solutions, such as the method of course finding to allow for drift in order to make good a desired track, and the method for finding the radius of action from an airdrome; the instruments used in flight, the instrumental aids to navigation, such as the drift indicator, the course solver, earth inductor compass and directional radio; and last but not least, meteorology and weather-map reading, at least to the extent that general weather and wind conditions can be predicted. A pilot having a very thorough grasp of all this has a valuable background of navigational knowledge that on occasion may prove invaluable. As long as he flies solo, he cannot do more today in the average airplane than get his drift by trial and error, fly by map and compass, and figure his ground speed as best he may; there is no room or provision for course solvers, drift and wind computers, or other gadgets, nor leisure to use them. Nevertheless, he is backed by the confidence of knowing what he is doing, and if he gets lost he at least will not cut aimless circles trying to relocate himself. It is much better to fly by compass than by instinct. As a passenger in a two-seater not long ago the author flew by instinct for a spell on a cloudy day, and was soundly berated by the pilot who awoke to find the plane going east when the desired course was west. It is only fair to say that there was no compass in the after cockpit.

In conclusion, instrumental aids and methods have reached a state of development that make air navigation almost as exact in skilful hands as marine navigation, and, the author believes, in any case sufficiently exact. The radio beacon is undoubtedly the outstanding development and improved methods and sextants are next, while many small but vitally important details such as air maps, charts, watches, etc., have been developed to fill a much-needed gap. We have only to use the equipment we now have, and refinement into something better must inevitably follow. As for the theoretical knowledge of navigation that should be stocked in the mental equipment of each type of aircraft pilot, the case is the same.

Discussion

Bradley Jones.[2] The author sums up the various phases of air navigation in a lucid manner and the status of the art is clearly stated. In connection with his comments on a second-setting watch, the desirability of having apparent time aboard the airplane is well put. One usually reads time, however, by noting the position of the hands and not by scanning the figures. Many large street clocks have no numerals, but merely dots at the hour points. A timepiece having a rotatable seconds dial cannot be read without close scrutiny or there is chance of error.

The average hourly difference in the equation of time is 0.56 seconds, so that if a watch is set on the average apparent time for a given day, the error in most cases will not be more than 6 or 7 seconds, even if the rate is not changed. Since at present flights of 24 hours and over are out of the ordinary and suitable preparations must be made for them, the labor of regulating a watch to nearly apparent time for such flights is insignificant. For the first Pacific flight the Army Air Corps set a watch on apparent time by momentarily stopping the escapement. Properly done, this does not injure the mechanism, and the added complication of the second-setting device is avoided.

The artificial horizon for a sextant is still a problem. No one would think of holding a tumbler full to the brim with water while riding in an automobile over a rough road without some of the water spilling. Yet it is a parallel case to attempt to hold a spirit level in an airplane and not have the bubble dancing about. By taking a great number of observations and averaging the results, this mean should theoretically be nearly correct. The camera sextant is designed to do this averaging quickly and easily. For several years the Army Air Corps, having despaired of obtaining an ideal artificial horizon with a single bubble, has been searching for combinations of spirit level and pendulum, spirit level and gyrostat, or pendulum and gyrostat, in which the effect of acceleration on one element is neutralized by the effect of the same acceleration on the other element.

Harold C. Gatty.[3] The need for accurate navigation in the operation of air lines is generally becoming recognized. Aviation has progressed, and paying air lines have been established so rapidly that navigation, or the need of it, has in the past received little attention from air-line operators. Efficient planes have been built, and engines have been improved, to a point where reliability is assured. Good business men are establishing air service on a sound business basis; slipshod methods and guesswork have been eliminated in all but one direction, and that is in the navigation of the plane.

Two pilots are carried on most transport planes. The only apparent excuse for the presence of the extra man is that in the event of the chief pilot dying of heart failure there is someone else capable of taking the controls. It is quite right that the passengers should be protected against all possible accidents, but there is no reason why the assistant pilot should not be actually prepared for other emergencies with regard to the navigation of the plane.

What would we think of a passenger vessel leaving port without an accurately checked compass and the necessary navigational equipment for the determination of the position of the ship by officers having the necessary knowledge? To be sure, it is quite likely that it may reach its destination, but we are not protected for any emergency that may arise.

Time in the air directly affects operating costs, and air-line operators should realize the great advantages of direct flight over railroad courses. Railroad courses may be all right for the man who is cruising for pleasure, but air lines are more concerned with cutting down the time in the air than in sightseeing. How many times in foggy or thick weather will a pilot keep close to the ground in order to recognize known landmarks beneath him, even though he knows full well that there will be no fog at his destination! Supposing, in this case, the pilot is equipped with an accurately checked compass, instruments for determining his position, and the necessary knowledge to make use of them. Would he not climb to a safe altitude above fog and clouds, make a direct course, and eliminate any danger of a crash in a mountain pass or into some unexpected obstruction, due to low-altitude flying? For the purpose of illustration let us assume we are making a flight from Los Angeles to Phoenix. There is a thick fog over Los Angeles and extending over the intervening mountains, but we know definitely that there would be no fog as far inland as Phoenix. The best route for the pilot to take is clearly indicated. Of course, this is one particular case, but there are numerous other instances where similar conditions prevail. It is characteristic of clouds that they form and cling around mountains, and therein lies the cause of an untold number of accidents.

[2] Army Air Corps, Wright Field, Dayton, Ohio.

[3] San Diego, Cal.

If in the air, and he sees the approach of a thunder storm, what does the pilot do? The storm area may be comparatively small, but on the other hand if it should be a line thunder storm, it may have a front extending for several hundred miles. There are two courses open to the pilot—one is to turn and run before the storm until a landing can be made, and the other is to fly through or over it. The higher the pilot flies, the more conspicuous are the tops of the thunderheads, and thus he can pick a corner over the smaller heads. If the top of the storm is so high that it is impossible to fly over it, and it is necessary to go through, the pilot should steer directly through the upper part of the cloud facing the storm.

Without the necessary instruments, and the knowledge of how to use them, the pilot can have no confidence in his ability to determine the course and his position without reference to the ground. Many thousands of miles have been flown by experienced pilots without the knowledge of navigation, just as ships in centuries past found their way to all corners of the globe with very crude navigation, but there is no reason why, from an economic and safety standpoint, these conditions should exist. There is a time in every pilot's life when he wishes that he knew just where he was; when a pilot needs navigation, he needs it badly.

The time-honored methods of marine navigation, entailing tedious calculations, have been found impracticable for use in the air, where speed and simplicity of operation are of prime importance. Great credit is due Lieutenant-Commander P. V. H. Weems for his foresight in realizing the need and for his successful development of a system of navigation which reduces calculation to a minimum and only requires of a student a knowledge of how to add and subtract. This method, though developed for aerial navigation, is equally applicable for marine use. Before outlining these developments, I would like to explain just what we mean by navigation.

There are four methods of navigation: piloting, dead reckoning, radio position finding, and celestial navigation; not one of which by itself will carry the pilot through at all times. A combination of the four methods will produce very accurate results under all conditions

"Piloting" is the method of navigating by known landmarks, or beacons, and though it is practically the only method in common use, it has decided limitations.

If flying over a fog bank or clouds, over unknown territory or at other times when we cannot see the ground, it is necessary to keep an accurate account of the course steered, the air speed, and the time flown on each course, in order to estimate our position. For instance, if we fly on a course due north at an estimated ground speed of 100 m.p.h., at the end of 30 minutes we should be 50 miles north of our point of departure. This is termed dead reckoning. Before dead reckoning can be used, we need a compass that has been compensated and checked for deviations. Air-line operators have given little thought to the importance of the aircraft compass, and it will be found that at the present time the compasses of many transport planes possess unchecked errors of 30 deg. and even more. The compass is the basic instrument of navigation, and as such deserves strict attention. A non-magnetic turntable such as Lieutenant-Commander Weems has devised should and will eventually be part of the standard equipment of every airport, for the compensating and checking of both magnetic and radio aircraft compasses. The weakness of navigation by dead reckoning alone is that one's estimated position depends solely on the accuracy of the account kept of the course and distance made good. Since we have no accurate or continuous means of keeping track of the course and distance made good, the position by dead reckoning is liable to be excessively in error after a few hours' flight. The importance of keeping a record of the dead reckoning cannot be stressed too much. Each time the course or speed is changed, this fact, together with the time, should be noted. In actual practice piloting is combined with dead reckoning, and there should be a careful running record of all possible data pertaining to both methods.

It is only by the knowledge and continued practice of the four methods of navigation during clear weather that we are prepared for bad weather.

In describing in detail the operations for setting a course, we make it seem harder than it really is. The main thing is to be able to change a true course as obtained from our map into a compass course by applying the compass errors, variation and deviation, without making any mistakes. Next, it is very important to have equipment handy for plotting the courses and distance on the map, and to be able to do this quickly and surely. Trying to do this on a board with crude parallel rules and with the map blowing about is very unsatisfactory. On the other hand, with a small drafting machine (or with an aircraft plotter) and with the map slipped under a celluloid sheet, to hold it in position, it becomes an easy matter to plot courses and distances. A convenient-sized board to fill the lap and still permit the use of the stick is about 10 by 16 in. If now we want to cover 600 to 800 miles without changing maps, the scale should be about 2 in. to a degree. This scale would permit fairly accurate work for plotting lines of position, courses, dead reckoning positions, and so on, while at the same time about 500 miles in latitude and about 800 miles in longitude could be included in a convenient-sized board.

Radio position finding is certainly of great value as an auxiliary method of navigation, but it does not, as some people would believe, solve the whole problem of navigation. Transport planes will undoubtedly be equipped with a receiving set and a transmitter, the main uses of which are to communicate with stations and other planes, to summon aid in an emergency, to receive radio beacon signals, to send signals, the bearings of which are picked up by two fixed radio compass stations, the plane's position being radioed back to the plane, and to take bearings of signals from sending stations by means of directional antennae. There are innumerable drawbacks to radio position finding, and disappointing results will be obtained if not used in conjunction with other methods of navigation. It is highly desirable for the plane to carry its own direction finder, for in this way the responsibility for navigating rests upon the shoulders of the pilot, where it undoubtedly should. A plane equipped with a built-in direction finder must head directly for the sending station, in order to get the bearing, yet if it continues to head directly for the station, no allowance is made for drift. In case there is considerable wind, a plane heading directly for the sending station would not travel the shortest route. Radio compasses may be carried either in the plane, and used to take bearings of signals from a known station, or located in a station and used to take bearings on the signals from the plane. In either case, the operation is the same. Bearings taken with a loop antennae or radio compass are subject to two sources of error. First, the metal in the plane may affect the incoming wave, so that the plane must be swung on the ground until signals have been received from all relative directions, and a table of corrections made out for each relative direction of incoming signals. Second, it must be remembered that radio signals follow a great circle course, and therefore, in distance over 50 miles, the direction of the signal is not the actual compass direction of the transmitting station, and a correction must be applied. Another type of radio direction aid is the radio beacon. The directional characteristics of a loop are always applicable to transmission as well as reception, to a limited

extent, and this principle is used in the most successful type of radio beacon yet developed. This type of beacon has been developed to the point where different signals operate lights on the pilot's instrument board, a white light showing as long as he is on the proper path, a green light if he is on the right of it, and a red one if he is to the left. At present the big drawback to the practical use of the radio for navigation is the fact that bearings become inaccurate at long ranges.

The onus of navigating the plane should rest on the pilot, and not upon the reliability of delicate instruments and dependence on people on the ground.

The method and equipment for celestial navigation developed by Lieutenant-Commander Weems enable a pilot to fix his position at night in one minute by simultaneous observation of two stars, by the use of the recently developed Bureau of Standards type sextant, an aerochronometer, rated to Greenwich sidereal (star) time, and Weems' "Star Altitude Curve Book." By placing a transparent sheet, graduated in longitude and latitude, over the curves and running the intersections of the altitudes of two stars, latitude and longitude may be determined without putting pencil to paper. The whole operation can be performed in less than a minute. This night method alone would justify the installation of this equipment in all transport planes. Night flying is becoming more common, and contrary to the average pilot's belief, navigation can be far more accurate at night than during the day. The rapid determination of position by the Weems star-curve method enables drift and ground speed to be calculated without any reference to the ground. Studding the entire country with beacons is both expensive and unnecessary. The most direct route over a lighted airway is not always the shortest. How many times, owing to weather conditions along the route, will a pilot find that by deviating from his course 50 or 100 miles he will not only avoid a strong head wind, but will actually get a tail wind. One of the main objections to beacons is the fact that in clear weather they are not required, while in a condition of fog or thick weather they cannot be seen.

Owing to recent developments by Lieutenant-Commander Weems, celestial navigation in the air may be used day or night, and all calculations in the air are eliminated by pre-computation. The principles of pre-computation have been made use of for many years, but the application of these principles were not entirely practical. The application of pre-computation by the Weems system is performed by the following procedure:

In order to show the practical use of the method for day navigation, let us take for example a flight from Seattle, Washington, to Sitka, Alaska, a distance of 740 miles over a region generally obscured by fog. The course is laid down on a Mercator chart, and found to be N. 40 deg. W. The time of departure is known, and the estimated time of crossing each degree of latitude and the corresponding longitude of the crossings noted. The altitude or angular height of the sun is calculated for those assumed positions, and corresponding times by means of Weems' "Line of Position Book." Lines at right angles to the bearing of the sun may be laid on the chart for the assumed positions, and estimated times at those positions marked on them. Curves of calculated altitude against time are plotted on cross-section paper. All that the pilot requires to do in the air is to take the sextant altitude of the sun, and the Greenwich civil time from his aero-chronometer, and he can, by plotting these on the curves, at once see how many miles toward or away from the sun his line of position is. If the sun be ahead, it will at once give him ground speed, and if abeam it will give him his course. It will not give him his actual position relative to his course unless the sun is abeam, but will give him a line of position relative to the earth's surface. For a great part of the year curves may be pre-computed for the sun and the moon, and at such times the plane's actual position may be obtained. This method is not dependent on taking the observations at specified times. Used by themselves, or in conjunction with radio, pre-computed curves by the Weems system are a valuable aid to aerial navigation. Pre-computation, but with different applications, has been used by Lieutenant Hegenberger and other Army fliers with considerable success. The use of mathematical tables in the air is not very practical, and the writer believes that Lieutenant-Commander Weems' methods are without doubt the solution of the difficulty of using celestial navigation in the air. Flights are becoming of longer duration, and will continue to do so.

The need for navigation is rapidly becoming realized as competition becomes keener and the days of hedge-hopping and railroad courses are near their end. The future will ever be grateful for Lieutenant-Commander Weems' far-sightedness and tireless work in the development of valuable aids to aerial navigation.

V. I. Zelov.[4] The paper covers well the existing status of air navigation, although on account of its purpose and scope, the author correctly does not present construction details of described apparatus.

It is believed that air navigation, successfully applied, requires comparatively considerable knowledge which the average pilot is hardly expected to acquire. Hence, a navigator is required whenever the responsible duty of course plotting or course maintenance is needed. To justify this, an aircraft should be of substantial size, carrying a score or more passengers, or equivalent, from the standpoint of business cargo. It is believed that with the advance of long-distance commercial flights and higher zones of air-travel routing, the present variety of devices will be standardized and their accuracy increased by means purposed to minimize errors of observation and calculation allowances, inherent in unstable aircraft. The majority of present means and devices, discounting their cost, do not appeal to business enterprises. In this direction the Government should contribute to the development of suitable and reliable equipments.

The much-heralded methods of airway illumination appear as a concession, since no absolute and independent means for blind flying or air navigation are available. By following beacons, the aircraft should be considered as ground-bound, limited from the best and most efficient use of favorable properties of its sphere, the air. From a military standpoint more attention should be paid to so-called absolute instruments, based on either magnetic or gyroscopic principles. Military aircraft obviously cannot depend for successful operation or navigation upon an easily confused and vulnerable ground system of beacons, light or radio signals, etc. Besides this, it is believed that independent or self-contained means of air navigation will prove, after all, the most economical and dependable. The less human element, or rather the less humans are involved in the matter, the better. Automatic signals or other radio-guiding arrangements would tend naturally to increase the factor of safety and confidence of the navigator. Properly organized and maintained radio systems offer powerful assistance in commercial flying of peace time, but until there is eternal peace, reliable means to get bearings derived from well-established directional properties of earth magnetism and gyroscope have to be made easily available to a good navigator.

The average pilot, and most of them will be in this class, will do well to know how to follow the indications of modern instruments. These are now reliable, and further improvements are rapidly being introduced into those available for aviation.

[4] Aircraft Control Corp., Philadelphia, Pa.

P. V. H. WEEMS.[5] No longer is it necessary for the pilot to give his attention solely to getting his plane in the air and keeping it there. Even handling the controls in modern planes does not occupy all of the pilot's attention. As actual flying conditions become less difficult, the navigation of the plane becomes relatively more important.

The ideal condition for navigation is in large planes on long flights where the relief pilot can navigate continuously. Under these conditions, using all the means at our disposal, there should be no delay in proceeding directly to the destination under all but the worst weather conditions.

For safe navigation the plane should be equipped with the usual panel instruments, radio, drift and ground-speed indicator, and celestial navigation equipment consisting principally of the sextant and the aero-chronometer. The radio not only affords definite information such as bearings, but it also affords a means of getting weather information and a means of calling for help in time of emergency.

Contrary to common belief, celestial navigation affords the most practicable means of navigation on long flights. Accuracy within the limits of tolerance for air navigation may be obtained from the plane by day and by night with practically the time and effort necessary to take drift observations or to transmit and receive radio information. Positions may be obtained in a minute at night by the use of the latest equipment, and "lines of position" on which the plane is known to be may be found every few seconds during the day. When the moon and sun are in the proper relative positions, positions may be determined during the day within a minute or so.

Efficient navigation within the plane results in (*a*) a saving of time and hence in cost of operation, (*b*) a decrease in the danger inherent in flying, and (*c*) more confidence in flying on the part of the passengers and hence more business to the carrier.

In single-seater planes efficient navigation becomes much more difficult. However, only on stunt flights will one-man planes remain for long periods of time in the air. In these cases a lone pilot may with certain limitations navigate a plane by dead reckoning, by radio, and by celestial navigation. Lieutenant Bromley, now training for a transpacific flight, is planning to do celestial navigation and to have radio.

The fact that air celestial navigation is associated with marine navigation, an old and heretofore difficult science, prevents many airmen from taking up celestial navigation for practical air work. This, combined with the fact that good bubble sextants have not been on the market, has prevented the needed progress in this important phase of aviation. As a matter of fact, advances in marine navigation within the past two years have reduced the time by more than 50 per cent and the labor correspondingly. This work may be still further simplified for air work, and if only there was no connection between sea and air navigation, the airman would doubtless take up celestial navigation with more avidity.

Not only is celestial air navigation practical and possible, but it has been demonstrated repeatedly. It only needs to be popularized, and the high cost of equipment be reduced by quantity production, to make it a valuable asset to aviation.

Some will think that extensive computations are necessary for celestial navigation. However, with proper equipment, the distance the plane is toward or away from the sun may be determined with a sextant and watch observation without putting pencil to paper. At night the latitude and longitude may be determined in a minute with only one subtraction of two times to get the longitude. The latitude is found without any computations whatever. The equipment used successfully by the writer consists principally of:

(1) Bureau of Standards type aircraft sextant
(2) A pair of aerochronometers
(3) The "Line of Position Book"
(4) Star altitude curves
(5) Charts, small drafting machine, pencil, dividers, and Nautical Almanac.

A special supplement to the Nautical Almanac will be published by the Naval Observatory on September 1, 1929, by means of which the moon may be used with as much facility as the sun.

Lest some qualified navigators will question the speed of day navigation, it might be stated that this is a new and special application of the old principle of "pre-computation" which has been used with success by the Army fliers, especially by Lieutenant Hegenberger.

AUTHOR'S CLOSURE

In general the author concurs with the opinions expressed in the discussions. It was his endeavor to stress the fact that the demand of air-navigation knowledge for pilots and the necessity for proper modern equipment depend upon the type of flying engaged in. To his mind the instruments now most needed for safe flying and position finding are better blind-flying instruments and drift indicators. He does not believe that commercial overland flying need ever be concerned about celestial navigation; light beacons and radio, expensive though they may be, are certainly not unnecessary as Mr. Gatty claims, and combined with well-organized weather information should suffice for overland commercial flying. On transwater flights, every known navigational means must be used with the very best of celestial navigation instruments, and it is this type of flying that will demand and get improvements in all branches of air-navigation instruments.

[5] Lieutenant-Commander, U.S.N.

AER-51-26

Recent Developments in Air Transport

By P. G. JOHNSON,[1] SEATTLE, WASH.

The author, after pointing out that there are 21,000 miles of established airways and 1600 airports in the United States, discusses the recent rapid growth of air-mindedness on the part of the public. As an index of this he estimates that 12,000 planes will be built in 1929 as against 622 in 1919, which latter were mostly military craft. He predicts greatly increased employment of planes for mail transportation, their speedy adoption for handling express matter, and their very general use for passenger travel.

TEN years ago a daily round-trip flight of government-owned and -operated mail planes, between Washington, D. C., and New York represented commercial air transport in the United States. Today commercial aviation, statistically expressed, is as follows:

There are approximately 21,000 miles of established airways over which mail-express and passenger planes, operating on regular schedules, will fly in excess of twenty million miles this year. Mail planes, now serving 108 cities in thirty-seven states, will transport more than 3000 tons of letters exclusive of express and passengers, and an estimated load of 300,000 lb. of express. The total mileage of civil flying was 40,000,000 miles in 1928, and that figure will be substantially exceeded this year.

There are 12,000 miles of lighted airways, and one American company, the Boeing System, which flies 46 per cent of its total mileage at night, does more night flying than all the European companies combined.

There are 1600 improved, or semi-improved, airports, the majority municipally owned. The value of existing air terminals is $100,000,000, and this year approximately $200,000,000 will be spent by 1100 airports for land, improvements, equipment, and supplies.

These figures are important because they express facts showing that commercial aviation has passed into the era of extensive operations, and that transportation of mail, express, and passengers by air already forms an important link in our national chain of communication and transportation.

Even more significant is the fact that practically all the development cited above has come about in two years. Commercial aviation progressed more rapidly in 1927 and 1928 than it did between 1903, when the Wright Brothers made the first successful flight with the powered airplane, and 1927. For a decade after the first successful flight, aviation was in the doldrums. It took a world war to hasten the development of the airplane and prove its possibilities, and also to recruit the personnel for peace-time use of the plane. But after the armistice, the aircraft industry paid the penalty exacted by reason of the sudden cessation of buying orders on military account, and, until 1923, the industry was at low ebb.

Then the Government stepped in to play a constructive part. The Department of Commerce assumed the sponsoring and regulation of air traffic and the Army and Navy embarked on a building program calling for 3200 planes over a five-year period. The Post Office Department, which had done a splendid job pioneering with the air mail, expanded its activities, and around the air-mail network this nation began to build its commercial air-transport system. In 1927 the Post Office Department discontinued its operation of the mail routes and private contractors took over the work and put into it the organization ability, initiative, and enthusiasm so characteristic of private enterprise.

Then came that which fanned the growing flame of aviation interest—the Lindbergh flight, followed by other transoceanic and transcontinental trips.

Recent Growth of Air-Mindedness

The public began to become air-minded and suddenly realized that the airplane was the latest and most effective transportation vehicle available in the ceaseless fight against time and space. People wanted to ride in airplanes and they began to send mail and express matter in an increasing volume. Airplane manufacturers and operators capitalized on this interest. Powerful banking groups looked into the future and made funds available for all legitimate expansions.

An index of how suddenly this air-mindedness has come about is seen in the following figures on production of planes in the United States: 1919, 622 planes; 1921, 302 planes; 1923, 587 planes; 1925, 789 planes; 1926, 1186 planes; 1927, 1962 planes; 1928, 4000 planes; and for 1929 an estimated production of 12,000 planes. The production in the early years cited was largely military, and in 1921 practically every plane sold was for Army or Navy use.

Other figures show the recent trends in commercial aviation and point to the future. The air mail carried in 1928 was 2.37 times that transported in 1927, and more striking still was the transportation of practically twice as much mail in the last five months of 1928 as during the first seven months of the year. Planes in commercial use over scheduled routes numbered 128 in 1927 and 227 in 1928, while the total of civil planes increased from 3731 in 1927 to 7800 in 1928.

Thus we see the aircraft industry was lifted from the doldrums by the war and dropped back into lean years after the armistice, with a revival in 1927, followed by the present rapid expansion. We are now in an era of keen competition featured by mergers and consolidations. Efficiency will count much more than it did in the experimental and colorful days of the past. The dominant survivors will be factories and operating companies which are well managed and financed, and fortified with designing and manufacturing staffs capable of keeping pace with the changes which we recognize will come in both the building and operating sides of this industry.

Some of the Definite Trends of the Future

The experience of the past indicates some of the definite trends of the future.

To date, the largest expansion has been in the carrying of mail and passengers, and the majority of lines have carried both on the same planes. However, air-mail loads have so increased and schedules of departures and arrivals are at such inconvenient hours in many cases, that there is a growing tendency to separate mail and passengers.

Different kinds of operations call for different types of planes. The air traveler pays more for transportation than the rail traveler, and he expects schedules and comforts which often cannot be given him when transportation of mail is the prime consideration.

[1] President, Boeing Airplane Co., Boeing Air Transport, Inc., and Pacific Air Transport. Assoc-Mem. A.S.M.E.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

Assistant Postmaster General Glover recently predicted that all first-class mail between certain communities will be carried by air. The Post Office Department averages daily 476 tons of first-class mail which must be given transit handling. So the future of the airplane as a mail transport can be visioned. Mr. Glover also predicted that within two years practically all cities of fifty thousand or more inhabitants will be located on air-mail lanes. He pointed out that the busy man has always been impatient regarding the speed with which his mail is handled, and will naturally demand that all first-class mail be given the speed that air mail now receives in the fast-flying planes.

It is not idle speculation to predict that the airplane companies will build mail planes which will have mail clerks aboard just as railway mail cars do, and that the mail planes will haul the bulk of their cargo between dusk and dawn. The airplane is so speedy that already it eliminates the loss of any business hours in transportation of mail and express between many cities. For instance, between Chicago and New York; between New York and Atlanta, Ga.; between Chicago and Dallas, Texas; between Chicago and Cheyenne, Wyo.; and between San Francisco and Los Angeles and Cheyenne, Wyo. The schedules can so be arranged that major cities within 1200 miles of each other can have their mail moved between dusk and dawn.

The Boeing System recently completed flying its two millionth mile of night flying and on May 1, when a double transcontinental schedule went into effect, planes were flying every hour of the night with mail between New York and San Francisco.

Airplane Transportation of Express

Air transportation must also be reckoned with in the transportation of express. Robert E. M. Cowie, President of the American Railway Express, in a signed article recently expressed his opinion of flying express when he wrote:

> We see the airplane as a builder of more business, basing the conviction on the history of transportation. Express companies were originally regarded as competitors of the railroad. Express tonnage is only a drop in the bucket compared to the volume and value moved in freight cars. Commercial air transport is the logical delivery system of hand-to-mouth buying, the instrument of the rapid turnover which has had so much to do with our prosperity.
>
> The airplane is virtually without limitations. It should open areas to settlement at present far beyond the economic reach of the motor truck and physically impossible to the railroads.
>
> What we need is planes in regular service with a capacity of 20,000 lb. and the ability to carry a pay load of at least five tons. When our air lines are equipped with such a unit, commercial aviation will follow the course of every other transportation system.

In this day of small stocks, quick turnover, little profit, and a growing demand for service—which costs money—competition has become keener and business is reaching out into new and larger trade zones. Air transportation, averaging 100 miles an hour, has already begun to revamp certain long-established distribution and selling methods, just as the automobile and train left their mark on our business structure. Recently the chamber of commerce of a prominent city in the Middle West complained that its banks and wholesale houses were losing long-established trade because competitive cities had direct overnight air-mail express service to commercial centers of the Southwest.

The American Railway Express estimates that approximately $25,000,000 worth of small merchandise leaves New York by express daily. New York City and its environs, said the statement, produce only one-twelfth of the small merchandise of the country. The express company fills up 500 railway express cars every night in New York alone, and in its 28,500 offices the American Railway Express handles approximately 1,000,000 packages a day, and speed is such a requisite that the coast-to-coast cars make faster time than most of the passenger trains. Airplanes of the future will have capacity for large shipments of light-weight emergency express.

But even the most optimistic commercial air transport officials do not claim that the airplane will cut into the freight revenues of the railroads. They believe, however, that the airplane, by speeding up our business structure, will so stimulate industry generally that the railroads will share in the greater prosperity which each faster form of transportation has always brought us.

Current Varied Commercial Uses of Airplanes

In addition to using airplanes operated on regular routes, on scheduled services, to handle their express and to transport their employees, corporations will buy airplanes for the expedition of their own business. Executives are already flying their airplanes in the conduct of their daily affairs. Oil companies, newspapers, manufacturing plants, wholesale houses, and other corporations have their own planes, ranging from single-motored to tri-motored craft. The airplane is used in such a wide variety of pursuits as aerial photography, airplane dusting of crops, forest patrol work, scientific studies, aerial advertising, spotting schools of fish, map making, geographical study, engineering surveys, carrying supplies to inaccessible regions, transporting emergency shipments of factory parts, and in many other ways. These instances indicate the expanding market for airplanes. Each day the air-transport companies hear of specific cases where the airplane has opened up new markets and speeded up business generally, which explains why it is being utilized by business in ever-increasing measure.

The experience of the past two years shows that the passenger side of air transportation can be expanded substantially and quickly. The public is ready to fly. The air-transport companies are prepared to supplement the present services so that, in a few years, major cities in the United States will be connected by air-transport services, and many of them will operate multi-motored planes with a cruising speed of 115 to 135 miles an hour. The public is willing to pay the higher rate charged for air transport.

Increased Passenger Travel by Airplane

The average charge, over the United States, for air travel is 11 cents a mile according to a compilation made by the traffic department of the Boeing system.

If a traveler purchased a ticket for a one-way trip over every line in the United States offering passenger service and operating on a regular schedule, he could travel 12,914 miles. For this air travel he would pay $1427, or 11 cents a mile.

The extra-per-mile charge for airplane travel over rail travel is more mathematical than real, however. The air mileage between cities is generally considerably less than the rail mileage charged for. There is no pullman or chair-car charge on airplanes or taxi fares to and from terminals, as these are included in the airplane ticket. Also, the airplane is three and sometimes four times as fast as train transportation; and if time is money, it is more expensive to travel by train than by plane.

Railroad executives with vision do not regard the airplane as an intruder, and many of the lines sell combination rail-plane tickets. Among these companies are the New York Central, Pennsylvania, Santa Fe, Illinois Central, Chicago & Northwestern, the Milwaukee System, Great Northern, Northern Pacific, Missouri Pacific, and others.

At what mileage air-transport companies can compete for traffic with railroads is a debatable question. European authorities, a year ago, thought 300 miles was the minimum, but Germany is proving that this figure is too high. There, with the aid of a federal subsidy which enables them to quote mileage rates quite comparable to rail tariffs, one German company has hauled

56,000 passengers in one year, or one-tenth of the first-class rail passengers of the German lines, and their average air haul was only 93 miles.

At the present time transport companies in this country find it difficult to secure sufficient passenger lists on routes less than 300 miles long. But the American's inborn desire for speed will probably assert itself when commercial aviation is a little older, when the companies have brought their passenger service to high efficiency, when airports are closer in, and when the public becomes a little more air-minded. In the future air-transport companies will not limit their operations to arbitrary distances, but will adjust the service to meet the demand, irrespective of the length of haul.

Passenger travel by airplane is certain to increase because the public embraces each form of faster transportation. Airplane travel is speedy, comfortable, and interesting, and the operators are bending every energy to make it as safe as any form of mechanical transportation.

The public pays an extra fare on faster trains like the Twentieth Century and the Broadway Limited. Speed is expensive, but the travelers are willing to foot the bill, especially when the airplane is three times as fast as rail transport. A plane with a cruising speed of 90 miles an hour is already becoming slow to some confirmed air travelers, and the airplanes of the future will undoubtedly be more speedy than any of those now flown on commercial routes.

Airplane engineers are planning larger, faster, and more comfortable transports.

The safety factor is increasing each year through improved equipment, more and better airports, more experienced pilots, better weather reporting, lighting and marking of airways, the directive radio, the radiophone, and other aids.

The manufacture and operation of airplanes is not merely a matter of mechanics, but of "humanics." To give the airplane its rightful place in our national system of communication, transportation, and distribution calls for vision and initiative, but the recent developments in air transport indicate clearly that we are making rapid progress toward our goal.

Author's Closure

The question has been asked, Will improvements in structural design ever overcome the inability of an airplane to fly through rough air without the jerky rising and falling caused by sudden contact with downdrafts or rising currents and what aerodynamical improvements have contributed to the stability achieved in modern airplane construction?

There is no possibility of building an aircraft which will not respond to the action of convection currents and to other aerological phenomena such as are encountered when flying at an altitude where two wind currents of different directions cross one above the other. The air is just as full of waves of various kinds as the ocean, and even the largest steamships occasionally run into conditions which will make them pitch and roll almost as badly as a small craft in less disturbed water.

There are two conditions which tend to minimize the effect of similar disturbances, these conditions being high speed and weight. An extremely fast airplane is affected less by rough air than a slow one, and a heavier airplane has so much inertia that it takes larger disturbances to change its direction quickly than will similarly affect a lighter one.

Wing loading has some effect also; but inasmuch as high wing loading is generally associated with high speed, these two might be classed in the same manner.

We cannot hope to eliminate the effects of rough air entirely. There are no aerodynamical improvements upon which one can place a finger and point them out as contributing specifically to the stability achieved in modern airplane construction. The ability to make an airplane stable is a result of long experience in design and flight testing; and, as a result of this experience, the aeronautical engineering profession has been able to devise certain empirical methods of calculation which permit the attainment of the desired amount of stability in design with a reasonable amount of probable error.

In aircraft of radical design, these empirical determinations are of questionable value, which is one reason why the development of aircraft must proceed by slow and easy stages. It is best to try the effect of one change in design and determine the limits within which our calculations will give us reasonable results.

Another question is, Does the size of an airplane have anything to do with stability?

The size of the airplane has nothing to do with the characteristic of stability. Stability is determined by the proper distribution of wing surface, center of gravity, dihedral angle, horizontal tail-surface area, vertical fin area, and the position of the thrust line of the motors with respect to the center of gravity. By a proper juggling of these factors, it is possible to obtain any degree of stability, providing time is available to make necessary calculations and tests. I would rather judge from the nature of the question that the word "stability," in this case, means the ability of the airplane to ride through bumps without severe disturbance.

Another question raised was, Does muffling an airplane engine necessarily result in appreciable loss of power due to back pressure and is a muffled airplane motor an improvement that can be reasonably expected to appear in the immediate future on regular passenger lines?

The loss of power incidental to muffling an airplane engine depends entirely on the amount of restriction which the muffler places on the exhaust system. Pratt & Whitney Aircraft Company is insistent that the back pressure in the exhaust system not exceed $\frac{1}{2}$ in. of mercury, and a muffler which would meet this condition would necessarily be so large and heavy as to preclude its use.

Even with the engine completely muffled, there is still the propeller, and propeller noises are of such intensity to make the muffling of the engine almost valueless. We have found that, with the proper design of exhaust manifolds, the separate explosions of the engine can be almost completely smoothed out into a steady hum, with the vibrations brought so low as to be entirely unobjectionable. As a consequence, no muffler which has been developed to date appears to us to offer any advantages. Propeller noises are of such a high frequency that these cause most of the trouble for the passenger. The only solution for this is apparently a slow-speed propeller.

The European engineers do not agree as to which is the most objectionable from the standpoint of noise: a geared engine with a slow-speed propeller or the direct drive with the high-speed propeller (see Third Guggenheim Foundation Report, December, 1928).

The author has been asked if sleeping accommodations are a feasible improvement that might be expected to appear generally in planes operated over long-haul or night routes within the next year?

There is much argument among operators as to whether sleeping accommodations will be provided on night planes. Unquestionably, the traveling public will eventually demand such accommodations. At the present time, however, the cubic feet of space per passenger is getting so large, even without sleeping accommodations, as to make carrying of passengers very uneconomical in view of the rates which can be charged. The provision of sleeping accommodations will make the re-

quirements of space per passenger even greater, and is a condition which cannot be met properly at the present time.

There is no doubt that the equivalent of Pullman accommodations will make their appearance eventually, but hardly within the next year, even though there is much talk and advertising matter put out with this end in view.

The serving of meals aboard passenger aircraft is a problem akin to that of the provision of sleeping berths. Equipment necessary for serving extensive meals together with the personnel necessary will represent a large item of non-paying load. Furthermore, the commercial transport with a range of more than six hours will not be economical because of the large amount of fuel which it is necessary to carry in order to obtain this endurance, and the pay load per horsepower is proportionately decreased.

Assuming that stops will be made every three or four hours, there is no justification for serving meals aboard. The exception to this will be a run which covers the four hours in the middle of the day, in which case it would be necessary to provide the passengers with lunches, which probably would be prepared ahead of time and carried on the run. This would eliminate the necessity for any extensive extra equipment in the plane itself.

If planes are carrying both mail and passengers, there would be objection to stops of sufficient length to permit the passengers to eat a meal because this would interfere with the mail schedule. It is believed that the tendency now is to segregate mail and passengers in order to relieve the mail pilot of the responsibility he feels when passengers are aboard. Probably, mail will be carried in passenger planes in the future only in an emergency, when the mail can be carried on the passenger schedule to a better advantage than leaving it for the next regular mail trip.

Another question is, What are some of the causes for delays in delivery of air mail to destination and how are these obstacles being overcome?

On rare occasions air mail is longer in transit than mail sent entirely by rail because of operating difficulties. In cases where the distance to be traveled is very short a handicap is placed on the air mail in that the advantage to be gained is so very slight as to be easily lost by minor delays. On air mail traveling longer distances the margin of advance possible is greater, and consequently minor delays may be absorbed and still place the air-mail service at an advantage over train service. Even on longer distance hauls there are sometimes cases where the train mail beats the air mail due to circumstances which are difficult to overcome, but which it will be possible to overcome.

When a plane is forced down at some remote point where train service is very infrequent and the local conditions are such as to make prompt transportation of the mail to a railroad center difficult, an important train connection may be missed and another not be available for a number of hours. A protracted delay in departure of the plane might in such a case permit the train to arrive at destination ahead of the air mail.

On long-haul mail it is believed this condition can be overcome by furnishing pilots with a schedule of train-departure time and destination, to apply to each railroad station in the vicinity of regular and emergency landing fields in terrritory through which they fly, and keeping this schedule corrected as changes in train schedules are made from time to time, and by specifying certain deadlines beyond which mail down at a specific point may not be held beyond train departure. This arrangement will make it possible for the mail to be dispatched by rail from the point where the ship is down, and if the weather or mechanical difficulty which interfered with the flight is overcome in time to enable the ship to overtake the train at some point further along the line, the mail may again be picked up and sent forward by plane. Otherwise it can be carried to the first point on the line where a plane is stationed and from which weather conditions will permit the flight to be continued, or delivered to destination by train without losing the time which had been gained up to the time the flight was interrupted.

The author had in mind as an auxiliary to the foregoing arrangement the plane entering into contracts with responsible parties in the vicinity of emergency fields, who would be under agreement to promptly call for and transport the mail to the nearest railroad station whenever required and thus eliminate any delay which might be encountered by the necessity of the pilot hunting up some one to handle the transportation. The operations department advises me that this feature of the case has already been taken care of. The preparation of the pilots' schedules referred to has already been started and will be completed just as soon as up-to-date Railway Mail Service schedules are received, which are needed before the work can be finished.

The arrangement outlined will enable us to realize our aim of being able to guarantee to the mailing public that the case will be unusual indeed when it will be possible for train mail to beat air mail.

AER-51-27

The Railroads and Passenger Air Transport

By C. E. McCULLOUGH,[1] WASHINGTON, D. C.

LINKING rail and air transport is an expression now commonly used to indicate the coordination of rail lines with commercial airways. It expresses, in a few words, the progressive action of the railroads in forging a new link in the chain of ever-improving transportation throughout the United States. It is most appropriate that the railroads of the United States should enter the commercial aviation field. They were the chief factors in the development of this country, through providing and constantly improving our transportation facilities.

Rapid transportation for goods as well as persons has been necessary for railroads to progress. It is generally recognized that without the railroads the United States could never have reached its present peak of industrial and commercial dominance and leadership in human progress during the comparatively few years which have brought us to the present day and age. The railroads realize that further development in transportation of any form will be helpful to the future of our country.

The development of the electric interurban car lines, and later the startling growth of the automobile and motor-propelled vehicles for the public highways, were looked upon by some railroads as threatening dangerous competition. But as time went on, it was soon demonstrated that any improvement in transportation, in any form beneficial to the public interests, aided the growth and prosperity of practically every line of business.

The railroads lost some travel and some freight shipments to the new forms of transportation which had grown up and thrived, but a careful analysis shows that these losses were more than made up by the increase in new classes of travel and shipments of new commodities and manufactured articles, resulting from the ensuing general prosperity and welfare of the country.

It has now become apparent to every thinking man that inevitably there must be in our scheme of life a constant improvement in all transportation facilities and in the development of new forms of transportation. Less than a century ago the battle for supremacy between canals and railroads was fought out. Men are still living who can recall it. Stage-coach service as a means of passenger travel, particularly in the West, survived into the present country. In 1850, a comparatively recent period as history measures time, by using the rail lines which had then been developed in the East and the overland stage routes in the West, it was possible to cross the continent in twenty-four days, which was thought to be a notable feat. The extension of the rail lines to the Pacific coast and the successive improvements which have been effected in their service now make it possible to go from ocean to ocean by train in a little less than four and one-half days, enjoying the luxury and comfort of the highest class American limited passenger-train service. The day is now upon us, however, when that time is to be still further shortened by the coordinating of rail service with the newest form of transportation, the airplane.

The desire for speed is not a mania, as it is too frequently called, neither are the persons who have striven, are still striving, and always will be striving in the future, to break speed records, in one manner or another, in one kind of vehicle or another, speed maniacs, who are merely seeking thrills or notoriety, or both. They may be rash, courageous, foolish, or adventurous in the eyes of many, but they are, nevertheless, pioneers. The very nature and progress of our civilization demand speed, and more speed, in transportation and communication. Faster railroad trains, both freight and passenger, faster automobiles and motor vehicles, more rapid means of communication, and the development of the long-distance telephone and the radio, all have brought the far corners of our country closer together. As we still progress still further, these far corners must be brought still closer together.

Possibilities of airplanes are being recognized. Progress cannot be stopped; we must keep abreast of it, or forfeit our place to others in the race. Hence the plane is being harnessed to the iron horse, and the Pennsylvania Railroad has taken the first steps toward linking rail and air transport. These first steps are through the means of coordinated rail and air service, in connection with Transcontinental Air Transport, Inc., for an ocean-to-ocean service which will make possible the trip across the continent in forty-eight hours, instead of four and one-half days or twenty-four days.

Air transport will be organized along lines quite similar to that of the larger railroads. For example, in addition to the executive officers, such as president, vice-president, and general manager, there will be a general superintendent and division superintendents, constituting the operating department. Pilots will receive orders from the division superintendent. The field manager can be likened, in railroad comparison, to a combination of master mechanic and station master. The traffic department will be in charge of a general traffic manager, and for purposes of convenient administration, the country has been divided into three regions, with a traffic manager in charge of each. The eastern traffic manager will be located in New York City, the central traffic manager in St. Louis, Missouri, and the western traffic manager in Los Angeles, California. These traffic managers will have under their jurisdiction certain prescribed territories, and men in charge of certain specific traffic work, with the titles of division traffic agents, traffic agents, and passenger representatives, whose duties will be the sale of this new and more rapid mode of transportation to the traveling and shipping public.

In addition to the pilot and assistant pilot, the crew of each plane will include a representative who will be known as a courier, that title having been selected as indicative of his duties, he being an attendant accompanying each trip for the purpose of being generally helpful to passengers, and giving them information of value and interest.

The railroads prospered, and the public benefited, as the modern luxurious Pullman cars were built and steadily improved, as dining cars gradually supplanted the old twenty- or thirty-minute train stop for meals, and as the modern day coach grew in comfort and up-to-date conveniences; likewise, as safety appliances were invented and adopted by the railroads, such as the airbrake, automatic signals, heavy steel rails, steel cars, and others too numerous to mention here, so the public confidence in railroad transportation grew steadily. Where only a generation or so ago it was considered a noteworthy adventure to make a long railroad trip, demanding quick advice to one's friends or relatives of the safe completion of the journey, now a railroad trip of hundreds or thousands of miles is undertaken on short notice and in a most matter-of-fact way.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

If commercial aviation and the coordinated rail and plane service is to be sold to the public, there must be a sound basis for instilling into the minds of prospective air travelers the fact that they can travel safely and comfortably, as well as speedily, in regularly scheduled commercial airplanes.

Severe or unfavorable weather is perhaps the greatest problem in organizing safe air transportation. Weather cannot be regulated, but precautions can be taken to avoid unfavorable weather conditions. The route of transcontinental rail-air service has been laid out through country where normal conditions are most favorable to flying. The train-by-night and plane-by-day idea was adopted with the view of using rail through the country least favorable for flying. In addition to these things, a most elaborate system of weather reporting and communications has been installed.

Regular United States Weather Bureau reports will of course be available and used, but in addition there will be a complete installation of regular Weather Bureau instruments in charge of expert meteorologists at all of the principal airports or landing fields across the continent. Twice daily two hundred United States Weather Bureau reports will be received at these stations, and weather maps drawn and studied. These particular reports will be augmented by a mid-day report from all observers. The United States Weather Bureau observations will of course cover the entire United States and provide general weather information, while the mid-day report will cover the territory traversed by the airplanes, and consequently furnish more concentrated data.

Observers will be stationed at seventy-two observation points, in addition to the airports and landing fields mentioned, these stations being not only on the airway across the country, about fifty miles apart, but on either side, off the route, about one hundred miles apart, and about seventy-five miles from the line of flight. As a further evidence of the linking of rail and air transport, most of these observers will be railroad operators at stations on the Pennsylvania and the Santa Fe Railroads; these operators or agents being not only equipped with the necessary instruments, but trained especially in making weather observations.

In order to provide for the prompt dissemination of these weather observations and reports, as well as the dispatching of planes and the necessary interchange of general information, a very complete communication system had to be installed. Naturally, the most modern methods are necessary; hence the network of telegraph, teletype, and radio which has been adopted. Seventy-two weather observers will make their observations when scheduled and send them by railroad telegraph to the railroad station operators in the cities where the fully equipped airports or landing fields are located. The railroad station operators at these cities will then relay the messages by teletype, especially adapted teletype machines having been installed in these offices. The main fields, having automatically and instantaneously received the observers' reports by teletype, then add their observations, and after careful checking and study, and comparison with United States Weather Bureau reports, they will be placed in the hands of the pilots before their departure.

Any important information received later, of a nature to cause a change of the course, will be sent by radio, through voice transmission, to the pilot while in flight. Radio, by voice transmission, or by code should conditions make voice transmission difficult or undesirable, will be used only between a ground station and a plane, or sometimes between two planes in flight. It will not be used for communication between two ground stations. The teletype system has been adopted for communication between ground stations where the telegraph or telephone is not used.

As an additional feature, for the convenience of passengers in flight, arrangements have been made through a hook-up with the American Telephone and Telegraph Company, so that in an emergency passengers can communicate quickly with their offices or homes, by either telephone or telegraph. This service will of course be restricted to the extent that it shall not interfere with the use of the radio for plane dispatching and operating.

Among other things, to insure the public of the safest possible air travel, only thoroughly trained pilots will be employed. They will fly at comfortable altitudes, going into the higher altitudes only to avoid rough air. The planes will take off and land easily. There will be no needlessly steep banking after taking off or when preparing to land. While this is not necessarily dangerous, it is perhaps disconcerting to passengers who have never flown before, or who are of a nervous temperament.

Only safe loads will be carried. The motors and all equipment will be inspected thoroughly and frequently with the same care as are railroad trains. No taking of chances with the safety or lives of passengers will be permitted at any time or in any way.

With a fully organized airway, the arrival, departure, speed, and handling of airplanes will be as carefully regulated and controlled as in the operation of railroad trains.

The future success, development, and progress of commercial aviation depend largely on the reliability with which operations are handled and the public informed. By this means only can the confidence of the public be won in this new form of transportation.

Discussion

STUART F. AUER.[2] Within the brief span of twenty-five years aviation has grown from the status of a dangerous novelty to an economic position with an influence far reaching both in scope and effect. It stands today where the railroad stood fifty years ago and where the automobile stood twenty years back. In the early days, because of mechanical imperfections and lack of aerodynamic knowledge, accidents were frequent. From the public point of view every accomplished flight was a marvel, and every accident was an occasion for an outburst. It is only natural, therefore, that under such circumstances the management of insurance companies was hesitant to assume the risks incident to aviation. The hazards of flying, until recently, were too great to attract much insurance capital. Many factors contributed to this condition of affairs.

First, in the early stages, a large percentage of airplane accidents resulted in the death of the occupants. In case that 50 per cent or more of the accidents attendant upon flying ended in death (and earlier statistics show such to have been the case), the company's loss would have been greater by far than the possible premium return.

Second, a large percentage of those not killed suffered severe injuries, which resulted in long periods of total disability or permanent partial disability which would have developed losses far in excess of any possible premium income.

Third, in surface transportation like railroads, steamships, buses, trucks, trolley cars, etc., travel was on a two-dimensional surface. In aviation, however, a third dimension is added, the hazards of which could not well be calculated because there was no background of previous experience. Prior to the Air Commerce Act no agency was charged with the investigation or recording of civil air accidents, and no reliable information was available upon which to base statistics.

In the year 1927 there occurred 200 reported accidents in which 164 persons lost their lives and 149 were injured, including both

[2] President, Auer, Inc., Milwaukee, Wis.

pilots and passengers. Of the 200 accidents, 34 occurred with licensed planes, 11 licensed pilots and 22 passengers being killed. The others were charged to unlicensed craft.

In scheduled flying over definite air routes there were six fatal accidents, six pilots and one passenger being killed. The other fatalities were in the course of miscellaneous flying, experimentation, contests, races, oceanic flights, etc. Deaths in the "experimental" classification accounted to 24 and are not properly chargeable to aeronautics as a means of public or private transport.

Thus the same Department of Commerce figures which show that aviation, under certain conditions, is extremely hazardous, also show that under other conditions it is extremely safe. For, in 1928, out of a total of 10,472,024 miles flown by commercial planes on a regular schedule (air-transport classification) there were but nine fatal accidents, in which nine pilots and thirteen passengers were killed. Available figures for the same year show 1062 accidents in all classes of flying, including air-transport classification, resulting in 215 deaths and 168 injuries as against the 200 accidents for all classes of flying, with 164 deaths and 149 injured, for 1927.

Although the pilots licensed for 1928 showed an increase of 150 per cent over 1927, the accidents increased over 500 per cent, probably as a result of the large number of inexperienced men taking up flying, yet the percentage of fatalities shows a marked decline as follows: In 1927 80 per cent of all the accidents resulted in fatalities, whereas in 1928 less than 21 per cent of all accidents resulted in fatalities, and there were only 19 more people injured in 1928 than in 1927.

The normal death rate in the United States is 11.4 persons per thousand of population. Deaths from airplane flights are but 0.024. Thus, it may be seen that, under certain conditions the pilot is safer in his plane than on the ground.

At first, insurance companies would not insure any perils of the air, and it was some years before they had sufficient information to estimate the rate for insuring the average careful pilot who was not participating in so-called stunt flying. The first policies were issued to such pilots on the basis of their previous records. Later it was recognized that if these pilots were a safe risk, their passengers were also, and insurance was extended to passengers under the same conditions.

Still later, available statistics made possible the study of flights about the airports, from which a rate was estimated to cover field hops of not more than a 25-mile radius, and issued for a definite period of time, usually less than half an hour.

Until quite recently aviation insurance has been in a truly pioneer state. However, with more reliable aircraft statistics, this condition is rapidly passing away, and more progressive companies are already working on comprehensive policies along the lines already in use in other fields of transportation.

Such insurance, of course, is divided in various classes. There are policies for pilots and for passengers, graded of course as to the length of flight. Recently policies are provided for student fliers, the status of whom has not yet been thoroughly determined. Some classify them as fliers, others as passengers. The maximum policy in any class is, at present, $5000. This is for pilots and passengers. Student policies are usually for $2500 or less.

The most recent passenger-accident policy is sold on the following basis: For each $1000 of insurance a premium is charged on the number of miles of flying away from the base airport; 50 cents for a trip of 200 miles or less; 75 cents for 500 miles; $1 for 1000 miles, and $1.50 to any point in the United States in excess of 1000 miles. Insurance of this form parallels very closely the type of insurance issued with railroad tickets and is now optional to passengers.

It is either automatic or optional, and is issued in accordance with the desires of the operator. If he wants automatic insurance of this type to be in force upon the purchase of tickets, he can secure such insurance. On the other hand, if he wishes it optional, with the passenger, he can secure this form also.

Group student-accident policies are a somewhat newer development, and are intended to meet the growing demand for protection by thousands who are learning to fly. Such a policy covers accidents during flight instruction, solo flights, or accidents to the ground during instruction, as in mechanics, repair, etc. The rate of such insurance is $20 for the first $1000 coverage, $17.50 for the second $1000, and $15 per thousand dollars of coverage for the next $500, making a total of $45 premium for the coverage of $2500 which is the maximum allowed.

This policy is good for one year or the length of stay in school within the limit of 12 months, after which a new policy must be taken out. It does not, however, cover any other form of accident away from the airport. Such a policy is non-cancellable to students to whom it has already been issued, and is open to new students provided 25 per cent of the class agree to take the coverage. The non-cancellable clause in this type of insurance has made it very popular.

This covers in a general way various classes of accident insurance written, and while it is impossible at present to underwrite all aircraft personnel or passengers as a group, indications are that, with the present rapid development of aircraft mechanics, safety measures, and additional statistical information, etc., such insurance may be available in the future.

Author's Closure

Referring to the comment of Mr. Auer, permit me to submit the following more definite description of the insurance arrangements of the Transcontinental Air Transport, Inc.:

The safety of air travel as it is offered by the regular operators using a fixed and definite schedule is further indicated by the fact that an insurance company whose experience is large in the transportation field has issued a blanket life insurance policy covering passengers while on planes of the Transcontinental Air Transport Company. This policy protects each passenger individually in the amount of $5000. This protection is included in the cost of the ticket. Passengers may secure additional protection through the purchase of accident insurance which is available at all Transcontinental Air Transport ticket offices and at most railroad ticket offices.

Installing and Servicing Aircraft Instruments

By JOHN D. PEACE, JR.,[1] BROOKLYN, N. Y.

Careful installation of instruments in airplanes is most essential in order that they function properly, and remain free from trouble. The author describes details of installation that have proved satisfactory for the principal types, and points out certain things that should be avoided.

Service work on instruments is divided into two classes, periodic overhauling and inspection, and adjustments done at service stations. Engineering service to the airplane builder is considered a very important part of the instrument manufacturer's duties as it insures proper installation of the product.

THREE essentials must be considered in the installation of instruments on aircraft. First, the method and conditions of installation, so that the instrument will function properly; second, the workmanship and detail which will insure that vibration and other factors will not cause an otherwise properly installed instrument to cease to function due to breakage of tubes and the like; and third, that the installation is arranged so that the instrument can be removed quickly and conveniently for servicing without having to remove an oil tank, the whole instrument board, or two or three other instruments. The responsibility for properly observing all these conditions rests primarily with the aircraft manufacturer together with the instrument company, whose business it is to see that the plane manufacturer is supplied with the necessary data and cooperation. Unfortunately the rapid increase in production in the last two years has at times made it inconvenient for the airplane company to assign a first-class man to the work of installations alone, not only as regards instruments, but with respect to numerous other accessories as well. Fortunately this situation has been or is rapidly being corrected in most of the larger companies where project engineers are giving considerable attention to the whole subject of the workmanship and quality of their product. The instrument manufacturer is working continually toward putting more instructive data in the hands of these project engineers and the designers.

Taken alphabetically, the first instrument is the air-speed indicator which is installed on all but the cheaper ships. This instrument is essentially a pressure gage calibrated in units of speed which measures the pressure difference created in a pitot and a static tube caused by the passage of the plane through the air. The indicator itself is mounted on the instrument board, the main consideration being that it should not be subjected to excessive and possibly destructive vibration. The pitot-static combination should be mounted where it will receive an undisturbed flow of air, and not be within the propeller wash or within the high- or low-pressure areas adjacent to a wing. There remains only the connection between these two units which is usually made with copper tubing of $^3/_{16}$ in. outside diameter, using a soldered cone and a nut at the joints. Aluminum tube is sometimes used to save weight which, however, requires the use of rubber tubing for connections unless Parker-type fittings are employed. There should, of course, be no leaks in the tubing. While this condition may be observed at the time of installation, it often happens that rubber tubing will deteriorate at some inaccessible joint within a wing, a long unsupported length may crack after continued vibration, or chafing against some structural or other member may cause a leak which will destroy the proper functioning of the instrument.

It is rather disconcerting to have to open up a wing to repair a poor rubber joint which was there just because there happened to be a piece of tubing left over at the factory which was not long enough to use without joining it to another piece. Lastly, the tubing should be arranged so as to minimize the possibility of water getting into the lines, and especially to the instrument. To accomplish this, there should be a definite rise of at least three to four inches from the pitot and static openings. The lines should then progress continuously downward from the high point to a point in the fuselage, preferably two to three feet below the level of the indicator. At this point the line from the wing should come into the stem of a T. The line from the instrument should come to the top of the T while the bottom should have a removable cap to facilitate drainage.

The altimeter installation is probably the simplest of any aircraft instrument. Roughly speaking, it involves merely its mounting on an instrument board. The advent of the cabin plane, however, introduced in certain cases a new condition. Certain ships used or do use the instrument board as part of the structure or fire wall separating the engine compartment from the cabin or cockpit. The old type of altimeter, with vent holes in the back of the case, was then subjected to the barometric pressure corresponding to the altitude of the ship plus a pitot pressure in the engine compartment due to the movement of the ship through the air. Thus an altimeter, reading zero when the plane was at rest on the ground, would register minus 100 to 200 ft. at the moment of takeoff, and would not come back to zero until the ship was 100 to 200 ft. above the ground. The error introduced would also vary with the speed of the ship so that the instrument readings were more or less unreliable except as approximations. Experiments showed that sufficient leakage could be provided around the glass to make the old vent holes at the rear unnecessary. This procedure automatically caused the altimeter to read pressures corresponding to the cabin instead of the engine compartment. Like any other instrument, the altimeter should not receive excessive vibration.

The installation of a climb indicator is almost as simple as that of an altimeter, for it involves merely the mounting of the indicator, its thermally insulated tank, and the connection of the two with a tube which is supplied with the instrument. Since its operation depends on rate of change of barometric pressure which is generally quite small, it is important that the instrument receive the true barometric pressure corresponding to the altitude of the plane, and not receive pressure changes due to other causes than changes of altitude. Where the instrument is mounted on an instrument board so that it receives engine-compartment pressure, it is necessary to connect it to a static line to the cockpit or cabin. If this is not done, changes in speed of the ship will cause climb or descent readings though the altitude of the plane is not changing. In some high-wing cabin monoplanes, pulling up sharply from level flight apparently caused an increased pressure in the cabin, especially with windows open, which would cause a momentary descent indication before the instrument showed a climb. This can be cured by venting the instrument to the rear of the fuselage or to the static line of the air-speed indicator.

The magnetic compass has probably been poorly installed more often than any other instrument. The earth's magnetic field is rather weak, and unfortunately the compass responds to

[1] Pioneer Instrument Co., Brooklyn, N. Y.

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the influence of any magnetic force present rather than responding to the earth alone, so that often one is confronted with quite a problem. In cabin ships with the pilot close to the motor, magnetos, generators, and electric starters on the rear of an engine, make things rather hard for the compass. There are two rules to follow. First, the compass should be placed in a location as far from magnetic disturbances as possible and second, as convenient as possible for the pilots observation. Often the two conditions conflict materially so that the best location may be a compromise of the two. As a general statement, a location which introduces more than 30 deg. maximum deviation before compensation should not be used. The compass should always be compensated and excessive vibrations should be avoided. The earth-inductor compass usually permits of a very advantageous location of the generator in the rear of the ship away from the motors. The indicator of this compass, which is a sensitive galvanometer, should be kept as free from vibration as possible.

Practically nothing need be said of the engine instruments except the tachometer. The tachometer shaft should be run as straight as possible without sharp bends. Shafts which bind or which have a number of sharp bends may cause a fluctuation in the instrument.

The turn-and-bank indicator rarely presents any trouble. A venturi tube which is mounted in the slipstream provides the power for operating the gyro wheel of this instrument. Failure of the instrument to operate can usually be traced to a poor location of the venturi tube. The general practice on mail planes at the present time is to mount the venturi somewhere on the exhaust pipe to prevent ice formation which might cause the instrument to cease working.

The service work on instruments may be divided into two classes, periodic air-line service, and occasional service as rendered at a service depot. Almost from the start the National Air Transport has had an instrument department which has made a careful study of the instrument problems on their lines, and has endeavored to keep their instruments in good working order at all times. Whenever a plane is in the shop for overhaul, all instruments are tested, not only to see whether or not they operate but also for accuracy, sensitivity, friction, etc. When the plane is checked out of the shop as being ready for operation, the instruments are all in first-class condition. This work is always done whether or not the pilot reports a faulty instrument. A plan worked out by National Air Transport some time ago is being adopted at the present time on several new ships. This consists in the main of mounting all of the flight instruments on a small demountable panel on the instrument board proper. This small panel can then be removed easily from the ship for testing during motor overhaul, and a spare can be carried on hand for replacement in a ship during an emergency. The overhaul of instruments consists of checking for calibration and friction, oiling, and replacing minor parts where necessary. As the need for them arises, service depots are being established which will make it possible for an individual to have any necessary work done on instruments. This will include the testing or checking of any instrument, minor repairs, compensating compasses, and all other details incidental to the proper maintenance of aircraft instruments.

Engineering service to a manufacturer is very important from the standpoint of insuring the proper installation of the product. This work includes the selection of proper locations for compasses and pitot tubes, advice as to types of instruments most adaptable to a customer's plane, provision for proper clearances, and the design of instrument boards. It may involve several visits to a factory to see that all the details of an installation are carried out properly. To generally improve their installations a number of companies have already sent men to spend some time at an instrument factory, and with the service department to improve their knowledge of the instruments which they place in a plane.

The past year has been the first in which there has been any amount of aircraft instrument servicing on a commercial basis. Each of the air lines has established or is arranging to establish this service for themselves. To the air line it is vital that their instruments function properly. This will undoubtedly be followed by service stations all over the country which will take care of the private individual in a manner similar to the automobile service station of today.

Discussion

W. G. Brombacher.[2] This paper calls attention to an important, and up to the present, a much neglected aspect of aircraft instruments. The conditions under which aircraft instruments operate are so severe that considerable care must be taken in their installation in order to obtain the most favorable operating conditions, and further, owing to this fact, more servicing is also found necessary.

The installation problems of the instruments ordinarily installed on aircraft have been fully presented by Mr. Peace. One additional point might be mentioned. If the aircraft operator for any reason is interested in obtaining the air speed accurately, a calibration of the completely installed pitot-static air-speed indicator must be made over a speed course or its equivalent. This is due to the fact that flow of air around the pitot-static tubes now commonly used is subject to interference from the structure of the aircraft. In order to eliminate this interference special pitot-static tubes must be used, and precautions observed which are impractical in the ordinary operation of aircraft. It is suggested as a possible requirement that the completely installed air-speed indicator of every individual ship be calibrated over a speed course at two speeds, one near the stalling speed and the other at the cruising speed. The results of this calibration should in some way be made available to the pilot, preferably by marking it permanently on the dial of the indicator.

The need for servicing aircraft instruments is similar to that now followed in servicing the speedometers of automobiles, service stations for which are maintained by manufacturers in every large city. Largely owing to the severe conditions of use, aircraft instruments are in frequent need of minor repairs, and often require adjustment in order to maintain their accuracy. As has been pointed out by the author of the paper, a start has been made by a few of the larger operating companies in organizing instrument-maintenance units. The organization of such units will be accelerated no doubt by the emphasis which is now being placed on "blind" or instrument flying during which complete dependence must be placed upon a certain group of instruments.

Author's Closure

The discussion calls attention to the fact that in order to obtain an accurate calibration of the air-speed indicator, it is necessary to run a plane over a speed course. While this is true as regards absolute accuracy, it is hardly necessary to go through such a procedure in the case of a production series of airplanes. Such tests would be of value on the first one or two of a production series in order to determine a good location of the pitot-static tube.

Dr. Brombacher also calls attention to the fact that not all aircraft operators can employ an instrument man among their personnel. To take care of this situation, there are already being established a number of service stations at strategic points throughout the country, which will be able to take care of any required service.

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Some Aerological Principles Applying to Airship Design and Operation

By F. W. REICHELDERFER,[1] LAKEHURST, N. J.

In this paper the author discusses some of the problems of wind structure and of vertical air currents. Data are presented on local wind fluctuations of great frequency, on less-frequent, wide-spread wind shifts, and on local vertical air movements. The discussion is not primarily of strong winds and gales as a navigational problem affecting cruising radius, but of those properties of wind structure which may be obstacles to air navigation because of the excessive forces which they may exert on airship structures.

THE physical properties of the atmosphere, especially its composition, distribution of pressure and temperature, and the average changes in these properties as they relate to airship design and operation are well known through the science of aerostatics. The forces relating to movements of powered aircraft through the air are treated in aerodynamics. In general, both of these sciences, in order to present their material systematically, regard the atmospheric elements as continuous phenomena, and overlook the marked discontinuities which occur; or, if they consider these abrupt variations in physical properties of the air with time and place, they treat them in their average form. In reality, however, the atmosphere rarely conforms to an average behavior. Its changes are not usually systematic in any simple manner. This characteristic is well demonstrated in the fickleness of the weather.

Rapid changes occur in practically all of the so-called weather elements—pressure, temperature, humidity, wind, clouds, and so on. All have their influence on every-day airship operations. The most capricious weather element, and the one which is most interesting in its possible influence on all branches of aeronautics, is the wind. The subject of wind structure, referring to the character of horizontal wind flow and sharp discontinuities therein, and to local vertical air flow in clear air and in thunderstorms where the most violent vertical air movements are found, is of importance to all types of aircraft, but it has special applications to lighter-than-air craft. This subject, while not so important during the experimental stage of lighter-than-air development, becomes increasingly important as commercial airship operation approaches.

Airships operating under commercial requirements will encounter wind conditions which ships under experimental operation could avoid. Reference here is not primarily to strong winds and gales as a navigational problem affecting cruising radius, but to those properties of wind structure which may be obstacles to air navigation because of the excessive forces which they may exert on airship structures.

Just now, it is important to find out what actually goes on in horizontal and vertical air movements. Some of the problems of wind structure and structure of vertical air currents have been attacked with some success from the theoretical and mathematical side, but in most cases it has been impossible so far to express as exact mathematical functions the large number of variables entering into atmospheric processes. (See reference A, and further references contained therein.) The data presented here pertain to (1) local wind fluctuations of great frequency (gusts); (2) less-frequent, wide-spread wind shifts (squall lines, discontinuities between air masses); (3) local vertical air movements.

(1) It is common observation that the wind blows in gusts. The speed of the wind, whether a gentle breeze or a gale, varies greatly from second to second. The wind direction, likewise, changes from moment to moment, swinging irregularly to one side or the other of the mean direction. The records of wind-recording instruments show that there is no simple periodicity about these fluctuations. For practical purposes they appear largely "accidental."

Fig. 1 shows a portion of a record of wind direction and wind speed at the naval air station, Lakehurst, N. J. It illustrates the irregularity of the wind. The lower graph gives wind speed,

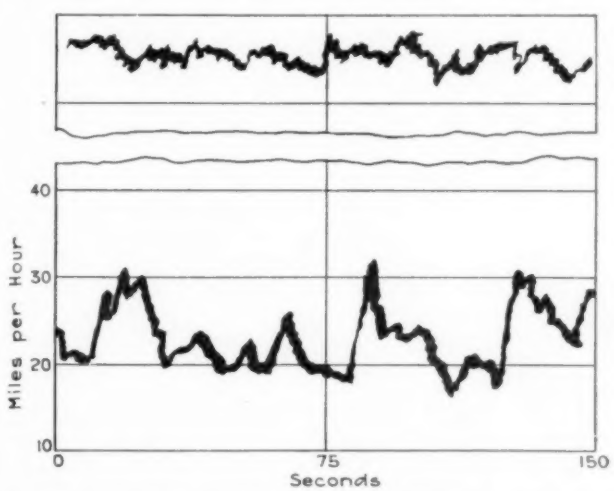


FIG. 1 GRAPHICAL RECORD OF GROUND WIND TAKEN BY AN ANEMOBIAGRAPH OF THE DINES PRESSURE-TUBE TYPE

each small ordinate representing two miles per hour (2.93 ft. per sec.). The upper graph gives wind direction, each ordinate corresponding to 20 deg., angular measure. The heavy ordinate, just below which most of the graph lies, is northeast (45 deg.). Direction and speed are plotted against time, the time interval between each vertical graduation being 75 sec. In Table 1 values are given for direction and speed of the wind during the 20 seconds beginning with the vertical line to the left in Fig. 1, and also for the 72, 73, and 74 seconds.

This example represents the usual character of the wind. It was taken at random. It shows the rapid changes in direction and speed which the ground wind passes through continually even over flat, unobstructed country. This record was made by an anemobiagraph of the Dines pressure-tube type. Doubtless this instrument fails to record some of the sharper and briefer fluctuations in wind speed and direction. Measurements by hot-wire anemometer show the existence of these more rapid fluctuations. Although there is some uncertainty as to the degree of accuracy of measurements of natural wind by hot-wire anemometer, owing to difficulties of exposure and instrumental errors, there is little doubt that this instrument is de-

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cidedly more sensitive than the anemometers commonly used for gust study. It appears to be a close approach to expressing the actual turbulence of wind flow near the ground. These very rapid fluctuations are tabulated, not because they are at present of direct practical value to airships, but because they are of interest in the study of wind structure.

TABLE 1 WIND FLUCTUATIONS, DINES ANEMOGRAPH

Wind (seconds)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	72	73	74
Direction (deg.)	60	62	55	60	50	52	51	50	49	43	43	50	49	44	61	59	59	59	72	65	67	90	35
Speed (m.p.h.)	23	23	22	21	20	19	20	20	20	21	20	19	18	20	22	26	28	23	28	30	19	18	18

TABLE 2 WIND FLUCTUATIONS, HOT-WIRE ANEMOMETER (REF. B)

Time interval	Character	Velocity fluctuations within time interval shown at left	Accompanying vertical fluctuations
0.5 sec.	Intensity above avg.	31 to 37 to 31 to 42 m.p.h.	Up to 16° incl.
0.5 sec.	Intensity above avg.	26 to 42 m.p.h.	+10 to −16 to +13
5.0 sec.	Wind "surge"	29–41–29–31–46–31 m.p.h.	+16 to −18 (1 s.)
0.3 sec.	Strong gust	26 to 46 m.p.h.	+16 to −16

The maximum wind acceleration measured by the hot-wire anemometer (ref. B) was at the rate of 24 meters per second per second (54 m.p.h. per sec.). The maximum change in wind direction was at the rate of 180 deg. per sec. It should be noted that the fluctuations did not actually pass completely over the maximum ranges mentioned, but that the fluctuations over a fraction of a second were at the rates stated. Further, fluctuations of this order of magnitude were not sustained. The element either returned quickly to about its mean value or went beyond toward the opposite extreme. These very large but transitory fluctuations from the "mean wind" are of little significance in airship design and operation. The cycle is passed before its maximum phase has exerted sufficient force to cause serious strains in any portion of the ship.

It is the fluctuation in wind speed or direction, or both simultaneously, which is sustained for several seconds that is a factor in airship handling. If the wind were steady in flow, it could blow with rather high velocities without having much effect in ground handling or in riding at the mast. It would be possible to estimate its effect and make provision for meeting it. With wide and sustained fluctuations in velocity and direction at irregular time intervals, however, it is difficult to provide handling lines and mooring devices which will meet the changing forces in strong winds. At the mast, airships have without difficulty ridden out strong winds. When the average wind speed is 35 or 40 m.p.h., and there are gusts up to 55 or 60 m.p.h., with the maximum value sustained for 5 or 10 sec., and at the same time a sustained change in wind direction, the forces exerted by the wind cause undesirable strains in the ship. Again, a sudden marked change in wind direction even though unaccompanied by an increase in wind speed may, if the speed averages around 50 m.p.h., produce large strains. This fact is not surprising when one considers the dimensions involved. When a ship 800 ft. long is moored to a mast so that its tail swings freely, the tail describes a circle about 5000 ft. in circumference. If there is a wind shift of 45 deg., the tail must move through an arc of about 600 ft. to place the nose directly into the wind. It requires appreciable time to overcome the initial inertia of the ship, and move it through the required distance, and during this time the wind exerts considerable forces against the side of the ship.

Most wind gusts fall well within the allowable limits of direction and velocity fluctuations, but occasionally there are winds with gusts which approach critical values. Some of the largest values obtained in gust records at Lakehurst are given in Table 3. These apply to fluctuations in gusts in the same general wind stream and not to the more violent wind shifts between air masses of different origin and density.

It is unlikely that gusts retain their individual form for very long periods of time or over considerable distances. An individual gust is constantly changing in form. There is little definite information about their usual dimensions in space and their rates of travel. They appear to travel for short distances at approximately the speed of the mean wind. Data at hand indicate that some gusts extend several hundred feet in the direction perpendicular to their line of travel. The height to which a strong surface gust ordinarily reaches is unknown. The approximate dimension of a gust in the direction of its travel can be computed on the assumption that it travels at the speed of the mean wind.

Using the first example in Table 3, in which the gust was of 90 seconds duration, and then returned to the mean value or slightly below, the front to rear dimension of the gust would be about one mile. The examples given do not, it is likely, mark the limit of intensity of gusts within the same wind stream (as distinguished from the gusts in wind shifts to be mentioned later). It is for the purpose of obtaining more information on the usual dimensions of gusts and their circulation and propagation that the gust measurements at Lakehurst, Cardington, and other places are being carried on. This information will apply in design of ground-handling equipment and methods.

Gustiness in the surface wind is largely the turbulence caused by the friction between the wind and the ground. This cause is aided during the day by convection resulting from the heating of the ground. The temperature distribution in the particular air mass constituting the wind is a factor in gustiness. If it is a north wind and is bringing colder air, the lagging of the surface layer will permit the cold air to come in more rapidly aloft, and result in instability due to the relatively greater density of the layers above. This leads to the breaking through of the air from above, giving the bursts or "surges" of downward gusts which characterize a cold north wind. A south wind, on the other hand, is usually less gusty in velocity because it brings warmer air aloft more rapidly than at the surface and results in thermal stratification and stability. There are a few other characteristics of gusts which may be referred to very briefly. An ocean wind is usually less gusty than a continental wind. The wind is less gusty at night than during the day, not only because of the absence of convection which would increase the mixing and turbulence, but also because a temperature inversion is often built up by radiation from the ground, thus increasing the vertical stability of the air.

As is to be expected from consideration of the causes of gustiness, no simple periodicity or regular structure is apparent either in wind-direction fluctuations or in wind-speed changes. It has been suggested by some meteorologists that air layers of different velocity generate a cylindrical motion in the air in their

TABLE 3 WIND CHANGES IN GUSTS AT LAKEHURST

Mean wind	Wind-speed change (m.p.h.)	Time interval	Rate, ft./sec.	Sustained speed and interval	Accompanying change in direction	Time interval of direction change
40 m.p.h.	27–45	4 sec.	7	40–45 for 90 sec.	veer 30°[1]	2 sec.
35 m.p.h.	24–44	6 sec.	5	40–45 for 75 sec.	v. 20° at end	3 sec.
35 m.p.h.	32–50	3 sec.	7	40–45 for 45 sec.	30° and held 2 sec.	1 sec. (30 sec. after crest)
35 m.p.h.	31–49	2 sec.	14	40 m.p.h. for 65 sec.	20° gradual	60 sec.
30 to 35 m.p.h.	20–42, then	10 sec.	3	40 m.p.h. for 180 sec.	N and NE, 35°	5 sec.
	42–29, then	12 sec.	1.5	40 m.p.h. for 180 sec.	backing, then	2 sec.
	29–36, then	3 sec.	3	40 m.p.h. for 180 sec.	10° veer, then	15 sec.
	36–34, then	3 sec.	1	40 m.p.h. for 180 sec.	45° veer, then	10 sec.
	34–55	7 sec.	4	40 m.p.h. for 180 sec.	steady 2 sec., then 45° back	... ...

([1] 30° veer momentarily)

boundary zones and that part of this motion is transmitted in gusts. There is apparent a very rough regularity of recurrence in direction and velocity changes in gusts. It varies in different winds, and from place to place. Computations of average variations as regards intensity and frequency give results of about the same order for different regions. Thus, gust measurements in Europe (ref. C) as well as in the United States show that the magnitude of the crests and lulls varies in rough proportion to the average wind speed. The variation is roughly 50 per cent of the mean wind. If the mean is 30 m.p.h., the gust peaks reach, on the average, 40 or 50 m.p.h., and the lulls, 10 or 15 m.p.h. The frequency of well-marked cycles (from one marked peak to the next) in wind speed is roughly 30 to 50 cycles per hour. Marked fluctuations in wind direction, usually in the form of an abrupt and momentary swing of 15 deg. or more from the previous direction, followed by a partial or complete return to the previous direction, occur with a frequency of three to six per minute, on the average. There are usually minor fluctuations in direction which average as many as forty or fifty per minute. These average values, obtained from Dines anemobiagraphs, are relative only. As stated before, the more sensitive hot-wire anemometer shows many sharp and momentary fluctuations not shown by the ordinary anemometer or anemobiagraph. The increase in wind speed in a gust often comes in surges, usually in the form of one or two momentary interruptions to the rise. Sometimes the surges take the form of double crests. Fig. 1 shows that there are numerous ripples superimposed on the larger waves. A characteristic which is often important in airship handling is the tendency for the wind after a gale to pass through three or four large cycles in mean wind speed and direction, each cycle covering 30 to 60 min., with the cycles gradually decreasing in amplitude. Another characteristic which is regularly used in airship docking and undocking is the well-defined sunset lull which occurs on most clear days. The wind decrease at sunset often remains through the night, but frequently the sunset lull is followed by an increase. The averages of wind velocity on clear days show a slight tendency for the wind to lull at sunrise also, but this lull is much more uncertain and ill-defined than the sunset lull.

The foregoing tables apply to surface winds. There is very little available in direct measurements of the gustiness at usual flying altitudes, that is, altitudes above 1000 ft. There are great difficulties in exposing gust-measuring instruments at high altitudes. It is known that gustiness in a general wind stream decreases rapidly with altitude, and practically disappears at a few hundred feet above flat country. Over mountainous country, surface gustiness in strong winds may extend to 1000 ft. or more above the ground. The vertical extent of the wind shifts between different air masses is referred to in Table 5. Turbulence having the effects of gusts is produced at high altitudes by thunderstorms. These two causes of turbulence aloft—namely, general wind shifts and violent convection as in thunderstorms—are by far the most important in producing sharp discontinuities at flying altitudes. Other causes of turbulence aloft, such as waves at the horizontal boundary between two air layers of different velocity, and interruption of air flow by rough terrain, usually have no appreciable effect other than to make the air "rough" for aircraft.

Some of the gustiest winds at Lakehurst, as regards changes in direction, occur when the station lies on the boundary between southerly winds associated with the general pressure gradient, and the southeast sea breeze. The boundary is often sharp, and the two wind currents sometimes blow side by side over the same locality for an hour or more. There are usually large temperature and humidity differences between the two currents, making the boundary turbulent. The instability of the air along this surface of discontinuity makes very rough going for aircraft. The surface wind speed under these conditions rarely becomes strong, seldom over 20 or 25 m.p.h., but the direction often swings through an arc of 135 deg. to 180 deg. within one or two seconds as a bulge, from first one air mass and then the other, passes over a given point. Usually the extremes of these direction fluctuations are held only momentarily, followed by a return to within 50 deg. or 60 deg. of the mean wind, but it is not infrequent to have an abrupt shift under these conditions in which 45 deg. to 90 deg. of the change is not momentary. The wind often continues in the new direction for several minutes.

(2) The sea breeze boundary effect has been described because it illustrates on a small scale the atmospheric distribution which gives rise to some of the most violent gusts to be encountered; namely, the gusts which occur at the boundary between general air masses of different density, usually the boundary between a warm, moist, and therefore relatively light air mass moving as a south or southwest wind, and a cold, dry, and therefore denser air mass moving as a northwest or north wind. These are the conditions commonly associated with a well-defined V-shaped "low" on the weather map. The warm southerly wind lies in front (eastward) of the trough line, and the cold northerly wind, just behind the trough. Under favorable conditions the cold northerly wind under-runs the warm southerly with the boundary so sharp as to give a wind shift from southwest to northwest within a very few seconds, and an accompanying velocity increase from 10 or 15 m.p.h. to 50 or 60 m.p.h., and occasionally to 75 m.p.h. or more within four or five seconds. The mass of cold air sweeps over a locality like a wall of water. These are the conditions which give the abrupt changes in temperature of 20 deg. to 40 deg. experienced occasionally in the United States. Usually such a violent under-running is accompanied by convection which leads to thunderstorms, and the violence of the wind shift between air masses is sometimes increased by the squall wind. There are lows in which this abrupt wind-shift line extends for hundreds of miles along the trough with severe wind squalls and thunderstorms over most of this distance. It sweeps across the country in a direction approximately at right angles to its length. This is a line squall. Frequently, in the summer time, a "heat" thunderstorm develops until it covers 500 sq. mi. or more and produces a wind shift which is an air-mass boundary wind shift on a small scale. The thunderstorm develops a cold-air mass on the ground in the region of heavy rainfall. This cold air under-runs the warm air in front of the storm producing a sharp surface of discontinuity and changes in wind direction and velocity quite as intense as those in a general wind-shift line. Viewed from a single point on the ground it may have all the appearances and produce all the effects of a general line squall, except that the wind often returns to its southerly or other

TABLE 4 WIND SHIFTS IN THUNDERSTORMS

Type	Change in wind speed (m.p.h.)	Time interval	Accompanying change in direction	Time int.	Remarks
Thundersquall	5 to 24, then stdy. 1–2 min., then 24–37	5 sec. 1–2 min. 5 s. or ls.	135° 45° return 90° return to 1st dir. 2 min. later, 135° op.	5 sec. 1–2 min. 5 sec.	Avg. wind speed 20 to 25 m.p.h.
Line squall	Calm to 35 m.p.h.	10 s. or ls.	90° S.S.E. to W.S.W.	1 sec.	10° temp. fall
Thunderstorm	4 to 18, then after several minutes	10 s. or ls.	.....		
	25 to 59	10 s. or ls.	.....		
Thunderstorm	12 to 53	8 s. or ls.	45°	1 sec.	.....
Thunderstorm	16 to 48, then ½ hr. later 30 to 74	Ls. than 1 min. 20 s. or ls.	No abrupt shift 135° in 10 min. Not abrupt		.....
Thunderstorm followed 2 hr. later by line squall	18 to 65, then 16 to 85, then 85 to 100 4 to 34	5 s. or ls. 5 s. or ls. 5 s. or ls. 30 s. or ls.	Uncertain, ls. than 45° 90°	.... Ls. than 5 s.	20° temp fall Few seconds betw. rises 6° temp. fall

previous direction, after the local thunderstorm has passed.

The cold air in the rear of a trough sometimes over-runs the warm southerly current instead of under-running it. In such case the resulting instability soon produces convection and thunderstorms.

Table 4 lists some of the violent wind changes which have been observed in connection with wind-shift lines, both line squalls and local thunderstorms. These data refer only to effects in the horizontal wind and not to the vertical currents in thunderstorms. While the violent effects of a local thunderstorm may frequently be avoided by detouring, the line squall of similar violence sometimes extends for long distances, making it impracticable to detour and necessitating use of some other means to avoid it.

The conditions described by the data in Table 4 are not infrequent. Wind shifts of this order of intensity occur once or twice a year in most portions of the United States. Many regions have them several times a year. Such shifts are not peculiar to the United States. Some of the examples are from European weather records. Most regions of the globe experience wind shifts either from line squalls or from thunderstorms, or both.

An interesting feature of the first example is that it occurred at night when the sky had been generally clear until the squall cloud arrived, and that the wind shifted through 270 deg. within a period of eight minutes. This occurred at Lakehurst. The last example, which occurred near Washington, D. C., is interesting in that this very violent thunderstorm came more than two hours before the line squall and general wind shift. The winds in the storm were extremely violent, and attained their full force within a very few seconds after the first burst of the storm. It is not implied that these data represent the most abrupt changes that occur in wind speed and direction. The most violent local wind conditions are those found in tornadoes. From the average diameter of these storms, their rate of travel, and wind speeds which are without doubt often in excess of 200 m.p.h., it is readily computed that a point directly on the line of travel must experience a wind increase from 30 m.p.h. or less up to 200 m.p.h. or more, and a complete reversal of wind direction within 15 sec.

The shifts in the surface wind, as tabulated, are of particular interest in connection with ground handling and mooring. Such shifts as these often extend to considerable altitude, and are therefore also of importance to airships in flight. Accurate data are scarce, but the observations in Table 5 give some indication of the vertical extent of wind-shift surfaces.

TABLE 5 WIND SHIFTS ALOFT (REFS. D AND E)

Wind shift	Wind speed before shift	Wind speed after shift	Minimum altitude to which shift was observed to extend	
E.S.E. to W.S.W., abruptly	30 m.p.h.	30 m.p.h.	3000 ft.	Both of these shifts were abrupt, probably less than 1 min.
S.S.W. to N.W.	33 m.p.h.	33 to 55	2500 ft.	

(3) While the shifts in horizontal air currents are important, they are not, in some respects, as important as the vertical currents associated with violent convectional phenomena. A sharp wind discontinuity imposes unusual forces on an airship, whether exerted laterally as in a wind-shift surface or vertically as in the convectional current of a thunderstorm, but the vertical current necessitates in addition that the ship be prevented from going too far aloft or too far downward. Direct measurements of vertical currents are difficult to obtain. There are numerous indirect measurements which demonstrate that vertical currents, especially ascending currents, may attain very high velocities.

The thunderstorm cannot be considered in detail here. The processes involved have been described elsewhere (ref. F). It is interesting to note, however, that saturated air ascending from 2000 or 3000 ft. to approximately 10,000 ft. would by virtue of its latent heat become "potentially" about 20 deg. warmer than the surrounding air at that level providing the temperature gradient is adiabatic. Under favorable conditions the thermal energy produced is available for formation of violent ascensional currents within the cloud. The thermodynamics of convectional processes is discussed briefly in reference A. Table 6 gives some of the observed and estimated vertical speeds of ascending and descending air currents.

A larger number of cases showing measurements of ascensional currents obtained like those in the table, more or less accidentally, could be listed. Circumstances indicate that these measurements do not represent extreme values. It is highly improbable that the balloons and recording instruments which supplied these measurements got into the most violent vertical currents. There is a mass of data indirectly substantiating the conclusion that the velocities listed are not infrequent in thunderstorms.

It is important to know something about the boundary of vertical currents typified by the foregoing examples. The airship designer needs to know whether the change from air without vertical motion to air with the excessive ascensional speeds mentioned is gradual or whether it occurs within 1000 ft., for example; also, whether the transition zone is turbulent or not; also, whether ascending and descending currents of considerable speed can exist in such proximity as to place the nose of an airship in the descending current while the tail is still in the ascending current, or vice versa. Information on this subject is far from complete.

An indication is afforded by the sharpness of boundary surface between air masses which give abrupt shifts in horizontal wind. If the second example in Table 5 is considered as a 90-deg. shift, from southwest to northwest, in which the average speed of each wind current is 30 m.p.h., giving an "easting" of about 21 m.p.h., and the interval between the full force of the two currents is 30 sec., the width of the boundary zone is 900 ft. or less. It is to be noted further that the two currents with respect to each other are blowing in opposite directions, instead of perpendicular to each other as it appears from a point on the ground, and that an airship passing directly across the boundary would experience a change in wind from south, 21 m.p.h., to north, 21 m.p.h., in a distance of 900 ft.

Evidence that similar sharp boundaries are found between vertical currents, especially under thunderstorm conditions, is given by observations of the behavior of cloud wisps in a line squall and of the circulation of the squall cloud in a thunderstorm. The squall cloud forms occasionally between the ascending and descending air near the front of the thunderstorm. Observations of the dimensions of squall clouds have shown that the distance between the strong ascending and the marked descending currents is often less than 1000 ft. This conclusion is substantiated by the experiences of aircraft which have been caught in this portion of a severe thunderstorm.

Another aspect of the study of wind discontinuities is the subject of means for identifying and locating intense discontinuities. There is, of course, no sharp division between the mild and visible

TABLE 6 VERTICAL CURRENTS IN THUNDERSTORMS

Type of current	Speed, met./s.	Speed, ft./s.	Source of estimate
Thunderstorm ascending	5	16	Aerograph on box kite which broke away
Thunderstorm ascending	8	26	Pilot balloon by range finder
Thunderstorm ascending	8	26	"End" velocity of largest raindrop
Thunderstorm ascending	10	33	Manned balloon
Thunderstorm ascending	16	53	Velocity required to support hailstone 2 cm. in diameter
Thunderstorm ascending	24	78	Velocity required to support hailstone 4 cm. in diameter
Thunderstorm descending	3	10	.....
Thunderstorm descending	5	16	.....

convectional current encountered as a "bump" on almost any clear summer day, and the very strong uprush of air in an intense thunderstorm. A light ascensional current of a meter per second may grow into a current rising at five to ten meters per second, still without clouds to indicate its presence. If a cloud is formed, and develops first into a shower, then into a thunderstorm with hail, the vertical currents within the cloud may reach the high speeds given in Table 6. Again, a thunderstorm associated with a suitable barometric depression may merge into a tornado with its extremely violent wind conditions. Obviously, aircraft of all kinds must avoid these extreme conditions as an ocean vessel avoids a derelict or an iceberg.

Cloud appearance is a rough indication of the presence and intensity of violent wind shifts. The more ugly the cloud appearance, and the greater the electrical activity in a thunderstorm, the more violent the wind conditions, as a general rule. Visible indications are not always reliable, however. Under certain conditions, ascending currents of five or more meters per second and wind shifts as violent as those given in Table 4 are unattended by clouds. Two other general methods of identification are available. They are both indirect. First, the use of the daily weather map to recognize atmospheric conditions which give rise to wind discontinuities. Second, the use of instruments which give an indication of violent conditions at a distance.

There is not space here to discuss in detail the use of the weather map in forecasting wind shifts and thunderstorm conditions. The comparatively recent development of methods of forecasting based upon air-mass analysis, rather than upon an objective survey of high and low areas on the map, opens the way to more definite and detailed information of storm structure of fundamental importance in airship operation. A single example is given to illustrate the aerological principles underlying this method. In winter and spring it is not unusual for a bulge of warm equatorial air from the Gulf of Mexico to ride up over a mass of cold northerly air overlying the southern states. This usually causes formation of an ill-defined but heavy rain-producing low. If the temperature-lapse rate of the southerly air exceeds the adiabatic rate for saturated air, thunderstorms develop, once condensation gets well under way. In such case, the turbulence of the thunderstorm is usually confined to the upper layers because the wedge of cold northerly surface air forms a barrier to the penetration of the thunderstorm circulation to the surface. An airship can, therefore, avoid the thunderstorm condition by keeping close to the surface, well within the cold wedge. An analysis of air-mass distribution on the weather map including upper air maps makes it possible to recognize this storm structure when it occurs. The procedure referred to here has been employed on one occasion for a storm of this type by the airship *Los Angeles*.

Three general types of instruments employing different principles for identifying severe thunderstorms or conditions giving rise to them are described briefly. All are still in the experimental stage. The first is use of a meteorograph or aerograph to obtain temperatures and humidities of air strata aloft. These, together with upper-wind data obtained by pilot balloon, are used to determine the thermodynamic structure of the air at a given time and place. This enables one to draw conclusions as to whether conditions are favorable for thunderstorm development (ref. H).

A second type of instrument for identifying severe thunderstorms is one which measures the potential gradient of the atmosphere. The gradient in the vicinity of a thunderstorm is usually very steep and distorted. To some extent, the electrical activity of a storm is proportional to the intensity of its vertical circulation. This instrument, like an electrometer in principle, makes it possible to determine when the gradient exceeds a certain critical value. The possibilities of this type of instrument are uncertain. The electrostatic effects of a thunderstorm probably do not extend to great distances from the storm, and the instrument, if successful at all, would be indicative only at close range.

The third type of instrument is the ceraunograph, or lightning and static indicator, which makes use of the electromagnetic effects of the thunderstorm. It is well known that "static" from a thunderstorm may be heard in radio receivers many miles away. A type of static indicator now under test at Lakehurst and Anacostia utilizes two fixed-loop antennae placed at right angles, each with its own receiver connected with the plates of a cathode-ray oscillograph. The deflection of the oscillograph beam gives not only a measure of the intensity and the form of the waves intercepted by the loops, but also the line of direction on which the wave is traveling. Simultaneous bearings from the instruments, one at Lakehurst, the other at Anacostia, make it possible to find the location of the static disturbance, and thus to locate a violent and isolated thunderstorm. This instrument has been used with success in England. Incomplete tests in this country indicate that it will find important applications in airship operations of the future.

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The Selection of an Airplane Engine

By LT.-COMDR. J. M. SHOEMAKER,[1] U. S. NAVY

The author states that the commonly accepted fundamental requirements of an aircraft engine are: minimum weight per horsepower; maximum dependability, durability, and economy; minimum cost; and maximum ease of maintenance. He discusses the relative importance of these requirements, from the point of view of the airplane designer, in the power plants of various types of airplanes, and adds to them another requirement, namely, the consideration of an airplane engine as an integral part of an airplane.

THE fundamental requirements of an aircraft engine have been enumerated many times. The commonly accepted requirements, set down without reference to their relative importance, are:

1 Minimum weight per horsepower
2 Maximum dependability
3 Maximum durability
4 Maximum economy
5 Minimum cost
6 Maximum ease of maintenance.

The author proposes to discuss the fundamental aircraft-engine requirements as to their relative importance in the power plants of various types of airplanes, and to amplify the foregoing list of requirements by the consideration of an airplane engine as an integral part of an airplane.

It is appropriate to define and discuss briefly the fundamental aircraft-engine requirements from the point of view of the airplane designer.

Minimum Weight per Horsepower

Up to the present time, the development of aircraft engines has been measured in terms of "pounds per horsepower." The specific weight of an airplane engine is one determinant of the gross weight of the airplane. The heavier the engine for a given power, the greater the wing area needed to support it. Greater wing area means heavier airplane structure, which requires more power for a desired performance. Considering the aircraft power plant as a whole, the engine must be charged with the weight due to the cooling system, necessary engine accessories, and propeller. From this viewpoint the air-cooled engine has a decided advantage over a water-cooled engine, as has the high-speed engine with its small propeller over a slow-turning or reduction-geared engine with its large, heavy propeller. Other requirements, as will be pointed out later, may operate to make the case against the heavy engine less conclusive.

Maximum Dependability

All heavier-than-air craft, depending as they do upon propeller thrust to overcome the force of gravity, must have power plants which are as nearly perfect as possible as regards dependability. Here again the power plant must be considered as a whole. Fuel- and lubricating-oil tanks and pipes, ignition, and cooling-system dependability are chargeable to the airplane power plant. Not only must the engine function perfectly throughout every flight, but the whole power-plant installation must be simple and rugged enough to obviate failures in the air. The water-cooled engine, with its cooling system, is again placed at a disadvantage as regards simplicity, and the extra plumbing involved must be carefully designed and maintained to insure dependability.

There has been a great deal of discussion among engine and plane builders as to the responsibility for power-plant failures outside of the engine itself. At present the engine manufacturer considers himself responsible for abnormal fuel or oil consumption and ignition and engine failures; the propeller manufacturer takes the responsibility for propeller failures; and the airplane manufacturer is blamed for any failure of engine mount, plumbing, or tanks. Failures which are due to excessive engine vibration should be charged to the engine, as should failures due to complicated plumbing (fuel, oil, or water) required by some engines because of poorly designed pumps and outlets.

Maximum Durability

Aircraft-engine durability is measured by the operating hours between overhauls. The durability of an airplane's power plant can be measured by that of the engine, as the other components of the power plant should all be more durable than the engine.

Maximum Economy

Specific fuel consumption, next to weight per horsepower, is probably the most important determinant of the efficiency of an aircraft engine. Most conventional engines are equal as regards specific fuel economy, although some of the less efficient designs of air-cooled engines require an abnormal fuel consumption for sufficient cooling. The impending entrance of an air-cooled Diesel engine into the family of successful aircraft engines requires more attention to specific fuel consumption as a component of specific engine weight. In other words, it may be highly desirable to consider the specific weight of an aircraft engine as installed as the sum of pounds per horsepower of complete engine with cooling system plus specific fuel consumption (multiplied by intended duration of flight).

It is somewhat of a problem to choose the percentage of engine maximum brake horsepower at which to compare specific fuel consumptions. The nature of the service expected of the airplane power plant varies widely with different types of airplanes. Military airplanes and commercial planes operating under large loads per horsepower must pay particular attention to fuel consumption at or near full power. Lightly loaded planes, on the other hand, will normally fly with the engines well throttled, and the specific fuel consumption under these conditions is of particular interest. Since the fuel-consumption curves of most engines are of approximately the same shape, it is believed that the specific fuel consumption at 60 per cent of full power on the propeller load curve (at which point r.p.m. = 0.843 of full-power r.p.m.) is probably as good a point for comparison as any.

A study of the power curves for various engines indicates that the full-power specific fuel consumption varies from 0.55 to 0.60 lb. per b.hp-hr., but that most of the engines now in service have a specific fuel consumption of 0.50 lb. per b.hp-hr. at 60 per cent of full power on the propeller load curve.

When a successful Diesel aircraft engine is available, it will probably have a fuel consumption in the neighborhood of 0.40 lb. per b.hp-hr. at 60 per cent power. If air cooled, such an engine will probably weigh about 3 lb. per hp. Comparing it

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

with existing air-cooled engines of 1.5 lb. per hp., we find that for flights of less than 15 hours the conventional air-cooled engine will have a lower weight of engine plus fuel. That is, an engine weight of 1.5 lb. per hp. plus a fuel consumption of 0.50 lb. per hp-hr. will be less than the Diesel weight of 3.0 lb. per hp. plus a fuel consumption of 0.40 lb. per hp-hr. until 15 hours has elapsed. Beyond 15 hours, the advantage is all with the Diesel.

Minimum Cost

The cost of an aircraft engine requires definition. Do we mean first cost, or first cost plus maintenance cost? In any airplane, power-plant dependability is of paramount importance. Dependability may be achieved by a combination of light weight with excellence of design, materials, and workmanship, or it may be obtained by a less refined design manufactured of cheaper but heavier materials. The first cost of the latter engine may be less, but the engine will undoubtedly have a higher specific weight and probably no better dependability, durability, or ease of maintenance.

The first cost of an engine is of course greatly affected by the rate of production. This being so, the popularity of an engine model will be the final determinant of price, and an engine which most closely meets all the other fundamental requirements will be deservedly popular and can be manufactured in sufficient quantity to insure minimum cost.

Maximum Ease of Maintenance

Ease of maintenance calls for simplicity of design and accessibility of the moving parts. Ease of maintenance is interlocked with durability. If, despite its complexity of design and relative inaccessibility of parts, an engine runs for several hundred hours between overhauls, ease of maintenance fades into comparative insignificance.

In addition to the above requirements, the author would like to present for consideration a requirement which, for want of a better name, he has called "maximum effective thrust per horsepower." This requirement has to do with the effective thrust available in an aircraft power plant.

An aircraft engine's direct output is Brake Horsepower, while the output of the airplane power plant (which includes the propeller) is Thrust Horsepower. The thrust horsepower developed is, of course, dependent on the efficiency of the propeller used. The propeller should be so designed as to deliver the maximum thrust horsepower possible with the given conditions. For purposes of comparison, it is assumed that this is the case—that all the aircraft-power-plant types discussed have the most efficient propellers possible.

Aircraft-engine design affects the effective thrust horsepower in two ways, directly and indirectly. The direct effect is the speed of rotation at which the engine develops its power. The efficiency of a propeller increases with V/ND, where V = speed of advance of the aircraft, N = speed of rotation of the propeller, and D = propeller diameter. Hence, a high-speed propeller is less efficient than a slower-turning propeller in the same aircraft. (The slow-speed propeller will be of slightly greater diameter, but the effect of change in r.p.m. will be much greater and hence the latter will have much more effect on propeller efficiency.) A high-speed direct-drive propeller may also suffer a loss of efficiency due to excessive propeller tip speed. There is very little definite information as to the tip speed beyond which propeller efficiency falls off, but we have not been far wrong in assuming that it is in the neighborhood of 900 ft. per sec.

We see that the effect of high crankshaft speeds is to lower thrust horsepower directly due to its effect on propeller V/ND and on propeller tip speed. Consequently, with high-speed engines it is necessary to incorporate reduction gearing between the crankshaft and the propeller shaft in order to reduce propeller speeds. This reduction gearing is an additional complication owing to its possible reduction of engine durability, dependability, and ease of maintenance and increase in engine weight per horsepower and engine cost. In addition, the transmission of engine b.hp. through gearing to the propeller shaft involves a slight loss in power, although this does not amount to more than one or two per cent.

Until recently, American practice in the use of reduction gears was to employ a 2:1 reduction ratio. The latest ideas as to the choice of the proper reduction-gear ratio take into account the increase in propeller diameter and propeller weight due to reduction gearing. The disadvantage of increased propeller weight is self-evident. Not only does increase in propeller diameter, while maintaining the same ground (or structure) propeller-tip clearance, require additional weight of structure, but the added length of supporting structure increases the parasite resistance of the aircraft. For these reasons the latest practice in regard to reduction-gear ratios is to choose one which will give a V/ND in excess of 0.007, where V = m.p.h., N = r.p.m., and D = propeller diameter in feet, and a propeller tip speed not greater than 900 ft. per sec. Working to these values will give a propeller efficiency somewhere near the maximum, and yet will insure that propeller diameter and weight are held to a minimum.

The indirect effect of engine design on the effective thrust horsepower of an aircraft power plant is the drag (air resistance) of the engine as installed. The thrust of the propeller overcomes the drag of an aircraft, of which drag that due to the engine is an important component. So if the engine is of such a shape or size that its inherent air resistance is high, the effective thrust is proportionately diminished. One of the drawbacks of the radial engine is its high drag. This can be cut to a minimum by totally enclosing the engine with cowling, but with an increase in weight of structure and an evident sacrifice of visibility (in a single-engine tractor installation).

The two factors—direct and indirect—which influence the effective thrust per horsepower of an aircraft engine are themselves conflicting. An engine which is designed to develop a certain b.hp. at high r.p.m. will be smaller and lighter than a larger-displacement, slower-turning engine of the same horsepower, and its air resistance in flight should be less. The high-speed engine will probably require reduction gearing for the purpose of obtaining reasonable propeller characteristics, and this will add to the power-plant weight and complication, and in air-cooled engines will call for additional care to insure proper cylinder and oil cooling.

The author has devoted a great deal of space to the subject of the effective thrust horsepower of an aircraft engine, for the reason that, though important, it has been very little discussed or considered in engine design. Builders of several well-known engines place emphasis on their small frontal area, but as pointed out above, this is only a part of the subject.

Discussion

Henry L. Brownback.[2] The selection of an airplane engine will naturally vary with the service for which it is being selected. I think that we might divide the different services as military, air-mail and passenger lines, training of pilots, privately owned small planes.

I am somewhat surprised upon returning to this country to see the tremendous stress being placed upon the manufacture of aircraft for established runs, which market must always have a very definite limit as opposed to privately owned planes, the

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market for which will eventually be extremely large. In fact, in certain quarters there seems to be an unspoken doubt that this market will materialize for some time to come.

There is no doubt that, with the exhaustion of the large supply of OX engines which permitted small planes to be turned out at normally fictitious values and the subsequent rise in price of small planes, the market will decline sharply until small planes are developed for smaller powers and engines suitable for quantity production are developed and placed on the market. Naturally these new engines will cost more than the OX as they must carry, not only the burden of cost of production, but development, advertising, sales, and general overhead costs, and a fair profit as well.

Engines for military small combat planes are a limited market possibility and a highly specialized product, which will probably be manufactured by a limited range of companies, but the large bomber engines can be made interchangeably, with certain detail differences, with commercial engines.

While this class of engine has weight limitations where long flights are desired, reliability and long life are essential. When one considers that engines in this class consume their weight in fuel and oil in approximately four hours' running, it will be seen that a slight increase in weight to gain other necessary and desirable qualities is not prohibitive.

In these engines, especially those having large cylinder bores, one of the determining factors in selection should be that of absence of localized high temperatures. While the European air lines have been remarkably free from accidents, due no doubt to excellent ground service and very conservative operation by the pilots, the same engines in military service have given a great deal of trouble from crash fires, due to the engines running from autoignition after the spark had been cut in preparation for the crash or even fires due to high volatile coming into contact with highly heated metal surfaces.

Another point to be carefully considered in the operation of aircraft in countries having high normal temperatures is the ability of the engine to operate on a stable fuel which can be transported over the desert and stored in tanks or the fuel wing tanks of the airplane without distilling off the highly volatile fractions, or in case of mechanical mixtures of several fuels, the more volatile constituents, leaving a poor grade of fuel to be fed to an engine which is not suited to operating on this grade of fuel and suffers accordingly.

While most English and American companies have jumped into high-speed engines which certainly give a higher power output per cubic inch, per pound weight, and per dollar cost, the French and the Germans have stuck to the wise course of the low-speed engine.

Lubrication and valve difficulties seem to increase with the square of the speed beyond a certain point, and while this is of secondary importance in a combat machine it is of primary importance in commercial and privately owned planes. I am frankly dubious of speeds in excess of 1700 to 1800 r.p.m. in the present stage of the art.

While both here and abroad there are murmurings about Diesel engines, they have not yet reached the stage of practical opposition. When they do, there will be certain things to be considered, such as the universal distribution of a suitable and uniform grade of fuel, a means of rapidly filtering this fuel, and the perfection of filling apparatus and handling apparatus so that none of it will be spilled upon or reach the plane in liquid form. Gasoline will evaporate if spilled, but fuel oil makes an awful mess.

Something outside of this discussion but interesting is the experience of one of the large European operators over a very long run. His business was falling off due to airsickness which could not be accounted for by rough air. It happened that the chief engineer of this company had a friend in the oil business, and to please this friend he substituted mineral-oil lubrication for castor on several trips. When the monthly survey was made, it was noticed that the airsickness rate for these trips was very low. The company now uses mineral-oil lubrication exclusively.

The cowling of radial engines has killed one of the arguments against this type of engine very effectively. The modern radial engine is a post-war development based upon an analysis of conditions, and to my mind it represents, up to certain sizes, the ideal cylinder arrangement. The vertical engine in small sizes is somewhat analogous to the OX situation as the original small four-in-line engines were designed to use up a vast store of war material and then continued to take advantage of the tremendous sales effort expended to sell the first engines.

In Europe I have laid out several radials and line engines, but all of the factors so favored the latter type that we always finished up by building it. The prospective buyer of the small plane will soon size his purchase largely on a viewpoint of ease of servicing.

As the number of planes increases there will be a corresponding decrease in the quality of available mechanics, and the servicing will be done by men approximating the average garage "grease monkey;" and when that time comes it will be necessary to have engines which can be renewed rather than repaired. If this were not so in the motor-car business, the service problem would be insurmountable.

With the advent of the cabin plane the engine designer is faced with the elimination of mechanical noise. The day is coming very rapidly when the public will not buy an engine which makes a horrible mechanical clatter to be carried by the metal portions of the plane to the cabin and the ears of the passengers; for noise, especially clashing noise, is very tiring.

The same thing holds true of rigid engine mounts, and the study of placing rubber or other shock-absorbing materials in the engine mount is well worth while.

Robert Insley.[3] While no news to those few aircraft designers who have had time and inclination to think beyond pounds and square feet, Commander Shoemaker's aircraft-engine requirement of "maximum thrust per horsepower" should be of considerable value to those aeronautical engineers who evaluate all engines in terms of "pounds per horsepower." Some such concise statement has been sorely needed since this aeronautical pandemonium made it possible for technical schools to make full-fledged chief aeronautical engineers out of high-school students in four years. The number of responsible engineers in the industry who consistently disregard the general purpose of the aircraft power plant and bless or condemn it for single more-or-less important characteristics is appalling.

I should like to believe that in his remarks Commander Shoemaker intended to convey the impression that the intrinsic weight of the power plant per se is becoming less important among the other requirements. Unfortunately each of the requirements of the aircraft engine conflicts with one or more of the others, but the only one which seems to conflict with all of the others is the requirement of minimum weight per horsepower. In the past, when all creditable designs were of military origin, when there was small commercial demand and little competition, cost was all but disregarded. Weight and performance were paramount considerations, with dependability close behind, and economy, durability, accessibility, and low cost desirable but by no means essential characteristics. In commercial operations economy, low cost, and durability assume more important positions. Dependability cannot be sacrificed, and it seems not unlikely that low weight per horsepower will have to yield place that the other

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characteristic may be improved. Also, as Commander Shoemaker explained, the intelligent application of the maximum-thrust requirement often will result in the selection of other than the engine with the lowest weight-horsepower ratio. For each particular installation the probable operating conditions will determine the relative importance of the various engine requirements, and the increasing variety of commercial applications of aircraft make the engine designer's task more difficult, because each new application arranges the engine requirements in a new order of importance.

Commander Shoemaker's remarks about the Diesel engine are particularly opportune. While the Diesel engine bids fair to play an important part in the development of aviation, there is no advantage in fooling ourselves about its probable blessings.

It seems to me that there is one more aircraft-engine requirement which in the past has been relatively unimportant but will take on rapidly increasing importance as air travel becomes more general, and that is "maximum comfort for passengers." A short flight in almost any of our passenger-carrying airplanes is convincing evidence that to advance passenger transport by air beyond the novelty or emergency stage there is much to be done for the comfort of the passenger, chiefly as regards noise and vibration. In few of our passenger-carrying airplanes is normal conversation possible in flight, and in many the vibration resulting from engine vibration or exhaust or propeller pulsations is distinctly uncomfortable. Engine noise, engine vibration, exhaust pulsations, and propeller noise all can be reduced enormously, but to do so we may have to rearrange some of our other engine requirements.

Author's Closure

Both Mr. Brownback and Mr. Insley have pointed out the advisability of judging engines as regards their freedom from mechanical noise. This subject was not discussed by the writer, due to the fact that the vibration and noise in an airplane arise from so many causes. Vibration may be largely due to the airplane structure. Noise in an airplane may be due to the plane itself, and particularly to the propeller, as well as to the exhaust pulsations and valve tappet and gear clatter of the engine. Propeller noise is a function of propeller-tip speed, which in turn depends on the rotative speed of the propeller. The question of propeller-tip speeds was discussed in the paper, but I might add that the lower the tip speed the less the propeller noise.

Mr. Brownback discusses the various classes of aircraft engines and their relative fuel requirements. He is referred to an article entitled, "The Problem of Engine Fuel Requirements," which appeared in *Aviation* of June 29, 1929, wherein the author set forth his views on that subject.

AER-51-31

Methods Used in Training Pilots for Commercial Flying

Three Papers Discussing Modern Flight-Training Courses and Advanced Methods Used at Flying Schools

The authors of these papers advocate use of more systematic methods for training pilots. Longer courses at flying schools with better and more uniform flight instruction will help materially for schools to discharge their duty to the aeronautical industry and to the flying public.

Greater safety in the operation of aircraft and a reduction of accidents are results that are being sought. More instruction and more varied experience for pilots before they enter the industry will do much to lessen the hazards of flying. Increased stabilization of the industry will come from providing a definite, satisfactory source of pilots and thus assure the public that satisfactory facilities are available for flight training.

Flying Schools and Flight Training

By CLARENCE M. YOUNG,[1] WASHINGTON, D. C.

THE FIRST natural division of flying schools and flight training might well be that of military schools and commercial schools. The efficiency and adequacy of the military school is unquestioned, therefore, the type of work done and the purpose of such flight training can be passed over with only a reference thereto. Before discussing the commercial schools as such, it should be pointed out, that until the last year the majority of the commercial pilots were men who had military training, that is to say, the largest source of trained pilots was from ex-service men who had had training in military schools, either during the war or since. On account of this available supply, the matter of the well organized commercial school and the demand for this phase of work has been slower in making its appearance on a large scale than other phases of commercial aviation. This is no longer true. The demand for pilots now far exceeds this source of supply. It also follows that for this reason, the commercial training which has been done and the commercial flying schools which have been operating have been inadequate both as to the amount and quality of instruction given. In addition, there has been a lack of uniformity in courses of instruction given by civilian schools.

During the first half of 1928, 17.69 per cent of all accidents in commercial aviation have been during the earlier solo stages of student instruction. This figure dropped to 9.65 per cent during the second half of 1928, but this is partially due to the fact, that there was a larger amount of other commercial operations during the second half of 1928 than there was during the first half.

For example, a group of 390 airplane accidents analyzes as follows: 76 involved pilots with less than 50 hours' flying time; 21 involved pilots having between 50 and 100 hours; 17 involved pilots having 100 to 150 hours; 20 involved pilots having between 150 to 200 hours; and 13 involved pilots having 200 to 250 hours' flying time, etc. This indicates, in the absence of adequate instruction, that three times as many accidents occur in the first 50 hours of a pilot's experience than in any other successive 50 hours.

As another example: Although, during the year 1928, 70 per cent of all pilots licensed were transport pilots, less than 20 per cent of accidents and fatalities involved pilots of this grade.

Again, as still another example: Only 9 pilots out of a total of 161 were involved in fatal accidents in 1928, while operating as part of a scheduled airway under an organized operation system. During this period, the scheduled airways did over 20 per cent of the total commercial flying, and approximately one-third of it was done at night. Further, in all the 9 cases referred to, weather was a contributing factor. This shows that properly trained pilots and properly maintained equipment, efficiently organized and operated, decidedly decreases the hazard of aeronautical operation. As a final illustration, the findings of the Aircraft Accident Board show that 12.64 per cent of the accidents in 1928 were due to errors of judgment on the part of the pilots, 29.80 per cent were due to poor technique, and a total of 55.79 per cent were chargeable to some class of pilot's error.

In the face of these figures, it is perfectly evident that anything which can be done to improve flying schools or flight training should correspondingly increase the safety of operation of aircraft and decrease accidents in the aviation industry.

The framers of the Air Commerce Act of 1926 had no more knowledge or idea of the tremendous development that has taken place in commercial aviation in the past two years than any one else. It is, consequently, remarkable that they covered the regulation of the industry as well as they did. The omission concerning the regulation of civilian flying schools has been corrected by the amendment to the Air Commerce Act approved February 28, 1929. This amendment states that the Secretary of Commerce shall, by regulation, provide for the examination and rating of civilian schools giving instruction in flying as to the adequacy of the course of instruction, the suitability and airworthiness of the equipment, and the competency of the instructors. The examinations and ratings thus provided for are made only upon the request of the owners or representatives of the air navigation facilities or schools.

The civilian schools now in operation can hardly be criticized for inadequacy of their courses of instruction. In fact, they are rather to be commended for the progress which they have made in this rapidly developing industry. Part of the inadequacy in flight training was undoubtedly due to the public demand and the popular idea that it was only necessary to get enough dual instruction to be able to solo an airplane in order to become a competent pilot. This is, as we all know, and as the past two years of commercial operation have shown, a distinct fallacy. If definite standards can be agreed upon, established and maintained, the present condition can largely be overcome.

Considerable pressure has been brought to bear upon the Department of Commerce to set up requirements for the regulation

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

of schools even before the amendment to the Air Commerce Act was passed; that is to say, public opinion now makes it imperative that steps be taken in this direction.

The school supplement to the Air Commerce Regulations, setting up these standards, following a conference with school representatives from the industry, has been formulated and is now available. It is expected that these regulations will result in the accomplishment of the following:

1 Better and more uniform flight instruction
2 Greater safety in the operation of aircraft and fewer accidents, due to pilots having had more instruction and a more varied experience before entering the industry
3 Increased stabilization of the industry by providing a definite, satisfactory source of pilots
4 Stabilization of the schools themselves by encouraging them to give a more complete course under the privileges granted to approved schools
5 Assurance to the public of satisfactory approved facilities for obtaining flight training.

A distinct attempt will be made to accomplish these results by means of rating and approval of schools which are qualified to give adequate flight training in accordance with the standards set up in the regulations just referred to.

The regulations prescribe for the rating and approval of schools under three separate headings; that is, (1) suitability of equipment, (2) adequacy of the course of instruction, and (3) competency of the instructors.

Under suitability of equipment, provision is made for a minimum-sized field for safe flying instruction purposes and hangar and shop facilities sufficient to maintain the airplanes in an airworthy condition.

A restriction is stipulated concerning the total number of planes permitted to operate for flying instruction purposes from a given field, that is, 10 planes per 100 acres. The restriction is also made as to the number of students that can be enrolled for each plane available for flying instruction purposes. Further provision of adequate classroom and ground-course facilities is required.

Under the heading of adequacy of the course of instruction, provision is made for time limits of the course which are established in order to assure a regularity of procedure of instruction.

The school is required to maintain individual records of each student, showing the progress through the course. Students' examination grades are to be made a part of this record. Schools are required to submit an outline of all courses to the Department of Commerce for its approval. The school is required to maintain a standard of instruction to assure that 9 out of 10 graduates who apply for license satisfactorily pass the Department's tests. Schools are required to give the students instruction in the recovery from stalls and spins prior to a first solo. The minimum curricula requirements are set up for both the flying courses and the ground courses. Under these minimum curricula requirements, the school is encouraged to give the student advanced instruction and check time by granting the graduates of the school the privilege to credit a portion of their dual and check time toward their solo-flying experience required for Department of Commerce license.

Under competency of instructors, provision is made for the licensing of flying instructors and ground instructors. The ground instructor will be required to pass examinations in the subject matter of the course he proposes to teach. The flying instructor will be examined as to his capability as a flying instructor, both by flight test and by a practical and theoretical examination on the explanation of flight maneuvers. The net result of this school rating should be, that an approved school with competent instructors, with sufficient suitable equipment, and with definite standard curricula, both in flying courses and ground courses, should be capable of graduating a class of pilots with guaranteed ability. This applies particularly to the schools for advanced grades of license.

At present schools will be approved as private-pilots' schools, limited-commercial-pilots' schools, and transport-pilots' schools. Students who desire to enter the aviation industry as a profession or vocation should unquestionably attend a limited-commercial-pilots' school or a transport-pilots' school. This would give them sufficient time for advanced instruction and experience on various types of airplanes as required in these courses, which should certainly qualify them better for their future work than is the case under the present civilian flying school condition.

In conclusion, our present situation might be likened to the fable of Daedalus and Icarus. Icarus, youthful enthusiast, overconfident but without experience, soared his way toward the sun on his almost untried wings, but, on account of his lack of experience, met with grief; whereas Daedalus, the older and more experienced of the two, realized and prepared himself for the emergencies which experience taught him might be encountered. Thus, today, the embryo pilot with few solo hours, unaware of the pitfalls in aviation as our statistics have shown, certainly encounters difficulty; whereas, if his period of training were of longer duration, under more competent instructors, and with a greater variety of experience, he would graduate better equipped to meet the emergency situations which all pilots encounter sooner or later in their experience.

Modern Flight Instruction

By TEX RANKIN,[1] PORTLAND, OREGON

FLIGHT instruction has undergone a number of changes and improvements since 1927. Practically all the better schools in the United States are giving a minimum of twenty-hours' flight instruction, ten of which are dual and ten solo hours. In the opinion of the author, most of these schools will adopt a minimum of fifty hours within the next two years, so regardless of whether a man wishes to fly for pleasure or for profit, he should have the complete flying course as well as a complete ground course.

The average twenty-hour course costs approximately $500 including the ground course, and the average fifty-hour course costs approximately $1200 including the ground course. There is a difference of $700 between the two which can be charged off under the heading of "Life Insurance." For the additional thirty-hours' flying instruction that the student receives for the additional $700, he will be assured of 150 per cent more aeronautical education, than he had at the expiration of twenty hours. Therefore, his chances of having an accident, in which he may damage an airplane or injure himself, have been lessened a great deal.

As far as can be learned by the author at the present time, not a single graduate of the fifty-hour flying course has had an accident, in which the pilot or an airplane has been injured. This is out of a total of thirty-six graduates of the fifty-hour course during the past two years.

It is necessary to give students a certain amount of stunt work, to assist in developing the feel of the ship and to insure confidence in their ability to fly the ship in an ordinary position. The student should be taught, in a practical way, accidental means of getting into spins and how to come out of them and,

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of course, how to avoid them. A number of solo students and embryo flyers have been killed by accidental spins, simply because they did not know how to avoid them and how to get the plane out of an accidental spin after they once got into it. The author recently investigated an accident in which a solo student pilot was killed. It is believed from testimony of eye-witnesses, that the pilot attempted to turn, using a 45 deg. bank and when he came back to center with the stick, he failed to center the rudder. This caused the nose to go down sharply, and apparently, an attempt was made to get the nose up by pulling back on the stick rather than by using the top rudder, which put the ship into a spin. Due to the student pilot's lack of knowledge concerning accidental spins, he attempted to hold the stick back, in an effort to get the nose up and consequently spun into the ground.

Learning to fly a ship and land it with a dead engine is another accomplishment that should be taught to students regardless of whether they are taking a twenty-hour course or a two hundred-hour transport course, for they will, undoubtedly, some day have engine trouble if they fly long enough. Dead-motor landings are actually more of a mental hazard than a real hazard. As soon as students become accustomed to flying without hearing the familiar sound of the engine, they will have little difficulty in making successful dead-motor landings. During the first few dead-motor landings that a student makes, the sight of a motionless propeller, in front of him, creates a mental condition in the student's mind that will upset his nervous system and reasoning powers completely and will cause him to make a poor landing or even to crash were an instructor not with him. The author has found it best not to permit students to make dead motor landing solo, until they can land a ship with a dead engine just as well as they can with the engine idling.

Simulated forced landings in strange fields should be featured in every good course and the student should have an opportunity to get close enough to the ground, to know actually whether he would make a good landing or not, before the instructor opens the throttle again. Of course in the case of a good field, it is a good plan to permit the student actually to make landings on that field.

In every good course, students should be given the opportunity of doing a great deal of taxiing around on the field. They should taxi down-wind, up-wind, and side-wind in a strong, brisk breeze as well as being able to taxi in a calm. Taxiing a plane is extremely difficult and it will improve the student's handling of the ship in the air a great deal. Taxiing down-wind is considerably different than taxiing up-wind and should be explained thoroughly to the student. The author has been on large metropolitan airports and has observed many students trying to taxi down-wind in the same manner in which they would taxi up-wind.

In the advanced course, a student should receive considerable cross-country flying, both as pilot and as navigator. He should receive night flying and a few hours of blind flying, in which he must depend entirely upon his instruments. The more blind flying the author does, the more he is convinced that this type of flying should be featured in every good fifty-hour flying course and also, at least a few lessons of it given, even in the twenty-hour course, not in an effort to teach the twenty-hour student in three or four lessons, that he can fly through a fog with instruments, but to convince him in three or four lessons that it is an utter impossibility for him to do so. Blind flying is a very deep subject and is as yet purely experimental. It is entirely different than ordinary flying and the old-time pilot that tries to fly blind by "feel" will find that he will have to learn almost to fly over again in order to fly entirely by instruments. It is necessary to sit in the cockpit, just the same as an engineer sits before his instruments in the hold of a large steamer. He must give no thought to the feel of the ship, but place confidence in the instruments and believe in them entirely. It is the opinion of the author after talking with Martin Jensen, winner of second place in the Dole flight to Hawaii, that the majority of the planes that were lost on the Hawaii flight, were lost because the pilots had more confidence in their "feel" of the ship than they did in their instruments. Mr. Jensen told the author that he went into two spins and barely got out before he struck the water, before he could believe in his instruments. He also stated that he had no further difficulty, after he devoted his attention to the instruments and quit trying to fly by the feel. Therefore, it appears that blind flying is a subject that demands a great deal of serious thought by the flying schools and pilots of the country at the present time.

Up to the present time, practically all of the schools have been using OX-5 motors in commercial planes capable of carrying two passengers and the pilot for instruction purposes. Each school found that the fuselage is entirely too wide to give the student the visibility he should have in taking off and landing. Therefore, it appears that one of the principal needs of flying schools at the present time is a good training plane that is built only for that purpose. Several manufacturers have recently turned their attention to this matter and undoubtedly before another year rolls by, we will see many types of training planes on the market at a reasonable price.

In the interests of safety, some type of collapsible-control stick should be placed in the student's cockpit of each training plane. One of the best developments along this line that the author has had the good fortune to see, is the Davis stick, which was invented by Mr. Davis after his son, an instructor, was killed by a student freezing on the control.

Instruction should be confined to private airports and municipal airports where no transport planes were coming and going, with the possibility of collision in mid-air. The Aeronautical Chamber of Commerce of America has done some very wonderful and constructive work in assisting the Department of Commerce in developing flying school regulations.

Modern Flying School Practices

By OLIVER L. PARKS,[1] ST. LOUIS, MO.

UNDER the old dispensation in flying instruction, there was a certain amount of grimly humorous formality attending the business of sending a student forth on his first solo flight. Old timers liked to picture the first flight alone as an event of great importance, which it was, but also as an event of danger. "You do or you do not," they would tell their students before they sent them off and frequently the latter alternative was accompanied by a significant gesture, expressive of an airplane crash. Frequently, too, the more hardened instructors would send their students off on their first flight with the admonition, "Now go out and kill yourself."

Times have changed since those days, and the first solo flight of the young student is now as safe as the first trip of the novice automobile driver through the city's traffic by himself. It is still an event in the young flier's life but a vast improvement in training methods has shorn it of its danger. The change in the attitude of the instructor toward the first solo flight of the student is indicative of the entirely new condition in the business of teaching young men to fly.

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Flying instruction has, until the last year or two, been a laggard in the swiftly growing aeronautical industry. The design of airplanes and methods of instruction have improved as aviation grew, but until the need for well-trained pilots became acute, the method of making flyers, particularly in the civilian schools, was a very haphazard, hit-or-miss proposition. The consequence is, that a large proportion of the experienced flyers operating in the United States are graduates of the service flying schools, particularly the army school, which has consistently given a well regimented and thoroughly systematized course of instruction. It is the adoption of this careful system of teaching the young men to fly which distinguishes the forward-looking commercial flying school of today from the school, let us say, of two years ago.

Possibly, the lack of suitable airports and the use of obsolescent military airplanes were partially responsible for the early methods of teaching in civilian flying schools. Then, too, the heritage of recklessness, that was left from the wartime days, may have had something to do with the methods by which young men were fitted for wings. In any event, the method of operation was roughly as follows:

The student arrived at the flying school full of ambition to become a flyer, signed up and was turned over to a pilot for instruction. Perhaps there were three or four pilots. If that was true, there were three or four methods of flying instruction in use. The instructor explained the workings of the controls, took the student aloft and gave a brief lesson. On following days, other lessons were given, until finally the student was able to take off and land the ship with ordinary student facility. If the instructor was extraordinarily careful, he told something about recovering from a spin and showed how to sideslip the ship for landings in tight fields. In most cases, however, the instructor was anxious to solo his student as quickly as possible in as much as he was paid a flat rate per student for instruction so that the student was, probably, sent off on a solo flight with the absolute minimum of instruction. If he succeeded in getting back on the field, all well and good. If he did not, it meant only a loss of a few dollars to the flying school since the ship was an old Jenny or a Standard, at the best, easily repaired, if the damage was slight, and capable of being junked without much loss, if the damage was great.

From that point on flying was largely self taught. Day by day the student took the air for solo flights, learning much by experience but getting into many dangerous positions as he tried out one maneuver after another. Here was one of the great dangers of early commercial flying instruction. The student was led to believe, that he had to learn only by experience and a certain amount of venturesomeness was necessary for the development of a well trained pilot.

Frequently the effort of self instruction paid its dividends in accidents with airplane injuries and fatalities. It was rarely that any student reached one hundred hours of flying under such methods of instruction, without cracking up at least one airplane. That first crash came to be regarded by flyers almost as a matter of course in the education of the student pilot.

At the end of fifty hours' instruction, however, the student found himself fairly well able to handle an airplane but from then on up until four hundred or five hundred hours, he frequently found himself in flying scrapes, from which he did not know how to extricate himself. In such contingencies he took the way out that seemed best from former experience and if it worked, all was well. If it did not he had just another crash.

At the time of entrance in the flying school, the student was told that a certain amount of Ground-School instruction would be given. After a few days at school he found that ground-school classes were occasional affairs. Probably one of the pilots was also ground-school instructor and on rainy days and other times when there was no flying, it was his custom to call the class together in the hangar or storeroom and lecture it on engines, aerodynamics, theory of flight, and other subjects that should go into the make-up of a good pilot. Possibly these lectures were competently given and, certainly, they were interesting to the young student. Their great failure, however, lay in the fact that they were given on no set system and on no established schedule. The consequence was that while some students drew a fairly complete course in ground school instruction, others had only a hodge-podge of aeronautical knowledge and lacked the academic background that every pilot should have.

The old system of commercial flying school instruction was dangerous, wasteful, and inefficient. It was foreordained that it would have to pass away, if instruction was to keep pace with the progress of the other phases of the aeronautical industry. A great change in flying-school instruction began more than a year ago when the more progressive schools perceived the fault in their system, saw how successful the army school was in turning out good pilots, and decided that in the interest of good business, as well as safety, it was necessary to improve methods.

The course in the commercial flying school, with which the author is connected, includes a system for organization routine and instruction so that no matter which pilot instructor takes charge of the student for flight instruction the student learns the same things in the same way. The instructor assigns the student to a definite flying period as, for example, from 9.30 a.m. to 10.00 a.m. daily and immediately the student's name is posted on the flight board under the name of his instructor and under the proper flight period.

On the day of arrival or the following day, he has his first hop. It begins with a lecture by the instructor as he and his student stand beside the ship. The pilot explains the workings of the controls, the use of flying instruments and the general theory of flight of an airplane. Every effort is made to make flying appear to the student as a normal activity for which almost any normal man is fitted.

The first flight is usually brief, not more than fifteen minutes. It is followed by successive flights, which increase in duration to half an hour and later to an hour. In each of them, the pilot is consistently watchful of the instruction of his student, attacking the problem of teaching him, according to the set schedule of instruction upon which the pilots and general manager have agreed as the best form of teaching.

Uniformity of flight instruction is obtained. The instruction plan is sufficiently flexible to allow the pilot to vary his methods somewhat with the needs of individuals. While system is necessary in the teaching of flying as it is in the teaching of any other activity, there is also a frequent need for variation in special cases, just as there is in the teaching of other branches.

Early flight instruction proceeds through straight flying, landings and takeoffs. At this point many of the older schools would set their students to solo, but before classes reach that stage, they are also taught how to recover from tail spins and how to sideslip, so that if they are suddenly confronted with the necessity of either recovering from a spin, or of landing on a small field, they are able to do so with the minimum of danger.

The average flying student solos in from seven to ten hours of instruction. His first solo flight is given quite casually, so that the student will suffer from no unnerving apprehension when he takes off and every effort is made to convince the student he is taking a very ordinary step.

The first flight is to fix in the student's mind the conviction that he can fly an airplane by himself. Immediately after the

solo hop, he is sent back to the dual instruction stage, where he remains for several hours longer while his instructor polishes off the rough features of his flying. This feature of instruction enables checking up of flying faults, which unquestionably would bring students to grief were they allowed to continue in them.

After twenty hours of dual instruction, the student is sent over to the solo stage. Here he flies alone daily but the pilot now in charge of his activities is still watchful for the faults that are bound to crop up in the first fifty hours of flying. The student is checked in his air work every day so that when he completes the fifty hours of instruction included in the primary and advanced flying courses, he has had a total of about twenty-five hours of solo flying and twenty-five hours of dual flying.

During his flight instruction, the student is also going through a course of instruction on the ground. Every day from three to six p.m., Saturdays excepted, he spends in the class rooms which are equipped with lecture chairs, blackboards, maps, globes, and the specific paraphernalia needed for instruction purposes. Every instructor takes his class through the same course of study and except for the phraseology of the instructors the lectures are identical.

The course covers theory of flight, meteorology, aerodynamics, theory of engines, carburetion, ignition, and navigation. The instruction in navigation and meteorology, incidentally, is sufficiently thorough to qualify students for the Transport Pilot written examination in those subjects. The ground school course runs for nine weeks with a written examination every Friday. The last three weeks is spent in the line shop in actual repair work upon the twenty-four airplanes which comprise the instruction fleet. This work is carried on under the careful supervision of licensed-airplane and engine mechanics.

Carefully systematized instruction is also carried out for airplane and engine mechanics. The course runs for seventeen weeks with classes held from 8.30 a.m. until 4.30 p.m. daily. A carefully laid out curriculum is followed. Class-room work and actual practice in the shop are combined. In the school for aerial photography, a course including details of photography, from the operation of the hand camera up to the making of aerial mosaics has been laid out. The course is practically taught and covers far more ground than would be possible under a hit-or-miss schedule taught without careful planning.

Only by systematic methods can any flying school hope to discharge its duty to the aeronautical industry and to the flying public. Undoubtedly, the methods now in use will be modified and improved. The Aeronautical Chamber of Commerce, by prescribing certain restrictions for flying schools, is doing admirable work in that direction.

Discussion

HARVEY H. HOLLAND.[2] These three papers cover very briefly one of the most important phases of aeronautical development. There is no single phase in the development of aeronautics which is more vital to the future of the industry than a high standard in the training of airplane pilots, especially those who propose to qualify as transport pilots.

The future development of the industry will be vitally affected by the standards which are adopted and maintained by the numerous flying schools which are rapidly springing into existence. It is gratifying to observe the trend toward the maintenance of high standards which is emphasized in these three papers.

Mr. Young's paper expresses clearly the necessity for the correct regulation of training schools by the Department of Commerce and the advantages to be derived therefrom. The new regulation requiring stalls and spins before allowing the student to solo is very important, and a few forced landings should also of course be required before the student ventures forth upon his own resources.

The subject of psychology is one of the important considerations in flight instruction that often is neglected and one in which the desired results are difficult to obtain. The proper function of the flying instructor is to lead the student, step by step, into the proper methods which he eventually will adopt involuntarily and habitually at the correct moment.

There has been an effort to standardize methods of instruction in the Army Air Corps by first training the instructor in the proper procedure of handling the student. Instruction in flying can be compared in a limited way to teaching a student to drive an automobile. Personalities often clash, and patience and consideration must be the rule.

It is believed that the tendency will be toward the attainment of higher standards in ground instruction, especially for those students who propose to enter the industry professionally.

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Radial Versus In-Line Engines

By GLENN D. ANGLE,[1] CINCINNATI, OHIO

In this paper the author, after recounting the history of the two types of engine under consideration, sets forth at some length reasons why the in-line type has been revived. Following this he compares the two types as regards such features of primary importance to the airplane manufacturer as weight per horsepower, cost, smoothness of operation, installation, visibility, cooling, and head resistance. He concludes that the radial engine is inherently superior on the basis of weight per horsepower, smoothness of operation, installation, service, and cooling. It also promises to excel in the matter of low manufacturing costs. The upright in-line engine, by virtue of its smaller frontal dimensions, is possibly better for visibility in some installations, although this advantage becomes of little consequence in certain types of airplanes. The inverted in-line type obviously excels in visibility, in almost any installation. The comparative resistance or drag of the radial and the in-line engine, however, is still a matter of opinion.

ENGINE development is of necessity a slow and expensive procedure, consequently engine production has not kept pace with the rapid growth of the aircraft industry. Because of the present increasing demand for airplanes of various types, there is a sellers' market in engines, a situation which may continue for some time. It is true that several new aircraft engines are being placed on the market, but it is hardly to be expected that this will greatly alter the situation for many months at least. Some of these engines are still to be more fully developed before it is really safe and advisable for their manufacturers to go into an extensive production program. There are even instances where the personnel of the organization attempting to market an engine have had little or no previous experience, and are merely copying other engines, including their mistakes. This is certainly an unhealthy condition, and it may reflect most unfavorably upon the industry if it continues. Still, it undoubtedly will continue as long as we have this sellers' market, and aircraft manufacturers will persist in selecting the engine for their ship on the basis of availability more than upon suitability.

Constructive criticism and accurate detailed reports on engine performance are always welcomed by the engine manufacturers who sincerely desire to improve their product. Unfortunately not all airplane manufacturers can spare the time or sufficiently appreciate the variety of problems involved in the development of aircraft engines to realize the importance of rendering this assistance. It is believed, therefore, that a better understanding of the engines available and a closer relationship between the airplane and engine manufacturer will do much to bring about better and safer airplanes upon which the successful growth of this new industry depends.

Discussion of the radial versus the in-line engine, is a subject of vital interest at the present moment, particularly among the aircraft manufacturers who are producing a sport or light commercial and training ship of approximately 100 hp. or less. Both types of engines are now available, and although some airplane manufacturers seem to have developed a marked preference for one or the other, it is apparently based upon some particular feature, and not the result of a thorough comparative study. The author was particularly impressed by this fact during recent discussions of the subject with a number of airplane engineers and manufacturers in the attempt to obtain material for this paper. For instance, among the few who preferred the in-line engine, the small frontal area seemed to be the only feature which could be referred to with any degree of certainty; while a few thought possibly the in-line engine could be produced at a lower cost than the radial, yet there were no cost figures or data of any kind at hand from which to draw such a conclusion. It would seem, therefore, that a comparative study might clarify the situation, or at least encourage investigations on such points as may still be regarded as doubtful.

History of the Two Types of Engine

But before entering into a discussion of the two types of engines under consideration, let us briefly review their history. Both types are older than the airplane itself, and it is rather interesting to study their rise or fall in popularity during the various stages of aircraft development.

The engine responsible for the first successful flight of man was, as we all know, a four-cylinder vertical or in-line type. The Wright Brothers followed a logical course in developing an engine of a type concerning which there was considerable information. Other constructors followed similar lines, taking the automobile engine in principle and general arrangement, and proceeded to refine and change its design in order to meet more fully the special requirements for use in aircraft.

However, most of these early four-in-line types were water-cooled, as the records show among the four-cycle engines a total of 99 different models as against 10 which were air-cooled. Of the two-cycle engines, ten were water-cooled and two were air-cooled. Besides being fewer in the number used, the air-cooled engines were on the whole inferior, mainly because of the lack of knowledge existing at the time relative to air-cooled cylinders.

The four-in-line types gradually gave way to the six-in-line, rotaries, and radials in the matter of popularity. There were several obvious reasons for this, among which perhaps one of the most important was the demand for higher power outputs, which experience had shown were impracticable with this type. There was also a need for engines with a lower weight per horsepower, the four-in-line never having shown to much advantage on this score. Vibration was a source of trouble, and it was desired either to eliminate or reduce it as much as possible. The inertia forces in the four-cylinder in-line engine are inherently unbalanced. Furthermore, attempts to dampen the fluctuations of torque on some of these early engines by the addition of a flywheel only added weight to an already heavy engine, and did not produce the improvement expected. In the early days of flying, then, the four-in-line engine had been found to be inherently heavy and rough, limited to relative low outputs, and was consequently abandoned for aircraft use.

On the other hand, the radial engine has, due more or less to the lack of design precedence, undergone a most arduous course of development. It has evidently survived a disappointing series of experiences only because of the inherent outstanding possibilities which the type possessed. Credit for originating the radial is generally given to Forrest, who built an engine of this type in the year 1888. However, the credit for the first radial aircraft engine goes to the late Charles M. Manly, who built a five-cylinder water-cooled engine for Professor Langley's experi-

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

mental airplane. This was a most remarkable contribution for the year 1901, and undoubtedly would have received greater acclaim if the Langley experiments had not met with such bitter disappointments.

Little attention was given to the radial engine until Anzani in France began building this type, following the construction of some fan types, one of which was used by Blériot in his flight across the English Channel in July, 1909. Aside from the Salmson engines, practically all of the early radial engines were air-cooled. None of these engines were noted for any degree of reliability, particularly if based on present day standards. Besides failures of valves, pistons, and the like, resulting from lack of air-cooling experience and unsuitable materials, the radial engine had several other common troubles. The crankpin-bearing loads were seldom, if ever, analyzed, and many failures resulted from overloading. Gas distribution was another problem that apparently was never satisfactorily solved. Yet the type offered sufficient advantages to inspire these early engine builders to carry on this development work.

It was during the war, when lighter and more powerful engines were in demand, that the first real scientific work in the development of air-cooled radial engines was undertaken. The British Government conducted a series of interesting investigations, the results of which have been invaluable in the development of the modern radial engine. Contracts were given to certain British firms by the Air Ministry for the construction of engines for military use. Following the war, the United States Army and Navy encouraged similar developments by placing contracts with some of the leading engine manufacturers, as well as conducting research work in their own laboratories. In due fairness, then, the development of the modern air-cooled radial engine must be credited mainly to the military services.

Commercial aviation, if the term may be used, was struggling along the best it could, using war-surplus engines and trusting faithfully in the future. The air-cooled radial developed for military use was still too expensive to be universally adopted, although as the war-surplus stock of engines was becoming gradually depleted, many began looking forward to this type as the logical possibility for commercial airplanes. Public interest in aviation was stimulated suddenly by a succession of long flights, particularly those made across the Atlantic Ocean, and with this came recognition of the fact that the radial air-cooled engine was perhaps the most reliable and best-suited type of power plant for commercial aircraft so far developed.

However, during this period of growing aeronautical interest, radial-type engines of lower horsepower were actually being developed in this country. It was nearly five years ago when the first strictly commercial engine was started, and there have been possibly 15 or 20 others designed since. Practically every one was an air-cooled radial. This indicates one of two things; that the designers were convinced of the advantages of the radial type, or were influenced by the popularity given it by the larger military engines.

Our last reference to the in-line type engines was that, for certain justifiable reasons, it had been abandoned for aircraft use. Why, then, has it been revived? If the facts were not known, one might be led to believe that some one had recently discovered some new and promising feature in this type that heretofore had escaped attention. However, this revival came as a result of rather unusual circumstances, and an attempt will be made to explain briefly from such reports as it has been possible to gather.

Why the In-Line Engine Has Been Revived

Great Britain had many trained flying men at the end of the World War, and was confronted with the problem of keeping these men in training, as well as training new men, in order to maintain a reserve flying force. It is understood that it was impossible to provide airplanes and equipment, as has been done by the U. S. Army Air Corps, so encouragement was given in the form of subsidies to the light-plane clubs. However, these subsidies were apparently contingent upon using equipment furnished by one of the government departments, the engines being built originally from salvaged or excess stocks of parts, and were of the four-in-line types, doubtless from necessity. This was indeed an economical measure, and we therefore cannot judge from the limited information at hand if there were any other reasons for choosing the in-line type of engine.

Recently the author read the following which is typical of some of the arguments for the in-line type of engine: "That insurance rates in England were very materially higher on commercial or flying-club airplanes with radial engines than on similar machines equipped with upright in-line engines, owing to the fact that a landing-gear collapse or mild crash does not result in wiping off cylinders from an upright engine, as it does the lower cylinders in the case of a radial." Such a mild argument might be taken seriously, if one did not understand these forced conditions under which the modern in-line was developed. Far greater damage can be done to the lower crankcase of an in-line engine than to the lower cylinders of a radial in event of a landing-gear collapse or mild crash. To substantiate this opinion, consider the condition of a certain radial engine which crashed into the ground at full throttle from approximately one hundred feet following the failure of the wing. The propeller was wiped off close to the hub, only four or five fins were broken on the two cylinders which had dug deep into the ground, and the crankshaft was sprung less than three thousandths of an inch, otherwise the engine was in perfect condition. Obviously there should be points of comparison between the two types of engines of far greater importance than the effects of a mild crash.

During the forced use of the four-in-line engine in Great Britain, it could reasonably be expected that a number of important or spectacular flights would be made. A list of these flights was given in some of the advertisements appearing in practically every current aeronautical magazine. If one were not familiar with light-plane performances or did not take the time to investigate the records, it would be only natural to assume that all the important flights had been made with the in-line engine. However, this is hardly the case as a more complete list of important light-plane performances will show.

Included in a list of officially recognized records of light planes, which was obtained from the National Aeronautic Association, the ones made by airplanes powered with in-line type engines are:

1 *Class C.* First category, two-seater, altitude record of 19,862 ft. made with D. H. Moth and 85-hp. Gypsy engine.
2 *Class C.* First category, two-seater, speed for 100 kilometers held in Germany by Gypsy 100-hp. engine, 119 m.p.h.
3 *Class C.* Third category, speed for 100 kilometers held in England by D. H. Moth 32, special Gypsy engine, 130 hp., speed 186.47 m.p.h.

Recognized records with radial engines are:

1 *Class C.* First category, two-seater, distance record over closed circuit, Avia using a Walter 60-hp. engine, distance 932 miles.
2 *Class C.* Third category, distance closed circuit, Walter 60-hp. engine, distance 1553 miles.
3 *Class C.* Third category, distance air line, Walter 60-hp. engine, distance 1249 miles.
4 *Class C.* Third category, altitude record 22,250 ft. held by Germany Baumer with Wright 60-hp. engine.

The radial engine, then, holds four officially recognized records, while the in-line engine is credited with three. It is believed that the first two records referred to in connection with the in-line engine have been broken unofficially in this country by

airplanes which use radial engines. At any rate there are airplanes capable of doing so. It is interesting to note that the American manufacturers of light planes have not as yet gone in for breaking records, but the author ventures to predict that during 1929 most of these records will be brought to this country, and the airplanes will probably be powered with radial engines.

Time will not permit listing all of the remarkable flights made with light planes using both in-line and radial engines. The flights of Lady Heath, Lady Bailey, and Bert Hinckler using in-line engines were especially brilliant. Those of Miss Bobbie Trout, Maryse Sebastie, and Mauler and Baud, in which radial engines were used, are equally spectacular. It may be interesting to know that the flight of Mauler and Baud from Paris to Cape Town and return, under the direction of the Paramount Film Co., is the outstanding flight through Africa in a light plane.

Nevertheless, through wide publicity, it has been possible to negotiate American manufacturing rights to help supply this present market; and even some American manufacturers, in their anxiety to get into the engine business, have suddenly taken up the in-line engine. What these engines will show in service is yet to be seen, as to date there are not enough in actual use in this country to form a definite opinion. The average American pilot forms his own opinion of the equipment he uses, and he can probably tell more regarding it a year from today.

The manufacturers of small radial engines have also boosted their products, and, quite frequently perhaps, a trifle more than actual conditions warranted. They seem to have taken for granted, however, that since practically all of the really outstanding flights of recent months have been made with radial air-cooled engines, there is no further need to boost this particular type of engine for the benefit of their present and future customers. But it appears that these manufacturers must continually parade engineering facts and performance figures of the radial engine before a fickle public, if they are going to meet the competition which the in-line propagandists are presenting. To illustrate, the following is quoted from a pamphlet published and distributed by a brokerage house which is offering the public stock in a company manufacturing an in-line engine. "Leading aeronautical engineers have stated that the day of the radial motor has about passed and that the in-line engine is most satisfactory for aviation, and the in-line motor that is at the same time air-cooled is now considered the acme in airplane motive power." This may be an excellent argument to encourage stock sales, but the author doubts very much if any one who could be classed as a leading aeronautical engineer would make such a positive statement, and surely not without including a few words of explanation.

The question of the suitability of either the radial or the in-line air-cooled engine for the lighter types of aircraft, can be determined only by investigations conducted along sound engineering lines. The opinion of one or more can have no great bearing on this question, because if personal opinions could settle the issue now, the radial engine would undoubtedly poll the larger vote, receptive subjects to in-line-engine propaganda notwithstanding. Neither will airplane-performance figures give us a solution, unless tests are conducted under comparable conditions. The advocates of both types of engines may justly point with pride to a number of noteworthy performances, but facts devoid of sales arguments are needed before a decision can be rendered.

Comparison of Various Important Features of the Two Engine Types

Let us now compare the two types of engines as regards features of primary importance to the airplane manufacturer. These features are weight per horsepower, cost, smoothness of operation, installation, service, visibility, cooling, and head resistance, all of which will be treated in the order named.

Weight per Horsepower. One needs only to study casually the radial and in-line engine to form an opinion of how the two compare for weight. It is quite unlikely that, for a given output and with similar standards of construction, a four-in-line engine could ever be built for the same or less weight than a corresponding radial engine of five or seven cylinders. In fact, the lightest in-line engines today generally show higher weight-per-horsepower values than the heaviest radials in their class.

The actual weights of existing engines of both types will not be included, but it may be of interest to mention that in-line engines of four cylinders are 20 per cent or more heavier than corresponding radials, and this in spite of no small advantage which the in-line engine possesses in the reduced weight of its cylinders. The weight of the bare engines is interesting, of course, from a design standpoint, but the weight completely installed is the figure of most importance. The author has been unable to obtain any accurate figures on the differences in weight between two such installations, but it is perfectly obvious that no reduction of any consequence could be expected, and it is more than likely that the long engine bearers, cowling, and extra air scoops would add more weight than the usual radial engine installation.

Cost. The loss determinate of all features under consideration is the question of cost. Naturally manufacturing cost figures of the various engines are not obtainable, and the present list or selling price is no criterion. Moreover, if actual cost figures were available, we should still be unable to draw a conclusion, unless the design, type of construction, and quality of workmanship and materials employed were much alike. The matter mainly resolves itself into the question of which type of engine can be purchased by the airplane manufacturer at the lowest price, assuming, of course, everything else to be equal. There will be no attempt here to prove that which it is only possible to determine through time and manufacturing experience, but in passing it may be well to refer briefly to a few of the more obvious features affecting the cost of production.

For a given type of cylinder construction, the four-in-line engine holds a distinct advantage, in that it has one or three less cylinders than the average radial of approximately the same horsepower. On the other hand, the crankshaft, crankcase, and associated parts are of larger dimensions, and it is naturally to be expected that both the material and machining costs will be higher. There are also several bearings for the crankshaft and camshaft in the in-line type that are not required in the radial engine. The assembly cost of any engine depends largely upon the design and type of construction, but experience so far indicates that a simple radial engine should require less time to assemble than an equally simplified four-in-line type.

A point has been raised that, since the in-line aircraft engine so closely resembles the automobile engine, high-speed-production machine-tool equipment can be employed. This is with particular reference to the crankcase, crankshaft, camshaft, and associated parts, which is undoubtedly a real factor to consider. It is believed that the use of this equipment should effect some saving in jigs and fixtures, as compared to those designed to fit standard machines, but the actual machining time would be nearly the same. Practically all standard machines are employed at present in the production of radial-engine parts, and it is not easy to predict how much saving would be possible on a large-production basis with machine-tool equipment especially designed for the purpose.

Smoothness of Operation. Nothing is so annoying to the senses of the airplane pilot or passenger as a rough engine. Any

engine which is not functioning properly, or has been fitted with an unbalanced or out-of-pitch propeller, can develop a considerable degree of vibration. These remarks, therefore, will be mostly confined to the inherent sources of vibration in the two types of engines under consideration during normal operating conditions.

The four-cylinder in-line engine, as previously stated, is not balanced for inertia. These unbalanced forces vary directly as the weight of the reciprocating parts and as the square of the engine speed. Modern engines employ higher speeds than the early four-in-line engines, which were considered so rough, therefore the fact must be recognized that the reduction in weight of the reciprocating parts in the modern four-in-line type has been of real consequence. This is the result of using aluminum-alloy pistons and forged duralumin connecting rods. Nevertheless the four-in-line engine will always be subject to some vibration from unbalanced inertia forces, regardless of any degree of refinement.

On the other hand, the single-crank radial engine of five or more cylinders can be quite nicely balanced, since the resultant of the primary inertia forces and the centrifugal force of the rotating masses are directed along the throw of the crankshaft. No regard need be taken of the secondary inertia forces except to avoid rough periods at idling, cruising, or full-throttle speeds. This can be accomplished for any given type of mounting by a slight variation in the size of the crankshaft counterweights. For all practical considerations, then, the radial engine is in nearly perfect inertia balance.

The radial engine of five or seven cylinders has some margin of advantage over the four-in-line engine in the matter of evenness of torque, due to the added cylinders. Also the radial engines to which we are referring should not become nearly so rough as the four-in-line engine in the event that one cylinder fails to fire or function properly. Crankshaft torsional deflections are of no consequence whatsoever in the short, rigid shaft of the single-row radial, while the vibrations from this source may become both noticeable and destructive in the four-in-line engine unless the crankshaft is especially rigid.

All radial engines cannot be less prone to vibrate than the four-in-line just because the type is inherently in better balance. Full advantage must be taken of the characteristics which the type offers, and when this is actually done the radial will unquestionably operate with less roughness than a four-in-line. We must not judge a type by one or more good or bad representatives.

Opinions as regards the smoothness of some of the modern four-in-line engines have been obtained from a number of pilots who have flown them. Naturally these opinions varied considerably, and several reported that the engines were not so rough as they expected them to be. This leads to the conclusion that all four-in-line engines are not objectionably rough, yet in no instance did any one venture the opinion that these engines were endowed with any inherent characteristics which would distinguish them for smoother operation than that of the radial engine.

Installation. There is really no feature quite so appealing to the average airplane designer as a simple engine installation. The author has actually encountered instances where the possibilities of a simple installation practically overshadowed all other features of the engine.

It appears that, in this particular feature, the radial engine holds the advantage by a considerable margin. In fact, simple installation has always been referred to as one of the outstanding features of the radial type, and comparisons, whenever drawn, have been with reference to the style of mounting required for the in-line type of engine.

Usually, with the modern welded steel-tubing construction, it is required only to provide forward extensions welded to and supporting a ring of the proper diameter. This ring has welded to it clips or brackets to receive studs in the engine crankcase. Aside from connecting engine controls and fuel and oil lines, it is only necessary to attach a few nuts, and the radial engine is installed. The ordinary cowling can be simple and of light weight, besides requiring only a few minutes for attachment.

The in-line engine, on the other hand, requires members extending the length of the crankcase, to form what is usually referred to as the engine bed. These members obviously need considerable bracing, because of the distance which they extend forward of the fuselage. The cowling required to enclose the engine is greater by far than the amount required for the radial engine, and seldom can the controls and other connections be arranged in such a simple manner. Briefly, there is hardly a single phase of installation in which the in-line can be shown to excel.

Service. Comparisons between the radial and in-line types of engine as to their possibilities for maintenance and service will be confined to the more obvious points of difference between the two. Naturally the more detailed questions of service depend upon the individual design and the type of construction employed.

With overhead valves operated through rocker arms and push rods common to both types of engine, there is really not much difference in the problem of service on cylinders. This is practically true of all units which can be readily removed and repaired without disturbing other parts of the engine. But in drawing comparisons with reference to the crankshaft and connecting rods there is a rather marked difference.

The radial engine, which uses articulated-type connecting rods, has but one crankpin bearing, while the in-line type under consideration has four. Burned-out crankpin bearings are not a common source of trouble in either type, yet the possibilities are four to one, and this is also the probable average ratio of the amount of work to be done on these bearings at the regular overhaul periods.

Ball or roller bearings are usually mounted on the main journals of a radial-engine crankshaft, and besides requiring no special fitting, are in only rare instances known to fail. Should replacement be necessary, it can be accomplished with both accuracy and speed. The four-in-line-engine crankshaft is usually carried in five plain bearings, with a ball bearing to carry the propeller thrust. If the ball bearing is arranged to carry radial loads as well, it is extremely difficult to maintain alignment with the plain bearings whenever take-up on account of wear in the latter bearings is found to be necessary. There are five possible sources of a burned-out main bearing in case of a failure in the lubricating system, none of which exist in the average radial engine. When a bearing burns out or must be replaced on account of wear, it is often necessary to fit all new bearings and line-ream them in place, in order to obtain a first-class job. Neither can one fail to take into account, with these long oil passages, the greater possibility of a clogged line, which invariably results in a bearing failure.

There are no apparent differences between the radial and the in-line engine when considered in connection with top overhauling and making routine adjustments, but if the trouble to be corrected is of a more serious nature, the radial-type engine will generally show to advantage. Very few major replacements can be made on an in-line engine unless it is taken into a repair shop, while some radial engines can be almost completely overhauled without being entirely removed from the airplane. This is a point worth considering, especially in a case where the airplane might possibly be grounded at a point some distance from a repair station.

No doubt dozens of points in contrast of lesser importance could be cited, but this really does not seem necessary, since the advantages of the radial engine from a service viewpoint are quite apparent.

Visibility. The general form of the in-line engine is conducive to excellent visibility, but in the particular power class which we have under consideration, the gain in this feature is possibly not as great as one might at first imagine. The upright in-line engine leaves an area for unobstructed vision forward, on either side of the cylinders, which is partly or wholly blocked when using a radial engine. The amount of obstructed vision, and the nature of its effects, depends upon the overall diameter of the radial engine and the seating arrangement in the airplane.

It is perfectly obvious that the diameter of any radial engine should be as small as possible in order to permit good forward visibility, and also to reduce head resistance to a minimum. Apparently some designers have not given this matter the attention it deserves, and consequently their engines do not compare favorably with the in-line type. But when the outside diameter of the radial engine of approximately 100 hp. or less does not greatly exceed 35 in., there are never any complaints about obstructed vision upon the part of those who use them. It is therefore rather doubtful if any great benefit would be derived by a slight increase in forward vision, which can be assumed for the moment would be gained by the use of an in-line engine.

The seating arrangement, however, must be taken into account. The pilot's seat, in airplanes using the in-line engine, is usually farther to the rear than in similar airplanes equipped with a radial engine. This is to balance a heavier engine whose center of gravity is located at a greater distance forward from the rear of the engine. In this position the pilot undoubtedly needs all the forward vision which the in-line engine affords, and therefore the ultimate gain in visibility resulting from reduced frontal area is questionable as compared to installations of the radial engines of small diameter.

The four-in-line engine, when mounted in an inverted position, really offers better visibility than either the upright four-in-line or radial engines, and moreover gives a higher line of thrust. In the author's opinion, this is the only arrangement which permits four-cylinder in-line engines to enjoy fully the few advantages which they possess. But up to this time the development has been slow, due to problems of inlet manifold design and distribution. One firm in Europe has spent over a year on these problems, and, according to latest reports, has not yet reached a solution. The author cannot, for obvious reasons, venture an opinion regarding the engineering phase of this situation.

The inverted four-in-line engine, when more fully developed, may be preferred to one designed for use in an upright position for reasons just given, but it otherwise offers no further advantages as compared to the radial type of corresponding output.

Cooling. It would be necessary to treat extensively several phases of air cooling in order to compare fully the cooling advantages of the several radial and in-line types of engine. Time will only permit mentioning some of the important contrasting features.

The radial cylinder arrangement is practically ideal from a cooling standpoint, in that an equal amount of air is directed against each cylinder. This also holds true with almost any type of symmetrical cowling that may be employed. When using well-designed cylinders it is unnecessary to provide baffles and deflectors to force the air in directions contrary to its general line of flow. Briefly, then, all cylinders of the engine share about alike in the cooling blast available under any set of conditions.

The air blast for cooling the in-line type of engine must be deflected at nearly right angles to the direction of flight. This requires a more or less complicated system of air scoops, baffles, and deflectors if anything like equal distribution of cooling air is to be made available for the cylinders. The correct size and shape of these parts making up the cooling system can only be approached after considerable experimentation. Another problem entering into the design is the arrangement of inlet manifold and exhaust pipes, so as not to interfere seriously with the free flow of cooling air around and between the cylinders. In a completed installation the in-line air-cooled engine is obviously made quite inaccessible.

The question which perhaps is of most interest is the comparative effectiveness of the two cooling systems. No accurate test results have been made available from which conclusions can be drawn. It would seem that the advocates of the in-line type of air-cooled engine would do well to present some data on the subject for the benefit of those who seriously doubt the effectiveness of their cooling system. The author has been unable to find any information which would lead to the belief that the cooling of the in-line type would in any way be comparable to that of the radial engine, and this appears to be the consensus of opinion among those who have really given the matter serious thought. It seems to be generally accepted, that the cylinders of an in-line engine do not operate at uniform temperatures. Perhaps this can be overcome by the proper use of deflectors, and if so, something should be done about it right away, and the proof of the accomplishment properly presented in order to relieve existing doubts. There are numerous reports of stuck pistons due to overheating, and one cannot deduce from such reports that the cooling is either uniform or adequate. The author has been reliably informed that last year at the Concours des Avions Legers in France, the only two airplanes powered with in-line engines to finish had cylinders and pistons changed almost every night during the tour, and this despite the fact that the engines were lubricated by a blended castor oil. For the heavy stunt work of this meet, some of these ships were powered with an air-cooled radial engine, and also the radial engines in the tour finished without repairs.

With the exception of a few spectacular flights of which we read and hear so much, the greater part of the flying with the in-line engine has been done about airdromes in England. In this country greater distances are flown, and it will be rather interesting to follow the success of the in-line-engine cooling system under more grueling conditions.

For the present, at least, it must be concluded that the radial engine, with its cylinders uniformly exposed to the cooling air, maintains a distinct cooling advantage over the four-in-line engine employing air scoops and deflectors of uncertain effectiveness.

Head Resistance. The small frontal area of the in-line engine appears to be its one outstanding feature, and we are naturally quite interested to know how much this actually reduces the head resistance. However, not too much importance should be attached to this phase of the comparison of radial and in-line engines, because speed in light planes is really secondary to other features, such as proper cooling and reliability.

The radial engine apparently offers considerable head resistance, the amount of which depends, of course, upon the speed of the airplane, the size and shape of the engine, and the manner in which it is cowled. It is believed that too frequently comparisons are carelessly drawn between the radial and the in-line engine on the basis of frontal area alone. If this in itself were a determining factor, how then could it be possible to develop our fastest military pursuit ship around a large radial engine? It is not intended to hold any brief here for the extremely large radial engine, but only a few of these installations need be con-

sidered to realize that comparisons cannot be correctly drawn on engine size alone.

Two radial engines of different design, but having approximately the same frontal dimensions, might reduce to quite different equivalent flat-plate areas. Therefore, how can one safely judge the resistance characteristics of an in-line engine by merely drawing comparisons with other engines on the basis of frontal dimensions? The drag or resistance can be measured only by placing full-scale models in the wind tunnel, and the author's opinion is that we may all be surprised when we learn how much resistance is developed by the air scoops and deflectors in turning the cooling air at approximately right angles to its normal direction of flow. It is sincerely hoped that the National Advisory Committee for Aeronautics, or some similar institution, will soon undertake an investigation of this kind. Whatever advantage in reduced drag that the in-line engine may enjoy by virtue of its small frontal area can be attained only with a narrow fuselage and a tandem seating arrangement. Even then it is not always possible to provide the best streamline forms. Many are of the opinion that side-by-side is the best arrangement for a two-seater airplane, both for instruction and private flying. If installed in an airplane of the latter type, the in-line engine must sacrifice any advantage that may be due to small frontal area, because a better streamline form can be developed about a larger nose.

Although reliable conclusions cannot be drawn at this time regarding the effects of small frontal area, its importance can be greatly overrated. One may assume for the moment that the in-line engine holds a fair margin of advantage in reduced drag over the usual radial-engine installation, in which the greater part of the cylinders are exposed to the slipstream. However, when these cylinders are enclosed by a cowling of the correct streamline form, it is necessary to regard comparisons in quite a different light.

Investigations conducted by the National Advisory Committee for Aeronautics disclosed some remarkable reductions in drag with an annular streamline form of cowling which completely surrounded the radial engine. This is generally referred to as the N.A.C.A. type of cowling, and several manufacturers have already adopted it with much success. The reports of these investigations have been carefully studied by the majority of aircraft engineers, and there is no need to even review the data.

Some engineers are rather reluctant to admit the advantages of the N.A.C.A. cowling, claiming instead that actual results fall short of the wind-tunnel investigations, and the engines are not properly cooled. If the cowling was improperly designed it could readily hinder the cooling efficiency of the engine. Moreover, the reduction in drag might fall short of the laboratory results if the installation was not correctly made.

Actual results in flight, however, have pretty well proved the advantages of the N.A.C.A. type of cowling applied to radial air-cooled engines. The maximum sea-level speed of the Curtiss AT-5-A airplane, powered by a Wright "Whirlwind" J-5 engine, was increased from 118 m.p.h. with the original cowling to 137 m.p.h. with complete N.A.C.A. cowling, a gain of 19 m.p.h. or 16 per cent. The Lockheed "Air Express," powered with a Pratt and Whitney "Wasp" engine, had its speed increased from 159 to 175 m.p.h. by the application of the N.A.C.A. cowling. This increase was accomplished with the same propeller and propeller setting, an increase of 75 r.p.m. of the engine being noted, and the cooling of the engine was reported satisfactory.

Such improvements in performance should remove doubts as to the effectiveness of the N.A.C.A. type of cowling. Moreover, it leaves us with still greater doubt in our minds as to the actual reduction in drag resulting from the smaller frontal area of the in-line type engine. The gain in speed by the use of the N.A.C.A. type of cowling is largely the result of reduced drag on the fuselage. While impinging along the airplane fuselage, this disturbed air flow caused by exposed cylinders apparently sets up considerable resistance, and therefore it is wondered if the cooling air leaving the in-line-engine cowling is not causing an even greater degree of drag. Wind-tunnel tests on full-scale models should tell the real facts.

Conclusions

The comparisons of the radial and in-line engine, which have just been made, are neither complete in themselves nor do they represent all the points upon which the two may be compared. An attempt has been made to treat the most important engineering features of the two types as completely as the limited time and data available would permit. As stated in the beginning, it is not expected that many of these questions will be brought to convincing conclusions, but it is sincerely hoped that they will at least encourage an analytical trend of thought on the subject. One hears so many amusing discussions pro and con these days, that it seems doubly important to confine investigations to the more fundamental characteristics.

It is desired to bring attention to the fact that these comparisons have been drawn on the basis of types without reference to any particular engine of either group. Moreover one should not, in all due fairness, compare a good radial with a poor in-line engine, nor a good in-line engine with a poor radial. With a view into the future we are perhaps more interested in the inherent possibilities of each type than we are in existing engines. We are now at a critical stage in aircraft development when problems of this nature should be given more than passing consideration.

From the views expressed, it appears that the radial engine is inherently superior on the basis of weight per horsepower, smoothness of operation, installation, service, and cooling. It also promises to excel in the matter of low manufacturing costs. The upright in-line engine, by virtue of its smaller frontal dimensions, is possibly better for visibility in some installations, although this advantage becomes of little consequence in certain types of airplanes. The inverted in-line type obviously excels in visibility in almost any installation. The comparative resistance or drag of the radial and the in-line engine is still a matter of opinion, and it is hoped that some investigations will be undertaken soon to settle the variety of ideas which now exist.

Discussion

H. L. Brownback.[2] This paper is particularly timely as aviation has reached a critical stage. A few years ago the industry was starved for money, with none in sight. Now companies are formed and merged, and their stocks played with for months, making and losing fortunes before they are even organized to produce. With these conditions prevailing, much of the advertising and quasi-scientific magazine articles originate in the advertising agencies of stock-selling concerns and in company sales organizations, rather than in the better-qualified technical departments. When the writer first entered aviation in 1912, the same malady in a milder form was epidemic, but the money was lacking to complete the "Florida boom" of today.

As the author points out, there is a sellers' market. Everybody who believes himself capable of producing an airplane and

[2] V. P. and Gen. Mgr., Brownback Motor Lab. Inc., Norristown, Pa. Assoc-Mem. A.S.M.E.

getting backing is "going to it" and crying for engines, and in less numbers the same thing is happening in the engine industry. There is one case in which a carpenter who turned garage man in the boom days of the motor car, is now selling himself at so much per share as a second Ford. In building his engine he has had no drawings. He simply took the pattern of a crankcase of some one else's motor, explained a few changes, then showed his mechanics some one else's cam crankshaft and connecting-rod layout, and ordered them to make something like that to fit the case as shown on the patternmaker's layout board which he kept as a drawing. Money was found for a beautiful little plant which is now going into production. This is only one case which has come to the writer's attention; perhaps there are many.

In considering radial versus in-line engines, if the discussion is to be confined largely to small engines, the desirability of either type will depend largely upon the points desired in the airplane.

In carrying mails, in contests, and in emergency flight, speed is of prime importance. In carrying passengers it has great importance, but in ordinary pleasure flying and aerial touring it is decidedly secondary to cost, ease of maintenance and repair, ease of inspection, ease of replacement, and comfort.

In very small airplanes, say, of the two- and three-passenger type which will use from 60 to 150 hp., at least two of the passengers will want to sit side by side and be protected, even though this be done at the cost of some visibility. This year, next year, and maybe for a few years to come, people in decreasing numbers will be willing to sit, swathed like mummies, far apart in the blast of air, noise, gas, and smoke in order to get air transport, but as time goes on and air travel becomes an ordinary means of transport, comfort and companionship in flight will supersede speed as the primary factor in determining the purchase of aircraft.

No one today will build a motor car having the comforts of the average present-day tandem airplane, and try to sell it to the public in competition with the small runabout or coupe; and one can well imagine the situation the average husband or beau would find himself in if he were to drive up to the house of the feminine half of the combination and say, "Here it is, let's ride in it."

These side-by-side protected airplanes are going to be more stub-nosed than the tandem plane, and with their general adoption the major talking point for the in-line engine will be blasted when it is compared with the radial type. Thirteen years with all kinds of engines, here and in France, has taught a lot of lessons, not the least of which is the realization that the pilot, when he crosses wide stretches of water, forests, deserts, mountains, etc., is staking his life on his confidence in the ability and integrity of the engineers who have designed his engine and the manufacturers who have built it, and it is up to us to keep faith.

At present the writer is interested in a radial project and an in-line project, so these remarks are in a way unprejudiced. About six months ago the design of a 55-hp. engine was started and the matter was approached in an open frame of mind. Two engines were laid out, one in-line and one radial, and they were analyzed. Today the designs for the radial are being finished. Why?

Any structure has a foundation, and upon the integrity of the foundation the life of the whole structure rests. The foundation of a gasoline engine with separate cylinders is the crankcase. The radial permits a short cylindrical or hemispherical crankcase enormously strong, with bearings accurately aligned, all parts flanged together, which is suspended in its mount in such a manner that distortion of the mount will not affect it, and can be made very light. The in-line engine on the other hand must have numerous crankshaft bearings accurately aligned and maintained in line by the crankcase structure. The case is generally long and mounted on a frail structure which it must help support rather than be supported by it. That means weight.

If the whole structure be gone through—the size and weight of the crankshaft of the in-line vs. the radial engine, the camshaft and its bearings vs. the floating cam ring of the radial, etc.—it will be found that were the in-line motor to be built with the same factors of safety as the radial, with the same compression ratio, and turned at the same r.p.m., it would be very considerably heavier. A glance by any experienced engineer at two of the three in-line engines offered generally on the American market, will instantly show the comparative flimsiness of the cam followers, rocker mounts, etc., of the two light ones, and the high total weight of the heaviest, as compared to the radial.

Furthermore it is the writer's belief that the high r.p m , that is to say, more than 1900–1950 with small engines and 1750 with medium-sized engines, are undesirable in private-ownership aviation, and the high rated speeds of the in-line engine in order to cut pounds per hp. are very apparent.

One feature criticized in the radial engine has been the connecting-rod structure. There are several solutions to the problem. One is use of the slipper-type rod which, if properly designed, gives good results up to five cylinders per crankpin, and the other is a mathematical solution of the articulated rod worked out and patented some years ago by A. Anzani, the father of the modern radial engine, with whom the writer was closely associated for some years. Mr. Anzani's scheme gives the same stroke on all rods. He also worked out a very ingenious rod for the big two-row 20-cylinder water-cooled radial which developed 840 hp. at 1400 r.p.m., and would develop more than 1000 hp. at 1600 r.p.m., which had large bearing surface on the knuckle pins, and yet was light and compact.

As to head resistance, the radial has more frontal area, but what is the resistance of the air conduits necessary to keep No. 3 cylinder cool on an in-line job, when climbing at full throttle, and what is the extra installation cost, the liability to damage of all of the sheet-metal work when the engine is being serviced, and the additional weight? An installation drawing of a well-known English in-line engine calls for cowling having $^{3}/_{8}$-in. passages between long sheets of sheet metal, which makes the engine appear somewhat burglar proof, and does not look as though it would remain efficient if handled roughly. Another thing: "It takes a pound to fly a pound," so every pound of unnecessary weight added means very approximately an additional pound of airplane structure, and as every 20 lb. means 1 wide-open or 1.2 cruising hp., it doesn't take much time to consume more hp. than a little additional head resistance will consume. Furthermore, because of its ease of installation and reliability, even in typical hay-wire installations, the radial has suffered a handicap until lately by just being hung on the nose of the fuselage, often totally uncovered. Judicious cowling will give good head resistance and temperature characteristics, while if one desires the last ounce of speed, the engine can be totally cowled and be no harder to get at than the in-line, but easier if a swing mount is used, and will have better temperature characteristics and little or no more penetration resistance.

One large maker of in-line engines modestly advertises that his engine powers the majority of light planes and made the light airplane possible, and produces an imposing performance record. That is sheer advertising "bunk." In the first place, several other countries, especially the United States and France, produce large numbers of planes of under 100 hp. Today most of the world's planes of under 100 hp. are powered by war-surplus OX-5 engines, and most of those powered by new-production motors carry radials. Last year France and Germany turned out immense numbers of low-power radial and low-

power air-cooled in-line engines, while in the United States the Department of Commerce bulletin of May 24, 1929, gives the 1928 production as: Engines under 150 hp., new-production radial 621; and all other types, 11.

As for records, a rival of the company mentioned and of the same nationality states in an advertisement: "The house of XYZ holds six of the seven international records now standing to the credit of . . . . . (Country) . . . . . of which three were gained by the XYZ, the only . . . . (Nationality) . . . . . light aero engine to hold world's records." All of the other world's light records are held by radials, one being endurance of 24 hours 37 minutes with but 40 hp., and as for the rest they simply enumerate remarkable flights. This would be impracticable with the radial as the compilation would fill a volume.

In the foregoing it is not the intention to be critical, but simply to impress upon the aeronautical world that its judgment should not be swayed by present-day high-pressure advertising.

The design and perfection of a really good in-line engine is a real job. Last year the company which made the first really good air-cooled eight-cylinder V-type engine started one, and with 18 years of experience in aero engines of 70 to 850 hp. back of it, the company has not yet marketed its engine.

In America some severe conditions are imposed on engines such as sleet, sandstorms, etc., all of which make at least a partial protection of the valve-operating mechanism desirable. This is comparatively simple in the radial engine but apparently difficult in the in-line type. The only 100-hp. in-line engine to pass the Department of Commerce tests, an American engine, was provided with a Mercedes-type overhead camshaft, and also has a unique gas-distribution control which governs the quality of the mixture by the angle which the manifold branch pipes present to the gas flow, giving a comparatively rich mixture to the cylinders, which normally run hot.

Another point which seems to have received more consideration in radial than in-line engines is lubrication. In this country we must use mineral oils and slow circulation, and wet-sump storage found in most in-line motors is not conducive to low oil temperature and consequent life, while the high-speed circulation and separate tank storage incident to most radial designs is ideal for oil cooling and long life.

As to cost, the radial engine properly designed is no doubt cheaper than the in-line engine of equal quality, and its installation and the structure necessary thereto are undoubtedly cheaper. Airplanes having radial engines will be cheaper than those carrying in-line engines, quality, profit, sales costs, etc. being equal in all cases.

For top overhauls the cost should be about the same in both types, but for complete overhauls the writer thinks the difference in man-hours will be in favor of the radial by at least 50 per cent, provided equal skill is used in the design of both engines.

The question of vibration is most important as an airplane structure is light, and in case of the popular welded steel-tubing fuselages, vibration coming from the motor can be most destructive. The ability to balance the vibration and fairly accurately to place the vibration periods through counterbalancing in a radial is very valuable. In both types more thought should be given to rubber vibration insulators in the mounts.

One thing in favor of the in-line engine, as far as sales resistance goes, is public familiarity with the type. The public, accustomed to seeing vertical steam engines and vertical gasoline engines, both marine and motor-car, understands the type and therefore feels at home with it. On the one hand, it does not understand the radial engine's multiple connecting-rod and valve-operating layout, and not understanding it, has a vague sense of resentment. On the other hand, mechanics who are accustomed to both types prefer to work on radials because of their simplicity.

There are two types of engines which should have a place in our minds. One is the two-row radial which, while it sacrifices some of the simplicity of the single-row and its cheapness of manufacture, has good points in the smallness of its cylinders for the same power, visibility and ease of cowling because of its small diameter, and its even torque due to its 6, 10, or 14 cylinders, and the other is the six-cylinder in-line engine which will undoubtedly come into a certain vogue with the perfection of Prestone cooling and the solution of the temperature difficulties now encountered. In large installations radial types, both air- and water-cooled, are on the gain.

In Europe where the Air Union has always used nine-cylinder water-cooled radials, the British 12-cylinder V- and W-type water-cooled, and the Germans six-cylinder vertical and 12-cylinder V engines, now at Le Bourget one sees nine-cylinder radials on the French and most German planes, and 14-cylinder radials on the British.

Owing to its structural advantages the writer looks forward to the increased use of large multi-row radials, using liquid cooling where 800 or more horsepower is desired. Both the radial and the in-line engines have their advantages and disadvantages, but as skill in designing eliminates some of the non-inherent faults of the present radials, he expects to see them gain ground in future installations, both high- and low-powered, even faster than they have in the past.

E. E. Wilson.[3] As the author points out, an analysis of this problem must necessarily be qualitative because of the present lack of reliable quantitative information. The author analyzes the two types of engines in a way that deserves the careful study of designers.

Our modern radials owe their superiority to fundamental advantages: (*a*) the single-row radial form, and (*b*) the direct or air-cooling principle. Considered from the points of view of light weight, low cost, easy maintenance, dependability, durability, and fuel economy, the advantages of these two features are outstanding. The in-line engines suffer by comparison.

As is pointed out in this paper, the in-line engine's only advantage lies in its improved forward vision. Even this advantage is greatly reduced in low-powered airplanes. The in-line engine's real advantage from this point of view is only attained when the engine is inverted. Designers will have to choose between the inverted in-line and the radial, and will have to weigh the advantages of improved forward vision against the disadvantages of increased weight, increased cost, and increased difficulty of maintenance.

The radials can be improved greatly through reducing their overall diameter and cleaning them up generally. This not only reduces drag but interference with forward vision. Experience to date indicates that the drag incident to cooling an engine is about the same in any good installation. A certain amount of heat must be dissipated in this way, and other things being equal, the cost in drag is about the same. It is clear that the in-line engine offers no advantage from the point of view of drag.

In general the radials have superiority from all points of view. Time will demonstrate to commercial operators what has already been demonstrated in naval aviation, namely, that the single-row radial is fundamentally superior.

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[3] Commander, U. S. Navy.



THE BELLANCA CH STANDARD MODEL BEING PRODUCED IN QUANTITY AT THE BELLANCA AIRCRAFT CORPORATION. POWERED WITH A WRIGHT WHIRLWIND ENGINE, THIS PLANE WON FIRST PLACE IN BOTH EFFICIENCY AND TRANSPORT RACES AT THE LAST NATIONAL AIR RACES

Development of the Commercial Airplane

An Outline of Some of the Factors That Have Contributed to Present-Day Success in Aviation With a Discussion of Single- and Multi-Engine Airplanes

BY G. M. BELLANCA,[1] NEW CASTLE, DEL.

AVIATION has recently made great advances in the eyes of the world, but while most of the spectacular accomplishments have come as surprises, they have really represented the culmination of the studies and experiments of the last ten years. Lindbergh, Chamberlin, Byrd, Brock, and many others contributed in the most popular and appealing manner to the development of this new science, but the steady work of constructors and designers must not be forgotten.

Progress in aviation has not always been steady and continuous but has deviated from time to time from a constant course.

The successes of the Wrights inspired the works of Voisin, Curtiss, and Farman; and through their influence the biplane had the official sanction of the aeronautic experts of the world. Subsequently, however, the Blériot monoplane made its entry into the field, followed by the Antoinette, and later by the Nieuport monoplane, and the early prominence of the biplane was threatened. After the crossing of the English Channel by Blériot, public opinion favored the monoplane. From that time on, we see the development of the airplane depending upon accidental or fortuitous successes instead of moving surely and steadily along in accordance with aeronautical laws.

In the books of five years ago, one finds a marked preference for the biplane. Today, in most publications, preference is shown for the monoplane. Typical is the instance of a certain author who, in rewriting and amplifying his book published four years before, changed his views in favor of the popular one.

The same uncertainty characterizes opinions pertaining to airplane power plants. Should they be single or multiple units? Opinion varies widely.

The feeling prevails that the multiple-engine plane offers a greater margin of safety than the single-engine plane. However, it is a fact that in most of the present designs the horsepower required for normal speed and load is about 60 per cent of the maximum available. Therefore, the advertised advantages of the multi-engined plane are limited, especially when we consider the fact that a single-engine plane is more efficient and offers fewer possibilities for engine troubles.

A similar situation prevailed for some time with the cooling system for airplane engines, and when the experts met during the war to give a more reliable engine to the airplanes of the allies, the water cooling system was preferred. This was in spite of the fact that most of the prizes in air contests staged in the first period of aviation were won by airplanes with air-cooled engines. Even now, after the impressive demonstration of reliability and economy given by the small air-cooled engines, the majority of European countries are using the water cooling system in their planes.

To add to this uncertainty and confusion of opinion there was also the practice, in aeronautical contests, of offering awards for superiority in spectacular attainments such as speed, climbing ability, and skill in landing, all of which diverted attention to the size and power of the engine with little or no regard to efficiency. It is worth while to remember that the limitations on the power plant which were laid on Germany by the Allies concentrated researches and experiments on the problems of airfoils and resistance and stimulated noteworthy advances in aviation.

These and other factors have led to uncertainty in the structural work and in aerodynamic design, diverting progress from a standard path, and creating types which are appealing to the eye rather than types which are based on the fundamentals of scientific design.

INFLUENCE OF MILITARY DESIGNS ON COMMERCIAL PLANES

Commercial aviation has been decidedly influenced in the last five years by military aviation. Whatever year book we open, either here or abroad, we see reproduced in commercial designs all the characteristics in structure, proportions, aerodynamic

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[1] President and Chief Engineer, Bellanca Aircraft Corp.

Presented by the Aeronautic Division at the Annual Meeting, New York, N. Y., Dec. 3 to 7, 1928, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

qualities, and power plant of military planes. England, one of the first countries that undertook commercial aviation, is slowly shaking off this influence. More slowly the other European countries are moving in the new direction. In America, a few years ago, we could not distinguish a commercial plane from a military airplane.

The development of the real commercial cabin airplane finally came as the result of the Philadelphia Air Meet held in 1926 in which a commercial airplane was considered the sensation of the contest. After the Philadelphia event, the commercial airplane began to be considered as a separate and distinctive design with functions no less important than those of the military plane.

The vogue at that time in commercial aviation was the same as that prevalent in military aviation—streamlining, with reduction of the section of the body to the lowest limit. Other conditions such as airfoils, outside bracing, controls, etc., remained unaltered. So also the disposition of the wing truss, the universal use of certain airfoils, the methods of attaching the wings, the type of the landing gear, and the distribution of load were not changed, while every deficiency in performance was remedied by increasing the power of the engine.

At this time the airplane as a new means of rapid transportation was moving along the lines first followed by the automobile in which the "sport" requirements were first considered and only later reliability and comfort. The commercial airplane, however, was designed with the idea of being economical and practical; hence maximum efficiency and comfort were desired.



THE "COLUMBIA" IN THE AIR ON ITS WAY TO GERMANY. ON THIS FLIGHT CHAMBERLIN AND LEVINE BROKE THE WORLD'S DISTANCE RECORD. IN THIS SAME PLANE, A FEW WEEKS BEFORE, CHAMBERLIN AND ACOSTA SET A WORLD'S ENDURANCE RECORD

Previous to the Philadelphia contest the cabin airplane for commercial purposes had made its appearance in 1922 in the Monmouth Air Races; but not withstanding the enthusiasm raised in aeronautical circles and in the press, the design did not have imitators.

The pay load of this airplane was 7.5 lb. per hp. whereas the pay load of the best and most representative foreign machines was at the most about 3½ lb. per hp. Here in America, the Air Mail D.H., rebuilt by the Government, had attained a pay load of only 1½ lb. per hp.

Up to that time the prevailing design had been that of the open cockpit or cabin with the least cross-section. But while the designer was sacrificing every personal comfort in the fuselage many parts were left exposed, reducing the performance considerably.

In commercial aviation, the airplane was designed to incorporate the latest results of researches and experiments. A machine was produced with all the space necessary for the comfort of the passengers; a machine in which practically all the essential elements of comfort were considered as well as lifting power and stability; a machine with a much greater safety factor and with faster flying speed and lower landing speed than machines built along military lines.

A NEW ELEMENT IN CONSTRUCTION

Some time ago, aerodynamic design was considered merely a mathematical exercise. Today, all factors, whether analytical or

based on laboratory expriments, have the main object of putting aviation at the service of the new industries. So rapid has been the study in this direction that we are wondering what new marvelous results are coming in the near future.

Today, in addition to efficiency, comfort is being considered, and with comfort, safety. Safety, to a certain extent, is a psychological phenomenon depending on the pilot's ability in maneuvering. The more at ease the pilot is, the better he controls his plane. Yet the pilot is only one factor, the others lying within the ship. When the ship is unstable and goes into a spin or stalls, the blame falls on the designer, not on the pilot. There is a case of a historical accomplishment attained with an unstable airplane, and there are also cases of good airplanes gone to destruction in the hands of poor pilots.

Single- Vs. Multi-Engine Planes

As previously mentioned, the multi-engine airplane is sometimes preferred, on the grounds of safety, to the single-motor one although the problem does not yet seem to be solved.

The problem of more engines has been considered since the beginning of aviation. Many experiments have been made but as the ships involved required the power of all the engines at the same time, the arrangement resulted in a great disadvantage, because the greater the number of engines, the greater were the possibilities of trouble and consequently forced landings. The multi-engine planes of today are much better, but other improvements in efficiency are necessary before they will be real successes.

To be effective, a multi-engine airplane should be able to fly on 50 per cent of its total power and carry all the load under normal weather conditions. In other words, a ship with two engines should be able to take off and cover its entire range of accomplishment with only one engine. There is, I believe, no airplane at present that meets this requirement.

These considerations bring us to the conclusion that the present uproar in favor of the multi-engine ship does not find justification in the facts. While there are advantages of safety for the multi-engined planes over the single-engined planes, the single-engined plane has its definite advantages in its own field.

Nor did the successful flights accomplished by the *America* and *Southern Cross* change the situation. These accomplishments prove the reliability of the single-engine plane for certain kinds of flights. The fact that three engines did not fail determined without a shade of doubt that the chances of trouble with one engine alone would have been proportionately less.

In one way only could the multi-motored airplane have proved superior and that is by making a long flight in spite of failure of one of the engines. But there is not a single case to support this. The only instance approaching this is that of Mrs. Grayson in the first-attempted flight of the ill-fated *Dawn*. A few hundred miles after the start, trouble developed in a cylinder of one of the twin engines, cutting down the engine speed to about 1500 r.p.m., partially reducing the total power. This was enough to compel the pilot to throw overboard part of the gas, to send out SOS signals, and to hasten back to the base. This isolated case, offered by one of the best multi-engine ships, shows we cannot derive a definite conclusion as to the present multi-engine airplane for oceanic flights.

The single engine has a creditable place in aviation and will keep it. Even after we shall have succeeded in designing a good and modern multi-motor airplane, the single engine will continue to be used in commercial airplanes of medium size.



The Latest Bellanca, the "Roma," a Sesquiplane Powered With a Pratt & Whitney 500-Hp. Hornet Engine. This Plane Took Off With a Gross Weight of 12,500 Lb.

American Supremacy in Commercial Aviation

In commercial aviation, America holds, and for a long time has held, one of the leading positions.

In a report made several years ago by the Joint Committee of the United States Department of Commerce and the American Engineering Council, the following conclusion was reached from a study of the economic characteristics of the commercial plane: namely, that "considering the pay-load capacity and the speed, American designs are ahead of foreign ones."

"As for example, while in America an airplane having a single 200-hp. air-cooled engine attains a maximum speed of 125 miles per hour, a cruising range of 400 miles at 100 miles per hour, and a pay load of 800 lb. or 4 lb. per hp., a German plane with water-cooled engines of the same horsepower and pay load attains a maximum speed of only about 100 miles." It was this economic characteristic that made possible the endurance record of April, 1927, and also the famous Atlantic flights.

Information Available to Designers

After these successes, aviation found itself advancing at a steady pace toward the conquest of the future. Today there is a veritable mine of information available to every designer of

airplanes. There is no problem which has been left untouched.

Even for the beginner, it is no longer a secret that complete success in aviation is a matter of compromise. The designer must balance conflicting factors in order to attain the complete unit that meets certain demands either in speed or in efficiency, maneuverability, or convenience.

The young engineer, depending largely on calculations, makes assumptions of matters of which he lacks exact knowledge. After the first disillusionment which costs him so dearly, he is forced to advance carefully, to be conservative, not to take chances, and not to leave the path marked by hundreds of others.

The Situation Today

Surveying the aeronautical situation today, we find many striking developments in the most important factors with no less important developments expected in the near future.

While the designer is remodeling his airplane in the light of the latest researches and experiments, the laboratory worker goes on incessantly, tackling new problems, pressing eagerly upward and onward, opening new fields of experiments, trying to follow the most intricate pathways, and contributing in every efficient manner to the advancement of science.

In this complicated work we have not lost sight of the main purpose; to develop an airplane which is aerodynamically safe for transportation and navigation; to develop an airplane which can be flown by the average man in almost any kind of weather. In consequence of this impulse, studies have been started all over the world. They cover the most varied fields.

From these studies, great advances have been made in air-cooled engines cutting down the weight, improving performance and reliability, and effecting great reduction in resistance and cost of maintenance.

In America we are following with great interest the experiments of the Wright Company on radial engines and on 12-cylinder air-cooled V-type engine of approximately 600 hp. Other interesting experiments are being made by the Curtiss Company with the "Hex" radial air-cooled engine of 600 hp. having two banks of cylinders; by the Allison Engineering Co. with the inverted air-cooled Liberty engine; and by the Packard Co., with radial and V-type engines as well as its Diesel engine.

The necessity of increased efficiency in flying at all altitudes has concentrated much effort on the supercharger, with the result that it has passed from the experimental stage to practical use.

Research on greater reliability has brought to experimental work the slotted wings of Handley Page and the slotted rear flaps of Göttingen; but while we are taking advantage of this undeniable improvement, we cannot say yet that the solution of the problem lies along these lines. Many structural difficulties are in the way of the arrangement.

Success has been achieved in fuel-injection engines to such an extent that we can already see what a great part it will play in the future airplane for the economical maintenance and in the elimination of inflammable and dangerous materials.

Another promising field of research is that of the variable-pitch air screw, the tests of which in the last two years have given wonderful results. During the tests conducted a few months ago by the Royal Canadian Air Forces, the blades maintained perfect track and no trace of failure or strain was found in the anchorage.

Because of the steady progress of the last two years, we are now engaged in a revision and refinement of theories. Excellent airplanes are in the air already and other better airplanes are coming. The year 1929 will see commercial aviation growing tremendously in every part of the world.

While making this progress, we should be careful not to be side-tracked from the main issue, a good reliable airplane structurally and aerodynamically sound. Every mechanical device should lead to greater efficiency and safety, for it must be remembered that no new ingenious device changes a poor airplane into a good one. It should be borne in mind that commercial aviation is for public service and ought to be safe. Cruising speed and landing speed should be determined by the general purpose of the design and not by the necessity of competition.

There are other questions which need answers; for instance, the magnitude and distribution of the forces acting on the ship; the mutual interference between propellers and the other parts of the ship; the effect of mutual interference between wings, fuselage, and landing gear. The designer, trying and experimenting, attacks these problems. To help him there are studies and experiments pertaining to every field; supercharging, metal construction, welding, strengthening of plates and tubes, fittings, woods and glues, metals and alloys, pressure distribution, wing and tail ribs, covering and dopes, propellers, oil sprays for fuel injection, engines, resistance and cooling power of the radiators, and ground effect.

From these united efforts, there is no doubt that many promising and permanent successes will result. One thing must be remembered; we must continue in the future to apply scientific knowledge and common sense to the problems of aeronautical design.

[The discussion of this paper was conducted jointly with that of the next paper by T. P. Wright and follows it.]

AER-51-33b

Relation Between Commercial Airplane Design and Commercial Uses of Airplanes

By T. P. WRIGHT,[1] GARDEN CITY, L. I., NEW YORK

The paper presents some of the problems of the airplane designers, together with an indication of a trend in airplane design which it is felt possibly may take place. The following topics are discussed: fundamental requirements, design elements, commercial uses of airplanes, and airplane designs.

Under the last item, an attempt is made to analyze the points discussed in the preceding ones, with a view to suggesting certain design arrangements which seem most logically to fulfil the requirements which the airplane so designed must meet.

I—FUNDAMENTAL REQUIREMENTS

A—Singleness of Function in Airplane Design

ONE FUNDAMENTAL consideration with which every airplane designer soon comes in contact is the importance of designing for a single purpose. During the development of a new design one is constantly faced with the necessity for compromise. It sometimes seems that there exists no element of design which does not conflict directly with every other element. Conflicts, requiring compromises, exist between such important elements as weight saving and structural strength; between weight saving and low production cost; high performance and commodious fuselage size; high cruising speed and low landing speed; high power loading and low operating expense; and many others. The difficulties of airplane design are further increased because of the necessity for dealing with many different branches of engineering, including mechanical, structural, aerodynamical, metallurgical, and naval architectural. It is because of these inherent compromises, difficulties, and complications that I feel the designer will do well in order to assure reasonable success in advance of construction to design with singleness of function in view. There must be only one major specific use for the airplane designed.

Up to the present time there have been comparatively few users of airplanes. It has been, therefore, almost a necessity that types produced be capable of performing several functions, in order that the market might be sufficiently broadened to result in enough sales of a given type to allow anything like production. This has not been conducive to rapid advance in the art, and it is felt that the anticipated increase in production possibilities, brought about both thorough introduction of new fields of activity for airplanes as well as demand within each field, will eliminate the necessity for designing multi-purpose planes, thereby permitting the design of single-function planes of far greater efficiency than were possible in the past. As has frequently been stated, it has been extremely difficult to convince the traveling public of the benefits to be derived from air transport, when the airplanes in which demonstrations had to be made were rebuilt, war-surplus, military ships. The same general condition exists in trying to adapt successfully a mail plane to a passenger carrier; a taxi-service ship to a transport; a night-flying transport to a day-flying transport; or even in adapting a ship which operates regularly from low-altitude fields to a ship for identical purpose but which operates from high-altitude fields. It should be noted that in other forms of transportation, wherein the basic problems are less difficult of solution, there is strict adherence to this principle of singleness of function in design. We have freight, coach, Pullman, parlor, dining, and express cars; passenger, freight, switching, and mountain locomotives; pleasure cars, delivery cars, and trucks, and, if you please, race horses, riding horses, truck horses, and plow horses. Now, therefore, that the field of aviation is enlarging and airplane uses are becoming more definite, it is particularly desirable to study carefully the probable trend of development in types of planes.

I believe we may anticipate a greater diversity of types of airplanes than exist in the older fields of transportation, because of the encroachment of aviation into all of these older fields. It is not to be expected that airplanes will supplant steamships, railroads, or automobiles; they will rather supplement and to some extent encroach on the fields of all three. The extent of and varying conditions found within the United States make this country particularly suited to the development of divergent types of airplanes. It is also likely that, once started, the achievement of airmindedness among our people will be rapid. This desired condition will be accelerated by designers if future airplanes are developed with singleness of function as a fundamental requirement.

B—Safety and Reliability in Aviation

The second and perhaps most important fundamental which must obtain in the design of all airplanes is safety and reliability. I feel that safety is in fact all-important. Designs which in any way lack this quality cannot be ultimately successful. It is, to me, the most important design requirement from the standpoint of usefulness in our crusade to make the public airminded. In order that airplanes may obtain a position comparable with that held by other carriers of men and goods, an equal degree of safety and reliability must be present.

As safety, by definition, is freedom from danger, some of the types of danger that at present exist in aviation should be outlined. Causes of accidents which may result either in loss of life or in schedule interruptions may be divided into two classes: (1) accidents brought about by causes not directly attributable to design deficiencies, and (2) accidents brought about by causes directly controllable in design.

(1) Accidents Brought About by Causes Not Directly Attributable to Design Deficiencies

In the first classification of accidents, those not attributable to design deficiencies are those due to errors in judgment of the pilot and those due to weather conditions.

Errors in Judgment of Pilot. Although at present some 50 per cent of accidents are marked against this cause, it is felt that better training schools will be developed rapidly and that in addition airplanes will become easier to fly and more foolproof, so that the percentage will decrease and in relatively short time reach an amount entirely comparable with that which exists in other forms of transportation. With proper training and equipment, there is no reason for expecting the personal equation to become a greater consideration in aviation than elsewhere.

Weather Conditions. Severe storms and fog are admittedly difficult conditions with which to cope. They cannot be con-

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trolled, but to a great extent they can be predicted. Their seriousness can be decreased greatly by the establishment of more and better meteorological stations and by the equipping of all our transport planes with reliable radios. Accompanying ground-control stations must be established. A solution of the problem of ice formation must and undoubtedly will be found. Also there must be further improvements in navigational instruments and equipment in order that the only hazard of fog flying may be one of gas supply. Improvements in airplane control and stability will also lessen the hazards of bad weather. It may be confidently expected that all these improvements will be developed so that here again present dangers which may exist will be to a great extent eliminated. In comparison with other means of transportation, I believe weather conditions will always be somewhat more of a source of danger in aviation, but I also believe that the percentage of accidents brought about by this cause will be small, approaching that which exists in rail travel and equaling that which exists in boat transportation.

(2) Accidents Brought About by Causes Directly Controllable in Design

Accidents brought about by causes directly controllable in design include structural failures, aerodynamic deficiencies, power-plant failures, and miscellaneous causes.

Structural Failures. The small number of accidents credited to this cause furnishes a record of which we may justly feel proud. An analysis of accident reports shows that only 8 per cent of all airplane accidents are caused by structural failure. Present-day stress analyses are accurately prepared, are based on sound assumptions, and make use of refinements unequaled in other fields of engineering. Further improvements may be expected, however, so that the occurrence of structural failures in the future will be so rare as to make them almost negligible.

Aerodynamic Deficiencies. There is a great need for improvement in this field. It is only comparatively recently that there has become general a realization of the importance of designing airplanes which are stable and easily controllable in all respects. Also a knowledge of the means of accomplishing this end has only just been made generally available. I believe the improvements to be expected in this direction are greater than in most others, but that ultimately the piloting of an airplane will be only slightly more difficult than with other means of transportation.

Miscellaneous Causes. There are a number of miscellaneous causes of accidents, such as fires, improper and insufficient airdromes, poor cabin arrangements, poor pilot's cockpit arrangement, and others, which are all appreciated by designers and engineers. The solution of the problems involved are not insurmountable and will, I feel, be overcome rapidly. The landing-field problem particularly is of the utmost importance. Its solution, however, will be almost automatic once the demand for air transportation is built up to such an extent that all communities are forced to provide such facilities if they hope to enjoy the benefits which an airminded people will have learned are attainable. In 1850 and 1860 the town which snubbed the railroad soon found itself overshadowed by its small neighbor who possessed greater vision.

Power-Plant Failure. I believe we can take an optimistic view in connection with the probable solution of all of the causes of accidents previously described, whether due to natural conditions not directly controllable in design, or to design deficiencies which are subject to improvement—at least to the extent of rendering such occurrences sufficiently infrequent as to make them of no greater importance in the public mind than comparable causes in other modes of transportation. There are still some people who do not ride in subways or cross the ocean because of the dangers involved! The effect is negligible, however, and I feel confident that a similar state of affairs will ultimately exist for air transportation. But with power-plant failures a somewhat different condition holds. Although we may expect to witness the development of airplane power plants of equal or even greater reliability than obtain for automobiles, it is nevertheless true that when failure does occur, the results, in the case of single-engine planes, are far more disastrous. It is difficult to convince the ordinary man that he should regularly use airplanes as a means of transporting himself, his family, or his goods, when he has in mind the possibly disastrous results which engine failure may bring about. And remember, he has had first-hand experience in the behavior of engines in his automobiles. The study of this particular problem has convinced me that the only possible solution is the use of multi-engined airplanes, so designed as to be capable of flying horizontally with one or more disabled power-plant units.

I am of the opinion that the fundamental requirement of safety and reliability will make the very general adoption of multi-engined designs in most fields of air transportation imperative. In another part of this paper the limitations involved in the above statement are discussed, and, in more detail, the most likely power-plant arrangements. The present discussion serves to emphasize that safety is a fundamental requirement in the design of every airplane. Every one actively engaged in aviation should constantly bear this in mind, as it is absolutely necessary to demonstrate that air transportation is safe before the country will attain complete airmindedness and regularly use airplanes to the extent which we feel is merited.

C—Economy of Air Transportation

Any commercial enterprise in this country must pay its own way. As applied to aviation, Wm. B. Stout says, "A commercial airplane is one than can support itself financially in the air." The whole political and economic growth of this country is closely related to the development of means of transportation. Air transportation is the latest development, and in order to carry on and parallel the good work of its predecessors, it must be placed on a sound economic basis. Fortunately we do not have the handicap of Government subsidies, either in the construction or operation of airplanes, which are present in most European countries. On this account our advance has been somewhat slower, but will be sounder economically and much more rapid hereafter.

In order that any transportation line may be successful, it must maintain its schedules. This means reliability. However, reliability of schedules alone is not sufficient. Speed also is necessary. It is the objective of air transportation to provide for greater speed with accompanying safety and reliability. The importance of safety and reliability has already been emphasized. It is now necessary to consider the fundamental necessity in air transportation for speed at reasonable cost. The elements, therefore, that must enter into the figure of merit for airplane designs are speed, pay load, and safety as the directly desirable attributes, with power and operational costs as the quantities which must be kept at a minimum. To attain speed the designer must produce a job which is both light and "clean," a term used in aviation to represent aerodynamic refinement. To make possible the transportation of a high pay load, the design must be light and must use efficient airfoil sections. To be safe, the plane must possess adequate load factor, a reasonably low landing speed, adequate navigational equipment, stability and controllability, and, in most cases, multiple power units. In order that such an airplane may be susceptible of economic operation, it must have the least possible power in its engines, must have efficient engines, and a construction which permits low maintenance costs. It is essential that each of these considerations be

analyzed in laying out each new design. Their relative importance will differ for each design depending upon its use or function. In order to arrive at a design which is economically practicable, it is necessary to study carefully the exact nature of its flying duties in conjunction with its design elements.

By paying strict attention to the fundamental requirements above discussed, I feel confident that designs of airplanes will be developed which will be of such excellence as to make certain the rapid adoption of the airplane as a vehicle of transportation equal in importance to the other means now established.

II—DESIGN ELEMENTS

Certain elements of design which the engineer must consider are discussed in the following paragraphs. The relative importance of each element will differ for each airplane used.

A—Load to Be Carried

In addition to its own weight, an airplane must carry the following items of useful load:

(1) *Crew*, consisting always of the pilot, and in addition, in some cases, assistant pilot, radio operator, mechanics, attendants. There is always the possibility of duplicating the duties of the personnel.

(2) *Fuel and Oil.* Considerations of range of operations with cognizance taken as to wind conditions prevalent for the particular route used, govern the decision on fuel supply.

(3) *Equipment.* Under this heading come such items as radio, lavatories, instruments, seats, and cabin appointments and sleeping accommodations.

(4) *Pay Load.* This item includes people, baggage, mail, express, or any other load for which payment is asked for transportation service rendered.

B—Performance Desired

The following are the main characteristics which come under the head of performance:

(1) *Cruising Speed.* This is the most economical speed of an airplane at which the miles traveled per gallon of fuel is greatest. It is the speed at which an airplane engaged in air transportation normally travels. It is usually about eight-tenths of the high speed.

(2) *Landing Speed.* This is the speed at which the plane will land. In order to avoid accidents when landing, it is essential that this characteristic be as low as possible. The landing speed is usually slightly less than the stalling speed or lowest speed at which the airplane can maintain horizontal flight.

(3) *Rate of Climb.* It is essential that a high rate of climb be characteristic of commercial airplanes as it represents a measure of the airplane's ability when taking off to clear obstacles which may bound the airdrome.

(4) *Service Ceiling.* This characteristic must be high in the case of airplanes which regularly operate in mountainous country or from high-altitude fields. It must also be high in the case of planes which are to be used for certain kinds of photographic and spotting work.

(5) *Range.* This characteristic represents the distance that the airplane can fly without replenishing its fuel supply. It is usually given either in hours or in miles at cruising speed.

C—Power Plant Available

The next element which must be completely determined by the designer is the power plant. A great many considerations must be studied in connection with this element, the chief of which are discussed in the following paragraphs.

(1) Power to Be Used

The total power on which the design is to be based must be selected with care. Very frequently designers are too optimistic as to their ability to construct to a low weight figure, and as a result find that when completed the only salvation for their plane is to add power to make up for such deficiency. The result can never be so economical a design as would have resulted had the other characteristics originally laid out materialized as planned. It must be remembered that in deriving the figure of merit the necessity for high power immediately tends to limit the plane as an economical paying transportation unit. The point is emphasized, as it has been so frequently forgotten.

(2) Number of Power Units

Decision as to the number of units into which the total power selected is to be divided is a question receiving much attention at the present time. As previously stated, multi-engine arrangements are essential in many types of airplanes. I believe that eventually the majority of air-transportation functions will be performed by this type of airplane. Assuming that a given type is to have more than one power unit, the next question is, how many? In a succeeding discussion, an answer to this question for each type of use described will be attempted. It is merely proposed here to state some mathematical facts which should aid in making a decision.

Assume that for any one power unit of the general type selected for adoption there is an average chance of completing 97 out of each 100 trips of a given scheduled run without failure. This means the chance of failure is 3 out of 100. (This can be expressed more conveniently as 9700 and 300, respectively, out of 10,000.) In Table 1 the chance of success and failure for airplanes equipped with multiple power units will be found, each unit having the same reliability as assumed above, and on the assumption that the airplane can maintain horizontal flight with one (or in one case, two) disabled power units.

TABLE 1

Number of power units	Least number of units with which horizontal flight can be maintained	Chances in 10,000—of success	of failure
1	1	9700	300
2	1	9991	9
3	2	9974	26
4	3	9948	52
4	2	9999	1

The figures of Table 1 are derived from well-known and reliable mathematical formulas dealing with probability and chance. A study of the failure column reveals at once the vast superiority of the multi-engined plane from the standpoint of reliability. A fact of great interest, perhaps not so well known, is the higher reliability of the two-engined plane that will fly with one engine when compared with the three-engine plane that will fly with two engines. It is important to keep this fact in mind when considering the power-plant arrangement in new designs for which multiple units are to be used. Attention is also drawn to the almost perfect reliability of the four-engined plane capable of flying on any two units.

(3) Type of Engine

Under this heading must be analyzed several considerations of almost as controversial a nature as that last described. These considerations are as follows:

(*a*) Air-Cooled Versus Water-Cooled Engines

The advantages of the air-cooled engine, as I see them, are the following:

Lower weight for a complete power unit

Better accessibility for maintenance, with particular reference to the radial type

Elimination of one potent source of trouble which may cause failure, namely, water system, including radiator and piping

Susceptibility to supercharging by increasing gear ratio of rotary induction system.

On the other hand, there are several important advantages of the water-cooled engine, which may be stated briefly as follows:

Lower head resistance, giving possibility of higher overall speed of the airplane in which it is installed

Greater reliability from the standpoint of the existence of a greater field of past experience and precedent on which to base design

More ready adaptation of mufflers and arrangements for diminishing noise

Possession of basic arrangement which allows for less vibration

Greater susceptibility to use of reduction gears.

In the above comparison, reference in the case of the air-cooled engine is particularly to the radial type. Other arrangements may be developed (and in fact some are now being used experimentally, such as the in-line "V" air-cooled, and the "Hex" type) which may eliminate the superiority of the water-cooled engine with particular respect to head resistance, noise, and vibration. Also as air-cooled engines are further developed and flown, the item "experience," now credited as a water-cooled-engine asset, will diminish in importance. It is also probable that eventually suitable gearing for air-cooled engines will be developed. It is not likely, however, that for some time to come any one air-cooled engine will be constructed which will possess all these possible improvements.

On the other hand, the importance of weight saving can not be overestimated, and in this respect it is believed that the air-cooled engine will always be superior. All things considered, it is my opinion that the air-cooled engine will predominate in aviation, never, however, entirely superceding the water-cooled engine for use in certain types of planes. Basically, the airplane, which requires relatively rapid forward motion for sustentation, is ideally suited for a motor cooled by the medium through which it is traveling. I believe it to be fundamentally sound to expect that eventually the airplane will be powered by air-cooled motors, just as marine craft use gasoline engines which are water cooled. For the present, however, and for some time to come, the designer must give careful consideration to the problem of motor selection from the standpoint of cooling medium, as there are many real advantages for each type, which may definitely indicate the use of one or the other depending on design function.

(*b*) Gearing

There is a very marked increase in overall propulsive efficiency by the use of a large-diameter propeller, as efficiency in general increases with decrease in propeller revolutions. By the use of reduction gears, it is possible to use such a large-diameter propeller, at the same time permitting high motor speed and attendant motor efficiency. The improvement in airplane performance is particularly noticeable in such characteristics as climb and take-off, which are often the most important qualities in a given design. Speed is also improved, as is also the ability of plane to fly on fewer power units than the total number supplied. The disadvantage of gearing is the increased weight, both of gears and propeller, and the imposition of an additional mechanical arrangement susceptible of failure or maintenance trouble. I believe that the use of reduction gears will become more general, particularly in larger types of airplanes.

(*c*) Supercharging

It is quite surprising to me that supercharging has not been more generally used in aviation engines than has been the case. When analysis is made of the marked improvement in all performance characteristics, even at altitudes of from 7000 to 12,000 ft., by the use of superchargers, it is difficult to account for the slow progress of design which has obtained in this line of development. I feel confident that some degree of supercharging will not only be general, but almost universal in the aviation engines of the not distant future.

(4) Type of Propellers

(*a*) Propeller Material

The very general adoption of duralumin for use in propellers, together with the numerous advantages inherent in its use, seem to point to the continued use of this material. The wooden propeller is lighter and cheaper but will not stand up in service so well and is not so easy to repair after minor crashes. Wood is also an ideal medium for satisfactorily absorbing vibration, and great care in the design of duralumin propellers from this standpoint is necessary. Steel has been successfully used for propellers in England, but from the standpoint of weight combined with fabrication difficulties it is not believed to represent the ultimate material for propeller use. Micarta is also successfully used and in large production will probably have an advantage in price over any other type.

(*b*) Propeller Arrangement

There are several designs of duralumin propellers now being successfully used. These include the single-piece type, made either from a rolled slab or forged blank, and the three-piece ground adjustable-pitch type, using a steel hub. The former types have the advantage of cheapness and simplicity, whereas the latter have the advantage of pitch adjustment. This advantage is important in connection with new airplane types in that it permits the ready correction of design errors. It also furnishes a convenient means of altering engine speed in the case of airplanes of a given type for use in varying functions. It is believed, however, that, in general, once a design of airplane is established on a production basis, a fixed-blade one-piece propeller will be found satisfactory, so that price is the determining factor.

I should like to call attention particularly to the controllable-pitch propeller. The advantages of this type are so great that I venture to predict the very general use of the type in the future on all multi-engined airplanes and on many single-engined types. The ultimate type selected may be of the automatically adjustable type, or of the type controllable as to pitch from the cockpit. I rather think the latter will predominate. The improvement in take-off, climb, and ability to fly on power units less than the total number provided is so marked as to render it inconceivable that eventually suitable designs will not be developed and adopted.

(5) Other Power-Plant Items

The designer has before him many installation problems, including gas- and oil-tank location, type of fuel system, installation of starting system, etc. Each must be thought out in regard to the others, and to the use of the airplane.

More and more attention is being paid to the matter of noise elimination. It is undoubtedly true that the noise now present in airplanes is a great handicap in making the public airminded. So much noise is not conducive to comfort when riding, and eventually planes must approach the freedom from noise which now exists in automobiles. The weight and power sacrifice involved in the use of mufflers is warranted in most types of plane. It should also be noted that the propeller noise can be lessened by using the slower turning propellers which accompany the use of engine reduction gears.

D—Arrangement

This design element requires for complete solution a decision on such questions as: biplane vs. monoplane; if the latter is favored, then internally braced vs. externally braced or low wing vs. high wing; disposition of tail surfaces; type of landing gear; open vs. cabin fuselage; satisfaction of requirements of visibility, and disposition of equipment and facilities inside the cabin. Below are given some of the factors which must be taken into account in arriving at decisions on these items of arrangement.

(1) Biplane or Monoplane

In general the biplane, because of better truss arrangement from the structural aspect, can be constructed lighter than the monoplane. It will also usually excel in the matter of airfoil efficiency. It will, however, fall far short of the monoplane both in general aerodynamic efficiency and in overall fineness. I am inclined to believe that, all factors considered, the performance of the monoplane, particularly the internally braced monoplane, will, when developed by equally competent designers, excel that of the biplane for like function and requirements. One of the outstanding advantages of the biplane is the compactness and reduced span made possible. This item will, I feel, become of increasing importance particularly in the large-plane field.

The externally braced monoplane is really a compromise between the biplane and internally braced monoplane wherein advantage is taken of the structural superiority of the former and the aerodynamic gains present in the latter. This development is truly American in both origin and adaptation, and one which has many features to commend it.

The so-called low-wing monoplane possesses advantages in stability and safety and certain performance characteristics, but has not come into general favor in this country, probably because of poor visibility from the passenger cabin. Its successful use in the Junkers series, however, indicates that its advantages may perhaps outweigh its disadvantages.

(2) Disposition of Tail Surfaces

The variations in arrangement include the conventional monoplane tail; the box or biplane tail; or the monoplane tail with duplicate fins and rudders. For single-engined ships the monoplane tail is desirable. However, for multi-engined planes, particularly when using two or four units, an arrangement having two fins and rudders is essential for proper control, particularly for use when flying with the outboard engines only. For very large airplanes the biplane arrangement will often justify itself.

(3) Landing-Gear Arrangement

The chief questions to be settled are tread of wheels and type of shock absorber, together with wheel motion. The tendency is toward large treads, oleo or pneumatic shock absorbers, and forward or combined forward and sideways wheel motion. The general adoption of oleo shock absorbers will undoubtedly greatly lessen the landing accidents which sometimes have resulted.

(4) Fuselage

I am convinced that except for a relatively small class of ships, the cabin plane will almost entirely supercede the open-cockpit type. By exercising proper ingenuity, the problems of vision, weight, and resistance, which are apparently of greater magnitude in the cabin plane, can be eliminated.

(5) Cabin Arrangement

The following are some of the problems which require very careful attention as they relate particularly to passenger comfort which must be present in transport airplanes before extensive patronage can be expected: seating arrangement; ventilation; toilet and lavatory facilities; elimination of gases and odors; heating facilities; elimination of noise and vibration; arrangements both for preventing airsickness and accommodating passengers once afflicted; provision for interesting passengers. As the need and demand for most of these items have only just come before us, their development is somewhat behind that of the airplane proper. They furnish, therefore, a fruitful item for the designer's attention, with particular reference to their provision at low cost in weight.

(6) Airplanes for Water Operations

Although not specifically discussed in this paper, it should be realized that many uses for airplanes call specifically for the selection of the seaplane, flying boat, or amphibian for proper solution. One of the greatest advantages of airplanes capable of operating from the water is the possibility thereby afforded of landing closer to the great centers of population in cities. This is a real problem in air transportation which may eventually find its best solution in this manner. It is also felt that even with multi-engined planes, long flights over water should be carried on only with airplanes capable of landing on the water.

E—Factor of Safety and Degree of Reliability

The selection of safety factor is not now a decision within the power of the designer, at least in so far as minimum factor is concerned, as it is prescribed by the Department of Commerce. Stress-analysis requirements are based upon a required load factor. The load factor represents the accelerations obtainable in a given maneuver when compared with the loads on the wing structure in unaccelerated horizontal flight. The prescribed load factors are sufficiently greater than those obtainable in maneuvers considered as reasonable for a given type, to allow for a real factor of safety of from $1^1/_2$ to $2^1/_2$. The smallness of this factor of safety, which must take care of defects in workmanship or materials, deficiencies of design, depreciation, and unreasonable maneuvering, makes it necessary that stress analyses be accurately prepared; and that the utmost in refinement of design and workmanship be adhered to throughout. And yet, because of the extreme necessity for weight economy, required load factors cannot be much acceded. The British system of permitting one gross weight for normal commercial flying, with another lower weight, and consequently lower useful load, for stunt flying, is believed to be of considerable advantage in permitting a combination of higher real factor of safety and accompanying low weight.

From the standpoint of operating reliability, consideration must be given to the relative degree of safety and schedule maintenance demanded by the particular service for which the particular plane under consideration is being designed. It is no violation of our fundamental requirement of safety to admit of degrees of safety for different types of airplanes. It is this consideration which makes it satisfactory to design single-engined ships for certain types of service.

F—Aerodynamic Considerations

The principal considerations of an aerodynamic nature which must be decided upon for each design are the following:

Fineness Ratio. All performance characteristics of the airplane are improved by "cleaning it up." This consists in eliminating protuberances as much as possible and fairing or streamlining those which must be outside. Expense, weight, and increased maintenance difficulties accompany the incorporation of such refinements. Nevertheless the whole tendency is toward increasing rather than decreasing this method of improving per-

formance, and I am of the opinion that progress will continue more rapidly hereafter to such an extent that in a few years airplanes in everyday use will have a degree of refinement of line and general fineness comparable to our racing ships of today.

Stability. This quality is important from the standpoint of making the airplane easier to fly and more foolproof; it is essential that it be present under many circumstances, such as cloud or fog flying. The ease with which a modern stable airplane can be flown is one of the most certain indications of the continued increase in popularity of flying as a sport. I believe the time is rapidly approaching when no airplane will be considered as at all satisfactory that is not positively stable laterally, directionally, and longitudinally under all flight conditions.

Controllability. It is essential that an airplane be not only stable but controllable. Although on some highly maneuverable military planes there may be some conflict between the two, it nevertheless appears that it is readily possible to have both qualities present to a marked degree in any commercial ship. Vast improvements in the proportioning and balancing of control surfaces have recently taken place, and further improvements of equal magnitude may be confidently expected. In order that the airplane may attain importance as a means of transportation for private individuals, it is necessary that it be capable of being flown by pilots of far less ability than most now operating and with a high degree of safety and dependability. In order that this condition may obtain, the qualities, stability, and controllability must be present in the design to a marked degree.

G—Weight

To a degree unrepresented in other engineering structures, weight is of importance in the airplane. It has been said with absolute truth: "Any airplane can be immediately judged a success or failure, when first it is placed upon the scales." The airplane which when completed is overweight to any appreciable extent had far better be stopped at once in production, rather than linger on to a certain death after money has been wasted in trying to save it. Too often at present such designs are continued with some degree of success by the expedient of adding power to compensate for the extra weight. This is not an economical process, nor will it result in a type which will long remain on the market.

H—Ease of Maintenance

For economy of operations it is essential that attention be paid to ease of maintenance, including inspection, assembly, repairs, and replacements. This is best accomplished by applying sound engineering principles to the design of details, accompanied by the use of materials selected with particular reference to the function in the airplane of the parts into which they are fabricated. I do believe, however, that for some time to come the importance of weight economy is so great that designers must use materials best suited from this standpoint, thus to an extent forcing on service personnel the necessity of finding methods of effecting repairs perhaps more difficult of accomplishment at first than in more commonly known materials.

I—Materials

I feel confident that the use of metal in airplane structures will increase steadily. Metal is better adapted than wood for use in the airplane from the standpoint of greater certainty of strength values and of inspection, smaller depreciation, and cheapness of construction in large production. In order to feel the advantages of this last factor, production must be considerably larger than it is now in any airplane factory. Wood in many parts of the structure can certainly be used conveniently in experimental models, particularly on relatively small planes. However, I believe the future will see most of our planes constructed principally of steel and duralumin.

III—COMMERCIAL USES OF AIRPLANES

In the foregoing discussion have been listed in considerable detail the design elements, each one of which must be considered and judged as to relative importance by the designer. As previously stated, almost every element conflicts in some respect with every other, thus necessitating compromises. It is the exercise of proper discretion and judgment in the case of these compromises which marks the successful engineer. It is only by carefully considering the use to which the finished airplane will be put, in conjunction with the element of design being considered, that one can expect to arrive at such a proper decision. The principal uses of commercial airplanes which are now foreseen will be discussed briefly.

Airplane Designs

There follows in as brief form as possible the design arrangement which to me offers the best solution of the many problems involved in designing an airplane for each of the uses mentioned. I wish to emphasize the fact, previously put forth, that the conflicting elements are so many as to render it really improper to give out any one arrangement as a "best solution." However, from the analysis made so far, many will agree that the arrangement submitted will be suitable for the purpose intended and will perhaps contain less serious compromises than some other.

A—Passenger Transport

Through Lines. The transportation of passengers in large airplanes is of particular importance and interest just now as the establishment of lines having this object in view has only just commenced. A few such lines are now operating on the Pacific Coast, and next summer will see many more in operation, including long transcontinental runs.

I feel that the following elements, in about the order named, are essential to the success of any airplane competing in this field: safety, high cruising speed, comfort, and low maintenance cost.

These requirements are best attained by using a biplane powered with two or four geared engines. At present this would require water-cooled engines. When developed to the same degree of reliability, I shall favor geared air-cooled engines. Also, when available, controllable-pitch propellers will be used.

The biplane arrangement favors safety because of the possibility of obtaining necessary large wing area for low landing speed without using a span which is so large as to make handling and hangar problems disproportionately great. The biplane also offers a more direct solution of the folding-wing problem than does the monoplane.

The analysis made in a previous section indicated the greater reliability of a two- or four-engined airplane than that of a three-engined ship, it being understood that in the case of the two-engined type, horizontal flight on one engine could be maintained, and in each of the other cases on two engines. This arrangement, then, also favors safety. The arrangement having no fuselage engine is believed desirable for other reasons, including increased comfort, performance, cabin capacity, and pilot's vision, with decreased hazard from fire. Comfort is increased by the reduction of noise and vibration and gasoline fumes; performance is increased because of reduced fuselage drag; cabin capacity is increased because with this arrangement the nose of the fuselage can be extended without adversely affecting balance; pilot's vision is improved by placing him ahead of everything in the airplane; and fire hazard is reduced by removing the only im-

portant source of fire, the gasoline tanks and engines, away from the fuselage. In this connection, it should be noted that there are a number of instances on record of safe landings made with raging fires in outer nacelle units, which, if present in the fuselage, would certainly have been disastrous.

In addition to the above considerations, it is essential that the large through-line airplane have a large degree of inherent stability and be readily controllable. In the future it is probable that supercharging will play an important part so that perhaps eventually we shall use planes with oxygen fed, sealed cabins, flying at 30,000 to 40,000 feet altitude at 200 to 300 miles per hour.

Feeder Lines. This type of service closely approximates the one just described, but on a smaller scale. It therefore will require a plane of the same general type. Here, however, as the size is smaller, the aerodynamic advantages of the monoplane can be used to advantage. A monoplane using two air-cooled engines would therefore indicate my solution of this problem.

Private Business Passenger Carrying. There is a good field for a plane capable of carrying five or six passengers, for the use of business firms transporting executives, officials, and associates. As the general requirements are identical for this and the feeder-line plane, it appears that in this case identical equipment can be used for the two functions.

Taxi Planes. A number of uses for this type of plane present themselves and have in fact already been demonstrated, as it is this class of service which is even now being used extensively. Such planes transport newspapermen and cameramen to the scenes of events having news value, such as regions devastated by storm or flood, fires, races, sporting events, and the like. There is also a use in carrying doctors and business men on important rush calls and, in fact, rendering service on all types of unscheduled passenger trips. This general type of plane finds a use in the movies and for advertising.

As the requirements for reliability are not so exacting as on scheduled passenger runs, it is felt that here the single-engined plane finds a field. It should be remembered that here the operator can exercise more discretion in the matter of both route and weather. The plane must be relatively small and easy to handle, and cheap to operate and maintain. The performance, however, must be good, in order to offer an inducement for its use in competition with other methods of transportation.

The single-engined cabin monoplane, either internally or externally braced, and with folding wings, will always find a wide usage in this field. However, as the particular service rendered becomes such as to require fairly long hops, it is conceivable that even in this case the two-engined machine will find a field. In all probability, flying services operating a taxi business will equip themselves with both types, using for any particular job the exact type best suited. I wish to emphasize here again the assumption made in all cases where two-engined planes are mentioned, that horizontal flight on one engine is presupposed. Without such possibility, reliability is far less than with the single-engined plane, although always the length of glide would be markedly increased. For taxi service the advantages of the amphibian plane should not be lost to view. The advantage of landing on water, which is practically always available near large cities, is very real, as it obviates the time loss of the intermediate automobile trip present in the case of most of the present flying fields. I feel certain, therefore, that flying services will equip themselves with some amphibian planes, to realize this advantage.

Sightseeing. In general it is believed that the same requirements here obtain as hold for the feeder-line ship. As safety, reliability, and good vision for pilot and passengers are essential, it appears that the twin-engined monoplane will be best adapted.

Short-Distance Transport to Centers of Population. The amphibian plane has already been mentioned as eminently suited for this kind of service. In addition the helicopter should be mentioned as offering a possible solution to the problem. The vertical ascent and descent features of the helicopter undoubtedly give the best answer, as by this means landing on buildings could be made, thus getting closer to passengers than could possibly be otherwise accomplished.

B—Goods Transportation

In the early part of this paper was indicated my belief that safety is a fundamental consideration in all phases of air transportation. There are, however, degrees of importance, and I feel that in the transportation of goods and mail and express a lesser degree of safety is necessary than in the carrying of passengers. Reliability is of course important, but relatively speaking, economy of operation is more important. Therefore the important design elements are arranged in general as follows: high percentage of pay load, safety, cruising speed, and low maintenance cost.

With these general requirements in view, let us determine the arrangement of plane which appears to be best suited for the types of goods transportation previously listed.

Mail. To date designers have favored the single-engined biplane, with pilot located aft of the mail compartment. I am inclined to believe that the development has been sound and the type will continue. Certainly the biggest percentage of pay load together with high cruising speed is thus obtained. However, I should like to urge the possibility of a future trend to three-engined planes for this service—on account of the added reliability thus attained, and on account of the probable future necessity for larger capacities because of the increased use of air-mail service even now indicated. I do not believe that the twin-engined plane will find a field in mail carrying, as to gain its superior reliability, a decreased percentage of pay load is necessary. It is true that it has a better arrangement for cargo carrying and with equal power will have a higher cruising speed; nevertheless I believe that if multi-engined machines are used, the three-engined type will predominate.

I see no particularly sound reason for the predominance of the biplane over the monoplane in this field. It can be built somewhat lighter, but will be inferior in fineness. It also possesses a disadvantage not yet overcome, namely, an arrangement which makes use of brace wires on which ice collects to such an extent under certain conditions as to force a landing or cause wire failure.

Express and Fast Freight. For this type of service the three-engined plane seems to offer an almost ideal solution. The monoplane arrangement also appears logical, at least until such time as sizes become so large as to make a biplane type imperative. However, in this service higher landing speeds can be allowed, so that the three-engined monoplane will predominate for some time to come.

Plant and Machinery. This heading includes the transportation of heavy equipment, particularly in undeveloped countries where transportation by ground methods is arduous and slow. There are many locations where, for example, oil-well equipment can be transported in a day, which it might well take weeks to transport by any other method. For this type of duty, reliability and capacity lead in importance, with performance a secondary consideration. A multi-engined ship is therefore clearly indicated. In all probability the three-engined plane will predominate because of high pay load carried in combination with excellent reliability and good cubic capacity. The latter consideration, however, would make the two- or four-engined plane superior. The two-engined plane, however, will not carry sufficient load (and still be capable of flying on one engine) so

that it does not appear to be so well adapted. The reliability of the four-engined plane capable of flying on any three engines is very good, and it may be that this will represent a type extensively used in this field.

Film and Newspaper Carrying. The requirements for film and newspaper carrying closely parallel those of the mail plane, when relatively small loads are carried, and of the express plane, when larger loads are present. It may therefore be expected that the lines of development of the carrying equipment will be similar.

C—Training Planes

The training of pilots should be in two stages. First, a primary training course, during which the student learns the fundamentals of flying and aeronautics, including the execution of maneuvers; and an advanced stage, during which training in more difficult maneuvers takes place, and in addition, practice is obtained in the flying of different types of airplanes. For the latter stage, all available commercial planes should be used. This stage is, of course, particularly necessary for the student who intends to take up piloting as a profession. The primary stage, essential for all students, may close the course of the man who contemplates private flying only.

I believe that the type of plane best adapted for primary training is the single-engined open-cockpit biplane, preferably of two bays in the wing cellule. The design must be relatively easy to fly, cheap, and easy to maintain. No great degree of engine reliability is necessary. All of these elements appear to call for a single-engined plane. The two-bay biplane is chosen as being safer in case of a crash and because of possessing the type of controllability which experience has indicated is best suited for the purpose. As performance is of minor importance and as intercommunication between instructor and student is of such great importance, it is surprising that more general use of the side-by-side seating arrangement has not taken place. This may be due to the fact that practically all present pilots were trained in a type descending from the *JN*4 or, in fact, in a *JN*4. On the other hand, it may be that a real advantage in training is gained by having the student alone in his seat. In my own case, I felt that the side-by-side seating arrangement would have facilitated matters, as the head-phone and hand-motion method used was, to me, somewhat confusing and undesirable. I therefore am inclined to believe that eventually, a side-by-side arrangement will be adopted. In such an arrangement the motor must be located sufficiently low to render the vision to both sides adequate.

D—Aerial Survey

As the name implies, the particular types of service rendered by the class of airplane here discussed indicate the use of a plane so arranged as to permit unrestricted view from varying altitudes, in some cases at least 15,000 ft. Reliability is absolutely essential as in general these airplanes are used over a type of country where forced landings would in many cases spell disaster. On the other hand, the load to be carried is relatively small, at least in so far as pay load is concerned. A fairly high fuel load is necessary, however. In many of the divisions of this general classification, the machine should be capable of landing either on the land or water, and it is therefore likely that the amphibian plane, based on a twin-float arrangement of pontoons, may find some usefulness here. The general order of importance of design elements for this class, therefore, seems to be the following, with detail variations as given below: vision, reliability, high ceiling, economy in fuel consumption, and stability.

Photography. Aerial photography has already been extensively used. The preparation of maps for tax appraisals, highway planning, investigation of public utilities such as water power and waterways, telephone and power-line extensions, and real-estate development, are but a few of the uses that one can name. The design elements given above as generally applicable to the whole class all seem to apply to aerial photography in detail. I do not feel that the single-engined airplane offers the proper solution except perhaps for the restricted use which obtains in the case of photographic projects carried on within normal gliding distance of an airdrome. Even here the vision of the pilot placed first in the list of important design elements is wanting. I therefore believe that the twin-engined monoplane offers an unusually good solution of all the problems involved. It furnishes at once an unexcelled view for pilot and observer and a high degree of reliability together with good performance characteristics. A properly designed plane of this type should find a wide usage almost at once, as the functions covered by aerial photography are not duplicated by any other means than the airplane.

Spotting. There are numerous specific uses which can be classified hereunder, such as fish, crop, and timber spotting, forest and border patrol, and coast-guard work. The characteristics required of an airplane for this type of duty seem to be almost identical to those just described for aerial photography. The same airplane, with slightly different equipment, can be used for each.

Exploration. Here again there seems to be no essential difference in airplane requirements when compared with the preceding divisions. Here, however, the field of operation is wider so that in certain cases other arrangements suggest themselves. In general a larger load-carrying capacity is required, and as vision is not perhaps of the same degree of importance, the three-engined monoplane may be best suited for exploration use when the period of absence from civilization is known to be long at the outset of the trip. It therefore seems logical that in this case no general type can be specified in advance as being susceptible of satisfactory use for all exploration projects. Certainly, however, a multi-motored airplane is necessary.

E—Industrial Uses

The possible uses of the airplane in the realm of industry have not been exploited to the extent which obtains elsewhere. We are all familiar with the use of planes for insecticide dusting, tree spraying, and certain forms of advertising. It is believed, however, that a great many other uses will come to light as the use of airplanes for transportation becomes more general. However, for the limited field above indicated it appears that the single-engined ship will be satisfactory, as in general landing fields will be available in the event of forced landing. Here again, however, I am inclined to believe that as the field broadens, and the more general use of airplanes in industry becomes a reality, the multi-engined type will come into its own.

F—Private Uses

The private individual has several uses for an airplane, including sport and touring. The former has become an important field in England where numerous flying clubs are in existence which, I understand, enjoy a large patronage. The country is in many respects better adapted for the use of the light plane than is the United States. Distances are shorter, and landing fields closer together. There are many who, I am sure, will not agree on this matter, but I am inclined to believe that in this country it will be some time before the private use of light planes will be general.

The type of plane which appears to be best adapted for this field is the single-engined biplane with folding wings and air-cooled engine. The British airplanes known as the *Moth* and the *Avion* are well-balanced designs of this type. These are two-place planes, eminently suited for sport use, having a low landing speed

and high degree of maneuverability. For long touring they do not have sufficient space or load-carrying capacity; nor do I feel that the tandem seating arrangement is suitable for this purpose.

Certainly for private use in making long trips, a side-by-side seating arrangement will be required. The American three-to-five-place externally braced cabin monoplane appears to me to be a more logical solution of the problem of design for touring use. In fact this entirely American arrangement, first brought to the front by its use in Colonel Lindbergh's tráns-Atlantic flight, represents a development of which we can be justly proud.

Conclusions

In conclusion I wish to stress the necessity of taking into full consideration, in every design of airplane, the fundamental requirements of safety and reliability, singleness of function, and economy. In order to bring about the general adoption of airplanes for use in the many fields of activity listed, the first and last fundamentals must be present in every design; and in order to accomplish such a high standard at all, the second basic requirement must be followed. Having given proper cognizance to the fundamentals, it then becomes necessary to study carefully the exact nature of the service to be rendered by the airplane which is receiving design consideration, with a view to compromising the conflicting detail elements of design so as to create an airplane which will compete successfully in its field of activity both with other airplanes and with other types of transportation. The problem is difficult, but the future for air transportation and service, which we all see before us, is well worth the effort.

Discussion

Robert J. McKay.[2] From the laymen's standpoint the conclusions as to the relative safety of the two-motored airplane as compared to a single- or three-motored type is particularly notable since, to the casual observer, it seems that there are many more of both single- and three-unit planes in the air today than of the two-unit type. The indicated possibilities for greater safety than is obtained at present would seem to have much public interest.

Another thought which might be useful in this discussion is the fundamental importance of materials in fuselage design. This thought is brought to my mind by the fact that the subject of materials has been relatively little discussed in this most interesting symposium. Again from the observers standpoint, it seems roughly that probably less keen study has been the case than in designing motors. But they are not less important in the case of the fuselage. With the present rapid development in alloys which are corrosion resisting, and those which have special mechanical and physical properties, possibly we may hope for future developments along this line which will make flying more efficient and safer.

Elmer A. Sperry.[3] This is a wonderful Division. Here is a section that is going 100 per cent, absolutely dealing with the most marvelous developments that are going on in this world today. It is fine to have such full attendance, and such upstanding and outstanding personnel as is here present! The Society is to be congratulated most heartily on this Division.

Now, the matter of fact is, the airplane is just about "110 per cent" mechanics. There isn't any chemistry about it, there aren't any miners or mining engineers who are going to have to go very deep to dig up an airplane, you know; the civil engineers give us a little bit better facilities for getting off, but they don't reach up very far in the air, so it is the mechanical engineer who has to do with this problem. It is delightful that such wonderful authorities as Mr. Wright and the others present have come out this morning to help. The best all can do together is none too good.

Points regarding design have been stressed. All engineering is a sort of compromise; the design problem not only has to fit into the picture, but the power plant has to be considered. The paper that was read by Captain Woolson, yesterday, in Chicago, touches on the power plant that is an aim in the right direction. The Diesel power plant has got to come. The marvelous increase in safety is only one incident.

Repeating what the writer said yesterday, downstairs, in the Diesel Division. How safe is Diesel oil? There is an oil that can be obtained on this market for $1.10 a barrel (42 gallons for $1.10, that is pretty cheap stuff), but it has 18,500 B.t.u.'s per pound. It has all the energy gasoline has. It must be used.

Touching on the safety of Diesel oil, Commander Cave-Brown-Cave (it is unnecessary to tell what country he came from) gave an impressive illustration. He took a metallic ladle, put some Diesel oil in it, and tried to touch it off, but it would not flame at all. He put it across the top of a Bunsen Burner, and when it boiled it would light, but when he took it off, it would go out. Then he cooled off the ladle and put a little gasoline in it, touched it off, and he wished he hadn't. It simply goes up and becomes a mass of flame pretty quickly. He cooled off the ladle again, put some more gasoline in, and touched it off again. Then he put some Diesel oil on and it put the flame out. Think of that. The first drop of Diesel oil will go into flame a little quicker than the first drop of gasoline. It has a slightly lower ignition point, but what should any one care about the first drop? It is the gush accompanying a smash-up that is the thing to consider. The following gush of gasoline needs no description as to what happens. The following gush of Diesel oil will not only put out the flame, but it will put out the red-hot steel as well. Why? Because its specific heat is five times that of the steel, and the flow takes care of the gravity difference. This is well worth considering. It is only the red-hot manifold that usually ignites anything, so the Diesel oil provides a perfectly safe fuel.

Now, the engine that Captain Woolson was talking about is just an ordinary, run-a-mile, Diesel radially disposed. He took a gasoline engine designed for the upper limits of preignition, or detonation, which every gasoline engine has to be designed to stand anyway, absolutely about 1100 to 1200 lb. per sq. in., put longer pistons in, put a clever little oil pump on, and started it up. His friends may have backed away, expecting the thing to go through the roof, but they discovered two things: First, it didn't go through the roof; second, its air-cooling was much easier than the air-cooling of the ordinary Otto-cycle engine. Why? Because it doesn't require as much cooling. No Diesel engine requires as much cooling as a gasoline engine.

The reason is so simple that a six-year old can understand it. Less fuel is used, and there is less heat produced and besides, there is more of the heat produced going into the power produced. To date only the surface of the Diesel has been scratched. By continuing the combustion somewhat so as to fatten out the cord, and get larger mean effectives will possibly be found just as applicable to the Diesel as it is to the Otto-cycle.

Then, supercharging. There is some trouble in getting a pump that will run along with 87 and 88 per cent efficiency at a speed of 2000 r.p.m. There is little trouble in the pump in getting rid of the condensed air, because a great deal less air has to be shoved out than is brought in. The great trouble, however, is inhalation. There are ways and means to approach almost perfect inhalation by bringing it in on every side, so the inhalation

[2] Supt. of Technical Service, The International Nickel Company, Inc., New York, N. Y.

[3] President A.S.M.E., Chairman Board of Directors, Sperry Gyroscope Co., Brooklyn, N. Y.

factor can be brought up to about where it belongs by having the mechanics right.

Improvements are going on. The biggest men in all countries are thinking on all these problems, which are all mechanical, as is this whole aviation proposition. It is on its home base here in the Society of Mechanical Engineers. Any demand should be resisted to have it removed and taken over by some other society, possibly, or some other division. It is mechanics from start to finish, and the great challenge should be met by rising to the situation and giving this Division all the support and service that lies within the power of this great Society to give.

Mr. Wright is working on the very latest development of an adjustable-pitch propeller. The propeller, which gives us the best level forward thrust, is not the best for climbing. The best propeller to climb with is not the best for regular projection through the air. Then again, at the high altitudes as suggested by Mr. Wright, the proper propeller is something different, especially for supercharging, but there is no trouble in getting very much better all-round performance through an adjustable-pitch propeller.

If a metal blade running at 2000 r.p.m. be taken, for instance weighing 30 lb., it tends to pull away from the hub. By what amount does it tend to pull away from the hub? Forty tons, that is all. To get a successful adjustable-blade propeller, the adjustment of the pitch of each blade must be within a tenth of a degree of the pitch of the other blades so that one blade doesn't try to do all the work, and that is no small problem. A great many minds have been working on it, but it is not on the shelf for ready purchase today. Mr. Wright is one of the leaders in the development, and when it comes to answering back to the cockpit, Mr. Wright has found how to secure the simplest answer. He says, "Look at the tachometer, simply adjust the loading of the engines entirely by the propeller; if it is right on the tachometer, the right adjustment of the propeller is obtained."

There is a lot ahead. More engineers are needed to study these problems, and give their assistance to these marvelous developments and possibilities.

Author's Closure

T. P. Wright. The reason that the two-motored biplane is favored by the author for passengers is that for so large a ship it is not economical from the standpoint of weight to build it in the form of a monoplane and to maintain a landing speed which is safe. In other words, a biplane can be built in the large size ship, and will be of shorter span, assuming the wing area gives a comparable landing speed. The two-engine ship is distinctly preferable for transporting large numbers of people, say fifteen to twenty passengers, because of the freedom of the fuselage from the engines, which is a consideration not appreciated as much as it should be.

Take the hazard of fire. It is eliminated by having the engines outside of the fuselage. There are on record at least three cases in which military bombers had the power plants catch fire and in which safe landings have been effected. If the power plant had been placed inside the fuselage, the plane probably would have been ruined and the people killed, as in the case of a three-engine ship with one of them in the center. That is why the two-engine ship for passengers, where safety is the all-important thing, is the most desirable, whereas for carrying goods, and where only the pilot has to be considered, the three-engine ship is suitable because the degree of safety is not so exacting, and also with that type of plane, a higher pay load can be carried, and still maintain approximately the same reliability as with a two-engine ship.

Generally the landing speeds are now higher than they should be, but there is probably reason for this, very largely due to the inefficiency of designers in meeting original weight estimates. When the weight goes up the landing speed goes up proportionately.

After a designer first puts the plane out for flight, if it is overweight the landing speed will go up. The designer generally resorts to the expedient, in making the plane successful, of putting in more power, which in turn runs up the landing speed a little bit more on account of the weight, that is one reason, incidentally, in connection with the multi-engine ship as against the single-engine ship, which partly counteracts the question regarding performance. It is legitimate for the multi-engine ship to land at higher speeds than the single-engine plane because it has a greater degree of reliability, and therefore can, presumably, in a great number of cases, reach an airport which is suitable for landing. In other words, if a motor fails on a multi-engine ship, assuming flight with a lessened number of power units, it will be able to make a designated airport, at which a higher landing speed is not particularly detrimental, whereas a single-engine plane must land at whatever point it happens to be passing over, where the higher landing speed is highly detrimental. Landing speeds are too high. For a single-engine ship they ought not to run over 50 m.p.h. and for a multi-engine ship not over 60 m.p.h.

Brakes are applicable to any type or size of ship, equally well. The condition of nosing over must be considered, but brake application is applicable to either type, and it must be considered in the design. At the present time, some wheels are placed slightly forward. The loads in the fuselage under brake-landing conditions are very severe, and run weights up appreciably. There is a torque relation in the fittings to which landing-gear is attached, which is quite a consideration.

The multi-engine ship does not need to be able to take off with the lesser number of power units. In flight when one power unit fails, gasoline should be dropped except perhaps enough for an hour's flight, and the flight can continue on the lessened number of units. There appears to be no condition where it would be necessary to take off (if it is a bi-motored ship) with one engine from an airdrome if the other engine isn't functioning. There is a period of twenty-five seconds just after taking off where, if the power unit fails, there will be danger, even in a multi-engine ship, but the degree of reliability is far superior to the single-engine ship provided it can actually maintain reasonable altitude on the lesser number of power units.

The laws of probability and chance show that the reliability of doubling up the units will decrease provided flying cannot be done on less than the total number of units. Where a two-engine ship can fly only on two engines, if one engine is disabled the ship has to land, and the two-engine ship is then far less reliable than the single-engine plane.

It must be possible to maintain at least three or four thousand feet altitude with the lessened number of power units for the mathematical formulas on the development of probability and chance to show the greater reliabilities. Assuming a 97 per cent efficiency for a single-engine plane, which was given in terms of 10,000 trips, the single-engine job will have 300 forced landings, whereas the two-engined ship which can fly on one engine will have nine forced landings. The three-engined ship which can fly on two engines will have 26 forced landings. The four-engined ship which can fly on three engines will have 52 forced landings, and the four-engined ship which can fly on two engines will have only one forced landing in 10,000 trips. Those are the figures, assuming our original premise is correct that the ship can fly on a lessened number of units. There is no controversy there at all, that is straight mathematics, but one must be able to fly on the lessened number of units. It is a point that is difficult to conceive because to date there have been not over two or three examples of planes built in the world that can fly with pay load

and drop the gas for one engine if they are two-engined ships. This puts it up to the designer to increase the efficiency of the ship. It can be done and it is going to be done, because it is going to promote aviation. It has been done on at least one bombing plane with two engines, which has flown with the other crippled, with all the gas dropped out except a sufficient amount for one hour's supply.

As to the use of materials of high-strength alloy steels, etc., there is no question but what they are going to be used very extensively in the future. The fact that for a time they have been disregarded is because commercial aviation has been developed quite largely by small concerns who are not in a position to put in heat-treating equipment, etc., which is essential for the successful use of the high-alloy steels. With 150,000 to 180,000 lb. per sq. in. of strength high-alloy steels will be used extensively in the airplanes of the future, and of course, duralumin will be used even more extensively.

Some author got up a paper two or three years ago showing that as the span goes up, there is a definite limit at which the weight goes up, disproportionately, so that there is no gain in going above a certain limit. Materials will undoubtedly be improved and therefore move that limit up all the time, but there is a definite limit above which it cannot be hoped to go.

Within the next two years there will be in successful operation, planes carrying twenty passengers and perhaps up to thirty or thirty-five, but it will be quite a while before there are planes carrying fifty passengers, perhaps ten years or so. It is very difficult to look into the future, but there is a definite limit in the size of airplanes. Aviation will not supplant other types of transportation. It merely supplements or aids in those fields to which it is particularly adapted. Higher speeds in aviation than in any other means of transportation will always be possible, but even at the increased cost and when safety is demonstrated, it will not supplant ocean, train or railroad transportation. So it appears that within the next five years, the limit will be about thirty passengers, and perhaps in ten years, fifty passengers. The total life of the plane will be probably in the order of 40,000 or 50,000 lb. gross weight. The four-engined plane will be most common. Undoubtedly, there will be six-engined planes, but four-engined planes is the next step. One manufacturer now is getting out a four-engined plane. The pay load now ranges from about 15 to 20 per cent of the gross weight. A two-engined ship which the writer's company is building has about 22 per cent of gross weight in pay load, and it will still be able to fly on one engine. There are single-engined planes carrying 25 per cent, and one three-engined plane that can carry or fly on about 27 per cent of its gross weight and still maintain altitude on two engines. That means if 30 passengers at 200 lb. apiece are carried, that would be 6000 lb. pay load, and a plane of about 30,000 gross pounds is a pretty good-sized ship. The NC-4 weighed 24,000 lb. when it took off, but it had a lot of power and it was quite efficient. Racing planes, having just sufficient gas for the particular course over which they travel, carry no pay load at all in the same sense that "pay load" is used in the other discussion. They have a pilot, gasoline, and oil, which is entirely part of the airplane, so that the speeds of commercial airplanes, which have economy as one fundamental requirement and safety as another, will be very decidedly less than racing planes probably in the order of half or perhaps less than half.

At the present time, there seems to be a trend away from the use of larger power units than 500 or 600 hp. so it is probable that in the single-engined commercial plane that carries a reasonable pay load, the speed will not go over 175 m.p.h. certainly, for some time to come. The Lockheed-Vega, one of the best in that direction, makes about 160 m.p.h. with a Wasp engine of 450 horsepower, carrying a reasonable pay load. With the Whirlwind engine it makes about 140 m.p.h. With a power plant on the order of 500 hp. the speed will be about 175 m.p.h. and 200 m.p.h. will be quite distant in the future for commercial aviation. That applies, of course, to sea-level speeds. That brings up the supercharging proposition, which is a different thing. As mentioned in the paper, if one supercharges at altitudes of from 7000 to 12,000 ft. the speed goes up tremendously.

The company with which the author is connected built two pursuit ships using the same power plant. The speed of the unsupercharged-engined ship was in a curve dropping off as the altitude went up, whereas the curve for the supercharged ship, starting (probably because of the lack of a controllable-pitch propeller) at a considerably lower speed at sea level, went up in exactly the opposite direction. The curves crossed at 6000 ft. altitude. At 20,000 ft., whereas the unsupercharged-engined ship made about 135 or 140 m.p.h., the supercharged-engined ship made 195 m.p.h., and that was with a very inefficient supercharger. With the present design, the ship can make 225 m.p.h. at 15,000 ft., and it is felt that in 10 or 12 years there will be transport planes flying at 40,000 ft. altitude with hermetically sealed cabins, and perhaps making 300 m.p.h. That is quite a way off, 10 or 15 years, but it is going to come. It is really the economic way to use airplanes, at 40,000 ft. altitude. It will be some time before the hop to Europe will be made with commercial airplanes. Perhaps the time will not come until there are high-flying, supercharged, hermetically-sealed ships. All of the flights which have been made to date have been made with zero pay load, simply with the weight of the pilots and gasoline sufficient to get them there, and when they did get there, they didn't have much gas left.

Commercially practicable flying cannot be expected with less than 10 per cent of pay load in relation to the gross. So it would be five or ten years before flying across the Atlantic, 2500 miles, can be done with sufficient reliability to make it a regular, established traffic. Probably it will be when there are high-flying ships that can gain by their increased speed such a marked amount over present-day airplanes.

But for the present and for the immediate future, there is no question but what hops will be made to Chicago from New York on the order of 1000 miles in one hop. They will not be very common, because they are hardly worth while. If a 500-mile hop can be made with a stop to gas up again, and thereby carry twice the pay load or half-again the pay load, why make a thousand miles? The loss with the stop will be but 20 min. or half an hour. It seems that the objective will be the flying across the Atlantic, and it is likely that will be some time to come with supercharged equipment.

Seaplanes and amphibians have been neglected in this country. The firm I am with is not building a good amphibian right now. I am sorry it has been neglected, because I think there is a wonderful market for it. It has the advantage of getting to centers of population, which is quite difficult with the conventional land plane, although there is the probability that the airport of the future will be the roof of the railroad station or something like that, so that trouble will be minimized.

However, the seaplane, to date, is not as efficient in both weight and resistance, which perhaps answers to a small extent, the reason that the development has been slower. The point is well taken that it is permissible to use higher landing speeds with seaplanes because of the vastness of the airdrome on which to land. One can land practically anywhere on the water, but the seaplane isn't nearly as efficient as the land plane, particularly because of its resistance and weight. The resistance is particularly great in amphibians unless the landing-gear is completely retracted. Mr. Bellanca has worked out such a construction about as well as any one in this country.

The seaplane will not be neglected in the next three or four years, and it will come into its own, although it never will be used probably as extensively as the straight land plane because of the fact that economics enter, and a very high pay load is required to make an airplane commercially pay its way, and as Mr. Stout says, "Support itself financially in the air."

In the small airplanes the problem of making a successful retractable landing-gear without increasing the weight too much is quite difficult. On the larger plane it can be done, although at best it is a difficult problem. As Mr. Sperry says, it is purely a problem of mechanics in working it out at a reasonably low weight, presumably on a low-wing monoplane rather than on a high-wing monoplane, because the thick, low wing would form an excellent place in which to retract the gear.

In connection with the use of the suction medium for changing the flow over a wing to increase lift, it was found that the blower takes quite a bit of power in the first place to form that suction, and the power required is only available in case the motor is running. In case of motor stoppage, the source of power is taken away at the very time when it is needed to lower the landing speed.

As to attaching several engines to a single propeller shaft, there is a real field for development. That proposition hasn't gone ahead since the days the Gallaudet Company was working on it. The problem never will be entirely solved until a controllable-pitch propeller is developed with the development of gears for shifting one or more engines to the same propeller shaft. Obviously, if the power unit is cut, it will be very inefficient for the single power unit. A controllable-pitch propeller is needed, and even there, the efficiency would drop off considerably. Undoubtedly in the future there will be in use several engines geared to the same shaft as it is merely a problem of mechanics.

The general relation of expense of the ground organization as compared to the total expense of air organization of a transport line runs somewhere in the neighborhood of 30 to 40 per cent. That is, for the airplane pilot's pay, depreciation of the plane and motor, together with gasoline and oil.

As to handling on the ground, at the present time, practically all ships are equipped with towing rings of some sort, and an ordinary two-wheeled dolly is used under the tail-skid. At all the larger airports, the hauling is done by tractors for the land planes. All seaplanes have towing rings provided at the nose of the pontoon or hull, which any kind of a tug can attach to. All seaplanes, at least for use on naval aircraft carriers, battleships, and other ships are equipped with hoisting slings so that the craft is hoisted on board much the same as davits are used for lifeboats.

On the larger airplanes there is a ring attached to the tail-skid, and a dolly has been developed which has a jack arrangement, a screw lifting device, which merely slips under the tail, puts a hook in it and hooks it up. That method will be used for some time, although the Transcontinental Air Transport Company now is spending a considerable amount of time in investigating this problem for large planes. It is building hangars especially to handle those particular ships, and so that the greatest efficiency can be secured it is building handling trucks with rails, and trying to arrange the best way of putting the planes into the hangars, either head first or sideways. The author understands that the sidewise method has been adopted, as more planes can be put in that way, although it would be different if the planes had folding wings.

As to the matter of loading, the problem is never going to be anywhere near as great as it is for other means of transportation. The asymptote established a while ago for loading one plane was in the nature of 8000 or 10,000 lb., which is, of course, just nothing to what is handled with other systems. Perhaps the matter of putting small packages into one container is a desirable feature, so as to facilitate checking, by letting all the checking be done indoors before the plane lands, and then putting a few trunks in the ship itself. At the present time, though, the maximum loads are from 2500 to 3500 lb. and the future loads, within the next five or ten years, are anticipated to be only 8000 or 10,000 lb., so the handling of cargo is not a terribly big problem.

The auto-gyro has for its principle, the production of lift through a rotating aerofoil rather than a stationary aerofoil. Its advantages include the attainment of a steeper-climb curve and a steeper-descent curve. It is not possible to land vertically with it, although Captain Richardson of the Navy advises that an auto-gyro appeared to be landing at a 45 deg. angle. That is very desirable from the standpoint of clearing an object, and of getting into a small airdrome.

The auto-gyro, as it is now designed in England by De Cierva, will not land vertically, although it is possible, because any rotating gear has a certain parachute coefficient while descending, that such a long oleo will enable it to land almost vertically, although the author is not competent to say just what angle that would be. Certainly it will land at a great deal steeper angle than is possible with an airplane, which is a great advantage in getting to small fields surrounded by high objects.

One must differentiate between an auto-gyro and a helicopter. The auto-gyro has a motor with a horizontal shaft and propeller, and it obtains its lift as a component of the forward motion, and does not have power go directly into the rotating blades themselves, whereas, a helicopter must have some arrangement of power so that it rotates the blades themselves, thus making possible ascent and descent absolutely vertical. Apparently, the auto-gyro has passed the experimental stage and is successful. It certainly is an interesting experiment in the way of improving landing possibilities.

AER-51-34

Management of an Airport

By MAJOR JOHN BERRY,[1] CLEVELAND, OHIO

The author gives details of the inception and development of this the first municipally owned and operated airport, and enumerates the facilities which it possesses. He further sets forth various precautions which should be observed in the operation of an airport which are based on his experience as manager of the airport since its operating.

EARLY in 1925, without precedent but with vision, W. R. Hopkins, City Manager of Cleveland, procured options on about 600 acres of the 1000-acre tract which the aeronautical authorities of the United States had determined to be the most suitable area from a weather standpoint, as well as the most fitted geographically within a ten-mile radius of the City of Cleveland.

The State Legislature was then requested to pass the necessary law to permit the raising of funds by bond issue. This was done and work was started on the development on June 1. On July 1 this port, the first to be owned and operated by a municipality, was opened, the first pursuit group from Selfridge Field, a bombing and operation group from McCook Field, the Ford freight line, the transcontinental air mail, and overnight New York-Chicago planes all participating.

The opening of the Cleveland Airport completed the transcontinental leg and made possible night service from the Pacific to the Atlantic.

On account of the condition of the field, the landing area on the opening night was extremely limited, yet America saw there the first night-flying "circus" ever organized, and 250,000 people stood spellbound as they gazed upon the greatest group of combat pilots up to that time assembled.

Development of the Airport

From a start of a single runway 400 × 2000 ft. this development has reached 70 per cent completion in four years, with approaches clear on all four sides and a landing area 3800 ft. square, all of which has been drained and made suitable for landings in all directions. On over a thousand acres of ground there was no place where there was a difference of more than 20 ft. in elevation.

In the exact center of the ultimate development there was built a 4-ft. diameter brick sewer 7000 ft. long. From this main artery were built 15-in. laterals every 400 ft., and into these laterals were carried the French drains, which were spaced 20 ft. apart and were adequate for the immediate disposal of water on the landing area. A duplication of this installation has just been completed on the south.

The Cleveland Municipal Airport is bounded by a series lighting system of eighty 600-lumen lights mounted on 24-in. standards, which are fed by a No. 8 steel armored cable buried in the ground. These standards are 200 ft. apart and surrounded the landing area. Green lights mark the approaches from the cardinal points of the compass, and white the remainder of the square. If, due to construction work, it is desired to protect the pilots from any hazard, red globes are substituted for white when needed.

A ceiling light for weather reports and two 24-in. beacons burn from dusk to dawn. A BBT floodlight, which lights the field as bright as day, is available whenever needed, and no pilot, whether or not he may be expected, is obliged to land in the dark. A landing tee equipped with Neon tubes gives the wind direction.

The Department of Commerce maintains the transcontinental lighted airway which east and west leads to the Cleveland Airport. This, together with the Louisville-Cleveland, Albany-Cleveland, Pittsburgh-Cleveland, and Detroit-Cleveland airways, completes the network radiating from the Cleveland Airport.

In addition to maintaining these aids to navigation, the Department of Commerce has installed and operates at its own expense a teletype service between New York and Chicago for the distribution of weather information. The stations for this service are the intermediate fields approximately 20 miles apart on the airway, and to supplement all this a radio station comprising radio beacon, radio telephone, and radio communication operates 24 hours daily.

The Department of Agriculture since 1925 has maintained an upper-air weather bureau where information may be had every hour in the day on the existing weather conditions on any course within a 500-mile radius of Cleveland, and no pilot is ever forced to "shove off" without full knowledge of the weather he is to encounter.

That these service adjuncts have played a vital part in the success of the port is vouched for by the fact that departures and arrivals, entirely exclusive of field "hopping," are about 1800 per month.

For the transient flier, use of the port is free. No charge for landing is made, and service may be had any hour of the day or night.

All operations for profit or hire are controlled by lessees who have built their own hangars on the field and pay a rental for the land only upon which they are built. To them also the use of the field is free. All construction and maintenance are provided for by the City of Cleveland.

The growth of activities on the Cleveland Airport is evidence of the value of this policy, as is borne out by the successful operations of the following leaseholders: National Air Transport, Boeing, operating the Stout Air Services, Universal Air Lines, Continental Air Lines, Colonial Air Lines, United States Air Lines, Thompson Aeronautical Corporation, Dungan Airways, Stewart Aircraft, Skyways, the Cleveland Institute of Aviation, and the 112th Aero Squadron of the Ohio National Guard.

Precautions to Be Observed in Airport Management

The experience acquired during the years the airport has been in operation give rise to the following deductions which the author would submit.

The phenomenal growth of commercial aviation and the diversified types of service, coupled with the countless student-flying activities, present today an operating problem for the airport executive to weigh carefully. Despite the care and thought given to the preparation of laws for navigation and particularly to those that apply to airport flying, by the Aeronautics Division of the Development of Commerce, casualties continue to multiply in an increasing ratio.

The great diversification in types of fields has necessarily led to the formulation of many rules for the governing of local conditions. Flying-traffic regulation is covered in its activity

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

by the air-commerce regulations, yet lack of standardization in airport construction gives leeway to the individual flier.

At the terminal ports today, with planes landing and taking off every few minutes, it is essential that a control station with unobstructed visibility be the point from which all departures are controlled. The inability of the pilot, in certain types of cabin planes, to see the air from all angles has frequently led him to take off just ahead of another one landing, and thus jeopardized the lives of the passengers in both planes.

The law giving the incoming plane the right of way automatically safeguards the entire traffic if control is exercised from a tower and signals used designating the area that is to take the air. The advancement of the art of navigation, as practiced by the pilots of the New York-Cleveland leg of the N.A.T. service, namely, that of flying "blind" and depending solely upon instruments, makes it extremely necessary that precautions be taken to protect the incoming flier, not only from collision but also from overshooting his landing area.

A four-way field with the perimeter outlined with boundary lights gives a sense of security to a pilot in bad weather if the entire area inside the lights is safe for a landing. This is particularly true in winter, when snow covers runways and the losing of the runway means nosing up, with resultant damage to plane and delay to cargo. Yet the weight increase in present-type transport planes makes hard surfacing seem essential, which surfacing must be over large areas rather than conforming to the runway type.

A far greater source of worry than all the flying activities is the absolute lack of precaution taken by the average spectator at an airport. Whether it is because of his interest in things strange and unfamiliar or whether he is merely exercising his inalienable rights as an American citizen, it seems impossible to impress upon him the thought of caution and the necessity for care.

The whirling propeller seems to be a natural attraction and the average spectator is drawn into danger by a lack of knowledge of this hazard. Naturally it may be asked, why not a fence? Primarily, our first reaction is that a fence high enough for the purpose will automatically become a hazard.

The average port today is municipally owned and for some years to come will be dependent upon the taxpayer for the money with which to operate. Complete exclusion of visitors from the port means lack of public support when funds are needed for further development and added necessities. The enclosed area at Tempelhofer Field seems from the airport managers' point of view an ideal arrangement, yet with eight to ten companies competing for the traffic and the removal of the few from the center of operations we should at once encounter the cry of favoritism and also stubborn resistance from the public.

A sufficient number of mounted police patrolling along a dead line has proved quite satisfactory as far as the sightseer is concerned, but once the latter buys a ticket and becomes a prospective passenger it is impossible to hold him on the line. He is in the hands of the operator, who, with limited personnel and the ever-present necessity of selling another prospect, seems to feel only a negative responsibility.

On single-motored biplanes the wing acts as a guard for the passenger entering and leaving, but the ever-present hazard of the starboard motor on tri-motored monoplanes constitutes what is probably the greatest hazard to the user of air travel. A rope fastened to the fuselage prior to the loading or unloading of passengers would, in a simple, inexpensive manner, reduce this hazard to a minimum.

In the life of every airport certain construction work is necessary. This in itself is a hazard and must be so marked that the pilot unfamilar with the port may know at a glance that the area is closed and unfit for service. The painting of tractors, trenchers, and graders with the standard black and chrome tector striping of the Department of Commerce sets out clearly these obstacles, and the complete surrounding of the area with a series of flags similarly marked and spaced about 20 ft. apart guarantees protection during the daylight hours.

It is exceedingly desirable and should be the practice of airport executives to see that all obstructions are removed before dark; but if the area itself is unsafe, red electric lights, not lanterns, should replace the flags at the same spacing. The insisting of the Aeronautic Division upon fire extinguishers in every ship has been an education for most operators. When a mechanic wheels a ship on the line it should be an inviolable rule that two hand fire extinguishers be mounted in a carrier and accompany him and be available for immediate use in case of backfire.

Every hangar should have spaced about the walls extinguishers that are ready for emergencies. These should be inspected at least once a week, and care should be taken to see that they are immediately refilled after having been used. The hand extinguisher should be supplemented by a forty- or fifty-gallon tank of the foam type, for further protection. The field itself must have an efficient fire department that is motorized and equipped with chemicals, pumps, hose, and first-aid equipment. In addition to this equipment it is the duty of the airport executive to supervise general conditions and, when carelessness and neglect are noticeable, to exercise his authority and eliminate the causes of both.

The duties and responsibilities of the airport manager are so many and of such wide variance that to attempt to cover the entire field would tax a superman. The supervision of flying is a minor incident in the day's work. The fact that the average pilot is a man of sound brain and approximate physical perfection guarantees to the manager strict compliance with the air code. This, of course, applies only to the experienced pilot. Student flying and the period after solo require more attention.

Printed rules mean but little; constant contact by the airport manager with the courses of instruction given by the various schools will educate the student in what is required of him on the particular port, both as to the hours of flying and the priority of flight, and the necessity to conform himself strictly to straight flying.

The care of the layman, is, as has been previously stated, a paramount worry. At Cleveland a week-end average crowd is about 75,000 people. There are from twenty to twenty-four ships in the air from daylight to dark. Each delay in loading and unloading lessens the day's receipts. The reputation of the port rests with the manager, and it is his duty to insure safety to spectator and passenger alike.

Constant supervision of equipment and close contact with the aircraft inspector are necessary in order that no ship not inspected and licensed may leave the ground. The service rendered to transients is bound to react on the reputation of the port. Strange pilots must be accorded every courtesy possible, and the quality of the gas and oil supplied must be unquestionable.

Mechanics who are not licensed are frequently the cause of forced landings, due entirely to their lack of experience, and it should be an inviolable rule that at least one member of each servicing crew be an experienced man and licensed by the Department of Commerce.

The information as to routes, weather, available fields, etc., belongs primarily to the manager's duties. This information or lack of it is a determining factor in the progress of the port.

Where aids to navigation are provided by the government, it little behooves any field manager to remain in ignorance of all conditions that pertain to flying within a 200-mile radius.

Finally, the base of all his work—the bedrock from which he builds—is the prime duty of every airport manager to know every hour in the day the exact condition of the entire flying area. During fair weather and foul the air traffic depends entirely for its very existence upon the airport. If a port is good it must be watched and maintained daily to keep it good. If it is ever to be good, only a thorough knowledge of its faults and a concentration of work on those faults will make it better. No port stays in shape, no matter how well built, unless it receives daily attention.

This close inspection should not be delegated to other than trained men, for upon this, more than upon any factor, does the port prosper or fall into disuse, and from the resultant reputation so fluctuates the efforts of the airport and the airport manager.

AER-51-35

Scientific Studies of Natural Flight

By MAURICE BOEL,[1] CHARLEROI, BELGIUM

In this paper the author gives the results of studies and experiments in bird flight with a view to bringing out features that can be used to advantage in future construction of airplanes. Birds can remain stationary in the air, they can rise and descend almost vertically, and can alight on a particular spot slowly. These are maneuvers not yet possible with airplanes.

The author has also developed a new theory of flapping flight. He renounces the analogy between wings and oars, and shows that flapping flight consists of a succession of short descents in gliding flight. The bird by flapping its wings is lifted a certain amount, and when released moves forward by gliding.

Various classes of flight by birds are discussed under four general headings. They are gliding flight, soaring flight, propulsion flight, and flapping flight. The mechanics, speed, stability, and useful power of the various classes are worked out by typical examples to show how the various types of birds are adapted to particular types of flying. The exterior anatomy of birds, problems of flight, and resistance of air are discussed, and examples are given to show the influence of the lines of the bird on its aerodynamic characteristics.

VALUABLE lessons in flying can still be learned from the birds. Studies of natural flight provide ideas for further progress in aeronautical construction. It can be predicted that some day airplanes will be constructed with retractable landing gear, and will have a speed range of at least ten to one. Birds have some superiorities over the best of present-day airplanes, such as their extreme suppleness, high efficiency, and particularly the very great speed range under their control. These advantages are of first importance, and many technical men are convinced that not only the safety but the future of air transportation is dependent to a large extent upon an increase in the range between flying and landing speeds. Birds capable of attaining speeds of 60 m.p.h. are able to alight on a fixed spot.

Exterior Anatomy of the Bird. From the aerodynamic point of view a bird consists essentially of body and wings, which unlike those of an airplane are inseparable, and ought never to be considered separately.

The Body. With birds best adapted for flight the claws are hidden under the down, and the tail is reduced to a minimum. Ornamentation lessens the ability to fly. Ornamented birds include many of the poultry yard like the cock and the peacock, and among the winged tribes of the tropical forest such as the bird of paradise or the lyre bird. In certain cases ornamentation is accompanied by complete inability to fly.

The exterior perfection of organs and forms is shown by simplicity of lines; a well-shaped body, wings narrow and long, presenting a continuous contour, and a form as nearly elliptical as possible. Each anomaly—each imperfection of line, too long a neck, too large a bill, the existence of which may be justified for other reasons—is a defect from the point of view of flying qualities.

For rapid flight the body must present the least possible resistance to forward movement, and the wings for a given weight of bird must show likewise as little drag as possible. These conditions are generally satisfied, or nearly so, in good fliers such as the swallow, pigeon, eagle, vulture, gull, petrel, seagull, and albatross. All have the body covered more or less thickly with down which fills up the space between head and body, and offers very little resistance to the air. This down is composed of a large number of small feathers which lap over from front to rear, each one consisting of a rib provided on each side with barbs of equal length.

The unit consisting of head and body, covered by the down, forms a sort of spindle, offering a resistance to the air which it is practically impossible to measure with accuracy on living birds, but which must be very small. These organs are constructed better than the fuselage of airplanes, which have sections often rectangular in shape, generally pierced with large holes and furnished with landing gear, the latter indispensable it is true, but whose weight and resistance absorb in flight as much as 25 per cent of the power.

The Wings. The wings, likewise covered with feathers, show the same anatomical features as the lower members of other vertebrates. The greater part of the bones constituting the skeleton of these members is recognized there—the humerus which corresponds to the arm, the radius and cubitus which form the forearm, the wrist, the metacorpus, and the phalanges of the fingers even, but modified by adaptation to their own function. Around these bones are found also, with some differences, the muscles and tendons, some of them, such as the pectoral muscle, considerably developed, and others either reduced in importance or increased in number.

In most birds certain of the important bones are hollow. It would seem that this fact could be interpreted as a lightening of the bird's weight. It is also possible that the cavities thus arranged in some parts of the skeleton have, besides, a physiological meaning. According to certain writers they play a role in respiration, but so far as is known, this function has never been proved with certainty.

In certain creatures, for instance the bat, and certain flying mammals, the wings consist of arm bones and muscles, with folds of skin which extend when the wing is opened. The skin is known as wing membrane, and is similar to that found between the arm and forearm of birds. In these creatures the membranous wings represent a sustaining surface which is totally inadequate. The surface is also increased in extent by the feathers, notably the coverts which form a sort of down, and in particular the wing feathers. Among the latter one distinguishes three kinds, which according to species are distributed in the wings in varying numbers. They are:

(*a*) The primary wing feathers mounted on the hand whose framework in flight is approximately perpendicular to the relative wind.

(*b*) Secondary wing feathers mounted on the forearm.

(*c*) Tertiary wing feathers mounted on the arm, and arranged in flight parallel to the bird's trajectory.

The part of the wing formed principally by the secondary and tertiary wing feathers is called the "fan" in contrast to the "tip" constituted by the extremity of the wing and the primary wing feathers.

The Feathers. All the feathers, whether of down, the coverings, the tail feathers, and the greater part of the wing feathers, are composed of a horny hollow tube prolonged by a shaft of rectangular section whose upper surface is called a scape, and is provided on both sides with two series of barbs of equal length

[1] Professor, Charleroi University.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

situated in the same plane, and inclined at 45 deg. nearly with respect to the axis of the stem which carries them. Each barb in turn is furnished with barbets which carry hooked rays or barbules which are entangled in the hooks of the neighboring barbs, and give to the whole the consistency of a textile fabric, which for the wing feathers is impenetrable to the air at the relatively low pressures which occur in flight, that is 5 to 20 kilograms per square meter or 0.5 to 2 grams per square centimeter.

The Wing Feathers. The carrying surface of these feathers formed by the barbs and their spread has only a slight thickness. The scape, on the contrary, which is the main member, and which must offer great resistance to bending, reaches habitually a depth in section of three or four times the greatest thickness in the plane of the barbs, and almost always projects beyond and below it. The upper surface of the feather is consequently flatter, and offers a smooth surface to the air, having very little resistance to forward motion. This is just the opposite to the lower surface, which broken up by the shaft is sacrificed to the necessity of resistance to bending.

In certain species of birds, notably the albatross, this defect is avoided as it is in some "soarers" whose first primary wing feathers are smooth on both sides, the shaft not projecting from the barbs of equal thickness.

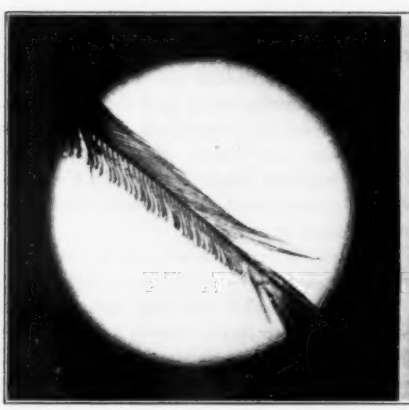


Fig. 1 Barb of a Feather With Its Barbules and Radioles



Fig. 2 Feather Structure With Intertwined Barbules

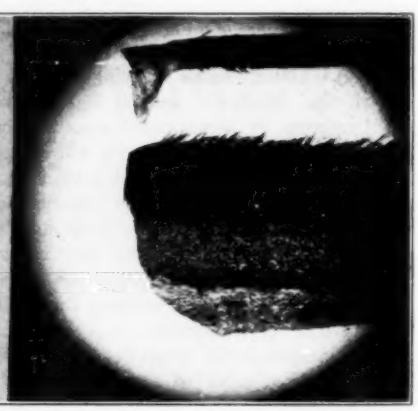


Fig. 3 Above, Barb From the Feather of a Vulture. Below, Barb From the Primary Feather of an Albatross

The primary wing feathers are distinguished from the feathers in general not only by their particular solidity but especially by the relative length of the two series of barbs, short to the front, long to the rear. This peculiarity is quite characteristic of the primary wing feather construction. It is found in the oldest ancestors of the real birds such as the archeopters, and it assures to these feathers the property of taking up, under air forces, an angle of incidence always the same for a given surface, such that the resultant force passes through the shaft of the feather, and which for a given wing feather depends on the center of pressure movement which is peculiar to it.

It is to this property of primary wing feathers that must be attributed the superiority of the birds over fliers without feathers, such as bats and flying fishes for example, as well as in the greater variety of attitudes and speeds that birds can take in flight.

The Tail. The tail of birds, formed by a fan of feathers called tail feathers, serves for their longitudinal and even lateral stabilization. This organ can not only be folded and unfolded, lowered or raised, but even twisted or warped. The shape of the tail is variable in one form or another even among the good fliers, with whom it is generally reduced to the minimum size. In certain cases it ends in two long points, as with the swallow, whose kind of life, especially in the chasing of insects to which it devotes itself morning and evening, imposes on it very rapid evolutions. In a bird which passes its existence in the woods, as the magpie, the tail is very long, giving the bird longitudinal mobility.

Finally, as observed by Leonardo de Vinci in the pigeon, the fan of the tail can be utilized as a lateral stabilizer by torsion in its plane, one point lifting when the other lowers, in a manner to present to the wind a twisted surface on which the air pressure produces a couple similar to that obtained in the airplane by manipulating the ailerons. This effect is evidently favored by the pointed tail already mentioned in the swallow, which owes to this arrangement its great agility and extraordinary maneuverability.

The Problem of Flight

From the mechanical point of view the problem presented by the flight of birds includes: (*a*) A rise from the ground or perch; that is to say, the effects of weight are overcome by having recourse to a counteracting force giving a component opposite to the weight, a force which in this case is the air resistance. (*b*) This result is attained at the expense of a minimum of power. (*c*) In addition to its equilibrium, translational motion must be obtained and the various maneuvers made which are necessary for the animal's subsistence.

With birds these various desiderata are satisfactorily obtained with a simplicity and efficiency of method that are truly remarkable. We admire the ease with which the birds go through the evolutions, the variety of maneuvers which they can perform, the extensive range of speeds at their disposal, the high speeds attained by most of them, and the ability to travel enormous distances out of all proportion to the speed and length of travel permitted to all other animals, land or sea.

It is evident that these results can be reached only by complete adaptation to the conditions which render aerial locomotion possible, and it is presumed that a sufficient knowledge of these conditions could be obtained only by study of the laws of air resistance as well as the construction of the organs of flight and the movements by which they are animated. It will be found useful to recall the essential principles of aerodynamics, limited to those which are indispensable for comprehending the observations and the experiments which follow.

Resistance of the Air

Air resistance is a force made evident when a body moves

through the air. It is manifested most often under the aspect of "resistance to further movement." That is to say, in order to keep a body immovable in moving air, a certain force must be expended, or to keep a body moving in the air, it is necessary to expend power continuously. The quotient of work done divided by distance covered is a force. This force, which can be measured in many cases, constitutes the reaction which is equal and opposed to the air resistance. To measure the force necessary to maintain a body in movement at constant speed relative to the air, means to measure the intensity of the resistance of the air on the body.

All methods of experimentation in this matter are based upon this statement. It is the measure of the intensity of the air resistance plus the determination of its direction, its kind, and its point of application under all possible conditions, which constitutes the object of aerodynamics. At the present time, and especially from the point of view of the present study, aerodynamics is largely an experimental science. Various physicists have been able to conclude from their tests within the limits of



FIG. 4 PRIMARY FEATHER OF A PIGEON SHOWING THE SHAFT AND RELATIVE LENGTH OF THE BARBS, SHORT IN FRONT AND LONG BEHIND

their experiments that, all other conditions being equal, (*a*) the resistance to the air is proportional to the density of the air; (*b*) the resistance to the air for like bodies is sensibly proportional to the surface of their projection on a plane perpendicular to the relative wind, and (*c*) the resistance to the air varies as the square of the velocity of the relative wind.

These laws known, the intensity of air resistance on a surface similar to that used in experiments can be calculated for a speed between the limits of speed obtained during the tests. The unit coefficient of air resistance is known to be different for each form of body. It is represented by the symbol K, which is the value of the air resistance under standard atmospheric conditions on a body of one square meter of frontal surface submitted to a relative wind of one meter per second. From which, taking into account the laws formulated, the intensity of air resistance F is given by the equation $F = K\frac{\delta}{\delta_0} SV^2$ in which δ represents the air density under the conditions of the problem, δ_0 the density of the air at a barometric pressure of 760 mm. of mercury and 15 deg. cent. temperature, S the frontal surface of the body expressed in square meters, and V the speed in meters per second.

Under normal conditions birds fly at low altitudes with the exception of certain birds of prey, so the relation $\frac{\delta}{\delta_0}$ is equal to one and the formula becomes $F = KSV^2$.

In practice it is preferable to introduce a non-dimensional number called the absolute coefficient of air resistance into this equation. This is usually represented by c and is related to K in the expression $K = c\frac{d}{2g}$ or $c = \frac{2kg}{d}$, in which d represents the specific weight of air (weight of a cubic meter, equal normally to 1.225 kg.) and g, the acceleration of gravity, considered constant and always equal to 9.81 m. per sec. per sec.

The formula of air resistance then takes the form $F = c\frac{d}{2g} SV^2 = c\frac{dV^2}{2g} S$ in which $\frac{dV^2}{2g} = g$ is no other than the dynamic air pressure which is registered directly by a pitot tube.

It is clear that c neither depends on air density nor nature of the system of units used. It varies with the form and relative dimensions of the body submitted to the air forces, very often also with the position of the body in the direction of relative wind, and must be determined experimentally in each case, hence the usefulness of aerodynamic laboratories.

Some values of c obtained in a tunnel on some characteristic forms of bodies are given in Table 1. (Values from Eiffel.)

TABLE 1 SIZE OF COEFFICIENT c FOR SOME BODIES

Form of body	Value of c
Hemispherical concave cup	1.400
Disk	1.097
Cylinder with plane bases (axis parallel to the wind)	0.805
Hemispherical convex cup	0.563
Cylinder with hemispherical bases (axis parallel to wind)	0.241
Sphere	0.161
Tapering body	0.145 to 0.113

From an examination of Table 1, all other factors being equal such as speed and frontal surface, the resistance of the air on two different bodies, a hemispherical concave cup and a streamlined body for example, are as 10 to 1.

From this point of view the bird whose body is well streamlined with the longitudinal axis parallel to the direction of the wind, with both frontal surface and coefficient c diminishing and with the claws hidden under the down, would appear to have every natural advantage, and ought to offer very small resistance to the air. It can then, while expending little energy, change its position very quickly. This distinctive characteristic is found to a remarkable degree in the flight of birds.

PROPERTIES OF WINGS

Of all bodies there is but one, the sphere, which always gives, whatever its position, a resultant parallel to the relative wind and in the same direction. All other bodies, even those symmetrical about some plane, such as disks of varying thicknesses, cylinders, spindles, etc., give a resultant sometimes parallel to the wind, at other times inclined to the direction of the wind. Experience shows that the resultant depends on the position of the body in relation to the wind. In most cases, however, the angle included between the resultant and the direction of the relative wind is so small that one neglects it and combines the resultant F with its projection on the line of relative wind.

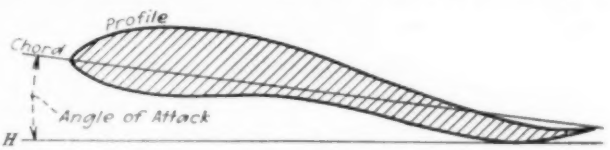


FIG. 5 PROFILE, CHORD, AND ANGLE OF ATTACK OF A WING

On certain bodies, however, when placed in certain positions, the angle included between the resultant and the trajectory can become very large, nearly 90 deg. Thin planes in particular show this property at small incidences as do also a large number of bodies called wings, characterized by their profile, form, and elongation.

PROFILE, CHORD, AND INCIDENCE OF A WING

The profile of a wing is the shape of a section of this wing in a plane parallel to the relative wind, and perpendicular to the line along which the spread of the wing is measured. In this profile, conforming to the definitions generally adopted at the present time, a chord is the straight line included between the front and rear points of the section.

The angle of attack is the angle included between the chord of the wing and the direction of relative wind. This angle

serves to define the position of the wing with respect to the trajectory. It is a conventional term whose value is related to the definition of the chord. It is important not to attribute to it any other significance, and especially, not to think that the lift of a wing is reduced to zero at zero degrees and becomes negative at negative angles. Experience, on the contrary, shows that, for curved profiles analogous to those of a bird's wing, and for a chord defined as above, for incidences of the order of —3 deg. and even —4 deg., positive lift may still result.

Form and Surface of the Wing

The form of the wing is called the projection of its contour when it is spread on a plane perpendicular to the plane of symmetry and passing through the principal chord. The surface limited by this contour is called surface S of the wing. This

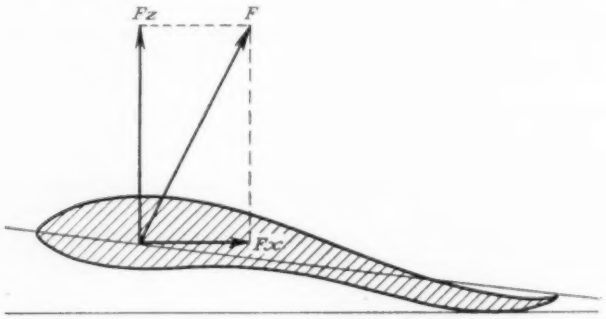


Fig. 6 Components of the Resultant Force on a Wing

definition should be contrasted with that other quite different one, of the surface of bodies other than wings, which should always be called frontal surface or area. In practice the wing is spread out on a plane, the contour of it is drawn, and the area of the figure thus limited is measured.

Spread and Elongation

The maximum transverse dimension with wings spread out is called span b, that is to say, the distance roughly included between the tip of one and the tip of the other.

The aspect ratio is the relative span, that is the relation of the span b to the mean width of the wing or the length of the mean chord l_m. With the usual irregular form of birds' wings, due to the difficulty of measuring the mean chord, and the relative facility of obtaining the wing area by the aid of a planimeter, for example, one finds the aspect ratio by the relation: $R = \frac{b^2}{S}$, since $S = bc$.

The Resultant F and Its Components F_x and F_z

Wings in general, at small incidences, possess the characteristic property of giving a resultant almost perpendicular to the relative wind. It is this property which benefits all flying organisms or mechanisms in dynamic flight, that is to say airplanes as well as birds. At small angles in fact, the resultant F of air resistance projected on two axes, one parallel and the other perpendicular to the trajectory, will give a small component F_x parallel to the relative wind and a component F_z, more important, perpendicular to the wind. (See Fig. 6.)

With birds, in contrast to airplanes, the body and wings being inseparable, one cannot do otherwise than consider them as complete systems in the same capacity as an isolated wing. As a result it is proper to determine not only the absolute coefficients c of air resistance at the various necessary incidences, but also the coefficients of lift c_z and of drag c_x on the organism

TABLE 2 RESULTS OBTAINED IN AN AERODYNAMIC TUNNEL BY MESSRS. J. L. NAYLER AND L. F. G. SIMMONS IN MEASURING THE EFFECTS OF AIR RESISTANCE OF A SWALLOW

i deg.	F_z kg.	F_x kg.	F_z/F_x (= L/D)
0	—0.0354	0.046	0.77
2	0.043	0.048	0.90
4	0.114	0.050	2.28
6	0.178	0.055	3.25
8	0.243	0.061	3.96
10	0.307	0.069	4.49
12	0.371	0.082	4.52
14	0.421	0.094	4.48
16	0.456	0.111	4.10
18	0.464	0.130	3.58
20	0.478	0.158	3.03
22	0.499	0.182	2.74
24	0.499	0.201	2.49
26	0.499	0.228	2.18

Extract of "Technical Reports of the Advisory Committee for Aeronautics" 1920–1921—R and M No. 708, page 915. Probable weight of bird 0.020 kg. Spread of wings from surface sketch dimensioned, 0.00364 sq. m. $W/S = \frac{0.020}{0.00364} = 5.500$ kg. per sq. m. = 1.12 lb. per. sq ft.

TABLE 3 RESULTS OBTAINED IN AERODYNAMIC TUNNEL BY W. L. LePAGE, IN MEASURING THE EFFECTS OF THE AIR RESISTANCE ON A CHEEL (PARIAH KITE)

i deg.	F_z kg.	F_x kg.	F_z/F_x (= L/D)
— 5	0.245	0.082	3.01
— 2	0.340	0.088	3.87
0	0.396	0.0965	4.11
1	0.425	0.103	4.125
2	0.450	0.1095	4.11
4	0.500	0.125	4.01
6	0.540	0.142	3.81
8	0.575	0.162	3.54
10	0.615	0.185	3.32
12	0.620	0.208	2.98
14	0.630	0.232	2.72
16	0.627	0.260	2.42
17	0.620	0.270	2.29

Extract from "Journal of the Royal Aeronautical Society," 1923.
The dimensions of the bird are not given.
Approximate dimensions: span = 5'9", mean chord = 10", length = 2'0".

TABLE 4 POLAR OF THE WING OF GÖTTINGEN NO. 426

i deg.	F_z kg.	F_x kg.	F_z/F_x = (L/D)
— 8.9	—0.144	0.0643	2.23
— 6	0.072	0.0212	3.30
— 4.6	0.172	0.0176	9.75
— 3.1	0.272	0.0187	14.60
— 1.6	0.376	0.0225	16.85
— 0.2	0.487	0.0271	17.90
1.3	0.586	0.0339	17.15
2.8	0.700	0.0418	16.70
4.2	0.800	0.0530	15.05
5.7	0.905	0.0640	14.00
8.6	1.090	0.0918	11.90
11.5	1.270	0.1300	9.75
14.5	1.280	0.1780	7.18

Extract from "Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen," 1ste Leiferung.
This wing has been tested at the aspect ratio 5.

TABLE 5 POLAR OF GLIDER FLORINE FURNISHED WITH A GÖTTINGEN WING, PROFILE 426, ELONGATION 12.80

i deg.	F_z kg.	F_x kg.	F_z/F_x = (L/D)
— 5	0.165	0.0340	4.88
— 3	0.350	0.0350	10.00
0	0.585	0.0380	15.40
22	0.760	0.0445	15.65
4	0.930	0.0525	17.75
6	1.085	0.0630	17.20
8	1.210	0.0790	15.30
10	1.290	0.1040	12.40
11.5	1.300	0.1200	10.80

Weight ready to fly, 180 kg. (396 lb.); carrying surface, 20 sq. m. (215 sq. ft.); wing spread, 16 m. (52.5 ft.); W/S = 9 kg. per sq. m. = 1.84 lb. per sq. ft.
Extract from descriptive notes (not published).

of the bird considered as a whole, which are given by the equations:

$$c_z = \frac{F_z}{\frac{dV^2}{2g} S} \qquad c_x = \frac{F_x}{\frac{dV^2}{2g} S}$$

Knowing two of the magnitudes F, F_z and F_x or c, c_z and c_x, one can always find the third as by definition:

$$F = \sqrt{F_z^2 + F_x^2} \quad \text{and} \quad c = \sqrt{c_z^2 + c_x^2}$$

Moreover if the magnitudes could be obtained with accuracy they would allow, as is done in aeronautics, the drawing of a polar diagram or "polar" of the bird, that is to say, a graduated curve of incidences, the projection of each point of which on two axes perpendicular would give: one the component F_x and the other the component F_z of air resistance, the latter always represented on graphical charts, by almost universal consent, in a scale either one-fifth or one-tenth the scale of F_x. From this "polar" on which F_x and F_z are taken proportional to c_z and c_x, can finally be deduced all the facts necessary for calculation of the conditions under which gliding flight and propelled flight are effected.

The study of "flapping" or ornithopter flight, in the light of this discussion, still presents very serious difficulties on account of the continual variations of surface, angle of attack, and speed

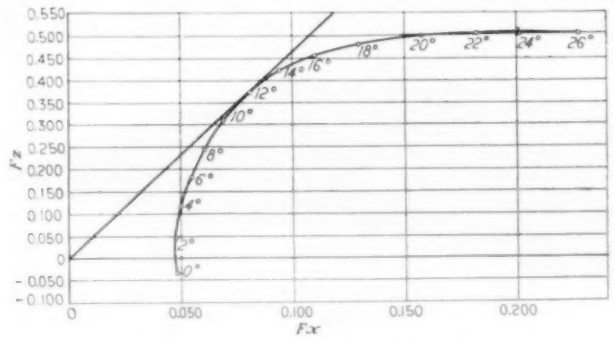


FIG. 7 TEST OF A SWALLOW IN A WIND TUNNEL

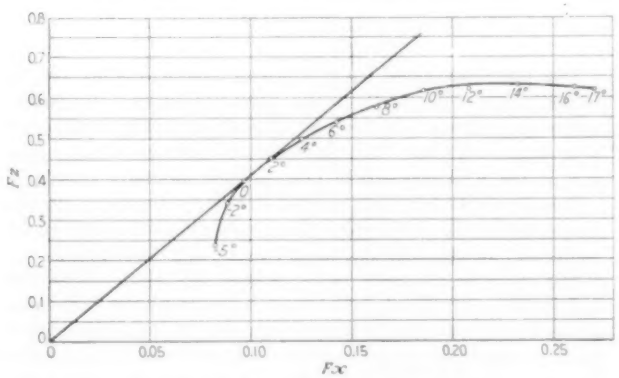


FIG. 8 WIND TUNNEL TEST OF A CHEEL

which for a given weight depend precisely on the surface and the incidence.

POLARS OF BIRDS

Various experimenters have attempted to draw the polar of a bird, notably the late J. L. Nayler and L. F. G. Simmons on the one hand and W. L. LePage (1923) on the other. The results obtained by these experimenters are given in tables and graphs, and are compared with the aerodynamic characteristics of a wing and of an efficient glider.

EFFICIENCY

In every case the importance is seen for the glider, living or mechanical, to provide for a given lift a small resistance to the air or, which means the same, that the ratio lift/drag (expressed L/D) be as large as possible. The value of this relation is independent of the units employed, and within the limits which interest us, of the dimensions of the glider and the relative velocity.

The figures of the fourth column of Tables 2, 3, 4, and 5 can then serve for a comparison of the two birds, the isolated wing and the glider. It is seen that each one of these presents a maximum L/D at a certain incidence, viz.:

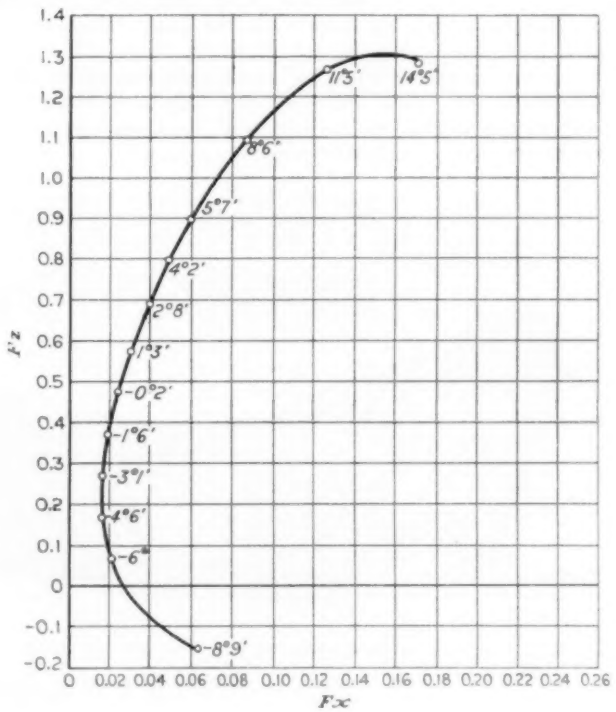


FIG. 9 POLAR DIAGRAM OF THE WING OF GÖTTINGEN NO. 426 SECTION

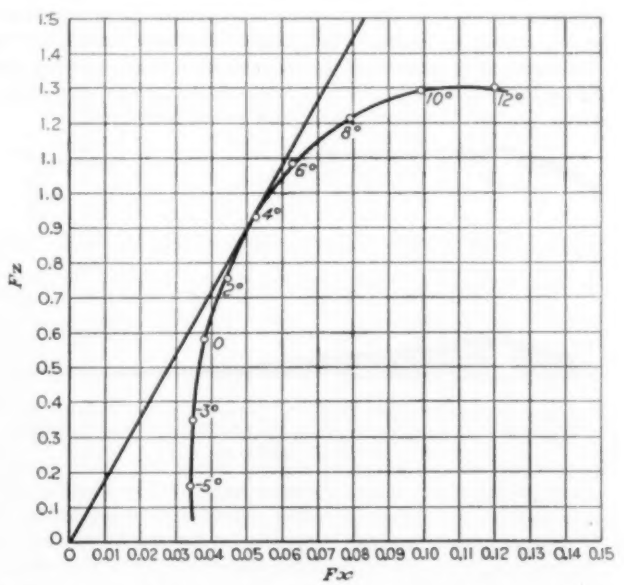


FIG. 10 POLAR DIAGRAM OF THE GLIDER FLORINE FURNISHED WITH A GÖTTINGEN NO. 426 WING SECTION

For the swallow	4.52	at	12 deg.
For the cheel	4.125	at	1 deg.
For the wing 426	17.90	at	—0.2 deg.
For the glider	17.75	at	4 deg.

On the diagrams Figs. 7, 8, 9, and 10, the angle of best L/D is found by drawing a tangent to the polar curve from the origin of the axes. The point of tangency gives the angle of maximum L/D.

The aerodynamic characteristics of the wing and of the glider are then far superior to those of the two birds exposed in the tunnel. Definite conclusions should not be drawn unless there is sufficient guarantee regarding the circumstances under which the graphs are drawn. Probably the tests were made with stuffed birds which do not have much in common with the conditions in which a living bird flies.

ing of the barbs, and in the albatross and petrel the defect does not exist. The primary wing feathers of these birds, the only ones which are exposed to the wind above and below, are smooth on both faces, the barbs near the scope having exactly the same height as the rib (Figs. 3 and 12). This peculiarity reduces air resistance, and consequently a certain improvement in efficiency results. Between a wing feather of common type and the first primary wing feathers of the albatross, there is all the difference which would exist between two airplane wings one with fabric only on the top surface and the other covered on both surfaces.

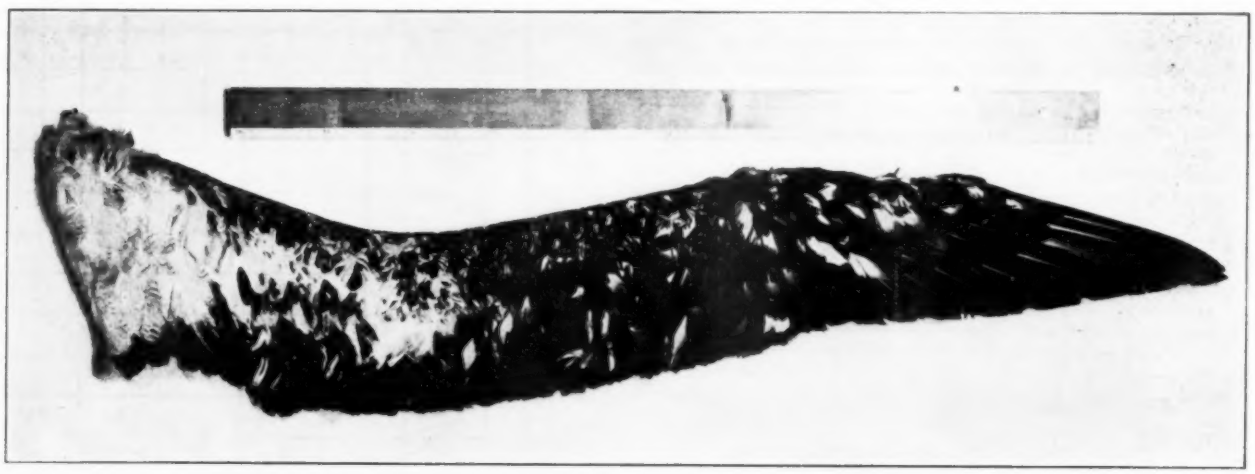


FIG. 11 OUTSPREAD WING OF AN ALBATROSS. THE SCALE IS ONE METER LONG

INFLUENCE OF THE LINES OF THE BIRD ON ITS AERODYNAMIC CHARACTERISTICS

It is on the lines, or expressed in a more general way, on the external aspects that the effect of air resistance on an apparatus or an organism depends. This aspect is characterized (*a*) by the degree of roughness of the surfaces in contact with the air, (*b*) by the shape of the body or fuselage, (*c*) by the form of the wings, (*d*) by their aspect ratio, and (*e*) by their profile.

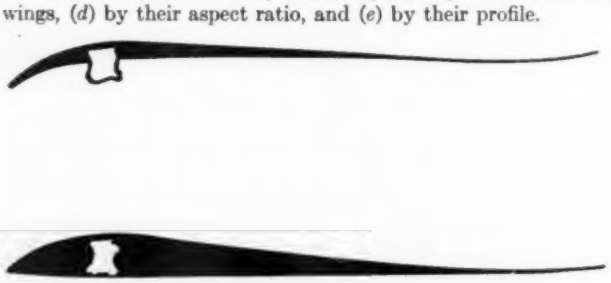


FIG. 12 ABOVE, SECTION OF THE USUAL TYPE OF PRIMARY WING FEATHER. BELOW, SECTION OF THE PRIMARY WING FEATHER OF THE ALBATROSS

The profile of the wings appears in general to be less than the other aerodynamic characteristics of birds. The lower face of the primary wing feathers, especially, is marred by the ribs of the feathers, veritable spars placed perpendicular to the relative wind, which certainly result in a definite increase in the horizontal component of the air resistance on the wings (Fig. 12). This defect is observed in most birds, including the condor and vulture, which, nevertheless, are reputed to be good soarers; also the swallow and the swift, birds that have always been considered as particularly fine examples of efficiency. In the duck, for example, this defect appears to be lessened by a slight thicken-

The more favorable these different characteristics are, the better will be the L/D of the given organism. Birds have bodies covered with feathers of which down is the smallest and most numerous form. The coefficient of resistance in the direction of the relative wind in normal flight is really very small, and varies little from one species of bird to another. For this reason particularly the good fliers can be considered as well equipped and even favored when compared with airplanes.

The shape of the body is an element of greatest importance and varies with the species much more than the consistency of the down. With migrating birds, and marine web-footed birds in particular, the shape is very nicely outlined, larger toward the front, tapered toward the rear, all by smooth, curved lines. The section between the head and the body is filled up by the down. With the sedentary species, on the contrary, these details are more or less neglected or sacrificed for other considerations. Often it appears for esthetic reasons that the birds belonging to this category possess plumages and appendages of an ornamental aspect. These obviously play no role in flight, and can only increase the resistance to forward motion.

The form of the wings in good fliers is, in general, very efficient. Experiments in the tunnel show that it is the elliptical wing which, all test factors being equal, has the best aerodynamic qualities, and that there are no great differences between one wing and the other, if one compares them with forms similar to the ellipse, such, for example, as trapezoidal wings with the smaller base on the outside and rounded corners. The greater number of wings of birds can be likened, when they are spread out, to ellipses, trapezoids, or triangles. These different contours result in an elliptical distribution of lift all along the span which seems to be as favorable for the carrying surfaces of airplanes as for natural wings.

The experimental researches of G. Eiffel, the theoretical works

of Joukovsky, Kutta, Prandtl, etc., show, on the other hand, the influence of aspect ratio. Their researches show that the drag diminishes when the aspect ratio increases, and that this drag must be considered as the sum of two partial resistances called drag of profile, and induced drag, such that the coefficient of total drag c_x must equal the sum of the coefficients of the profile drag c_{xp}, and of the induced drag c_{xi}, that is to say that

$$c_x = c_{xp} + c_{xi}$$

Experience shows that c_{xp} is connected with the form of the profile of the wing, and that for small incidences it remains constant for all aspect ratios. When the profile is very smoothly curved c_{xp} is small. The biconvex sections, for example, give the smallest values of the profile drag.

The induced drag is determined by distribution of the lift which depends on the form of the wing through interference of other parts nearby, and in particular by the aspect ratio. For a given value of c_z it diminishes with increase of R. For a certain aspect ratio the values of c_{xi} are given by the formula:

$$c_{xi} = \frac{1}{\pi} \cdot c_z^2 \frac{S}{b^2}$$

In this equation S/b^2 is the reciprocal of the aspect ratio, and consequently, is equal to I/R which represents the ratio of the span to the mean chord. It shows that if the aspect ratio becomes infinitely large, c_{xi} disappears. At this point there remains the profile drag, which can therefore be defined as the drag of a wing of infinite aspect ratio, or replacing the drags by their absolute coefficients,

if $$c_{xp} + c_{xi} = c_x$$

then $$c_{xi} = 0$$

These deductions compare very favorably with experimental evidence at the angles used by flying organisms, and other things being equal, the total drag of the bird will be inversely proportional to its aspect ratio, i.e., birds with the highest values of aspect ratio will have the highest L/D ratio. One may also conclude that for different values of R with the same weight, form, profile, etc., the angle of attack is also different, the angle being smaller for the higher aspect ratios. It has also been determined experimentally that the angle of best L/D as well as the angle for minimum power is larger for the higher aspect ratios and increases with an increasing difference between these two angles.

Finally, it is seen that an organism should fly at the angle of best L/D in order to offer the least possible air resistance (in aeronautics this angle is called angle of minimum power required), and that birds with very long, narrow wings will support a unit load larger than the others at equal speeds.

These many properties and advantages of high-aspect ratios will be mentioned several times in the course of this study, it being understood that it is due primarily to them that certain birds can claim superiority over others, for example, the swallow over the pigeon and the albatross over the sea gull.

Work and Useful Power

Let us imagine a bird following a horizontal trajectory at a constant speed V. In this case $F_x = W$ and the resistance $F_x = \frac{W}{L/D}$.

We shall see further on that the useful power P_u represented by the motion of the bird ought to be the product of the resistance F_x and the speed, let it be:

$$P_u = F_x V = \frac{W}{L/D} V$$

At a given speed P_u will then vary with F_x. That is to say, the L/D ratio will be better, and the bird will fly at a more favorable angle. One can also consider F_x as the product of the dynamic pressure by the sum of the drags which signifies that, for a given weight, the resistance to the air will be small when its parallel components are small, that is, $qSc_a = D_a$ the parasite drag, $qSc_{xp} = D_{xp}$ the profile drag, and $qSc_{xi} = D_{xi}$ the induced drag. The useful work which the horizontal movement of a bird requires per second can then be expressed by

$$P_u = V(D_a + D_{xp} + D_{xi})$$

It results from this that each fault in the line of the body upon which depends D_a, each factor in the construction of the wing feathers that might affect the profile of the wing on which D_{xp} depends, and insufficient aspect ratio which increases D_{xi} will make the bird more easily fatigued, for fatigue is linked with the useful power and also to the efficiency. This term efficiency, employed in matters relative to animal mechanics, brings us into a field that is still extremely mysterious and raises problems which, from our point of view, must be considered as unsolved at the present time. There will be no longer any question, then, but that of the motive power of living creatures, since they expend their energy in movement, lifting or propelling themselves. The work which one causes them to do many times for the profit of man, for example, and in relation to which their efficiency is calculated, is furnished by the excess of power with which all animals are endowed, and which in a moment of strain can reach a very high value, while it diminishes progressively in proportion to the duration of effort, and to the approach of fatigue. It is well understood that the idea efficiency thus understood has not much in common with the relation P_u/P_m, which would interest us if it were possible to define and determine the motive power P_m.

To relate these ideas to the polar diagram, the useful power which the flight of the swallow represents at different angles of incidence in moving itself at a given speed would be equal if the bird weighed 0.020 kg., probable weight, and possessed a supporting surface of 0.00364 sq. m., to:

$$P_u = F_x V = \frac{W}{L/D} \sqrt{\frac{W}{c_z \frac{d}{2g} S}}$$

which gives, for various incidences, the values shown in Table 6.

TABLE 6 USEFUL POWER REPRESENTED BY THE FLIGHT OF THE SWALLOW AT VARIOUS INCIDENCES, IN KG-M. PER SEC.

Incidence	4°	6°	8°	10°	12°	14°
Kg-m., P_u per S_k	0.2440	0.1375	0.0967	0.0775	0.0685	0.0637
Incidence	16°	18°	20°	22°	24°	26°
Kg-m., P_u per S_k	0.0680	0.0773	0.0897	0.0972	0.1068	0.1220

From these figures one can deduce that under those conditions which appear most favorable, that is to say, at the incidence of 14 deg. which appears to be that of the minimum of power, P_u would be equal to 0.0637 kg-m. per sec., the loading per horsepower being only

$$\frac{0.020 \times 75}{0.0637} = 23.5 \text{ kg.} \qquad (51.7 \text{ lb.}) \text{ per hp.}$$

greater than the extremes of loading used in airplanes, but less than the usual loads of living motors per useful horsepower. In fact it is known that there is not a vertebrate animal capable of developing by continuous effort such power for such a relatively small weight. It would therefore appear that the re-

sistances given by the polar diagram must be considered as excessive. For this incidence of 14 deg. the mean speed of the bird would be of the order of

$$V = \sqrt{\frac{0.020}{0.421 \times 0.062 \times 0.00364}}$$

$$= 14.50 \text{ m. per sec. (32.4 m.p.h.)}$$

which is certainly too low, the speed of the swallow being at least from 22 to 23 m. per sec. (49.3 to 51.5 m.p.h.).

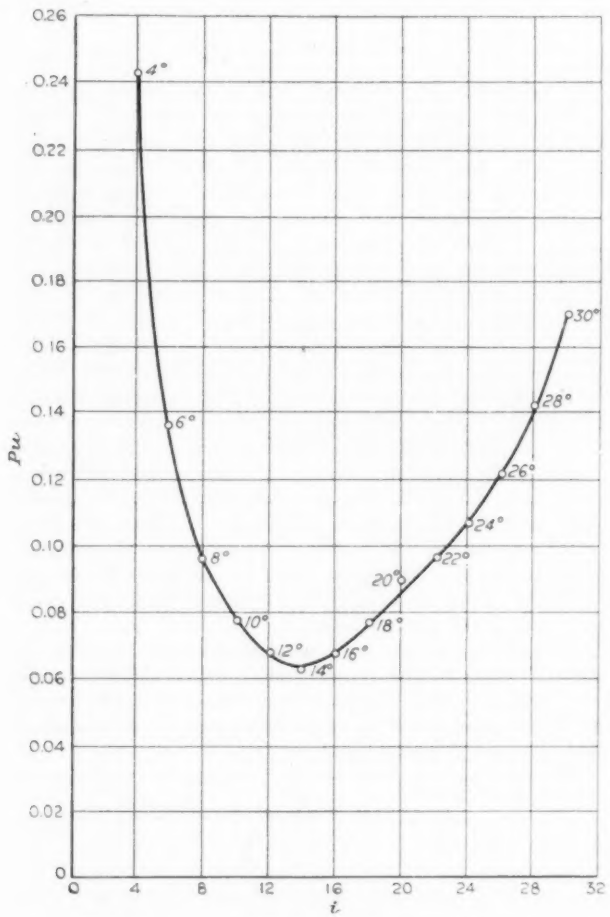


FIG. 13 GRAPH OF POWER EXERTED BY A SWALLOW

It is true that at small angles, eight degrees for example, the power curve admits of greater speeds, of the order of

$$V = \sqrt{\frac{0.020}{0.243 \times 0.062 \times 0.00364}}$$

$$= 36.30 \text{ m. per sec. (81.3 m.p.h.)}$$

which, this time, appears greater than the normal speed of the bird.

Table 6 shows that for this law the useful power is from 0.0967 kg-m. per sec., which represents only 16.6 kg. (36.6 lb.) per hp. of useful power, a value absolutely false, that is to say, much too small for a living motor. A swallow of the same weight and surface as that exposed in the tunnel, but which would present the aerodynamic characteristics of the glider of which Fig. 13 shows the polar diagram, would utilize for P_u the values of power shown in Table 7.

TABLE 7 USEFUL POWER REPRESENTED BY THE HORIZONTAL FLIGHT OF AN IMAGINARY BIRD WHICH WOULD HAVE THE SAME POLAR DIAGRAM AS THE GLIDER FLORINE

i	—5°	—3°	0°	2°	4°	6°	8°	10°	11° 30′
P_u	0.0462	0.0317	0.0162	0.0138	0.0119	0.011	0.0112	0.0136	0.01525

A comparison of these figures with those of Table 6 shows that from the point of view of the necessary power, the superiority of the glider is even greater than was indicated by the values of total resistance at different incidences and that the P_u minima of the glider and the swallow are between them as $\frac{0.011}{0.0637} = \frac{1}{5.79}$, that is to say, that the swallow submitted to the draft by Messrs. Nayler and Simmons would have to exert under the same conditions and under the most favorable circumstances, nearly six times the power of a good glider of the same dimensions. A swallow as well adapted for flight as that one, could, as an extreme case, present a load per useful horsepower of

$$\frac{0.020 \times 75}{0.011} = 136 \text{ kg.} \qquad (300 \text{ lb.}) \text{ per hp.}$$

which appears much more nearly normal. By way of comparison, a man weighing 75 kg. (165 lb.) who could furnish 10 kg-m. per sec. of useful power in a sustained effort and 40 kg-m. per sec. in an exceptional case, would represent a horsepower loading of 562.5 to 140 kg. per hp. (1240 to 308 lb. per hp.).

POSITION OF THE RESULTANT OF AIR RESISTANCE

It is known that the resultant of air resistance on a wing or a system of wings changes its point of application when the angle of attack varies. Occasionally, some rare profiles show, at small incidences, a constant position of the center of pressure. According to G. Eiffel, the center of pressure is a certain point in space through which the resultant passes. In our case, we may designate by center of pressure, the point where the resultant force crosses the mean chord of the wing profile, a point which is necessarily situated in the vertical plane of symmetry of the bird. In considering an isolated feather the same junction chord and resultant force line is taken.

The phenomenon of center of pressure movement is very important for the reason that its behavior determines the stable or unstable equilibrium of the part affected by the air forces. Experiments have shown that while the position of the resultant depends on incidence, the movement of it is related in a wing, to the profile, and in a system of wings, to the profiles, but also especially to the relative incidence of the front and rear wings or as it is sometimes called, the longitudinal dihedral angle.

Given the infinite number of profiles and combinations possible between wings and tails, it is not possible to express the movement of the center of pressure by a law. Each case is a special case. Nevertheless, among the possible movements and within the limits of incidences used in flight, one can distinguish three kinds, viz.:

(*a*) The resultant approaching the leading edge when the angle of attack diminishes and vice versa,

(*b*) The resultant retreating from the leading edge when the angle of attack diminishes and vice versa,

(*c*) The resultant not moving when the angle of attack is varied.

In the first group (*a*) is found: thin plates, the profiles in *S* form or double curvature, the greater number of thick or incurving profiles, but only when utilized at negative incidences in slotted wings and systems having a positive longitudinal dihedral. (Fig. 17.)

In the second group (*b*) are included incurved or thick profiles, but at small incidences only.

In the third group (*c*) are found certain unusual profiles, hardly interesting from point of view of lift and efficiency.

The graphs of Figs. 15, 16, and 17 give some examples of the movement of center of pressure on various profiles and a stable wing system.

Stability or Instability of Static Equilibrium of a Wing or Wing System

In the study of natural flight it is necessary to consider the character of equilibrium in the case of a complete bird in gliding

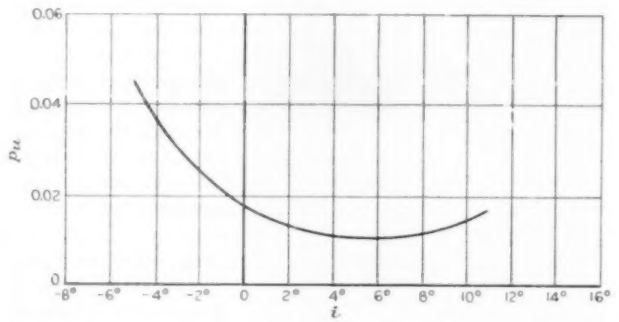


Fig. 14 Graph of Useful Power Exerted by a Hypothetical Bird Having a Polar Similar to That of the Glider Florine

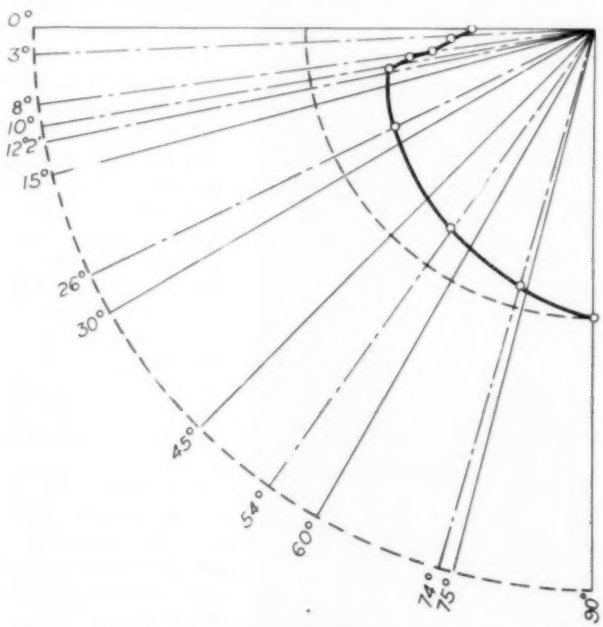


Fig. 15 Center of Pressure Movement on a Plate 85 × 15 Centimeters

flight, and in the case of an isolated feather submitted to air forces.

Let us apply to these two cases the general condition of equilibrium of bodies which is satisfied when the sum of the moments with relation to some axis is zero, where the position of the body or of the feather with relation to the direction of relative wind is constant.

In the case of a bird in gliding flight two forces are to be considered in the same plane, gravity and resultant air force, but gravity acts as though the total weight were concentrated at the center of gravity; therefore, the bird will not be subjected to any couple since the resultant will pass through the center of gravity, and there will be equilibrium only at one definite angle of attack. To put it differently, in the case of a glider or a bird descending in gliding flight, the angle of attack for a given wing position depends upon the position of the center of gravity.

If the resultant does not take up this position of equilibrium with the range of angles used in flight, the glider is unstable and

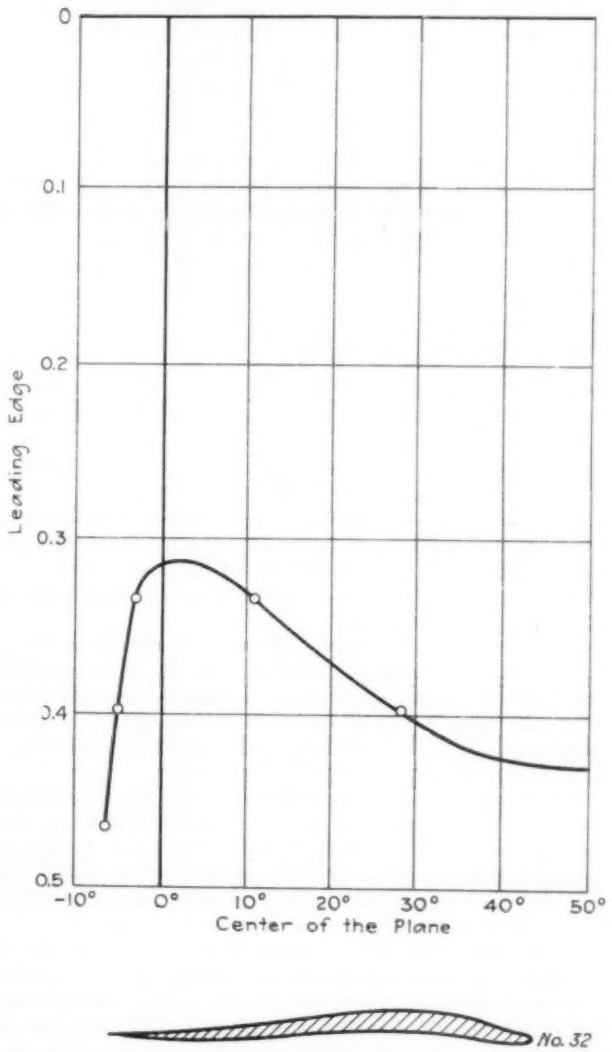


Fig. 16 Graph of Center of Pressure Travel on the Eiffel No. 32 Section

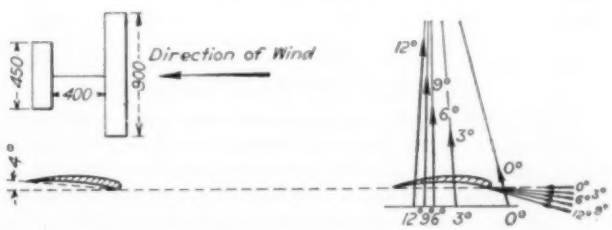


Fig. 17 Portions of Resultant Air Force on a System With Positive Longitudinal Stability

practically useless. In the second case in order that the feather may not have any tendency to turn about its shaft in a relative wind of constant direction, it is necessary that the sum of the moments with relation to the axis of this shaft shall be zero. There can be only the moment of air resistance and the moment

of gravity with relation to this axis. The moment of gravity is zero when the center of gravity is on the axis and is negligible in all other cases on account of the light weight of the feathers. There remains practically then with relation to the shaft only the moment of air resistance which is also zero when the resultant cuts the axis. From this it can be seen that it is the position of the shaft with respect to the leading edge of the feather which determines its angle of attack. There persists still, nevertheless, another moment, with respect to the insertion or fitting of the feather, but this is balanced by the resistance of the tissue itself and of the shaft serving as a strut.

In every case, the angle of equilibrium imposed by the distance between the leading edge of the supporting surfaces and a transverse axis will be stable or unstable according to the behavior of the center of pressure movement on the wings or the feather.

If the resultant advances when the angle diminishes, that state of equilibrium is stable. It is easy to see in fact, that any variation in the angle produces a couple which tends to bring the feather of a bird back into its original position. This is the case for all the profiles and systems of group (*a*). It is also seen that if the resultant recedes when the angle of attack diminishes, the least change in that angle, either increase or decrease, will bring into play a couple tending to turn the wing still further in that direction, rendering the system unstable. The profiles of group (*b*) are included in this category.

When the resultant does not change its position in the range of angles used in flight, the state of equilibrium does not depend upon the condition of coinciding center of pressure and center of gravity, or for the feather, on the center of pressure passing through the axis of the shaft.

For a condition of stable equilibrium, for a glider as well as for a wing alone, when the resultant advances as the incidence diminishes, the angle of attack will be smaller when the center of gravity (case of a bird) and center of the shaft (case of the feather in general and primary wing feathers in particular) is near the leading edge of the supporting surface.

For example, if an airplane can fly at small angles, it is because a vertical line through the center of gravity passes, as a rule, through a point on the wing, between 25 per cent and 33 per cent of the chord length, measured from the leading edge. One presumes that in a normal position of gliding flight, the situation ought to be almost the same with the bird flight.

On the other hand, if at a certain height, a covert, which is a symmetrical feather, is released, it descends along a vertical trajectory, while a primary wing feather whose rear barbs are longer than the front barbs, descends in a spiral, each element of the feather describing a helical trajectory which has the effect of reducing the angle of attack of each of its elements, and allows them to glide at the small angle related to the position of the center of gravity.

Experimental Verification of the Stability of Equilibrium of Birds and Primary Wing Feathers

It is relatively easy to prove and even verify qualitatively, the stability of primary wing feathers taken individually and birds considered as a unit during gliding flight. For this it is sufficient to take in hand a wing feather coming from some large bird and move it to and fro at arm's length, quite rapidly, allowing the extremity of the shaft, called the stem, which is circular in section, to turn freely about its axis. It will be noticed that the feather assumes in each direction an angle of attack which is fixed as soon as the direction of motion has changed, a proof of stability. The angle, which is constant for each face of the feather, appears very small if one is using the first of the primary wing feathers whose front barbs are relatively the shortest with all birds. It is due directly to this property that the primary wing feathers tend, when they are subjected to air forces, to take up an angle of attack which is always the same; at which angle the resultant passes through the shaft of the feather, the actual angle being determined for each feather by the center of pressure movement peculiar to it.

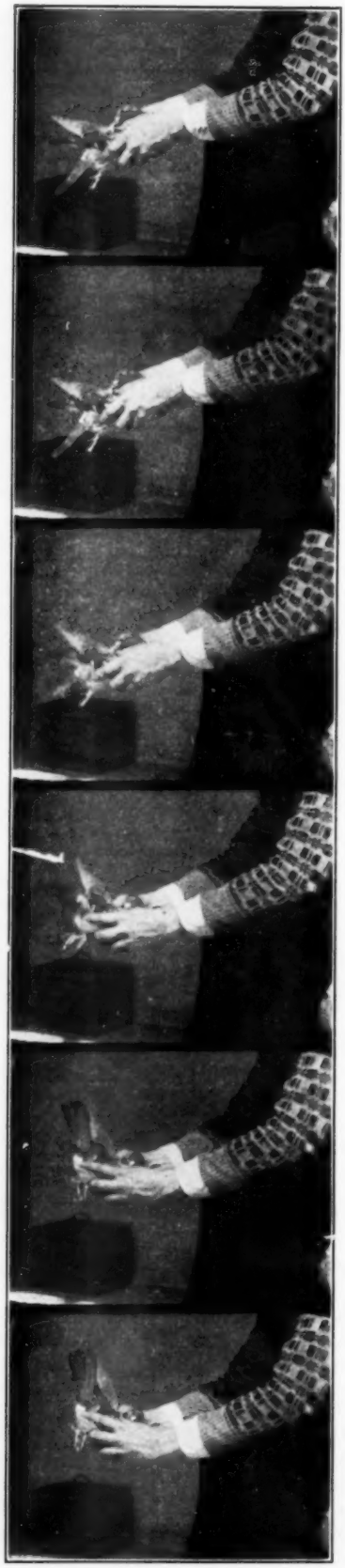


Fig. 18 Section of Film Showing the Reactions of a Pigeon When Suddenly Placed in a Diving Attitude. The Movement of the Tail Should Be Noted. The Center Views Give an Idea of the Quickness of the Reaction

Moreover when a pigeon, for example, descends in gliding flight, it carries the wings forward little by little with the intention not only of spreading them, but especially of increasing their angle of attack. If the birds were unstable they would evidently make the opposite movement. In still more direct fashion, when a pigeon is taken in one's hand and oscillated briskly about a transverse axis, it spreads the tail fairly strongly, and lifts it with a quick movement if it is put at a negative angle or drops it with a movement no less rapid if it is put at a positive angle. These reactions are too quick not to be instinctive. They are representative of the actual movements of the bird in flight and these movements are those of a stable glider. (Fig. 18.) It was evident, besides, that the movements of the resultant air force must have had a stabilizing effect.

Variation of Angle of Attack in Flight

As an example of application of the preceding discussion, imagine a bird in gliding descent.

If the resultant did not pass through the center of gravity a couple would come into play which would have the effect of turning the bird about a horizontal transverse axis. If the incidence of the bird does not vary spontaneously, it means that the sum of moments with relation to this axis can be constantly brought back to a zero value.

Suppose that the bird at a given moment draws back its wings, for example, by turning the hand through several degrees in its own plane, articulated about the wrist bone. In this case, not only is the center of gravity of the whole bird moved back through a very small distance, but also the center of pressure which passes behind the vertical line through the center of gravity is moved back very perceptibly. Under the effect of the couple thus produced, the bird dives, its angle of attack diminishes, the resultant moves forward, and finishes by passing again through the center of gravity. Equilibrium is at once established but at the price of a change in position of the body which is shown by the decreased angle of attack.

Similar reasoning shows that a displacement of wings toward the front would have produced an increase of angle. Following this reasoning it may be said that, with birds, a rearward movement of the leading edge of the wings produces a diminution of the angle of attack, and a forward movement of the wings produces an increase of angle of attack. Moreover, it should be noted that the angle of equilibrium of a feather is small when the zone situated in front of the shaft is narrow with respect to the surface located behind it.

Gliding Flight

There exist many more organisms making use of gliding flight than appear at first sight. In fact, among creatures which use the glide constantly, are found not only birds, but many insects, some mammals such as bats and different kinds of squirrels, even winged seeds and pollen dusts. In principle there are no flying organisms which cannot at least make a gliding flight. It is true that to use the term gliding flight to describe the movements of all these creatures, it is necessary to generalize and to give to this expression a definition broad enough to make it include all possible varieties of descent at constant speed following trajectories more or less inclined to the horizontal.

It is known that in space, bodies fall vertically with an acceleration which is the same for all, about 9.81 m. per sec. per sec. As a result all bodies dropped from a certain height, whatever their specific weight, form, or size, would reach equal speeds at the end of a certain time were it not for the resistance of the air. It is the air resistance that, in our atmosphere, limits the speed of bodies and imposes on them a trajectory which is vertical in only a few cases. In the broadest sense, gliding flight is a descent in which the resistance of the air modifies the circumstances of the fall as well as the speed. As usually referred to, gliding flight is the descent of a body at constant speed following a trajectory somewhat inclined to the horizontal. Gliding flight can further be distinguished from falling by the stability which characterizes gliders and parachutes, and permits them to keep a definite position with relation to their trajectory. It is true that under this definition the descent of a projectile falling from a sufficient height for it to reach a constant speed before grounding is a gliding flight. That, however, is an abuse of the definition. Nevertheless, in the field, of discussion, where we consider only lightly loaded organisms, the term can be used in its broadest sense, and under the term "gliding flight," can be listed the movement of all bodies possessing the facility of descending at constant speed, and following a trajectory of any inclination whatever, even vertical, as for example that of tufted seeds.

There are many bodies in the vegetable world which practice all kinds of gliding from vertical descent, to gliding along a rectilinear trajectory or a spiral. Among birds there are cases of gliding flight, from descent like a parachute to the glide at constant speed following a trajectory more or less inclined to the horizon.

Speed of Birds Descending in Gliding Flight

Imagine a bird in gliding flight with wings extended. For some given position its speed is influenced from the aerodynamic point of view, by its weight, its supporting surface, which is a variable, the position of the center of gravity, a polar diagram, and a curve of movements of the center of pressure, all given. For the speed to be constant, it is necessary that the sum of the forces acting on the bird be zero, that is, it is necessary that the air resistance shall equal the weight $D = W$, and that the forces be opposite. This equilibrium is only reached at a certain speed given by the relation:

$$V = \sqrt{\frac{W}{c\,\frac{d}{2g}\,S}}$$

in which taking c, d, and g to be constants, it is seen that the speed depends on $\sqrt{\frac{W}{S}}$, that is to say on the square root of the specific loading on the wings.

TABLE 8 AERODYNAMIC DIMENSIONS OF SOME BIRDS AND ANIMALS

Name of animal	Weight "W" 16	Surface "S" sq. ft.	Wing spread b ft.	Wing loading $\frac{W}{S}$	Aspect ratio $\frac{b^2}{S}$
Condor	22.04	11.84	9.68	1.86	7.40
Albatross	16.58	5.08	9.88	3.31	19.30
Vulture	11.02	7.21	8.20	1.53	9.32
Percnopter	4.96	5.88	7.05	0.843	8.45
Andes Eagle	4.40	4.94	5.68	0.893	6.50
Raven	1.35	0.33	3.12	1.01	7.40
Wood Pigeon	1.31	0.78	..	1.68	..
Pigeon	0.99	0.625	2.04	1.59	6.60
Grey Petrel	0.95	..	3.64	0.921	12.95
Magpie	0.44	0.55	1.76	0.805	5.60
Jay	0.385	0.505	1.77	0.756	6.20
Flying Fish	0.31	0.131	1.10	2.34	9.20
Dactylopter	0.22	0.086	0.968	2.56	10.90
Swallow	0.066	0.073	..	0.903	..
Sparrow	0.055	0.154	1.115	0.358	8.10
Bat	0.040	0.126	1.15	0.315	10.50
Martin	0.066	0.142	1.315	0.540	12.07

The conclusion is that a given bird can glide with a speed inversely proportional to its specific wing loading, and this speed, for a given angle of attack, will be proportional to the square root of the weight lifted. It is evident, moreover, that a bird can choose its speed between two limits, one a minimum when the wings are spread out to the maximum, the other a maximum when the wings are completely folded to the body.

Table 8 gives aerodynamic dimensions of some flying organisms measured to conform to conventions actually used in aerodynamics.

It is interesting to compare these figures with the following. A school airplane is loaded generally with 20 to 25 kg. per sq. m. (4.5 to 5.1 lb. per sq. ft.); a commercial airplane reaches 50 kg. and more per sq. m. (10 lb. per sq. ft.) and certain record planes exceed 100 kg. (22 lb.).

The albatross has the highest aspect ratio, its value not being approached by any others. A single advantage results from this, it gives a better L/D to the bird. It is reasonable to think the body itself is modeled so as to offer a minimum resistance to the air, under the same influences which causes the increase of relative wing spread, and one is even more strongly convinced of this when sea gulls, petrels, and albatrosses are observed, whose lines are exceptionally good, and whose first primary wing feather,

at least in the case of the albatross, is made with the idea of reducing the drag of the wing. It may therefore be said that birds endowed with the highest aspect ratio are at the same time most efficient and the best soarers. In this class the albatross holds first place.

Equilibrium of the Gliding Bird

Because of the constant position of the bird with relation to its trajectory, one may conclude that the sum of the moments of gravity and air resistance applied to the whole organism is zero. This condition is satisfied only when the resultant of air resistance passes through the center of gravity and this position is only reached at a certain angle of attack. As the position

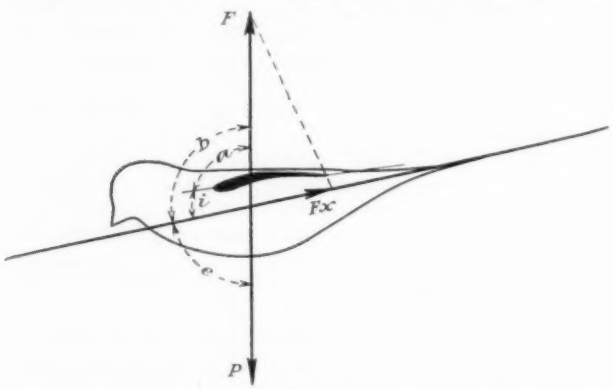


Fig. 19 Diagram of the Forces on a Bird in Gliding Flight

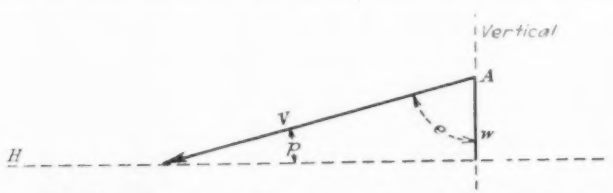


Fig. 20 Trajectory and Speed of a Bird in Gliding Flight

of the center of gravity is practically invariable, the angle of attack depends on the position of the wings, and this angle becomes smaller as the wings are moved back. Given that the size of c varies with the angle, following a law which for small angles can be considered as linear, the speed is also proportional to the square root of the angle but in an inverse ratio.

Slope of Trajectory

The air resistance must counteract the weight, for D is inclined at a certain angle to the chord of the wing. On the other hand the angle i, included between the chord and the line of flight, also has a definite value so that in order for D to be vertical the angle b included between the vertical and the line of flight is equal to the sum of the angles a and i. ($a + i = b$.) In practice the angle of inclination of the line of flight to the horizontal can be denoted by e, supplement of b, such that $e = 180° - b$. (Fig. 19.) This angle is opposite the vertex and is consequently equal to the angle between D and D_z (F and F_z) whose tangent is no other than L/D, that is to say, the efficiency. So that L/D expresses the tangent of the angle e, and the angle of gliding p, complement of e will decrease as e increases. From this it appears that the best gliders will be birds whose relative resistance is the smallest, that is, birds with well-formed bodies, long wings, and with an efficient wing profile.

Vertical Speed w

Examination of Fig. 20 shows that vertical speed in gliding is expressed by $w = V \cos e$. This speed w will vary directly with V and inversely as the angle e whose tangent is no other than L/D, the efficiency at the given angle of attack. Another effect of the high value of L/D is to reduce the height lost per unit time for any given gliding velocity.

Mechanics of Gliding Flight

When a bird descends in gliding flight it takes up a gliding angle which depends on the position of the leading edge of the wings with respect to the center of gravity. It adopts a trajectory inclined at angle e to the vertical, such that its tangent expresses the L/D of the glider at this flight attitude.

On this trajectory it maintains a speed at which its air resistance is equal to its weight. This speed depends on the square root of the wing loading, and does not depend directly either on the will or the vigor of the bird. The speed and gliding angle are therefore determined entirely by two factors, the weight per square meter and the L/D of the bird.

It should be noted, nevertheless, that the bird can modify its supporting surface on which the wing loading depends, by opening or closing the wings, and its angle of attack on which its L/D depends, by displacement of the leading edge of the wings with respect to the center of gravity. To these changes are due the variety of speeds a bird can take in gliding flight, from descents at low speed where the gliding angle is often less than five degrees, to vertical descents at high speed, wings folded (swoops of birds of prey), not to mention the parachute descents of certain vultures observed by Dr. Ahlborn in Bulgaria, during which these birds bring their wings forward so that the center of the horizontal projection of their supporting surfaces and the center of gravity of the bird are brought one above the other. The gliding angle then becomes a right angle, that is to say, the descent is along a vertical path. (Fig. 21.) In other words, a given bird, depending on the wing position and for each particular position, constitutes a glider whose weight alone is constant and whose other characteristics, surface and L/D on which depend its velocity and gliding path, are variables, although of course both of them have a maximum value.

Soaring Flight

A bird or a glider, therefore, in calm air, if it does not want to expend the necessary energy for maintaining altitude can only descend. However perfect its form may be from the aerodynamic point of view, it will inevitably come to the ground at the end of the time t, which may be expressed as the quotient of its altitude at the start h divided by its vertical speed w:

$$t = \frac{h}{w} = \frac{h}{V \cos e}$$

However, observations made by meteorological stations and conclusions formed in recent works on wind phenomena, show that air currents are rarely horizontal or of constant velocity. On the contrary at any one place, the currents change continually in speed and direction. They also increase with altitude but in decreasing fashion. Certain kinds of topography such as long slopes in the right direction, or cliffs of sufficient height, can produce oblique rising currents, which are very steady. A swell which raises the sea with vertical speeds of from 75 centimeters to 1.5 meters per second draws along a thin layer of air above it. Heating by the sun of sandy or rocky spaces devoid of vegetation or humidity can produce local ascending winds.

Now, although the horizontal current at constant speed is useless from the point of view of soaring flight, the rising currents whose velocity changes with altitude, and currents undergoing horizontal variations of speed, can furnish gliders with the

energy necessary for sustentation and enable them momentarily to maintain their height, and even to rise for a few moments without any expenditure of their own energy. The passive character of this form of flight is manifested externally by the absence of flapping which usually accompanies active flight.

Static Soaring Flight

The problem of soaring flight on an ascending current so often observed and described, notably by Mouillard, has been explained quantitively by Colonel Dorand in the Jan., 1922, number of *Aeronautique*. The theoretical principle can be formulated as follows: The speed of a glider with respect to the earth is the resultant of its own speed and the speed of the wind. Therefore in order for a bird to keep its altitude without expenditure of energy, it is necessary that the vertical component of the wind be upward and at least equal to the vertical component of the speed of the bird. The greatest speeds observed of birds being about 25 m. per sec. (case of the albatross) and their L/D being about 20, it is found that the vertical upward component of velocity of the air w_v necessary for keeping such a bird from descending will be

$$w_v = \frac{25}{20} = 1.25 \text{ m. per sec.}$$

It is evident that those birds which have learned to use ordinarily favorable atmospheric conditions, rarely find ascending currents with the exact velocity suited to their characteristics. When w_v is greater than w, there is an excess of up current over that required to sustain the bird, and it can rise with a speed, with respect to the earth, equal to $w_v - w = w_s$. When w_v is smaller than w, the bird must descend with a speed $w - w_v$ or else furnish in the form of useful work the relatively small quantity of energy necessary to lift its own weight to a height equal to w_s.

This process, called static-soaring flight, is used involuntarily and automatically by dust, inert winged organisms, and (consciously, no doubt) by animal fliers, even if flying is not their natural form of locomotion, who find themselves in the air at the time and place where the ascending current exists. Descending air currents exercise the same influence on gliders by reducing the time of falling by adding to the velocity of descent. Birds most likely to profit from this are those that have least speed and best L/D and those that by appropriate maneuvers succeed in keeping up the longest time in a favorable zone.

Vultures in the neighborhood of Dakar do this. In the morning they perch on the enormous baobabs or collect at the tops of the dunes which the wind raises along the shore from Cape Vert to Rufisque. They rise, flapping their wings laboriously, and make for other trees or ridges. This flight shows a peculiar character which distinguishes it from the flapping flight of birds. Toward mid-day when the hot sun has heated the earth they seek convection currents and congregate on this "ascending resting place," as said P. Idrac, where with wings extended they can rise and remain in the air for hours, but always above the same spot. Through the burning sunny days they can be seen thus assembled one above the other, 100, 200, 300 meters high often, turning as in an aerial merry-go-round, in a column of rising air. When the sun is hidden they voluntarily stay perched or fly from one tree to another, flapping and gliding. In the evening if frightened, they rise flapping the wings and go to find a resting place elsewhere. These birds use alternately soaring flight and active flight with flapping wings.

It is the same with the albatross and petrel whose life makes it necessary to stay in the air from morning until evening, and at sea for weeks or months at a time, perhaps. Under such conditions it may be supposed that from necessity they have developed a mode of flying which requires a minimum of power. It is noticed, however, that in the regions where they are found, for example about 42 deg. south latitude and 15 deg. east longitude at 550 kilometers from the nearest land, there are also terns, these swallows of the sea, so graceful and swift in their flight, which is identical with that of our swallows. It is hard to believe that the albatross and petrel are essentially different from any other bird submitted to the same necessities as they are. In reality the flight of the one differs less from the other than it appears, and the little terns do not expend relatively more energy than the large albatross.

With the albatross, it is true, there is more evidence of a certain skill and constant care to profit by the least circumstances favorable to sustentation. It was noticed that they came near a sail boat with 3000 sq. m. of sail rising to a height of 50 m., and here they found ascending currents which generously sustained them. Level with the water and using only the end of their primary wing feathers, they cover hundreds of meters in some seconds, wings extended, rising on the slope of the swell into the region of upward currents caused by the displacement of the wave.

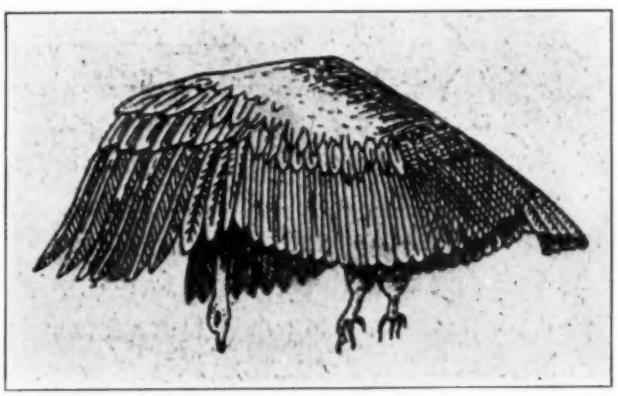


Fig. 21 Vulture Descending as a Parachute

It may be concluded, therefore, that in reality the birds best adapted for flight are those that are able to take advantage of the smallest ascending currents with the expenditure of a minimum of energy, so that they can maintain, gain, or recover altitude as the case may be, with a minimum of fatigue. Just how far they are from flying without expenditure of energy in any form cannot be measured at the present time.

Dynamic Sailing Flight

Observers, however, such as Dr. Hankin (see "Animal Flight" by E. H. Hankin, London, 1913) who studied the flight of vultures in the Indies, and P. Idrac, whose work in the South Seas preceded this study state: first, "that at Calcutta and at Agra flights of considerable duration without loss of height or speed have been observed in the absence of any wind of appreciable velocity;" second, that in the absence of swell, albatrosses maneuver for hours without flapping their wings, in successive ascents and descents from 8 to 15 meters height combined with sharp turns at top and bottom of their trajectories.

Dr. Hankin confessed his inability to penetrate the mystery of the phenomena that he observed. Idrac on the contrary has been able to establish that the albatross utilized the variations of wind velocity by continually changing altitude. "The bird maneuvers in such a manner as to place itself in layers of air having different velocities," he writes. This trick consists in creating its own relative wind in turning face into the wind when

rising (in a rising current) and tail into the wind for descending (in a descending current).

The author then analyzes the phenomenon quantitatively and concludes: The mathematical analysis of the trajectories observed, together with laws of the growth of air currents which have been determined experimentally, permit the prediction of certain phenomena in the maneuvers. Thus they require birds

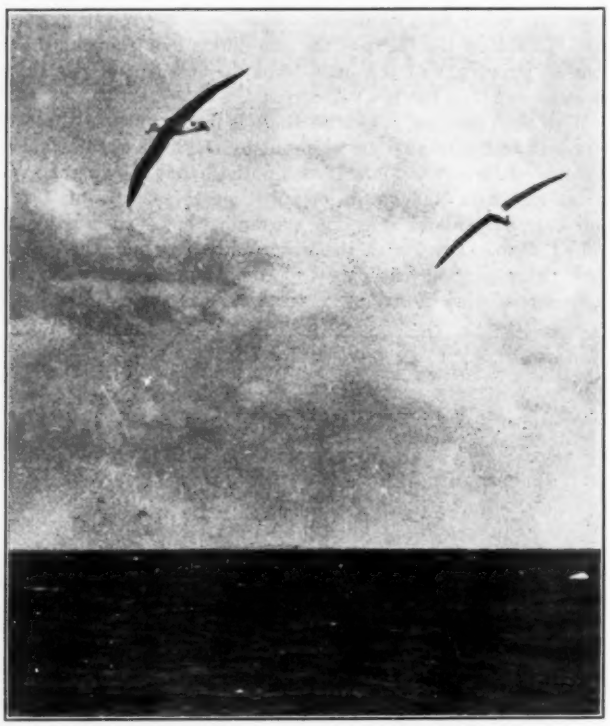


FIG. 22 GLIDING FLIGHT OF ALBATROSSES

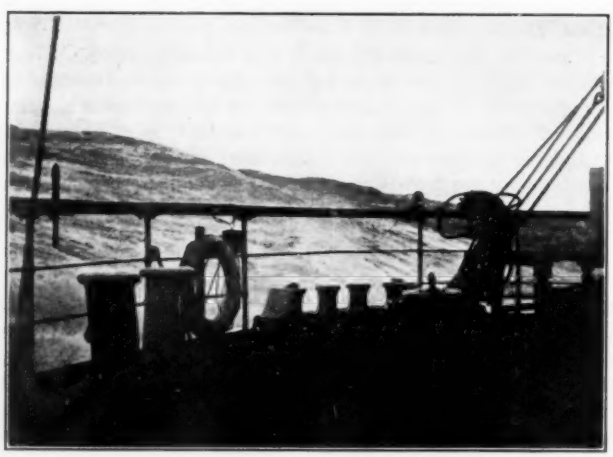


FIG. 23 THE SLOPE OF THE SWELL IN SOUTHERN SEAS PRODUCES UPWARD AIR CURRENTS WHICH ARE TAKEN ADVANTAGE OF BY BIRDS IN GLIDING FLIGHT

capable of high aerodynamic speeds, hence with strong lifting ability. The minimum theoretic wind necessary for soaring flight is of the same order of size as the observed wind. Finally the altitudes reached and the theoretical trajectories for maximum efficiency are precisely those which the birds use. (*La Technique Aeronautique*, Dec., 1924.)

In this study of a certain form of soaring flight different from static soaring flight, there is nothing new to take up. In the course of the maneuvers described, the albatross and petrel search for an altitude where the occasional changes in acceleration of the air currents in a horizontal plane may be utilized. In the latter case their flight was analyzed by Colonel Alayrac in *L'Aeronautique* of March, 1922. The changes in acceleration are taken to mean changes in the relative wind, that is to say, with respect to the bird. That does not mean necessarily an increase of real velocity of the wind. With the wind from behind, for example, a retardation of the air velocity constitutes an acceleration of the bird's aerodynamic speed.

Let A (Fig. 24) be a bird in flight under the acceleration of gravity g and also, for an instant of time, under the influence of a variable wind, from either before or behind, having a relative acceleration $+a$, i.e., in the direction of the line of flight. In this simple case, the direction of the wind being in the vertical plane of flight, the bird is affected by the resultant acceleration:

$$a_r = \sqrt{g^2 + a^2}$$

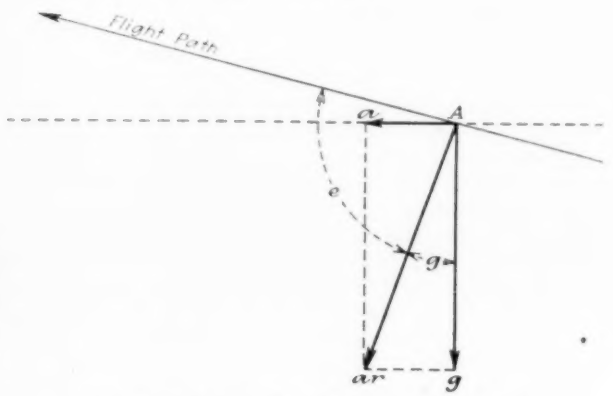


FIG. 24 TRAJECTORY OF A BIRD IN AN AIR GUST

inclined with respect to the vertical at an angle q such that $\tan q = \frac{a}{g}$.

This is just as if the direction of a_r, thus determined, constituted for the bird an apparent trajectory along which it could glide without expending its own energy, at the angle e defined previously. If the sum of the angles $(e + q)$ is smaller than a right angle, the bird descends with an angle equal to 90 deg. — $(e + q)$. If $(e + q) = 90$ deg., the trajectory is horizontal and if $(e + q) > 90$ deg., the trajectory will be inclined upward with respect to the earth and the bird will gain altitude, its line of flight being at an angle $(e + q) - 90$ deg. with the vertical. The conditions necessary in order that the trajectory at a given moment may be inclined upward or horizontal are, then, that the acceleration be positive and that the angle q be equal or larger than the angle p which is the complement to e, i.e.,

$$\tan q \geq \tan p$$

or, again, in replacing $\tan q$ and $\tan p$ by their values, that

$$\frac{a}{g} \geq \frac{1}{L/D}$$

For example, with a possible L/D of 12.5, the horizontal acceleration must be such that

$$\frac{a}{9.81} \geq \frac{1}{12.5}$$

in other words, it is necessary that

$$a \geq \frac{9.81}{12.5}$$

or 0.785 m. per sec. per sec. at least. The principal criterion of a good soarer is therefore a high value of L/D.

The accelerations will not often be positive for a bird unless it maneuvers rapidly with many turns and changes in altitude, and on this account it is necessary for the bird to possess great speed, a conclusion similar to that reached by P. Idrac and a justification of the high wing loading characteristic of the albatross.

In each case the aerodynamic speed at each instant will be



FIG. 25 CAPTIVE ALBATROSS

equal, by substituting for the actual weight its apparent value W_1:

$$W_1 = m\sqrt{g^2 + a^2}$$

$$V = \sqrt{\frac{m\sqrt{g^2 + a^2}}{c\frac{d}{2g}S}} = \sqrt{\frac{ma_r}{c\frac{d}{2g}S}}$$

Observers of these birds in their native seas are surprised to see them already in the air at break of day and to notice them in the evening during the very long twilight which, in summer, ends the day, in 42 deg. latitude. It is imposible to suppose that they could regain the land during the night, for it is often 2000 kilometers distant. One must conclude that they sleep on the water rocked by the swells as in a cradle.

In the air all day in the course of their monotonous existence, they are forced to expend the least amount of energy possible, and they adapt their flight to momentary conditions, seeking the ascending winds wherever they can be found. In between times, perhaps once a minute, especially in light winds, and always while soaring, they work hard for a few moments. When circumstances are not so favorable, they increase their speed to the necessary value by a slight effort, the flapping being so slight that it can be detected only at the wing tips, where there is sufficient movement to cause a disinterested observer to attribute to the wing tip an essential role in the normal flight of these birds.

Often during their descent, they perform this maneuver, thus acquiring a greater speed which enables them to climb vertically higher if they wish it than the point from which they started.

Thus the vultures and their cousins, the condors, as well as the albatross and petrel, divide their time between a kind of active flying and soaring which is a gliding flight practiced under favorable circumstances. Obviously it is necessary for them to expend at irregular intervals a certain amount of energy to make up for unfavorable atmospheric conditions. None of them would succeed in keeping in the air if they did not have recourse from time to time, oftener than it appears, either to propelled flight or to flapping flight, these being the active process of flight.

FLAPPING FLIGHT

This method of flying appears to be most ingenious, and perhaps the most efficient process of aerial locomotion known. It is moreover the only one, which so far man has not succeeded in imitating or interpreting, not because it has not been attempted, but because the mechanics of it are not understood.

Flapping birds do not use a propeller, and researches to discover a similar organ in the gull, for example, have been futile. The author renounced the analogy which has been established between wings and oars, and has developed a theory perhaps a little paradoxical in certain respects but which is quite rational and promises to give excellent results.

From a study of documents collected by E. J. Marey, and additional observations of birds in flight through use of slow motion pictures made at the laboratory of Rhode St. Genese and at the Institute of the Mechanics of Animals at Boulogne on the Seine, the latter by Mr. P. Nogues, it appears that there are only two essentials; (*a*) that which gives a sinuous motion to the center of gravity of the bird, and (*b*) that movement which, with reference to some fixed point, is characterized by a trajectory in the form of a closed curve at the end of the wing. (Fig. 27.)



FIG. 26 FLAPPING FLIGHT OF A GULL

Other phenomena which accompany the flapping flight such

as the marked bending of the wing, just as it begins to rise from its lowest position, are minor matters, easily explained, and not necessary for the existence of flapping flight. The essentials (*a*) and (*b*) are linked up with this kind of flight so that they constitute, in one way, a definition of it which should be called an ornithopter. This name should not apply to any system of flapping wings, but to all apparati without a propeller whose trajectory in a vertical plane is sinuous and whose angle of attack is always positive and alternate rhythmically between two angles not very different in value, the definition applying even when these results are obtained without flapping the wings.

It is easily established that, in the movements described by

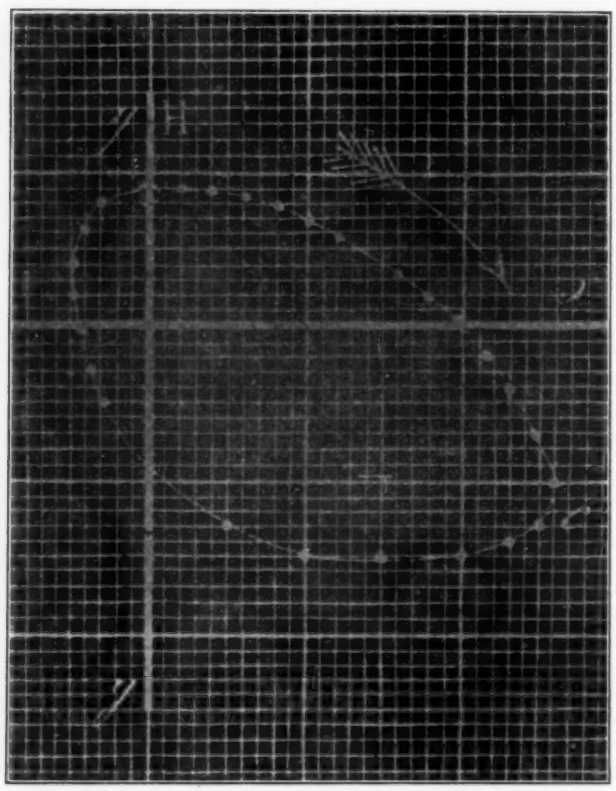


FIG. 27 PATH OF A POINT ON THE WING TIP ABOUT A FIXED POINT. THE POINTS ON THE CURVE ARE SEPARATED BY EQUAL PERIODS OF TIME

Marey, there is a clear relation of cause and effect. The trajectory of the end of the wing in the form of a closed curve is a result of the combination of two different movements which the bird gives to its wings. They are flapping in a plane nearly perpendicular to its longitudinal axis and moving the wings alternately from front to rear in their own plane. The first is very apparent and the second appears clearly on the end of a film taken of some pigeons passing overhead. Although the motion of the wings changes continually the position of the center of gravity of the bird, the variation is slight and almost negligible. The weight of the wings represents on the average only a seventh or an eighth of the total weight of the bird.

It follows that the double movement of a point in front of the wing for example, can be treated with respect to the center of gravity, considered as a fixed reference point, and the effects of the longitudinal displacement of the wings during flight are easily visualized. In fact, this particular motion of the wings from front to rear in their own plane can have only one important effect, that of varying the angle of attack. At least, that would be the effect on a glider, if one were able to move the main planes longitudinally. Upon moving the wings farther back the angle of attack would decrease, and vice versa, within certain limits. In the case of the bird, however, whose wings may be spread out or folded like a fan, there is not only a variation in the angle of attack, but also a variation of the supporting surface. That is to say, the angle of attack increases as the wing is opened and decreases as it is folded. Since the air resistance is expressed by the equation:

$$F = c\frac{d}{2g}SV^2$$

in which, taking V, d, and g as constant, R varies directly with c and S, and it is seen that for a given speed, F is dependent upon the changes of size, surface, and angle of attack which largely determine the value of the coefficient c.

It is therefore concluded, that the resistance of the outspread wings carried forward is greater than that of folded wings carried backward. Now, on looking over Marey's curve and the various films it is seen that the different phases of the flapping are distributed in such a manner that the descent of the wing, which takes place while it is in the forward position, and consequently at a high angle of attack, coincides with the rising of the body, and that the wing which is lifted, being in the rearmost position, at a smaller angle of attack, accompanies the curve in the trajectory of the center of gravity. To state that the bird mounts or descends periodically with respect to a mean horizontal trajectory, establishes the fact that the resistance to the air, in flight, increases and decreases in the same period, and its variations follow very closely the changes in angle of attack and supporting surface, being directly proportional to them. Under these conditions flapping flight can be represented as a succession of short descents in gliding flight, the bird sacrificing, upon raising its wings, all or part of the height gained in the active period preceding. In the first case the mean trajectory is horizontal; in the second case it is inclined upward.

In other words, the bird progresses like a glider that is lifted up to a certain height, released for a certain period, lifted up anew and so on, and which will succeed in covering considerable distances in this manner without being drawn or pushed. It follows that in flapping flight, the movements of the wings and their functions are not comparable to the movement and role of the oars on a boat. The former accomplish only a sustentation, the latter only a propulsion. Therefore in order to practice flapping flight it is necessary for the flying organism to be furnished with supporting surfaces capable of performing the double movement of flapping and oscillating observed by Marey. The nature of the structure of these supporting surfaces matters little; they may be either continuous membrane or independent feathers. It suffices if they are light and strong. This is why flapping flight can be practiced by animals other than birds, bats and flying fish for example, which are nevertheless unable to make a getaway. In January, 1925, the author made a voyage to the tropics during which he was able to observe some flying fish and some dactylopters. When they are taken in hand, living, they wriggle and struggle vigorously as in the water, without ever using their wings. These wings are made of a very fragile membrane, not at all rigid but crossed by narrow radial cartilaginous ribs.

Moreover, the sailors have found that the flying fish, when it has fallen on the deck, does not succeed in getting away, in spite of the bewildered flapping of its fins, which perform the function of wings. The bats, for their part, have anterior members, a posterior member, and the tail joined by a continuous wing-like membrane. They are incapable of taking flight from the ground. These two types of fliers have to be given a start, in order to

fly. The fish obtains its high initial velocity in the water by violent movements of its caudal fin, and the bats by simply letting themselves fall from the beams where they hang for the day's rest. Their usual active flight is flapping flight, identical with that used by birds. The latter at the getaway use another process, which enables them to make a point take off.

The Ornithopter-Glider

It ought to be possible to verify directly by experiment the mechanism of flapping flight. Let us imagine an ornithopter consisting essentially of a wing capable of performing the double movement as described, that is, rotation about an axis parallel to the body and of oscillation along this axis. Provided that certain other conditions were satisfied such as maximum admissible weight, sufficient power, and a good value of L/D, the apparatus could reproduce mechanically the flapping flight of the bird, without, however, being able to make a take off. It would have to be launched like a glider. In the same way one can conceive of a second apparatus, somewhat similar to the first with wings capable of a double movement with respect to the fusilage alternating from high to low and from front to rear, not by oscillation around a point but remaining constantly parallel to each other. Their longitudinal displacement could even be replaced by motions of their elevators, this being also capable of providing the necessary variations in angle of attack.

Thus the beating of wings, a relatively complex motion, could be reduced to some movements of the elevators and to the alternate displacements of the wings, with relation to the fuselage, in a plane perpendicular to its longitudinal axis, or to a similar displacement of the fuselage with relation to the wings which would give the same results. This movement, whose effect is due entirely to the consequent vertical displacement of the center of gravity relative to the plane of the wings, could finally be substituted, in an ordinary glider, with fixed wings. A periodic displacement of weights in the interior of the fuselage, following a suitable rhythm, would necessarily cause changes of position of the center of gravity. So it might happen, in perfectly calm air favorable for gliding, that forward motion might be effected, or even an upward motion, in a very light, efficient glider. The pilot would have to furnish a certain amount of muscular work by rhythmic bending of his legs, a further condition being that the movements of the pilot and the oscillation of the elevator should be coordinated so that the apparatus would glide at a certain angle when the pilot is seated and at a greater angle if he stood up.

As an initial experiment this result could be obtained by the use of a model, consisting of a glider and a clock-work mechanism as a motor. This last would be arranged so as to make itself turn about its own axis situated at the distance $D/2$ from its own center of gravity, and on a vertical through the center of gravity of the complete machine. The motor would be at its lowest position in the position of horizontal flight with an angle of attack included for example between the angles of $(L/D)_{max}$ and of P_{min}.

Let the expression

$$\frac{\text{weight of glider + weight of motor}}{\text{weight of motor}} = \rho$$

and let it be assumed that the object to be attained (work to be furnished) is to lift, with each revolution, the center of gravity of the whole machine to a height at least equal to W (vertical component of the speed of the apparatus at incidences included between the incidences of $(L/D)_m$ and of P_{min}), and greater if possible; it is necessary that the diameter D of the circle described by the center of gravity of the clock-work mechanism shall be equal to or greater than

$$\rho = \frac{w}{n}, \quad \text{that is} \quad D \geq \rho \frac{w}{n}, \quad \text{or} \quad \frac{D}{\rho} \geq \frac{w}{n}$$

n being the number of wing flappings per second, for the bird, and the revolutions of the motor in the case of the ornithopter-glider.

With this condition satisfied, the mechanism, passing from the lowest to the highest point of its trajectory, will lift at each revolution the center of gravity of the machine to a height

$$h = \frac{D}{\rho}$$

Looking more closely at the problem, the moving motor should be raised from the rear of the axis and lowered from the front. In this manner it will also displace the center of gravity of the machine longitudinally displacing it alternately in front (descending at a small angle of attack) and toward the rear (rising at a large angle of attack) from its neutral position. In the case of the glider, it is also necessary to take account of the relation which must exist between its own period of oscillation around a transverse axis xx passing through the center of gravity and the duration of one revolution of the motor.

The oscillations of the apparatus being efficiently damped by the resistance of the air on the fins and on the wings themselves, it will be perfectly safe to reduce to a minimum the moment of inertia of the apparatus around this axis xx, so as to render the period of oscillation much smaller than the duration of one revolution of the motor, to which would correspond the period of a forced oscillation. The fuselage and the fins must therefore be very light; in this manner it will be possible for the apparatus to follow with a minimum of lag, the variation of angles which the longitudinal displacement of its center of gravity will impose upon it, and there will be no need of fearing an amplification of the oscillations by resonance. In certain cases, nevertheless if one cannot succeed in constructing a glider presenting a sufficiently small period of oscillation around the axis xx, one could always increase the duration of one motor revolution without changing the power output, by increasing the torque.

Again, due to the lack of exact information as to the conditions under which birds fly and ornithopters function, it is impossible at present to ascertain or guess the precise limiting angles between which it is advantageous for such an apparatus to fly. These angles, moreover, depend upon the type of performance desired. By taking a range of angles low in value greater speeds would result, and taking a range of large angles, would give smaller speeds, but in each case at the expense of a greater expenditure of power. Now, as in the apparatus proposed it is necessary to be economical with the available power, which in any case is on the small side, even for horizontal flight. One can take as limiting angles $(L/D)_{max}$ which corresponds to the smallest gliding angle and a relatively high speed and the angle of P_{min}, which causes the least loss of height per unit of time in gliding flight.

The range between these two angles varies directly with the aspect ratio, and the angles are always in the same relative position, the angle of $(L/D)_{max}$ smaller than the angle of P_{min} which is favorable to the objective. It is necessary, in fact, that the machine should not lose too much speed at small angles or too much height at large angles. In certain cases the range between these angles may not be great enough. One must then choose some limiting angles from the curves of P, either side of the incidence of P_{min}, so long as one does not go beyond the angle of maximum vertical speed. With these conditions in mind, let us imagine a glider weighing about one kilogram, half of which, namely 500 grams, being the motor weight, whose speed should not, if possible, exceed 10 m. per sec. at

angles in the neighborhood of 338, the apparatus being balanced in such a way that it takes up the minimum angle of attack called for when the motor is in its forward position. With a sufficiently high aspect ratio one obtains at this angle coefficients c ranging from 0.75 to 1.15. For a mean value, $c = 0.95$, one can deduce an approximate value for the necessary supporting surface:

$$S = \frac{W}{c\frac{d}{2g}V^2} = \frac{1}{0.95 \times 0.062 \times 10^2} = 0.17 \text{ sq. m.}$$

Assume the following dimensions:

$$L = 1.80 \text{ m.} \quad l = 0.10 \text{ m.} \quad S = 0.18 \text{ sq. m. and}$$

$$R = \frac{1.8}{0.1} = 18$$

with a wing load of $1/0.18 = 5.5$ kg. per sq. m.

One could then anticipate, in the range of flying angles, an L/D which could not be better than 0.06, and a speed W that would not exceed a maximum calculable value. In fact, L/D



FIG. 28 FLYING FISH

$= tg\ p$ and, since at small enough angles, the sign and the tangent are approximately equal, if $V = \sqrt{\frac{1}{1.95 \times 0.062 \times 0.17}}$ = 10 m. per sec.

$$w = 0.06 \times 10 = 0.600 \text{ m. per sec.}$$

These figures represent a useful power

$$P_u = 1 \times 0.6 = 0.600 \text{ kg. m. per sec.}$$

to be furnished by the motor whose weight was estimated at 500 g., that is to say, it must be able to lift its own weight at 0.600/0.500 = 1.2 m. per sec., or per revolution

$$d = \frac{1.2}{n}$$

The general formula is

$$d = \frac{\rho w}{n}$$

from which

$$n = \frac{\rho w}{d}$$

If ρ is equal to 2, w to 0.600 m. per sec. and in taking d = 0.10 m., one finds

$$n = \frac{2 \times 0.600}{0.10} = 12 \text{ per sec.}$$

This result is not suprising since the frequency of the wing flappings of the pigeon is about 10 to 15 per second. This frequency could be reduced by a reduction of ρ.

If this term has a value 1.5

$$n = \frac{1.5 \times 0.6}{0.1} = 9 \text{ per sec.}$$

It is understood that it would be difficult to construct a clockwork motor capable of furnishing one-half kg-m. per sec. of

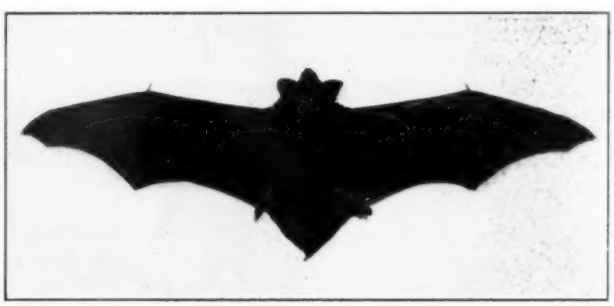


FIG. 29 BAT

power, even for 60 or even 30 sec., and in spite of the fact that 500 g. for 0.600 kg. represents about 62.5 kg. steam horsepower.

Horizontal flight is therefore not likely to be attained by the ornithopter glider described above, but whatever may be the available useful power, the maximum duration in gliding flight will be different from the maximum duration of ornithopter flight, and it is these durations that it would be useful to measure and interesting to interpret. No doubt it would be better to be able to measure flight distances with relation to the ground, these values being more easily used, but experience of model glider contests shows that it is very difficult to keep the machines flying in a straight line. It therefore becomes practically impossible to measure the distances covered, and it is worth while to take account, as proposed, of the comparison of durations of flight. From the practical point of view, the only advantage of the new process is its high efficiency, and it would be accompanied by various inconveniences, among which the most obvious are the sinuous trajectory of the apparatus and the impossibility of ever making a take-off.

SPEED OF FLAPPING BIRDS

The mechanism of power flight as described shows the similarity of the bird to a glider, and to a certain extent to an airplane. One might suppose that flapping wings fill a role comparable to that of the sustaining screws of the helicopter, and indeed, this is often the case. It is sometimes the one, and sometimes the other, but in flapping flight it is indisputably a glider. It is known that in the case of a helicopter, the speed of the center of gravity of the apparatus is independent of the relation of supporting surface to weight lifted, otherwise called the wing loading. The latter has no influence except on the speed of

rotation of the screws, to which, in the bird, would correspond the frequency of wing flapping.

In the glider, on the contrary, this speed is determined only by the angle of attack and the supporting surface and at a given angle by the square root of the wing loading only. From this fact it is possible to deduce a new experimental verification of the theory of flapping flight. One would simply take some birds having known speeds, modify their wing loading and register the influence of this modification on the speed of flight. If this variation of speed is zero or negligible in value, there would be good reason to doubt the foregoing conception of flapping flight. On the other hand, if the surface reduction was accompanied by a measurable increase in speed, the theory would be better established.

In order to perform such experiments there is in Belgium a large number of carrier pigeons, probably several millions, and one can benefit also by the accuracy of the data accumulated by the fanciers themselves on the occasion of the numerous races which they organize every year, in calculating the distances covered, the elapsed time and the average speeds.

It has been determined as the result of a very great number of experiments made in all sorts of weather that the speed of a carrier pigeon in calm air is about 66 km. per hr. (41 m.p.h.), and that a maximum rate is attained on shorter trips diminishing in proportion as the length of the journey increases. It remains to determine the effect on the speed of a modification of area of the supporting surface. There appeared in 1905 in the "Travaux de l'Institut Marey" a very accurate account of the experiments made by Messrs. Ch. Richet and P. Noguès, respectively director and chief of the Institute Laboratory, a summary of which follows:

The authors admit, with Harting (1862), that there is a constant relation, or nearly so, between the square root of the supporting surface and the cube root of the weight of birds capable of flight, such that the value of the relation $\frac{\sqrt{S}}{\sqrt[3]{P}} = R$ varies very little. The extreme values found after a large number of measurements varied from 2.5 to 6, the average value for pigeons being 3.5. By modifying the wing surface of some pigeons in various ways, the experimenters arrived at the following results. Reduction of S according to Fig. 30 (parallel clipping)

For R = 3.1 the pigeon flies admirably
3.0 the pigeon flies quite well
2.95 the pigeon flies well
2.90 the pigeon flies a little
2.85 } the pigeon still flies a little
2.80 } (limiting value)
2.75 } the pigeon does not fly

Reduction of S according to Fig. 31 (clipping of the tertiary and secondary wing feathers)

For R = 2.85 the pigeon flies admirably
2.75 the pigeon flies well
2.60 the pigeon can still fly a little

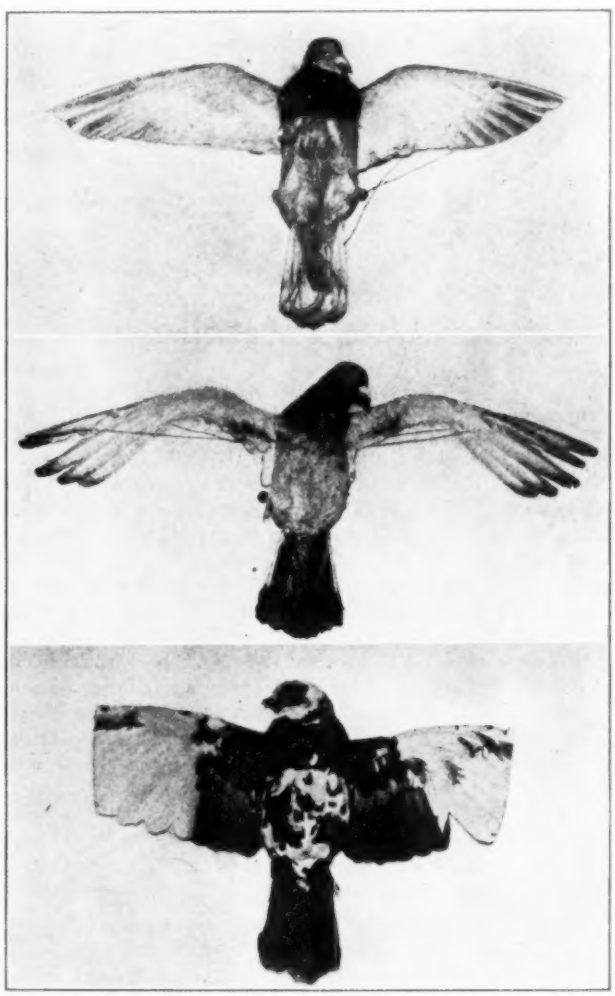
Reduction of S according to Fig. 32

For R = 3.20 the pigeon flies
3.15 the pigeon flies a little
3.10 the pigeon flies a little (limiting value)
3.05 the pigeon does not fly

The authors add another method of determining the limits of the ratio R to increasing the animal's weight in place of reducing its wing surface, but owing to the difficulty of dividing up the weight in the right way, this system would be difficult to carry out. Brazilian vultures have a high value of R, about 5, as do all birds capable of soaring in light winds. By putting lead in plates around the body, the ratio was lowered to 4.5 and that was sufficient to prevent the vulture from making a prolonged flight.

From these tests it is concluded that pigeons are least affected by reductions in the supporting surface, when the reduction is in width, i.e., the length of the tertiary and secondary wing feathers (Fig. 31); for that operation increases the aspect ratio, with the beneficial results already discussed.

Other conditions being equal, V varies as $\sqrt{\frac{W}{S}}$, that is to say that at least in gliding flight, by reducing the supporting surface



MODIFICATIONS OF THE WINGS OF PIGEONS: FIG. 30 AT TOP, PARALLEL CLIPPING. FIG. 31 IN CENTER, TERTIARY AND SECONDARY WING FEATHERS CLIPPED. FIG. 32 AT BOTTOM, ENDS OF WINGS CUT

of the pigeon V is increased, W being considered constant. In reality the formula of speed is $V = \sqrt{\frac{W}{c_z \frac{d}{2g} S}}$ in which the factor c_z may also be varied at the will of the bird, by modification of the angle of attack. It is therefore almost certain that in the pigeon it is not possible to keep all the conditions constant, and it may be said that the variation of speed of flight will not

follow strictly the variations of $\sqrt{\frac{W}{S}}$. In flapping flight moreover, the problem is complicated by the fact that useful power grows with the speed, that an increase of speed lessens the excess of power, and that in any case an increase in speed gained by reduction of S may take place only up to a certain limit, probably a little less than the total excess of power available at first.

For the purpose of verifying these deductions, in 1924 the author made a large number of measurements of pigeons, in order to obtain the mean of the weight P, the carrying surface S, and the wing spread L. The weight W was obtained by weighing the pigeon in a paper bag, when possible in the morning before feeding, the weight of the bag being balanced by a similar bag placed in the other pan of the scale.

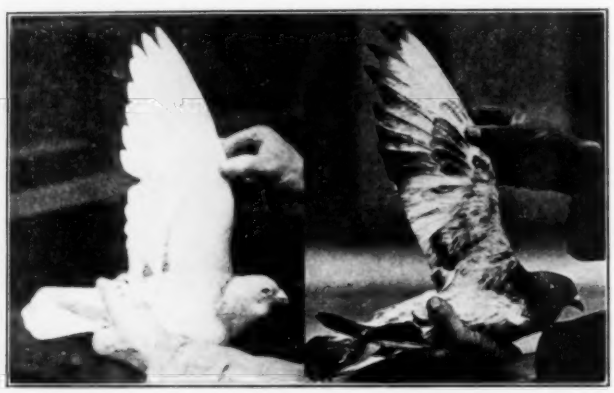


FIG. 33 · WING OF A PIGEON IN ITS NATURAL FORM

FIG. 34 WING OF A PIGEON WITH REDUCED SURFACE

To obtain S it is necessary to put the pigeon on the table, claws upward, and one of the wings spread out as far as the separation at the end of the primary wing feathers. The outline of the wing is then drawn on a piece of paper. The extremities of the curve thus obtained are joined by a straight line and the surface of the figure taken by a planimeter, which gives S with even more accuracy than is strictly necessary.

The wing spread is taken from the pigeon held in the hands, the two wings spread out as in drawing the outline of one of them. From these values the aspect ratio (R) and wing loading ϕ may be found. As a result of a large number of measurements it is found that in the carrier pigeon the extreme values of the wing loading vary from 5.8 kg. to 10 kg. per sq. m. (1.2 to 4.0 lb. per sq. ft.) and the aspect ratio from 6.7 to 9. Table 9 gives some dimensions taken from carrier pigeons of good stock, among which the numbers 8, 9, and 15 belong to the same breeder, the birds having won the second, first, and sixth places, respectively, in a competition at Dax.

TABLE 9 DIMENSIONS OF CARRIER PIGEONS

No.	W lb.	S sq. ft.	$\frac{W}{S}$ lb. per sq. ft.	b ft.	$\frac{b^2}{S} = R$
1	8.60	0.544	1.496	2.00	6.90
2	8.27	0.545	1.517	2.17	8.60
3	10.63	0.651	1.538	2.23	7.20
4	9.15	0.594	1.538	2.13	7.65
5	9.92	0.625	1.589	2.13	7.30
6	8.60	0.524	1.640	2.13	8.70
7	9.59	0.583	1.640	2.17	8.00
8	10.80	0.614	1.700	2.07	6.95
9	10.14	0.582	1.741	2.13	6.70
10	10.14	0.582	1.741	1.97	7.80
11	10.14	0.646	1.741	2.13	7.00
12	11.21	0.569	1.752	2.10	7.80
13	9.92	0.447	1.762	2.07	9.00
14	8.27	0.538	1.843	2.23	7.75
15	9.92	0.631	1.843	1.97	6.75

Similar observations which were confirmed several times quite naturally led to the theory that if pigeons capable of remarkable performances are loaded normally from 8.3 to 9 kg. per sq. m. then those loadings suit the birds better than extreme loadings of 6 and 10.

Systematic experiments were undertaken. Various pigeon fanciers loaned pigeons for single tests. One of these pigeons in particular, a bird 3 years old which had never taken a prize, but had a habit of returning regularly, which is a sign of a well-developed sense of direction, was weighed and measured. Values found were $W = 0.460$ kg. and $S = 0.0696$ sq. m. The surface was then reduced to 0.0600 sq. m., an increase from 6.60 to 7.75 kg. per sq. m. The bird, sent to Vendome and released June 1, 1924, was placed twenty-first out of 3176 competitors.

Several tests of the same kind gave similar results on each occasion. In order to proceed in a methodical way the cooperation of the military dove cotes was obtained. A movable pigeon cote (wagon No. 14) of 120 pigeons was placed at the disposal of the aerotechnical laboratory about May 15, 1925, and maintained at Rhode St. Genese up to the beginning of July. To accustom the pigeons to the country each day they were moved from their home and several baskets at a time were released. The distance from the base was raised progressively to 15 kilometers and was maintained at this distance. Releases were finally made basket by basket and later single birds were released at 2- or 3-minute intervals according to atmospheric conditions. In the course of these numerous tests those pigeons having the least regularity or apparently endowed with the least sense of direction were removed from the experiments and this process of selection finally reduced the number to 24 pigeons. From these the six swiftest were chosen as examples and the six slowest for those intended to be modified.

TABLE 10 DIMENSIONS OF PIGEONS TESTED

No. of pigeon	W lb.	S sq. ft.	$\frac{W}{S}$ = wing load	b ft.	$\frac{b^2}{S} = R$	After modification		
						S_1	Wing load	R_1
1048	0.855	0.645	1.32	2.23	7.70	0.491	1.74	9.95
1026	0.922	0.650	1.41	2.30	8.15	0.528	1.75	10.00
1129	0.805	0.508	1.58	2.07	8.40	0.523	1.75	9.30
1050	0.765	0.559	1.36	1.97	6.90	0.441	1.73	9.05
1037	0.904	0.656	1.37	2.20	7.25	0.517	1.74	9.35
1002	0.925	0.607	1.52	2.04	6.80	0.528	1.75	7.85
1047	0.870	0.622	1.40	2.13	7.30	Pigeons		
1060	0.915	0.656	1.39	2.26	7.85	retained		
1041	0.870	0.620	1.40	2.10	7.15	intact as		
1012	0.904	0.630	1.43	2.10	7.00	examples		
1018	0.893	0.593	1.50	2.07	7.20			
1040	0.867	0.605	1.41	2.13	7.50			

In the tests the times of pigeons taking more than one hour to return to their cote were neglected, these birds being considered as having wandered en route. Table 10 gives the dimensions of 12 pigeons retained for speed measurements.

In the course of seven tests on the same route performances given in Table 11 were obtained, the speed being those relative to the ground.

TABLE 11 PERFORMANCE OF PIGEONS IN TEST

No. of pigeon	Speed in meters per second							Mean speed of pigeon m. per sec.	Mean speed ft. per sec.	Class
	1st test	2d	3d	4th	5th	6th	7th			
1047	12.5	17.9	12.5	14.7	19.2	16.7	14.7	15	49.2	1
1060	16.7	14.7	16.7	15.6	14.7	14.7	12.5	14.95	49.1	2
1041	16.7	14.7	11.9	15.6	13.9	11.3	14.7	13.9	45.6	3
1012	15.6	15.6	11.9	11.3	13.1	15.6	13.9	13.7	45.0	4
1018	13.1	16.7	8.6	14.7	13.9	17.9	..	13.2	43.3	5
1040	13.9	11.9	11.3	11.3	13.9	11.3	14.7	12.5	41.0	6
1048	13.9	13.9	8.9	13.2	11.9	13.10	..	12.0	39.4	7
1026	9.6	10.0	9.6	..	17.9	10.00	..	10.4	34.2	8
1029	7.8	14.7	7.8	8.6	17.9	11.90	9.6	10.1	33.2	9
1050	11.9	8.2	10.8	..	7.8	12.50	12.5	10.1	33.2	10
1037	13.1	6.9	8.6	..	..	7.80	..	8.65	28.4	11
1002	6.2	5.2	6.2	..	8.2	7.70	9.3	6.9	22.6	12

The six slowest pigeons then had their wings reduced in a manner to give W/S the mean value of 8.5 kg. per sq. m. (1.87 lb. per sq. ft.). Table 12 gives the time and speed for ten tests

TABLE 12 RESULTS OF TEN TESTS MADE WITH TWELVE PIGEONS

No. of pigeon	1st		2d		3d		4th		5th		6th		7th		8th		9th		10th		Mean speed ft. per sec. after modification
	t	V	t	V	t	V	t	V	t	V	t	V	t	V	t	V	t	V	t	V	
1047	20	12.5	14	17.9	20	12.5	17	14.7	13	19.2	15	16.7	17	14.7	19	13.1	26.5	9.4	13	19.2	
1060	15	16.7	17	14.7	15	16.7	16	15.6	17	14.7	17	14.7	20	12.5	24	10.4	15.5	16.15	13	19.2	
1041	15	16.7	17	14.7	21	11.9	16	15.6	18	13.9	22	11.3	17	14.7	22	11.3	31.5	7.95	15	16.7	
1012	16	15.6	16	15.6	21	11.9	22	11.3	19	13.1	16	15.6	18	13.9	22.5	11.5	29.5	6.35	15	16.7	
1018	19	13.1	15	16.7	29	8.6	17	14.7	18	13.9	14	17.9	..	..	18	13.9	15	16.7	21	11.9	
1040	18	13.9	21	11.9	22	11.3	22	11.3	18	13.9	22	11.3	17	14.7	17	14.7	23	10.8	24	10.4	
																After reduction of S					
1048	18	13.9	18	13.9	28	8.9	19	13.2	21	11.9	19	13.1	..	..	..	..	16	15.6	21	11.9	45.2
1026	26	9.6	25	10	26	9.6	..	..	14	17.9	25	10	..	..	..	..	..	..	11	22.7	74.5
1029	32	7.8	17	14.7	32	7.8	29	8.6	14	17.9	21	11.9	26	9.6	17	14.7	12	20.8	14	17.9	57.3
1050	21	11.9	31	8.2	23	10.8	..	..	32	7.8	20	12.5	20	12.5	16	15.6	11.5	21.6	9	27.5	70.8
1037	19	13.1	36	6.9	29	8.6	..	..	..	..	32	7.8	..	..	..	..	16	15.6	17	14.7	49.8
1002	38	6.2	49	5.2	38	6.2	..	..	31	8.2	34	7.7	27	9.3	18	13.9	17	14.7	12	20.8	54.0

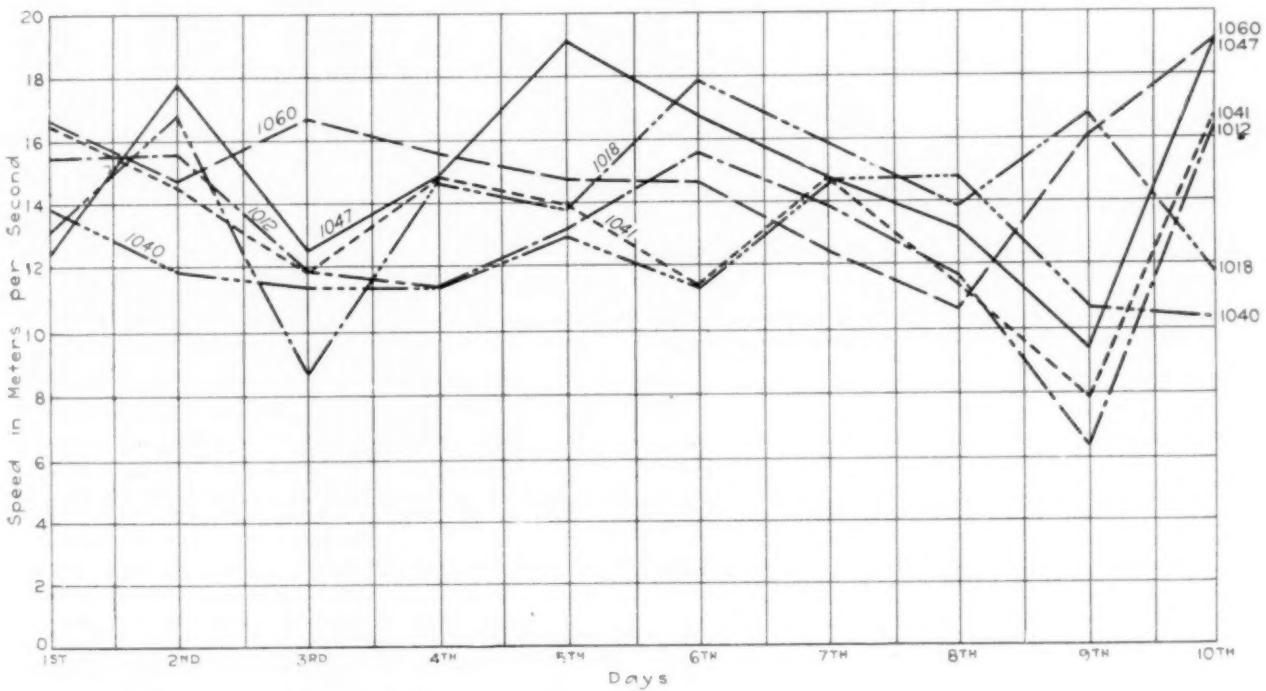


FIG. 35 SPEED ATTAINED BY STANDARD PIGEONS IN TEN-DAY TEST

by the 12 pigeons of which numbers 1048, 1026, 1029, 1050, 1037 and 1002 had their carrying surface modified as shown in Table 10 after the seventh trial.

These figures have been expressed in curves in the accompanying diagrams. Fig. 35 gives the speed realized each day by each one of the six standard pigeons; Fig. 36 the daily speed of each one of the pigeons treated before and after changing the wings and Fig. 37 the daily mean of the speeds realized by the six standard pigeons and the mean of the speeds of the treated pigeons before and after modification. The effect of increasing W/S is shown in a most striking manner. For example, in the course of trials 5, 6, and 7 (before modification), the mean speed of the six treated pigeons is 10.40 m. per sec. and in trials 8, 9, and 10 (immediately after modification), 16.95 m. per sec. The standard pigeons in the same trials gave means of 14.10 m. per sec. and 12.40 m. per sec.

The benefit derived from the operation is indisputable and the performances of the modified pigeons showed marked superiority over the performances of the standard pigeons. It is certain that further increases in wing loading would have resulted in still higher speeds. The figures in Table 9 especially show that pigeons normally loaded with 8.3 to 9 kg. per sq. m. could be classified first, second, and sixth on the Dax–Brussels distance, and that among a distinguished group of contestants.

INFLUENCE OF SPEED ON THE FATIGUE OF THE BIRD

Although increased wing loading is unfavorable to speed the power necessary for the pigeon increases at least proportionally to the speed, whatever method of flight is used, providing the weight and L/D remain the same. Birds do not possess motors whose power remains virtually constant, at least up to the wearing or breaking of parts, but they have a motive system whose power at the end of several hours of effort, falls according to the laws formulated by Prof. Mosso (see "La Fatigue Intellectuelle et Physique" by A. Mosso, Professor at Turin University).

As an example take pigeon number 2 of Table 9, which weighs 350 grams and has a surface of 0.0506 sq. m. Its unit load is $\frac{0.375}{0.0506} = 7.4$ kg. per sq. m. The mean speed of this pigeon on short distances must be nearly 16.60 m. per sec. (54.5 ft. per sec.). This value is low which is only natural in such a lightly loaded bird. On the contrary, the largest wing load shown by a pigeon in its natural state is 10. The useful limit appears to be 9.

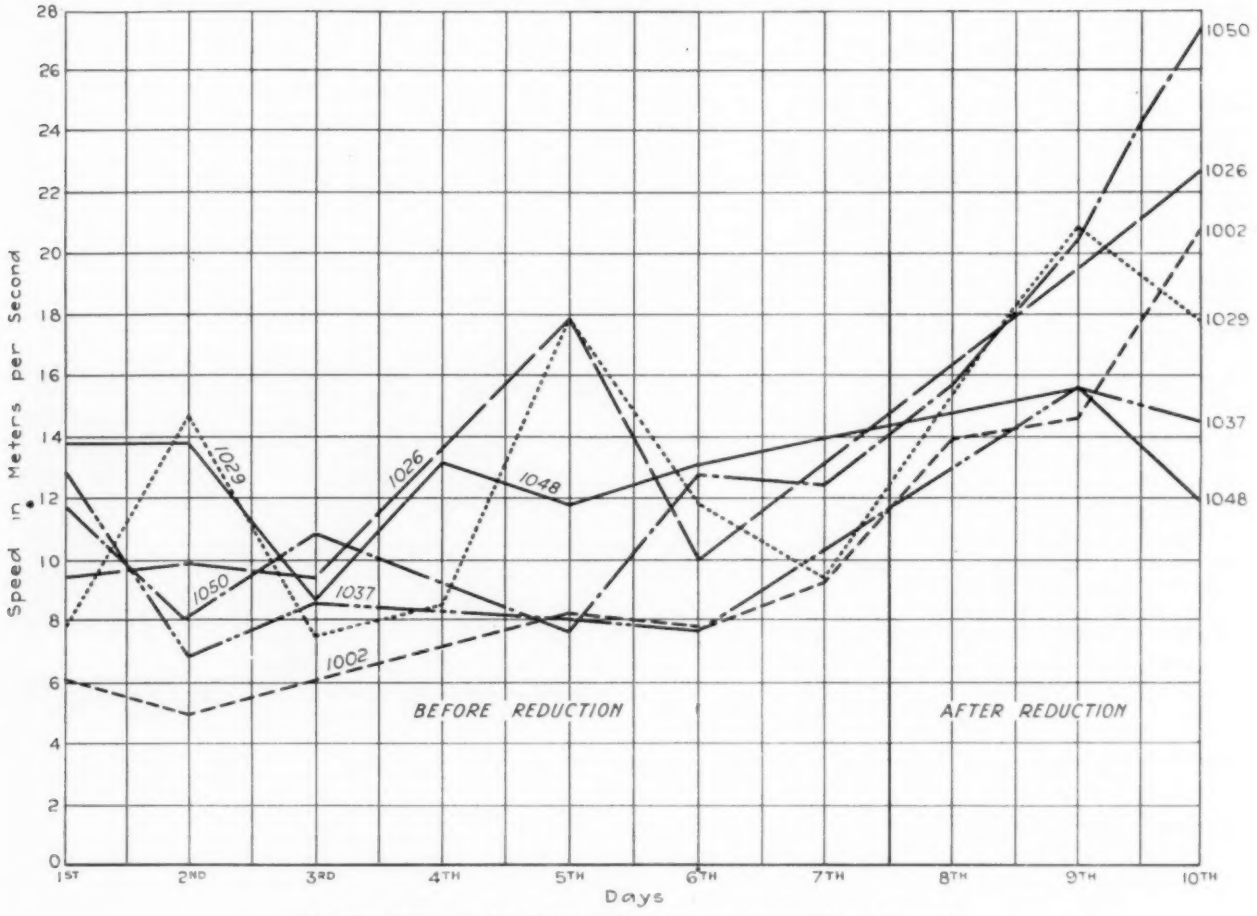


FIG. 36 SPEED OF SIX PIGEONS BEFORE AND AFTER WING CHANGES

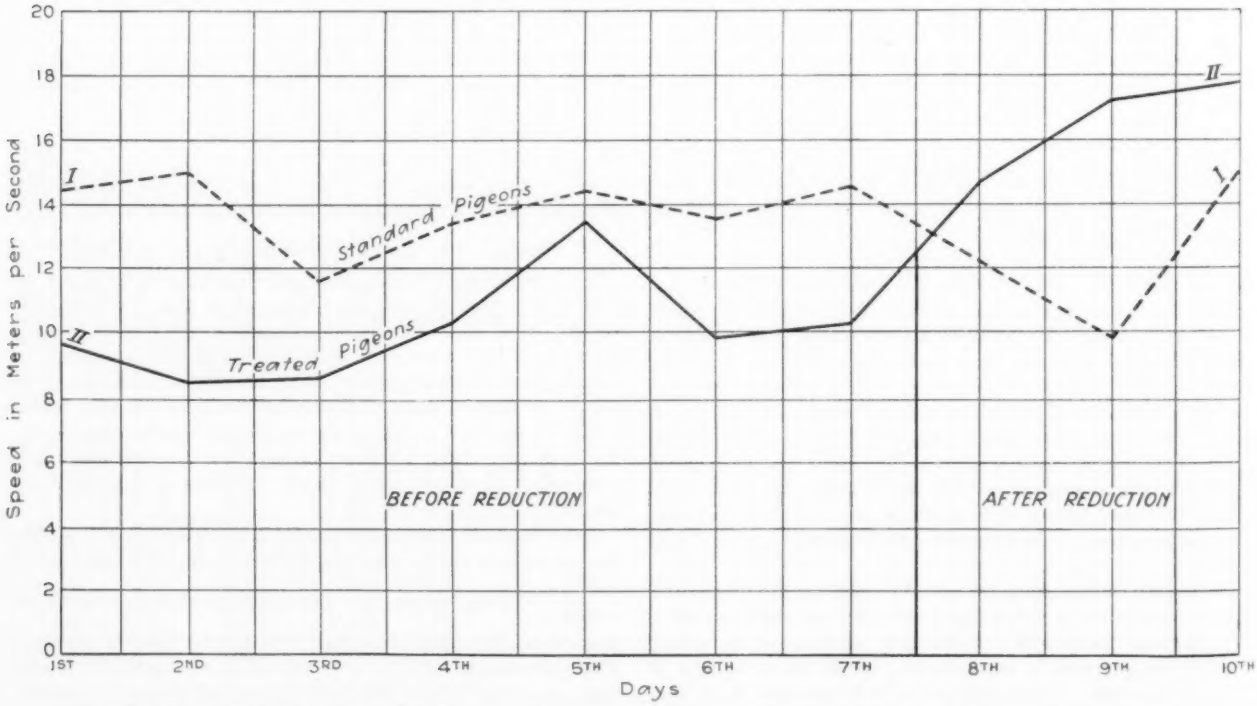


FIG. 37 DAILY MEAN SPEED OF SIX STANDARD PIGEONS AND TREATED PIGEONS BEFORE AND AFTER MODIFICATION

By reducing the wing surface the maximum load of this pigeon is calculated. Conforming to the data given this becomes $\frac{16.60 \times \sqrt{10}}{\sqrt{7}} = 19.85$ m. per sec. (65.2 ft. per sec.), a speed superior to the best observed average. The necessary power, which is proportional to the speed, will then change from 1 to $\frac{19.85}{16.60} = 1.2$. This is a high value, considering that it must be maintained for some hours.

In reality things would not happen in quite this way. When the bird perceives that the effort required is too great it probably modifies its mean angle of attack by longitudinal displacement of the wing, thus compensating in a certain measure, by increasing c, for an excessive reduction of S. Although still obliged to fly rapidly it no longer has to fly at the calculated speed. The result is that the effort and fatigue are lessened even if the L/D was decreased with the angle of attack.

Useful Power of the Flapping Bird

The work expended in the course of flapping flight, which is the product of the motive power times the time, cannot be measured or calculated exactly. The useful power, on the contrary, should be capable of expression. If it is considered that the efforts of the flapping bird have the effect of lifting it by a certain amount at each flapping

$$P_u = Whn = \frac{Wh}{t}$$

in which W equals the weight, n the number of wing strokes in a unit of time, and h the height that the bird will lose in descending by gliding, with the angles of attack and surfaces successively adopted during the full period of a wing flapping.

The height h lost depends on the speed and the L/D. At each instant it is equal to $h = V \sin p$. It will therefore vary directly as the speed and inversely as the L/D at each angle of attack.

Finally, it is seen that P_u will then become smaller as W and h or W and V are smaller, and L/D larger, or S and b^2/S larger and W smaller. The conclusion is that in all cases a bird should have as high an L/D as possible, and one will recall that this quality is closely related to the aspect ratio, the profile of the wings and the form of body not varying to any great extent.

The Speed Range of Birds

It is important to remember that flapping or ornithopter flight cannot supply a start, and that those animals which use flapping only, such as flying fish and bats, must find some special way to gain an initial speed, since they are incapable of taking off from the ground. Without their primary wing feathers, birds would be in the same situation, but the feathers which can be moved to and fro in any direction give an important component of resistance to the air perpendicular to the plane of flapping or oscillation.

The result is that when a bird starting from rest wishes to rise vertically, or when at the end of its flight wishes to alight, it is not obliged either to run or to swim in order to gain horizontal speed or to land by gliding at a speed proportional to the square root of the wing loading. In order to gain the desired result which is summed up in being able to remain stationary in the air over one spot, and descend or rise slowly almost vertically, maneuvers not possible to airplanes, the bird simply flaps the wings in a horizontal plane, spreading them to induce the separation of the primary wing feathers. The alternating movement obtained under these conditions produces on each primary wing feather, which is now an independent unit, a force F having a vertical component F_z. The sum of these F_z components, acting on the primary wing feathers, if equal to the bird's weight, will sustain it, and if greater, it will raise it with an acceleration equal to $a = \frac{\Sigma F_z - W}{m}$, and if less, it will allow a descent with an acceleration of similar order, $a_1 = \frac{W - \Sigma F_z}{m}$ always, evidently a small value, which will quickly become zero in every case, as soon as the air resistance reduced by the ascent or increased by the drop becomes equal to the weight.

These considerations are verified in a photograph of a vulture taken in the Antwerp Zoological Garden at the instant when it launched itself from the rock which served as its perch. The small size of its cage did not allow it to reach the ground by gliding so it had to come down vertically by flapping its wings to reduce its speed. An examination of the photograph taken shows that the axis of the body is evidently vertical, that the plane of flapping of the primary wing feathers is horizontal, that the wing feathers are separated one from another, and that the amplitude of movement of the tips is greater than the dihedral angle described by the main portion of the wing. This phenomenon had also been explained by E. J. Marey, who recorded it on his chronophotographs of birds in flight, and stated in particular that ducks flapped their wings much faster when taking off than in horizontal flight and that at the take-off there were large spaces between the primary wing feathers. These birds, which are very heavy and have a small supporting surface, perform this maneuver so vigorously that their feathers have become adapted to this special condition, and in order to reduce their resistance, the barbs are thickened near the rib, not as carefully as in the albatross, but in a characteristic manner nevertheless.

The mechanics of helicopter flight appears also very well on slow motion pictures showing vertical take-offs of pigeons and alightings of the same birds. At the take-off the wings always flap faster and in a horizontal plane. In certain cases the separation of the wing feathers can be seen. On alighting the birds arrive in flapping flight, with the axis of the body nearly tangent to the trajectory. As they approach the landing shelf, the body straightens up, the flapping becomes more rapid, passing from 10 to 15 per second in a plane more and more approaching the horizontal, while the amplitude of flapping increases up to 180 deg. It sometimes happens that pigeons having commenced this maneuver change their mind, and still flapping their wings stop an instant above the perch without alighting, then bringing back the axis of the body toward the horizontal, lessening the rapidity of the flappings and decreasing their amplitude, they set off again in flapping flight.

The same theory must apply to the little birds which rise vertically, like the lark, or begin their flight from one spot, as the swallow at the entrance to its nest or the humming bird before the corolla of a flower. The phenomenon has been accurately observed by the Duke of Argyll and reported as follows in "The Flight of Birds" by Marey:

"The humming bird, when it hovers over one spot, holds the tail open, fan-wise, and the body almost vertical. The wings are seen only as a dim shape, very vague because of the rapidity of their movement." It is this which gives birds the valuable range of speeds with which they are endowed, allowing them to land vertically with a horizontal speed either zero or negligible, and makes it unnecessary for them to seek open areas for landings and take-offs, such as the plane requires. The faculty of being able to hover over one spot constitutes, then, in a way, the limit of speed variation, the goal toward which airplane constructors should strive, and which would be attained by a successful helicopter.

Propulsive Flight

Because of the faculty which all birds possess of utilizing their primary wing feathers to sustain them by oscillating movements, there are some species which use this process for horizontal translational motion, either by that remarkable vibration that may be readily observed in the sparrow hawk doing the "saint esprit," or else by flapping the whole wing, though with an amplitude which increases considerably toward the tips like the rotation of the hand about the wrist. This method of flying requires that the primary wing feathers be independent, so as to be able to oscillate freely about the axis of their shafts. Now it happens that the wings of birds using this special form of flight



Fig. 38 Primary Wing Feather of a Bird Fitted Especially for Propelled Flight



Fig. 39 A Gathering of Percnopters Near Cape Verde, Senegal

give the appearance of having been cut off at the ends, the wing feathers being separated one from another by spaces which sometimes are of several feather widths. Taken individually these feathers show a marked narrowing some distance from the point of insertion. (Fig. 38.) Also, the ribs which have a rectangular section for a large part of their length, and in a tube of circular section which is evidently designed to facilitate changes in the angle of attack.

These peculiarities, which are complementary, can have no other effect but to adapt the primary wing feathers to their function of propellers. Again, there are some birds that actually move the tips of their wings in one or another of these ways, and this movement is indispensable for their sustentation and translational motion. At first sight this seems easy to demonstrate. It should be sufficient, in fact, to remove from one of these birds the front edge of its primary wing feathers, an operation which would result in taking away the precious property which they possess of balancing themselves at small angles of attack, and would deprive the bird of the use of its propellers, like screws whose blades had been straightened, and the pitch reduced to zero. It would be necessary to apply such a test to a bird having wings specially equipped for propelled flight, and incapable of flying otherwise or of adapting itself to the situation as the sparrow hawk for example, which is a strong flapper.

Experimental Verification of the Mechanism of Propelled Flight

In this class are found only certain birds of prey such as the condor, the vulture, and the eagle, perhaps also some other birds of great weight and stature, as the pelican and the stork. A living vulture was captured having the following dimensions: weight 2.25 kg. (4.95 lb.), carrying surface 0.546 sq. m. (5.87 sq. ft.), wing spread 2.15 m. (7.05 ft.), $\frac{W}{S} = 4.120$ kg. per sq. m. (0.845 lb. per sq. ft.), aspect ratio 8.45.

The bird was in perfect flying condition before modification of its primary wing feathers. (Figs. 40 and 41.) When taken to the coast where it had been captured and given its liberty, it faced the wind and began to run, flapping its wings in its normal manner. At a distance of 150 ft. perhaps, the bird tried vainly to rise and finally disappeared behind a ridge of

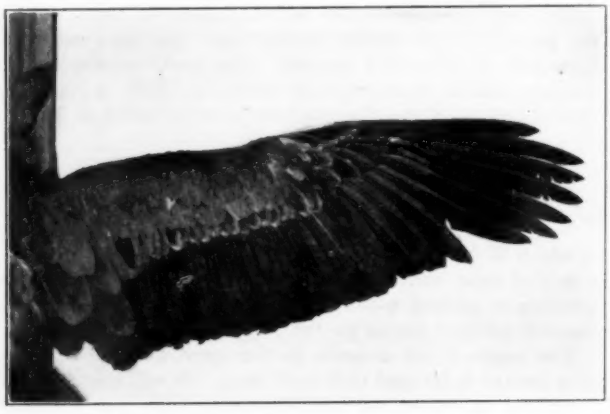


Fig. 40 Wing of a Vulture in Its Natural State

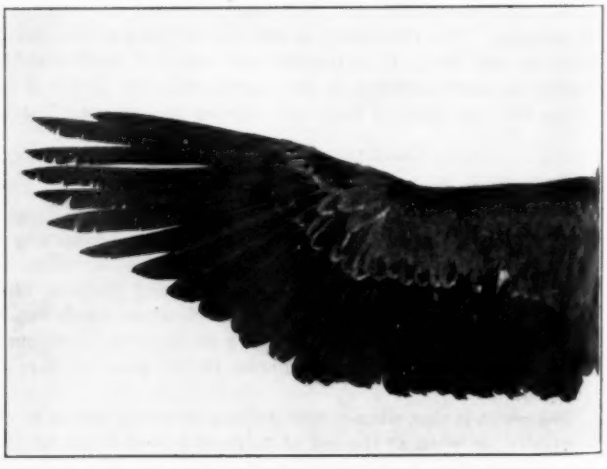


Fig. 41 Wing of a Vulture With Modified Primary Wing Feathers. Note the Front Edge of the Tip Feathers

rocks. It was found on the sand below with wings folded. When thrown into space toward the sea, free again in the air, and having some initial speed, the vulture flapped its wings in vain, without succeeding in maintaining its height. Seemingly discouraged, it soon ceased its fruitless efforts and resigned itself to descending in a glide.

A pigeon, under the same treatment, does not show any immediate effects. It takes flight without appearing to notice any difference. This is because it is accustomed to flapping flight and can take-off even in gliding flight, the change from gliding to flapping taking place without difficulty after the bird attains a sufficient speed of translation.

Messrs. Demonty and Poncelet, director and shop superintendent at the S.A.B.C.A., the most important of our airplane manufacturing plants, have produced a kind of scull or oar with constant angle of attack based on the principle of wings in equilibrium at an angle determined by the position of an axis. On a tube of duralumin 25 mm. inside diameter and 1 mm. thick they slipped 20 profiled elements, biconvex of 170 mm. depth, 20 mm. width (dimension along the wing spread), and 32 mm. thick. The blade thus constructed presents a surface of $0.170 \times 0.40 = 0.0680$ sq. m. (0.731 sq. ft.). Maneuvered alternately from left to right, this oar is deformed as a helicoid and adopts, both in going and coming, an incidence which remains constant, whatever may be the variations in the rapidity of oscillation and speed of translation of the vehicle being propelled. When tried out with a canoe, this apparatus worked better than an oar used as a scull, not only in point of speed attained but also in respect to the effort exerted by the oarsman.



FIGS. 42, 43, AND 44 ALBATROSS IN GLIDING FLIGHT. AT LEFT WINGS SPREAD OUT. AT RIGHT WINGS BENT SLIGHTLY

This particular oar plays the same part relative to the boat, that is, an oscillating, propelling movement, as do the primary wing feathers in birds such as the vulture, and the mechanics of it is identical.

EXTENT OF THE THRUST IN PROPELLED FLIGHT

Certain examples of flight where the primary wing feathers, acting as sustaining surfaces or as propellers, are of first importance, can be readily observed by aid of moving pictures. The use of the process by the vulture, even when it seems to be flapping, cannot be doubted after the experiment just related. Moreover, the birds of prey, from sparrow hawk to condor, appear to have recourse to it each time when, from the heights, they spy their prey or when, conditions not being favorable for gliding flight, they have to supply a certain effort. It is only rarely that they employ pure flapping flight, that is, the flight of marine soarers, and the greater number of birds best described as ornithopter flight.

This mode of support can be compared to the flight of helicopters or of airplanes, according as the plane of flapping or oscillation of the wing feathers approaches a vertical or a horizontal plane. In the first case the component of resistance perpendicular to the plane of flapping must be opposite, that is to say vertical and equal or superior to the weight of the bird.

This fact does not affect the possibility of the phenomenon. There is even less reason to doubt it in the case of horizontal flight, which is that of the hawk, the eagle, the vulture, and the condor, and which requires in comparison, only a very small lift of the order of $^1/_{15}$ or of $^1/_{20}$ the weight of the bird, since in propelled flight, the thrust T necessary for horizontal flight, equal to the resistance of the air, is expressed by $T = F_x = \frac{W}{L/D}$, where L/D represents as before, the efficiency of the bird at that angle of flight. It is seen from this that in the condor, which is the heaviest bird practicing this form of flight (an example of great size captured in 1925 at Uspallata in the Cordillera Andes weighed 10 kg. or 22 lb.), has approximately: $F_x = \frac{W}{20} = \frac{10}{20} = 0.500$ kg. (1.102 lb.), or, since there are 14 primary wing feathers for the two wings, $\frac{0.500}{14} = 0.036$ kg. (0.0793 lb.) per wing feather.

To be perfectly accurate, having given that this force of 36 grams is the value of only one of the components of the air resistance on the feather, it is possible, using an L/D of 20, to find the size of the average force applied on each wing feather, that is,

$$F_{x1} = \sqrt{\overline{0.036^2} + \sqrt{\frac{0.036^2}{20}}} = 0.0365 \text{ kg. } (0.0804 \text{ lb.})$$

The difference is negligible and the figures obtained are quite within the realm of possibility.

USEFUL POWER IN BIRDS PRACTICING PROPULSIVE FLIGHT

As in the case of flapping flight, this well-defined conception of propulsive flight permits one to deduce a clear expression of the useful power represented in horizontal flight. Knowing the resistance to the air, the useful power consumed, as in an airplane whose flight is strictly comparable to the above, is simply the product of F_x by the speed, as

$$P_u = F_x V = \frac{W}{L/D} V$$

In the condor, whose mean speed is about 15 meters per second (50 ft. per sec.) $P_u = 0.5 \times 15 = 75$ kg-m. per sec., that is to say, that this bird seems to weigh $\frac{15 \times 75}{75} = 150$ kg. (330 lb.) per useful hp., a normal enough value compared to that of other living creatures.

This phenomenon whose existence is indicated by the present knowledge of aerodynamics, which instantaneous photography and slow motion pictures reveal to perfection, which is proved by experiments as simple as that at Dakar, and which mathematics has shown to be practicable, may be observed in many different places, even in the flight of insects, for which it constitutes the essential mechanism, and in widely different species of birds. Some, specifically those which have the primary wing feathers separated, appear better adapted for it than those

provided with wings of continuous contour or not serrated. The latter, that is to say the majority of the flapping birds, from pigeon and the marine web-footed birds to the albatross, often have recourse to it on special occasions such as in take-off and particularly in landing, in order to increase their range of speeds.

The existence of propelled flight can no longer be doubted, and one can conclude that it exists in the nature of parachutes, gliders, and also naturally ornithopters but equally helicopters and airplanes.

Conclusions

The theory of natural flight as discussed under the several headings of gliding flight, flapping flight, and propelled flight, does not explain all the attitudes that birds adopt in the air. It describes the mechanics of flight and enumerates the essential phenomena, some of which are used to the exclusion of others by certain birds but which are all used by many birds, the processes succeeding each other according to the circumstances or necessities of the moment.

It appears certain that an exclusively gliding bird or soarer does not exist. There is none using only propelled flight or flapping flight. There are some birds better adapted than others to one or the other of these processes but that is all.

It is evident that all birds can glide, but the majority do not use this method except at rare intervals or in descending, because they are not particularly suited for gliding, lack sufficient L/D, or have too little supporting surface. Often they do not acquire the habit, because they do not live where atmospheric conditions are generally favorable to gliding. Species that live in favorable regions and have learned to lose but little height in a unit of time, use it as often as they are able, and the fact that they fly almost always with the wings extended, has earned them the name of soarers.

Birds with separated wing feathers generally show marked preference for propelled flight, while flapping flight is practiced in particular by birds with continuous contour to the wings. Small birds such as the swallow use the two methods alternately flapping to effect horizontal forward motion, and using helicopter flight to remain stationary, to alight or to rise vertically.

Birds of medium size, such as the pigeon and the sparrow hawk, might use either form, but the anatomy of their wings and feathers shows the effect of a preference in the pigeon toward flapping flight, and in the sparrow hawk toward propelled flight. The former in fact has no recourse to the properties of primary wing feathers except when taking off and alighting. In case of necessity usually a pigeon prefers to take off with a glide. The hawk, on the contrary, often uses propelled flight even when it is flying horizontally.

Finally, it should be noticed that flapping flight is that used by migratory birds that cover long distance, while the sedentary birds such as birds of prey, which never leave a certain region, prefer propelled flight. Birds of very large size having wing spreads of two or even three meters such as the eagle and condor, are so sensitive to the removal of the forward barbs of the primary wing feathers, that they cannot fly after an operation which has left intact the precious faculties of the flapping birds.

Only fliers not furnished with wing feathers use flapping flight as their sole mode of locomotion in the air. The albatross flies with the whole wing but by alternations, irregularly, with all movements voluntarily reduced to a minimum and on account of the increase of tangential speed along the wing span, apparent only toward the tips of the wings.

AER-51-36

The Metalclad Airship[1]

By CARL B. FRITSCHE,[2] DETROIT, MICH.

This paper sets forth the advantages to be obtained from the employment of all-metal construction in dirigibles, and discusses at length costs and other economic questions involved. Extended particulars are then given regarding the design, all-metal construction, erection, and assembly of the U. S. Navy Dirigible ZMC-2, together with data on dimensions, weight arrangement, shape and stress analysis, and performance, as well as conclusions on the test flights.

IT IS ALMOST impossible to present a description of the metalclad airship without comparing it with the well-developed Zeppelin art in rigid-airship construction. However, it should be clearly understood that where such comparison does occur in this presentation, it is resorted to in order to clarify the description and to stimulate engineering inquiry into the art as a whole. Therefore it is hoped that no one will misinterpret any technical comparison as indicating an attempt on the author's part to speak in disparaging fashion of the wonderful contribution the entire personnel of the Zeppelin organization has made to lighter-than-air development.

Rising again and again above disappointment and failure, which would have crushed the ordinary man, Count Ferdinand Zeppelin gave to the world the rigid airship. He and his associates have personified the indomitable spirit of progress. Obstacle with him spelled opportunity. His fine courage has been a constant inspiration to airship engineers and constructors in all countries. His spirit was truly that of the philosopher who is sure of his ground, unconquered and unconquerable. Disaster with him meant only temporary delay in progress. It did not stop progress.

Many of Count Zeppelin's associates and his co-adventurers in other countries have been the victims of tragedy, which always takes its toll along new frontiers of science. On each occasion the world at large has said that men have gone too far in daring exploration. But invention has always to convince skeptics. This was well illustrated in May (1929) when the *Graf Zeppelin* turned back to Toulon on its attempted second flight from Germany to America. Public comment generally was unfavorable. The fact that no lives were lost was attributed to luck. The airship was denounced most bitterly in some papers, while the airplane was pointed to with pride.

Unfair Criticism

Now newspaper editors, as a general rule, are men of scholarly attainment and possessed of exceptional power of analysis. Yet only a few on this occasion differentiated between the failure of four engines and the extraordinary performance of the *Graf Zeppelin* in landing safely without a scratch under the power of its one remaining engine. And still fewer newspapers commented on the tragedy that would inevitably have happened to an airplane if four out of five engines had failed in a storm over the Mediterranean.[3]

This illustrates how deceptive and inaccurate is the public's understanding of the safety of airships and their ability to fly independently of engine power in emergency. The burden of the proof, therefore, still rests with the designers, builders, and operators of airships.

Orderly Development

Progress is made in any line of endeavor, first, by retaining the best of the past; second, by developing the good of the present; third, by encouraging the development of that which may be better in the future. The metalclad airship belongs to the last category. In the middle classification belong the big fabric-covered airships such as the U. S. S. *Los Angeles*, the *Graf Zeppelin*, the British *R-100* and *R-101*, and the two large airships (*ZR-4* and *ZR-5*) now under construction for the Navy by the Goodyear-Zeppelin organization.

To put it another way, there are four major stages in any new scientific development: (1) Invention, (2) Engineering, (3) Production, and (4) Distribution. The metalclad airship is classified under the second stage, namely, engineering. The "Zeppelin" fabric-covered airship, on which, of course, engineering improvement is possible, has very nearly reached the third stage.

Airship Lines to Encircle Globe

Those who have had the good fortune to study the craft must be impressed with the immensely useful field in aerial transportation open to the dirigible airship. With its large weight-carrying capacity; its safety; its increasing speed; its long range; its improvement in efficiency with size; its comparative economy in operation and the fact that the air is "open" in all directions; it is no idle prophecy to visualize a network of airship lines encircling the globe, supplementing our present land and water transport facilities and penetrating remote regions of the world now little known to civilized man.

The sheer economy of this service will dictate its accomplishment. Think of a fuel cost per passenger for crossing the Atlantic in a super airship of only $20 and of a fuel cost per passenger for flying from New York to Chicago of only $5, or little more than that required for touring in a $4000 automobile.

Airships More Economical Than Airplanes

The commercial airplane and the flying boat have demonstrated their useful field of activity, but until much greater progress is made in their design and motive power, their useful radius of economic operation is limited to about 500 miles and 800 miles, respectively. On the other hand, the modern airship possesses

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[1] Author's Note. Building an airship is not a one-man job. Any comprehensive description of a new type, particularly, requires the cooperation of the key men responsible for its development. In the preparation of this paper the author desires to acknowledge his indebtedness to the following members of the Aircraft Development organization for their technical aid and for their personal contribution to this unique project: namely, Edward J. Hill, William E. Kepner, W. A. Klikoff, V. Pavlecka, S. A. U. Rasmussen, Arthur G. Schlosser, and Ralph H. Upson.

[2] Vice-President of the Detroit Aircraft Corporation, and President of its subsidiary, the Aircraft Development Corporation. Mem. A.S.M.E.

Presented at the Third National Meeting of the A.S.M.E. Aeronautic Division, St. Louis, Mo., May 27 to 29, 1929. Revised and brought up to date of September 20, 1929, after the *ZMC-2* had completed its test flights, and been delivered to Lakehurst, N. J.

[3] The most extraordinary achievement of Dr. Hugo Eckener and his associates, starting at Lakehurst, August 8, 1929, and encircling the globe in the *Graf Zeppelin* in 21 days, revives the old adage, "Seeing is believing." Editorial writers in all lands vied with each other, and rightfully so, in singing his praises. It should be recalled that out of nine attempts to fly the Atlantic in both British and German rigid airships, all but one have been successful, and on this one occasion, May, 1929, the *Graf Zeppelin* returned safely to its base in spite of serious engine trouble. Likewise, the one attempt to conquer the Pacific, by the *Graf Zeppelin* in August, 1929, was successful.

a radius of economic operation which exceeds the width of the Atlantic or the Pacific in distance. For example, let us compare the performance of a 57-ton flying boat with that of a 75-ton airship on a mail route between a California port and Hawaii, a distance of 2540 statute miles. In the light of present development, the flying boat would be able to transport about 1000 lb. of mail while the airship could transport 14,000 lb. The power requirement for the flying boat would be 6000 hp. and for the airship, 4000 hp. Flying non-stop in still air, the fuel cost per pound of mail transported in the flying boat would be about $1.85, while in the airship it would be only 10 cents for the entire distance. Amazing? Not at all when further examination reveals that the pay load in the flying boat is only $2^2/_3$ oz. for each horsepower as compared to $3^1/_2$ lb. in the airship.

Then why this enormous advantage in favor of the airship? Because searching investigation by leading scientists and qualified engineers proves conclusively that there are inherent limitations in the performance of heavier-than-air craft. It is a matter of common knowledge in engineering circles that the efficiency of an airship increases very rapidly with increase in volume, while in heavier-than-air craft there is little increase in efficiency with size, and in extremely larger sizes there may be an actual decrease.

Airship's Fuel Cost Low

It is realized that the cost of fuel represents in itself only a small portion of the cost of operation. However, for the purpose of simple comparison, fuel cost is a vital consideration for the reason that it is a direct index of the cost of power-plant operation, maintenance, replacement, and depreciation, which collectively, in long-range transport, represent at least 50 per cent of all operating charges.

Attention is invited to the fact that the 75-ton airship used in the above comparison has a displacement of only 2,500,000 cu. ft., which is small as airships go. On the other hand, the 57-ton flying boat, according to some authorities, seems to approach the economic limit in size, with little hope for improvement in performance except by resorting to the very questionable expedient of increasing the wing loading per unit of area, which in turn increases the landing speed and thereby reduces the operating safety factor. Of course, any improvement in the use of fuel or in the thermal efficiency of engines will be of value, but this same improvement will aid the rigid airship also.

Consider also that the field of safe activity of the airplane is limited to land and of the flying boat to water. The airship, however, can combine both land and sea in a voyage, and any large industrial center, no matter how far inland it may be located, may become a seaport, so to speak, and be connected directly with some other large industrial center that is inland in some other country.

Schwartz's Airship

These are some of the considerations which explain why a group of Detroit men, identified principally with the automotive industry, turned their attention as early as 1921 toward the possibility of constructing airships built entirely of metal, hoping thereby to develop a distinctly American type of dirigible applicable to both commercial and military uses. Roger Bacon had dreamed of this achievement in the thirteenth century, but it was not until the latter part of the nineteenth century that an Austrian named Schwartz actually built in Berlin a rigid airship covered with thin aluminum. This ship was 155 ft. long, had a maximum diameter of 44 ft. and a displacement of about 130,000 cu. ft. In its first flight (1897) the driving belts slipped off their pulleys on the crude engine, and the powerless craft, with one man on board, was carried by the wind to a forced landing in a forest. The ship was badly damaged and it was soon dismantled by curio seekers and the task passed on to succeeding generations having available better engines, stronger aluminum alloys, and a higher degree of technical skill.

Metalclad Construction the Obvious Step

Encouraged by the availability of dependable engines, of light, thin alloys, and of more seasoned engineering talent, the Aircraft Development Corporation was organized in 1922 to modernize the rigid airship, already a metal-framed structure since 1900, and make it all-metal like the hull of a seagoing vessel, which seemed an obviously desirable step.

For five years engineering research and study was carried on at a total expense exceeding $300,000. Trial-and-error methods disclosed that while the Zeppelin development was extremely valuable from the standpoint of the definition of technical problems, still the methods of solution were in most cases different. This is due to the fact that the entire theory of the accommodation of shear stresses in the metalclad ship is different from Zeppelin practice. In the metalclad the shear stresses, and to a large extent the tensile stresses, are carried in the metal hull covering, the plating of which also very definitely reinforces the internal frame members, thus reducing their design weight requirements. In the Zeppelin the shear stresses are carried through a complex system of diagonal wiring in the planes of the ship's surface. The fabric covering serves mainly to fair the surface and to resist the outer air pressure in flight. It may, under favorable conditions, carry minor shear stresses, but designers do not include it in their calculations.

In the course of the development work, hypothetical metal ships of different sizes were designed in considerable detail and a thorough stress analysis, involving even the integration of all forces as applied to each square foot of the hull surface, was worked out carefully. A large metal water model built to a 1:30 scale was tested at different angles of pitch to verify the theory of the design; of the main bending and shearing actions; and of the stress safety factors involved. So gradually, through years of painstaking and systematic effort, the general theory of the metalclad airship was mathematically and experimentally proved by an excellent and patient engineering staff.

Metalclad No Heavier than Fabric Construction

It was discovered that not only can an airship be built, possessing all of the obvious advantages of all-metal construction, but contrary to common belief, metalclad construction in the larger sizes does not add any considerable weight in excess over fabric construction. This surprising development is due largely to the fact that, as previously described, the shell or hull plating of the metal airship itself carries the tensile and shear stresses, and also reduces the internal framing necessary. To illustrate more fully, let us examine the actual physical structure of the two different types.

The surface of the fabric-covered ship consists of five layers, (exclusive of framing) from outside to inside: namely,

Outer fabric cover with doped coating
Shear wiring
Gas pressure wiring
Cord netting (eliminated in the *ZR-4* and *ZR-5*)
Gas-cell fabric lined with goldbeaters' skin.

These five layers in the fabric-covered rigid airship collectively perform the functions of:

Fairing the surface
Protection against atmosphere
Transmission of shear stresses

Accommodation of pressures
Retention of buoyant gas.

The single metal surface in the metalclad ship performs all these functions and performs each of them with less construction cost.

In the fabric ship the rigid frame members receive little substantial support from the fabric, due to its varying tension with change in weather conditions.

Metal Plating Carries Direct Stresses

The metalclad ship is a single structural unit in which the surface plating carries a considerable portion of the direct stresses. The metal plating, which is the outer cover, is the principal strength member and is also the gas container.

All-metal construction seems, therefore, to assure fulfilment of the world-wide demand for a commercial airship which is:

Fireproof
Weatherproof
Durable and permanent in structure
Navigable in practically all kinds of weather
Economical in the use of buoyant gas and ballast
Capable of high speed—up to at least 100 miles per hour, and
Limited in cruising range only by the expanse of the broadest ocean.

The *ZMC-2* Too Small for Commercial Use

No doubt all will understand that these ideal specifications do not apply to the first experimental unit, the *ZMC-2*, which is entirely too small to comply in detail with such high standards. In load capacity it is similar to the non-rigid fabric blimp, but in performance characteristics it is superior. In the magnitude of its importance as an engineering achievement, it is believed it equals that of any rigid airship ever built.

Its size of 200,000 cu. ft. was determined upon as sufficient to fulfil Government experimental requirements. Prior, however, to submitting a definite proposal to the Government, various systematic experiments were conducted.

Tests made in three different wind tunnels indicated that the new hull form and fin arrangement constitute an improvement in aerodynamic qualities.

The results of the large-size metal water-model test checked with our stress formulas and gave practical proof of the remarkable gain in strength obtained with the metalclad design.

Fins, girders, and other full-size parts were similarly built and tested. Tests made at the Bureau of Standards and in the corporation's own laboratory showed satisfactory resistance of the thin sheet and seams to vibration and diffusion.

The Automatic Seam-Riveting Machine

Paralleling the scientific work was the mechanical development and finally the completion and demonstration of a successful automatic riveting machine, a remarkable achievement and an economic necessity in the production of large-size metalclad airships.

This machine (Fig. 1) has successfully driven about 3,500,000 rivets of 0.035 in. diameter in the *ZMC-2*, with only one-third of one per cent of defective rivets. In its operation three strands of wire are fed like thread into the machine and three rows of rivets are "sewed" simultaneously. The machine shears off the wire in rivet lengths; these tiny wire lengths are punched through the two sheets of metal and revolving cams head them up into rivets.

The spacing of these rivets is also automatic. With this machine two men are able to accomplish as much work in a given time as 128 men working by hand. It inserts and completes 140 rivets per minute or about 40,000 rivets in an eight-hour shift. The cost per lineal foot of seam is about the same as the cost of sewing, cementing, and taping one foot of fabric seam. Thus we were prepared to answer the all-important question, "How are you going to join these thin sheets of plating together?"



Fig. 1 Metalclad Dirigible "ZMC-2" Taking Off on Her First Flight
(Grosse Isle Airport, August 19, 1929. Flight lasted 46 min.)

Congressional Appropriation for Building Experimental Unit

On the basis of the foregoing experimental work, in 1926 Congress was asked for an appropriation of $300,000 to be added to the Navy Bill for the construction of the first experimental unit. It was explained that this amount would be less than one-half the cost of the ship, which was estimated to be—including preliminary development—about $700,000. Actual experience to date indicates a total cost in excess of $750,000.

After many committee hearings, during which various objections were answered, Congress appropriated the money and the United States Navy awarded the corporation the contract to build the *ZMC-2*. It is true that this first unit, with a displacement of only 200,000 cu. ft., is too small in size to have any extended military or commercial value. It is not designed to accommodate passengers; it was not intended for that purpose. This small size was selected in order to prove the theory of all-metal construction with the least expenditure of capital, labor material, and time. On the experience thus gained it will be entirely feasible to base the design and undertake the construction of the larger and more useful sizes.

The *ZMC-2*, as the Navy has designated this first unit, made

its first trial flight August 19, 1929. It was delivered to Lakehurst on September 12, and on September 16 it completed successfully its final test, exceeding all contract specifications. It is of exceptional interest to relate that the actual finished construction weight permits a useful load of 127 lb. in excess of contract requirements. The original weight estimates were calculated in 1925. We believe this compares favorably with any experience in forecasting weight estimates of large airplanes. The general characteristics and performance data are given in Tables 1 and 2.

TABLE 1 GENERAL CHARACTERISTICS OF THE "ZMC-2" AIRSHIP

Length of hull	149 ft. 5 in.
Diameter of hull (max.)	52 ft. 8 in.
Fineness ratio	2.83
Displacement of hull	202,200 cu. ft.
Total ballonet displacement	50,600 cu. ft.
Front ballonet displacement	22,600 cu. ft.
Rear ballonet displacement	28,000 cu. ft.
Ratio of ballonet volume to hull volume	25 per cent
Thickness of skin	0.0095 in.
Length of car	24 ft.
Width of car	6 ft. 6 in.
Number of air valves	3
Number of gas valves	2
Number of fins	8
Total fin area	440 sq. ft.
Total elevator area	190 sq. ft.
Total rudder area	95 sq. ft.
Total automatic rudder area	95 sq. ft.
Engines (Wright Whirlwind J-5)	2
Power at 1800 r.p.m.	440 hp
Propeller diameter (all metal)	9 ft. 2 in.
Lineal feet of seam	17,600 ft.
Surface area	19,436 sq. ft.

TABLE 2 PERFORMANCE DATA OF THE "ZMC-2" AIRSHIP

Gross lift (100 per cent inflation with 92 per cent pure helium at 60 deg. fahr. and 29.92 in. Hg)		12,242 lb.
Weight empty		9,115 lb.
Useful load		3,127 lb.
Crew (three)	600 lb.	
Fuel (200 gal.)	1,200 lb.	
Oil (25 gal.)	200 lb.	
Ballast (50 gal.)	420 lb.	
Passengers and cargo	707 lb.	
Range with 250 gal. (cruising speed)		680 miles
Maximum possible range (still air)		1,000 miles
Maximum speed at 440 hp		62 ±2 m.p.h.
Cruising speed at 220 hp		50 ±2 m.p.h.
Static ceiling		9,000 ft.

A more detailed description of the *ZMC-2* structure is contained in Appendixes Nos. 1–5. Consideration is also given in these to tests performed on the structure; principles of the basic shape and of stress analysis; shop facilities and methods used; construction problems overcome; and method of inflation.

Assuming, from the experience thus gained from this initial unit, that we can proceed confidently to build the larger and more useful sizes, what use will they be commercially? How will they compare in economic performance with the large airplane or the super flying boat? This is an important question and deserving of consideration in this discussion.

With the experience in design, construction, and operation of the *ZMC-2* available, we have concluded that we can safely proceed with the design and construction of a metalclad airship of approximately 2,500,000 cu. ft. displacement (plus or minus) as the next succeeding unit in size. A ship of this size inflated with helium will have a gross lift of about 75 tons. Its general characteristics and performance are given in Table 3. It is designated *MC-25*.

Proposed 75-Ton Airship for the California-Hawaii Route

This ship would not be large enough for commercial operation across the Atlantic to Europe, but it would be practicable for operation on the shorter route from California to Hawaii, a distance of 2540 statute miles. This latter route is very desirable for any pioneering effort in commercial-airship transport for several reasons. First, it would save between four and five days of the time now required by Pacific steamships. Thus the first-class mail to Hawaii, which averages about 5000 lb. per week each way, would be greatly expedited. Furthermore meteorological conditions on the Pacific are much more favorable to the operation of smaller airships than they are on the Atlantic, for the simple reason that head winds are considerably less violent. Monthly pilot charts compiled from maritime records during the past 21 years indicate head winds with a force of 8 (Beaufort's scale, 39–46 m.p.h.) on an average of only twice per month between California and Hawaii. From the experience thus gained in operation and in the training of personnel, the development of larger and more useful airships applicable for continuous flight across both the Atlantic and the entire Pacific could follow in the natural course of events.

TABLE 3 GENERAL CHARACTERISTICS AND PERFORMANCE OF THE PROPOSED 75-TON METALCLAD AIRSHIP "MC-25" ON THE CALIFORNIA-HAWAII ROUTE

Displacement (actual gas-cell volume)	2,540,000 cu. ft.
Gas, helium 95 per cent pure, 60 deg. fahr., 29.92 in. Hg	62.5 lb. per 1000 cu. ft.
Lift, 95 per cent full	150,800 lb.
Weight empty	89,300 lb.
Speed, maximum	100 m.p.h.
Speed, cruising	70 m.p.h.
Horsepower, maximum	4,000 hp.
Horsepower, cruising	1,360 hp.
Range	2,540 statute miles = 2,200 nautical miles
Time required at cruising speed	36 hours
Fuel and oil consumption per hour (cruising)	750 lb.
Fuel required for range	27,000 lb.
50 per cent fuel margin	13,500 lb.
Total fuel	40,500 lb.
Mail and express	10,000 lb.
Passengers (10)	4,000 lb.
Crew (28)	7,000 lb.
Useful load, 40.7 per cent	61,500 lb.
Skin thickness	0.016 in.
Surface area	120,000 sq. ft.
Fineness ratio	4.5
Length	472 ft.
Diameter	105 ft.

In the construction of this larger ship, the same principle of utilizing the metal skin as outside cover, as the principal strength member, and as the gas container will be retained. The girders will be located so as to carry the loads in most advantageous manner. The interior structure will consist of deep transverse frames rigid enough to maintain the circular shape without any wire bracing. In addition, longitudinal members will be provided for supporting the skin and carrying the stresses, which the skin is not designed to transmit. The thickness of the skin will be determined as a function of dynamic and operating pressures and of the ship's dimensions. The frame dimensions will be determined by the external loads that they are required to carry.

A spacious corridor at the bottom of the hull will be provided the whole length of the ship. The passengers will be housed directly above the control and lounge cabin. The frames will be large enough so that all crews' quarters, fuel, cargo, and probably the engines, can be located inside. The outside appendages such as fins, control cars, and handling lines will be attached to the main frames.

Gas Compartments. The gas space will be subdivided by transverse fabric partitions into separate cells. The fabric partitions at the bottom of the cells will divide the hull into gas and air compartments. These partitions will be so arranged that the whole hull may be filled with either air or gas. This feature of cell arrangement will simplify the inflation and will take care of any possible deflation of a cell or a partial loss of gas in flight. It will also compensate for the expansion and contraction of the buoyant helium.

The hull will be maintained during flight under pressure by utilizing scoops providing the necessary internal pressure to counterbalance the dynamic pressure and by auxiliary blowers located in the ship and driven by separate engines. The hull will be designed to maintain its shape at slow speeds without internal pressure, so that it can be safely maneuvered in the air in case of temporary lack of pressure.

The water-recovery system developed by the U. S. Navy which condenses water vapor from exhaust engine gases can be very easily adapted to this metalclad ship. The metal plating will provide a perfect radiating surface so that for each pound of fuel consumed, one pound of water ballast will be recovered. This will effect a large saving in helium gas, which otherwise would have to be valved to compensate for the loss of fuel weight.

This ship will be constructed on the same principle as *ZMC-2*, by building it vertically in several sections and joining them together in horizontal position.

Cost. The estimated capital investment required for the building of two metalclad airships for the California-Hawaii route is set forth in Table 4. It is suggested that each airship would make one round trip per week. With a pay load of 14,000 lb. these two ships would transport 1456 tons or 2,912,000 lb. of pay cargo per annum.

Inasmuch as these ships would be inflated with helium, it is interesting and important to investigate the annual operating cost of helium for two ships flying on a schedule of one round trip per week each. Table 5 contains a study of these helium costs, based on the best available experience to date. The annual operating charge for helium on two ships, based on a cost of $40 per 1000 cu. ft., is $328,000. Surprising as it may seem, this is only about $50,000 more than the annual cost for fuel.

TABLE 4 CAPITAL INVESTMENT FOR AIRSHIP EQUIPMENT—CALIFORNIA-HAWAII ROUTE

2 metalclad airships (75 tons gross lift each) at $2,500,000	$5,000,000
Spare parts, reserve engines, gas cells, helium, etc.	500,000
1 hangar	1,750,000
2 mooring towers, fuel tanks, and utilities	400,000
Helium purification plant and gasometers	400,000
Repair shops	150,000
Crews' quarters and miscellaneous buildings	200,000
2 terminal buildings and office equipment	250,000
Land	250,000
Electrical equipment, radio stations, etc.	200,000
Unforeseen items	900,000
Working capital	1,000,000
Total capital investment required	$11,000,000

TABLE 5 HELIUM NECESSARY FOR ONE YEAR'S OPERATION OF TWO CALIFORNIA-HAWAII AIRSHIPS

(Based on experience with fabric-covered airships of 2,500,000 cu. ft. capacity)

Initial inflation of one 2,540,000-cu. ft. airship to 95 per cent of its displacement with 98 per cent pure helium (2,413,000 cu. ft.)	
The purity will drop to 94 per cent the first month with a loss of	96,520 cu. ft.
The loss will continue at the rate of 2½ per cent per month, or 60,325 cu. ft. per month, which for 11 months shows a loss of	663,575 cu. ft.
Repurification of the entire contents of each gas cell once per annum will result in a loss of about 9 per cent gross volume, or	217,170 cu. ft.
Total loss for one airship for one year's operation	977,265 cu. ft.
This constitutes a loss of 40½ per cent of the gross volume per annum.	
Under the proposed Pacific schedule each ship will make 104 trips per annum. Let us assume for the purposes of conservative estimate that 1 per cent of the volume of helium gas will be valved on each trip. This is contrary to experience, but this expense is included in order to provide against any emergency that might arise.	
104 per cent of the gross volume of helium (2,413,000 cu. ft.) would show a loss from this source per annum of	2,509,520 cu. ft.
Total helium required per annum per ship	3,486,785 cu.ft.
This constitutes 144½ per cent of the gross volume. At a cost of $40 per 1000 cu. ft., this shows an annual operating charge per ship for helium, of $139,471.	
For two ships operating on the Pacific schedule, the helium cost per annum would be	$280,000
To this must be added the cost of repurification of 2,413,000 cu. ft. for each ship. This is estimated at $10 per 1000 cu. ft. or $24,130 for one ship, and for two ships let us say	48,000
Total operating charge for helium per annum	$328,000

Economy of Operation as Compared With a 57-Ton Flying Boat

The economy in airship operation can be emphasized in no better way than through comparing the operating cost of this size of ship with the largest flying boat in the world. It was built in Germany.

This super flying boat has a gross lift of about 57 tons. It is equipped with engines developing 6000 hp., and its cruising speed is announced to be 115 m.p.h. Its useful load minus crew, according to its designer, is 56,815 lb. Assuming that the entire useful load capacity would be available for fuel and pay cargo, let us endeavor to ascertain how much pay load it would be able to transport on a route from California to Hawaii, a distance of 2540 miles.

At a speed of 115 m.p.h. it will require twenty-two hours for this boat to negotiate the voyage in still air. 2100 lb. of fuel will be required per hour at crusing speed. Twenty-two hours' flying will require 46,200 lb. Allowing for 20-mile head wind, which is the minimum allowance that would be safe to consider, would require additional fuel of 9615 lb. The total fuel required would then be 55,815 lb. Subtracting this figure from a useful load of 56,815 lb. would leave only 1000 lb. carrying capacity for pay load.

Assuming that each boat would make a round trip every four days, this would allow approximately 24 hours at the end of each voyage for adjustment and repairs. It would require 17 of these boats to transport the same weight of pay cargo per annum that two 75-ton rigid airships, previously described, could transport.

Power Ratio. The amount of power required to transport this pay cargo by flying boat is so extraordinarily large that it is ridiculous. Each boat is capable of carrying 1000 lb. of pay load. Each boat has engines developing 6000 hp. One horsepower is equivalent to 33,000 ft-lb. of energy per min. Applying this formula to the flying boat and utilizing 70 per cent of its gross power at cruising speed, this means that for each minute approximately 138,000,000 ft-lb. are developed. Inasmuch as the boat carries only 1000 lb. pay load, it is a matter of simple arithmetic to determine that for each pound of pay load transported, 138,000 ft-lb. of energy are required each minute.

Contrast this with the small amount of energy required for the operation of the rigid airship. Each airship carries 14,000 lb. pay cargo. Its engines generate 4000 hp. At crusing speed approximately 67,000,000 ft-lb. of energy are developed each minute. Therefore, for each pound of pay load transported, only 4800 ft-lb. of energy are required, or a power ratio of almost 29 to 1 in favor of the airship.

In order to carry an actual pay load equivalent to that of the two 75-ton metalclad airships, these seventeen 57-ton flying boats would have to make 2912 trips one way per annum, as compared with only 204 trips negotiated by the airships. The estimated capital investment for these flying boats, equipment, etc., is given in Table 6.

TABLE 6 CAPITAL INVESTMENT FOR FLYING-BOAT EQUIPMENT, CALIFORNIA-HAWAII ROUTE

17 flying boats (57 tons gross lift each) at $425,000	$7,225,000
Spare parts, reserve engines, etc.	1,500,000
Hangars, dry docks, and marine railways	600,000
2 repair shops	225,000
Docking facilities, fuel tanks, and utilities	500,000
2 terminal buildings and office equipment	250,000
Crews' quarters and miscellaneous buildings	200,000
Land and harbor frontage	250,000
Electrical equipment, radio stations, etc.	200,000
Unforeseen items	1,050,000
Working capital	1,000,000
Total investment required	$13,000,000

With the capital-investment estimates before us for the two types of craft performing the same volume of work per annum, it is important to compare their respective operating costs per pound of pay cargo carried. These estimated costs are not guaranteed as perfect. There is no authentic experience at this time on which to base a reasonably exact estimate. However, the author has had access to the related experience found in the actual

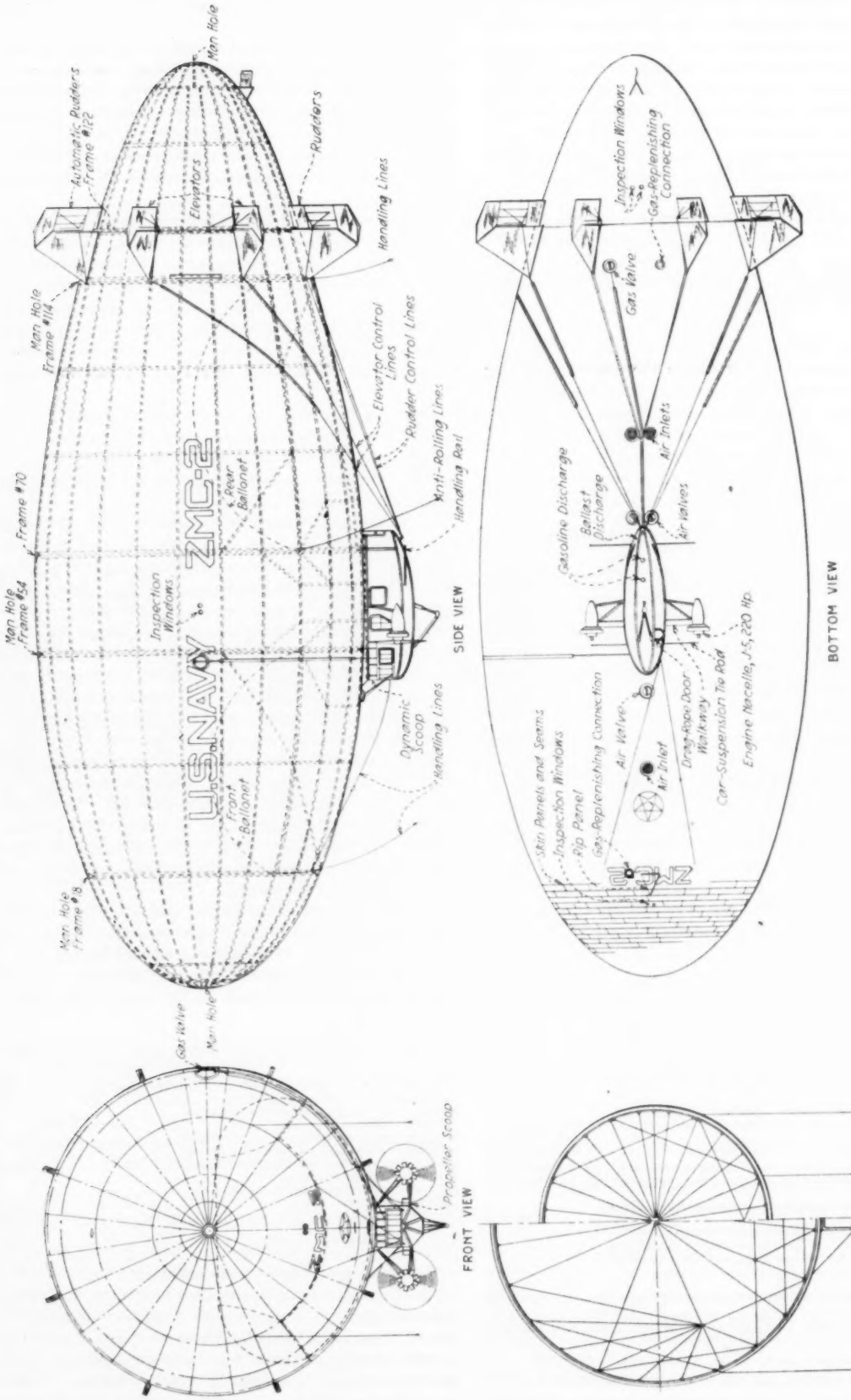
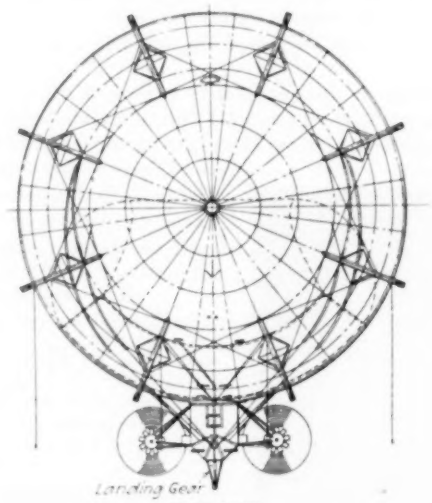


FIG. 2 SIDE, BOTTOM, AND FRONT VIEWS OF THE METALCLAD DIRIGIBLE "ZMC-2"
(For rear view, see Fig. 2a. For dimensions, other characteristics, and performance data, see Tables 1 and 2.)

operation of the larger and more important air-mail transport lines in the United States. Therefore no important or essential items have been overlooked and, while future experience may disclose that the individual figures are somewhat in error, still the totals are reasonably certain for purposes of comparison.



REAR VIEW

FIG. 2a REAR VIEW OF "ZMC-2"

Table 7 shows that the total estimated cost for carrying one pound of pay cargo in the flying boat from California to Hawaii is \$8.34, while in the airship it is only \$2.63.

TABLE 7 COMPARISON OF OPERATING COSTS ON THE CALIFORNIA-HAWAII ROUTE—17 FIFTY-SEVEN-TON FLYING BOATS VS. 2 SEVENTY-FIVE-TON METALCLAD AIRSHIPS

(All figures based on cost per pound, assuming that 100 per cent of pay cargo capacity is utilized. Total volume, 2,912,000 lb. pay cargo, per annum for both types of craft. Fuel consumption based on operation in still air.)

	Item	Cost per lb. Flying boat	Cost per lb. Airship
1	Executive expense	\$0.03	\$0.03
2	Administration: general overhead	0.07	0.07
3	Traffic: expense of sale of service and handling of passengers	0.05	0.05
4	Transportation, including crews' pay, gasoline, oil (and helium for airship)	2.62	0.43
5	Operation of terminals, including taxes, labor, salaries, etc.	0.10	0.10
6	Maintenance of equipment, including aircraft, engines, motor cars, tools, etc.	1.03	0.26
7	Communication, including teletype, radio, etc.	0.14	0.07
8	Insurance, including flying personnel, passengers, and equipment	0.26	0.17
9	Depreciation of flying equipment (aircraft 20 per cent per annum and engines on basis of 1000 hr.)	3.00	0.50
10	Depreciation of buildings and terminals	0.03	0.07
11	Unforeseen items	0.12	0.12
12	Profit, assumed, because of pioneering hazard, at 20 per cent on total capital investment	0.89	0.76
	Total cost per lb.	\$8.34	\$2.63

In analyzing Table 7 it is apparent that cost items of lesser importance for the two types of craft are reasonably close together. However, in the major items there is a wide difference as is illustrated in the cost of transportation, which includes crews' pay, fuel, and, for the airship, the additional cost of helium. In order to carry the assumed pay cargo, the flying boats would consume \$5,380,000 in fuel per annum. The airships would consume only \$275,000 in fuel plus the additional cost for helium amounting to \$328,000. In other words, by spending \$328,000 for helium, the airships save \$5,105,000 in fuel. Subtracting the helium cost from this latter figure we have an indicated net saving of \$4,777,000 in fuel. Consequently each dollar spent on helium saves about \$14.50 in operating charges for fuel.

Tariff Charge per Pound. It is recognized, of course, that no vehicle engaged in commercial transport consistently carries a 100 per cent load. Therefore, for the purpose of estimating a fair and minimum price per pound to charge for such service, it is assumed that the average cargo for each type of craft will approximate 75 per cent of its pay-load capacity. On this basis the rate per pound for the flying boat would be \$11.12, and for the airship it would be \$3.50. The ratio is 3.2:1 in favor of the airship.

It is important to emphasize that this differential in cost between the two types of craft does not begin to disclose the economic advantage of the airship over the flying boat, for the simple reason that the flying boat herein described, according to the conclusion of eminent authorities, seems to approach its economic limit in size. On the other hand, the 75-ton airship is relatively small as airships go, and if a super airship of 300 tons gross lift were employed, the cost per pound of cargo would be reduced to approximately \$1.25. Therefore, in comparing the super flying boat with the super airship, the differential in tariff in favor of the airship has a ratio of 9 to 1. It is believed also that approximately the same ratio in favor of the airship will prevail in the operation of any modern dirigible of equal size.

The Airship's Fuel Reserve Greater. Furthermore it should be emphasized that the airship carries a 50 per cent fuel reserve

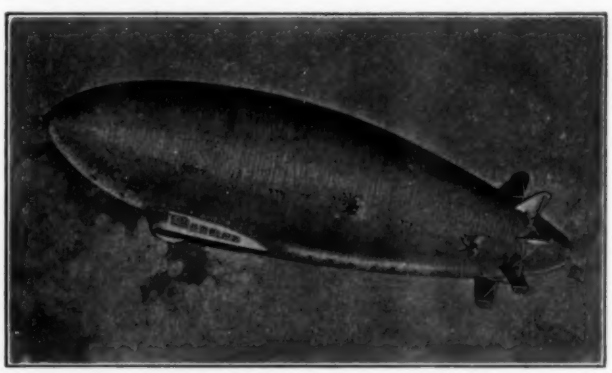


FIG. 3 PROPOSED METALCLAD AIRSHIP "MC-25"
(Gross lift, 75 to 90 tons; helium inflated; speed, 100 m.p.h.)

against head winds, which conforms to standard practice in the older forms of transportation. On the other hand, the flying boat used in the comparison carries only 21 per cent fuel reserve. If it should be loaded with a 50 per cent fuel reserve, not only would it be devoid of pay cargo, but it is doubtful if it would be able to get off the water under the most favorable conditions.

CONCLUSION

It is true that the present paper is intended primarily to describe the metalclad airship. However, in discussing the commercial utilization of this type of craft it is possible to do so only by comparing its performance with something with which the general public already has an intimate acquaintanceship. Therefore the flying boat has been used as a yardstick for this purpose. It is clearly realized that both the large flying boat and the large airplane possess practical advantages on shorter routes over which it would not be economical for the large airship to operate. Consequently this comparison should not be interpreted as an attempt to discourage the development of heavier-than-air craft but rather to designate the range of activity in which they can operate most efficiently. It is very clear, therefore, in the face of this staggering array of evidence, that the conquest of the Atlantic and Pacific oceans by commercial aircraft lies within the province of the large dirigible more than in that of the airplane or the flying boat.

Now, the question will be asked, "If all these advantages in favor of the airship are true, why has not its development progressed more rapidly?" This question to some is not easily answered.

In the first place, prior to the World War, practically all rigid-airship development was confined to Germany. At the conclusion of the war the Versailles Treaty halted this excellent development, and for a period of about six years this highly specialized talent remained dormant.

In the second place, a world-wide survey of aircraft activities reveals that for each $50 spent on airplane development only $1 has been spent on airship development. In the United States for each $100 spent on airplane development only $1 has been spent on rigid airships.

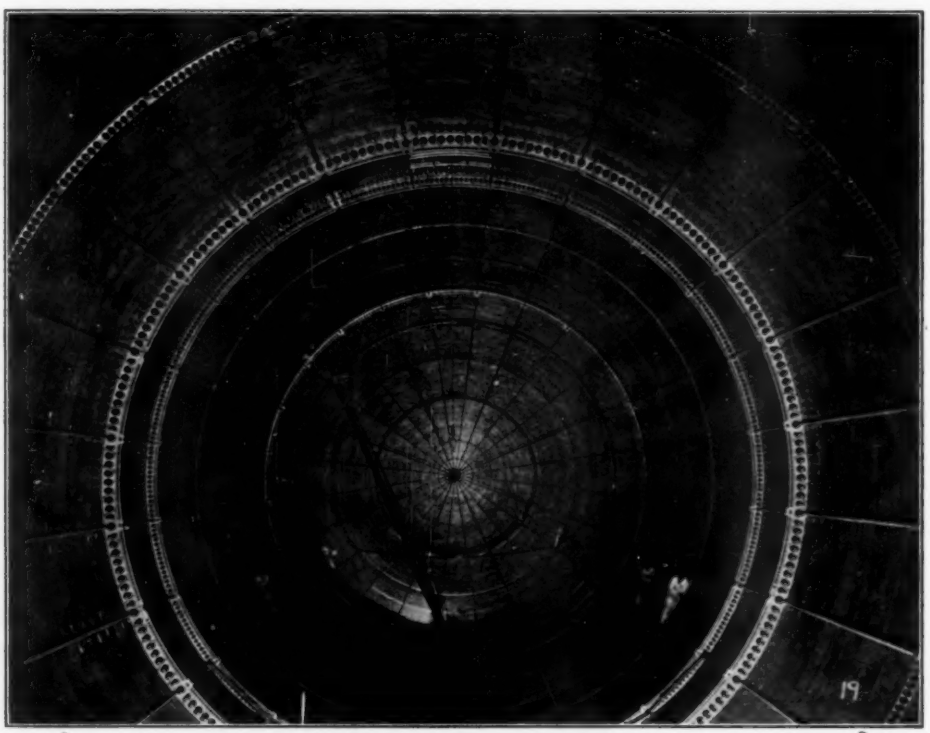


FIG. 4 INTERIOR VIEW OF THE "ZMC-2"
(Note men at work.)

In the third place, when one attempts to develop the rigid airship he must think in terms of millions of dollars instead of thousands for a new type of airplane. His effort must cover a period of years instead of a period of a few months.

In the fourth place, each disaster which has occurred in the experimental development of airships has caused a decided drawback in so far as securing additional funds for prosecuting the work at hand was concerned. The cumulative effect of these disasters has been more serious to airship development than have the most tragic airplane disasters to the development of heavier-than-air craft. This is due to the fact that the enormous size of the airship so captures the public imagination that the influence of any accident is multiplied accordingly.

Finally, the lack of adequate Government support in the past has retarded the development of the airship. However, the early enactment of federal legislation providing for the award of exclusive long-term contracts to commercial-airship companies for the transport of ocean mail at a price that will yield a fair return on the investment, will give the same encouragement to airship development that the air-mail contract act of 1925, as applied to transcontinental service, has given to airplane development.

Coupled with this legislation it would be entirely justifiable to amend the Jones-White Bill, extending its provisions to commercial airships so that the United States Shipping Board will be authorized to loan up to 75 per cent of the construction cost of new commercial airships on the same basis as it is now authorized to loan funds for the purpose of the upbuilding of our merchant marine.

Either phase of this program of legislation will give private capital the courage to go ahead. The full enactment of this program will place the United States in the forefront of all nations probably for generations to come. The goal ahead is the capture of the international trade routes.

TIME DEVOTED TO DEVELOPMENT OF METAL-CLAD AIRSHIP

The equivalent of 65 man-years has been spent by the engineering staff of the Aircraft Development Corporation in solving the problems involved in the design of the *ZMC-2* metalclad airship. The approximate details follow:

	Problems	Man-hours
1	Development of basic theories of stress distribution in metal hull	12,600
2	Extensive original researches made in aerodynamics and static theory of airships	6,090
3	Aid in development of the successful rolling of wide, thin duralumin sheet for hull	2,320
4	Development of suitable practical internal rigid structure of metal-airship hull	2,540
5	Fabrication and testing of many types of lattice and rolled girders	1,780
6	Development of semi-portable automatic riveting machine which sews plating of hull together	9,430
7	Devising practical methods of erecting and inflating metal airships	4,810
8	Development of several different successful sealing compounds for making seams gastight	2,720
9	Development and testing of protective coatings against weathering and corrosion	1,930
10	Fabrication and load testing of fins and various control mechanisms	2,450
11	Construction and testing of first large metal-airship water model	4,310
12	Development of hand method of riveting thin sheet metal	2,400
13	Mooring methods studied and developed	5,390
14	Vibration tests	1,580
15	Development and construction of machine for laying out large radii	800
16	Lightning and fire tests made	590
17	Design and test of all-metal car	7,040
18	Study and development of ballonet and methods of attachment	3,360
19	Seam design	810
20	Detailed design of structure including shop drawings	43,200
21	Building and testing assembled structure	19,400
22	Design and testing inflation methods and equipment	12,300
23	General	10,000
	Total time (man-hours)	157,850

The total given above does not include time devoted to problems by outside cooperating organizations, nor by the Corporation's very able Technical Committee, Messrs. Alex Dow, C. F. Kettering, W. B. Mayo, and Warren Noble, whose seasoned judgment and pioneering faith contributed so much to the success of the enterprise.

Appendix No. 1

DESCRIPTION OF THE "ZMC-2"[4]

The Hull

THE *ZMC-2* hull is 149 ft. 5 in. long from bow to stern, and 52 ft. 8 in. in maximum diameter. Its covering consists of 0.0095-in. thin Alclad alloy sheets sewed together by the Aircraft Development Corporation riveting machine. Each of the rings of sheet metal composing the hull surface has the shape of the frustum of a cone; together the cones closely approximate the curvature of the hull. There are 142 such conical surfaces from bow to stern.

The skin contains lifting gas, consequently the seams and rivets have to be gasproof. This is achieved by a gas-sealing compound which is applied on the seams, is drawn in by capillary action, and seals the lap seam.

Inside the hull the skin is held firmly by rivets to a supporting structure built up of transverse frames and longitudinals. There

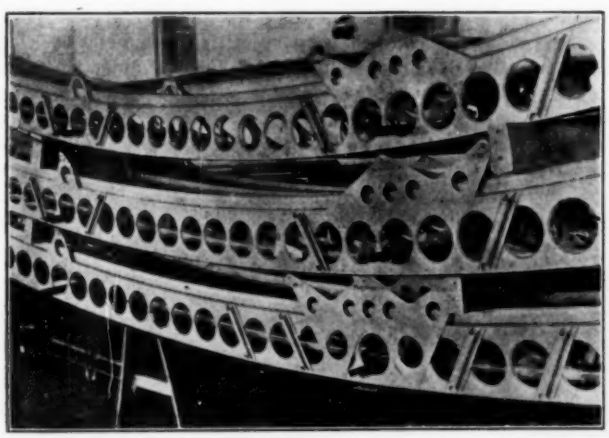


FIG. 5 GIRDERS READY TO BE ASSEMBLED AS PART OF FRAMEWORK

are 24 equally spaced longitudinals running from the bow to frame No. 145, and 16 from this frame to the stern. In the bow and the stern is a rigid ring from which the longitudinals radiate. Incidentally, these rings serve also the purpose of manhole frames, as will be mentioned later.

The cross-section of the longitudinals is of the "capital omega" type (Ω). The flanges of the section are attached to the skin by rivets. Two sizes of sections are used. In the extreme bow and stern, where the longitudinals are crowded, the section is only 1 in. high, while the rest of the hull has longitudinals 2 in. high. The latter are lightened and strengthened by flanged holes.

The thickness of longitudinals varies from 0.014 in. to 0.032 in. according to the intensity of compressive loads in any particular portion of the hull.

Transverse Frames

There are altogether 12 transverse frames, numbered according to their distance from the bow of the hull. Of these, frames Nos. 18, 54, 70, 114, and 122 are of the heavy built-up type, the remainder being of simple triangular cross-section.

[4] Prepared by Vladimir Pavlecka and S. A. U. Rasmussen, Detroit Aircraft Corporation, Detroit, Mich.

The main frames are also of a substantially triangular cross-section, having all three apexes formed by channels and mutually connected by light web plates and a base plate. Flanged holes are used liberally in the channels, webs, and base plates. In cross-section the frames are 8 in. high on center line and from 8 to 10 in. wide at the base, depending on the base slant, which corresponds to the angle of the skin at the location of the frame. The

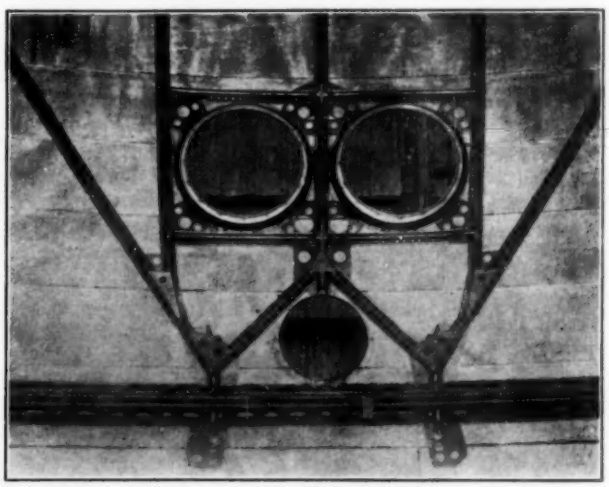


FIG. 6 REINFORCING AROUND VALVE SEATS IN HULL

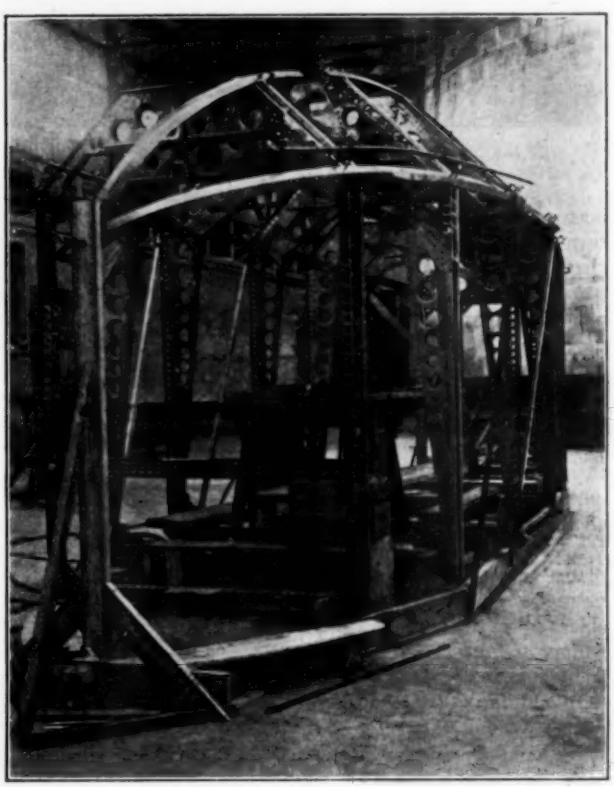


FIG. 7 FRAMEWORK OF CAR

channels are 0.025 in. in thickness, the webs, 0.020 in., and the base plate, 0.014 in.

Each frame is built up of a number of bowed sectors spliced end to end. The main frames take primarily all concentrated loads,

such as landing and handling lines, weight of the car, fin loads, etc. They are diagonally wired by hard-drawn aircraft wire and tie rods. Wires are spliced to eyebolts screwed into Lynite cast-

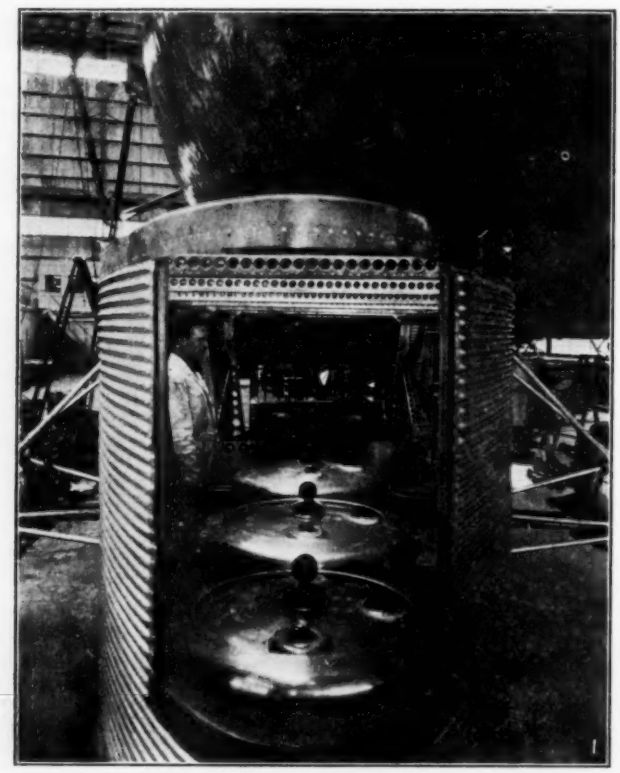


Fig. 8 Interior View of Car

ings as filler blocks, which in turn are anchored in the apex channel of the frames. Gusset plates or, as in the fin attachments, tubular spacers are employed to transmit the external loads from the base of the frames to the apex.

The outer bolts, on which the outside loads are applied, pass through heavy reinforcing plates on the inside of the hull skin; in addition, cast bearing blocks hold the bolts on these base plates against tangential loads. Gastightness at such joints is achieved by a gasket of canton flannel treated with the same compound as the seams.

In addition to the load wiring, frames 54 and 70 have, in their lower halves, auxiliary wiring to take the loads imposed by the ballonet diaphragm.

The frames are continuous, while the longitudinals stop at the frames and are connected through the frames by special straddling splices of formed sheet. In addition to the main longitudinal and transverse structure described above, there are rigid diagonals at places of considerable shear stresses as on the side of the hull, and auxiliary ribs in locations where stiffness is desired, as on the bottom of the hull. All these reinforcements employ a 2-in. channel section, and all are rigidly connected to the rest of the structure by means of base plates and splices.

The average spacing of rivets attaching the skin to the structure is 2 in. Flat-headed rivets of $^3/_{32}$ in. diameter with round edge are used, which eliminates cutting of the skin by the sharp edge of customary round-head rivets. All rivets are of 17ST aluminum alloy.

The bow and stern are fitted with manholes, covered normally with a metal shield plate, screwed down. Similar manholes are placed at all main frames on top of the hull, to facilitate entrance into the hull for adjustment or repairs. These manholes, 5 in all, are provided with a fabric seal against leakage. The rear ballonet can be entered by a similar manhole in the bottom of the hull.

Valves

The hull is equipped with 5 valves, automatically as well as hand operated, of the Gammeter type, 20 in. in diameter and having a 4-in. water column maximum blow-off pressure. Two of the valves are for gas and three for air, the latter being at the bottom of the hull. One of the gas valves is on the port side of the hull and the other in the stern at the bottom of the hull.

Two gas-replenishing connections of 15 in. diameter each are built into the bottom of the hull, one in the bow, the other in the stern. All of these holes and valve seats require secondary structure for local reinforcement, and 2-in. channel is used for this purpose.

The largest unsupported area of the hull skin occurs at the maximum diameter and is 6 ft. wide by 15 ft. long.

Three pairs of glass inspection windows are located in the bow, port side, and the stern of the ship, respectively.

On the underside of the bow is a triangular rip panel, which can be torn open by the pilot in an emergency; it operates on the principle of shearing the skin against the edges of two thick strips.

Control cables to the gas valves run in Bowden casings, securely



Fig. 9 Side View of Car and Motor Mount

cemented to the outside surface of the hull. The air resistance of this type of controls is negligible.

Handling Lines

The *ZMC-2* is equipped with four sets of steel-cable handling lines; each set has two lines, attached to the hull symmetrically to the ship's center line. Each of these lines is designed for 5600 lb. ultimate strength.

One set, on the bow, is intended for landing the ship. Two sets, one on the bow and the other on the stern, are intended for handling the ship on the ground. The last set, located amidships, is designed for holding the ship against rolling when on the ground.

The landing lines are each 40 ft. long, with cotton-rope extensions. In addition, a third landing rope is carried; this is a $^3/_4$-in.-diameter manila, 250 ft. long, which can also be used as a drag rope. This rope is attached to the bottom of the control car and can be dropped at the pilot's will. By a quick-release device it may be swung to a bridle on frame 18 of the hull.

The Car

In plan view the car has a smooth curve, which provides most floor space for the general car dimensions. In cross-sectional view the bottom is a semi-octagon with its vertical sides extended up to the top of the car. The car is 24 ft. long and 6.5 ft. wide at maximum section. With scoops, tanks, controls, radio, drag rope, blower, and a chart table installed inside, it still is quite roomy for two pilots, one mechanic, and about four student pilots.

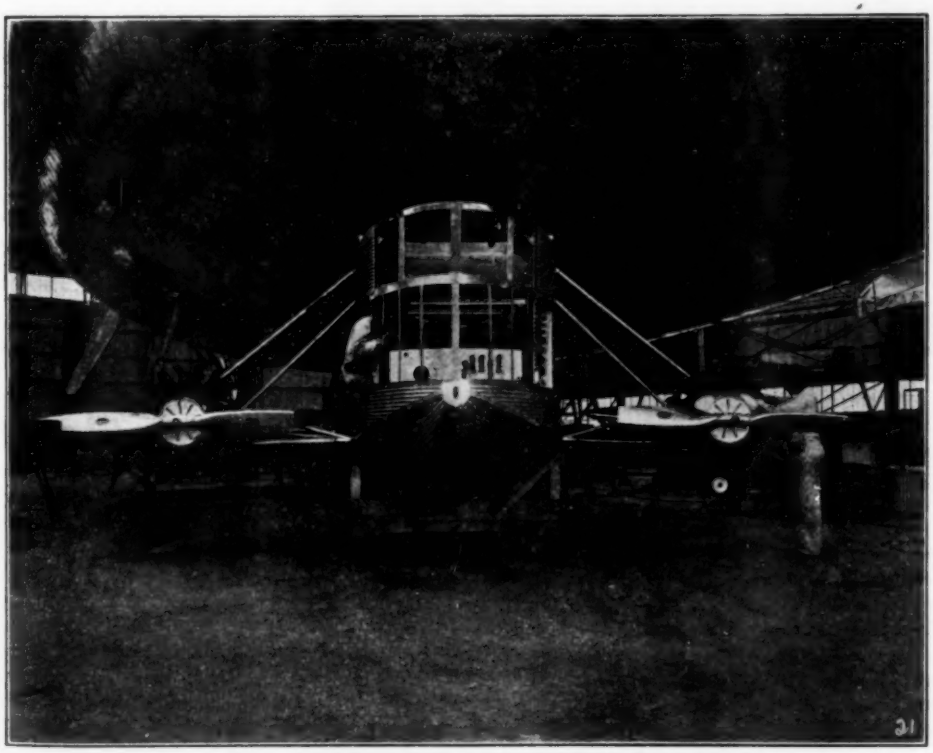


Fig. 10 End View of Car Showing Mounting of the Two 220-Hp. Wright "Whirlwind" J-5 Engines

The structure of the car consists of eight transverse frames, a number of longitudinals, and diagonal braces. It is a bridge-like structure and has been analyzed as such.

The covering is of 0.014-in. corrugated Alclad sheet. The chines, 4 in all, are very rigid, and together with the plating add considerably to the strength of the structure.

All frames are box girders, trussed with inside channels, and each frame is designed for the particular loads it takes. The heaviest frame weighs 33 lb., and the highest load occurring in it is 17,000 lb. compression.

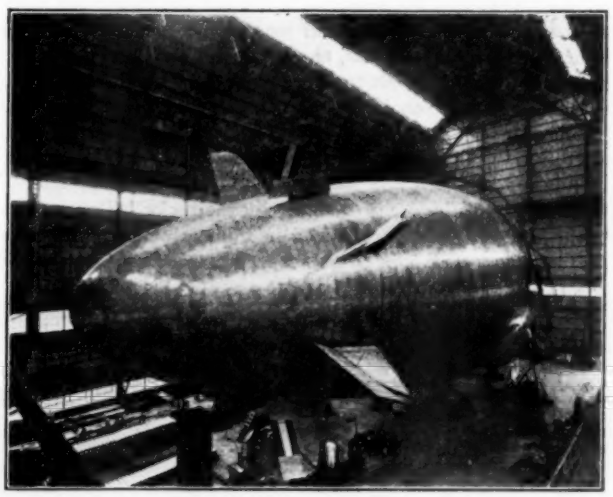


Fig. 11 Showing Series of Control Fins, Embodying a New Feature in Airship Design

The basic rolled section used in the car is a U-channel with flanges, 2 in. wide and $2^1/_2$ in. high. Flanged holes are used profusely in all except the tension members.

The car is suspended to the hull by means of eight attachments; of these four are rigid joints and four streamline rods. Either of these two sets of joints is designed to carry the car independently with all loads in it. Furthermore, the rigid joints are called upon to take the thrust of the power plants. The factor of safety of all joints is 6 on the highest load each joint might ever be required to carry.

The pilots sit side by side in the bow of the car, with all controls and instruments on panel in front of them. Directly above their heads is a compartment containing a dynamic scoop.

The altitude pilot, seated on the port side, can operate a radio set immediately behind him on a desk. The direction pilot, on the starboard side, can read maps on a chart table placed opposite the radio desk.

The engine outriggers are attached to the car aft of the pilots, with the plane of the propellers behind the pilots' seats. In the middle portion of the car are propeller scoops and vertical ducts, one on each side of the car. In this section also is the mechanic's post with a board of engine instruments and controls.

All engine-control rods are carried in the double bottom of the car and are easily accessible.

The pilots can control engine speed by their own set of throttles in addition to the mechanic's controls.

The entrance door is on the starboard side, behind the propeller scoops. Opposite the entrance is a hand air blower for maintaining the hull pressure in an emergency. Besides the main door there are four hatches, one for each pilot and two for the mechanic, to gain access to the engines during flight. A walkway is built on the engine outriggers to connect the car with the nacelles.

Gasoline Tanks on Special Cradles

The stern of the car is occupied by the tanks. Two gasoline tanks, 100 gal. each, are placed in tandem on special cradles. The rearmost tank is of 50 gal. capacity, to be used as a water-ballast container. It can, however, be connected to the fuel system and used as a gasoline tank for long flights.

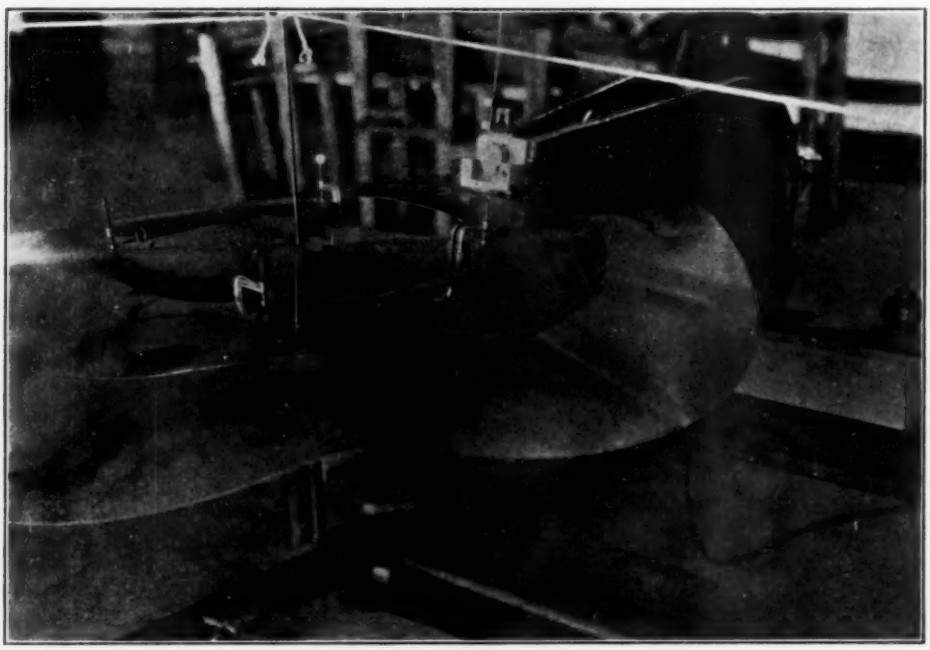


Fig. 12 The First Step in Building the Metalclad Dirigible "ZMC-2"—Riveting the Nose Plates

All tanks have dump valves.

Special balsa plywood flooring is laid in the car and fastened solidly to the structure, except in a few places where it is removable to gain access to control rods and cables.

Electric light is supplied by a 12-volt storage battery.

Power Plants

The engines are carried on tubular outriggers, one on each side of the car. Wright "Whirlwind" J-5-A engines are used, developing a maximum of about 220 hp. each. They operate tractor metal propellers. Each power plant has independent fuel, oil, and ignition systems and controls.

The oil tank, with a glass volume gage, is located inside the streamline nacelle behind each engine. Oil pressure and temperature gages are on the mechanic's board inside the car. All controls, tubes, and ducts to the engines are inside a walkway leading to each engine. A push-and-pull system is used for engine controls.

Each outrigger can be detached from the car by removing four bolts and disconnecting the instrument leads, pipes, and controls from the car. The engines have inertia starters, hand operated from inside the car through the hatches.

The Landing Gear

A bumper bag for taking landing shocks was out of the question in the case of the *ZMC-2*, due to the low position of the engines and excessive height of the car above the ground.

The landing gear is an inverted tripod with one leg as a shock absorber of the Oleo type; the other two legs are streamline duralumin tubes. The impacts are received by a streamline hollow steel shoe at the apex of the tripod.

The streamline struts are calculated for a factor of safety of only 2. In case of a severe shock these struts are expected to collapse, and in doing so check the inertia of the hull, thus eliminating the danger of damaging the car. This appears to be the only practical solution of the problem of protecting the car from ground impact.

The Ballonets

Two ballonets, one of 22,600 cu. ft. and the other of 28,000 cu. ft. capacity, are located between frames 18 and 54, and 70 and 114, respectively. These ballonets are made as complete fabric bags laced to the above-mentioned main frames and also to the longitudinals. The bags are so designed that practically all loads due to surging and to pressure are transmitted to the main frames.

The ballonets are made of two-ply rubberized fabric weighing 9 oz. per sq. yd. The seams are palled $^3/_4$ in.—cemented and taped but not sewed. The ends and sides of the ballonets are attached to the main frames and longitudinals by lacing strips with grommets. The bottoms are tied to the intermediate crossings of longitudinals and frames in order to prevent the possibility of gas getting under the ballonet and lifting it while surging. Around the various openings leading into the ballonets through the bottom of the hull, the ballonets are cemented to the hull with "Vulcalock."

Ballonets Filled by Air Scoops

Air is supplied to the ballonets through a scoop located in the forward end of the car and two propeller scoops. For ordinary conditions of level flying the car scoop should be sufficient, as it automatically increases the air pressure in the ballonets with the increase of the speed of the ship. The scoops are so controlled that air can be let into either ballonet independently or into both ballonets at the same time. An auxiliary hand blower with a maximum capacity of 2200 cu. ft. per min. is provided in the car. This blower should take care of a descent of 250 ft. per min.

One automatic and hand-operated air valve and one automatic emergency air inlet are provided for the forward ballonet and two of each for the rear ballonet. The air valves are standard 20-in.

Gammeter airship valves. The air inlets are inward-opening fabric flaps. To prevent leakage they are sealed with easily ripped fabric covers.

A rip panel is located in the rear end of the forward ballonet and another in the forward end of the rear ballonet, to open ballonets into the gas space.

Fins

A group of eight fins and control surfaces is located at frame No. 122, all surfaces being equally spaced around the hull. The

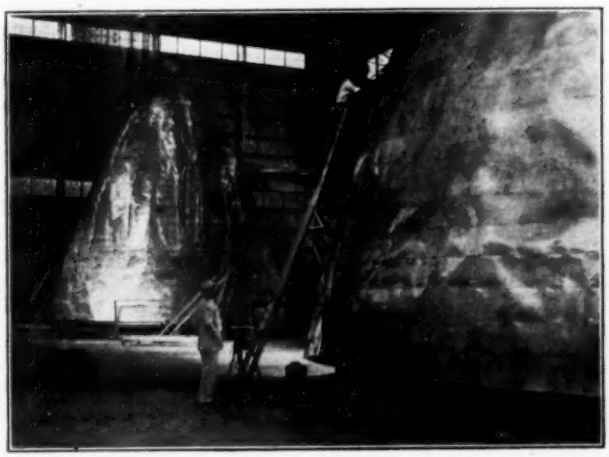


Fig. 13 Partially Completed Sections of "ZMC-2" During Construction in Hangar at Grosse Isle Airport

control surfaces consist of four elevators, two service rudders at the bottom, and two fins. The latter two are normally provided with means of locking rudder and fin together to form a fixed surface, or may be arranged as automatic rudders. All surfaces are mutually interchangeable by means of minor modifications. The elevators are limited to approximately 20 deg. movement, and the lower rudder to 30 deg. each way, by means of stops on the control cables in the car. All movable surfaces are counterbalanced. Each fin is externally braced by a pair of streamline wires.

The fin and control-surface frames are made of aluminum alloy and covered with corrugated Alclad of an average thickness of 0.008 in.

Controls

The rudders as well as the elevators are operated by wheels. To move the elevators from extreme up to extreme down position requires two complete turns of the wheel. The extreme movement of the rudders requires three complete turns. The control cables are brought from the wheels in the forward end of the car beneath the floor to the rear end of the car in such a way that they are easily accessible for inspection. From an outside "pulley bridge" at the rear end of the car the elevator cables lead through Bowden casings to the elevators, while the rudder cables lead over pulleys at the leading edge of the fins direct to the rudders.

TABLE 8 GENERAL WEIGHT STATEMENT, "ZMC-2" AIRSHIP

	Pounds	Pounds
Hull		4,625
Frames	965	
Longitudinals	420	
Skin	2,700	
Seam rivets	30	
Seam dope	70	
Closing seam	30	
Ribs and diagonals	55	
Attachments	355	
Ballonets		780
Fin group		650
Car		1,310
Power plant		1,750
Total weight empty		9,115
Useful load		3,127
Crew	600	
Fuel	1,200	
Oil	200	
Ballast	420	
Cargo and passengers	707	
Gross lift at 92 per cent pure helium at 60 deg. fahr. and 29.92 in. Hg		12,242

The rudder cables from the two rudders are joined together just abaft the car and carried through to the control wheel as a single cable. At the point where they are joined an auxiliary hand line is attached which enables the controls to be operated in case of breakage of cables in the car. As the elevators and rudders on each side of the ship are operated independently (the independent elevator cables being carried right through to the control wheel),

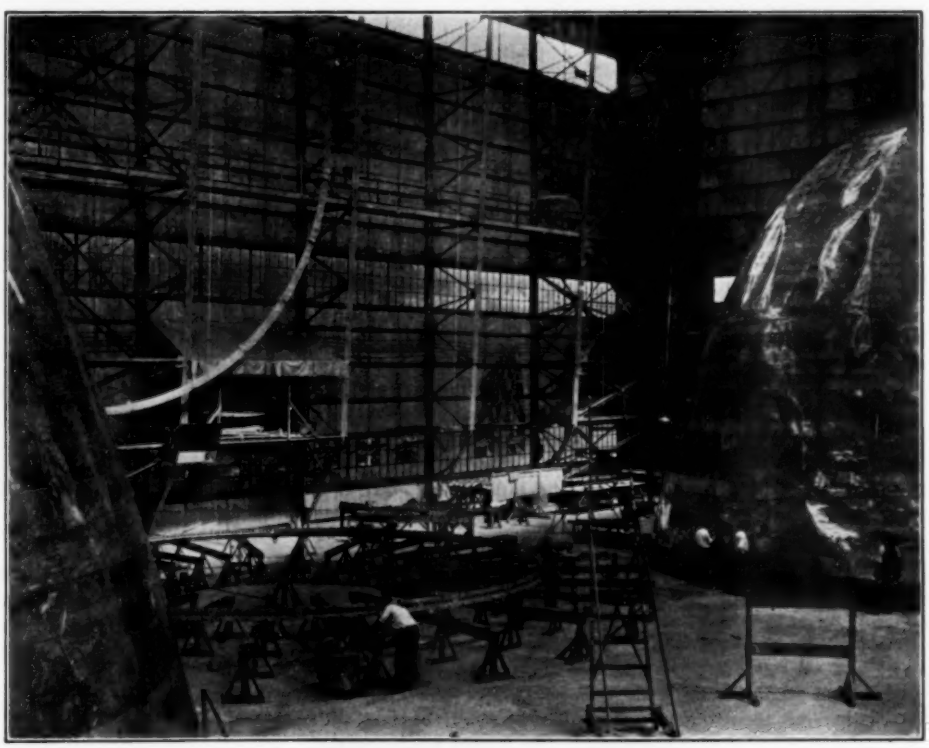


Fig. 14 Assembly of Main Frame Member of "ZMC-2"

the breakage of the cables on one side still allows the ship to be controlled by the surfaces on the other side. The control cables throughout are $^1/_8$-in. diameter, 7 × 19 extra flexible steel cable.

Appendix No. 2

BASIC SHAPE AND STRESS ANALYSIS[5]

THE hull is a double curved surface of revolution based on the geometrical *EH* curve, composed of elliptical and hyperbolic surfaces.

All characteristics of the curve are expressed as functions of

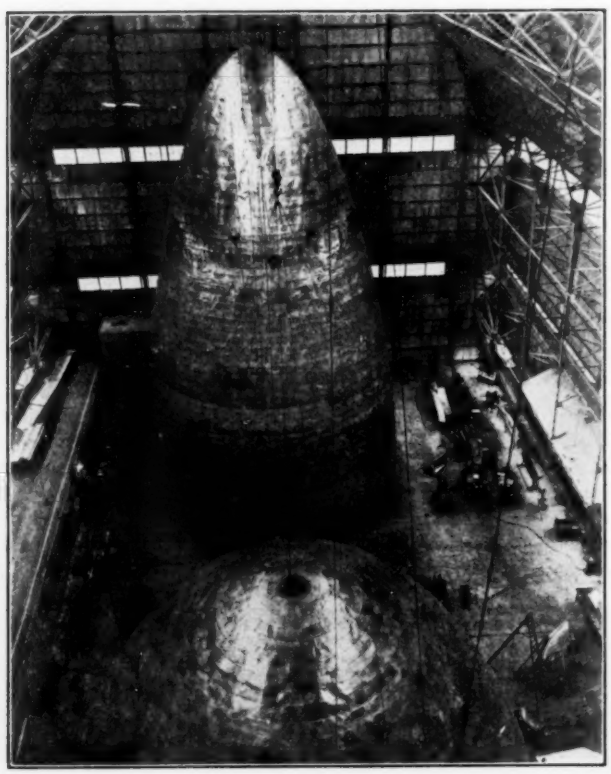


FIG. 15 SHOWING VERTICAL METHODS OF ERECTION USED DURING CONSTRUCTION OF THE "ZMC-2"

the semi-major and semi-minor axes of the bow ellipsoid, and can be easily applied to any volume and fineness ratio.

The particular feature of this purely geometrical curve is that it facilitates the very accurate plotting of elements of the hull structure and allows the determination of aerodynamic loads on the hull, applying the hydrodynamic theory of pressure distribution to the ellipsoidal shapes. This distribution was found to approach very close to the actual condition in the bow, where the aerodynamic loads are most important.

The general load distribution on the hull of the *ZMC-2* is unique because major concentrated loads are carried first to circular transverse frames and from them by pure shear forces into the surface plating. Each of the main frames consists of a rigid continuous girder, riveted directly to the inside of the surface plating and cross-braced by wires in its own plane.

Each frame was analyzed by assuming the concentrated load carried by the frame to be balanced by the system of shear forces, acting tangentially to the hull. For simplicity these tangential forces were assumed to act at the intersection of the ring with internal wiring and the longitudinals. The frames usually carried a system of wiring which was redundant; to simplify the computations the wires which are most liable to be subjected to compressive loads were assumed as not acting, thus making the structure determinate. The maximum load found to be applied at the span of the girder between two wire terminals was taken as the design load for the girder and the girder designed accordingly. The portions of duralumin girders built of rolled shapes, and plates of light gages of the length equal to the arc of the circle between two wire terminals (usually $^1/_{24}$ of circumference) were tested, and finally the present girders were developed. The smallest factor of safety in these girders is 4.7 (disregarding the effect of gas pressure).

It can be shown mathematically that any cylindrical section of a flexible envelope supported in shear by circular bulkheads remains circular at all positive pressures without any tendency to distort, provided the envelope is full of gas and has a uniform weight per square foot. This fact was proved in the *ZMC-2* case by water-model tests. Only secondary forces tend to distort the section from its circular shape, and these are taken care of by the double curvature of the hull's surface and absence of straight elements.

Small longitudinal girders built of rolled sections 2 in. deep of very light gages were introduced to support the skin when the ship is deflated and to take care of compressive stresses occurring in the plating under extreme conditions. The longitudinals were neglected in calculations of general skin stresses, but if compressive loads were found in any direction in the skin, these compressive loads were then investigated and were assumed to be carried by the longitudinals. All these cases, such as bow ribs, suction on lower part of hull, and compressive loads were investigated separately.

EQUATION OF STRESSES

The calculation of stresses in the envelope is based on the

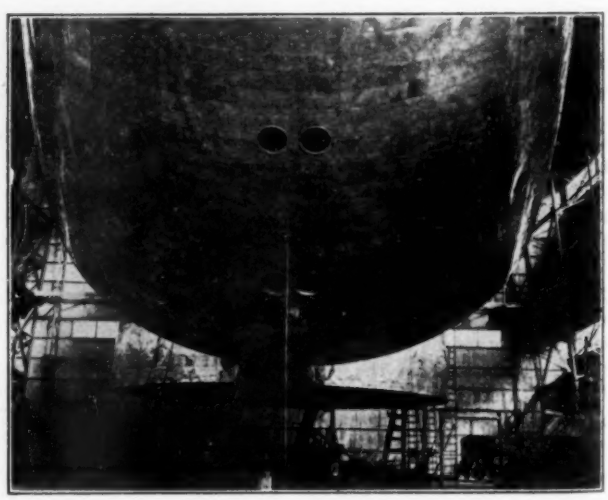


FIG. 16 PLACING HULL SECTION IN POSITION FOR JOINING

fundamental equation of stresses in containers of double curvature subjected to internal pressure, namely,

$$p = \frac{T}{R} + \frac{t}{r}$$

where p = pressure on any section, lb. per sq. ft.
T = longitudinal tension, lb. per lin. ft.
t = transverse (or hoop) tension, lb. per lin. ft.
R = longitudinal radius of curvature, ft.
r = transverse radius of curvature, ft.

This equation is indeterminate, containing two unknown tensions, T and t. Assuming the pure bending theory to apply,

[5] Prepared by W. A. Klikoff, Detroit Aircraft Corporation, Detroit, Mich.

T can be determined from bending-moment theory, making a correction for longitudinal loads. By introducing T in the general equation, t can be determined. The unit value of shear can be computed, and consequently the principal stresses in the skin can be found.

The computation of the skin stresses used was not entirely satisfactory, because the conventional beam conditions were assumed to be applied. The elastic deformations upon which the beam theory is based are small compared to the possible normal deflections, and can even be neglected. The problem was to determine the distribution of stresses in a curved non-extensible surface. A series of formulas of various forces and skin stresses on the whole cross-section of the ship as well as on a small element can be written. The solution of them is a very complicated mathematical problem if all the items are taken in consideration, and as long as the beam theory was found to be closely applicable to the actual practice and the formulas used very simple and easily applied, no complete calculations of these conditions were made but a possibility for further research is left open.

The following limiting conditions were investigated and stresses in the skin in most critical places computed.

Case I. Hull fully inflated; full speed 70 miles per hour; zero yaw and pitch; maximum internal pressure (blow-off pressure at the valve) 100 mm. above the outside air pressure at the valve section.

Case II. Minimum pressure for full speed above atmosphere assumed as one-half of maximum hydrodynamic pressure. Other conditions same as Case I. (Analyzed for nose stiffness only.)

Case III. Speed 50 miles per hour; pitched downward 10 deg. relative to air, but horizontal relative to ground; minimum pressure above atmosphere one-half of maximum hydrodynamic pressure.

Case IV. No power. Hull fully inflated at zero angle; internal pressure at bottom 0 mm.

Case V. Speed 45 miles an hour; pitched upward 10 deg. relative to air, but horizontal relative to ground; (*a*) minimum pressure one-half of maximum hydrodynamic pressure; (*b*) maximum pressure same as Case I.

Case VI. Speed 45 miles per hour; pitched 25 deg. downward relative to ground and 4 deg. downward relative to air. Minimum pressure one-half of maximum hydrodynamic pressure. (This case analyzed for nose stiffness and scoop capacity only; ballonet level full.)

Case VII. Same as Case IV except with ballonet level full (19,000 cu. ft. of air).

Case VIII. Same as Case III except with ship momentarily pitched down by the nose 10 deg. relative to air and 30 deg. relative to the ground.

Case IX. Same as preceding except with ballonet level full.

Case X. Same as Case IV except with surplus load causing descent of 500 ft. per min., plus 500 ft. per min. downward air current.

Case XI. Same as Case IV except with 33,500 cu. ft. of air in ballonet. Upper air level 10.5 ft. below ship's axis. Ballonet diaphragm was assumed to be level across the hull at all sections.

Case XII. Same as Case IV except ship is helium filled.

Case XIII. Same as Case XI except ship is helium filled.

Case XIV. Same as Case II except ship in a gust suddenly pitched 25 deg. down relative to the air, horizontal relative to the ground.

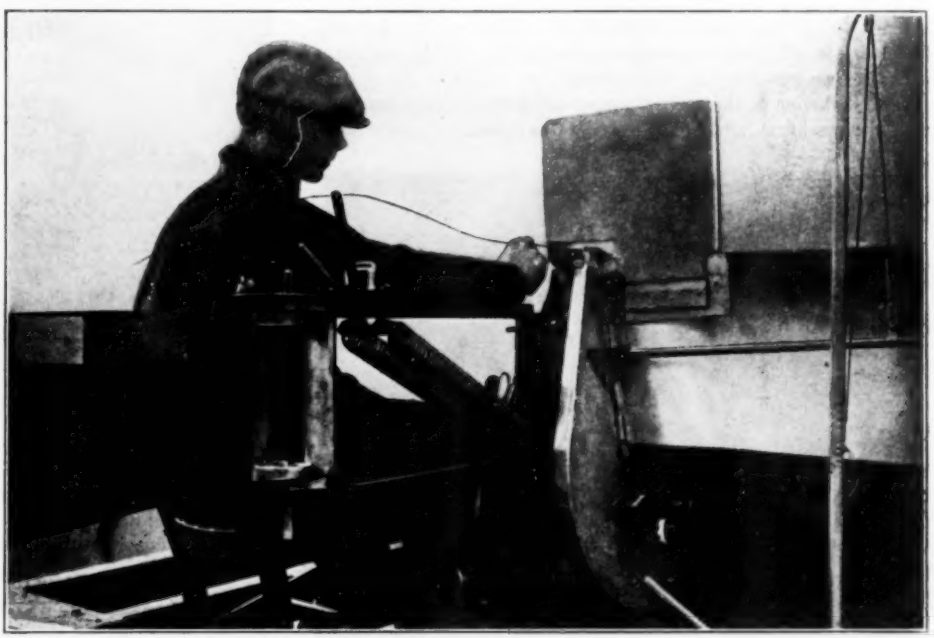


FIG. 17 AUTOMATIC RIVETING MACHINE AT WORK ON HULL OF METALCLAD DIRIGIBLE "ZMC-2"
(This machine "sews" 5000 rivets per hour and put 3,500,000 rivets in the "ZMC-2" during its construction.)

Case XV. Same as Case II except elevators suddenly thrown up 20 deg.

The maximum tension on the skin was found in Case V (*b*) and is equal to 610 lb., giving a factor of safety of 5 on the yield point of the material.

The comparison of the longitudinal strength of the *ZMC-2* with that of long, slender ships like the *Shenandoah* is very favorable. The worst bending moment considered in the design data of the *Shenandoah* was reached at an angle of pitch of 6 deg. relative to the air at a speed of 46 m.p.h., with a blow-off pressure of 10 mm. The factor of safety in the top longitudinal was only 1.68. The *ZMC-2* under the same conditions and blow-off pressure of 100 mm. shows a longitudinal factor of safety of over 10.

A complete analysis was made on the control surfaces and followed by a sand-load test.

The car was analyzed on the same principle as an airplane fuselage under static, flying, and landing conditions.

The capacities of valves, scoops, and all other outlets and inlets were computed. Miscellaneous small computations such as snap diaphragm action and effect of holes and small irregular forces were investigated.

All joints and fittings were carefully investigated with respect to their ability to sustain the loads imposed. Higher safety factors were used in this case than in the main structure.

The various formulas used for calculation of stresses in the skin are given in a paper by R. H. Upson: "Metalclad Rigid Airship

Development," *Journal* of the Society of Automotive Engineers, February, 1926.

Stern-Section Test

Considering all the tests made previous to flight tests, the most instructive one and the one that was done on a larger scale than any other was the leakage, inflation, and side load test on the stern portion of the *ZMC-2* hull.

The metal covering of the hull was changed from duralumin 0.008 in. thick to Alclad 0.0095 in. thick, which necessitated scrapping the work previously done, and it was decided to use the fully completed stern portion 29 ft. high and 34 ft. in diameter at the base for this experiment. This section was embedded in a concrete ring weighing 50,000 lb., which was made airtight by a water seal at the bottom.

The behavior of the section under different internal pressures was observed first, and no air leakage was found.

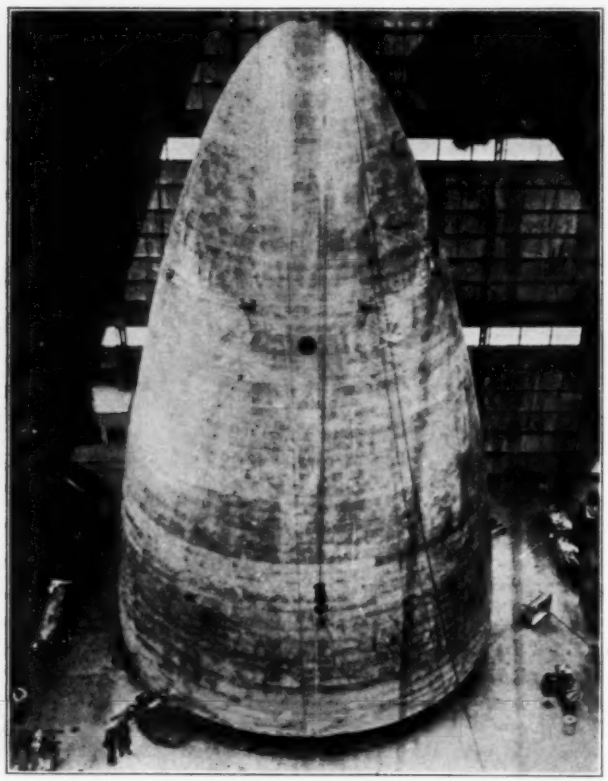


Fig. 18 Showing Vertical Method of Erection, With Automatic Riveting Machine at Work on Hull

After that the section was inflated with carbon dioxide by introducing it at the bottom of the hull and letting the air exhaust at the top. When the section was filled with carbon dioxide, hydrogen was introduced at the top, displacing carbon dioxide flowing out at the bottom. When the exhaust mixture was found to contain 10 per cent of hydrogen it was passed through the scrubbing apparatus, which contained caustic soda solution and absorbed carbon dioxide out of the gas mixture, and the repurified hydrogen gas was forced back into the hull. By continuing this process the hull was filled with pure hydrogen. The reason this method was adopted was the ease with which carbon dioxide is absorbed when mixed with hydrogen, compared with expensive apparatus used to freeze air out of a mixture with hydrogen.

The section filled with hydrogen was allowed to stand for a day, and no loss of purity and no leakage at the seams were observed.

When the gastightness and the correctness of the inflation process had been proved, the hydrogen was then displaced by carbon dioxide and the hull section subjected to a side load equal to 2000 lb. This was done to approximate the condition of bending occurring in the actual ship. The interior pressure was varied from 0.2 in. of water below to 13 in. above atmospheric pressure.

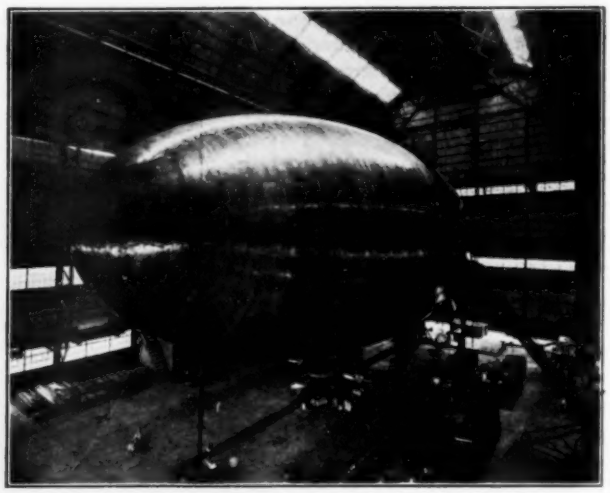


Fig. 19 The "ZMC-2" Ready for Inflation With Helium

At this latter pressure the concrete ring was lifted from the floor, the water seal was broken, and the experiment discontinued.

This test conclusively proved:

1 That an all-metal hull can be made gastight
2 That the method of inflation using carbon dioxide as the intermediate medium is sound
3 That the hull can safely sustain the loads for which it was designed
4 That the hull can stand even pressures slightly below atmospheric.

Appendix No. 3

Construction, Erection, and Assembly of the "ZMC-2"[6]

The assembly and erection methods of the *ZMC-2* were given prime consideration even during the preliminary design of the ship. The building of rigid airships is not only a design problem but a construction problem of the first magnitude. It was not entirely necessary that the ship be constructed with the skin plating and the main seams running transversely; it could have been constructed with the plating and main seams running longitudinally.

The assembly of the metal plating into a gastight hull was accomplished very efficiently by building it in two halves, erecting each half with its major axis vertical to the floor, and then turning them up into a horizontal position and joining them together into a complete hull. The hull plating was built up of a series of transverse rings, each of which was the surface of a frustum of a cone. Each ring was composed of a number of equal panels, the panels being cut to the proper curvature from flat sheets and riveted together end to end to make up a curved band which became one of the transverse rings when riveted in place.

[6] Prepared by Edward J. Hill and Arthur G. Schlosser, Detroit Aircraft Corporation, Detroit, Mich.

A manhole placed at the bow and stern of the hull was the starting point of each half. The first ring of plating was riveted to the manhole by hand and suspended by a single cable through the center of the manhole in such a manner that the manhole and ring were parallel to the floor. The suspension was by means of a winch at the hangar roof so that the height above floor could be easily adjusted.

The automatic riveting machine, described more fully below, was then set up on a carriage. The carriage was mounted on concentric rails which were placed in the floor directly under each half of the hull. The riveting machine was then adjusted to the proper angle of the hull plating. The manhole with the first plating ring attached was then adjusted to the level of the riveting-machine jaws and the second plating ring was riveted to the first, thereby completing the first transverse seam in the hull plating. A gaging device was located on the riveting machine, giving a correct and uniform overlap of the plates. When the transverse seam was completed the two ends of the transverse band of plating were riveted by hand, thus closing the band into a complete ring.

The assembly was then raised by means of the winch, the angle of the riveting machine again adjusted, and the next transverse band was riveted to the last one in a like manner, the shape of the panels with the proper overlap determining the form of the hull.

As the riveted transverse seam emerged from the riveting machine it was inspected and repaired. Any defective rivets were drilled out and replaced with hand rivets. It should be noted that the defective and missed rivets amounted to only about one-third of one per cent of the total number used.

After a thorough check on the seam by an inspector, it was given his approval and the sealing compound applied to the seams to make them gastight.

The Frame Assembly

As one band after another was added, the thin plating was supported by a light falsework of wood until it reached the point where the first transverse frame went in. This frame, having previously been assembled on the radial jig, was expanded tightly into the plating and riveted in its proper place with respect to a given plating ring. After riveting in the transverse frame the longitudinal members connecting the frame to the manhole were forced tightly into the plating and riveted in place. The whole assembly of plating and internal structure was then given a thorough inspection.

The assembly of the plating then commenced again and proceeded down to the point where the next transverse frame was put in, and the longitudinal members connecting it to the previous frame were put in place as before.

Thus the assembly proceeded progressively from frame to frame and as the maximum distance between frames was 18 ft., the work was always within 20 ft. of the floor and all the scaffolding required were ladders about 16 ft. high. All work on the hull was completed and passed by the inspector while in reach of these ladders.

While the structure was being placed in one of the halves, the skin plating was being assembled to the other and vice versa, causing the two operations to dovetail perfectly. There was very little lost time from having one crew wait until the other was out of the way.

As the weight of the assembled hull increased, the suspension was changed from one single cable at the center of the manhole to a bridle which distributed the load from the single cable to eight clips provided for the purpose on one of the main transverse frames. The hull was guyed from the sides of the hangar by cables to prevent it from swaying and turning. The operation of sewing the plating was continued, and main rings and longitudinals were put in place until all plating was riveted. Each half was then completed by the addition of a small angle riveted on with one leg inward.

Joining the Hull Sections

The point at which the two halves were joined together was at the maximum section, 63 ft. from the bow and 86 ft. from the stern. Thus the stern section was 23 ft. longer than the bow section. While still in the vertical position the two sections were given a final inspection, and when passed were ready to be turned into the horizontal position for joining together.

The turning was accomplished by a system of equalizing bridles and winches. Two turning bridles were used for each section, one being attached at the lowest main transverse frame and the other at the highest. The bridles were attached to the frames at four points, and by running over pulleys they equalized the loads. They were then brought up to a single cable which wound on a winch mounted on a trolley on the monorail at the apex of the hangar roof.



Fig. 20 Showing Inflation Tubes Over Top of Hull

By taking up on the bridle connected to the lowest frame, the lower end of the hull section was slowly raised and the upper end in a like manner slowly lowered. The weight of the section was gradually transferred from the original suspension bridles to the two turning bridles. A point occurred in this operation where the center of gravity of the section passed under the upper turning bridle and the total weight of the section was then supported entirely by four points on the one main frame. This was the critical point, and once passed, the lower end was raised and the upper end lowered until it was in a horizontal position.

When the two halves were in the horizontal position they were each suspended by means of the bridles at two of the main frames. The bridles were attached to cables which wound on the winches, which in turn were mounted on trolleys, so that each section could be raised, lowered, tilted, or moved back and forth along the trolley rail into any position, and by this means the two halves were brought together and lined up for the joining.

The Closing Seam

Before any riveting was done on the final joint, the plating was thoroughly clamped in position. It required many clampings and reclampings before a satisfactory job was obtained. In this clamping and subsequent riveting it was necessary for one workman to work inside and one outside at any point on the seam. To accomplish this an opening was cut in the bottom of the hull and an adjustable extension ladder put through this opening up into the inside of the hull. From this it was possible to reach any point on the inside of the seam. For the outside riveting of the final seam a workman was suspended on a sling from the hangar roof over the top of the hull, and from a ladder on the sides and bottom.

When the two halves had been satisfactorily clamped together by the boundary angles the actual riveting of the joining seam began. A lap strip or metal band about two inches wide was laid over the joint and a double row of rivets joined this band to each of the two sections. This was entirely a hand-riveting job and the workmen were given every facility for speeding up the work, yet it took two men the equivalent of two months at eight hours a day to complete the seam, a job which the automatic riveting machine could have done in six hours. After riveting, the seam was sealed and the connecting longitudinal structural members riveted in place.

While the final seam was being riveted, the fins and rudders were bolted to the fittings, which extended out from the main frames designed for that purpose, and the control lines run down to the car through Bowden casings which were cemented to the outside of the hull plating.

The Internal Structure

The internal structure was built up entirely of rolled and flat-plate sections riveted together. The five main transverse frames are of the triangular type having a rolled channel member at each corner riveted to flat plates which make up the sides. The channel members were rolled to fit the circumference of the hull at that point, and the flat-plate sides were cut on the proper curvature to make a complete girder having the required curvature and angle of the hull.

Wooden formers were built for sections of each frame to the exact shape and curvature required, and into them were clamped the channels and plates making up the girders to be riveted together. When completely riveted, the sections rigidly maintained their shape and curvature. They were then taken out of the formers and assembled into a complete ring on the radial jig.

On the radial jig the sections were laid out into a true circle and leveled up. The joints connecting them into a complete ring were then riveted and the radial wiring put in place and adjusted while the frame was perfectly round. The wiring was removed before placing the frame in the hull, and later replaced and adjusted to the exact position it had been in while in the radial jig. This insured the frame's being round.

Longitudinal members were in all cases a single rolled channel section. Each one was rolled to fit a former representing the longitudinal curve of the hull plating. Flanged lightening holes were extensively used in the channel sections and side plates of the frames to lighten and strengthen the structure.

All the material in the internal structure was built of Aluminum Company of America's aluminum alloy 17 ST. All the sections were rolled and worked in the hard state with one exception, namely, where the curvature was so great that heat treating was necessary. The rivet used to build up the structure was the same flat-head rivet that was used to rivet the plating to the structure, the sizes used being mostly $^3/_{32}$ in. in diameter. At joints and highly stressed places $^1/_8$-in. diameter rivets were used, and in a few exceptional places $^3/_{16}$-in. rivets were necessary.

Cutting the Plating Panels

The cutting of the plating panels was done very accurately as the hull was constructed by building from one ring of plating to the other, gaging from the edges of the plating in all cases. Naturally any errors in the cutting of the panels would accumulate. It should be remembered that the shape of the hull was entirely the result of the shape of the panels, and therefore any errors would affect the hull to the extent that the internal structure would not fit.

The panels were cut to various radii, ranging from a few inches at the extreme bow and stern to over three miles at the maximum diameter. For scribing all radii under 20 ft. a specially constructed beam trammel was used. For larger radii recourse was had to an indirect method based on the geometric principle that the locus of all points which jointed to two fixed points form a constant angle, is a circle.

A metal-topped table was constructed with two fixed metal pins spaced 14 ft. apart. Two straight edges 14 ft. long were joined together with a pivoted point, adjustable to any angle. These straight edges were very heavy and were mounted on rolling balls to allow them to roll over the table top freely. With the straight edges rigidly clamped to a given angle, they were rolled across the table with each straight edge in sliding contact with one of the fixed pins. This caused the apex of the angle formed by the two straight edges to scribe the arc of a circle, and the length of the radius of each circle was controlled by setting the angle of the straight edges to the calculated angle.

The sheet to be scribed was clamped to the surface of the table, then the scribing point of the arc machine was brought down in contact with the sheet. The machine was then drawn from left to right by hand operation over the pins until a satisfactory pattern was scribed. The sheet was then removed from the table and trimmed around the scribe line, and the final panels were then made from this pattern.

The Gastight Sealing Compound

The development of the sealing compound was given much thought. A compound had to be found that would adhere to the metal and seal the seams perfectly, yet it had to be a substance that would remain flexible and pliable at winter temperatures and also would not become fluid at the higher summer temperatures. A satisfactory bitumastic compound was finally arrived at. This was applied to the inside of the seams immediately after the riveting had been inspected and approved.

The first coat was thinned out with naphtha until it was very fluid, and was then flowed into the seam between the overlapped plates by means of a rubber-bulb syringe with a very fine nozzle. Capillary action caused the fluid to penetrate into the seam, and in this it was greatly aided by gravity as it was applied in such a manner that it always flowed downward into the seams. This thin application dried very quickly, due to the rapid evaporation of the naphtha, and as soon as shrinkage was apparent, more compound of the same consistency was applied.

In this way the seam was kept full of the fluid until every void was filled by the building up of the sealing compound as the naphtha evaporated. When the minute spaces between the overlapping plates were thus thoroughly filled and no more fluid could be made to flow into the seams, a heavy application of very thick compound was applied over the edge of the plate exposed on the inside of the hull.

This application was thinned by heating to about 200 deg. fahr. This was accomplished by means of an electrically heated pressure gun which forced the heated compound through a nozzle with an opening about $^1/_{16}$ in. in diameter. By this means a ridge of the compound was laid over the inner exposed edge of the plating, covering it completely. This compound cooled very

rapidly in contact with the metal and thus did not fall from the seam of its own weight. While this application was still damp, it was dusted with aluminum powder to facilitate inspection and to harden the "tacky" surface.

The Automatic Riveting Machine

The development and perfection of the automatic riveting machine played a very important part in the construction of the metal hull. It would have been practically impossible to build the hull without such a machine. The machine riveted a very highly efficient seam composed of three rows of staggered rivets, the rows being $^3/_{32}$ in. on centers and the rivets being spaced $^1/_4$ in. in the rows, the overlap of the plating being $^7/_{16}$ in.

Aluminum-alloy wire 0.035 in. in diameter was fed into the riveting machine. The wire was cut to length and forced through the overlapping sheets into hardened steel dies, cutting a clean, round hole. The soft wire was able to punch a hole in the hard sheet because, being confined, it could not expand when the pressure was applied but had to punch through the sheet. When the wire was through the sheet, it was headed first on the under side and then on the top, making a complete rivet. The machine then spaced the next set of rivets automatically.

Three rows of rivets were driven at a time and at the rate of 140 (or 1 ft. of seam) per min. while operating steadily. In actual operation the machine was able to rivet up to 200 ft. (or 28,800 rivets) in eight hours; the greatest number of rivets driven in eight hours was 40,000, not including time for set-up. Concentric circular tracks were laid on the floor, whose common center was at the axis of the hull assembly. A car mounted on these tracks carried the riveting machine around the hull. The riveting machine was mounted on the car by means of an arm with a universal movement and balanced by springs so that it could be easily moved to any position independently of the position of the car, within a limit of about two feet.

The riveting machine automatically pulled itself along the sheet by its own feeding mechanism. A ratchet mechanism conveniently placed for the operator was used to propel the car around the hull. Inasmuch as the forward movements of the riveting machine and car were independent within certain limits, it was not necessary for these two operations to be synchronized.

Appendix No. 4

INFLATION OF THE METALCLAD AIRSHIP "ZMC-2"[7]

DUE to the fact that it is very difficult to separate helium from air, the gas inflation of the *ZMC-2* was carried out in two stages. In the first stage carbon dioxide displaced the air in the ship. The carbon dioxide was let in at the bottom and the air forced out of the top of the hull. In the second stage, the process was reversed. Helium was let in at the top while carbon dioxide was forced out of the bottom of the hull. The carbon dioxide-helium mixture was circulated through a caustic scrubber to remove the carbon dioxide and the purified helium was then returned to a point 10 ft. above the bottom of the hull. Chemical control tests were conducted to show at all times the exact progress of inflation in the various parts of the ship and to give the purity of the gas after inflation.

Gas Sampling and Analysis

The gas was sampled at five stations, at the bow, stern, at each side of the ship, and at the exhaust line. Each station, except the exhaust-line station, controlled three sampling tubes from the top, equator, and bottom of the hull, respectively, and samples from the same level were analyzed at the four stations simultaneously. Samples were drawn through small tubes of rubber. These tubes were always open to insure a fair sample from the ship at all times. The gas was sampled at ten-minute intervals with the result that the time between two successive analyses at any level was one-half hour.

The gas analysis was run by means of A. H. White's modification of the Hempel burette and the carbon dioxide absorbed in a pipette filled with caustic solution in the usual manner. Exactly 100 cc. of gas was drawn into the pipette and the contraction after the gas was run into the caustic pipette was taken as the volume of carbon dioxide. Since exactly 100-cc. samples were always taken, the volume of this contraction is read in percentage of carbon dioxide directly. A sample run follows:

Original sample	100 cc.
After caustic pipette	78 cc.
Contraction	22 cc. or 22 per cent CO_2

During the first stage the carbon dioxide percentage indicated directly the progress of inflation. During the second stage (inflation with helium) the percentage of carbon dioxide subtracted from 100 per cent was taken as the percentage of helium in the ship, as it was impossible to measure the helium directly with the apparatus at hand. This method, undoubtedly, is not very much in error as the final results of the carbon dioxide inflation showed 99.4 per cent of carbon dioxide. Final tests for oxygen were also taken on the inflated ship and showed not over 0.2 per cent oxygen, so undoubtedly the inert gas was very nearly as pure as the helium run in from the containers.

The gas analyses were made by three squads of five men, one squad for each eight-hour shift.

Final tests were made after inflation with the following average results: Per cent oxygen, not over 0.1 per cent; CO_2, 2.3 per cent; carbon monoxide, 0 per cent. This final test was run by means of an Orsat apparatus, absorbing the carbon dioxide in caustic solution, the oxygen in alkaline pyrogallate solution, and the carbon monoxide in cuprous chloride solution in the manner ordinarily used in gas analysis.

Separation of the Carbon Dioxide and Helium

An attempt was made to carry out the inflation in such a manner as to allow one gas such as the carbon dioxide to displace the other gas (air) in the ship without either gas mixing with the other. The chief causes of mixing of the gases were two, namely, diffusion, and turbulent flow of gas from the containers. Turbulent flow is caused mainly by the velocity of flow of the fluid. The velocity of the input of the carbon dioxide was kept within the region of quiet or viscous flow and never exceeded 12,000 cu. ft. per hour. Diffusion depends upon various factors and may be expressed by the following equation:

$$m = \frac{KA(d_2 - d)t}{h}$$

Where m is the mass of substance diffused, K the coefficient of diffusion, A the cross-section through which diffusion takes place, d_2 and d the respective densities of the diffusing substances, t the time, and h the thickness of the diffusing layer.

During the carbon dioxide inflation it was apparent that the amount of diffusion was very small and that stratification of the carbon dioxide and air was quite complete. This was borne out by gas analysis and was visible to some degree from the fact that the moisture in the air condensed on the surface of the cold carbon dioxide as it lay as a level layer of fog which rose as the carbon dioxide rose in the ship. This stratification was probably due, as indicated above, to the slow rate of inflow of carbon

[7] Prepared by Dr. A. R. Carr and E. C. Good, Detroit Aircraft Corporation, Detroit, Mich.

dioxide and to the fact that the differences in density of air and carbon dioxide is not abnormally great.

However, stratification during the second stage of inflation was not nearly so marked. This is also apparent from the gas analysis. The reason for this non-stratification is the fact that the rate of diffusion of helium with carbon dioxide is much greater than that of the carbon dioxide with air as the densities of carbon dioxide and helium are as 44 to 4. Therefore, when over 40 per cent of helium began to appear in the exhaust, the gas from the exhaust was led to the carbon dioxide scrubber.

After the carbon dioxide was scrubbed out the helium was returned from the scrubber to the hull. The proportion of caustic (NaOH) in the scrubbing solution was checked by chemical analysis and kept around 5 or 6 per cent. This checking for

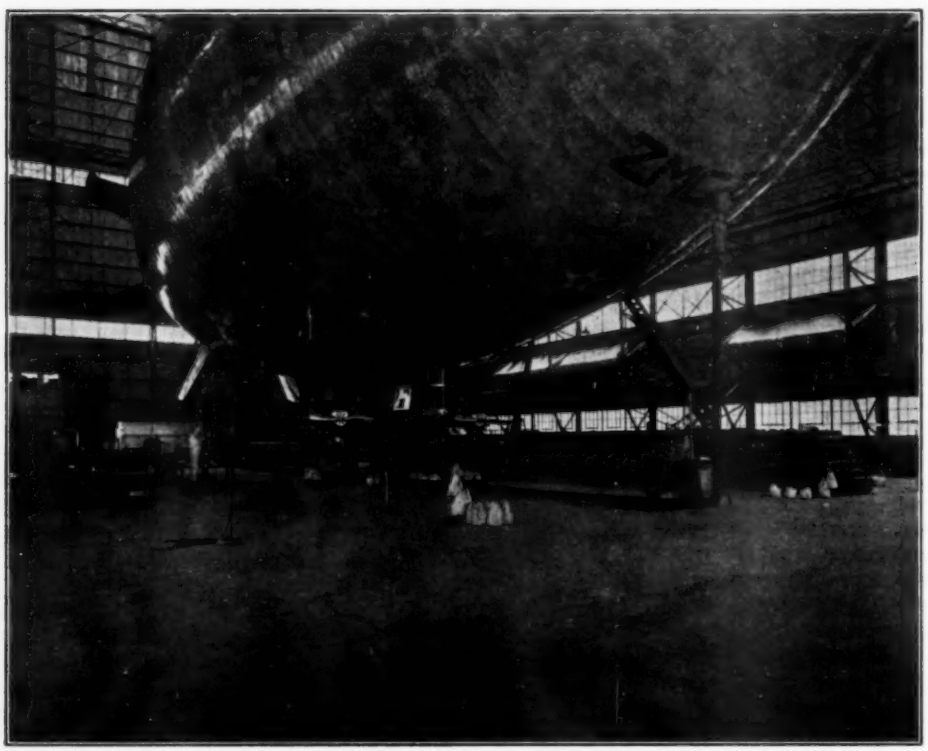


FIG. 21 ARRANGEMENT FOR INFLATION OF DIRIGIBLE WITH HELIUM

NaOH was done by a double titration with standard sulphuric acid, using phenolphthalein as an indicator for the first end point and then continuing to the final end point with methyl orange. This was necessary since the end point for the caustic titration occurred in a basic solution. The first titration gave the amount of caustic plus one-half of the sodium carbonate; the last titration with methyl orange gave the other half of the carbonate. From these two titrations the available hydroxide in the scrubbing solution was calculated and thus controlled. The temperature of the scrubbing solution was kept below 100 deg. fahr., by means of ice placed in the dissolving tank.

Scrubbing of the gas was continued until the content of CO_2 was reduced to 3.4 per cent.

During the scrubbing some moisture was carried over into the hull due to insufficient drying facilities. This was apparent from the water which condensed in the intake pipe and in the ship itself. There was a slight indication of the presence of alkali, as tests showed a faint pink in the presence of phenolphthalein. The condensate, measuring 250 cc. in all, which was drained from the ship at various times was analyzed for total alkalinity. The only means available was by titration with $^1/_{100}$ normal acid. This was of course not an absolutely accurate method, but it gave a very good idea of the magnitude of the total alkalinity. One of the first samples of condensate analyzed 0.0084 per cent NaOH. The second draining gave 0.0031 per cent NaOH. Apparently from the small amount of condensate and also from the very small amount of caustic in the condensate, this should cause little concern.

CONCLUSIONS

The results of the gas analyses show that the two stages of inflation of the *ZMC-2* were successful. In the first stage the carbon dioxide almost completely displaced the air. In the second stage the helium, aided by the carbon dioxide scrubber, displaced the carbon dioxide. The method could be improved by placing a drier in the line from the caustic scrubber. However, the results show that the method employed is a relatively simple, efficient one for inflating this type of ship.

Appendix No. 5

CONCLUSIONS FROM THE TEST FLIGHTS[8]

THE METAL SKIN

THE hull of the *ZMC-2* is an accurately tailored piece of work and unusually free from "stock wrinkles." The patterns give every evidence of having been perfectly cut, and nearly as perfectly joined together.

The rivets and seams are perhaps the most remarkable part of the entire construction. Each line of rivets runs around the ship, acting in a way as a small intermediate transverse ring.

The diffusion during actual operation showed an apparent loss of less than 100 cu. ft. of helium per 24 hours. This is excellent, and indicates that the compound used on the seams is a success.

On the flight to Lakehurst, two punctures occurred. Apparently some one shot a bullet through the hull. Examination of the entry hole showed no signs of enlarging or additional opening beyond what was probably the exact size of the bullet. The exit hole was not round. Rather it might have been a hole made by a flattened soft bullet. If it did enlarge at all the amount was so small as to be negligible, as far as tearing was concerned. The pressure ran as high as $3^1/_2$ in. with these holes in the hull. In fact, the ship at one time was nosed down at an angle of 22 deg., which would put at least 4 in. of pressure on the skin in the vicinity of the holes. Therefore the holes afforded an excellent test of possible tear and opening from an accidental skin break. The result justifies the conclusion that no alarm need be felt over this possibility.

The strength of the metal skin is of course very much greater than that of a fabric covering. The best evidence of this was

[8] Prepared by Capt. William E. Kepner, U. S. A.; Test Pilot.

when, with 0.5 in. pressure, 17 workmen were inside and supported by the skin between frames 54 and 70 before inflation. Needless to say, it is a great satisfaction for the pilot to be able to realize that there is comparatively no danger in letting the pressure run too high.

With a reasonable amount of air in the ship the ballonets act very well for trimming, even at high speed. The damper doors must be forcibly held shut and locked as the elastics tend to pull each door half open, thus equalizing the air in both ballonets, with the consequence that this particular air system can be made to act as a single ballonet, while it is in fact a two-ballonet system. This makes the automatic-pressure idea a thoroughly practicable one, because a ship with a single ballonet is always in a natural static trim.

The air-system controls are easily worked and, once understood, are fairly simple.

The opening for regular entry and inspection of the ballonets is not necessary, as it is much easier for a man to enter the distribution chamber through one of the air-riser ports. This also makes an excellent place to enter the helium compartment, and this on several occasions has been done. This ability to get into the helium compartment and inspect the interior of the ship is a valuable feature, and the apparatus developed by the Aircraft Development Corporation which permits it, adds much to the efficient operation of airships.

The air breathers are at present sealed and are equipped with a pulling-rip cord, that is, a cord that can be pulled from the car should the ship ever approach zero pressure. This has been tested and found perfectly safe.

Only one place has shown any skin flutter, and that was directly in the plane and above the right propeller. It was fixed by putting in a small lattice. In all flying so far there has been no evidence of snap diaphragm action.

Transverse Rings

The main transverse ring frames at Nos. 18, 54, 70, 114, and 122 are held rigidly by radial wiring. The wiring apparently holds these rings to a positively true form whether on the ground or in the air, as far as can be judged from visual observation.

Longitudinals

The convergence of the longitudinals makes a very stiff nose, and even at the lower pressure, i.e., $\frac{1}{2}$ in., the shape is always good from frame 18 forward and from frame 114 back. The nose will undoubtedly stand considerable external pressure, even with very little pressure inside. However, the ship should always be flown at some pressure unless in case of an emergency. This rigidity is simply a safety factor. The recommended flying pressure is 2 in. for an air speed of 50 miles; at higher speeds than 60 miles, $2\frac{1}{2}$ in. to 3 in. is recommended.

Fineness Ratio

The feature of an extremely low fineness ratio affords an excellent opportunity for a practical test of this new shape. The *ZMC-2* is satisfactory as to stability, but it is believed that larger ships flying at higher speeds will have more inherent stability if the fineness ratio is increased.

Controls

Not enough flying has been done yet to determine the exact value of the eight fins and controls. However, it is certain that on this particular ship four fins would not work nearly so well as the present arrangement of eight.

The present set of eight fins, together with the low fineness ratio, make the *ZMC-2* the most maneuverable ship the writer has ever flown. The elevators quickly take care of the most violent angles and in smooth air hold the altitude exceptionally steady. They also force the ship into very quick descent and ascent when so used. The rudders turn the ship in approximately one-half the area necessary for a "TC" or "J" ship.

The Car

The motors furnish plenty of power and the maximum speed is probably 68 to 70 miles an hour. It has been estimated that on the three-hour endurance flight at 1740 r.p.m. the air speed was about 65 miles per hour, which is in excess of contract requirements.

The outriggers are satisfactory and apparently absorb a major portion of the engine vibration.

Sundry Conclusions

There is very little vibration anywhere in the car, and the general arrangement of the interior is very satisfactory.

The air scoops are excellent and, once understood, are easy to operate; in fact, much easier than the conventional drop-pipe scoop. The dynamic scoop is excellent for automatic pressure at speeds of about 1500 r.p.m.

The hand blower is perfectly satisfactory from the standpoint of efficiency. It is particularly valuable for maintaining pressure while the ship is being maneuvered in and out of the hangar.

The fuel system functions satisfactorily.

The handling lines required care to avoid hitting the propellers when letting them out. But throttling the motor on one side while the line is put out on that side, takes care of this. The drag rope functions properly.

The landing skid has excellent shock-absorbing qualities. Probably a wheel would be much better.

The suspension cables are apparently all right; they do not seem to vibrate at all.

The thrust-point fittings are fitted snugly, and show no signs of vibration.

The general location of miscellaneous articles such as tables, stools, seats, fuel pumps, motor power controls, fire extinguishers, shut-off cocks, control cables, handles, etc., seems satisfactory, though each pilot's opinion will differ on this subject.

The writer's general impression is that as an experimental development this ship should have an enthusiastic reception and careful consideration. He for one will follow its operating career with the greatest of professional interest, and believes it to be the forerunner of a long line of super airships.

Discussion

J. C. Hunsaker.[9] The author has made a very competent and stimulating study of the commercial uses of airships and airplanes, but naturally in an airship paper he must choose a 2000-mile route where the two types of aircraft are not at all competitive. For long voyages, and particularly overseas, the airship appears clearly to have only the steamship as competitor. Here the airship offers speed and the steamship comfort. The safety and reliability of airships will eventually be established. Steamships have had a century to reach their present state of development and airships about a quarter of that time. Nevertheless steamships have even today reached only what the writer would call a "commercial degree" of safety. The *Vestris* accident and the deliberations of the International Conference for Safety of Life at Sea now going forward in London, serve to remind us that absolute safety is yet to be reached.

As regards comfort, the writer doubts whether the airship can really compete with the steamship. Large airships will be more

[9] Vice-President, Goodyear Zeppelin Corporation, Akron, Ohio.

comfortable than small steamers or very fast steamers from the point of view of those liable to sea sickness, but if a few knots speed be sacrificed, steamships of reasonable size can be built of extreme comfort and steadiness. Again, airships cannot economically compete with steamships for the transport either of large numbers of passengers or quantities of freight.

Consequently the writer looks upon airships as a new transportation medium that competes neither with the airplane nor the steamship but offers a high-speed service, for a price, to a relatively small quantity of passengers, mails, and express. The speed of the airship cannot be met by steamers, and the carrying capacity and duration of non-stop flight cannot be met by the airplane. For short distances, the airship cannot compete with the airplane (or flying boat) any more than the *Leviathan* could compete successfully on a New York-Boston run with the Sound steamers. Terminal charges and cost of turn around are too great.

For overland work, the airship can carry passengers non-stop in comfort for a long voyage, while the airplane can carry them in considerably less comfort with intermediate stops. For this phase of competition one thinks of the relative comfort of motor buses and railways for long trips.

In general, the picture as we see it now shows airships as offering a super first-class service overseas as a supplement to existing steamship services. A few passengers, first-class mails, and urgent express matter would make up the pay load. The volume of passengers, mail, and freight going by steamer will not be reduced, but will rather tend to increase by virtue of the stimulus to international business afforded by more rapid communications.

The airships likewise supplement the continental and coastal airplane services, for whose routes they provide the necessary extensions overseas.

The Society is to be congratulated on Mr. Fritsche's paper, which contains so much meat. The description of his metalclad airship is most interesting, and is suggestive of the improvements in the art which American ingenuity and skill always bring forward when commercial conditions make development work possible.

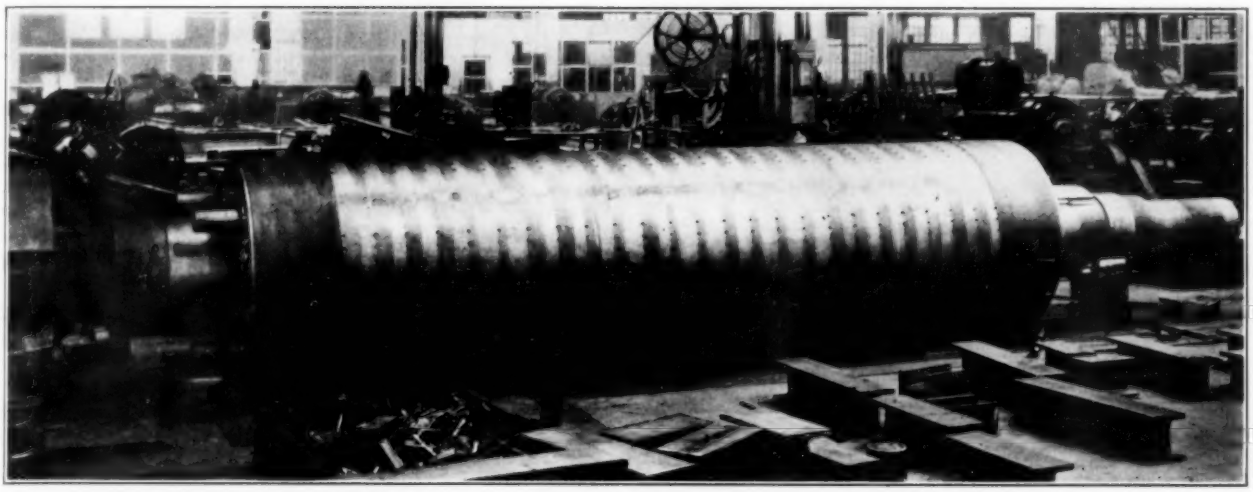


FIG. 1 PLATE ROTOR

The Mechanics of Plate Rotors for Turbo Generators

BY J. P. DEN HARTOG,[1] EAST PITTSBURGH, PA.

With the rapid development of turbo-generators during the last five years, the length of the rotors has practically been doubled. More or less elaborate mechanical calculations, especially of the "plate rotors," have been evolved and experiments have been carried out in order to place the design of this apparatus on a rational basis. In this paper some of the mechanical problems which have come up in connection with this development are discussed in detail.

THIS paper deals with some of the mechanical problems which were encountered during the past few years in the construction of plate rotors. By "plate rotor" is meant a particular construction which the Westinghouse Electric and Manufacturing Company uses for its turbo-generator rotors and which was initiated by Behrend and Field.[2] It consists of a stack of plates between two end-forgings held together tightly by means of four or six heavy bolts (Figs. 1 and 2). These bolts are stressed to about 40,000 lb. per sq. in., so that the stack of plates forms a rigid body.

The advantages of this construction as compared to solid forgings are as follows:

1 For the largest machines of the present time single-piece forgings cannot be bought from the steel companies, so that the rotor has to be made up of several pieces.

2 Better material can be obtained in plate form than in large solid forgings. This is of great importance, since large stresses, due to centrifugal forces, are set up at the roots of the teeth.

3 The ventilation of the rotor can be effected in a simple manner by machining down four sections in every plate (*A* in Fig. 3). This offers greater difficulties for a solid forging. Either circumferential grooves have to be machined, which diminishes the bending rigidity considerably, or recourse has to be taken to drilling holes from the outside into the rotor, which does not secure as good a ventilation.

The material of the plate rotor is more efficiently utilized, so that its bending rigidity is even larger than for a solid rotor. Though the plate rotor has been in use successfully for over 15 years, it is only comparatively recently that refined mechanical calculations on it have become necessary, due to the phenomenal increase in its size.

The investigations dealt with in this paper cover only such subjects as are peculiar to the plate rotor, as distinguished from the solid rotor, namely: 1 stresses and deformations of the structure due to bending under its own weight, 2 the stresses in the bolts and their screwthread, and 3 the effect of centrifugal forces on the bolts. These three aspects of the problem will now be discussed in detail.

THE PLATE ROTOR IN BENDING

Before investigating the stresses appearing in the rotor due to bending, it will first be seen what the stresses in the structure are before any outside loading is imposed on it.

Fig. 3 represents a typical plate of a four-pole generator designed to run at 1800 r.p.m. The shaded area shows the existence of direct contact between neighboring plates. Without bending from outside, this area will be subjected to a uniform compressive stress. The four bolts will carry a certain tensile stress, such that the product of stress and cross-sectional area for the bolts equals that of the plates.

When the structure is bent, this relation remains valid, though both the stresses in the bolts and the pressure between plates cease to be uniform over the section. For the calculation two assumptions have been made:

1 Plane cross-sections remain plane after bending (which is the usual assumption for bending).

2 There is no longitudinal friction between bolts and plates.

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[1] Mechanics Section, Research Department, Westinghouse Electric and Manufacturing Company.

[2] A. B. Field, Journal A.I.E.E., vol. 54 (1915), p. 65.

Contributed by the Applied Mechanics Division and presented at the New England Industries Meeting, Boston, Mass., October 1 to 3, 1928, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

From the second assumption it follows that the stress in any bolt must be constant over its entire length. The bending moment taken by the bolts, therefore, also has to be constant along the rotor.

Fig. 4 shows the bending moment diagram of the rotor, due to its own weight. This bending moment M is taken partly by the bolts M_b and partly by the plates M_p. The magnitude of M_b will be found from the condition that the total elongation of the

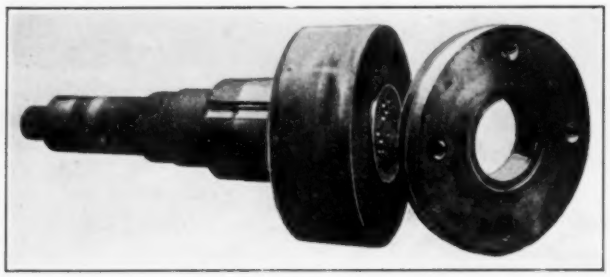


FIG. 2 END FORGING AND PLATE

bolt, due to the bending stresses, is equal to the elongation of the corresponding fibers of the plates at the same location.

The elastic elongation of the bolts can be calculated from its stress very simply. However, for the plates, this is another question.

A stack of plates is compressed more under a certain load than a solid piece of the same size and material under the same load (Fig. 5).

Tests have been carried out in the laboratory on test-pieces of the same thickness, 2 in., and of the same surface finish as the actual rotor plates.

It was found that the first load-compression curve differs considerably from the second and following cyclic curves (Fig. 6). The difference between the second and third loading curves and also between the up-going and down-going branches of these curves is very small. This change in character of the curve is due to microscopic plastic deformations at the contact surfaces. After the first cyclic loading, the unevennesses of the surfaces match fairly well. It was also found that taking the blocks apart and loading them up again produces the "first loading curve," independent of the previous history of the plates. It is evident that, after restacking, the test pieces have undergone a change in relative position, which, however small it may be, is always great in comparison with the unevennesses of the surface. The tests were made on material of three different heat treatments, but the influence of this on the load compression curves was found to be negligible.

The average load-compression curve after permanent set had fully taken place (the third loading curve) is shown in Fig. 7.

If the stack is compressed to a certain stress, and this stress varies in a cyclic manner by a small amount, the corresponding elongation or contraction is determined by the *slope* of the load-deflection curve at that stress.

In this manner the modulus of elasticity of a stack of plates was determined. It was found that this modulus increases with the pressure on the plates, so that it is desirable to have the plate pressure fairly high in order to secure a modulus of elasticity which is not far removed from 30×10^6 lb. per sq. in.

With this in mind, the area of contact between the plates was purposely diminished. The four areas A in Fig. 3 serve ventilation purposes, and the inside circle, which does not add appreciably to the section modulus but decreases the unit pressure between the plates, has been entirely relieved.

Until 1926 all rotor plates were made 2 in. thick. At the present time, however, this practice has been abandoned in favor of 8-in. thick plates, since a metallurgical procedure has been worked out which secures uniform material of good properties and without internal stresses.[3]

The 8-in. plates have the additional advantage of raising the modulus of elasticity of the stack.

In Fig. 7, AB is the compression in the solid material and BC the compression due to the surfaces of the touching plates. With the 8-in. plates no compression tests have been carried out, but since the number of faces is one-fourth of that with the 2-in. plates, it can be concluded that the deformation BD due to this also will be 25 per cent of BC.

Then if E_s and E_2 denote the moduli for a solid piece and for a stack of 2-in. plates, the modulus for the 8-in. plates E_8 can be calculated from

$$E_8 = \frac{4\,E_2 E_s}{3\,E_2 + E_s} \qquad [1]$$

STRESSES IN THE BOLTS

Until a few years ago the bolts of the plate rotors were tightened by means of a wrench of about 4 ft. length, pulled by a crane

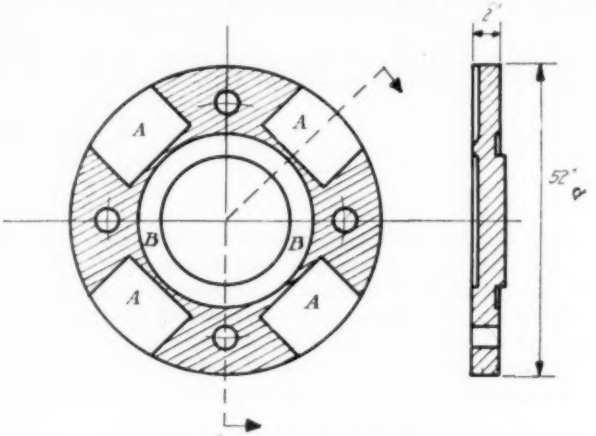


FIG. 3 TYPICAL PLATE

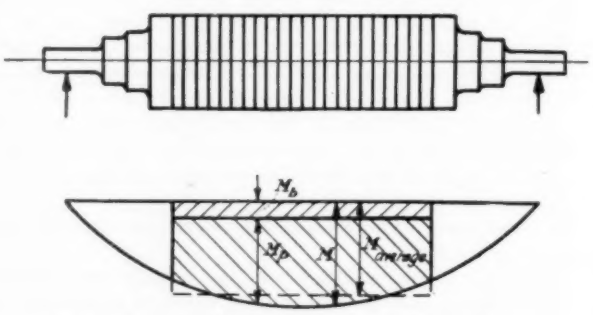


FIG. 4 BENDING MOMENT DIAGRAM OF PLATE ROTOR

overhead. The force with which the crane pulled was read on a spring scale and taken as an indication for the bolt stress. Later the stress in the bolt was not judged by this torque, but determined by measuring the overall length of the bolt.

The torque exerted by the wrench on the nut overcomes friction, which is located partly in the threads of the nut and partly under its seat. If r be the mean radius of the bolt thread, R the mean radius of the seat, f_t and f_s the coefficients of friction in the thread and under the seat, we have for the torque exerted by the wrench

[3] W. J. Merten, Trans. A.S.S.T., 1928, vol. 13, p. 18 ff., annual convention.

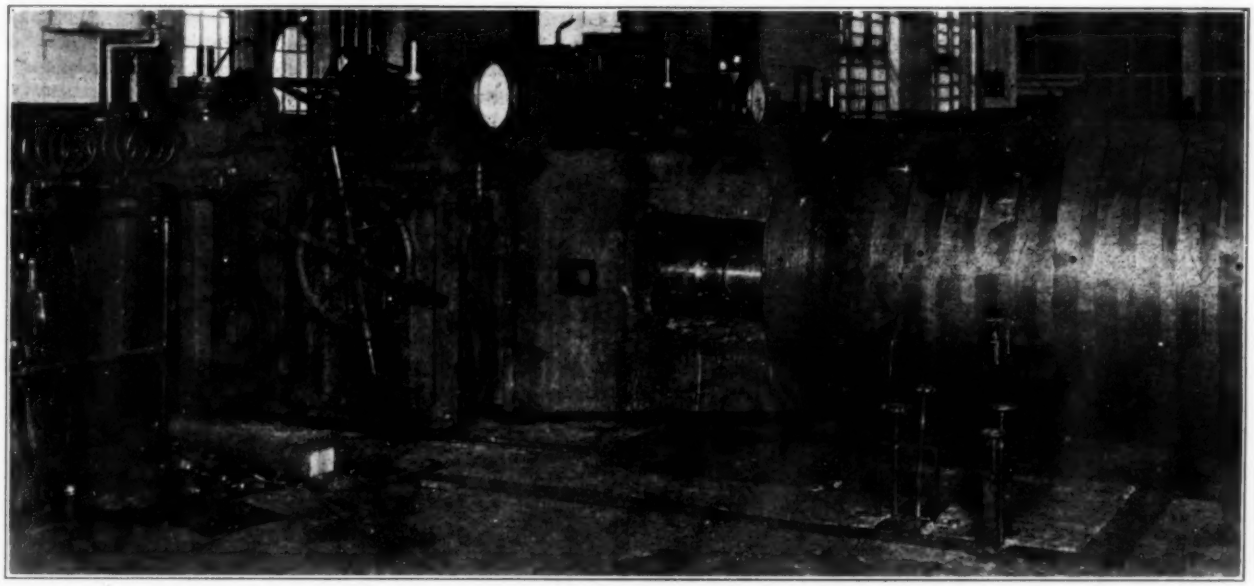


FIG. 8 PULLING MACHINE

$$M = f_t Pr + f_s PR \quad \ldots\ldots\ldots\ldots\ldots\ldots [2]$$

In this equation the coefficient f_t is supposed to include the effects of the shape of thread used. The second term in [2] may under certain conditions gall the seat of the nut; the first term transmits a part of the torque into the bolt. The ratio between f_t and f_s is not known, but it is a reasonable assumption that M is divided about equally over the seat and the threads of the nut. The torque M in one of the rotors was measured to be 6000 lb. × 4 ft. = 288,000 inch-pounds. It is assumed that 144,000 inch-pounds was transmitted into the bolt. The diameter of this bolt was 5 in., so that an extra shear stress of 6000 lb. per sq. in. was imposed.

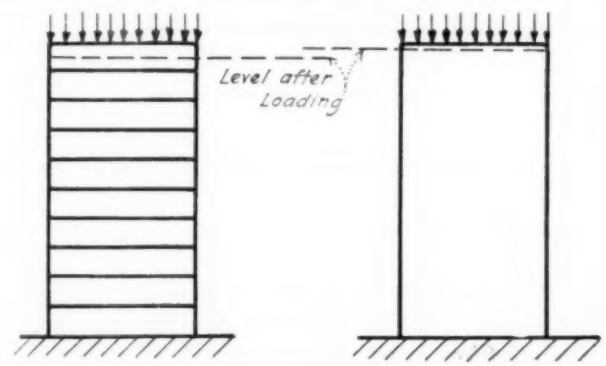


FIG. 5 COMPRESSIBILITY OF A STACK OF PLATES AND OF A SOLID BODY

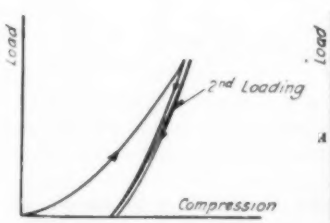


FIG. 6 LOAD-COMPRESSION CURVES FOR STACK OF PLATES

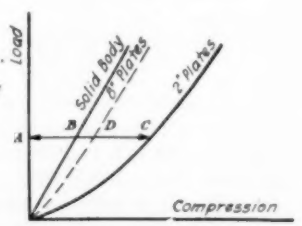


FIG. 7 LOAD-COMPRESSION CURVES AFTER THE PERMANENT SET

This is not a high figure, especially when considered in conjunction with the tension of 40,000 lb. per sq. in., existing in the bolt, but nevertheless it represents an undesirable increase in stress.

In order to avoid this and also to prevent galling of the thread or the seat of the nut, the plate rotors now are assembled by means of a hydraulic pulling machine, suggested by G. M. Eaton. The rotor is first stacked up horizontally and assembled with temporary bolts. One of the bolts is removed and the hole reamed very accurately. The final bolt is put in place and tightened by two men. This is done with all the bolts successively, whereupon the rotor goes to the pulling machine (Fig. 8). Two diametrically opposite bolts are stretched hydraulically, the reaction being applied toward compressing the plates. The two nuts which have become loose due to this are now tightened with a hand-wrench as far as they will go and the hydraulic pull is released. The greater part of the stress in the bolt will remain and be carried by the nut. The rotor is then turned 90 deg., and the operation is repeated for the two other bolts. The stress in the first two bolts is then increased, and so on, going from one to the other in several steps. The stress existing in the bolts after each pulling is measured by a depth gage. Since the pulling machine grabs one end of the bolt, it is not possible to measure its overall length. Therefore, a fairly long hole of small diameter is drilled in the other end of the bolt, in which the depth gage is inserted.

The last pulling brings the bolt stress up to about 45,000 lb. per sq. in., which when the nut is run on and the hydraulic pull is released comes down to about 40,000 lb. per sq. in. This is the usual stress in operation. The material of the bolts is a molybdenum nickel steel with a yield point of over 100,000 lb. per sq. in. It is clear that this method of assembling avoids both galling the threads or the seat of the nut and imposing a torsional stress on the bolt.

STRESS CONCENTRATION IN THE THREADS OF BOLT AND NUT

If a bolt and a nut of equal pitch are loaded in the ordinary manner, so that the bolt is in tension and the nut in compression, the bolt will elongate and the nut will shorten, with the obvious

result that the pitches are not the same any more when loaded. This remark raises a suspicion that the load will be rather unequally distributed over the various threads. Since the bolt threads are a vital part of the rotor construction, an analysis and some tests were made in order to find the stress distribution. The analysis, which is given fully in Appendix No. 2, shows that

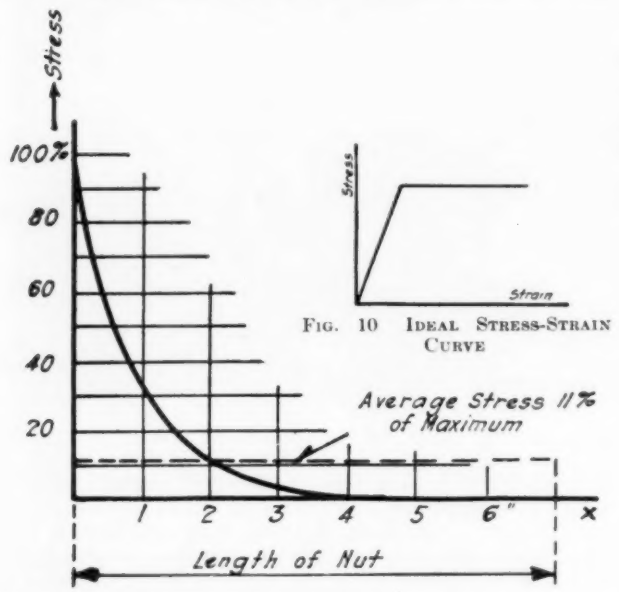


FIG. 9 STRESS DISTRIBUTION ALONG THREADS, SHOWING LARGE STRESS AT SEAT OF NUT

FIG. 10 IDEAL STRESS-STRAIN CURVE

the stress distribution in the threads can be represented by the formula

$$\text{stress} = S = Ce^{-Bx} \quad \ldots\ldots\ldots\ldots\ldots\ldots [3]$$

where x is the distance from the seat of the nut and B is a constant depending on the dimensions of the bolt, nut, and thread.

For a particular bolt and nut of 5 in. diameter and 7 in. length, as used on the rotor, the result [3] is shown in Fig. 9. It was calculated under the assumption of a non-yielding material that the peak stress at the seat of the nut is nine times as large as the "average stress," that is, the stress calculated under the assumption of an equal-load distribution along the bolt. The first two turns of the thread ($^1/_7$-in. bolt length) take 45 per cent of the load. It is clear that this is a case of very pronounced stress concentration. The maximum stress at the first thread is hardly increased by shortening the nut considerably. Since it does not affect the maximum stress whether the nut is 7 in. or 5 in. long, the first result of the analysis was to reduce the length of the nut by 2 in.

However, the derivation underlying the result [3] is based on the assumption that the proportional limit of the material is not exceeded. Now, with such an enormous stress concentration this will happen in practically every existing screw connection. The material in the highest stressed threads will then yield, so that the more remote threads can carry more of the load. If we assume an idealized stress-strain curve as shown in Fig. 10, it is seen that a definite maximum is imposed on the stress, so that the diagram of Fig. 9 is replaced by the one shown in Fig. 11.

Instead of the stress, the ordinates represent the quantity q, that is the load carried by 1 in. length of the bolt, which is proportional to the stress. The part AB follows the law [3]

$$q = q_{\text{yield}}\, e^{-Bx}$$

and is the same as the original curve CD shifted to the right.

The depth of penetration of the yielding into the nut AF can be found from the consideration that the total area of the curve equals the load P_0. Thus from the condition that

$$\text{area } FAEO = \text{area } HCGO$$

the distance FA can be easily calculated.

The yielding of the threads necessary to change the stress distribution from Fig. 10 to Fig. 12 is exceedingly small. Therefore it does not constitute any dangerous condition; the threads merely change their shape slightly so as to readjust themselves to the new circumstances.

It may be thought that if the pitch of the thread on the bolt be made slightly different from that of the nut, such that the bolt when stretched has the same pitch as the unloaded nut, a more

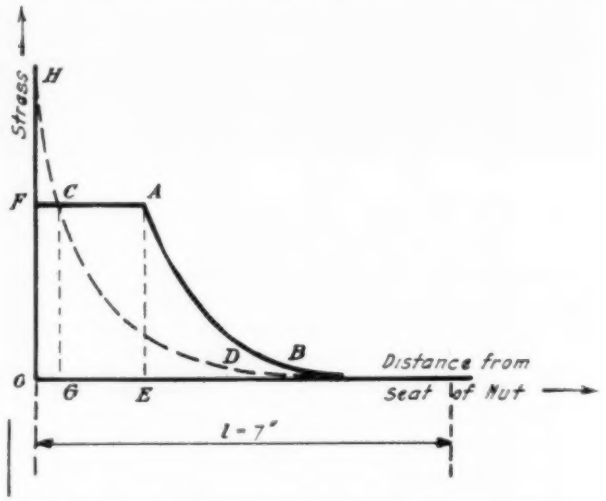


FIG. 11 STRESS DISTRIBUTION IN THREADS AFTER YIELDING

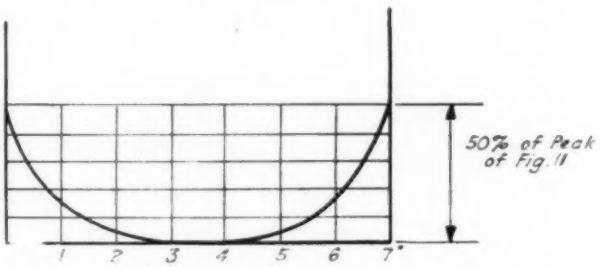


FIG. 12 STRESS DISTRIBUTION ALONG THREADS OF BOLT AND NUT WITH DIFFERENT PITCH

favorable distribution will be the result. This is true to a certain extent; an analysis of this case shows that the maximum stress is cut in half and that two peaks appear at the two extremities of the nut (Fig. 12).

Though this is quite an improvement, it is by far not ideal yet. It is shown in Appendix No. 2 that a perfectly uniform stress distribution along the length of the nut can be obtained by shaping the nut parabolically at the inside.

EXPERIMENTS WITH BOLTS

An experimental confirmation of the result [3] has been obtained with the photoelastic method by Mr. R. V. Baud. A celluloid model of the shape shown in Fig. 13 was very carefully machined and subjected to tension. The two lower "threads" were seen to take nearly the entire load, while the upper

ones were practically not stressed at all. Other tests were made with steel bolts of 1 in. diameter. These bolts were pulled, and the strain was measured directly between a section of the bolt very near to the nut and the seat of the nut by means of the very sensitive Martens extensometer. Though the yield in the first thread was calculated by [3] to occur at a pull of about 7000 lb., no curvature in the stress-strain curve could be detected below 28,000 lb. This confirms that the effect of local yielding in the threads is of such a small magnitude that it cannot be detected on the bolt structure itself. The tests were made with various lengths of nut. No appreciable difference in the yield load was found for nut lengths ranging from $^3/_4$ to $1^1/_2$ times the bolt diameter.

Finally the bolts were pulled to destruction. For nut lengths above 0.6 times the bolt diameter, failure took place in the bolt. With 0.6 times the diameter of the nut the threads of the bolt

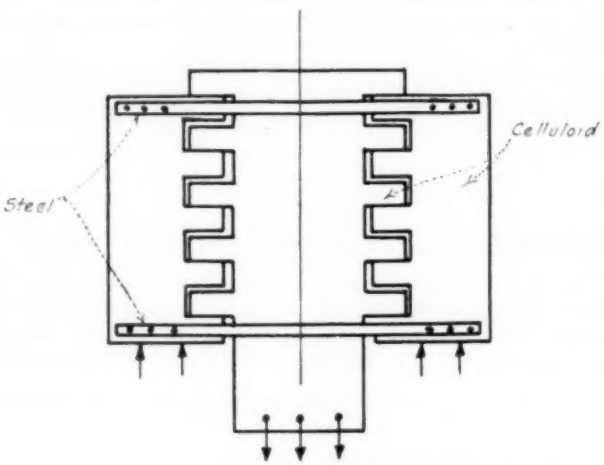


Fig. 13 Celluloid Model for Photoelastic Test

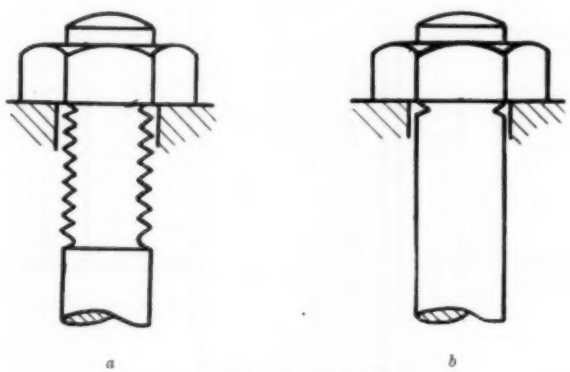


Fig. 14 Diagrams of Bolts Tested

were sheared off. This shows conclusively that in a construction with a nut height of $^3/_4$ times the diameter or more, the weak point is in the bolt rather than in the threads.

The length of the thread under the seat of the nut also has its influence on the final strength and the type of fracture of the bolt. With a sufficient length of nut the construction of Fig. 14*a* will fracture at a slightly lower load than Fig. 14*b*. However, Fig. 14*a* is to be preferred to Fig. 14*b*, since the last construction breaks without hardly any yielding. The fracture looks like that of a brittle material. Fig. 14*a* shows considerable elongation and necking before it fails. The photographs (Fig. 15) show these conditions sufficiently clear. Since a condition as in Fig. 14*b* is particularly dangerous under shock loads, it has been carefully avoided in our plate rotors.

Effect of Centrifugal Force on the Bolts

If there exists some clearance between the bolts and the plates, the bolts will be subjected to bending stresses due to the centrifugal forces acting on them. Let δ be the clearance, that is the

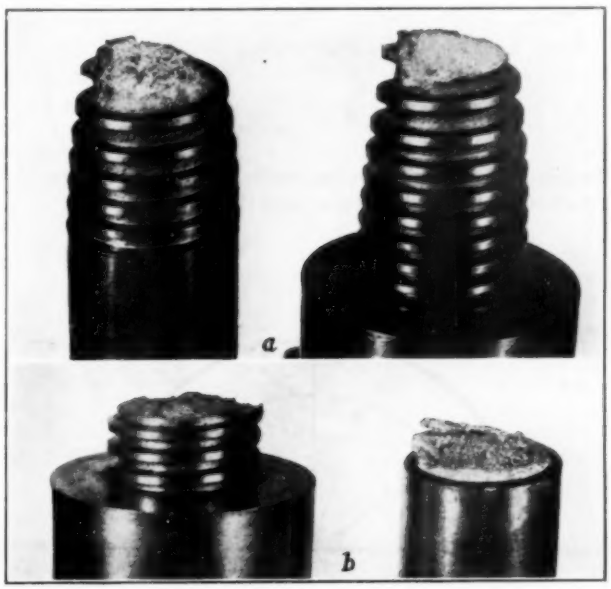


Fig. 15 Types of Fractures of Bolts

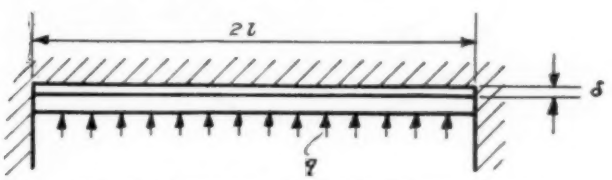


Fig. 16 Rotor Bolt Under Centrifugal Load

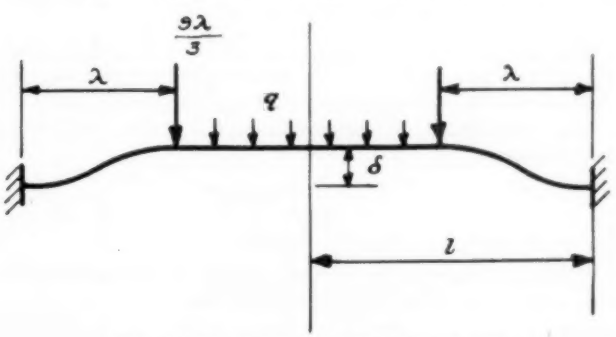


Fig. 17 Reaction From Plates on Bolt

difference between the diameters of the hole in the plate and of the bolt. The most unfavorable case which may happen is that the clearance for *all* the plates is situated on the outside of the rotor. Then there is the problem of a bolt or beam clamped at both ends parallel to a rigid wall (of plates) at a distance from it (Fig. 16) and loaded with a uniform (centrifugal) load of q lb. per in.

At very low speeds, when q is sufficiently small the bolt will not touch the plates at all on the outside. When q becomes larger than $24EI\ \delta/l^4$, the deflection in the middle would become

larger than the wall permits, consequently there will be a point contact between the wall and the bolt there. With increasing q, the reaction of the plate in the middle increases, and the curvature and bending moment of the bolt at that point decrease gradually. This bending moment can be easily calculated to

$$M_0 = \frac{6EI\delta}{l^2} - \frac{ql^2}{12}$$

When this becomes zero, that is when

$$q = \frac{72EI\delta}{l^4}$$

the bolt is again straight in the middle. For larger q's than this a part of the bolt will lie flat against the plate (Fig. 17).

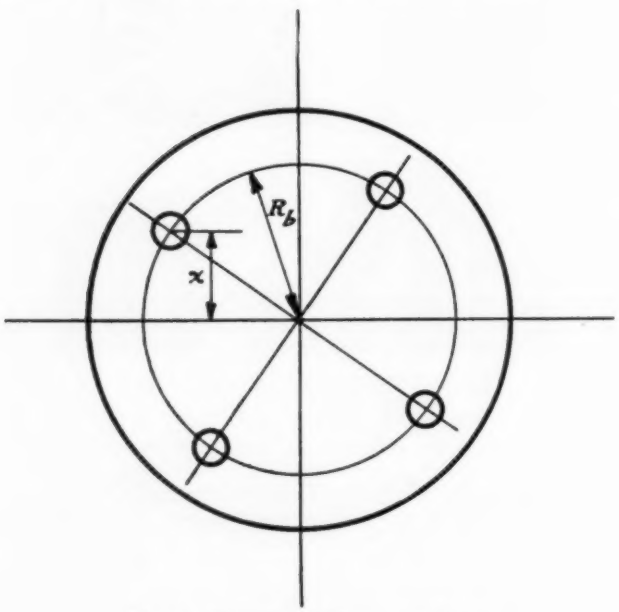


FIG. 18 POSITION OF HOLES FOR BOLTS

The reaction from the plates is calculated to consist of a uniformly distributed reaction q on the straight part and two concentrated forces $q\lambda/3$ on the extremities of the straight part.

The "free" length will diminish when q increases, the relation being expressed by

$$\lambda = \sqrt[4]{\frac{72EI\delta}{q}} \quad \ldots\ldots\ldots\ldots\ldots\ldots [4]$$

The maximum bending moment appears at the clamped section at the end of the bolt and has the value

$$M_{max} = \sqrt{2EIq\delta} \quad \ldots\ldots\ldots\ldots\ldots\ldots [5]$$

It has been calculated that for a clearance equal to 0.004 in. at a speed of 1800 r.p.m. for the usual construction, this bending moment causes a stress of 22,000 lb. per sq. in. It should be noted that the underlying assumptions of the calculation are excessively severe. The condition of clamping at the end will be only partly fulfilled. Then in case the clearance is all on the outside, the centrifugal force acting on the bolt and nut will overcome the friction under the seat of the nut and slip the bolt so that the clearance shifts to the inside of the rotor.

At any rate this fairly high-stress figure points to the desirability of small clearances in the bolt holes and of an overspeed run to shift these clearances to the inside. Therefore, it is practice to ream the bolt holes in the (temporarily) assembled rotor accurately with 0.004 in. clearance.

After final assembling and machining, the rotor is given a run with 20 per cent overspeed, which will overcome the friction under the nut seat and create a condition in which hardly any stresses, due to centrifugal action on the bolts, can exist.

CONCLUSIONS AND ACKNOWLEDGMENT

The various problems regarding stresses and deformations characteristic of plate rotors are discussed in this paper. It is shown that, in bending, the stiffness can be calculated from some test data on the compressibility of stacks of plates. This stiffness is hardly less than with solid rotors. Various angles of the problem of the stress distribution in the bolts are reviewed, and it is shown that the stresses in any part of the structure are kept well below safe limits by means of special assembly methods in the shop.

The author is indebted to Mr. W. F. Shirk for the compressibility tests on plates; to Mr. R. V. Baud for the photoelastic test, and especially to Dr. S. Timoshenko, not only for the analysis of the bending rigidity of the rotors, but more so for many valuable suggestions and personal encouragement.

Appendix No. 1—Bending of the Plate Rotor

In the following analysis the subscripts $_b$ or $_p$ will refer to bolts and plates, respectively. Let e be the unit elongation of a bolt, at a distance R_b from the neutral axis (Fig. 18) due to bending only. The unit elongation of the bolt at any other location x will then be $\frac{x}{R_b} e_b$. Since the stress in the bolts becomes $\frac{x}{R_b} e_b E_b$, the bending moment M_b is

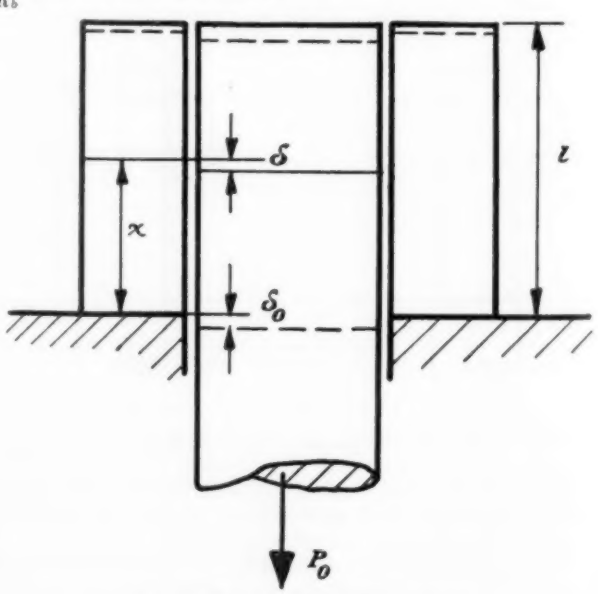


FIG. 19 BOLT AND NUT CONNECTION

$$M_b = \int \left(\frac{x}{R_b} e_b E_b\right) x\, dA = \frac{eE_b}{R_b} \int x^2\, dA = \frac{e_b E_b I_b}{R_b} \ldots [6]$$

where I_b is the moment of inertia of the cross-section of the four bolts with respect to the neutral axis.

In the same manner an equation for the plates can be obtained

$$M_p = \frac{e_p E_p I_p}{R_b} \quad \ldots\ldots\ldots\ldots\ldots\ldots [7]$$

where e_p is the unit elongation in the plates at a distance R_b away from the neutral axis. It should be noted that e_b is constant but that e_p varies along the length of the rotor. The total elongation of a bolt has to be equal to the total elongation of the plates in the same location. Thus

$$e_b l = \int_0^l e_p \, dx$$

Substituting (1) and (2) in this relation, we obtain

$$\frac{M_b R_b}{E_b I_b} l = \frac{R_b}{E_p I_p} \int_0^l M_p \, dx$$

Since $M = M_p + M_b$ and M_b is independent of x, this can be written as

$$M_b = \frac{E_b I_b}{E_b I_b + E_p I_p} \cdot \frac{\int_0^l M dx}{l} \quad \ldots\ldots\ldots\ldots [8]$$

or

$$M_b = \frac{E_b I_b}{E_b I_b + E_p I_p} \cdot M_{\text{average}} \quad \ldots\ldots\ldots\ldots [8a]$$

For the smaller rotors, where the ratio of stack length to overall

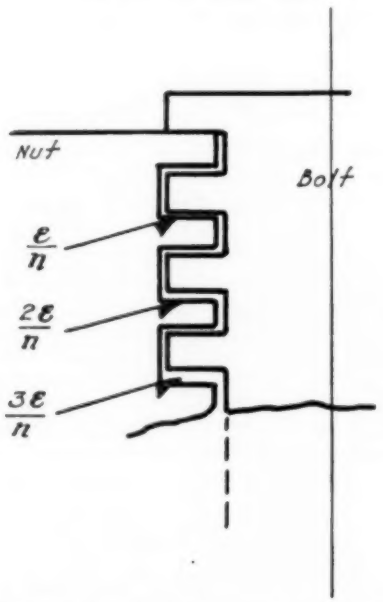


Fig. 20 Bolt and Nut With Different Pitch, Unstressed

length is small, the moment M will not vary much along the stack of plates. In that case we may make the approximation $M = M_{\text{average}}$. Then the moment M_p carried by the plates is also constant, and the central part of the rotor will be bent circularly. We do not need to calculate the plates and bolts separately any more, but may find the deflection as if it were a solid rotor with a stiffness

$$EI = I_b E_b + I_p E_p \quad \ldots\ldots\ldots\ldots\ldots [9]$$

However, for longer rotors, as they are being built now and especially for those which probably will be built in the future, this simplification cannot be made. Then the deflection has to be found graphically. Formula [8] is correct, so that M_p can be found as is shown in Fig. 4. With this moment we construct graphically the deflection curve of a bar with a stiffness of $I_p E_p$.

It will be noticed that even this is only approximate. The bolts, transmitting a constant moment M_b, will bend circularly, while the plates will deform in some other curve. Since, however, the stiffness against bending of the plates is about five times that of the bolts, it is reasonable that the plates will force the bolts in the same deformed shape as they assume themselves. This entails bending stresses in the individual bolts, which, however, are so small as to be entirely negligible.

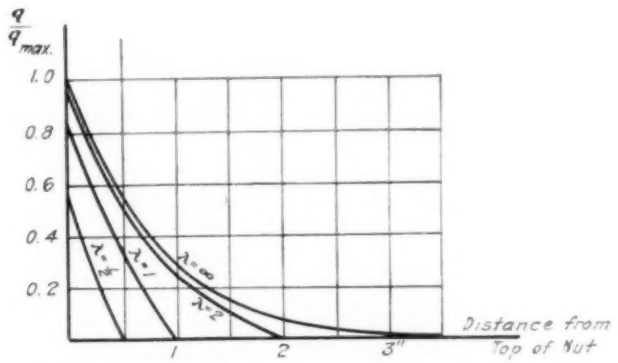


Fig. 21 Stress in Threads for Different Bolt Pulls

The foregoing theory holds only in case the compressive stresses in the plates on the convex side of the rotor do not decrease to a small value, and certainly it will cease to be applicable when these stresses become zero, so that the rotor gaps. However, with the size of rotors now used or under construction, this limit is by far not yet reached.

The periods of natural vibration in bending of a plate rotor, which we want to know in connection with the determination of the critical speeds of operation, are found graphically on the basis of a deflection curve constructed in this manner.

Appendix No. 2—On the Stress Distribution in Screw Threads

Let, in Fig. 19

x = distance from seat of nut
P = tensile force in bolt at any section $P = f_1(x)$
q = force transmitted by the thread from bolt to nut per unit length of the nut $q = f_2(x)$
A_b, A_n = cross-sectional areas of bolt and nut
α = spring constant of the thread, which is the displacement in inches between two originally coinciding cross-sections of bolt and nut, due to unit q.

The total force P in the bolt which is P_0 outside the thread will diminish gradually with increasing x, till for $x = l$, $P = 0$.

The decrease in P is transmitted to the nut, thus

$$\frac{dP}{dx} = -q \quad \ldots\ldots\ldots\ldots\ldots [10]$$

The elongation of an element dx of the bolt due to the tensile force P is

$$\frac{P dx}{A_b E}$$

Due to the loading, a cross-section x of the bolt comes down the amount

$$\delta_0 - \int_0^x \frac{P dx}{A_b E}$$

Due to the compression in the nut, a section x of it gets a downward displacement of

$$\int_0^x \frac{P dx}{A_n E}$$

The relative displacement of two initially coinciding sections of bolt and nut therefore is

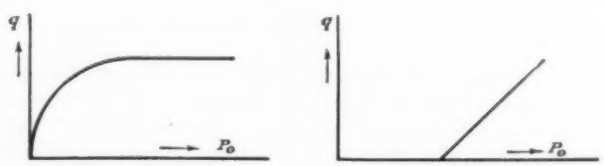
$$\delta = \int_0^x \frac{P dx}{A_n E} + \int_0^x \frac{P dx}{A_b E} - \delta_0$$

This deflection is taken up by the threads, which can be expressed as $-\alpha q$. Thus

$$\int_0^x \frac{P dx}{A_b E} + \int_0^x \frac{P dx}{A_n E} + \delta_0 = -\alpha q$$

Differentiating this equation, and remembering (10), we get:

$$\frac{d^2P}{dx^2} = \frac{P}{\alpha E}\left(\frac{1}{A_b} + \frac{1}{A_n}\right)$$



a b

FIG. 22 LOADING ON TOP AND BOTTOM THREAD AS FUNCTIONS OF BOLT PULL, DIFFERENT PITCH

a shows the loading on top thread and *b* shows the loading on bottom thread.)

or introducing a constant B, such that

$$B^2 = \frac{1}{\alpha E}\left(\frac{1}{A_b} + \frac{1}{A_n}\right) \quad \ldots\ldots\ldots\ldots [11]$$

the equation becomes

$$\frac{d^2P}{dx^2} = B^2 P$$

Its solution is

$$P = Ae^{Bx} + Be^{-Bx}$$

or with the boundary conditions

$$x = 0, P = 0, \quad \text{and} \quad x = l, P = P_0$$

we have

$$P = P_0 \frac{e^{Bx} - e^{B(2l-x)}}{1 - e^{2Bl}} \quad \ldots\ldots\ldots\ldots [12]$$

Now for the bolts with Acme thread as used in these rotors, the flexibility α was calculated by considering the threads as cantilever beams of variable cross-section. Deformation due to bending and due to shear was considered. In this manner it was found for the value of B: 1.27 inches^{-1}. For any length of nut over 4 in. it is seen that only a very small error is made by writing, instead of [12]

$$P = P_0 e^{-Bx} \quad \ldots\ldots\ldots\ldots [13]$$

The load on the thread and consequently the stress is characterized by q

$$q = \frac{-dP}{dx} = P_0 B e^{-Bx} \quad \ldots\ldots\ldots\ldots [14]$$

which is the same as [3] and is shown in Fig. 9. If the pitches of bolt and nut are slightly different, a similar analysis can be applied; only for simplification it is desirable to measure downward from the top of the nut, and to denote by δ_0 the distance between the bolt and nut sections at the *top* of the nut, instead of at the seat (Fig. 19).

Let ϵ be the difference in pitch of bolt and nut per unit length, the pitch of the nut being greater than that of the bolt. If n be the number of threads per unit length, the clearances between consecutive threads in unstressed conditions are ϵ/n, $2\epsilon/n$, ...

The clearance at x inches from the top in unstressed condition is ϵx.

The load q on the threads becomes

$$q = \frac{1}{\alpha}(\delta - \epsilon x) \quad \ldots\ldots\ldots\ldots [15]$$

which formula holds only for $q > 0$.

The displacement δ between two sections is

$$\delta = \delta_0 + \frac{1}{E}\left(\frac{1}{A_b} + \frac{1}{A_n}\right)\int_0^x P dx \quad \ldots\ldots\ldots\ldots [16]$$

Differentiating (15) and substituting (16), the differential equation becomes

$$\frac{d^2P}{dx^2} = BP - \frac{\epsilon}{\alpha} \quad \ldots\ldots\ldots\ldots [17]$$

the general solution of which is

$$P = Ae^{Bx} + Be^{-Bx} + \frac{\epsilon}{\alpha B_2} \quad \ldots\ldots\ldots\ldots [18]$$

When P increases from zero up, more and more threads will come in contact and will help carry the lead. We denote by λ the length from the top of the nut down, where the threads are in contact. From the derivation of [18] it is evident that it holds only for

$$0 < x < \lambda$$

For determining the arbitrary constants A, B, and λ we have the three boundary conditions

$$x = 0 \qquad P = 0$$
$$x = \lambda \qquad P = P_0$$
$$x = \lambda \qquad \frac{dP}{dx} = 0$$

This leads to

$$P = \frac{\epsilon}{\alpha B_2}\left[1 - \frac{e^{Bx}}{1 + e^{2Bx}} - \frac{e^{-Bx}}{1 + e^{-2Bx}}\right] \quad \ldots\ldots\ldots\ldots [19]$$

and

$$\cosh B\lambda = \frac{\epsilon}{\epsilon - \alpha B^2 P_0} \quad \ldots\ldots\ldots\ldots [20]$$

The maximum value of the thread pressure $q = \frac{dP}{dx}$ occurs at $x = 0$, the top of the nut. Letting this maximum value be q_0, we have from [19]

$$q_0 = \frac{\epsilon}{\alpha B} \cdot \tanh B\lambda \quad \ldots\ldots\ldots\ldots [21]$$

Eliminating λ between [20] and [21] we have the following relation between q_0 and P_0

$$q_0^2 = \frac{2\epsilon}{\alpha} \cdot P_0 - B^2 P_0^2 \quad \ldots\ldots [22]$$

For a certain value of P all the threads along the length of the nut will be in contact ($\lambda = l$). For the conditions prevailing in turbo bolts, $q = \frac{dP}{dx}$, calculated from [17] becomes very nearly equal to

$$q_0 = \frac{\epsilon}{\alpha B} e^{-Bx} \quad \ldots\ldots [23]$$

The relations [19], [20], [21], and [22] have been plotted to curves. Figure 21 shows the pressure distribution for various pulls.

It is seen that the peak pressure does not increase much by increasing the contact λ from 2 in. to 7 in. (or ∞ in.). The curve for $\lambda = \infty$ is represented by [23]. As soon as all clearances have been taken up, the Equations [15] to [23] cease to apply and the conditions expressed by the Equations [12] to [14] prevail. From then on, therefore, pressure starts to be built up from the bottom end of the nut following the law [11], which is the same as [23] except that it slopes down from the bottom of the nut instead of from the top. When P_0 reaches the value $\frac{2\epsilon}{B^2\alpha}$ the peaks at the two ends of the nut become equal (Fig. 12). The most favorable pitch difference at which this happens therefore is

$$\epsilon = \frac{P_0 \alpha B^2}{2} \quad \ldots\ldots [24]$$

The relation between the pull P_0 and the pressure (or stress) q_0 is shown clearly in Fig. 22. Fig. 22*a* represents the stress at the top of the nut and Fig. 22*b* the stress at the bottom of the nut for increasing P_0.

The question may come up whether a bolt can be made in which the pitch varies along its length according to a certain law, such that the stress is distributed uniformly all along the length of the nut. In order to investigate this, let $\epsilon = f(x)$ be the vertical clearance between the threads of bolt and nut both in the unstressed state. At the top of the nut ($x = 0$) the clearance $\epsilon = 0$, it increases when going down. Again x is now measured from the top of the nut down.

The relative displacement between bolt and nut at a section x in this case is

$$\delta = \delta_0 + \frac{1}{E}\left\{\frac{1}{A_b} + \frac{1}{A_n}\right\} \int_0^x P dx$$

This relative displacement is taken up partly by the clearance ϵ and partly by the elastic deformation αq

Thus

$$\epsilon + \alpha q = \delta_0 + \frac{1}{E}\left\{\frac{1}{A_b} + \frac{1}{A_n}\right\} \int_0^x P dx$$

It is desired to have uniform stress distribution, in other words to have $q = \text{constant} = \frac{P_0}{l}$ and $P = P_0 \frac{x}{l}$.

Substituting this in the foregoing equation, integrating and noting that $\delta_0 = \alpha q$, it is found that

$$\epsilon = \frac{P_0}{2El}\left\{\frac{1}{A_b} + \frac{1}{A_n}\right\} x^2 \quad \ldots\ldots [25]$$

in other words, the clearance has to increase parabolically.

Now suppose a bolt and nut fit exactly together without any clearances whatsoever. If the inside diameter of the nut could be increased, it would be easy to create clearances. Then it is clear that by giving the inside of the nut the shape of a paraboloid of revolution, the clearance distribution [25] can be realized.

For the 5-in. diameter Acme-threaded bolts used in the plate rotors, loaded to a stress of 40,000 lb. per sq. in. and equipped with nuts of 5-in. length, the distance a in Fig. 23 was calculated to

$$a = 0.021 \text{ in.}$$

This can be machined by guiding the tool by means of a suitable

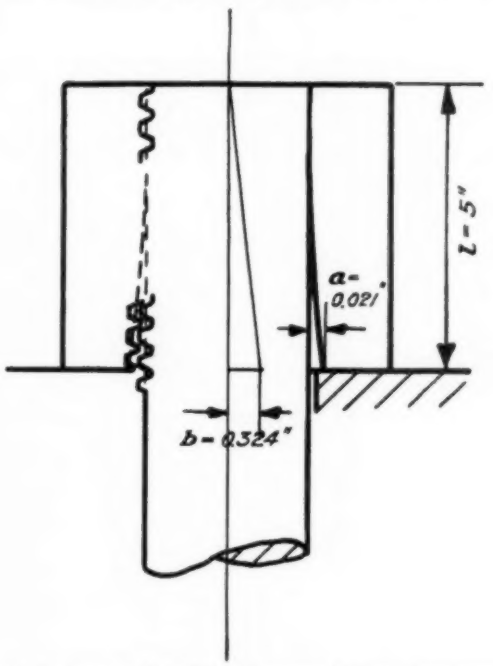


FIG. 23 PARABOLIC NUT WITH UNIFORM STRESS DISTRIBUTION

templet. Since for such small ratios a/l a paraboloid and a hyperboloid are practically coinciding, another method of machining would be by feeding the tool along a generatrix of the hyperboloid. This does not necessitate a templet, and the feed is also much greater ($b = 0.324''$ in Fig. 23). Though this latter method is probably easier to carry out, it has the disadvantage that the *shape* of the thread varies along the nut, due to the fact that the plane of the normal section of the tool does not contain the axis of the nut, but is only parallel to it. Though this change in shape is very small, it has been calculated that it annihilates completely the desired effect on the stress distribution.

Of all these calculations it can be said that they are true only if the machining errors in making the thread are small in comparison to the elastic deformations. With the present state of the art this is not possible as a commercial proposition. Therefore, it was decided not to use the parabolical nuts for the present time, because they represent no improvement when machining errors of 0.0001 in. are possible. The natural ductility of the material takes care automatically that excessive stresses are distributed more uniformly along the length of the bolts.

Discussion

B. A. BEHREND.[4] It may be of interest to recall the reasons which prompted me to propose in 1909 the plate-rotor construc-

[4] Consulting Engineer, Wellesley Hills, Mass. Mem. A.S.M.E.

tion which we developed in the Westinghouse Company and which is now their standard type for all the larger sizes of their turbo-generators.

Twenty years ago, when this work was initiated, large carbon and nickel-steel forgings were very popular. Great stress was laid on the elastic limit and the ultimate strength. The question of ductility of the material was considered of little importance. This state of affairs underwent a change when a number of notable failures called attention to the importance of ductility as measured roughly but most conveniently by the elongation in a 2-in. test bar and by its reduction of area. Another rather old-fashioned but very useful way of getting a conception of the ductility of the material is the cold-bending test in use for boiler plate, which should accompany all conventional tests.

In reviewing the opposition to this construction it is now curious to recall that the importance of the direction in which the test bar was taken from the material could hardly be brought home to the designers and makers. It had been customary to take tests from a small projection of the forging, called the coupon, altogether without regard to the direction in which the material was subjected to stress. We began to call for "radial" and "tangential" test bars, instead of the conventional "axial" tests, and the result of these tests showed very clearly the unsuitability of large forgings, whether of nickel steel or carbon steel, for radially and tangentially stressed rotors.

In the designs as more recently adopted by the Westinghouse Company, there is to be noted a departure from the original design on one salient point. It has been the underlying principle in the designs worked out by A. B. Field and myself to use rolled or comparatively thin forged plates without a hole or other perforation in the center. Such plates were made for these designs on different occasions by the armor-plate department of the Carnegie Steel Company, and it was demonstrated that a small percentage of nickel about one to one and a half per cent greatly improved the plate over plain carbon steel. What excellent results were obtained in this manner has been discussed elsewhere.[5]

Mr. Den Hartog's paper discusses principally the elastic theory of the bolt construction, to which it is a most valuable contribution. In the main I concur with its statements. However, the use of thick forged plates with a large hole in the center appears to me a retrograde step, for two reasons. First, it sacrifices the advantage obtained by a continuous unperforated plate by increasing the stress at the center to about double the stress for the unperforated plate; secondly, mandrel-forged thick plates have, by the very nature of the forging process, better properties axially where they are not required than radially or tangentially, where they are required.

As it has been possible to secure rolled plates with excellent properties, rolled at right angles, if desired, of two or three inches thickness, it would seem deplorable to give up two of the best features of the plate-rotor construction while retaining those features which must be considered the least desirable features of the design which I originated. I should be tempted to consider a solid rotor forging the equal of or superior to a rotor made up of rings of great thickness.

A solid rotor forging with a large central hole has the same radial and tangential stresses as the rotor construction described in the paper. The heat treatment can be carried through in the same manner, as the hole through the center permits this readily. A periscope can be used for inspection of the inside, thus the probability that a flaw may remain undiscovered is likely to be the same in both types. I used to state that the plate rotor as developed by Mr. Field and myself greatly reduced the chance that a flaw would remain undiscovered. Assuming a 2-in. plate and a 100-in. solid rotor, the probability of a flaw remaining undiscovered in the two types is 1 to 2500. This brings home the argument in behalf of the plate construction. Part of this great benefit is lost by going to much thicker plates unless these plates are forged axially of an alloy steel. This latter appears essential to safety in view of the tendency of carbon-steel forgings to develop almost invisible hair cracks.

[5] B. A. Behrend, Trans. A.I.E.E., 1917, p. 883 et seq.

A. L. Kimball.[6] The writer is glad to see that Mr. Behrend has taken up the cause of the solid rotor in single units, because one thing the writer was going to take exception to in this paper is the statement, "For the largest machines of the present time single-piece forgings cannot be bought from the steel companies, so that the rotor has to be made up of several pieces," and the statement, "Better material can be obtained in plate form than in large solid forgings."

The author mentioned among a number of large machines of this type which were built the 100,000-kva. machine which was being built by the General Electric Company. This machine, except for a pair of comparatively small end pieces, is to be made of a single solid forging. The same is true of a 160,000-kva. machine now in the shop. Experienced engineers still use solid forgings, and we have an instance of one of the very largest machines ever built made of such a forging.

Author's Closure

It is particularly gratifying to have a discussion from Mr. B. A. Behrend, the originator of the plate-rotor construction. Mr. Behrend gives a valuable historical review of the development of the rotor and a synopsis of the reasons which led him to propose a plate rotor rather than a solid one as early as 1909. The construction has persisted in its original form for more than 25 years, which certainly is a great tribute to Mr. Behrend's foresight.

However, when the size of the rotors became about three times as large as in 1909 some slight modifications became desirable on account of troubles experienced. It is to two of these modifications that Mr. Behrend takes exception—first, that a hole is made in the plates and, second, that the thickness of the plates is increased.

It is true that the stress at the inside of the plate is increased to more than twice its value due to the influence of the central hole, but since the stress at the roots of the teeth is still higher, this is of no importance. After all, it is only the weakest link in a chain that determines its strength. On the other hand, there are two distinct advantages of the central hole, for which it was adopted, namely, the possibility of a more thorough heat treatment and a superior method of ventilating the rotor. Especially the latter point is of fundamental importance, but since it does not belong to our subject we refer to a very complete paper by S. L. Henderson and C. R. Soderberg, Trans. A.I.E.E., 1928, vol. 47, pp. 549–578.

Mr. Behrend states that the properties of the material in 8-in. plates are necessarily inferior to those of the thinner ones. If that were the case, the Westinghouse Company certainly would have hesitated very much in adopting the thicker plates. Tests have been made showing that in the 8-in. plate the yield point was 65,000 lb. per sq. in., the elongation 25 per cent, and the reduction in area 50 per cent, which is better than anything obtained before with rolled 2-in. plates. These tests were consistent and showed practically no difference between the tangential and radial directions. The behavior of these plates in regard to residual stresses was also far superior to the 2-in. plates

[6] Research Engineer, General Electric Company, Schenectady, N. Y. Assoc. A.S.M.E.

or to solid forgings. The material is more expensive than that of the rolled 2-in. plates, on account of the heat treatment; but this is easily compensated for by the saving in machining obtained by reducing the number of contact surfaces.

Therefore, though Mr. Behrend's discussion has brought out many valuable points, I cannot agree with this view that the latest developments are a "retrograde step," but I am rather inclined to believe that they are a positive step in the direction of improvement.

Mr. Kimball's discussion takes exception to the two opening statements of my paper. The first statement still stands as it is, as Mr. Kimball agrees to himself in the latter part of his discussion. The "comparatively small end pieces" mentioned are equivalent to the end forgings shown in Figs. 1 and 2 of my paper. For the rotor shown in Fig. 1 the end forgings weigh 22,000 lb. each, while the plates together weigh 98,000 lb. The General Electric Company builds its large rotors of three pieces; two end forgings and one central forging taking the place of our plates. The three pieces are joined by means of a shrink connection and a comparatively great number of short bolts. Since such a joint is not easy to make, and since the smaller rotors of the General Electric Company consist of a single forging, it is evident that it was not possible to get a single forging for the larger sizes. In the case of Fig. 1 it would mean the difference between 98,000 lb. and 142,000 lb.

The paragraph in my paper that is questioned should possibly read: "The Westinghouse Company has not been able to secure from any steel manufacturer in the country single solid-rotor forgings of the same quality as obtained in its rotor plates."

APM-51-2

Tests on Belleville Springs by the Ordnance Department, U. S. Army

By D. A. GURNEY,[1] WASHINGTON, D. C.

BELLEVILLE springs are dish-shaped springs, as shown in Fig. 1, which have long been used where relatively large load capacity, small deflection, and a limited closed height are necessitated by other controlling features of design. Recent demands for larger Belleville springs than those previously used disclosed the absence of any reliable design data; and the investigation here presented was undertaken to supply this lack of information. After experimentation and investigation, the following formula was evolved for purposes of design:

$$\delta = \frac{4W}{3\pi Et^3}\left(R_2^2 - R_1^2 + R_2R_1 \log_e \frac{R_2}{R_1}\right) \ldots\ldots\ldots [1]$$

where

δ = deflection, in
W = load, lb.
E = modulus of elasticity, lb. per sq. in.
R_1 = inside radius, in.
R_2 = outside radius, in.
t = thickness, in.

A method of plotting this formula in such a way that designs may be made directly from the plot is given in Fig. 2. Springs may be designed in accordance with this formula and plot; and accurate design is thereby possible with a maximum error of 7 per cent.

The investigators also found that springs made of SAE 6145, which was the steel used throughout these tests, would not be stressed above the elastic limit if designed with the following ratios of outside radius to thickness and with the corresponding dish angles:

$R_2 = 6t$, dish angle = 4 deg.
$R_2 = 9t$, dish angle = 5 deg.
$R_2 = 12t$, dish angle = 6 deg.

where the dish angle is the angle between the sides of the spring and the plane on which it rests. No spring should be made with a ratio of R_2/t less than 6 and a dish angle of 6 deg. is sufficient for any spring.

In addition, the following conclusions were reached after correlation of the results of tests, of mathematical analysis (not here given), and of a photoelastic analysis made by Prof. Paul Heymans of Massachusetts Institute of Technology:

a The maximum compressive fiber stress occurs at the upper inner edge and the maximum tensile fiber stress occurs at the lower outer edge. The former is numerically the greater.

b The neutral axis is about one-third the width of the spring from the inner edge and does not move appreciably during flattening.

c Rotation of lines normal to the surface, which occurs during flattening of the spring, causes most of the stress. This rotation moves the upper surface inward and the lower surface outward.

d The compressive stress due to the sum of this rotation and the radial stress at the inner edge is the limiting stress.

e The upper surface of the spring is under tangential compression throughout.

f Considering a section midway between the inner and outer edges, the resultant tangential strain (top and bottom surfaces) is zero at this point. Inside this point the tangential strain is compression, and outside of it, tension. The radial strains are compression throughout, small at the edges and maximum at the point where the tangential strain is zero. Inside this point both the radial and tangential strains are compression, and this part of the spring tends to remain conical in flattening. Outside this point, radial strain is opposed to tangential, more bending takes place, and the surface becomes warped under load.

g Just before the spring becomes flat the outer point of application of the load moves inward because of this warping, or, in other words, the effective width of the spring is decreased: this explains the shape of the load-deflection curves just before the spring becomes solid.

h Grinding off the surface of a spring decreases very rapidly the load that a spring will carry.

i Grinding off either the inner or outer edge increases this load much less rapidly.

j A relatively thick spring has a straight load-deflection curve until solid height is approached, but the allowable dish is small.

k A relatively thin spring will not have a straight-line load-deflection curve, but the load per unit deflection will drop off as the spring flattens until this curve breaks sharply upward on approaching solid height.

l The load per unit deflection varies as the cube of the thickness.

m Springs of various sizes but of the same proportions obey the following laws:

1 The load per unit deflection varies as the first power of the size ratio.

2 The load at solid height varies as the square of the size ratio.

By making use of these relations, springs of various sizes may be reduced to a unit basis and compared.

n In specifying tests for Belleville springs, loads at two different heights should be given in order to obtain the proper load-deflection rate, or the free height may be specified together with the deflection per spring for a given load. This load should be from two-thirds to three-quarters of the spring capacity. The working load should not exceed two-thirds of the flattening load.

History of Belleville Springs

Belleville springs were used by the Ordnance Department as counter-recoil springs for 7-in. howitzer and 12-in. mortar carriages designed about 1890. The production of these springs at that time proved to be a difficult problem. No private factory would undertake to produce them, and extensive experiments were necessary before the Watertown Arsenal was able to make those necessary for the carriages then under construction. They were finally replaced by helical springs, which were found to be cheaper and easier to fabricate.

During the World War the Ordnance Department obtained

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[1] Ordnance Engineer, U. S. Army.

Contributed by the Sub-Committee of the A.S.M.E. Special Research Committee on Mechanical Springs and presented at the Applied Mechanics Session at the New England Industries Meeting, Boston, Mass., October 1 to 3, 1928, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

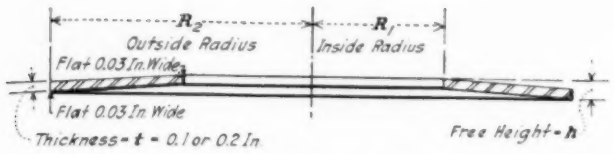


FIG. 1 SHAPE OF BELLEVILLE SPRINGS TESTED FOR DEVELOPMENT OF FORMULA AND DATA

(By arranging these springs in stacks and by alternating them so that two concave and two convex sides are adjacent, almost any desired spring characteristics may be obtained.)

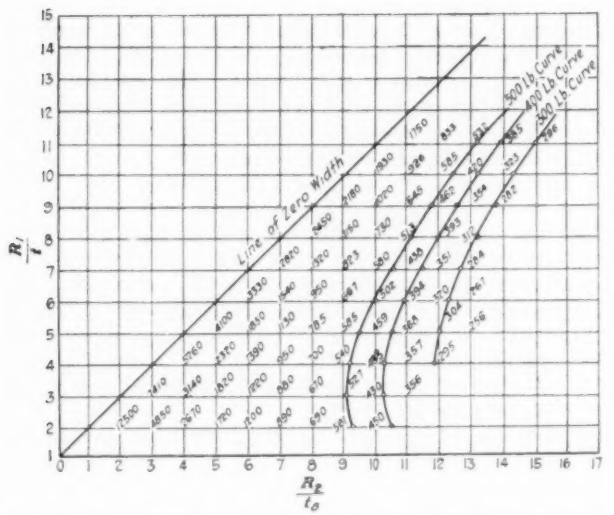


FIG. 2 CURVES OF FORMULA [1] FOR SPRING DEFLECTION PLOTTED FOR USE OF DESIGNERS

(For these curves, t = 0.1 in. and E = 23,300,000. Ordinates are values of R_1/t and abscissas are values of R_2/t. The curves show values of load per 0.01 in. of deflection. To use the plot to find the load per 0.01 in. of deflection for any spring, divide the inside and outside radii of the given spring by the thickness; and, using these values as ordinate and abscissa, respectively, locate the point. By interpolation between curves find the load per 0.01 in. deflection for a spring of unit size, t = 0.1 in. Multiply this value by the ratio of required spring to unit spring to obtain the required result. If the modulus of elasticity is different, i.e., if a different spring steel is used, this result should be corrected in the direct proportion of the two moduli.)

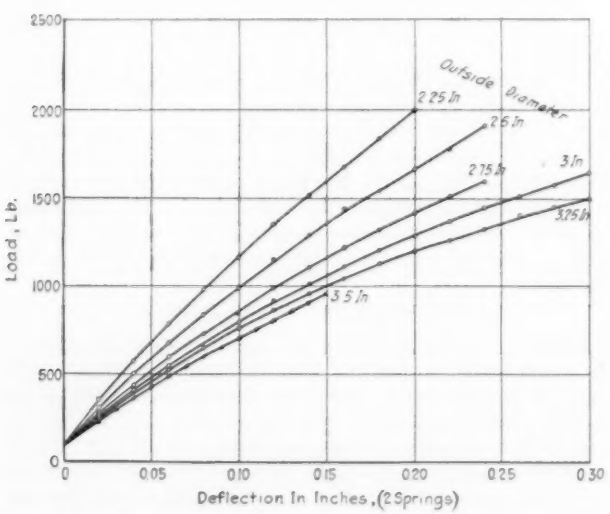


FIG. 3 DEFLECTION CURVES OF BELLEVILLE SPRINGS

(SAE 6145 chrome-vanadium steel springs 0.1 in. thick and 1 in. inside diameter. E = 23,300,000 and tensile strength = 102,000 lb. per sq. in.)

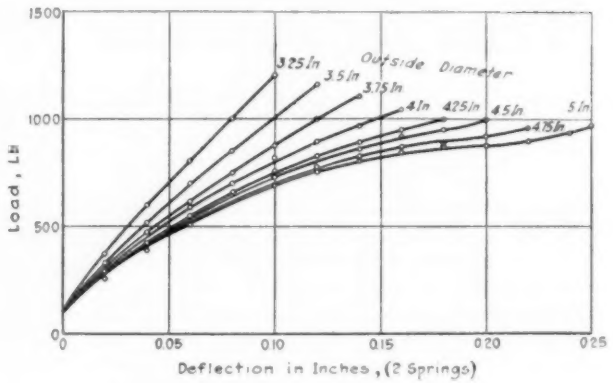


FIG. 4 DEFLECTION CURVES OF BELLEVILLE SPRINGS

(SAE 6145 chrome-vanadium steel springs 0.1 in. thick and 2 in. inside diameter. E = 23,300,000 and tensile strength = 102,000 lb. per sq. in.)

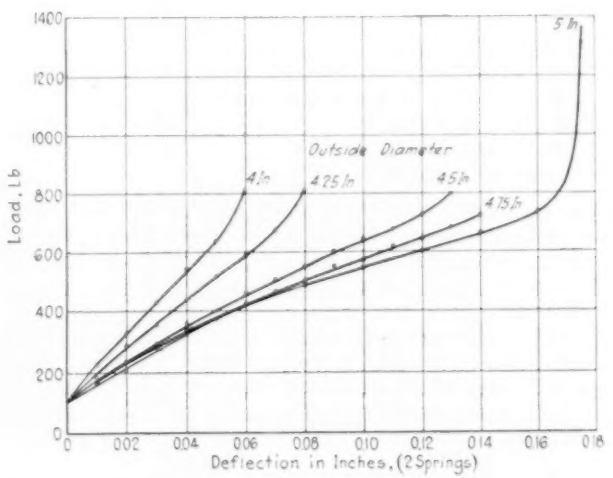


FIG. 5 DEFLECTION CURVES OF BELLEVILLE SPRINGS

(SAE 6145 chrome-vanadium steel springs 0.1 in. thick and 3 in. inside diameter. E = 23,300,000 and tensile strength = 102,000 lb. per sq. in.)

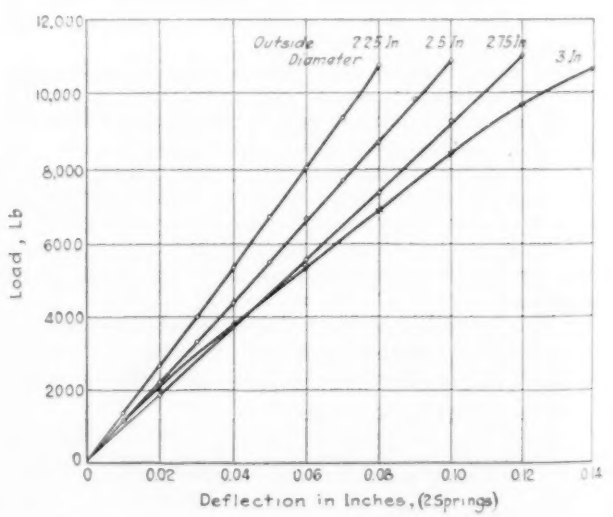


FIG. 6 DEFLECTION CURVES OF BELLEVILLE SPRINGS

(SAE 6145 chrome-vanadium steel springs 0.2 in. thick and 1 in. inside diameter. E = 23,300,000 and tensile strength = 102,000 lb. per sq. in.)

from the French Government designs of gun carriages such as the 75-mm. Puteaux and the 155-mm. G.P.F., in which Belleville springs were used for placing initial compression on the packings which must hold compressed air or nitrogen indefinitely under a pressure of more than 100 atmospheres. This type of spring is admirably suited for this purpose. It therefore became necessary to manufacture great quantities of these springs for use in the recoil mechanisms of these carriages. A considerable number were also used as rebound springs between the frame and chest of caissons for carrying ammunition. Some difficulties were experienced in the manufacture of these springs, but these were overcome and did not delay the manufacturing program. Many of these springs have, however, broken in service and in storage, and there is always the danger that a broken spring may ruin the expensive recoil mechanism in which it is assembled.

After the war, the Department undertook the design of more powerful guns, involving larger recoil mechanisms and consequently Belleville springs having different dimensions and characteristics from those theretofore used. Attempts were made to design the springs by means of a table which had been obtained from the French, with indifferent success. Occasionally a spring would be found to test out as designed, but more often it was necessary to cut and try several times before a satisfactory spring could be produced. A limited amount of research work on these springs was performed during the years 1919 to 1921, and finally, on May 19, 1922, a program for an investigation of the manufacture of Belleville springs was approved. The work has since been carried on in a systematic manner at the Rock Island Arsenal under the supervision of Capt. R. S. Barr, Ord. Dept. U.S.A. A number of partial reports have been submitted, a digest of which is contained in this paper.

Description of Tests

The material used throughout the tests was chrome-vanadium sheet steel SAE 6145, having the following analysis: carbon, 0.43; manganese, 0.62; chromium, 1.10; and vanadium, 0.18. Springs of this material were fabricated by the following operations: shear into square plates; bore inside diameter; turn outside diameter; grind to thickness, 0.1 or 0.2 in.; form hot; heat treat; oil quench from 1650 deg. fahr.; draw at 650 deg. fahr., 30 min. in niter bath. The foregoing heat treatment was found to be best by a number of tests made on strips of the material. The physical properties after heat treatment are: tensile strength, 102,000 lb. per sq. in.; E, 23,300,000; scleroscope hardness, 75 to 80.

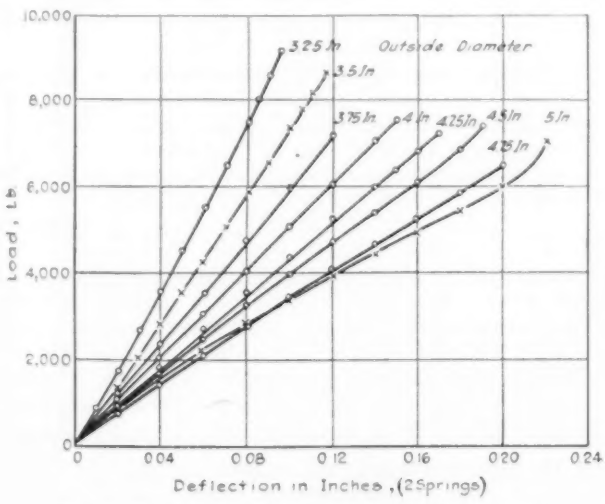


Fig. 7 Deflection Curves of Belleville Springs
(SAE 6145 chrome-vanadium steel springs 0.2 in. thick and 2 in. inside diameter. E = 23,300,000 and tensile strength = 102,000 lb. per sq. in.)

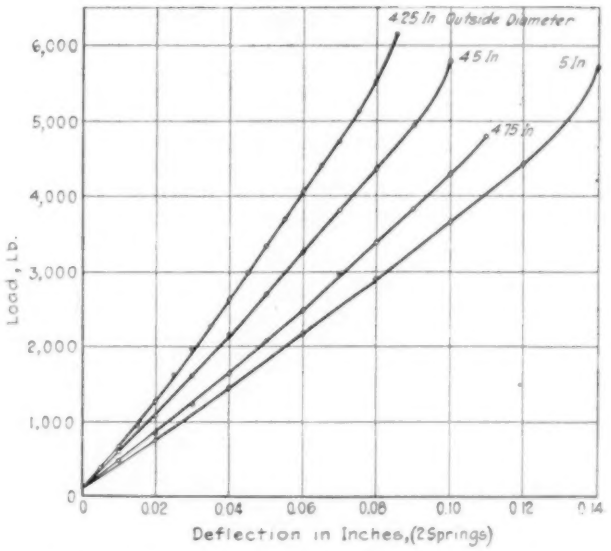


Fig. 8 Deflection Curves of Belleville Springs
(SAE 6145 chrome-vanadium steel springs 0.2 in. thick and 3 in. inside diameter. E = 23,300,000 and tensile strength = 102,000 lb. per sq. in.)

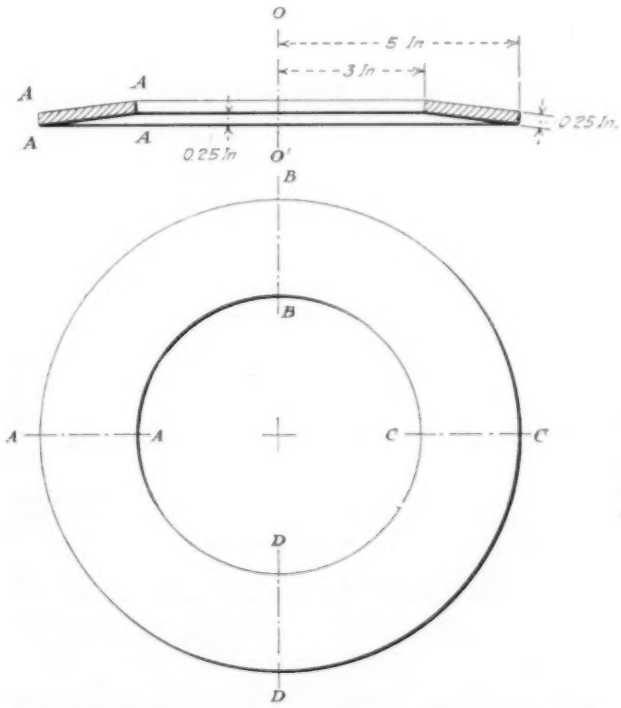


Fig. 9 Celluloid Belleville Spring for Photoelastic Test

In testing, a pair of springs of like dimensions were assembled with their concave sides together and loaded with an initial load of 100 lb. This was used as the starting height or zero point. They were then compressed by successive intervals of 0.01 in. and the load recorded, after which the zero point was again registered and the load noted. In no case were the springs compressed to such a point that the permanent set exceeded 0.001 in. per spring. After completing the test on a pair of springs the

outside diameter was reduced by increments, generally of 0.25 in., by means of Stellite tools, and after each reduction the tests were repeated. This was a great saving in time and labor, and had the advantage of producing sets of springs of varying characteristics and identical material and heat treatments. In most of the tests a dish angle of 7 deg. was used, but a number of springs of varying dimensions were also made and tested having a dish angle of 10 deg. in order to determine the maximum allowable dish. The results of most of the tests are shown graphically in Figs. 3 to 8.

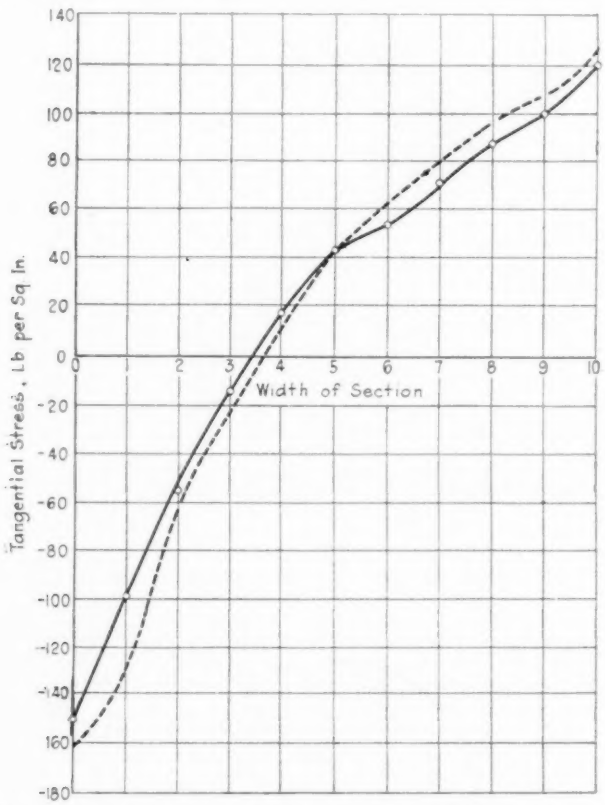


FIG. 10 DISTRIBUTION OF TANGENTIAL STRESS IN RADIAL CROSS-SECTION OF PHOTOELASTIC MODEL

(Abscissas are ten equal divisions of a radial cross-section, the point 0 being at the inside radius of the spring and the point 10 at the outside radius. Solid line shows stresses caused by change of load from 0 to 25 lb. Dotted line shows stresses caused by change of load from 25 to 50 lb.)

TABLE 1 VERTICAL DEFLECTION OF PHOTOELASTIC MODEL IN HORIZONTAL POSITION FOR THREE DIFFERENT LOADS

Point	Height in cm. above base line of cathetometer for loads of 0 lb.	25 lb.	50 lb.	Point	Height in cm. above base line of cathetometer for loads of 0 lb.	25 lb.	50 lb.
	Section *AA*				Section *BB*		
1	79.4830	79.4642	79.4500	1	79.3402	79.3240	79.3067
2	79.5500	79.5269	79.4840	2	79.4162	79.3878	79.3458
3	79.6326	79.5778	79.5232	3	79.5095	79.4525	79.3876
4	79.7102	79.6285	79.5572	4	79.5792	79.5023	79.4248
5	79.7847	79.6888	79.5922	5	79.6660	79.5682	79.4638
6	79.8466	79.7324	79.6257	6	79.7540	79.6492	79.5417
7	79.9311	79.7970	79.6692	7	79.8366	79.7018	79.5514
8	80.5302	80.3768	80.2231	8	80.4601	80.3298	80.1412
	Section *CC*				Section *DD*		
1	79.3648	79.3415	79.3240	1	79.4130	79.3980	79.3742
2	79.4584	79.4132	79.3674	2	79.4918	79.4540	79.4084
3	79.5332	79.4796	79.4008	3	79.5731	79.5179	79.4482
4	79.6093	79.6911	79.7655	4	79.6520	79.5693	79.4819
5	79.6772	79.7719	79.8514	5	79.7141	79.6145	79.5071
6	79.7581	79.6289	79.4971	6	79.8096	79.6839	79.5552
7	79.8326	79.6898	79.5358	7	79.8662	79.7210	79.5818
8	80.4500	80.2658	80.1094	8	80.4693	80.3016	80.1343

Each radial cross-section (see Fig. 9) is divided into 8 equal parts, point 0 being at the outside radius of the spring and point 8 at the inside radius. All point 8 readings for the inside edge are measured to the top of the loading plate.

A photoelastic stress analysis was also made by Prof. Paul Heymans of Massachusetts Institute of Technology, using a celluloid spring of the dimensions shown in Fig. 9. The distribution of tangential and radial stresses along a radial section (such as *AA* in Fig. 9) of this celluloid model was obtained from the

TABLE 2 THE CHANGE IN DIAMETER FOR FOUR POINTS ON UPPER SURFACE OF THE PHOTOELASTIC MODEL FOR THREE LOADING CONDITIONS

Diameter	Point	Change of diameter in cm. Change of load 0 to 25 lb.	Change of load 25 to 50 lb.
AD	1–1	−0.0019	−0.0083
	3–3	−0.0086	−0.0160
	5–5	−0.0113	−0.0187
	7–7	−0.0144	−0.0206
BC	1–1	−0.0017	−0.0068
	3–3	−0.0055	−0.0119
	5–5	−0.0093	−0.0168
	7–7	−0.0142	−0.0214

The minus sign signifies that the diameter becomes smaller as load is applied. For system of numbering points see Table 1.

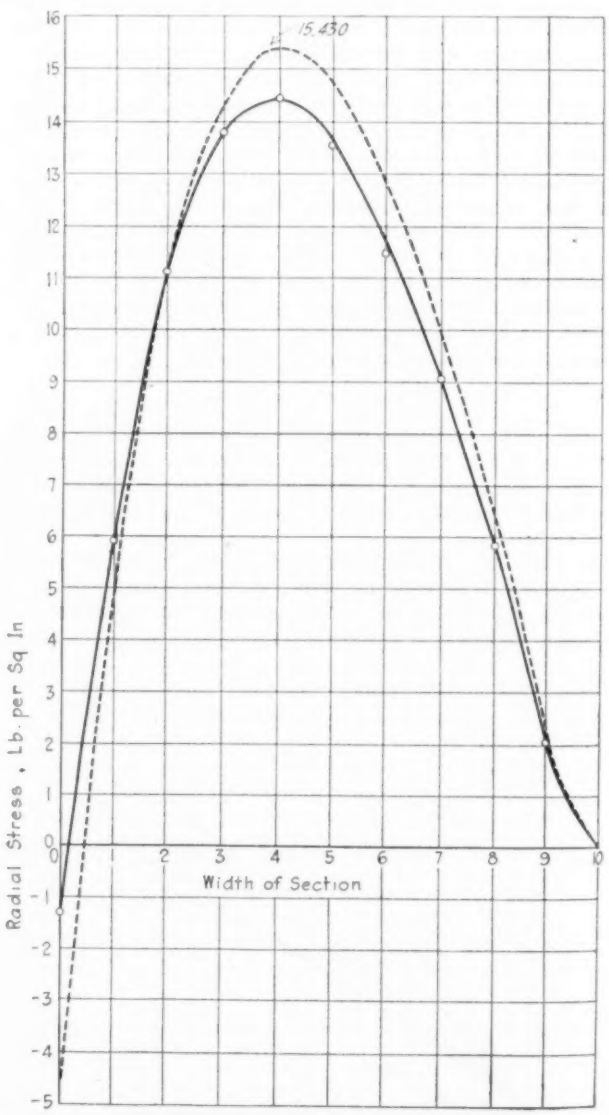


FIG. 11 DISTRIBUTION OF RADIAL STRESS IN RADIAL CROSS-SECTION OF PHOTOELASTIC MODEL

(Abscissas are ten equal divisions of a radial cross-section, the point 0 being at the inside radius of the spring and the point 10 at the outside radius. Solid line shows the stresses caused by change of load from 0 to 25 lb. Dotted line shows stresses caused by change of load from 25 to 50 lb.)

photoelastic measurements and is plotted in Figs. 10 and 11. These stress values were obtained first for a change from zero load to a medium load of 25 lb. and then for a change from 25 lb. to a heavy load of 50 lb. Deflections of various points of the surface of the spring for 25- and 50-lb. loads measured during the photoelastic tests are given in Table 1; and changes of diameter observed during the same tests are listed in Table 2.

Discussion

E. O. Waters.[2] The tests on Belleville springs have interested the writer on account of his having carried on an investigation of stresses in rings for pressure vessels a few years ago. At first sight the problem of the Belleville spring and that of the plain pipe flange or ring might appear to be practically identical, the emphasis being placed on deflection in the one case and stress in the other.

Closer inspection, however, shows certain marked differences. In the first place, the writer found it possible to analyze the stresses and strains in flat pipe flanges according to the classical theory for thick plates set forth by St. Venant, Foppl, Bach, and others, and obtain calculated deflections that agreed perfectly with test results, within the limits of experimental error. This means that the meridian plane, so called, of such flanges underwent no sensible distortion under load. The dish angle, at the point where the extreme fibers began to reach the limit of proportionality, was of the order of 0.5 deg. or less in most of the tests.

On the other hand, the present article specifies possible dish angles of 4 to 6 deg.; this is partly due to the low E of the steel used, but it appears that it is caused chiefly by the thinness of the disks relative to their outside diameter. In other words, the distortion of the meridian plane can no longer be neglected, and the condition becomes one of thin plates rather than thick plates.

So far as the writer is aware, the theory of thin plates, in which the deflection may be of the same order of magnitude as the thickness, has never been discussed at these meetings. He would be interested to know if it was employed in the derivation of Equation [1], or whether the equation was based on the classical thick-plate theory, modified to agree with the tests. It is interesting to note that this equation is identical with the one derived for the classical theory, with the exception of the R function, which is almost one-third less in Equation [1] for all values of R_2/R_1 from 1.4 to 4.0. This is in agreement with two of the pipe-flange tests, where the thickness was only $^1/_{20}$ to $^1/_{15}$ of the outside diameter, and the dish angle at the yield point was twice as great as in the other tests.

Evidently, the thick-plate formula breaks down in the region of high R_2/t, the ring becomes more rigid than called for by the theory, and finally the load-deflection line becomes curved as the ring enters the thin-plate field and a "toggle-joint" action comes into play.

Joseph Kaye Wood.[3] This investigation, which is a result of war-time experience with Belleville springs used in French guns, deserves worthy mention not only as a valuable research work on this particular type of spring, but also as an example of procedure for the investigation of certain other types of springs. Thus the author provides the designer with a deflection formula established by experiment and with the limitations of that formula for a given standardized material, namely, SAE 6145 steel. The competence of the designer with respect to this problem is further increased by providing him with a clear picture of the stress gradient under various loadings as determined by a photoelastic analysis. Mr. Gurney's paper should likewise be commended for its general educational value, since previous to this publication very few engineers in this country knew of the Belleville type of spring as such, either theoretically or practically.

This type of spring is particularly useful in mechanisms which require the storage and release of elastic energy at a relatively large load-deflection rate. It should be of interest and value therefore to know how the Belleville spring compares with other types of springs, particularly those having relatively large load-deflection rates. In making this comparison I shall refer to "A Code of Design for Mechanical Springs"[4] in which I devised a method for determining the efficiency of any general type of spring.

This method uses a bar under pure tensile load as the basis for computing elastic efficiency, it being considered as a theoretical spring of 100 per cent efficiency. A bar under torsion as in a helical spring is about 50 per cent efficient, under uniform flexure is 25 per cent efficient, and under "cantilever" flexure it is only 8 per cent efficient. By this method any type of spring can be rated for stress efficiency provided the form of the stress gradient does not change appreciably under different loads. Since Mr. Gurney has found that the neutral plane in the stress gradient of the Belleville spring does not alter appreciably under different loads, it should be possible with the aid of Figs. 10 and 11 in Mr. Gurney's paper to estimate the general efficiency of this type of spring. I made this estimation and found it to be somewhere between 30 and 35 per cent.

Belleville springs are therefore not as efficient as helical springs, but since a given volume of material in the former may be wholly contained within a smaller space than the same volume of the latter at solid loading, somewhat higher load-deflection rates may be obtained more easily with Belleville springs. Even in the case of the ring[5] type of spring, which approaches very closely to having 100 per cent stress efficiency, it is doubtful whether its overall dimensions are less bulky than those of the Belleville spring at solid loading. The preponderance of efficiency in the ring spring in addition has a large frictional hysteresis loop superimposed on its elastic characteristic.

In computing the efficiency of the Belleville spring it was found that the lever ratio (distance of neutral plane from maximum outside fiber stress to moment arm) was relatively large, which is the reason for the high-load-deflection rate of this type of spring. This fact may be ascertained by referring to Formula [1], page 715, in the code of design previously mentioned.

As a matter of information I would like to know whether the "diameter" designations in Table 2 are correct.

A. L. Kimball.[6] There is a factor which comes into play which tends to prevent the existence of a large difference between the stress in the top and in the bottom sides of the spring, namely, the hoop tensions and compressions in the circular elements of the spring.

Such a spring depends largely for its stiffness upon these hoop tensions and compressions, bending playing only a minor part.

It would seem, therefore, that the photoelastic method should give good results, provided that the springs are relatively thin compared with their diameter.

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[2] Associate Professor of Mechanical Engineering, Yale University, New Haven, Conn. Assoc-Mem. A.S.M.E.

[3] Consulting Engineer, New York, N. Y. Chairman of the Special A.S.M.E. Research Committee on Mechanical Springs. Assoc-Mem. A.S.M.E.

[4] *Mechanical Engineering*, vol. 47, no. 9, September, 1925, pp. 713–718.

[5] "The Ring Spring," by O. R. Wikander, *Mechanical Engineering*, vol. 48, no. 2, February, 1926, pp. 139–143.

[6] Research Engineer, General Electric Co., Schenectady, N. Y. Mem. A.S.M.E.

M. F. Sayre.[7] I have been very much interested in this paper with its description of a spring which has been in use for some time past, but which has not been well known. The Belleville spring certainly possesses very attractive features in the form of economy of space; that is, in making it possible to place a very large weight or volume of material in a relatively small space, which is often highly desirable in a spring. I should like to congratulate the author on the very careful and very valuable work which he has done.

There are some questions that I would like to ask. The first is as to the modulus of elasticity, which is given as 23,000,000 lb. per sq. in. Was that obtained from the formula or from tension tests? It seems a surprisingly small value. I have not tested any chrome vanadium steel, but I have tested a good many samples of high-chrome steel and obtained values of about 31,000,000 lb. per sq. in. as the modulus of that material. I do not remember ever having seen any figures on steel that even approached 23,000,000.

Professor Hoadley and I have been carrying on some tests on springs which are to be reported at the December meeting of the Society, and we have been very much interested in the hysteresis question. I should like to know if you have any information on the efficiency of the springs, that is, the per cent of energy returned on the release of the load.

Another question, Were these deflection curves taken on first loading or after the material had been loaded a number of times? I should expect that this would make some difference. Also, judging from the result given by Mr. DenHartog in his paper on "The Mechanics of Plate Rotors for Turbo Generators," I should expect that there would be sufficient yielding at the contact between the successive springs to affect the curve considerably. If, on release of load, the springs shifted on each other even a small distance, a different stress strain would be obtained on the next loading.

J. P. DenHartog.[8] The tests carried out on the Belleville springs are a valuable contribution and should be very useful to design engineers. Some caution has to be exercised, however, in the application of the results of the tests on celluloid models.



Fig. 12

The photoelastic method of investigating stress distributions is inherently applicable only to two-dimensional problems. However, the dished Belleville spring, loaded by forces perpendicular to its plane, certainly is a three-dimensional problem. This is especially clear in the limiting case of a spring with zero dish angle; in other words, a flat plate loaded with a uniformly distributed load along its outer and inner circular boundaries. (See Fig. 12.)

At the inside boundary there will be compression in the tangential fibers at the top of the plate and tension at the fibers at the bottom. The average stress will be zero, and a photoelastic investigation would show zero stress even when the model is nearly failing.

With dished plates these relations will not be so pronounced as with a flat plate; with a sufficient dish angle the tangential stress at the inner boundary will be compressive everywhere, but the compression at the top will always be greater than at the bottom. The photoelastic test gives the average of the two extremes, and the maximum stress will be considerably greater than this average for dish angles of the order of 10 deg.

In connection with this, Mr. Wood's calculations of the efficiency of these springs have to be used cautiously, since they are based on the photoelastic test results. Since the stresses are actually higher than found by this method, Mr. Wood's figures for the efficiency of Belleville springs are too high.

Author's Closure

It is believed that Mr. Kimball's criticisms are in general correct. However, bending may play a considerable part in thin springs.

Professor Sayre is undoubtedly correct in questioning the modulus of elasticity as 23,000,000 lb. per sq. in. The testing facilities at Rock Island Arsenal were not very good, and the Ordnance Department is not entirely satisfied with this figure. There is considerable hysteresis in a Belleville spring, and this was measured in a few cases. All tests were made after working the springs from 20 to 50 times to minimize this effect.

Answering Mr. Waters, the thin-plate theory as given by A. E. Love in chap. xxiv of his "Mathematical Theory of Elasticity" was used in deriving the deflection-load formula. However, an assumption had to be made that lines normal to the surface remain straight and normal under all loads. This is probably inaccurate and accounts in some respects for the difference between the formulas and tests. There is a large shearing action between layers of the spring as the spring flattens. The stress formula derived from this theory was not at all in agreement with the tests for this reason, and the ones given were derived entirely empirically.

Mr. DenHartog is correct, except that the photoelastic test gives the mean stress through the whole thickness of the spring and not the average between the top and bottom.

Mr. Wood's criticism compares the efficiency of the Belleville springs with other springs based on the results of the photoelastic tests. Unfortunately the data obtained from the celluloid spring tested by Professor Heymans are not conclusive because the stresses were very small. Celluloid has a modulus of elasticity in the neighborhood of one one-hundredth of that of steel, and accordingly a celluloid Belleville spring to be stressed close to the yield point would require a much steeper dish angle. In answer to Mr. Wood's question in regard to the data given in Table 2 of the original paper, these data appear to be correct.

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[7] Associate Professor of Mechanical Engineering, Union College, Schenectady, New York. Mem. A.S.M.E.

[8] Research Laboratory, Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pa.

APM-51-3

Journal Running Positions

By H. A. S. HOWARTH,[1] PHILADELPHIA, PA.

The classic theory of bearing lubrication propounded by Reynolds and extended by Sommerfeld has shown that when a journal runs in a full bearing with a complete film, the journal center will run opposite the center of the bearing when the journal load acts directly downward against the bearing. The classic theory has, the author believes, also shown that the journal will never rise higher than opposite the bearing center, if the bearing measures 180 deg. long and if the load line bisects that angle.

Consequently the impression has gained ground that under no circumstances will the journal rise higher than the bearing center in any clearance bearing—if the classic theory is correct. The present investigation was therefore undertaken to find out whether that current impression was reasonable. A study of journal running positions was undertaken for clearance bearings whose resultant pressures do not bisect the bearing angle.

The 120-deg. bearing was chosen for the present study because it approximates the bearings widely used in steam turbines. Other angular lengths will be studied later if the results of the present study indicate that an extension of the work is desirable, much of the preliminary work already having been done.

The result of our analysis proves that the current impression is erroneous. The journal may run with its center higher than that of the bearing, if the load, speed, viscosity, clearance ratio, and α/β ratio are compatible therewith. This must be so if the classical theory is correct.

The present analysis indicates that the load direction upon a partial bearing has an amazing influence upon the running position of the journal. It also indicates that no matter how high above the bearing center the journal may run at any finite speed, it will return to run concentric with the bearing at infinite speed.

This study disclosed other interesting phenomena that are explained in the text. Side leakage was neglected. The viscosity was assumed constant. Influence of friction upon the direction of the resultant pressure was estimated. Consequently the present work is readily comparable with that of Reynolds, Sommerfeld, Harrison, and with previous work of the author.

FOR SEVERAL years past the question of the stability of the running position of the journal upon its film has been in my mind as a problem deserving further study. It has presented itself in the form of these questions: (1) Can the journal run with its center above the center of the bearing, that is, with its center above a horizontal line through the center of the bearing, assuming the direction of the load is such that the journal presses vertically downward against the bearing? (2) Is there more than one stable running position for the journal for any one combination of load, speed, viscosity, and running clearance? (3) Along what path does the axis of a journal rise from rest to its running position as its speed increases?

In order to answer these questions, we examined our previous analytical work[2] on journal bearings to see what light, if any, it would throw thereon. A 120-deg. partial bearing was chosen for study because it is so commonly used. Such a bearing is illustrated in Fig. 1. It was assumed to be provided with running clearance.

Symbols Employed

The center of the journal in every figure is at O, while that of the bearing is at O'. The symbols used in the figures and formulas are listed below. Lengths are in inches.

- a = radius of journal
- $a + \eta$ = radius of bearing when provided with running clearance
- c = eccentricity factor
- $c\eta$ = eccentricity at which journal runs in a bearing provided with running clearance
- h = film thickness at angle θ
- h_1 = film thickness at point of maximum pressure
- l = axial width of bearing
- N = r.p.m. of journal
- P = w, which see
- q = vertical lift of journal from lowest position to running position
- R = resultant pressure of journal against bearing film
- R' = resultant pressure of bearing against film
- w = $R/2a = W/2al$ = load on bearing in pounds per inch of journal diameter and per inch of axial width of bearing
- W = total load on journal (lb.) = $Rl = 2wal$
- Z = viscosity of the lubricating film in centipoises
- α = angle from leading edge of partial bearing to position of resultant pressure R measured in direction of journal rotation
- β = angular (circumferential) length of partial bearing
- ϵ = eccentricity at which journal runs in a fitted bearing
- η = radial clearance
- θ = angle to point where film thickness is h and pressure is P
- θ_1 = angle to point of maximum pressure
- μ = coefficient of viscosity (absolute), inches, pounds, seconds
- ϕ = angle from line of action of resultant pressure R to line passing through the centers of bearing and journal, measured in direction of rotation of journal.

The path of the journal center as the journal increases its speed of rotation from rest can be determined if we assume the bearing position to be fixed with respect to the load direction and if we also assume the film to be complete from end to end. Under these conditions the ratio α/β of the leading angle to the bearing arc (see Fig. 1) must be constant. Our previous work gives the data on only one such bearing, that for $\alpha/\beta = 0.5$. It is true that we studied another set of bearings called the offset series, but that series does not have a constant α/β.

This present analysis, like our previous work, has the same limitations as the work of Reynolds, Sommerfeld, and Harrison, namely, it is based upon constant viscosity, no side leakage, and neglect of the initial pressure that would be present at the leading edge of a high-speed bearing. It also neglects the influence of friction upon the direction of the resultant pressure. All conclusions drawn from this analysis are subject to these limitations. Experimental verification is needed for the purpose of obtaining correction factors.

The question at once arose as to how the data should be plotted in order to visualize fully what takes place. It appeared that

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[1] Vice-President, General Manager, and Chief Engineer, Kingsbury Machine Works. Mem. A.S.M.E.

[2] "A Graphical Study of Journal Lubrication," Part I, Trans., A.S.M.E., vol. 45, p. 921; Part II, vol. 46, p. 809; Part III, vol. 47, p. 1073.

Contributed by the Special Research Committee on Lubrication and presented at the Annual Meeting, New York, N. Y., December 3 to 7, 1928, of The American Society of Mechanical Engineers.

the journal positions could profitably be plotted in a circle having as radius the radial clearance η, i.e., the difference of radius between the bearing and its journal. On such a figure concentric circles could be drawn to show the values of the eccentricity factor c from zero to 1.0, or even higher if such a running position should be found. Fig. 2 illustrates such a chart. The vertical line through the center O' of the circles represents the line of action of the bearing pressure R', whereas the journal pressure R

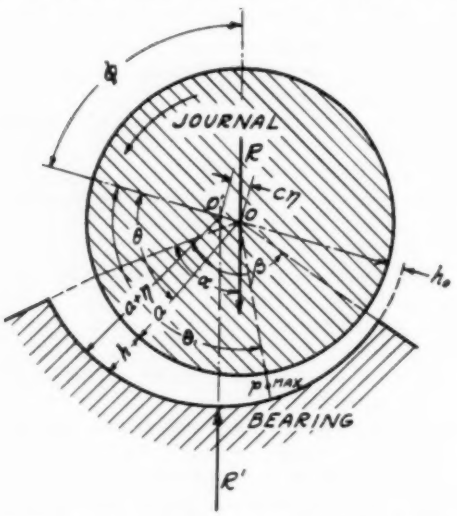


FIG. 1 SET-UP OF 120-DEG. BEARING WITH SYMBOLS USED IN TEXT

acts downward through point O against the bearing. The bearing arc is represented by GH. The running position of the journal is at some point O. If the journal runs concentric with the bearing both centers would be at O'. If metallic contact takes place the journal center would usually be somewhere on arc GDH. When the journal runs at any point O the diameter through $O'O$ makes an angle ϕ with the load line $O'D$. The angle ϕ is measured from the load line in the direction of journal rotation. The curved arrow in Fig. 2 represents the rotation of the journal. Hence the coordinates of point O on the chart Fig. 2 will be known when the eccentricity factor c and the angle ϕ are both known.

Curve B on Fig. 2 was drawn to show the path of the center of the journal for the central bearing, in which $\alpha/\beta = 0.5$. There is also drawn the curve A to show running positions for the offset series, in which α/β is a variable. Curve B therefore represents the only true path of the journal in Fig. 2.

When the journal speeds up from rest in such a 120-deg. central ($\alpha/\beta = 0.5$) partial bearing, its center presumably starts at D directly below the bearing center O'. Before the film is formed the journal will roll around inside the bearing until its center reaches some point E, determined by the coefficient of starting friction. Slip will then take place and if the acceleration is rapid a thin film will form at once. The journal center will at the same time slip over to some point near F. Its exact location has not been determined, because at this time we were more concerned with the journal path at the higher speeds.

When this slip takes place the coefficient of friction will fall off to a very low value, and there may be some oscillation of the journal across its normal position for the speed at which it is rotating. As the journal gathers speed this oscillation should cease and the journal center should rise along a path like curve B toward the bearing center O, as the oil film thickens. Our analysis indicates that at infinite speed the journal and bearing centers will coincide. This statement appears to be true if the effect of friction upon the direction of the resultant pressure be neglected. We have not yet completed our study of the influence

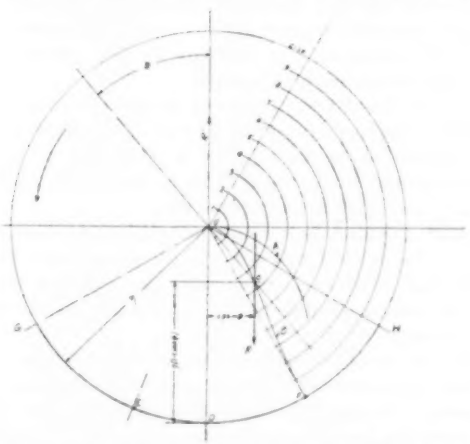


FIG. 2 CLEARANCE CIRCLE FOR 120-DEG. BEARING WITH PLOT OF DATA OBTAINABLE FROM "GRAPHICAL ANALYSIS"

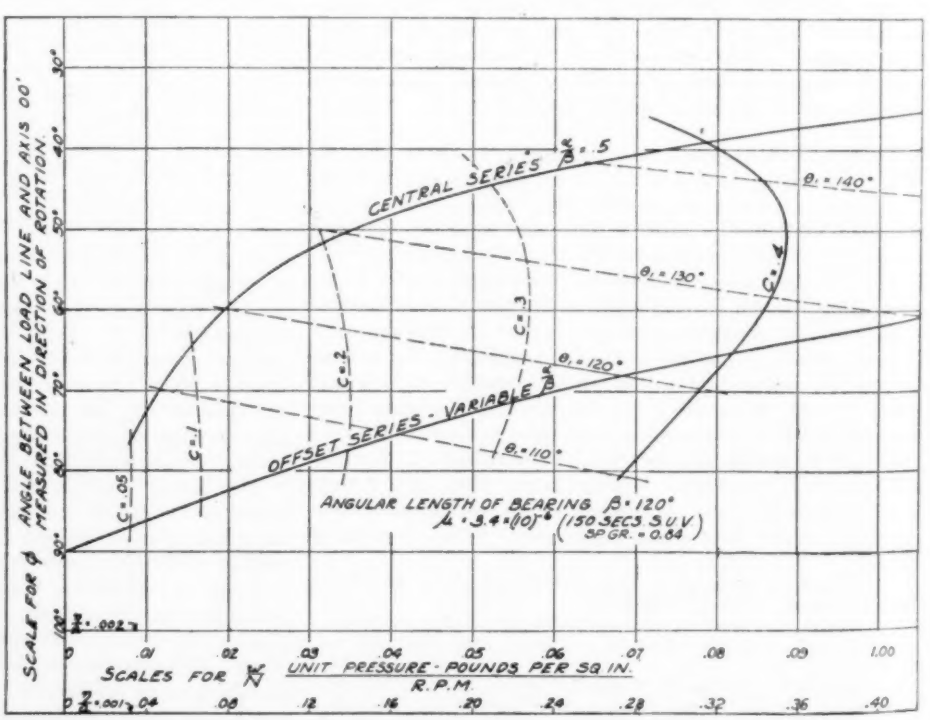


FIG. 3 PLOT OF CHARACTERISTICS OF 120-DEG. BEARING OBTAINABLE FROM "GRAPHICAL ANALYSIS"

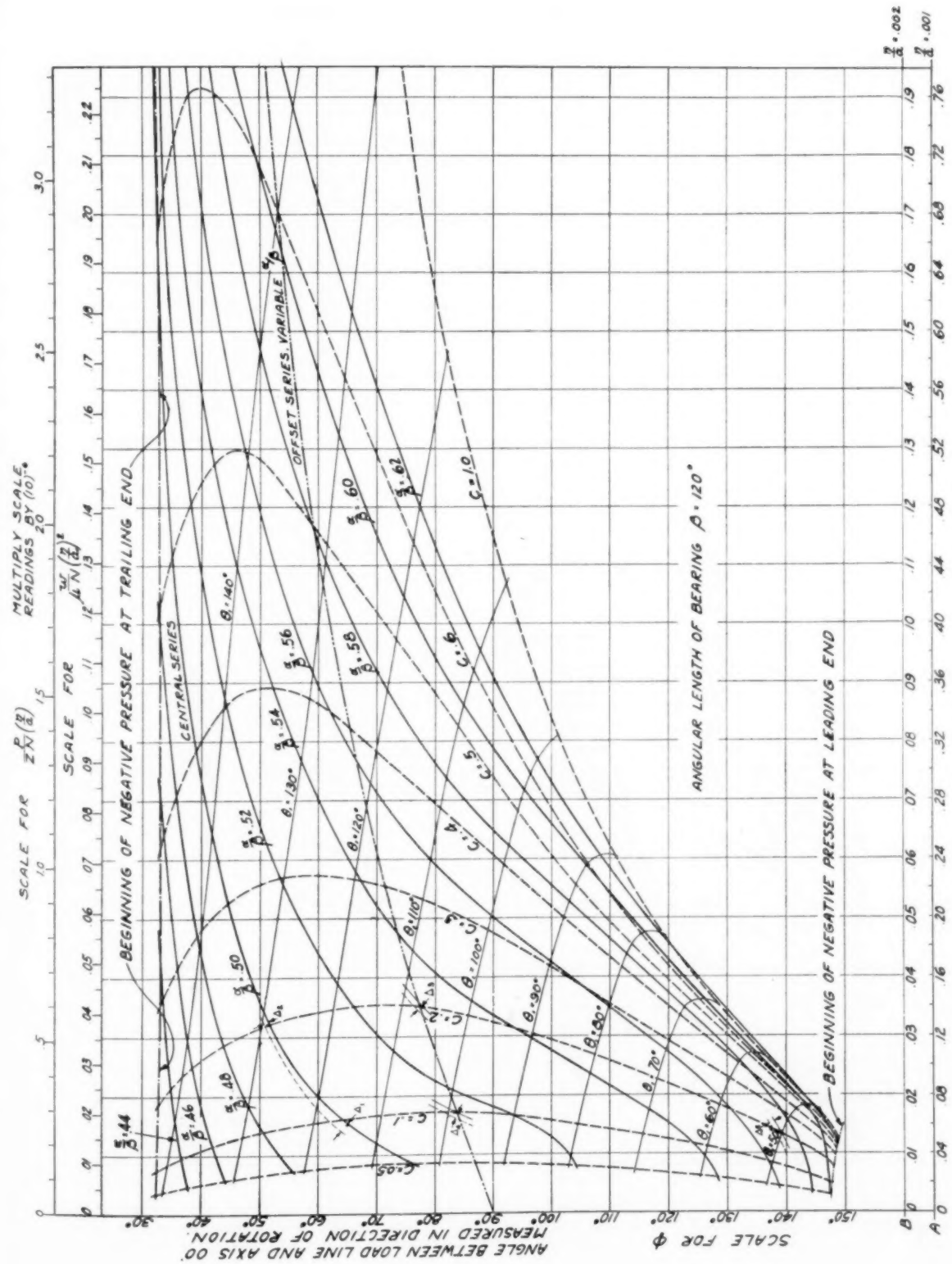


FIG. 4 CHARACTERISTICS OF 120-DEG. BEARING SHOWING RELATION OF ECCENTRICITY, RUNNING POSITION, MAXIMUM-PRESSURE LOCATION, AND LEADING-ANGLE RATIO, TO THE LOAD-SPEED-VISCOSITY-CLEARANCE FACTOR

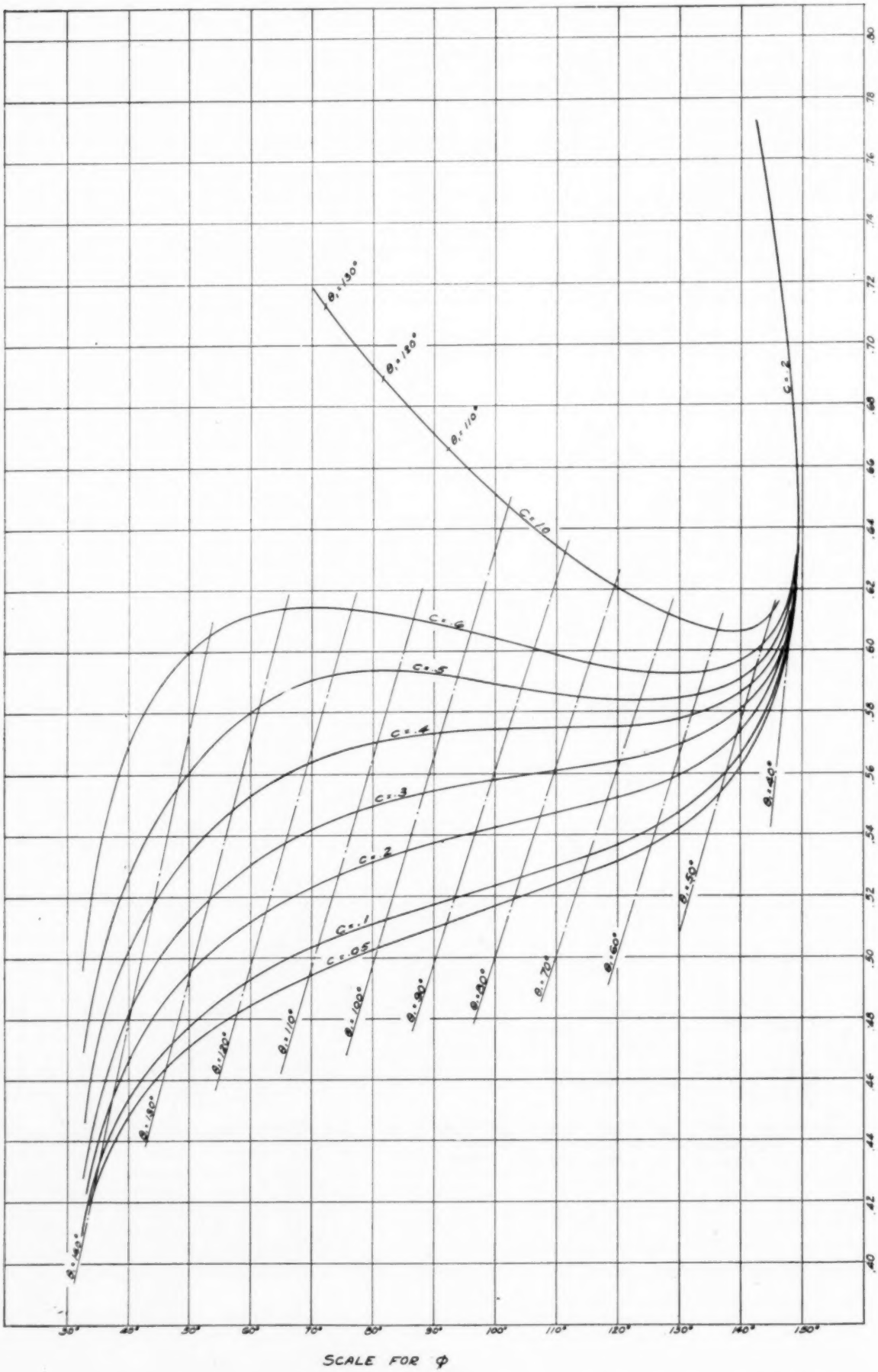


FIG. 4a CROSS-PLOT OF PART OF THE DATA IN FIG. 4

of friction upon the resultant pressure direction, and therefore cannot yet say definitely whether concentricity is the ultimate position when friction is taken into account. (See p. 9, however.)

Bearings are not all of the central type discussed above. Hence it is desirable to obtain data on the path of the journal for values of α/β other than 0.5. The inadequacy of our available data became evident when it was plotted on a chart like Fig. 3 whose coordinates are w/N and ϕ. Upon this chart we built up curves for constant θ_1, constant eccentricity factor c, and constant α/β, as far as they could be drawn from data underlying the "Graphical Study of Journal Lubrication,"[2] published in 1923, 1924, and 1925. Upon such a chart we could also show the influence of friction upon the position of the resultant pressure with respect to the bearing, if such data were available. The curves in Fig. 3 were so few, so uncertain, and so limited as to be obviously inadequate for the purpose of the present study.

In order to get the additional data required, it was necessary to go back to Harrison's fundamental Equation [20] (see "A Graphical Analysis of Journal Bearing Lubrication"[3]) and to plot curves like Figs. 38 and 39 of that analysis, for a wide range of values of c and θ_1. This necessitated an immense amount of careful graphical and tabulating work, extending over a period of nearly a year. This work was done by S. J. Needs, who aided the author so materially with similar work in the preparation of Parts II and III of "A Graphical Study of Journal Lubrication."[2] Up to the present time we have investigated the field covered by Fig. 4. This extends from the upper line marked "Beginning of Negative Pressure at Trailing End" to the lower line marked "Beginning of Negative Pressures at Leading End." Between these limits the film pressures are always positive. How far Fig. 3 fell short of our present needs is seen by comparing it with Fig. 4, on which we have plotted all the important data thus far obtained.

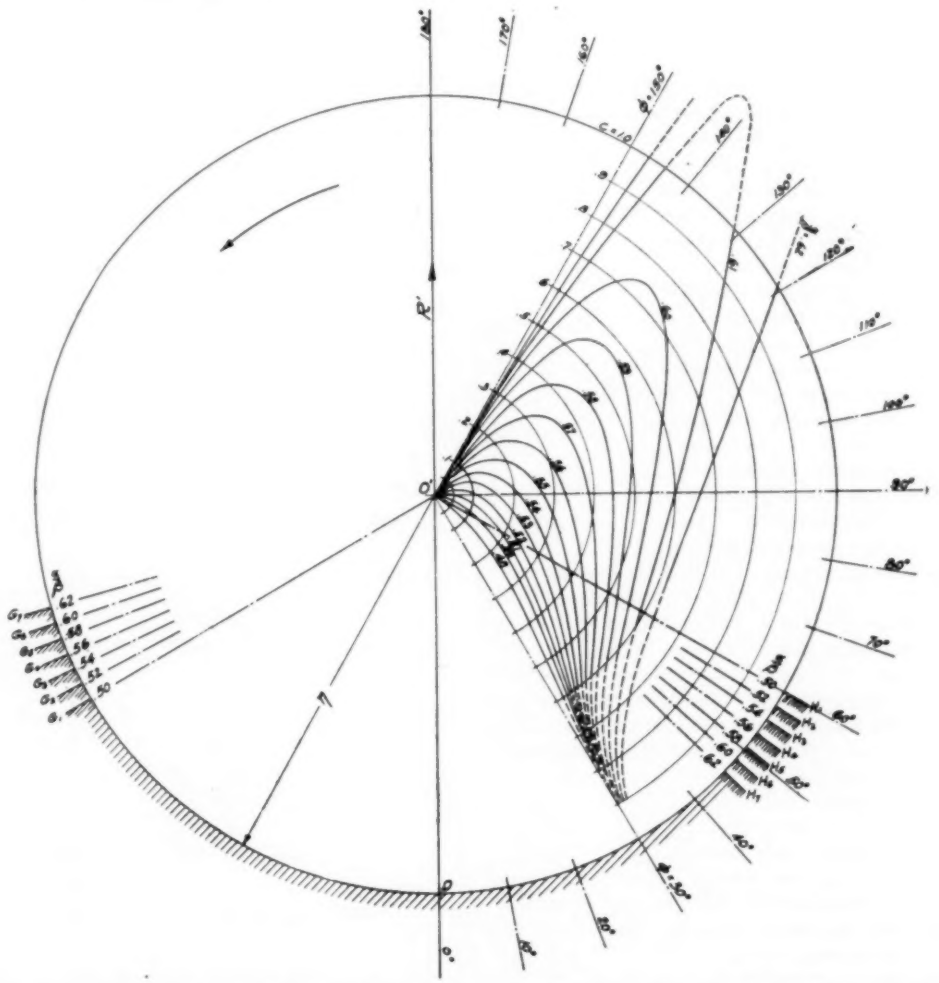


FIG. 5 CLEARANCE CIRCLE SHOWING JOURNAL RUNNING POSITION LOCI FOR DIFFERENT LEADING-ANGLE RATIOS FOR 120-DEG. BEARING

On Fig. 4 there are constant α/β lines for values ranging from below 0.50 to above 0.60. There are also constant maximum-pressure lines ranging from $\theta_1 = 50$ deg. to $\theta_1 = 140$ deg. The constant eccentricity factor lines vary from $c = 0.05$ to $c = 1.0$. In Fig. 4 therefore a much larger field of bearing running conditions is shown than has heretofore been available.

For convenience in working with Fig. 4 there are four horizontal scales. One for $(P/ZN)(\eta/a)^2$ is placed at the top. Just below it there is a scale for $(w/\mu N)(\eta/a)^2$. At the bottom there are two scales for w/N alone, assuming $\mu = 3.4 \times 10^{-6}$. The lower one covers bearings in which $\eta/a = 0.001$, while the upper of the two covers those in which $\eta/a = 0.002$.

From Fig. 4 and an intermediate Fig. 4*a* it is now a simple matter to plot Fig. 5 which, similar to Fig. 2, shows the path of

[3] This pamphlet, published by the Society in 1926, combines parts I, II, and III of the "Graphical Study of Journal Lubrication" in a form better suited for convenient reference.

the journal center within a circle whose radius is η. Fig. 5 can be interpreted in connection with Figs. 4 and 4*a*, and vice versa, referring both to Fig. 1. The dotted portions of the curves on Fig. 5 have not yet been fully verified. They show, however, the general character of the journal paths.

On the lower half of the circle whose radius is η we have marked at the left with G_1 to G_7 the leading edges of the bearing for different values of α/β. At the right the corresponding trailing edges are marked H_1 to H_7. The angular length of each bearing is 120 deg. Hence it is possible to visualize the position of the bearing with respect to the load line for each proportion being examined.

Practical Applications

The above analysis of the 120-deg. clearance bearing has given us Figs. 4, 4*a*, and 5 for use in seeking practical application of the results. There may be added to this material such data on the 120-deg. and other fitted bearings as are contained in Fig. 14 of "A Graphical Analysis of Journal Bearing Lubrication."[3] This figure is reproduced herein with same number for convenience.

The direction of the load against a bearing is usually known in the case of important machinery, such as horizontal steam turbo-generators. In these the load is vertical, if we assume perfect mechanical and electrical balance. With geared or belted machinery the bearing-load direction is not so obvious. In the case of motor drives through belts, chains, or gears, the load direction cannot be predicted by the motor manufacturer. Therefore, no matter what the bearing maker may expect, it is quite possible in some cases that the bearing will be called upon to take its load in an unexpected direction. These facts make it necessary to examine the unfavorable as well as the favorable running conditions.

The running position of a journal with relation to its bearing will depend upon the bearing design, its load, the viscosity of the oil, and the speed. Under "bearing design" we must consider (*a*) whether it is full or partial; (*b*) the position of the bearing surface with respect to the load line; (*c*) the running clearance provided on the working bearing face; (*d*) the clearance, shape, and position of the bearing cap; and (*e*) the method of applying the lubricant. The following bearings will be studied in the examples discussed below:

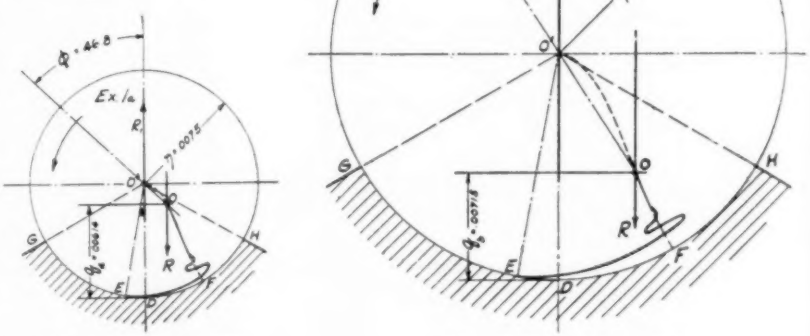
(1) 120-deg. fitted bearing, (*a*) without cap; (*b*) with caps of various designs.

(2) 120-deg. clearance bearing, (*a*) without cap; (*b*) with caps of various designs.

Example 1. A turbine journal, 15 in. in diameter and 30 in. long, exerting a vertical load of 67,500 lb., runs at 1500 r.p.m. in a 120-deg. clearance bearing, central type ($\alpha/\beta = 0.5$), *without a cap*. What will be the running position on a light oil (S.U.V. = 150 sec. at 100 deg. fahr.) whose operating temperature is 130 deg. fahr., assuming (*a*) the clearance ratio $\eta/a = 0.001$, and (*b*) the clearance ratio $\eta/a = 0.002$?

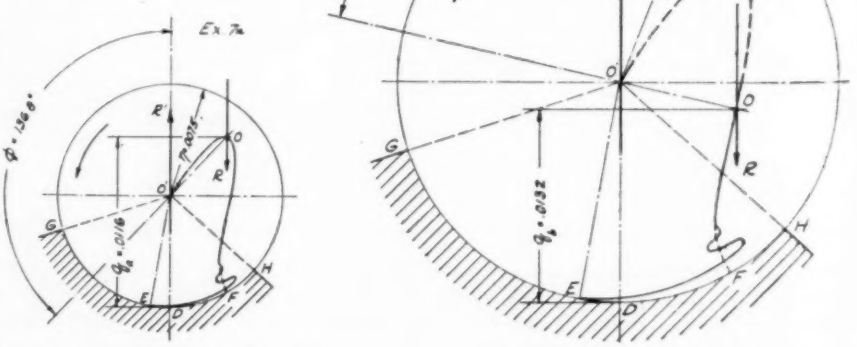
Example 2. Same as 1, except that load is 33,750 lb.

Example 3. Same as 1 except that a medium oil is used, S.U.V. = 300 sec. at 100 deg. fahr.



$\alpha/\beta = 0.50$; $\eta/a = 0.0001$ $\alpha/\beta = 0.50$; $\eta/a = 0.002$

FIGS. 6 AND 7 PLOTS OF JOURNAL RUNNING POSITION FOR EXAMPLE 1



$\alpha/\beta = 0.60$; $\eta/a = 0.001$ $\alpha/\beta = 0.60$; $\eta/a = 0.002$

FIGS. 8 AND 9 PLOTS OF JOURNAL RUNNING POSITION FOR EXAMPLE 7

TABULATION OF EXAMPLE DATA WITH SOLUTIONS

Example No.	α/β	Load, lb.	Oil S.U.V. at 100 deg. sec.	Oil S.U.V. at 130 deg. sec.	Equiv. abs. viscosity 130 deg.	w_1/N_1	w/N	$\eta/a = 0.001$ ϕ	θ_1	c	q_a	$\eta/a = 0.002$ ϕ	θ_1	c	q_b
1	0.5	67,500	150	85	1.87×10^{-6}	0.10	0.182	46.8	134.6	0.266	0.00614	32.6	153.5	0.62	0.00715
2	0.5	33,750	150	85	1.87×10^{-6}	0.05	0.091	57.5	122.5	0.15	0.0069	37.4	145.5	0.44	0.0098
3	0.5	67,500	300	150	3.4×10^{-6}	0.10	0.19	55.8	124.5	0.16	0.0068	36.2	146	0.46	0.0094
4	0.5	33,750	300	150	3.4×10^{-6}	0.05	0.05	69	111.0	0.080	0.0073	45.5	136.0	0.29	0.0120
5	0.6	67,500	150	85	1.87×10^{-6}	0.10	0.182	118	78.5	0.65	0.0098	49.1	143.5	0.60	0.0091
6	0.6	33,750	150	85	1.87×10^{-6}	0.05	0.091	139.0	55	0.75	0.0118	82.0	113	0.53	0.0139
7	0.6	67,500	300	150	3.4×10^{-6}	0.10	0.10	136.8	57.5	0.75	0.01160	77.0	118.0	0.54	0.01320
8	0.6	33,750	300	150	3.4×10^{-6}	0.05	0.05	146.5	43	0.34	0.00965	113.9	82.2	0.64	0.01898

Example 4. Same as 2 except that a medium oil is used, S.U.V. = 300 sec. at 100 deg. fahr.
Example 5. Same as 1, except $\alpha/\beta = 0.60$
Example 6. Same as 2, except $\alpha/\beta = 0.60$
Example 7. Same as 3, except $\alpha/\beta = 0.60$
Example 8. Same as 4, except $\alpha/\beta = 0.60$

Solution of Example 1.

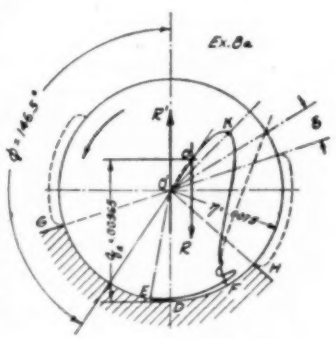
$$w_1 = \frac{67{,}500}{15 \times 30} = 150 \qquad \frac{w_1}{N_1} = \frac{150}{1500} = 0.1$$

The oil viscosity at operating temperature (130 deg. fahr.) will be approximately 85 S.U.V. ($\mu_1 = 1.87 \times 10^{-6}$). Since the scale of chart Fig. 4 is based on a viscosity of 3.4×10^{-6} we must enter it with $w/N = 3.4 \times 0.1/1.87 = 0.182$. Hence from Fig. 4 we find (for $\eta/a = 0.001$) that $\phi = 46.8$, $c = 0.266$, and $\theta_1 = 134.6$. If $\eta/a = 0.002$ we find that $\phi = 32.6$, $c = 0.62$, and $\theta_1 = 153.5$. Therefore the two running positions will be as indicated in Figs. 6 and 7, by points O. The lifts vertically from rest position D would be $q = \eta(1 - c\cos\phi)$. Lift (a) will then be $q_a = 0.00614$ (see Fig. 6); $q_b = 0.00715$ (see Fig. 7).

Solution of Example 7.

15 in. × 30 in. journal, $\alpha/\beta = 0.6$, $w_1/N_1 = 0.1$, S.U.V. = 300 at 100 deg. fahr., S.U.V. at 130 deg. fahr. will be 150 sec. = 3.4×10^{-6}. Enter Fig. 4 with $w/N = 0.1$ and find that $\phi = 136.8$, $\theta_1 = 57.5$ deg., and $c = 0.75$ or greater. Enter with $w/N = 0.1$ when $\eta/a = 0.002$ and get $\phi = 77$ deg., $\theta_1 = 118$ deg., $c = 0.54$. The lifts will be as follows:

$$q_a = 0.001 \times 7.5 \times (0.75 \times 0.731 + 1) = 0.0116$$
$$q_b = 0.002 \times 7.5\,(1 - 0.54 \times 0.225) = 0.0132$$



$a/\beta = 0.60$; $\eta/a = 0.001$

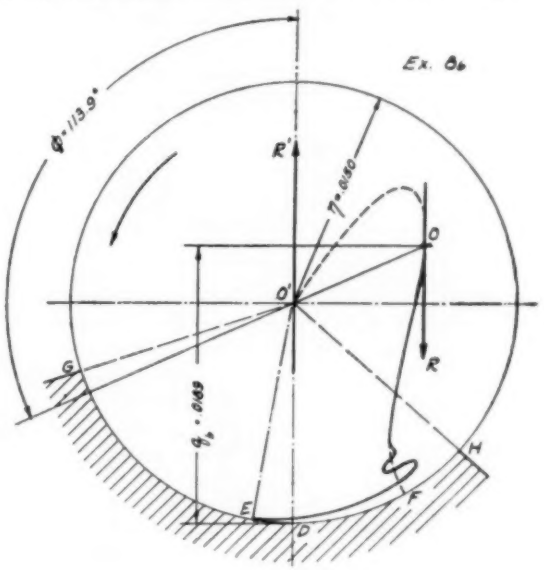


Fig. 11

FIGS. 10 AND 11 PLOTS OF JOURNAL RUNNING POSITION FOR EXAMPLE 8

Solution of Example 8.

This will have maximum lift because of the lighter load and higher viscosity. 15 in. × 30 in. journal, $\alpha/\beta = 0.6$, $w/N_1 = 0.05$, S.U.V. at 100 deg. fahr. = 300 sec., S.U.V. at 130 deg. fahr. = 150 sec., $\mu_1 = 3.4 \times 10^{-6} = \mu$. Entering Fig. 4 with $w/N = 0.05$ we find that $\phi = 146.5$, $c = 0.34$, $\theta_1 = 43$ deg., for $\eta/a = 0.001$; while for $\eta/a = 0.002$ we find $\phi = 113.9$ deg., $c = 0.64$, $\theta_1 = 82.2$ deg.

$$q_a = 1 - (-284) = 1.284 \times 0.0075 = 0.00965$$
$$q_b = 1 - (-261) = 1.261 \times 0.015 = 0.0189$$

But in getting to $q_a = 0.00965$ the journal has to pass over a point whose lift is about 0.0116.

When examining the tabulated data and results for examples 1 to 8, it must be remembered that certain data are the same throughout, for example, journal diameter = 15 in. × 30 in.; speed = 1500 r.p.m.; operating temperature = 130 deg. fahr.; and the bearing has no cap.

The above eight examples refer to bearings without caps or those whose caps are provided with extra clearance so that no appreciable interference may be expected from them.

When the running clearance of the bearing is $\eta/a = 0.001$ the radial clearance will be 0.0075. Hence "over center" running would occur in examples 5*a*, 6*a*, 7*a* (see Fig. 8) and 8*a*, whereas practically "opposite center" running would occur for examples 2*a*, 3*a*, and 4*a*. For the larger clearance ($\eta/a = 0.002$) over center running occurs only for example 8*b*, whereas practically "opposite center" running occurs for example 4*b*, 6*b*, and 7*b* (see Fig. 9).

Comparing only the *lifts*, we find in example 1 about the same lift for both clearances, as will be clear from Figs. 6 and 7. This is also approximately true for examples 5, 6, and 7.

Hence for a given bearing the lift will be increased if the load is reduced, or the oil viscosity increased, except in cases where the running position O lies on the curve at the left of the point of maximum lift (see Fig. 5) as it does for example 8*a* (see Fig. 10).

Clearance Bearings With Caps

Bearing-cap construction and position both play an important part in the operation of the journal. The cap clearance should be partly determined by the method of applying oil to the bearing. The cap may exert pressures upon the journal that will partly determine its running position. Three distinct types of caps will be considered.

Cap *A*. The surface of this cap is concentric with the bearing.

Cap *B*. The surface of this cap is concentric with running position of the journal.

Cap *C*. The surface of this cap is offset and not concentric with either the bearing or journal.

We shall examine these three types of caps with reference to a 120-deg. partial bearing for which $\alpha/\beta = 0.6$ and whose running clearance is $\eta/a = 0.001$. We shall assume that the cap is also 120 deg. long and that the oil channels at the sides are each 60 deg. long.

Cap *A*. This problem is illustrated by Fig. 12. The load, speed, and viscosity are assumed to be such that the journal would run at O if there were no cap. If a cap is then placed in position, and it has the same radius as the bearing, pressures would be set up in the cap film. Their net effect would be to draw the journal toward the cap provided the natural negative pressures in the film are not impossibly high. Presumably the negative pressures could not exceed 14.7 lb. per sq. in. If not, the film under the cap would break when that negative value was reached, and then the positive pressures under the cap might dominate, thereby opposing closer approach of the journal. Obviously this needs experimental verification.

Cap *B*. This problem is illustrated by Fig. 13. The natural running position of the journal is assumed to be at O if no cap is used. In this case if a cap is added and so set that its center

is also at O, no pressures would be set up in the cap film if the cap radius exceeded that of the journal.

Cap C. This problem is illustrated by Fig. 15. Obviously if it is desirable to reduce the natural lift of the journal a cap could be so placed as to generate only positive pressures that would oppose the normal lift. Fig. 15 is therefore made by shifting the cap sideways to some position such that with zero load the journal would run halfway between the bearing and cap. To better illustrate this case we shall solve the following problem.

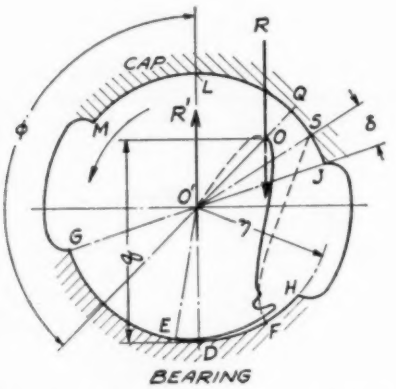


FIG. 12 CLEARANCE-CIRCLE REPRESENTATION OF 120-DEG. BEARING AND CAP, CONCENTRIC AND WITH SAME RADIUS

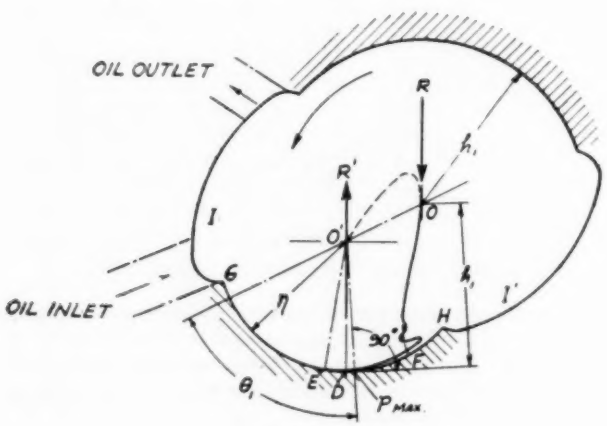


Fig. 13 Cap B

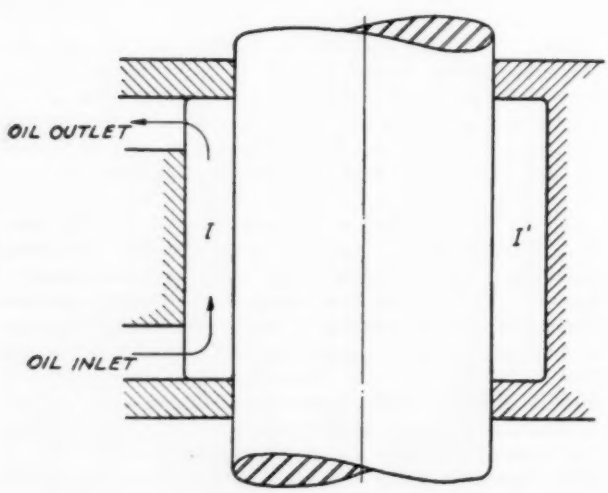


Fig. 13a

FIGS. 13 AND 13a CLEARANCE-CIRCLE REPRESENTATION OF 120-½DEG. BEARING WITH CAP SET OVER TO BE CONCENTRIC WITH RUNNING POSITION OF JOURNAL

Example 9.

How far to the right should the cap be shifted in order that its downward pressure will so balance the lifting force of the bearing film that the journal will never rise above the horizontal line $O_B'O_C'$ through the bearing center? Example data are the same as in example 8, except that $\alpha/\beta = 0.62$ and $\eta/a = 0.001$. In Fig. 15 the cap has been assumed to be as effective as the bearing; and the journal path with respect to it, if the load were upward, has been shifted with the cap toward the right until it coincides approximately with the path of the journal with respect to the bearing. The w/N values were marked on both curves and then projected horizontally to scale from a vertical line PQ. The lower half of the line representing w/N for the bearing is ST, and that representing w/N for the cap is TV. They cross at T on the line of centers $O_B'O_C'$. The horizontal distances between lines TV and ST show net upward capacities (w/N) of this bearing-cap combination for different lifts of the journal. The lift is measured from the horizontal line through D on which the w/N scale is laid off for curves ST and VT.

For a net value of $w/N = 0.2$ the lift of the journal in this shifted cap bearing will be only to point O_2 which gives $q_2 = 0.0065$ as compared with $q_1 = 0.0105$ when the cap was not shifted and the journal ran at O_1.

The side shifts of the cap to bring about this increased stability of the journal would be $S = 0.0093$.

FITTED BEARINGS WITH AND WITHOUT CAPS

When consideration is given to the *fitted bearing* without a cap, it is found that α/β cannot be less than $77.4/120 = 0.645$. In fact the only data we have on fitted partial bearings cover this case. The running clearance is zero in a fitted bearing. Hence the circle of radius η used heretofore would become a point. Fig. 16 represents a fitted 120-deg. partial bearing. Line $O'O$ represents the path of the journal center. Just where the journal center will lie for a given set of data can be found from chart, Fig. 14.

Example 10.

Taking the same data as for example 7, but with $\eta/a = 0$ and $\alpha/\beta = 0.645$, we have $w_1/N_1 = 0.1 = w/N$. Hence $\epsilon/a = 0.0031$. $\epsilon = 7.5 \times 0.0031 = 0.0232$. The lift will be $q = \epsilon\cos(180 - \phi)$ which for this case becomes $0.0232 \times 0.85 = 0.0197$. This lift is much more than we had in example 7, e.g., 0.0116

TABLE 2 COMPARISON OF CHARACTERISTICS OF 120-DEG. BEARINGS WITH AND WITHOUT THE INFLUENCE OF FRICTION THEREON

		Without friction	With friction
Partial bearing	ϕ	65.7°	65.357°
$c = 0.095$	α	60°	60.343°
$\beta = 120$ deg.	α/β	0.50	0.5029
	w/N	0.0602	0.06049
Partial bearing	ϕ	51.5°	51.313°
$c = 0.2$	α	60°	60.187°
$\beta = 120$ deg.	α/β	0.50	0.5016
	w/N	0.1272	0.127013
Partial bearing	ϕ	77.80°	77.64°
$c = 0.2$	α	63.70°	63.86°
$\beta = 120$ deg.	α/β	0.5308	0.5322
	w/N	0.140	0.14007
Partial bearing	ϕ	84.0°	83.68°
$c = 0.1$	α	61.8°	62.12°
$\beta = 120$ deg.	α/β	0.5150	0.5177
	w/N	0.0692	0.06924
Partial bearing	ϕ	139.28°	138.906°
$\beta = 120$ deg.	α	68.72°	69.094°
$c = 0.2$	α/β	0.5727	0.5758
$\theta_1 = 50$ deg.	w/N	0.0531	0.053213

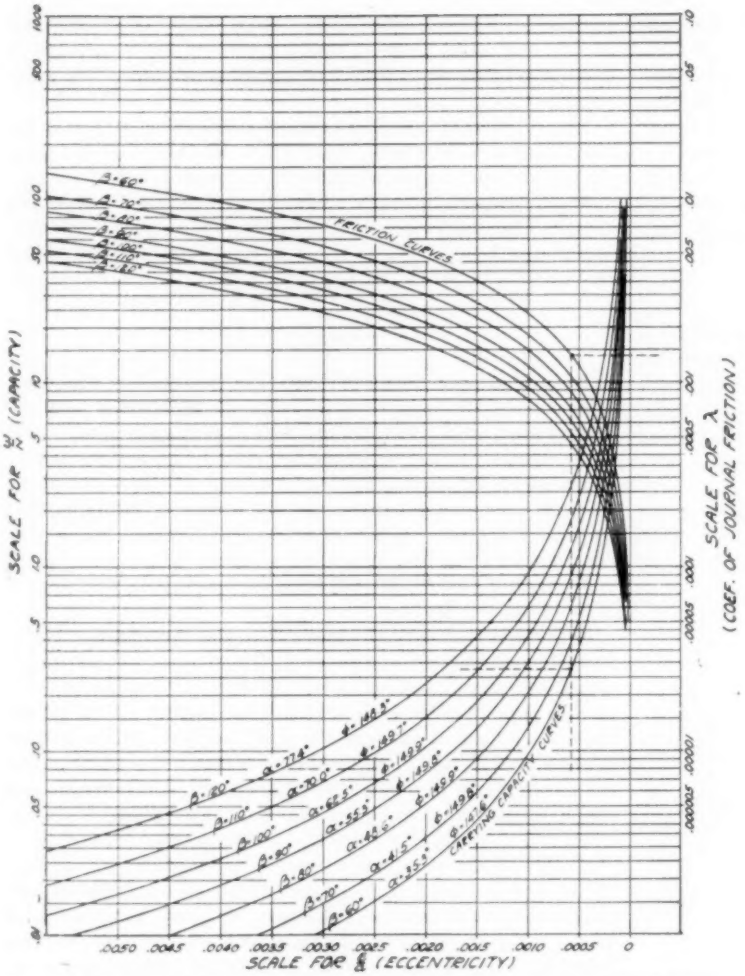


FIG. 14 CHARACTERISTICS OF FITTED PARTIAL BEARINGS OF MINIMUM FRICTION $\mu = 3.4 \times 10^{-6}$

and 0.0132. If, however, the curves for the clearance bearings were extended to a value of $\alpha/\beta = 0.645$, it is quite likely that the lift for such a case would not be far different than for the fitted bearing.

It is very evident that the cap of a fitted 120-deg. bearing, if of types *A* or *B*, would have to be bored with large clearance so as not to interfere with the natural running position of the journal. Otherwise instability might result. It is obvious also that the cap might be made of type *C* so as to reduce the journal lift to a desired minimum.

OIL SUPPLY FOR CAPPED BEARINGS

Assuming the bearing to have a cap of type *B*, let us examine the available locations of oil inlet and outlet for the bearing. Since we know the running position of the journal, the place where the pressure is maximum, and the film thickness there, we can estimate the volume of oil passing through the working film. We must be able to supply this amount at the leading end of the bearing and to get rid of it at the trailing end. For a bearing like Figs. 13 and 13*a*, the cap clearance should equal or exceed the film thickness at the point of maximum pressure, if the oil passing through the film is to return to the starting point without setting up a pressure in channel *Z*. Any increase in the cap clearance will reduce the friction. Any decrease in cap clearance would increase the friction and at the same time set up pressure in channel *Z* that would disturb the running position and require a new estimate thereof.

If the channel at *Z* were connected with the discharge line and the inlet were left at *I*, the cap clearance would have no material influence upon the journal position. If the inlet were at *Z* and the outlet at *I*, all the oil for the film would have to pass through the cap clearance before it reached the bearing. Then after passing through the film it would have to pass under the cap again before being discharged. In that event the cap clearance would have to be twice the film thickness at point of minimum pressure. The discharge opening would preferably be restricted.

If the inlet were at one end of *Z* and the outlet at the other end, the cap clearance would only need to be equal to the film thickness at point of maximum pressure, to avoid disturbing the assumed equilibrium.

EFFECT OF FRICTION ON RESULTANT PRESSURE DIRECTION

The influence of friction on the position and value of the resultant pressure upon the bearing for a given film has been studied for a few critical positions, with the results set forth in Table 2.

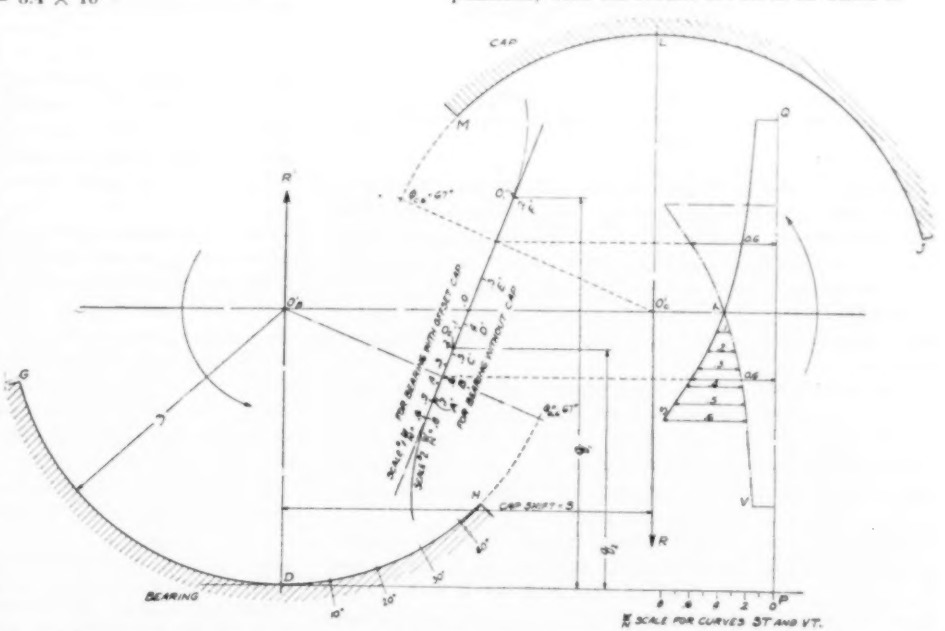


FIG. 15 CLEARANCE-CIRCLE REPRESENTATION OF CHARACTERISTICS OF BEARING WITH ITS CAP SET OVER SO AS TO PRODUCE COUNTER PRESSURES THAT WILL REDUCE THE JOURNAL LIFT IN A 120-DEG. BEARING

These points corrected for friction are shown in Fig. 4 by x's connected by small dotted lines with the uncorrected points. It is interesting to note that the positions of the x's vary but little from the points previously placed upon Fig. 4. The influence of friction upon the α/β lines is more pronounced, however. We have therefore drawn, with reference to these x-points, corrected curves for α/β. These calculations prove that we were justified in neglecting the influence of friction upon resultant position throughout the range of the present investigation.

Conclusion

Having applied these new charts to the solution of several hypothetical problems, one will realize that it is a serious handicap to have charts only for the 120-deg. bearing. We shall therefore endeavor to have this analytical work extended to include bearings with different angular lengths. The amount of labor required for this extension will be much less per bearing than for the first one covered by this paper, because much of the preliminary work has now been done.

The present paper indicates that the journal lift, under adverse conditions, may be much greater than generally expected. This

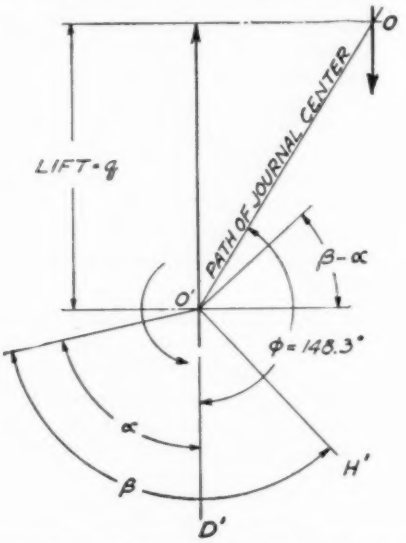


Fig. 16 Path of Journal Center, as Speed Increases in a Fitted Partial Bearing of Series Whose Characteristics Are Given in Fig. 14

may result in rubbing against the cap. The design of the bearing cap therefore becomes important. The methods of providing and removing the lubricant are dependent upon the cap design. The cap may be fixed in an offset position so as to maintain a pressure film that will reduce the journal lift. Such a cap would be suitable for one direction of rotation. If made so as to adjust itself automatically it would be suitable for either direction of rotation.

The journal lift will not be serious if the oil is not heavy for the *load-speed-clearance* combination employed. In other words a bearing that is too lightly loaded for the *oil-speed-clearance* combination may give trouble by rubbing against the cap. Again, if the speed is too high for the *oil-load-clearance* combination, similar rubbing may take place.

High-speed bearings will require further study to take into account the initial pressure at the leading end of the bearing. In general it may be expected that high-speed bearings will have journal lifts that correspond with longer bearings of greater α/β value. Hence when a journal runs in a 120-deg. central bearing at high speed its lift will follow more nearly one of the paths for $\alpha/\beta > 0.5$. The greater the speed the higher the value of the equivalent α/β. These predictions are suggestive of what is to be expected from studies taking account of the initial pressure.

The side-leakage effect in journal bearings[4] is still open for rigorous analytical investigation, as is also the influence of viscosity decrease as oil passes through the bearing film.

Much experimental work is under way from which useful factors for designing engineers may result. Important in this class is the work being done by the U. S. Bureau of Standards under the direction of M. D. Hersey.

Discussion

E. O. Waters.[5] The author has pictured in very commendable manner the history of a journal in its bearing from the instant that motion commences until it gets up to speed. The polar diagram certainly brings out clearly the various journal positions for different speeds and loadings, and shows to what a surprising extent the journal may rise when the bearing is offset only a few degrees. It is to be hoped that the study will be continued for partial bearings of other angular lengths, as there is certainly a wide field of application for bearings in the range of β = 160 to 180 deg.

A problem that naturally arises from this investigation is that of the restraining effect of caps. It would seem that the author could well afford to elaborate on this at much more length than he has permitted himself in the present paper. He cites three cases only (caps A, B, and C), and leaves the writer with the impression that the first of these may or may not be stable at low w/N values; the second is somewhat neutral, and to resume real stable running it is desirable to offset the cap a distance equal to twice the horizontal eccentricity of the journal. Needless to say, this would be a very sensitive and difficult adjustment. Actually, however, a much smaller offset would serve just as well. Take the case of cap B as an example. Suppose that the journal is running at O and that there is a small film of uniform thickness between it and the cap. Then, if the speed is increased or the load decreased, it will tend to rise and crush this film. As soon as this occurs and there is actual pressure on the cap, the journal and cap will act as an inverted bearing and the journal center will take up some position in the region between O and O' such that the algebraic sum of the pressures produced in the lower and upper films will equal the external load. At an infinite speed, the journal center would be exactly midway between O and O'. Such being the case, it would seem that the amount of offset for the cap would not be such a sensitive adjustment after all.

The writer regrets that he has not had sufficient time to work out a sample problem to illustrate his point. Briefly outlined, one method of procedure would be to assume the amount of cap offset for a given-sized bearing; then assume arbitrarily a location for the journal center relative to the bearing center, leading edge, and trailing edge. This would also define the journal position relative to the cap center, leading edge, and trailing edge. With these data, previous formulations and charts could be used to determine the magnitude of the carrying capacity and w/N direction of the resultant film pressure between the journal and bearing and the journal and cap; and the vector sum of these carrying capacities would determine the net carrying capacity

[4] A. G. M. Michell's study of side leakage for inclined flat surfaces with rectilinear motion is the only sound analysis at present available. It indicates what may be expected for curved surfaces in short arc partial bearings. See "The Lubrication of Plane Surfaces," by A. G. M. Michell, Melbourne, *Zeitschrift für Mathematik und Physik*.

[5] Assistant Professor of Mechanical Engineering, Yale University, New Haven, Conn. Assoc-Mem. A.S.M.E.

of the capped bearing. By repeating the process for several locations of O, data would be obtained for plotting contour lines of carrying capacity for any given direction of external load.

GEORGE B. KARELITZ.[6] The paper gives a very interesting analytical study of the motion of a shaft in the bearing shell. It would be of importance to arrange for experimental verification of the results obtained. Some measurements of journal positions are on record, and a number of people will probably be engaged in the future in such investigations. A few of the methods of measurement will be mentioned. V. Vieweg (Germany) used a beam of light just skimming the journal. This allowed microscopic measurement of the journal motion; a photographic record was taken on a moving film (*Petroleum Zeit.*, vol. 18, 1922, p. 1405). He found vibrations of the journal during the first moments of rotation, as described by the author. Ch. Hummel (Germany) attached a pivoted mirror to the end of a journal and studied the motion of the shaft by observing the deflection of a beam of light from the mirror (*Forschungsarbeiten*, No. 287, V.D.I., Berlin, 1926). Mr. Schering (Germany) measured the thickness of the oil film by the variation in capacity of the condenser formed by the bearing and journal (*Instrumentenkunde*, June, 1927, p. 286). In our company we made accurate tests in

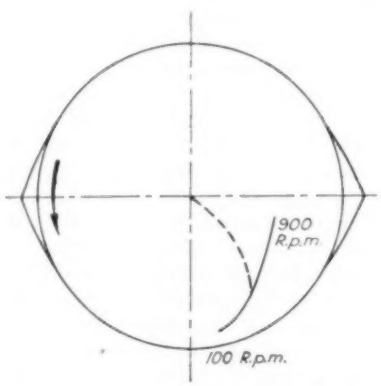


FIG. 17 POSITIONS OF JOURNAL FOR SPEEDS FROM 100 TO 900 R.P.M.

connection with a study of oil-film vibration; the work was done by Mr. M. Stone of our Power Engineering Department. A disk was attached to the shaft of a 9-in. bearing outside the bearing housing. The disk was ground concentric with the journal. Four small electromagnets were placed around the disk, two on the horizontal and two on the vertical diameters, with a very narrow clearance between their armatures and the disk. Motion of the shaft caused electromotive forces in the coils which were recorded by an oscillograph. Proper interpretation of the oscillograms gave a complete picture of the journal movement in the bearing clearance.

Using the clearance circle proposed in the paper, the positions of the journal were found for various speeds from 100 to 900 r.p.m., as approximately shown in Fig. 17. The center of the journal did not approach the center of the bearing, as should be the case for a central-portion bearing. This was caused, no doubt, by an oil pressure on the down side of the journal, as mentioned in the paper, which moved the journal over to the right.

In his investigations Mr. Howarth assumes that there is a positive pressure all over the bearing surface. The writer in his analysis ("Charts for Studying Oil Films in Bearings," Trans. A.S.M.E., 1925) assumed that the pressure is zero at the point of nearest approach, the pressure being also zero in the divergent part of the oil film. This was done on basis of observations on large turbo-generator bearings which are machined with utmost care and accuracy. Invariably from 2 to 6 in. of vacuum is found in the divergent part of the bearing surface; this vacuum is negligible when compared with the pressures existing in the load-carrying oil film, justifying the assumption of zero pressure. Comparing the data in Fig. 4 of the paper and in the diagram Fig. 5 of the above-mentioned "Charts," the values of $(w/\mu N)(\eta/a)^2$ for a 120-deg. bearing are:

Eccentricity c	0.1	0.2	0.3	0.4	0.5	0.6
Value from the paper	0.019	0.038	0.061	0.093	0.132	0.196
Value from the "Charts"	0.019	0.039	0.063	0.091	0.123	0.172

Taking into consideration the inaccuracies of the graphical computations employed, we can say that up to $c = 0.4$ the data coincide, while at higher values of c, i.e., for heavier loads, the carrying capacity of the bearing appears to be less than is represented in Fig. 4 of the paper. It would be very valuable to have reliable experimental work done on this question.

It is interesting to note from Figs. 6 and 7 that an increase of the clearance, all other conditions being equal, affects but little the thickness of the oil film at the point of nearest approach. This appears to be true for all partial bearings of ordinary design.

A. G. M. MICHELL.[7] The writer hopes that the author will extend his investigations to bearings of smaller angular lengths. In the bearing of 120 deg., treated in detail in the paper, there may be considerable portions of the oil film diverging in the direction of relative motion, and such a condition is usually inadmissible in bearings of finite width.

The author's discussion of the possible variations of the position assumed by the journal relatively to the bearing is paralleled to a certain extent in R. O. Boswall's recently published work, "The Theory of Film Lubrication." The criterion for determining the stability of the journal in one or other of such positions does not appear, however, to have been satisfactorily established. The fact brought out by the author that the center of the journal may rise above the center of the bearing surface is not to be regarded as paradoxical. Such a condition necessarily occurs in "pivoted" journal bearings, in which the curvature of the bearing surfaces is made the same as that of the shaft, and the same effect is to be expected, under certain conditions, in bearings formed with a clearance.

M. D. HERSEY.[8] An investigation has recently been completed by R. Wolff in the laboratories of the German Government Railways at Göttingen, in which the optical-interference method was used for measuring displacements of the bearing relative to the journal. This method is extraordinarily sensitive.

The conclusion reached was that the laws of fluid-film lubrication would not fully account for the observed results and that the oiliness of the lubricant must be considered as a rather large factor. It will be interesting, when we can get the details of these experiments, to compare them with Mr. Howarth's curves and results.

To my mind the significance of this German work is the interest that it gives once more to the experimental study of oiliness. We are indebted to Mr. Kingsbury for calling the attention of the profession to the importance of oiliness in a paper presented before the Society in 1902. The Special Research Committee on Lubrication has had on its program, since its organiza-

[6] Research Engineer, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa. Assoc-Mem. A.S.M.E.

[7] New York, N. Y.

[8] Chief of Friction and Lubrication Section, U. S. Bureau of Standards, Washington, D. C. Mem. A.S.M.E.

tion in 1915, the attempt to study oiliness as thoroughly as the viscous-film effects analyzed by Mr. Howarth.

A. E. Flowers.[9] I want to express heartfelt appreciation for the worth-while theoretical study of lubricated bearings that the author has been conducting. It is particularly important, it seems to me, that it should have been carried out as it has as a theoretical study, because that puts it at once in a form such that designers can make use of it.

Attention should again be called to the research program of the Lubrication Committee and to the fund it is proposed to raise for these research studies. It is hoped that the sum of approximately $25,000 may be raised in the near future in order to conserve the results, the equipment, and the personnel that are now available for actively prosecuting these studies, and particularly for prosecuting the highly important studies upon the unknown or x-factor of lubrication, called unknown simply because, although we know it is present, we do not know to what it may be due or how it may be controlled.

I think there is no other single project now before our Society or the profession of greater importance or more worth while than the study of the lubrication conditions, and particularly this so-called unknown factor.

Author's Closure

It is certainly gratifying that this paper has provoked so much discussion by such competent persons. It is very interesting indeed to know what Mr. Michell, Mr. Hersey, and Mr. Karelitz have pointed out, that so many investigators are studying experimentally the running position of the shaft in a journal bearing, and it is hoped that these experimenters will be able to show exactly how much the true paths of the journal axis deviate from the theoretical ones shown in the paper.

Mr. Karelitz has plotted a curve representing results of experiments made by him in the Westinghouse Research Laboratory. It is to be hoped that he will continue these experiments and present a paper to the Society giving all the data necessary for a complete understanding of the results obtained. When he compares values obtained from my Fig. 4 with the Fig. 5 taken from his charts,[10] he should state that there is only one point on my Fig. 4 that represents a bearing agreeing with the series covered by his charts. The bearings covered by my curves are assumed to have complete films. Those illustrated by my Fig. 4 are all assumed to have positive pressures in the films. A curve can be drawn on my Fig. 4 representing a series of bearings in which $\phi = \beta - \alpha$. This curve, about straight, would pass approximately through the point where $\phi = 60$ deg. on the curve for $\alpha/\beta = 0.5$, and the point for $\phi = 50$ deg. on the curve for $\alpha/\beta = 0.58$. Bearings of this series would have the ϕ-line, which passes through the journal and bearing centers, coinciding with the trailing end of the bearing. This new curve would represent a series of 120-deg. bearings with complete films, no portions being discarded as ineffective. It would cut the various α/β curves and would have one intersection with each. It would cut the curve for $\alpha/\beta = 0.5$ where $\phi = 60$ deg. and where the eccentricity factor $c = 0.12$. For this point on Fig. 4 the value of $(w/\mu N)(\eta/a)^2$ is about 0.023. Hence in the series of comparisons made by Mr. Karelitz, between his work and mine, one would expect agreement at that point only. This is confirmed by the fact that he shows agreement when the eccentricity factor $c = 0.1$.

For all values of c, other than $c = 0.12$, Mr. Karelitz has made his comparisons with the series represented by my curve $\alpha/\beta = 0.5$, which assumes the oil film to be complete throughout the whole bearing angle β. The series covered by Mr. Karelitz's chart Fig. 5 include only those bearings in which the oil film is assumed to be complete throughout an angle $\alpha + \phi$. Since he neglects pressures between the ϕ-line and the end of the bearing and has determined his resultant position on that assumption, the comparisons between his work and mine merely indicate deviations resulting from different points of view. My work has been rigorously true to the Reynolds theory, whereas Mr. Karelitz has arbitrarily located his point of zero pressure at the point of closest approach where he believes experiments will prove it to lie in bearings where side leakage is taken into account. I believe he has jumped at a conclusion that will not be supported generally by the results of experiments covering the field of practical bearing proportions.

Referring to the comments by Professor Waters, it was not my intention to convey the impression that the journal would be unstable with any particular form of cap, but merely to show that the freedom of the journal to rise along any path, or particularly upon the path determined by the bearing alone, would be interfered with by any cap under which a counter pressure would be generated. His statement that an offset cap need not have as much eccentricity as illustrated by my Fig. 15 does not seem reasonable if one wishes to preserve full films in both the bearing and the cap. For a lesser cap eccentricity than shown, there would undoubtedly be pressures developed under the cap that would oppose the normal lift of the journal from the bearing. It therefore seems most natural and correct to shift the cap until the journal path for it, if the load were upward, would coincide as nearly as possible with the journal path for the bearing, assuming that the load is downward. This assumption avoids the necessity of a detailed analysis for a particular cap and bearing combination with a lesser or a greater cap offset.

I hope we shall find time to extend this investigation to bearings of smaller angular lengths than 120 deg., in accordance with Mr. Michell's suggestion. He is interested in the shorter bearings because of his subdivision of the bearing surface into small sections about 40 deg. long in the pivoted-block journal bearing. If our preliminary investigation of short-arc bearings, comparing them with longer ones,[11] had shown the short arc superior to the longer ones, we would perhaps have made our preliminary study for the shorter ones. It is our belief, however, that the longer-arc bearing is bound to remain superior to the more complex group of shorter ones. We therefore explored the 120-deg. bearing quite fully, as shown in this paper.

I have not yet examined R. O. Boswall's recent book and am therefore not familiar with his discussion of stability of the journal. In Part III of my "Graphical Study," in the discussion of Fig. 44 showing capacity and friction coefficient curves for a 120-deg. fitted-journal bearing, I found a number of pairs of journal running positions for the same load, speed, and viscosity factor—each, however, with a different eccentricity. The friction coefficients were lower for the greater eccentricity than for the lesser. An analytical investigation I conducted at that time led me to conclude that the journal would run in the position where the friction was least.

It seems reasonable enough, now that the investigation has been made, that the journal should be expected under certain conditions to rise above the center of the bearing surface in a bearing with running clearance, especially when the clearance is small. The slant of the path of journal center in Fig. 16 is seen

[9] Engineer in Charge of Development, De Laval Separator Co., Poughkeepsie, N. Y. Mem. A.S.M.E.

[10] "Charts for Studying the Oil Film in Bearings," by George B. Karelitz, presented at the Annual Meeting of the A.S.M.E., December, 1925.

[11] See example 9, part III, "A Graphical Study of Journal Lubrication," presented in December, 1925; and letter entitled "A Graphical Study of Journal Lubrication," published in Mechanical Engineering for April, 1927.

to be quite similar to the slant of the path in Fig. 5 for $\alpha/\beta = 0.62$. When the α/β is the same in both cases, we should expect the corresponding paths in the two figures to approach one another as the clearance ratio approaches zero.

It seems desirable to point out that in the various analytical investigations I have conducted, including the present paper, there is complete consistency in assumptions, methods, and results. That seems to be the best way to lay the foundation for an experimental determination of deviations in actual bearings from conditions that our theoretical study with its limitations would lead us to expect. This experimental check is under way, but is very far from complete. When it is far enough along the time will be ripe for a fuller discussion of certain suggestions made by Mr. Michell and Mr. Karelitz concerning the location of positive and negative pressures in the bearing films, with relation to relative inclinations of the surfaces, and the point of closest approach.

Dr. Flowers has pointed out the need of funds for carrying out the investigations fostered by the Special Research Committee on Lubrication, of which Mr. Hersey is chairman. Several important projects are under way in laboratories that have inadequate funds available; for example, Pennsylvania State College and the U. S. Bureau of Standards. If adequate funds were on hand, important work could be carried forward in those and possibly in other laboratories. Investigation is not standing still, however. For example, some important work is being carried on in the Westinghouse laboratory at East Pittsburgh under the direction of Mr. Karelitz. Further investigations are being conducted under Mr. Kingsbury's supervision at his laboratory in Philadelphia. Neither the Westinghouse nor the Kingsbury laboratory requires financial assistance from the Society. For this reason the Special Research Committee on Lubrication has been able to make a fair showing on very slim appropriations from the Society. Nevertheless the committee's work should not drag along too slowly, and adequate funds are therefore very much needed.

Erratum

Paper No. APM-50-3 ("Measurement of Flow of Air and Gas in Nozzles," by S. A. Moss): In the last equation on page 9, second column, the denominator or in the final parenthesis should be p_1 instead of p_2. As corrected the equation will read

$$\sqrt{1-\left(\frac{D}{D_p}\right)^4\left[1-\frac{2}{k}\left(\frac{p_1-p_2}{p_1}\right)\right]}$$

Combustion at High Pressures[1]

By L. C. LICHTY,[2] NEW HAVEN, CONN.

In the author's experiments hydrogen-air mixtures ranging from 1 H_2 + 0.85 Air to 1 H_2 + 6.5 Air were investigated, first, by varying the length of combustion chamber, holding the initial temperature constant at 80 deg. fahr., and second, by varying the initial temperature from 80 to 400 deg. fahr., holding the length of combustion chamber constant. Records were obtained from which the effect of mixture on maximum pressure was studied, as well as the effect of surface-volume relationships on maximum pressure, effect of mixture on time of pressure rise, rate of flame propagation, heat loss during pressure rise, etc.

THE object of these experiments, which have recently been made at Mason Laboratory, Yale University, was to study the process of combustion. Air-hydrogen mixtures were chosen for the first experiments, due to the rapid rate of combustion of the correct mixture.[3] High pressures were used so that the apparatus when developed would be satisfactory, with small change, for low-pressure experiments.

The investigation is divided into two parts with variables as follows:

a The length of combustion chamber was varied while the initial temperature was held at 80 deg. fahr. for ten mixtures ranging from one explosive limit to the other.

b The initial temperature was varied from 80 deg. fahr. to 400 deg. fahr. while the length of combustion chamber was held constant for eleven mixtures from one explosive limit to the other.

All experiments were conducted at an initial mixture pressure of 1000 lb. per sq. in. gage, the combustion taking place at constant volume.

Description of Apparatus

A Kipp generator with zinc and dilute sulphuric acid was used to obtain the hydrogen. After generation the gas was washed through two warm solutions of potassium permanganate and one solution of lead acetate.

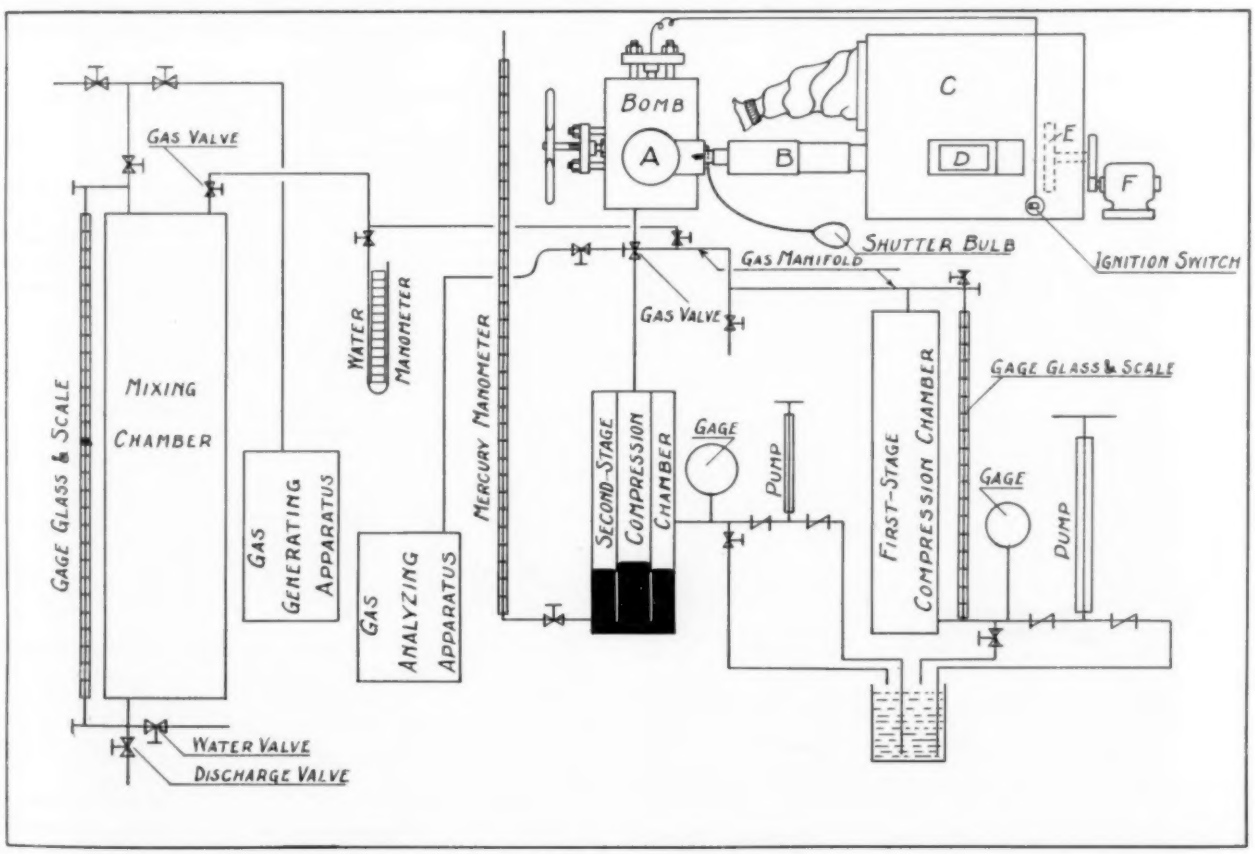


Fig. 1 General Arrangement of Apparatus

The air was washed through a cleaning solution while being drawn into the mixing chamber. The mixing chamber, 10 in. in diameter and $3\frac{1}{2}$ ft. high, provided enough volume for the mixture required for one series of experiments to be completed in one day. A motor-driven fan mixed the gases and maintained uniformity of mixture throughout any series of experiments.

The compression of the combustible mixture was accomplished in two stages. In each stage a hand-operated water pump was used to compress the charge. In the first stage the compression

[1] This work was done under the direction of Prof. Walter J. Wohlenberg.

[2] Assistant Professor of Mechanical Engineering, Sheffield Scientific School, Yale University. Assoc-Mem. A.S.M.E.

[3] The correct mixture is one in which the combining constituents are in the proper proportions (1 H_2 + 2.39 Air).

occurred over water, and in the second, over mercury. (See Fig. 1.)

The combustion chamber was formed inside of a piece of 8-in. shafting. The shape of the combustion chamber was cylindrical except for the upper part, which was conical in form. The cylindrical combustion chamber was chosen since it made possible various volume experiments in which only the length of the flame travel varied. Spherical combustion chambers, while having less heat loss per unit volume, present a changing cross-section to flame travel.

The temperature of the bomb was maintained with an electrical heating element embedded in magnesia and surrounding the bomb.

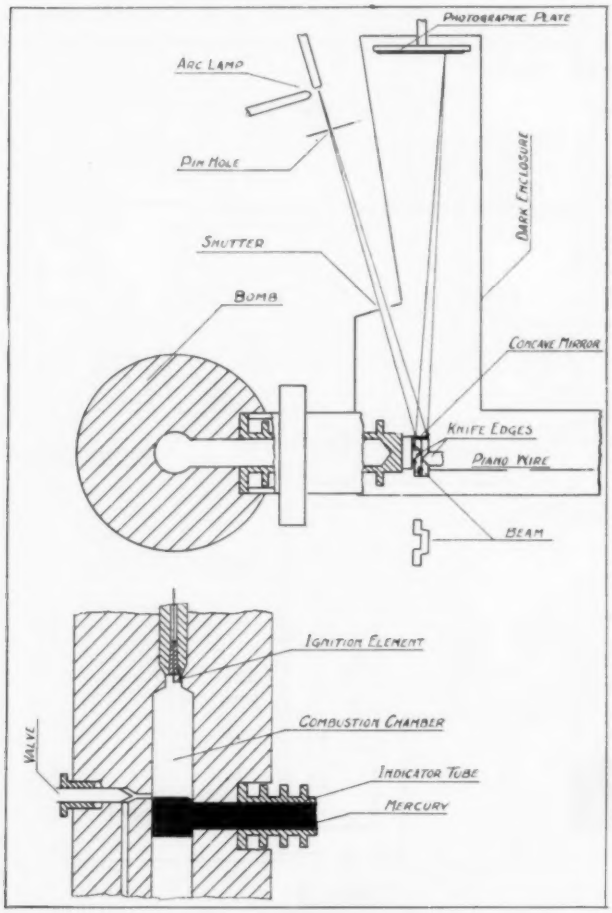


Fig. 2 Bomb and Photographic Apparatus

Three connections were made into the combustion chamber. The lower valve trapped the charge in the chamber. The upper valve contained the ignition element and also permitted the combustion chamber to be scavenged with mercury. The third connection was for the optical indicator.

For low-pressure work the pressure element of an optical indicator is usually a steel diaphragm.[4] The compression of a steel tube has been used extensively for high pressure experiments.[5] The tendency to buckle and to introduce friction to the movement are the two principal disadvantages to this type of element. The pressure element in this work (Fig. 2) was a $3\frac{1}{2}$ per cent nickel-steel tube subjected to tension. With this type of element, friction and buckling were eliminated. The tube was heavily reinforced with integral flanges to make it approximately equally strong both longitudinally and circumferentially.

The movement of the tube is transferred to a beam held between two knife edges by means of a No. 19 piano wire stretched almost to the elastic limit. The outer knife edge and beam are so designed that both knife edges and wire support are in the same plane. A slot in the support of the outer knife edge permits an adjustment whereby the leverage ratio of the magnifying apparatus may be changed.

The optical and photographic apparatus consisted of an arc lamp, shutter, mirror, and revolving plate. Immediately in front of the carbon-arc lamp was located a piece of copper with a minute aperture. This formed the point source of light. The cone of light from this source was directed toward a mirror on the beam of the optical indicator. The light reflected from the mirror was focused on a revolving photographic plate. The camera enclosure surrounded all of the optical apparatus except the point source of light, the light being admitted at the desired time by a photographic shutter.

Experiments With Various Volumes and Mixtures

In this series of experiments ten hydrogen-air mixtures were used, ranging from 1 H_2 + 0.53 Air to 1 H_2 + 7.00 Air.

The photographic records shown in Figs. 3 and 4 are taken from the various mixtures and show the differences in rates of pressure rise and the effect of different lengths of combustion chambers. Table 1 gives information regarding the conditions under which these records were made, along with maximum pressure and time of pressure rise for each record.

TABLE 1

No.	H_2-Air ratio	Vol. of comb. chamber, per cent	Plate speed, r.p.m.	Max. pressure, lb.	Time of pressure rise, sec.
1	1:0.53	40	300	4775	0.1060
2	1:0.57	10	300	5000	0.0280
3	1:0.57	80	300	5125	0.1200
4	1:1.25	100	300	7100	0.0170
5	1:2.41	10	900	6700	0.0015
6	1:2.41	100	900	7900	0.0121
7	1:3.6	100	900	6700	0.0125
8	1:5.0	20	300	4850	0.0090
9	1:5.0	80	300	5300	0.0320
10	1:6.0	40	300	5025	0.0075
11	1:7.0	10	300	2400	0.0394
12	1:7.0	40	300	4400	0.2000

The inner circle of each record represents atmospheric pressure. The circle around it represents the compression pressure of 1000 lb. per sq. in. gage. The beginning of combustion is noted by a line leaving the outer circle and going rather quickly to the maximum pressure, where it merges with the spiral cooling curve.

In general, near both explosive limits the process of combustion appears to be similar, although at the lean end of the mixture a much slower rate of combustion exists. Away from the explosive limits a marked difference is noted. With rich mixtures the rise of pressure ends almost abruptly, while for lean mixtures the end of the pressure rise is well rounded into the cooling curve.

Records Nos. 2 and 3 show the effect of combustion-chamber volume on the process. Being a rich mixture with a slow rate of burning, the process reaches maximum pressure much more quickly with the small volume and in a much different manner.

Record No. 4 is typical of all the records taken with this mixture in which the air supplied was about 50 per cent of that required for the correct mixture.

Record No. 5 shows the effect of inertia of a large mass of mercury in the combustion chamber and should not be thought to represent the combustion process. Record No. 6, with the

[4] Kratz and Rosecrans, Eng. Exp. Station, University of Illinois.

[5] Petavel, "Manometer for High Pressures," *Phil. Mag.*, 1902; W. A. Bone and others, "Gaseous Combustion at High Pressures," *Phil. Trans. A*, vol. 215; *Proc. Roy. Soc. A*, 1921–22, 1923, 1924, 1925.

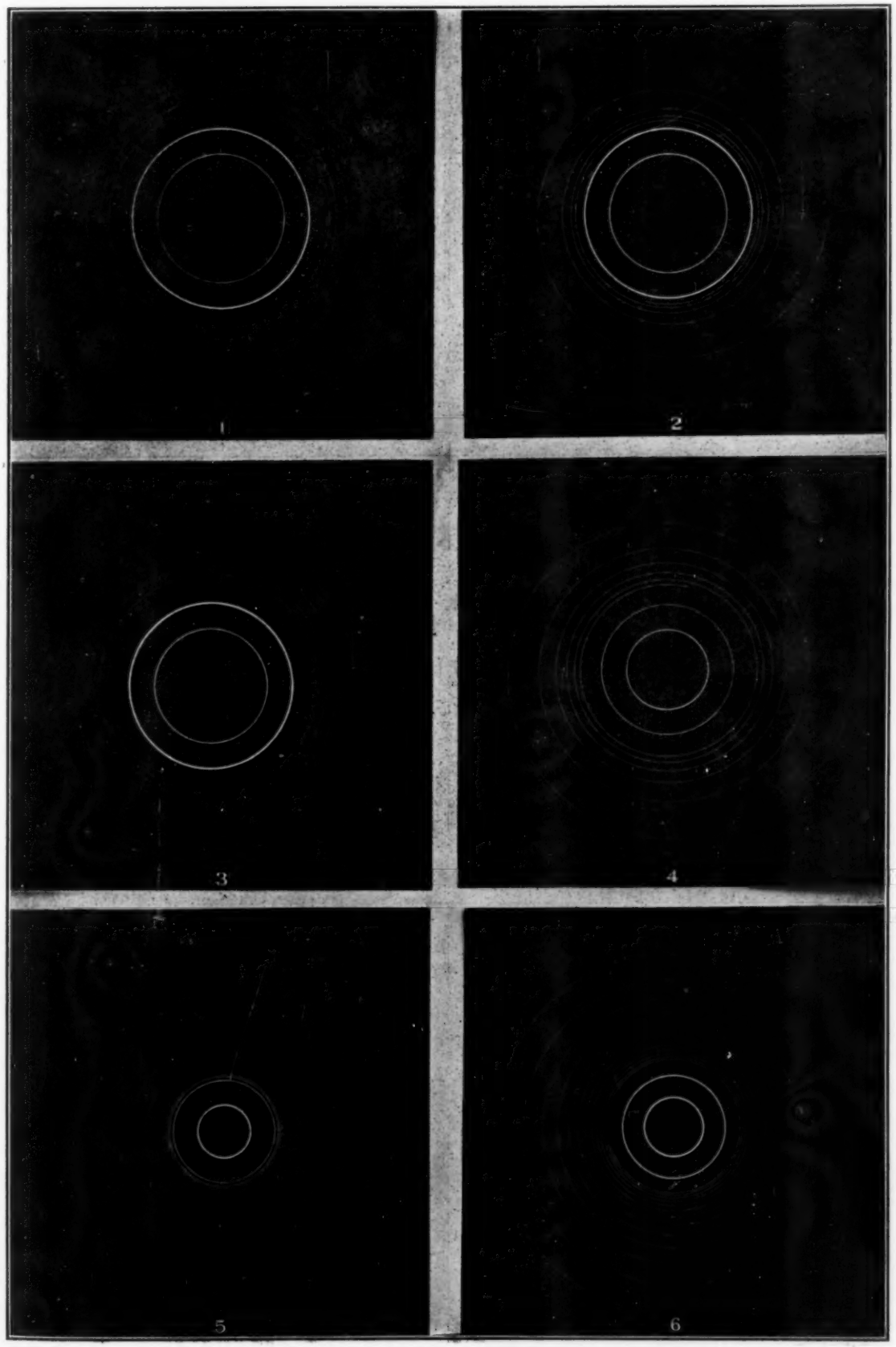


FIG. 3 AIR-HYDROGEN COMBUSTION RECORDS

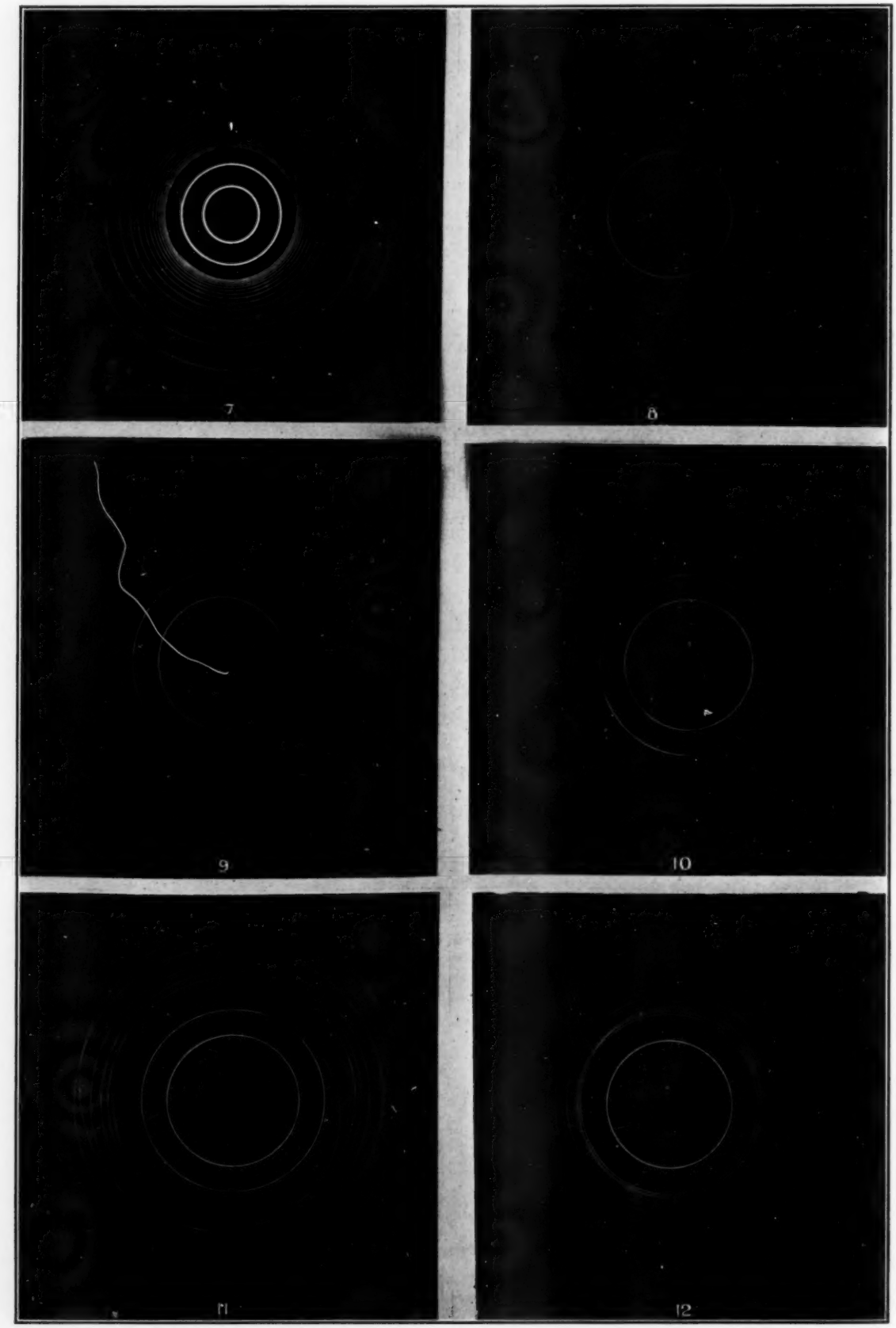


FIG. 4 AIR-HYDROGEN COMBUSTION RECORDS

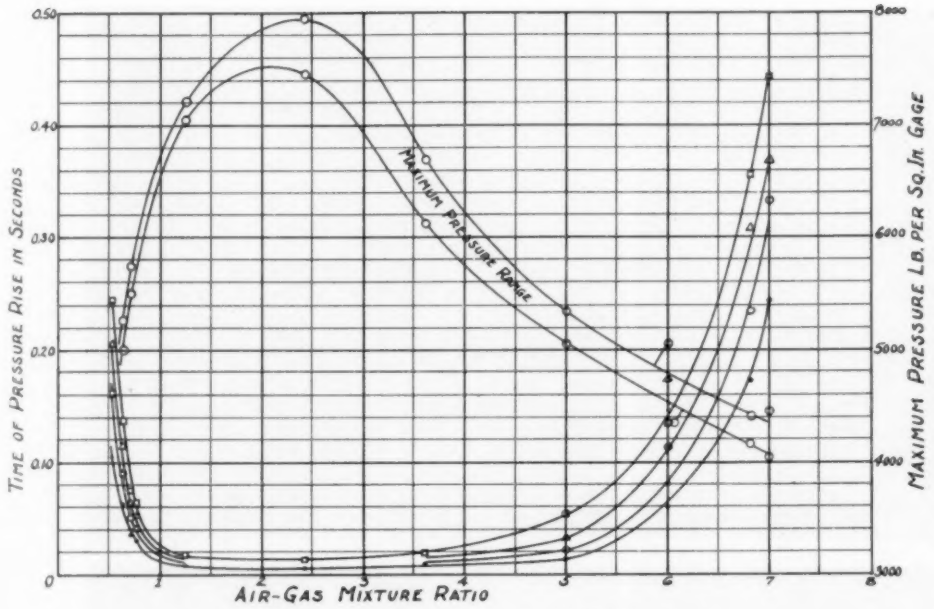


FIG. 5 EFFECT OF MIXTURE ON MAXIMUM PRESSURE

same mixture but with larger combustion chamber, is typical of the other records taken with this mixture. In the original of this record the natural frequency of vibration of the indicator tube could be seen. It appears to be of the order of 6000–7000 vibrations per second.

Record No. 7 shows the speeding-up and slowing-down tendency of the combustion process which occurred with the lean mixtures.

Records Nos. 8 and 9, with the same mixture but with small and large volumes, show the process speeding up in the case of No. 9 until it resulted in detonation, while in No. 8 the process speeded up and slowed down several times without detonation. At no time when detonation occurred did the pressure rise to an abnormal amount.

Record No. 12 is of particular interest, for in this case the rate of energy liberation was not very much greater than that of energy dissipation, with the result that only a small pressure rise resulted. Record No. 11, with the same mixture but with a larger volume, shows a much larger pressure rise.

EFFECT OF MIXTURE ON MAXIMUM PRESSURE

The highest and lowest maximum pressures for each mixture are plotted in Fig. 5, no reference being made to the volume corresponding to the maximum pressure. The maximum pressure occurs with the correct mixture, maximum pressures being considerably lower for leaner and richer mixtures.

EFFECT OF SURFACE-VOLUME RELATIONSHIPS ON MAXIMUM PRESSURE

The maximum pressure for each of the mixtures was not obtained with the same volume. For mixtures near the correct mixture the maximum pressure occurs with large volumes and small surface-volume relationships (Fig. 6). For both the richer and leaner mixtures maximum pressures were obtained with smaller volumes and larger surface-volume relationships. In general, slow-burning mixtures will reach maximum pressures in the smaller combustion chambers where flame travel is shorter, while the faster-burning mixtures will reach maximum pressures in the larger combustion chambers. Thus the combination of the rates of energy liberation and energy dissipation fix the maximum pressure for a given mixture in a definite length of combustion chamber.

This relationship between fuel-air mixture, rate of flame propagation (and resultant energy liberation), and rate of energy dissipation during the combustion process should be considered in the design of the combustion chamber of an internal-combustion engine. These experiments show that no one combustion chamber will result in maximum pressures for all mixtures of fuel and air; and that when the fuel-air mixture is such that the flame propagation is slow it is better to provide a combustion chamber with small flame travel even though the surface-volume relationship is undesirable, from a heat-dissipation standpoint.

EFFECT OF MIXTURE ON TIME OF PRESSURE RISE

The time of burning is low for correct and near-correct mixtures, being below 0.02 sec. for mixtures ranging from 1.25 air to 3.60 air per volume of hydrogen. The correct mixture burned the fastest.[6]

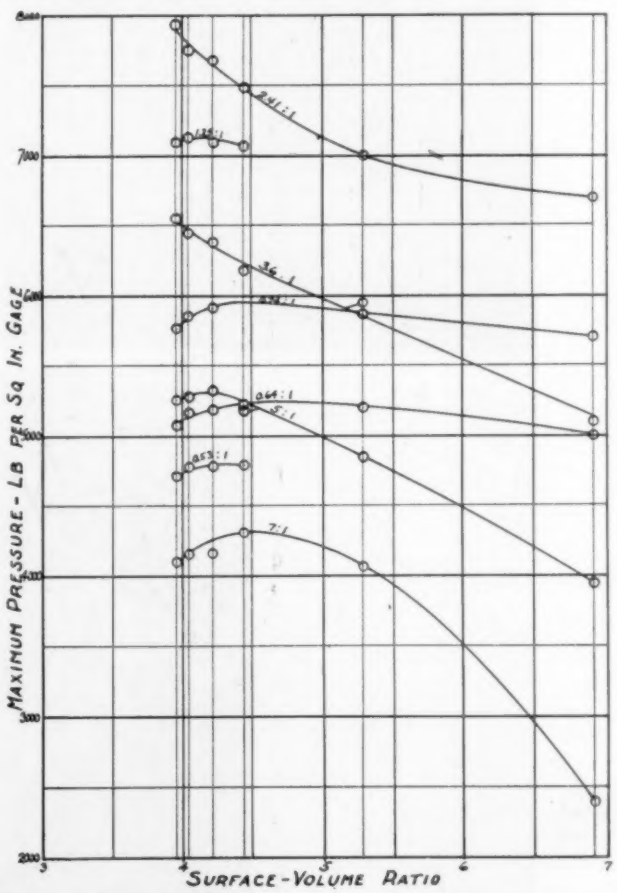


FIG. 6 EFFECT OF SURFACE-VOLUME RATIO ON MAXIMUM PRESSURE

[6] See second set of experiments.

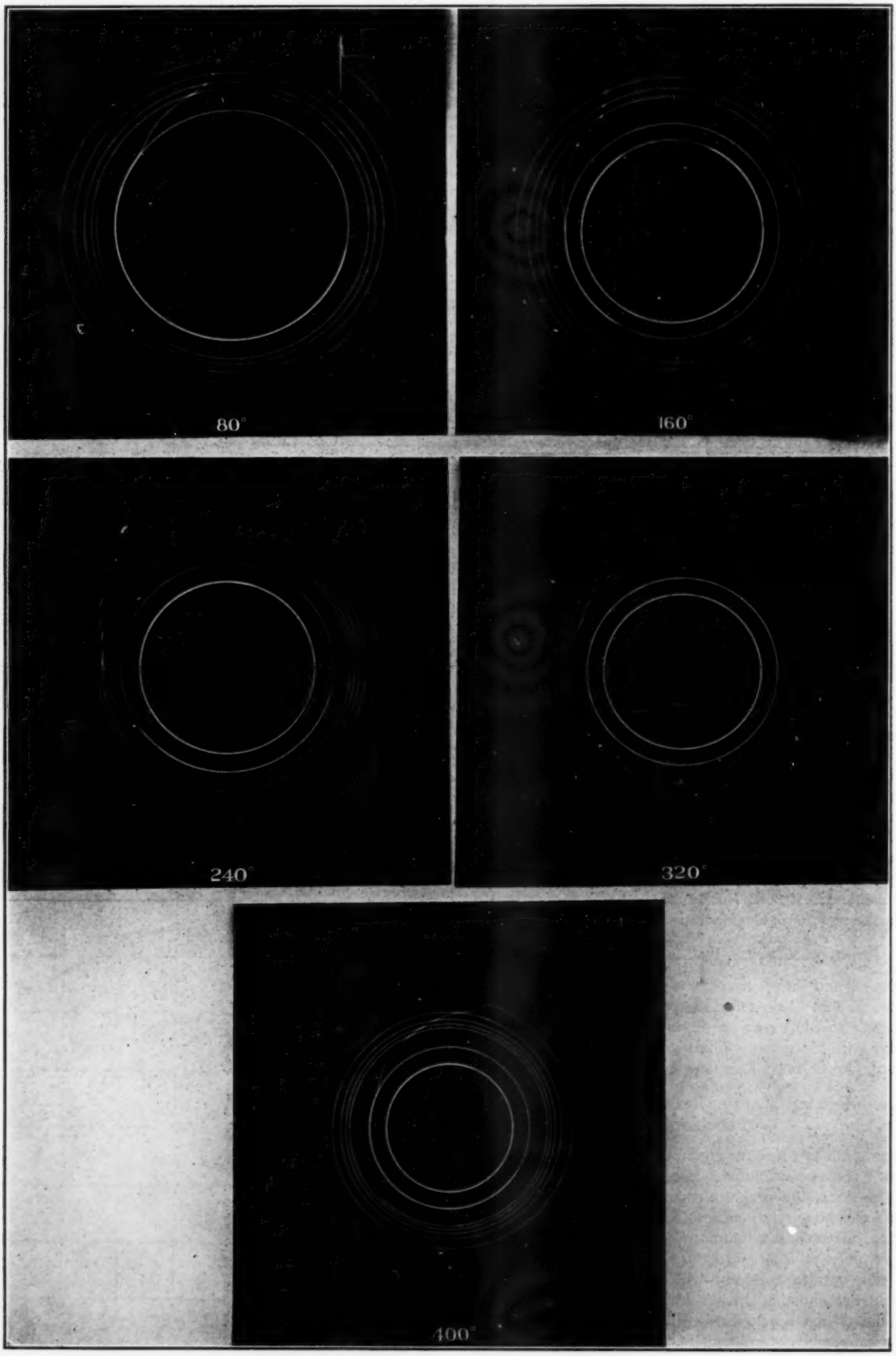


FIG. 7 RECORDS FOR 1 H_2 + 4.78 AIR MIXTURE

For any mixture, the smaller the volume the lower the time of pressure rise, indicating that the length of flame travel directly affects the time of pressure rise.

Explosive Limits

The explosive limit is apparently independent of the volume for this apparatus, and was found to be 1 H_2 + 0.5 Air on the rich side, and to lie between 1 H_2 + 7 Air and 1 H_2 + 8 Air on the lean side, probably quite close to the 1 H_2 + 7 Air.

Flame Propagation

The average rate of flame propagation was determined from the time to reach maximum pressure and the length of flame travel. The lowest rate was 0.11 ft. per sec. with the leanest mixture and the smallest volume, while a rate of over 30 ft. per sec. was found with the correct mixture. The length of flame travel had no apparent effect on the rate of flame propagation in some mixtures, while in others the rate increased or decreased with the length of flame travel.

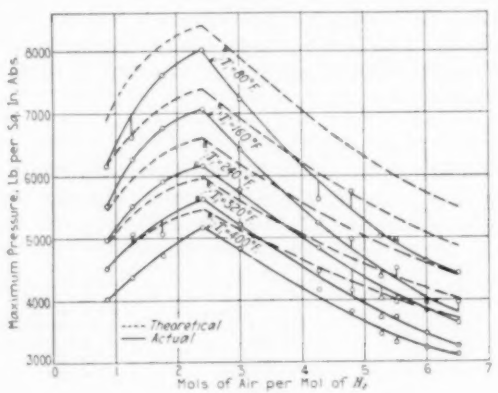


Fig. 8 Maximum Pressures

The average rates of flame propagation are given in Table 2.

TABLE 2 FLAME VELOCITIES, FEET PER SECOND

Length of travel, in.	Air-hydrogen mixture ratios 0.53	0.64	0.74	1.25	2.41	3.6	5.0	6.0	6.8	7.0
0.52	...	1.6	3.3	..	28.9	15.5	4.3	1.0	0.53	0.11
0.88	...	2.1	3.3	..	26.2	14.1	8.2	1.5	0.46	0.32
1.61	1.3	2.1	4.1	14.2	23.5	12.7	10.3	1.7	0.78	0.58
2.29	1.2	2.1	4.4	16.7	26.6	19.6	9.1	1.7	0.82	0.59
2.98	1.2	2.2	4.4	17.8	30.3	22.2	7.8	1.4	0.82	0.68
3.66	1.3	2.3	4.5	17.9	26.3	20.3	6.1	2.3	0.88	0.66

Experiments With Various Initial Temperatures and Mixtures

In this series of experiments eleven hydrogen-air mixtures were used, ranging from 1 H_2 + 0.85 Air to 1 H_2 + 6.5 Air, some of the mixtures being chosen to lie between those used in the first series of experiments. The records taken with 1 H_2 + 4.78 Air are shown in Fig. 7. Attention is called to the type of pressure rise, which is similar to Record No. 8 in Fig. 4, this record being made with approximately the same mixture. The records for the other ten mixtures in this series exhibit pressure rises with the same characteristics as found in the first series for similar mixtures, showing that the mixing of the fuel and air was complete in all cases. The ease with which records could be duplicated was considered rather remarkable.

Theoretical and Actual Results

The results of this series of tests are shown in Figs. 8 and 9. The theoretical results were obtained by an analysis based upon the work of Goodenough and Felbeck.[7] This work takes into account variable specific heats and the effect of dissociation, but does not include correction for the deviation from perfect-gas laws. For this reason, only the shape of the theoretical curves are of particular consequence. The actual pressures attained decrease with an increase in initial temperature of the mixture. This was to be expected, since theoretically

$$\frac{P_0}{P_1} = \frac{T_0}{T_1}$$

and for a practically constant energy liberation the difference between T_1 and To would vary only a small amount, so that

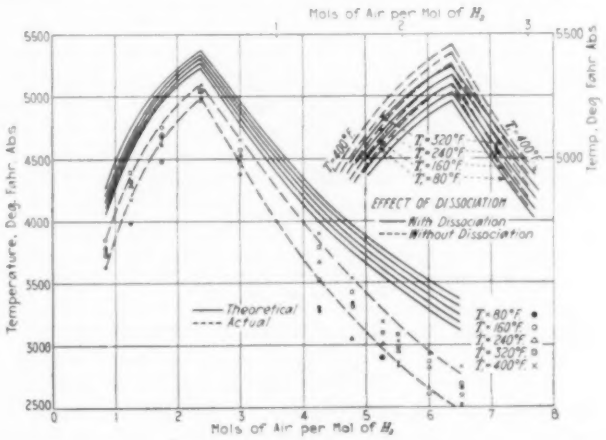


Fig. 9 Maximum Temperatures

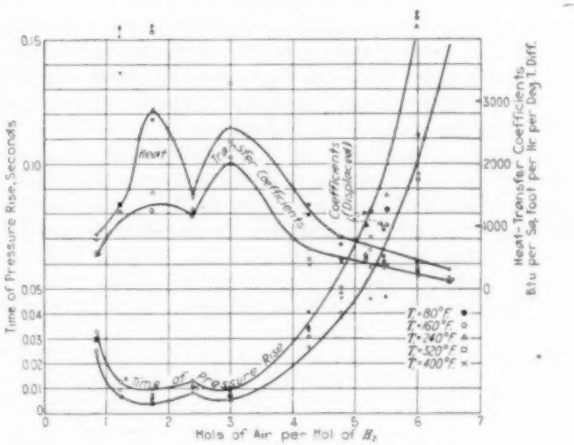


Fig. 10 Curves of Time of Pressure Rise

for higher initial temperatures (T_1) the ratio of pressures would be lower.

The curve of time-of-pressure rise (Fig. 10) shows two minimums, one on each side of the correct mixture. The experiments of R. W. Fenning[8] show the same effect on the rich side but not on the lean side.

The possible number of combinations of molecules of hydrogen with oxygen for lean mixtures and of oxygen molecules with hydrogen for the rich mixtures is greater than for the correct mixture. This would indicate the longest time of burning at the correct mixture and the shortest time at the explosive limits. This combination effect is offset by that of high mean temperature at the correct mixture and low mean temperature

[7] Bulletin No. 139, Eng. Exp. Station, University of Illinois.

[8] "Air-Hydrogen Explosions in Closed Vessels." Aeronautical Research Com. (England), 1923–24.

at the explosive limits. At the explosive limits the speed of the molecules is low compared with the speed at the correct mixture. The higher the molecular speed the greater the number of collisions per unit of time and the greater the possible number of combinations. For these experiments the above two factors apparently combined to give the time-mixture curve the two minimum points, one on each side of the correct mixture.

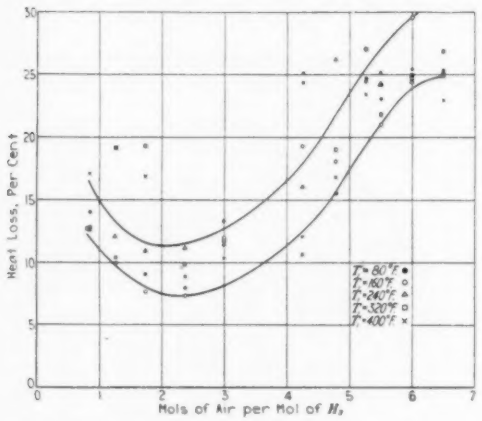


FIG. 11 HEAT LOSSES DURING PRESSURE RISE

HEAT LOSS

The heat loss during the pressure rise is indicated by the difference between the temperatures computed from the actual pressures and the theoretical maximum temperatures. Since differences of values based upon these temperatures approximately eliminate the deviation from perfect-gas laws, it is thought that the area between the curves (Fig. 11) indicates rather closely the range within which the heat losses for these experiments should lie. The heat loss ranges from 7.3 per cent at the correct mixture to 29.6 per cent at the 1 H_2 + 6 Air mixture.

In order to determine the heat-transfer coefficients it was assumed that the time rate of pressure rise was a straight line, making one-half of the temperature rise the mean temperature difference. With this assumption the heat-transfer coefficients shown in Fig. 10 were computed. The outlying points have been neglected, it being believed that enormous coefficients of heat transfer indicate leaky experiments. It is interesting to note that at the correct mixture the heat-transfer coefficients average above 1300 B.t.u. per sq. ft. of surface per hour, per degree difference in temperature. From the high rates of heat transfer determined it is quite evident that rapid combustion is accompanied by extraordinary movement of the mixture. A comparison of all the curves in Fig. 10 indicates that the heat-transfer coefficients during pressure rise vary inversely with the time of pressure rise.

Heat-transfer coefficients during the first part of the cooling process were also computed. In all cases in a very short time the coefficients of heat transfer dropped to values that would be considered normal. For example, in less than 0.2 sec. after maximum pressure the coefficients of heat transfer for the correct-mixture experiments had dropped to an average value of 115. At atmospheric pressure this would be equivalent to 1.6. For still air at atmospheric pressure the heat-transfer coefficient is about 1.4, indicating in this case only a small movement of the gas 0.2 sec. after maximum pressure.

The effect of radiation on heat loss during pressure rise is of considerable interest in that it was found to account for less than 1 per cent of heat loss for the experiment resulting in the highest temperature. This would indicate that in high-pressure combustion work conduction and convection account for practically all the heat loss.

FLAME PROPAGATION

There appeared to be no definite relationship between the rate of flame propagation and the initial temperature of the mixture. Values of rates of flame propagation given in Table 3 are the average values for all experiments with each mixture.

TABLE 3 AVERAGE RATES OF FLAME PROPAGATION

Mixture, mols of air per mol H_2	Avg. time of pressure rise, sec.	Rate of flame propagation, ft. per sec.
0.85	0.0281	10.8
1.25	0.0095	32.1
1.75	0.0066	46.2
2.39	0.0104	29.3
3.00	0.0068	44.8
4.25	0.0320	9.5
4.78	0.0447	6.8
5.25	0.0321	9.5
5.50	0.0747	4.1
6.00	0.1293	2.4
6.50	0.236	1.3

The maximum rate of flame propagation (82.3 ft. per sec.) was attained in an experiment with 1 H_2 + 1.75 Air at an initial temperature of 320 deg. fahr.

APM-51-5

Fatigue and Corrosion-Fatigue of Spring Material[1]

By D. J. McADAM, JR.,[2] ANNAPOLIS, MD.

BY NUMEROUS investigations of fatigue of metals certain facts and relationships have been well established. These will be briefly mentioned and defined.

Fatigue failure of metals is due to the repeated application of a range of stress that is insufficient to cause failure on single application. The range of stress may be between zero and a maximum, or between a maximum and minimum value, or there may be complete or incomplete reversal of stress per cycle. Fatigue depends more on the range of stress than on the extremes of the range. It has been definitely established that, below a limiting value of the stress range, fatigue failure does not occur even after many million applications of the stress range. This limiting range has been called the "endurance range."[3] A greater range causes failure after a number of cycles that decreases rapidly with increase in the range. When the endurance range involves complete reversal of stress per cycle, as in a rotating beam or cantilever, the half range is known as the "endurance limit" or "fatigue limit." The object of the great majority of endurance tests of metals has been to determine the "endurance limit."

The effect of varying the position of the endurance range with reference to the elastic range has been investigated for steel, but much further investigation is needed. Under many conditions of service, especially for springs, the stress range is between zero and a maximum or between a minimum and maximum. Under such conditions the endurance range may or may not be somewhat less than if it involved complete reversal of stress per cycle. Since information about this important subject is at present very incomplete, however, chief attention must here be given to the endurance properties of spring material under complete reversal of stress per cycle, with some tentative conclusions as to the endurance properties under actual service conditions.

In this paper the term "endurance limit" will be applied only to values obtained with specimens designed so as to avoid stress concentration due to abrupt change of section. With specimens so designed, carefully machined and polished, and tested under conditions as free as possible from corrosion, the endurance limit presumably represents an inherent property of the metal or alloy. The nominal values obtained with notched specimens, or with specimens subjected to simultaneous corrosion and fatigue, will be called "fatigue limits." In this paper attention will first be given to endurance limits. The influence of abrupt changes of section and the broad subject of corrosion-fatigue will be considered later.

Relation of the Endurance Limit to Other Physical Properties

The endurance limit seems to be more closely related to the tensile strength than to any other physical property except the Brinell number. The endurance limit seems to have very little relationship to elastic limit or to ductility. The ratio of the endurance limit (as obtained by rotating beam or cantilever) to the tensile strength, which has been called the endurance ratio,"[4] is for most normal metals between 0.4 and 0.55. For some non-ferrous metals the endurance ratio is lower, and may be as low as 0.25.

In the great majority of all fatigue tests the specimen has been tested as a rotating beam or cantilever. Some tests have been made under a range between direct tension and compression. The difficulty of avoiding eccentric loading in direct tension-compression tests has made many of the results of such tests of little value. The weight of evidence seems to indicate that if the loading is concentric the endurance limit as obtained by direct stress agrees closely with the limit obtained by rotating beam or cantilever, at least for metals and alloys whose endurance limit is below the elastic limit. The endurance limit in repeated shear or torsion is less than in tension-compression. The ratio of the endurance limit for torsion to the endurance limit for tension-compression varies from 0.38 to 0.84. These ratios for various kinds of material are given in Tables 5 and 6.

For commercial carbon steel of lowest carbon content the endurance ratio is about 0.5. For carbon steels of medium strength the endurance ratio is about 0.45. For high-carbon steels the endurance ratio may be somewhat less. For alloy steels the endurance ratio is usually about 0.5, unless the steel has been heat-treated to exceptionally high strength. For spring steels of highest strength the endurance ratio is usually less than 0.5 and may be little more than 0.4. If, however, a specimen of such steel is subjected to gradually increasing cyclic stress starting at a point well below the endurance limit, the endurance limit may be raised to about half the tensile strength. This seems to indicate that the lower endurance ratio obtained with the hardest material by ordinary endurance tests is due to internal stress.

Endurance Properties of Steels

In Figs. 1 and 2 are stress-cycle graphs illustrating the endurance properties of steels as determined by rotating-cantilever tests at the Naval Engineering Experiment Station. Ordinates represent the half range of stress and abscissas represent cycles on a logarithmic scale. Each set of graphs has its own scale of abscissas, which is indicated by numerals adjacent to the graph. Some of these graphs have been assembled from various papers[5,6,7,8] by the author. Others represent new data. For graphs that have been published elsewhere, the results of individual tests have been omitted and the stress-cycle relationship has been represented by lines only; for graphs representing new data, both individual results and the stress-cycle lines have been included. The solid lines represent results of rotating-cantilever tests of oiled specimens in air. The dotted and broken lines represent results of tests of notched specimens and results of tests of specimens under simultaneous corrosion and fatigue. These will be discussed later.

The unnotched specimen used in these experiments is shown in Fig. 6. The taper of this specimen is such that the maximum stress is in a plane $^3/_4$ in. out from the fillet, and the stress varies only about $1^1/_2$ per cent over a length of $1^1/_2$ in.

Beside each stress-cycle graph there are indicated some of the

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[1] Presented by permission of the Secretary of the Navy.

[2] Metallurgist, U. S. Naval Engineering Experiment Station.

[3] For this and the following references see list at end of paper.

Contributed by the Research Special Committee on Mechanical Springs and presented at the Annual Meeting, New York, N. Y., Dec. 3 to 7, 1928, of The American Society of Mechanical Engineers.

strength values as determined by tension test. These are represented by the following symbols: tensile strength, T. S.; Johnson's limit, J. L.; proof stress, P. S.; elastic limit, E. L.; proportional limit, P. L.

Each zigzag line in these figures represents the course of a fatigue or corrosion-fatigue test in which the stress was raised at intervals until the specimen failed or was removed. The lower end of the zig-zag line represents the original stress and the number of cycles endured at this stress. The course of the line represents the stress increases and the cycle intervals at which these increases were made.

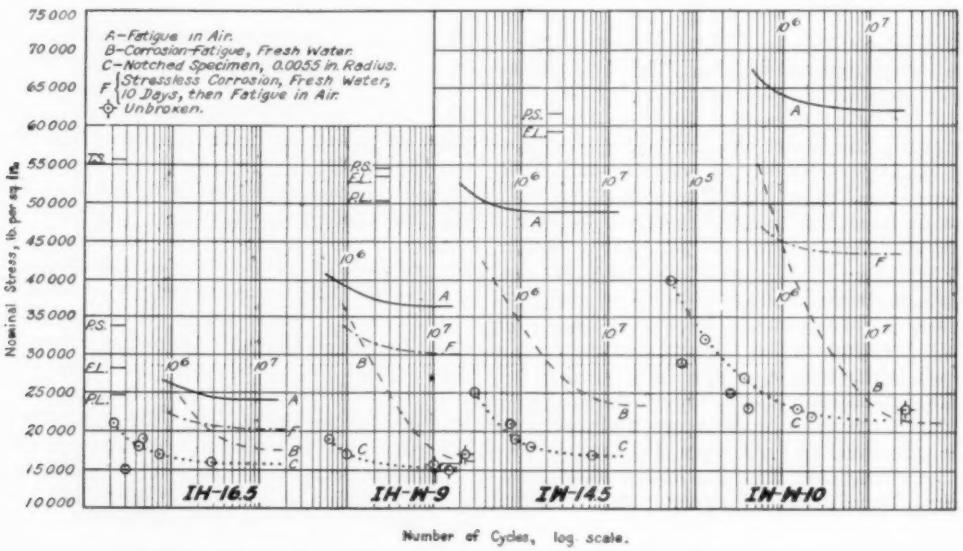


FIG. 1 FATIGUE AND CORROSION-FATIGUE OF CARBON AND NICKEL STEELS

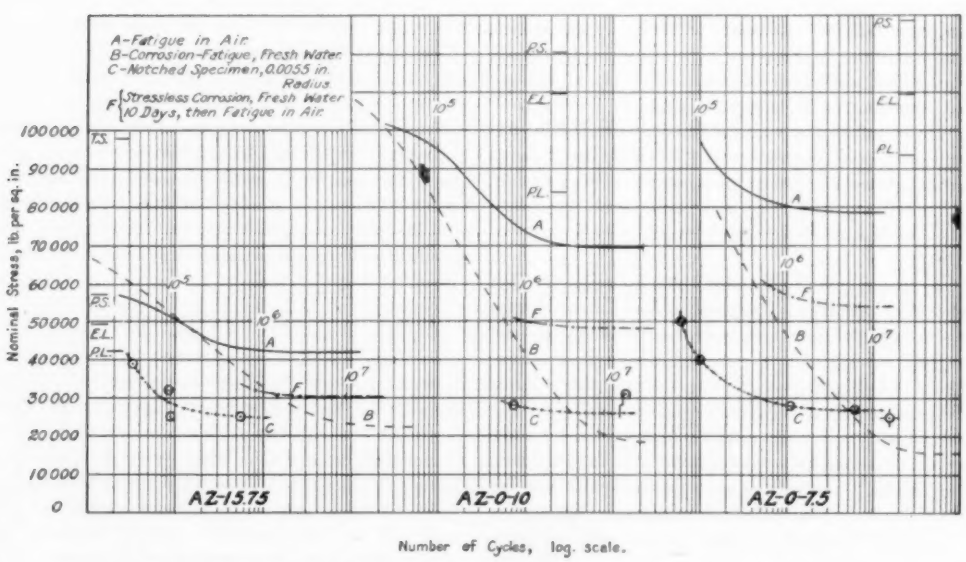


FIG. 2 FATIGUE AND CORROSION-FATIGUE OF CHROMIUM-VANADIUM STEEL

In Fig. 1 are shown two pairs of graphs representing results obtained with a 0.25 carbon steel and with a $3^1/_2$ per cent nickel steel, respectively. In each pair the graph on the left represents fully annealed material and the graph on the right represents quenched-and-tempered material. In Fig. 2 are shown three groups of graphs representing a chrome-vanadium steel in three different degrees of hardness as produced by heat treatment. The group on the left represents fully annealed material and the group on the right represents material of spring temper.

As illustrated by the solid lines in these figures, each stress-cycle graph is a curve approaching a horizontal asymptote, the ordinate of which is the endurance limit. A comparison of the endurance limits as obtained in this way with the corresponding tensile strengths will show that the endurance limit is roughly proportional to the tensile strength. With sufficient upward extension of the stress-cycle graph, there is usually reversal of curvature. As shown in a previous paper by the writer,[9] this reversal is due to rise of temperature of the specimen when tested at stresses above the endurance limit.

Endurance limits obtained in this way for various steels are listed in Table 5. Chemical composition is given in Tables 1 and 2 and details of heat treatment in Tables 3 and 4.

ENDURANCE PROPERTIES OF NICKEL-COPPER ALLOYS

Monel metal is the only nickel-copper alloy that is extensively

used for springs. As some other nickel-copper alloys, however, would be practically as suitable for springs it has seemed desirable to discuss briefly the endurance properties of the entire series of nickel-copper alloys as affected by chemical composition, cold working and annealing.

Heat treatment is the usual method of hardening steels. Some non-ferrous alloys such as duralumin can be hardened by heat treatment. Heat treatment can also be used to harden some characteristic of all fully annealed alloys made up of only one microconstituent (alpha solid solutions). During an endurance test, however, the elastic limit in the surface layers of a fully annealed specimen is undoubtedly raised by the cold working.

A comparison of all the graphs for fully annealed material shows that with increase in nickel content the endurance limit rises rapidly until the percentage of nickel is about 45 per cent,

TABLE 1 CHEMICAL COMPOSITION OF STEELS

(All Values are Averages of at Least Four Determinations)

Alloy	Designation	Per cent C	Mn	P	S	Si	Ni	Cr	V	Mo	Cu
0.24% carbon steel	*IH*	0.24	0.57	0.005	0.036	0.006	..	...	..	..	..
0.24% carbon steel	*O*	0.24	0.45	0.009	0.051	0.007	..	...	..	..	..
0.38% carbon steel	*T*	0.38	0.57	0.033	0.048	0.04	..	...	..	..	..
0.81% carbon steel	*S*	0.81	0.32	0.012	0.031	0.16	..	...	..	..	..
1.09% carbon steel	*EM*	1.09	0.33	0.023	0.015	0.28	..	...	..	..	..
Silicon-manganese steel	*M*	0.51	0.66	0.029	0.045	1.96	..	...	..	..	..
3.35% nickel steel	*Q*	0.31	0.64	0.026	0.028	0.13	3.35	...	..	..	..
3.50% nickel steel	*EJ*	0.32	0.60	0.280	0.023	0.21	3.47	...	..	..	..
3.60% nickel steel	*P*	0.42	0.70	0.017	0.016	0.12	3.60	...	..	..	..
3.70% nickel steel	*IW*	0.28	0.50	0.150	0.360	0.19	3.70	0.26	..	..	..
5% nickel steel	*CA*	0.52	0.41	0.022	0.009	0.32	5.37	...	..	..	..
Silicon-nickel steel	*HG*	0.47	0.87	0.013	0.015	1.58	3.11	0.32	..	..	..
Nickel-chromium steel	*L*	0.33	0.46	0.006	0.051	0.19	3.16	0.95	..	..	..
Nickel-chromium steel	*BC*	0.28	0.69	0.014	0.016	0.21	1.51	0.73	..	..	..
Chromium steel	*A*	0.08	0.07	0.017	0.007	0.11	..	11.78	..	..	0.05
Chromium steel	*G*	0.21	0.59	0.017	0.017	0.79	0.13	13.31	..	..	..
Chromium-vanadium steel	*AU*	0.34	0.71	0.016	0.022	0.21	..	1.05	0.16	..	..
Chromium-vanadium steel	*AZ*	0.46	0.69	0.007	0.010	0.24	..	0.88	0.14	..	..
Chromium-molybdenum steel	*IAA*	0.42	0.53	0.032	0.023	0.25	..	1.04	..	0.09	..
Chromium-molybdenum steel	*AL*	0.33	0.71	0.005	0.017	0.13	..	0.93	..	0.15	..
Nickel-molybdenum steel	*H*	0.42	0.47	0.380	0.020	0.25	1.68	...	..	0.12	..
Stainless steel	*I*	0.42	0.27	0.016	0.058	0.56	0.20	12.26	..	..	0.04
Stainless steel	*GS*	0.38	0.26	0.017	0.012	0.20	0.23	14.50	..	..	0.07
Stainless steel	*J*	0.40	0.28	0.017	0.058	0.59	0.18	15.21	..	..	None
Chromium-copper steel	*B*	0.61	0.39	0.018	0.035	0.03	0.24	15.81	..	..	1.10
High-chromium, low-copper steel	*GQ*	0.19	0.31	0.018	0.015	0.55	0.17	20.94	..	..	0.92
Medium-chromium, high-nickel steel	*D*	0.45	0.49	0.012	0.022	1.39	28.20	8.38	None	..	0.67
Medium-chromium, high-nickel steel	*FK*	0.39	1.10	0.002	0.011	0.15	34.70	10.89	..	..	..

TABLE 2 CHEMICAL COMPOSITION OF NON-FERROUS ALLOYS

Alloy	Designation	Per cent Ni	Cu	Sn	Fe	Zn	P	C	Mn	S	Si
Nickel, cold-drawn	*IT*	99.17[a]	0.16	...	0.42	...	0.003	0.085	0.08	...	0.09
Nickel, cold-rolled	*CT*	99.07	0.20	...	0.47	...	...	0.100	0.07	0.018	0.07
Nickel, cold-rolled	*EO*	98.96[a]	0.17	...	0.57	...	0.006	0.098	0.03	0.011	0.09
Nickel, cold-rolled	*A*	98.95	0.12	...	0.50	...	...	0.250	0.10	0.175	0.06
Monel metal, cold-rolled	*EP*	67.51[a]	29.54	...	1.76	...	0.015	0.159	0.95	0.017	0.05
Constantan, hot-rolled	*CE*	44.68	53.77	None	0.52	...	...	...	1.14	...	...
Monel metal, hot-rolled	*AA*	66.78	29.54	...	2.10	...	0.019	0.160	1.44	0.039	0.02
40:57 nickel-copper, hot-rolled	*FH*	40.29[a]	56.63	...	1.09	...	...	0.160	1.77	...	0.06
29:67 nickel-copper, cold-rolled	*HZ*	28.66[a]	67.11	...	2.82	...	0.008	0.040	1.35	0.006	0.002
21:78 nickel-copper, cold-worked	*HE*	21.24[a]	77.92	...	0.51	...	0.007	0.031	0.29	0.011	0.005
20:79 nickel-copper, cold-worked	*HV*	19.80[a]	79.47	...	0.43	...	0.005	0.020	0.27	0.008	0.004
Copper, electrolytic, cold-worked	*HU*	....	...	...	0.008	...	...	...	..	...	...
Copper, electrolytic, cold-worked	*HF*	....	...	...	0.004	...	...	...	..	...	...
Copper, electrolytic, cold-rolled	*BO*	....	...	...	0.0052	...	...	...	..	...	...
Copper, electrolytic, hot-rolled	*EE*	....	...	...	0.004	...	...	...	..	...	...
Copper, electrolytic, hot-rolled	*BU*	....	...	...	0.0058	...	...	...	..	...	...
95:5 copper-tin, cold-drawn	*CK*	....	95.00	5.06	....	...	0.026	...	..	...	...
95:5 copper-tin, cold-drawn	*EA EAA*	....	94.55	5.42	0.07	...	0.149	...	...	...	...
92:8 copper-tin, cold-drawn	*ED EDA*	....	91.71	8.20	0.07	...	0.131	...	..	...	...
90:10 copper-tin, cold-drawn	*EC ECA*	....	89.41	10.49	0.07	...	0.131	...	..	...	...
Copper-nickel-zinc, hot-rolled	*CD*	19.75	74.01	...	0.34	5.17	...	...	0.73	...	...
Copper-nickel-zinc, cold-drawn	*CH*	17.63	65.30	...	0.23	17.15	...	...	..	...	...
Copper-nickel-zinc, cold-drawn	*CG*	10.89	60.08	...	0.20	29.05	...	...	..	...	...

[a] By difference

of the alloys of copper and of nickel. The most generally useful method for hardening non-ferrous alloys, however, is cold working. For this reason, in discussing the endurance properties of non-ferrous spring material, prominence must be given to the effect of cold working.

In Figs. 3 and 4 are shown stress-cycle graphs for a series of nickel-copper alloys ranging from pure copper to commercial nickel.[5,9,10] These graphs are in pairs. The graph on the left of each pair represents fully annealed material and the graph on the right represents material in a severely cold-worked condition. As illustrated by these pairs, cold working raises the endurance limit of nickel-copper alloys about in proportion to the increase in tensile strength.

It will be observed also that for fully annealed material the nominal endurance limit is above the elastic limit and even above the yield point as represented by proof stress. This seems to be which corresponds to the alloy constantan. With further increase in nickel content, the endurance limit changes relatively little. The endurance limit of monel metal (which has about 70 per cent nickel) is possibly a little higher than that of alloys having lower or higher nickel content. If monel metal is superior to constantan, however, the superiority is slight.

As shown in these graphs and in Table 6, it is possible to obtain cold-rolled nickel-copper alloys in rods 1 in. in diameter, with tensile strength of 135,000 to 140,000 and endurance limit of slightly over 50,000. To obtain an endurance limit of this amount, however, the cold-worked material must be given an annealing treatment (as indicated in the case of monel metal in Table 4) to relieve internal stress. In smaller rods or in wire, it should be possible to obtain even higher fatigue limit, if the severely drawn material is properly annealed so as to relieve internal stress with practically no loss in tensile strength.

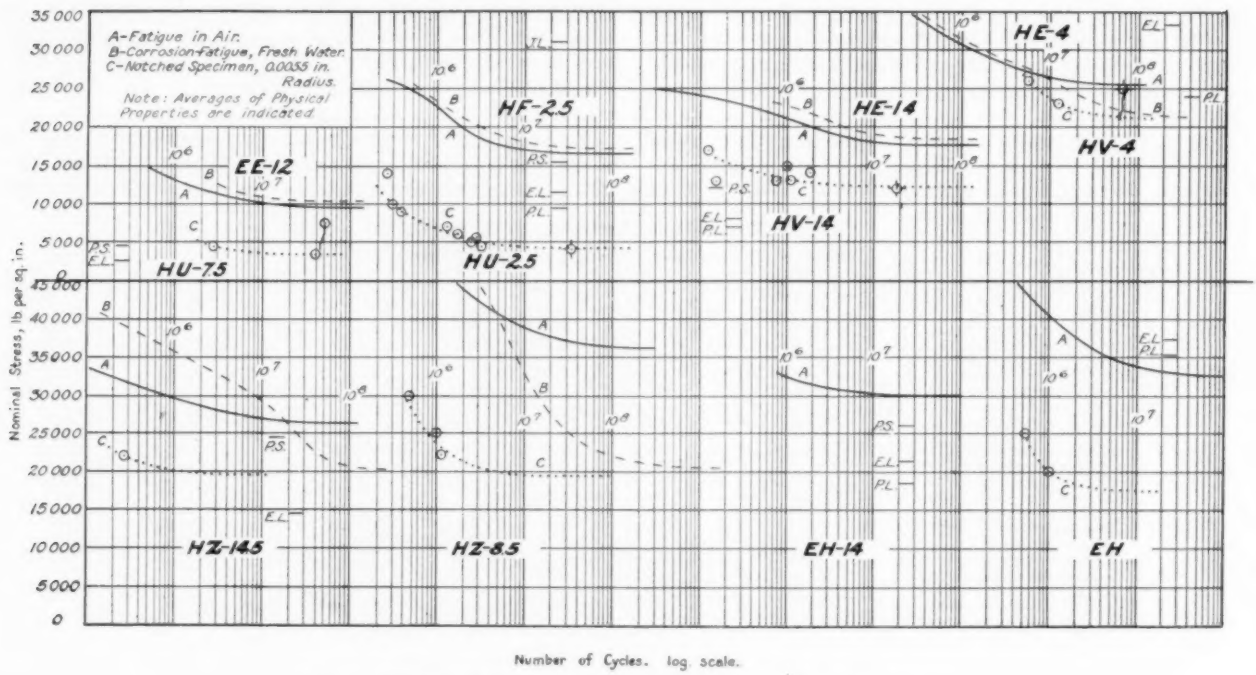


FIG. 3 FATIGUE AND CORROSION-FATIGUE OF NICKEL-COPPER ALLOYS

ENDURANCE PROPERTIES OF COPPER-TIN ALLOYS

In Fig. 5 are shown graphs illustrating endurance properties of three copper-tin alloys with tin content ranging from about 5 to 10 per cent. Each horizontal row of graphs represents one of these three alloys in three different degrees of hardness imparted by cold working. In each row the graph on the left represents fully annealed and the graph on the right represents most severely cold-worked material. As in previous figures, the solid-line graphs represent results of ordinary endurance tests.

As illustrated by the lower row of graphs, the endurance limit of the alloy containing 5 per cent tin is improved somewhat by moderate cold working. With increase in the severity of cold working, however, the endurance limit is raised little, if at all. As illustrated by the middle row of graphs, the endurance limit of the 8 per cent alloy is improved slightly by moderate cold working. With further increase in the severity of cold working, however, the endurance limit is lowered. As illustrated by the upper row in this figure, either severe or moderate cold working lowers the endurance limit of the alloy containing 10 per cent tin.

Comparison of the three kinds of fully annealed material seems to indicate that addition of tin beyond about 5 per cent causes little if any improvement in the endurance limit of a copper-tin alloy. Comparison with the endurance limit of annealed copper, as shown in Fig. 3, however, shows that the addition of tin up to about 5 per cent makes the endurance limit of the alloy more than double that of pure copper.

The results obtained with copper-tin alloys seem to indicate that cold working is less effective in raising the endurance

TABLE 3 HEAT TREATMENT OF STEELS

Alloy	Designation	Heated to deg. fahr.	Heated Time held, min.	Heated Cooled in	Reheated to deg. fahr.	Reheated Time held, min.	Reheated Cooled in
0.24% carbon steel	*IH-W-9*	1650	60	Water	900	120	Air
0.24% carbon steel	*IH-16.5*	1650	60	Furnace			
0.24% carbon steel	*O-1*			As received			
0.38% carbon steel	*T-6*	1550	30	Oil	800	30	Furnace
0.38% carbon steel	*T-3*	1700	120	Oil	1550	30	Furnace
0.81% carbon steel	*S-1*	1450	30	Furnace			
1.09% carbon steel	*EM-14.75*	1475	60	Furnace			
Silicon-manganese steel	*M-2*			As received[d]			
Silicon-manganese steel	*M-1*			As received[d]			
3.35% nickel steel	*O-7*	1475	30	Oil	1050	30	Furnace
3.35% nickel steel	*O-5*	1475	60	Furnace			
3.5% nickel steel	*EJ-W-10*	1450[a]	60	Water	1000	120	Furnace
3.5% nickel steel	*EJ-14.5*	1450	60	Furnace			
3.6% nickel steel	*P-8*	1450	60	Water	900	60	Furnace
3.6% nickel steel	*P-2₂*	1450	60	Furnace			
3.7% nickel steel	*IW-W-10*	1450[b]	60	Water	1000	120	Furnace
3.7% nickel steel	*IW-14.5*	1450	60	Furnace			
5% nickel steel	*CA-W-11*	1425	60	Water	1100	120	Furnace
5% nickel steel	*CA-14.25*	1425	60	Furnace			
Silicon-nickel steel	*HG-O-7.5*	1500	60	Oil	750	60	Air
Silicon-nickel steel	*HG-14.75*	1475	60	Furnace			
Nickel-chromium steel	*L-2*			As received[d]			
Nickel-chromium steel	*L-1*			As received[d]			
Nickel-chromium steel	*BC-W-10*	1550	60	Water	1000	120	Furnace
Chromium steel	*A-O-9*	1800	90	Oil	900	120	Air
Chromium steel	*A-O-10*	1800	90	Oil	1000	120	Air
Chromium steel	*G*			As received[d]			
Chromium-vanadium steel	*AU-O-10*	1575[c]	60	Oil	1000	60	Furnace
Chromium-vanadium steel	*AZ-O-7.5*	1575[c]	60	Oil	750	60	Furnace
Chromium-vanadium steel	*AZ-O-10*	1575[c]	60	Oil	1000	60	Furnace
Chromium-vanadium steel	*AZ-15.75*	1575[c]	60	Furnace			
Chromium-molybdenum steel	*IAA-2*			As received[d]			
Chromium-molybdenum steel	*AL-O-10*	1575	60	Oil	1000	60	Furnace
Nickel-molybdenum steel	*H-2*			As received[d]			
Nickel-molybdenum steel	*H-1*			As received[d]			
Stainless steel	*I-10*	1650	60	Water	1000	60	Air
Stainless steel	*I-12*	1650	60	Water	1200	60	Air
Stainless steel	*GS-O-10.75*	1800	60	Oil	1075	120	Air
Stainless steel	*GS-O-12*	1800	60	Oil	1200	120	Air
Stainless steel	*GS-17*	1700	60	Furnace			
Stainless steel	*J-9*	1650	60	Water	900	60	Air
Stainless steel	*J-re*			As received			
Chromium-copper steel	*B-9*	1650	60	Water	900	60	Air
Chromium-copper steel	*B-rc*			As received			
High-chromium, low-copper steel	*GQ*			As received			
Medium-chromium, high-nickel steel	*D-r*			As received			
Medium-chromium, high-nickel steel	*FK*			As received			

[a] Previously heated to 1675 deg. fahr., held 30 min. and cooled in air.
[b] Previously heated to 1675 deg. fahr., held 60 min., and cooled in air.
[c] Previously heated to 1700 deg. fahr., held 60 min. and cooled in air.
[d] Heat-treated by manufacturer. Heat treatment unknown.
[e] Annealed by manufacturer.

limit of these alloys than in raising the endurance limit of nickel-copper alloys. Even by moderate cold working, the endurance limit of copper-tin alloys is increased less than in proportion to the increase in tensile strength. This in spite of the fact that the cold-worked copper-tin alloys had been subjected to a low-temperature anneal (as indicated in Table 4) to relieve internal stress. As shown in previous papers[6,24] copper-zinc alloys in this respect behave like the copper-tin alloys.

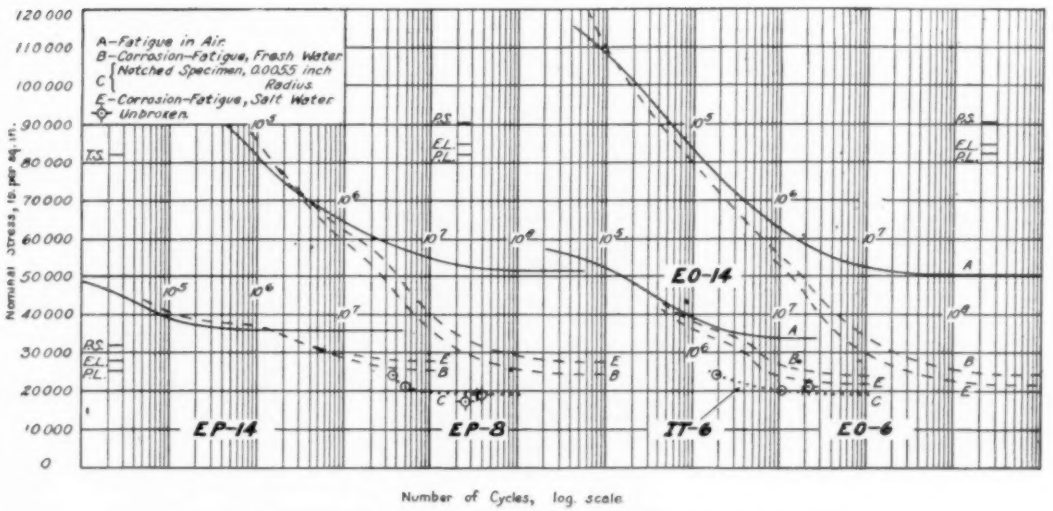


FIG. 4 FATIGUE AND CORROSION-FATIGUE OF MONEL METAL AND NICKEL

Possible reasons for the fact that the endurance limit of the copper-zinc alloys is apparently little improved by cold working were given in a previous paper.[5] It was there suggested that possibly the cold working has caused minute discontinuities in the metal. Such discontinuities might behave like notches and thus reduce the fatigue limit of the cold-worked material. Such internal notches would affect the fatigue limit more than they would affect the tensile properties. They would affect the fatigue limit of the severely cold-worked material more than they would affect the fatigue limit of material that had been subsequently annealed. It is possible, therefore, that the behavior of copper-tin and copper-zinc alloys toward cold working is due to the same cause, namely damage to the alloy in the cold-working process.

TABLE 4 HEAT TREATMENT OF NON-FERROUS ALLOYS

Alloy	Designation	Heated to deg. fahr.	Time held, min.	Cooled in
Nickel, cold-drawn	*IT*-6	600	120	Furnace
Nickel, cold-rolled	*CT*-16	1600	60	Furnace
Nickel, cold-rolled	*EO*-6	600	120	Furnace
Nickel, cold-rolled	*EO*-14	1400	60	Furnace
Nickel, cold-rolled	*A*-11	1100	60	Furnace
Monel metal, cold-rolled	*EP*-8	800	180	Furnace
Monel metal, cold-rolled	*EP*-14	1400	60	Furnace
Constantan, hot-rolled	*CE*-1	1450	60	Furnace
Monel metal, hot-rolled	*AA*	As received		
40:57 nickel-copper, hot-rolled	*EH*	As received		
40:57 nickel-copper, hot-rolled	*EH*-14	1400	60	Furnace
29:67 nickel-copper, cold-rolled	*HZ*-8.5	850	180	Furnace
29:67 nickel-copper, cold-rolled	*HZ*-14.5	1450	60	Furnace
21.78 nickel-copper, cold-worked	*HE*-4	400	180	Furnace
21.78 nickel-copper, cold-worked	*HE*-14	1400	60	Furnace
20.79 nickel-copper, cold-worked	*HV*-4	400	180	Furnace
20.79 nickel-copper, cold-worked	*HV*-14	1400	60	Furnace
Copper, electrolytic, cold-worked	*HU*-2.5	250	180	Furnace
Copper, electrolytic, cold-worked	*HU*-7.5	750	120	Furnace
Copper, electrolytic, cold-worked	*HF*-2.5	250	180	Furnace
Copper, electrolytic, cold-rolled	*BO*.	As received		
Copper, electrolytic, hot-rolled	*EE*-12	1200	60	Furnace
Copper, electrolytic, hot-rolled	*BU*	As received		
95:5 copper-tin, cold-drawn	*CK*	As received		
95:5 copper-tin, cold-drawn	*EA*-3.75	375	180	Oil bath[a]
95:5 copper-tin, cold-drawn	*EA*-4	400	180	Oil bath[a]
95:5 copper-tin, cold-drawn	*EAA*-3.75	375	180	Oil bath[a]
95:5 copper-tin, cold-drawn	*EAA*-4	400	180	Oil bath[a]
95:5 copper-tin, cold-drawn	*EA*-12	1200	60	Air
95:5 copper-tin, cold-drawn	*EAA*-12	1200	60	Air
92:8 copper-tin, cold-drawn	*ED*-3.75	375	240	Oil bath[a]
92:8 copper-tin, cold-drawn	*ED*-4	400	240	Oil bath[a]
92:8 copper-tin, cold-drawn	*EDA*-2.5	250	240	Oil bath[a]
92:8 copper-tin, cold-drawn	*EDA*-3.5	350	240	Oil bath[a]
92:8 copper-tin, cold-drawn	*EDA*-4	400	240	Oil bath[a]
92:8 copper-tin, cold-drawn	*ED*-12	1200	60	Furnace
92:8 copper-tin, cold-drawn	*EDA*-12	1200	60	Furnace
90:10 copper-tin, cold-drawn	*EC*-4	400	180	Furnace
90:10 copper-tin, cold-drawn	*ECA*-3.5	350	180	Furnace
90:10 copper-tin, cold-drawn	*ECA*-4	400	180	Furnace
90:10 copper-tin, cold-drawn	*EC*-12	1200	60	Furnace
90:10 copper-tin, cold-drawn	*ECA*-12	1200	60	Furnace
Copper-nickel-zinc, hot-rolled	*CD*	As received		
Copper-nickel-zinc, hot-rolled	*CD*-1	1400	60	Furnace
Copper-nickel-zinc, cold-drawn	*CH*	As received		
Copper-nickel-zinc, cold-drawn	*CG*	As received		

[a] Oil bath, instead of furnace, used in heat-treating this material.

As shown in a previous paper,[24] the endurance limit of cold-worked copper-zinc alloys is considerably affected by internal stress. The low-temperature annealing treatments given to the copper-tin alloys were for the purpose of removing, or at least diminishing, internal stress. In these annealing treatments, as determined by preliminary tests, the temperatures were the highest that could be used (for the times indicated) without decided decrease of the alloy's strength. It should be noted that the alloys that were heat-treated for three hours at 250, 350, and 375 deg. fahr. had practically the same tensile properties and endurance limits as the alloys that were heat-treated at 400 deg. fahr. It does not seem practicable, therefore, by heat treatment to cause any further reduction of internal stress. Any internal stress remaining in the bars, moreover, would be still further reduced by the machining of the specimens from the full-sized bars. For these reasons it seems probable that the low endurance limits of the severely cold-worked alloys containing 8 and 10 per cent tin are due chiefly to internal flaws in the hard material rather than to internal stress. As will be shown later, it would be possible for material with such internal flaws to show lower endurance limits in the cold-worked condition than when afterward annealed. It is possible also that the slight improvement of the fatigue limit of the copper-tin and copper-zinc alloys is due to some inherent property of these alloys. This subject needs further investigation. It is important to know definitely whether or not the endurance limit of these alloys in the form of spring wire is much or little greater than that of fully annealed material.

EFFECT OF VARYING THE POSITION OF THE ENDURANCE RANGE WITHIN THE ELASTIC RANGE

The results presented in Figs. 1 to 5 and in Tables 5 and 6 all are results of endurance tests with complete reversal of stress per cycle. Relatively few experiments have been made with other than complete reversal of stress per cycle. These cannot here be discussed in detail. Complete discussions of the relation between the endurance range and the mean stress of the cycle are given in a book by H. J. Gough[11] and in a book by H. F. Moore and J. B. Kommers.[12]

Moore and Jasper[13] investigated the effect of varying the mean

TABLES 5 AND 6 SOME PHYSICAL PROPER

(All values are averages of at

TABLE 5 SOME PHYSICAL

1	2	3	4	5	6	7	8
Alloy	Condition	Designation	Tensile strength, lb. per sq. in.	Johnson's limit, lb. per sq. in.	Proof stress, lb. per sq. in.	Elastic limit, lb. per sq. in.	Proportional limit, lb. per sq. in.
0.24% carbon steel	Quenched and drawn	*IH-W-9*	78,700	55,000	54,300	53,700	50,000
0.24% carbon steel	Annealed	*IH-16.5*	55,600	33,300	34,000	28,000	24,700
0.24% carbon steel	As received	*O-1*	60,400	38,200	.....	.....	37,600
0.38% carbon steel	Quenched and drawn	*T-6*	91,500	60,000	.....	.....	60,000
0.38% carbon steel	Quenched and drawn	*T-3*	72,000	40,000	.....	.....	39,000
0.81% carbon steel	Annealed	*S-1*	89,800	34,000	.....	.....	32,300
1.09% carbon steel	Annealed	*EM-14.75*	103,400	60,300	60,600	55,800	49,500
Silicon manganese steel	As received	*M-2*	157,500	101,000	.....	.....	100,000
Silicon manganese steel	As received	*M-1*	111,800	60,000	.....	.....	57,500
3.35% nickel steel	Quenched and drawn	*Q-7*	158,500	134,500	.....	.....	124,500
3.35% nickel steel	Annealed	*Q-5*	103,900	53,500	.....	.....	50,500
3.5% nickel steel	Quenched and drawn	*EJ-W-10*	124,800	105,400	106,700	102,800	79,600
3.5% nickel steel	Annealed	*EJ-14.5*	92,500	61,000	61,600	54,400	51,400
3.6% nickel steel	Quenched and drawn	*P-8*	154,200	142,000	.....	.....	139,000
3.6% nickel steel	Furnace	*P-22*	113,600	70,300	.....	.....	69,500
3.7% nickel steel	Quenched and drawn	*IW-W-10*	130,400	118,000	118,700	115,000	90,700
3.7% nickel steel	Annealed	*IW-14.5*	90,500	60,800	61,200	59,300	49,600
5% nickel steel	Quenched and drawn	*CA-W-11*	132,700	93,000	96,000	84,300	80,000
5% nickel steel	Annealed	*CA-14.25*	121,800	57,000	56,300	47,700	46,300
Nickel-silicon steel	Quenched and drawn	*HG-O-7.5*	251,300	201,700	203,000	189,700	155,000
Nickel-silicon steel	Annealed	*HG-14.75*	137,500	75,000	76,000	67,500	41,500
High nickel-chromium steel	As received	*L-2*	133,400	108,800	.....	.....	105,000
High nickel-chromium steel	As received	*L-1*	103,000	52,500	.....	.....	50,000
Low nickel-chromium steel	Quenched and drawn	*BC-W-10*	138,300	122,000	123,000	119,700	95,700
Chromium steel	Quenched and drawn	*A-O-9*	166,500	35,000	.....	24,500	24,500
Chromium steel	Quenched and drawn	*A-O-10*	164,600	72,000	.....	60,000	25,000
Chromium steel	As received	*G*	97,500	61,300	.....	.....	61,300
Chromium-vanadium steel	Quenched and drawn	*AU-O-10*	139,500	117,500	118,000	112,500	95,000
Chromium-vanadium steel	Quenched and drawn	*AZ-O-7.5*	179,300	128,600	129,200	110,000	93,300
Chromium-vanadium steel	Quenched and drawn	*AZ-O-10*	146,800	120,100	121,000	109,900	83,700
Chromium-vanadium steel	Annealed	*AZ-15.75*	97,800	56,200	57,100	50,000	42,100
Chromium-molybdenum steel	As received	*IAA-2*	141,400	115,000	.....	.....	112,500
Chromium-molybdenum steel	Quenched and drawn	*AL-O-10*	117,600	90,700	91,500	88,000	81,000
Nickel-molybdenum steel	As received	*H-2*	133,400	102,500	.....	.....	100,000
Nickel-molybdenum steel	As received	*H-1*	104,700	56,300	.....	.....	51,300
Stainless steel	Quenched and drawn	*I-10*	171,500	88,000	.....	.....	27,500
Stainless steel	Quenched and drawn	*I-12*	121,500	83,000	.....	.....	56,000
Stainless steel	Quenched and drawn	*GS-O-10.75*	178,100	126,700	119,000	101,700	86,700
Stainless steel	Quenched and drawn	*GS-O-12*	117,400	53,200	56,000	45,000	41,200
Stainless steel	Annealed	*GS-17*	94,400	40,000	39,800	37,500	37,500
Stainless steel	Quenched and drawn	*J-9*	164,300	112,000	.....	111,000	72,500
Stainless steel	As received	*J-r*	110,000	47,500	.....	32,000	22,500
Chromium-copper steel	Quenched and drawn	*B-9*	199,500	133,000	.....	.....	119,000
Chromium-copper steel	As received	*B-r*	99,800	50,000	.....	.....	25,000
High-chromium, low-copper steel	As received	*GQ*	87,900	48,800	47,800	40,500	40,000
Medium-chromium, high-nickel steel	As received	*D-r*	111,100	52,500	.....	.....	22,800
Medium-chromium, high-nickel steel	As received	*FK*	112,300	67,200	65,600	57,600	46,600

TABLE 6 SOME PHYSICAL PROP

1	2	3	4	5	6	7	8
Nickel, cold-drawn	Low anneal	*IT-6*	84,600	47,000	48,000	42,000	40,000
Nickel, cold-rolled	Fully annealed	*CT-16*	72,400	*a*	11,900	9,600	*a*
Nickel, cold-rolled	Low anneal	*EO-6*	131,700	95,200	92,000	86,000	85,000
Nickel, cold-rolled	Fully annealed	*EO-14*	77,600	22,000	21,800	19,300	19,000
Nickel, cold-rolled	Low anneal	*A-11*	142,600	86,300	70,300	52,700	50,000
Monel metal, cold-rolled	Low anneal	*EP-8*	127,200	95,000	90,000	84,500	82,000
Monel metal, cold-rolled	Fully annealed	*EP-14*	81,900	31,000	31,900	28,200	25,000
Monel metal, hot-rolled	As received	*AA*	90,200	56,200	60,000	51,500	42,700
Constantan, hot-rolled	Annealed	*CE-1*	70,300	27,500	26,500	19,000	14,000
40:57 nickel-copper, hot-rolled	As received	*EH*	78,000	47,300	47,300	37,300	35,300
40:57 nickel-copper, hot-rolled	Fully annealed	*EH-14*	71,800	25,500	26,500	21,300	18,800
29:67 nickel-copper, cold-rolled	Low anneal	*HZ-8.5*	97,800	71,700	72,300	59,300	51,700
29:67 nickel-copper, cold-rolled	Fully annealed	*HZ-14.5*	70,500	24,500	24,800	14,500	10,300
21:78 nickel-copper, cold-worked	Low anneal	*HE-4*	62,400	44,700	44,700	34,300	24,700
21:78 nickel-copper, cold-worked	Fully annealed	*HE-14*	47,300	11,000	11,700	9,200	8,100
20:79 nickel-copper, cold-worked	Low anneal	*HV-4*	61,000	50,000	47,000	32,000	24,000
20:79 nickel-copper, cold-worked	Fully annealed	*HV-14*	47,500	12,000	12,500	7,300	5,800
Copper, electrolytic, cold-worked	Low anneal	*HU-2.5*	46,900	30,800	20,000	11,000	9,900
Copper, electrolytic, cold-worked	Low anneal	*HU-7.5*	30,300	*a*	4,700	2,000	*a*
Copper, electrolytic, cold-worked	Low anneal	*HF-2.5*	46,500	31,000	21,200	12,000	9,200
Copper, electrolytic, cold-rolled	As received	*BO*	40,400	*a*	13,000	5,000	*a*
Copper, electrolytic, hot-rolled	Fully annealed	*EB-12*	31,200	*a*	4,500	3,000	*a*
Copper, electrolytic, hot-rolled	As received	*BU*	31,400	*a*	6,500	3,700	*a*
95:5 copper-tin, cold-drawn	As received	*CK*	62,900	*a*	25,000	7,500	*a*
95:5 copper-tin, cold-drawn	Low anneal	*EAA-3.75*	81,300	60,000	55,500	47,800	46,300
		EAA-4.00					
95:5 copper-tin, cold-drawn	Low anneal	*EA-3.75*					
		EA-4.00	63,000	47,500	48,000	42,500	37,500
95:5 copper-tin, cold-drawn	Fully annealed	*EA-12*					
		EAA-12	48,000	16,800	16,750	14,000	9,500
92:8 copper-tin, cold-drawn	Low anneal	*EDA-2.50*	96,000	61,000	60,000	47,500	40,000
		EDA-3.50	94,600	65,000	57,500	40,000	37,500
		EDA-4.00	94,000	65,000	59,000	45,000	37,500
92:8 copper-tin, cold-drawn	Low anneal	*ED-3.75*	81,000	48,800	43,300	34,000	31,500
		ED-4					
92:8 copper-tin, cold-drawn	Fully annealed	*ED-12*					
		EDA-12	55,400	19,800	19,800	18,800	18,000
90:10 copper-tin, cold-drawn	Low anneal	*ECA-3.5*					
		ECA-4.00	91,300	56,800	47,600	35,900	35,300
90:10 copper-tin, cold-drawn	Low anneal	*EC-4.0*	86,700	*a*	46,500	34,000	*a*
90:10 copper-tin, cold-drawn	Fully annealed	*EC-12*					
		ECA-12	64,300	22,200	22,800	19,800	14,700
Copper-nickel-zinc, hot-rolled	As received	*CD*	57,100	*a*	32,000	21,500	*a*
Copper-nickel-zinc, hot-rolled	Annealed	*CD-1*	51,100	*a*	12,300	6,500	*a*
Copper-nickel-zinc, cold-drawn	As received	*CH*	62,400	33,500	38,000	22,500	20,300
Copper-nickel-zinc, cold-drawn	As received	*CG*	58,700	28,500	31,500	22,500	12,500

a Stress-strain graph curved from origin.

TIES OF STEELS AND NON-FERROUS ALLOYS

least two determinations)

PROPERTIES OF STEELS

9	10	11	12	13	14	15	16	17	18	19	20	21
						Endurance Limit		Ratio	Ratio	Ratio	Rotating cantilever corrosion-fatigue limit	
Elongation in 2 in., per cent	Reduction of area, per cent	Shearing strength, lb. per sq. in.	Torsional strength, lb. per sq. in.	Brinell hardness number	Charpy impact value, ft-lb.	Rotating cantilever, lb. per sq. in.	Alternating torsion, lb. per sq. in.	Col. 16 to Col. 15	Col. 15 to Col. 4	Col. 16 to Col. 4	Fresh water, lb. per sq. in.	Salt water, lb. per sq. in.
26.2	68.1	.....	.....	...	42.8	37,000	....	..	0.47	..	16,000	....
39.0	59.7	.....	.....	...	12.1	24,000	....	..	0.43	..	17,000	....
39.0	63.7	.....	53,400	...	..	25,500	14,000	0.55	0.42	0.23	....	....
29.0	58.0	.....	57,700	...	..	33,500	20,500	0.61	0.37	0.23	....	....
30.5	47.0	.....	54,800	...	..	29,500	16,000	0.55	0.41	0.23	....	....
22.5	34.4	.....	70,800	...	..	31,500	19,000	0.60	0.35	0.21	....	....
29.5	50.8	.....	.....	...	12.9	42,000	....	..	0.41	..	21,000	13,000
16.5	40.0	.....	102,600	306	3.7	62,000	....	..	0.39	..	....	....
24.5	40.3	.....	77,900	226	3.2	49,100	....	..	0.44	..	....	....
16.0	42.0	.....	111,400	...	..	59,500	....	..	0.38	..	....	....
22.5	44.5	.....	.....	...	..	49,500	28,000	0.57	0.48	0.27	....	....
21.6	65.8	.....	.....	...	37.9	69,000	....	..	0.55	..	25,000	....
29.9	58.5	.....	.....	...	29.6	49,000	....	..	0.53	..	29,000	....
15.0	55.0	.....	91,400	...	..	74,000	47,000	0.64	0.48	0.31	....	....
22.0	43.5	.....	77,800	...	..	51,000	22,500	0.44	0.45	0.20	....	....
19.3	61.3	.....	.....	...	29.0	62,000	....	..	0.48	..	22,000	16,000
29.6	59.3	.....	.....	...	..	49,000	....	..	0.54	..	23,000	....
18.7	40.5	.....	.....	...	16.7	66,000	....	..	0.50	..	19,000	....
18.0	25.8	.....	.....	...	7.6	55,000	....	..	0.45	..	21,000	....
8.3	46.5	166,500	.....	...	8.6	108,000	....	..	0.43	..	12,000	....
22.5	46.4	90,600	.....	...	3.7	65,000	....	..	0.47	..	22,500	....
20.7	58.2	.....	88,400	260	29.8	60,000	....	..	0.45	..	....	....
27.0	49.5	.....	71,400	208	33.7	39,900	....	..	0.39	..	....	....
19.3	57.7	.....	.....	...	30.6	68,000	....	..	0.49	..	16,000	14,000
18.0	58.5	.....	.....	...	..	74,000	....	..	0.44	..	....	....
17.0	61.6	.....	.....	...	..	77,000	....	..	0.47	..	.. .	....
30.5	59.8	.....	72,700	194	15.2	47,000	....	..	0.48	..	....	....
17.5	59.0	.....	.....	...	..	.....	....	..	..	..	....	....
14.0	47.8	109,200	.....	...	7.6	78,000	....	..	0.44	..	16,000	....
16.6	54.4	94,500	.....	...	17.7	69,000	....	..	0.47	..	16,000	....
28.1	53.8	66,400	.....	...	17.1	42,000	....	..	0.43	..	22,000	....
18.5	50.5	.....	93,100	279	29.1	67,500	....	..	0.48	..	....	....
20.6	65.1	.....	.....	...	39.6	58,000	....	..	0.49	..	20,000	....
19.8	51.3	.....	90,300	245	22.5	57,000	....	..	0.43	..	....	....
21.3	41.0	.....	68,400	211	10.3	46,600	....	..	0.44	..	....	....
11.0	41.0	.....	110,500	...	..	88,000	43,000	0.49	0.51	0.25	....	....
15.0	52.2	.....	83,300	...	..	62,500	37,000	0.59	0.51	0.30	....	....
13.0	40.2	.....	.....	...	4.9	88,000	....	..	0.49	..	32,000	....
21.0	52.3	74,400	.....	...	13.4	53,000	....	..	0.45	..	37,500	27,000
29.8	61.3	.....	.....	...	26.8	52,000	....	..	0.55	..	36,000	36,000
11.5	38.4	.....	110,500	...	..	84,500	40,000	0.47	0.51	0.24	....	....
24.0	53.5	.....	70,600	...	..	49,500	24,000	0.48	0.45	0.22	....	....
9.0	13.0	.....	118,700	...	..	114,500	47,000	0.41	0.57	0.23	....	....
28.0	48.5	.....	76,100	...	..	54,500	22,000	0.40	0.55	0.22	....	....
27.9	52.2	71,900	.....	...	2.4	47,000	....	..	0.53	..	40,500	30,000
24.0	48.9	.....	72,800	...	..	58,500	22,500	0.38	0.53	0.20	....	....
28.8	46.6	.....	.....	...	29.6	57,000	....	..	0.51	..	41,000	22,000

ERTIES OF NON-FERROUS ALLOYS

9	10	11	12	13	14	15	16	17	18	19	20	21
37.0	66.7	.....	.....	...	..	.....	....	..	..	..	....	....
53.7	76.4	51,900	75,200	109	..	26,000	22,000	0.84	0.36	0.30	....	....
16.2	34.1	78,400	.....	...	65.8	50,500	....	..	0.38	..	26,000	23,500
49.3	75.2	.....	.....	...	..	33,000	....	..	0.42	..	24,000	22,000
16.3	24.8	83,700	66,100	255	..	49,000	28,500	0.58	0.34	0.20	....	....
20.6	57.8	67,700	.....	...	51.4	52,000	....	..	0.41	..	25,000	28,000
48.9	73.6	.....	.....	...	..	36,000	....	..	0.44	..	26,000	28,000
35.9	69.7	59,400	69,000	...	..	38,000	19,000	0.50	0.42	0.21	....	....
48.8	79.3	50,400	60,200	89	..	34,500	17,000	0.49	0.49	0.24	....	....
32.7	60.2	51,800	.....	...	44.8	32,500	....	..	0.41	..	....	....
47.0	66.6	48,800	.....	102	45.5	30,000	....	..	0.41	..	....	28,000
18.0	54.2	50,400	.....	...	38.0	36,000	....	..	0.37	..	21,000	....
45.3	71.4	48,500	.....	...	65.8	26,500	....	..	0.32	..	21,000	....
22.5	68.2	38,200	.....	...	34.7	25,500	....	..	0.40	..	21,500	....
50.0	74.7	32,700	.....	...	47.2	17,800	....	..	0.38	..	18,500	18,500
24.5	68.5	38,200	.....	...	36.8	.....	....	..	..	..	....	....
49.3	72.4	33,100	.....	...	55.6	.....	....	..	..	..	....	....
15.8	63.4	25,500	.....	...	24.7	.....	....	..	..	..	....	....
58.0	74.8	21,900	.....	...	42.1	.....	....	..	..	..	....	....
14.3	55.8	27,100	.....	...	18.0	16,500	....	..	0.35	..	....	16,500
24.5	66.0	25,800	3,400	...	..	.....	6,000	..	..	..	....	....
73.8	52.8	22,300	.....	...	29.4	10,000	....	..	0.32	..	....	10,000
58.7	72.2	23,900	29,900	...	..	.....	4,000	..	..	..	....	....
32.5	73.8	.....	50,700	...	..	27,000	12,000	0.44	0.43	0.19	....	....
25.0	64.5	54,500	.....	132	24.1	23,000	.....	..	0.28	..	22,000	22,000
37.5	74.0	43,900	.....	107	36.8	23,000	....	..	0.43	..	23,000	....
71.5	79.6	39,900	.....	56	49.8	20,000	....	..	0.41	..	....	....
19.5	56.5	66,400	.....	...	15.4	.....	....	..	....	..	....	....
26.0	59.0	66,000	.....	...	20.6	17,000	....	..	....	..	....	....
26.0	59.5	66,900	.....	143	16.7	.....	....	..	0.20	..	....	....
37.5	74.0	59,800	.....	121	33.7	22,000	....	..	0.27	..	18,000	....
84.3	77.8	41,700	.....	55	47.0	21,000	....	..	0.38	..	....	19,000
32.0	65.5	67,800	.....	158	21.0	17,000	....	..	0.19	..	....	....
35.5	56.0	63,900	.....	121	26.6	17,000	....	..	0.20	..	....	....
77.2	73.1	45,900	.....	67	44.2	20,000	....	..	0.31	..	....	....
35.8	69.8	38,300	45,300	...	..	23,000	....	..	0.40	..	....	....
47.3	72.7	.....	.....	...	..	20,800	13,000	0.62	0.40	0.25	....	....
28.0	49.6	.....	.....	...	..	22,000	....	..	0.35	..	....	....
49.5	59.0	.....	.....	...	..	17,000	....	..	0.29	..	....	....

stress of the tension-compression range. Specimens were subjected to rotating-bend test and at the same time to additional tensile stresses throughout the entire cross-section. By this means the endurance range was investigated with various ratios between its superior and inferior limits. These authors conclude that, as the endurance range is moved away from the position involving complete reversal of stress, it decreases at an increasing rate. Other investigators have studied this subject and come to similar conclusions. Formulas, more or less elaborate, have been evolved to represent the change of the endurance range with change in the ratio between maximum and minimum stress of the range. These formulas have all been developed as a result of experiments with the tension-compression range.

The effect of varying the position of the torsional endurance range within the elastic range was discussed by the writer in a

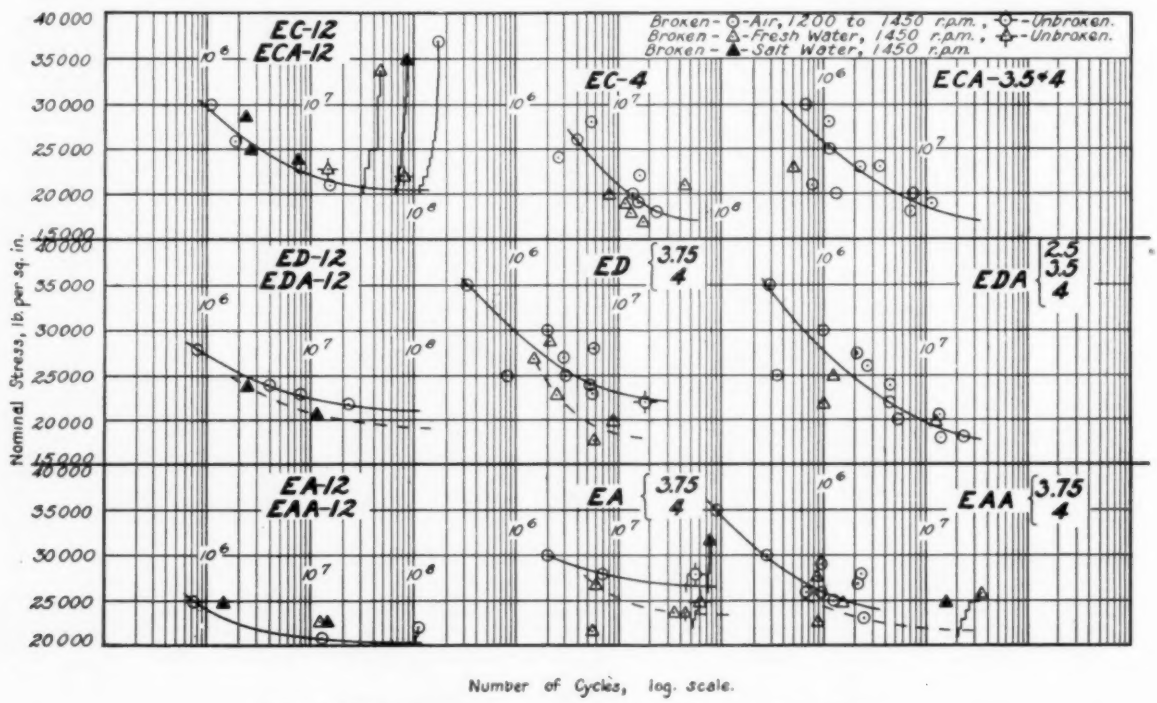


FIG. 5 FATIGUE AND CORROSION-FATIGUE OF COPPER-TIN ALLOYS

previous paper.[3] It was there shown that the torsional endurance range for steel decreases not more than about 5 per cent as the position of the stress range is moved from the middle to the tensile end of the elastic range. Even if the superior stress of the range is increased nearly to the yield point, the endurance range is only slightly less than for complete reversal of stress per cycle. These results indicate that a high elastic limit is of considerable value in spring steel. A high elastic limit makes it possible to vary considerably the position of the endurance range within the elastic range. As stated previously, however, the magnitude of the endurance range depends not on the elastic limit but on the tensile strength.

Almost simultaneous with this paper was a bulletin by H. F. Moore and T. M. Jasper[14] in which similar conclusions were reached as to the approximate constancy of the torsional endurance range within the limits, or even slightly beyond the limits, of the elastic range. More recently these conclusions have been confirmed in a paper by G. A. Hankins.[15] He says also: "In addition the present results show that the variations are small when the superior stress limits are well outside the original elastic or proportional ranges. In the case of mild steel and the normalized 0.6 per cent carbon steel, the safe ranges are less than 10 per cent below those for reversed stresses even when the superior limits of stress approach the ultimate stress of the material. In the case of hardened and tempered spring steels, there are marked decreases in the safe ranges at the high mean stresses." The conclusion seems to be well established, therefore, that the torsional endurance range for steel varies only slightly with variation in its position within the elastic range.

The constancy of the torsional endurance range for steel within the elastic range suggests the possibility that further experiments may show that there is a similar constancy of the tension-compression endurance range within the elastic range. The author in the before-mentioned paper[3] discussed the results obtained by Moore and Jasper[13] and suggested that these results seem to indicate an actual constancy of the tension-compression endurance range within the elastic range. It is worthy of note that nearly all who have investigated this subject have attempted to work out a formula that would apply whether or not the primitive elastic range is exceeded. While such formulas are useful, it appears to the writer that a knowledge of the effect of varying the position of the tension-compression endurance range within the elastic range is of more practical importance. Such knowledge is especially needed for application to spring design.

For steel springs subjected to a torsional range of stress, therefore, it is well established that the endurance range does not vary appreciably provided the elastic range is not exceeded. For steel springs subjected to a range of bending stress, it seems possible that the same law applies. Further investigation of this subject, however, is necessary.

For non-ferrous metals, the endurance range has been investigated only with complete reversal of stress per cycle. Investigation of the endurance range for non-ferrous metals with other ratios between the superior and inferior limits is much needed. For application to springs, a knowledge of the endurance range when the inferior limit is zero, or even when the range is between

a maximum and a minimum in the same direction, would be of great use in design.

Effect of Abrupt Changes of Section

The endurance properties discussed in the previous sections have been those obtained with specimens as free as possible from abrupt change of section. In springs, however, the surface is usually much less smooth than in the endurance test specimens. There are frequently also surface defects and internal flaws due to processes of manufacture of the metal and the forming of the spring. It is necessary, therefore, to consider the effect of abrupt changes of section on the nominal fatigue limit.

This subject has been investigated by Stanton and Bairstow,[16] Eden, Rose, and Cunningham,[17] R. R. Moore,[18,19] H. F. Moore and J. B. Kommers,[20] W. Norman Thomas,[21] and others. Theoretically, the stress concentration depends on the ratio between the notch depth and the radius of curvature at the bottom. In general, the conclusion reached by these investigators is that the reduction of fatigue strength of steel by notches is less than the theoretical reduction. The softer the material, the greater the difference between the observed and the theoretical reductions.

The writer has made fatigue tests on notched specimens of most of the materials that are represented in Figs. 1 to 4. All these tests were made with the same type of notch. The form and dimensions of this notch are shown in Fig. 6 in comparison with the unnotched tapered specimen. As shown in this figure, the notch has an angle of 60 degrees and the radius at the bottom is 0.0055 in. (0.14 mm.). To obtain the proper radius the tool was ground with frequent checking by microscopic measurement at a magnification of 50.

Results of these tests with notched specimens are shown in Figs. 1 to 4 in comparison with the previously described results of tests with unnotched specimens.

A comparison of the fatigue limits for the notched specimens of various steels in Figs. 1 and 2 shows that the nominal fatigue limit increases with increase in the strength of the steel. The percentage increase in the fatigue limit for the notched specimen, however, is much less than the percentage increase of the endurance limit of the metal (as obtained with unnotched specimens). The fatigue limit of the notched specimen ranges from 16,000 for annealed mild steel to 27,000 for chrome-vanadium steel of spring temper, whereas the corresponding endurance limit ranges from 23,500 to 78,000. The fatigue limit of the notched specimen, therefore, varies about 69 per cent, while the endurance limit varies 239 per cent.

Evidently, the greater the ratio of depth to radius of notch, the less influence does the strength of the material have on the fatigue limit of the notched steel specimen. The results would seem to indicate that with an even sharper notch the fatigue limit of the hardest steel would be no greater or even less than that of the mildest steel. This illustrates the great importance of avoiding abrupt changes of section and surface roughness in spring steel.

In Figs. 3 and 4 are shown results obtained with notched specimens of nickel-copper alloys in comparison with the previously described results obtained with unnotched specimens. The ductility of a metal as determined by tension test is not a reliable index of the damaging effect of notches. The fatigue limit of copper, for example, a metal with high ductility, is greatly affected by notches. This was pointed out by H. F. Moore and S. W. Lyon.[22] This fact is also illustrated in Fig. 3. The fatigue limits of both cold-rolled and fully annealed copper are reduced to about the same low value, although the ordinary endurance limit of the cold-worked metal is nearly twice that of the fully annealed metal.

With increase of nickel content to about 20 per cent, as illustrated by material *HE*-14, the fatigue limit of notched specimens is much greater than that of pure copper, although the endurance limit of the 20 per cent nickel alloy is only slightly greater than that of the cold-worked copper, *HF*-2.5. Comparison of the graphs for materials *HE*-14 and *HE*-4 shows that cold working improves the fatigue limit of notched specimens of the 20 per cent alloy. A comparison of the graph for material *HE*-4 with the graphs in the lower row of Fig. 3 and with the graphs in Fig. 4 indicates that the fatigue limit of specimens having this type of notch is practically unaffected by chemical composition or cold working, within a range of composition from 20 to 99 per cent nickel, although within this range the tensile strength and endurance limit vary greatly.

Evidently, therefore, for nickel-copper alloys as for steels, abrupt changes of section, surface defects, internal flaws, etc., tend to neutralize the effect of increased strength on the fatigue limit. If the notch is sharp enough, the effect of strength is neutralized for alloys having 20 to 99 per cent nickel. With even sharper notch, it is possible that this complete neutralization would prevail over an even wider range of composition.

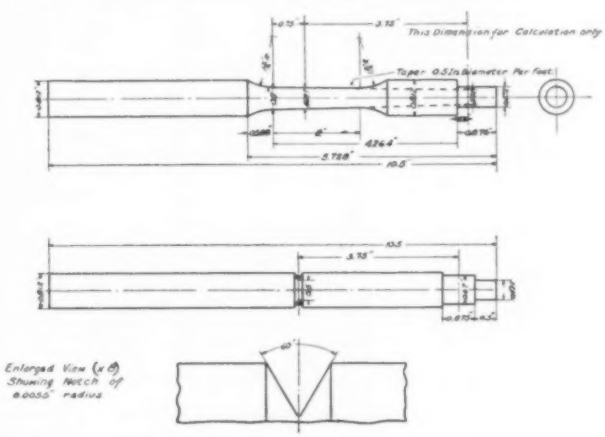


Fig. 6 Rotating-Cantilever Specimens Used in Fatigue Tests

The importance of surface condition for spring material, therefore, can hardly be overemphasized. On account of its hardness, spring material is especially susceptible to the effect of surface defects. Such material in service is never notched as deeply as were the specimens illustrated in Fig. 6. The shallower surface defects that occur in service, however, may be as damaging if the ratio of notch depth to radius is as great. That such defects often exist in springs is shown in a paper by F. C. Lea.[23] His fatigue tests on helical springs showed that the fatigue limit of the spring may be much lower than the endurance limit of the same material in the form of ordinary fatigue test specimens. He also showed that under such conditions the fatigue limit of a spring of high-strength steel may be little, if any, higher than that of a spring of mild steel. Lea's tests on springs, therefore, as well as the results of tests on notched specimens as shown in Figs. 1 to 4, show the importance of freedom from surface defects and internal flaws in spring material.

Corrosion-Fatigue of Spring Steels

In 1917 Haigh showed that the fatigue limit of brass in contact with ammonia is lower than that of brass tested in air. He also showed that the damage is greater if the fatigue and corrosion are simultaneous than if the corrosion is prior to the fatigue. In recent papers by the writer,[5,6,7,8,9,10] the combined influence of corrosion and fatigue has been discussed. It has been shown that even slight corrosion simultaneous with fatigue may cause

failure at nominal stresses far below the ordinary endurance limit. It was also shown that for most metals severe stressless corrosion prior to fatigue is much less damaging than even slight corrosion simultaneous with fatigue. The term "corrosion fatigue" has been used to designate the simultaneous action of corrosion and fatigue. The action of corrosion followed by fatigue has been called "prior-corrosion fatigue."

A stress-cycle graph representing corrosion fatigue is a curve approaching a horizontal asymptote, which may be called a "corrosion-fatigue limit." In Figs. 1 to 5 are shown corrosion-fatigue graphs for most of the alloys whose endurance properties in air have already been discussed.

As illustrated by the broken-line graphs in Fig. 1, the corrosion-fatigue limits for the two mild steels, and for the nickel steel in two degrees of hardness, differ very little. As illustrated by the three graphs in Fig. 2, the corrosion-fatigue limit for the steel of spring temper is actually lower than for the fully annealed steel. As shown in Fig. 3 of another paper,[6] for a steel of even greater strength the corrosion-fatigue limit is only 12,000, whereas for the same steel in the fully annealed condition the corrosion-fatigue limit is more than twice as great.

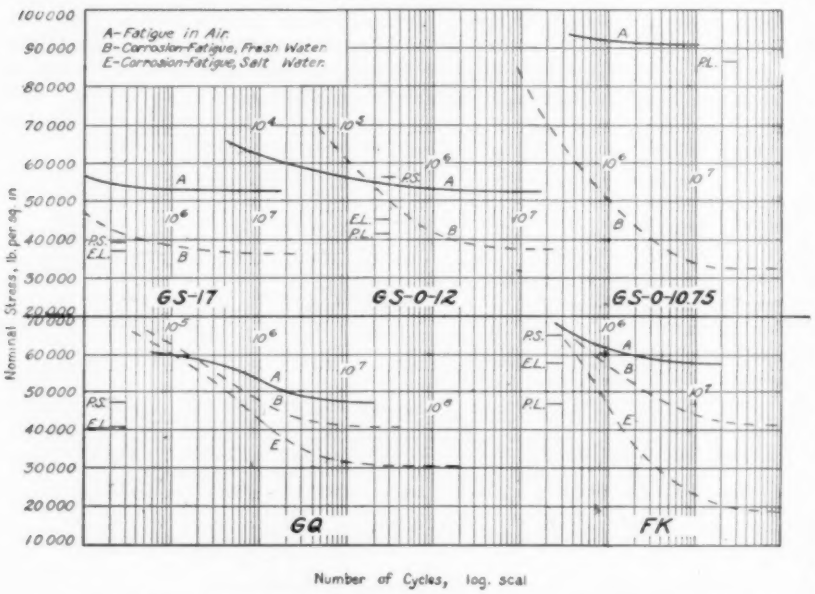


FIG. 7 FATIGUE AND CORROSION-FATIGUE OF CORROSION-RESISTANT STEELS

For steel of spring temper, as shown in Fig. 2 of the present paper and in Fig. 3 of a previous paper, the corrosion-fatigue limit in fresh water may be only one-fourth to one-ninth the endurance limit. In salt water, the corrosion-fatigue limit is still lower.

The rapidity with which corrosion fatigue manifests itself in these spring steels, moreover, is remarkable. As illustrated by the graph for *AZ*-O-7.5, in Fig. 2, in contact with water, failure would occur in 3 hours at a stress of 70,000, in 12 hours at a stress of 45,000, and in five days at a stress of about 20,000.

The fresh-water corrosion-fatigue limit for carbon and ordinary alloy steels ranging in tensile strength from 45,000 to 250,000 varies only from about 12,000 to 24,000. It is little affected by composition or heat-treatment unless such composition or heat-treatment affects the corrosion-resistance of the steel.

Results of prior-corrosion fatigue tests are also shown in Figs. 1 and 2. Graphs *F* represent results of fatigue tests of specimens that have been previously corroded for 10 days in fresh water. Comparison of these graphs with the corrosion-fatigue graphs shows that the effects of 10 days' stressless corrsioon followed by fatigue are much less than the effects of simultaneous corrosion and fatigue. It should be noted also that the prior-corrosion fatigue limit obtained under these conditions increases with increase in tensile strength, differing in this respect from the corrosion-fatigue limit. The phenomena of corrosion-fatigue and prior-corrosion fatigue as illustrated by the graphs in Figs. 1 and 2 suggest the idea that the effects are due to pits or notches in the surface, and that the notches produced by simultaneous corrosion and cyclic stress are usually sharper than the notches produced by stressless corrosion. This subject has been discussed in recent papers[7,8] and results have been presented showing the effect of stress, time, and number of cycles on the depth and sharpness of corrosion pitting. These results throw much light on the mutual effect of corrosion and cyclic stress. Much further investigation, however, is needed.

If sufficient chromium with or without other elements is added to increase the corrosion resistance of steel, the corrosion-fatigue limit is raised considerably above that for carbon or ordinary alloy steels. As shown in recent papers[5,6,7,8] the fresh-water corrosion-fatigue limit for corrosion-resistant steels is more than twice as great as for ordinary steels. It has been shown in recent papers,[7,8] however, that ordinary steels are severely pitted at a stress of only about 8000 lb. per sq. in. This stress, which has been called a "notching limit," is well below the corrosion-fatigue limit. It depends somewhat on time and number of cycles. It was also shown in these papers[7,8] that corrosion-resistant steels are not severely pitted at stresses below the corrosion-fatigue limit. For corrosion-resistant steels, therefore, the notching limit is about four times as great as for ordinary steels.

For many of the corrosion-resistant steels the elastic limit and proportional limit are low. This is a decided disadvantage for spring material. The steels represented in Fig. 7 have been selected from among those with highest elastic limit.

The upper row of this figure shows graphs for stainless steel in three degrees of hardness due to heat treatment. The group of graphs on the left represents fully annealed material. The group on the right represents material of spring temper. The elastic limit, proportional limit, and endurance limit are fairly high. As shown by the three broken-line graphs, the corrosion-fatigue limit of this material is little affected by heat-treatment. It is more than twice the corrosion-fatigue limit of ordinary steels.

The two groups of graphs in the lower row represent other corrosion-resistant steels with fairly good elastic limit. The graph on the right is a medium-chromium, high-nickel steel. This type has a higher elastic limit than does the high-chromium, low-nickel type, and hence would be more suitable for spring material. In view of the susceptibility of ordinary steels to corrosion fatigue, it is possible that the use of corrosion-resistant steels for this purpose may increase.

CORROSION-FATIGUE OF NICKEL-COPPER ALLOYS

Results of corrosion-fatigue tests of nickel-copper alloys in fresh water are shown in Figs. 3 and 4 in comparison with results obtained by endurance tests in air. As shown in Fig. 3, the corrosion-fatigue limit of copper and of the fully annealed alloy

HE-14 is not below the ordinary endurance limit. With increase in the endurance limit, due to cold working or to increase in the nickel content, the endurance limit rises above the corrosion-fatigue limit, as illustrated by the graph for alloy *HE*-4, by the groups of graphs in the lower row of Fig. 3, and by the graphs in Fig. 4. It should be noted also that the corrosion-fatigue limit of alloys having 20 to 99 per cent nickel is practically the same, although throughout this range of composition the endurance limit varies greatly. Under corrosion fatigue, therefore, most of the advantage of high strength due to cold working or chemical composition is lost.

As shown in recent papers,[7,8] for nickel-copper alloys the corrosion-fatigue limit corresponds to a fairly definite pitting limit. At lower stresses these alloys are not appreciably pitted. The superiority of nickel-copper alloys to ordinary steels under corrosion-fatigue, therefore, is somewhat greater than would be indicated by a comparison of their corrosion-fatigue limits as obtained under ordinary short-time tests. The salt-water corrosion-fatigue limit of nickel-copper alloys, moreover, is slightly higher (whereas the corresponding limit for steels is much lower) than the fresh-water corrosion-fatigue limit.

Corrosion-Fatigue of Copper-Tin Alloys

Corrosion-fatigue tests of copper-tin alloys, as illustrated in Fig. 5, gave somewhat erratic results, especially for the alloys of higher tin content. Results obtained with the 5 per cent alloy as shown in the lower row of the figure, however, seem to indicate that the intrinsic corrosion-fatigue limit of this material is about 21,000 or 22,000. The endurance limit of the fully annealed material is not quite high enough to reach this intrinsic corrosion-fatigue limit. The endurance limits of the cold-worked material, however, rise well above the corrosion-fatigue limit.

For the 8 per cent alloy, as shown in the middle row, the corrosion-fatigue limit seems to be slightly below 20,000. For the 10 per cent alloy, as shown in the upper row, the results are too erratic to permit definite conclusions. They seem to confirm the above-expressed tentative conclusions, however, that cold working has caused structural damage to this hard alloy.

On the basis of the results given in Fig. 5, therefore, the cold-worked 5 per cent alloy known as phosphor bronze seems to be the most suitable copper-tin alloy for spring material. Unless ways are found of avoiding damage due to cold working, the alloys containing 8 per cent and 10 per cent tin would be too erratic in behavior to be suitable as spring material. The corrosion-fatigue limit for the 5 per cent alloy is little, if any, lower than for the nickel-copper alloys. The salt-water corrosion-fatigue limit is practically the same as the fresh-water corrosion-fatigue limit.

Metallic Protective Coatings for Spring Material

In view of the results presented in Figs. 1 to 5, it appears evident that corrosion-fatigue is a frequent cause of failure of springs in service. Uncoated springs, therefore, exposed to even brief and intermittent contact with water, should be designed with the corrosion-fatigue limit in view. For such purposes corrosion-resistant steels may be more suitable than ordinary steels of higher physical properties. The possibility of raising the corrosion-fatigue limit of spring material by means of protective coatings, however, deserves careful consideration. An investigation of this subject has been started at the Naval Engineering Experiment Station.

Experiments have been made with a heat-treated chrome-vanadium steel, plated directly with chromium. Specimens so coated were subjected to corrosion-fatigue test in fresh water, and the results were compared with the results obtained with uncoated specimens of the same material. The results indicate that the corrosion-fatigue limit for coated specimens is little, if any, higher than for uncoated specimens. Experiments will be made to determine the effect of electroplating, first with copper or nickel, then with chromium.

Similar experiments were made with specimens electroplated with cadmium. The cadmium plating was done at the Washington Navy Yard. Two specimens were given a coating 0.0002 in. thick, and two other specimens were given a coating 0.0004 in. thick. The results are shown in Fig. 8 in comparison with the ordinary endurance and corrosion-fatigue graphs obtained with uncoated specimens.

As shown in the group of graphs on the left in Fig. 8, specimens coated 0.0002 in. thick have a corrosion-fatigue limit of about 35,000. This is more than twice the corrosion-fatigue limit of uncoated specimens. As shown in the group of graphs on the right, specimens given the thicker coating have a still higher corrosion-fatigue limit. Possibly if the corrosion-fatigue graphs for coated specimens were extended further by additional experiments for longer time, the difference between the corrosion-fatigue limits for the two thicknesses would grow less. Nevertheless, there seems to be an advantage in use of the thicker coating.

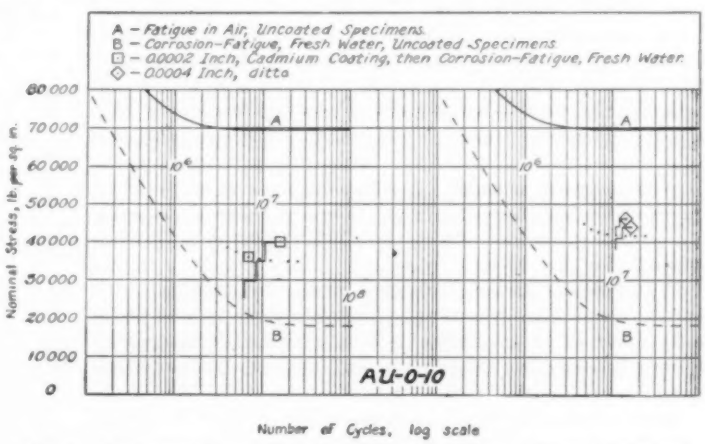


Fig. 8 Corrosion-Fatigue of Chromium-Vanadium Steel Specimens Plated With Cadmium

These preliminary experiments, therefore, indicate that cadmium plating, when properly applied, is of considerable value in raising the corrosion-fatigue limit of spring steel in contact with fresh water. This does not mean that steel springs, so coated, could be used for long periods under water. It does indicate, however, that cadmium plating would prevent damage due to corrosion-fatigue of springs subjected for short intermittent periods to the action of moisture. Investigation of the effect of metallic protective coatings of spring material is continuing.

Effect of Cycle Frequency on Fatigue and Corrosion Fatigue

The effect of cycle frequency on the endurance limit for a range of frequency between about 100 and 5000 cycles per minute is negligible. At much higher frequencies there is some evidence that the endurance limit is slightly higher. For cycle frequencies below 100 per minute, few determinations of the endurance limit have been made. It is possible that for frequencies below one per minute the endurance limit may be lower. At such low frequencies the time required for endurance tests to destruction is so long as to be almost prohibitive.

The effect of cycle frequency on corrosion-fatigue is of great importance. The interrelationship of the stress, time and number of cycles in causing penetration of metal under corrosion is still being investigated.

Summary

For spring steels the ratio of the endurance limit in tension-compression or repeated bend to the tensile strength is from 0.4 to 0.5. The ratio of the torsional endurance limit to the tensile strength is from about 0.2 to 0.3. The torsional endurance range is about twice the torsional endurance limit and is practically constant for any position within the elastic range. It seems possible, also, that in tension-compression or repeated bend the endurance range is practically constant within the primitive elastic range. Further investigation of this subject is needed.

Monel metal and other nickel-copper alloys can be obtained, by cold-working, with tension-compression endurance limit more than 50,000 lb. per sq. in., and with torsional endurance limit about 35,000 lb. per sq. in. No information is available as to the effect of varying the ratio between the superior and inferior stress of the endurance range. This subject needs investigation.

For copper-tin alloys the 5 per cent alloy, known as phosphor bronze, can be obtained with tension-compression endurance limit of about 27,000 lb. per sq. in. in the form of 1-in. round bars. In the form of smaller rod or wire, the endurance limit may be somewhat higher. Cold working, however, does not raise the endurance limit of this material in proportion to the increase in tensile strength. Nothing is known as to the effect on the endurance range of varying the ratio between the superior and inferior extremes of the range. This subject needs investigation.

The above-mentioned endurance limits are obtained with smooth specimens. The fatigue limit is greatly influenced by surface defects. Even shallow defects, if sufficiently sharp, may practically neutralize the effect of high strength on the fatigue limit.

Spring steels subjected to a range of stress while in contact with fresh water fail at a stress range only one-fourth to one-ninth the ordinary endurance limit. In contact with salt water the endurance range is even smaller. Under such conditions the advantage of high strength due to composition or heat-treatment is lost. The corrosion-fatigue limit depends more on electrochemical than on physical properties.

The corrosion-fatigue limit of corrosion-resistant steels is about twice that of ordinary steels. Corrosion-resistant steels are available with elastic limit high enough to be suitable for spring material. Electroplating ordinary steels with cadmium and probably with other metals more than doubles the corrosion-fatigue limit.

Nickel-copper and copper-tin alloys are subject to corrosion-fatigue, but are not appreciably pitted at stress ranges below the corrosion-fatigue limit. Under corrosion fatigue the advantage of high physical properties is neutralized.

The importance of protecting spring material against corrosion, especially when under a stress range, can hardly be overemphasized.

Acknowledgments

Acknowledgment is due to the following personnel of the Naval Engineering Experiment Station: To Captain H. C. Dinger, U. S. N., officer in charge, for encouragement received in this work; to Messrs. George F. Wohlgemuth and Lawrence Thompson, associate metallurgists; W. C. Stewart, assistant metallurgist; A. P. Vandermast, laboratorian; and John K. Amoss, leadingman-machinist, for assistance in the experiments.

Acknowledgment is also due to three manufacturers, The Firth-Sterling Steel Company, the International Nickel Company, and the American Brass Company, for generous cooperation by furnishing material.

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Discussion

R. Eksergian.[25] The author has given us very valuable data as well as an interesting interpretation as to the properties of spring materials.

It appears from Tables 5 and 6 that the endurance limit for torsion averages somewhat less than 50 per cent of the endurance limit for tensile or bending stresses. That is, the torsion endurance limit is roughly from 20 to 25 per cent of the tensile strength of the materials. The remark that the endurance limit appears primarily dependent on the tensile strength and hardness alone and is quite independent of the ductility or elastic limit concurs with the works of Prof. H. F. Moore.

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[25] Engineer, Baldwin Locomotive Works, Philadelphia, Pa. Mem. A.S.M.E.

A spring is subjected to a considerable variation in cycles of stress, and the stress range may take place between various minimum and maximum values. A very important question in the selection of a material is therefore the effect of unequal stress cycles on the endurance limit and the effect of alternations of stress about different mean values of stress on the endurance range.

The author's statement that the endurance range remains practically constant when the upper limit of stress does not exceed the elastic limit, particularly for torsion and very likely for direct stress, is of considerable importance in the selection of the proper working stress of a spring.

From observations made by the writer on oscillations of railway truck springs, it appears that a 35 per cent overload and 15 per cent underload is the maximum average stress range for an ordinary proportional spring. This means a stress range of 50 per cent of the work loading so that the endurance range should be equal to or greater than this value. That is, if f_w is the working stress and f_v the ultimate tensile stress, then, assuming the endurance range for torsion at $0.20\,f_v$, we have:

$$0.50\,f_w = 0.20\,f_v$$

therefore

$$f_w = \frac{0.20}{0.50} f_v = 0.40\,f_v$$

Thus the working stress would be about 40 per cent of the ultimate, and is also sufficiently below the elastic limit to prevent the average pack stress from passing into the plastic range and thus preventing a gradual settling of the springs. If the stress range is reduced, it would be desirable to increase the ratio of elastic limit to ultimate strength, so that surge loads will not greatly pass over the elastic limit.

Undoubtedly, in the selection of a satisfactory working stress the estimation of the concentration of stress is of importance, since the stress range should be based on the actual peak values.

The fact that the endurance limit is primarily dependent on the tensile strength alone, opens the question as to the importance of ductility. Since both plate and coiled springs are subject to occasional large overloads and deflections as well as considerable stress concentration, it appears to the writer that the ductile properties serve as a sort of safety valve for relieving the stress concentration and tending toward a better distribution of stress. Very often the major part of stress alternations for certain service is of relatively low amplitude, while occasionally the spring would be subjected to extreme amplitudes. In such a case the ductility of the material may become a factor of importance.

Thus we see the endurance strength of the spring demands a high ultimate strength, and for reducing peak loadings in the plastic range a high ratio of elastic to ultimate strength. This is consistent with a high-carbon steel. On the other hand, to allow for stress concentration, a lower carbon or plastic material is desirable. Finally, the flexibility of a spring is proportional to the square of the working stress, so that a high working stress is very desirable. This would indicate that a high ultimate strength and high elastic limit with maximum ductility are the primary qualifications of a good spring material.

J. M. Lessells.[26] This paper is further evidence of the continued interest being shown in the phenomenon of the fatigue of metals by a cyclic stress. In the present case we have a paper devoted entirely to spring material.

I shall restrict my remarks purely to the points dealing with the fatigue of metals and omit all reference to the corrosion fatigue data since I have had occasion in the past to discuss this entirely new field uncovered by Dr. McAdam.

I agree that the position of the endurance range with reference to the elastic range needs further investigation. This is a phase of fatigue of especial interest in spring applications since most of the stresses to which springs are subjected are never completely reversed. We are at present, in the Westinghouse Research Laboratory, investigating the condition where a steady stress has a variable one superimposed.

I agree that in tests between direct tension and compression it is necessary to take special precautions to avoid eccentric loading. This was done by P. L. Irwin[27] and is described fully elsewhere. In this connection it is well to point out that not only was good agreement found between the endurance limit by direct stress and by bending, for materials where the endurance limit was below the elastic limit, but this also obtained in the case of copper, nickel, and monel metal where the endurance limit was in excess of the elastic limit.

Material	Proportional limit, lb. per sq. in.	Endurance limit, lb. per sq. in. Direct stress	Bending
Annealed copper	1875	10000	11000
Annealed nickel	16000	38000	41000
Annealed monel metal	14000	40000	40000

Referring to the endurance limit as determined by bending and by torsion as given in Tables 5 and 6, columns 15 and 16, I note that the ratio of these values varies over quite a wide range, i.e., 0.84 to 0.38. On the basis of the maximum shear theory, this ratio should be 0.5. Mason,[28] using hollow specimens, found such to be true for low-carbon steel. I wonder if Dr. McAdam will comment on this point. This variation may be due to the use of solid specimens in torsion.

Dr. McAdam states that the endurance range has only been investigated for the complete reversal of stress for non-ferrous metals. It should be noted that Haigh[29] worked on the fatigue of brass for conditions of stressing other than complete reversal. I thoroughly agree, however, that more work is necessary.

Wheeler P. Davey.[30] Dr. McAdam mentions the remarkable rapidity with which corrosion fatigue manifests itself in spring steels. This corrosion fatigue is apparently tied up with the embrittling effect on steel of atomic hydrogen. This effect was described by T. S. Fuller of the Research Laboratory of the General Electric Co. in the Transactions of the American Electrochemical Society, vol. 32, p. 247 (1917) and his work has been confirmed by C. A. Edwards in the Journal of the Iron and Steel Institute, vol. 110, p. 9 (1924).

The crystal structure and inter-atomic spacings in iron and steel are such that at room temperature molecular hydrogen cannot pass through the crystals of the metal. However, it is possible for atomic hydrogen to pass through single crystals of iron or steel quite readily so that it is possible to measure on a manometer the pressure of molecular hydrogen formed by the recombination of the atoms of atomic hydrogen after they have passed through the crystals and have emerged from the surface. The presence of atomic hydrogen causes the iron or steel to become very brittle, and this brittleness apparently persists until the hydrogen has been driven out by heating the metal. Any source

[26] Research Department, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa. Mem. A.S.M.E.

[27] "Fatigue of Metals by Direct Stress," by P. L. Irwin, Proceedings A.S.T.M., vol. 26 (1926), part 2, p. 218.

[28] "Alternating Stress Experiments," by William Mason, Proceedings Institution of Mechanical Engineers, 1927, p. 121.

[29] "Experiments on the Fatigue of Brasses," by B. P. Haigh, Journal Institute of Metals, 1917, part 2, p. 55.

[30] Professor of Physical Chemistry, The Pennsylvania State College, State College, Pa.

of atomic hydrogen is sufficient to produce the effect. The action of acids on the metal yields nascent (i.e., atomic) hydrogen. The corrosive effect of water or steam will also produce nascent hydrogen. It is inevitable that atomic hydrogen is produced at the surface of the steel during those electroplating processes which require the steel to be negatively charged in the electrolytic bath. I believe that T. S. Fuller took out a patent on a method of coating spring steel in such a way as to avoid the entrance of atomic hydrogen.

Of course, all of the foregoing has to do with the embrittling of the crystals themselves, and is in addition to such embrittling as may be caused by corrosion along inter-crystalline boundaries. The effect of inter-crystalline corrosion is to decrease the effective cross-section of the specimen. The effect of atomic hydrogen on spring steels is to cause brittleness of the crystals themselves.

H. W. Gillett.[31] Dr. McAdam's tentative conclusion that severely cold-worked copper-tin and copper-zinc alloys are damaged as to endurance through internal flaws rather than wholly through internal stress is most interesting, although since such flaws may doubtless be submicroscopic and still be effective, it will be difficult to establish their presence by other than indirect methods.

Internal stress in quenched spring steels, according to work reported by Mack and the writer in 1924,[32] appears to have considerable effect in lowering the endurance limit, since release of stress by prolonged tempering at such temperatures that the hardness and strength were either not at all affected or were actually slightly decreased resulted in marked improvement in endurance limit. The obvious conclusion was that for best endurance of heat-treated spring steels tempering times far in excess of those normally used would be of real advantage. It would be of interest to learn whether Dr. McAdam has had occasion to verify or disprove this conclusion.

The work reported to the American Society for Testing Materials and just referred to, very definitely corroborates Dr. McAdam's findings as to the effect of notches in hard spring steel. Un-necked bars tested in the Upton Lewis endurance machine, and so gripped that there was stress concentration at the grip analogous to that at a notch, gave fatigue limits (using the nomenclature Dr. McAdams suggests, to differentiate between "endurance limit" of a specimen without stress concentration and "fatigue limit" of one with stress concentration) of about 30,000 lb. per sq. in., no matter whether the tensile strength was 100,000 or 200,000 lb. per sq. in. Only the very hardest steels tested fell materially above this figure, and only the very softest one (of the heat-treated steels tested, materially below it. The true endurance limits on properly necked bars were of course about 50 per cent of the tensile strength. The 30,000 lb. per sq. in. figure agrees closely with Dr. McAdam's maximum of 27,000 lb. per sq. in.

Still earlier, Stribeck (*Z.V.D.I.*, vol. 67, 1923, p. 631) gave fatigue limits of 27,000 to 34,000 lb. per sq. in. for specimens gripped so as to set up local stress concentration for steels varying from 78,000 to 140,000 lb. per sq. in. in tensile strength. There seems to be no question that notches are more evil in very hard steel than in very soft steel. This fact leads to the further conclusion that spring steel ought to be as clean as it can be procured, since inclusions act as internal notches.

In regard to the behavior of cadmium coatings, it would be of interest if Dr. McAdam would state whether the coatings at a stress of 35,000 lb. per sq. in. are not above their own endurance limit, and what the tested surfaces look like.

In some analogous work at the Bureau of Standards with duplex metals, the softer metal being on the surface, there is indication that a crack or scratch in the soft surface is not readily propagated through the harder base, even though the coating is in good metallic contact with the metal underneath.

The effect of varying the position of the endurance range is something that must be settled before ordinary rotating-beam endurance tests can be applied to springs with complete certainty. Indeed, before that point can be effectively studied it is necessary to know more about the relation between rotary-beam tests and axial-loading tests with completely reversed stress. There is lack of agreement between some American and some English investigators on this point, and the subject is being studied at the Bureau of Standards. Our experience on that probelm to date leads us to feel that there is a very little amount of data on axial-loading tests in existence which have been carried out on apparatus in which sufficiently true axial loading was obtained to give data on which much reliance can be placed. The variation in endurance range with change in position of the range is generally found to be within 10 per cent, and deviation from axiality of loading that would not make errors of 10 per cent quite probably is more likely to be the rule rather than the exception. Hence we feel that Dr. McAdams torsional-endurance data are practically the only data on change of position of the range that are reliable enough to justify drawing conclusions therefrom.

Author's Closure

Replying to Dr. Wheeler P. Davey, I would say that the suggestion that the phenomena of corrosion fatigue may be due to influence of nascent hydrogen has been made by Dr. Speller and by Mr. Fuller in discussing previous papers[5,7] by the writer. It appears probable, however, that nascent hydrogen is not responsible for the effect of water in lowering the fatigue limit of steel. In the two-stage tests described by the author in previous papers,[7,8] the interval between the corrosion stage and the fatigue stage varies from a few hours to several days. This variation has no effect on the resultant fatigue limit. Damage persists, therefore, after the nascent hydrogen has had opportunity to escape.

A recent paper by T. S. Fuller[33] confirms the conclusion that corrosion fatigue of steel in water is not due to nascent hydrogen.

Corrosion fatigue affects practically all metals. The phenomena, therefore, must be explained by hypotheses that are applicable to all metals. As shown by the author in recent papers[7,8,34] the phenomena of corrosion fatigue may be attributed to the fact that cyclic stress accelerates the penetration of metal due to corrosion.

Dr. Gillett's conclusion that internal stress in quenched spring steels has considerable effect in lowering the endurance limit is in accord with results obtained at the Naval Experiment Station.

An investigation of the effect of cadmium coatings on spring steel is being continued. We have not yet determined whether a coating on steel that is stressed to 35,000 lb. per sq. in. is above the endurance limit of the cadmium. On account of the lower modulus of elasticity of the cadmium, however, the stress in the coating is much less than in the steel.

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[31] Chief, Division of Metallurgy, Department of Commerce, Bureau of Standards, Washington, D. C.

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APM-51-6

Combined Stresses in Thick-Walled Cylinders[1]

By EARLE B. NORRIS,[2] BLACKSBURG, VA.

The paper describes some tests which were made on 11 thick-walled cylinders of high-quality gun steel, ranging in bore from 3 in. to 9.5 in., and with ratios of outside to inside diameters of 1.5, 2, and 3. To the author the evidence seems to warrant the conclusion that the maximum-strain theory of St. Venant is correct for cases of tension and compression in mutually perpendicular planes where the tension stress is the greater of the two.

FOR SEVERAL years past the Army Ordnance Department has been conducting an extensive research program at Watertown Arsenal upon thick-walled cylinders subjected to internal pressure, both within and beyond the elastic limit of the material. In addition to the information sought by the Ordnance Department, the data accumulated afford an opportunity to study the effects of combinations of tension and compression upon mutually normal planes and to test the various theories relative to the elastic strength of material subject to such a combination of stresses.

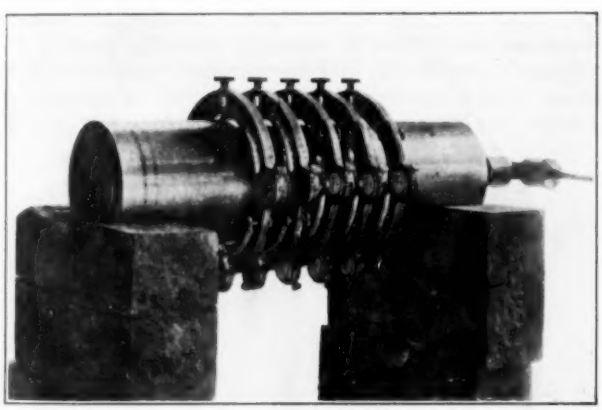


FIG. 1 TEST CYLINDER WITH GAGE RINGS SET UP FOR TEST

Many contributions have been made to the technical literature on this subject, some of them philosophical discussions and others based upon experimental data. We are still, however, in the dark on many phases of this subject of combined stresses. This paper is offered merely as one more contribution to the mass of literature in this complex field. It relates only to the case of tension upon one plane in combination with compression upon a plane normal thereto. In view of the apparent conflict of theories and of many of the experimental studies, any conclusions to be derived from this paper should be considered as being applicable only to conditions similar to those used in these tests.

DATA ON CYLINDERS TESTED

The particular series of tests with which this paper is concerned was made upon a series of 11 steel cylinders of high-quality gun steel, ranging in bore from 3 in. to 9.5 in. and with ratios of outside to inside diameters of 1.5, 2, and 3. The serial numbers of the cylinders, their dimensions, and the elastic limits of the steel in tension, as determined from tangential specimens, are as given in Table 1.

METHOD OF TESTING

A photograph of one of the cylinders set up for test is shown in Fig. 1. This cylinder is shown with five gage rings, each carrying three Ames dial gages. One of the gage rings with the dial gages is shown separately in Fig. 2. The sum of the readings of the two gages in the horizontal plane gave the diametral change in that plane. The change in the vertical plane was given directly

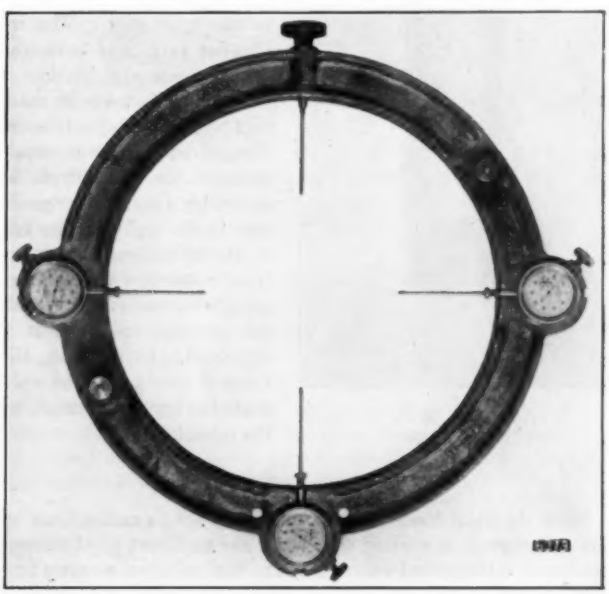


FIG. 2 GAGE RING WITH DIAL GAGES

TABLE 1 ELASTIC LIMIT OF STEEL IN CYLINDERS TESTED

Cylinder no.	Diameters, inches, In.	Out.	Overall length, in.	Length subjected to pressure, in.	Tangential elastic limit, lb.
1	3	× 4.5	19	12	75,000
2	3	× 6	19	12	75,000
3	3	× 9	19	12	75,000
4	6	× 9	31	24	75,000
5	6	× 12	31	24	75,000
6	6	× 18	31	24	60,500 to 68,650 65,000 mean
7	8	× 12	40.5	32	67,000 to 91,000 77,125 mean
8	8	× 16	40.5	32	66,000 to 85,000 73,750 mean
9	8	× 24	40.5	32	64,750 to 76,350 70,750 mean
10	9.5	× 14.25	48	38	69,750 to 73,750 71,100 mean
11	9.5	× 19	48	38	64,700

by the single gage at the bottom of the ring, the adjustable screw at the top being necessary to carry the weight of the ring and to provide a vertical adjustment. In certain of the tests only three gage rings were used, all measuring dilations of the outside of the cylinders. In other tests each gage ring was used to determine the dilation of the cylinder at a particular annulus, the gage pins being inserted into radial holes drilled into the cylinder to the desired depth. These dial gages read directly to 0.0001 in. The observers on the tests were able to record readings to ap-

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proximately one-fourth of a division, or to 0.000025 in. On a diameter of 10 in. there would therefore be indications of unit deformations read to approximately 0.0000025 in. per in.

The cylinders were closed by steel spools having a slight mechanical clearance in the bore and fitted with self-sealing rubber packings bearing against the bore and held out by the pressure of the water. In the first tests an annular rubber sack was used between the spool and the bore of the cylinder, but this was later replaced by the use of C-ring packings. Fig. 3 shows one of the spools assembled, while Fig. 4 shows it disassembled.



FIG. 3 ASSEMBLY OF STEEL SPOOL USED FOR SEALING TEST CYLINDERS

Pressures were established by means of a Watson-Stillman high-pressure pump supplying water to an intensifier of 18:1 ratio. Pressures were determined by a Crosby test gage in the low-pressure line to the intensifier. The intensifier ratio was corrected for the mass and friction of the ram. The pressure readings thus obtained were checked at regular intervals against the pressures as shown by a coil of manganin wire in the high-pressure line to the cylinders. The electrical resistance of this manganin wire varies directly with the pressure to which it is subjected. In addition, the Crosby test gage was calibrated at regular intervals by the manufacturers.

STRESSES PRODUCED

Since the axial forces are carried by the spools rather than by the cylinders, it is evident that there are no direct axial stresses produced in the cylinders. There are two principal stresses produced in the metal, a radial pressure produced by the internal pressure and a tangential tension.

Referring to Fig. 5, in which

P_0 = unit internal pressure
R_0 = internal radius
R_1 = external radius
r = any radius at which the stress is desired
t = tangential tensile stress produced, and
p = radial compressive stress produced,

$$t = \frac{P_0 R_0^2}{R_1^2 - R_0^2}\left(\frac{R_1^2}{r^2} + 1\right) \quad \ldots\ldots [1]$$

$$p = \frac{P_0 R_0^2}{R_1^2 - R_0^2}\left(\frac{R_1^2}{r^2} - 1\right) \quad \ldots\ldots [2]$$

The derivation of these equations, which are generally credited to Lamé, will not be repeated here, but may be found in any standard text on mechanics of materials, or in Tschappat's "Ordnance and Gunnery," pp. 228–231. Being based upon theories of elasticity, they are applicable only within the elastic limits of the metal and assume that the cylinders are free from initial stress. These two stresses are the ones which are concerned in this study, and it is the purpose of this paper to show the relation of these two stresses to the elastic strength of the cylinders tested.

Since the cylinders were longer than the length subjected to internal pressure, it is well to examine the effect of this "overhang." It is clear that the ends of the cylinders projecting beyond the packing rings were not directly stressed, and that these ends would exert a restraining effect upon the expansion of the cylinders for a certain distance inside of the packing rings.

The deformation of these ends is shown on a grossly exaggerated scale in Fig. 6. The line A-B represents the original position of an element of the bore before application of pressure. The line A'-B' represents its approximate position under pressure. The deviations of A'-B' from the original position A-B can be taken as representing the radial forces exerted by the cylinder in resisting the hydraulic pressure. Obviously, if the cylinder is long enough, there will be a length from some point p at the right of the packing to a corresponding point at the other end in which the cylinder will expand freely and will exert a radial resistance in full equilibrium with the internal pressure. To the left of p, as we approach the packing, the overhang exerts a restraining influence and the radial resistance of the cylinder is not adequate to balance the internal pressure. This deficiency is transferred (by shear) to the overhang. At the extreme end we find that the cylinder has actually contracted from its original diameter, thus exerting an outward force. There is, evidently, a bending stress in this overhang, due to the fact that the reactions or radial resistances are not directly opposite the applied pressures. It remains only to find by test the extent of this end effect and to be sure that, for the purposes of these tests, the measurements of expansions of the cylinders are taken well within the straight sections of the cylinders.

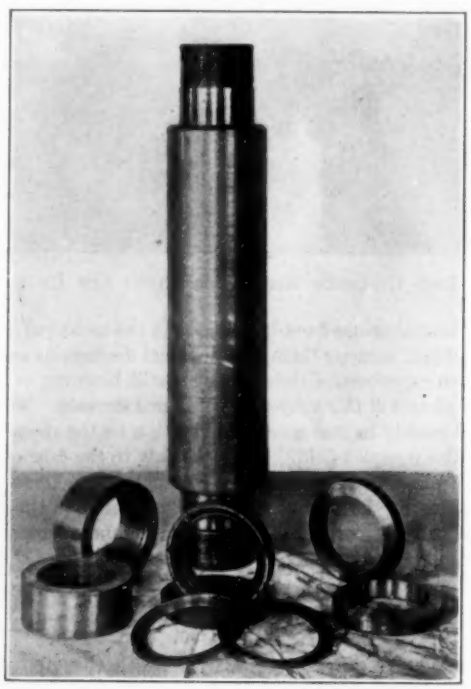


FIG. 4 STEEL SPOOL TAKEN APART TO SHOW PACKING

That length of the cylinder in which the elements remain straight under pressure must be free from bending stress, since there could be bending only where there was a resultant curvature of the elements.

Experimental Verification of Lamé's Laws

Lamé's Equations [1] and [2] should give the tangential tension and the radial compression at any radius r of a cylinder under a given pressure P_0. (As these equations are presented they both give positive values, although one is tension and the other compression.) The equations show that the tangential stress is the greater, but that both produce a shortening of a radial element and a lengthening in the tangential direction. Using the customary E to represent the modulus of elasticity and $1/m$ for Poisson's ratio, the tangential unit deformation e_t produced by the stresses of Equations [1] and [2] will be

$$e_t = \frac{t}{E} + \frac{p}{mE} \quad \ldots\ldots\ldots\ldots\ldots\ldots [3]$$

By the use of the arrangement of gages shown in Figs. 1 and 2, it is possible to measure the diametral enlargement of any annulus of a cylinder and to compare it with the theoretical enlargement given by Equation [3], provided the stresses are within the elastic limits, since these equations are based on the theory of elasticity. In the set-up of Fig. 1 each gage ring has its gage pins inserted into holes drilled to a given depth in the cylinder walls. Thus with five gage rings it is possible to observe the changes of diameter at five different annuli. This study was made of a number of cylinders at various pressures within the elastic limits of the metal, but the calculations and observations for a single case will serve to show the agreement between the observations and Equation [3]. Table 2 gives this comparison for cylinder

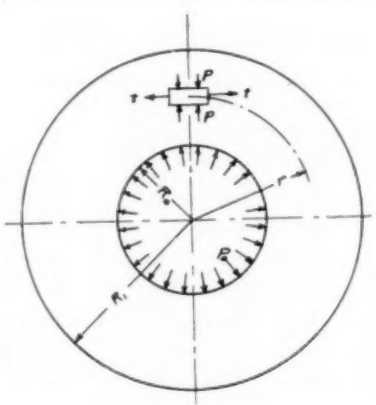


Fig. 5 Stresses in Thick-Walled Cylinders

No. 6 of Table 1 when subjected to an internal pressure of 40,000 lb. per sq. in.

TABLE 2 CALCULATED AND OBSERVED DIAMETRAL STRAINS FOR 6-IN. × 18-IN. CYLINDER UNDER AN INTERNAL PRESSURE OF 40,000 LB. PER SQ. IN.

	$r = R_0$ = 3 in.	$r =$ 3.5 in.	$r =$ 4.5 in.	$r =$ 6 in.	$r =$ 7.5 in.	$r =$ 9 in.
t from Eq. [1].	50,000	38,050	25,000	16,250	12,200	10,000
p from Eq. [2].	40,000	28,050	15,000	6,250	2,200	0
$\frac{t}{E} + \frac{p}{mE}$	0.00211	0.00158	0.00100	0.000611	0.000431	0.000333
Observed strain.	(a)	0.00154	0.00105	0.000610	0.000428	0.000336

(a) No measurements of enlargement were obtainable at the bore.
E is taken as 30,000,000.
$1/m$ is taken as 1/3.

Table 2, which is typical of the results obtained from all the cylinders, shows a maximum deviation of the observations from Lamé's laws of 5 per cent and may be taken as a substantiation of his theories, if any such experimental proof were needed.

There are, however, several conflicting or varying theories as to how these two "true" stresses should be used to determine the limit of elastic resistance. There is tension in a tangential direction with compression exerted simultaneously normal thereto, or in a radial direction. Both produce a shortening of radial elements and a lengthening of tangential dimensions.

Theories of Combined Stresses

Rankine's Theory. Rankine, Lamé, and Clapeyron advanced the maximum-stress theory that, regardless of what the stresses acting in mutually perpendicular planes might be, elastic failure would occur only when one of the stresses reached the elastic

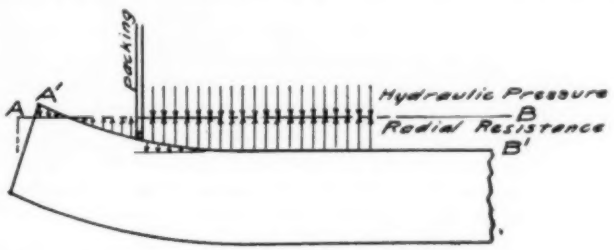


Fig. 6 Deformation at Ends of Test Cylinders Due to Unstressed Portion Beyond Packing Rings

limit of the material. This Rankine theory, so-called, has no following at the present time.

St. Venant's Theory. St. Venant advanced the maximum-strain theory that the material would remain elastic as long as the strain, or unit deformation, in any direction did not exceed the permissible strain as determined by the elastic limit of the material under simple stress. In the cylinders under consideration t is the greater of the two stresses, and the greater strain would be produced in the direction of t, i.e., tangentially. The combined strain thus produced by t and p is given by Equation [3]. According to St. Venant's theory this combined strain must be within the strain shown by a simple tensile test at the elastic limit, and, when that strain has been exceeded, the material is permanently deformed. Then if θ represents the elastic limit of the material in tension, the elastic resistance of these cylinders is reached when at any annulus

$$\frac{t}{E} + \frac{p}{mE} = \frac{\theta}{E}$$

or when

$$t + \frac{p}{m} = \theta \quad \ldots\ldots\ldots\ldots\ldots\ldots [4]$$

Equations [1] and [2] are so written as to give positive values, although the one is tension and the other compression. Equation [4] is written with this in mind. Those who prefer to consider compression as negative in value may change the factor in the parenthesis in [2] to read $\left(1 - \frac{R_1^2}{r^2}\right)$, thus securing a negative value for this stress. This must be accompanied by a change of Equation [4] to read $t - \frac{p}{m} = \theta$.

Guest's Theory. Coulomb, Hartmann, and Guest, and more recently Duguet and Mohr, have held that elastic failure is in the nature of a shear or sliding, and that inelastic deformation will occur under a combination of stresses when the resultant shearing stress becomes equal to the shear produced by a simple stress at the elastic limit of the material. In a simple tension or compression test the maximum shear is in a plane making an angle of 45 deg. with the axis of loading, and its unit value at the elastic limit is

$$S_s = \frac{\theta}{2}$$

By the same analysis, the maximum shearing stress produced by tension and compression at right angles, as in the case under consideration, is

$$S_s = \frac{t + p}{2}$$

Hence, if the maximum-shear theory of Guest is applicable to this case, the elastic limit of the material would be reached when

$$t + p = \theta \quad \ldots\ldots\ldots\ldots\ldots\ldots [5]$$

Some variations of this theory have been proposed. Coulomb attempted to consider the effect of internal friction upon the angle and value of the maximum shearing stress. Mohr's modification takes into account the fact that limiting stresses (elastic limits) in tension and compression are quite different for some materials. When these elastic limits are the same, Mohr's theory becomes the same as Guest's. This may be assumed to be the case with the material of these cylinders with which we are dealing.

Comparison of Test Data With Shear Theories

The experimental work done at Watertown Arsenal upon these eleven cylinders affords an opportunity to check these three principal theories with the data secured. Without reproducing all the observations and data taken, which would require many pages, an explanation of the methods used and the general results will be given.

As a preliminary step, the permissible pressures within the elastic limits of the cylinders were calculated by St. Venant's maximum-strain theory, which is the theory followed in American ordnance practice. These permissible pressures were then applied to the cylinders eight successive times, while readings of the external dilations were taken at pressure increments of 5000 lb. per sq. in. and at similar decrements as the pressures were removed.

Under this program, cylinders Nos. 1, 7, 8, and 10 showed a slight flow or permanent deformation at the pressures to which they were subjected. The other cylinders showed complete recovery, and plans were made to carry them to higher pressures. Cylinders 4 and 5, however, were then annealed for purposes which have no relation to this paper. They were therefore lost as far as any more definite location of their pressure limits is concerned. It is known that they supported elastically the pressures given in column 5 of Table 3.

Cylinders 2, 3, 6, and 9 were carried to higher pressures by increments of 2000 lb. On cylinder 11, unfortunately, there are no readings between pressures of 35,000 and 40,000, between which the elastic limit of pressure was reached. In carrying cylinders 2, 3, 6, 9, and 11 to higher pressures, holes were drilled into the walls and readings taken of the expansions at different annuli, as was explained in connection with Table 2.

Table 3 gives in column 5 the pressures at which the elastic limit of the metal was reached, as nearly as they can be located. Columns 2, 3, and 4 give in comparison the pressures at which the elastic limits would be reached according to the three general theories of combined stresses.

In calculating these, it will be noted from Equations [1] and [2] that tension and compression are greatest at the bore, i.e., when $r = R_0$. By substituting R_0 for r, expressions are obtained for the true tension and compression at the bore. According to Rankine's theory, permanent deformation will impend when t reaches the elastic limit θ of the material. Solving for P_0, according to this theory,

$$P_0 = \theta \frac{R_1^2 - R_0^2}{R_1^2 + R_0^2} \quad \ldots\ldots\ldots\ldots\ldots [6]$$

Under St. Venant's theory the stresses in tension and compressions at the bore must be combined by Equation [4]. Using a value of $^1/_3$ for $1/m$, this gives for the permissible pressure:

$$P_0 = \frac{3}{2} \theta \frac{R_1^2 - R_0^2}{2 R_1^2 + R_0^2} \quad \ldots\ldots\ldots\ldots\ldots [7]$$

By Guest's theory Equations [1] and [2] must be combined by means of Equation [5], which gives

$$P_0 = \theta \frac{R_1^2 - R_0^2}{2 R_1^2} \quad \ldots\ldots\ldots\ldots\ldots [8]$$

In applying these equations, the mean elastic limits of the cylinders were used. Reference to Table 1 reveals that for some of the larger cylinders there was a considerable variation in elastic limits as shown by different specimens. This is especially noticeable in cylinders 7, 8, and 9. There may be some question as to the wisdom of using the mean value instead of the minimum. However a cylinder could not deform permanently at one point without its being evident also at other points in the circumference. The higher elastic limit at one point will certainly exert some restraining influence upon sections of lower elastic limits. This question may be considered in interpreting the results of the tests on those cylinders.

Discussion of Table 3

A casual comparison of column 5 with the calculated values in columns 2, 3, and 4 will convince the reader that the St. Venant theory, as represented by column 3, comes nearest to the actual pressures sustained by the cylinders without inelastic flow. In

TABLE 3 THICK-WALLED STEEL CYLINDERS UNDER INTERNAL PRESSURE; PRESSURES AT ELASTIC LIMIT OF MATERIAL

(For dimensions of cylinders and elastic limits of material, see Table 1)

(1)	(2)	(3)	(4)	(5)	(6)
Cylinder no.	Pressure to reach elastic limit according to Rankine	St. Venant	Guest	Observed pressure at elastic limit	Remarks
1	28,850	25,570	20,830	25,000 −	Slight flow at 25,000 lb.
2	45,000	37,500	28,130	36,000 +	No flow at 36,000 lb.
				38,000 −	Slight flow at 38,000 lb.
3	60,000	47,370	33,330	50,000 +	First flow at 52,000 lb.
				52,000 −	
4	28,850	25,570	20,830	25,000 +	No flow at 25,000 lb.
5	45,000	37,500	28,130	37,000 +	No flow at 37,000 lb.
6	52,000	41,050	28,890	46,000 +	First flow at 48,000 lb.
				48,000 −	
7	29,660	26,280	21,420	24,000 −	Slight flow at 24,000 lb.
8	44,250	36,880	27,660	36,500 −	Slight flow at 36,500 lb.
9	56,600	44,690	33,670	43,200 −	Slight flow at 43,200 lb.
10	27,350	24,240	19,750	23,500 −	Slight flow at 23,500 lb.
11	38,820	32,350	29,260	35,000 +	No flow at 35,000 lb.
				40,000 −	

five of the cylinders (Nos. 1, 7, 8, 9, and 10) there was evidence of a slight inelastic flow at pressures somewhat below those indicated by St. Venant's theory. In the cases of cylinders 7, 8, 9, and 10 it will be noted from Table 1 that the physical tests showed a lack of uniformity in the metal. Since mean values of elastic limits have been used, it is possible that soft streaks as shown by the physical tests may account for the flow of these cylinders at lower pressures. Not only these cylinders but some of the others showed decided departures from circular sections when stressed considerably beyond their elastic limits. In cylinders 4 and 5 the elastic pressure limit may be above that indicated by St. Venant's theory but cannot be much, if any, below. These two cylinders were not tested to higher pressures than those given in column 5. Cylinder 2 is in agreement as closely as can be determined. Cylinders 3, 6, and 11 supported higher pressures than would be indicated by St. Venant's theory.

Altogether five cylinders failed to support without permanent flow the pressures indicated by St. Venant's theory; three supported higher pressures; and three were uncertain, but with the probabilities that two were higher and one lower. Thus the probabilities are six below and five above St. Venant's theory.

As another method of comparison, the sum total of the pressure limits for the eleven cylinders according to Rankine's theory is 456,380. For St. Venant's theory it is 379,000. For Guest's it is 291,900. In attempting to sum up column 5, the author has deducted 1000 from the values given for cylinders 1, 7, 8, 9, and 10. He has similarly added 1000 to the values for No. 4 and 5 and has used the means of the two limiting pressures given for Nos. 3, 6, and 11. This gives a total for column 5 of 373,700. This differs from the total for the St. Venant theory by only 1.4 per cent.

Summary

To the author this evidence seems to warrant the conclusion that the maximum-strain theory of St. Venant is correct for cases of tension and compression in mutually perpendicular planes where the tensile stress is the greater of the two. Certainly the evidence shows the theories of Rankine and Guest to be beyond possible consideration for stress combinations of this type.

Other studies indicate that the shear theory, as developed by Guest and Mohr, may be the most reliable for certain cases, namely, those where fracture can be definitely shown to be due to shear or sliding. In this connection, Fig. 7 will be of interest in showing the type of fracture in one of these cylinders which was carried to incipient rupture. These evidences of fracture show no relation whatever to shear. They rather show actual separation by a tangential movement or strain, the planes of fracture being generally normal to the direction of the tangential stresses. These fractures give supporting evidence to the probability of the correctness of the St. Venant theory in this case.

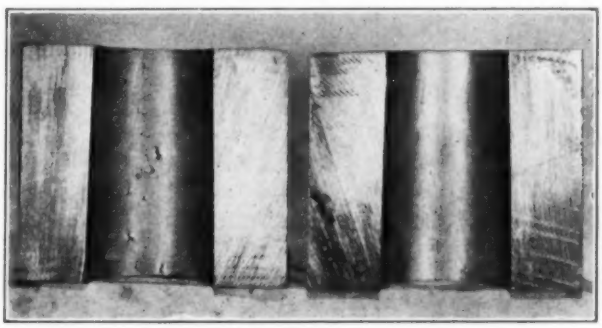


Fig. 7 Type of Fracture in One of the Test Cylinders

APM-51-7

Short Flanged-Tube Cantilevers Under Concentrated Radial Load

By WENDELL P. ROOP,[1] WASHINGTON, D. C.

The author gives results of tests of a short thin-walled tube fixed at one end and reinforced at the free end by a flange, which was loaded radially at the free end. The elastic deflections and ultimate strength were observed, and the author devises formulas for their expression. The strength and stiffness of an elastic ring are discussed mathematically in an appendix.

THE usual formulas for strength and stiffness of a cantilever become inapplicable when the length of the cantilever is not great compared with its depth, and when the external load is not distributed over the end section in accordance with the assumptions of simple beam theory.

Tests have been made on a simple type of cantilever structure consisting of two elements: namely, a thin-walled tube of a length eight-tenths its diameter, reinforced at its free end by a flange. This structure was loaded radially by means of a hook, and elastic deflections and ultimate strength were observed.

As a first approximation these two elements may be considered separately. The flange (or ring) may be supposed to be supported only at the ends of a diameter transverse to the load. Theoretical analysis of the action of a ring so loaded is given in an appendix. The tube may be regarded as a simple cantilever whose load is applied to its free end by the ring, supposed to be stiff enough to realize the distribution assumed by the simple theory. The first approximations thus obtained have been used to obtain nominal values with which to compare the experimental data.

Ring Stiffness. It is found that the ring and tube are not independent of each other. The ring receives support from the tube which is relatively greater as the ring is smaller. The change in transverse diameter under load P is

$$\delta = \frac{P}{ER} \frac{1}{[17\ (I/R^4) + 0.02\ (t/R)]}$$

R being the radius, assumed equal in tube and flange, I the sectional moment of inertia of the ring, and t the thickness of the tube wall.

Strength of the Tube. On the other hand, the ultimate strength of the tube is strongly affected by the stiffness of the ring, being

$$Q = 14{,}500 \left[1 - \frac{21}{[(P/10^3\delta) + 5]^2}\right] \text{lb. per sq. in.}$$

Q is the average shear in the tube section, obtained by dividing the load by the sectional area. The nominal shear, approximating the actual maximum shear stress, is twice as great.

These two formulas summarize the quantitative results of the tests. They evaluate the stiffness of the ring and the strength of the tube.

Strength of the Ring. As to the strength of the ring, no complete failure was obtainable, but departures from proportionate deflections began at loads that were small compared with the strength of the tube. The strength of the ring (in terms of nominal value of maximum stress) is of no special significance, as overstressing locally leads simply to local yield and consequent increase of area within which stresses moderately exceed the elastic limit.

Upon removal of a load thus locally exceeding elastic limits, stresses in the region overstressed are reversed, so that when reapplication of loads begins a complex pattern of initial stress already exists. The characteristic behavior is this: Up to loads not exceeding the previous maximum the structure gives deflections proportional to loads (aside from moderate hysteresis), and repeats this performance with little or no cumulative deflection, provided the maximum has not exceeded a certain value. If the load is increased beyond the previous maximum, the course of the first deflection curve is resumed and a new maximum established. The systematic determination of the critical loads, beyond which repeated application might lead to cumulative deflection, has not been undertaken, but it is clear that the ring is able to resist without cumulative deflections repeated loads leading to nominal bending stresses considerably exceeding elastic limits as determined in a tensile test specimen.

Cantilever Deflections. The cantilever deflections of the tube are found to vary considerably and unsystematically. Whether this is due to errors of observation or to irregularities in the action of the structure is not clearly established. An approximate or nominal deflection is found by adding the nominal bending deflection to *twice* the deflection in shear, obtained by taking the shear stress equal to its average value, or total load divided by sectional area of tube. The factor 2 may be roughly justified by the fact that in a tubular section in combined bending and shear the maximum shear stress is twice the average. Observed cantilever deflections are referred to such nominal values simply for purposes of comparison. It is found that observed values exceed the nominal values by as much as fourfold, though in many cases by less than this, and in one case the observed value is even less than the nominal.

There appears to be no reason why the results of these tests with models should not be applicable to large structures approximating the form and proportions of the models, similarly loaded, except in so far as continuity of structure is broken by riveted joints.

As to structures departing from the proportions of the models, it is naturally not possible to speak with like assurance. The governing action appears to be the stiffening effect of the ring in preventing wrinkling. At the ratio of length to depth adopted in the models, viz., $^8/_{10}$, the support received by the tube from the ring depends on the stiffness of the ring; if this is sufficient the tube develops a resistance to maximum shear approximating 30,000 lb. per sq. in. It is unlikely that this figure could be increased by a closer spacing of ring stiffeners.

Thus in a shorter cantilever the shear would be unchanged, and unless the shorter length of tube between ring stiffeners did have an influence, we should expect the equation as given for ultimate strength of tube to remain valid. On the other hand, as the lengths considered become greater, a proportion is undoubtedly soon reached at which bending is the predominant action, and buckling leads immediately to collapse. It is clear that longer cantilevers are unable to make effective resistance to bending beyond the buckling point, though that is exactly what is observed in shorter ones. Thinking always of the mode of ultimate failure, we see that there is presumably a critical length beyond which tubular cantilevers of a given diameter will begin to fail in bending. It is impossible to infer

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from the data what that critical length may be or how it would depend on the diameter. The choice between failure in shear and failure in bending is not simply a matter of comparing the nominal shear stresses and compressive stresses with the values at which they would cause failure in flat plates of similar proportions of length and thickness. More extensive tests are required on the point, but it is certain that within the rather wide

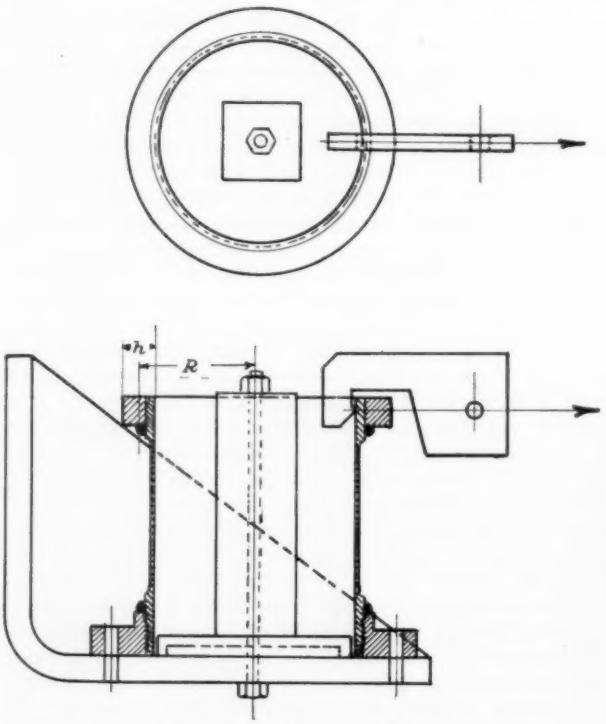


FIG. 1 LAYOUT AND LOADING OF FLANGED TUBE

range of wall thicknesses and ring strengths tested, tubular cantilevers of a length eight-tenths the diameter fail by wrinkling in the region of maximum shear. The shear intensity at which failure occurs depends on the ring stiffness as shown in the formula. Until better information is available, this formula may be used up to the limits, wherever they are, within which failure occurs in shear.

DETAILS OF THE TESTS

The chief results will now be taken up and justified in detail by reference to the data and the methods used.

In all, twelve models of varying ring stiffness and wall thickness have been tested. The deflections directly observed were simply those of four radii of the ring, taken from a post supported at the axis of the tube by rigid connections with the base, as indicated in Fig. 1. Two of these observed radii were transverse to the load; the sum of their increments gives the diametral deflection. That these two radii were not always exactly equal shows that the post did not function perfectly as a point of reference. But this does not affect the observations of diametral deflection.

These radial deflections were observed with calibrated "Last Word" dials, and the results plotted as a function of load.

The Diametral Deflection showed quite consistent values, with relatively little hysteresis or other departure from linear variation with load, up to certain elastic limits. In spite of the relative certainty with which each value of ring stiffness was determined, the spots in Fig. 3 are rather scattered, and obscure any moderate variation with h/R which might exist. It would be expected that ring stiffness would fall somewhat under the values calculated for the case of pure bending on account of the additional deflection in shear; this would be relatively greater at greater values of h/R.

The theoretical value of ring stiffness, or load per unit diametral deflection, is

$$\frac{P}{\delta} = 19.7\, E\, \frac{I}{R^3}$$

if the ring is thin in proportion to its radius, and if the load and supporting forces are concentrated at points. As soon as we have to deal with a ring attached to a tube, there is uncertainty as to where ring stops and tube begins, reflected in uncertainty as to the value of I. It is here assumed that the tube extends through the ring. In calculating I, no material enclosed between the two cylinders bounding the tube was included. The support afforded to the ring by the tube is accounted for by the term $0.02\, Et$, which is dimensionally correct but empirically determined. In case of a jointed connection between ring and tube the joint would form the boundary.

The addition of the term in tube-wall thickness gives us calcu-

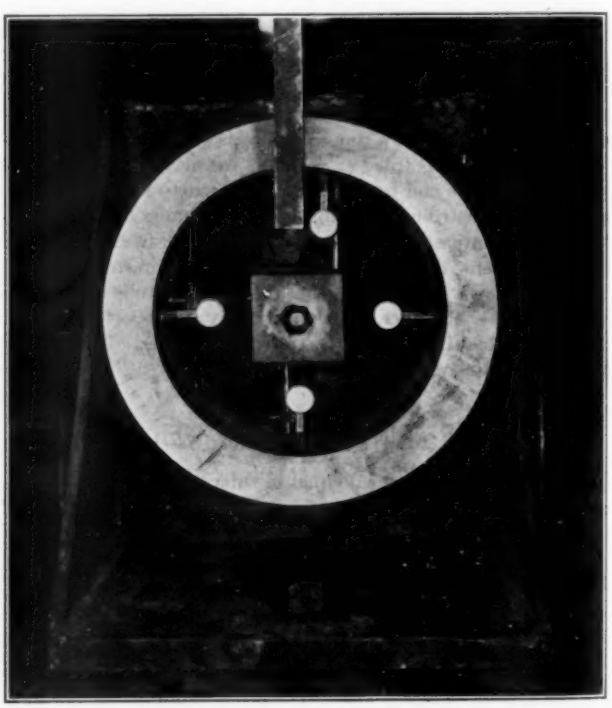


FIG. 2 TOP VIEW OF LAYOUT

lated values of ring stiffness which agree fairly well with those observed. The formula which represents the observed data best is

$$\frac{P}{\delta} = E\left[17\,\frac{I}{R^3} + 0.02\, t\right]$$

In Fig. 3 is plotted the ratio of observed ring stiffness to that found from this formula.

In order to make this formula applicable to similar structures of different size, it may be written

$$\frac{P}{\delta} = ER\left[17\,\frac{I}{R^4} + 0.02\,\frac{t}{R}\right]$$

Since the quantity in brackets is of zero dimensions, no confusion in units can arise. The above comment is all directed to the diametral deflection taken transverse to the load. The relative precision with which these deflections were observed extends only to certain limits of load, beyond which deflections exceed values proportionate to the load. In models 9 to 12 closely spaced load increments made it possible to follow the deflections rather closely as they varied with load, and it was found that diametral deflections are affected to a certain extent by whatever affects radial deflections parallel to the load, especially with respect to the proportionality limits. These limits, in the case of diametral deflections, need not indicate that bending stresses at the side points have become excessive, but may only show that deflections have occurred elsewhere in the ring or in the tube which have altered the stress distribution. This matter will be discussed further in connection with ring strength and cantilever deflections.

The rather close agreement of the empirical formula with that obtained analytically for the case of pure bending under concentrated external forces, indicates that the conditions under which the ring worked in the tests did not depart greatly from those assumed in the mathematical study. This agreement may be accepted as also affording partial confirmation of the theoretical results for load divided between front and rear points, although such divided loads were not applied to the steel models. It is

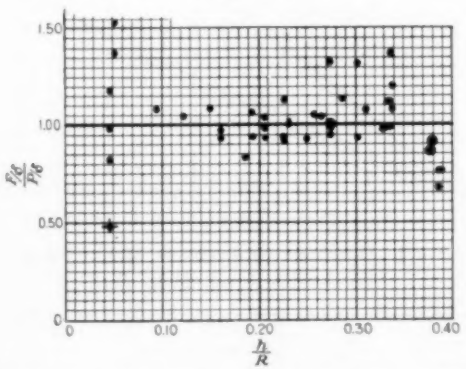


FIG. 3 RING STIFFNESS. LOAD PER UNIT TRANSVERSE DIAMETRAL DEFLECTION

$$\text{Ring stiffness } \frac{P}{\delta} = E\left[17\,\frac{I}{R^3} + 0.02l\right]$$

$\frac{F/\delta}{P/\delta}$ = ratio of observed to calculated ring stiffness

h/R = ratio of width of ring to radius

worth noting that the stiffness of the ring under divided load is much greater than under undivided load.

Ultimate Tube Strength. Each model was ultimately loaded to the maximum it would carry. The normal ultimate failure was caused by wrinkling in the sides of the tube wall and so gave a value of ultimate tube strength. In various cases high values of load were reached which fell short of the ultimate wrinkling load. All the data are plotted in Fig. 4. Large and permanent deflections of the ring occurred in all cases before the wrinkling load was reached. On the other hand, the failure in wrinkling was essentially of unstable mode, and up to the moment at which wrinkling occurred no distress in the tube was apparent. The wrinkling loads were very definite. The ordinate in Fig. 4 is the wrinkling load divided by the sectional area of the tube; to obtain the nominal maximum value of shear stress in the tube section, this should be doubled. In a purely empirical way it is noted that this shear intensity at the wrinkling load depends on ring stiffness. The evidence as to the functional relation is contained in Fig. 4. The values of ring stiffness, in load per unit diametral deflection, are observed values; calculated values might have been used on models 1 to 8, but in models 9 to 12 the rings were so slender that the wrinkling load could hardly be approached unless the ring were somehow supported. The support used consisted of a transverse brace like the stud in a link of anchor chain. This reduced the diametral deflections to about one-third or less what they would have been without

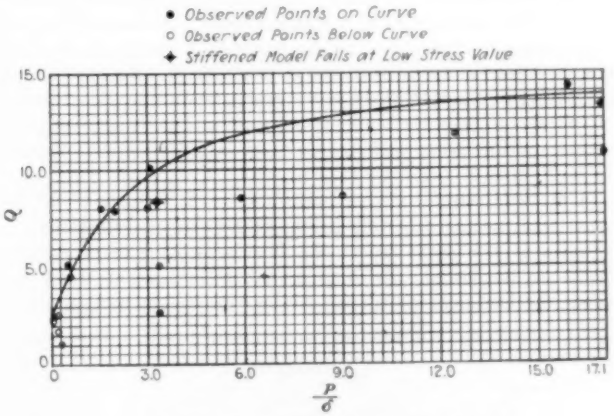


FIG. 4 ULTIMATE STRENGTH OF TUBE

$$Q = 14{,}500\left(1 - \frac{21}{[(P/\delta) + 5]^3}\right)$$

P/δ = ring stiffness, load per unit diametral deflection in 100,000-lb. units

Q = wrinkling load ÷ tube-section area in 1000-lb. units

it; this mode of stiffening the ring operates to increase the effectiveness of the resistance of the tube to shear just as well as a heavier ring section. As an example of this take model 10. With the ring bare it could not be loaded beyond an average shear stress of 2600 lb. per sq. in. without excessive ring deflections, and, according to Fig. 4, near approach to wrinkling. When a heavy cast-iron ring was slipped over the flange ring and wedged tight, the average shear stress was run up to 10,700 lb. per sq. in. with no distress anywhere; the diametral deflection was substantially zero. With a light transverse brace, diametral deflections were about one-sixth those of the bare ring and the tube wrinkled at 8000 lb. per sq. in.

In Fig. 4 ultimate strengths are plotted on ring stiffness. Since in a general way those models with the heaviest rings also had the thickest walls, it might be supposed that the variation with ring stiffness is spurious, and that it is the wall thickness that really determines the ultimate value of shear stress. The case of model 10 already cited, and other similar cases, offer convincing proof that the ring stiffness is actually the controlling factor.

Ring Strength. Rupture of the ring was in no case obtainable, but departures from proportionality of deflections began at very small load. Radial deflections in line with the load were subject to error due to motion of the datum post, but again this did not affect diametral deflections. The diametral deflection parallel to the load was compared with that transverse to the load. The calculated values of the first is about 10 per cent greater than that of the second, and this figure is verified on the average, but shows larger values in thick rings, probably due to deflections in shear, which would be especially great in way of the point of application of the load. It is at this point that the ring is most heavily stressed and that the strength limit will first be manifest.

The load at the strength limit of the ring cannot be as accurately defined as the two quantities already dealt with. Long before there is any question of rupture, we run into ring distortions which completely alter the form and hence the stress

distribution in both ring and tube. This is true even of the heaviest ring sections used in the tests, so that rupture of the ring was not obtainable except in one case in which small cracks opened in a weld, and set a limit to the load that could be carried. (See Fig. 5.) However, the rings tested were all of solid rectangular section; doubtless sections stiffened by coring would act differently.

The conditions of loading of the ring are believed not to depart greatly from those of a straight beam, so far as failure at the section of maximum bending moment is concerned, so that study of rings of special sections is unnecessary. Sections which would fail in a stable mode in a straight beam may be expected to fail also in a stable mode in the ring, and with rings of such sections ultimate strength has no exact significance with respect to structures of the type studied.

A strength limit based on appearance of permanent deflections also meets with difficulties, as permanent deflections, especially at the point where the load acts, appear practically from the beginning. Data on radial deflections at the point where the load acts are subject to error due to the motion of the

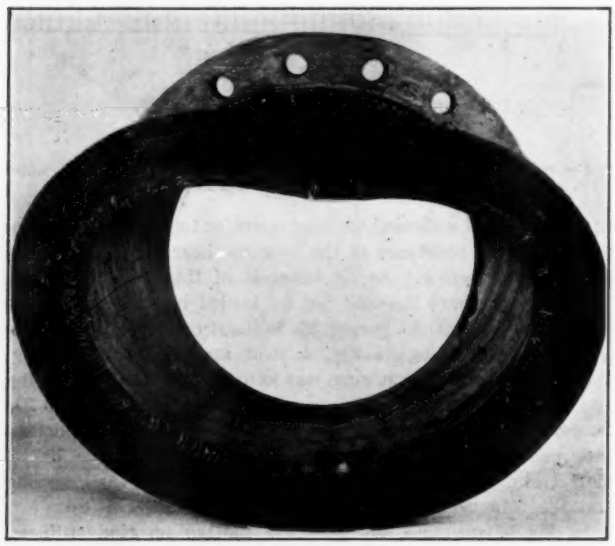


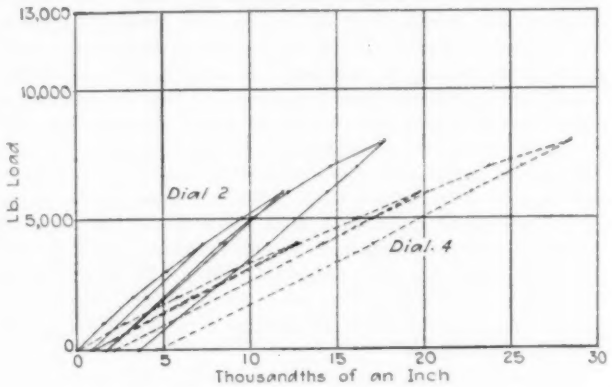
FIG. 5 RUPTURE OF RING DUE TO SMALL CRACKS OPENING IN WELD

datum post. No measurements of a single radius have been used in reducing data for quantitative results. In a qualitative way, however, radial deflections can tell us something about what is happening, especially as the deflections at the load point are large compared with the probable motion of the datum post.

The general nature of the behavior of all models may be illustrated by No. 6; the data for this model were especially clear and consistent, yet the features indicated were typical of all models. Deflections on the initial loading are shown in Fig. 6. The transverse diametral deflections are consistent and proportional throughout; the parallel diametral deflections are almost equally so, but show a value almost twice the transverse one instead of 10 per cent greater as calculated.

The radial deflections at the point of load and opposite (marked stations 4 and 2) are plotted separately. They show that both points moved in the direction of the load at rates that increased with increasing load. Removal and reapplication of the load gives the characteristic behavior already described.

The flange in this first run was 2.42 in. wide by 0.95 in. thick, and the wall thickness 0.05 in. After the first run the flange was reduced in width. The removal of this material had the effect of partially restoring the original unstressed condition, so that subsequent first runs after machining showed similar



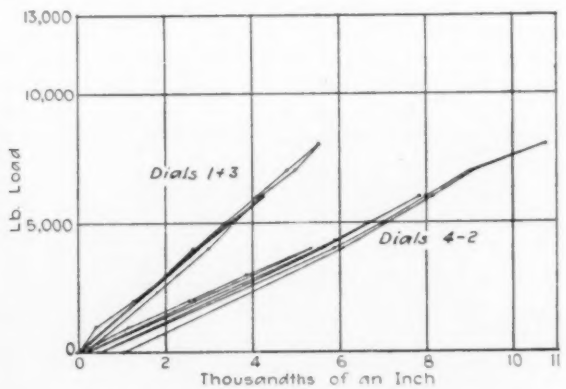
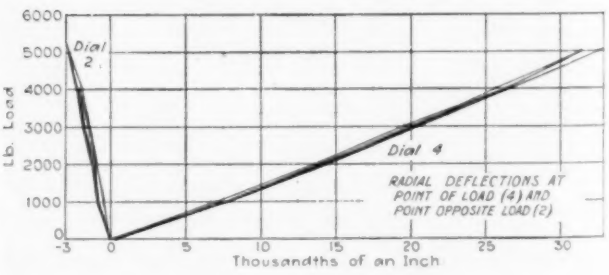


FIG. 6 OBSERVED DEFLECTIONS, MODEL NO. 6
(Initial loading; flange width, 2.42 in.)



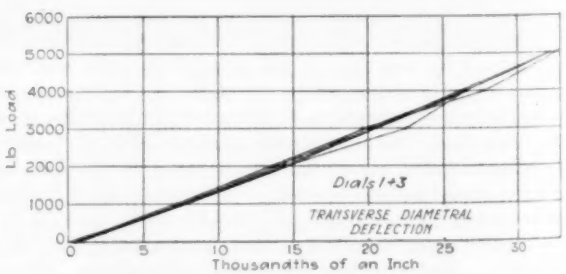


FIG. 7 OBSERVED DEFLECTIONS, MODEL NO. 6
(Flange width, 0.90 in.; deflections within proportional limits on enlarged scale)

effects, but with departures from a straight line not quite so pronounced. A test in compression instead of tension showed loops that were much wider, and even the transverse diametral deflec-

tions were to some extent affected. Test with a wide hook showed rather irregular results.

After these special loadings, the model was retested in tension as in the initial run, but with flange width of 0.90 in. Efforts were made to improve the fixity of the datum post, but without affecting it much. Its motion is believed not to have amounted to more than about 10 per cent of the radial deflections at sta-

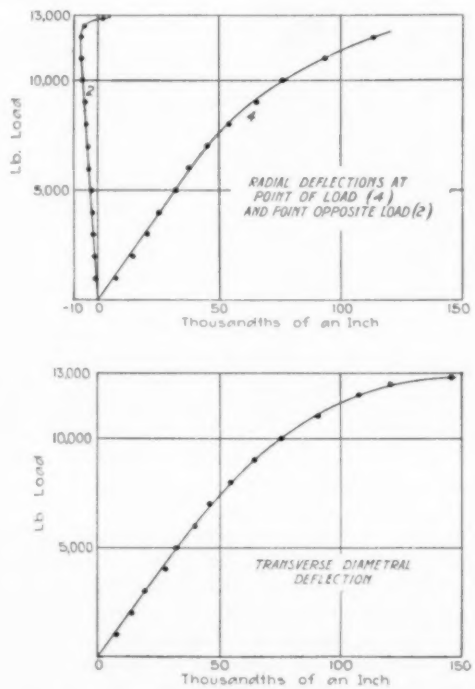


FIG. 8 OBSERVED DEFLECTIONS, MODEL NO. 6
(Failed at 13,930 lb.; flange width, 0.90 in.)

tion 4. Up to the previous maximum all deflections were consistent and linear, as shown in Fig. 7. Ring stiffness now being greatly reduced as compared with that in initial test, the motion at station 2 is opposite to the load in direction. The point of yield of the cantilever is clearly marked in Fig. 8 by the reversal of the deflection at station 2 (see below). But no such clear indication of ring-strength limit is to be found.

Up to the limit to which the load was carried in removal and reapplication, it was found that after the initial loading the deflections were substantially linear. Doubtless there is a critical load beyond which each successive reapplication of the same load would result in increased deflection. Such a point, however, has not been found, and for the immediate purpose it is unnecessary to find it. If ring stiffness is in itself of no great significance, the ring can be depended on to withstand any load that the tube will withstand, and the limiting action would be that of local stress in way of the load. In the commoner case in which ring stiffness is the controlling feature, the formula for ring stiffness given above will serve as a guide.

The only case not covered by these observations is that of a lightened ring section of great stiffness and small strength, which, by failure through instability, might establish a definite limit to ring strength. Such a section would be handled in design as in a straight beam, and the maximum bending moment is calculated to be

$$M = \frac{FR}{4}$$

which is just what it would be in a straight beam of length $2R$ with fixed ends.

In order to obtain further confirmation of the ring formula, and to obtain data on the stress distribution in the ring, especially under divided load, a photoelastic study was made of two rings 4 in. in diameter; they were supported at the two ends of the horizontal diameter, and loaded with equal weights at top and bottom. Under these circumstances analysis shows the bending moments to be zero at the points of support, and this is verified experimentally; the maximum bending moments and stresses appear to occur at the two load points, but the maximum is rather sharp, and the highly stressed area adjoining is not extensive. Under these circumstances it is not surprising to find that observed maximum moments and stresses are only about half the calculated values. A feature not studied analytically is the distribution of moment and stress around the ring. An extended region around the quarter-points is seen to be rather highly stressed, though the maxima at the load points are definitely higher. The isochromatics, lines of stress, and curves of tangential stress are shown in Figs. 10, 11, and 12. They serve to emphasize the difference between the loading used and loading in ordinary tension at opposite points, as in a chain. The latter case is much simpler and has been studied repeatedly. In one feature, however, the two cases are similar, viz., both show actual strength limits which are higher than stresses calculated from bending loads would indicate. A rather broad analogy with the action of a gun which has been subjected to auto-frettage exists.



FIG. 9 AXIAL SECTION OF MODEL NO. 6

The photoelastic study was made by the Heat and Light Division of the Naval Research Laboratory, Belleview, D. C.

In connection with ring strength we may finally note again the interdependence of deflections in different parts of the ring as soon as elastic limits are exceeded locally. Radial deflections at the load point and point opposite show a marked tendency to pass proportional limits at the same load, although the calculated stress at the first point is about four times that at the second. The photoelastic study, although it dealt with a loading different from that used in the steel models, nevertheless indicated in general that the stress in way of point of load reaches a rather sharp maximum, but also has high intensity over extended regions elsewhere. The departure of deflections from proportionality does not become appreciable until local stresses about the point of maximum stress have exceeded the elastic limits of the material over a moderately extended region. But when the stresses about the quarter-point reach excessive values, the non-elastic yield tends to become general throughout the structure, because the maxima there are flatter. Undoubtedly the tube enters into this matter also, so that the photoelastic results have only a general bearing on this point. It is mentioned in detail in order to give an example of the inapplicability of analysis based on elastic action in a complex structure to questions of strength involving stresses exceeding elastic limits.

Cantilever Stiffness is indicated by the data obtained with less certainty than any other quantity investigated. Cantilever

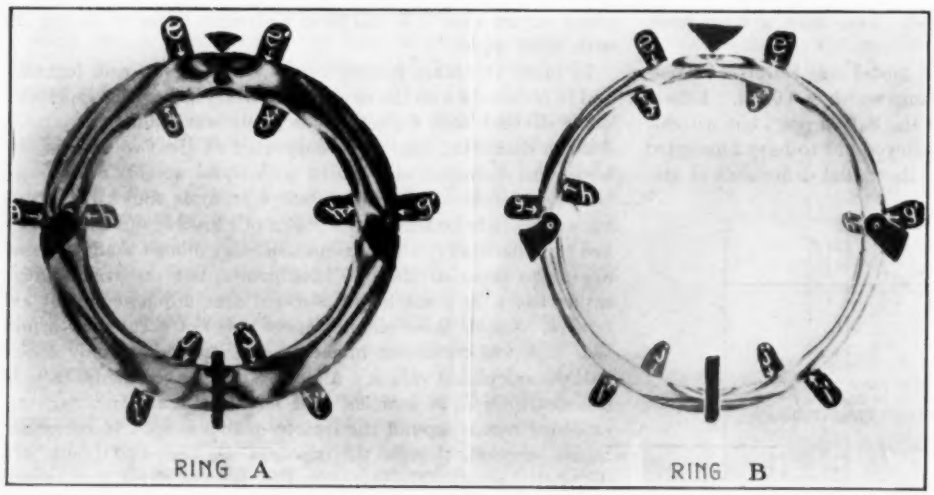


FIG. 10 ISOCHROMATICS UNDER DIVIDED LOAD

is about 15 per cent. The motion of the datum post is more difficult to estimate, but changes were made in the hope of reducing this motion. The first of these changes effected a great improvement; it consisted in inserting a plate under the post as shown in Fig. 1, recessed so as to bear only around the circumference. This eliminated warping of the back plate as a source of error. Later all connection with the back plate was dispensed with and the post supported by a diaphragm plate welded to the inside of the base of the tube. This eliminated the threaded connection of tube to flange and the bolted connection of flange to back plate, but no improvement in data appeared to result. The motion of the post is believed to be small because after these changes the connection of the post with the base of the tube is as direct as it is possible to make it, because the stress on the post and diaphragm on which it rests is practically nil, and because the transverse radial deflections were observed to be very nearly equal.

Nevertheless, the cantilever deflections obtained were very erratic, and no generalizations about them are permissible. It is worth noting, however, that they are not inconsistent with the assumption of a rather heavy shear concentration, such as would cause deflections greater than nominal without altering the general nature of the action of the structure as failure is approached. Such a shear concentration may be supposed to depend on wall thickness and on accidental non-uniformities

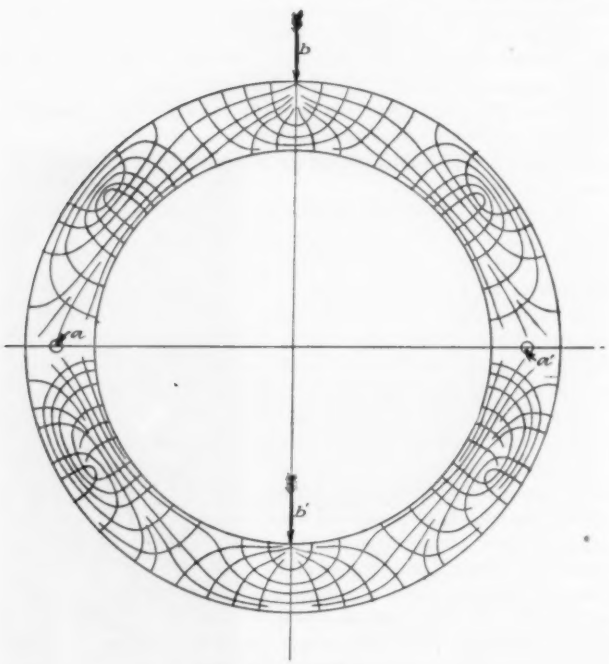


FIG. 11 STRESS LINES
(Ring loaded at b and b'; supported at a and a'.)

deflections were not observed directly at the ends of the transverse diameter, but were inferred from the radial deflections at station 2, the point opposite to the load. Under tensile load this point, of course, moves outward with respect to the center of the ring. Since the center of the ring moves in the direction of the load, station 2 may move either with or against the load, depending on whether the cantilever deflection is greater or less than the ring deflection.

The ring deflection at station 2 is calculated to be 0.384 times the transverse diametral deflection. This ratio is applied to the observed diametral deflection, and the result combined with the observed radial deflection at station 2. Errors in the cantilever deflection so found may arise in motion of the datum post, and in departure of the actual ratio from its calculated value. The error in the ratio is probably less than the departure of actual from calculated transverse diametral deflections, which

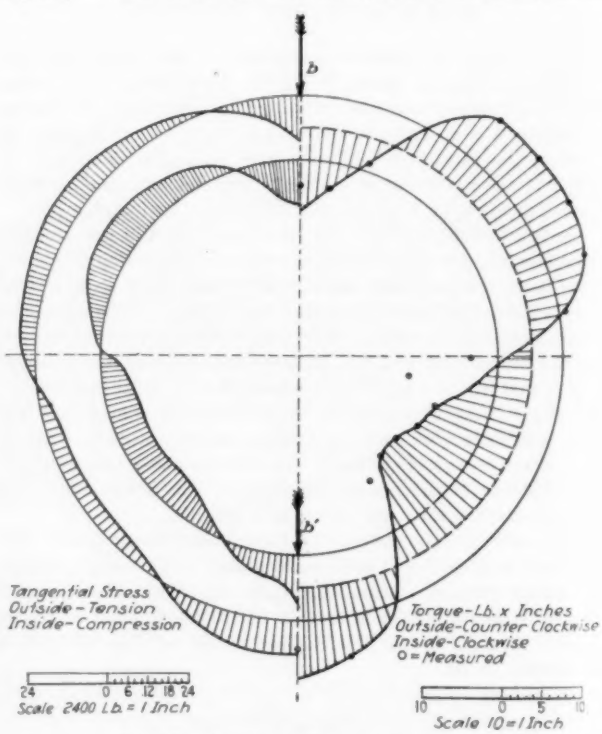


FIG. 12 CURVES OF TANGENTIAL STRESS

of various sorts such as would determine the pattern in which wrinkling will occur on failure. A feature of importance is the buckling at the point of maximum compressive stress in bending. The formula given above for ultimate strength of the cantilever takes no account of the bending load; the justification for this lies in the facts about buckling. Failure of the cantilever was due to wrinkling in the sides and was unstable



FIG. 13 MODEL NO. 6, SIDE VIEW

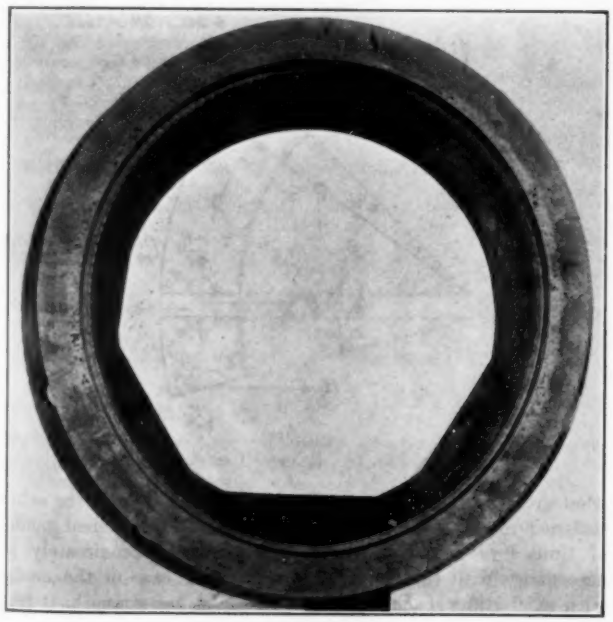


FIG. 14 MODEL NO. 6, END VIEW

in mode. Ultimate loads actually carried were many times those which consideration only of the compressive stresses at the buckling point would indicate to be possible. The slenderness ratio of an axial section of one side of the tube is very high, as may be appreciated by looking at Fig. 9. The curvature of the tube wall undoubtedly assists it to withstand buckling, but the loads at which buckling actually became noticeable were still less than the ultimate wrinkling load. In addition, buckling was not marked by any distinct discontinuity in the cantilever stiffness. Taking all these facts into consideration, it is clear that at loads approaching the limit of cantilever strength the action in buckling is of little or no direct influence.

Indirectly, buckling probably does have some effect on ultimate strength by initiating departure from the cylindrical form, and furnishing a starting point for the wrinkles. The tube falls into a series of folds which spread from the center where buckling occurs. When cantilever stiffness is plotted as a function of load, a roughly periodic variation occurs which suggests that the stiffness drops away when a new fold begins to form, and recovers as the stress is redistributed in consequence, until the wrinkling point is reached. Effects of this sort are probably quite sufficient to explain the erratic character of the data on cantilever stiffness.

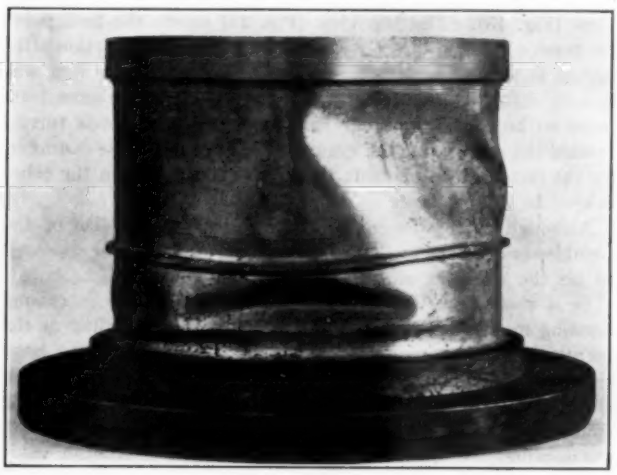


FIG. 15 MODEL NO. 10, SIDE VIEW

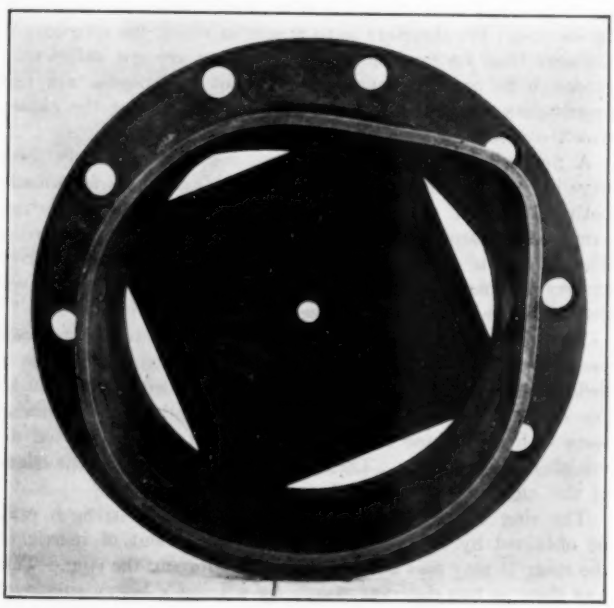


FIG. 16 MODEL NO. 10, END VIEW

The tests show, however, that these effects are unrelated in a large way to the ultimate loads, which depend consistently on values of average shear in the tube section, regardless of the buckling loads.

Figs. 13 to 16 have been chosen from the photographs which were taken of all models after failure, to illustrate typical patterns. In each of these cases the deflections have been carried considerably beyond the point at which the load passed its maximum

value, in order that the nature of the folds might be made more apparent. This partly obscures the facts as they develop under increasing load, which are as follows: The first sign of distress in the tube is a slight bulge at the base, in way of point of loading if in tension. This does not disturb the deflection curves in any unmistakable way, and the buckling does not immediately increase much. As the load goes on increasing, however, yield begins to be apparent in the deflections, the buckling increases strongly, and shortly wrinkles appear suddenly, after which the maximum load will not again be accepted.

In No. 6 the buckle appears at the center of a symmetrical series of wrinkles, turned about 30 deg. to the left in the side view (Fig. 13). The top view (Fig. 14) shows the load point (in tension) turned directly downward; the ring distortion after failure is seen to be small. In No. 10 (Fig. 15) the ring was greatly distorted by the ultimate load, and would have been more so but for the transverse brace. The buckle is turned toward the reader, but the symmetry of the wrinkles is disturbed by the fact that on one side they come above and on the other below the ring stiffener.

Although there was considerable variety in the size of the wrinkles in different models, the essentials of the action appeared to be the same in all.

It is clear that the tube continues to resist the increasing bending moment after buckling has occurred. Buckling at the point of greatest nominal compression must somehow have the effect of shifting the compressive load to material better adapted to carry it; it cannot, of course, eliminate bending effects, which are inherent in the manner of loading. In fact, the compressive stresses must have a greater maximum value after buckling has shifted the load to fibers nearer the neutral axis. Nevertheless the fact is that the structure withstands loads up to several times that at which buckling becomes apparent to the eye. We therefore have a case in which the structure is stronger than its weakest part. The data are not sufficiently extensive to permit generalization about such cases, nor has opportunity occurred for a detailed investigation of the causes which control the form of the wrinkle pattern.

A number of incidental variations were tried. Early tests were in compression, later ones in tension. Six tests were made both in tension and compression. The diametral deflection averaged the same in both within limits of observational error. The cantilever deflection appeared to be about one-quarter greater in tension, but in view of the variability of cantilever deflections, no great significance is attached to this result.

In three tests a wide hook was used, which spread the load over a sector of about 60 deg. This did not alter the cantilever deflections, but diminished the diametral deflections about 15 per cent on the average. In the photoelastic studies similar tests with distributed load were made, and those indicated diminished stress values. The maximum stress came at the edges of the surface through which the load was applied.

The ring stiffness necessary to develop tube strength may be obtained by increasing the sectional moment of inertia of the ring; it may also be obtained by reinforcing the ring. This was done in two different ways: by placing a heavy ring over the ring flange and wedging the two into intimate contact; and by inserting a cross-piece transverse to the load, like the stud in an anchor-chain link.

All the models were turned from seamless drawn tubing. The heavier ring flanges were welded on, but otherwise the models were all of unbroken metal, with one exception. In all but three the tubes were simple cylinders; in those, three types of tube reinforcement were introduced. In one, the tube-wall thickness was increased for two-fifths the height above the base. In one, a circular stiffener was placed at two-fifths height. In one, axial stiffeners were riveted to the uniform tube wall, and this was the only case in which the continuity of the tube wall was interrupted by rivet holes or joints.

With respect to cantilever strength, the increased thickness at the base and the circular stiffener gave results not different from those with the unstiffened tubes; the riveted model, however, showed diminished rather than increased strength. Cantilever stiffness, though more variable, was on the whole somewhat greater in the case of the axial stiffeners than in the other stiffened models, but greatest of all in the unstiffened model of similar proportions. Ring stiffness was approximately in accordance with the formula, except in the case of the model with axial stiffeners, in which, as in cantilever strength, it had a low value rather than a high one.

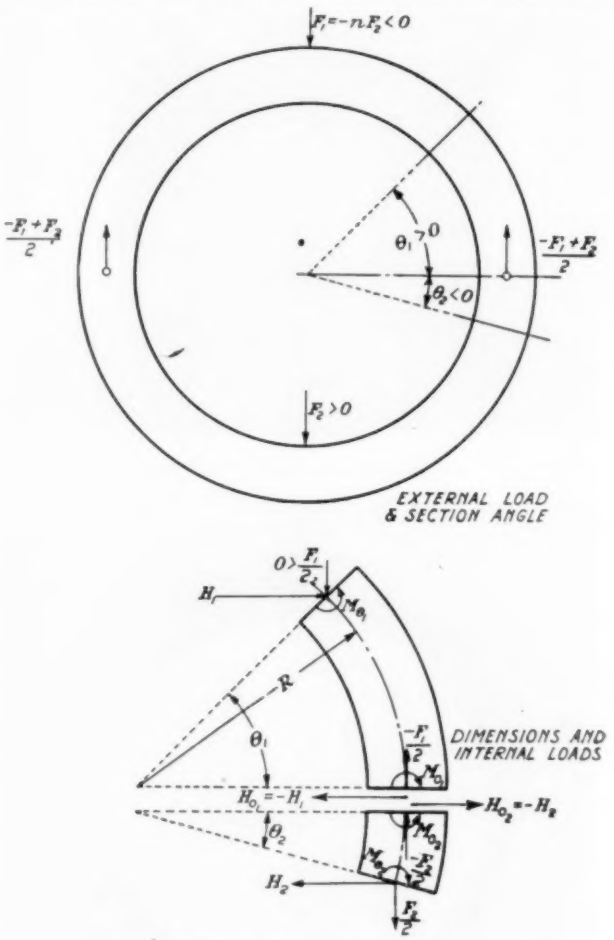


FIG. 17 ELASTIC RING

ACKNOWLEDGMENT

The models described were made at the U. S. Experimental Model Basin, and tested with the assistance of Dr. A. H. Stang at the Bureau of Standards.

Appendix

STRENGTH AND STIFFNESS OF AN ELASTIC RING

THE procedure will be to calculate bending moment M and deflection δ in a ring (Fig. 17) of sectional moment of inertia I loaded with a force F_1 at the front point, and $F_2 = -F_1/n$, at the rear point, so that the total load is $F = F_2 - F_1 = F_2 (1 + n)$, the support being equally divided between the two side points. The load

F_1 is considered negative because it acts radially inward and F_2 positive because outward. The calculations will be made on the curved-beam theory, and separately for the front half of the ring (subscript 1) and the rear half (subscript 2). Points on the ring are referred to the angle θ, measured positively counterclockwise from the right side point (3 o'clock) looking down on the ring. Deflections are measured radially, positive outward; angular deflections α of elastic lines are positive when increase in θ accompanies increase in δ. Bending moments acting across a radial section at angle θ are reckoned positive clockwise; at each such section there is an action and a reaction of opposite sign. M_θ denotes the moment at section θ acting on the sector between zero and θ, and M_0 the moment at section zero acting on the same segment. At the radial section at angle θ there is also a force whose components will be called V, parallel, and H, perpendicular, to the line of action of the applied forces F. Following the same convention as for the moments, V_θ and H_θ denote the components of force applied to the sector between zero and θ, and V_0 and H_0 those applied to the same sector at θ = zero. In the case of the forces we have immediately

$$V_\theta = -V_0$$

and

$$H_\theta = -H_0$$

On account of symmetry it is clear that

$$V_0 = -\frac{F}{2}$$

By reason of symmetry, $\alpha = 0$ when $\theta = \pi/2$ and when $\theta = -\pi/2$.

Upper Half, Subscript 1

$$F_1 < 0,\ H_{01} < 0,\ \theta_1 > 0$$

Considering equilibrium of the sector between $\theta = 0$ and $\theta = \theta_1$,

$$M_{01} + M_{\theta 1} - H_{01} R \sin\theta_1 + F_{1/2} R (1 - \cos\theta_1) = 0 \ldots [1]$$

H_{01} being the transverse (radial) force exerted on the surface at $\theta = 0$, reckoned positive outward.

Since $\alpha_{\pi/2} = 0$,

$$\alpha_\theta = \frac{R}{EI}\int_{\pi/2}^{\theta} M_\theta \, d\theta$$

Solving Equation [1] for $M_{\theta 1}$, substituting and integrating,

$$\alpha_{\theta 1} = \frac{R}{EI}\left[\frac{F_1 R}{2}\left(\sin\theta_1 - \theta_1 + \frac{\pi}{2} - 1\right) + M_{01}\left(\frac{\pi}{2} - \theta_1\right) - H_1 R \cos\theta_1\right] \ldots [2]$$

Now in general $\delta_\theta - \delta_0 = R\int_0^\theta \alpha_\theta \, d\theta$

and by substituting the value of α and integrating

$$\delta_{\pi/2} - \delta_0 = \frac{R^2}{EI}\left[\frac{F_1 R}{2}\left(1 - \frac{\pi}{2} + \frac{\pi^2}{8}\right) + M_{01}\frac{\pi^2}{8} - H_1 R\right] \ldots [3]$$

The condition that the length of the sector from 0 to $\pi/2$ is invariable is now introduced; this amounts to neglecting the effects of tension in the ring as compared with bending. This gives us the condition

$$\int_0^{\pi/2} (\delta_\theta - \delta_0)\, d\theta = -\frac{\pi\delta_{01}}{2}$$

Evaluating the integral,

$$\frac{\pi\delta_{01}}{2} = -\frac{R^2}{EI}\left[\frac{F_1 R}{2}\left(-1 + \frac{\pi}{2} - \frac{\pi^2}{8} + \frac{\pi^2}{24}\right) + M_{01}\frac{\pi^2}{24} - H_1 R\right] \ldots [4]$$

Lower Half, Subscript 2

$$F_2 > 0,\ H_2 > 0,\ \theta_2 < g$$

The equations corresponding to those given above are

$$M_{02} + M_{\theta 2} - H_2 R \sin\theta_2 - \frac{F_2 R}{2}(1 - \cos\theta_2) = 0 \ldots [1]$$

$$\alpha_{\theta 2} = -\frac{R}{EI}\left[\frac{F_2 R}{2}\left(-\sin\theta_2 + \theta_2 + \frac{\pi}{2} - 1\right) - M_{02}\left(\frac{\pi}{2} + \theta_2\right) - H_2 R \cos\theta_2\right] \ldots [2]$$

$$\delta_{-\pi/2} - \delta_0 = \frac{R^2}{EI}\left[\frac{F_2 R}{2}\left(1 - \frac{\pi}{2} + \frac{\pi^2}{8}\right) - M_{02}\frac{\pi^2}{8} - H_2 R\right] \ldots [3]$$

$$\frac{\pi\delta_{02}}{2} = \frac{R^2}{EI}\left[\frac{F_2 R}{2}\left(1 - \frac{\pi}{2} + \frac{\pi^2}{8} - \frac{\pi^3}{24}\right) + M_{02}\frac{\pi^3}{24} + H_2 R\right] \ldots [4]$$

Combined Halves

Equating δ_{01} and δ_{02} and noting that $H_1 = -H_2$ and $F_1 = -nF_2$,

$$H_2 = F_2\frac{n+1}{4}\left(\frac{\pi^3}{24} - \frac{\pi^2}{8} + \frac{\pi}{2} - 1\right) = F_2(n+1) \times 0.157256. \ [5]$$

Equating δ_{01} and α_{02}, and noting in addition that $M_{01} = -M_{02}$,

$$M_{02} = F_2 R\frac{1-n}{4}\left(1 - \frac{2}{\pi}\right) = F_2 R(1-n) \times 0.0908450 \ldots [6]$$

The radial deflections at the three stations are as follows:

$$\delta_1 = \delta_3 = \delta_0 = F_2\frac{R^3}{EI}(1-n)\left(\frac{1}{2\pi} - \frac{1}{4} + \frac{\pi}{48}\right)$$

$$\frac{1}{2\pi} - \frac{1}{4} + \frac{\pi}{48} = 0.025395$$

$$\delta_4 = \delta_{-\pi/2} = F_2\frac{R^3}{EI}\left[\frac{1}{\pi} - \frac{\pi}{12} + (1+n)\left(-\frac{1}{2\pi} + \frac{1}{2} - \frac{5}{24}\pi + \frac{\pi^2}{16} - \frac{\pi^3}{96}\right)\right]$$

$$\frac{1}{\pi} - \frac{\pi}{12} = 0.056510;\quad -\frac{1}{2\pi} + \frac{1}{2} - \frac{5}{24}\pi + \frac{\pi^2}{16} - \frac{\pi^3}{96} = -0.019789$$

$$\delta_2 = \delta_{\pi/2} = F_2\frac{R^3}{EI}\left[\frac{1}{\pi} - \frac{\pi}{12} + (1+n)\left(-\frac{1}{2\pi} - \frac{1}{2} + \frac{7}{24}\pi - \frac{\pi^2}{16} + \frac{\pi^3}{96}\right)\right]$$

$$-\frac{1}{2\pi} - \frac{1}{2} + \frac{7}{24}\pi - \frac{\pi^2}{16} + \frac{\pi^3}{96} = -0.036721$$

The moments at the same three stations are:

$$M_1 = M_3 = M_0 = F_2 R(1-n)\frac{1}{4}\left(1 - \frac{2}{\pi}\right)$$

$$\frac{1}{4}\left(1 - \frac{2}{\pi}\right) = 0.090845$$

$$M_4 = M_{-\pi/2} = F_2 R\left[\frac{1}{\pi} + (1+n)\left(-\frac{1}{2\pi} + \frac{1}{2} - \frac{\pi}{8} + \frac{\pi^2}{32} - \frac{\pi^3}{96}\right)\right]$$

$$\frac{1}{\pi} = 0.318310;\quad -\frac{1}{2\pi} + \frac{1}{2} - \frac{\pi}{8} + \frac{\pi^2}{32} - \frac{\pi^3}{96} = -0.066412$$

$$M_2 = M_{\pi/2} = F_2 R\left[-\frac{1}{\pi} + (1+n)\left(\frac{1}{2\pi} + \frac{1}{2} - \frac{\pi}{8} + \frac{\pi^2}{32} - \frac{\pi^3}{96}\right)\right]$$

$$\frac{1}{2\pi} + \frac{1}{2} - \frac{\pi}{8} + \frac{\pi^2}{32} - \frac{\pi^3}{96} = 0.251898$$

As a check on this work we may substitute the value $n = -1$, which should give us values of deflection and bending moment of a ring in simple tension, as in a chain. We find

$$\delta_{-\pi/2} = \delta_{\pi/2} = \frac{FR^3}{EI}\left[\frac{1}{\pi} - \frac{\pi}{2}\right]$$

$$M_{-\pi/2} = -M_{\pi/2} = FR \times \frac{1}{\pi}$$

These results are in agreement with known values for thin rings. In terms of total load F we have, finally,

$$\delta_1 = \delta_3 = 0.0254\frac{1-n}{1+n}\frac{FR^3}{EI}$$

$$\delta_4 = \left(\frac{0.0565}{1+n} - 0.0198\right)\frac{FR^3}{EI}$$

$$\delta_2 = \left(\frac{0.0565}{1+n} - 0.0367\right)\frac{FR^3}{EI}$$

$$M_1 = M_3 = 0.091\frac{1-n}{1+n}FR$$

$$M_4 = \left(\frac{0.318}{1+n} - 0.066\right)FR$$

$$M_2 = \left(-\frac{0.318}{1+n} + 0.252\right)FR$$

APM-51-8

Telephone-Apparatus Springs

A Review of the Principal Types and the Properties Desired of These Springs

By J. R. TOWNSEND,[1] NEW YORK, N. Y.

THE proper functioning of telephone-apparatus springs depends upon careful design and selection of material. In many instances the springs must occupy small space and maintain delicate adjustment throughout the life of the apparatus with a minimum of attention. Whereas the physical size of these springs is small, the forces are sometimes necessarily large due to space limitations and this requires careful choice of material. It is believed that the use of these small springs is not unique to the telephone art and a discussion of materials and methods of test used should have broad interest and application.

There are three general classes of springs employed in telephone apparatus. For the purposes of this discussion these springs may be classified as follows:

Sheet non-ferrous-metal springs
Clock-spring-steel springs
Helical springs.

Sheet Non-Ferrous Springs

Sheet non-ferrous springs usually consist of punched and formed parts made from brass, nickel silver, or phosphor bronze. These serve as electrical-contact carrying members that are deflected or operated either electromagnetically or mechanically. Such springs are essentially cantilever springs clamped at one end and bearing near the other end one or more precious-metal contacts that are spot-welded in place. The precious-metal contacts are employed to reduce contact resistance and the destructive effects of arcing when circuits are broken. The apparatus employing such springs are keys of the switchboard type, relays, jacks, and interrupters. Certain apparatus known as switches employ these springs as brushes or wiping members. Here precious-metal contacts are not usually employed, but the ends of the springs must serve as contacting and wearing parts. Other springs are employed statically or, in other words, are required to maintain constant pressure for long periods without interruption. All springs that are attached to, or form part of, electrical circuits are soldered to connecting wires. This is usually done by soldering the wire or connection to a lug or projection that forms part of the spring. These lugs in most instances are on the opposite end of the clamped area from the operated end. These springs are almost always clamped between strips of phenol fiber because of its good insulating properties, mechanical strength, and permanency of form. A typical example of the use of the more common type of these springs is illustrated by Fig. 1 showing the familiar switchboard key.

The properties required of these small springs which are numerous and vary for each type of application are summarized as follows:

The proportional limit must not be too high to prevent the spring's being adjusted by flexing it with a tool to the point where it takes a set and occupies a position where it provides the desired operating pressure. In other words, there must be room enough to flex the spring to the point where it will take a set within the space provided. A spring of high proportional limit such as one of clock-spring steel may be bent nearly double without being permanently deformed. Obviously, such a spring could not be adjusted in the key shown by Fig. 1.

The modulus of elasticity should be within the range of 12,000,000 to 20,000,000 lb. per sq. in. in order that the load-deflection rate will not be too steep to permit reasonable ease of adjustment by hand.

The endurance limit must be high enough to permit satisfactory operation throughout the life of the apparatus. In cases where space limitation is a factor, reference must be made to a stress-cycle graph made for the material to determine if the spring may be expected to stand up in service.

Creep or deformation under sustained load must not take place since the material will lose tension. Brass, nickel silver, and phosphor bronze may be expected on the basis of years of experience to hold adjustments when stressed up to approximately their proportional limits. Other materials when considered for telephone apparatus springs are investigated to determine their "creep" characteristics.

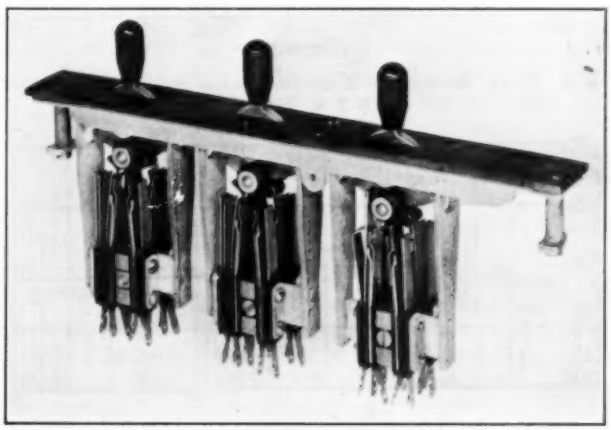


Fig. 1 Common Type of Switchboard Key Illustrating Use of Sheet-Metal Springs

Season cracking takes place with highly stressed brass under severe atmospheric conditions, and for this reason springs that are required to hold their pressures without being deflected for long periods are not made from this material. Nickel silver will also season-crack under still higher stress and phosphor bronze least of all. In designing these springs, generous fillets and easy curves are employed to prevent the building up of localized high stresses that may lead to season cracking under sustained load and fatigue failure under repeated flexure.

When these springs are used as wipers in electrical circuits where arcing can occur, brass and nickel silver are not employed for the reason that the heat of the arc breaks down the material, volatilizes the zinc, and disintegrates the material. For this reason, phosphor bronze, which does not contain zinc and has superior wear resistance to brass and nickel silver, is employed.

In addition to the foregoing properties, these springs must be resistant to atmospheric corrosion and capable of being readily soldered with soft solder. Nickel silver, as may be seen from Table 2, is superior to brass in its mechanical properties and in ad-

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[1] Bell Telephone Laboratories. Assoc-Mem. A.S.M.E.

Contributed by the Research Special Committee on Springs and presented at the Annual Meeting, New York, N. Y., Dec. 3 to 7, 1928, of The American Society of Mechanical Engineers.

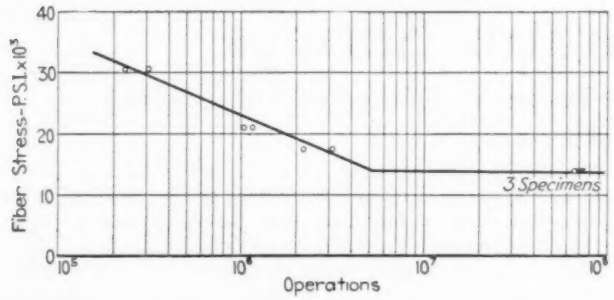


FIG. 2 FIBER STRESS VS. FATIGUE—BRASS SHEET—24 GAGE—4 NOS. HARD

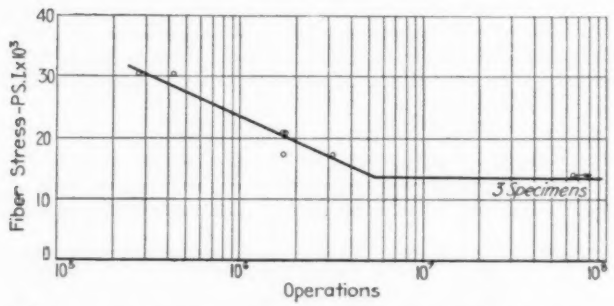


FIG. 3 FIBER STRESS VS. FATIGUE—BRASS SHEET—24 GAGE—10 NOS. HARD

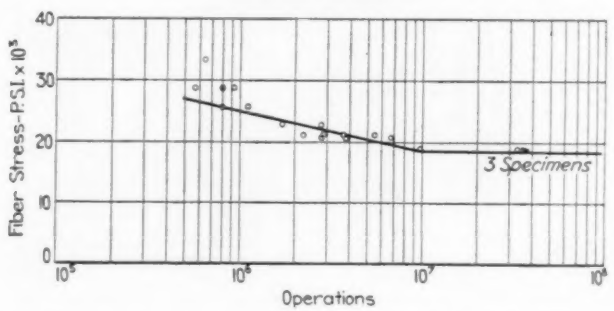


FIG. 4 FIBER STRESS VS. FATIGUE—NICKEL-SILVER SHEET—24 GAGE—4 NOS. HARD

dition may be readily spot-welded, and as a result of years of experience it has been found that springs made of this material are capable of maintaining adjustment in a satisfactory manner in service. Phosphor bronze has still greater wear resistance than brass or nickel silver and superior spring properties.

The chemical compositions of brass, nickel-silver, and phosphor-bronze spring materials used for telephone-apparatus springs are shown in Table 1 and the mechanical properties are given in Table 2. The range of tensile strength and hardness shown comprises the specification limits. A more detailed description of the properties of brass has been given elsewhere.[2]

FATIGUE PROPERTIES OF SHEET NON-FERROUS SPRING MATERIALS

Because of the important bearing of fatigue on telephone-apparatus springs this property has been carefully studied. The stress-cycle graphs shown by Figs. 2 to 4 give typical results. The design of specimen is shown by Fig. 7. These specimens were alternately stressed at the rate of 700 cycles per minute.

[2] "Physical Properties and Methods of Tests for Sheet Brass," by H. N. Van Deusen, L. I. Shaw, and C. H. Davis. Proceedings A.S.T.M., 1927.

CLOCK-SPRING-STEEL SPRINGS

These springs are used as vibrating elements in interrupters, where a high fatigue endurance is required, and for springs that must be worked at high pressures and built up. The material has the chemical composition shown by Table 3 and mechanical properties given by Table 4. This is a carbon-steel, heat-treated and then cold-rolled, and is by nature brittle. To guard against excessive brittleness and at the same time provide a high-strength cold-rolled steel, a bend test has been developed.

The bend test requires that when the material is bent back parallel to itself to form a "U" and further compressed between flat parallel surfaces (for example, between the jaws of a vise), at a rate not to exceed 0.3 in. per min., the material shall break along a straight line making approximately a 90-deg. angle with the axis of the strip when the distance between the inside of the legs of the "U" is 25 to 16 times the thickness of the material. It must not break before the distance is reduced to 25 times the thickness of the metal. This test can be conveniently applied

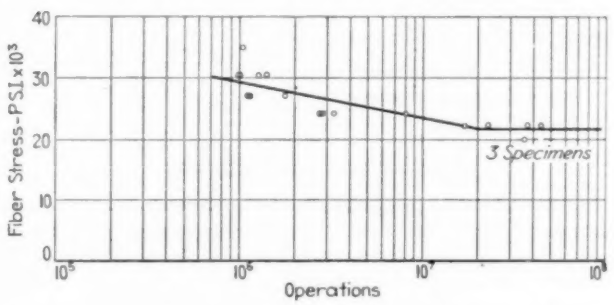


FIG. 5 FIBER STRESS VS. FATIGUE—NICKEL-SILVER SHEET—24 GAGE—10 NOS. HARD

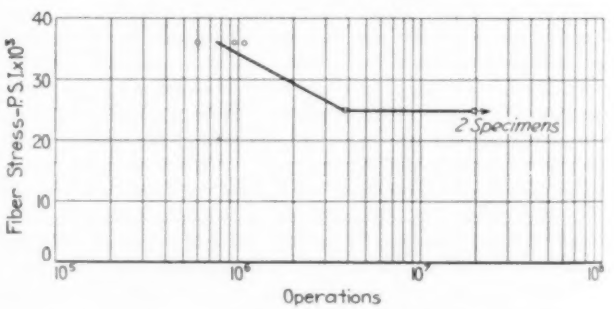


FIG. 6 FIBER STRESS VS. FATIGUE—PHOSPHOR-BRONZE SHEET—24 GAGE—10 NOS. HARD

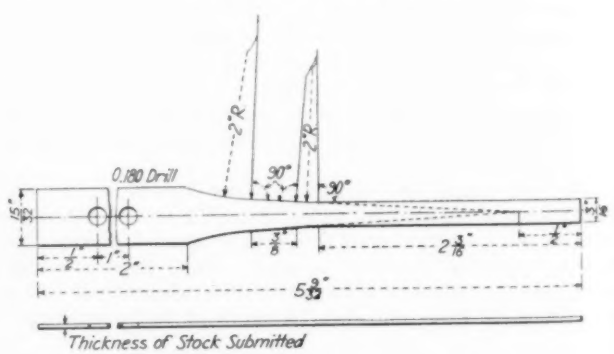


FIG. 7 TEST SPECIMEN FOR FATIGUE STUDY

(The piece must be free from scriber and tool marks. The 2-in. radius fillets must join the tapered portion without an abrupt break. The intersection of the tapered sides produced must be 1/32 in. from the end.)

TABLE 1 CHEMICAL COMPOSITION NON-FERROUS SHEET SPRING MATERIALS

	Min., per cent	Max., per cent
Brass		
Copper	64.5	67.5
Lead	0.0	0.3
Iron	0.0	0.05
Zinc	Remainder	
Nickel Silver		
Copper	53.50	56.50
Nickel	16.50	19.50
Zinc	25.50	28.50
Iron	...	0.35
Phosphor Bronze		
Copper	91.0	...
Tin	7.50	8.50
Phosphorus	0.05	0.25
Iron	...	0.10
Lead	...	0.02
Antimony	...	0.01
Zinc	...	0.20

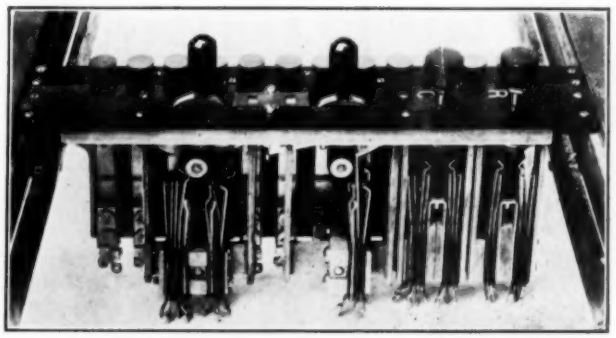


FIG. 8 SWITCHBOARD KEYS EMPLOYING SHEET-METAL SPRINGS AND HELICAL COMPRESSION SPRINGS

TABLE 2 MECHANICAL PROPERTIES NON-FERROUS SHEET SPRING MATERIALS

Thickness	Temper	B. & S. gage nos. hard	Tensile strength Min.	Tensile strength Max.	Rockwell hardness "B" scale Min.	Rockwell hardness "B" scale Max.	Proportional limit	Modulus elasticity
Brass								
0.40 in. and thicker	Hard	4	68,000	78,000	78	85	30,000	14×10^6
Below 0.40 in.	Hard	4	68,000	78,000	75	83		
0.040 in. and thicker	Spring	8	86,000	95,000	88	92	30,000	14×10^6
Below 0.040 in.	Spring	8	86,000	95,000	85	89		
0.040 in. and thicker	Extra	10	89,500	98,500	89	93		
Below 0.040 in.	Spring	10	89,500	98,500	86	90		
Nickel Silver								
0.40 in. and thicker	Hard	4	92,000	106,500	69	77	60,000	20×10^6
Below 0.40 in.	Hard	4	92,000	106,500	66	75		
0.040 in. and thicker	Extra hard	6	102,000	115,000	75	82	60,000	20.0×10^6
Below 0.040 in.	Extra hard	6	102,000	115,000	72	79		
0.040 in. and thicker	Spring	8	108,000	120,000	78	84		
Below 0.040 in.	Spring	8	108,000	120,000	75	81		
0.040 in. and thicker	Extra spring	10	111,000	123,000	80	85		
Below 0.040 in.	Extra spring	10	111,000	123,000	77	82		
Phosphor Bronze								
0.040 in. and thicker	Extra hard	6	97,000	111,500	76	82	55,000	15×10^6
Below 0.040 in.	Extra hard	6	97,000	111,500	73	79		
0.040 in. and thicker	Spring	8	105,000	118,500	79	85		
Below 0.040 in.	Spring	8	105,000	118,500	76	82		
0.040 in. and thicker	Extra spring	10	109,500	122,000	81	86		
Below 0.040 in.	Extra spring	10	109,500	122,000	78	83		

NOTE: The proportional limit and modulus figures are representative. They are only slightly changed by cold work.

TABLE 3 CHEMICAL COMPOSITION OF CLOCK-SPRING STEEL

	Min., per cent	Max., per cent
Carbon	0.85	1.15
Manganese	0.25	0.60
Silicon	..	0.22
Sulphur	..	0.025
Phosphorus	..	0.03
Nickel, chromium, tungsten	..	0.10
Vanadium	Optional	0.25
Iron	Remainder	

TABLE 4 MECHANICAL PROPERTIES OF CLOCK-SPRING STEEL

Ultimate strength,	250,000 to 290,000 lb. per sq. in.
Proportional limit,	160,000 to 215,000 lb. per sq. in.
Modulus of elasticity,	27.6×10^6

by drawing the looped material through two of a series of graduated slots.

MUSIC-WIRE SPRINGS

Tinned and plated music wire is extensively used for compression springs in telephone apparatus. Here the spring is in the form of an open helix. Fig. 8 shows a type widely used. These springs are either tinned or plated with nickel. It has been observed the plating baths adversely affect the fatigue characteristics of music wire and this is under investigation at the present time.

The average tensile properties of the music wire employed are: proportional limit, 217,000 lb. per sq. in.; ultimate tensile strength, 350,000 lb. per sq. in. No chemical or tensile requirement is placed on this wire, the main feature of concern being brittleness. The elongation measured in an 8-in. length must be between 1 and 4 per cent for wire up to and including No. 27 gage and $1^1/_2$ to 7 per cent for No. 28 gage wire and heavier. The wire is also subject to a kinking test in which No. 30 wire and smaller shall kink without breaking. This last test indirectly controls the tensile properties of the wire.

SUMMARY

A review has been given of the more common type of telephone-apparatus springs. Vast quantities of these springs are employed in the telephone plant and the numerous factors that must be considered with regard to selecting material have been reviewed. The materials most generally used have abundant competitive sources of supply and the quality is carefully controlled by tests that are designed to be easy to apply and effective in evaluating the properties desired.

APM-51-9

The Piston-Crosshead Motion of the Oilgear Pump

By ELEK BENEDEK,[1] MILWAUKEE, WIS.

The Oilgear pump is a variable-delivery radial pump which converts mechanical power into hydraulic power in the form of an oil stream under pressure. An oil motor is used to reconvert the power into mechanical form and apply it to the work to be done. For services requiring greater power than the applications so far made, improvements are needed which will reduce noise and vibration. The present paper derives equations for determining the path, velocities, and accelerations of any point in the piston-crosshead unit of the Oilgear pump, as investigation has shown that an accurate knowledge of the accelerations of the mechanical parts and of the oil in the cylinders and ports is necessary as a basis for further improvements in design.

THE analysis given herein derives equations for the determination of the path, velocities, and accelerations of any point in the piston-crosshead unit of the Oilgear variable-delivery radial pump. During the last seven years this pump has been widely used in previously undeveloped fields of power transmission where variable speed control is required. The pump converts mechanical power into hydraulic power in the form of an oil stream under pressure. An oil motor, which may be of the same construction as the pump, is used to reconvert the power into mechanical form and apply it to the work to be done.

The Oilgear applications so far made have been below 65 hp. In this field the units have shown a ruggedness and durability which strongly suggests that others may be built suitable for driving Diesel locomotives, or for similar variable-speed applications requiring much greater power. Certain improvements will be required for such service, notably the reduction of noise and vibration, which might be excessive in large units on the present basis of design. Sufficient investigation work has already been done to indicate that an accurate knowledge of the accelerations of the mechanical parts and of the oil in the cylinders and ports is necessary as a basis for further improvements in design.

The equations obtained are useful in determining the forces resulting from the accelerations, which are of considerable importance in the design of pumps of this type, particularly of large units and units operating at high speeds. They can cause both wear and vibration unless suitably provided for in the design. The behavior of the fluid in the cylinder chambers and in the passages to and from the cylinders also depends on the velocities and accelerations of each piston-crosshead unit, and a knowledge of these values is thus required in order to determine proper areas and contours for the fluid passages and cylinders.

The elements of the pump mechanism considered in this paper are shown in Fig. 1. The actual construction of a pump is shown in the sectional drawings of Fig. 2, 1*a* and 1*b*. For a description of the operation of the pump, see the paper entitled "Applications of Hydraulic Transmissions Variable-Speed Drive to Machine Tools and Manufacturing Processes," by Walter Ferris, presented at the annual meeting of the A.S.M.E., December 4, 1922, and published in *Mechanical Engineering*, vol. 45 (1929), p. 238.

Referring to Fig. 1, the assembly consists of a driving member C, a driven member A, and several interconnecting members B. The driving member is called the driver, the driven member is the cylinder block, and the interconnecting members are the piston-crosshead units. The motion is imparted to the pistons by the relative eccentricity of the rotating parts C and A, which is adjustable so that the piston strokes may be changed at will. The driving member C rotates at uniform speed around the fixed center O_2. The driven member A also rotates at uniform speed around the adjustable center O_1. At any given time, however, the center O_1 is fixed. (It is only shifted when it is desired to change the stroke.) This driven member or cylinder block is forced to rotate at the same angular velocity as the driver by the interconnecting piston-crosshead units which carry it around with the driver. The piston-crosshead units slide tangentially in the slots in the driver and radially in the cylinder bores as the driver and cylinder block rotate on their separate centers. The construction of the piston-crosshead unit is such that the axis of the piston is always perpendicular to the flat surfaces of the crosshead. Thus the axes of the cylinders, which are radii of the cylinder-block bores, must always be perpendicular to the grooves in the driver, and therefore parallel to radii of the driver which are perpendicular to the driver grooves. As the radii of cylinder block and driver are thus always mutually parallel, it follows that the cylinder block and driver must rotate with the same angular velocity. The geometry of the assembly is the same as that of an Oldham coupling, except that in the Oilgear pump there are several interconnecting members instead of only one.

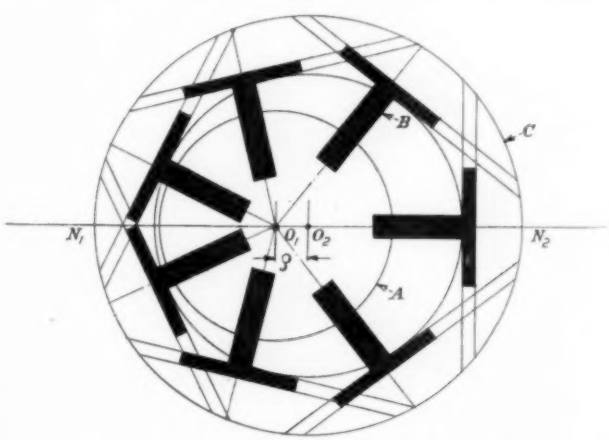


FIG. 1 ELEMENTS OF OILGEAR PUMP MECHANISM

Every point in the cylinder block A rotates around the center O_1. Thus the paths of all these points are circles around the center O_1. Every point in driver C rotates around the center O_2. The paths of these points are therefore circles around the center O_2.

The points in the piston-crosshead units B have a composite motion about the centers O_1 and O_2. To describe this motion in a general way, it can be stated that relatively to the center O_1 all these points move first radially toward the center and then away from it during the rotation of the pump. Relatively to

[1] The Oilgear Company.

center O_2, all these points move tangentially to one side and then to the other side of a radius of the driver C, which represents the center line of a piston-crosshead unit in its neutral or dead-center position in the driver. The exact motion of any point of the groove of the driver in which this piston and crosshead slides, is selected for the x-axis of a rotating coordinate system. The coordinates of point O in the stationary coordinate system are x_0,y_0. Point P is an arbitrary point lying anywhere in the

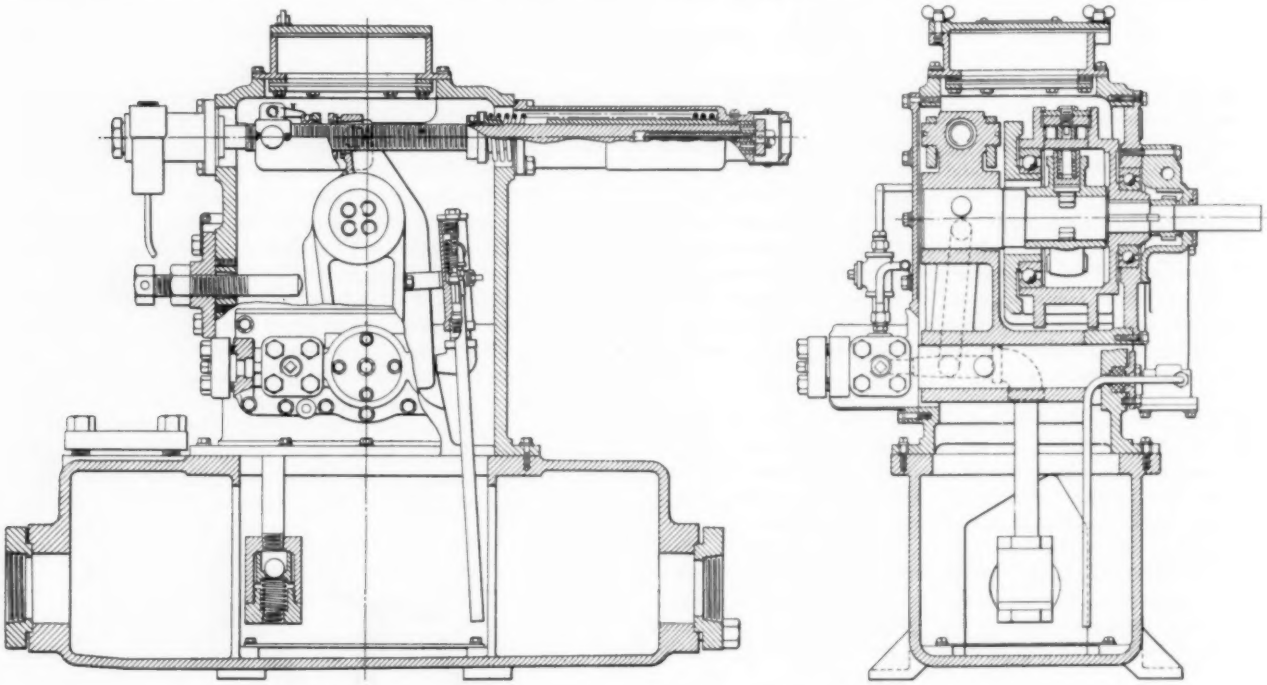


FIG. 2 VERTICAL CROSS-SECTIONS OF ACTUAL PUMP, SHOWING CONSTRUCTION

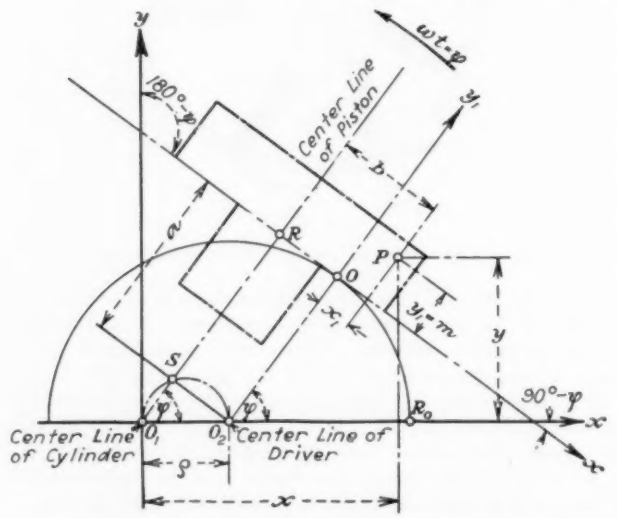


FIG. 3 ANALYSIS OF MOTION OF A POINT IN PISTON-CROSSHEAD UNIT

in a piston-crosshead unit will be described exactly by the mathematical derivations which follow.

Referring to Fig. 3, the neutral line of the pump is the line O_1, O_2, intersecting the cylinder center O_1 and the driver center O_2. This line is chosen as the x-axis of a stationary coordinate system to which all motions will finally be referred. The center line of a piston-crosshead unit always intersects O_2 and is shown as having rotated in a counterclockwise direction through the angle φ. The line Ox_1, which represents the bottom piston-crosshead unit. Its coordinates in the stationary system are x,y, and in the rotating system x_1,y_1. The motion of the point P in the moving coordinate system is a simple harmonic motion. If the motion of a point is known in a moving coordinate system which has a uniform linear or angular motion with reference to a stationary coordinate system, then the motion of the point in the stationary system can be found by the transformation of its coordinates in the moving system into coordinates in the stationary system. The motion of point P will be expressed in the moving system first and then in the stationary system. The following notation will be used:

- α_1 = angle of the positive x_1 in regard to the positive x
- β_1 = angle of the positive x_1 in regard to the positive y
- α_2 = angle of the positive y_1 in regard to the positive x
- β_2 = angle of the positive y_1 in regard to the positive y
- ρ = eccentricity of pump
- φ = phase angle of the piston, measured from the positive x-axis
- ω = angular velocity of the piston
- t = time of the rotation of piston, measured from the positive x-axis
- a = radius of the path of O
- b = distance of P from the center line of piston
- k = distance of P from the center line of driver.

TRANSFORMATION EQUATIONS

$$\left.\begin{aligned} x &= x_0 + x_1 \cos \alpha_1 + y_1 \cos \alpha_2 \\ y &= y_0 + x_1 \cos \beta_1 + y_1 \cos \beta_2 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots [1]$$

From Fig. 3 it is obvious that

$$\left.\begin{aligned} \alpha_1 &= (90 - \varphi)^\circ \\ \alpha_2 &= \varphi^\circ \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots [1a]$$

and

$$\left.\begin{aligned}\beta_1 &= (180 - \varphi)^\circ \\ \beta_2 &= (90 - \varphi)^\circ\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots [1b]$$

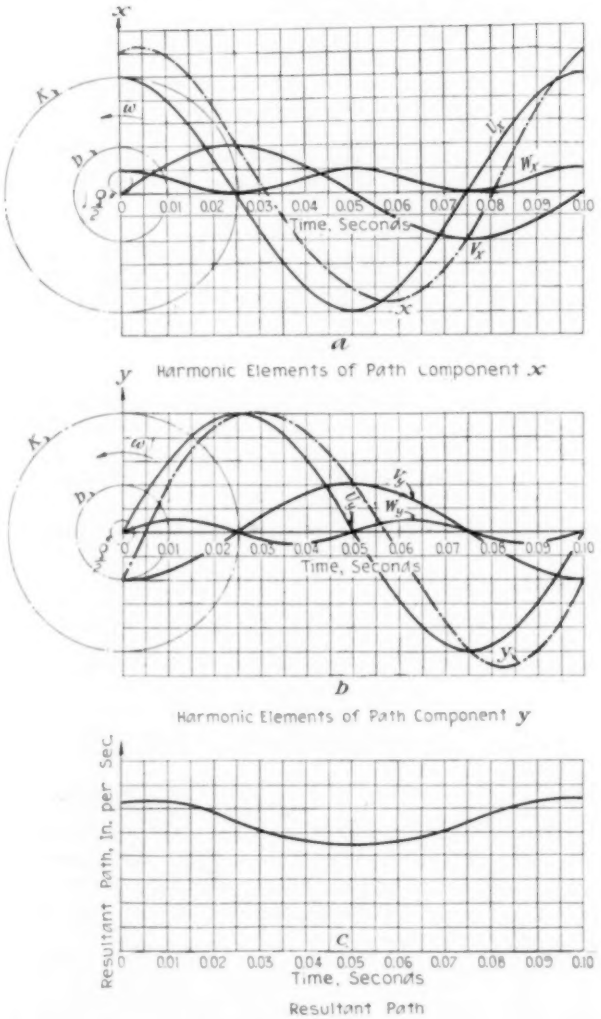


FIG. 4 HARMONIC ELEMENTS OF PATH COMPONENTS x AND y, AND RESULTANT PATH DIAGRAM

Furthermore the coordinates of O in the stationary system are given by

$$\left.\begin{aligned}x_0 &= \rho + a\cos\varphi \\ y_0 &= a\sin\varphi\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots [1c]$$

DERIVATION OF THE PATH OF POINT P IN THE STATIONARY COORDINATE SYSTEM

The coordinates of P in the rotating system are given from Fig. 3, and can be expressed as functions of the phase angle as follows:

$$\left.\begin{aligned}x_1 &= b - \rho\sin\varphi \\ y_1 &= m = \text{constant}\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots [2]$$

The motion expressed by Equation [2] can be transformed by Equation [1]. We shall then have the motion of P in the stationary system O_1xy. By substituting [1a], [1b], [1c] and [2] in Equation [1], we obtain the coordinates of point P in the stationary system:

$$\left.\begin{aligned}x &= \rho + a\cos\varphi + (b - \rho\sin\varphi)\cos(90 - \varphi) + m\cos\varphi \\ y &= a\sin\varphi + (b - \rho\sin\varphi)\cos(180 - \varphi) + m\cos(180 - \varphi)\end{aligned}\right\}$$
$$\ldots\ldots\ldots [3]$$

By adding the homogeneous members and designating $a + m$ by k, the path components of the motion are expressed by the equations

$$\left.\begin{aligned}x &= k\cos\varphi + b\sin\varphi + \rho\cos^2\varphi \\ y &= k\sin\varphi - b\cos\varphi + \frac{\rho}{2}\sin 2\varphi\end{aligned}\right\} \quad \ldots\ldots\ldots [4]$$

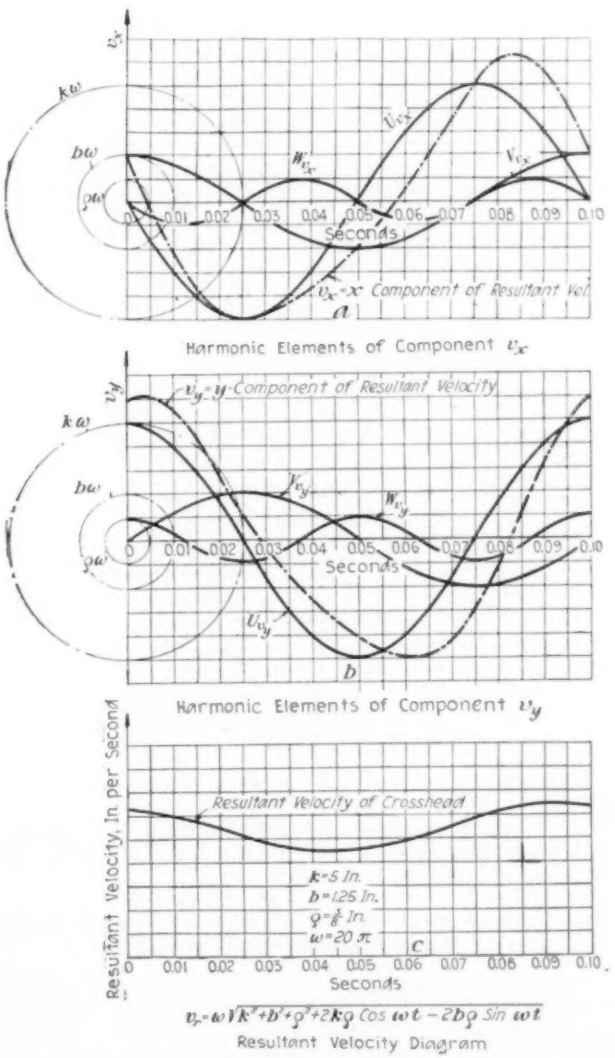


FIG. 5 HARMONIC ELEMENTS OF COMPONENT VELOCITIES v_x AND v_y, AND RESULTANT VELOCITY DIAGRAM

Equation [4] expresses the motion of the path components of P in the stationary system O_1xy.

If we designate by r the hypothenuse O_1P of the right triangle whose other two sides are the coordinates of P, then

$$r = \sqrt{x^2 + y^2} \quad \ldots\ldots\ldots\ldots\ldots [5]$$

By susbstituting x and y from Equation [4], the radius vector r to point P will be

$$r = \sqrt{(k + \rho\cos\varphi)^2 + b^2} \quad \ldots\ldots\ldots\ldots [5a]$$

Equation [5a] is the polar equation of the path of P in the stationary coordinate system. From [5a] it follows that if $k = b = 0$, the path of P is a circle. If $b = 0$ and $k = \rho$, the path of P is a cardioid. The path expressed by [5a] is the so-called Pascal limacon.

Fig. 4 shows at (a) and (b) the actual diagrams of the right-hand members of [4]. We call these members the harmonic elements of the path components x and y, and designate them

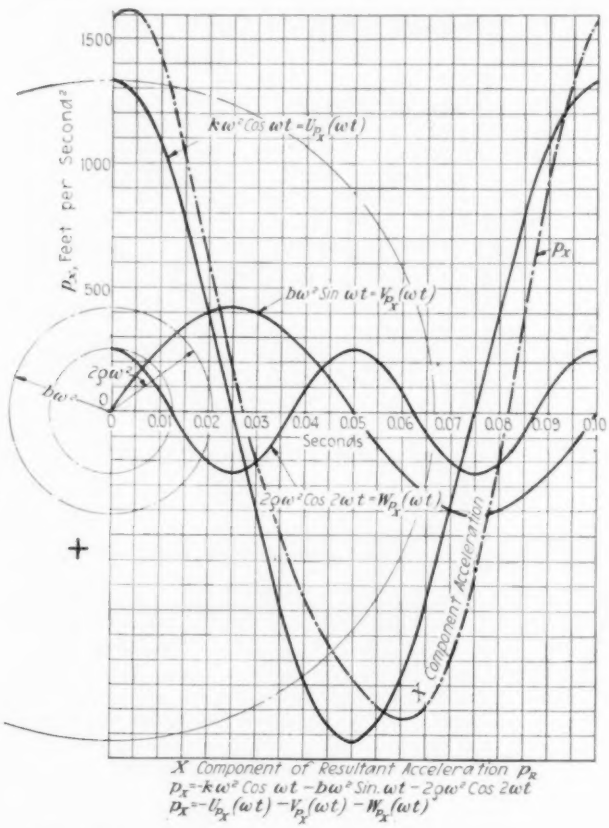


Fig. 6(a) x-Component of Resultant Acceleration p_r

by U, V, and W. Fig. 4(c) shows the diagram of the resultant path. The characters and significance of the elements will be discussed at the end of this paper.

Derivation of the Resultant Velocity of Point P in the Stationary Coordinate System

According to Equation [4], the path components are simple harmonic functions of the phase angle. The phase angle itself is a function of the time: thus $\varphi = \omega t$.

The velocity can be derived from the path by the differentiation of the path equation according to time.

Since

$$\left.\begin{aligned} \frac{dx}{dt} &= v_x \\ \frac{dy}{dt} &= v_y \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots [6]$$

Equation [6] gives the x- and y-components of the resultant velocity v_r.

Therefore

$$\left.\begin{aligned} v_x &= \omega(-k \sin \varphi + b \cos \varphi - \rho \sin 2\varphi) \\ v_y &= \omega(k \cos \varphi + b \sin \varphi + \rho \cos 2\varphi) \end{aligned}\right\} \quad \ldots\ldots [6a]$$

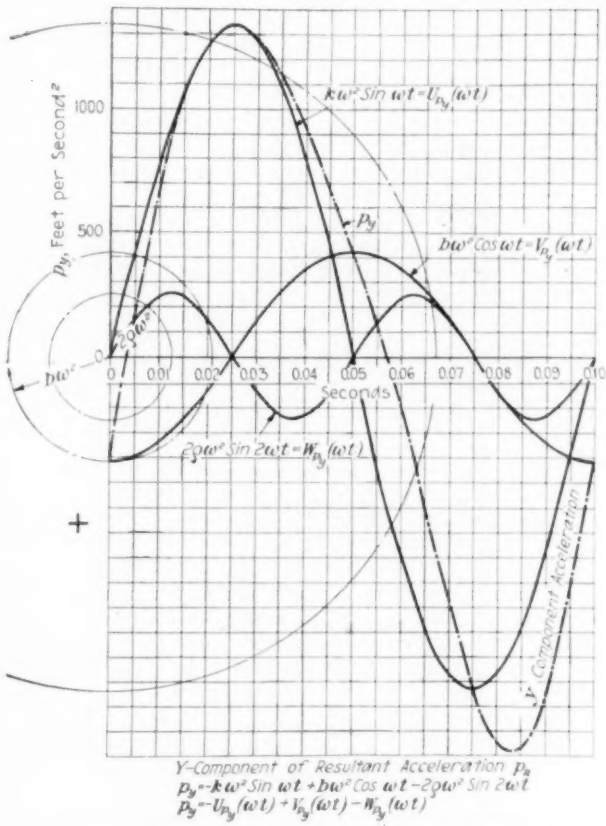


Fig. 6(b) y-Component of Resultant Acceleration p_r

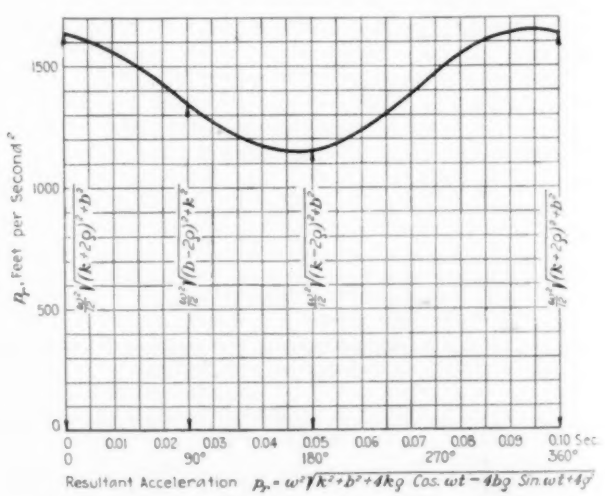


Fig. 6(c) Resultant Acceleration p_r

and the resultant velocity in the stationary coordinate system will be

$$v_r = \sqrt{v_x^2 + v_y^2} \quad \ldots\ldots\ldots\ldots\ldots [6b]$$

By substituting v_x and v_y from [6a], we shall have

$$v_r = \omega\sqrt{(k + \rho \cos \varphi)^2 + (b - \rho \sin \varphi)^2} \quad \ldots\ldots [7]$$

Equation [7] shows that the velocity v_r can be represented also by the hypothenuse of a right triangle, the sides of which are proportional to $(k + \rho \cos \varphi)$ and $(b - \rho \sin \varphi)$, respectively.

The component velocities v_x, v_y, and the resultant velocity v_r are shown at (*a*), (*b*), and (*c*) Fig. 5, as well as the harmonic elements of the components, which are the members of the right-hand side of Equation [6*a*]. The resultant velocity is obtained by the graphical superposition of the curves of the elements in the right-hand side of Equation [7].

Derivation of the Resultant Acceleration of the Point *P* in the Stationary Coordinate System

The components of resultant acceleration can be derived from the components of velocity by differentiating them according to time.
Thus

$$\left.\begin{aligned}\frac{dv_x}{dt} &= p_x\\ \frac{dv_y}{dt} &= p_y\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots [8]$$

By carrying out the differentiation, the components of the resultant acceleration become

$$\left.\begin{aligned}p_x &= \omega^2(-k\cos\varphi - b\sin\varphi - 2\rho\cos 2\varphi)\\ p_y &= \omega^2(-k\sin\varphi + b\cos\varphi - 2\rho\sin 2\varphi)\end{aligned}\right\} \quad \ldots [8a]$$

The resultant acceleration is

$$p_r = \sqrt{p_x^2 + p_y^2} \quad \ldots\ldots\ldots\ldots\ldots [9]$$

which, by substituting p_x and p_y from [8*a*], becomes

$$p_r = \omega^2\sqrt{(k + 2\rho\cos\varphi)^2 + (b - 2\rho\sin\varphi)^2} \quad \ldots [10]$$

The component accelerations of p_x and p_y and the resultant acceleration p_r are shown by Figs. 6(*a*), 6(*b*), and 6(*c*). These diagrams also show how the components are graphically superposed from the different elements according to Equation [8*a*], to give the resultant accelerations.

Equation [10] shows that the resultant acceleration can be considered as the resultant of two vectors which are perpendicular to each other and proportional to the expressions $(k + 2\rho\cos\varphi)$ and $(b - 2\rho\cos\varphi)$, respectively.

Analysis of the Motion of a Piston With Reference to the Cylinder Block

In order to analyze the motion of the oil in a cylinder, the path of a point R in the piston with reference to the cylinder itself must be known. The travel of the point R (at the intersection of the piston center line and the lower surface of the crosshead) during the rotation of the piston at an angle from center O_1 is equal to $O_1R_0 - O_1R = (\rho + a) - (\rho\cos\varphi + a)$

$$= \rho - \rho\cos\varphi$$
$$= \rho(1 - \cos\varphi)$$

If we designate this travel by s, then

$$s = \rho(1 - \cos\varphi) \quad \ldots\ldots\ldots\ldots\ldots [11]$$

This is the equation of a pure harmonic motion. A piston therefore reciprocates in its cylinder with a pure harmonic motion, without such modifying factors as might result from connecting rods or cams in other types of construction.

Harmonic Analysis of the Resultant Path, Velocity, and Acceleration of the Point *P* in the Stationary Coordinate System

The analytical Equations [5*a*], [7], and [10] express the motion of an arbitrary point of the piston-crosshead, and the diagrams of its motions are constructed from these equations. Curves are drawn for each of the harmonic elements in the equations; these curves are superposed, and the values of the resultant path, velocity, and acceleration are obtained graphically. These elements are designated by U, V, and W on the diagrams and are functions of some of the dimensions of the pump. For instance, U is a function of k, the distance of the point P from the center line of driver, and the phase angle of the piston. V is a function of b, the distance of the point from the center line of the piston, and the phase angle of the piston. W is a function of the eccentricity of the pump and the phase angle of the piston.

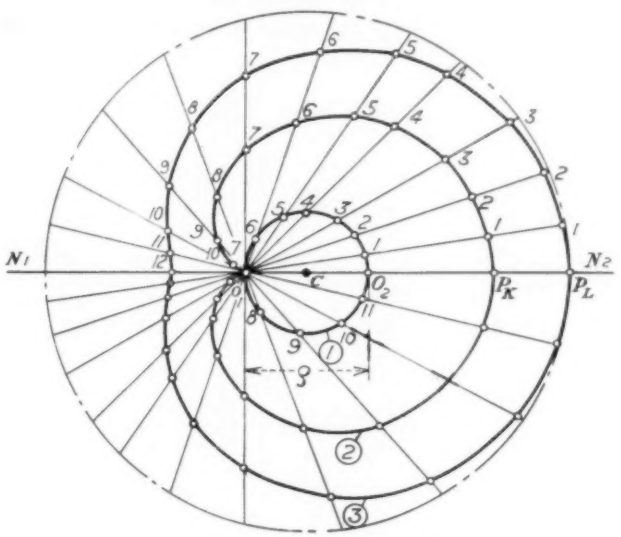


Fig. 7 Paths of Piston Points
(1) Path of the harmonic piston point O_2 is a circle.
(2) Path of the piston point P_K is a cardioid.
(3) Path of the piston point P_L is a limaçon.

According to the resultant diagrams of path, velocity, and acceleration, every one of them consists of three elements of a pure harmonic nature. These elements show the following characteristics:

1 Every harmonic element represents the motion of a point rotating on a circle with a constant angular speed.

2 The path elements U and V increase with the dimensions of the pump.

3 The velocity elements are directly proportional to the speed of the pump.

4 The acceleration elements increase with the square of the angular speed.

5 The U and V elements represent the pure harmonic motion of points which rotate at the angular speed of the pump.

6 The W elements represent the pure harmonic motion of a point which rotates twice as fast as the pump.

7 Every general point of the piston unit has a motion which can be resolved into three pure harmonic elements.

8 The only points of the piston unit, the motion of which can be resolved into two pure harmonic motions, are those which are on the center line of the piston. For these points, $b = 0$, and consequently the V harmonic element is zero.

9 The piston unit has only one point the motion of which can be resolved into one pure harmonic motion. This point is on the center line of the piston and coincides with the center line of the driver when the piston is in the outstroke dead center; see point O_2 in Fig. 3. For this point the equations of the resultant path, velocity, and acceleration are respectively:

$$\left.\begin{aligned} r &= \rho \cos \varphi \\ v &= \rho\omega \\ p &= 2\rho\omega^2 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots [12]$$

These results follow directly from the general equations of the motion by substituting in them $k = b = 0$. We call this point the harmonic point of the piston.

10 The harmonic point of the piston rotates with constant angular speed and acceleration on a circle of radius $\rho/2$.

11 The W harmonic element is identical with the resultant motion of the harmonic point of the piston.

12 The U harmonic element is the motion of a point rotating on a circle of radius k with the angular speed of the pump.

13 The V harmonic element is the motion of a point rotating on a circle of radius b with the angular speed of the pump.

14 The harmonic point of the piston is the only one which rotates on a circle. The remainder of the piston points rotate on different curves, such as the Pascal limacon or the cardioid. (See Fig. 7.)

15 Each piston reciprocates in its cylinder with a pure harmonic motion. As the piston motion determines the fluid motion, it follows that the resultant fluid motion consists of a series of pure harmonic impulses.

APM-51-10

Deflection of a Round-End Strut Subjected to a Constant Moment or a Transverse Force at the Middle

By JAMES E. BOYD,[1] COLUMBUS, OHIO

When a loaded strut is subjected to a transverse force or any other bending moment in addition to that caused by the longitudinal load, the additional deflection is greater than it would be if the additional moment were applied to the strut as a beam. In this paper an equation is derived for the deflection of a loaded round-end strut subjected to a transverse force at the middle and a moment which is constant throughout the length. To experimentally investigate the theory, tests were made on a cold-rolled steel strut 60 in. long and 1.25 in. in diameter with transverse forces of 5 lb. and 10 lb. at the middle. Measurements were taken for a series of longitudinal loads with positive, negative, and nearly zero eccentricity. Other experiments were made with a constant moment alone.

For both sets of experiments the additional deflections agree so closely with the theory as to leave no doubt in regard to the validity of the equations.

For the application of the theory, a formula which gives the ratio of the additional deflection of a loaded strut to its deflection as a beam in terms of the ratio of the longitudinal load to Euler's critical load is derived for each of these two moment conditions. A similar formula is derived for a uniformly distributed transverse force. Tables and curves are given which show the relation of these ratios with sufficient accuracy for practical computations.

THIS paper gives an account of the experimental determination of the deflection of a round-end strut subjected to a transverse force at the middle. The theoretical equations are derived, and the deflections calculated from these equations are shown to agree with the experimental results. Since the design of the apparatus used in the tests was such that the transverse reactions were applied at a small distance beyond the ends of the strut, instead of exactly at ends as would be assumed under the ideal conditions, the bending moment caused by the transverse force was made up of a constant moment throughout the length of the strut in addition to the moment as a beam supported at the ends and loaded at the middle. For this reason the theoretical deflection includes one term for a moment which increases uniformly from each end to the middle, and a second term for a moment which is constant.

For the convenience of the practicing engineer, a formula is derived for each of these two moment conditions which gives the ratio of the additional deflection of the loaded strut when this moment is applied to its deflection as a beam in terms of the ratio of the longitudinal load to Euler's critical load. For instance, when the longitudinal load on the strut is one-fourth of Euler's critical load, the additional deflection when either moment is applied is about one-third greater than it would be with no longitudinal load. When the longitudinal load is 50 per cent of Euler's critical load the additional deflection is twice as great, and when the longitudinal load is 90 per cent of Euler's critical load the additional deflection is 10 times as great as either moment would produce on the strut as a beam with no longitudinal load. Tables are calculated and curves plotted which give the relation of these ratios with sufficient accuracy for practical computations.

[1] Professor of Mechanics, The Ohio State University. Mem. A.S.M.E.

Contributed by the Applied Mechanics Division and presented at the Annual Meeting, New York, N. Y., December 3 to 7, 1928, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

EXPERIMENTAL INVESTIGATIONS

APPARATUS AND MATERIAL

The experimental measurements were made on a strut of cold-rolled steel 60 in. long and 1.25 in. in diameter. The round-end condition was secured by means of semicylindrical heads of hardened steel which rolled on hardened plane surfaces. To measure the deflection a wooden bar was supported parallel to the strut by means of yokes which were attached to the axes of the semicylindrical heads by cone bearings. An Ames dial at the center of this wooden bar measured the deflection at the middle of the strut, and two other dials at 10 in. from the top and bottom measured the deflection at these positions. (Bulletin No. 25, "Tapered Struts," of The Ohio State University Engineering Experiment Station, gives a detailed description of the construction and use of this apparatus.)

The loads were applied by means of an Olsen 100,000-lb. universal testing machine, using the 10,000-lb. poise. A pointer was attached to the beam and moved in front of a graduated scale fastened to the frame. With this arrangement it was possible to balance to within 2 lb.

The transverse force was applied by a cord at the middle of the strut. This cord passed horizontally to a pulley and carried a load Q at the lower end. The shaft of the pulley was ground to a knife edge at the axis at each end, as shown in Fig. 1. These knife edges were supported inside hardened rings inserted in the bracket. The bracket was fastened to the frame of the weighing table of the testing machine before the zero adjustment was made, and the weight Q was taken into account in reading the poise. The Ames dials were of the usual type, graduated to thousandths of an inch. All readings were taken to the ten-thousandth of an inch.

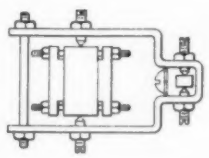
ADJUSTMENT OF ECCENTRICITY

The strut was loaded and the deflections were read on the upper and lower dials. The load was then reduced to about 400 lb. and the strut was shifted by means of clamp screws attached to the compression heads. The amount of shift at each end was obtained from the change of the readings of upper and lower dials. The load was then raised several thousand pounds. When the deflection of the two dials for this increased load became equal, the eccentricity was known to be the same at both ends. The ends were then shifted equally a couple of times and the corresponding readings were taken. By interpolation and extrapolation the shift of the strut for any desired eccentricity could be approximately determined.

TESTS FOR THE EFFECT OF TRANSVERSE FORCE

To determine the effect of initial eccentricity upon the additional deflection caused by a transverse force, readings were taken with considerable positive eccentricity, with considerable negative eccentricity, and with practically zero eccentricity

Table 1 gives part of the log of two consecutive runs made March 21, 1928. Euler's critical load had previously been found to be 10,025 lb. Since the theoretical deflection may be calculated in terms of this load, the longitudinal load was applied in



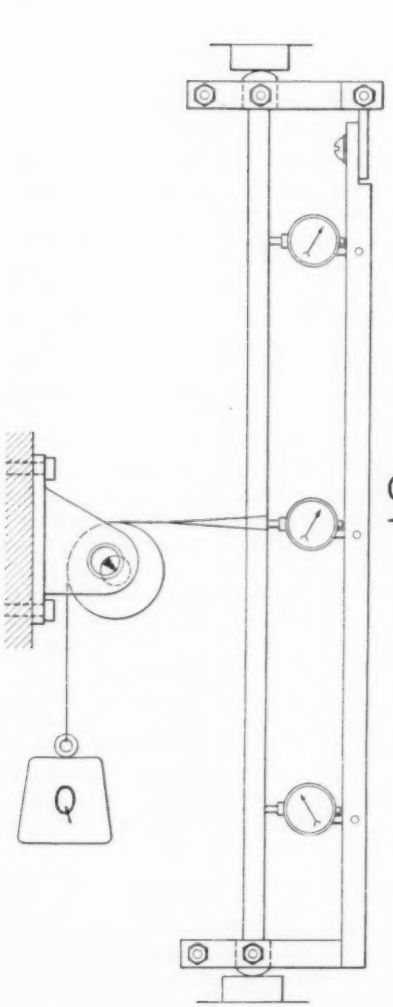


FIG. 1

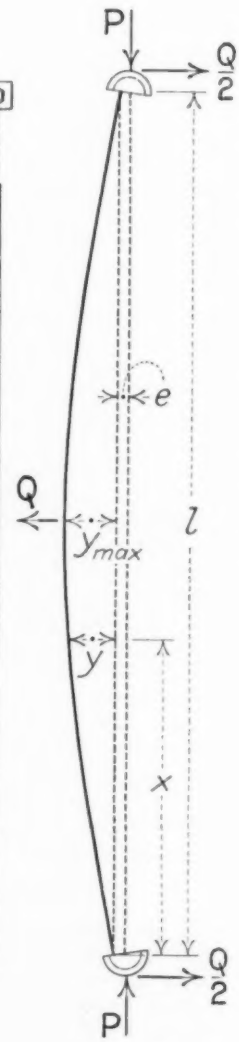


FIG. 2

increments of 5 per cent of 10,025 lb. The initial load was 501 lb. Since the deflection apparatus was made of wood, brass, and steel and the room temperature varied considerably during the day, the load was brought back to the initial value after each test. This precaution was not necessary to get the additional deflection caused by the addition of transverse forces, since these forces were applied immediately after the readings were taken for the deflection caused by the longitudinal load. It was desirable, however, in order to check the deflection caused by the longitudinal load, to calculate the eccentricity, and to detect any permanent deformation.

The figures in the deflection column of Table 1 in line with the applied load and the zero transverse force measure the deflection as a column when this load was applied. For instance, when the load of 3007 lb. was applied the reading of the middle dial was 3980. The initial reading at 501 lb. was 3874. The additional deflection as a column when the load was changed from 5 per cent of 10,025 to 30 per cent of 10,025 was 106 ten-thousandths of an inch. The deflection at 10 in. from the top was 63 and at 10 in. from the bottom was 66 ten-thousandths. With a different negative eccentricity the same change in load caused a negative deflection of 136 at the middle gage, 55 at the upper gage, and 70 at the lower gage. This deflection will be designated as the "eccentricity deflection."

The figures in the deflection columns in line with the applied load and 5 lb. transverse force measure the change in deflection caused by this force. For instance, with a load of 3007 lb. and zero transverse force the reading at the middle was 3980. When the transverse force of 5 lb. was applied the reading fell to 3886, a change of 0.0094 in. When the transverse force of 10 lb. was applied the deflection was 3980 — 3791 = 189. This will be designated as the "transverse-force deflection."

(With the apparatus as set up the additional deflection caused by the transverse force was always negative. The negative sign is omitted in Table 1 for these deflections.)

The "eccentricity deflection" at the lower dial was somewhat larger than at the upper dial when the eccentricity was positive, and considerably larger when the eccentricity was negative. This difference was caused by some difference in the eccentricity at the two ends. The "transverse-force deflections" were nearly equal, and the sum of the deflections at the upper and lower gages was very nearly the same as the deflection at the middle.

When the eccentricity was positive, as in the upper half of Table 1, the "eccentricity deflection" was positive. The trans-

TABLE 1 PART OF THE LOG OF TESTS OF A ROUND-END STRUT SUBJECTED TO A LONGITUDINAL LOAD AND A TRANSVERSE FORCE AT THE MIDDLE

(Readings in ten-thousandths of an inch)

Longitudinal load, lb.	Transverse force, lb.	Upper Dial Reading	Upper Dial Deflection	Middle Dial Reading	Middle Dial Deflection	Lower Dial Reading	Lower Dial Deflection
501	0	1564	...	3873	....	3507	...
501	5	1532	32	3805	68	3470	37
501	10	1500	64	3735	138	3439	68
501	0	1569	..	3874	..	3509	...
3007	0	1632	63	3980	106	3575	66
3007	5	1588	44	3886	94	3530	45
3007	10	1542	90	3791	189	3485	90
501	0	1569	..	3876	..	3509	...
5012	0	1746	177	4198	322	3696	187
5012	5	1679	67	4069	129	3623	73
5012	10	1621	125	3940	258	3561	135
501	0	1564	...	3872	...	3505	...
8020	0	2323	759	5346	1474	4278	773
8020	5	2170	153	5034	312	4120	158
8020	10	2010	313	4712	634	3962	316
501	0	1561	...	3869	...	3504	...
8521	0	2648	1087	5992	2123	4602	1098
8521	5	2442	206	5580	412	4396	206
8521	10	2235	413	5157	835	4182	420
501	0	1562	...	3872	...	3506	...
Compression Heads Shifted to Change Eccentricity to Negative							
501	0	1039	...	3296	...	2988	...
501	5	1004	35	3231	65	2956	32
501	10	972	67	3160	136	2921	67
501	0	1038	...	3286	...	2980	...
3007	0	983	− 55	3150	− 136	2910	− 70
3007	5	939	44	3059	91	2860	50
3007	10	899	84	2972	178	2812	98
501	0	1040	...	3286	...	2981	...
5012	0	914	−126	3002	− 284	2826	−155
5012	5	849	65	2876	126	2762	64
5012	10	790	124	2749	253	2698	128
501	0	1040	...	3283	...	2980	...
8020	0	556	−484	2270	−1013	2435	− 545
8020	5	400	156	1953	317	2280	155
8020	10	250	306	1632	638	2118	317
501	0	1039	...	3281	...	2970	...
8521	0	348	−691	1846	−1435	2225	− 745
8521	5	140	208	1420	426	2007	218
8521	10	Off scale	...	992	854	1780	445
501	0	1038	...	3280	...	2980	...

verse force in the negative direction reduced the deflection and made the strut more nearly straight. This straightening increased the pressure on the heads and raised the beam of the testing machine. With the larger loads it was necessary to run the movable crosshead of the testing machine upward a little to balance after the transverse force was applied. With negative eccentricity the transverse force increased the negative deflection and reduced the pressure on the weighing table. With the larger loads it was then necessary to run the movable crosshead of the testing machine downward a little to balance after the transverse force was applied.

Euler's Critical Load

After the second run of Table 1 the heads were again shifted to bring the eccentricity to zero at each end. A set of readings for "eccentricity deflection" and "transverse-force deflection" was taken up to a load of 8521 lb. After the dial readings at 501 lb. had again been taken, the load was increased to 8521 lb., and then continued, with "eccentricity deflection" readings only, up to the critical load. On account of some little crookedness of the strut, these deflections were negative at first. At the upper and lower dials the deflection changed to positive below 9400 lb.; at the middle dial the zero deflection was between 9400 and 9600 lb.

The results of Table 2 are characteristic of the behavior of a round-end strut with extremely small eccentricity. Under this condition the deflection at the maximum load can be changed from positive to negative by a very small transverse force, and the maximum longitudinal load is practically the same in either position. In this experiment the maximum load with positive deflection was 10,025 lb. and with negative deflection was 10,016 lb. On the basis of this and other previous experiments 10,025 lb. has been adopted as the critical load in the computations which follow. (This strut with the same mounting was tested July 18, 1927, on the 400,000-lb. Riehlé machine of the Department of Civil Engineering of Cornell University. The critical load was there found to be 10,050 lb. A series of struts made of steel of high elastic limit might well be used for calibrating testing machines.)

From Euler's formula, $P = \pi^2 EI/l^2$, the value of the product of the modulus of elasticity multiplied by moment of inertia is found to be

$$EI = \frac{10{,}025 \times 60 \times 60}{9.8696} = 3{,}656{,}700$$

This value is a factor in the theoretical calculations which follow.

Theoretical Deflection

The deflection formula is derived from the equation $EId^2y/dx^2 = M$. From Fig. 2, at a distance x from the end,

$$EI\frac{d^2y}{dx^2} = -Py - Pe - \frac{Qx}{2} - M \quad \ldots\ldots [1]$$

in which y is the deflection at a distance x from the end, e is the eccentricity measured from the line of the longitudinal load to the center line of the strut, Q is the transverse force at the middle, and M is a moment, constant throughout the length of the strut. This moment M is the product of one-half Q multiplied by the radius of the cylindrical head. The differential equation may be written

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = -\frac{Pe}{EI} - \frac{Qx}{2EI} - \frac{M}{EI} \quad \ldots\ldots [2]$$

The solution is

$$y = A\cos\sqrt{\frac{P}{EI}}\,x + B\sin\sqrt{\frac{P}{EI}}\,x - \frac{Qx}{2P} - e - \frac{M}{P} \quad \ldots [3]$$

$$\frac{dy}{dx} = -A\sqrt{\frac{P}{EI}}\sin\sqrt{\frac{P}{EI}}\,x + B\sqrt{\frac{P}{EI}}\cos\sqrt{\frac{P}{EI}}\,x - \frac{Q}{2P} \quad \ldots [4]$$

In Equation [3], when $x = 0$, $y = 0$, hence

$$A = e + \frac{M}{P}$$

(It is evident that M/P is equivalent to an additional eccentricity. See "Equivalent Eccentricities Due to Secondary Stresses," by Cecil Napier Ross, Trans. Inst. of Engineers, Australia, vol. 7, 1927.)

When $x = l/2$ in Equation [4], $dy/dx = 0$, hence

$$B = \left(e + \frac{M}{P}\right)\tan\sqrt{\frac{P}{EI}}\,\frac{l}{2} + \frac{Q}{2P}\sqrt{\frac{EI}{P}}\sec\sqrt{\frac{P}{EI}}\,\frac{l}{2} \quad \ldots [5]$$

Substitution of the values of the arbitrary constants, A and B, in Equation [3] gives

$$y + e + \frac{M}{P} = \left(e + \frac{M}{P}\right)\left(\cos\sqrt{\frac{P}{EI}}\,\frac{l}{2} + \tan\sqrt{\frac{P}{EI}}\,\frac{l}{2}\sin\sqrt{\frac{P}{EI}}\,x\right) + \frac{Q}{2P}\sqrt{\frac{EI}{P}}\sec\sqrt{\frac{P}{EI}}\,\frac{l}{2}\sin\sqrt{\frac{P}{EI}}\,x - \frac{Qx}{2P} \quad \ldots [6]$$

Equation [6] may be written

$$y + e + \frac{M}{P} = \left(e + \frac{M}{P}\right)\left[\sec\sqrt{\frac{P}{EI}}\,\frac{l}{2}\cos\sqrt{\frac{P}{EI}}\left(\frac{l}{2} - x\right)\right] + \frac{Q}{2P}\sqrt{\frac{EI}{P}}\sec\sqrt{\frac{P}{EI}}\,\frac{l}{2}\sin\sqrt{\frac{P}{EI}}\,x - \frac{Qx}{2P} \quad \ldots [7]$$

At the middle, where $x = l/2$,

$$y_{max} + e + \frac{M}{P} = \left(e + \frac{M}{P}\right)\sec\sqrt{\frac{P}{EI}}\,\frac{l}{2} + \frac{Q}{2P}\sqrt{\frac{EI}{P}}\tan\sqrt{\frac{P}{EI}}\,\frac{l}{2} - \frac{Ql}{4P} \quad \ldots [8]$$

$$y_{max} = e\left(\sec\sqrt{\frac{P}{EI}}\,\frac{l}{2} - 1\right) + \frac{M}{P}\left(\sec\sqrt{\frac{P}{EI}}\,\frac{l}{2} - 1\right) + \frac{Q}{2P}\sqrt{\frac{EI}{P}}\tan\sqrt{\frac{P}{EI}}\,\frac{l}{2} - \frac{Ql}{4P} \quad \ldots [9]$$

In these equations y_{max} is the total deflection at the middle.

The term $e\left(\sec\sqrt{\frac{P}{EI}}\,\frac{l}{2} - 1\right)$ gives the eccentricity deflection, the first deflection figure of the middle gage at each load of Table 1. The remaining terms give the additional deflection which is produced when a transverse force is applied. In this experiment $M = Qr/2$. The transverse force Q therefore appears in the first degree in each of the last three terms. The additional deflection is directly proportional to the transverse force.

The term $\frac{M}{P}\left(\sec\sqrt{\frac{P}{EI}}\,\frac{l}{2} - 1\right)$ expresses the additional deflection caused by the constant moment. For an ideal pin-end column with frictionless pins the resultant reactions would pass

TABLE 2 DEFLECTION FROM VERY SMALL ECCENTRICITY

(Deflections in ten-thousandths of an inch)

Longitudinal load, lb.	Upper Dial		Middle Dial		Lower Dial	
	Reading	Deflection	Reading	Deflection	Reading	Deflection
501	1202	...	3517	...	3270	...
8521	1193	−9	3471	−46	3257	−13
9000	1200	−2	3490	−27	3264	−6
9400	1209	7	3503	−14	3272	2
9600	1220	18	3528	11	3284	14
9800	1231	29	3548	31	3292	22
9900	1255	53	3596	79	3319	49
9950	1280	78	3640	123	3342	72
9980	1302	100	3686	169	3368	98
10000	1360	158	3812	295	3428	158
10020	1540	338	4200	683	3620	350
10025	1740	538	4605	1088	3830	560
10025	1880	678	4884	1367	3972	702
1 Lb. Transverse Force Applied. Machine Not Moved						
11100	1790	...	4710	...	3890	...
2 Lb. Transverse Force Applied. Machine Not Moved						
9875	330	...	1710	...	2344	...
Transverse Force Removed. Machine Moved to Balance						
10016	448	−754	2054	−1463	2516	−754
501	1204	2	3520	3	3271	(1)

through the axes of the pin and there would be no constant moment term in the deflection formula.

Calculation of the Additional Deflection Caused by a Transverse Force at the Middle

For an ideal round-end strut which ends with knife edges or cylinders of infinitely small radii, or is loaded through frictionless pins, the additional deflection caused by a transverse force is given by the last two terms of Equation [9]. If y_1 represents this deflection,

$$y_1 = \frac{Q}{2P}\sqrt{\frac{EI}{P}}\tan\sqrt{\frac{P}{EI}}\frac{l}{2} - \frac{Ql}{4P} \quad \ldots\ldots\ldots [10]$$

Since the tangent series is

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{.15} + \text{etc.},$$

$$\tan\sqrt{\frac{P}{EI}}\frac{l}{2} = \sqrt{\frac{P}{EI}}\frac{l}{2}\left(1 + \frac{Pl^2}{12EI} + \frac{2P^2l^4}{240E^2I^2} + \text{etc.}\right)$$

$$\frac{Q}{2P}\sqrt{\frac{EI}{P}}\tan\sqrt{\frac{P}{EI}}\frac{l}{2} = \frac{Ql}{4P} + \frac{Ql^3}{48EI} + \frac{QPl^5}{480\,E^2I^2} + \text{etc.}$$

$$\frac{Q}{2P}\sqrt{\frac{EI}{P}}\tan\sqrt{\frac{P}{EI}}\frac{l}{2} - \frac{Ql}{4P} = y_1 = \frac{Ql^3}{48EI} + \frac{QPl^5}{480\,E^2I^2} + \text{etc.} \ldots [11]$$

The first term after the equality sign in Equation [11] gives the deflection of the strut as a beam supported at the ends with a concentrated load Q at the middle. The terms which follow form an infinite series in ascending powers of $Pl^2/4EI$. For small values of $Pl^2/4EI$ the sum of this series may be neglected and $Ql^3/48EI$ may be taken as the entire deflection. For larger loads the deflection may be many times greater than that of a simple beam.

Computation of the Deflection

The additional deflection of an ideal round-end strut may be calculated by Equation [10]. Since Euler's critical load is known, it is not necessary to substitute in the expression $\sqrt{\frac{P}{EI}}\frac{l}{2}$ to get the corresponding angle. For Euler's critical load the secant of this angle is infinite and the angle is 90 deg. For any given load the corresponding angle is 90 deg. multiplied by the square root of the ratio of this load to Euler's load. A load of 8120 lb., for instance, is 0.81 of 10,025 lb., and the angle $\sqrt{\frac{8120}{EI}}\frac{l}{2}$ is 90 deg. multiplied by $\sqrt{0.81} = 90° \times 0.9 = 81°$. To get the additional deflection caused by a transverse force of 10 lb. at the middle when the load is 8120 lb. the formula gives

$$y_1 = \frac{10}{2 \times 8120} \times \sqrt{\frac{3{,}656{,}700}{8120}} \times 6.31375 - \frac{10 \times 60}{4 \times 8120}$$

$$= \frac{1}{1624} \times 21.221 \times 6.31375 - \frac{15}{812}$$

$$= 0.08250 - 0.01847$$

$$= 0.0640$$

The addition deflection caused by the constant moment is

$$y_2 = \frac{M}{P}\left(\sec\sqrt{\frac{P}{EI}}\frac{l}{2} - 1\right)$$

Since the radius of each semicylindrical head used in the experiments was $^{15}/_{16}$ in., the moment for a transverse force of 10 lb. is $^{75}/_{16}$ in-lb. For a longitudinal load of 8120 lb.,

$$y_2 = \frac{75}{16 \times 8120}(\sec 81° - 1) = \frac{75}{129{,}920}(6.39245 - 1)$$

$$= 0.0031$$

The total additional deflection is $0.0640 + 0.0031 = 0.0671$.

Table 3 gives the additional deflection for a transverse force of 5 lb. and for a transverse force of 10 lb. for positive, negative, and zero eccentricity. The lower part of the table, beginning with a load of 501 lb., is from the tests of March, 1928, which are given in part in Table 1. The positive eccentricity, calculated from the "eccentricity deflection" from 501 lb. to 8521 lb., was about 0.03 in. The negative eccentricity was about 0.02 in. The upper part of the table is from measurements taken in September, 1927. The positive and negative eccentricities were each about 0.02 in. The run with zero eccentricity was made a few days before the others, and with somewhat less care. To reduce the labor of computation the tests of the upper part of the table were made with equal increments of the square root of the ratio of the load to Euler's critical load. Those of the lower part were made with equal increments of the ratio.

For the smaller loads the readings indicate that the additional deflection when eccentricity was positive (and the additional deflection negative) was a trifle larger, on the average, than when the eccentricity was negative or zero. This slight difference may be caused by crookedness of the strut which affects the "eccentricity deflection" most at the lower loads. Within narrow limits, however, the table shows that the magnitude of the additional deflection is independent of the eccentricity.

Fig. 3 represents Table 3 graphically. The ordinates give the ratios of the longitudinal loads to Euler's critical load. The abscissas are the additional deflections. The curves are drawn from the theoretical deflections. The circles represent the experimental points. A preliminary test with a timber strut gave equally close agreement with the theory.

Experiment With Constant Moment Alone

For the experiments of Tables 1 and 3 the additional moment was that of a beam supported at the ends and loaded at the middle, together with a relatively small constant moment which was equal to the transverse reaction at the end multiplied by the radius of the semicylindrical head. Since the additional de-

flection caused by this constant moment was less than 5 per cent of the total additional deflection, it was deemed advisable to measure the deflection caused by a larger constant moment acting alone. To apply this moment a light horizontal arm was fastened to each semicylindrical head. A 10-lb. mass was hung from the upper arm at a distance of 8.45 in. from the axis of the strut. A cord attached to the lower arm at the same distance from the axis of the strut ran vertically upward, passed over the knife-edge pulley shown in Fig. 1, and supported a second 10-lb. mass on the descending end.

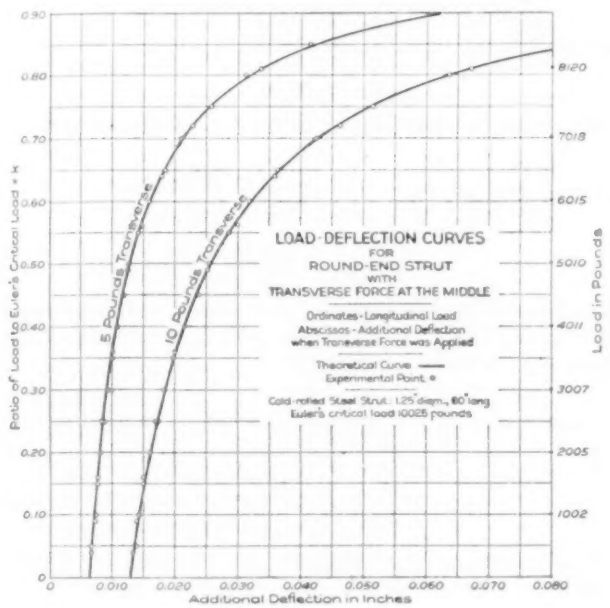


FIG. 3

TABLE 3 ADDITIONAL DEFLECTION OF A ROUND-END STRUT CAUSED BY A TRANSVERSE FORCE AT THE MIDDLE

(Deflections in ten-thousandths of an inch)

Longitudinal load, lb.	Ratio k	Transverse Force 5 Lb. Eccentricity +	−	0	Average deflection	Transverse Force 10 Lb. Eccentricity +	−	0	Average deflection	Computed deflection
401	0.04	67	68	66	67	135	134	132	134	134
902	0.09	73	71	72	72	144	136	136	139	141
1604	0.16	79	77	76	77	153	149	146	149	153
2506	0.25	89	78	83	83	170	174	163	169	171
3609	0.36	102	105	97	101	200	204	196	200	200
4912	0.49	127	127	120	125	256	257	244	252	251
5639	0.5625	152	152	140	148	316	304	280	300	293
6416	0.64	181	185	168	178	367	362	346	358	355
7243	0.7225	232	230	224	229	472	468	452	464	460
8120	0.81	337	354	317	336	676	708	659	681	671
501	0.05	68	65	69	67	138	136	142	139	136
1002	0.10	77	73	71	74	149	140	140	143	143
1504	0.15	80	74	71	75	157	146	146	150	151
2005	0.20	88	78	78	81	171	156	157	161	161
2506	0.25	91	85	85	87	178	174	168	173	171
3007	0.30	94	91	92	92	189	178	181	183	183
3509	0.35	95	107	101	101	202	196	196	198	197
4010	0.40	109	109	105	108	222	219	209	217	214
4511	0.45	123	119	113	118	244	240	229	238	233
5012	0.50	129	126	124	126	258	253	252	254	256
5514	0.55	146	145	139	143	293	292	281	289	285
6015	0.60	160	162	155	159	322	319	312	318	320
6516	0.65	189	184	178	184	368	367	358	364	365
7018	0.70	213	210	208	210	432	418	426	425	427
7519	0.75	274	250	251	258	535	505	508	516	511
8020	0.80	312	317	310	313	634	638	636	636	638
8521	0.85	412	426	415	418	835	854	856	848	850

The results of this test are given in Table 4. The moment arms were so placed as to make the deflection positive. The second column of the table gives the additional deflection at the middle when the eccentricity was about +0.015 in., and the third column gives the deflection at the middle when the eccentricity was about —0.015 in. The fourth column gives the average of these two deflections, and the fifth column gives the computed value. When the load was 2506 lb., for instance,

$$y_2 = \frac{84.5}{2506}(\sec 45° - 1) = \frac{84.5 \times 0.4142}{2506} = 0.0140 \text{ in.}$$

The average experimental value at this load was 0.0138 in.

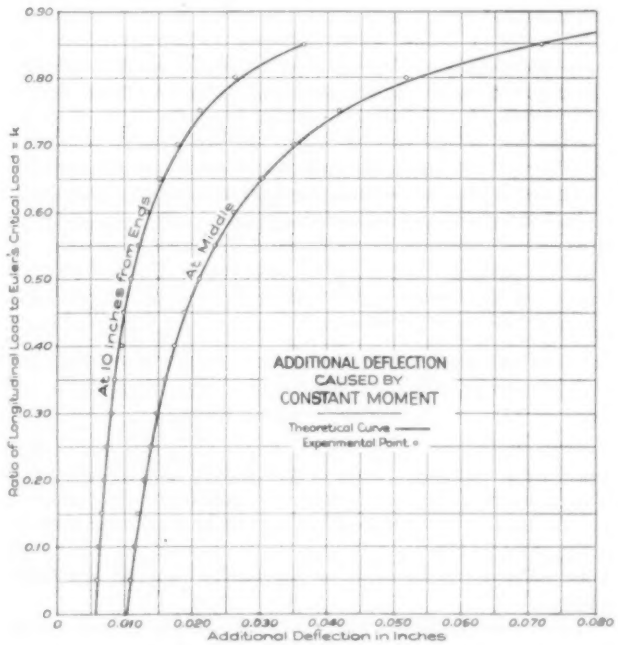


FIG. 4

TABLE 4 ADDITIONAL DEFLECTION OF A ROUND-END STRUT CAUSED BY AN ADDITIONAL MOMENT OF 84.5 IN-LB.

(Cold-rolled steel strut 60 in. long, 1.25 in. in diameter. Deflections in ten-thousandths of an inch)

Longitudinal load, lb.	Middle Gage Eccentricity +	−	Average	Computed	Upper Gage Eccentricity +	−	Lower Gage Eccentricity +	−	Upper and Lower Average	Computed
501	106	112	109	110	56	58	59	57	57.5	61
1002	116	114	115	116	61	59	63	59	60.5	64
1504	121	119	120	123	67	61	68	63	65	67
2005	128	129	128.5	131	69	68	70	68	69	71
2506	137	139	138	140	72	69	74	73	72	76
3007	146	150	148	150	80	76	79	79	78.5	81
3509	160	160	160	162	88	80	84	88	85	87
4010	173	173	173	175	95	99	92	97	96	93
4511	190	184	187	192	102	94	98	100	98.5	102
5012	207	215	211	211	110	99	115	110	108.5	111
5514	237	231	234	235	117	120	127	119	121	124
6015	261	263	262	265	130	132	137	133	133	139
6516	305	301	303	303	145	154	153	153	151	158
7018	357	347	352	354	177	176	179	178	177.5	183
7519	423	411	417	426	210	211	213	210	211	219
8020	514	519	516.5	533	262	269	260	262	263	273
8521	752	683	712.5	712	389	344	374	347	363	362

Table 4 also gives the deflection at 10 in. from the top and at 10 in. from the bottom of the strut. The additional deflection from Equation [7] is

$$y = \frac{M}{P}\left[\sec\sqrt{\frac{P}{EI}}\frac{l}{2}\cos\sqrt{\frac{P}{EI}}\left(\frac{l}{2} - x\right) - 1\right]$$

At 10 in. from the ends, $x = \frac{l}{6}$, $\frac{l}{2} - x = \frac{l}{3}$, and $\sqrt{\frac{P}{EI}} \times \left(\frac{l}{2} - x\right)$ is two-thirds of $\sqrt{\frac{P}{EI}}\frac{l}{2}$.

For a load of 2506 lb., for instance, $\sqrt{\frac{P}{EI}}\frac{l}{2} = 45°$,

$$\sqrt{\frac{P}{EI}}\left(\frac{l}{2}-\frac{l}{6}\right)=30^\circ \qquad y=\frac{84.5}{2506}\left(1.4742\times 0.8660-1\right)$$

$$y=\frac{84.5\times 0.2247}{2506}=0.00758 \text{ in.}$$

Fig. 4 is plotted from Table 4. The abscissas are additional deflections and the ordinates are the ratios of the longitudinal loads to Euler's critical load. There is a close agreement between experiment and theory. For the smaller loads the experimental deflections are slightly smaller than the theoretical values, and the relative differences is greater for deflections at 10 in. from the end. This may be due to the small elastic resistance to rolling of the semicylindrical heads on the plane surfaces. A very slight deviation of the stress-strain diagram of the cold-rolled steel from an absolutely straight line can explain the better agreement of theory and experiment at the larger loads.

Taken together, Figs. 3 and 4 and Tables 3 and 4 may be regarded as a verification of Equation [9], well within the limits of accuracy of the experiments.

Ratio of the Additional Deflection of a Round-End Strut Caused by a Transverse Force at the Middle to Its Deflection as a Beam

While the additional deflection of an ideal round-end strut caused by a transverse force at the middle may be calculated from Equation [10], it is better to transform the equation to give the *ratio* of the additional deflection of the loaded strut to its deflection as a beam. With this ratio known, the calculation of the additional deflection of any given strut may be performed quickly.

Euler's critical load is π^2EI/l^2. For any given load on the strut, $P=k\pi^2EI/l^2$, in which k is the ratio of the given load to Euler's critical load. The angle which is represented by $\sqrt{\frac{P}{EI}}\frac{l}{2}$ is $(k^{1/2}\,90^\circ)$ and Equation [10] may be written

$$y_1=\frac{Ql^3}{2EI}\times\frac{\tan(k^{1/2}\,90^\circ)}{k^{3/2}\,\pi^3}-\frac{Ql^3}{4k\pi^2EI}$$

$$=\frac{Ql^3}{2EI\pi^2}\left(\frac{\tan(k^{1/2}\,90^\circ)}{k^{3/2}\,\pi}-\frac{1}{2k}\right)\qquad[13]$$

If r_1 is the ratio of this deflection to the deflection as a beam

$$r_1=y_1\div\frac{Ql^3}{48EI}$$

$$=\frac{24}{\pi^3k}\left(\frac{\tan(k^{1/2}\,90^\circ)}{k^{1/2}}-\frac{\pi}{2}\right)\qquad[14]$$

In this equation $24/\pi^3=0.774037$, and $\pi/2=1.57080$. For $k=0.81$, $k^{1/2}=0.9$, $k^{1/2}\,90^\circ=81^\circ$, and $\tan 81^\circ=6.31375$, whence

$$\frac{6.31375}{0.9}=7.01528, \qquad \frac{7.01528-1.57080}{0.81}=6.72158,$$

and

$$r_1=0.774037\times 6.72158=5.20275$$

When the longitudinal load on a round-end strut is 81 per cent of Euler's critical load, the additional deflection when a transverse force is applied at the middle is 5.20275 times as great as the deflection which this force would produce if there were no longitudinal load.

For the strut which was tested, the calculated deflection as a beam with a load of 10 lb. at the middle is

$$y=\frac{10\times 60\times 60\times 60}{48\times 3{,}656{,}700}=0.012306 \text{ in.}$$

when the longitudinal load is 8120 lb. The calculated additional deflection for a transverse force of 10 lb. at the middle of the tested strut for that part of the moment which increases uniformly from the ends to the middle is

$$y_1=0.012306\times 5.20275=0.0640 \text{ in.}$$

Ratio of the Additional Deflection of a Round-End Strut Caused by a Constant Moment to Its Deflection as a Beam

The additional deflection caused by a constant moment is

$$y_2=\frac{M}{P}\left(\sec\sqrt{\frac{P}{EI}}\frac{l}{2}-1\right). \quad \text{If } P=\frac{k\pi^2EI}{l^2}, \text{ then}$$

$$y_2=\frac{Ml^2}{k\pi^2EI}\left(\sec(k^{1/2}\,90^\circ)-1\right)\qquad[15]$$

The deflection at the middle of a beam under a constant moment is $y=Ml^2/8EI$. The ratio of the additional deflection of the strut to its deflection as a beam is

$$r_2=y_2\div\frac{Ml^2}{8EI}=\frac{Ml^2}{k\pi^2EI}\left(\sec(k^{1/2}\,90^\circ)-1\right)\div\frac{Ml^2}{8EI}$$

or

$$r_2=\frac{8}{\pi^2k}\left(\sec(k^{1/2}\,90^\circ)-1\right)\qquad[16]$$

In this equation $8/\pi^2=0.810569$. For $k=0.81$, $k^{1/2}=0.90$, and $k^{1/2}\,90^\circ=81^\circ$, whence

$$r_2=\frac{8}{\pi^2}\times\frac{\sec 81^\circ-1}{0.81}=0.810569\times\frac{6.39245-1}{0.81}$$

$$=0.810569\times 6.65735$$

$$=5.39625.$$

When the longitudinal load on a round-end strut is 81 per cent of the critical load, the additional deflection caused by a constant moment is 5.39625 times as great as the deflection which this moment would produce if there were no longitudinal load.

For the strut which was tested with a transverse force of 10 lb., $M=75/16$ and $EI=3{,}656{,}700$, whence

$$y=\frac{Ml^2}{8EI}=\frac{75\times 60\times 60}{16\times 8\times 3{,}656{,}700}=0.0005768 \text{ in.}$$

The additional deflection at the middle caused by the constant moment is $0.0005768\times 5.29625=0.0031$ in. The *total* additional deflection is $0.0640+0.0031=0.0671$, which agrees with the results previously obtained by direct substitution in Equation [9] and is one thousandth of an inch smaller than the average of three experiments as given in Table 3.

Additional Deflection of a Round-End Strut Caused by a Uniformly Distributed Transverse Force

While no experiments were made with uniformly distributed transverse forces, the ratio of the additional deflection of the strut to its deflection as a beam is here given for comparison.

If w is the transverse force per unit length tending to deflect the strut in the positive direction, the additional deflection has been found to be[2]

$$y_3=\frac{wEI}{P^2}\left(\sec\sqrt{\frac{P}{EI}}\frac{l}{2}-1\right)-\frac{wl^2}{8P}\qquad[17]$$

[2] *Engineering News*, April 11, 1907, p. 404.

If Equation [17] is written by means of the secant series, the first two terms are

$$y_3 = \frac{5wl^4}{384\,EI} + \frac{61\,Pwl^6}{46{,}080\,EI} + \text{etc.} \quad \ldots\ldots\ldots [18]$$

The first term of the second member of Equation [18] is the well-known expression for the deflection of a beam supported at the ends and uniformly loaded. The terms which follow form an infinite series in terms of ascending powers of $Pl^2/4EI$ multiplied by w. The additional deflection is directly proportional to the transverse force.

To find the ratio of the additional deflection of the strut to its deflection as a beam, $k\pi^2EI/l^2$ is substituted for P in Equation [17], whence

$$y_3 = \frac{w\,EIl^4}{k^2\pi^4E^2I^2}\left(\sec\left(k^{1/2}\,90°\right) - 1\right) - \frac{wl^4}{8k\pi^2EI}$$

$$= \frac{wl^4}{\pi^4kEI}\left(\frac{\sec\left(k^{1/2}\,90°\right) - 1}{k} - \frac{\pi^2}{8}\right)$$

and

$$r_3 = \frac{384}{5\pi^4k}\left(\frac{\sec\left(k^{1/2}\,90°\right) - 1}{k} - \frac{\pi^2}{8}\right) \quad \ldots\ldots\ldots [19]$$

in which r_3 is the ratio of the additional deflection of the loaded strut caused by the uniformly distributed transverse force to $5wl^4/384EI$. In this equation $384/5\pi^4 = 0.788427$ and $\pi^2/8 = 1.233700$. When $k = 0.25$, $k^{1/2}\,90° = 45°$, sec $45° - 1 = 0.414214$, $0.414214/0.25 = 1.65686$, and

$$r_3 = 0.788427 \times \frac{1.65686 - 1.23370}{0.25}$$

$$= 0.788427 \times 1.69264$$

$$= 1.3345.$$

Table 5 gives the values of r_1, r_3, and r_2 for equal increments of k, and also equal increments of the square root of k. Of these ratios, r_2 is the largest and r_1 is the smallest. When $k = 0.81$, for instance, $r_1 = 5.2028$, $r_2 = 5.3962$, and $r_3 = 5.2792$, whence

$$\frac{r_2 - r_3}{r_2 - r_1} = \frac{0.1170}{0.1934} = 0.605$$

Except for very small values of k, for which the differences are too small for accuracy, the ratio $(r_2 - r_3)/(r_2 - r_1)$ is found to be about 0.6. The reason for this difference may be seen from a consideration of the moment diagrams of a beam for the three kinds of loading. For a beam under a constant moment the diagram is a rectangle. The moment is as large at the ends as at any other sections. The slope and deflection increase rapidly near the ends. In a strut a considerable deflection at one-tenth of the length from each end, for instance, means that eight-tenths of the length is subjected to an additional moment which is the product of this deflection multiplied by the longitudinal load. For a beam supported at the ends and loaded at the middle, the moment diagram is a triangle. (If the deflection at the middle is equal to that caused by a constant moment the maximum at the middle is three-halves as great as the constant moment.) The slope changes slowly near the ends, and the maximum deflection as a beam is equal to the deflection under the constant moment only at the middle. It is evident, therefore, that the effect of this moment upon the additional deflection when the longitudinal load is applied would be less than for a constant moment. For a beam supported at the ends with a uniformly distributed load, the moment diagram is a parabola. (If the deflection at the middle is that caused by a constant moment, the maximum at the middle is six-fifths as great as the constant moment.) The deflection as a beam is less than that caused by a constant moment and greater than that caused by a concentrated load at the middle (except at the middle, where they are equal) and is nearer the latter.

Fig. 5 shows the ratio of the additional deflection of a round-end strut to its deflection as a beam. The entire curve is drawn for r_1, which is the ratio when the bending moment is that of a beam supported at the ends and loaded at the middle. The curve for a constant moment begins at $k = 0.4$.

Fig. 6, with the abscissas on a larger scale, gives all three ratios up to $k = 0.50$, which is beyond the limits of structural struts.

Application to Practice

The engineer who has the problem of calculating the maximum unit stress in a round-end strut which is subjected to a longitudinal load and some other bending moment may first calculate the deflection of the strut as a beam under the action of this

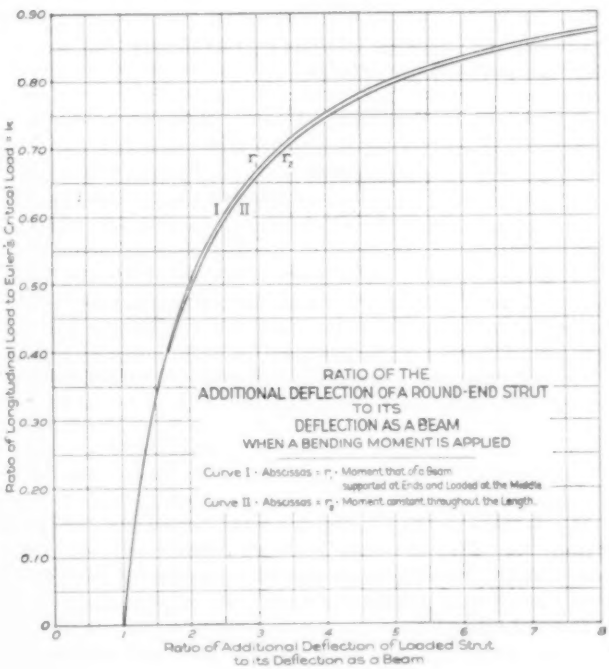


Fig. 5

TABLE 5 RATIO OF THE ADDITIONAL DEFLECTION OF A ROUND-END STRUT WHEN A BENDING MOMENT IS APPLIED TO ITS DEFLECTION AS A BEAM

r_1 = ratio when the bending moment is that of a beam supported at the ends and loaded at the middle.
r_2 = ratio when the bending moment is constant.
r_3 = ratio when the bending moment is that of a beam supported at the ends and uniformly loaded.
k = ratio of the longitudinal load on the strut to Euler's critical load.

Equal Increments of k				Equal Increments of $\sqrt{k}$				
k	r_1	r_2	r_3	$\sqrt{k}$	k	r_1	r_2	r_3
0.05	1.0519	1.0541	1.0528	0.05	0.0025	1.0025	1.0027	1.0026
0.10	1.1096	1.1143	1.1115	0.10	0.0100	1.0100	1.0104	1.0101
0.15	1.1741	1.1815	1.1771	0.15	0.0225	1.0227	1.0237	1.0231
0.20	1.2467	1.2572	1.2509	0.20	0.0400	1.0411	1.0428	1.0418
0.25	1.3289	1.3430	1.3345	0.25	0.0625	1.0658	1.0685	1.0669
0.30	1.4228	1.4411	1.4301	0.30	0.0900	1.0976	1.1017	1.0992
0.35	1.5312	1.5543	1.5404	0.35	0.1225	1.1378	1.1436	1.1401
0.40	1.6576	1.6864	1.6691	0.40	0.1600	1.1880	1.1959	1.1911
0.45	1.8070	1.8425	1.8211	0.45	0.2025	1.2505	1.2612	1.2548
0.50	1.9863	2.0299	2.0036	0.50	0.2500	1.3289	1.3430	1.3345
0.55	2.2054	2.2591	2.2267	0.55	0.3025	1.4279	1.4463	1.4352
0.60	2.4792	2.5455	2.5055	0.60	0.3600	1.5549	1.5790	1.5644
0.65	2.8313	2.9139	2.8640	0.65	0.4225	1.7217	1.7533	1.7342
0.70	3.3007	3.4051	3.3419	0.70	0.4900	1.9476	1.9895	1.9643
0.75	3.9578	4.0929	4.0113	0.75	0.5625	2.2680	2.3245	2.2904
0.80	4.9434	5.1248	5.0151	0.80	0.6400	2.7530	2.8320	2.7843
0.85	6.5860	6.8452	6.6889	0.85	0.7225	3.5670	3.6839	3.6133
0.90	9.8712	10.2847	10.0346	0.90	0.8100	5.2028	5.3962	5.2792
0.95	19.7266	20.6047	20.0728	0.95	0.9025	10.1239	10.5491	10.2916

moment. He will next find the ratio of the proposed longitudinal load to Euler's critical load. From Table 5 or from Fig. 6 he can read off the ratio which, multiplied by the deflection as a beam, gives the additional deflection. Methods of computing

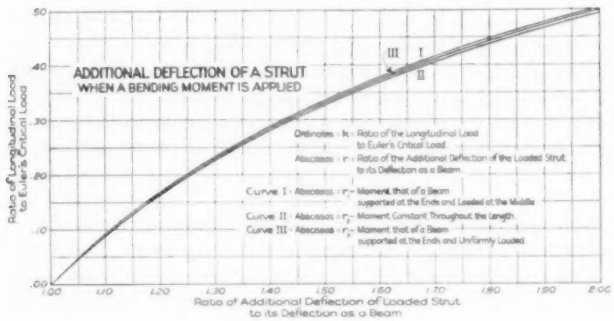


FIG. 6

stress when the deflection is known may be found in treatises on structures and technical journals.[3]

MAGNITUDE OF THE CORRECTION FOR COMMERCIAL STRUTS

The strut used in these experiments, with a slenderness ratio of 192, reaches Euler's critical load while the stress at the concave surface at the middle is far below the yield point of the material. With a factor of safety of 2.5 the value of k is 0.40, for which the additional deflection caused by a bending moment is about 67 per cent greater than the deflection calculated by the beam formulas. For short struts, which fail by compression before the critical load is reached, the coefficient k is smaller and the additional deflection differs less from the deflection as a beam.

For material with a modulus of elasticity of 29,000,000 the ultimate unit load by Euler's formula is

$$\frac{P}{A} = \frac{9.8696 \times 29{,}000{,}000}{(l/r)^2} = \frac{286{,}220{,}000}{(l/r)^2}$$

If the slenderness ratio is 100 the ultimate load is 28,622 lb. per sq. in. The allowable load per square inch on structural steel for this slenderness ratio by the American Institute of Steel Construction formulas is 11,571 lb. This is 40 per cent of the critical load for a round-end column, and the additional deflection caused by a bending moment is 67 per cent greater than the deflection as a beam.

For $l/r = 80$, Euler's load is 28,622 ÷ 0.64 = 44,720 lb. per sq. in. The American Institute of Steel Construction allows 13,279 lb. per sq. in., which is nearly 30 per cent of the critical load. From Table 5 the additional deflection when $k = 0.30$ is 43 per cent greater than that of a beam subjected to the same moment.

These examples show that the additional deflection of a strut caused by a bending moment is considerably greater than that given by the beam formulas, even for relatively short members. The calculation of the exact deflection by the formulas is laborious. This labor may be avoided by means of Table 5 or Fig. 6.

Discussion

A. M. WAHL.[4] The writer was interested in applying an approximate method to the solution of the problem of the deflection of a column subjected to a transverse load. This method consists in assuming the deflection curve of the strut in the form of a trigonometrical series and so determining the constants of this series that the work done by the external forces is equal to the stored potential energy of the system. The solution of the problem in this manner for a hinged-end strut is given in "Applied Elasticity," Timoshenko and Lessells, page 161. From this solution, the deflection at any distance x from the ends, the notation of the author's paper being used, is:

$$y = \frac{2Ql^3}{EI\pi^4}\left(\frac{\sin\frac{\pi c}{l}\sin\frac{\pi x}{l}}{1-k} + \frac{\sin\frac{2\pi c}{l}\sin\frac{2\pi x}{l}}{2^4-2^3k} + \dots\right) \quad [20]$$

Since this is a rapidly converging series, especially for practical cases where k is small, it will usually be sufficiently accurate to use only the first term of the series in practical work.

Using one term of Equation [20], the writer has computed values of r_1 given in Table 5 of Professor Boyd's paper. In the case of a transverse load at the middle, the deflection, as given by the author, is:

$$y = r_1\frac{Ql^3}{48\,EI} \quad [21]$$

Using one term of the above series, Equation [20], we find the deflection is:

$$y = \frac{Ql^3}{48EI}\left[\frac{96}{\pi^4}\frac{\sin\frac{\pi c}{l}\sin\frac{\pi x}{l}}{1-k}\right] \quad [22]$$

Since for the case considered $c = \frac{l}{2}$, $x = \frac{l}{2}$, we have by comparing [21] and [22]

$$r_1 = \frac{96}{\pi^4(1-k)} \quad [23]$$

This is the equation used by the writer in computing Table 6, wherein values of r_1 computed in this way are compared to those computed by Professor Boyd and given in Table 5 of his paper.

TABLE 6 VALUES OF r_1 AS COMPUTED BY THE USE OF EQUATION [23]

K	r_1 Computed from Equation [23]	r_1 (Computed by Prof. Boyd)
0.1	1.093	1.109
0.2	1.231	1.247
0.3	1.408	1.422
0.4	1.641	1.657
0.5	1.970	1.986
0.6	2.462	2.479
0.7	3.280	3.300
0.8	4.925	4.943
0.9	9.850	9.871

From this table it may be seen that the writer's values agree within about 1.5 per cent with those given by Professor Boyd. This suggests that in practical work it may not be necessary to use the complicated Equation [14] given by the author and used by him in the computation of Table 5 of his paper. For practical work the following approximate equation, obtained directly from [22] when

$x = \frac{c}{2}$, $c = \frac{l}{2}$, may be used:

$$y = \frac{Ql^3}{48.7\,EI}\left(\frac{1}{1-k}\right) \quad [24]$$

Of course, the computation would be facilitated by the use of the curves given by the author.

[3] E.g., E. W. Rettger, "Add-Area Method for Columns With Eccentric and Lateral Loads," *Engineering News-Record*, October 23, 1926, p. 662.

[4] Mechanics Section, Research Department, Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pa. Mem. A.S.M.E.

Equation [22] suggests the possibility of obtaining the approximate deflection of a strut under longitudinal loading and subjected to a transverse load at any point distant c from one

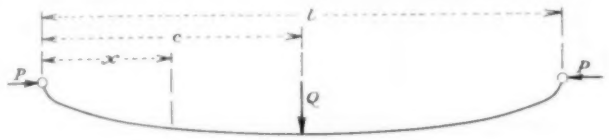


FIG. 7

end. The deflection at the middle, using [22] and [23] and remembering $x = \frac{l}{2}$, would be:

$$y = \frac{Ql^3}{48.7EI}\left(\frac{\sin\frac{\pi c}{l}}{1-k}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots [25]$$

Oftentimes it may be desirable to determine the deflection directly under the transverse load Q. In this case, $x = c$, and [22] becomes:

$$y = \frac{Ql^3}{48.7\ EI}\left(\frac{\sin^2\frac{\pi c}{l}}{1-k}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots [26]$$

These equations are more general than those given in the author's paper, since they apply to a transverse load at any point of the strut, while the author's equations hold only for a lateral load in the middle.

It must be noted, however, that these are approximate equations obtained by using one term of Equation [20]. In case of doubt, however, additional terms of [20] may easily be calculated; in this way a determination as to the accuracy of the foregoing solutions may be made for the particular case in hand.

It should also be noted that the foregoing Equations [20], [22], [23], [24], [25], and [26] apply not only to longitudinal compressive forces acting on the strut, but also to longitudinal tensile forces. In such cases, it is only necessary to change the sign of k in these equations. They thus show that where longitudinal tensile forces are acting the deflection is less than would be the case where no longitudinal forces are acting.

S. TIMOSHENKO.[5] The problem of the bending of a bar submitted to the simultaneous action of lateral and axial loads is of considerable practical importance and has been the subject of numerous investigations.[6] For many cases, there exist not only equations for calculating the deflections and moments but also tables giving directly the deflections and moments for various values of the ratio of the longitudinal force to the critical load given by Euler's formula. The cases discussed by J. E. Boyd are also well-known. They can be found in various textbooks on the strength of materials and also in such well-known handbooks as Hütte.[7]

The investigations show that if all the lateral forces have the same direction, the maximum deflection of a strut can be obtained, with an accuracy sufficient for practical applications, from the following approximate equation:

$$\delta = \frac{\delta_0}{1-\frac{P}{P_{cr}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots [27]$$

in which δ_0 denotes the maximum deflection produced by lateral loads alone and $\frac{P}{P_{cr}}$ is the ratio of the compressive force to Euler's critical load. Assuming, for instance, $\frac{P_{cr}}{P} = 0.81$, we obtain, from Equation [27]

$$\delta = \frac{\delta_0}{1-0.81} = 5.26\delta_0 \quad \ldots\ldots\ldots\ldots\ldots\ldots [28]$$

This result is very close to the exact solution in the case of a uniformly distributed lateral load. In the case of a lateral load concentrated at the middle, the error of the approximate solution, [28] is about 1 per cent. In the worst case, where constant bending moments are applied at the ends, the error is about $2^1/_2$ per cent. This error also cannot be considered as excessive if we take into consideration that the quantity EI in our equations is known with an accuracy which usually is not higher than 3 per cent. Having this in mind it seems meaningless to make the numerical calculations with five and six figures as is done in the paper.

With regard to laboratory investigation, struts submitted to the simultaneous action of an axial load and a constant bending moment have been the object of numerous experiments.[8] The results of these experiments are in good agreement with the elementary theory within the elastic limit. Beyond the elastic limit the problem becomes more complicated, but here too the theory of the buckling of struts has been developed,[9] and comparison shows a satisfactory agreement between the theory and experiment. It has been shown that in the case of a shorter strut, when the critical load is beyond the yield point of the material, a very small eccentricity in the application of the compressive forces reduces considerably the magnitude of the maximum compressive load which can be carried by the strut. This fact is of great practical importance in the design of compression members of engineering structures.

The theory shows also that, in the case of materials with a sharply defined yield point, instability occurs when the compressive stress approaches the yield point. This justifies the German practice in column design, which takes the yield point as the critical stress for all members for which $\frac{l}{i}$ (the ratio of the length to the radius of gyration) is smaller than 60.

E. O. WATERS.[10] The paper shows conclusively that the accepted flexure theory as applied to beams, columns, and a combination of the two, is correct. While the tests were all run on pin-ended struts, the fact that one series was run with no transverse load, but with a constant moment of large magnitude, and gave results checking closely with theory, is an indication that fixed-end struts also act in accordance with the theory.

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[5] Professor of Mechanical Engineering, University of Michigan, Ann Arbor, Mich. Mem. A.S.M.E.

[6] See, Tolle, *Z. D. Ver. deutsch. Ing.*, 1897, p. 855. Van der Fleet, Bulletin of the Society of Engineers of Ways of Communication, 1900–1903, S. Petersburg. H. Kayser, *Zentrabl. Baur.*, 1910, p. 304. S. Timoshenko, Annales des Travaux Publics de Belgique, 1914, p. 263. A. Morley, *Phil. Mag.*, June, 1908.

[7] See, Hütte, 25 Edition, pp. 643–646, Berlin, 1925.

[8] The references to English papers may be found in the paper by Andrew Robertson, "The Strength of Struts," Selected Engineering Papers, The Institution of Civil Engineers, London, 1925.

[9] See Th. V. Karman, Forschungsarbeiten, No. 81, Berlin 1910; M. Ros, Zürich International Congress for Applied Mechanics, 1926, Proceedings, pp. 364–367, H. M. Westergaard and Wm. R. Osgood, Am. Soc. Mech. Eng. Trans., vol. 50, 1928.

[10] Asst. Prof. M. E., Yale University, New Haven, Conn.

It is true that experimental verifications of stress analyses are best made by means of deflection measurements; but for design purposes, some function of the member in question other than deflection is generally more useful. In the present case, the writer suggests that a curve of k versus "ratio of additional moment" should be plotted. Then, to find the stress in a transversely loaded strut, it is only necessary to multiply the moment due to the transverse load alone, by the proper factor, and substitute in the equation

$$p = \frac{P}{A} \pm \frac{My}{I}$$

For the case of a concentrated load Q,

$$M = EI\frac{d^2y}{dx^2} = -\frac{Q}{2}x - Py,$$

the solution of which is

$$M = Qj\left(\frac{\sin l/2j}{\tan l/j} - \cos\frac{l}{2j}\right)\sin\frac{x}{j}, \text{ where } j = \sqrt{\frac{EI}{P}}$$

$$= Qj\frac{\tan L/2j}{2} \qquad \text{when } x = \frac{l}{2}$$

According to the author's notation, $P = k \times$ Euler load $= \frac{K\pi^2EI}{L^2}$ and therefore $\frac{1}{j} = \frac{\pi}{l}\sqrt{k}$.

Then

$$M = -\frac{QL}{2\pi}\sqrt{\frac{1}{k}}\tan\frac{\pi}{2}\sqrt{k}$$

For transverse loading alone,

$$M = -\frac{Ql}{4}$$

hence

$$R_1 = \frac{2}{\pi\sqrt{k}}\tan\frac{\pi}{2}\sqrt{k}$$

For a constant moment $-M_1$,

$$M = EI\frac{d^2y}{dx^2} = -M_1 - Py$$

the solution of which is

$$M = m, \sec\frac{l}{2j} \qquad \text{when } x = \frac{l}{2}$$

Making the same substitution for $1/j$ as before, $M = M_1 \sec\frac{\pi}{2}\sqrt{k}$ and $R_2 = \sec\frac{\pi}{2}\sqrt{k}$.

If the transverse load is uniformly distributed,

$$M = EI\frac{d^2y}{dx^2} = -\frac{wlx}{2} + \frac{wx^2}{2} - Py$$

the solution of which is

$$M = -wj^2\left(\sec\frac{l}{2j} - 1\right) \text{ when } x = l/2$$

$$= -\frac{wl^2}{\pi^2k}\left(\sec\frac{\pi}{2}\sqrt{k} - 1\right)$$

M for transverse load only $= -wl^2/8$, hence $R_3 = \frac{8}{\pi^2}\left(\frac{\sec\frac{\pi}{2}\sqrt{k} - 1}{k}\right)$.

By plotting the equations for R_1, R_2, and R_3 in a form similar to Fig. 5 and 6, they can be put into shape suitable for general designing purposes.

In the case of the concentrated transverse load, it is interesting to note the effect of shifting the point of application away from the mid-point of the strut. Suppose that Q is applied at one-quarter the strut length; then the ratio of the maximum moment in the transversely loaded strut (not to be confused with the moment at the point of application of transverse load) to the moment for the transverse load only, is 1.563, for $k = 0.49$. If Q is applied at the middle, then $R_1 = 1.786$ for the same value of k. In other words, it appears that the load can be shifted anywhere along the second and third quarter-lengths of the strut without greatly affecting the value of R_1, and the error is on the side of safety.

Author's Closure

The principal object of this investigation was the comparison of experiment with theory. For this purpose exact theoretical equations are better fitted than approximations. Exact theory also furnishes a standard for the accuracy of approximate or empirical equations. For instance, it is possible to state that Equation [23] for a transverse force at the middle is correct to within 1.5 per cent and that Equation [27] for any additional moment is correct to within 2.5 per cent by comparison with the exact equations of this paper or with other similar equations previously published.

In these problems, as in many others, it is easier to derive the exact equations than the approximate ones.

Mention has been made of the case of a transverse force which is not applied at the middle. This is covered indirectly by the tables and curves. For r_1 the diagram of the additional moment is a triangle. For r_2 the diagram is a rectangle. Almost any other loading will give a diagram between the triangle and the rectangle, and the corresponding deflection will fall in the small space between curves *I* and *II* of Fig. 5. A concentrated force which is not in the middle will evidently give a deflection very close to that of curve *I*.

Emphasis has been placed on the deflection rather than on the maximum moment or maximum stress because it was desired to keep as close as possible to the quantities actually measured. For design purposes the moment or stress would be preferable.

Mention has also been made of the unnecessary five-place accuracy of the tables. The point is well taken. For practical design Equations [23] or [27] are sufficiently accurate for most purposes. For plotting of curves and comparison with experiment the tables are carried one place farther than necessary. On the other hand, a technical paper is a matter of record which may be used for purposes other than those which the writer has in mind. In the use of published material it is much easier to cut off significant figures when too many are given than it is to calculate them when too few are given. The writer of a technical paper owes it to his readers to give his data as complete as possible, to make his calculations accurately, and to carry the results as far as any future reader is likely to need. Again, if time permits, the degree of accuracy of any computation should always be greater than that of the data. If there is a possible error of 2 per cent in the data and an equal error in the calculations, an

error of 4 per cent is possible in the final result, while if the error in the calculations is only one-tenth of 1 per cent, the error in the final result is practically that of the data.

Dr. Timoshenko has given a number of references to previous publications, a number of which are not available to the author. A very complete discussion is that of Prof. John Perry in Philosophical Magazine of March, 1892. Professor Perry derives several equations in practically the same form as those given in this paper. He even anticipates Professor Ott's explanation of the fall of the temple of Gaza.

APM-51-11

Design of Columns of Varying Cross-Sections

By DR. A. DINNIK,[1] DNIEPROPETROVSK, UNION OF SOVIET SOCIALIST REPUBLICS

(Translated by M. Maletz,[2] Milwaukee, Wis.)

IN THIS article the author develops formulas for the design of columns of varying cross-sections. Tables and graphs and many illustrative problems facilitating the correct use of these formulas are given.

1 In designing compression members for use as booms and masts for derricks, connecting rods for engines, airplane struts, etc., it is desirable to reduce their weight to a minimum and yet maintain stability. To meet these conditions it is the usual practice to make these members with a varying cross-section.

It is very common practice to use Euler's formula when investigating members of varying cross-section for buckling. While this is correct for constant cross-section members, it is entirely wrong to use this formula for compression members of varying cross-sections.

It is the purpose of this article[3] to present formulas and coefficients of stability which will enable the designer to select the proper section for a given column without setting up the differential equations and their integration.

2 Let us consider Euler's theory. Let a column of constant cross-section be compressed by axial forces applied at its ends. If these forces are comparatively small, the column remains in a straight line, but if the forces are increased beyond the so-called critical value, the column will begin to buckle.

Let us consider the more important cases.

(A) Column with both ends hinged (Fig. 1):

The differential equation[4] of the elastic curve is

$$EIy'' + Py = 0 \quad \ldots\ldots [1]$$

From the conditions at the ends, the critical load is found to be

$$P_e = \frac{\pi^2 EI}{l^2} \quad \ldots\ldots [2]$$

which is the so-called Euler's critical load.

(B) A column with both ends fixed:

The differential equation of the elastic curve for this case is

$$EIy'' + Py = n \quad \ldots\ldots [3]$$

Integrating and taking into consideration the conditions at the ends, we obtain the critical load

$$P_e = \frac{4\pi^2 EI}{l^2} \quad \ldots\ldots [4]$$

where

l = length of column
E = modulus of elasticity
I = moment of inertia of the cross-section
P_e = critical load (Euler)

The Equations [3] and [4] are commonly used for designing columns of any cross-section.

Let us next consider cases where the rigidity of columns varies according to an exponential law and a law of powers.

The Rigidity of the Column Varies According to an Exponential Law

3 The rigidity in bending is called the product EI, where E denotes the modulus of elasticity and I the moment of inertia of the cross-section of the member.

Let us consider a column (Fig. 2) the cross-section of which decreases from its middle to its ends according to a law of exponents. If I denotes the moment of inertia of the mid-section (largest) and i the moment of inertia of the end cross-section, then the moment of inertia of any cross-section distant x from the middle of the column will be

$$I_x = Ie^{-\frac{mx}{l}} \quad \ldots\ldots [5]$$

i.e., the moment of inertia changes according to an exponential law. The coefficient m characterizes the degree of change of the rigidity of the column. When the value of m is very small, the column will be slightly tapered toward its ends and will differ very little from a column of uniform cross-section.

With an increasing value of m the column becomes more tapered until the ends become points.

The elastic-curve equation for such a column with both ends hinged is

$$EIe^{-\frac{mx}{l}}y'' + Py = 0 \quad \ldots\ldots [6]$$

and the critical load is then

$$P = K\frac{EI}{l^2} \quad \ldots\ldots [7]$$

where K is the so-called coefficient of stability (for its derivation see Appendix No. 2). Table 1 contains values of K for different ratios of $i:I$; from Fig. 3, other intermediate values of K may be obtained.

TABLE 1

$i:I$	K
0.01	1.27
0.2	5.68
0.4	7.34
0.6	8.42
0.8	9.23
1.0	9.87

For columns with both ends fixed, the coefficients of stability for different ratios $i:I$ are given in Table 2 and Fig. 4.

TABLE 2

$i:I$	K
0.01	3.29
0.2	17.57
0.4	24.92
0.6	30.55
0.8	35.29
1.0	39.48

Example 1. A tapered airplane pine strut is 6 ft. long and has an elliptical cross-section. The axes of the largest (middle) cross-section are $2a = 4$ in. and $2b = 1.5$ in.; the axes of the smallest (end) ellipses are $2a = 2.75$ in. and $2b = 1.25$ in. Assuming an allowable stress of 1200 lb. per sq. in. and taking $E = 1{,}200{,}000$, examine this strut for buckling.

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[3] For a complete treatise on this subject the reader is referred to the author's articles in the bulletins of the Ekaterinoslav Institute of Mines (Russian). Some results are also given in Timoshenko-Lessells' "Applied Elasticity," p. 176, Pittsburgh, Pa., 1925.

[4] The solution of this equation can be found in any text on mechanics of materials.—Translator.

Contributed by the Applied Mechanics Division and presented at the New England Industries Meeting, Boston, Mass., Oct. 1 to 3, 1928, of The American Society of Mechanical Engineers.

Solution. We compute first the moment of inertia of the middle cross-section about its axis of the smallest rigidity, i.e., where the danger of buckling is the greatest.

$$I = \frac{2\pi \times (0.75)^3}{4} = 0.663$$

For the end section the moment of inertia $i = 0.264$ and $i : I = 0.4$. For the case of hinged ends we find in Table 1 $K = 7.34$ and from Equation [7] $P = 960$ lb. The compressive stress is

$$S = \frac{960}{\pi \times 1.375 \times 0.625} = 355 \text{ lb. per sq. in.}$$

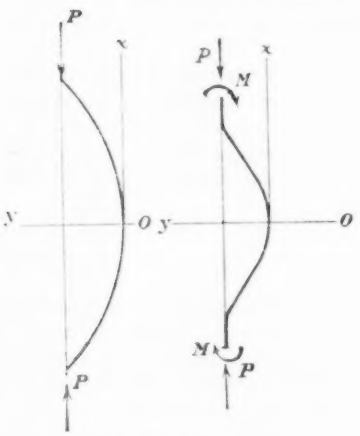


Fig. 1

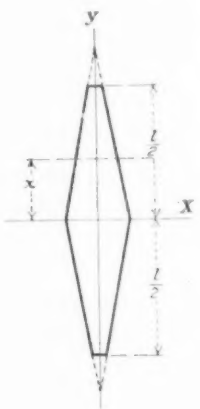


Fig. 2

which is below the allowable stress. (Note that in calculating S we used the cross-section area of the small end section.)

For the case of fixed ends, we find in Table 2 that $K = 24.92$, and from Equation [7] $P = 3210$ lb. and the stress $S = 1190$ lb. per sq. in. Using a factor of safety 5, the allowable load for hinged ends will be 192 lb., and for fixed ends it will be 642 lb. In both cases the stress is below the maximum stress allowed.

The Rigidity of the Column Varies as a Law of Powers

4 Let the moment of inertia of the cross-section of a column change along its length from the middle toward its ends according to a law of some power

$$I_x = I \left\{ 1 - (1 - k^2) \frac{x}{l} \right\}^m \quad \ldots\ldots\ldots\ldots [8]$$

where

$$k^{2m} = \frac{i}{I} \quad \ldots\ldots\ldots\ldots\ldots\ldots [9]$$

In these equations I denotes the moment of inertia of the mid-section, and x the distance from the middle to any section whose moment of inertia is I_x (Fig. 2).

This case is more important than that of Equation [7] for the following reasons: first, for $m = 0$ we get a column of constant cross-section; second, for $m = 2$ we obtain a braced pyramidical column; third, for $m = 3$ a hollow cone, and for $m = 4$ a solid cone, etc.

The differential equation of the elastic curve has the following form

$$EI \left\{ 1 - (1 - k^2) \frac{x}{l} \right\}^m y'' + Py = 0 \quad \ldots\ldots [10]$$

This equation can be readily integrated by means of Bessel's functions for any value of m, except for $m = 2$ and $m = 4$, when common (elementary) integration is used (see Appendix No. 1).

The Pyramidical Column

5 The coefficients of stability K for both hinged and fixed-end columns for different ratios of $i : I$ are given in Table 3[5] and Fig. 5, the general form of the formula for the critical load being

$$P = K \frac{EI}{l^2}$$

TABLE 3

$i : I$	K_1 (hinged ends)	K_2 (fixed ends)
0.0	1.00	1.00
0.1	5.40	14.39
0.2	6.37	18.43
0.4	7.61	25.54
0.6	8.51	30.79
0.8	9.24	35.35
1.0	9.87	39.48

A Conical Column

6 For a column with both ends hinged, we obtain for the critical load

$$P = K_3 \frac{EI}{l^2} \quad \ldots\ldots\ldots\ldots\ldots [11]$$

and for a similar column with ends fixed

$$P = K_4 \frac{EI}{l^2} \quad \ldots\ldots\ldots\ldots\ldots [12]$$

Table 4 and Fig. 6 give the coefficients of stability for these two cases.

General Case

7 For all other values of m, Equation [10] is integrated by means of Bessel's functions, and from conditions at the ends

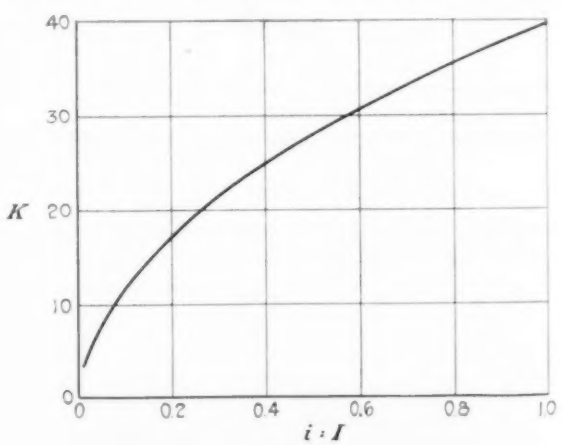


Fig. 3

[5] Bleich in his new treatise on bridge design, "Theorie u. Berechnung der eisernen Bruecken," Berlin, 1924, pp. 136–142, considers the case of a pyramidical column, the rigidity of which varies as the law of squares, i.e., when $m = 2$. The coefficients obtained by Bleich are practically the same as those in Table 3, except for small ratios of $i : I$. For instance, for $i : I = 0.04$ Bleich gives $K = 4.72$, whereas in Table 3 we find $K = 4.44$; for $i : I = 0$ Bleich gives $K = 3.32$, whereas in Table 3 $K = I$. The discrepancy is due to Bleich's making use of Ritz's method, which is an approximate one only, and computes the first two terms only, which are not sufficient to obtain accurate results for small ratios of $i : I$. The author solves the same problem by integrating the differential equation for flexure.

we can calculate the critical load. The coefficients of stability for values of $m = 1$ are given in Table 5 and Fig. 7 and $m = 3$ in Table 6 and Fig. 8.

TABLE 4

$i:I$	K_3 (hinged ends)	K_4 (fixed ends)
0.0	0.00	0.00
0.1	4.31	13.30
0.2	6.02	18.23
0.4	7.48	25.23
0.6	8.47	30.68
0.8	9.23	35.33
1.0	9.87	39.48

TABLE 5

$i:I$	K_5 (hinged ends)	K_6 (fixed ends)
0.0	3.78	5.78
0.2	7.01	20.35
0.4	7.87	26.16
0.6	8.61	31.03
0.8	9.27	35.42
1.0	9.87	39.48

TABLE 6

$i:I$	K_7 (hinged ends)	K_8 (fixed ends)
0.0	0.00	0.00
0.2	6.14	18.44
0.4	7.53	25.34
0.6	8.50	30.71
0.8	9.23	35.32
1.0	9.87	39.48

Example 2. An airplane pine strut is 6 ft. 6 in. long and has a rectangular cross-section. The thickness of the strut is $^3/_4$ in. and remains constant throughout its entire length. The width varies according to a straight-line law, and is 4 in. in the middle and 2.4 in. at the ends; the modulus of elasticity $E = 1{,}200{,}000$. Examine this strut for buckling.

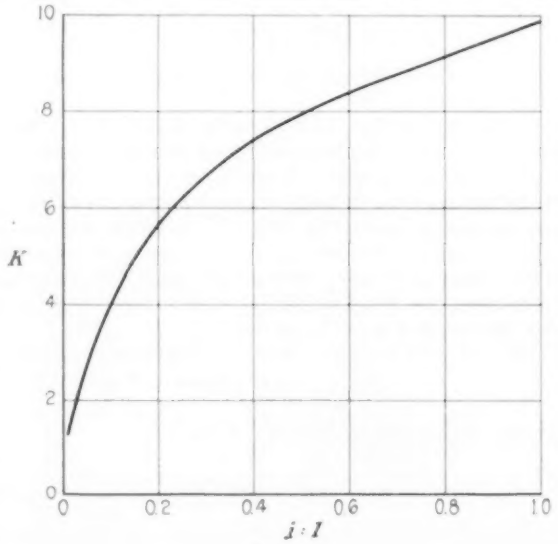


FIG. 4

Solution. The rigidity of the strut varies as a straight line, i.e., $m = 1$, the ratio of $i:I = 0.6$, the coefficient of stability for hinged ends from Table 5 is $K = 8.61$, and the critical load $P = 239$ lb. The maximum stress is about 323 lb. per sq. in. This is below the allowable stress. If the ends of the strut are fixed, then $K = 31.03$, $P = 860$ lb., and the maximum stress will be 478 lb. per sq. in., which is also below the allowable stress.

Example 3. A supporting column in a mine structure which is 65 ft. high carries a load of 66,000 lb. It is built up of four equal leg angles connected with lattice bars. Select the size of the angles and the distances between them, assuming an allowable compressive stress of 7000 lb. per sq. in. and a factor of safety 4.

Solution. Considering the column under compression only, it is found that the area of each angle must be 2.36 sq. in. This corresponds to a $3^1/_2 \times 3^1/_2 \times {}^3/_8$ angle, the area of which is 2.48 sq. in. Then the compressive stress will be 6650 lb. per sq. in.

Let us now select the distance between the angles so that the column will not buckle. If the width of the column is taken as 12 in., then the moment of inertia of the section will be (according to the well-known theorem of moments of inertia about parallel axes)

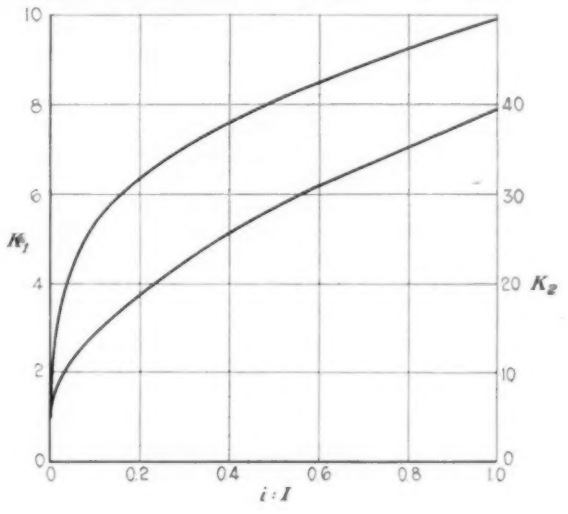


FIG. 5

$$i = 4 \times 2.9 + 4 \times 2.48 \times (6 - 1.01)^2 = 258.6 \text{ in.}^4$$

where

2.9 = the moment of inertia of each angle about the axis through its center of gravity

1.01 = the distance from the back of the angle to its center of gravity. (All figures are taken from Carnegie's Handbook.)

If the section of the column is uniform, the critical load in accordance with Euler's theory will be $P = 126{,}000$ lb.; i.e., the factor of safety is less than 2. In order to increase the stability we shall make the column wider in the middle only. Let us space the angles in the middle of the column $20^1/_2$ in. back to back. The moment of inertia of the new section consisting of the same angles will then be

$$I = 4 \times 2.9 + 4 \times 2.48 \times (10.25 - 1.01)^2 = 859.5 \text{ in.}^4$$

and the ratio $i:I = 0.3$.

As the moment of inertia varies along the length very closely as the law of squares, the coefficient of stability may be taken from Table 3. For the ratio $i:I = 0.3$, $K = 7.05$, and the critical load will be $P = 295{,}000$ lb.; i.e., by increasing the width of the column in its middle we have increased its stability more than twice with a factor of safety of more than 4.

It is evident that by just widening the column in its middle we can increase the stability of the column without increasing the area of the angles provided; that the connections between the angles are rigid enough to prevent any relative displacement of the angle so that they will act as a whole.

Example 4. A cast-iron supporting column of a transmission line has the form of a hollow double cone. The diameter of the middle (largest) cross-section is 10 in. and that of the end is $7^1/_2$ in., the thickness of the wall is $^3/_4$ in., and the height of the column is 23 ft. 9 in. Examine the column for buckling.

Solution. The moment of inertia of the largest cross-section is $I = 234.7$ and that of the smallest end is $i = 91.7$ and the ratio $i:I = 0.395$, say 0.4. The moment of inertia of the cross-sections in a cone varies approximately as the law of cubes. Hence from Table 6 ($m = 3$ and $i:I = 0.4$) we find that $K = 7.53$ and $P = 283{,}000$ lb., the average modulus of elasticity for cast iron having been taken $E = 13{,}000{,}000$ lb. per sq. in. Using a factor of safety 6, the column can safely carry 47,150 lb., and the maximum compressive stress will be 2965 lb. per sq. in. (When this column was actually designed a compressive stress of 5000 lb. per sq. in. was allowed.)

Example 5.[6] Design a wooden conical strut for a hydroplane when the load is 500 kg. The strut is 2 m. long and of circular cross-section. Assume an allowable stress of 50 kg. per sq. cm. and a factor of safety of 5.

Solution. Allowing a compressive stress of 50 kg. per sq. cm., the smallest cross-sectional area of the strut will be 10 sq. cm., its diameter $d = 3.56$ cm., and its moment of inertia will be $i = 8$ cm.[4] The moment of inertia of the larger section when a factor of safety 5 is allowed will be

$$I = \frac{Pl^2}{KE} = \frac{5 \times 500 \times 40000}{K \times 100000} = \frac{1000}{K}$$

Substituting the corresponding coefficient of stability from Table 4 for different ratios of $i:I$ we compute and tabulate a new Table 7, containing the following values:

1 The moments of inertia I of the largest cross-section (middle)
2 The largest moments of inertia i of the end sections
3 The corresponding diameters D and d of the middle and end cross-sections
4 The ratio of the weight of the designed strut Q to the weight of a cylindrical strut Q_0 designed according to Euler's formula.

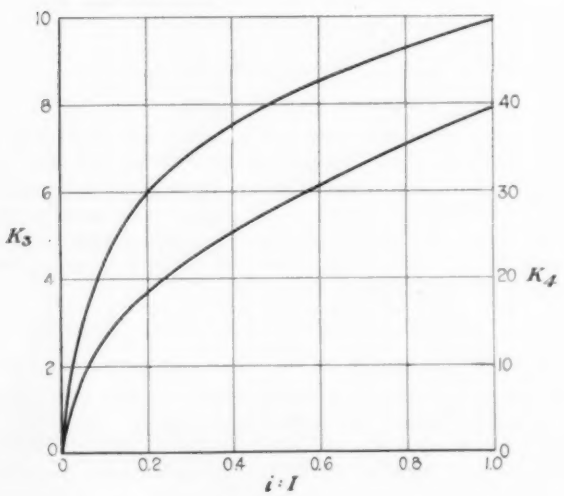


FIG. 6

The last ratio shows at a glance that by making the strut conical in shape with a ratio $i:I = 0.1$ its weight can be reduced to 63 per cent of that of a cylindrical strut. Further reduction is impossible as the material will be stressed beyond the allowable range of stresses.

TABLE 7

$i:I$	I cm.[4]	i cm.[4]	D cm.	d cm.	$Q:Q_0$
0.1	208	21	8.0	4.5	0.63
0.2	166	38	7.5	5.1	0.71
0.4	134	54	7.2	5.7	0.81
0.6	118	71	7.0	6.2	0.88
0.8	108	86	6.8	6.6	0.97
1.0	101	101	6.7	6.7	1.00

[6] Values in metric units.

The same procedure may be followed when the cross-section is other than circular, e.g., an elliptical (elliptical cone). The moment of inertia should be taken about the major axis, i.e., about the axis of the smallest rigidity.

Conclusions

The method of using Euler's formula for designing tapered columns is shown as being incorrect. Coefficients of stability

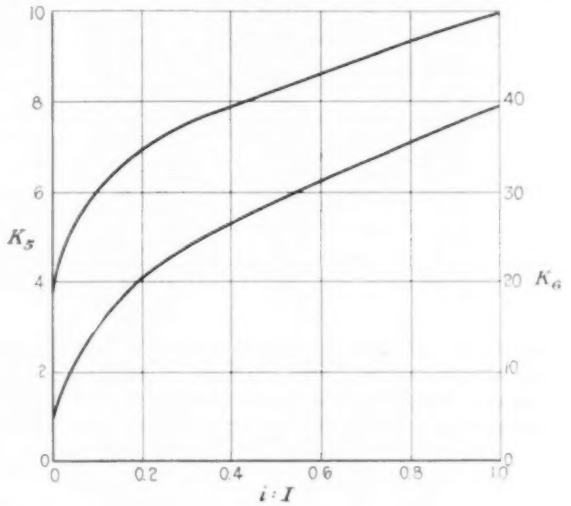


FIG. 7

for columns the rigidity of which varies according to a law of exponents or a law of powers are given in Tables 1 to 6, and their use is facilitated by graphs. A column of varying cross-section being lighter than one of constant cross-section is evidently of great advantage in cases where reduction in weight is important. Should the rigidity of a column vary according to some other laws than those mentioned in this article, Tables 1 to 6 may also be used, and the designer should then decide which of the laws in this article is closer to his case.

Appendix No. 1

Solution of Equation [6]

$$EIe^{-\frac{mx}{l}}y'' + Py = 0 \quad \ldots\ldots\ldots\ldots\ldots [6]$$

This equation can be readily integrated by using Bessel's (cylindrical) functions. Its integral is[7]

$$y = AJ_o(u) + BN_o(u) \quad \ldots\ldots\ldots\ldots\ldots [13]$$

where

$$u = \frac{2l}{m}\sqrt{\frac{P}{EI}}\, e^{+\frac{mx}{2l}} \quad \ldots\ldots\ldots\ldots\ldots [14]$$

From the conditions at the ends it is found

$$\Delta(U) = J_o(U)N_1(Ue^{\frac{m}{2}}) - N_o(U)J_1(Ue^{\frac{m}{2}}) = 0 \ldots\ldots [15]$$

[7] For a definition of the terms in the solution of these equations the reader not familiar with Bessel's functions is referred to some treatise. Tables of cylindrical functions for calculating the roots of Equations [15] and [16] are given in G. N. Watson's "Theory of Bessel's Functions," Cambridge, 1922.

and the critical load

$$P = [mUe^{\frac{m}{2}}]^2 \frac{EI}{l^2} = K \frac{EI}{l^2} \quad \ldots\ldots\ldots\ldots [7a]$$

For a column with both ends hinged the roots of Equation [15] for different ratios $i:I$ are given in Table 8.

TABLE 8

$i:I$	U
0.01	2.448
0.20	3.31
0.40	4.68
0.60	7.33
0.80	15.22
1.00	∞

For columns with both ends fixed, U may be calculated from the following equation, obtained from the conditions at the ends,

$$\Delta(U) = J_1(U)N_1(Ue^{\frac{m}{2}}) - N_1(U)J_1(Ue^{\frac{m}{2}}) = 0 \quad \ldots\ldots [16]$$

The roots U of this equation are given in Table 9.

TABLE 9

$i:I$	U
0.01	3.94
0.20	5.82
0.40	8.61
0.60	13.97
0.80	29.76
1.00	∞

Solution of Equation [10]

$$EI\left\{1 - (1 - k^2)\frac{x}{l}\right\}^m y'' + Py = 0 \quad \ldots\ldots\ldots [10]$$

1 In case $m = 2$ (for a pyramidical column)

$$y = \sqrt{u}\,\{A \cos(\mu \log_e u) + B \sin(\mu \log_e u)\} \quad \ldots\ldots [17]$$

where

$$u = 1 - (1 - k^2)\frac{x}{l}$$

$k^{2m} = i:I$, A and B are constants of integration

and

$$\mu^2 = \frac{Pl^2}{EI(1 - k^2)^2} - \frac{1}{4} \quad \ldots\ldots\ldots\ldots [18]$$

In the case of a column with both ends hinged, we obtain from the conditions at the ends

$$P = \frac{(1 + 4\mu^2)(1 - k^2)^2}{4l^2} EI = K \frac{EI}{l^2} \quad \ldots\ldots [10a]$$

where 2μ is the root of equation

$$\tan(2\mu \log_e k) = 2\mu \quad \ldots\ldots\ldots\ldots [19]$$

For a column with both ends fixed, instead of Equation [19] we have

$$2\mu \log_e k = \pi \quad \ldots\ldots\ldots\ldots [19a]$$

2 In case $m = 4$ (conical column), the integral of Equation [10]

$$y = A \frac{\cos u}{u} + B \frac{\sin u}{u} \quad \ldots\ldots\ldots\ldots [20]$$

where

$$u = \sqrt{\frac{Pl^2}{EI(l - k^2)}} \quad \frac{1}{1 - (1 - k^2)\frac{x}{l}} \quad \ldots\ldots [21]$$

and from the conditions at the ends we obtain for a column with both ends hinged

$$P = 4(1 - k^2)^2 U^2 \frac{EI}{l^2} = K \frac{EI}{l^2} \quad \ldots\ldots\ldots [22]$$

where U is the root of equation

$$\tan \frac{1 - k^2}{k^2} U + U = 0 \quad \ldots\ldots\ldots\ldots [23]$$

For a column with both ends fixed U is the root of equation

$$(k^2 + U^2) \tan \frac{1 - k^2}{k^2} U = (1 - k^2)U \quad \ldots\ldots\ldots [24]$$

3 For a general case, for any value of m except for $m = 2$ and $m = 4$, as stated before, Equation [10] is integrated by means of Bessel's function. The integral of Equation [10] is

$$y = \sqrt{1 - (1 - k^2)\frac{x}{l}}\left\{AJ_{\frac{1}{2-m}}(u) + BJ_{-\frac{1}{2-m}}(u)\right\} \quad \ldots [25]$$

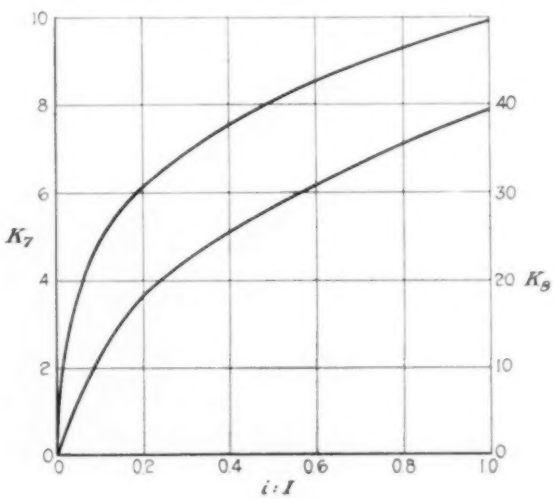


FIG. 8

where

$$u = \frac{2l}{(2 - m)(1 - k^2)}\sqrt{\frac{P}{EI}}\left\{1 - (1 - k^2)\frac{x}{l}\right\}^{\frac{2-m}{2}} \quad \ldots [26]$$

Appendix No. 2

To find the coefficients of stability, one proceeds as follows:

1 From the given moments of inertia of the end and mid-sections find k from Equation [9].

2 Inserting this value of k in Equation [19], calculate its first root (2μ) different from zero; and finally

3 Substituting k and 2μ in Equation [10a] obtain K.

This procedure should be followed in computing other coefficients in all other cases.

Discussion

JAMES E. BOYD.[8] The author has derived the equations for the ultimate strength of five forms of columns of varying cross-section and has computed the constants for both fixed ends and hinged ends. He has given these constants numerically and graphically and has applied the results to several examples of practical design. The paper represents a considerable contribution to the theory of tapered compression members.

[8] Professor of Mechanics, The Ohio State University, Columbus, Ohio. Mem. A.S.M.E.

Since the equations for hinged-end columns with rigidity varying as a law of powers have been derived by the writer,[9] he is naturally interested in comparing his work with that part of the author's paper which covers these forms.

In order to make these comparisons, it is necessary that the numerical results be reduced to the same units. The author expresses the relative strength by the letter K, which is a constant to be used in Euler's formula in place of 9.87. The writer uses the "strength ratio." This is the ratio of the strength of the tapered column to the strength of a uniform column of cross-section equal to the maximum of the tapered column.

Table 10 gives the author's values of K for hinged-end columns from Tables 5, 3, 6, and 4 (for m = 1, 2, 3, and 4 respectively) together with the corresponding values of the strength ratio, calculated by dividing K by 9.87.

TABLE 10

	m = 1		m = 2		m = 3		m = 4	
i:I	K_5	Str. ratio	K_1	Str. ratio	K_7	Str. ratio	K_9	Str. ratio
0.0	3.78	0.383	1.00	0.101	0.00	0.00	0.0	0.0
0.1			5.40	0.547			4.81	0.487
0.2	7.01	0.710	6.37	0.645	6.14	0.622	6.02	0.610
0.4	7.87	0.797	7.61	0.771	7.53	0.763	7.48	0.758
0.6	8.61	0.872	8.51	0.862	8.50	0.861	8.47	0.858
0.8	9.27	0.939	9.24	0.936	9.23	0.935	9.23	0.935

(Table 4 as printed gives 4.31 for K_3 when i:I = 0.1. Since 1000 ÷ 208 in Table 7 gives 4.81 and a recalculation by the writer results in the same value, he assumes there has been a typographical or clerical error.)

The author uses i:I, the ratio of the moment of inertia at either end to the moment of inertia at the middle, as the second variable. His expressions for moment of inertia are given in terms of the distance of the section from the middle of the column. The writer takes his origin of coordinates at the point beyond the end of the column at which the moment of inertia would be zero if the surfaces were extended to it. His expressions for the moment of inertia at a section at a distance x from the origin are $I = Cx$, Cx^2, Cx^3, and Cx^4, corresponding to m = 1, 2, 3, and 4 of the author's equations. These expressions apply to one-half of each column. The two halves are symmetrical with respect to

TABLE 11

		Strength ratio:				Strength ratio:	
k	i:I	Theoretical	Experimental	k	i:I	Theoretical	Experimental
($I = Cx$, m = 1.0)				($I = Cx^3$, m = 3)			
0.01	0.01	0.5952		0.01	0.000001	0.0142	
0.05	0.05	0.6257		0.05	0.000125	0.0687	
0.10	0.10	0.6570		0.10	0.001	0.1300	
0.20	0.20	0.7101		0.20	0.008	0.2419	
0.30	0.30	0.7560	0.750	0.30	0.027	0.3459	0.367
0.40	0.40	0.7974	0.790	0.40	0.064	0.4452	0.467
0.50	0.50	0.8357	0.824	0.50	0.125	0.5416	0.530
0.60	0.60	0.8717	0.865	0.60	0.216	0.6359	0.624
0.70	0.70	0.9058	0.897	0.70	0.343	0.7284	0.700
0.80	0.80	0.9382	0.932	0.80	0.512	0.8194	0.825
0.90	0.90	0.9684	0.963	0.90	0.729	0.9085	0.912
1.00	1.00	1.0000	1.000	1.00	1.000	1.0000	1.000
($I = Cx^2$, m = 2)				($I = Cx^4$, m = 4)			
0.01	0.0001	0.2176		0.01	0.00000001	0.0004	
0.05	0.0025	0.2841		0.05	0.00000625	0.0090	
0.10	0.01	0.3492		0.10	0.0001	0.0326	
0.20	0.04	0.4497		0.20	0.0016	0.1071	
0.30	0.09	0.5341		0.30	0.0081	0.2108	
0.40	0.16	0.6107		0.40	0.0256	0.3067	
0.50	0.25	0.6821		0.50	0.0625	0.4170	0.414
0.60	0.36	0.7480		0.60	0.1296	0.5306	0.527
0.70	0.49	0.8152		0.70	0.2401	0.6463	0.641
0.80	0.64	0.8784		0.80	0.4096	0.7632	0.754
0.90	0.81	0.9399		0.90	0.6561	0.8814	0.878
1.00	1.00	1.0000		1.00	1.0000	1.0000	1.000

(The data for Table 11 are from Table XXXVII and from Tables XXXIII, XXVI, and XVII of Bulletin No. 25 of The Ohio State University Engineering Experiment Station.)

the middle section. For instance, I = Cx^4 represents a cone with the origin at the vertex. One half of the column is a frustum of this cone extending from $x = kd$ to $x = d$. In the derivation of the equations for the ultimate strength the writer uses the ratio k as one variable and gives most of his results with it instead of i:I. In Table XXXVII of Bulletin No. 25, however, he has summarized his results for the four forms in terms $\sqrt{\frac{i}{I}}$, and has plotted the corresponding curves in Fig. 22. Table 11 gives this summary, using the ratio i:I in place of the square root of the ratio.

A comparison of Table 10 with Table 11 for m = 1 shows that three of the strength ratios are identical, while the figures for i:I = 0.8 differ by less than unity in the third place. The only difference is for i:I = 0, for which the author's figure is 0.383, while the writer's, which is not given in Table 11, is 0.586. Since a value of less 0.1 for i:I would never be used, this discrepancy is unimportant.

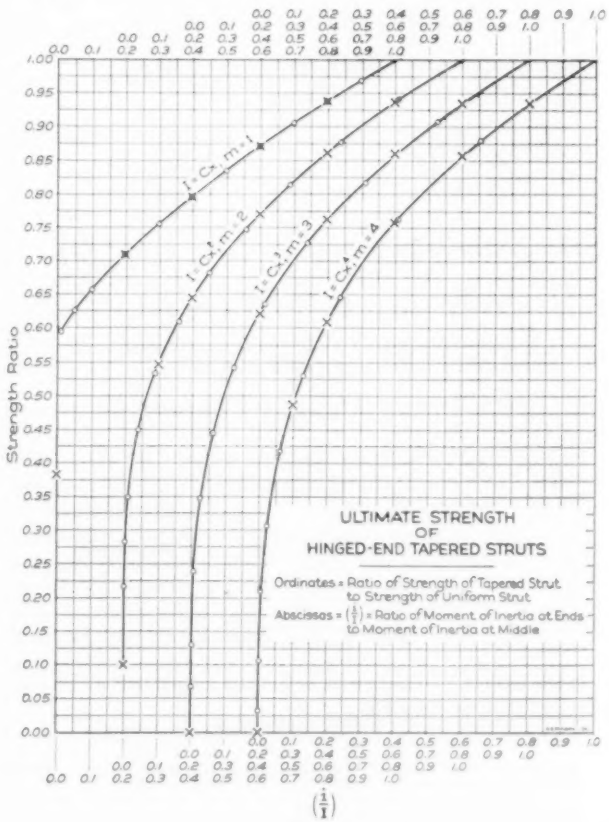


FIG. 9

For the other ratios of i:I the tables may best be compared by means of curves. This is shown on Fig. 9. The circles represent the writer's results as given in Table 11, while the crosses show the author's values from Table 10. The two sets of points for each value of i:I lie on the same curve.

While the fundamental mechanics and mathematics for both sets of results are the same, the methods of solution are considerably different. The exact agreement of the final numerical results as shown by the curves proves the correctness of both. The curves of Fig. 9, therefore, may be used wth confidence as representing accurately the relation of the strength ratio to the ratio of the moment of inertia at the ends to the moment of inertia at the middle.

In order to compare theory with experiment, the writer ran a series of tests on three types of tapered struts made of cold-rolled steel. The deflection of each strut was measured for a number of

[9] Bulletin No. 25, The Ohio State University Engineering Experiment Station, September 15, 1923; and pp. 24–36, Technologic Paper No. 152, U. S. Bureau of Standards.

loads at a known eccentricity, and compared with the theoretical value. The strut was then adjusted to a very small eccentricity for the determination of Euler's critical load at which it could be deflected a relatively large amount by a slight touch applied transversely with the little finger.

These ultimate loads from Tables XXXIII, XXVI, and XVII of the bulletin are included in Table 11, and are plotted on Fig. 10. The agreement of the experiments with the accurately drawn theoretical curves adds confidence in the theory. A comparison of the experimental deflections at the middle and at one-sixth the length from the ends with the theoretical values is equally satisfactory.

The author calculates in Table 7 the saving of material by the use of a tapered strut. The last column of this table, however, does not give the ratio of the volume of the tapered strut to the volume of a uniform cylindrical strut of equal strength. It does give the ratio of the volume of the tapered strut. For

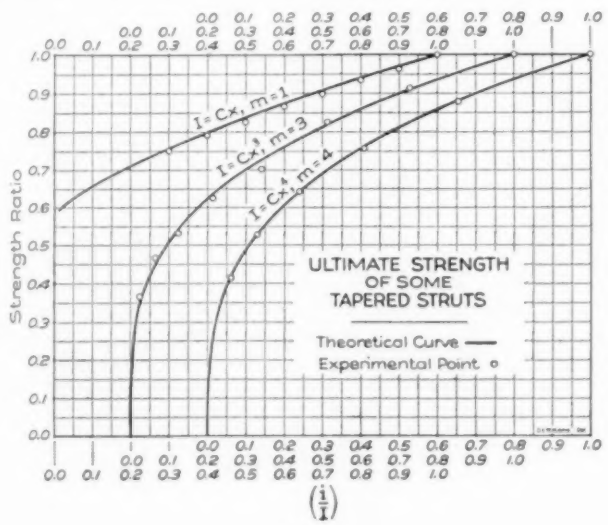


FIG. 10

instance for $i:I = 0.1$ the maximum diameter from Table 7 is 8.0 cm. and the diameter at the ends is 4.5 cm. The average diameter squared for the frustum of the cone is

$$\frac{8\times 8 + 8\times 4.5 + 4.5 \times 4.5}{3} = \frac{120.25}{3} = 40.08$$

The quotient of 40.08 divided by 64 is 0.626, the author's figure. The diameter of the uniform strut which will carry the same load is given in the last line of Table 7 as 6.7 centimeters. The quotient of 40.08 divided by the square of 6.7 is 0.896. The saving of material by using a tapered strut of this ratio is approximately 10.4 per cent instead of 37 per cent.

In place of the volume ratio, which is the ratio of the volume of the tapered strut to that of a uniform strut of equal strength, the writer has used the relative efficiency which he defines as the ratio of the strength of the tapered strut to the strength of a uniform strut of equal volume. For struts with all sections similar the moment of inertia varies as the square of the area while the volume is directly proportional to the area. The relative efficiency is the square of the reciprocal of the volume ratio. The writer has given the relative efficiency for five types of tapered struts in Tables XXXVI and XXXVII of Bulletin No. 25, and has plotted the curves in Fig. 23. Inspection shows that a conical strut, $I = Cx^4$, has a maximum relative efficiency of 1.243 when $i:I = 0.115$ approximately, and $d:D = 0.58$. The volume ratio is the square root of the reciprocal of 1.243, which is $\sqrt{0.8045} = 0.897$.

For $I = Cx^3$ the maximum relative efficiency is 1.249 at $i:I = 0.11$ and $d:D = 0.57$. The volume ratio is 0.895, and the maximum saving of material with this form 10.5 per cent.

For $I = Cx^2$ the maximum efficiency is 1.264 at $i:I = 0.09$ and $d:D = 0.56$. The volume ratio is 0.889, and the maximum saving in material is 11.1 per cent.

For $I = Cx$ the greatest efficiency is 1.315 when $i:I = 0$, which, of course, cannot be used. When $i:I = 0.1$ the relative efficiency is 1.277, the volume ratio is 0.885 and the maximum saving of material is 11.5 per cent.

For solid tapered struts with all sections similar, with length sufficiently great to fail by flexure before the ultimate compressive stress is reached at the ends, and with area at the ends about one-third the area at the middle, the possible saving of material varies from 10 to 11.5 per cent. Forms have been designed which give a saving of 12 per cent.[10]

A. M. WAHL.[11] In looking over this paper, the writer was wondering how the formulas given would apply to columns where the axis of minimum moment of inertia of the middle cross-section is in a plane different from that of the axis of minimum moment of inertia of the end cross-sections; in other words, how does it apply to twisted columns? Certain machine members might quite conceivably have this form of column. Presumably an approximation would be obtained by using the minimum value of moment of inertia at the middle section, and taking this as I in the formula of the paper. On this basis, the value of moment inertia at the ends of the column about an axis in the plane of the axis of I would be taken for i. The safest course, however, would be to take the minimum value of moment of inertia at the ends of the column for i, and use this value in conjunction with the value I of minimum moment of inertia at the middle.

The writer was also wondering whether any data exist in the literature regarding the strength of columns of variable cross-section under lateral loading. This would be of practical importance, for example, in the case of airplane struts under lateral load due to wind pressure. Such data exist for the case of columns of constant cross-section.[12] In the case of columns of variable cross-section, laterally loaded, it is possible that Ritz's approximate method may be simpler to apply than the more exact method of integrating the differential equations of flexure used by Dr. Dinnik for his case.

It would also be of interest to find the buckling strength for columns of variable cross-section when loaded beyond the proportional limit. Where the cross-section is constant this problem has been solved for columns having rectangular cross-section.[13]

E. O. WATERS.[14] Dr. Dinnik's paper is sure to be of value in many applications of machine design where one of the standard column formulas has been used in the past. It also places due emphasis on an error that has long been neglected, namely, the use of the simple Euler formula for tapered columns. This prac-

[10] "Strength of Materials," John Case, Longmans, Green & Co., p. 339.

[11] Research Department, Westinghouse Electric & Mfg. Co., East Pittsburgh, Pa. Jun. A.S.M.E.

[12] See, for example, "Applied Elasticity," by Timoshenko and Lessells, p. 161. Also see paper by H. M. Westergaard, Proc., A.S.C.E., vol. 47, p. 455.

[13] See Von Karman's article "Untersuchungen über Knickfestigkeit," Mitt. u. Forschungsarberten auf dem Gebeite des Ingenieurwesens, vol. 81 (1910). See also "Strength of Steel Columns," by H. M. Westergaard and Wm. R. Osgood, Trans. A.S.M.E., vol. 50, paper No. APM-50-9, p. 65.

[14] Associate Professor of Mechanical Engineering, Yale University, New Haven, Conn. Assoc-Mem. A.S.M.E.

tice has been largely due, no doubt, to the lack of widespread knowledge as to the proper method of correcting the Euler theory for tapered columns; probably, too, it has been justified by a tacit assumption that the mid-section of a column, like that of a uniformly loaded beam, was "critical" and hence was the determining factor in the strength of the entire member.

As a matter of fact, however, the column presents a strain, as distinguished from a stress, phenomenon. In a beam, we can determine the stress at any section by investigating the moment of inertia of that section alone; but to find the deflection, we must know the moment of inertia at all sections. And likewise in a column, the critical load is a function of the deflection, and hence of the moment of inertia at all sections.

The tables in this paper lead to some interesting comparisons in economy of design. Consider, for example, a series of hollow conical columns, all having unit diameter at the ends, and varying from unity to 1.8 times unity diameter at the middle. Then, according to the uncorrected theory, the buckling load varies according to curve (1) in Fig. 11 and the buckling load per unit weight varies as curve (2). For the corrected theory, the corresponding curves are (3) and (4). Suppose that the maximum compressive load is twice the buckling load of a cylindrical column of unit diameter (see line ABC); then, according to the old theory, the mid-diameter should be increased 26 per cent (point B) in order to develop the full strength of the column, giving an efficiency factor 80 per cent better than the maximum possible with a cylindrical column (point D). However, according to the corrected theory, the mid-diameter would have to be increased 38½ per cent (point C) and the efficiency of utilization of material would be only 70 per cent greater (point E).

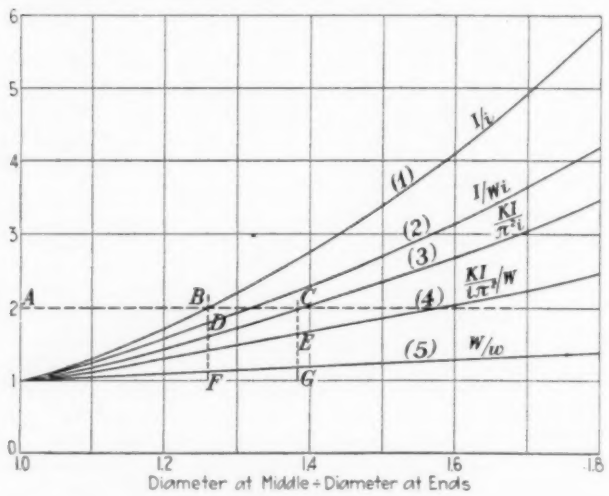


FIG. 11

In mentioning the braced pyramidal column, and in Example 3, the author seems to have overlooked a point which is quite as serious as the correction of the elastic-curve equation for variation in cross-section. I have reference to the fact that open, built-up structures have considerably less rigidity than is obtained by calculation, using the appropriate deflection formula in the paper. All such formulas assume that the material is homogeneous, and that there are no discontinuities of form. Stated in other words, the shear and direct stresses are supposed to vary by infinitesimal increments from point to point along any section. Now, as is well known, the so-called shear and direct loads in a pin-connected truss are practically divided between the web and flange members, respectively; even if the flange members are continuous they do not carry much of the shear, due to their small I in comparison with the fictitious I of the whole section (cf. 4×2.9 with 258.6, in Example 3). The proviso at the end of Example 3, "that the connections between the angles are rigid enough to prevent any relative displacement of the angle so that they will act as a whole," is almost always a practical impossibility.

In order, therefore, to make Dr. Dinnik's refinements of value for latticed columns, the rigidity function EI should be reduced by a factor taking account of the spacing, size, and method of fastening of the lattice bars.

L. B. TUCKERMAN.[15] Dr. A. Dinnik's paper is interesting in contributing another special solution of Euler's differential equation, applicable to slender columns of varying cross-section. So far as I can find from my bibliography, the case when the moment of inertia (or Euler's "moment of stiffness") varies exponentially along the column has never previously been given.

The wording of the paper, however, leaves the impression on the casual reader that recognition that the treatment must be modified in the case of varying cross-section is unusual. A study of the literature shows, however, that the difference in treatment necessary was recognized by Euler, and by many later writers. Unfortunately for the development of the subject, few of the later authors considered it necessary to find out what had previously been done or to refer to the original sources. As a consequence we find a host of special solutions worked out independently when a general solution was already known.

Euler's original paper on the calculus of variations[16] contains the famous appendix (pp. 245–310) "Additamentum I. de curvis elasticis"[17] in which the problem of the elastica is treated in great detail not only for elastica of constant stiffness but for those of varying stiffness. Incidental to this investigation he notes, (Par. 37, "de vi columnarum," p. 267) in passing, the peculiar properties of straight elastica under axial load, giving the formula $P = \frac{\pi\pi}{aa} Ekk$ (now commonly called Euler's formula) as the minimum load under which the column would bend. The existence of this critical load puzzles Euler and he returns to it again and again in his published papers, first in 1757[18] and later in three papers in 1778[19] seeking a satisfactory explanation of this phenomenon which "ne paroitra pas peu paradoxe" and "est difficile a concevoir." He never reached a satisfactory explanation because of his failure to see clearly the effect of end constraint which was first cleared up by Lagrange[20] to whom we owe our present conceptions of "round" and "fixed" ends and the critical loads of higher order corresponding to multiple lobed flexure. However, in these investigations Euler formulates clearly[17] the differential equation for columns of varying stiffness (par. 19, p.

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[15] Assistant Chief, Division of Mechanics and Sound, Department of Commerce, Washington, D. C.

[16] Leonhardt Euler, "Methodus inveniendi lineas curvas maximi minimive proprietate gaudentes sive solutio problematis isoperimetrici latissimo sensu accepti." Lausanne et Genevae apud Marcum-Michaelem Bousquet et Socios, 1744.

[17] Translated into German in Ostwald's "Klassiker der exakten Wissenschaften," Nr. 175, Leipzig, 1910.

[18] Leonhardt Euler, "Sur la Force des Colonnes." Mémoires de L'Academie des Sciences et Belles-lettres (Berlin), vol. 13, pp. 252–282, for the year 1757, published in 1759.

[19] Leonhardt Euler, "Determinatio onerum quae columnae gestare valent." "Examen insignis paradoxi in theoria columnarum occurrentis." "De altitudine columnarum subproprio pondere corruentium," Acta. Acad. Petropolit., vol. 2, pars. 1, pp. 121, 146, and 163, for the year 1778, published in 1780.

[20] Lagrange, "Sur la force des colonnes," Mélanges de Philosophie et de Mathématique de la Société Royale de Turin (Miscellanea Taurenensia), vol. 5, 1770–1773. Reprinted in Oeuvres de Lagrange, vol. 2, pp. 125–170. Gautier-Villars, Paris, 1868.

262) in the form $\frac{Ekk}{P} X ddy + y dx^2 = 0$, where X is an arbitrary function (une fonction quelconque) of x, notes that they are soluble for cases where (in modern notation) $I_x = I(a + bx)^m$ and m has certain rational values (Bessel's functions were then unknown) and gives the solution for the cases $I_x = I(a + bx)^4$ which corresponds to a truncated cone or pyramid (par. 20, p. 263) and $I_x = I(a + bx)^2$ which corresponds to a braced pyramidal column (par. 30, p. 271) and discusses approximate solutions for other cases. With the end conditions clearly stated by Lagrange[20] a theory of slender columns of varying cross-section was already in existence in all its essentials as early as 1770.

The failure of Euler's theory to represent satisfactorily the behavior even of ordinary structural columns of uniform stiffness led to a neglect of this theory. After the work of Engesser and Considère had commenced to clear up the subject of medium-length columns and the need arose for long slender columns to which Euler's theory was applicable, interest in tapered columns revived and a number of investigators independently worked out these same special cases and a number of others without troubling to find out what others had done. I find in my files references[21] to closed solutions for the following:

$$I_x = I\left(1 - \frac{x^2}{l^2}\right), I_x = I\,\frac{4(lx - x^2)}{2}, I_x = \frac{Ix}{l},$$

$$I_x = I\,\frac{(l + x)^4}{16l^4} I, {}_x = I(l - x), I_x = I\,\frac{x^2}{l^2}$$

and $I_x = I\left(\frac{x}{l}\right)^n$

It would be interesting to know how may other people have worked out independently for special purposes special forms of Euler's more general case[17]

$$I_x = I(a + bx)^m.$$

A critical discussion of the relative advantages and disadvantages of these particular forms for particular purposes would be of considerable value.

Engesser in his second paper[21] introduces an idea, familiar in other lines, from which theoretically an infinite number of forms of closed solutions may be obtained. By assuming the deflection (y) as any arbitrary function of x, $y = f(x)$, a solution is obtained for the case

$$I_x = \frac{P}{-E}\,\frac{f(x)}{f''(x)}$$

He discusses merely the simple case of a parabolic deflection

$$y = c(l^2 - x^2)$$

which gives the special case already noted

$$I_x = I\left(1 - \frac{x^2}{l^2}\right)$$

The usefulness of the method depends of course on the type of variation found.

Paralleling these exact analytic solutions are numerous methods for successive analytic, graphic, or tabular approximations, or for graphic or tabular solution of the differential equation for arbitrary variation including discontinuities of section.[22] In addition, several authors, beginning with Lagrange, have attacked the problem of the most economical shape of a long, slender column.[23]

There is thus a considerable body of material published on the theory of slender columns of varying cross-section which would well repay a critical study.

However, a theory of the medium-length tapered column, corresponding to the theory worked out by Engesser, Considère, Jasinsky, Kármán, and Southwell (although touched upon by Engesser[21]) is yet to be developed. The problem of the tapered column can not be considered settled until that has been done.

G. B. Karelitz.[24] The tables and diagrams given in the paper are of value for a designer, where weight comes in as an important consideration. It is quite natural that most examples are taken from airplane structures. However, the Equation [8] is difficult to apply in a general case, since the ratio $i:I$ is insufficient to find both k and m. The author himself in his examples chooses first the value of m by assuming a definite shape of his strut. A sentence should be added, therefore, to the Appendix No. 2 reading: "First, choose a value for the coefficient m."

Since the sections are generally close to those discussed in the paper, the practical value of the paper is but slightly impaired by the difficulty of finding m for a strut of an arbitrary shape.

M. F. Sayre.[25] In dealing with Euler's formula, it is important to realize its limitations. These are possibly better visualized if we think of it as a special case of the more general formula

$$S = \frac{P}{A} + \frac{ePc}{I} \sec. \sqrt{\frac{Pl^2}{4EI}},$$

giving unit stresses in terms of the load P and the eccentricity e. For columns, whether short or long, where the load is applied off-center on the column, this general formula should be used. It is valid up to the point where the computed unit stress S exceeds the proportional limit of the material.

In developing Euler's formula, it is tacitly assumed (1) that there is a slight eccentricity e present, but that this eccentricity

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[21] The following references are probably far from complete. They were noted incidental to a search on a related subject: Fr. Engesser, Zeit. d. Oesset. Ing. u. Arch. Verb., vol. 45, pp. 506–508, 1893; ... pp. 545–548, 1909.

Arthur Morley, *Engineering* (London), vol. 97, pp. 566–568, Apr. 24, 1914, and vol. 107, pp. 295–298, Sept. 21, 1917.

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James E. Boyd, Bureau of Standards Tech. Paper No. 152, pp. 24–35, 1920.

[22] Fr. Engesser, loc. cit.

L. Vianello, *Z.V.D.I.*, p. 1436, 1898.

Arthur Morley, "Critical loads for ideal long columns," *Engineering* (London), vol. 97, pp. 566–568, Apr. 24, 1914.

L. Bairstow and E. W. Stedman, "Critical loads for long struts of varying sections," *Engineering* (London), vol. 98, p. 403, Oct. 2 1914.

John Case, "An approximate graphical treatment of some strut problems," *Engineering* (London), vol. 98, pp. 699–700, Dec. 20, 1914.

Arthur Morley, "Critical loads for long tapering struts," *Engineering* (London), vol. 104, pp. 295–298, Sept. 21, 1917.

[23] Lagrange, loc. cit.

Clausen, "Ueber die Form architektonischer Säulen," Bull. physico-math. de l'Acad. (Petersburg), vol. 9, pp. 368–379, 1851.

L. Bairstow and E. W. Stedman, "The design of a strut of uniform strength," British Adv. Comm. for Aero. R. & M., no. 158, Nov., 1914.

H. Blasius, "Träger Kleinster Durchbiegung und Stäbe grösster Knickfestigkeit bei gegebenem Materialverbrauch," *Zeit. für Math. u. Physik*, vol. 62, pp. 182–197, 1914.

T. R. Darnley, "Design of a Strut of Uniform Strength," British Adv. Comm. for Aero., T. 1186, June, 1918, Akimoso Ono, loc. cit.

[24] Research Department, Westinghouse Electric & Mfg. Co., East Pittsburgh, Pa. Mem. A.S.M.E.

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is so small that taken by itself it would be impossible to measure, or at least negligibly, and (2) that the column is of such length that e sec. $\sqrt{\frac{Pl^2}{4EI}}$ which measures the deflection at the mid-point of the column, will become of appreciable magnitude (i.e., the column buckles) before the computed unit stress S reaches the elastic limit of the material. This naturally limits the application of Euler's formula to long columns. There is another theoretical limitation in that in an ideal case where the eccentricity was actually zero, the last term of the general formula would disappear, and the unit stress would equal P/A. This, however, is a case of unstable equilibrium. Within these limitations, the theoretical and practical errors in Euler's formula are very small; in fact, it is by far more accurate than the formulas which we are compelled to use in the design of short or medium-length columns. I understand that, because of this fact, European designers have extended its use well beyond its theoretical limitations, but with appropriately large safety factors. The story goes that at one place in South America an attempt was made to combine this European formula with American safety factors, with disastrous results.

I might second Dr. Waters' suggestion about the need of care in applying this formula to built-up columns, because of the low apparent stiffness of these columns. I am inclined to think, however, that this low stiffness is due to the large shearing deflections in the column. We have had some experiments which show us that these shearing deflections are not always as negligible as we have considered them. For instance, in timber beams, the shearing modulus of elasticity parallel to the grain is so low that bending deflections are much greater than would be expected based on the usual bending formulas, and on moduli of elasticity obtained in tension or compression tests. In built-up latticed columns, the shearing deflections in the latticing will again have an important effect upon the apparent lateral stiffness of the column. Allowance must be made for this fact when using Euler's formula, and it is likely that for built-up columns values of E appreciably less than 30,000,000 lb. per sq. in. should be used.

APM-51-12

Calculation of Flywheels for Air Compressors

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The authors present a short history of the application of low-speed synchronous motors direct connected to ammonia compressors. It is shown how the use of XY curves has brought about a practical solution of the flywheel problem and how in practically all present-day cases the necessary flywheel effect can be incorporated in the rotor of the synchronous motor. A short history of the application of synchronous motors to air compressors is also given, and the special problems encountered due to the unbalanced weights of reciprocating parts and to part-load operation of air compressors are discussed.

A derivation of the fundamental differential equation of the XY curve is given and a simple method presented for its solution. An example illustrating the method of making XY curves is given. Curves and data are given which materially reduce the labor involved in making XY curves. A set of eight XY curves for two-cylinder double-acting two-stage air compressors is included. These curves cover full-load, no-load, and six different part-load conditions. A second set of two XY curves is presented covering full-load and no-load operation for single-cylinder double-acting single-stage compressors.

The paper has two appendixes. Appendix No. 1 presents formulas for calculating the theoretical indicated horsepower for single- or for two-stage multi-cylinder compressors. Appendix No. 2 presents methods of calculation of the theoretical torque curves for compressors. An expression for the inertia torque due to the reciprocating parts of a compressor is given. Tables are presented giving the results of the harmonic analysis of the torque curves due to the gas forces at full load, half load, and no load for the high-pressure and low-pressure cylinders. It is shown how these harmonics of the separate cylinders can be combined to give the harmonics in the torque of the complete machine when operating at full load or under part-load conditions.

VERY few practical problems involving an application of simple mathematics are more fascinating than the flywheel problem in synchronous motors direct connected to reciprocating apparatus. It is in reality the old problem of forced vibrations in a mechanical system, or, to use an electrical analogy, it is the problem of the simple resonance circuit—not the transient but the steady-state condition.

Any engineer will write for the current in the simple alternating-current circuit

$$I = \frac{E}{Z}$$

almost more readily than he will write the equation

$$E = ir + L\frac{di}{dt} + \int \frac{1}{C}\, idt$$

of which it is the steady-state solution. For a fact he usually knows the solution better than the differential equation. The flywheel problem involves the same equation and the same solution just dressed up differently. It is more interesting, however, because the impressed force instead of being a well-behaved sine wave, like the voltage, is now the compressor torque, which involves not only a fundamental sine wave, but also three additional harmonics of importance and many others so small as to be disregarded. In other words, the impressed force is a Fourier's series and the solution must likewise be a Fourier's series but of the same form as

$$I = \frac{E}{Z}$$

and no more difficult.

Another thing about this problem which stirs one's enthusiasm is the fact that it can easily be checked experimentally. The solution to be sure is a Fourier's series and it actually represents the current pulsation in the line of the synchronous motor. The pulsation is hence not a sine wave but an irregular wave which can be obtained by adding together the several harmonics in the series. One needs only to make an oscillogram of the current in the line to obtain experimentally the actual irregular wave of current pulsation which will check remarkably well with the calculated wave. For some actual comparisons of this kind the reader is referred to the third item in the bibliography at the end of this article. The comparisons are truly remarkable.

There is no gainsaying that the work involved in working curves for the calculation of flywheels is tedious. It is very tedious and time-consuming and yet so fascinating is the problem that the writers have kept at it intermittently for several years.

History of the Flywheel Problem in Ammonia Compressors

One object of this paper, in addition to the presentation of a set of flywheel curves for air compressors, is to arouse interest in the subject and to show how vitally it is connected with the development of the compressor industry.

The application of direct-connected synchronous motors to compressors is but little more than twelve years old. At first it was not altogether successful. Sometimes the motors would hunt severely and some cases occurred where it was actually impossible to hold the motors in step. It was soon found, however, that by getting a flywheel of the right size the operation was entirely satisfactory. Increasing the size of the flywheel often bettered the operation, but many cases occurred where increasing it resulted more or less disastrously.

The problem was up to the motor manufacturers and they found that it submitted to very simple analysis. Methods were devised for making flywheel curves, called *XY* curves, by means of which the correct amount of flywheel effect could be easily and quickly determined. At first the work of making these curves was so laborious as to be almost prohibitive, but later on methods were developed which shortened the work. The invention of an instrument called the "harmonic curve tracer" by one of the present writers did much to put the making of these curves on a workable basis.

The early *XY* curves which were made for ammonia compressors showed clearly that in many cases the necessary flywheel effect could be included in the motor rotor. They showed also that if the speeds of compressors could be increased to some extent, it would be possible to put the flywheel effect in the rotor in the great majority of cases. What was the result? The first machines with flywheel effect in the rotor were put into

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

operation. The compactness of the direct-connected unit and the elimination of the flywheel appealed to the compressor user. The idea spread rapidly and in the short space of two years there was such an insistent demand for flywheel effect included in the rotor that practically all installations were sold on that basis. And it had its effect on the design of compressors and motors alike.

Compressor builders knew that the old style heavy valves used in their machines would not stand the increase in speed necessary, in many cases, for flywheel effect in the rotor. For this reason many of them stood out firmly against any increase in compressor speeds. But the more enterprising manufacturers set to work to develop new lighter valves which would stand the higher speeds. The modern feather-weight valve was the result and compressor speeds are now considerably higher than they were ten years ago.

The problem of the motor builder was to incorporate the maximum flywheel effect possible in his machines. At first it was customary to build the motor with a frame of larger diameter than would ordinarily be used. This resulted in a heavier, more expensive machine, but it had higher efficiency and lower exciter capacity and the customer was usually willing to pay for it. Gradually the standard motor design tended more and more to that required by the compressor. Larger diameters, narrower cores, lower exciter capacity, and higher efficiency resulted for the standard motor.

The constant speed of the synchronous motor also brought new problems to the ammonia-compressor industry. In the old steam-driven plants the amount of refrigeration could be regulated as desired by merely changing the speed of the steam engine. With the constant-speed synchronous motor there was no immediate method of changing the amount of refrigeration. Several schemes were devised, but the principal idea was to divide the required refrigeration between several units so that a customer might perhaps purchase three small units for his plant instead of one larger one as he would have done in the past. This then would give him a three-step variation in refrigeration capacity. It also had other advantages. The efficiency was just as good at two-thirds or one-third capacity as at full capacity. It also made a much more dependable plant as an accident to one compressor would not render the whole plant inoperative. This was an important point and one which appealed strongly to the ultimate user. And so it was that soon after the advent of the synchronous motor compressors began to increase in number and decrease in size.

Another method used to change the amount of refrigeration was to disconnect one connecting rod of a two-cylinder double-acting machine, and operate it on one cylinder. This was rather a ticklish method because the flywheel effect had to be almost exactly right. A trifle too much would produce trouble at full load and a trifle too little would cause trouble at half load when operating on one cylinder. It was not at all satisfactory until XY curves were made covering this condition of operation. With these curves the exact amount of flywheel effect could be determined and this method came into quite general use.

Another scheme which has come into favor involves the use of two two-cylinder single-acting machines placed end to end with the motor between. The motor rotor is arranged to slide back and forth in such manner as to operate either one of the compressors singly or both together with the cranks of one at right angles to the cranks of the other. Still another method, which has been used to some extent, employs hand-operated clearance pockets on the head ends of a two-cylinder double-acting compressor. Every one of these methods requires an XY curve for the calculation of the required flywheel.

There is at the present time a budding development in the refrigeration industry known as automatic refrigeration. This is simply a refrigeration plant which can be started automatically from a thermostat. Ordinarily compressors are unloaded at starting by hand-operated bypasses, but in the automatic refrigeration plant this will have to be eliminated and the compressors started with full gas pressure in the cylinders. This means that a starting torque of 175 to 200 per cent is required. This development has been anticipated and unity power-factor synchronous motors are available with these unusual starting torques. Probably installations of this type will come into general use for small sizes which are becoming popular for use in delicatessen stores and meat and fruit markets.

Another line along which the refrigeration industry is developing is that of theater refrigeration. The building restrictions in most states prevent the use of ammonia for this purpose and carbon dioxide has proved fairly successful although it has some disadvantages. The principal one is the enormous pressures required which are about 450 lb. at suction and 1000 lb. at discharge. Carbon-dioxide compressors are usually of the two-cylinder double-acting type, but their torque curves are quite different from those of ammonia compressors of the same type. This is due to the fact that the ratio of absolute discharge to absolute suction pressure is altogether different and also to the fact that the piston rod is very large in diameter so that the work done in the crank end of a cylinder is considerably less than that in the head end. Very little work has been done in calculating flywheel requirements of carbon-dioxide compressors, and recommendations based on ammonia-compressor XY curves have not always proved successful. In one case where trouble occurred it was found advantageous to set the cranks at 180 deg. instead of 90 deg. The effect of different crank angles in machines of this type should be thoroughly investigated. XY curves for the various conditions studied should be made as these will show what combination is most desirable from the standpoint of flywheel requirements.

History of the Flywheel Problem in Air Compressors

The history of the air-compressor industry in its relation to the synchronous motor is an altogether different story. The outstanding development of the refrigeration industry, which was the incorporation of the flywheel effect in the rotor, did not happen in the case of the air compressor. The reason for this was that the torque curves of air compressors, especially when operating at partial loads, were so irregular and unbalanced that the amount of flywheel effect required for satisfactory operation was usually far in excess of what could be put in the rotor. Even at full load the unbalance created by the difference in weight between the high-pressure and low-pressure pistons made it impossible to use the flywheel which would ordinarily be used for an ammonia compressor of the same type, without excessive current pulsation. The problem of developing methods of operation at partial loads which would give more balanced torque curves seemed difficult of solution. The result was that the customer who demanded flywheel effect in the rotor for his compressor was simply told that it was impossible and he was obliged to be content with the extra wheel.

The condition still exists to some extent, but it is a safe prediction that there will be some change in the future. The new valves developed for higher speed ammonia compressors were used in air compressors and resulted in a similar increase in speed. Even this did not reduce the flywheel effect required to the point where it could be put in the rotor, but it did result in higher volumetric efficiency and hence a lower priced air compressor for the same capacity.

The real solution to this problem is going to come about by the development of more balanced methods of unloading and a

more balanced full-load condition. So far only one compressor builder has developed a five-step method of unloading which is balanced enough to permit the flywheel effect to be put in the rotor. It is not surprising that this same manufacturer is the one who has cooperated most with the motor builders in the matter of making *XY* curves and studying the flywheel problem. It would be very strange indeed if other balanced methods of unloading cannot be devised. They can without doubt, but it is going to be necessary for the compressor manufacturers to get an understanding of the flywheel problem and of *XY* curves.

A few installations without external flywheel have already been made and are giving entire satisfaction. The demand for this type of installation may possibly increase just as it did in the case of the ammonia compressor.

Some suggestions might be made for producing more balanced conditions in air compressors, but they are so obvious that compressor designers must have thought of them many times. The full-load condition can be greatly improved by making the weights of the high-pressure and low-pressure pistons the same. This can easily be done by using some aluminum alloy or other light-weight material for the low-pressure piston and adding some extra weight to the high-pressure piston. The work done by the high-pressure and low-pressure cylinders should also be balanced. Strange as it may seem, many compressors are so designed that the work is equally divided between the cylinders when the discharge pressure is about 120 lb. instead of 100 lb. which is the usual discharge pressure. This condition can easily be rectified by changing the ratio of cylinder diameters slightly. One can never be sure that the work will be absolutely balanced between the cylinders because the interstage temperature may vary to some extent, but it can be at least very nearly balanced. Now with balanced gas forces and balanced inertia forces it would be a logical thing to try the cranks at 180 deg. instead of the customary 90 deg. This would eliminate completely the first harmonic and would probably permit the use of a flywheel calculated on the basis of an $x = 10$ or 12 which could in most cases be put in the rotor.

A three-fourths load obtained by the use of clearance pockets on either the head ends or the crank ends of each cylinder would still preserve the balance and would give a condition at least no worse than the full load. Similarly a balanced half load could be obtained by opening the suction valves of either both head ends or else both crank ends to atmosphere. The one-quarter load would be similar to the three-quarter condition except that the ends of the cylinders opposite the clearance pockets would be unloaded to atmosphere. The no load would be as at present, that is, all ends open to atmosphere.

It seems very probable that some such arrangement as this could be made to work out satisfactorily, but one can only tell by making an *XY* curve covering this condition. No doubt compressor designers will be able to devise other schemes, but the test of all of them will be *XY* curves. It is evident that such designers should understand the theory of the *XY* curves and how to make them if they are to cooperate with the motor manufacturers in solving this problem of incorporating flywheel effect in the rotor for air compressors. For this reason the theory of the *XY* curve is given in this article together with an example illustrating a short method of making the curves.

Another solution to this whole problem which has been adopted by one manufacturer is the entire elimination of all the part-load operations so that the compressor operates all the time at either full load or no load. Possibly this is the best method after all. It is certainly the simplest and by balancing the inertia and gas forces the flywheel effect can easily be put in the rotor. However, there must be some demand for part-load operation or so many other manufacturers would not be spending so much time and money in developing and selling their multiple-step unloading devices.

Theory of the *XY* Curve

In the olden days when automobiles were cranked by hand, the man who cranked exerted exactly the same torque on the engine as the engine did on the man. The engine's torque was due to the gas forces in the various cylinders and also to the effect of the flywheel. The man's torque, however, was due to the flexure of the muscles of his arm and back. But the point is that the two torques were equal. Exactly the same thing is the basis of the fundamental differential equation governing the action of a synchronous motor direct connected to a compressor. The motor has two muscles—one the synchronizing torque T_s which is proportional to its angular displacement, and another much smaller muscle, the damping torque T_d, which is proportional to the rate of change of the angular displacement. T_s is given in units of foot-pounds per mechanical radian displacement and T_d in foot-pounds per mechanical radian per second slip. If ψ is the angular displacement of the rotor, the motor torque is

$$T_s\psi + T_d p\psi \quad \ldots\ldots [1]$$

where

$p = \dfrac{d}{dt}$ and denotes differentiation with respect to time.

The torque of the compressor is

$$f(t) - Ip^2\psi \quad \ldots\ldots [2]$$

where $f(t)$ is some function of time which denotes the torque due to the gas and inertia forces of the cylinders. The expression $p^2 = \dfrac{d^2}{dt^2}$ denotes the second derivative with respect to time. The term $p^2\psi$ is thus acceleration and $Ip^2\psi$ has the units of moment of inertia multiplied by acceleration, and equals the torque required to accelerate the flywheel.

Now Equation [1] must be exactly equal to Equation [2] so that

$$T_s\psi + T_d p\psi = f(t) - Ip^2\psi$$

or

$$Ip^2\psi + T_d p\psi + T_s\psi = f(t) \quad \ldots\ldots [3]$$

The permanent solution of Equation [3] can be obtained in the usual way by putting $p = j\omega$, in which ω is the frequency or angular velocity of $f(t)$ in radians per second, so that

$$\psi = \frac{f(t)}{(T_s - I\omega^2) + jT_d\omega} \quad \ldots\ldots [4]$$

Now $f(t)$ is the compressor torque and as shown in Appendix No. 2 it can be expressed in the form of a Fourier's series of four harmonics. For any particular harmonic

$$\left.\begin{aligned} f(t) &= a_n + jb_n \\ \omega &= n\omega \end{aligned}\right\} \quad \ldots\ldots [5]$$

and,

n being the number of the harmonic.

Where,

a_n = sine harmonics in the torque curve
b_n = cosine harmonics in the torque curve
ω = angular velocity of the first harmonic.

Substituting Equation [5] in [4] gives

$$\psi_n = \frac{a_n + jb_n}{T_s - In^2\omega^2 + jT_d n\omega} \quad \ldots\ldots [6]$$

or, taking account of all harmonics.

$$\sum \psi_n = \sum \frac{\sqrt{a^2_n + b^2_n}}{\sqrt{(T_s - In^2\omega^2)^2 + T^2_d n^2\omega^2}} \sin\left(n\omega t + \tan^{-1}\frac{b_n}{a_n} - \tan^{-1}\frac{T_d n\omega}{T_s - In^2\omega^2}\right) \quad [7]$$

Multiplying Equations [7] through by $\frac{T_s}{a_0}$, where a_0 is the theoretical average torque of the compressor as described in Appendix No. 2, gives

$$\sum \frac{\psi_n T_s}{a_0} = \sum \frac{\sqrt{A^2_n + B^2_n}}{\sqrt{\left(1 - \frac{In^2\omega^2}{T_s}\right) + T_d{}^2 n^2\omega^2}} \sin\left(n\omega t + \tan^{-1}\frac{B_n}{A_n} - \tan^{-1}\frac{T_d n\omega}{T_s - In^2\omega^2}\right) \quad [8]$$

where $A_n = \frac{a_n}{a_0}$ and $B_n = \frac{b_n}{a_0}$.

These are the values of the harmonics given in the Tables 5 and 6 of Appendix No. 2.

It will be found convenient to put in this equation.

$$\left.\begin{aligned} Y &= \sum \frac{\psi_n T_s}{a_0} \\ Z &= \frac{I\omega^2}{T_s} \\ \lambda &= \frac{T_d\omega}{T_s} \end{aligned}\right\} \quad [9]$$

which gives

$$Y = \sum \frac{\psi_n T_s}{a_0} = \sum \frac{\sqrt{A^2_n + B^2_n}}{\sqrt{(1 - n^2Z)^2 + (n\lambda)^2}} \sin\left(n\omega t + \tan^{-1}\frac{B_n}{A_n} - \tan^{-1}\frac{n\lambda}{1 - n\,Z}\right) \quad [10]$$

This is the equation of the XY curve[2] and as such it merits special attention. In the first place it will be noticed that $\Sigma\psi_n T_s$ represents with very close approximation the torque pulsation of the motor, since $\Sigma\psi_n$ represents the pulsation in the angular displacement in radians and T_s is the torque per radian. The motor damping torque which is $\Sigma T_d \frac{d\psi_n}{dt}$ has been neglected since it is small and the power drawn by it from the line adds at right angles in time to the power drawn by the main torque pulsation $\Sigma\psi_n T_s$. So that Y, which is Y of the XY curve and which is equal to $\sum \frac{\psi_n T_s}{a_0}$, represents the torque pulsation in percentage based on the theoretical average torque of the compressor. Since the speed of the motor is practically constant Y is also the power pulsation in percentage based on the indicated horsepower calculated from the theoretical indicator cards described in Appendix No. 1. Assuming the power-factor constant during the pulsation, the current pulsation in percentage is equal to the power pulsation. Usually, however, the desired current pulsation is expressed in percentage of the name-plate rated current so that the value of Y required for a given current pulsation is

$$Y = \frac{\text{Motor rated horsepower}}{\text{Theoretical i.hp.} \times \text{power factor}} \times \text{(Desired current pulsation in per cent of name-plate rated current. Usually 66 per cent.)} \quad [11]$$

where power-factor is the power-factor of the motor.

From Equation [11] it is very easy to calculate Y for any given case corresponding to any desired current pulsation. Then with an XY curve available the corresponding value of X can be obtained and from X the actual WR^2 required for a given motor may be quickly obtained. It will be shown in Equation [17] that after X is determined the required WR^2 is the product of X multiplied by the flywheel constant of the motor called F_c. It will be shown that F_c is a characteristic constant of the motor alone involving its speed, frequency, horsepower, and natural frequency constant.

The XY curve, on the other hand, is fundamentally a performance curve of a reciprocating machine just as a speed-torque curve or an efficiency curve might be performance curves of a motor. The type of XY curve obtained for a given machine is determined by the harmonics in its torque curve. These harmonics enter in Equation [10] in the numerator and also in the phase angle $\tan^{-1}\frac{B_n}{A_n}$.

It probably will be wondered where X is in Equation [10]. As a matter of fact it will be shown that $X = 9.25Z$, so that Z is really taking the place of X in this equation. The meaning of Z can be seen at once by writing

$$Z = \frac{I\omega^2}{T_s} = \left(\frac{\omega}{\sqrt{\frac{T_s}{I}}}\right)^2 \quad [12]$$

ω is the frequency of the lowest harmonic expressed in radians per second, corresponding to the number of revolutions per minute of the machine and $\sqrt{\frac{T_s}{I}}$ is the natural frequency. Z is thus the square of the ratio of the revolutions per minute to the natural frequency in cycles per minute and X is 9.25 times this value.

The use of X came about in the following way. It is customary for some manufacturers to list a natural-frequency constant P_0 which is related to T_s by the formula

$$T_s = \frac{2625 \times \text{hp.} \times P_0 \times p}{s \times \text{kw.}} \quad [13]$$

where

s = r.p.m.
p = number of poles
hp. = rated horsepower of motor
kw. = rated kilowatts of motor.

P_0 is defined as the input kilowatts per electrical radian displacement of the rotor. With this in mind definition [13] follows at once. Substituting Equation [13] in [12] and solving for I

$$I = \frac{Z}{\omega^2} \times \frac{2625 \times \text{hp.} \times P_0 \times p}{s \times \text{kw.}} \quad [14]$$

But $I = \frac{WR^2}{32.2}$, $p = \frac{f\,120}{s}$ where f is the electrical frequency and $\omega = 0.1048\,s = \frac{2\pi}{60}\,s$.

[2] In Equation [10], Z is taking the place of X. Later it will be shown that $X = 9.25Z$.

Making these substitutions and solving for WR^2 gives

$$WR^2 = \frac{9.25Z \times \left(\frac{\text{hp.}}{100}\right) \times \left(\frac{f}{10}\right) \times \left(\frac{p_0}{\text{kw.}}\right)}{\left(\frac{s}{100}\right)^4} \times 10^3 \quad \ldots\ldots [15]$$

It is easily seen that if X is put for $9.25Z$ and the curves plotted against X instead of Z, one slide-rule operation is saved in calculating the WR^2 required from Equation [15]. It should be noted, however, that the use of X originated from a formula employing P_0 so that X and P_0 are really tied up together and

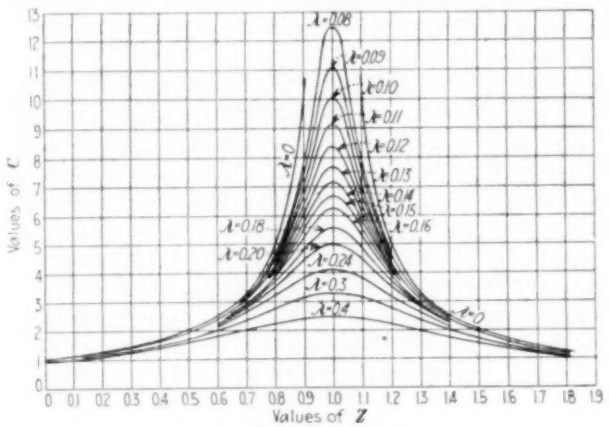


FIG. 1 FIRST HARMONIC

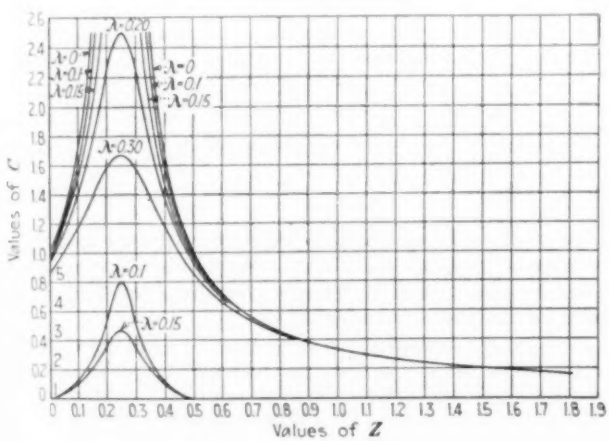


FIG. 2 SECOND HARMONIC

Formula [15] is useful only where the natural frequency constant is given as P_0.

Motor manufacturers should standardize on some flywheel constant which would permit of a simple calculation for the WR^2 required in a compressor installation. XY curves are in such general use and have been presented in so many different papers that it seems undesirable to change abscissa X to any other value although it would be possible to use an abscissa not directly related to the natural frequency constant P_0.

With this in mind the Westinghouse Company has adopted a flywheel constant called F_c defined as follows:

$$F_c = \frac{\frac{\text{hp.}}{100} \times \frac{f}{10} \times \frac{P_0}{\text{kw.}}}{\left(\frac{s}{100}\right)^4} \times 10^3 \quad \ldots\ldots [16]$$

It should be noted that F_c is a constant which depends entirely on the rating and electrical characteristics of the motor and its value can therefore be listed in the price book with each motor

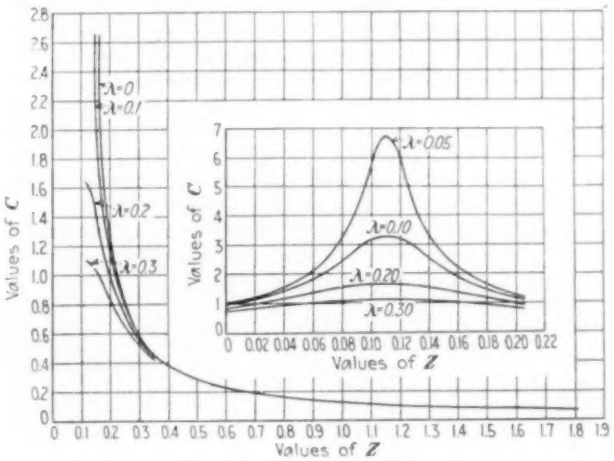


FIG. 3 THIRD HARMONIC

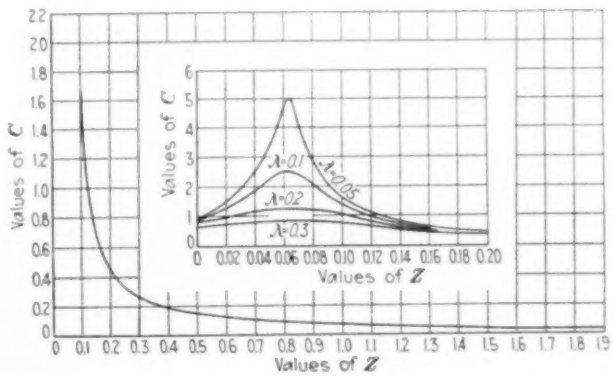


FIG. 4 FOURTH HARMONIC

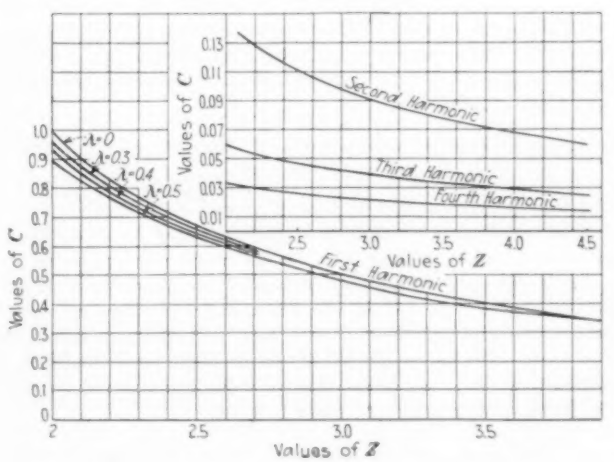


FIG. 5 ALL HARMONICS, HIGHER VALUES OF Z

rating. The formula for calculating the required flywheel effect becomes simply

$$WR^2 = XF_c \quad \ldots\ldots [17]$$

It would be to the advantage of the compressor builders if

all the electrical manufacturers would list the constant F_c defined as in the foregoing.

Returning to Equation [10] the term λ must still be explained. It will be seen by studying Equation [10] that λ is a variable, depending on the damping torque and speed of the motor but independent of Z. Hence an XY curve is not entirely independent of the speed since λ varies with the speed. The effect of λ is, however, very small except in the regions around the resonance points so that XY curves for different values of λ will be very nearly coincident except in these regions. Fortunately those portions of an XY curve in the resonance region are not used for making flywheel recommendations since they would correspond to conditions of extremely high current pulsation. It was for this reason that the early XY curves presented by Dr. Stevenson[3] were not plotted through the resonance points. Later on, however, one of the present writers suggested plotting the curves through the resonance points with an average value of λ. The value $\lambda = 0.1$ was found to be a good average and since then all XY curves which could be classed as standard have been plotted with a value of $\lambda = 0.1$.

For rapid solution of Equation [10] the term

$$\frac{1}{\sqrt{(1 - n^2Z)^2 + (n\lambda)^2}} = C$$

can be plotted as a family of curves for each value of n corresponding to the first four harmonics. These families of curves are shown in Figs. 1, 2, 3, 4, and 5. They are plotted as function of Z for several different values of λ.

Similarly the phase angle $\tan^{-1} \dfrac{n\lambda}{1 - n^2Z} = y_n$ can be plotted as function of Z, a family of curves being obtained for each harmonic and each curve being for a different value of λ. It has been found that for all values of X and λ within reasonable limits, the fourth harmonic y_n is practically 180 degrees and will be taken as such, so that no family of curves is given for the fourth harmonic. Figs. 6, 7, and 8 give these families of curves for y for the first three harmonics.

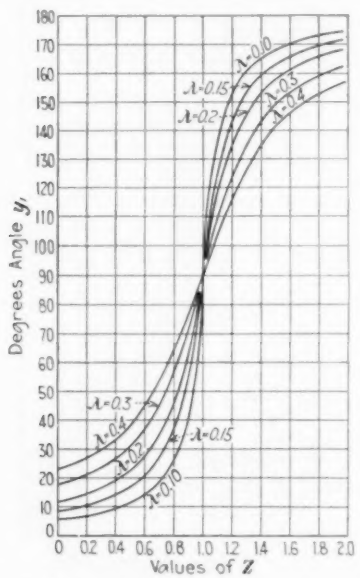


Fig. 6 First Harmonic Angles

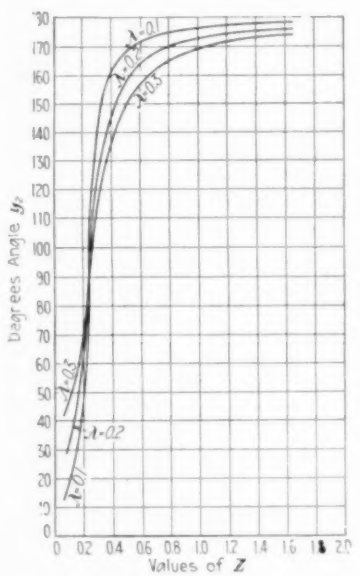


Fig. 7 Second Harmonic Angles

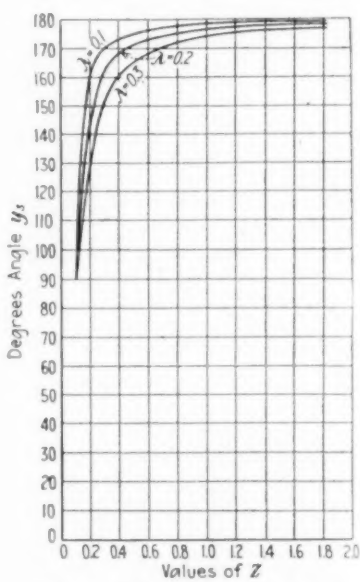


Fig. 8 Third Harmonic Angles

Example Illustrating Method of Making an XY Curve

The construction of an XY curve consists in solving Equation [10] for different values of Z or X, since $X = 9.25Z$. Each solution thus gives a value of Y corresponding to an assigned value of X and from these data the XY curve can be plotted.

As an example of the construction of such a curve consider the case of a two-cylinder double-acting two-stage air compressor operating at three-fourths load and having a value[4] of $q = 18.4$. The harmonics of the torque curve for a compressor of this type operating under this condition are given in column 2 of Table 5, Appendix No. 2, as follows:

$A_1 = +4.6$	$B_1 = +15.9$
$A_2 = -18.6$	$B_2 = +2.4$
$A_3 = +25.8$	$B_3 = +45.5$
$A_4 = +11.8$	$B_4 = -1.7$

[3] See third reference in bibliography.

[4] q is a factor depending on the weight of reciprocating parts. See Appendix No. 2.

With the information given in these curves and a knowledge of the harmonics of the compressor-torque curve one is ready to solve Equation [10] for different values of X, thereby obtaining the XY curve. In making standard XY curves, as stated before, an average value of $\lambda = 0.1$ will be used. Experience has shown that to insure greatest accuracy and simplicity in solving for Y repeatedly, it is advisable to use a blank similar to the one shown in Table 1. The blank is arranged to record $Q_n = \sqrt{A^2_n + B^2_n}$ and the phase angle $x_n = \tan^{-1} \dfrac{B_n}{A_n}$. These figures are the same for any value of Z so they are recorded once at the top of the blank. The remainder of the blank consists of eight similar rectangles, one for each value of Z. In each rectangle there is a column for C which is read from the curves in Figs. 1 to 5, for $\lambda = 0.1$. The column of products is CQ_n. The values of y_n are read from the curves in Figs. 5, 6, and 7 and the last column is as indicated the angle $x_n - y_n$ in degrees. The figures in the column "Prod." are the magnitudes of each harmonic and the angles in the column $x_n - y_n$ are the phase angles of the respective harmonics. The four harmonics must be plotted to scale and at their respective phase angles for one revolution of the crank. If they are added

a true picture of the current pulsation of the motor is obtained. The distance between the maximum points on the positive and negative side of the zero axis is the value of Y as given by Equation [10].

The space shown in Table 1 has been properly filled out with data covering the illustrative example. Fig. 9 gives the four harmonics for $Z = 0.5$, drawn to scale at their proper phase

TABLE 1

n	1	2	3	4
A_n	+ 4.6	−18.6	+ 25.8	+11.8
B_n	+15.9	+ 2.4	+ 45.5	− 1.7
$Q_n = \sqrt{A^2_n + B^2_n}$	16.5	18.75	52.3	11.9
B_n/A_n	∓ 3.46	∓ 0.129	∓176	∓ 0.144
$x_n = \tan^{-1} B_n/A_n$	74°	173°	60°	352°

$Z = 0.3$, $X = 2.78$ — $Z = 1.1$, $X = 10.2$

	C	Prod.	y_n	$x_n - y_n$		C	Prod.	y_n	$x_n - y_n$
1st	1.4	23.1	8°	66°	1st	7.0	115.5	138°	296°
2nd	3.7	69.5	110°	63°	2nd	0.29	5.45	177°	356°
3rd	0.58	30.4	170°	250°	3rd	0.041	5.75	179°	241°
4th	0.26	3.1	180°	172°	4th	0.065	0.77	180°	172°
	$Y = 167$					$Y = 238$			

$Z = 0.5$, $X = 4.63$ — $Z = 1.3$, $X = 12$

	C	Prod.	y_n	$x_n - y_n$		C	Prod.	y_n	$x_n - y_n$
1st	1.9	31.4	11°	63°	1st	3.15	52	161°	273°
2nd	0.96	18	168°	5°	2nd	0.235	4.4	178°	355°
3rd	0.29	15.2	175°	245°	3rd	0.09	4.7	179°	241°
4th	0.14	1.67	180°	172°	4th	0.055	0.65	180°	172°
	$Y = 98$					$Y = 113$			

$Z = 0.7$, $X = 6.47$ — $Z = 1.5$, $X = 13.9$

	C	Prod.	y_n	$x_n - y_n$		C	Prod.	y_n	$x_n - y_n$
1st	3.15	52	19°	55°	1st	1.95	32.2	168°	266°
2nd	0.55	10.3	174°	359°	2nd	0.20	3.75	178°	355°
3rd	0.19	9.95	177°	243°	3rd	0.08	4.18	179°	241°
4th	0.10	1.19	180°	172°	4th	0.045	0.54	180°	172°
	$Y = 113$					$Y = 73.5$			

$Z = 0.9$, $X = 8.33$ — $Z = 2$, $X = 18.5$

	C	Prod.	y_n	$x_n - y_n$		C	Prod.	y_n	$x_n - y_n$
1st	7.0	115.5	40°	34°	1st	1.0	16.5	175°	259°
2nd	0.38	7.12	176°	357°	2nd	0.142	2.66	179°	354°
3rd	0.14	7.33	178°	242°	3rd	0.059	3.1	180°	240°
4th	0.08	95	180°	172°	4th	0.032	0.38	180°	172°
	$Y = 244$					$Y = 40$			

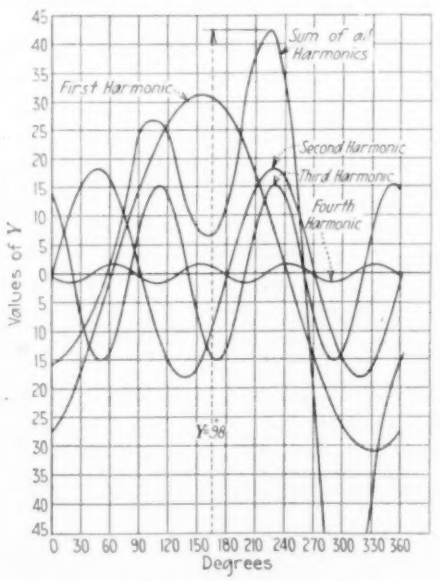


FIG. 9 FOUR HARMONICS FOR $Z = 0.5$

angles. The maximum points are indicated so that the value of Z can be read directly. Referring to Table 1, the behavior of Y should be particularly noted in that the value decreases as Z increases and then increases for larger values of Z. This increase is due to resonance with the first harmonic of the compressor torque curve. Beyond this resonance point Y decreases with increased values of Z. If one were to investigate the values of Y for values of Z smaller than 0.3, it will be found that there is a resonance peak for the second, third, and fourth harmonics.

An idea of the amount of work involved in making an XY curve can be gained when one remembers that it is necessary to draw up a Fourier's series such as the one exhibited in Fig. 9 for each value of X or for each one of the rectangles in Table 1. Fortunately the "harmonic curve tracer" shown in Figs. 10 and 11 reduces the drawing of these Fourier series to a matter of a few minutes. The instrument is provided with two ad-

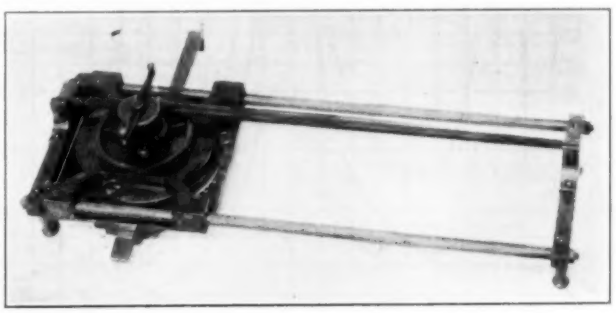


FIG. 10 HARMONIC CURVE TRACER

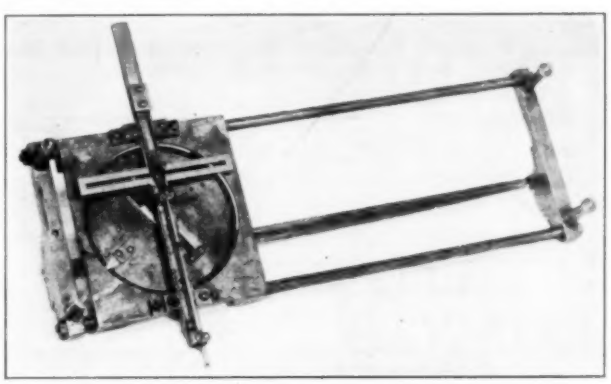


FIG. 11 HARMONIC CURVE TRACER

justments, one for magnitude and one for phase angle. Each harmonic is drawn separately, but only two or three minutes is required for a series of four. One can tell by looking at the series about where the maximum points will occur, and by adding up the curves with the dividers in about three points the maximum and minimum values are readily determined. The sum of these values is Y as shown in Fig. 9. Without the curve tracer the work involved in making XY curves is almost prohibitive.

A SET OF XY CURVES FOR AIR COMPRESSORS

Figs. 12 to 19 inclusive exhibit a set of eight XY curves for two-cylinder double acting two-stage air compressors with cranks at 90 deg. In addition to the full-load and no-load curves, two curves each are given for the part-load conditions at $^3/_4$, $^1/_2$, and $^1/_4$ load. The methods of unloading are described on each curve. The number on each curve is the same as that of the corresponding Fourier series listed in Tables 5 and 6 in Appendix No. 2. This is the most common type of compressor and is built in medium and large sizes.

Figs. 20 and 21 exhibit a similar set of XY curves for single-cylinder single-stage machines. Only full-load and no-load curves are given as these machines are built only in small sizes and are seldom if ever equipped with multi-step unloaders.

Example Illustrating Use of XY Curves

As an example illustrating the use of XY curves for the calculation of the proper WR^2 to limit the current pulsations to 66 per cent of rated current, consider the following case:

Two-cylinder double-acting two-stage compressor; cylinder sizes 26 and 15 × 21 in.; 5-step unloader giving the partial

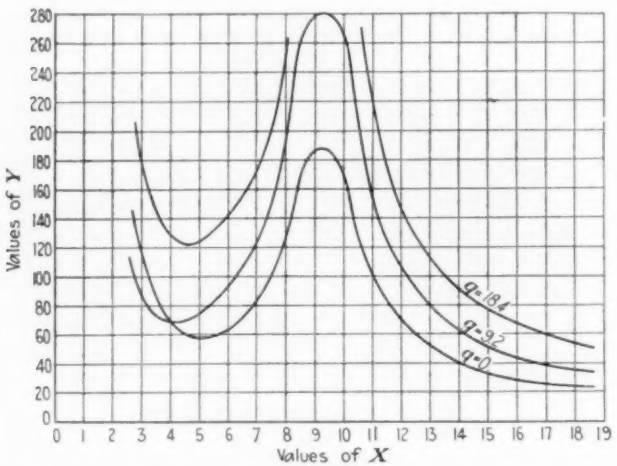


Fig. 12 Standard Two-Stage Air Compressor, Full Load, M = 0.75 C. P. or W. Method

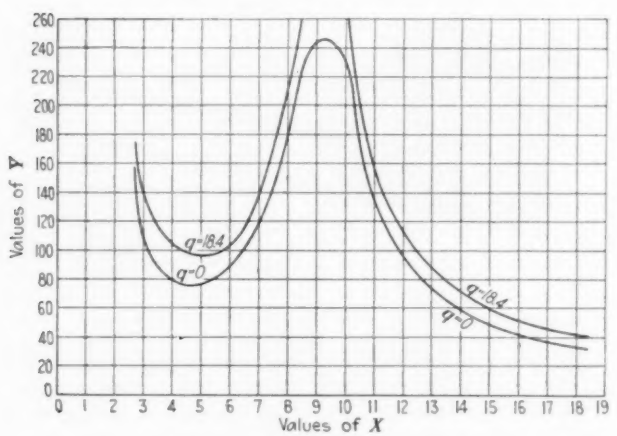


Fig. 13 Standard Two-Stage Air Compressor, Three-Quarter Load, M = 0.75 C. P. Method

(Head end high pressure and crank end low pressure on clearance pockets; opposite ends normal.)

load conditions described in Table 4, Appendix No. 2, Nos. 1, 3, 5, 7, and 8. Weight of reciprocating parts, including half the connecting rod weight, for the low-pressure cylinder is 1500 lb.

Motor, 450-hp., three-phase, 60-cycle, 200 r.p.m., 1.00 power factor, 361 kw., P_0 = 615 kw.

The directions will be followed through step by step:

First Step. Calculate the indicated horsepower of the compressor by the standard formula given in Equation [18], Appendix No. 1, which is

$$\text{i.hp.} = 8.99 \times K \times \left(\frac{V}{1000}\right) \times \frac{s}{100}$$

where K = a constant = 2 for this type of compressor. See Appendix No. 1 for K

V = volume of low-pressure cylinder in cu. in.

s = r.p.m.

$$\text{i.hp.} = 8.99 \times 2 \times 11.15 \times 2.0 = 401$$

Second Step. Calculate Y from Equation [11]

$$Y = \frac{450}{401} \times 66 = 74$$

Third Step. Calculate q from the formula in Appendix No. 2 which is

$$q = \left(\frac{s}{100}\right)^2 \times r \times w$$

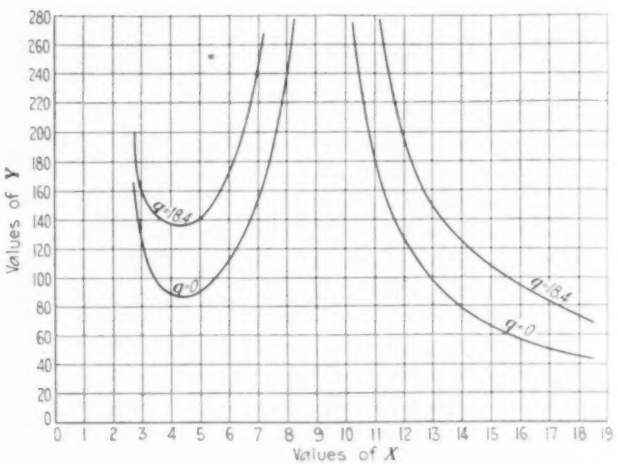


Fig. 14 Standard Two-Stage Air Compressor, Two-Cylinder Double-Acting, M = 0.75, $^3/_4$-Load

(Crank ends on clearance, head ends normal, W. method.)

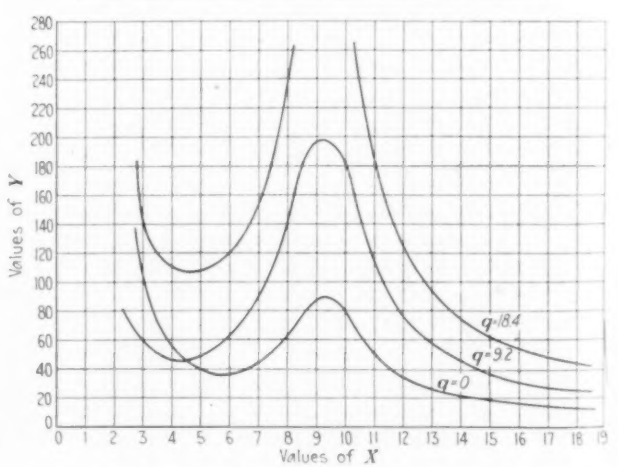


Fig. 15 Standard Two-Stage Air Compressor, Two-Cylinder Double-Acting, $^1/_2$ Load, M = 0.75

(Head end and crank end of both cylinders on clearance, C. P. method.)

where r = crank radius in feet

w = weight of reciprocating parts in lb. per sq. in. of piston area

$$q = (2)^2 \times 0.875 \times \frac{1500}{530} = 9.9$$

Fourth Step. Examine the XY curves given in Figs. 12, 14, 16, 18, and 19 which cover the five different conditions for this

case. It will be noted that one-half load (Fig. 16) is the worst condition, that is, the one which requires the largest value of X and hence the largest value of WR^2. For $q = 9.2$ which is almost the same value as calculated it is seen that $x = 25.5$ for $Y = 74$.

Fifth Step. Calculate the WR^2 required by Formula [15]

$$WR^2 = \frac{25.5 \times 4.5 \times 6 \times 1.7}{(2)^4} \times 10^3 = 73{,}500 \text{ lb-ft.}^2$$

This value of WR^2 is just sufficient to limit the current pulsation to 66 per cent at half load. At quarter load the pulsation will be nearly as great but at the other loads it will be considerably less as the other XY curves show.

Appendix No. 1—Indicated Horsepower

In determining the size of a synchronous motor required to drive an air compressor the following formula for the indicated horsepower of the compressor will be found convenient

$$\text{. . .} = 8.99K\left(\frac{V}{1000}\right)\left(\frac{s}{100}\right) \text{. . . . . [18]}$$

where

s = r.p.m.
V = volume of one cylinder. For two-stage machines V is the volume of the low-pressure cylinder
K = 2 for single-cylinder double-acting single-stage compressors
= 4 for two-cylinder double-acting single-stage compressors
= 2 for two-cylinder double-acting two-stage compressors.

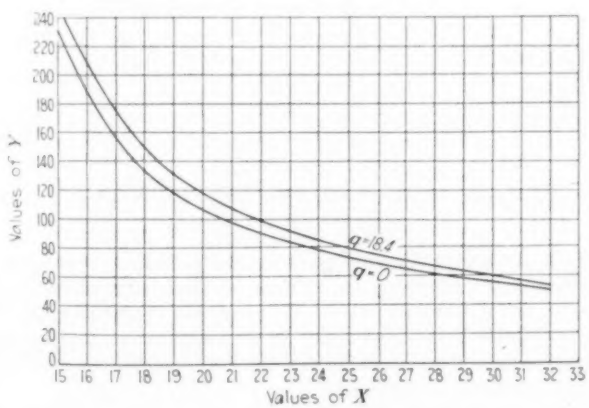


FIG. 16 STANDARD TWO-STAGE, TWO-CYLINDER, DOUBLE-ACTING AIR COMPRESSOR, 1/2 LOAD; $M = 0.57$
(Both crank ends with suction valves held open opposite ends normal.)

Experience has shown that in a great majority of cases, compressor manufacturers will recommend a motor the rated horsepower of which is from 5 to 15 per cent greater than the indicated horsepower calculated by Formula [18].

This simple formula was derived by calculating the area of the theoretical indicator card shown in Fig. 22. In constructing this card a polytropic exponent of 1.4 was used for both the compression and expansion lines. Compression was taken between atmosphere and 100 lb. gage, and a clearance of 4 per cent was used. For the purposes of this article it will be convenient to speak of this theoretical single-stage card as the "standard air cycle."

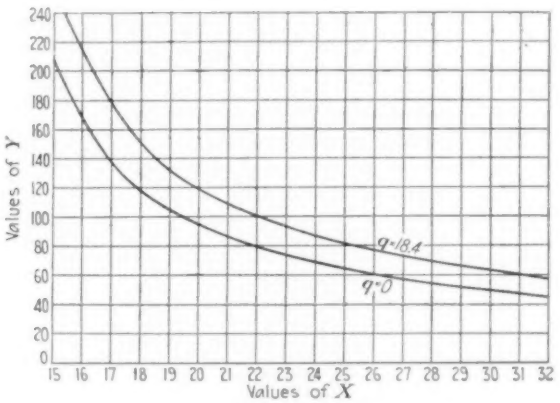


FIG. 17 STANDARD TWO-STAGE, TWO-CYLINDER, DOUBLE-ACTING AIR COMPRESSOR, 1/4 LOAD, $M = 0.75$
(Head end high pressure and crank end low pressure suction valves held open, opposite ends operating with clearance pockets.)

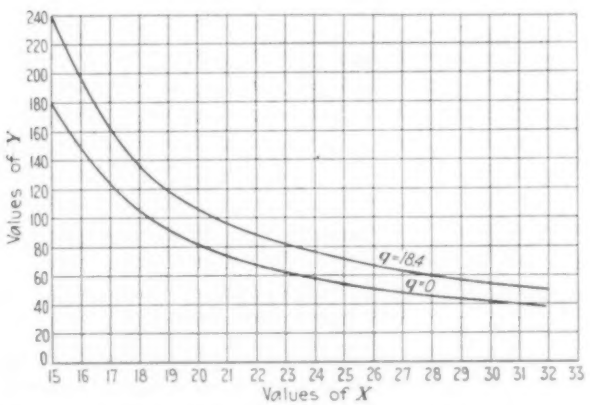


FIG. 18 STANDARD TWO-STAGE, TWO-CYLINDER, DOUBLE-ACTING AIR COMPRESSOR, 1/4 LOAD, $M = 0.75$
(Both crank ends with clearance pockets and both head ends with suction valves held open.)

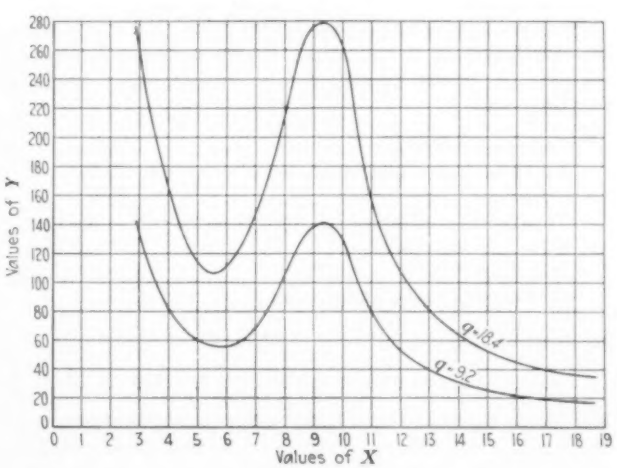


FIG. 19 STANDARD TWO-STAGE AIR COMPRESSOR, $M = 0.75$
(No load, all valves open.)

It is often necessary to calculate the indicated horsepower under different conditions than those on which Formula [18] is based, that is, different pressures or different clearances. For this purpose the following formula, in connection with the family of curves given in Fig. 23, may be used.

$$\text{i.hp.} = 0.252\, A_0 P_s h \left(\frac{V}{1000}\right)\left(\frac{s}{100}\right) \quad \ldots\ldots\ldots [19]$$

where

A_0 = constant given by curves in Fig. 23
P_s = absolute suction pressure
h = number of active cylinder ends
V = volume of one cylinder
s = r.p.m.
r = ratio $\dfrac{\text{Absolute discharge pressure}}{\text{Absolute suction pressure}}$
α = percentage of clearance.

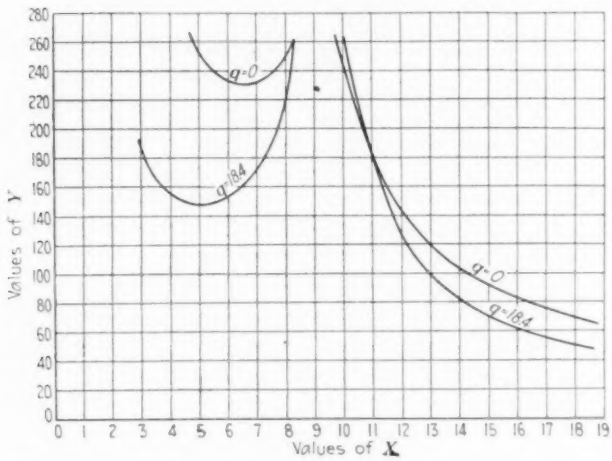


FIG. 20 SINGLE-CYLINDER DOUBLE-ACTING SINGLE-STAGE AIR COMPRESSOR, FULL LOAD

In applying this formula to two-stage compressors, the low-pressure and high-pressure cylinders must be figured separately. This is also advisable because the clearance of the high-pressure cylinder is usually larger than that of the low-pressure due to the fact that the same size valves are used in both cylinders but the high-pressure cylinder is smaller in diameter than the low-pressure cylinder.

As an example of the use of Formulas [18] and [19] consider the calculation of the indicated horsepower of the following two-cylinder, double-acting, two-stage compressor operating at 150 r.p.m.

Diameter low-pressure cylinder	37 in.
Diameter high-pressure cylinder	22 in.
Stroke	27 in.
Clearance low-pressure	4 per cent
Clearance high-pressure	8 per cent
Pressures	0 to 100 lb. gage
Interstage pressure approximately	30 lb. gage

By Formula [18]

$$\text{i.hp.} = 8.99 \times 2 \times 29 \times 1.5 = 781$$

The motor recommended by the compressor manufacturer for this machine was 875 hp. which is 12 per cent greater than by [18].

By Formula [19]

Low-pressure cylinder i.hp. $= 0.252 \times 1.24 \times 14.7 \times 2 \times 29 \times 1.5 =$ 400

High-pressure cylinder i.hp. $= 0.252 \times 1.0 \times 44.7 \times 2 \times 10.25 \times 1.5 =$ 346

Total i.hp. = 746

The lower value of indicated horsepower obtained by Formula [19] is due largely to the fact that Formula [18] assumes single-stage compression while Formula [19] takes into account automatically the greater efficiency of two-stage compression.

If the interstage pressure is not given by the compressor manufacturer, it can be calculated from the fact that the ratio of the absolute interstage pressure to the absolute suction pressure must be approximately equal to the ratio of the volume of the low-pressure cylinder to that of the high-pressure cylinder. This cannot be exactly true since it would assume that both cylinders have the same clearance and that the air in the intercooler is reduced to the same temperature as in the intake. Obviously the first condition depends on the construction of the compressor and the second on the amount and temperature of the cooling water used. The lower the interstage temperature the lower will be the interstage pressure and the greater the efficiency of compression. It is obvious that the gain in efficiency of two-stage over single-stage compression depends on the relative temperature of the gas in the intercooler and in the intake. In air compressors the gas in the intercooler is reduced to practically the temperature of the intake without difficulty, but in two-stage ammonia compressors the temperature of the gas in the intercooler is reduced only to that of the cooling water while the temperature of the gas in the suction line is below freezing. Hence for ammonia compressors there is very little gain in efficiency of two-stage over single-stage compression. This accounts for the very limited use of two-stage ammonia compressors.

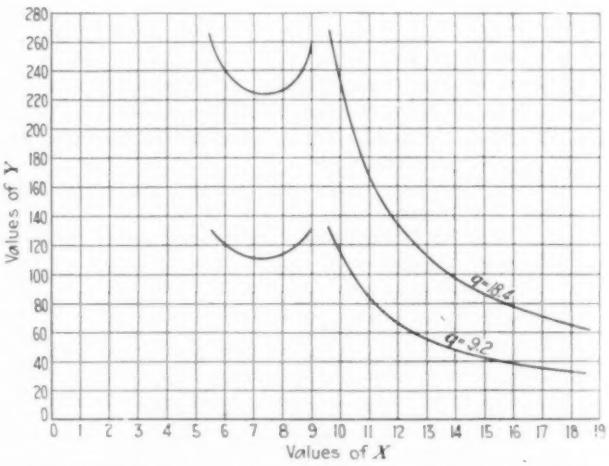


FIG. 21 SINGLE-STAGE SINGLE-CYLINDER DOUBLE-ACTING AIR COMPRESSOR, NO LOAD

In the calculation of the above example by Formula [19] it may be noted that the indicated horsepower of the high-pressure cylinder is somewhat less than that of the low-pressure cylinder. This does not indicate a wrong interstage pressure but is due to the fact that most compressors are designed to give balanced work between the two cylinders at a discharge pressure of from 115 to 120 lb. gage. Under ordinary conditions of operation the work done by the two cylinders is slightly unbalanced, being larger for the low-pressure cylinder.

Appendix No. 2—Theoretical Torque Curves

The calculation of a set of theoretical torque curves which can be applied quite generally to the different types of air compressors can at best be only a good approximation, because the details of construction differ with the various manufacturers.

Such items for instance as clearance, ratio of connecting-rod length to crank length, ratio of the weights of reciprocating parts of high-pressure and low-pressure cylinders, and others, vary over small ranges and have some effect on the crank effort of the machine. The best one can do is to choose a middle path and proceed with average values and this is what has been done in this paper. The resulting torque curves will therefore not apply absolutely to any one make of compressor but will apply approximately to all makes of the types discussed in this paper. They will be accurate enough to form a basis for flywheel calculations and they will show the relative merits of the different methods of unloading.

The torque required by a compressor may be separated into two parts, one due to the gas forces, and the other due to the

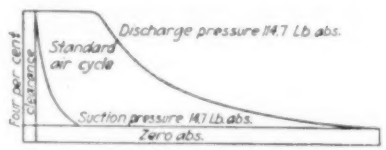


FIG. 22

inertia. It will be found convenient to calculate these entirely separately, since the inertia forces depend on the speed of the machine as well as the weights of reciprocating parts. Instead of actually calculating and plotting torque curves, the torque will be expressed in Fourier's series. In this work the coefficients of the several harmonics will be given in percentage based on the average torque of the compressor or rather on what would

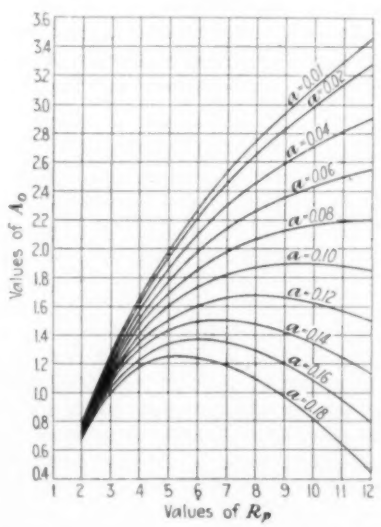


FIG. 23 INDICATED HORSEPOWER OF AIR COMPRESSOR

where i.hp. $= 252 A_0 P_s h \ (g/10^3)(s/10^3)$
A_0 = constant, read from curve
P_s = absolute suction pressure
h = number of active cylinder ends
V = cylinder volume in cubic inches
R_p = Ratio = absolute suction pressure / absolute discharge pressure
a = clearance, per cent

be the average torque if the indicator cards were all standard air cycles as shown in Fig. 22. This will be true of the inertia torque as well as the gas torque.

INERTIA TORQUE

It is not within the scope of this paper to present in detail the theory underlying the calculation of the inertia torque of a reciprocating machine such as is shown in Fig. 24.

Perhaps the best method of calculating this torque, however, is to obtain an expression for the kinetic energy stored in the moving system, in terms of the angle θ through which the crank has moved *from its head-end position*. Differentiating this

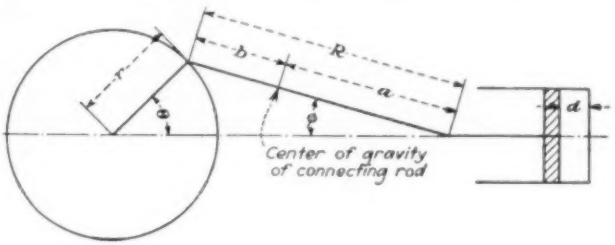


FIG. 24

energy expression with respect to the angle θ will give the torque required for the inertia as follows:

$$T = \frac{r^2\omega^2}{4g}\left\{-\frac{1}{l}\left[W_r + \frac{b}{R}W_c\right]\sin\theta + 2\left[W_r + W_c\left(1 - \frac{K^2}{R^2}\right)\right]\sin 2\theta + \frac{3}{l}\left[W_r + \frac{b}{R}W_c\right]\sin 3\theta + \frac{1}{l^2}\left[W_r + \frac{b^2}{R^2}W_c\right]\sin 4\theta\right\} \quad \ldots [20]$$

where

r = crank length in feet
R = connecting rod length in feet
b = distance from the center of gravity of the connecting rod to the axis of crank pin in feet
K = radius of gyration of connecting rod about its center of gravity in feet
l = ratio of connecting rod length to crank length = R/r
ω = angular velocity of crank in radians per sec.
g = acceleration due to gravity
W_r = weight of reciprocating parts, that is, piston, piston rod, etc.
W_c = weight of connecting rod.

Of the four terms in the above series the second is by far the largest and the fourth much the smallest. Investigations of the design of several connecting rods show that the ratio K^2/R^2 is very nearly equal to $1/2$ in most cases. Also b/R is approximately $1/2$. Hence with good approximation Equation [20] can be written.

$$T = \frac{r^2\omega^2}{4g}W\left\{-\frac{1}{l}\sin\theta + 2\sin 2\theta + \frac{3}{l}\sin 3\theta + \frac{1}{l^2}\sin 4\theta\right\} \quad \ldots [21]$$

where W is taken as the weights of the reciprocating parts plus half the weight of the connecting rod.

Let w = weight of reciprocating parts plus half the connecting rod weight per sq. in. of piston area.

A = piston area

then $W = Aw$, and the coefficients of the four harmonics in [21] may be written

$$\left.\begin{aligned} a_1 &= -\frac{r^2\omega^2}{4g}\frac{Aw}{l} & a_3 &= \frac{r^2\omega^2}{4g}\frac{3Aw}{l} \\ a_2 &= +\frac{r^2\omega^2}{4g}2Aw & a_4 &= \frac{r^2\omega^2}{4g}\frac{Aw}{l^2} \end{aligned}\right\} \quad \ldots [22]$$

The factor $\frac{r^2\omega^2 A}{4g}$ which occurs in all of these terms can be reduced as follows:

$$\frac{r^2\omega^2 A}{4g} = \frac{Ar^2(0.1048s)^2}{4 \times 32.2} = \frac{85.272}{10^6} r^2 s^2 A \quad \ldots\ldots [23]$$

The problem is now to express the harmonics in [22] in percentage based on the average torque of a standard air cycle. It has been shown that the indicated horsepower of the standard air cycle is

$$\text{i.hp.} = 8.99 \times \left(\frac{V}{1000}\right) \times \left(\frac{s}{100}\right) \quad \ldots\ldots [24]$$

where

V = volume of the cylinder in cu. in.
s = r.p.m.

But $V = A \times 2r \times 12 = 24\,Ar$ ..... [25]
since r is the crank radius in feet.
Substituting [25] in [24] gives

$$\text{i.hp.} = \frac{8.99}{10^5} \times 24\,Ars = \frac{2.1576}{10^3} Ars \quad \ldots\ldots [26]$$

The average torque in ft-lb. corresponding to this i.hp. is

$$a_0 = \frac{5250}{s} \times \frac{2.1576}{10^3} rAs = 11.327rA \quad \ldots\ldots [27]$$

Dividing the a's in [22] by a_0 gives the harmonics in per cent based on the average torque of one standard cycle. For instance

$$A_1 = \frac{a_1}{a_0} = -\frac{w}{l} \frac{85.272 \times r^2 s^2 A \times 10^2}{10^6 \times 11.327\,rA}$$

$$= -\frac{7.528}{l}\left[rw\left(\frac{s}{100}\right)^2\right] \quad \ldots [28]$$

Particular attention should be called to the expression in the bracket which will be called q. It is this factor which determines the inertia torque of a compressor. One should note that it involves only the crank radius in feet, the weight of the reciprocating parts per square inch of piston area and the speed.

Putting q in place of the bracket gives

and similarly

$$\left.\begin{aligned} A_1 &= -\frac{7.528}{l} q \\ A_2 &= +15.056\,q \\ A_3 &= +\frac{22.58}{l} q \\ A_4 &= +\frac{7.528}{l^2} q \end{aligned}\right\} \quad \ldots\ldots [29]$$

These are the inertia forces for a single cylinder in per cent based on one standard cycle.

For a single-cylinder double-acting single-stage compressor, the above values would have to be divided by 2 to get the percentages based on the average torque of the whole compressor since two standard cycles would be involved. Similarly for a two-cylinder double-acting single-stage compressor with 90 degrees between cranks, it would be necessary to add two Fourier's series like [29] at right angles and divide the result by 4 since there would be four standard cycles involved. Percentages would then be obtained based on the average torque of the whole compressor.

This gives for the two cylinder double acting single stage

$$\left.\begin{aligned} A_1 &= -\frac{1.88}{l} q & B_1 &= +\frac{1.88}{l} q \\ A_2 &= 0 & B_2 &= 0 \\ A_3 &= +\frac{5.65}{l} q & B_3 &= +\frac{5.65}{l} q \\ A_4 &= +\frac{3.76}{l^2} q & B_4 &= 0 \end{aligned}\right\} \quad \ldots\ldots [30]$$

The B's in [30] are coefficients of the cosine terms which occur when the two series are added at right angles.

For two-stage compressors with balanced inertia forces, that is, with low-pressure and high-pressure pistons of the same weight, the values given by [29] must be multiplied by 2 since in this case the percentages must be based on two standard cycles instead of four.

It is only in special cases that two-stage compressors are made with high-pressure and low-pressure pistons of the same weight. Ordinarily the high-pressure piston has about three-quarters the weight of the low-pressure piston.

In general let

$$m = \frac{\text{Weight of reciprocating parts of h-p. cylinder}}{\text{Weight of reciprocating parts of l-p. cylinder}}$$

Assume also that the high-pressure cylinder is leading the low-pressure cylinder by 90 degrees. This is the customary method of operation for two-stage machines and all the torque curves will be worked out on this basis.

If [29] is taken as the inertia torque of the low-pressure cylinder, and m is defined as above, the inertia torque of the high-pressure cylinder on the same basis is

$$\left.\begin{aligned} A_1 &= -\frac{7.528}{l} qm \\ A_2 &= +15.056\,qm \\ A_3 &= +\frac{22.58}{l} qm \\ A_4 &= +\frac{7.528}{l^2} qm \end{aligned}\right\} \quad \ldots\ldots [31]$$

Adding the series [31] to series [29] and 90 degrees ahead of it and dividing by 2 the result gives the inertia torque of the two-cylinder double-acting two-stage compressor in percentage based on the average torque of the compressor as follows:

$$\left.\begin{aligned} A_1 &= -\frac{3.76}{l} qm & B_1 &= +\frac{3.76}{l} q \\ A_2 &= -7.53\,q(1-m) & B_2 &= 0 \\ A_3 &= +\frac{11.3}{l} qm & B_3 &= \frac{11.3}{l} q \\ A_4 &= +\frac{3.76}{l^2} q(m+1) & B_4 &= 0 \end{aligned}\right\} \quad \ldots [32]$$

Table 2 has been prepared showing numerical values of the inertia torque for various types of compressors. A value of $m = 0.75$ has been used for the two-stage machines as this represents a fair average. The ratio of connecting-rod length to crank length which is l, has been taken as 5. Some compressors have ratios as high as 5.5 but 5 is a good average.

TABLE 2 INERTIA FORCES

	Single-cylinder double-acting, single-stage		Two-cylinder double-acting, single-stage		Two-cylinder, double-acting, two-stage, balanced inertia		Two-cylinder, double-acting, two-stage, $m = 0.75$	
	$q = 9.2$	$q = 18.4$	$q = 9.2$	$q = 18.4$	$q = 9.2$	$q = 18.4$	$q = 9.2$	$q = 18.4$
A_1	− 6.95	− 13.9	− 3.46	− 6.92	− 6.92	−13.84	− 5.2	−10.4
A_2	+69.5	+139	0	0	0	0	−17.3	−34.6
A_3	+20.75	+ 41.5	+10.4	+20.8	+20.8	+41.6	+15.6	+31.2
A_4	+ 1.4	+ 2.8	+ 1.38	+ 2.77	+ 2.77	+ 5.54	+ 2.5	+ 4.9
B_1	0	0	+ 3.46	+ 6.92	+ 6.92	+13.84	+ 6.9	+13.8
B_2	0	0	0	0	0	0	0	0
B_3	0	0	+10.4	+20.8	+20.8	+41.6	+20.8	+41.6
B_4	0	0	0	0	0	0	0	0

Calculations have been made for two values of q, 9.2 and 18.4. The value 9.2 represents an average case while experience has shown that a value of 18.4 represents an outside limit. Complete torque curves are given for three values of q, 0, 9.2, and

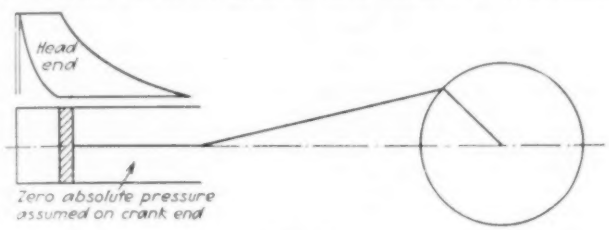


FIG. 25

18.4 in the body of the paper and flywheel calculation curves presented on the same basis. This gives a curve for the case of no inertia, another for the case of maximum inertia, and a third with average inertia. In any particular case the actual value of q can be calculated and final results obtained by interpolation between the curves given.

TORQUES DUE TO GAS FORCES

It has been found a decided advantage to obtain the gas torque due to each indicator card, that is, each end of a cylinder separately. There are fairly few types of indicator cards involved in most compressors even under partial load conditions. By obtaining Fourier's series for these component parts of the gas torque they can be combined in a variety of ways to give the complete torque of different types of compressors under different conditions of loading. As an illustration of this method consider a single-stage single-cylinder machine compressing air from atmosphere to 100 lb. gage. For this case the indicator card will be a standard air cycle as shown in Fig. 22.

The ratio R_p, which is the ratio of absolute discharge pressure to absolute suction pressure is 7.8. Assuming 4 per cent clearance and a ratio of connecting rod length to crank length of 5, a torque curve could be constructed due to a standard air cycle in the heat end of the cylinder as shown in Fig. 25. This torque curve could be analyzed in a Fourier's series and each harmonic divided by $a_0 = 11.327$ the average torque of the standard air cycle. The resulting harmonics would then be obtained in percentage based on one standard air cycle and would be as follows:

$$\left.\begin{aligned} A_1 &= -287 & B_1 &= +109.1 \\ A_2 &= -171 & B_2 &= -32.5 \\ A_3 &= -80.1 & B_3 &= -83.5 \\ A_4 &= -22.3 & B_4 &= -68.6 \end{aligned}\right\} \quad \ldots\ldots [33]$$

Similarly if the standard air cycle is put in the crank end of the cylinder and the same operations performed, the results for a crank-end card would be

$$\left.\begin{aligned} A_1 &= +302.1 & B_1 &= -84.3 \\ A_2 &= -124 & B_2 &= -69.7 \\ A_3 &= +37.4 & B_3 &= +86.6 \\ A_4 &= +6.2 & B_4 &= -43.2 \end{aligned}\right\} \quad \ldots\ldots [34]$$

In making both of the foregoing analyses, time has been counted from the instant the piston is in its head end position. Time was counted in the same way in making the series for the inertia forces and it is of course necessary to maintain the same convention throughout.

If the series [33] and [34] are added directly and divided by 2, the torque for a single-cylinder double-acting single-stage machine is obtained. In this manner it can be seen that a great variety of torque curves can be made up at once if one has the analyses corresponding to all the different indicator cards involved.

For the part-load conditions discussed in this paper half-load indicator cards are required. These are obtained in practice by the use of clearance pockets of such size that the area of the card is reduced to half that of the normal card. Such a card for single-stage compression is shown in Fig. 26.

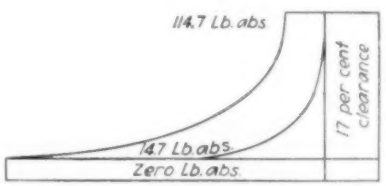


FIG. 26

The no-load conditions are obtained by opening the ends of the cylinder to the atmosphere. The indicator card for an end open to the atmosphere is simply a straight line at a pressure = 14.7 lb. The harmonics due to this type of a no-load card in percentage based on one standard cycle are easily shown to be

Crank End	Head End
$A_1 = +130$ (+124)	−130
$A_2 = +13$ (+12.4)	− 13

(Values in parenthesis are reduced 4.5 per cent to compensate for that portion of the piston area taken up by the piston rod.)

In making the cards for two-stage compressors an interstage pressure of 30 lb. gage has been assumed. This makes a ratio of absolute discharge pressure to absolute suction pressure of 3.04 for the low-pressure cylinder and 2.57 for the high-pressure cylinder. These cards must be made up for both full load and half load so that in all four are required. These four cards have to be analyzed for both head- and crank-end positions giving in all eight analyses for two-stage compressors as shown in Table 3.

All the harmonics in Table 3 are given in per cent based on one standard air cycle.

By properly selecting and combining the series in Table 3 and Table 2, torque curves for the eight different conditions of operation of two-stage machines listed in Table 4 are obtained. It will be noted that in addition to full-load normal operation, two methods each are given for $^3/_4$, $^1/_2$, and $^1/_4$ load and one method for no-load. These are not all of the part-load methods of operation in use but they are among the most important and will be found to represent the cases most needed for flywheel calculations.

In Table 5 the harmonics are given for the eight conditions of operation listed in Table 4. The set for $q = 0$ represents the gas forces alone, since if $q = 0$, the inertia forces are zero. Similarly the set for $q = 9.2$ represents the combinations of the gas forces and an average value of inertia forces. The set for $q = 18.4$ represents the extreme case of very large inertia forces.

Table 6 is similar to Table 5 except that it covers the cases for single-stage single-cylinder machines. Only full-load and no-load conditions are given, as machines of this type are usually

TABLE 3

Full Load, Two-Stage, Low-Pressure Cylinder
$R_p = 3.04$ $\alpha = 4$ per cent $l = 5$

	Crank end		Head end
A_1 =	+219	(+209.1)[a]	−215
A_2 =	− 22.8	(− 21.8)	− 68.4
A_3 =	− 16.6	(− 15.9)	+ 0.5
A_4 =	+ 4.2	(+ 4.0)	+ 10.2
B_1 =	− 25	(− 23.9)	+ 39.9
B_2 =	− 52.4	(− 50.0)	− 40.0
B_3 =	+ 23.5	(+ 22.4)	− 37.5
B_4 =	+ 4.9	(+ 4.7)	− 7.8

Full Load, Two-Stage, High-Pressure Cylinder
$R_p = 2.57$ $\alpha = 4$ per cent $l = 5$

	Crank end		Head end
A_1 =	+204.1	(+194.9)[a]	−200.7
A_2 =	− 11.1	(− 10.6)	− 53.7
A_3 =	− 15.7	(− 15)	+ 4.4
A_4 =	− 0	(0)	+ 6.2
B_1 =	− 17.5	(− 16.7)	+ 30.0
B_2 =	− 44.4	(− 42.4)	− 35.2
B_3 =	+ 14.4	(+ 13.8)	− 26.6
B_4 =	+ 5.3	(+ 5.1)	− 2.8

Half Load, Two-Stage, Low-Pressure Cylinder
$R_p = 3.04$ $\alpha = 43$ per cent $l = 5$

	Crank end		Head end
A_1 =	+216.3	(+206.6)	−209
A_2 =	− 46.8	(− 44.7)	− 84.9
A_3 =	+ 9.6	(+ 9.2)	− 29.8
A_4 =	+ 5.6	(+ 5.3)	− 4.7
B_1 =	− 9.1	(− 8.7)	+ 16.6
B_2 =	− 30.2	(− 28.8)	− 24.2
B_3 =	+ 12.6	(+ 12.0)	− 19.6
B_4 =	+ 1.8	(+ 1.7)	− 5.0

Half Load, Two-Stage, High-Pressure Cylinder
$R_p = 2.57$ $\alpha = 54$ per cent $l = 5$

	Crank end		Head end
A_1 =	+201.5	(+192.4)	−196.2
A_2 =	− 33.5	(− 32.0)	− 70.6
A_3 =	+ 5.0	(+ 4.8)	− 21.3
A_4 =	+ 4.8	(+ 4.6)	− 1.8
B_1 =	− 5.9	(− 5.6)	+ 12.5
B_2 =	− 26	(− 24.8)	− 21.7
B_3 =	+ 8.5	(+ 8.1)	− 15.7
B_4 =	+ 3.5	(+ 3.3)	− 2.3

[a] The values in parenthesis are corrected for 4½ per cent piston-rod area. In other words it is assumed that the crank-end pressure is reduced to 95.5 per cent of what it would have been were there no piston rod.

TABLE 4 TWO CYLINDER DOUBLE ACTING, TWO-STAGE AIR COMPRESSOR TORQUE CURVES, CRANKS SET AT 90 DEGREES (SEE TABLE 5)

1 Compressor operating at full load.
2 Compressor operating at ¾ load with head-end high-pressure and crank-end low-pressure operating with clearance pockets and opposite ends normal.
3 Compressor operating at ¾ load with both crank ends operating with clearance pockets and head ends normal.
4 Compressor operating at ½ load with all ends operating with clearance pockets.
5 Compressor operating at ½ load with both head ends normal and both crank ends with suction valves held open.
6 Compressor operating at ¼ load with head-end high-pressure and crank-end low-pressure suction valves held open and opposite ends operating with clearance pockets.
7 Compressor operating at ¼ load with both crank ends operating with clearance pockets and both head ends with suction valves held open.
8 Compressor operating at no load with all suction valves held open.

TABLE 5 TWO-CYLINDER DOUBLE-ACTING TWO-STAGE MACHINES

A	1	2	3	4	5	6	7	8
Torque Curves for $q = 0$								
A_1	+ 5.1	+15	+11.5	+ 2.05	−60.5	−39.7	−41.1	
A_2	+13	+16	+13.7	+13.5	− 7.35	−13.8	− 6.35	
A_3	+ 2.3	− 5.4	+17.4	− 4.45	−13.0	−10.9	+ 8.65	
A_4	+10.2	+ 6.9	+13.2	+ 1.7	+ 8.2	− 0.05	+ 4.95	
B_1	+ 9.6	+ 2.1	+16.4	+ 4.65	−18.4	+39.5	+26.8	
B_2	+ 6.2	+ 2.4	+ 4.4	+ 3.25	+37.6	+ 0.3	− 2.0	
B_3	−14.1	+ 3.9	− 4.4	−14.1	−20.9	−12.2	+ 3.6	
B_4	− 0.4	− 1.7	− 2.8	− 1.15	− 5.3	− 0.85	+ 2.5	
Torque Curves for $q = 9.2$								
A_1	− 0.1	+ 9.8	+ 6.3	− 3.15	−65.7	−44.9	−46.3	− 5.2
A_2	− 4.3	− 1.3	− 6.6	− 3.8	−24.6	−31.1	−23.6	−17.3
A_3	+17.9	+10.2	+33	+11.1	+ 2.6	+ 4.7	+24.2	+15.6
A_4	+12.7	+ 9.4	+15.7	+ 4.2	+10.7	+ 2.5	+ 7.4	+ 2.5
B_1	+16.5	+ 9.0	+23.3	+11.5	−11.5	+46.4	+33.7	+ 6.9
B_2	+ 6.2	+ 2.4	+ 4.4	+ 3.25	+37.6	+ 0.3	− 2.0	0
B_3	+ 6.7	+24.7	+16.4	+ 6.7	− 0.15	+ 8.6	+24.4	+20.8
B_4	− 0.4	− 1.7	−2.8	− 1.15	− 5.3	− 0.85	+ 2.5	0
Torque Curves for $q = 18.4$								
A_1	− 5.3	+ 4.6	+ 1.1	− 8.35	−70.9	−50.1	−51.5	−10.4
A_2	−21.6	−18.6	−20.9	−21.1	−41.9	−48.4	−40.9	−34.6
A_3	+33.5	+25.8	+48.6	+26.7	+18.2	+20.3	+39.8	+31.2
A_4	+15.1	+11.8	+18.1	+ 6.6	+13.1	+ 4.9	+ 9.8	+ 4.9
B_1	+23.4	+15.9	+30.2	+18.4	− 4.6	+53.3	+40.6	+13.8
B_2	+ 6.2	+ 2.4	+ 4.4	+ 3.25	+37.6	+ 0.3	− 2.0	+ 0
B_3	+27.5	+45.5	+37.2	+27.5	+20.7	+29.4	+45.2	+41.6
B_4	− 0.4	− 1.7	− 2.8	− 1.15	− 5.3	− 0.85	+ 2.5	0

TABLE 6 SINGLE-CYLINDER DOUBLE-ACTING SINGLE-STAGE AIR-COMPRESSOR TORQUE CURVES FOR DIFFERENT VALUES OF q

9 Compressor operating at full load
10 Compressor operating at no load with all suction valves held open.

	$q = 0$		$q = 9.2$		$q = 18.4$	
	9	10	9	10	9	10
A_1	+ 0.8	...	− 6.15	− 6.95	−13.1	− 13.9
A_2	−144.7	...	−75.2	+69.5	− 5.7	+139
A_3	− 22.2	...	− 1.45	+20.75	+19.3	+ 41.5
A_4	− 8.2	...	− 6.8	+ 1.4	− 5.4	+ 2.8
B_1	+ 14.6	...	+14.6	0	+14.6	0
B_2	− 49.5	...	−49.5	0	−49.5	0
B_3	− 0.2	...	− 0.2	0	− 0.2	0
B_4	− 54.9	...	−54.9	0	−54.9	0

built only in small sizes and are not provided with multi-step unloading devices.

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Discussion

F. K. Brainard.[5] The authors are to be congratulated on giving such a clear exposition of their method of computing XY curves. However, the writer would like to ask if they can tell approximately how much error is introduced by making the approximations used in the paper. To begin with, Equation [3] is approximate only, and later in the paper the item $\Sigma T_d \frac{d\psi_n}{dt}$ is neglected as the authors state. (See place a few lines below Equation [10].) Also no mention is made of the variation in the synchronizing power of the motor with frequency which Mr. Putman stressed so much in an A.I.E.E. paper in 1926. At that time, the writer felt that this last phenomenon was due to the approximation in Equation [3] just mentioned and submitted an equation and its solution which fitted the case a little better and which seemed to render the idea of the synchronizing power's varying with the frequency unnecessary, although Mr. Putman did not agree with this.[6] In the present paper the authors neglect this point entirely as they make no use either of the method derived in the paper referred to above or the substitute method proposed by the writer. Probably the method used in this paper is sufficiently accurate for all practical purposes and the complication resulting from the other methods is not justified, but the writer would like to have the authors' opinion on this point.

Also there seems to be some discrepancy in the definition of the quantity X (or C as it is sometimes called).

Stevenson's definition of X (and the writer believes it was he who first proposed it) was as follows:[7]

$$X = \frac{0.80\, WR^2 \text{ r.p.m.}^4}{10^6\, P_0 f}$$

This, as stated by him, reduces to

[5] Electrical Engineer, Allis-Chalmers Manufacturing Company, Milwaukee, Wis.

[6] See Trans. A.I.E.E., vol. 45 (1926), p. 1124.

[7] See *Refrigerating Engineering*, vol. 2 (1924), p. 128.

$$X = 10\left(\frac{\text{r.p.m.}}{F}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots [a]$$

where F = natural frequency in oscillation per minute
f = frequency of circuit in cycles per second, and
r.p.m. = revolutions per minute.

This seems a very logical definition since resonance with the fundamental occurs at $X = 10$. However, Stevenson's curves published in the same paper were apparently based on

$$X = 9\left(\frac{\text{r.p.m.}}{F}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots [b]$$

Now the present authors propose

$$X = 9.25\left(\frac{\text{r.p.m.}}{F}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots [c]$$

Hence it would seem quite desirable to standardize on some definition of X and the corresponding "flywheel constant" and not leave it to the individual to select his own definition.

The A.I.E.E. have adopted the following formula for natural frequency:

$$F = \frac{266{,}500}{\text{r.p.m.}}\sqrt{\frac{P_s f}{WR^2}}$$

where P_s = synchronizing power, which is defined as the rate of change of power with respect to the displacement angle expressed in kilowatts per electrical degree. Hence it would seem desirable to use P_s in this work also rather than P_0.

If the above value for F is substituted in Equation [*a*] and the resulting expression solved for WR^2

$$WR^2 = \frac{71.0\ P_s f X}{\left(\frac{\text{r.p.m.}}{100}\right)^4}$$

and the authors' flywheel constant F_c becomes

$$F_c = \frac{71.0\,P_s f}{\left(\frac{\text{r.p.m.}}{100}\right)^4} \quad \ldots\ldots\ldots\ldots\ldots\ldots [d]$$

If Equation (*b*) or (*c*) is used instead to define X, then F_c will become $\dfrac{79.0\ P_s f}{\left(\frac{\text{r.p.m.}}{100}\right)^4}$ or $\dfrac{76.8\ P_s f,}{\left(\frac{\text{r.p.m.}}{100}\right)^4}$ respectively.

Any of the above expressions will do, of course, but some one should be used, and that only.

The writer would also like to ask if there is any advantage in defining the synchronizing power in terms of power measured electrically at the motor terminals, as the authors have defined P_0, rather than mechanically at the motor shaft as P_s is presumably defined by the A.I.E.E. In the first case it results in the efficiency of the motor (or the ratio of the hp. output to kw. input which is proportional to efficiency) entering into the expression for F_c (see Equation [16] of the paper), while in the second case it does not and the expression for F_c is simplified correspondingly. (See Equation [*d*] above.)

M. C. Davison.[8] The writer was in very close touch with Mr. Putman and also with Mr. Stevenson of the General Electric Co. while they were both working very intensively on this flywheel problem. His company, he believes, is the only one making compressors that has gone far enough into the problem to get together sufficient information to solve its own flywheel problems. However, he has reached the conclusion that the present solution of the flywheel problem is entirely practical and dependable.

In cases where the writer's company has changed flywheels on machines which were put out before this problem was solved (there have probably been 25 or 30 of these), it has almost invariably been found that the trouble was due to the incorrect value of P_0 as given by the electrical manufacturers at the time the motor was purchased. It is further interesting to note that, in so far as it has been possible to determine, the new flywheels furnished on these old jobs have in every case remedied the trouble. Furthermore, the writer does not know of a single case of flywheel trouble the company has had since it has been working on the basis outlined in the paper.

There is one point brought out by the authors which the writer is inclined to question: They indicate that the ammonia-compressor manufacturers were the first to use plate valves and operate the reciprocating compressors at the higher speeds. As a matter of fact, the writer believes it will be found that compressor manufacturers had standardized synchronous-motor-driven plate-valve air compressors of the so-called "high speed" type before there was any appreciable movement among the ammonia-compressor builders toward the development of the synchronous-motor-driven direct-connected ammonia compressor.

M. N. Halberg.[9] The authors comment on the fact that their XY curves were plotted through the resonance points of the motor while this was not done by Dr. Stevenson in his original XY curves.

The writer questions the value of plotting these curves through resonance except for cases where the current pulsation is relatively low. The differential equation which is used in obtaining the XY curves is based on the assumption that the synchronous torque of the motor is directly proportional to the angle between the rotor and rotating magnetic field. For ordinary values of current pulsation this approximation is quite close, but if the pulsation is extremely high at resonance, as is the case with many air compressors, the assumption will be far from correct and the results obtained will have little meaning. On the other hand, if the current pulsation is relatively low at points near resonance, it might be of advantage to have the entire curve plotted, since values of X may be chosen quite close to the resonant point. However, in this case it would not be advisable to use an average value of the parameter λ for all the XY curves as was suggested in this paper, since the effect of variations in this parameter is quite large at points near resonance.

In discussing the application of synchronous motors to carbon dioxide compressors, the authors mention that little work has been done in calculating the flywheel requirements of this application. A paper entitled "Some Problems in the Application of Direct-Connected Synchronous Motors to Carbon Dioxide Compressors," by D. W. McLenegan, appeared in the January, 1927, issue of *Refrigerating Engineering*. The author of this paper discussed both the torque and flywheel requirements of synchronous motors direct-connected to a CO_2 compressor. A typical XY curve was shown for each type of compressor in general use. The advantages of 180-deg. cranks over 90-deg. cranks for certain types of CO_2 compressors were also pointed out and explained.

D. W. McLenegan.[10] The authors have presented a very useful paper. It is well to have this method of analysis com-

[8] Compressor Engineering Department, Ingersoll-Rand Co., New York, N. Y.

[9] General Electric Co., Schenectady, N. Y.

[10] Ibid.

pletely described and explained so that it may be applied more widely than it has been. The writer feels that they have made a valuable contribution by their data on compressors, both as to the suggested method of capacity reduction for air compressors, and by the method presented for taking account of the relative weights of high-pressure and low-pressure reciprocating parts.

The XY-curve method is not new but has been in constant commercial use for fully seven years, and has been checked by many oscillograph tests on various compressor installations. After the general solution of the flywheel problems had been worked out and presented to this Society in 1920 (Item 1 of the author's bibliography), the method was applied to a large number of compressors by calculating the current pulsation for various assumed values of WR^2 and plotting the results. However, this method was too slow and laborious for commercial use. It was found that these curves were of remarkably similar shape for all compressors of a given class. By dimensional analysis, Dr. A. R. Stevenson worked out the parameter X, by means of which the peaks of these curves could be superposed; and after correcting for the varying ratios of compressor-horsepower to motor rating, the curves for various compressors of the same class fell very close together. The remaining differences were explained by correcting for variations in weight of reciprocating parts and ratio of connecting-rod length to crank length.

Mr. Putman then redeveloped this same parameter X by the method described in this paper, and added a number of valuable short cuts which have been useful in constructing new XY curves—especially his resonance curves for the amplitude factor C, and the phase-shift curves. His curve tracer has also been very useful, and the original model is still in use at Schenectady.

The authors' suggestion regarding automatic operation of compressors, although apart from the main subject of the paper, is very interesting to the writer. A number of large automatic air compressors with synchronous drive are already operating, and he believes that similar arrangements will be worked out for ammonia compressors. However, he thinks that this will eventually be done by means of automatic bypassing rather than by high-torque motors. Motors of such characteristics would involve greater first cost, and also somewhat greater starting inrush, which would probably not be acceptable to power companies in most cases.

Standardization of a flywheel constant, as proposed by the authors, is very desirable. However, the writer thinks that this should be based on a motor constant such as P_0 or P_s, rather than the inclusive value F_c. P_s (which has been adopted by the A.I.E.E. in connection with the expression for natural frequency of oscillation of a synchronous machine) represents the kilowatts per electrical degree displacement of the rotor.

Either P_0 or P_s will have an approximate relationship to the kilowatt rating of the motor. This relationship is sometimes very useful as a means of estimating the value of P_0 or P_s. If the formula for flywheel effect were made to include X as a compressor factor and P_0 or P_s as a motor factor, then the remaining factors, namely, speed and frequency, could be incorporated in a single constant which could then be tabulated for all standard frequencies and synchronous speeds. The formula for required flywheel effect would then be:

$$WR^2 = X \times P_s \times E$$

where E would be taken directly from the proposed tabulation.

Authors' Closure

Mr. Goss and Mr. Putman. Answering Mr. Brainard's questions, we do not know what approximation is involved in the assumption of a constant value of T_s for a given case of current pulsation. All the oscillographic measurements of current pulsation which have been made have checked so close to the calculated values that we feel that whatever approximation is involved must be negligible. It is true that T_s varies with the frequency, and to get the correct value one must know the frequency of the principal pulsation in the particular case being investigated.

The omission of $\sum T_d \frac{d\psi_n}{dt}$ in calculating the torque pulsation corresponding to ψ_n is not at all necessary. In fact, this term was actually used in making many of our standard XY curves. The effect of damping on the shape of XY curves was clearly shown by Stevenson. (See Bibliographical Reference 3.)

After all, this paper presents nothing new so far as the fundamental theory is concerned. It is only an additional contribution to a well-known problem. It presents for general use the results of many hours of tedious work in applying known methods to the making of a standard set of XY curves.

Mr. M. C. Davison, in his discussion, says, "The writer does not know of a single case of trouble the company has had since it has been working on the basis outlined in this paper." Mr. D. W. McLenegan says in his discussion, "The XY curve method is not new, but has been in constant commercial use for fully seven years and has been checked by many oscillograph tests on various compressor installations." Such testimony should satisfy the most skeptical about the sufficiency of the fundamental solution.

Regarding the definition of "X" there should be no confusion. Dr. Stevenson's Equation [2] (Bibliographical Reference 3) for "X" is identical with the authors' Equation [15]. "X" is defined as 9.25 times the square of the ratio of the r.p.m. divided by the natural frequency in cycles per minute. This is the formula on which Stevenson's curves were based (Bibliographical Reference 3), not nine times as stated by Mr. Brainard. Stevenson's formula $X = \frac{0.80\, WR^2(\text{r.p.m.})^4}{10^8\, P_o f}$ is an approximate form assuming an efficiency of 93 per cent. The other formula $X = 10\left(\frac{\text{r.p.m.}}{F}\right)^2$ is an approximate form in which the efficiency is neglected entirely.

The P_s used in the A.I.E.E. natural frequency formula is supposed to be defined as the rate of change of input power with respect to the displacement expressed in kilowatts per electrical degree. This definition is the same as that of P_s except that displacement is expressed in electrical degrees instead of in electrical radians.

We are glad to have Mr. Davison's correction of our statement regarding the development of the high-speed compressor valves.

We wish to apologize for omitting reference to Mr. McLenegan's excellent paper entitled, "Some Problems in the Application of Direct-Connected Synchronous Motors to Carbon-Dioxide Compressors."

Mr. Putman. Mr. Halberg has questioned the value of plotting XY curves through the resonance points. Both Dr. Stevenson and the writer thought this was desirable, but the problem was to do it and still keep the XY curve a characteristic curve of a compressor type only; that is, keep it independent of the motor speed. It was for this reason that the writer suggested that an average value of $\lambda = 0.1$ be used in plotting the curves. It was recognized, of course, that such curves would not be accurate near the resonance points. When it was necessary to investigate a case of operation near resonance, it was necessary to calculate a curve based on the correct value of λ.

I am glad to hear from Mr. McLenegan that my original curve tracer is still in use in Schenectady. The curve tracer pictured

in the paper is the second one designed, and its mechanical features have been considerably improved. I will be glad to supply further information regarding the construction of this curve tracer to any one desiring to build one.

I like Mr. McLenegan's suggestion for a standard flywheel constant. I have often felt that our F_c had a decided disadvantage in that it bears no simple relation to the horsepower or kilowatt rating of the machine. I always did like P_o, because one could guess its value so easily from the kilowatt rating of the machine. I suggest that Mr. McLenegan place before the N.E.M.A. his suggested tabulation and formula for calculating the WR^2 since this association now has under consideration the adoption of standard values of X for different compressor applications.

APM-51-13

Design of Ellipsoidal Heads for Pressure Vessels

By T. W. GREENE,[1] NEW YORK, N. Y.

This paper, after dealing theoretically with the stresses set up in cylindrical pressure vessels provided with ellipsoidally dished heads, gives results of tests made on a propane storage tank 7 ft. in diameter, 38 ft. 4 in. long, and designed for a working pressure of 200 lb. per sq. in., the heads being ellipsoidal with a 2-to-1 ratio of axes. From these results the author concludes that (1) heads with dish and large knuckle radii proportioned to closely approximate true ellipsoidal shapes with a ratio of axes of 2 to 1, are far superior to the flat dished heads with small knuckle radii commonly used in pressure vessels; (2) that the stress distribution in such heads closely approximates that set forth by theory, and that in consequence the formulas presented in the paper can be safely used for design.

DURING the last few years, advantage has been taken of the construction of a large number of pressure tanks to make an intensive study of the design and stress distribution in pressure vessels with the object of developing correct procedure and methods of construction for oxyacetylene-welded vessels. This has been necessitated by the high test pressure generally specified for welded equipment, which, in accordance with standard practice, is three times the working pressure or a maximum test fiber stress of 24,000 to 27,000 lb. per sq. in. These high test pressures, abnormally high compared to test pressures for other types of construction, have shown that extremely high stresses and excessive straining far out of proportion to those in the shell, exist in certain parts of the heads. High stress concentration has especially been noted around standard nozzle and saddle-reinforced manholes, and at the sharp knuckles of flat-dished heads commonly used in pressure vessels where the heads are dished to a radius equal to the tank diameter. These high stresses have, in fact, been of such magnitude as to cause failure of one manhole head of a large ethylene tank designed for 300 lb. working pressure when tested several times hydrostatically to 1000 lb.[2]

The conditions brought out by this failure and subsequent investigations have led to the adoption of heavily reinforced openings and to the change in head designs from the commonly used flat dished heads to an ellipsoidal shape with an axis ratio of 2 to 1. Theoretically, the stresses in any part of such ellipsoidal heads should not be in excess of those in the shell.[3] Very good results have been obtained with these designs in the construction of over a hundred oxyacetylene-welded pressure vessels of various sizes, typical ones of which have been investigated by taking strain-gage measurements during test. This has led to the adoption of the new-design head as standard for welded construction. Practical considerations in regard to the design and stresses measured in these ellipsoidal heads are presented below, as it is believed they will be of interest to engineers and fabricators, for they permit economical construction of pressure vessels of practically equal strength in all sections and with the same factor of safety throughout.

[1] Development Section, Engineering Department, The Linde Air Products Company. Mem. A.S.M.E.

[2] See "Stresses in a Large Welded Tank Subjected to Repeated High Test Pressures," *Mechanical Engineering*, February, 1927, p. 124.

[3] See "Design of Dished and Flanged Pressure-Vessel Heads," by A. B. Kinzel, *Mechanical Engineering*, June, 1927, p. 625.

Contributed by the Applied Mechanics Division and presented at the Annual Meeting, New York, N. Y., December 3 to 7, 1928, of The American Society of Mechanical Engineers.

Stresses in Ellipsoidal Heads—Theoretical Consideration

The stresses in any portion of a pressure vessel, according to the common approximate theory, are of three kinds: (1) transverse or hoop tension, tending to cause rupture along an element of the vessel; (2) radial or longitudinal, tending to cause rupture at right angles to the first; and (3) radial to the center of the tank due to difference of internal and external pressure. The last one is usually considered negligible, since generally it is relatively small.

In a spherical head the stresses and strains are theoretically independent of direction, and the stresses are found from the common formula

$$S = \frac{PR}{2t}$$

where S is the stress, P the pressure, R the radius, and t the thickness of the head. This formula, however, is not correct for heads with the large radius of dish and small knuckle radius commonly used in pressure vessels. Such heads do not act as sections of spheres except at relatively low pressure. Under high pressure, they appear to act more as dished plates supported at the rim. At the knuckle, where there is an abrupt change in curvature and directional change in the forces acting, high bending stresses are set up, the head tending to protrude at the center and flatten or contract at the knuckle. If the elastic limit of the material is exceeded, which occurs at relatively low pressures, the head tends to take a new shape and to assume that of an ellipsoid of revolution. The magnitude of the stresses and strains developed around the knuckle are given in a paper on "Stresses in a Large Welded Tank Subjected to Repeated High Test Pressure," in *Mechanical Engineering*, February, 1927.

Huggenberger,[4] starting with the stress-strain relations in a solid of revolution and assuming that the head is an ellipsoid of revolution and neglecting bending, has developed the following equations for stresses in ellipsoidal heads:

$$T_r = \frac{P}{2t}\left[(Rk)^2 + x^2\,(1 - k^2)\right]^{1/2} \quad \ldots\ldots [1]$$

$$T_t = 2T_r\left(1 - \frac{1}{2}\,\frac{R^2}{R^2 + x^2\left(\frac{1}{k^2} - 1\right)}\right) \quad \ldots\ldots [2]$$

where T_r = radial tension
T_t = hoop tension
$R = a$ = radius of cylindrical shell ($^1/_2$ the major axis)
b = semi-minor axis
k = ratio of major to minor axis ($= a/b$)
x = distance from minor axis to any point (abscissa)

[4] *Zeitschrift des Vereines deutscher Ingenieure*, vol. 69, no. 6, Feb. 7, 1925, pp. 159–162. For review in English, see "Design of Dished and Flanged Pressure Vessel Heads," by A. B. Kinzel, *Mechanical Engineering*, June, 1927, p. 625.

P = internal pressure, and
t = thickness of plate.

It can be shown from the foregoing formulas that if the depth of the dish is one-quarter the diameter of the tank or the ratio of the major to the minor axis is 2 ($k = a/b = 2$), the stresses in the head at any point will not exceed those in the shell of a vessel with the same plate thickness. This ellipsoidal shape having a depth one-quarter the diameter of the tank is a practical design

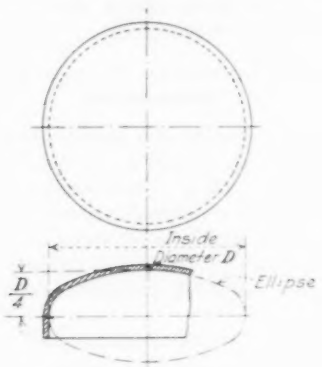


FIG. 1 ELLIPSOIDAL HEAD IN COMMERCIAL PRODUCTION

and is the one which we now use. The stress distribution in such a head, as can be shown from the foregoing formulas, is given in the following formulas:

$$T_r = \frac{P}{2t}\sqrt{4R^2 - 3x^2} \quad \ldots\ldots [3]$$

$$T_c = T_r\left(2 - \frac{4R^2}{4R^2 - 3x^2}\right) \quad \ldots\ldots [4]$$

where T_r = radial stress at any point, and
T_c = hoop stress at any point.

A concrete illustration will show the theoretical stress distribution in such a head. Consider a tank 7 ft. in diameter, for 200 lb. working pressure, plate thickness 1 in. These are the dimensions of the tank investigated for the data in this paper.

Theoretically, from [3] and [4] the maximum radial stress occurs at the center of the crown where $x = 0$. The stress is tension and is 8400 lb. per sq. in. at 200 lb. design working pressure. The radial stress gradually decreases from the center to one-half its value or 4200 lb. per sq. in. tension at the boundary of the knuckle and cylindrical shell. The hoop stresses, theoretically, are of the same magnitude as the radial stresses at the center of the head, and decrease to zero and change to compression around the knuckle. The maximum compressive stress occurs at the boundary where $x = R = 42$ in. and has the same magnitude as the maximum tensile stress, or 8400 lb. per sq. in. under design working pressure. The point where the hoop stress is zero is where $x = \sqrt{2/3}\,R$ or about 0.8 the distance from the center of the tank to the shell. The theoretical radial and hoop-stress distributions in ellipsoidal heads are shown in Fig. 7.

PRACTICAL DESIGN OF ELLIPSOIDAL HEADS

Ellipsoidally shaped heads having a depth of dish one-quarter of the tank diameter are now commercially produced. At least one large fabricator can supply these heads at very little additional cost over other heads, the cost of the dies having been absorbed by the company. This company's general drawing for this shaped head is shown in Fig. 1.

TABLE 1 HEAD DESIGN TO APPROXIMATE ELLIPSOIDAL SHAPE HAVING DEPTH OF DISH ONE-QUARTER THE DIAMETER OF TANK[5]

Diameter of tank, in.	Radius of dish, in.	Knuckle radius, in.	Straight flange length, in. (recommended for welding)
36	30	5.65	4
48	42	8.0	5
60	54	10.3	5
72	60	11.25	5
84	72	13.65	5

A close approximation to the ellipsoidal design can be obtained in a very practical way with a basket-shaped head formed by the combination of two radii, namely, the radius of dish of the head and that of the knuckle. This can be checked on any large-scale drawing. Table 1 gives data for use in designing ellipsoidal heads with a depth of dish equal to one-quarter the diameter of the tank for commonly used sizes. These heads have been quite generally used and can be supplied by several fabricators at practically the same cost as the old-type heads.

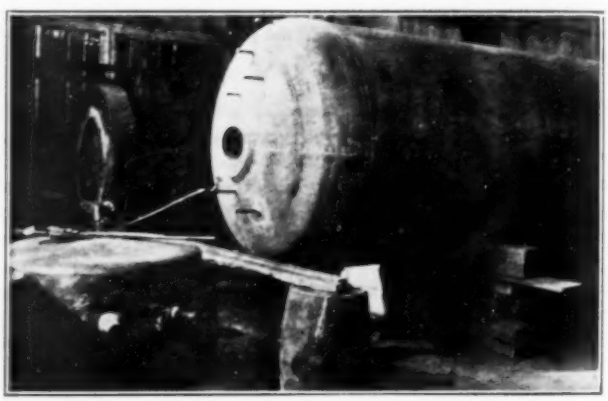


FIG. 2 7 FT. × 38-FT. TANK WITH ELLIPSOIDAL HEAD DESIGNED FOR 200 LB. AND TESTED TO 600 LB. PER SQ. IN.
(The short white lines show locations of 2-in. strain-gage lines for measuring the strains throughout the head.)

DESCRIPTION OF VESSEL TESTED

The experimental data concerning ellipsoidal heads presented in this paper were obtained from a typical test of one of a number of large tanks used for the storage of propane, on which complete strain investigations were made. Test and strain measurements on other pressure tanks of various sizes showed equally satisfactory results.

These propane storage tanks were 7 ft. in internal diameter by 38 ft., 4 in. long, designed for 200 lb. working pressure. The shell thickness was $^{15}/_{16}$ in. and the design fiber stress 9000 lb. per sq. in. Fig. 3 shows the general design of the vessels. The shell was designed and constructed according to the usual practices. Ellipsoidal heads were used with ratio of axes 2 to 1, that is, the depth of dish was one-fourth the diameter of the tank. The radius of dish of the head was 6 ft. and the radius of knuckle $13^3/_4$ in., which gives a very close approximation to the true ellipsoid having a 2-to-1 ratio. The heads were 1 in. thick, or $^1/_{16}$ in. thicker than the shell plate, in order to take care of any possible decrease in thickness due to flanging. No thickness was added to the manhole head for the manhole opening, as the stresses due to the opening were provided for by the manhole design. For the 11-in. × 15-in. opening, a ring-reinforced manhole was welded into the head. The ring was designed to take care of the enormous stresses around the manhole and reduce their intensity to equal that in the shell, the design being based

[5] "Procedure Control in Pressure-Vessel Welding," by H. E. Rockefeller, *Journal American Welding Society*, March, 1928.

upon the theory of a hollow cylinder subjected to an external tensile force equal to the stresses in the head.[6]

The whole tank was annealed after fabrication in order to anneal the weld joining the ring to the head. This weld is of the rigid type, since, unlike a seam weld, its contraction is rigidly resisted by the position and mass of the surrounding metal, so it is necessary to anneal it to relieve any locked-up stresses that may exist.

The Test

The tank was tested hydrostatically to three times the working pressure or to 600 lb. per sq. in. This gave a maximum test fiber stress of 27,000 lb. per sq. in. At 400 lb. pressure all seams were vigorously hammered with a 10-lb. hammer. The strains in the ellipsoidal heads were measured by means of a 2-in. Berry strain gage. For taking the measurements, small holes were drilled 2 in. apart with a No. 56 drill. These holes were drilled just deep enough (slightly less than $^1/_{16}$ in.) for the parts of the strain gage to seat and in no way weakened the plate. A complete set of measurements were taken extending from the end of the major axis of the manhole to the shell. These were taken in both a radial and transverse direction, that is, both the radial and hoop strains were measured. Fig. 2, 5, 6, and 7 show the location of the strain-gage readings. The strains were measured at 0, 200, 400 and 600 lb. test pressure and again at 0, after the pressure was released, to determine the permanent sets. A second test was then made and similar sets of readings were taken to determine the stress distribution after the slight irregularities of shape had been ironed out by the first test. These fabrication irregularities are always present and superimpose local bending strains on the principal pressure strains.

Strain readings were also taken on the inside of the tank before and after the test, at positions opposite to those taken on the outside, to measure the permanent sets. These determinations are important in interpreting and explaining any bending or overstrain that may take place.

[6] For a theory of design of ring-reinforced manholes see "Design of Dished and Flanged Pressure-Vessel Heads," by A. B. Kinzel, *Mechanical Engineering*, June, 1927, p. 621.

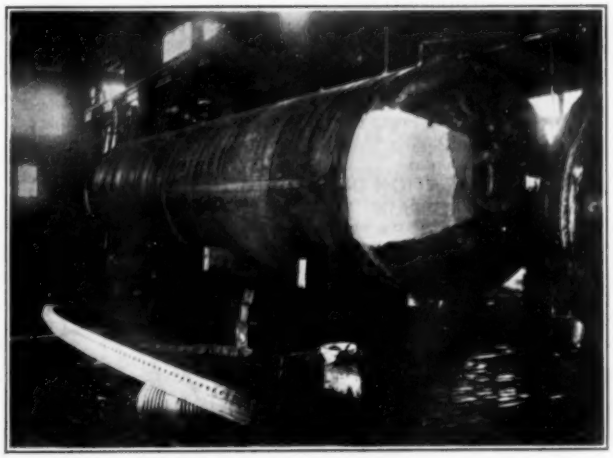


Fig. 4 7-Ft. × 38-Ft. Tank with Ellipsoidal Head
(Note absence of Luder lines or scaling under test to 600 lb. or three times the working pressure.)

Measured Stresses in Ellipsoidal Heads

The results of the 40 strain-gage measurements taken 2 in.

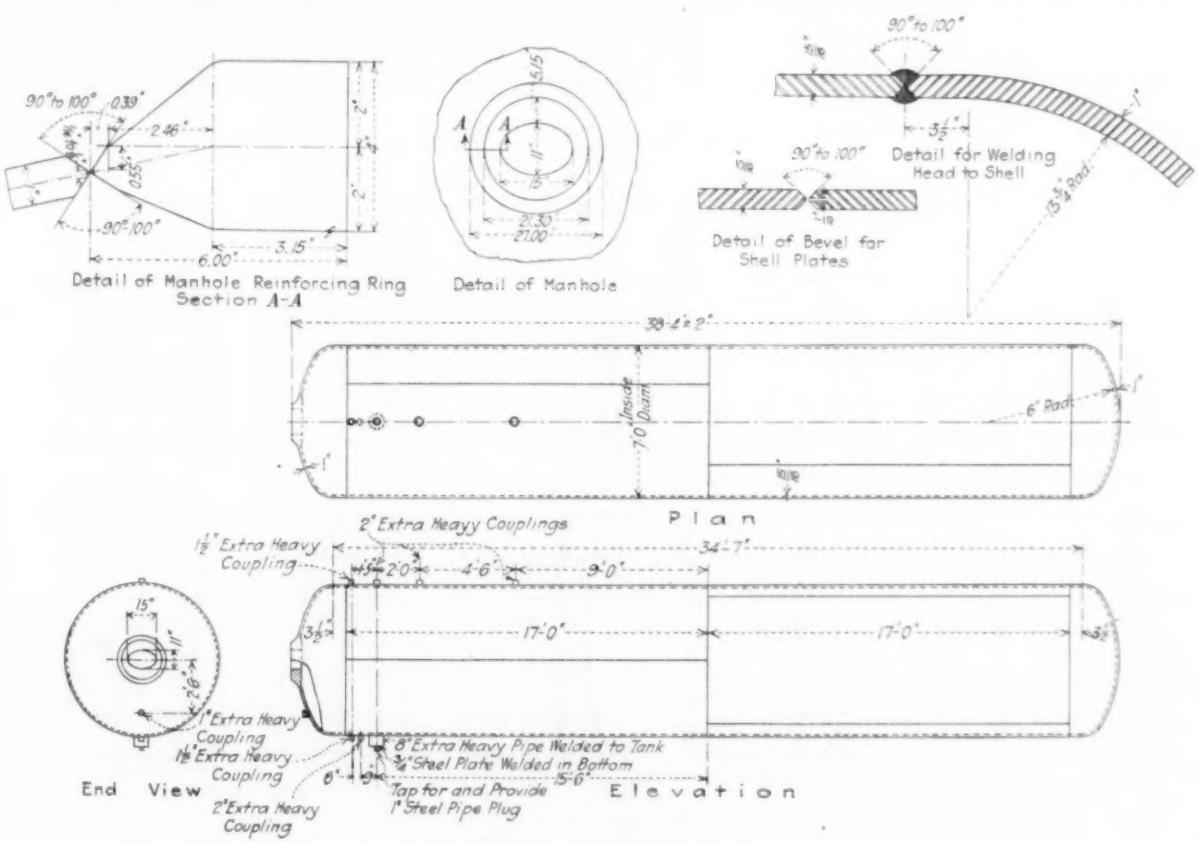


Fig. 3 Showing General Design of Propane Storage Tank Tested

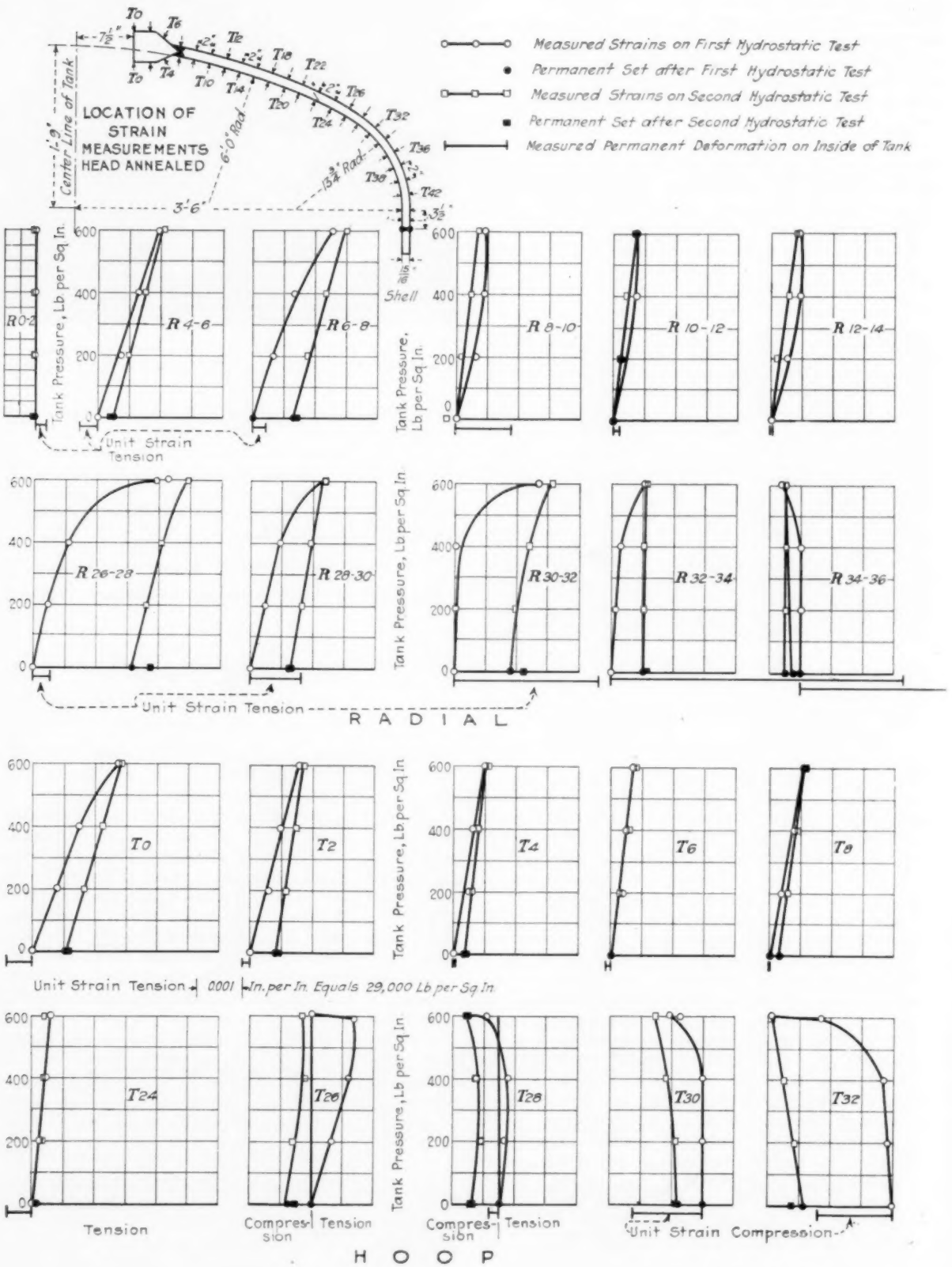
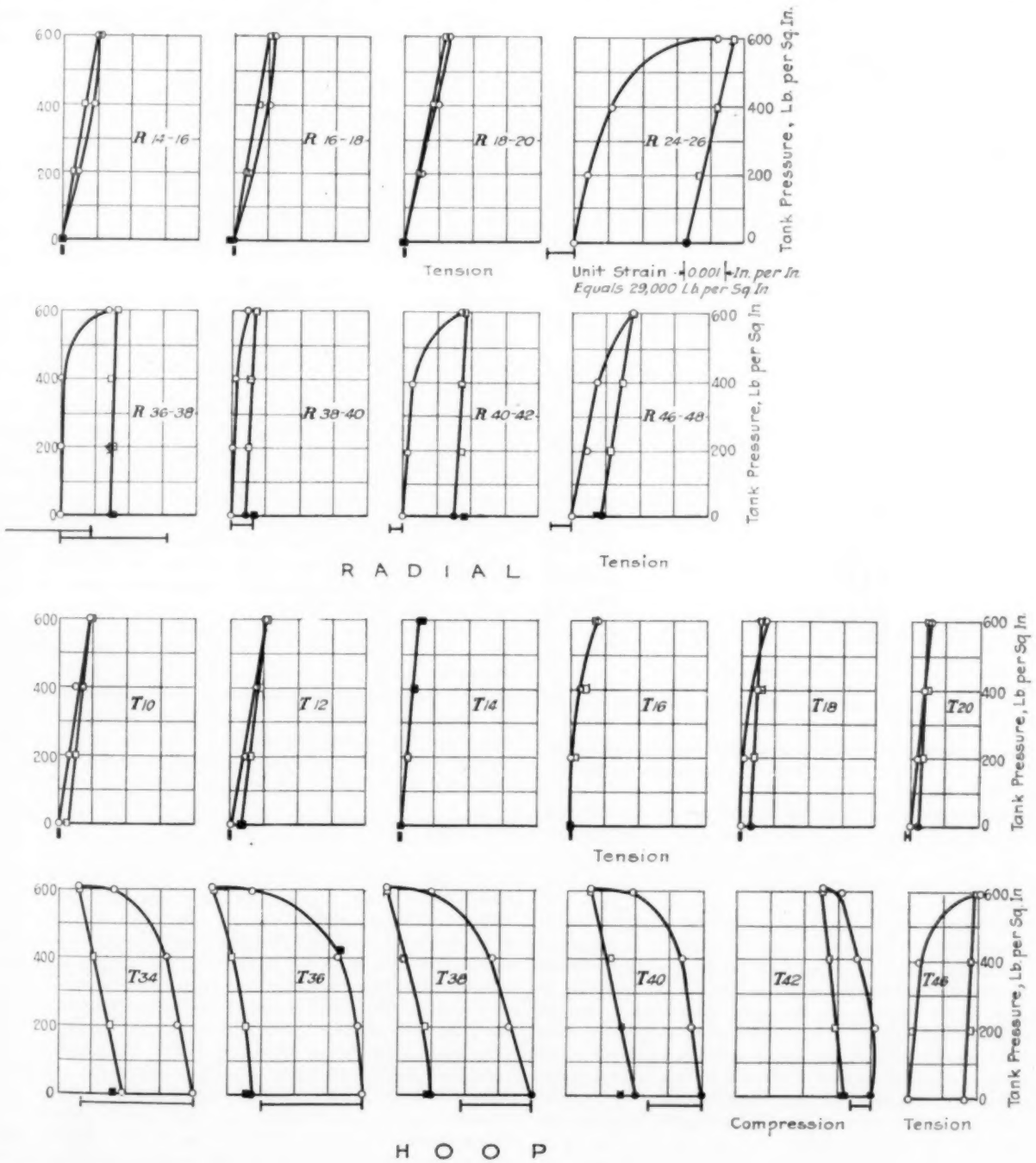


FIG. 5 UNIT STRAINS IN

apart on the ellipsoidal head are shown in Fig. 5, the locations being indicated in the sketch at the top. These curves show the complete distribution of the stresses, both radial and hoop, throughout the head from the edge of the major axis of the manhole to the shell, for both tests. The permanent sets measured on the inside of the tank are shown below the curves.

It will be seen the strains and stresses are practically normal in all parts of the head, and that at no place are they excessive. Due to irregularities in the head, the stresses are slightly higher than they should be at certain points. The head, just inside and next to the knuckle, was somewhat flat and the effect of this irregularity is seen in the stress distribution of the gage lines 24 to 32. The pressure, it was noticed, tended to iron out this flat spot, which superimposed bending strains and increased the total strain at this point. The radial stresses are tension throughout the head, as would be expected. The hoop stresses are tension in the dished section of the head and change to compression at the knuckle, as theoretically they should. The point where the stresses reverse is about gage line 28, which is approximately eight-tenths the distance from the center of the tank to the shell, as indicated by the theory. The permanent sets, measured on both the outside and inside of the tank, were very small, no



Ellipsoidal Head of Tank

permanent set or overstraining occurring at most of the points. The highest overstraining was measured in a radial direction at the knuckle on the inside of the tank where the contraction of the knuckle causes the tension from bending to be added to the normal tension in the head. These strains were not large when it is considered that the tank was tested so near the yield point of the material. In fact, they are negligibly small compared with those measured in heads with sharper knuckle radii and tested at much lower pressures.

The stresses in the second test to 600 lb. were normal and proportional to the pressure, with no increase in the permanent

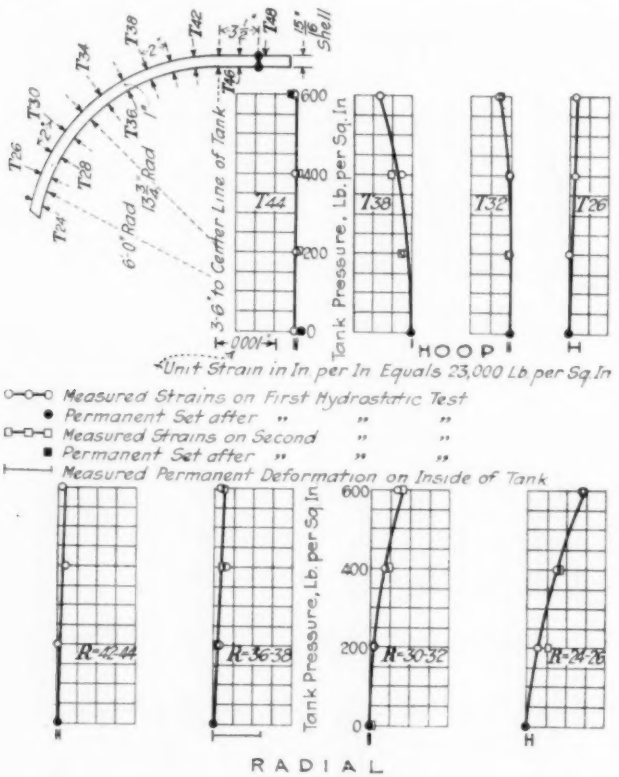


FIG. 6 UNIT STRAINS IN PLAIN HEAD OF ANNEALED TANK

set. The irregularities that had superimposed bending strains and caused the slight overstraining at a few points in the first test had been eliminated.

The stresses measured in the plain head are shown in Fig. 6. These are of the same general character as those in the manhole head but of less intensity. It is remarkable that there was no overstraining around the knuckle, even at three times the working pressure.

The stresses shown in Figs. 5 and 6 are those corresponding to the actual deformations of the material which were measured with the strain gage. These strain stresses are evaluated from the measured unit strains by multiplying by the modulus of elasticity. They are not the design or theoretical stresses in ellipsoidal heads given earlier in the theoretical analysis for these heads, because the strain in any one direction in a material under two or more principal stresses is affected by the lateral deformation caused by the stresses in the other directions. The stresses corresponding to the design or theoretical stresses, however, can be determined from the measured strains. By the theory of elasticity, the following is the relationship between the principal strains and principal stresses:

$$T_r = \frac{mE}{m^2 - 1}(me_r + e_c) \quad [5]$$

$$T_c = \frac{mE}{m^2 - 1}(me_c + e_r) \quad [6]$$

where T_r = radial stress
T_c = hoop stress
e_r = radial strains (measured with strain gage)
e_c = hoop strains (measured with strain gage)
$1/m$ = Poisson's ratio or factor of lateral contraction.

Taking $1/m = 1/3$ as Poisson's ratio, the radial and hoop stresses in ellipsoidal heads in terms of the measured strains are

$$T_r = 3/8\,(3\,e_r + e_c) \quad [7]$$

$$T_c = 3/8\,E\,(3\,e_c + e_r) \quad [8]$$

These measured stresses are shown plotted and compared to design stresses in Fig. 7. These are plotted for 400 lb. test pressure, or twice the design working pressure. For working pressure, they are only one-half as great as those shown. The stresses at three times the pressure were not plotted because, due to irregularities in the heads, the stresses in the first test were slightly above the elastic limit at a few places, and the stress-strain relation, of course, is not proportional. After the irregularities were ironed out, however, the stresses at three times the working pressure in the second test, although not shown, did not pass the elastic limit and were proportional to those plotted at 400 lb. for this test.

This curve, Fig. 7, shows the measured stresses to be in close agreement with both the radial and hoop theoretical stresses. The agreement is not so close on the first test, due to the irregularities in the head. There is a very close check in the second test. The radial stresses are a maximum at the center of the head and decrease to a minimum at the boundary of the head and shell. The hoop stresses are a maximum in tension at the center and change to compression at the knuckle, the maximum value of both the tensile and compressive hoop stresses being the same as the maximum radial stress.

These results show that heads dished to correct curvature and with a large knuckle radius, to approximate an ellipsoidal head with ratio of the axis 2 to 1, are far superior to the flat-dished, sharp-knuckle heads commonly used in pressure vessels, and that for design the stresses can be accurately calculated by the theoretical formulas for ellipsoidal heads. The ring-reinforced manhole, when the whole head is properly annealed, adequately takes care of the enormous stresses around such openings.

COMPARISON OF STRESSES IN ELLIPSOIDAL AND STANDARD DISHED HEADS

The extent to which the stress distribution in heads has been improved by using the elliptical shape, and the superiority of this shaped head over the standard flat-dished head, may be appreciated by comparing the stresses computed from measurements of the two types. A comparison of the maximum stresses measured around the manhole and knuckle of both types is shown in Table 2. The strain measurements in the flat-dished heads are those taken on the ethylene tank and which were published in *Mechanical Engineering*, February, 1927. The heads of this tank were dished to a radius equal to the diameter and had a $4^3/_4$-in. knuckle radius. The two heads are not exactly comparable in size as the ethylene tank was 5 ft. in diameter and $1^1/_8$ in. thick. The test shell fiber stress, however, was the same in both cases, and the relative difference in the stress intensity in the two types is approximately that shown in the table. It can

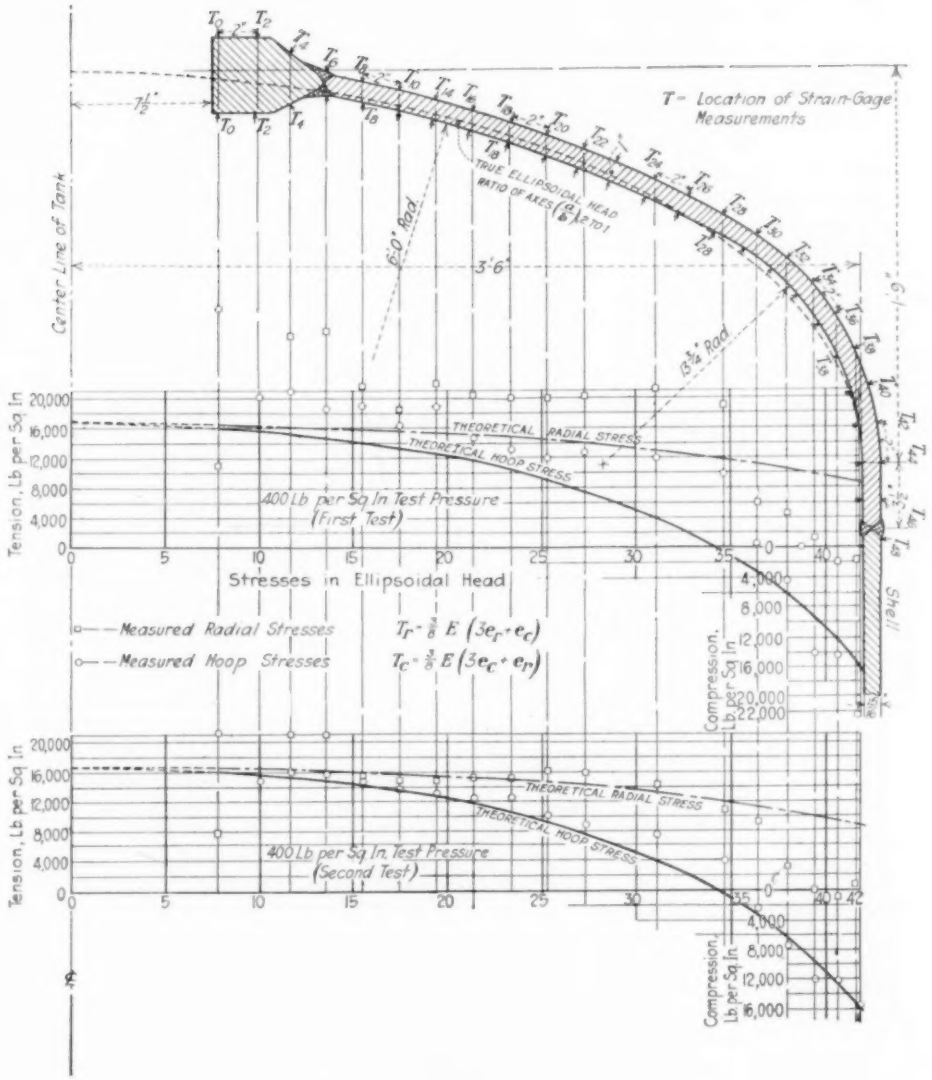


FIG. 7 COMPARISON OF MEASURED AND DESIGN STRESSES IN ELLIPSOIDAL HEAD

TABLE 2 COMPARISON OF MAXIMUM STRESSES IN ELLIPSOIDAL HEAD WITH MAXIMUM STRESSES MEASURED IN FLAT-DISHED HEAD[1]

	Tensile stresses near 11 × 15-in. manhole		Compression hoop stresses around knuckle		Tensile radial stresses on inside of head at the knuckle	
	Ring reinforcement of manhole in ellipsoidal head	Standard manhole saddle in flat-dished head	Ellipsoidal head	Flat-dished head	Ellipsoidal head	Flat-dished head
At design working pressure..	13,600	44,000	−11,000	−11,600	....	....
At twice working pressure..	29,000	0.0056[1]	−21,000	−0.0015[1]	....	....
At three times working pressure.....	0.0018[1]	0.0296[1]	−0.0028[1]	−0.0068	....	....
Permanent set after test to three times working pressure.....	0.0007	0.0258	−0.0020	−0.0062	0.0060	0.0206

[1] Measured unit deformation or strain, inches per inch. Below elastic limit 0.001 unit strain equals 29,000 lb. per sq. in. stress.

be seen from the table that the ellipsoidal heads are far superior and that the high stress concentration found in flat-dished heads does not exist with ellipsoidal heads, the stresses in the elliptical-shaped heads being practically normal throughout, even under very high pressure.

CONCLUSION

The results of the foregoing investigation and test of ellipsoidal heads for pressure vessels show that:

1 Heads with dish and large knuckle radii proportioned to closely approximate true ellipsoidal shapes with a ratio of axes of 2 to 1, are far superior to the flat-dished heads with small knuckle radii commonly used in pressure vessels.

2 The stress distribution in such heads closely approximates the theory, and the formulas for ellipsoidal heads reviewed in the paper can be safely used for design.

Discussion

W. M. COATES.[7] The paper contributes to engineering experience additional measurements of the strain in pressure-

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vessel heads. From the strains (Fig. 5), the stresses are computed on the assumption that there is no resistance to bending; i.e., each element of the head is in equilibrium under normal forces in the directions of the meridians and the parallel circles (formulas [5] and [6]). These stresses are then compared with those computed from formulas [3] and [4], which give the stresses on an element of an ellipsoid which is under uniform internal pressure, and offers no resistance to bending (Fig. 7).

The hypotheses underlying the development of formulas [1] and [2] contain two elements which are not realized. The bending stresses are negligible and the surface has a continuously varying radius of curvature.

The exact solution to the problem, taking full account of both these elements, is not yet in a state in which it can be offered to the engineer.[8] However, approximate solutions of a form sufficiently simple to permit practical application have been developed for full heads.[9] These have been checked experimentally both in the laboratory and in the plant.[10] The qualifications of the desirable full head are those stated in this paper, namely, that the pressure vessel shall be practically of equal strength in all sections.

There is nothing to be said in favor of the flat-dished head with small knuckle radius. It gives a very high stress-concentration factor on the inside surface at the knuckle. The hemispherical head does not represent the most economical use of material, for the head is underworked, and the maximum stress is concentrated in the drum. A sample computation shows stresses in the head from 27 to 50 per cent lower than those in the drum.[11]

It is possible to design full heads of ellipsoidal or quasi-ellipsoidal shape (i.e., with two, three, etc., radii of curvature) in which the stress is approximately the same in the drum, the knuckle, and the crown, if proper care is given to the dimensions of knuckle and crown radii for a given ratio of the depth of the head to its base radius. The true half-ellipsoid of revolution with this ratio equal to $^1/_2$ is not the most favorable form.[12]

The approximate method mentioned can be applied to this last group of heads, to take account of the bending stresses and the geometric discontinuities of the surfaces, such as at the junction of head and drum, or of two sections with different radii of curvature (e.g., near point T_{26} of Fig. 7). At such places, stresses are developed which are not negligible, not only in the head, but also in the drum. Even for the favorable case of an ellipsoidal head with axis ratio $^1/_2$, stresses are set up in the cylinder which are some 13 per cent higher than those computed from the membrane Equations [3] and [4], and as the knuckle radius decreases, the difference exceeds 100 per cent. In the head itself, a hoop shearing stress is developed at the junction of about the same magnitude as the crown stress, and a meridianal bending stress whose maximum is about 60 per cent of that magnitude, at a point about $^1/_{10}$ the distance to the crown, measured along the meridian away from the junction. Such stresses due to discontinuities of the surfaces joined are of periodic character, and are damped out rapidly as the distance from the junction increases. Outside the region of their action, the so-called "membrane" formulas are good, and the design can be based upon a superposition of the two systems of stresses: membrane, and discontinuity.

To check computations, measurements similar to those of this paper have been made, supplemented by measurement of the angular deformation of normals to the head. From the latter the bending stress may be computed, and thus the stresses at the inner surface of the head are obtained.

The introduction of a manhole into the head brings new stresses at its rim. With designs favorable for full heads, these are greatly in excess of the stresses at the inner surface of the knuckle. The problem here is not in the same stage of refinement as that of the full head, and awaits a more thorough investigation of these discontinuity stresses.[13]

The measurements given in this paper are in line with expectations. Referring to Fig. 7, first test, there is a large percentage error in the radial (called meridianal) stress from that predicted by use of the membrane equations, enough to cause a change in character from tension to compression near the junction of head and drum. This is due to the discontinuity in the radii of curvature of the surfaces joined here; i.e., the cylinder and the quasi-ellipsoid. The compressive stress on the outer surface due to bending plays an important role, and since a pressure of 600 lb. per sq. in. should not make much difference in the radii of curvature (for cylinder and $13^3/_4$ in. for head) this stress is not changed greatly. The second test verifies this. The junction of the surface with radius of curvature $13^3/_4$ in. to that of radius of curvature 6 ft., sets up new discontinuity stresses which are large in the first test. As is to be expected, the second test shows a marked decrease in these stresses, since the difference in the radii has been diminished. Similar discussion holds throughout. All of these stresses are of oscillatory character and are damped out as they leave the junction at which they are set up. They can be computed from the data given. However, an experimental check demands the mentioned additional measurements of angular deflection of normals.

These are some points in the paper concerning which, it is felt, elaboration or correction would be advisable. In Fig. 5, the unit strain is not indicated on the axis of abscissas. There are two values indicated in two different places, neither of which coincides with the unlabeled unit used in the graphs. How were the permanent deformations on the inside of the tank measured (with check of the scale here also, in view of the last sentence on the fifth page that they are very small)? What is the maximum error to be expected in the strain measurements? The lettering of the gage points in Figs. 5 and 7 seems to be in error. In Fig. 7 the stress-distribution curves obtained by joining the points are, of course, for the outer surface. They will show a different character for the inner surface.

Furthermore, there are places where revision of expression should be made. The first statement on the second page cannot be demonstrated. The surface constructed from a meridian with two radii of curvature should not be called an "ellipsoid." On the fifth and sixth pages there should be more care in distinguishing "stress" and "strain." On the sixth page under Fig. 6, "the irregularities, etc.," is not borne out in Fig. 7.

In conclusion, it may be stated that greater progress can be expected in the head design of pressure vessels if experimental investigations are planned to get not only the membrane stresses, but also the discontinuity stresses, especially in the neighborhood of manholes and other breaks in the surface. There is reason to expect that the approximate methods of computation may be ap-

[8] Bolle, "Festigkeitsberechnung von Kugelschalen," Füssli, Zürich, 1916.

[9] Meissner, "Zür Festigkeitsberechnung von Hochdruck—Kesseltrommeln," Schweitzerische Banzeitung, vol. 86, p. 1, July 4, 1925.

[10] Geckeler, "Über die Festigkeit achsensymmetrischer Schalen," Forschungsarbeiten, V.D.I., No. 276, 1926.

[11] Hohn-Huggenberger, "Über die Festigkeit der gewölbten Böden und der Zylinderschale." Part II by Huggenberger. Springer, Berlin, 1927.

[12] Huggenberger, "Über die günstigste Gestalt des vollen, gewölbten Bödens, etc." Schweitzerische Banzeitung, vol. 91, p. 203-217. Apr. 28, May 5, 1928. This is an abstract and a continuation of the work of reference 3.

[13] Siebel, "Über die Beanspruchung und das Verhalten von Dampfkesselböden," Stahl und Eisen, vol. 46, p. 1181, Sept. 2, 1926. This is an abstract of work done at the Kaiser-Wilhelm Institute für Eisenforschung at Düsseldorf, which appears in more detail in the "Mitteilungen der K. W. I., etc," vol. VII and VIII.

plied which hold for favorably shaped full heads, although here it may be necessary to find a new favorable shape.

G. M. Eaton.[14] The advantage of having equal tension and compression is only a matter of words unless it is true that the tank is just as liable to fail in compression as it is in tension. In determining the ratio of minor to major axis on a really reliable basis, it would be essential to test tanks to destruction and find whether or not compression failure is going to occur. In many cases overstress in compression is not feared, but if this head is liable to fail first from collapse then it is essential to adjust the stress so that the section is as nearly balanced as possible.

H. E. Rockefeller.[15] When need was first found for a head having a depth of dish equal to one-quarter of the diameter of the tank, it was learned from the head fabricators that the depth of dish desired could be secured and an approach to a true elliptical head by using a large knuckle radius smaller than standard crown radius. Consequently the design of heads for various diameters was changed so as to include the crown radius and knuckle radius which would give the closest practical approach to the truly ellipsoidal shape.

Within the past year, however, one of the head manufacturers has produced dies for forming true ellipsoidal heads for some of the diameters of the heads which are required. As truly ellipsoidal heads have become available they have been used, and undoubtedly when the ellipsoidal dies for the full range of sizes for heads are available, the writer's company will probably eliminate the deep-dished type of head which it is now using in general.

A. L. Kimball.[16] The curves in Fig. 7 do not take account of the variation of stress on outside and inside radius. Of course, a case of this sort will only arise where there is quite a sharp curvature and Fig. 7 was not intended to represent this case.

In justification of the photoelastic method, the writer disagrees with one speaker that it cannot be used. In this case it would be better not to use the photoelastic method because the variation of stress in the cross-section is so large, but where the contour is gradual, and the thickness of the plate is relatively small, the method of polarized light could be used very well on the assumption that the stress is substantially uniform from one side to the other of the cross-section. A very useful check could thus be obtained by the photoelastic method although it might not be 100 per cent accurate.

G. O. Carter.[17] This work started with extensometer readings on the actual vessels with short-radius knuckles where failure occurred, and Mr. Greene obtained marked differences in compression and tension at very close points. That started a good deal of this investigation, and to obtain confirmation several boiler accidents were investigated.

Some engineers talked about caustic embrittlement and things of that kind as causing failures, but when they checked what had been done on new equipment stressed to three times working pressure, they agreed that there were localized stresses of terrific magnitude. The stresses thus concentrated at one point have probably caused thousands of boiler and pressure-vessel accidents that were attributed to fatigue of metal or caustic embrittlement, assuming that the design of the vessel was correct.

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[15] Development Engineer, Linde Air Products Co., New York, N. Y. Jun. A.S.M.E.

[16] Research Engineer, General Electric Co., Schenectady, N. Y. Mem. A.S.M.E.

[17] Consulting Engineer, Linde Air Products Co., New York, N. Y.

W. Trinks.[18] A question was asked by one of the speakers about the fact that compression exists at the end of the long axis of the ellipse. The writer can fully understand why compression should exist at that point, but in going from there to the immediately adjacent cylindrical part, there should be, at the end of the ellipse, a very large amount of compression, and immediately adjacent to it in the cylindrical part only tension. Now there certainly must be a transition from one to the other. It is unthinkable that at a thousandth of an inch to one side of the imaginary division line 20,000 lb. per sq. in. compression is encountered and a thousandth of an inch to the other side of that line there is 10,000 lb. per sq. in. tension.

That means that at the important line, where the ellipsoidal head becomes a cylinder, the stresses which are given by the author's theory must be departed from and the question comes up whether the ellipsoid is not used solely because the stresses in it can be figured. In other words, certain shapes are used because they allow of analytical solution and engineers think they have solved the problem by using the stresses computed from the simple shape. To the writer it appears that all are being fooled about the applicability of the ellipsoid because something else is going on at the junction and, therefore, the stresses figured from the ellipsoid are seriously modified at that point. All of this brings up the question whether, to the author's knowledge, somebody may have succeeded in using a graphical method of figuring stresses in dished heads of any shape. If that were so the old method of a circular arc for the center of the head with smaller circular arcs joining the cylindrical might be adopted.

B. L. Newkirk.[19] Where a small number of points are involved, as in this case, it is doubtful whether the line which would be indicated by the theory of least-squares is the best representation. For example, if another observation added to the observations we have already would change the position of the line materially, then it is not right to say that even these observations are best represented by the line that the theory of least-squares indicates. The theory of least-squares rests on the supposition that that representation is the best which makes the sum of the squares of the residuals a minimum. There is no proof that this supposition is correct when the number of observations is small.

It is important that observations wherever possible should be represented graphically. The application of the theory of least-squares can mislead very decidedly if observations are discussed by the theory of least-squares to arrive at the values of certain unknowns without making a graphical representation. The least-square treatment often covers up things that would appear from the graphical representation. The equation used in applying the theory of least-squares represents only a few of the phenomena involved in any problem, one or two or three unknowns. The number of unknowns is actually infinite. It is hoped that the unknowns considered are dominant, that the equation represents the dominant phenomena or the dominant phenomenon involved. It may be that other considerations which are overlooked affect the observations. If the observations are represented graphically, they are much more likely to disclose such a situation, but if treated by the theory of least-squares, and without a graphical representation, it practically never would be found out.

In that connection, when a graphical representation is made, one should distinguish carefully between large discrepancies and inaccuracies. The writer includes under large discrepancies departures from the curve that are too large to be accounted for

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[19] Engineer, Research Laboratory, General Electric Co., Schenectady, N. Y. Mem. A.S.M.E.

as inaccuracies. If points occur too far off the line to represent inaccuracies of observation, warning is given that either a mistake has been committed or that something occurs that the theory does not take account of.

RICHARD A. E. DE JONGE.[20] As far as I know, the first experiments with ellipsoidal-shaped heads were made by Mr. Höhn, chief engineer of the Swiss Boiler Inspection Association, and by his assistant, Dr. Huggenberger. The latter has given a complete mathematical analysis of the whole problem which is very interesting. He takes all stresses into consideration. Mr. Höhn has made experiments with dished heads as well as with ellipsoidal-shaped heads. He has developed a simple method of finding the two radii called for in basket-shaped heads.

It is the present custom in Europe to shape boiler drums for high-pressure boilers out of one solid piece, and to dish the heads on directly in preference to riveting or welding. This has been done especially for boilers with steam pressures of 1200 lb. per sq. in. and above, where riveting or welding is simply out of the question, Mr. Höhn has also given information on how to weld boiler shells and heads where this is still practicable so as to avoid additional stresses.

Is it not that designers of heads, who send with their orders their recommendations to the manufacturers, are not educated to the fact that the ellipsoidal head is the best, that they still use a radius at the throat and a radius at the center. It would be best to refer the matter back to the standardization committee, and make it compulsory to use the ellipsoidal head, because that is the logical step in the development from the spherical head which has no bending stresses, and the flat head which has the highest bending stresses. The ellipsoidal head thus represents an intermediate stage.

Barraclough and Gibson in Australia made some interesting investigations on a boiler under actual load conditions. They placed a large number of little mirrors along the meridian line of the boiler, and from actual measurements on these mirrors with an optical instrument they found that the stresses did vary so as to show strong compression in the interior of the head near the throat. They made the experiments on an existing boiler under pressure, and observed how the stresses vary when the boiler is under no load and when the load is applied. They used water pressure instead of steam, but for the purpose of stresses this is immaterial. They have shown the actual stress distribution. The stress distribution indicated uniform tensile stress in the cylindrical part of the boiler, and in the head, a sharp decrease to almost zero at the joint. From there on there is a very strong compressive stress started which reaches its maximum at the point of least radius of curvature at the throat. This then drops to zero a short distance from the point of greatest curvature, and becomes a tensile stress again which increases to a maximum at the crown of the head. This refers to the outer elements of the head. The inner ones showed exactly the opposite stresses. Since this investigation was on a working model, the three-dimensional process has been taken care of.

L. H. BURKHART.[21] Nearly all manufacturers making dished and flanged heads have a large quantity of forms or dies on hand, probably an investment of thousands of dollars, and they are generally made for spherical-dished heads, and they do not always suit the diameter of the dished head required. The dishing blocks may be smaller than the diameter of the head, yet it is always possible to dish a head much larger than the diameter of the block, if it is spherical, because the head can always be shifted so that every part of the head covers the die, and as the radius of the curvature in a spherical head is the same, a smoothly dished head will result. It is not possible to do that with an ellipsoidal head because the radius of curvature changes in the different proportions of the head so the die must cover the entire surface of the head if it is to be dished to the ellipsoidal shape.

Again, the ellipsoidal head, when the ratio of one to two is used, is about twice the depth of dish of the spherical head where a radius equal to the diameter of the head is used. On thin plate it becomes impossible to dish the plate to that depth without buckling the plate. It is possible in thicker plates to make a deep dish. But manufacturers have occasion to use heads $^1/_4$ in. and up, and the thinner the heads the more difficult to dish without buckling.

The problem seems to work out best by taking the standard spherical-radius die, and turning a larger fillet radius on the male die so that it will approximate the ellipsoidal head. The strains are much less in the knuckle of a large radius than they would be if a shorter radius were used, as has generally been the practice. The ellipsoidal head has advantages of uniform stress over the spherical head, but the users of vessels employing dished heads ought to go guardedly against specifying ellipsoidal heads until manufacturers are equipped and means have been worked out for producing these heads.

The company the writer represents is a large user of dished heads and has recently requested quotations from several mills that are provided with spinning machines for producing these heads. They come back with prices of equipment which seem exorbitant and cannot be charged into the cost of the job. In some cases some mills refuse to quote as they are not equipped to produce ellipsoidal heads. These are the difficulties that a manufacturer encounters.

AUTHOR'S CLOSURE

In Fig. 7, the dotted line, which is outside the inside surface at the crown, but on the inside of the surface near the knuckle, is the true ellipsoid of revolution in which the major axis is twice the minor axis. The basket-shaped head, with the radius of 6 ft. and a knuckle radius of $13^3/_4$ in., deviates from the true ellipsoidal shape as shown in the curve, which is drawn to scale. The deviation is not more than $^1/_2$ in. to $^3/_4$ in., going outside of the true ellipsoidal shape at the knuckle and slightly inside near the crown. It is about the closest approximation that can be obtained to the true ellipsoidal shape with any two combinations of radii.

The question has been raised as to why a true elliptical head has not been recommended in place of one with two radii as used for the head investigation reported in this paper. I do not want to give the impression that this head is recommended in preference to the true elliptical one or vice versa. Practical considerations will govern the type of heads eventually used. It was brought out in the discussion that one company, as head fabricators, preferred to have the true ellipsoidal shape. One fundamental reason for this is that it is possible to approximate the true ellipsoidal shape by a basket-shape design using slightly varying radius of dish and slightly different knuckle radius. Some purchasers, for instance, might specify a 68-in. radius of dish and 12-in. knuckle radius instead of the 72-in. and $13^1/_4$-in. radii used for the heads investigated in this paper, which, incidentally, would fairly approximate a true ellipsoidal shape. It can readily be appreciated from a fabrication standpoint that if they were to be required to have numerous dies of different sizes, the cost of fabricating these heads would become prohibitive. On the other hand, several fabricators are standardizing on certain combination of radii, so that the basket-shape heads are becoming more or less stock sizes.

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[21] Chief Engineer, Struthers-Wells Co., Warren, Pa. Mem. A.S.M.E.

As time goes on, there probably will result a condition in which the basket-shape heads will be standard with some fabricators, while with others the true ellipsoidal heads will be standard. The difference between the two is small, and is more a practical consideration of fabrication.

The data given in this paper are not intended to present a brief for either head, but are intended to show the advantage of the ellipsoidal head, either true or approximate, over the older types of heads, and to present the theory and engineering data of stress distribution for such heads.

The question has been raised regarding the stresses in the head and cylinder at the junction of the head with the shell of the tank, as to whether it is possible to have compression in the head and tension in the adjacent metal of the cylindrical shell. At this point the condition of boundary limits governs and the exact theory breaks down. This upset of theory at the boundary condition is best illustrated in a tank with true semi-spherical heads. The stresses in a semi-spherical head are one-half the stresses in the cylindrical shell of equal plate thickness. Thus we have a condition at the junction of the semi-spherical head and shell where the stresses in the head by theory are one-half the stresses in the adjacent cylindrical shell. The theory, therefore, breaks down at this boundary condition. What actually happens is that the stresses change gradually in their intensity with no sudden demarkation. Likewise, with an ellipsoidal head there will be found a gradual change from the hoop compression stresses in the head to the cylindrical tensile stresses. It is possible to notice with the older type heads, where the hoop compression stresses are very intense at the knuckle, that the forces will deform the cylindrical shell inward for a distance of 6 to 12 in. if the pressure is carried up to or slightly in excess of the yield point of the tank material. Huggenberger's theoretical analysis of the ellipsoidal head neglects both bending and the forces of the attached cylinder which he assumes to be comparatively small.

The subject of a true ellipsoidal head versus the basket-shaped head has been discussed so extensively that it seems advisable to add a few comments in regard to this subject.

The question as to which is the better type of head or the one that should be specified, as pointed out before, is purely a matter of practical fabrication. The situation is in the process of evolution. As these heads are more extensively adapted, and more manufacturers specify their use, a greater number of fabricators will fall into line to make them. Some of these have already gone to true ellipsoidal shape. Others, as has been pointed out, have had to compromise with the basket-shape design, using a combination of two radii.

A little discussion of Huggenberger's formula may throw some light upon this subject. Huggenberger's formula, which is reviewed in the paper, is based on an ellipsoid revolution, and is applicable to ellipsoidal shapes other than the one in which the major axis is twice the minor axis.

With an elliptical shape, in which the depth of the dish is less than one-quarter that of the tank diameter, the stress distribution is somewhat different from that indicated in the paper. The radial tensile stresses are maximum at the crown and decrease slightly toward the knuckle. The hoop stresses, on the other hand, are of maximum tension at the crown, but change to very high-compression stresses at the knuckle, the intensity of these stresses depending upon the shape of the ellipse. With increased depth of the dish, the stresses decrease in intensity until a shape is reached in which the ratio of the axis is one; in other words, a semi-spherical head. In this latter case, the stresses are theoretically and actually at a minimum, their intensity being one-half the stresses in the shell.

The ellipsoidal shape, in which the depth of the dish is one-quarter that of the tank diameter or the ratio of the axis two to one, presents a compromise between shape and cost of fabrication. With this shape, according to Huggenberger's theory, the maximum stresses will not exceed those in the shell; in other words, a design of equal strength.

In the use of such heads, as the stresses are very much reduced below the older standard heads, a decrease in thickness is permitted, which of course is an item of economy. Opposed to that is the increase of forming due to die costs. This is a new type of head and as such demands special dies. In the evolution attending its use, as the heads are more extensively adopted, the cost of forming will naturally decrease and the tendency in the meantime will be to use, in one case, a basket shape and in another case a true ellipsoidal shape, and it will be up to the boiler code to more or less standardize type of head and its design. With standardization, the cost of forming will not be the important item that it is today.

The Boiler Code Committee is considering specifying the use of either the true ellipsoidal shape or the basket shape, provided the latter does not materially deviate from the first.

Mr. Huggenberger in his theoretical analysis and derivation of formulas for the stress distribution in ellipsoidal heads has neglected bending. The analysis becomes extremely complicated if this is taken into consideration. In the investigation an attempt was made to evaluate the bending, if such occurred, by taking strain-gage measurements on the inside as well as outside. It was, of course, impossible to measure the strains on the inside under load. Strain measurements, however, were taken inside the tank prior to putting in water, and after the test was completed, which of course determined only the permanent set but from which the nature of the bending could possibly be evaluated.

Unquestionably there is some bending imposed in an ellipsoidal head which somewhat affects stress distribution. The agreement of the measurements taken during the first test was not as close as those obtained in the second test, due, no doubt, to slightly reshaping the head to give a better stress distribution. The disagreement, however, between actual measurements and theory seems to be of little importance when compared to the improvement of stress distribution with these heads over the older type, and it is questionable whether a more accurate theoretical analysis would add any practical advantages over the approximate analysis of Huggenberger.

APM-51-14

Analysis of the Uniform-Rise and Uniform-Pressure-Angle Cam Curves

A Method of Designing Cams by Means of Prepared Charts

By R. F. MALLINA,[1] CAMDEN, N. J.

The practical design of cams sometimes requires a thorough investigation of the practical limits to which a designer may go without greatly changing the leverage ratio from cam shaft to cam-lever shaft and without putting unnecessary strain on the follower stud.

It is the purpose of this paper to point out methods of obtaining such limits and to provide tables and charts which will enable the draftsman to determine certain cam curves with a minimum of effort.

The results of this investigation are as laid down in two appended charts.

DISREGARDING cams with uniform acceleration which are ordinarily used when cams revolve with great velocity, two of the principal cam curves are the uniform-rise curve and the uniform-pressure-angle curve. The graphical method of laying out the uniform-rise curve is simple, accurate, and direct. The graphical method for the uniform-pressure-angle curve, however, is of a complicated nature, inaccurate, and indirect. Engineering handbooks and textbooks do not mention it, and this is probably the reason few design engineers are acquainted with it.

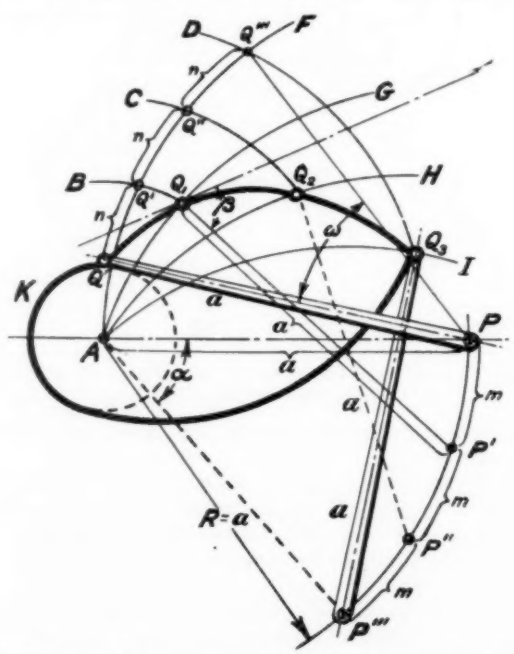


FIG. 1 GRAPHICAL METHOD FOR LAYING OUT UNIFORM-RISE CURVE

The mathematical method of determining the uniform-pressure curve is of course correct, but it is not simple. The equation of the curve is a logarithmic function, rather long and expressed in parametric form.

The only practical way to make use of the uniform-pressure-angle curve is to develop a family of curves for a series of pressure angles, plot them in a chart, and designate the leverage at certain intervals. The designer has then but to copy the particular curve in which he is interested and reduce or enlarge the scale to suit his given conditions.

In what follows the author will describe graphical methods of determining the cam curves, giving the equations employed and also explaining the use of the charts derived.

THE UNIFORM-RISE CURVE

From the standpoint of maintaining the torque or the leverage between the cam shaft and cam-lever shaft constant, the uniform-rise cam is ideal. Such a cam mechanism acts similarly to a pair of circular gears, and the angular-displacement ratio or leverage $L = \alpha/\omega$ is constant.

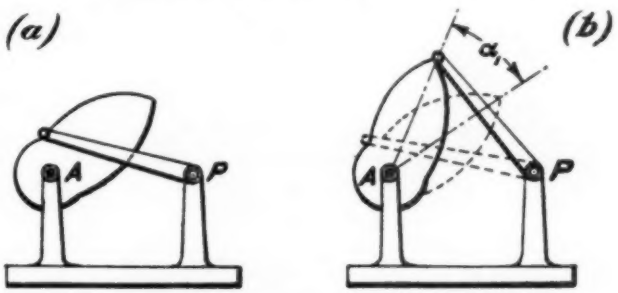


FIG. 2 PHYSICAL CONCEPTION OF A CAM SYSTEM
(Cam A is rotating and cam lever pivot P is stationary.)

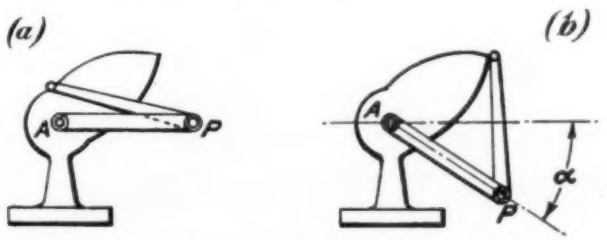


FIG. 3 MATHEMATICAL CONCEPTION OF A CAM SYSTEM
(Cam A is stationary and cam lever pivot P is rotating around cam axis.)

Fig. 1 illustrates the graphical method in general use. Assuming the base circle K, the maximum-lift circle D, and the cam-displacement angle α as given, the curve may be laid out in the following manner: Using the follower axis P as a center, strike arc F with the cam-lever length $a = PA$ as radius. A is the axis of the cam. Arc F intersects circle K at point Q, and circle D at Q'''. Angle QPQ''' is the cam-lever displacement angle ω. With A as center, strike arc PP''' and make angle PAP''' equal to α. Divide arc QQ''' into equal parts: $QQ' = Q'Q'' = Q''Q''' = n$ and arc $P'P'''$ in the same number of equal parts: $PP' = P'P'' = P''P''' = m$. Strike arc B through Q' and arc C through Q'' with A as center. With P', P'', and P''' as centers, strike arcs AG, AH, and AI with radii a, which intersect arcs B, C, and D at points Q_1, Q_2, and Q_3. The points Q, Q_1, Q_2, and Q_3 then represent the uniform-rise

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curve. The method is extremely simple and direct because the curve is not obtained by a trial-and-error method.

Uniform-rise cams can be used only within certain limits. The nearer the origin Q is to the cam center A, the more the pressure angle β approaches 90 deg., causing a friction moment greater than the cam lifting moment, with a consequent binding of the mechanism.

The Uniform-Pressure-Angle Curve

The logical method to avoid binding action is to design a cam curve with a uniform pressure characteristic; that is, a curve such that the angle between the lever and the tangent to the cam curve is constant and less than 90 degrees.

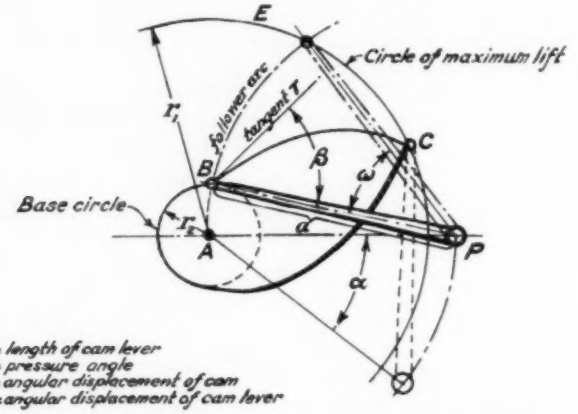


Fig. 4 Graphical Method for Laying Out Uniform-Pressure-Angle Cam Curve

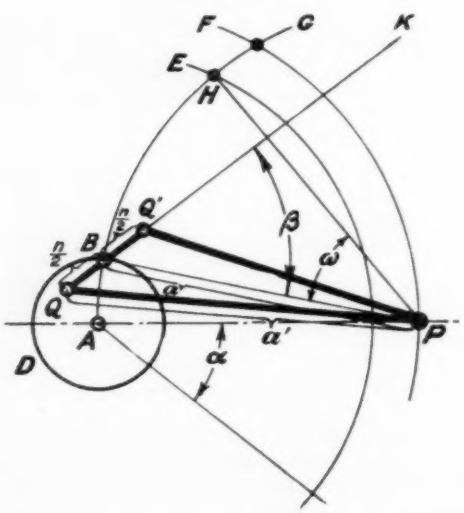


Fig. 5 Characteristic Triangle Used for Graphical Layout of Uniform-Pressure-Angle Cam Curve

The graphical method for determining the cam curve is illustrated in Figs. 2 to 6, inclusive.

Fig. 2 shows the general arrangement of cams. The cam pivot A and the cam-lever pivot P are parts of the frame. The angle α_1 is the displacement angle of the cam or the angle through which it must move to cause the follower to rise from the base circle of the cam to its extreme outer point.

For the graphical as well as for the mathematical treatment, it is more convenient to consider the cam stationary and let the cam-lever pivot P rotate around the cam pivot A as in Fig. 3.

For the same rise of the follower the angle α_1 or α is called the displacement angle or the angular displacement of the cam.

Fig. 4 shows the relationship between the angular displacement α of cam and the angular displacement ω of the cam lever.

The given values in a cam problem are as a rule the angle α the cam-lever length a, and the radii r_1 and r_2. In automatic machines α is part of a cycle within which a certain operation has to be performed.

From Fig. 4 it is obvious that the angle ω and the points B and C are determined by the values mentioned. The angle β is called the pressure angle and is the angle formed by the tangent T to the curve and the cam lever PB (see Fig. 12). If β is 90 deg. the cam cannot revolve, and if $\beta = 0$ deg., the follower cannot be lifted from the base circle. In order to keep the friction in the various elements constant, the curve between B and C must be such that β remains constant.

The graphical method of laying out the constant-pressure-angle cam curve as illustrated in Figs. 5 and 6 is a trial-and-error method.

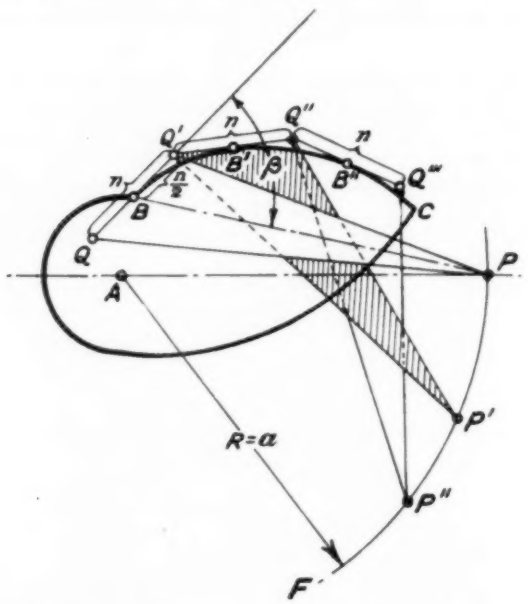


Fig. 6 Consecutive Positions of Characteristic Triangle for Laying Out Uniform-Pressure-Angle Cam Curve

A pressure angle β must be assumed and the curve laid out section by section. If the cam angle α proves to be too small or too large for a given cam-lever displacement angle ω, the pressure angle β must be changed until α approaches the required magnitude.

The curve is laid out as follows: Referring to Fig. 5, draw base circle D, circle of maximum lift E, and cam-lever-fulcrum circle F around center A. Then with P as a center, strike follower arc AG. Follower arc AG intersects the base circle D at point B. Assume a pressure angle $\beta = PBK$, and on the side BK of this angle locate points Q and Q' equidistant from point B. Distance BQ is chosen arbitrarily. Point B is to become a point of tangency in the layout of the first section of the cam curve. With points P, Q, and Q' established, the characteristic triangle PQQ' of the cam curve is obtained.

Keeping point P on the arc F (Fig. 6), rotate the characteristic triangle until point Q falls on point Q'. This gives a new point Q'', as well as a new point of tangency B'. The process should be repeated until the cam curve passes through point C.

The envelope of the elements is the uniform-pressure-angle curve.

The most convenient manner for the draftsman to lay out the uniform-pressure curve is to draw the characteristic triangle on a small piece of transparent paper and add the elements together by marking the points Q, Q', Q'', etc., while rotating P on circle F.

The smaller the lengths n, the more accurate the completed cam curve will be.

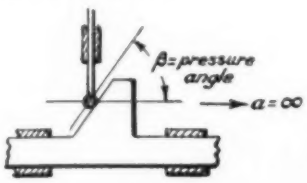


Fig. 7

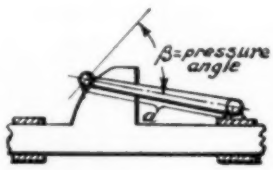


Fig. 8

FIG. 7 CAM WHERE RADIUS a OF CIRCLE DESCRIBED BY THE FOLLOWER IS OF INFINITE LENGTH AND WHERE RADIUS ρ OF CIRCLE DESCRIBED BY A POINT OF THE CAM IS ALSO OF INFINITE LENGTH

FIG. 8 CAM WHERE CAM LEVER RADIUS a IS OF FINITE LENGTH AND CAM RADIUS ρ OF INFINITE LENGTH

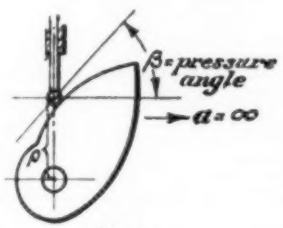


Fig. 9

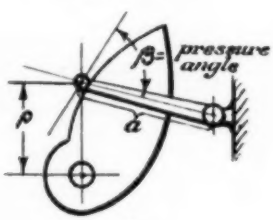


Fig. 10

FIG. 9 CAM WHERE CAM LEVER RADIUS a IS OF INFINITE LENGTH AND CAM RADIUS ρ IS OF FINITE LENGTH

FIG. 10 CAM WHERE BOTH CAM LEVER RADIUS a AND CAM RADIUS ρ ARE OF FINITE LENGTH

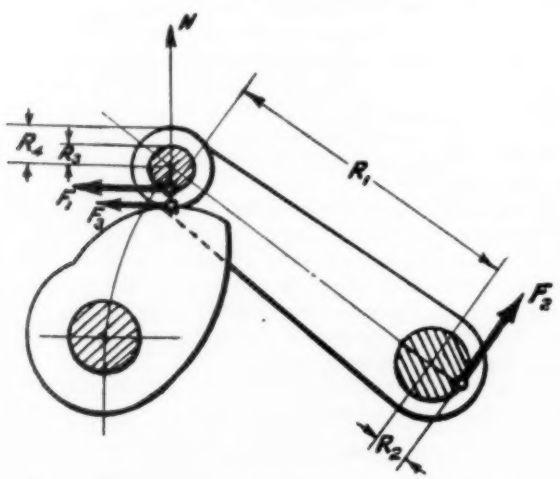


FIG. 11 FRICTIONAL FORCES ACTING ON A CAM SYSTEM

TYPES OF UNIFORM-PRESSURE CAMS

The four types of cams in general use are shown in Figs. 7–10.

Fig. 7 shows a cam with the cam radius and cam lever of infinite length. In this case the cam is both a uniform-pressure cam and a uniform-rise cam.

Fig. 8 shows a cam with an infinite cam radius and a cam lever of finite radius.

Fig. 9 shows a cam with cam-lever radius of infinite length and a cam radius of finite length.

In Fig. 10 both the cam radius and the cam-lever radius have finite dimensions.

DEFINITION OF MAXIMUM PRESSURE ANGLE

The maximum pressure angle or binding angle is the angle at which the follower begins to bind on the cam. The binding takes place when the friction moment, acting on the cam lever, is equal to the lifting moment on the cam lever.

The total friction moment is caused by three component forces (see Fig. 11):

a The sliding friction F_1 of the follower on the stud,
b The sliding friction F_2 of the cam lever shaft in the cam lever bearing, and
c The rolling friction F_3 of the follower on the cam curve.

DETERMINATION OF MAXIMUM PRESSURE ANGLE

To illustrate the influence of each of the three aforementioned forces on the binding angle, four different cases will be calculated

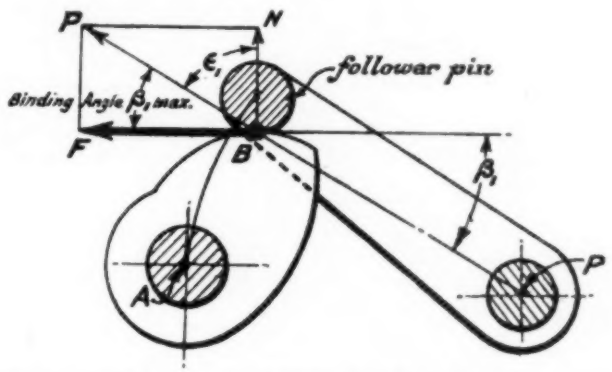


FIG. 12 ILLUSTRATION OF BINDING ANGLE WHERE ONLY THE SLIDING FRICTION BETWEEN FOLLOWER AND CAM SURFACE IS CONSIDERED
(Binding angle $\beta_1 = 78°$.)

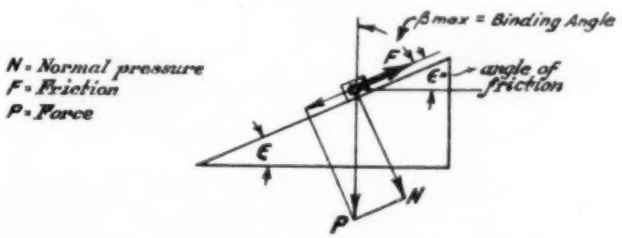


FIG. 13 DIAGRAMMATIC ILLUSTRATION OF BINDING ANGLE

with the following values given (Fig. 11):

Angle of sliding friction, steel on steel (dry),

$$\epsilon = 12°$$

where $\tan \epsilon$ = coefficient of sliding friction,

Angle of rolling friction, steel on steel,

$$\delta = 0.1°$$

where $\tan \delta$ = coefficient of rolling friction,

$R_1 = 2$ in. $R_3 = 1/8$ in.
$R_2 = 3/16$ in. $R_4 = 1/4$ in.

Case 1. The follower is a pin and the sliding friction F acting on the pin is the only friction considered. (See Fig. 12.) The follower binds on the cam when

$$\beta_1 \text{max.} = 90° - \epsilon_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

$$\epsilon_1 = \epsilon = 12°$$

$$\therefore \beta_1 \text{max.} = 78°$$

Equation [1] will be more easily understood by constructing a vector diagram of a force P acting on an inclined plane as shown in Fig. 13.

Case 2. Fig. 14 represents a cam mechanism with a follower roller. The rolling friction F between the follower roller and the follower stud is the only friction under consideration.

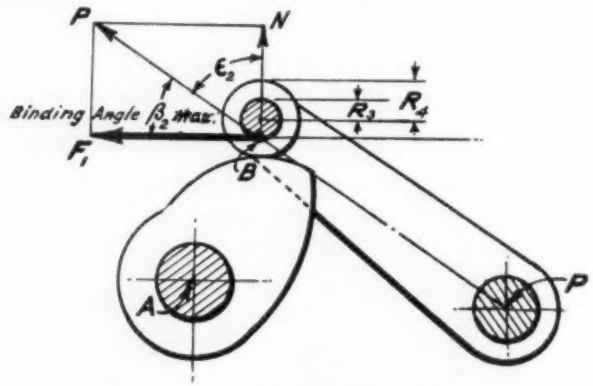


FIG. 14 ILLUSTRATION OF BINDING ANGLE WHERE ONLY THE SLIDING FRICTION BETWEEN FOLLOWER AND FOLLOWER STUD IS CONSIDERED
(Binding angle $\beta_2 = 83° 56'$.)

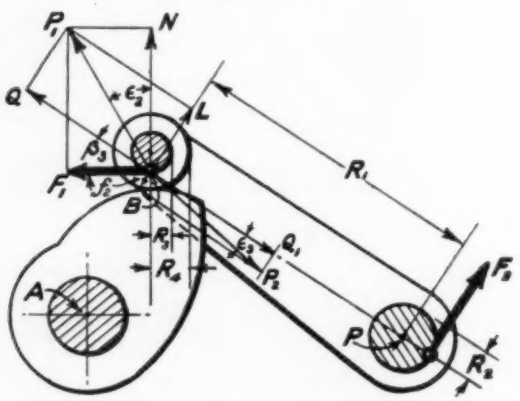


FIG. 15 ILLUSTRATION OF BINDING ANGLE WHERE ONLY FRICTION BETWEEN FOLLOWER AND FOLLOWER STUD AND THE FRICTION IN CAM LEVER BEARING ARE CONSIDERED
(Binding angle $\beta_3 = 82° 47'$.)

The follower binds on the cam when

$$\beta_2 \text{max.} = 90° - \epsilon_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots [2]$$

$$\tan \epsilon_2 = \frac{R_3}{R_4} \tan \epsilon \quad \ldots\ldots\ldots\ldots\ldots\ldots [3]$$

$$= \frac{1}{2} \tan 12° \text{ or } \epsilon_2 = 6° 4'$$

$$\therefore \beta_2 \text{ max.} = 83° 56'$$

Case 3. Here the friction F_2 of the cam-lever shaft on its bearing in addition to the friction F_1 is considered.

Referring to Fig. 15, the two forces acting on the follower are the normal force N and the friction F_1. N and F_1 are components of P_1. The force P_1 may be divided into the lifting component L and the cam-lever force Q. The reaction Q_1 of the force Q is equal to the latter magnitude and opposite in direction. This force Q_1 produces a friction F_2 which, reduced to a force acting on radius R_1, results in $f_2 = F_2 \frac{R_2}{R_1}$. The two forces Q_1 and f_2 are components of P_2. If P_1 and P_2 lie in the same line and are opposite in direction there will be no lifting force and the follower will bind on the cam.

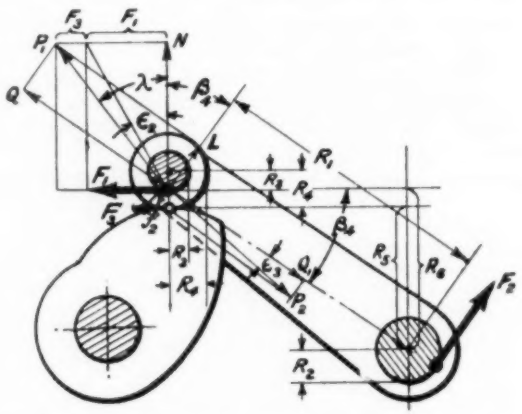


FIG. 16 ILLUSTRATION OF BINDING ANGLE WHERE ALL FRICTIONS (SLIDING AND ROLLING) ACTING ON THE SYSTEM ARE CONSIDERED
(Binding angle $\beta_4 = 82° 41'$.)

The forces P_1 and P_2 are in line when:

$$\beta_3 \text{max.} = 90° - (\epsilon_2 + \epsilon_3) \quad \ldots\ldots\ldots\ldots [4]$$

$$\tan \epsilon_2 = \frac{R_3}{R_4} \tan \epsilon \quad \ldots\ldots\ldots\ldots\ldots\ldots [3]$$

$$\tan \epsilon_3 = \frac{R_2}{R_1} \tan \epsilon \quad \ldots\ldots\ldots\ldots\ldots\ldots [5]$$

Since $\epsilon_2 = 6° 4'$ (see *Case 2*)

$$\tan \epsilon_3 = \frac{3}{16}, \quad \tan 12° \text{ or } \epsilon_3 = 1° 9'$$

$$\therefore \beta_3 \text{max.} = 82° 47'$$

Case 4. This case includes all friction forces acting on the cam lever (Fig. 16).

The three forces acting on the follower are the normal pressure N, the sliding friction F_1, and the rolling friction F_2. The rolling friction F_3 is very small compared with F_1, and the difference between R_5 and R_6 is also very small. For practical purposes F_3 may therefore be assumed to be in line with F_1. The force P_1 is determined by the normal pressure N and the sum of the friction forces $F_1 + F_2$.

The follower binds when P_1 is in line with P_2 and when P_1 and P_2 act in opposite directions, or when

$$\beta_4 \text{max.} + \lambda + \epsilon_2 = 90°$$

$$\tan \lambda = \frac{F_1 + F_2}{N} = \frac{N \tan \epsilon_2 + N \tan \delta}{N} = \tan \epsilon_2 + \tan \delta$$

$$\tan \epsilon_2 = \frac{R_3}{R_4} \tan \epsilon \quad \ldots\ldots\ldots\ldots\ldots\ldots [3]$$

$$\tan \lambda = \frac{R_3}{R_4} \tan \epsilon + \tan \delta \quad \ldots\ldots\ldots [7]$$

$$\tan \lambda = {}^1/_2 \tan 12° + 0.1 \text{ or } \lambda = 6° 10'$$

$$\tan \epsilon_3 = \frac{R_3}{R_4} \tan \epsilon \quad \ldots\ldots\ldots\ldots\ldots\ldots [5]$$

$$\tan \epsilon_3 = \frac{3}{16} \times \frac{1}{2} \tan 12° \text{ or } \epsilon_3 = 1° 9'$$

$$\therefore \beta_4 \text{max.} = 82° 41'$$

Summarizing, the binding angles in the four cases are as follows:

Case 1, $\beta_1$max. = 78° Case 3, $\beta_3$max. = 82° 47′
Case 2, $\beta_2$max. = 83° 56′ Case 4, $\beta_4$max. = 82° 41′

As will be noted, the rolling friction is practically negligible, the difference between the binding angle in Case 3 and Case 4

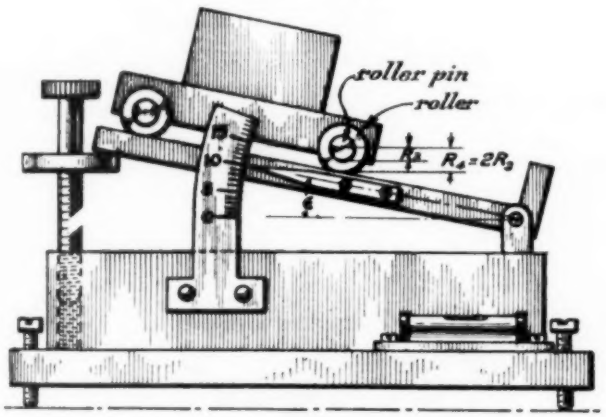


FIG. 17 APPARATUS TO DETERMINE EXPERIMENTALLY THE BINDING ANGLE OF CAMS USING DIFFERENT METALS AND LUBRICANTS

being very small. The effect of the friction in the cam-shaft bearing which is represented by the difference of Case 2 and Case 3 is also negligible for practical purposes.

For all practical purposes, where a follower roller is used, Case 2 will give results sufficiently accurate. In order to have a correction and safety factor, however, it is advisable to increase the friction angle ϵ_2 by 25 per cent. In the examples given this safety factor would cause the binding angle in Case 2 to closely approach the real conditions as given in Case 4.

Taking Case 2 with the corrected angle of friction

$$\epsilon_0 = \epsilon + 0.25\epsilon = 1.25\,\epsilon$$

and substituting ϵ_0 for ϵ in Equation [3],

$$\tan \epsilon_2 = \frac{R_3}{R_4} \tan \epsilon_0$$

$$\tan \epsilon_2 = \frac{R_3}{R_4} \tan 1.25\,\epsilon \quad \ldots\ldots\ldots\ldots [8]$$

The determination of the maximum-pressure angle or binding angle β max. by Equations [2] and [8] will serve for all engineering purposes.

EXPERIMENTAL DETERMINATION OF MAXIMUM-PRESSURE ANGLE

To check the computed binding angle βmax., a carriage on interchangeable rollers with adjustable friction angle was used (Fig. 17). The values obtained are given in Table 1.

DETERMINATION OF A PRACTICAL PRESSURE ANGLE

The maximum-pressure angle cannot be used as a practical pressure angle since no power can be transmitted. Assuming

TABLE 1 VALUES OF β MAXIMUM

Material of			Bearing of roller on pin	Lubricated or dry	Composite friction angle ϵ	Binding angle βmax.
Roller pin (stud)	Roller (follower)	Sliding surface (cam)				
Steel	Steel (locked to skid)	Steel	Locked to skid	Dry	11½°	78½°
Steel	Steel (locked to skid	Steel	Locked to skid	Lubricated	11°	79°
Steel	Steel (free to revolve)	Steel	Plain bearing	Dry	6½°	83½°
Steel	Steel (free to revolve)	Steel	Plain bearing	Lubricated	5¾°	84¼°
Steel	Steel (free to revolve)	Steel	Ball bearing	Dry	1½°	88½°
Steel	Steel (free to revolve)	Steel	Ball bearing	Lubricated	1°	89°

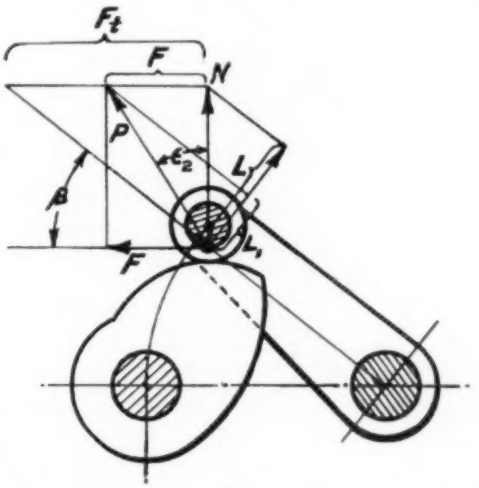


FIG. 18 ILLUSTRATION TO DEFINE FRICTIONAL LOSS OF A CAM

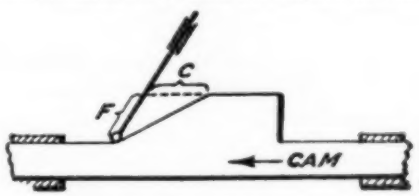


FIG. 19 ILLUSTRATION TO DEFINE LEVERAGE OF A STRAIGHT-MOTION CAM

or allowing a certain percentage of frictional loss in the cam mechanism, the pressure angle can be computed. It is simpler, however, to assume a pressure angle, see whether it is sufficiently large or small to do a certain work in a certain time, and then figure the frictional loss.

Example:

$R_3 = \frac{1}{8}$ in. $\epsilon = 12°$
$R_4 = \frac{1}{4}$ in. βmax. = 82° 22′ (by [8] and [2])

Assume that the practical pressure angle $\beta = 60°$.

The frictional loss η is the ratio $(L - L_1)/L = F/F_t$. (See Fig. 18.)

At $\beta = 82° 22'$ the friction $F = F_t$ and $\eta = F/F_t = 1$, or the frictional loss = 100 per cent.

At $\beta = 60°$ the ratio

$$\eta = \frac{F}{F_t} = \frac{N \tan \epsilon_2}{N \cot \beta} = \frac{\tan \epsilon_2}{\cot \beta}$$

By Equation [8] $\tan \epsilon_2 = \frac{R_3}{R_4} \tan (1.25\epsilon)$

$$\eta = \frac{\tan (1.25\,\epsilon)}{\cot \beta} \times \frac{R_3}{R_4} \quad \ldots\ldots\ldots\ldots\ldots\ldots [9]$$

or

$$\eta = \frac{\tan(1.25 \times 12°)}{\cot 60°} \times \frac{1}{2} = 0.232$$

or the frictional loss $\eta = 23$ per cent.

The Leverage on the Uniform-Pressure-Angle Curve

On the uniform-pressure-angle curve the leverage or the ratio of the angular velocities or displacements of the cam and the cam lever is not constant. Since the lifting power is dependent upon this ratio, it is important to determine it for different points of the curve.

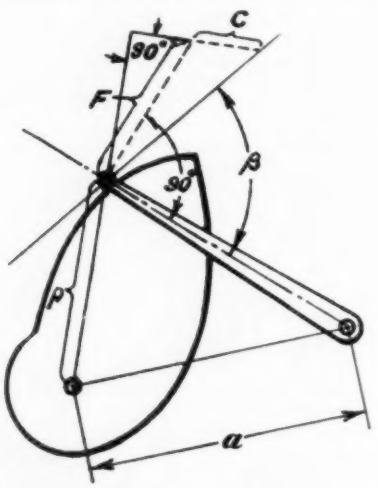


Fig. 20 Illustration to Define Leverage of a Circular Cam

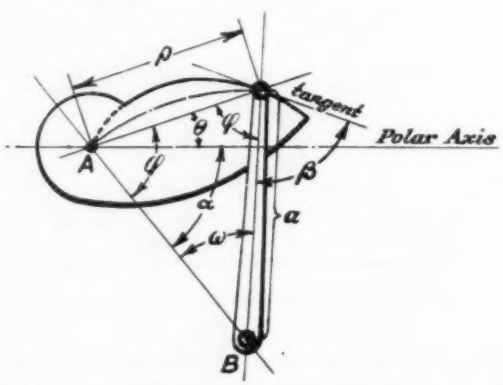


Fig. 21 Illustration to Define Symbols Used in the Equations of the Uniform-Rise Curve

Fig. 19 shows the displacement of cam and follower on a straight-line-motion cam mechanism.

When the cam is displaced by the distance C and the follower displacement is F, the leverage L is the displacement ratio C/F.

On a circular-arc cam small angular displacements have to be taken into consideration. Referring to Fig. 20, the angular displacement of the cam is $d\alpha = dC/\rho$ and the angular displacement of the follower is $d\omega = dF/a$.

$$\text{The leverage } L = \frac{d\alpha}{d\omega} = \frac{dC}{dF} \times \frac{a}{\rho} = \frac{C}{F} \times \frac{a}{\rho}$$

The quotient C/F is determined graphically and then multiplied by a/ρ.

The Uniform-Rise-Curve Equation

As previously mentioned, the uniform-rise-curve cam transmits motion like a pair of gears with a constant ratio or constant leverage L. The variable in such a cam is the pressure angle β.

When the polar axis is so chosen that $\omega = 0$ for $\alpha = 0$, the mathematical expressions are (Fig. 21):

$$L = \frac{\alpha}{\omega} = \text{leverage} = \text{constant}$$

$$\begin{cases} \theta = (1 - 2\,L)\,\varphi + K \\ \rho = 2a \cos \varphi \end{cases}$$

$$\tan \beta = \frac{\cot \dfrac{\alpha}{2L} + (1 - 2L)\tan \dfrac{\alpha}{2L}}{2L}$$

where the integration constant $K = L \times 180°$.

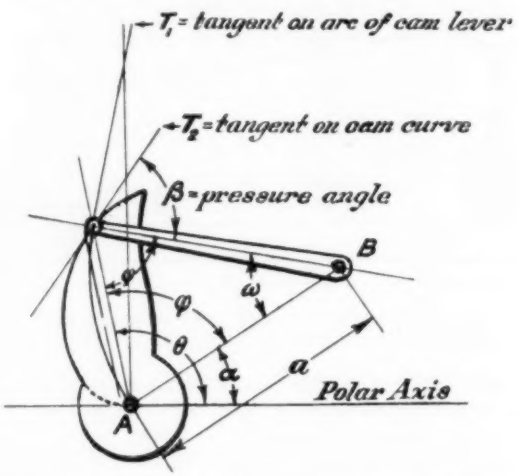


Fig. 22 Illustration to Define Symbols Used in the Equations of the Uniform-Pressure-Angle Curve

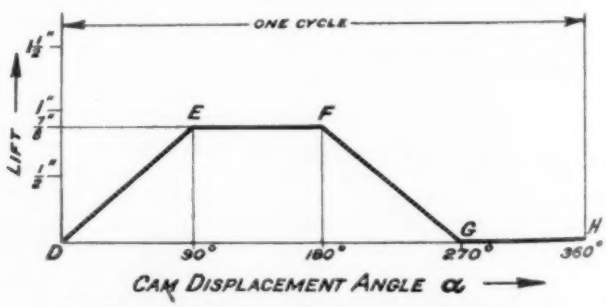


Fig. 23 Cycle Diagram of Cam as Shown in Fig. 24

The Uniform-Pressure-Curve Equations

This cam curve transmits motion at a constant pressure angle β and a variable leverage L.

The mathematical expressions for an arbitrarily chosen polar axis are (Fig. 22):

$$\begin{cases} \beta = \text{pressure angle} = \text{constant} \\ \theta = \varphi \mp \cot \beta \log_e (\tan \beta \tan \varphi \pm 1) + K \\ \rho = 2\,a \cos \varphi \end{cases}$$

$$L = \frac{d\alpha}{d\omega} = \frac{[e^{\mp \tan \beta(\alpha - K)} \mp 1]^2 + \tan^2 \beta}{\mp 2 \tan^2 \beta e^{\mp \tan \beta(\alpha - K)}}$$

The leverage L may be more simply computed as a hyperbolic function.

$$L = \frac{1}{-2}\{\cosh[\tan\beta(\alpha - K)] \mp \sinh[\tan\beta(\alpha - K)]\} + \cot^2\beta\{1 \mp \cosh[\tan\beta(\alpha - K)]\}$$

where K is an integration constant and e the base of the Napierian logarithmic system. The upper signs refer to cam and cam lever when rotating in the same direction. The lower signs refer to cam and cam lever when rotating in opposite directions.

Use of the Charts

As previously mentioned, the uniform-rise curve can be accurately laid out and it may not be necessary to copy the curve from Chart I. However, it provides a convenient means of selecting a suitable curve from a region where the pressure angles are such as not to cause excessive frictional loss.

The uniform-pressure-angle curve cannot be laid out directly to suit a given cam displacement angle, and in most cases it will be desirable to copy it from Chart II.

An example will illustrate the manner of laying out the cam curve from this chart.

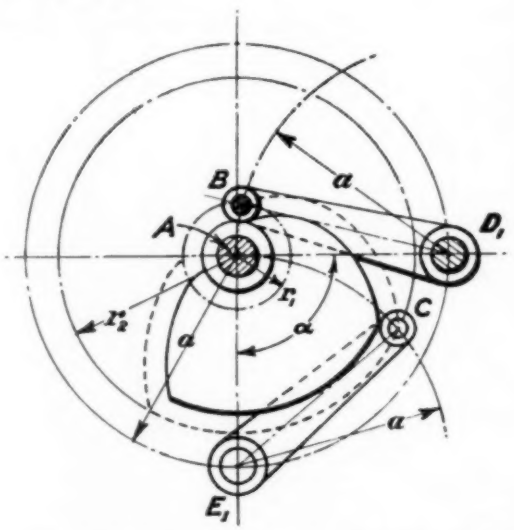


Fig. 24 Uniform-Pressure-Angle Cam, Showing the Use of Chart II and Made According to Cycle Diagram Fig. 23

Given: the diagram as shown in Fig. 23.
Assumed: (See Fig. 24).

Length of cam lever	$a = 1^1/_2$ in.
Radius of base circle	$r_1 = {}^3/_8$ in.
Radius of circle of maximum lift	$r_2 = 1^1/_4$ in.
Follower lift	$r_2 - r_1 = {}^7/_8$ in.
Radius of follower stud	$R_1 = {}^1/_{16}$ in.
Radius of follower roller	$R_2 = {}^1/_8$ in.

To find the uniform-pressure-angle curve suited for the diagram, proceed as follows:

The line *DE* in Fig. 23 represents the rise of the follower, and for simplicity is shown as a straight line. For a uniform-pressure-angle cam curve it would in reality be a curve.

The graphs of the uniform-pressure-angle curve on the chart are laid out for a cam-lever length of $a = 1^1/_2$ in. On the assumption that $a = 1^1/_2$ in. the enlargement ratio of the cam curve on the chart to the actual cam curve 1:1, that is to say, the coordinates (polar or rectangular) must be multiplied by 1. If the enlargement or reduction ratio is other than unity, the curve must be correspondingly enlarged or reduced.

In order to copy the curve conveniently, the layout is made on transparent paper to the scale of the chart. The procedure illustrated in Fig. 24 is as follows:

1 Describe circles with radii $r_1 = {}^3/_8$ in., $r_2 = 1^1/_4$ in., and $a = 1^1/_2$ in.

2 Choose point D_1 on the circle whose radius is a which corresponds to point D on diagram (Fig. 23) and determines the zero value for the cam displacement angle α.

3 Determine point E_1 90 deg. from point D_1 and describe arc AB with center D_1 and arc AC with center E_1. The intersection points B and C are two points on the cam curve.

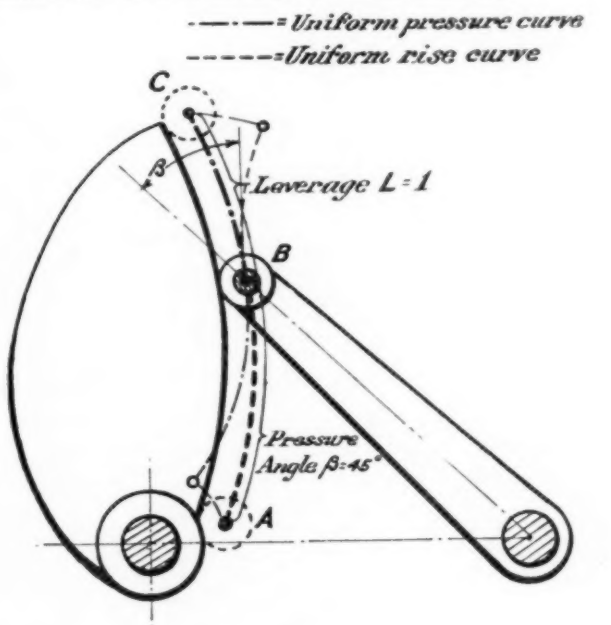


Fig. 25 Combined Uniform-Pressure-Angle and Uniform Rise Cam, Showing the Use of Chart I and Chart II

4 Place this layout on Chart II so that center A coincides with A_1 on the chart and rotate the layout around A_1 until a curve is found which passes through B and approaches C.

In this way it will be found that the pressure angle of the curve in the example is 50 deg.

The frictional loss of the cam mechanism corresponding to this pressure angle, is, according to Equation [9],

$$\eta = \frac{\tan\ (1.25\epsilon)}{\cot\ \beta} \times \frac{R_2}{R_1} = 0.16$$

or frictional loss $\eta = 16$ per cent.

Assuming that $\epsilon = 12°$ (for steel on steel), the frictional loss η may be plotted in terms of R_2/R_1 as shown at the bottom of both charts.

In cases where the leverage on the uniform-pressure-angle curve decreases undesirably, it is sometimes advisable to combine the uniform-pressure-angle curve with the uniform-rise curve.

Referring to Fig. 25, at point B, the uniform-pressure-angle curve AB reaches a leverage of $L = 1$, and from this point to the point C the curve is a uniform-rise curve.

Conclusion

Cams with uniform-pressure angles have been built in accordance with the foregoing methods and employed in light automatic machines with very satisfactory results. They have proved

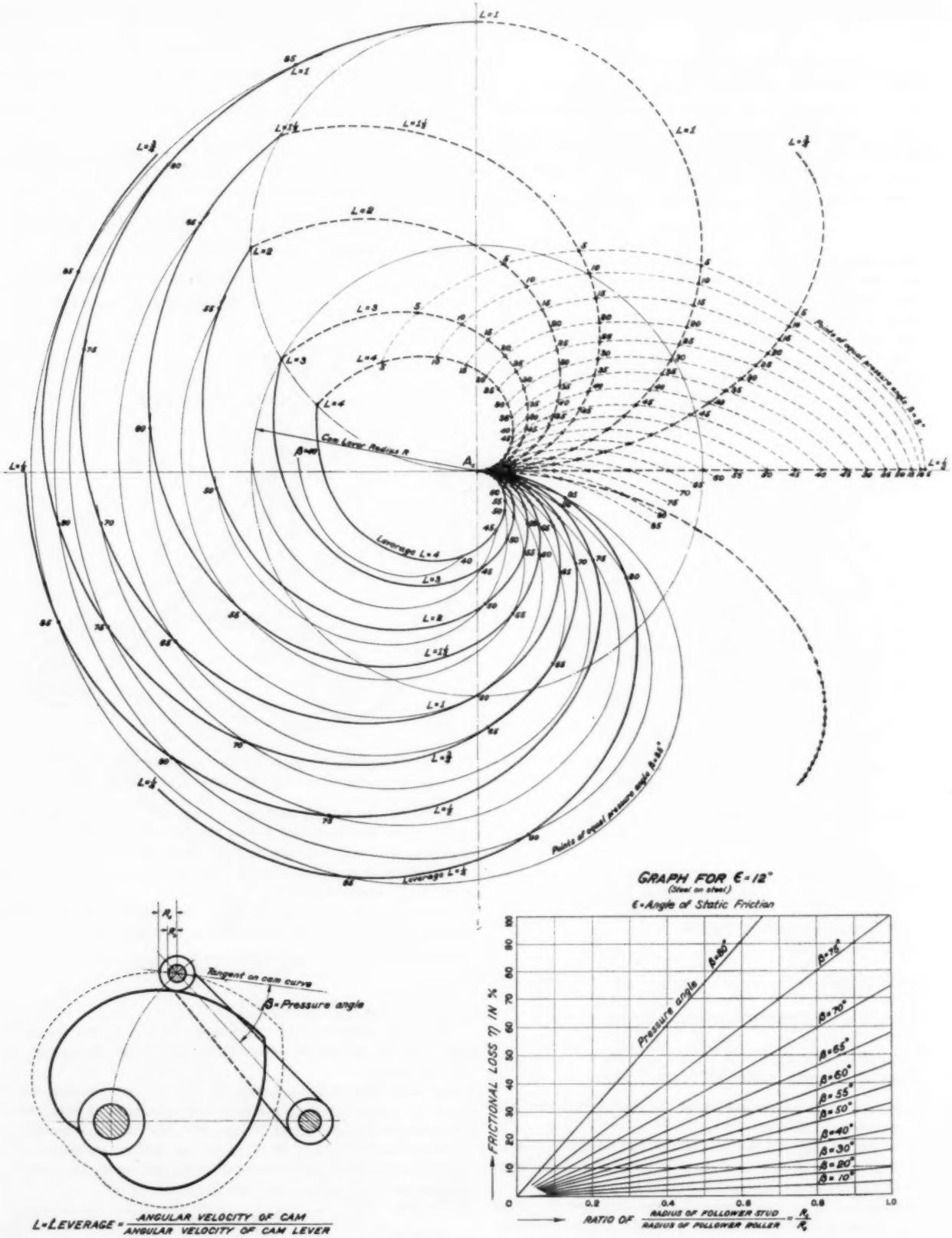
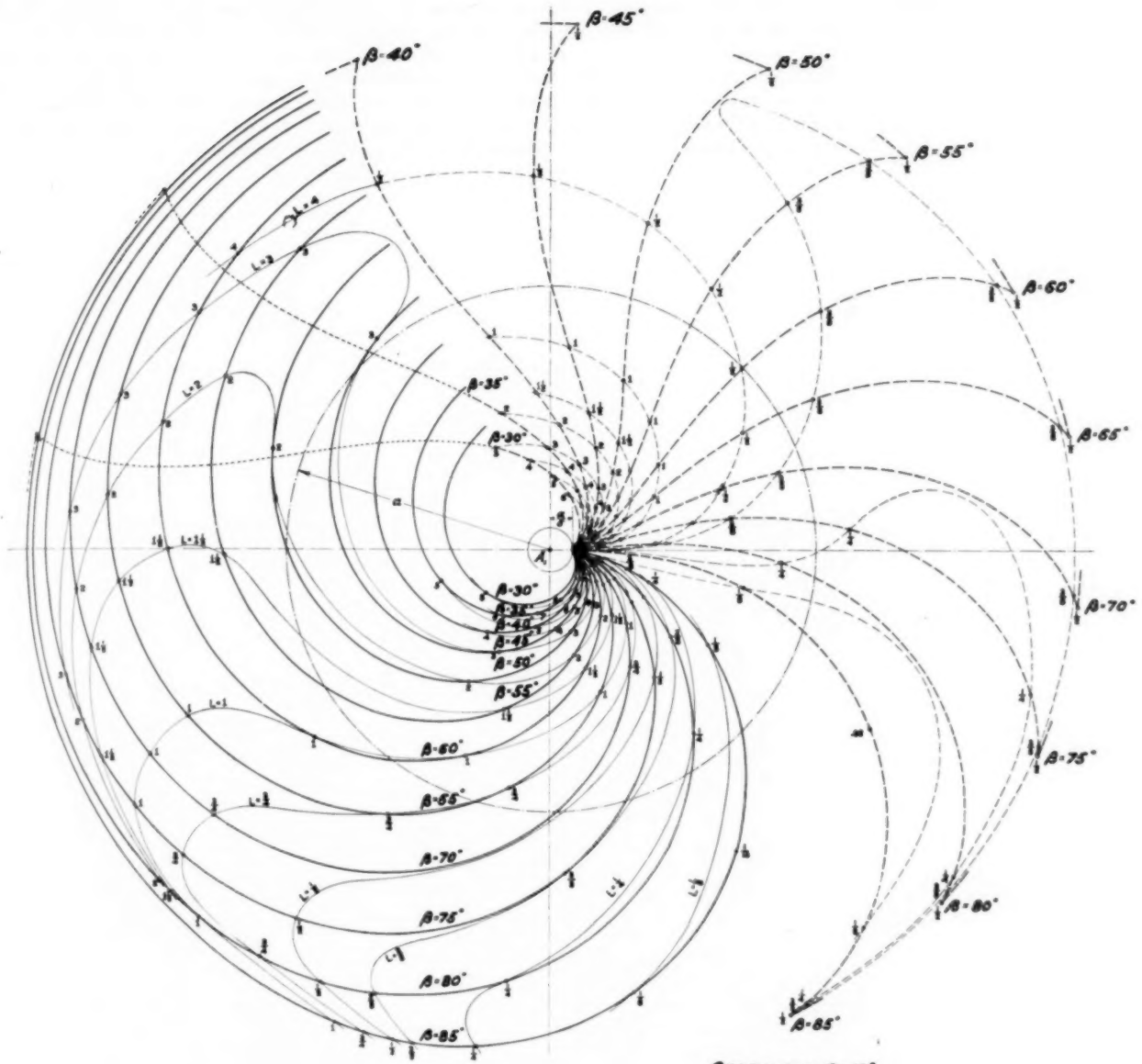
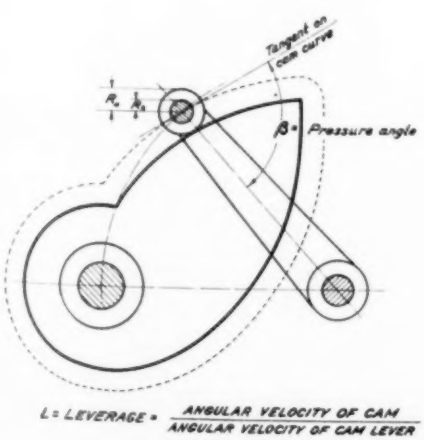


CHART I—UNIFORM-RISE CAM CURVE

NOTE: For cam and cam lever rotating in *same direction*, the heavy dash-lines are the cam curves; the fine dash-lines indicate points of equal pressure angle.

For cam and cam lever rotating in *opposite directions*, the heavy lines are the cam curves; the fine lines indicate points of equal pressure angle.





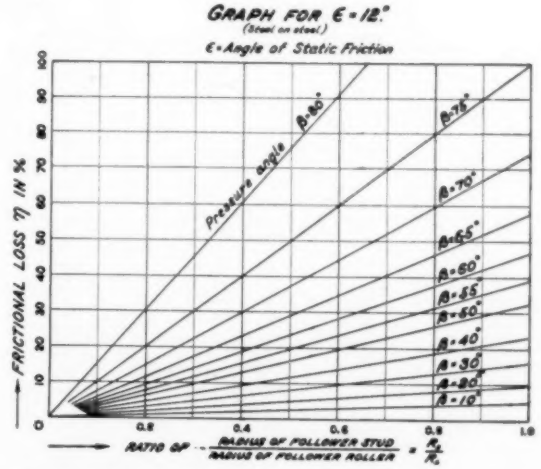


CHART II—UNIFORM-PRESSURE-ANGLE CAM CURVE

NOTE: For cam and cam lever rotating in *same direction*, the heavy dash-lines are the cam curves; the fine dash-lines indicate points of equal leverage.

For cam and cam lever rotating in *opposite directions*, the heavy lines are the cam curves; the fine lines indicate points of equal leverage.

to be of decided advantage where it has been necessary to construct a cam mechanism in a minimum of space and with a maximum of efficiency.

If no limitations of space were dictated in the design of cam structures, it would probably be best to employ the uniform-rise curve and make the cam diameter large enough to obtain a negligible pressure angle. However, in most cases it is necessary to make the parts as small and light as possible. On a small cam the base is always in very close proximity to the cam shaft, and the cam angle is very steep. It is then obvious that best results are obtained by a cam design wherein the cam stud, cam lever, etc., are under uniform stress and uniform wear throughout the cam cycle. A cam curve of such character is necessarily of the uniform-pressure-angle type.

APM-51-15

Friction of Journal Bearings as Influenced by Clearance and Length

By S. A. McKEE[1] and T. R. McKEE,[2] WASHINGTON, D. C.

An investigation at the Bureau of Standards is described, the object of which was to determine the effects upon the frictional resistance of small-bore full-journal bearings of changes in the length-diameter and clearance-diameter ratios. A journal-bearing friction machine provided a method for measuring the frictional resistance when operating under different conditions of load on the bearing, speed of the journal, and viscosity of the lubricant. By a suitable correlation of these factors a measure of the effects of changes of clearance and length with bearings of the same diameter was obtained. The results indicate that changes in the length-diameter ratio as well as in the clearance-diameter ratio have marked effects upon the frictional characteristics of journal bearings in normal operation. Suitable corrections to a theoretical equation for journal friction are derived from these experiments for bearings of all normal clearances and lengths. In the conclusion, further steps toward a more complete understanding of journal-bearing performance are indicated.

THE factors that determine the performance characteristics of journal bearings might conveniently be divided into two classes, operating factors and factors of construction. Among those classed as operating factors (i.e., those that are dependent upon the particular conditions of operation) are the load on the bearing, the speed of the journal, the temperature of operation, and the viscosity, oiliness,[3] and amount of the lubricant. Among the factors of construction (i.e., those that are built into the bearing itself) are (1) the absolute size as represented by the journal diameter; (2) L/D, the ratio of the length of the bearing to the diameter of the journal; (3) C/D, the ratio of the diametral clearance to the diameter of the journal; (4) the condition of the bearing surfaces, which is dependent upon the materials of which the journal and bearing are made, their smoothness, and their deviation from true cylindrical form; and (5) the means for applying or retaining the lubricant, such as oil holes, oil grooves, and relieved spaces.

The object of this study is to determine the effects of changes in two of the most important factors of construction in a normal bearing—namely, the L/D and C/D ratios—in the hope that the results may be of value in the formulation of rational methods for bearing design.

Method for Comparing Bearing Performance

The method chosen for showing the effect of changes in clearance and length was to measure the frictional resistance of a set of bearings having a given length and clearance under a wide range of operating conditions and to repeat the measurements with other sets of bearings having different lengths and clearances, the same journal being used in all the tests.

A convenient graphical method for evaluating the results obtained is to compare the respective coefficient-of-friction curves of bearings having different lengths and clearances, plotting the coefficient of friction f against the generalized operating variable $\mu n/p$ which denotes the product of the viscosity of the lubricant by the number of revolutions per unit time divided by the bearing pressure (load per unit projected area). The justification and advantages of the generalized variable in place of the separate operating factors have been previously discussed. The particular choice of units and notation employed are the same as were used in three previous investigations[4] and consist in expressing the viscosity in centipoises, the speed in revolutions per minute, and the bearing pressure in pounds per square inch, and in denoting the respective quantities by Z, N, and P when so expressed. The graphical method of this paper may therefore be very simply described as a comparison of the f-(ZN/P) curves for bearings of the same absolute size having widely different values of L/D and C/D.

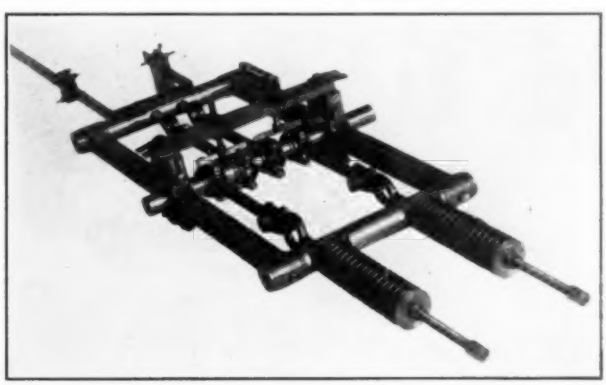


FIG. 1 FRICTION MACHINE, ESSENTIAL PARTS

The Friction Machine and Its Operation[5]

A photograph of the friction machine used in this and preceding work is shown in Fig. 1. The machine was originally designed to use four connecting rods of an automobile engine, with the babbitt-lined "big end" bearings serving as test bearings, mounted on a steel shaft set in a lathe. These rods were used in the tests where the L/D ratio was 1.00, but in all the others solid-bronze babbitt-lined bearings, clamped in special holders of the same general shape as the connecting rods, were used. A photograph of these bearings with a holder is shown in Fig. 2. These

[1] Assistant Mechanical Engineer, Bureau of Standards.

[2] Research Associate, Bureau of Standards (A.S.M.E. Special Research Committee on Lubrication).

[3] Two oils are said to differ in the property of oiliness if they possess the same viscosity at the temperature of the film and yet give different amounts of friction in the bearing; cf., Kingsbury, A., "A New Oil-Testing Machine and Some of Its Results," Trans. A.S.M.E., vol. 24 (1903), pp. 143–160; Report of Special Research Committee on Lubrication, A.S.M.E., *Mechanical Engineering*, vol. 41, June, 1919, p. 537; Herschel, W. H., "Viscosity and Friction," *Journal S.A.E.*, vol. 10 (1922), pp. 31–41.

Presented by the Research Special Committee at the Annual Meeting, New York, N. Y., December 3 to 7, 1928, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS. Published by permission of the Director of the Bureau of Standards of the U. S. Department of Commerce.

[4] McKee, S. A., "Effect of Kerosene on Oiliness of Lubricating Oils," *Journal S.A.E.*, vol. 19 (1926), pp. 356–360; "Performance of Journal Bearings When an Abrasive Is in the Lubricant," *Journal S.A.E.*, vol. 20 (1927), pp. 3–6; "Effect of Running in on Journal-Bearing Performance," *Mechanical Engineering*, vol. 49, December, 1927, pp. 1335–1340.

[5] For a more detailed description see papers by S. A. McKee (previously cited), *Journal S.A.E.*, vol. 20 (1927), pp. 3–6, and *Mechanical Engineering*, vol. 49, December, 1927, pp. 1335–1340.

four rods are fastened to a frame by means of eyebolts and clevises in such a manner that they lie in a horizontal plane with the two end bearings on the opposite side of the shaft from the two in the middle. The frame is suspended by two equalizers mounted on a cross-beam fastened to the frame just above the shaft. Load is applied to the bearings by compressing the calibrated coil springs mounted on the sliding eyebolts fitted to the end rods.

Oil is fed to the center of the unloaded sides of the bearings by lines of rubber tubing from a glass bottle. The oil is forced from the bottle by the house air pressure of about 7 cm. of mercury.

The working temperature of the bearings is measured by copper-constantan thermocouples fastened to the loaded sides of the second and fourth bearings. When using the regular automotive connecting rods where the bearing shell was thin, the thermocouples were soldered in the center of the outside of the bearing cap. When using the special bearings, however, con-

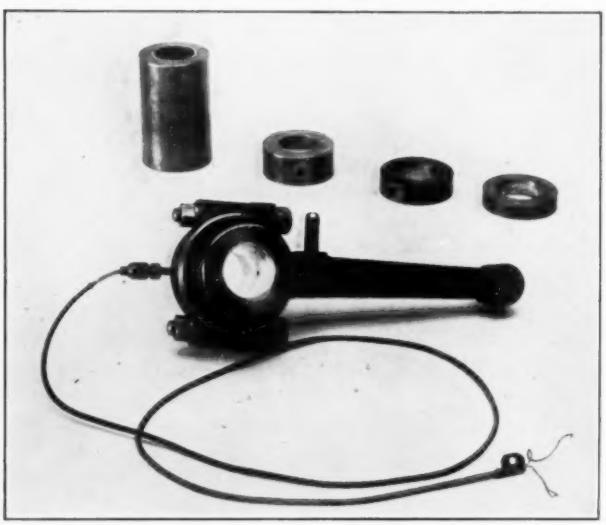


FIG. 2 SPECIAL BEARINGS AND HOLDER, WITH THERMOCOUPLE AND CONNECTOR

nectors were made, which, when screwed in the longitudinal center of the loaded side of the bearing shell, brought the thermo-junction to within about $^1/_{16}$ in. of the babbitt lining. These connectors were found to be advantageous in another respect in that they facilitated the changing of a couple from one bearing to another. It was found advantageous also where frequent bearing changes were necessary, as in these tests, to protect the wires from becoming kinked and broken by winding each couple with spring-steel wire. A couple with its armor and connector is also shown in Fig. 2.

Temperature control is provided by circulating water through the hollow test shaft by means of a small motor-driven gear pump connected to a 20-gal. supply tank.

In making a test run oil is fed to the bearings and the machine brought to steady conditions under a predetermined load and speed to give the desired value of ZN/P.

DESCRIPTION OF THE BEARINGS AND JOURNAL TESTED

In order to conserve both material and labor and yet cover the full range of C/D and L/D ratios desired, the same test shaft was used for all the tests, and but one set of bearings for all the tests with a given L/D ratio. The procedure was to run the tests for a given L/D ratio with successively increasing clearances, starting with the smallest clearance desired and reaming the bearings out to the next larger size after each test was completed.

Tests were made upon bearings having the following L/D and C/D ratios:

L/D = 2.80, with C/D = 0.00075, 0.00100, 0.00200, and 0.00455

L/D = 1.00, with C/D = 0.00046, 0.00065, 0.00100, 0.00200, 0.00400, and 0.01000

L/D = 0.75, with C/D = 0.00043, 0.00093, 0.00200, and 0.00400

L/D = 0.50, with C/D = 0.00048, 0.00100, and 0.00400

L/D = 0.25, with C/D = 0.00038, 0.00091, and 0.00191, and 0.00400.

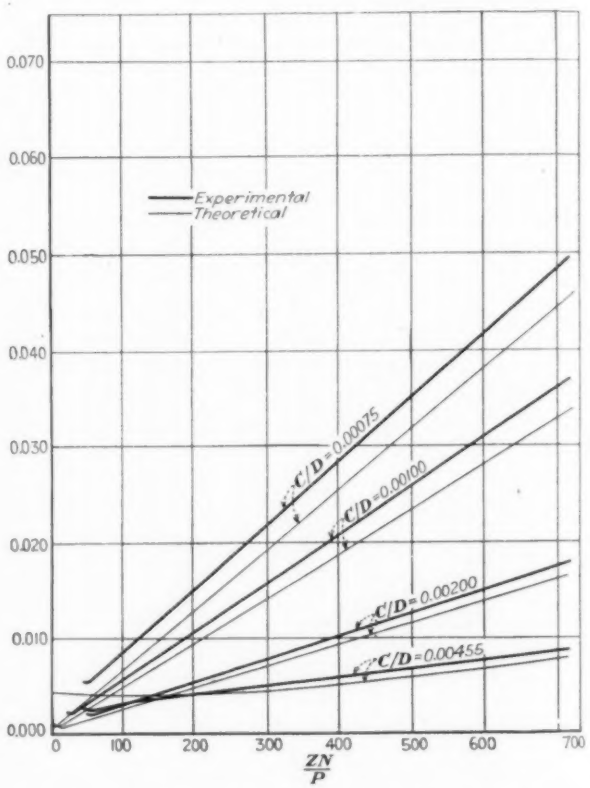


FIG. 3 JOURNAL-FRICTION CURVES FOR VARIOUS C/D RATIOS; L/D = 2.80

For the set with the L/D ratio of 1.00, the automotive connecting rods were used. These were specially prepared with oil grooves omitted and the babbitt lining cast in one piece so that they formed solid bearings. The approximate composition of the babbitt-metal lining was 85 per cent tin, 7.5 per cent copper, and 7.5 per cent antimony. The babbitt-metal lining in the special solid bearings used in all the other tests was of the same composition.

The ends of all the bearings were faced off square and their lengths were correct to within ±0.001 in. to give the required L/D ratio.

The bearing surfaces were finished with a special type of commercial reamer which provided a smooth, accurate bearing surface. The proper diameter for each bearing was based upon the average diameter of that part of the shaft at which it was located, and the diameter of no single bearing was allowed to deviate more than ±0.0002 in. from that required to give the average C/D ratio of the whole set of four. The measurements of diameter were made with a special type of commercial internal micrometer and were accurate to within ±0.0001 in.

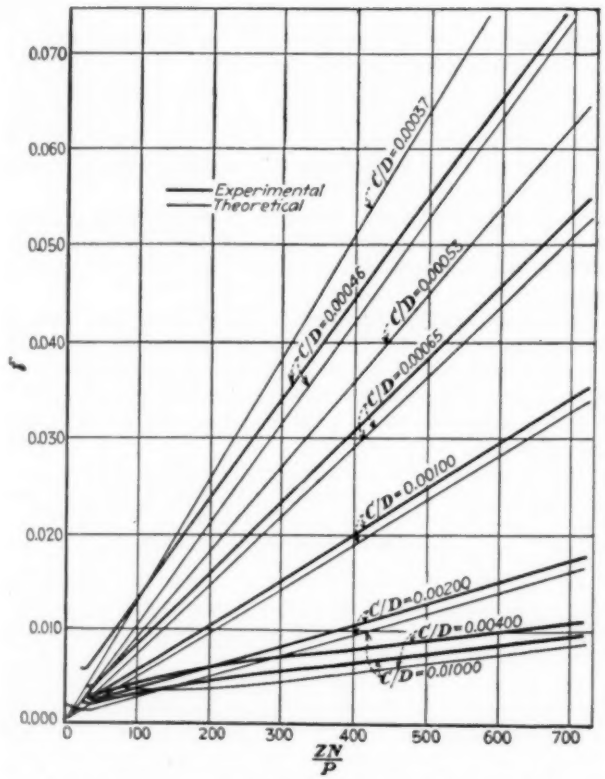


FIG. 4 JOURNAL-FRICTION CURVES FOR VARIOUS C/D RATIOS; $L/D = 1.00$

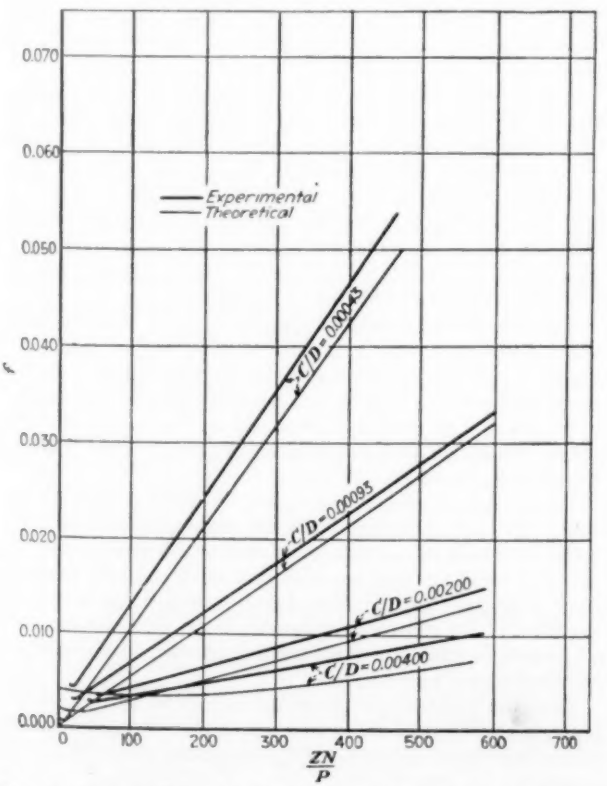


FIG. 5 JOURNAL-FRICTION CURVES FOR VARIOUS C/D RATIOS; $L/D = 0.75$

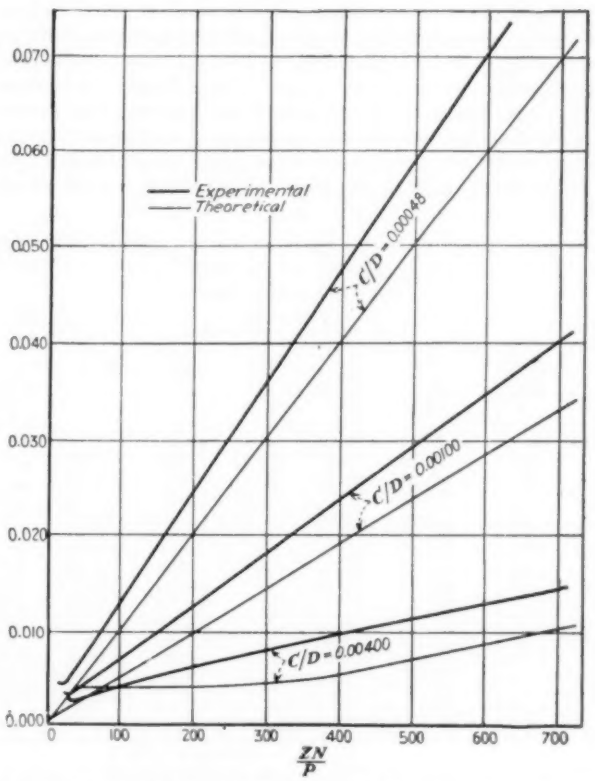


FIG. 6 JOURNAL-FRICTION CURVES FOR VARIOUS C/D RATIOS; $L/D = 0.50$

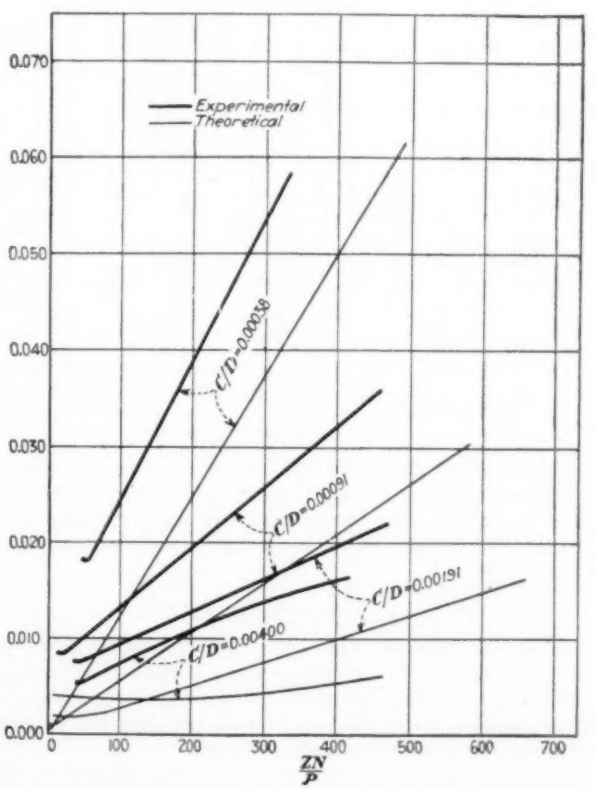


FIG. 7 JOURNAL-FRICTION CURVES FOR VARIOUS C/D RATIOS; $L/D = 0.25$

The test shaft used in these tests was made of a high carbon-tungsten tool steel, its heat treatment being: heated at 1500 deg. fahr., held for 30 min., quenched in oil, reheated at 850 deg. fahr. for 45 min., and air-cooled. It had a Brinell hardness of about 179. After treatment it was ground and lapped. The average diameter at the journals was dependent upon the location and length of the bearings, the maximum value being 1.25054 in. and the minimum 1.25043 in., these values being correct within ±0.00005 in.

TABLE 1 TYPICAL DATA FROM A JOURNAL-BEARING TEST

Bearing Set No. 2, L/D = 1.00
Test No. 3, C/D = 0.00100

Observation No.	Average speed of journal, N r.p.m.	Total load per bearing, PLD, lb.	Temperature of bearings, deg. cent.	Viscosity of lubricant, Z, centipoises	Frictional torque per bearing, lb-in.	Coefficient of friction, f	Value of ZN/P
1	548.0	51.30	27.05	37.8	1.002	0.0312	630.0
2	420.5	51.30	26.40	39.0	0.770	0.0240	498.0
3	307.0	51.30	26.05	39.7	0.577	0.0180	371.0
4	234.0	51.30	25.65	40.3	0.456	0.0142	286.0
5	168.0	51.30	25.30	41.1	0.341	0.0106	210.0
6	110.0	51.30	25.20	41.5	0.241	0.0075	139.5
7	253.0	100.70	25.60	40.5	0.497	0.0079	159.0
8	193.0	100.70	25.60	40.5	0.392	0.0062	121.0
9	156.0	100.70	25.55	40.7	0.333	0.0053	98.5
10	110.5	100.70	25.50	40.8	0.257	0.0041	70.0
11	85.5	100.70	24.95	41.9	0.222	0.0036	55.8
12	62.5	100.70	25.00	41.8	0.187	0.0029	40.7
13	46.0	100.70	25.05	41.7	0.163	0.0026	30.0
14	36.0	100.70	25.00	41.8	0.163	0.0026	23.6
15	28.0	100.70	25.00	41.8	0.187	0.0029	18.3
16	20.5	100.70	24.95	41.9	0.368	0.0058	13.4
17	17.5	100.70	24.95	41.9	0.561	0.0089	11.5
18	15.0	100.70	24.95	41.9	0.811	0.0128	9.7

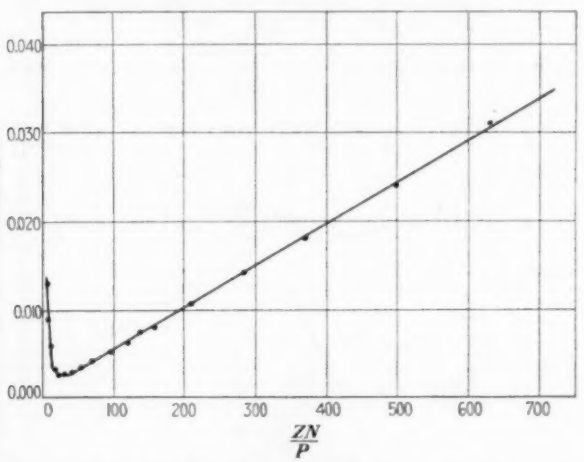


FIG. 8 TYPICAL PLOT OF JOURNAL-FRICTION OBSERVATIONS; L/D = 1.00, C/D = 0.00100

THE LUBRICANT USED

The lubricant used in all the tests was a mineral spindle oil having a viscosity of 126 sec. Saybolt Universal at 100 deg. fahr. The average running temperature was about 77 deg. fahr., the absolute viscosity of the oil at this temperature being about 42 centipoises.

PROCEDURE IN TESTING

All of the bearings were tested without being run in. The runs were made in the decreasing order of ZN/P. The first point of a run was obtained by using a light load combined with a high speed, and the succeeding points were obtained either by decreasing the speed or increasing the load.

The total load per bearing ranged from 25 to 200 lb., and the speed range was from 15 to 700 r.p.m. A typical data sheet is shown in Table 1.

GRAPHICAL PRESENTATION OF RESULTS

The results of these tests plotted in the form of f-(ZN/P) curves are shown by the heavy lines in Figs. 3, 4, 5, 6, and 7. Each figure contains curves for bearings of the same length, but with different clearances. Curves obtained from theoretical equations,[6] in which longitudinal oil flow and end effects are neglected, for the corresponding C/D ratios are shown by the light lines in these figures.

Intercomparison of the curves on a single figure indicates the effect of changes in the C/D ratio and shows the amount that the practical bearing deviates from the theoretical for any given change.

Intercomparison of the figures for different L/D ratios with special attention to the curves having comparable C/D ratios indicates the degree of departure of the experimental from the theoretical values of f which results from different values of L/D; i.e., from differences in the longitudinal oil flow which depends upon this ratio.

Fig. 8 is given as a typical example showing the magnitude of the deviations of the observed points from a smooth curve.

GENERAL DISCUSSION OF RESULTS

(*a*) *Effects at High Values of* ZN/P. As a matter of convenience, the discussion of the effects shown by the curves is divided into two parts, the first being a consideration of the results at the high values of ZN/P.

It will be noted immediately that, in general, the experimental curves for all the tests are more comparable to their corresponding theoretical curves at the higher values of ZN/P than they are at the lower values. This is consistent with the fact that at the higher values of ZN/P the journal runs more nearly concentric with the bearings. Thus there is a greater likelihood that under these conditions the friction of a given journal and bearing is dependent entirely upon the viscous shear of the lubricant, which is a condition upon which the theoretical equations depend.

In considering the effect of the various C/D ratios, it is seen that in general the experimental curves are higher than the theoretical curves for the corresponding C/D ratio, the amount of the deviation being dependent upon both C/D and L/D. The one exception, the curve for C/D = 0.01000 with L/D = 1.00, shown in Fig. 4, can be attributed to the possibility that this clearance was so large that the lubricant did not entirely fill the space between journal and bearing and the resulting action was like that of a partial rather than a full bearing.

When the effect of length is considered, it will be noted, where direct comparison with a given C/D ratio can be made, that except in a few cases there is a general tendency for a progressive departure from the theoretical curves as the length of the bearing is decreased. An obvious explanation for this is that the effect of end-leakage becomes proportionately greater as the length of the bearing decreases.

The exceptions are the two curves for bearings with L/D = 2.80 and C/D = 0.00075 and 0.00100, respectively. Effects that might have had some influence in causing this departure of the performance of the long bearings with small clearance from the general trend are that the effect of shaft deflection may be greater, the effect of the relative irregularities of the surfaces taken as a whole may be greater, and the relative accuracy of the measurements of diameter may be less.

That the effect of the irregularities of the surfaces of the journals and bearings taken as a whole may be greater for the longer bearings is brought out by comparing the C/D ratio of

[6] Sommerfeld, A., "Zur Hydrodynamischen Theorie der Schmiermittelreibung," *Zeit. für Math. u. Phys.*, vol. 50 (1904), pp. 97–155.

the upper curve in Fig. 3 to that of the upper curve in Fig. 7. These C/D ratios represent approximately the condition of smallest clearance with which the bearings of these respective lengths could operate without having an initial binding effect due to the rubbing together of the high spots on the journals and bearings. In other words, at clearances smaller than are represented approximately by these C/D ratios, the friction of the bearings of these respective L/D ratios at high values of ZN/P is not dependent upon the viscous shear of the lubricant alone, but also upon the rubbing together of the high spots.

Too much weight should not be given the curves for the bearings with very small clearances, since the effect of a given error in measuring the diameters becomes proportionately greater as the clearance is decreased. As an illustration of this effect, the theoretical curves for bearings with C/D ratios of 0.00037 and 0.00053 are plotted in Fig. 4, these being the approximate C/D ratios for bearings having clearances only 0.0001 in. less and 0.0001 in. greater than those with the C/D ratio of 0.00046.

(*b*) *Effects at Low Values of* ZN/P. At the lower values of ZN/P the important question to consider is the effect of changes in the C/D and L/D ratios upon the location of the point of minimum friction, since this point can be considered as a basis for the determination of the factor of safety[7] under which a bearing is operating.

It will be seen that the locations of the points of minimum friction in the experimental curves do not have the same definite relation to the value of C/D as is shown by the minimum points of the theoretical curves. In fact, the experimental points of minimum friction lie so much at random that about all that can be determined concerning them is that they lie at or below $ZN/P = 50$ and that for a given L/D ratio the minimum coefficient of friction is highest for bearings with the smallest clearance.

The large deviation between the experimental and theoretical curves near the the point of minimum friction is emphasized in the Appendix, wherein the curves for some of the bearings are plotted on a chart suggested by Howarth.[8]

That there should be no very definite correlation between the experimental and theoretical curves near the point of minimum friction is easily understood when it is considered that the theoretical curves are based upon the assumption that the friction of the bearing depends on the viscous shear of the lubricant alone and that there are no end effects. In the actual bearings, especially at the larger clearances, it is reasonable to expect that the lubricant does not entirely fill the clearance space, and previous experiments[9] have shown that the location of the point of minimum friction is dependent also upon other factors such as oiliness and the condition of the bearing surfaces.

An Equation Representing the Friction of Small-Bore Journal Bearings

The results at the higher values of ZN/P are so consistent that it appears worth while to represent them by a simple equation whereby the frictional torque of a given bearing may be estimated for most normal operating conditions.

The results shown by Fig. 4 indicate that for bearings having an L/D ratio of 1 or thereabouts, the Sommerfeld[10] or Harrison[11] equations will give results accurate enough for practical purposes where the value of ZN/P is greater than 100 and where the coefficient of friction is greater than 0.01. Thus the charts based upon Harrison's equation for bearings of various nominal C/D ratios in Howarth's graphical analysis[12] are suitable for bearings of this type. These equations, however, do not take end effects into account and thus can be considered fair approximations only where L/D is greater than about 0.75.

In developing corrections for various L/D ratios Petroff's[13] equation, owing to its simplicity, appears to be most suitable to use as a basis. While this equation is based upon the assumption that the journal is at all times concentric with the bearing, the deviations between it and the more rigorous Sommerfeld or Harrison equations are negligible at values of ZN/P above 100 and values of f above 0.01.

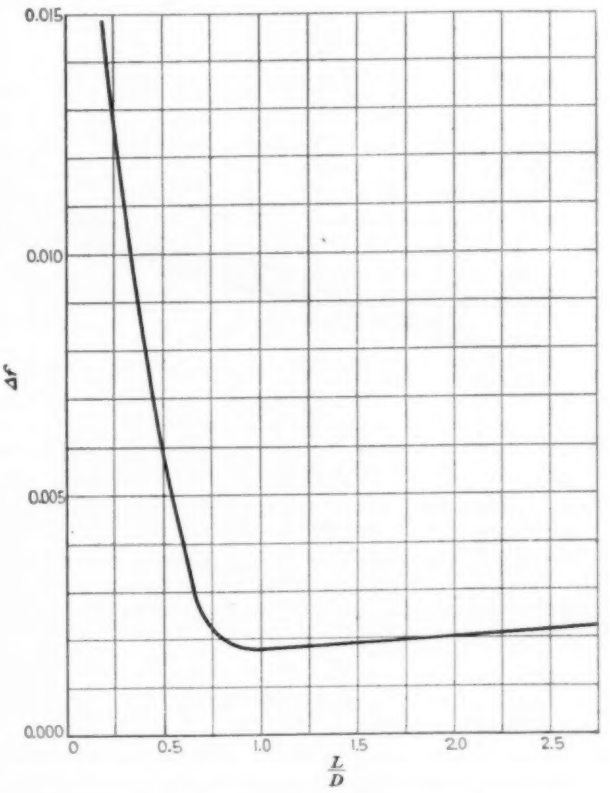


Fig. 9 Correction to Petroff's Equation for Various L/D Ratios

This equation when expressed in the units employed in this paper becomes

$$f_P = 473\left(\frac{ZN}{P}\right)\left(\frac{D}{C}\right)10^{-10} \quad \ldots\ldots\ldots\ldots [1]$$

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[7] See paper by S. A. McKee (previously cited), *Mechanical Engineering*, vol. 49, December, 1927, pp. 1335–1340.

[8] See H. A. S. Howarth's discussion of paper on "The Effect of Running-in on Journal Bearing Performance," by S. A. McKee, *Mechanical Engineering*, vol. 50, July, 1928, pp. 528–533.

[9] Barnard, D. P., 4th, Meyers, H. M., and Forrest, H. P., "The Effect of Oiliness on the Behavior of Journal Bearings," *Industrial and Engineering Chemistry*, vol. 16 (1924), pp. 347–350. McKee, S. A. (two papers previously cited), *Journal S.A.E.*, vol. 19 (1926), pp. 356–360; *Mechanical Engineering*, vol. 49, December, 1927, pp. 1335–1340.

[10] See paper (previously cited) by A. Sommerfeld, *Zeit. für Math. u. Phys.*, vol. 50 (1904), pp. 97–155.

[11] Harrison, W. J., "Hydrodynamical Theory of Lubrication," Trans. Cambridge Philosophical Society, vol. XXII (1913), pp. 39–54.

[12] Howarth, H. A. S., "A Graphical Study of Journal Lubrication," Trans. A.S.M.E., vol. 45 (1923), pp. 421–448; Part II, vol. 46 (1924), pp. 809–832; Part III, vol. 47 (1925), pp. 109–114; reprinted by The American Society of Mechanical Engineers, 29 West 39th St., New York, N. Y.

[13] "Neue Theorie der Reibung," St. Petersb. Acad., 1883; German trans., Hamburg, L. Voss, 1887; French Trans., Rev. de Mécanique, vol. 7 (1900), pp. 571–602.

From the nature of the equation it will be seen that the f_P versus (ZN/P) curves for various C/D ratios are straight lines passing through the origin and that the slope of the line depends upon the C/D ratio.

The equation chosen to correct for the L/D ratio is

$$f = f_P + \Delta f \quad \ldots\ldots [2]$$

where f = coefficient of journal friction for an actual bearing
f_P = coefficient of friction as determined by Petroff's equation
Δf = L/D correction.

This assumes that all the experimental curves for a given L/D ratio are parallel to and equidistant from the Petroff curves for the same C/D ratios. This of course is not exact, but the values obtained are a fair approximation throughout the whole range covered.

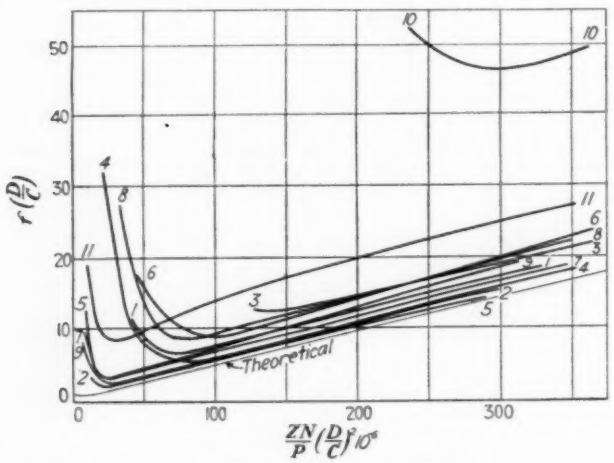


FIG. 10 FRICTION CURVES PLOTTED ON HOWARTH'S CHART; ALL L/D RATIOS; SMALL C/D RATIOS

The curve in Fig. 9 shows the average value of Δf for the various L/D ratios. The maximum error in using this method is about 16 per cent, the average being about 4 per cent. Thus the equation

$$f = 473\left(\frac{ZN}{P}\right)\left(\frac{D}{C}\right)10^{-10} + \Delta f \quad \ldots\ldots [3]$$

should be reasonably accurate for the practical determination of friction losses in small-bore full journal bearings provided ZN/P is greater than 100 and f greater than 0.01. It is of course understood that the speed and load are fairly constant and that an adequate supply of oil is maintained at a relatively low pressure and that there are no oil grooves in such a location on the bearing surface as to materially affect the pressure distribution in the film.

CONCLUSION

The limit of the scope of this investigation is to provide a reasonably accurate experimental indication of the behavior of small-bore full-journal bearings of various clearances and lengths, under normal working conditions.

While the results of the tests at the higher values of ZN/P are fairly consistent with theoretical equations based upon the viscous shear of the lubricant, it would not be justifiable to predict that these results would be a reliable indication of the characteristics of large bearings operating at high speeds and high loads, since it is reasonable to expect that under the latter conditions the effects of such factors as journal deflection, distortion caused by uneven heating, and changes in viscosity due to temperature gradients in the oil film may be of considerable magnitude.

The inconsistencies shown by the results at the lower values of ZN/P are an indication that considerable caution should be used if the carrying capacity of a particular bearing is determined by any method based upon the theoretical value of the minimum thickness of the oil film.

Further steps toward a better understanding of the performance of full-journal bearings suggested by these tests are (1) to make similar tests of the effect of changes of clearance and length using larger diameter bearings under heavier loads and at higher speeds and (2) to determine the carrying capacity of bearings by a study of the effect of changes in bearing metal, lubricant, and over-all dimensions upon the point of minimum friction of bearings finished by the usual commercial methods.

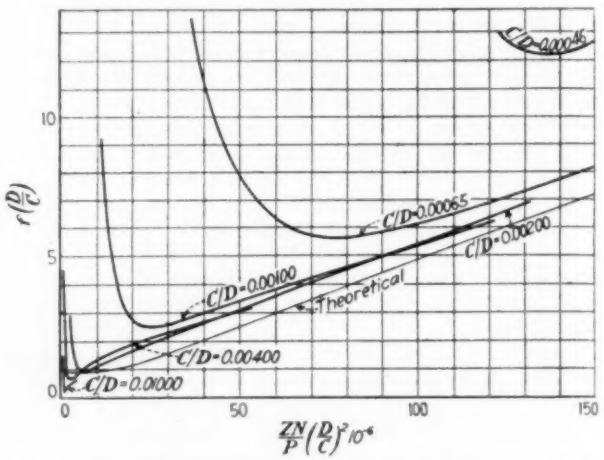


FIG. 11 FRICTION CURVES PLOTTED ON HOWARTH'S CHART; L/D = 1.00; ALL C/D RATIOS

APPENDIX—CURVES PLOTTED ON HOWARTH'S CHART

By Howarth's method of plotting, which uses $f\,(D/C)$ as ordinates and $(ZN/P)\,(D/C)^2$ as abscissas, a single theoretical curve can be drawn to represent bearings of all C/D ratios. In Fig. 10 this theoretical curve is represented by the light line, while the heavy lines numbered from 1 to 11 are the curves for bearings with L/D = 2.80, C/D = 0.00075 and 0.00100; L/D = 1.00, C/D = 0.00046, 0.00065, and 0.00100; L/D = 0.75, C/D = 0.00043 and 0.00093; L/D = 0.50, C/D = 0.00048 and 0.00100; and L/D = 0.25, C/D = 0.00038 and 0.00091, respectively. To avoid crowding the diagram the curves for the bearings with large clearances have been omitted, since all are closely bunched near the origin. Intercomparison of curves having the same L/D ratio but different C/D values (i.e., curves 1 and 2; 3, 4, and 5; 6 and 7; 8 and 9; 10 and 11) shows that there is a tendency for the curves to move progressively upward and to the right as the clearance is decreased. This is shown in Fig. 11, where the curves for the various C/D ratios with L/D = 1.00 are plotted. Upon intercomparison of curves having approximately the same C/D ratio but with different L/D values (i.e., curves 2, 5, 7, 9, and 11) a less pronounced tendency is shown for the curves for the shorter bearings to move upward. These effects indicate that some of the factors treated as negligible in the theoretical equation are having a pronounced influence upon the friction of the bearings.

Discussion

H. WEISS.[14] I should like to ask two questions which an examination of the results suggested to me: (1) In examining the curves representing the results of experiments one is impressed

[14] Assistant Director, L'Ecole Nationale Superieure du Petrole, University of Strassbourg, Strassbourg, Germany.

at seeing that the gross values of ZN/P in the theoretical and experimental curves are two straight lines coming less closer together as the ratio L/D decreases. In particular for L/D = 0.25, these straight lines are nearly parallel, as if there were a systematic wear, a constant friction which has to be added to the friction predetermined by the theory. Do the authors believe that this form of wear is due to disturbances brought by the edges of the bearings which become gradually preponderant, in view of the fact that the formula is derived for a bearing of infinite magnitude, or do they, on the contrary, believe that with errors of the geometric order, such as the size of the bearing, the centering becomes more and more important?

(2) Modern theories on lubrication seem to gain more and more in precision through a separation of two distinct sets of conditions: (*a*) Perfect lubrication, which is of dominating influence at high speeds and at low pressures. The viscosity or the internal properties of the liquid play a preponderant part. The equations of Petroff and the more exact ones of Sommerfeld are applicable to bearings of large size. (*b*) The so-called improved lubrication which is preponderant at low speeds and high pressures. The surface phenomena at the surface of separation between the lubrication of the metal (oiliness) are then of primary importance.

The preceding formulas are no more applicable. We ar edealing here with the friction of solid against solid and it is the Coulomb laws that govern. There is naturally a gradual passage from one process of lubrication to the other as the speed decreases and the pressure increases. I see that in the experiments of Mr. McKee the speed of the shaft was decreased to 15 r.p.m. for a load of 100 lb. I notice that it is in the case of small values of ZN/P that the most important relative deviations are found. In that case, are we not then working within the region of improved lubrication? Experiments of that kind would be suitable to discover under what conditions the influence of surface phenomena shows itself in an appreciable manner.

A. R. Roberts,[15] L. R. McCurdy.[16] The value of this paper would be increased greatly if space could be found for a more complete tabulation of results. In most of the diagrams, an important part of the field, containing probably a majority of the observations, is crowded into a corner where the relative accuracy of scaling is low. With tabulated data, other means of presenting the results than those adopted in the paper are open to any one interested.

If, for example, the data in Table 1 are plotted to a larger scale or to log scales, it will be found that while the points may be fitted fairly well by a single curve they may be fitted much better by two distinct curves, one for each test load, suggesting a "load effect." This feature is noted merely as an example of the possibilities of further analysis of the results afforded by tabulated data.

The factor Δf is found to be independent of C/D; that is, as shown by Fig. 11, the departure of the experimental curves from the reference curve is greatest with the fine clearance. This result is probably not due to end-leakage so much as it is to other factors such as relative smoothness of the surfaces. When "run-in," the friction curves will be modified not only near the point of minimum friction but throughout most of the range.

H. Bateman.[17] These researches go a long way toward the elucidation of a subject in which further mathematical progress is difficult. Even when the question of the effect of the L/D and C/D ratios is treated from the simple viewpoint of Petroff, in which the journal and bearing are regarded as concentric, it presents great difficulties.

Some idea of the effect of a variation of the C/L ratio may be obtained by neglecting curvature. The problem is then equivalent to that of an infinite strip of breadth L, moving with constant velocity V, parallel to a stationary infinite plane at distance C from it. This problem, however, is equivalent to the electrostatic problem of a parallel plate condenser in which the plates are at a distance $2C$ apart, are of finite breadth L, but of infinite length. This problem has been solved by Michell, Bromwich and Love with the aid of elliptic functions.[18]

The charge on one plate of the condenser, which corresponds to the viscous drag of the hydrodynamical problem, is proportional, when C/L is small, to the quantity $VL(1 + \epsilon)/C$ where

$$\epsilon = \frac{2C}{\pi L}\left(1 + \log \frac{\pi L}{C}\right)$$

and is thus greater than the value VL/C obtained by neglecting the end effect. When L/C = 200 the quantity ϵ is of magnitude 0.0237. In the experiments made by the authors of this paper L/C has values between 62.5 and 1867. Some idea of the effect of curvature might, perhaps, be obtained by finding the electrostatic capacity for different values of the ratios L/D and C/D of a condenser whose plates consist of the journal and bearing. Though there is no true analogy between the electrical problem and the hydrodynamical problem, the end effect will probably be of the same order of magnitude in the two cases.

E. O. Waters.[19] It seems that three outstanding conclusions may be drawn from this paper. First, and most obvious, is the fact that decreasing L/D ratios give higher coefficients of friction, as would naturally be the case on account of end leakage. But apparently this is only true for L/D between 1 and 0. The increase in f for values of L/D greater than 1 does not seem logical, and certainly does not tally with Michell's theoretical results for flat bearings, in which f increases continuously from a low value for an infinitely wide bearing to a value more than twice as great for a square bearing and a value about twenty-seven times as great for a bearing of width equal to one-third the length.

The writer agrees with the authors that this peculiar behavior is probably due to shaft deflection, surface irregularities that eluded measurement, etc.—in other words, causes that are in the nature of experimental errors rather than the consequence of high L/D per se. Such being the case, the question may be raised as to whether this phenomenon can be legitimately recognized, when establishing values for Δf. There is danger that the curve in Fig. 9, or at least the upward-sloping part of it, may be valid only for bearings of the same size and tested under exactly the same conditions as those upon which the authors experimented.

In the second place, it is noteworthy that the experimental and theoretical lines agree very closely, in the range from L/D = 0.75 to L/D = 1.00. Moreover, as stated by the authors, Petroff's equation is applicable except in the region of minimum friction, and gives a simple basis for computing the friction of a bearing of infinite length, for any values of Z, N, P, and D/C. For bearings of finite length, the additive correction indicated in Equation [2] would seem to be accurate enough, although it has no analytical background. Exactly the same result is ob-

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[16] Lecturer, Department of Mechanical Engineering, McGill University, Montreal, Canada.

California Institute of Technology, Pasadena, Calif.

[18] The most complete treatment is that of A. E. H. Love in *Proc. London Math. Soc.* [2] 22 (1924), 337.

[19] Assoc. Prof. M.E., Yale University, New Haven, Conn.

tainable graphically on Howarth's chart of generalized coordinates, Fig. 11, in which $X = ZN/P(D/C)^2\ 10^{-6}$ and $Y = f(D/C)$, and the basic relation between X and Y can be approximated very closely by a straight line, namely, the asymptote to the theoretical curve.[20] The difference between $f(D/C)$ given by the asymptote and the curve is less than 6 per cent for all values of $ZN/P(D/C)^2\ 10^{-6}$ greater than 5.

As soon as Howarth's coordinates are related by a straight line passing through the origin, D/C may be cancelled out of both X and Y, giving what would seem to be the most simple and practical means for determining f for general design and operating purposes.

Thirdly, it is clear that the experimental results are at variance with the hydrodynamic theory under two distinct conditions: very small values of (ZN/P) (D/C) for all values of C/D tested, and small values of C/D for all values of (ZN/P) (D/C). In both cases, the tests show a marked increase in f above that computed by theory, just as was apparent from S. A. McKee's investigation of the effect of running in on journal bearing performance.[21] The writer believes that this may be explained as follows: In the second case, the minimum film thickness is of the same order of magnitude as the permissible variations and combined errors in measurement of the journal and bearing diameters, and this is true even if the eccentricity is practically zero. The shape of the cross-section of the film space, which when developed is theoretically bounded by a straight line on one side and a cosine curve on the other,[22] determines the relation between $f(D/C)$ and (ZN/P) $(D/C)^2$. When these bounding surfaces are subject to variations in form, that are comparable in magnitude with the average distance between them, the shape of the film space cross-section is profoundly altered and it is to be expected that the theory built up on the simple shape will fail. This explanation does not assume any metal-to-metal contact or breakdown of the oil film.

In the first case, the same effects are manifest, but owing to the high eccentricity of the journal position the cosine curve has a larger relative amplitude and variations in this curve will only affect the running conditions in the short region, where the curve is in close proximity to the developed journal surface. It is therefore to be expected that there will be a much more abrupt transition from the condition of fluid friction to that of solid or semi-solid friction.

It would be interesting if a law could be determined relating ZN/P and D/C with the point of minimum friction, but it is doubtful if this can ever be done, as the relation is of necessity extremely sensitive to conditions of bearing surface that are practically impossible to measure. The present tests show the inadequacy of the hydrodynamic theory in the region of low f, but do not establish anything definite to take its place.

H. A. S. Howarth.[23] This paper represents a very commendable effort to develop rules for taking account of the influence of side leakage on the friction of full bearings. One is apt to be misled, however, by the apparent close agreement between theoretical friction and actual friction. The authors apparently have this thought in mind when they state that errors of 0.00005 in. in the measurement of their diameters would have a large influence upon the position of the theoretical reference curves, shown by light lines in all the figures except Figs. 8 and 9. Such an error would be of more importance for the small clearances than for the large ones.

Although the authors have compared their test friction coefficients with those to be expected in full bearings, they have given us no assurance that their test films were complete. It is quite reasonable to suppose, therefore, that for the high speeds, small clearances, and large L/D ratio, the films might be fairly complete. For small values of L/D, large clearances, and low speeds, it is quite likely that the useful portions of the films, i.e., those in which substantial pressures were present, extended over small angles, possibly not more than 30 deg. If such is the case, it would have been better to have determined the useful length of the bearing in each case and to have plotted the theoretical curve for it instead of assuming that the films were complete 360 deg. in all cases. If this suggested procedure had been followed, it is quite likely that none of the experimental curves would have extended below the theoretical ones, as they do in several of the figures.

In their Fig. 10 the authors have plotted their data using coordinates that make necessary the use of only one theoretical curve. Not all the data is plotted in that chart. There is enough, however, for the present purpose. One wonders perhaps why D/C appears in both coordinates of this chart. One reason for this is the necessity of using the same scale of abscissas with an eccentricity factor scale. It is suggested, therefore, that Fig. 12 of my discussion be placed directly below Fig. 10, making the base scales identical. The theoretical eccentricity of the journal in a full bearing for any abscissa in Fig. 10 may then be found by reading its intersection with the curve in Fig. 12, by means of the eccentricity scale at the left. The figures I have marked on the curve of Fig. 12 indicate the ratio of theoretical friction, per given eccentricity, to the friction when the journal is running concentric. It is therefore evident that when the abscissa value is higher than 50, the friction in the bearing might just about as well be compared with the friction for concentric running, as proposed by Petroff, as with eccentric running as proposed by Sommerfeld.

Another fact we note when referring the curves in Fig. 10 to the eccentricity curve in Fig. 12 is this—the McKee curves Nos. 10, 6, 3, 8, 4, and 1 begin to turn upward at the left when the theoretical eccentricity is less than 10 per cent of the radial clearance. This must mean that the actual eccentricities are considerably greater than the theoretical ones. We would expect the actual eccentricity to be greatest for the very narrow bearing for which $L/D = {}^1/_4$ and when the clearance ratio is greatest, i.e., $C/D = 0.00091$. Curve 10, however, which represents $L/D = {}^1/_4$ and $C/D = 0.00038$, has its lowest point where the abscissa of Fig. 10 is 300. Metallic contact must therefore have taken place when the theoretical eccentricity was approximately 3 per cent. Curve 11, however, which also represents this very short bearing, but which has a greater clearance ($C/D = 0.00091$), does not turn upward until the abscissa is about 30, which corresponds with the theoretical eccentricity of 25 per cent. This is contrary to our expectation. We would expect the closer approach of the surfaces to have been obtained with the bearing having the larger clearance. To this extent the test results appear inconsistent. One explanation for this condition appears to be the lack of uniformity of finish of the bearing surfaces.

A similar eccentricity curve is illustrated in Fig. 13 with the same scale of abscissas as given in the authors' Fig. 11. If Fig. 13 is placed directly below Fig. 11, the theoretical eccentricities corresponding with any points on the curves on Fig. 11

[20] Taking the parametric equations for X and Y in terms of the eccentricity ratio c, it is easily proven that the asymptote passes through the origin and has a slope of $\pi^2/30$ when the viscosity is expressed in lb. sec. per sq. in. One lb. per sq. in. = 68950 dynes per sq. cm.; hence, changing the units to centipoises and introducing 10^{-6} in the X scale, the slope becomes $(\pi^2/30)(10^4/6{,}895{,}000) = 0.0477$. Cf. the constant in the authors' equations [1] and [3].

[21] Reported in *Mechanical Engineering*, December, 1927.

[22] Howarth, "A Graphical Study of Journal Lubrication," Part I, Fig. 2.

[23] Vice-Pres. and Genl. Mgr., Kingsbury Mach. Wks., Philadelphia, Pa. Mem. A.S.M.E.

can be determined from Fig. 13. All of the Fig. 11 curves are drawn for $L/D = 1$. We would expect the greatest deviation from the theoretical eccentricity when the running clearance ratio is greatest; that is, we would expect the thinnest theoretical films when the clearance ratio is greatest. When examined with this idea in view, the curves appear inconsistent. The low point indicating metallic contact on the curve for $C/D = 0.00046$ occurs when the abscissa is about 140; that is, when the eccentricity is only 5 per cent. Metallic contact, as indicated by Fig. 11, occurs at progressively greater eccentricities as the clearance ratios increase; for example, 0.05, 0.09, 0.28, 0.80, 0.93, 0.99, respectively. These theoretical eccentricities lead to minimum film thickness as follows: 0.00028, 0.00037, 0.00045, 0.00025, 0.00018, 0.00006. We would expect these minimum film thicknesses to decrease uniformly. Instead, however, there is first an increase and then a decrease. Nevertheless, the minimum film thickness apparently occurs as expected when the running clearance is greatest. The result of this analysis, just as that given in a previous paragraph, leads us to believe that the perfection of finish is not the same for all bearings. This explains inconsistencies in the test results, which might be shown in other ways, but the foregoing examples will suffice.

Following up the idea expressed in a previous paragraph, that the useful portions of the test films in the authors' bearings were in some cases much less than 360 deg., it is desirable to find out how low the coefficient of friction could be expected to fall if the test bearings were assumed to be partial. Perhaps we will find lower theoretical coefficients for partial bearings than McKee found for his test bearings. If so, we can believe that McKee's bearings were partial in some cases at least, and that if he had used for comparison theoretical friction curves for such partial bearings, his test coefficients would always have been higher than the theoretical values.

The viscosity employed by the McKees averaged about 40 centipoises. This corresponds in the English system with about 5.85×10^{-6} reyns (inches-pounds-seconds). The curves in the charts published in my "Graphical Analysis of Journal Bearing Lubrication" were drawn in almost every case only for a viscosity of 3.4×10^{-6} reyns. With this in mind, my charts can be used for the present purpose, as explained below. The low points on the McKee curves are obtained for the most part by lowering the speed, as will be evident from the sample of his data given in Table 1. To obtain values of w/N for use in my charts, we divide 40 by his abscissa, for Figs. 3 to 7. It will be noticed that his low points lie in the region of $ZN/P = 50$ or less. We might therefore assume them to be 40, then $w/N = P/N = 1$. This must be corrected for entering my charts by multiplying it by the viscosity ratio 3.4/5.85. We will therefore assume $w/N = 0.58$, and in every case read to the curve drawn for the viscosity 3.4×10^{-6}.

Referring now to figures in my Graphical Analysis and assuming in every case that the clearance ratio is 0.001, we find from Fig. 3 an eccentricity of 0.17 and a coefficient of journal friction of 0.0021, for the full bearing. For the central partial bearings, whose characteristics are given in Fig. 8, we find eccentricities and coefficients of friction varying with the angle length β of the bearing, as follows:

β	Eccentricity factor	Friction coefficient
180 deg.	0.38	0.00134
160 deg.	0.434	0.00130
140 deg.	0.5	0.00128
120 deg.	0.57	0.00128
100 deg.	0.63	0.00128
80 deg.	0.694	0.00128
60 deg.	0.772	0.00128

For partial bearings of this kind, we find theoretical coefficients of friction less than half the lowest test value obtained by the authors. It therefore appears that, as previously stated, if the authors could have measured the useful length of their films, that is, the length in which appreciable pressure existed, they would have found bearings much less than 360 deg. in length; and if they had used the theoretical curves of such bearings as reference curves, their test coefficients of friction would have lain above such reference curves.

There is some question also whether the viscosity of the oil within the film had the average value assigned to it by the authors. Any error in the determination of this viscosity would have a directly proportional influence upon the plotted abscissa of the point for which the friction was measured. If the actual viscosities were lower than the authors estimated, the difference between the theoretical curves and the actual curves would be greater than the authors show. If the actual viscosities were greater than estimated, the differences between these curves

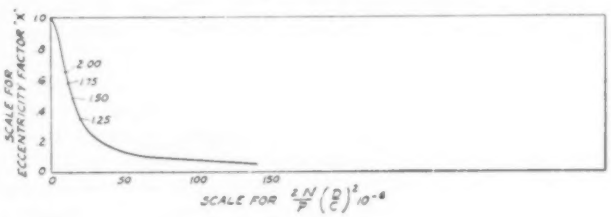


FIG. 12

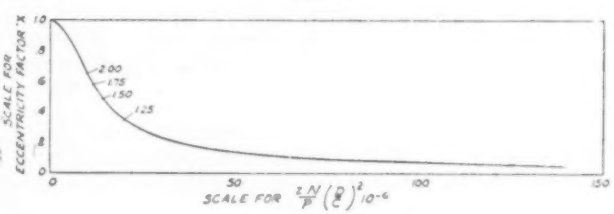


FIG. 13

would have been less. In fact it is quite conceivable that if the authors had determined the actual mean oil viscosity, all their points would have fallen below the theoretical curves for the full bearing. The film temperature was controlled by water-cooling the shaft within the film, and the viscosity was measured in the bearing surrounding the film. Hence it is quite possible that the film was cooler than indicated by the thermocouple.

The net result of this discussion indicates the desirability of extreme care in the design, preparation, and use of apparatus for such experiments; the need for testing larger bearings; the need for higher loads and speeds; and also the necessity for knowledge of the useful length of the films. If the points of nearest approach could be determined also, that would be another long step forward. Many tests by various persons, and with different apparatus, are necessary before authorities can decide how nearly the test results can be expected to compare with our theoretical data. Then only will it be possible to set up dependable factors for determining probable film thicknesses, friction coefficients, and useful proportions of actual bearings. The work of the authors is most commendable, and I hope that they will be able to continue the useful investigations in which they have been engaged for some time past.

G. B. KARELITZ.[24] The paper deals with a series of very important and interesting experiments. Very few systematic

[24] Research Engineer, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa. Mem. A.S.M.E.

tests were made on the question of bearing friction to date, which makes the work published by the authors quite a valuable one.

However, it should be noted that the diameter of the test journal is very small and the machining must be exceedingly accurate if a reliable control of clearances is to be obtained. Under these circumstances it would be advisable to record in the paper all dimensions, as actually measured, of the journals and bearings employed.

The foregoing remark is not intended to cast any doubt on the results obtained. On the contrary, the findings were compared with the available data on friction in large sleeve bearings of the turbo-generator type. The coefficients of friction check quite closely. They also check with measurements made by Lasche in Germany in 1918 on a journal 8 in. in diameter.

It is interesting to note that the coefficient of friction plotted against $\frac{ZN}{P}$ is practically the same for bearings with a ratio of length to diameter from 0.75 to 2.80. This may be taken as an indication that the conditions in the oil film for these lengths of the shell are the same in so far as the influence of end leakage is concerned. For this range of length, commonly employed in bearings, the longitudinal pressure distribution is nearly parabolic. In shorter bearings the end leakage is comparatively worse and the oil film for the same value of P is appreciably thinner, causing higher friction. This means that the running position of a journal in its shell can be conveniently estimated by the standard methods and diagrams for bearings of length equal to the diameter or larger, while for shorter bearings the estimate is more difficult. The running position determines the minimum oil film thickness in the bearing and thus defines the safety of the bearing.

M. D. Hersey.[25] I would like to mention a rather curious confirmation of the McKee paper as regards absolute values. Of course, the paper would be justified if it gave only relative values.

About twenty years ago I undertook an investigation at the Massachusetts Institute of Technology, in the Department of Mechanical Engineering, which was published a few years later in the Journal of the Washington Academy of Sciences, November 19, 1914. This paper gave the experimental results in the form of an empirical equation. I believe these were the first experiments on journal friction in which the results were plotted and interpreted by the dimensional analysis method. This empirical equation gave a line having an intercept on the coefficient of friction axis. The horizontal coordinate is the dimensionless variable, viscosity times speed in revolutions per minute divided by the so-called bearing pressure. The points were rough in these early experiments and scattered, but although three different oils were used, sperm, lard, and mineral oil, there was no systematic difference due to the different oils. There was no doubt at all that they gave this peculiar line standing definitely above a line drawn through the origin and separated from it by a constant amount.

These results have remained under suspicion because of that intercept which, according to Petroff's equation, should not occur.

Now, along comes the McKee paper with an intercept again. Upon converting my empirical equation into the same units the authors of this paper are using, and plotting it on the same chart using appropriate L/D and C/D ratios, a surprisingly good agreement was found. It remains now for some one to explain the physical meaning of the unexpected constant term.

[25] Chief, Friction and Lubrication Section, U. S. Bureau of Standards, Washington, D. C. Mem. A.S.M.E.

Authors' Closure

Mr. Howarth's interesting analysis of the results obtained tends to emphasize the difficulties involved in obtaining a complete solution to the problem of bearing performance when operating under conditions of low values of ZN/P. He substantiates two points already pointed out in the paper: namely, that in actual bearings, especially at the larger clearances, it is reasonable to expect that the lubricant does not entirely fill the clearance space and that the location of the point of minimum friction is dependent upon the condition of the bearing surfaces. The presence of the latter factor was one of the chief reasons for not attempting to make a definite analysis of the operation of the bearings at low values of ZN/P. The data were obtained without "running in" the bearings, and previous experiments[26] had indicated that even a slight amount of running in would materially affect the point of minimum friction of a given set of bearings.

The question as to the publishing of the complete data for all the bearings, as was suggested by Messrs. Karelitz, Roberts, and McCurdy, was considered at the time of publication, but it seemed preferable to omit all but a sample sheet because of the large amount of space required.

It was realized, as was pointed out by Mr. Karelitz, that errors in measurements of diameter are of greater significance when using a small rather than a large journal. The particular size chosen, however, was most advantageous from the standpoint of the facilities available to carry on the work. It should be noted that a statement is made in the paper limiting the scope of the investigation to small-bore bearings and also that the precision of measurement is probably far greater than usually occurs in commercial practice.

Referring to the question by Mr. Weiss regarding the cause of the increase in deviation between the theoretical and experimental curves as the length of the bearings decreased, it seems probable that the effect of end leakage in changing the lines of flow and pressure distribution in the oil film was the chief factor.

At the small values of ZN/P below the point of minimum friction, such as that mentioned by Mr. Weiss, previous experiments[27] have shown that oiliness has a measurable influence upon bearing friction. This tends to indicate that under these conditions the bearings are operating in the region of boundary or, as he calls it, "improved" lubrication.

Referring to the question of Professor Waters, the large scale for Δf used in Fig. 9 perhaps overemphasized the increase in Δf with an increase in the L/D ratio above $L/D = 1.00$. It should be noted that Equation [3] is limited to cases where f is greater than 0.01, and thus from the practical standpoint of obtaining an estimate of the friction of a given bearing, the upward slope of the curve has little significance. For example, suppose the case of a bearing with $L/D = 2.75$ and $f_P = 0.01$. Then Δf as determined by Fig. 9 would be 0.00225 and f0.01225. Assuming the curve as drawn parallel to the base line, Δf becomes 0.00180 and f becomes 0.01180, the difference between the two results being approximately 4 per cent. Thus even should it be more proper for the curve to have a downward slope, the difference would not be very great.

The authors' basis for drawing the upward slope to the curve was that this curve was at best an empirical approximation based solely upon the results of these tests.

[26] S. A. McKee, "Effect of Running-in on Journal Bearing Performance," *Mechanical Engineering*, vol. 49, December, 1927, pp. 1335–1340.

[27] C. P. Barnard, 4th, H. M. Myers, and H. P. Forrest, "The Effect of Oiliness on the Behavior of Journal Bearings," *Ind. & Eng. Chem.*, vol. 16 (1924), pp. 347–350. S. A. McKee, "Effect of Kerosene on Oiliness of Lubricating Oils," *Jl. S.A.E.*, vol. 19 (1926), pp. 356–360.

In considering the conditions during the tests, the test bearings were spaced close together, the loads were comparatively light, and the journals and bearings were probably more nearly true cylinders than is usually found in practice. This would seem to indicate that unless the construction were of great accuracy and rigidity it is not unreasonable to expect that bearings in actual service might show this tendency to a greater extent than was indicated by these tests.

APM-51-16

Stress Distribution in Rotating Disks of Ductile Material After the Yield Point Has Been Reached[1]

By A. NADAI,[2] GÖTTINGEN, GERMANY, and L. H. DONNELL,[3] ANN ARBOR, MICH.

This paper covers a study of the distribution of stress in a rotating disk of ductile material (of uniform thickness, with and without a central hole) after the yield point has been passed. Curves are computed showing the growth of the "plastic" region at different speeds, until the whole disk has become plastic. General methods are developed, and applied to examples for finding the complete distribution of stresses and strains in such disks.

While the conditions assumed are seldom exactly realized, the results should throw light on many practical problems as well as form a starting point for a more complete investigation of more complex problems of plastic flow in ductile materials.

WORKERS in many branches of science and engineering are interested in the question of plastic flow of solid bodies. (By plastic deformation in contrast to elastic deformation is meant a permanent and considerable change in the shape of a body which occurs without appreciable weakening of the material.) The various manufacturing processes used to change the shapes of bodies by cold or hot working, such as rolling, drawing, forging, etc., belong to this class of problems in the mechanics of the plastic stage of materials, and several attempts have been made to analyze, with the methods of applied mechanics, the distributions of stresses in such processes.

Another interesting field for study in mechanical engineering is concerned with the improvement of our knowledge of what takes place in various machine elements of ductile material after the yield point has been passed and plastic deformation ha begun to develop. That such problems have practical importance is indicated by the fact that engineers have long found it necessary to specify a high degree of ductility in the materials used for many such machine elements.

This paper is an attempt to make such a study for the comparatively simple case of a disk of uniform thickness stressed by centrifugal forces due to rotation about an axis through its center and perpendicular to its plane. As Stodola mentions in his "Steam and Gas Turbines,"[4] attempts have been made to improve the stress distribution in rotating disks with a central hole by running them during manufacture at a speed such that the inner part of the disk is permanently stretched. This question is also considered by Hencky,[5] by Lasche,[6] and in a report of the Brown-Boveri Co., of Baden.[7] Laszlo[8] has considered an interesting question of some practical importance, namely, that of the stability of the quasi-static equilibrium of a rotating disk stressed above the yield point; i.e., whether there is a certain speed at which a small increment of speed causes the disk to stretch a comparatively large amount, resulting in the destruction of the disk.

Notation

b	= radius of the disk, inches.
r,a,c	= radius of the point considered, of the hole, and of boundary of the plastic region, inches.
ρ,α,γ	= r/b, a/b, c/b, respectively.
S_y	= yield-point stress of material in a simple tensile test, lb. per sq. in.
S_t,S_r	= tangential and radial stresses in the plastic region, lb. per sq. in.
S'_t,S'_r	= same for the elastic region.
$\sigma_t,\sigma_r,\sigma'_t,\sigma'_r$	= S_t/S_y, S_r/S_y, S'_t/S_y, S'_r/S_y, respectively.
S_1,S_2,S_3	= principal stresses in plastic region (= S_t, S_r, 0 in the present discussion).
e_y	= unit strain at yield point in a simple tensile test
e_t,e_r	= tangential and radial unit strains in plastic region.
ϵ_t,ϵ_r	= e_t/e_y, e_r/e_y, respectively.
V	= peripheral speed of disk, ft. per sec.
ν	= Poisson's ratio for the material (= 0.3 for steel).
λ	= density of material, lb. per cu. in. (= 0.282 for steel).
g	= acceleration of gravity, ft. per sec. per sec. (= 32.2).
Φ^2	= $\lambda V^2/gS_y$.

Yielding Under Combined Stress

In recent years a great deal of valuable work has been done to determine the laws of plastic flow in the principal materials of construction, the ductile metals; and before endeavoring to establish the assumptions for finding the stress distributions, it is necessary to consider this.

According to the so-called maximum-shear theory,[9] yielding takes place when the greatest shear stress exceeds a certain value. In a thin rotating disk, where there are no appreciable stresses in the axial direction, the maximum shear stress will depend only on the maximum tensile stress (which is always the tangential normal stress S_t) just the same as in a simple tensile test. Hence, yielding will take place when $S_t = S_y$.

Newer tests seem to indicate that the condition of yielding in the ductile metals under normal temperatures is dependent on the value of all three principal stresses. Simple torsion tests have shown that yielding under simple shear stress takes place, not when the shear stress = 0.5 S_y as it should according to the maximum-shear theory, but when the shear stress is equal to

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[1] This work has been carried out at the Research Department of the Westinghouse Elec. & Mfg. Co. at East Pittsburgh.

[2] Professor, University of Göttingen.

[3] Assistant Professor, University of Michigan.

[4] English translation by L. C. Loewenstein, vol. 2, p. 1080.

[5] H. Hencky, *Zeitschrift für Angewandte Mathematik und Mechanik*, 1924, p. 331.

[6] Lasche-Kieser, "Konstruktion und Material in Bau von Dampfturbinen und Turbodynamos," p. 55. Julius Springer, Berlin.

[7] *Brown Boveri Revue*, 1919, p. 260.

[8] F. Laszlo, *Zeitschrift für Angewandte Mathematik und Mechanik*, 1925, pp. 281–293.

Contributed by the Applied Mechanics Division and presented at the Annual Meeting, New York, N. Y., December 3 to 7, 1928, of The American Society of Mechanical Engineers.

[9] S. Timoshenko and J. M. Lessells, "Applied Elasticity," Westinghouse Technical Night School Press, East Pittsburgh, 1925, p. 501. James E. Boyd, "Strength of Materials," McGraw-Hill Co., New York, 1924, p. 325.

H. M. Westergaard, in the *Journal of the Franklin Institute*, May, 1920, gives a very comprehensive comparison of the conditions of yielding in ductile metals.

about 0.6 S_y, or a little less than this.[10] This and tests made by W. Lode[11] and later by Eichinger and Ros,[12] seem to indicate that plastic flow takes place when the quantity $(S_1 - S_2)^2 + (S_1 - S_3)^2 + (S_2 - S_3)^2$ reaches a certain value characteristic of the material. This can be interpreted as meaning that a material can absorb elastically a certain amount of energy in being distorted (as distinct from the energy stored up due to change of volume), and will yield only when this is exceeded.

The problem has been analyzed on the basis of both of these theories and it is found that the maximum-shear theory permits, in the special case of the rotating disk considered here, of simpler analysis. The stresses figured on this basis (shown by dotted lines in the figures) differ at most by 16 per cent from those

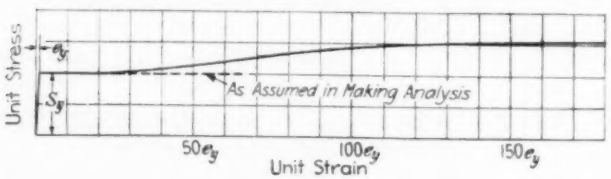


FIG. 1 TYPICAL STRESS-STRAIN DIAGRAM FOR SIMPLE TENSION FOR STEEL

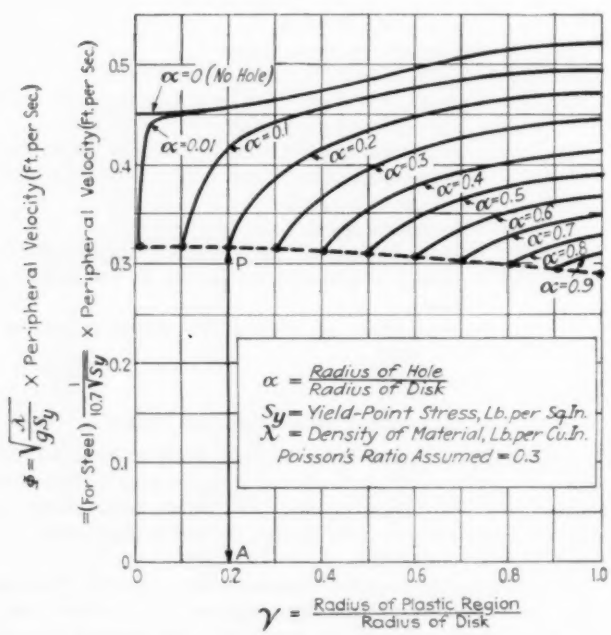


FIG. 2 PROGRESS OF PLASTIC REGION OUTWARD FROM THE CENTER WITH INCREASING SPEED IN A ROTATING DISK ACCORDING TO EXACT THEORY

figured from the more exact theory (shown by full lines), and hence can be considered a good approximation.

ASSUMPTIONS MADE

For the sake of simplicity we shall assume at first:

1 That the disk is of uniform thickness.

2 That the thickness is so small that the stresses do not vary through the thickness. In practice the results can be applied without great error to any disk of constant thickness whose radius is several times the thickness.

We introduce two further assumptions:

3 That the stress-strain curve of the material under simple tension is as shown in Fig. 1, with a sharp corner at the yield point, neglecting the rounded portion of the stress-strain curve which may be detected by very exact measurements of the strains when the plastic deformation starts to develop. In other words, we assume that the material of the disk has a well-defined yield point. In practice, if the material does not have a very well-defined and sharp yield point the stress conditions will not be radically altered, as this will merely affect the abruptness of the transition between the plastic and the elastic regions of the disk (provided that the stress-strain curve is not of the form shown, for example, by soft annealed copper, having a very continuous transition between elastic and plastic deformation).

The last assumption is:

4 That the horizontal portion of this stress-strain curve extends indefinitely to the right as shown by the broken line, Fig. 1. This assumption will be practically correct and is legitimate for most steels, so long as the unit strains do not exceed a few per cent. It is shown later that the maximum strains do *not* exceed such a value until the whole disk has become plastic.

RESULTS

In any rotating disk, conditions are worse nearest the center. As the speed is increased the material at the center will become plastic (i.e., pass the yield point) first, giving a central disk or ring of material in the plastic state surrounded by an outer ring of material still elastic. As the speed is further increased, this plastic region will gradually spread outward, until finally the whole disk is in the plastic state.

SPREAD OF PLASTIC REGION OUTWARD WITH INCREASING SPEED

This is shown by Fig. 2. Here the different curves are for disks with different-sized holes; the abscissas represent the ratio of the radius of the boundary circle of the plastic region to the radius of the disk, and the ordinates are proportional to the peripheral velocity of the disk. The points P on the dotted curve indicate by their ordinates PA the peripheral velocity at which the yield point is reached on the inner boundary of the disk. The corresponding abscissas OA show the ratio of the radius of the hole to the radius of the disk. The curves shown in solid lines may be explained by taking an example. Thus the curve marked "$\alpha = 0.01$" is for a disk with a central hole having a radius of one-hundredth of the radius of the disk, and shows that the material around the hole first passes the yield point at a peripheral speed of about $3.4\sqrt{S_y}$. However, the speed can then be increased, without much more material becoming plastic, until it is equal to the speed at which yielding first takes place in a disk *without* a hole, about $4.8\sqrt{S_y}$. If the speed is increased beyond this the plastic region spreads very rapidly for both the disk with the small hole and for a disk without a hole, the whole disk becoming plastic with an increase in speed of only about 17 per cent, at a speed of about $5.6\sqrt{S_y}$.

This would seem to indicate that while elastic theory shows the stress to be doubled when a small hole is made in the center of a solid disk, actually this is of no great importance when the material is ductile.

DISTRIBUTION OF STRESSES AND RESIDUAL STRESSES

These are shown in Fig. 3 for a solid disk under the following conditions: just before the first yield takes place; when the

[10] A. J. Becker, *Bulletin of the University of Illinois*, 1916, no. 85, Eng. Exp. Sta.

[11] "Der Einfluss der mitteleren Hauptspannung auf das Fliessen der Metalle." Forschungsarbeiten V.D.I., Berlin, heft 303, 1928. See also the report of one of the authors of this paper in the Proceedings of the Second International Congress of Applied Mechanics, Zurich, 1926, p. 336; and A. Nádai: "Der bildsame Zustand der Werkstoffe," J. Springer, Berlin, 1927, p. 47.

[12] Proc. Second International Congress of Applied Mechanics, Zurich, 1926, p. 315.

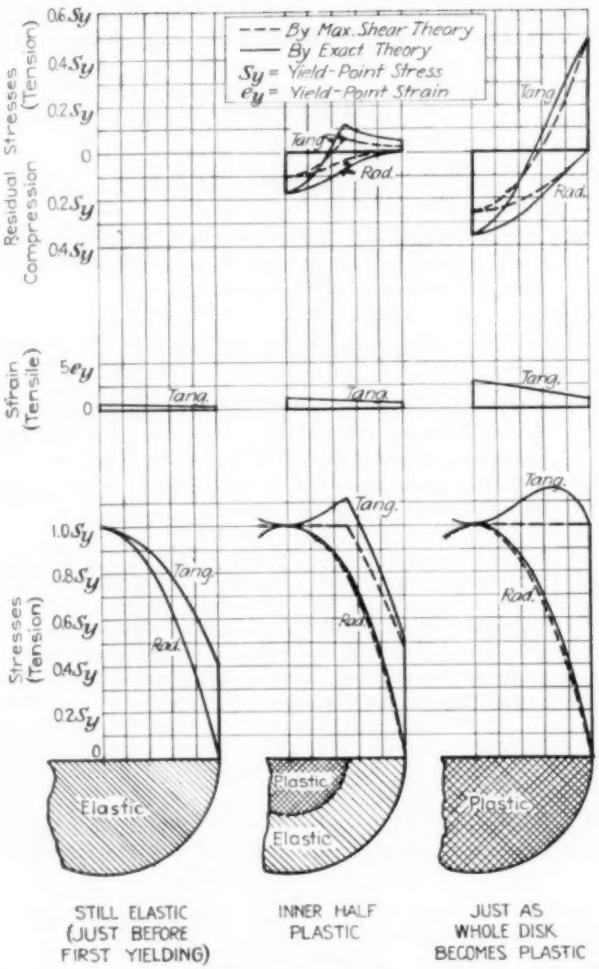


FIG. 3 STRESS DISTRIBUTION, STRAINS, AND RESIDUAL STRESSES IN A SOLID DISK

plastic region is half-way across the disk; and when the plastic region has just reached the outer edge. Fig. 4 shows the distribution for the particular case of a disk with a hole of one-fifth the outer radius. It is worthy of note that the residual stresses at the edge of the hole are just about at the yield point in one case, and well over it in the other.

STRAINS

The unit strains in the plastic region are, like the stresses, greatest in the tangential direction. In a solid disk they are a maximum at the center, but are never greater than three times the unit strain at the yield point (e_y) until after the whole disk has become plastic, as shown in Fig. 3.

The unit strains may be somewhat larger in a disk with a hole, especially at the edge of the hole. Fig. 4 shows how the strains vary along the radius in a typical case, rising to a maximum at the hole. This maximum value is still larger in the case of a very small hole, but never exceeds about 15 e_y until after the whole disk has become plastic. A comparison of this with the unit strains occurring in an ordinary tensile test, Fig. 1, indicates that, for most steels, the assumption that the stress-strain curve remains horizontal after the yield point causes no appreciable error until the whole disk has become plastic.

METHOD FOR FINDING STRESSES

In a given problem the stress at the yield point S_y and the ratio $\alpha = a/b$ of the inner to the outer radius will be known.

On the Basis of the Maximum-Shear Theory the quantity Φ (and hence the peripheral speed V, if desired) can then be determined for any value of $\gamma = c/b$ (c = radius of plastic region, b = radius of the disk) from the following equation:

$$\Phi^2 = \frac{\lambda V^2}{gS_y} = \frac{2 - \alpha(\gamma + 1/\gamma)}{9 + 3\nu - (2 + 6\nu)\gamma^2 + (1 + 3\nu)\gamma^4 - 4\alpha^3(\gamma + 1/\gamma)} \quad \ldots [1]$$

and

$$\Phi^2 = \frac{2}{9 + 3\nu - (2 + 6\nu)\gamma^2 + (1 + 3\nu)\gamma^4} \text{ for a solid disk} \quad \ldots [2]$$

The stresses at any point $\rho = r/b$ in the plastic region are then found from the following:

$$\left.\begin{aligned} \sigma_t &= 1 \\ \sigma_r &= 1 - \frac{\alpha}{\rho} - 4\Phi^2\left(\rho^2 - \frac{\alpha^3}{\rho}\right) \end{aligned}\right\} \quad \ldots [3]$$

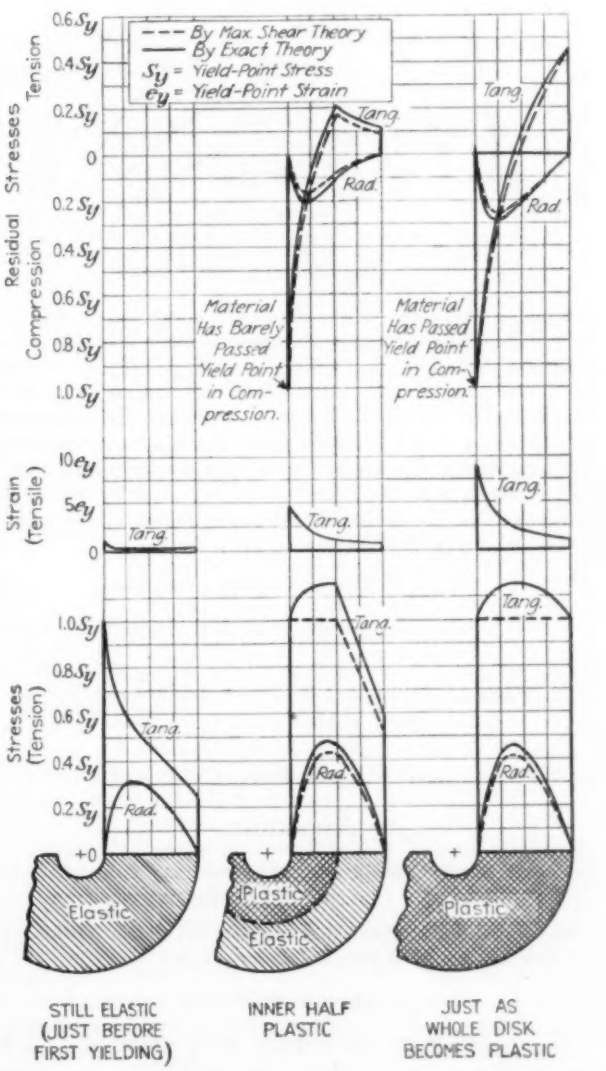


FIG. 4 STRESS DISTRIBUTION, STRAINS, AND RESIDUAL STRESSES IN A DISK WITH A 1-TO-5 HOLE

and

$$\left.\begin{aligned}\sigma_t &= 1\\ \sigma_r &= 1 - 4\Phi^2\rho^2 \text{ for a solid disk}\end{aligned}\right\} \quad \ldots\ldots\ldots [4]$$

On the Basis of the More Exact Theory, Φ is to be determined for any given value of α and γ from Fig. 2, as previously explained. The stresses at any point ρ in the plastic region are then determined from Fig. 5. In this figure the different curves are for disks with different-sized holes. From the known values

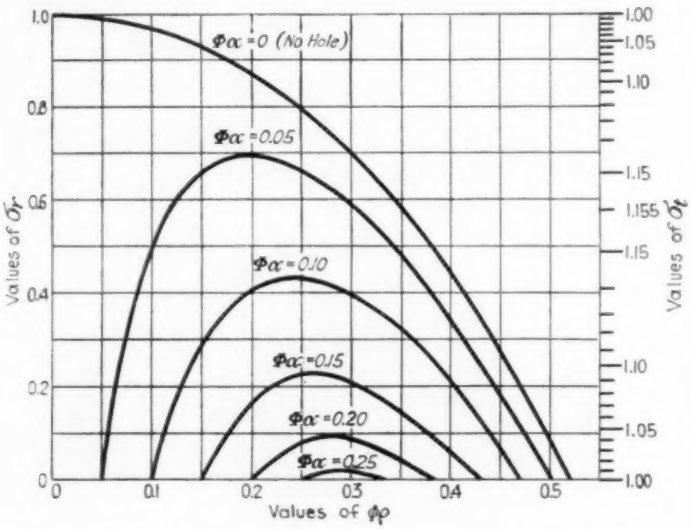


FIG. 5 CHART FOR FINDING STRESS DISTRIBUTION IN PLASTIC REGION ACCORDING TO EXACT THEORY

of Φ and α, and hence of $\Phi\alpha$, the proper curve can be selected. Then if it is desired to know the stress at some point, i.e., some particular value of ρ, it is only necessary to calculate $\Phi\rho$ and locate the point on the curve which has this as its abscissa. The ordinate of this point gives the stress (both radial and tangential, according to the two scales given at the sides of the chart).

Having found the value of σ_t at the boundary of the plastic region (which will be designated by $\sigma_{t(\rho = \gamma)}$) by one of the above methods, the stresses in the elastic region at any point ρ are obtained from the following:

$$\sigma t' = M + N/\rho^2 - 1.5(1 + 3\nu)\Phi^2\rho^2 \ldots\ldots [5]$$

$$\sigma'_r = M - N/\rho^2 - 1.5(3 + \nu)\Phi^2\rho^2 \ldots\ldots [6]$$

where

$$M = [\gamma^2\sigma_{t(\rho = \gamma)} + 1.5\,(1 + 3\nu)\Phi^2\gamma^4 + 1.5\,(3 + \nu)\Phi^2]/(1 + \gamma^2)$$

and

$$N = [\gamma^2\sigma_{t(\rho = \gamma)} + 1.5\,(1 + 3\nu)\Phi^2\gamma^4 - 1.5\,(3 + \nu)\Phi^2\gamma^2]/(1 + \gamma^2)$$

Appendix

THEORY WORKED OUT UNDER THE ASSUMPTION THAT THE YIELD STRESS IS INDEPENDENT OF DEFORMATION

The radial and tangential stresses in the elastic portion of a rotating disk are given from well-known theory by:

$$\sigma'_t = -\tfrac{1}{8}\,(1 + 3\nu)\,\Phi^2\rho^2 + k_1 + k_2/\rho^2 \ldots\ldots [7]$$

$$\sigma'_r = -\tfrac{1}{8}\,(3 + \nu)\,\Phi^2\rho^2 + k_1 - k_2/\rho^2 \ldots\ldots [8]$$

where k_1 and k_2 are integration constants to be determined from boundary conditions.

The yield point will be first reached at a peripheral speed V given by the following equations easily derived from the above:

$$\frac{\lambda V^2}{gS_y} = \frac{1}{3[3 + \nu + (1 - \nu)\alpha^2]} \ldots\ldots [9]$$

and

$$\frac{\lambda V^2}{gS_y} = \frac{2}{3(3 + \nu)} \text{ for a solid disk} \ldots\ldots [10]$$

In the plastic region, Fig. 6 shows the forces on an infinitely small element. The condition of equilibrium of these forces is expressed by the equation:

$$\frac{d}{d\rho}(\rho\sigma_r) - \sigma_t + 12\,\Phi^2\rho^2 = 0 \ldots\ldots [11]$$

(*a*) If σ_t is taken = 1, according to the maximum-shear theory, Equation [11] can be readily integrated and the integration constant determined, giving Equations [3] and [4]. From [4], [7], and [8] and the boundary conditions, one finds for the solid disk:

$$\frac{\lambda V^2}{gS_y} = \frac{24}{9 + 3\nu - (2 + 6\nu)\,\gamma^2 + (1 + 3\nu)\gamma^4} \ldots\ldots [12]$$

Through this the peripheral speed V is obtained for any solid rotating disk, if the radius c of the plastic region (or $\gamma = c/b$) is given. If $\gamma = 1$, that is, if the plastic region just reaches the outer circle:

$$\frac{\lambda V^2}{gS_y} = \frac{1}{4} \ldots\ldots [13]$$

Comparison of [10] and [13] yields a result of practical importance: that is, that in a solid disk the peripheral

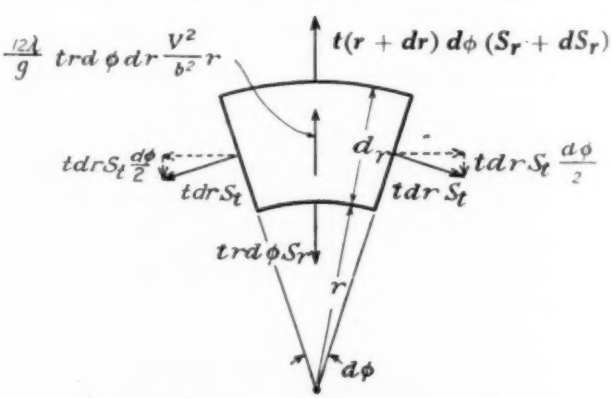


FIG. 6 FORCES ON AN ELEMENT OF DISK

speed V_2 when the disk just begins to stretch permanently on its outer circle is:

$$V_2 = \sqrt{\frac{3(3 + \nu)}{8}}\,V_1 \ldots\ldots [14]$$

or, with $\nu = 0.3$, $V_2 = 1.11V_1$, which is only 11 per cent higher than the speed at which the first yield occurs.

For the *disk with a hole* the expression for the peripheral speed becomes a little more complicated and is:

$$\frac{\lambda V^2}{gS_y} = \frac{2 - \alpha\,(\gamma + 1/\gamma)}{9 + 3\nu - (2 + 6\nu)\gamma^2 + (1 + 3\nu)\gamma^4 - 4\alpha^2\,(\gamma + 1/\gamma)} \quad [15]$$

Yield first occurs now at a speed V given by:

$$\frac{\lambda V^2}{gS_y} = \frac{1}{3[3+\nu+(1-\nu)\alpha^2]} \quad [16]$$

and the plastic region spreads to the outer boundary at a speed given by:

$$\frac{\sigma V^2}{gS_y} = \frac{1}{4(\alpha^2+\alpha+1)} \quad [17]$$

(*b*) According to the more exact theory the quantity

$$(S_1-S_2)^2+(S_1-S_3)^2+(S_2-S_3)^2$$

must always have the same value when yielding takes place. Thus it must have the same value when the material yields under a simple tensile test, in which case it becomes $[(S_y-0)^2+(S_y-0)^2+(0-0)^2] = 2S_y^2$. Hence in a plastic region (before cold working takes place):

$$(S_1-S_2)^2+(S_1-S_3)^2+(S_2-S_3)^2 = 2S_y^2$$

which in our case, as $S_1 = S_t$, $S_2 = S_r$, and $S_3 = 0$, reduces to

$$\sigma_t^2-\sigma_r\sigma_t+\sigma_r^2 = 1 \quad [18]$$

or

$$\left(\frac{\sigma_t+\sigma_r}{2}\right)^2+3\left(\frac{\sigma_t-\sigma_r}{2}\right)^2 = 1 \quad [19]$$

which is now the condition of plasticity. This makes σ_t a variable and when its value is substituted in [11], integration seems to be impossible. To reduce the variables we put

$$\sin\theta = \frac{\sigma_t+\sigma_r}{2} \qquad \cos\theta = \frac{\sigma_t-\sigma_r}{2}\sqrt{3} \quad [20]$$

whence

$$\left.\begin{aligned}\sigma_t &= \sin\theta+\frac{\cos\theta}{\sqrt{3}}\\ \sigma_r &= \sin\theta-\frac{\cos\theta}{\sqrt{3}}\end{aligned}\right\} \quad [21]$$

Substituting [21] in the differential Equation [11] and letting $\Phi^2\rho^2 = w$ we obtain the following:

$$\frac{d\theta}{dw} = \frac{\cos\theta-6\sqrt{3}\,w}{2w\cos(\theta-\pi/6)} \quad [22]$$

where $w = \Phi^2\rho^2$.

One set of values of θ and $\Phi^2\rho^2$ is given by the inner boundary conditions and [21]: $\rho = 0$, $\sigma_r = \sigma_t$, $\Phi^2\rho^2 = 0$, $\theta = \pi/2$ or for a disk with a hole, $\rho = \alpha$, $\sigma_r = 0$, $\Phi^2\rho^2 = \Phi^2\alpha^2$, $\theta = \pi/6$ (from [21]).

Building up from these values by numerical integration we can obtain for any value of $\Phi^2\alpha^2$ (including 0, when there is no hole) a series of values of θ and $\Phi^2\rho^2$ satisfying [22]. From each set of values of $\Phi^2\alpha^2$, $\Phi^2\rho^2$, and θ thus obtained we can calculate the value of $\Phi\alpha$, $\Phi\rho$, and σ_r, σ_t (from θ, using [21]) and these will also satisfy [22]. These values are plotted in Fig. 5.

Each point in Fig. 5 then represents a set of values of $\Phi\alpha$, $\Phi\rho$, σ_r, or, if we wish to so regard it, of $\Phi\alpha$, $\Phi\gamma$ and $\sigma_{r(\rho=\gamma)}$ (the value of σ_r at the boundary of the plastic region) which satisfy conditions in the *plastic* region.

From [7], [8], and [18] and the boundary conditions of the elastic region we can get the following equation:

$$\phi^2\gamma^2 = \frac{(3+\gamma^2)\,\sigma_{r(\rho=\gamma)}+(1-\gamma^2)\sqrt{4-3\sigma_{r(\rho=\gamma)}^2}}{6[(3+\nu)/\gamma^2-(2+2\nu)-(1-\nu)\gamma^2]}$$

from which, if we know ν, we can obtain (by assuming γ and $\sigma_{r(\rho=\gamma)}$ and solving for $\phi\gamma$) sets of values of γ, $\phi\gamma$, and $\sigma_{r(\rho=\gamma)}$ which satisfy conditions at the boundary of the *elastic* region. If we plot from this a new set of curves on Fig. 5, of $\phi\gamma$ plotted against $\sigma_{r(\rho=\gamma)}$ for various values of γ, then every intersection of one of these new curves with one of the old ones will give us a set of values of $\phi\gamma$, $\sigma_{r(\rho=\gamma)}$, $\phi\alpha$, and γ satisfying conditions for both the plastic and elastic regions. For each set of these values we can calculate ϕ, α, and γ. In this way Fig. 2 was plotted, for the particular case of $\nu = 0.3$.

Residual stresses were estimated on the assumption that when the disk is slowed down, stresses are removed according to Hooke's law—i.e., the stresses which would exist at the given speed if the material were perfectly elastic (obtained from [7], [8], and the boundary conditions) were subtracted from those previously obtained. Where the residual stresses reached the yield point in compression, Fig. 4, they were simply cut down to the yield-point value; and as this involved only a very small area, the effect on the remaining stresses was neglected.

(*c*) According to accepted theory, in a plastic region the direction of the principal stresses coincides with the direction of principal strains, and these are related as follows:

$$\frac{2S_2-S_1-S_3}{S_1-S_3} = \frac{2e_2-e_1-e_3}{e_1-e_3} \quad [23]$$

Also, the volume remains practically constant, so

$$e_1+e_2+e_3 = 0 \quad [24]$$

These reduce, in the present case, to

$$e_r/e_t = (2S_r-S_t)/(2S_t-S_r) \quad [25]$$

As in the elastic theory, u being the radial displacement,

$$e_t = u/r,\ e_r = du/dr$$

therefore

$$\frac{de_t}{e_t} = \left(\frac{e_r}{e_t}-1\right)\frac{dr}{r} = \frac{3(\sigma_r-\sigma_t)\,d\rho}{(2\sigma_t-\sigma_r)\rho} \quad [26]$$

whence, using [3] and [4],

$$\frac{d\epsilon_t}{\epsilon_t} = -\frac{3(\alpha+4\Phi^2(\rho^3-\alpha^3))d\rho}{[\rho+\alpha+4\Phi^2(\rho^3-\alpha^3)]\rho} \text{ for a disk with a hole} \quad [27]$$

and

$$\frac{d\epsilon_t}{\epsilon_t} = -\frac{12\Phi^2\rho d\rho}{1+4\Phi^2\rho^2} \text{ for a solid disk} \quad [28]$$

The latter equation is easily integrated:

$$\epsilon_t = k/(1+4\rho^2\Phi^2)^{3/2} \quad [29]$$

and when $\rho = \gamma$, $\epsilon_t = \sigma_t-\nu\sigma_r = k/(1+4\Phi^2\gamma^2)^{3/2}$,

$$\epsilon_t = \sqrt{\left(\frac{1+4\Phi^2\gamma^2}{1+4\Phi^2\rho^2}\right)^3}\times(\sigma_t-\nu\sigma_r)$$

which cannot exceed 3 until the whole disk becomes plastic, as $(\sigma_t-\nu\sigma_r) \lesssim 1$ and the maximum value of $4\Phi^2\gamma^2$ is 1, when the whole disk is plastic (from [13]).

For a disk with a hole Equation [27] can be integrated, but it was found easier to use numerical integration to obtain an approximate solution. The starting point is given as in [29]: $\rho = \gamma$ and $\epsilon_t = \sigma_t-\nu\sigma_r$, which has a value so close to 1.0 under all conditions (as can be seen from Figs. 3 and 4) that this value was assumed. Thus the values of ϵ_t were found over the whole of the plastic region for various values of α and γ, Φ being determined from [1], with the results previously described.

Method for Computing Stress in a Solid Disk Stretched Permanently Under the Assumption That the Yield Stress Depends on the Deformation

Here the authors will finally try to indicate how it seems to be possible to consider the effect of cold work in their calculation. As is well known from an ordinary stress-strain diagram, the stresses within the plastic region depend upon the deformation and increase with the unit elongation. Until now no general law has been known by means of which the effect of cold work *under combined stress* could be expressed. The authors will introduce again certain simple assumptions which are more or less evident, some of them, derived from results of tests, having been previously used in this paper. It will be assumed that when plastic flow takes place in steel:

1 The volume is not changed.

2 Mohr's diagram of the three principal stress circles is a figure geometrically similar to the three principal strain circles.[12]

3 The effect of cold work will be expressed by assuming that the maximum shear stress is a given function of the maximum shearing deformation which we shall take to be a linear function.

4 The change of the dimensions through the plastic deformation remains small when compared with the dimensions of the stressed disk.

Assumption 1 is expressed by the equation:

$$e_r + e_t + e_z = 0 \qquad [30]$$

where e_z denotes the principal extension in the direction perpendicular to the plane of the disk. Assumption 2 is expressed by the proportion:

$$\frac{e_t - e_r}{e_t - e_z} = \frac{S_t - S_r}{S_t - S_z} \qquad [31]$$

Now by using [30] and taking $S_z = 0$, [31] reduces to:

$$\frac{S_r}{S_t} = \frac{e_t + 2e_r}{e_r + 2e_t} \qquad [32]$$

From this proportion we conclude that S_r and S_t can be expressed as follows:

$$\left.\begin{aligned} S_r &= k(e_t + 2e_r) \\ S_t &= k(e_r + 2e_t) \end{aligned}\right\} \qquad [33]$$

where k is an unknown function of the coordinate ρ.

The maximum shearing deformation at an arbitrary point ρ is[14]

$$\gamma_{max} = e_t - e_z = e_r + 2e_t \qquad [34]$$

and the maximum shear stress is $S_t/2$.

Assumption 3 can now be expressed by taking $S_t/2$ to be a linear function of the maximum shearing deformation γ_{max}:

$$S_t = S_0(1 + c\gamma) \qquad [35]$$

where c is a constant and S_0 is a constant stress which is apparently very nearly the stress at which the first yield occurs.

Calling e_0 the tangential unit elongation where the first yield occurs in pure tension, then

$$e_r = e_z = -\frac{e_t}{2} = -\frac{e_0}{2}$$

$$\gamma = \gamma_0 = e_r + 2e_t = -\frac{e_0}{2} + 2e_0 = \frac{3e_0}{2}$$

γ_{max} can never be smaller than $3e_0/2$. By using [33] and [34], Equation [35] takes the form:

$$S_t = \frac{kS_0}{k - cS_0} \qquad [36]$$

It is now necessary to connect these equations with the condition of equilibrium of the disk already given, namely,

$$\frac{d(\rho S_r)}{d\rho} - S_t + S_0\beta\rho^2 = 0, \quad \beta = \frac{\lambda V^2}{gS_0} \qquad [37]$$

and with the condition of compatibility of the strains:

$$\frac{d(\rho e_t)}{d\rho} - e_r = 0 \qquad [38]$$

Equations [33], [36], [37], and [38] are sufficient to determine the stress distribution.

If the constant c, which was introduced in the idealized stress-strain diagram to express cold work (Equation [35]), be taken equal to zero, we obtain the case already worked out in case (*a*) at the beginning of the appendix, and c can only be positive. The actual value of c can be estimated from the stress-strain diagram for pure tension. For example, if in a tensile test the stress S_t should increase by 100 per cent for 10 per cent unit plastic elongation (that is, for $\gamma = 3e/2 = 0.015$), it follows from [35] that

$$c = \frac{S_t - S_0}{S_0\gamma} = \frac{1}{0.15} = \frac{20}{3}$$

It seems impossible to give the exact functions in finite form which would satisfy all the prescribed conditions. But it is possible to find an approximate form of the distribution of the radial and tangential stresses by functions similar to those found for the case where $c = 0$. We indicate the calculation only for the case where the *entire disk is in the plastic stage.*

Assume for the radial stress the function:

$$S_r = CS_0(1 - \rho^2) \qquad [39]$$

where the constant C has to be determined. It is easy to find the corresponding tangential stress to this radial stress distribution which establishes equilibrium, by deriving it from [37]:

$$S_t = S_0[C(1 - 3\rho^2) + \beta\rho^2] \qquad [40]$$

From [33] we now find that

$$\left.\begin{aligned} e_r &= \frac{S_0}{3k}[C(1 + \rho^2) - \beta\rho^2] \\ e_t &= \frac{S_0}{3k}[C(1 - 5\rho^2) + 2\beta\rho^2] \end{aligned}\right\} \qquad [41]$$

and from [36]:

$$k = S_0c\,\frac{C + (\beta - 3l)\rho^2}{C - 1 + (\beta - 3l)\rho^2} \qquad [42]$$

$$\text{For } \rho = 0,\ k = k_0 = S_0c\,\frac{C}{C - 1} \qquad [43]$$

$$\text{for } \rho = 1,\ k = k_1 = S_0c\,\frac{\beta - 2C}{\beta - 2C - 1} \qquad [44]$$

The boundary conditions are: on the outer circle $\rho = 1$, $\sigma_r = 0$; on the boundary of the plastic region $\rho = \gamma$, $\sigma_r = \sigma'_r$, $\sigma_t = \sigma'_t$; and in the middle of the disk $\rho = 0$, $\sigma_r = \sigma_t$ for a solid disk; or for a disk with a hole $r = a$, $\rho = \alpha$, $\sigma_r = 0$.

[12] See "Der bildsame Zustand der Werkstoffe," J. Springer, Berlin, 1927, p. 51.

[14] The letter γ in Equation [34] has another meaning from that used earlier in the paper.

but from [40], for $\rho = 1$, $S_t = S_0(-2C + \beta)$ . . . . . [45]

and from [35], for $\rho = 1$, $S_t = S_0(1 + c\gamma_1)$ . . . . . [46]

The equality of the right-hand members of [45] and [46] gives finally the boundary condition for $\rho = 1$, namely,

$$-2C + \beta = 1 + c\gamma_1 \text{ . . . . . } [47]$$

Until now the condition of compatibility of the strains [38] has not been used. By taking the values of e_r and e_t from

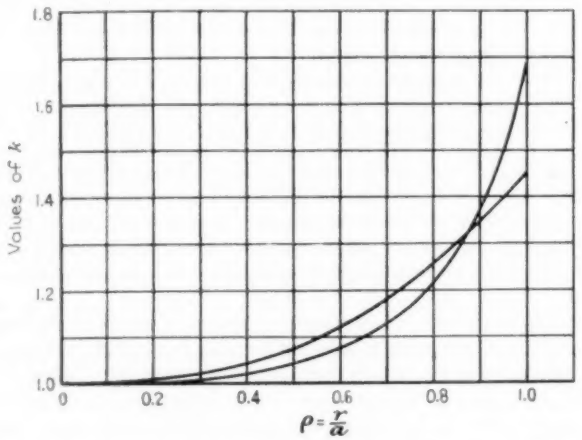


FIG. 7 FUNCTIONS OF k CORRESPONDING TO EQUATIONS [20] AND [25] FOR $\beta = 5.5$

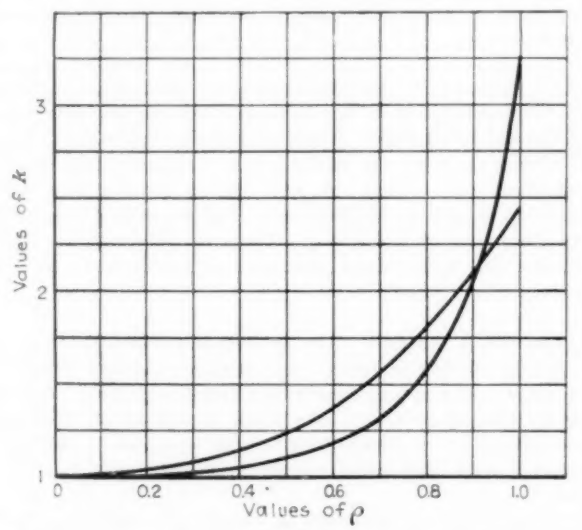


FIG. 9 CURVES OF k CORRESPONDING TO EQUATIONS [20] AND [25] FOR $\beta = 12.5$

[41], one obtains from [38] a differential equation for the function which can be readily integrated. This gives a second expression for the function k, namely,

$$k = k_0(1 + A\rho^2)^m \text{ . . . . . } [48]$$

where

$$m = \frac{8 - \beta + 8c\gamma_1}{5 - \beta + 5c\gamma_1} \qquad A = \frac{5 - \beta + 5c\gamma_1}{\beta - 1 - c\gamma_1} \text{ . . . . . } [49]$$

Had the exact function [39] been chosen for the radial stress S_r, both expressions [42] and [48] for the unknown function k would have been found equal. As this will not be the case, it is now possible for given values of γ and c to find a value of β for which expressions [42] and [48] for k differ the least. It was indeed possible to find such values of β for which the errors seemed to be not too serious, so that the expressions for the radial and tangential stresses S_r and S_t in [33] are exact enough practically to account for the effect of cold work.

It is not necessary to calculate the value of β by the method

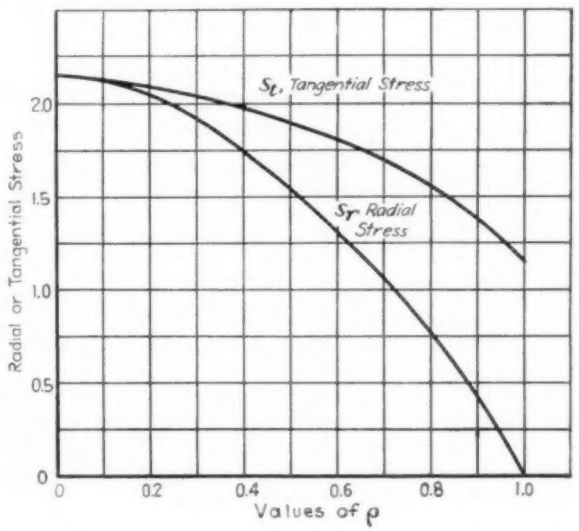


FIG. 8 RADIAL AND TANGENTIAL STRESSES IN A SOLID DISK STRETCHED BEYOND THE YIELD POINT FOR $\beta = 5.5$, $V = 1.5V_0$

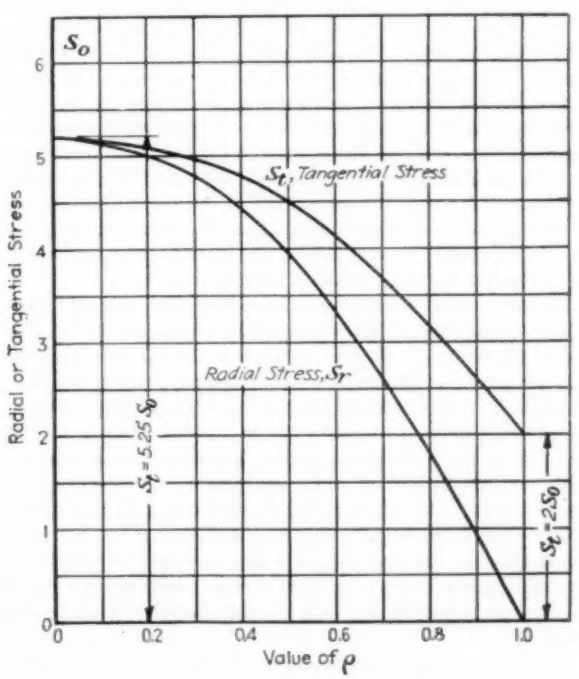


FIG. 10 RADIAL AND TANGENTIAL STRESSES IN A SOLID DISK STRETCHED BEYOND THE YIELD POINT FOR $\beta = 12.5$, $V = 2.24\ V_0$

of least squares, for by trial with several values of β one finds easily the approximate values required.

The results of two calculations for a solid disk are as follows: The value of β when the first yield occurs is

$$\beta = \frac{8}{3 + \nu} = 2.43$$

as was shown above. The circumferential speed V is then

$$V = \sqrt{2.43}\frac{gS_0}{\lambda}$$

If we call this value of $V = V_0$, then for a disk stretched on its circumference permanently by 2 per cent in a tangential direction,

$$\beta = 5.5 \qquad V = 1.5V_0$$

and for a disk stretched by 10 per cent on its circumference,

$$\beta = 12.5, \qquad V = 2.24\,V_0$$

For these two cases the corresponding distribution of the radial and tangential stresses are shown in Figs. 8 and 10. Figs. 7 and 9 show the two curves for the function k, which curves should coincide, if the assumed form for the stress distribution were exact.

As one can see, for example, from Fig. 8, the tangential stress on the circumference is 1.2 times the yield stress S_0 and at the middle point of the disk both the radial and tangential stresses are 2.15 times the yield stress S_0.

The authors wish to express their great thanks to S. M. Kintner, Manager of the Research Department of the Westinghouse Electric & Manufacturing Co., and to Mr. J. M. Lessells, in charge of the Mechanics Section, for the opportunity given them to work out these cases of plastic flow.

Discussion

S. Timoshenko.[15] It is now a general practice to run high-speed rotors, after assembly, at overspeed so as to produce a certain plastic flow at the inner bore of the rotor. In this manner not only will the yield point of the material at the bore be raised, but also certain residual stresses will be created, so that when these are combined with the stresses due to inertia forces, a more favorable stress distribution in the rotor will be obtained. There is no doubt that an analytical solution which establishes the relation between the magnitude of the speed and the radius of the region where plastic flow of metal takes place will be of practical interest in establishing the most favorable conditions for the overspeed trials of these large rotors.

The paper is of interest also from the more general point of view of stress analysis when a structure is partially under condition of plastic flow. Such conditions are not rare and sometimes they are produced at service speed, principally due to stress concentration. Till now there has not been a complete solution of the question, at what condition the flow of metal begins, although the authors speak of "exact theory" when comparing the theory based on consideration of strain energy with the usual maximum shear theory. The writer would like to suggest further development of the work in an experimental direction for clearing up this question. There is no doubt that such a work will be very beneficial, and will help very much toward establishing a rational basis for determining working stresses for each kind of stress condition.

As another desired development of the theory presented by the authors, the writer suggests looking into the stress analysis of long rotors where the problem can be considered as a two-dimensional one. There is no doubt that the overspeed treatment is more important in the case of long rotors than in the case of rotating disks.

[15] Professor of Engineering Mechanics, University of Michigan, Ann Arbor, Mich. Mem. A.S.M.E.

M. Stone.[16] In various engineering applications stresses exceed the yield point, but work is done under these conditions, because of the extreme localization of the effects. Such conditions usually occur where "stress concentrations" are present as in bolt threads, tension members with holes, dovetails, and even in some shrink fits. The case that the authors have treated has always been one where it was doubtful to judge the seriousness of the non-uniform stress distribution. Elastic considerations could never go far enough. Certain results that the authors have arrived at should be emphasized.

1 Only a 17 per cent increase in speed after the yield point is exceeded at the center will place the whole disk above the yield-point stress.

2 A disk with a hole, although resulting in a maximum elastic stress twice that of a solid disk, will hardly precipitate general failure any quicker than a solid disk.

The methods of analysis of the authors using a two-line stress-strain curve, considering the failure of material under three-dimensional stress, assuming plastic changes to be isometric in volume, etc., are all critical ones and are pertinent in making this very complex problem capable of analysis and generalization.

G. B. Karelitz.[17] The material given in the paper is very important, the stresses in large rotors being fairly high at present. It would be of interest to continue the analysis for cases where the radial stress at the rim is not zero, as is assumed in the paper. It is true that in a rotating disk the radial stress is zero at the outer rim; when, however, a turbine rotor with blades or a turbo-field with coils or poles is considered, the rim stresses are appreciable. It would be well worth while for the authors to amplify their paper covering this boundary condition, which is frequent in practice.

A. L. Kimball.[18] An interesting thing which appears in this paper is that only one of two component stresses produces yielding, the tangential stress. The yield is in all cases in the tangential direction, while the radial stress remains purely elastic. This shows quite clearly that the yielding actually must have been a sort of shearing displacement. Failures in ductile material take place through a shearing. The shearing planes are often irregular, but nevertheless they account satisfactorily for the yielding.

Alphonse A. Adler.[19] The problem of stress distribution in a rotating rim having teeth is not simple. Machine designers who made similar assumptions in the design of gear teeth[20] showed that by making models of gear teeth of transparent materials and studying stress distribution by means of Nicol prisms and polarized light, the usual assumptions were open to objections.

In looking over the literature one notes such phrases as apparent stresses, true-apparent stresses, etc. Each time it was thought that a more exact theory resulted. The paper speaks of the maximum-shear theory and also of a more exact theory. What is this exact theory?

The Society published the Shafting Committee's report. The writer served with Professor Beyer on that committee. A careful examination was made of the work of A. J. Becker, which

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[19] Consulting Engineer, New York, N. Y. Mem. A.S.M.E.

[20] Heymans and Kimball (*Mech. Eng.*, Mar., 1924, p. 129).

s referred to in this paper. Becker discarded the maximum-shear theory after making a large number of tests on shafting. This was believed to be an error, since conclusions drawn from torsion tests on solid bars are apt to be misleading. The committee took for its reason the fact that the less stressed inner layers of the bar assist the more highly stressed outer layers with the result that the shearing resistance is augmented.

Professor Beyer plotted several reported results of torsion tests on tubes. These, under twist, are likely to have a more uniform shear-stress distribution particularly when the wall thickness is small. It appeared, from the curve so obtained, that as the wall thickness of the tube became vanishingly small, the ratio of shear to tensile strength at elastic failure approached 0.5 as a limit. Becker's tests on solid bars showed this ratio to be more nearly 0.6, a value believed too high.

The writer's personal opinion is that shear failure is the only type of failure which really exists. Such notions as tensile and compressive strength are, at best, convenient through inaccurate concepts.

J. M. Lessells.[21] The writer would like to ask Professor Donnell if this theory has been worked out for a material which has no definite yield point. Suppose, for example, an alloy steel were used in the heat-treated condition where the yield point might not be so well defined. Would the same conditions apply?

With regard to Mr. Adler's remarks, it should be borne in mind that the maximum-shear theory is only applied to ductile materials. When dealing with a material like cast iron, having tensile characteristics entirely different from compression, some other theory must be applied. On these grounds the Mohr theory, as developed in Germany, which may be called a special case of the maximum-shear theory, is better. A more exact theory has since been discussed by B. P. Haigh. One cannot talk about any theory as being strictly exact. The remarks of Professor Boyd are interesting, but if the writer's understanding is correct, the maximum-energy theory is midway between the maximum-shear theory and the maximum-strain theory.

[21] Engineer, Mechanics Division, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa. Mem. A.S.M.E.

Authors' Closure

The use of the word "exact," to describe one of the theories as to the conditions under which yielding takes place, is undoubtedly open to criticism. This theory has the most evidence in its favor at the present time (of which Becker's work is only a part) but it is perhaps premature to speak of it so confidently. A great number of tests under combined stresses with thin tubes, which have been made by W. Lode and others during the last years, seem to indicate clearly that the ratio of the maximal shearing stress under pure shear to the yield stress under pure tension is for ductile materials not 0.5, as after the theory of maximum shear, but about equal to 0.56. As Dr. Timoshenko points out, this is an excellent field for experimental investigation at the present time.

It can hardly be questioned that materials can fail due to overcoming their cohesive forces, that is, due to a pure tensile failure and that brittle materials, and even ductile materials under certain conditions, do fail in this way. If the three principal stresses are equal and all three are tensile stresses, there is no shear. Hence, in this case failure cannot be attributed to shear. Therefore Mr. Adler's remark that shear failure is the only type of failure cannot be maintained. But plastic deformation, yielding, is undoubtedly a purely shearing phenomenon, as both theories used in the paper assume. The difference between them is merely as to whether shearing depends only on the maximum shearing stress or is influenced to some extent by the shearing stresses in other directions.

The same method of analysis could be used to extend the theory to include cases where the radial stress at the rim is not zero. The only difference would come at the start of the analysis in eliminating the integration constants from Equations [7] and [8] by use of the boundary conditions of the elastic region. Extension of the theory to the case of long rotors is more difficult, but is an attractive field for investigation.

APM-51-17

Stresses in Heavy Closely Coiled Helical Springs

By A. M. WAHL,[1] EAST PITTSBURGH, PA.

In this paper more exact formulas are derived for computing stress in heavy closely coiled helical springs of the type used frequently in railway work. These formulas indicate that the maximum stress in such springs may, in many practical cases, be from 40 to 60 per cent greater than the stress computed by the use of ordinary helical-spring formulas. The new formulas are verified by strain measurements, using special extensometers on semi-coils of actual springs loaded so as to simulate the axial loading of a complete spring, and on complete springs loaded in compression. The work indicates that the frequent failures of this type of spring in service may, at least in part, be due to the existence of higher stresses than has been thought possible on the basis of ordinary spring formulas.

IT IS common knowledge that heavy closely coiled helical springs, especially those having a comparatively large diameter of wire relative to the coil diameter, often fail in service under loads which are well below the safe load determined by the use of ordinary helical-spring formulas. In this paper it will be shown both analytically and experimentally that the true maximum shearing stress in springs of this type may in many practical cases be from 40 per cent to 60 per cent greater than that calculated by these ordinary formulas, the amount of discrepancy depending on the ratio of mean coil diameter to wire diameter. This maximum stress occurs at points on the wire at the extremities of the inside diameter of the coil, that is, at points a' (Fig. 1*a*). On the other hand, at points of the wire at the extremities of the outside diameter of the coil, such as points a, the stress is much smaller and may be only from one-half to one-third that existing along the inside surface of the coil at a'. For example, in the case of the two springs tested the stress on the inside of the coil was found by actual strain measurements to be respectively 2.4 and 2.3 times the stress on the outside. The fact that failure of these springs always starts along the inside of the coil at a' (Fig. 1), that is, at the point of maximum stress, is, therefore, not surprising. Although this fact has often been attributed largely to faulty heat treatment,[2] it is quite possible that the unexpectedly high concentration of stress here occurring may be sufficient to cause failure even though the metal at this point were up to standard.

A photograph showing a typical fracture in service of a heavy helical spring of the type considered in this paper is shown in Fig. 1*c*. It will be noted that failure starts from a fatigue crack which begins along the inside of the coil at a', that is, near the point of maximum stress as mentioned above and progresses at an angle of about 45 deg. to the axis of the wire.

The purposes of this paper are therefore twofold:

1 To derive formulas suitable for the practical determination of maximum stress in heavy closely coiled helical springs. These formulas will take into account both the difference in fiber length along the inside and the outside of the coil and the direct shear due to the external load.

2 To check up these formulas as far as possible by actual strain measurements on practical springs. To this end the paper will be divided into two major parts, i.e.

Part 1 The derivation of more exact formulas for stress in heavy closely coiled helical springs.

Part 2 Experiments and comparison of experimental data with the results obtained by the new formulas.

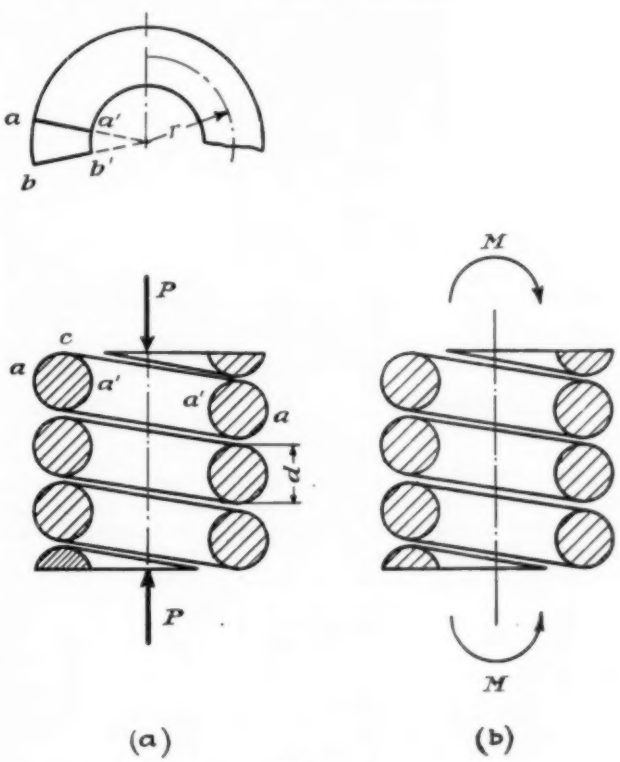


FIG. 1

PART 1 THE DERIVATION OF MORE EXACT FORMULAS FOR STRESS IN HEAVY CLOSELY COILED HELICAL SPRINGS

Consider a heavy closely coiled helical spring of mean coil diameter $2r$ subjected to an axial load P as shown in Fig. 1. The forces acting on any element $aa'\ b'b$ cut out by two neighboring radial planes may be resolved into a twisting moment Pr acting in a radial plane and a direct shearing force P acting in the direction of the axis of the spring.

The twisting moment Pr will cause rotation of the two sections aa' and bb' with respect to each other through a small angle which we will call $d\alpha$. But since the length of the fiber $a'b'$ is much less than the length of the fiber ab it is clear that the shearing strain and hence the shearing stress for any given relative angular rotation of the two cross-sections will be much greater in $a'b'$ than in ab. The stress at a' is further increased by the direct

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[2] The need for an investigation of the kind described in this paper was noted in a paper by G. M. Eaton, of the Molybdenum Co. of America, "Design from a Heat Treating Standpoint," Trans. A.S.S.T., Nov., 1927.

Contributed by the Research Special Committee on Mechanical Springs and presented at the Annual Meeting, New York, N. Y., December 3 to 7, 1928, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

shearing stress due to the axial load P. This stress corresponds to the shearing stress at the neutral axis of a cantilever beam of circular cross-section loaded with a load P.

The shear stress distribution, due to the twisting moment Pr, along a transverse diameter of the wire is somewhat as shown by the shaded area of Fig. 3, the stress at a' being much greater than that at a. To this stress at a' must be added the direct shear stress due to the external load P.

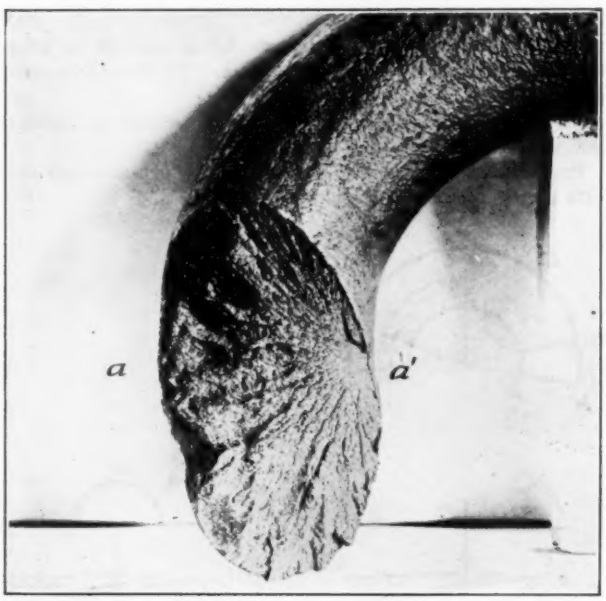


FIG. 1c

MORE EXACT HELICAL-SPRING FORMULAS

In line with the above considerations, a more exact formula for determining the maximum shearing stress at a' for the case of a helical spring axially loaded (Fig. 1) was derived. (See Appendix.) This formula is

$$S = \frac{16\,Pr}{\pi d^3}\left(\frac{4c-1}{4c-4} + \frac{0.615}{c}\right) \quad \ldots\ldots\ldots\ldots [1]$$

In this formula:

P = axial load on spring
d = diameter of wire
r = mean radius of coil
S = maximum shear stress, lb. per sq. in.
c = ratio $\frac{2r}{d} = \frac{\text{mean coil diameter}}{\text{wire diameter}}$

Evidently we have here simply the ordinary formula for stress in helical springs $16Pr/\pi d^3$ multiplied by a certain factor k which depends on the ratio c of mean coil diameter to wire diameter. The formula thus is

$$S = \frac{16Pr}{\pi d^3}\,k \quad \ldots\ldots\ldots\ldots\ldots\ldots [2]$$

where

$$k = \frac{4c-1}{4c-4} + \frac{0.615}{c}$$

We call k the stress multiplication factor.

In Table 1 are given numerical values of k for various integral values of the ratio c computed from this formula.

TABLE 1

c = mean coil diameter ÷ wire diameter	3	4	5	6	8	10
k = stress multiplication factor	1.580	1.404	1.310	1.252	1.184	1.145

Using the data of Table 1 the curve of Fig. 2 was plotted to simplify the practical application of Equation [2].

In order to apply this formula to any practical case of a helical spring loaded axially it is only necessary to compute the stress using the ordinary helical-spring formula and multiply the resulting value of stress by the stress multiplication factor k obtained from the curve of Fig. 2.

A formula for computing the stress on the outside of the coil, i.e., at a, Fig. 1, was also derived (see Appendix). This formula is

$$S' = \frac{16\,Pr}{\pi d^3}\left(\frac{4c+1}{4c+4} - \frac{0.615}{c}\right) \quad \ldots\ldots\ldots\ldots [3]$$

In case a pure bending moment M acts on the spring as shown in Fig. 1*b*, the formula for computing the maximum shearing stress is (from Equation [11] of the Appendix)

$$S = \frac{16\,M}{\pi d^3}\left(\frac{4c-1}{4c-4}\right) \quad \ldots\ldots\ldots\ldots\ldots [3a]$$

This is likewise the ordinary formula for shearing stress in a rod of circular cross-section subjected to a twisting moment M multiplied by a factor $\frac{4c-1}{4c-4}$ which depends only on the value of the ratio of mean coil diameter to wire diameter.

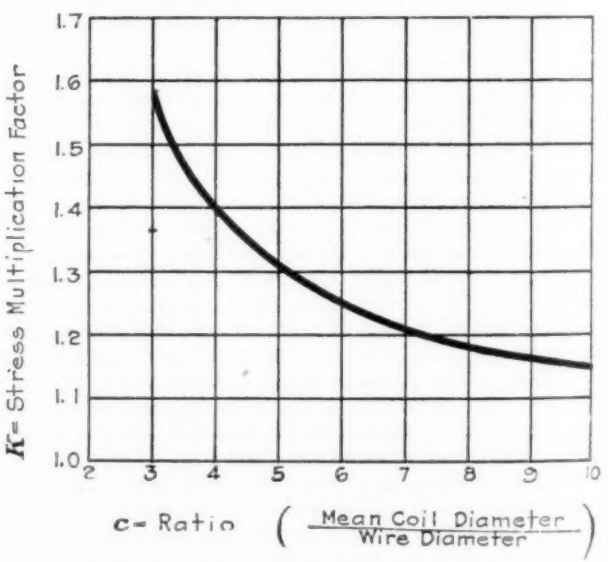


FIG. 2 CURVE FOR DETERMINING THE STRESS MULTIPLICATION FACTOR K

PROCEDURE IN DESIGN

In the practical design of heavy closely coiled helical springs, Equation [2] may be applied for the determination of stress due to axial loads. When, in addition to these axial loads, bending moments such as shown in Fig. 1*b* are acting on the spring, the stresses due to these must be superimposed on those due to the axial load. These stresses may be determined by applying Equation [3*a*]. When the load is not axially applied, as is the case in Fig. 4*a*, the spring may be considered subjected to a direct axial load P and a bending moment Pe where e is the eccentricity of the load. As mentioned above the stresses due

to this bending moment Pe should be superimposed on those due to the axial load P.

PART 2 EXPERIMENTS AND COMPARISON OF EXPERIMENTAL DATA WITH THE RESULTS OBTAINED BY THE NEW FORMULAS

In order to check the correctness of the new helical-spring formulas given in Part 1, strain measurements were made on heavy helical springs of the type used frequently in railway work. The springs tested had a ratio of mean coil diameter

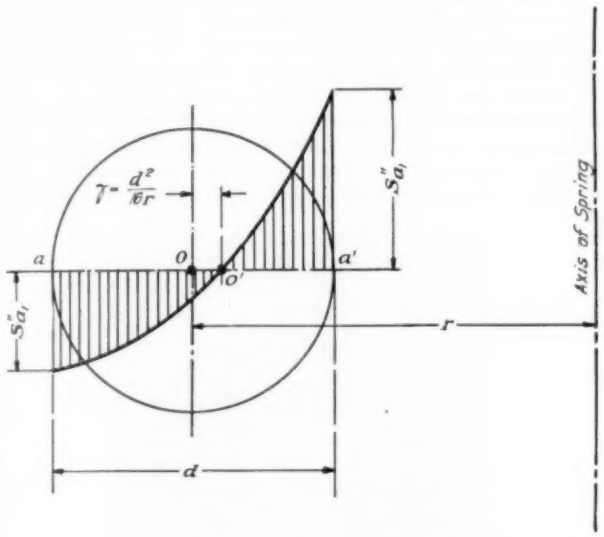


Fig. 3 Stress Distribution Along a Transverse Diameter a-a' Assuming Rotation About the Point O'

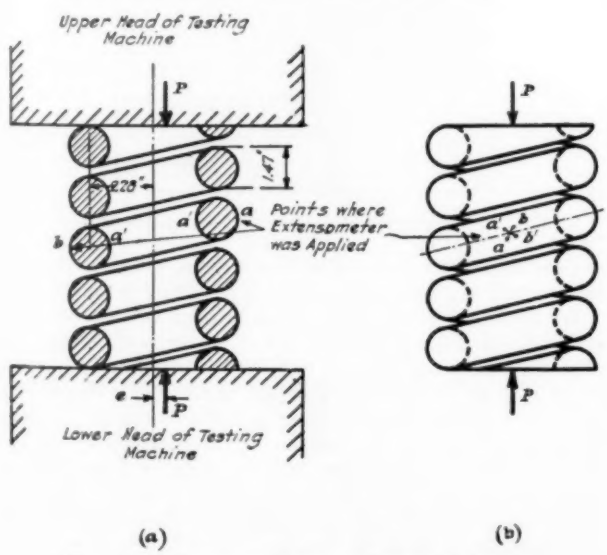


Fig. 4 Compression Test of Complete Spring

to wire diameter of approximately 3, that is, $c = 2r/d = 3$. While this ratio is somewhat lower than that existing in the great bulk of springs used, nevertheless there exist certain applications where springs of these dimensions are used. By testing a spring having a small value of the ratio c, the validity of the new formulas may be more conclusively shown, since it is in such cases that the stress determined by these formulas (which are derived by approximate methods) will deviate a maximum from the true value. Hence, if the new formulas are found approximately correct for $c = 3$ we may assume that they will also be practically correct for larger values of c, since

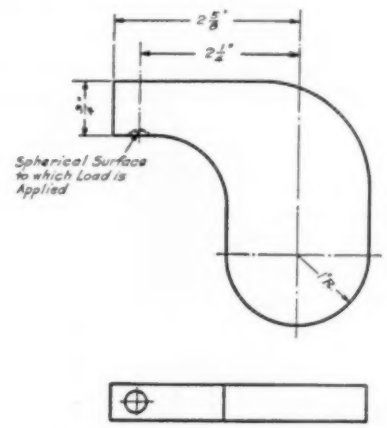
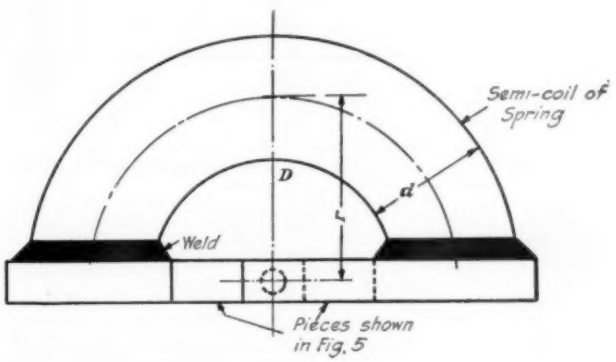


Fig. 5 Steel Arm for Welding to Semi-Coil of Spring



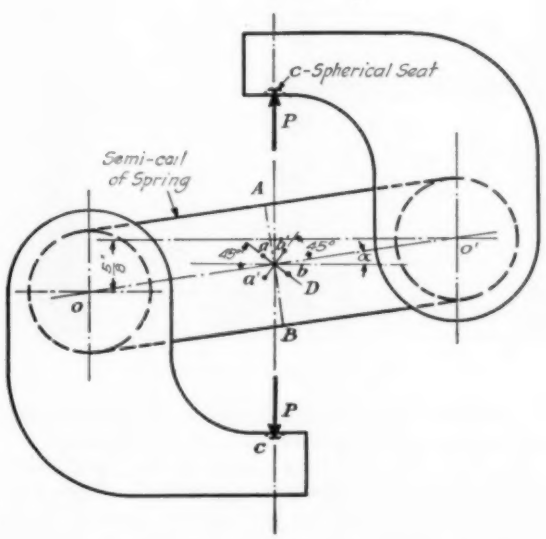


Fig. 6 Semi-Coil Spring Test Arrangement

as c becomes larger the new formulas more nearly approximate the ordinary helical-spring formulas and the assumptions on which they are based are more nearly exact.

The tests comprised strain measurements on semi-coils of

actual springs as well as strain measurements on full or complete springs. All springs tested had approximately $1^1/_2$-in. diameter wire and had a mean coil diameter of about $4^1/_2$ in.

Tests on Semi-Coils of Actual Springs

Since the dimensions of the springs tested were such that it was impossible to apply an extensometer on the inside surface of a complete spring, that is, at points a' (Fig. 1*a*), semi-coils were

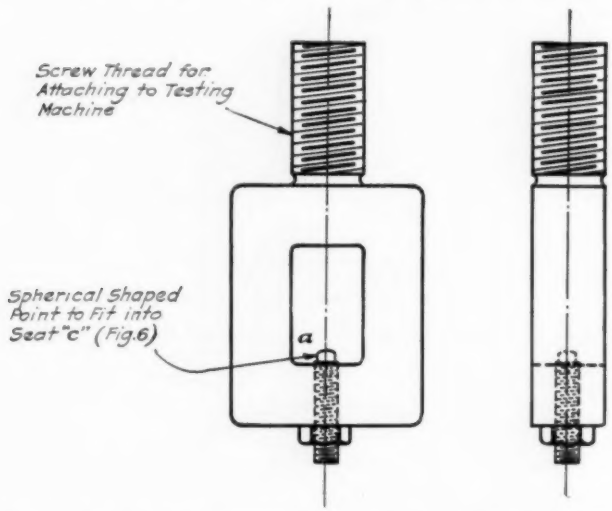


Fig. 7 Eyebolt for Applying Load to Semi-Coil of Spring

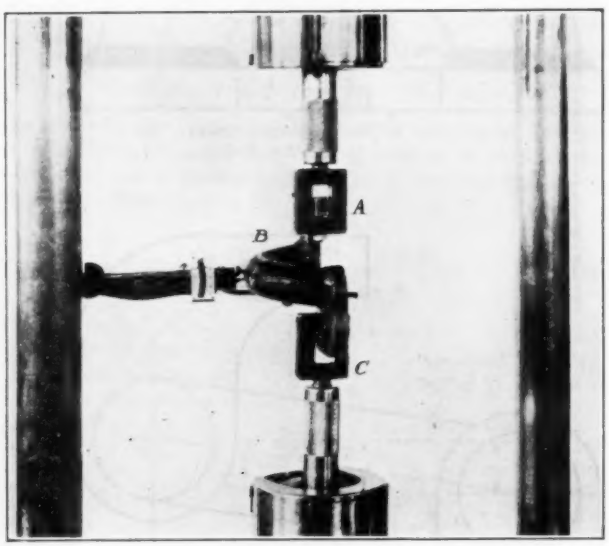


Fig. 8 Semi-Coil Test Arrangement

cut from actual springs and loaded in such a way as to simulate the actual loading of a helical spring under an axial load.

For this purpose two steel arms having the shape and dimensions shown in Fig. 5 were welded to each end of a semi-coil cut from an actual spring to form the arrangement shown in Fig. 6. The load P was applied to this arrangement through the intermediary of two eyebolts (Fig. 7) which are attached to the two heads of a testing machine. The screws a having spherical points fit into the spherical seats c (Fig. 6). In this manner two equal and opposite loads P are applied axially, as in the actual spring, while at the same time there is space for applying special extensometers along the inside surface of the coil at position D, Fig. 6. This could not have been done if a complete spring were used. The helix angle of the semi-coil was made so as to correspond approximately with the helix angle of the actual spring when loaded.

A photograph showing the test arrangement in an Amsler testing machine is shown on Fig. 8. Here the eyebolts A correspond to Fig. 7 while the semi-coil B corresponds to Fig. 6. In this case the extensometer is in position for measuring the stress on the outside of the coil, corresponding to points a (Fig. 1*a*). Fig. 9 shows the extensometer in position for measuring the stress on the inside of the coil corresponding to points a' (Fig. 1*a*) while Fig. 10 shows the position of the extensometer for measuring the stress corresponding to point c (Fig. 1*a*).

In all cases the gage length of the extensometer was 1 cm. The points were applied first at a and b, and then at a' and b', (Fig. 6) so that the strains at an angle of 45 deg. to the axis of the wire and to the line OO' were measured. Because there is a slight slope to the wire, there acts over the cross-section AB not only a twisting moment $M = Pr \cos \alpha$ where α is the helix

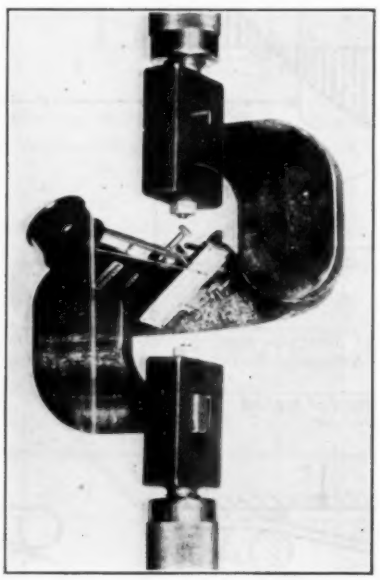


Fig. 9 Test With Extensometer on Inside of Coil

angle of the coil and a shearing force $P \cos \alpha$, but also a bending moment $M' = Pr \sin \alpha$ which acts in the plane of the coil.

We consider first the effect of the twisting moment $M = Pr \cos \alpha$ and the shearing force $P \cos \alpha$ acting across the section AB (Fig. 6). This moment and force will produce in an element at D a condition of practically pure shear. This means that we have a tensile stress S, equal in value to the shear stress, acting at 45 deg. to the direction of the shear stress, that is, in the direction of the fiber $a'b'$. Likewise for pure shear an equal compressive stress S acts along the fiber ab. The unit elongation along $a'b'$ will be[3] (from the ordinary formula for two-dimensional stress conditions):

$$e = \frac{S}{E} + \frac{1}{m}\frac{S}{E} = \left(1 + \frac{1}{m}\right)\frac{S}{E} \quad \ldots\ldots\ldots\ldots [4]$$

In this E is the modulus of elasticity and $1/m$ = Poisson's ratio for the material. Moreover, for this case (pure shear) the unit compression along ab is of the same value as the extension along $a'b'$ and is likewise given by Equation [4].

[3] See "Applied Elasticity," Timoshenko and Lessells, p. 12, regarding this formula.

On the strains produced by the twisting moment $Pr \cos \alpha$ and the shearing force $P \cos \alpha$ we now superimpose those produced by the bending moment $M = Pr \sin \alpha$ acting in the plane of the coil. This bending moment produces a relatively slight but definite elongation of both the fibers ab and $a'b'$ and of equal amounts. When these elongations are superimposed on the pure shearing strains discussed above the result is that the total elongation of the fiber $a'b'$ is slightly increased while the total compression of the fiber ab is slightly reduced by an equal amount. This was actually found to be the case in the springs tested although the difference between the measured elongation and the measured compression was never greater than about 10 per cent. In order to get the strains due only to the twisting moment $Pr \cos \alpha$ and the shearing force $P \cos \alpha$ we must, therefore, take the average between the absolute value of the compressive strain along ab and that of the tensile strain along $a'b'$. For practical purposes this is the strain due to a twisting moment Pr and an axial force P since $\cos \alpha$ for closely coiled springs has a value very nearly unity. This value of strain is then used in Equation [4] to determine the shearing stress.

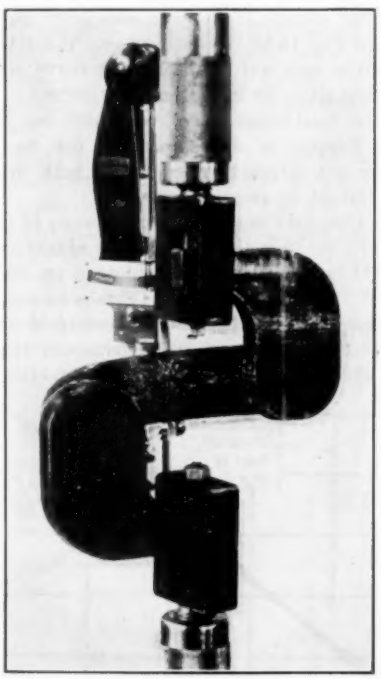


FIG. 10 TEST WITH EXTENSOMETER AT MEAN POSITION

Two semi-coils were made and tested in the following manner. Load was applied to the semi-coils in the testing machine (Fig. 8) up to a maximum of 3000 lb., readings of the extensometer being taken at 500 lb. increments. This load produced a maximum shearing stress in the coil of about 18,000 lb. per sq. in. It was not thought advisable to go to higher loads because of the possible failure of the welded joint and because an excessive pressure might be produced on the point a of the eyebolt (Fig. 7) which had a diameter of only $^{5}/_{16}$ in.

The unit compressive strain along ab (Fig. 6) and the unit tensile strain along $a'b'$ were thus determined by means of the extensometer for each 500-lb. increment of load. In line with the considerations mentioned above, the average value of the compressive strain and the tensile strain was taken as a basis for determining the value of shearing stress at each load. Values of $E = 29 \times 10^6$ lb. per sq. in. and $1/m = 0.3$ were used. The load-stress curves thus obtained for the inside and for the outside of the coil, corresponding to points a' and a (Fig. 1) respectively, are shown in full lines in Figs. 11 and 12, Fig. 11 representing semi-coil 1 and Fig. 12 semi-coil 2.

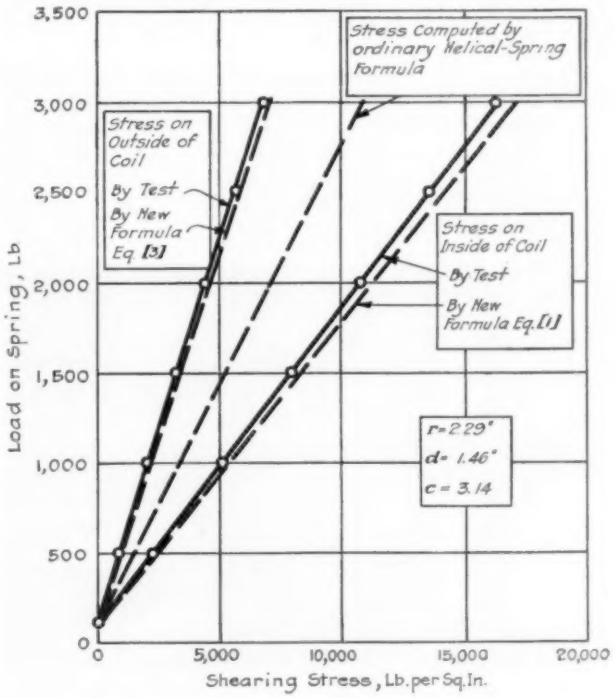


FIG. 11 LOAD-STRESS CURVES FOR SEMI-COIL 1

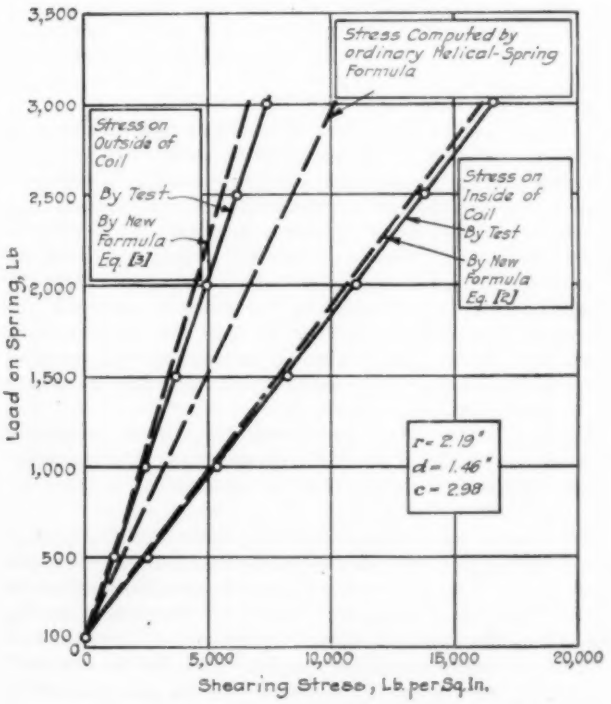


FIG. 12 LOAD-STRESS CURVES FOR SEMI-COIL 2

For comparison with the test load-stress curves, the theoretical load-stress curves computed for the inside of the coil by Equation [2] and for the outside by Equation [3] are shown

by dotted lines. It will be noted that these curves agree within a few per cent with those obtained experimentally. On the other hand, the load-stress curve computed by means of the ordinary helical-spring formula, which curve is also shown dotted, gives approximately a mean value of stress existing between inside and outside of the coil but differs widely from the true value of stress at these points.

For comparison, load-stress curves were taken also with the extensometer in the position of Fig. 10 corresponding to the measurement of stress at point c (Fig. 1). It was found that the stress at this position had approximately a mean value between that on the inside and that on the outside of the coil, and corresponded quite closely with that calculated by the use of the ordinary helical-spring formula.

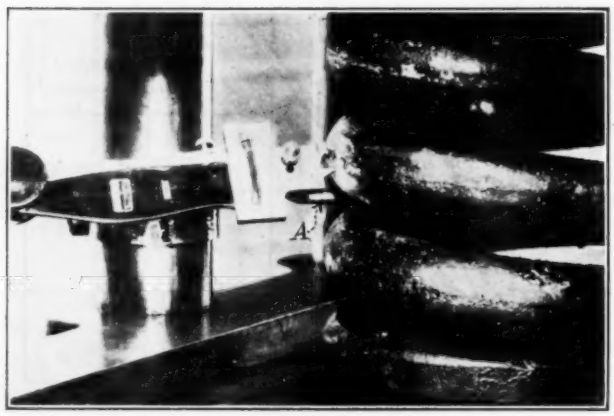


FIG. 13

These tests thus indicate that the true maximum stress in helical springs of this type axially loaded may be determined with sufficient accuracy for practical purposes by Equation [2].

TESTS ON COMPLETE SPRINGS

In order to show that the above-described tests on semi-coils were representative of tests on complete springs axially loaded, a complete spring was tested in compression as shown schematically in Fig. 4*a*. As before extensometers having a 1-cm. gage length were applied at the outside of the coils so as to measure strains at 45 deg. to the axis of the wire, the points of the extensometer having positions represented by the points ab and $a'b'$ (Fig. 4*b*). A photograph showing the spring in position in the testing machine is shown in Fig. 13. Here the clamp A which holds the extensometer in place is shown projecting between the coils of the spring and serves to hold the points firmly against the surface of the coil.

Readings of the extensometer were taken at increments of 1000 lb. up to 10,000 lb. load, corresponding to a maximum shearing stress of about 60,000 lb. per sq. in. on the inside of the coil and 25,000 lb. per sq. in. on the outside.

It was found, as in the case of the semi-coils tested, that because of the helix angle of the coil the tensile strain between the points at a and at b (Fig. 4*b*) was slightly greater than the compressive strain between the points at a' and at b'. To get the shearing stress from the measured strains between a and b and between a' and b' we proceed exactly as was done above for the semi-coils by taking the average of the values of both the compressive strain between a' and b' and the tensile strain between a and b and using this value in Equation [4] with a modulus $E = 29 \times 10^6$ lb. per sq. in. and Poisson's ratio $1/m = 0.3$.

The load-stress curves thus obtained at points a and b (Fig. 4*a*) (which were on opposite sides of the spring) are shown in Fig. 14, the stress at b being slightly larger than that at a. The load-stress curve, computed by Equation [3] for stress on the outside of the coil in a spring loaded axially and using the dimensions given in Fig. 4*a* is shown by the dotted line.

The reason the stress at b is slightly greater than that at a may be explained by the fact that it is almost impossible to load a helical spring exactly centrally because a slight amount of play in the guides of the testing machine and because of irregularities in the flattened ends of the spring. Hence the resultant load P on the spring is not axial but is displaced a slight amount ϵ from the axis as shown in Fig. 4*a*. This loading condition is equivalent to an axial force P and a bending moment $M = P\epsilon$. The axial load P will produce shearing stresses at a and b which may be computed by Equation [3]. In addition, the bending moment $P\epsilon$ will produce a slight stress which may be determined from Equation [3*a*]. This stress will increase the stress at b by a small amount and decrease that at a by an equal amount. To get the value of shear stress due only to the axial load P, we must therefore take the mean value of shear stress at a and b. This was done for each load and the resulting load-stress curve is given on Fig. 15. For comparison the load-stress curve for stress on the outside of the coil, computed using Equation [3], is shown on Fig. 15 by the dotted line. It will be noted that this curve agrees very well with the test curve, and indicates that the formula given by Equation [3] is correct. As a matter of interest the load-stress curve computed by the ordinary helical-spring formula is also shown. It can be clearly seen that while the new formula agrees closely with the test curve, the ordinary formula is greatly in error.

Another fact brought out by an examination of the test load stress curve of Fig. 15 is that it coincides almost exactly up to a load of 3000 lb. with the curve obtained on semi-coil 1 for the outside of the coil (Fig. 11). (This semi-coil had nearly the same values of r and d as did the complete spring.) The coincidence of these curves shows conclusively that the test-arrangement shown in Fig. 8 simulates the condition of a helical

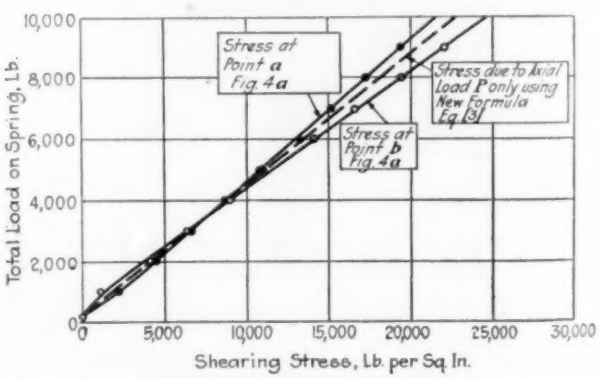


FIG. 14 LOAD-STRESS CURVES ON COMPLETE SPRING LOADED AS IN FIG. 4*a*

spring axially loaded. The important point to note is that had it been possible to take strain measurements along the inside of the complete spring at points a' (Fig. 4*a*), there is every reason to believe that at a given load we would have found the same stress as obtained on the inside of semi-coil 1, that is, a stress from 50 to 60 per cent greater than that computed by the ordinary helical-spring formulas, but in good agreement with the stress computed by Equation [2].

THE PRACTICAL SIGNIFICANCE OF THE NEW FORMULAS

Let us consider what the new formulas given in this paper mean in the practical design of springs. Take, for example,

the case of railway passenger-car springs. These are often designed so that the shearing stress at their working load computed by ordinary helical-spring formula is about 65,000 lb. per sq. in. Applying the more exact formulas given in this paper (see Equation [2]) to the case of these springs which commonly have a ratio of mean coil diameter to wire diameter of 4, we find that the actual shearing stress is 90,000 lb. per sq. in. at the working load. On this stress at the working load may be superimposed a vibratory stress due to a vibrating load which may possibly at times, as when passing over track irregularities, be sufficient to compress the spring nearly solid. These springs are often designed for a shearing stress when compressed solid of 110,000 lb. per sq. in. computed by ordinary formulas, or 155,000 lb. per sq. in. computed by the more exact formulas (Equation [2]). This does not mean that this stress may actually exist in these springs when they are compressed solid, since some yielding of the material at the point of maximum stress may take place and when the load is removed residual stresses of opposite sign will be induced. However, these residual stresses will be such that the range of stress between no load and solid load is approximately 155,000 lb. per sq. in., even if the maximum is not. Since even good spring steel does not have an endurance limit[4] greater than about 60,000 lb. per sq. in. in torsion or a total range of 120,000 lb. per sq. in. it seems not surprising to the writer that these springs so frequently fail in service.

Furthermore, the coil diameter of helical springs is often reduced with the idea of decreasing the stress. The formulas brought out in this paper show that there is a limit to the gain which may be possible by this procedure. For example take the case of a spring where the ratio c of mean coil diameter to wire diameter is 4. By decreasing the coil diameter so that $c = 3$,

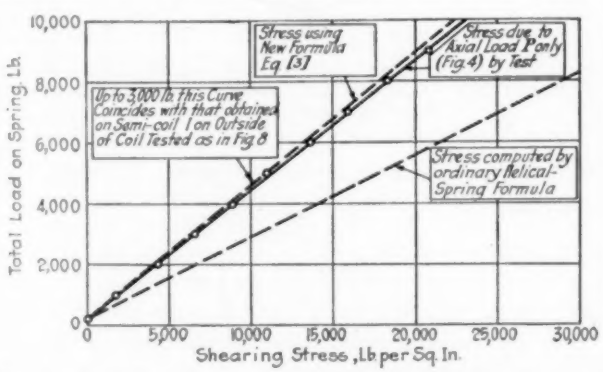


FIG. 15 LOAD-STRESS CURVES FOR COMPLETE SPRING LOADED AS IN FIG. 4*b*

the twisting moment Pr is reduced about 25 per cent but at the same time the stress multiplication factor k is increased about 15 per cent. Consequently the net reduction in stress is not 25 per cent as would be expected using ordinary spring formulas, but considerably less. At the same time, reducing the diameter has the serious disadvantage of greatly reducing the flexibility of the spring, the flexibility being proportional to the cube of the coil diameter.

SUMMARY AND CONCLUSIONS

In this paper it has been shown that the true shearing stress in heavy closely coiled helical springs axially loaded may in many practical cases be from 40 to 60 per cent greater than that calculated by ordinary helical-spring formulas. Certain more exact formulas have been derived for facilitating the practical calculation of maximum shearing stress in springs of this type. These new formulas have been checked by means of strain measurements on semi-coils of actual springs and on full springs loaded in compression.

It should be noted that the new formula herein given applies not only to the calculation of heavy springs having a small ratio c of mean coil diameter to wire diameter, but also to light springs where the value of c is large. In any case the new formula applies with greater accuracy than the ordinary helical-spring formula; however, where the ratio of diameters c is greater than 6, the ordinary formula is accurate within 25 per cent as may be seen by reference to the curve of Fig. 2.

Since these new formulas have been thus shown to be correct, it is the writer's opinion that they should be applied in the practical design of helical springs of this type. It is also the writer's belief that the frequent failures of heavy helical springs may at least in part be explained by the existence of much greater stresses than thought possible, on the basis of ordinary spring theory.

The writer desires to thank Drs. Timoshenko and Nadai and other Westinghouse engineers for the suggestions made.

Appendix

DERIVATION OF FORMULAS FOR STRESS IN A HEAVY CLOSELY COILED HELICAL SPRING

This derivation[5] is based on several assumptions which are not rigidly correct, but which, it is believed, will give results accurate enough for practical purposes.

In the first place, since the springs with which we are dealing are closely coiled, it will be sufficiently accurate to assume that the helix angle of the spring is zero. Furthermore it may be shown that a change in slope from zero to 10 deg. does not change the resulting maximum shear stress more than a few per cent.

Consider an element of an axially loaded helical spring (Fig. 16). This element is formed by two neighboring cross-sections aa' and bb' and having a mean radius of curvature r.

In a closely coiled helical spring axially loaded, we may resolve the forces acting on this element into a twisting moment $M = Pr$ acting in a radial plane and a direct axial shearing force P. We will consider first the stresses set up by this twisting moment, and on these will later superimpose the stresses due to the direct shear load P.

Under the action of this twisting moment M the two cross-sections aa' and bb' (Fig. 16) rotate with respect to each other, through a small angle $d\alpha$. Since the length of the fiber ab is much greater than the length of the fiber $a'b'$, the shearing deformation per unit length of the fiber $a'b'$ will be greater than that existing in the fiber ab. Hence the stress in the fiber ab will be less than that in the fiber $a'b'$.

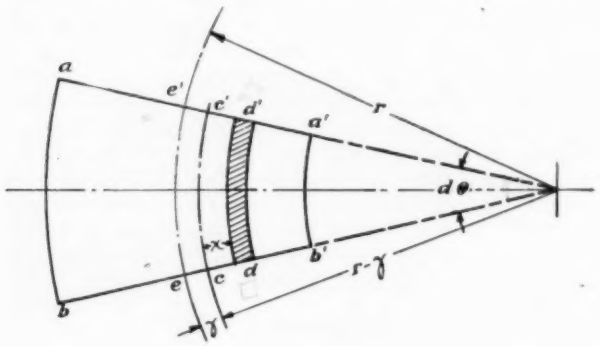
We now consider the shear stresses S acting over the cross section to be divided into two components, one, which we denote by S_a, in a direction parallel to the axis of the spring and the other, which we denote by S_t, in a direction perpendicular to the axis of the spring (Fig. 16*b*).

If we assume that the two neighboring cross-sections aa' and bb' rotate relative to each other and about an axis ee' perpendicular to their surfaces and passing through their centers O,

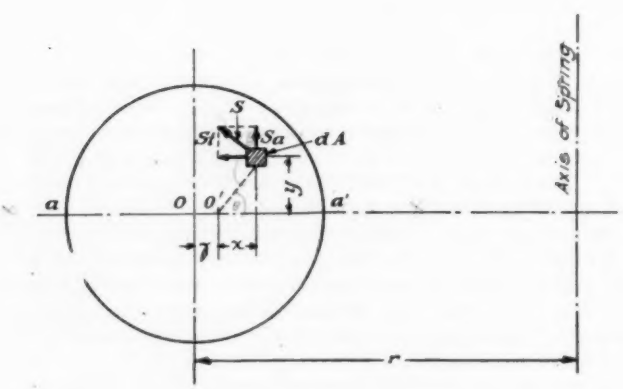
[4] See "The Fatigue of Metals," by H. J. Gough, p. 169, and University of Illinois Bulletin No. 142, Engineering Experiment Station, by H. F. Moore and T. M. Jasper, p. 49, for data on endurance limits in torsion for high-carbon and nickel-chromium steels which are commonly used for springs.

[5] The approximate solution of this problem here given differs in several particulars from an approximate derivation given by A. Röver, "Beanspruchung Zylindrische Schraubenfederr mit Kreisquerschnitt," Zeitschrift des Vereines deutscher Ingenieur, 1913, p. 1907. The final result is also different but agrees within 0.7 per cent with the results of a more exact but very complicated solution worked out by Röver for the particular case of a spring where the ratio c of mean coil diameter to wire diameter was 4.

the distribution of the axial components of stress S_t over the cross-section will be somewhat as shown by the shaded area of Fig. 17. It is clear that since we are considering only the effects of a twisting moment M this stress distribution would be impossible, since the area to the right of O is greater than that to the left of O and hence a resultant external force is produced. However, if we assume that rotation occurs about some point O' (Fig. 3) displaced toward the axis of the spring, instead of about point O, we obtain a stress distribution which consists of a pure couple only and which may be in statical equilibrium under the action of the moment M. From considerations of



(a)



(b)

FIG. 16

symmetry the transverse stress components S_t will be in statical equilibrium when rotation occurs about any point on the axis xx. We now determine the point O' as follows.

Under the assumption of rotation about O' the stress S acting on any element dA (Fig. 16*b*) having the coordinates x and y may easily be found. When the sections aa' and bb' have rotated through an angle $d\alpha$ with respect to each other, the relative movement of the ends of the filament dd' corresponding to the element dA will be $d\alpha\sqrt{x^2+y^2}$ and since the length of the filament is $(r-\gamma-x)d\theta$, the shearing stress S acting on this element will be:

$$S = \frac{Gd\alpha\sqrt{x^2+y^2}}{(r-\gamma-x)d\theta} \quad \ldots\ldots [5]$$

The axial components Sa of the stress S will be (Fig. 16*b*), using Equation [5]

$$S_a = S\frac{x}{\sqrt{x^2+y^2}} = \frac{xGd\alpha}{(r-\gamma-x)d\theta} \quad \ldots\ldots [6]$$

Since y does not appear in this equation the values of S_a depend only on the value of x and are independent of the value of y. This equation is identical with the equation of stress distribution in a curved bar[6] and the distribution of the components S_a is hyperbolic. We may, therefore, apply curved-bar theory to this problem. The distance γ must be determined as in curved-bar theory so that the integral of $S_a dA$ taken over the cross-section is zero. For γ we use the known approximation:[7]

$$\gamma = \frac{d^2}{16r}\left(\frac{1}{1+\frac{d^2}{16r^2}}\right) \approx \frac{d^2}{16r} \quad \ldots\ldots [7]$$

We neglect $\frac{d^2}{16r^2}$ since $\frac{d}{2r}$ is seldom greater than $\frac{1}{3}$ in practical

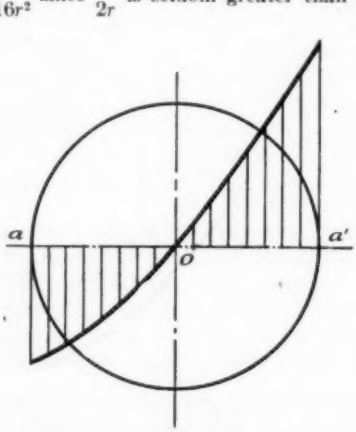


FIG. 17 SHEARING-STRESS DISTRIBUTION ALONG A TRANSVERSE DIAMETER a-a' ASSUMING ROTATION ABOUT THE CENTER POINT O OF THE CROSS-SECTION

cases; hence, $\frac{d^2}{16r^2}$ is small compared to unity. Putting [7] in [6] we get:

$$S_a = \frac{xGd\alpha}{\left(r-\frac{d^2}{16r}-x\right)d\theta} \quad \ldots\ldots [8]$$

From the ordinary formula for angle of twist of circular sections, the value of $d\alpha$ is:

$$d\alpha = \frac{32Mrd\theta}{G\pi d^4} \quad \ldots\ldots [9]$$

Putting this in [8] we get:

$$S_a = \frac{32xMr}{\pi d^4\left(r-\frac{d^2}{16r}-x\right)} \quad \ldots\ldots [10]$$

It is clear that the maximum value of S_a will occur when $x = \frac{d}{2}-\frac{d^2}{16r}$ that is, at point a' Fig. 16.

Putting this value in [10] and also putting $c = 2r/d$ we get:

$$S'_{a1} = \frac{16M}{\pi d^3}\left(\frac{4c-1}{4c-4}\right) \quad \ldots\ldots [11]$$

[6] See for example "Applied Elasticity," Timoshenko and Leasells, p. 217.

[7] *Loc. cit.*, Equation [216], p. 224.

Likewise the stress at point a (Fig. 16) will be (putting $x = -\frac{d}{2} - \frac{d^2}{16r}$ in [10] and taking $c = \frac{2r}{d}$)

$$S'_{a2} = \frac{16M}{\pi d^3}\left(\frac{4c+1}{4c+4}\right) \quad [12]$$

We here take the absolute value of S'_{a2}, in other words, we drop the negative sign, which signifies merely that the direction of the stress S'_{a2} is opposite to that of S'_{a1}.

These are the shearing stresses which would be produced by a pure bending moment M acting on the spring as in Fig. 1*b*.

In the case of an axial load P acting on the spring we may, as mentioned above, replace the external forces acting over the cross-section of Fig. 3 by a twisting moment $M = Pr$ and a direct shear force P. The stress, due to the moment M, may be found by substituting $M = Pr$ in Equations [11] and [12]. We then obtain:

$$S''_{a1} = \frac{16\,Pr}{\pi d^3}\left(\frac{4c-1}{4c-4}\right) \quad [13]$$

$$S''_{a2} = \frac{16\,Pr}{\pi d^3}\left(\frac{4c+1}{4c+4}\right) \quad [14]$$

S''_{a1} and S''_{a2} are thus the shear stresses at points a' and a (Fig. 3), respectively, due only to the twisting moment. On these we must superimpose the shearing stress at a and a' due to the axial shear load P (Fig. 3). For this purpose we may use the value given by the theory of elasticity of shearing stress at the outer edges of the neutral surface of cantilever having a circular cross-section of diameter d and loaded with a load P. This exact value is[8] $\frac{1.23\,P}{\pi d^2/4}$. In this connection we note that this shear stress acts in the same direction as the stress S''_{a1} but in an opposite direction to the stress S''_{a2}. We therefore add and subtract this value to Equations [13] and [14] respectively, to obtain the true resultant values of stress at a' and a. The maximum stress at a' then becomes:

$$S_{a1} = \frac{16\,Pr}{\pi d^3}\left(\frac{4c-1}{4c-4} + \frac{0.615}{c}\right) \quad [15]$$

while the stress at a is:

$$S_{a2} = \frac{16\,Pr}{\pi d^3}\left(\frac{4c+1}{4c+4} - \frac{0.615}{c}\right) \quad [16]$$

Discussion

R. Eksergian.[9] The author is to be congratulated on a very interesting analysis and carefully conducted experimental verification of the nature of stress concentration in helical springs.

From an examination of the upper diagram of Fig. 1, we might anticipate with a relative rotation of sections $a - a'$ and $b - b'$, that a necessarily greater shear must take place along the inner edge, and this condition will increase with a smaller radius of helix and larger wire diameter. The condition would suggest the analogy to the bending action of a curved bar and corresponding stress concentration at the inner edge.

The ingenious feature of the analysis is Equation [6] and the recognition that this equation is identical in form to the equation for the tensile stress in a curved beam. The location of the offset center of rotation corresponding to the neutral axis of a curved beam follows in a way analogous to the curved-beam theory, because, since we have pure torsion, the small vertical shear being superimposed later, the summation of the vertical components of the shear is nil. That is,

$$\int S_a dA = \frac{Gd\alpha}{d\theta}\int \frac{x}{r'-x}\,dA = 0$$

where $r' = r - \gamma$ = the radius to the neutral axis.

It is interesting to compare the similarity of the equations for bending of a curved beam and for torsion of a curved wire. In both cases we assume plane sections to remain plane after bending and torsion, respectively. If $\Delta d\phi$ is the change in angle for a slice of a beam subtending an angle $d\phi$, x is the distance from the neutral axis, and r' its radius from the initial center of curvature, then for a curved beam we have:

$$\int p dA = \frac{E\,\Delta d\phi}{d\phi}\int \frac{x dA}{r'-x} = 0 \quad [17]$$

$$\int py dA = \frac{E\,\Delta d\phi}{d\phi}\int \frac{x^2 dA}{r'-x} = M_b \quad [18]$$

From [17] we may obtain r' or r, which substituted in [18] permits us to solve for $E\frac{\Delta d\phi}{d\phi}$, and thus we may determine the stress, since

$$p = E\frac{\Delta d\phi}{d\phi}\frac{x}{r'-x}$$

For the torsion of a curved wire, we have approximately:

$$S = \frac{Gd\alpha}{d\theta}\frac{\sqrt{x^2+y^2}}{r'-x}$$

with radial and tangential components

$$S_r = -\frac{Gd\alpha}{d\theta}\frac{y}{r'-x}$$

and

$$S_t = S_a\frac{Gd\alpha}{d\theta}\frac{x}{r'-x}$$

The corresponding equations are:

$$\int S_t dA = \frac{Gd\alpha}{d\theta}\int \frac{x}{r'-x}\,dA = 0 \quad [17a]$$

$$\int (S_t x + S_r y)\,dA = \frac{Gd\alpha}{d\theta}\int\left(\frac{x^2}{r'-x} + \frac{y^2}{r'-x}\right)dA = M_t \quad [18a]$$

Substituting r' from [17*a*] in [18*a*], we obtain $\frac{Gd\alpha}{d\theta}$ and thence the corresponding torsional shear.

The location of the neutral axis follows from the solution of the integral

$$\int \frac{x}{r'-x}\,dA = 0$$

so that we may use the solutions already worked up for the ordinary curved-beam theory.

Comparing Equation [18*a*] with Equation [18], we note an additional term in the integral, so that the evaluation of $\frac{Gd\alpha}{d\theta}$

[8] This value may be derived using the formulas given by Love, "The Mathematical Theory of Elasticity," chapter XV.

[9] Engineer, Baldwin Locomotive Works, Philadelphia, Pa. Mem. A.S.M.E.

for torsion is more complicated than the evaluation of $\frac{E\,\Delta d\phi}{d\phi}$ for bending.

A considerable simplification has been made by the author in Equation [9] by assuming the angle of twist of adjacent sections to be approximately the same as if the neutral axis were at the center. This assumption is apparently justified by the close experimental verification.

Thus, to recapitulate, the two simplifying assumptions the author has made which differ from a more rigid analysis are (1) plane sections remain plane in torsion and (2) the effect of the small shift of neutral center from geometric center of wire for ordinary proportions of springs permits the assumption of calculating the angle of torsion from the simple torsion formula for round wires.

In considering the experimental phase of this work the problem resolves itself to measure the diagonal tensile and compressive stresses (which are numerically equal to the shear stresses) for a differential cube at the inner and outer portions of the

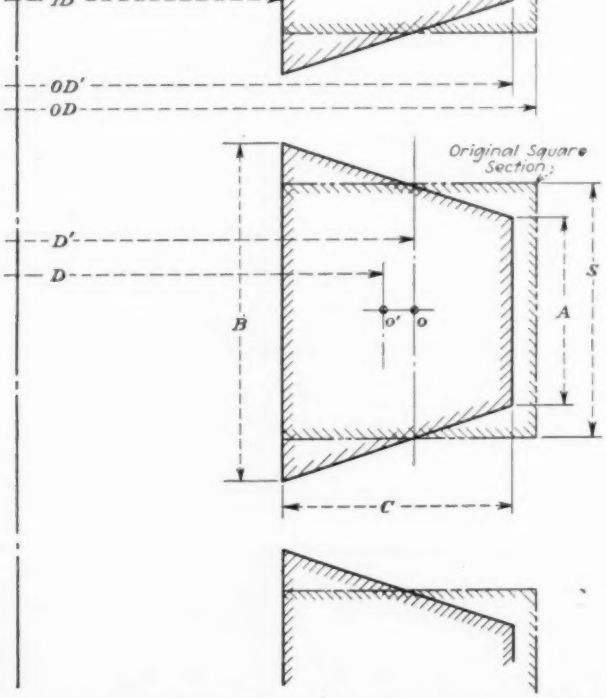


FIG. 18 DEFORMATION IN HELICAL SPRING MADE OF SQUARE WIRE

wire of the helix. Obviously, the choice of a small gage length is desirable, since with increasing gage lengths serious errors due to curvature and variation of stress would vitiate the results. On the other hand, the smallest satisfactory measurement experienced by the writer of a mechanical strain gage appears around 0.00005 in. With a 2-in. gage length, the stress can be measured to 750 lb. per sq. in., with a 1-in. gage length to 1500 lb. per sq. in., and with a half-inch gage length to 3000 lb. per sq. in. Of course, by interpolations and repeated workings, a closer accuracy could undoubtedly be obtained. The selection of an inch gage length would seem to the writer to place a limitation of 1000 lb. per sq. in. minimum error for the gage readings. With workings at 18,000 lb. per sq. in., this introduces an error of approximately 5 per cent, due to the instrument readings.

Another factor is the proper value of Poisson's ratio. In comparing the ratios of inner and outer stresses, this factor would not be so important.

The simplest procedure of increasing the accuracy of the experimental work is to operate with much larger stresses. This incidentally would be a very good test for a welded joint for the extension arms.

On the whole, this paper is a very creditable piece of work. The paper moreover represents a procedure that should be very much encouraged and that is a sufficient approximation in an analysis combined with experimental verification, and finally the boiling down of the work to suitable design formulas.

The formulas and tables suggested should be incorporated in the A.S.M.E. Spring Code, since the results appear sufficiently accurate for the proportions used in ordinary spring proportions.

T. C. RATHBONE.[10] The author's analysis of maximum stresses in helical springs and his substantiation of theory by actual experiment leave no other alternative than to accept the fact that the usual spring formulas are inadequate. His correction to be applied to the usual formulas is so simple and direct that it may well be adopted in future handbooks and textbooks.

It would be interesting to learn how this correction applies to such springs of square section. When closely coiled springs are made of square wire, a considerable deformation occurs, the section becoming trapezoidal, as shown in Fig. 18. The inner side is widened while the outer side is shortened. As the maximum stress occurs on the inside of the coil, the widening from deformation may, partially at least, prevent the unequal stress distribution as shown for round wire in Fig. 3, and thus lessen the correction to be applied.

M. STONE.[11] The question of the action of a closely coiled spring has always been the subject of analyses and researches concerning deflection characteristics, elastic characteristics, stress distributions, failures under various types of loading, and similar problems. Nevertheless, the action of this important mechanical element is not completely known. Such considerations have included variation of pitch angle, ratio of wire diameter to coil diameter, ratio of height of coil to coil diameter, lateral stability, type of wire section, and allied considerations.

The author's thesis is one that has been attracting the attention of engineers and designers of heavy coil springs for locomotive suspension, shock absorbers, and the like for some few years. The recognition of a hyperbolic distribution of shear stress in coil springs was first made by A. Röver,[12] although some consideration of stress distribution in closely coiled springs was made earlier by Siebeck, also in Germany, Röver worked out an approximate analysis of the problem, and justified his assumptions by giving a rigorous solution from the mathematical theory of elasticity. His work must be considered for its full worth, and particularly must he be credited with the recognition of the problem, which is in the nature of a major advance in thinking on the subject.

Mr. Wahl's experimental verification of the existence and quantitative measurements of the Röver distribution of shear stress is a valuable extension of the subject. Such work brings the more theoretical consideration out of the realm of hypothesis into the field of reality, and forces it to the attention of designing

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[10] Experimental Engineer, Westinghouse Elec. & Mfg. Co., South Philadelphia, Pa.

[11] Research Mechanical Engineer, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

[12] "Beanspruchung Zylindrischer Schraubenfedern mit Kreisquerschnitt," *Z.V.D.I.* 1913, p. 1906.

engineers. The tests are ingeniously devised, the use of the Heaggenberger extensometers demonstrating the value of such instruments.

A more practical and perhaps a more important test of the existence of high stress concentrations on the inside of the coil springs would be to subject such springs to fatigue loading, placing the spring in a Haigh fatigue machine, or some more suitably designed apparatus. Any high stress always shows up in a fatigue test; at the same time, the test would give some definite information on how highly we may stress the springs. It must be recognized that a knowledge of the stress distribution and that of failure are ideas not always simply connected. Complexity is introduced in *this* problem immediately when it is seen that although the high shear-stress values are on planes perpendicular to the axis of the wire, the spring failure that Mr. Wahl shows in Fig. 1c occurs on a plane 45 deg. to the axis. There are many other similar instances.

In the appendix, Mr. Wahl has derived an expression for the stress at the inner and outer fiber. His approximate method differs somewhat from that given by Röver, but the results are close. In any approximate method, it is always a question of what are the conservative assumptions. In the author's transition from Equation [8] to [10] he uses Equation [9] which is derived for straight bars under twist. To maintain consistency, the similar expression derived from consideration of the curved bar should be used. This becomes, when we realize that the stresses integrated over the cross-section around the center of twist must balance the external moment,

$$M = \int\int S\sqrt{x^2 + y^2}\,dx\,dy$$

$$= \frac{Gd\alpha}{d\theta}\int_{-\frac{d}{2}-\gamma}^{\frac{d}{2}-\gamma}\int_{-\sqrt{\frac{d^2}{4}-(\gamma+x)^2}}^{+\sqrt{\frac{d^2}{4}-(\gamma+x)^2}} \frac{x^2+y^2}{r-\gamma-x}\,dx\,dy$$

The integration of this will give an expression similar to that of [9] but which may depart by more than 5 per cent for certain values of c.

Although all derivations have been carried through with the pitch angle assumed to be zero, any departure from this will increase the magnitude of the stresses. For a ratio of $\frac{d}{2r} = \frac{1}{3}$, the pitch of the helix can be no less than $\tan^{-1} 0.17 = 10$ deg., and in Röver's original work, this corresponds to an increase in stress of about 10 per cent. It is significant to note, however, that the stress never exceeds more than 25 per cent, even for what we ordinarily call "open coil springs."

Joseph K. Wood.[13] This paper is one of the most valuable contributions made in recent years to the mechanical-spring art. In this instance, the testing specialist has been called in and he has successfully diagnosed what might be called the spring-index ill of the art. The situation regarding the spring index (ratio of mean coil diameter to bar diameter) previous to the presentation of Mr. Wahl's paper, was as follows:

a It was known for many years that springs with low indexes were stressed considerably beyond the calculated values due to such causes as the relative increase in vertical shear. For example, it has been known that vertical shear in a helical spring having an index of 3 is equal to 16 per cent of the regular torsional shear, while in a spring having an index of 8 the vertical shear is equal to only 6 per cent of the torsional shear. This increase in stress, however, was insufficient to account for breakages obtained in service, and consequently it has been considered good practice to avoid indexes in design less than 5, and preferably 7. In cases where low indexes have been unavoidable, due to space and other limitations, the best practice has been to use low stresses as calculated by the usual formula, but never as high as 65,000 lb.

b The complete solution of the "index" problem has been delayed for two reasons, namely, the limited use of the advanced theory of elasticity in this country, and to the lack of adequate testing equipment for the confirmation of the theory. It might be opportune at this point, to suggest the adoption of a regular course on the advanced theory of elasticity by the various technical universities throughout the country. Regarding the theoretical solution of the "index" problem, A. Röver in 1913 accomplished this important task by a slightly different and somewhat more direct method than that described in Wahl's paper. Röver obtained practically the same curve illustrated in Fig. 2 of Wahl's paper, which gives the relation between the index c and the stress multiplication factor k.

It took a period of 15 years since the publication of Röver's results, to produce a testing genius like Wahl to prove the truth of Röver's theoretically deduced curve, by the proper set-up of equipment. The results of this research are particularly gratifying to the writer because they represent a response to an appeal made by him in his paper "A Code of Design for Mechanical Springs" several years ago, to the effect that the constants in the present spring formulas be more accurately determined. To quote from this paper, "the entire question of whether the constants in the present spring formulas are sufficiently accurate in all cases, is in need of considerable research."

Wahl's derivation of the new formulas, differing in certain respects from Röver's, gives an expression for k dissimilar in the arrangement of the terms, but with both yielding practically the same calculated results.

The discovery which led to the solution of the "index" problem, and which is the basic assumption in the derivation of the new formula, was that for a given angle of twist the fiber on the extreme inner side of the coil is very much shorter than the fiber on the extreme outer side of the coil, which gives rise to differences of appreciable magnitude in the unit deformations for those fibers. Röver deserves full credit for this discovery and for the resulting formula. It required experimental confirmation of Röver's work, however, to provide that confidence which industry requires for the practical application of new formulas. Wahl without question has provided the required experimental proof of Röver's results. In attempting to improve upon Röver's derivation of a formula, Wahl resorts to the indirect use of the flexural curved-beam formula to overcome certain mathematical difficulties. From a mathematical point of view this is perfectly proper, but in the average case, something is likely to be lost in the process from the physical point of view. For example, it is conceivable that the flexural curved beam could be in error, so far as physical concepts are concerned. Apparently Wahl's difficulty was encountered when he attempted to obtain an expression for the amount of offset of the neutral axis from the geometrical axis of the wire. Röver ignored this offset, and consequently was in error, but only to a relatively small extent since Wahl obtains practically the same result as he does. Wahl differs with Röver in the evaluation of pure vertical shear, the former using 1.23 of the vertical load. The writer does not agree with Röver on this point and until Love's value of 1.23 is confirmed by experiment, fails to see why this value should be used in preference to unity.

Since neither Röver nor Wahl have provided the deflection formula on the basis of the former's discovery, the writer will

[13] Consulting Engineer, New York, N. Y. Assoc-Mem. A.S.M.E. Chairman, A.S.M.E. Special Research Committee on Mechanical Springs.

endeavor to make this derivation. Before making this derivation, however, a simplified derivation of the stress formula will be given somewhat along the lines first laid down by Röver.

According to old and well-established principles we have by definition:

$$G = \frac{\text{stress}}{\text{strain}} = \frac{S}{\epsilon}$$

or

$$S = \epsilon G \quad \ldots\ldots [19]$$

Referring to Fig. 19

$$\epsilon = \frac{\delta\alpha d}{2}\frac{1}{\delta z_0} = \frac{d}{2}\frac{\delta\alpha}{\delta z_0} \quad \ldots\ldots [20]$$

Now for small strip δz_0,

$$S_{s0} = \frac{d}{2}\frac{\delta\alpha}{\delta z_0} G = \frac{d}{2}\frac{Pr}{J}$$

whence

$$G = \frac{\delta z_0}{\delta\alpha}\frac{Pr}{J} \quad \ldots\ldots [21]$$

Also for small strip δz (neglecting pure vertical shear)

$$S_s = \frac{d}{2}\frac{\delta\alpha}{\delta z} G \quad \ldots\ldots [22]$$

Now simply impose upon Formula [22] the new condition $\delta_s = \delta z_0 \frac{r-x}{r}$ obtained geometrically from Fig. 19

$$S_s = \frac{d}{2}\frac{\delta\alpha}{\delta z_0}\frac{r}{r-x} G \quad \ldots\ldots [23]$$

and substituting G from [21] we get

$$S_s = \frac{d}{2}\frac{\delta\alpha}{\delta z_0}\frac{r}{r-x}\frac{\delta z_0}{\delta\alpha}\frac{Pr}{J} = \frac{d}{2}\frac{r}{r-x}\frac{Pr}{J} \quad \ldots\ldots [24]$$

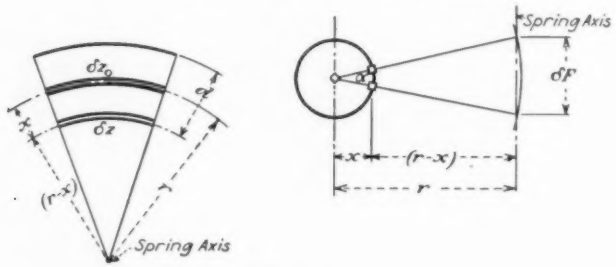


FIG. 19

Now to obtain maximum stress at z the direct vertical shear must be added to [24]

$$S_s = \frac{d}{2}\frac{r}{r-x}\frac{Pr}{J} + \frac{4P}{\pi d^2} \quad \ldots\ldots [25]$$

Reducing and substituting the value of $J = \frac{\pi d^4}{32}$ and $x = \frac{d}{2}$ (approximate) we get

$$S_{s(\max.)} = \frac{16\,Pr}{\pi d^3}\left(\frac{c}{c-1} + \frac{1}{2c}\right) \quad \ldots\ldots [26]$$

in which

$$c = \frac{2r}{d}.$$

According to this derivation, the calculated value of $k = \left(\frac{c}{c-1} + \frac{1}{2c}\right)$ is practically equal to Röver's or Wahl's value of k, the slight difference in each case being due to the value of the vertical shear loads used.

Derivation of Deflection Formula

By definition

$$G = \frac{\text{stress}}{\text{strain}} = \frac{S}{\epsilon}$$

Considering strip δz where stress is maximum we have

$$\epsilon_s = \frac{d}{2}\frac{\delta\alpha}{\delta z} \text{ (approximately)} \quad \ldots\ldots [27]$$

and since

$$\delta z = \delta z_0 \frac{r-x}{r}$$

$$\epsilon z = \frac{d}{2}\frac{\delta\alpha}{\delta z_0}\frac{r}{r-x} \quad \ldots\ldots [28]$$

Substituting $\delta\alpha = \frac{\delta F}{r}$ and $\frac{\delta F}{\delta z_0} = \frac{F}{l}$, we get

$$\epsilon z = \frac{d}{2}\frac{F}{rl}\frac{r}{r-x} \quad \ldots\ldots [29]$$

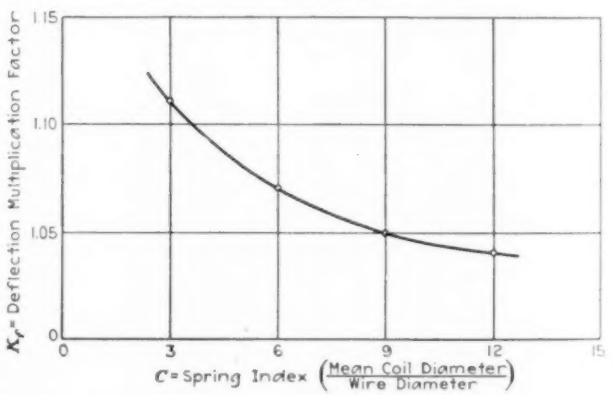


FIG. 20 CURVE FOR DETERMINING THE DEFLECTION MULTIPLICATION FACTOR K_f

Now,

$$S_s = \frac{d}{2}\frac{r}{r-x}\frac{Pr}{J} + \frac{4P}{\pi d^2} \text{ (approximately)} \quad \ldots\ldots [30]$$

therefore,

$$G = \frac{S_s}{\epsilon_s} = \left(\frac{d}{2}\frac{r}{r-x}\frac{Pr}{J} + \frac{4P}{\pi d^2}\right) \times \left(\frac{2}{d}\frac{rl}{F} \times \frac{r-x}{r}\right)$$

$$= \frac{Pr^2 l}{FJ} + \frac{8Prl}{\pi F d^3} \times \frac{r-x}{r} \quad \ldots\ldots [31]$$

Substituting $J = \frac{\pi d^4}{32}$ in [31] we get,

$$G = \frac{32Pr^2 l}{\pi F d^4} + \frac{8Prl}{\pi F d^3} \times \frac{r-x}{r} \quad \ldots\ldots [32]$$

or deflection

$$F = \frac{32Pr^2 l}{\pi G d^4}\left(1 + \frac{d}{4r} \times \frac{r-x}{r}\right) \quad \ldots\ldots [33]$$

Letting $x = \frac{d}{2}$ (approximate) and $\frac{2r}{d} = c$ we get

$$F = \frac{32Pr^2l}{\pi Gd^4}\left(\frac{2c^2+c-1}{2c^2}\right) \quad \ldots\ldots\ldots\ldots\ldots [34]$$

Calling $\left(\frac{2c^2+c-1}{2c^2}\right)$ the deflection multiplication factor K_f the curve in Fig. 20 of this discussion has been plotted in much the same manner as was done for the stress multiplication factor k. This correction factor for the deflection formula gives slightly larger deflections than those calculated by the old formula, a result which is somewhat unexpected. The writer hopes that Wahl will give the same attention to the deflection formula as he has to the stress formula by making appropriate tests to prove or disprove the writer's theoretical deductions.

Attention is drawn to Wahl's use of fatigue endurance limit in his discussion of this problem. It is the writer's opinion that the regular torsional endurance limit should not be applied to this problem until further research is made to determine actual endurance limits for the type of stress gradient involved.

W. M. Austin.[14] The writer has made several attempts to solve the helical-spring problem by the general method employed by the author. He has worked with both polar and rectangular coordinates. With rectangular coordinates he found an equation connecting the author's value γ, the radius of the wire, and the mean radius of the coil. This equation represented a very complicated surface. By dividing through by the mean radius he got an equation of two variables, one of which was the radius of wire divided by coil radius and the other was the author's γ divided by the wire radius.

The equation was still so involved that it took several hours to establish, by cut and try methods, one point on the curve. He finally abandoned this method of attack for the more general, but in this case simpler, method outlined in the following.

Let a force W act along the axis of a helical spring whose mean radius is r and wound of wire of radius a. (See Fig. 21.) Let CC be a line perpendicular to the axis of the spring and intersecting the axis of the wire at o. At some point g on CC the stress due to the force W is zero. This point is at o, or nearer the axis of the spring than o, or farther from the axis than o. I have drawn it farther from the axis mainly because that is the way the solution results, that is e is $+$.

The equation to the circle Fig. 21 of radius a referred to point Q as origin, the axis of the spring as the y-axis and line CC as the x-axis, is $(x-r)^2 + y^2 = a^2$

or

$$y = \pm\sqrt{a^2-(x-r)^2}.$$

The moment of inertia about g of the area of a thin section of the wire between two planes intersecting in the axis of the spring, each elementary area being divided by the ratio which its thickness bears to the thinnest part of the section, is

$$MI = (r-a)\int\frac{(x-r-e)^2+y^2}{x}\,dX\,dy \ldots\ldots [35]$$

The limits of y are $+\sqrt{a^2-(x-r)^2}$
and $-\sqrt{a^2-(x-r)^2}$
The limits of x are $r+a$
and $r-a$

$$MI = 2\pi(r-a)\left[\frac{2}{3}r^3 + e^2r + 2er^2 - a^2e - \sqrt{r^2-a^2}\left(\frac{2}{3}r^2 + e^2 + 2er + \frac{1}{3}a^2\right)\right] \ldots\ldots [36]$$

[14] Engineer, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa. Mem. A.S.M.E.

The section modulus is

$$\frac{2\pi(r-a)}{a+e}\left[\frac{2}{3}r^3 + e^2r + 2er^2 - a^2e - \sqrt{r^2-a^2}\left(\frac{2}{3}r^3 + e^2 + 2er + \frac{1}{3}a^2\right)\right] \ldots\ldots [37]$$

If S = the maximum shearing stress, the torsional moment is

$$\frac{2\pi S(r-a)}{a+e}\left[\frac{2}{3}r^3 + e^2r + 2er^2 - a^2e - \sqrt{r^2-a^2}\left(\frac{2}{3}r^2 + e^2 + 2er + \frac{1}{3}a^2\right)\right] = (r+e)W \ldots [38]$$

$$W = \frac{2\pi S(r-a)}{(a+e)(r+e)}\left[\frac{2}{3}r^3 + e^2r + 2er^2 - a^2e - \sqrt{r^2-a^2}\left(\frac{2}{3}r^2 + e^2 + 2er + \frac{1}{3}a^2\right)\right] \ldots\ldots [39]$$

Now let us take the axial component of the stress of expression [35]. We have

$$\frac{S(r-a)}{a+e}\int\frac{(x-r-e)}{x}\,dxdy + W = 0$$

using the same limits as in expression [35] we have

$$\frac{2\pi S(r-a)}{a+e}\left[-\left(r^2+re-\frac{a^2}{2}\right) + (r+e)\sqrt{r^2-a^2} + W = 0\right] \ldots\ldots [40]$$

Substituting for W its value in [39] and dividing the equation by

$$\frac{2\pi S(r-a)}{(a+e)(r+e)}$$

and simplifying we get

$$e = \frac{\frac{2}{3}(r^2-a^2)^{3/2} + a^2r - 2/3\,r^3}{a^2} \ldots\ldots [41]$$

let $e/a = u$ and $a/r = V$ and Equation [41] reduces to

$$u = \frac{2}{3}\left(\frac{1}{v^2}-1\right)^{3/2} + \frac{1}{v} - \frac{2}{3v^3}$$

$$u = \frac{1}{4}v + \frac{1}{24}v^3 + \frac{1}{64}v^5 + \frac{1}{128}v^7 + \ldots\ldots [42]$$

giving to v the values

$$\text{if } v = \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{8}, \text{ and } \frac{1}{10}$$

$u = 0.1308, 0.0849, 0.0632, 0.0504, 0.0418, 0.0313, 0.0257.$

If we divide the second member of Equation [39] by r^2 and multiply it by a^2/v^2 we get

$$W = \frac{2\pi Sa^2(1-v)}{2v^2(1+uv)(1+v)}\left[(1-\sqrt{1-v^2})\left(\frac{2}{3}+u^2v^2+2uv\right) - uv^3 - \frac{1v^2}{3}\sqrt{1-v^2}\right] \ldots\ldots [43]$$

let $d = 2a$

$$W = \frac{\pi d^2S(1-v)}{2v^2(1+uv)(1+v)}\left[(1-\sqrt{1-v^2})\left(\frac{2}{3}+u^2v^2+2uv\right) - uv^3 - \frac{1}{3}V^2\sqrt{1-v^2}\right] \ldots\ldots [44]$$

The expression

$$\frac{\pi(1-v)}{2v^2(1+uv)\,(1+v)}\left[(1-\sqrt{1-v^2})\left(\frac{2}{3}+u^2v^2+2uv\right)-uv^3-\frac{1}{3}\,v^2\,\sqrt{1-v^2}\right]$$

can be reduced to

$$\pi\left[\frac{1}{8}\,v-\frac{1}{4}\,v^2+\frac{59}{192}\,v^3-\frac{35}{96}\,v^4+\frac{115}{256}\,v^5-\frac{205}{384}\,v^6+\frac{4969}{9216}\,v^7-\frac{5036}{9216}\,v^8+\ldots\ldots\right]\ldots[45]$$

by expanding $\sqrt{1-v^2}$, substituting for u its value in [42] and dividing by the denominator.

Tables and curves can be made up for the expression [45] for many values of v. Then all that will be necessary in calculating the safe load on a spring is to get v by dividing the diameter of the wire by the mean diameter ($2r$) of the coil; get the value of [45] from the table or curve, then multiply by the stress desired and by the square of the diameter of the wire.

Very few approximations are made in the derivation of Formula [44]. I have assumed that the length of the various helical elements to be π times the diameter of the particular helix times its number of turns. I have also assumed that a plane section of the wire through the axis of the spring remains a plane when the spring is loaded. This is nearly, but probably not exactly true. I have also not taken into account the complicated nature of torsional shear. It is presumed that experiments can be made on straight bars of spring material to determine the torsional modulus, the proportional limit, the elastic limit, and ultimate strength, and the values thus obtained used without modification in the calculation of the spring.

The writer made an attempt once at a graphical solution of this problem. While he did not get the solution he found an interesting condition governing the stress distribution over the wire section. He found that lines of equal stress were conic sections having the point g of Fig. 21 for a common focus and the axis of the spring for a directrix.

If the inside diameter of the spring is greater than the diameter of the wire, there conics are all ellipses, but if the inside diameter is smaller than the wire diameter there will be one parabola and outside of that hyperbolas with ellipses inside of the parabola.

On account of the consequent shear longitudinally of the wire these lines of equal stress no doubt deviate from the true conic section near the surface and intersect the circumference of the wire radially, or nearly so.

To calculate the deflection of the spring,

$$F=\frac{2S\pi(r-a)\,(r+e)}{G(a+e)}=\text{the deflection per turn}$$

because S/G = strain on inside diameter of spring on unit length, $2\pi(r-a)$ = length per turn and $\frac{r+e}{a+e}$ = the ratio of distance of torsional axis from spring axis divided by distance of torsional axis to the inside of spring. Dividing both numerator and denominator by r we have

$$F=\frac{2\pi S(1-v)\,(1+uv)r}{G(v+uv)x}$$

$$=\frac{2\pi SV(1-V)(1+uV)a}{G(V+uV)V}=\frac{\pi S(1-V)(1+uV)d}{G(1+u)V^2}$$

$$=\frac{\pi S}{G}\left[\frac{1}{v^2}-\frac{1^1/_4}{v}+\frac{9}{16}-\frac{83}{192}\,v+\frac{155}{768}\,v^2-\frac{403}{3072}\,v^3+\frac{3168}{36864}\,v^4-\ldots+\right]\ldots\ldots[46]$$

This solution is presented as an approximation to the true conditions of shear in the spring, not as substitute or criticism of the author's formula. They are both approximations. One may prove to be better in certain situations while in other situations the other may come nearer to the truth.

It is presented as added evidence that the old spring formula is far from accurate and that a more economical and safer design of springs can be had by using one of the improved formulas.

Formulas [44] and [46] should permit accurate calculation of the stress and deflection in springs that have little or no internal stress when free. It is well known to spring makers that they can, by surging a spring several times beyond the elastic limit cause the spring to take up a shorter free length. It is at once noticed in such a spring that the deflection for a given load has increased. The reason for this is that the point g, Fig. 21, is now farther from the axis of the spring.

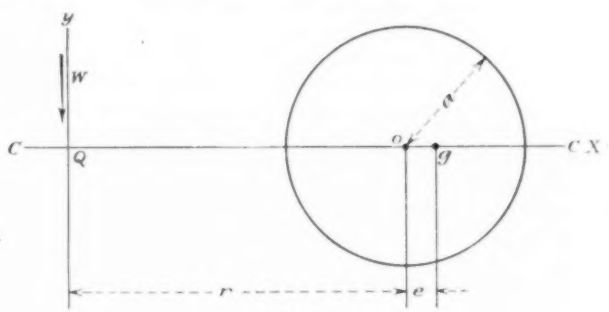


FIG. 21

Several years ago I had 12 springs made by cold-winding $^1/_4$ in. wire with a spring approximately 2 in. in diameter. These springs all had the same diameter but varied progressively in pitch of turn. Two of the springs when surged solid did not take any set. The two longest springs while differing considerably in original height were nearly the same height when surged and offered nearly this same resistance when compressed nearly solid.

Figs. 22 and 23 show the springs before and after surging.

The load sustained by the longest springs was approximately 29 per cent greater than that required to produce set in all of the ten longer springs. The deflection per pound per turn was measurably greater than it was before surging. Only about one-fourth to one-third of this greater deflection could be accounted for by the slightly increased outside diameter after surging.

If as an approximation we assume that Hooke's law holds up to the elastic limit, and that after permanent set takes place the shearing stress in the outer portions of the wire increases much more gradually but does increase as the deflection increases and that then the load is decreased slightly, we shall be able to find a deflection that will accompany a uniform (or nearly uniform) shearing stress in the outer portion of the wire surrounding a stress gradually decreasing to zero at the axis. Starting at the point g, Fig. 21, the point of zero stress moves farther and farther from the axis of the spring as the set goes farther and farther into the wire. It is of course impossible with any spring material to get the set to go clear to the axis; rupture would

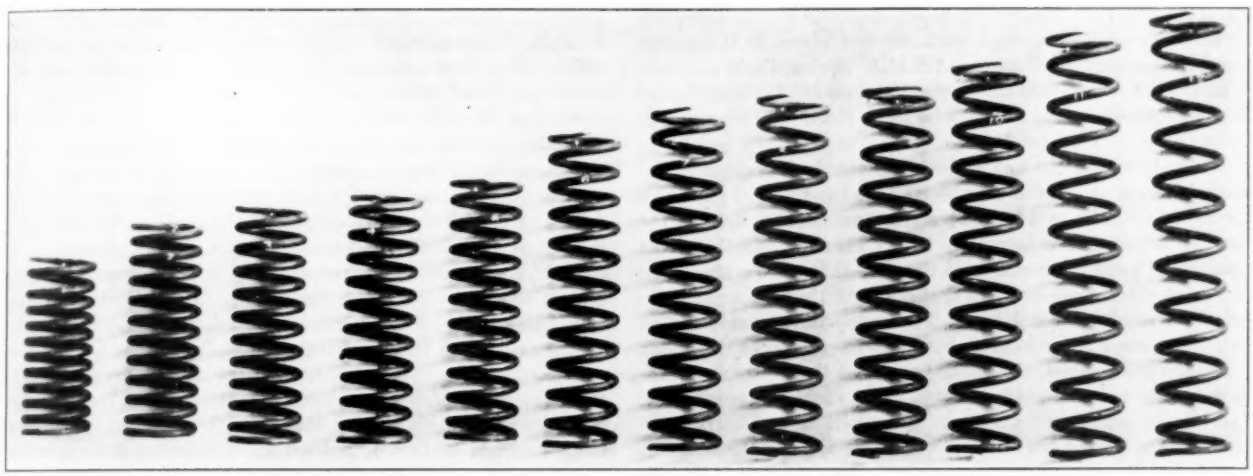


FIG. 22

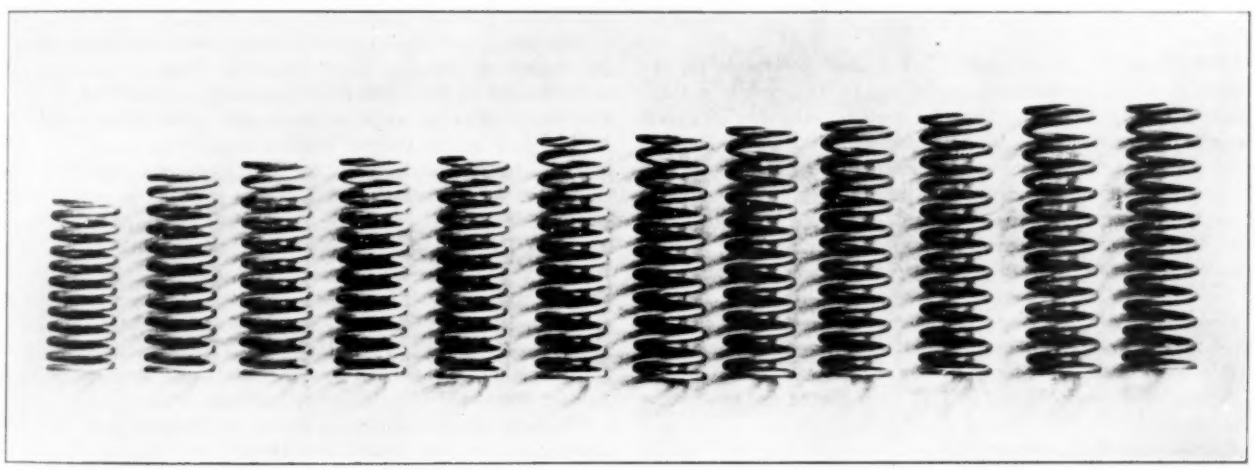


FIG. 23

occur first. However, we can by assuming set clear to the axis find the limit beyond which we cannot go.

For the above conditions we have:

$$W(r+e) = 2S \int\int_{\substack{t=0,\ \theta=0}}^{t=-e\cos\theta+\sqrt{a^2-e^2\sin^2\theta},\ \theta=\pi} t^2 dt d\theta$$

The limit of t is gotten from the polar equation to the circle

$$(t\cos\theta + e)^2 + t\sin^2\theta = a^2$$

from which

$$t = -e\cos\theta + a - \sqrt{a^2 - e^2\sin^2\theta}$$

we use the + sign since t is always positive.

Again, let $\frac{e}{a} = u$; then

$$W = \frac{\pi S a^3}{r+e}\left[\frac{2}{3} + \frac{1}{2}u^2 - \frac{1}{32}u^4 - \frac{1}{384}u^6 \ldots\right] \ldots [47]$$

For the vertical component of this stress we have the relation

$$2S \int\int_{\substack{t=0\ \theta=0}}^{t=-e\cos\theta+\sqrt{a^2-e^2\sin^2\theta},\ \theta=\pi} t\cos\theta\, dt d\theta + W = 0$$

Integrating and substituting for W its value from [47] and letting $\frac{a}{r} = v$ and solving for v we have

$$v = \frac{u - \frac{1}{8}u^3 - \frac{1}{64}u^5 \ldots}{\frac{2}{3} - \frac{1}{2}u^2 + \frac{3}{32}u^4 \ldots} \ldots [48]$$

This equation can be plated alongside the other equation between u and V, for any value of V in a surged spring the value of u will lie somewhere between the two curves near the first curve if only a little surging has been done and near the second if very severe surging has been resorted to.

There is no gain in severely surging a spring that has to operate from zero load to maximum load very often. If the maximum load seldom occurs or if the minimum load is one-half or more of the maximum load, great economy in space and material can be secured. It is almost an axiom that large stresses should not be used in any spring, the failure of which would cause loss of life or injury or damage to property. In these cases liberally designed springs are cheap insurance.

AUTHOR'S CLOSURE

As was suggested in the discussions by Messrs. Eksergian

and Rathbone, it appears desirable that one of the improved formulas for helical springs, such as that given in the paper, should be incorporated in the A.S.M.E. Spring Code and also in future handbooks and textbooks. It does not matter whether Röver's formula or the author's be used, since both give practically the same final results.

The only reason the author included the derivation of a new formula in the paper was that he believes the physical reasoning involved in its derivation was clearer than that involved in the derivation of Röver's formula. Röver assumes, in effect, that rotation of adjacent sections of the wire takes place about the center of these sections. He realized that this assumption involved a resultant axial force; consequently he *assumed* that *half* the vertical shear load was used up in producing this force.

On the other hand, the author assumed that rotation of adjacent sections took place about a point displaced toward the axis of the spring, producing a hyperbolic stress distribution similar to a curved bar in bending. This type of stress distribution would be produced by a pure torque moment. On this the shearing stress due to the vertical shear load was superimposed. This gave a simple solution and one which agreed within 0.7 per cent with Rover's exact solution for a spring with an index $c = 4$.

Mr. Wood, in his discussion, has derived a formula for the stress in which he assumes, as did Röver, that rotation takes place about the center of the wire section. On this he superimposes the *total* stress due to the vertical shear load. He thus neglects the fact that some of the vertical shear load is used up in producing a stress distribution compatible with the assumption of rotation about the center of the wire. His solution is still, however, another approximation, which differs from Röver's exact solution by about 3 per cent for a spring with an index $c = 4$. This compares with 0.7 per cent difference for the author's or about 1 per cent for Röver's approximate solutions. It should be noted, however, that even 3 per cent is a comparatively small amount, especially when applied to spring design.

The author used the value 1.23 as a factor for evaluating the stress due to the vertical shear load since this value and its method of derivation are generally accepted by authorities on the theory of elasticity.

The author did not pay much attention to the question of the exactness of the deflection formula ordinarily used for computing helical springs since it was his opinion that the difference between the ordinary deflection formula and a more exact one was quite small. Mr. Wood's derivation of a new deflection formula, in the author's opinion, gives an approximate formula which will give results probably nearer the truth than will the commonly used deflection formula. However, it should be noted that the difference in the results calculated by Mr. Wood's deflection formula and those computed by the ordinary deflection formula will not be greater than 11 per cent for nearly all springs used in practice which have indices greater than 3. This difference of 11 per cent could possibly be nullified by variations in spring-coil diameter and wire diameter of only slightly more than $1^1/_2$ per cent. At present, tests are in progress by the author to determine the correctness of Mr. Wood's formula, and if this is shown to be correct, he believes it should also be incorporated into future spring tables.

Regarding Mr. Stone's discussion, the author agrees that to maintain consistency the stresses should be integrated over the cross-section, as Mr. Stone suggests. The author attempted to do so when he first considered the problem, but on account of the mathematical difficulty involved he made the assumption mentioned by Mr. Stone, relative to the angular twist of adjacent sections. He believes this assumption gives sufficiently accurate results for practical work.

The author believes actual fatigue tests of springs should be carried out, as Mr. Stone suggests. Work of this kind is now planned by the Research Department of the Westinghouse Company. Further spring tables should, in the author's opinion, be based on actual fatigue tests such as these.

The question brought up by Mr. Rathbone regarding the correction factor for springs of square wire is one which is at present being investigated by the author at the Westinghouse Research Laboratory. Preliminary results indicate that the correction factor for square wire is approximately the same as that for round wire. As in the case of the round wire, the stress on the inside of the coil was found to be approximately 2.5 times that on the outside, even though the section was somewhat trapezoidal, as pointed out by Mr. Rathbone. A paper on this subject is planned when the work is completed.

The approximate solution to the spring problem given by Mr. Austin is based on somewhat different assumptions than those made by the author. However, Mr. Austin's solution gives results not greatly different from those obtained using Röver's or the author's formula, and, as Mr. Austin points out, furnishes added evidence that the ordinary spring formulas are inadequate.

APM-51-18

Graphical Methods for Least-Square Problems

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IT IS a well-known fact that engineering test data are commonly presented in the form of curves. Simultaneous observations are made of two varying quantities, such as temperature and time, speed and horsepower, or payroll and production; these observations are plotted as points on a plane chart which affords an x-value and a y-value for each point; and a curve is then drawn through or near the points with the tacit assumption that there is some sort of relation between these x- and y-values. If the points are exactly traversed by the curve, this relation is evidently a simple mathematical equation in x and y. If, as is much more often the case, the curve is merely adjacent to the points, it is still assumed that the mathematical relation exists, but that the discrepancy is due to unavoidable errors of observation or to unexplained factors which the chart is supposed to suppress rather than emphasize.

Naturally, therefore, the curve should be drawn so as to reduce these errors to a minimum. It may be located by placing a transparent straight edge or curved rule on the plot and moving it about until the errors *appear* to be as small as possible, the eye of the draftsman being the sole guide to the fit of the curve, or the points may be separated into groups, the mean of each group located, and a line drawn through these mean points. In this case an exact calculation has been substituted for visual guesswork. But the question may fairly be asked: Is this line the "best" one that can be drawn? Obviously, the answer is ambiguous, since different groupings give different averages and the resulting curves would be far from identical.

In any case the best curve will be that one which reduces the standard error to a minimum, the standard error being defined as the root-mean-square of the errors of all the plotted points.[2] If the x-observations are perfect and the y-observations are inexact, these errors will be the differences between the y-values of the plotted points and the y-values of the curve for corresponding x-values. If both x- and y-observations are inexact and to an equal degree, the errors are the distances from the plotted points to the curve. No proof of these statements is given here, as ample substantiation may be found in most treatises on statistics or the theory of probability. An obvious corollary is this: by making the number of coefficients in the equation relating x and y equal to the number of plotted points, the standard error may be reduced to zero. But the snakelike curve resulting would defeat its own purpose by obscuring the relationship that the chart is intended to prove. Generally speaking, the nearer the plotted curve approaches a straight line—i.e., the flatter it is—the more suitable is the diagram for giving a mental picture of the results. Very often a straight line is the most satisfactory one that can be drawn.

Unfortunately, the least-square method has, in the minds of most engineers, one great drawback: the involved computations required. The slide rule cannot be used efficiently, because of the mixture of multiplication and addition; furthermore, it is not accurate enough, because the solution of the normal equations usually brings in small differences of large products, and the observational errors of the slide rule may easily overshadow those of the original data. A simple and accurate device for applying the least-square method without calculation should therefore meet with ready acceptance. Such a method will be outlined in the following paragraphs. The process is essentially a graphical one, using the planimeter to perform additions and multiplications simultaneously. A polar planimeter with adjustable arm, such as is used for averaging indicator cards, is most convenient, but a plain planimeter will do about as well.

The following type cases will be discussed, as being those most frequently encountered in engineering practice. However, they are merely representative, and many others could be added before the list would be complete.

- Case 1. Straight line, x-observations perfect, y-observations subject to error
- Case 2. Straight line, x- and y-observations subject to error
- Case 3. Conditioned hyperbola
- Case 4. Parabolic curve, symmetrical about y-axis or origin, exponent has any value
- Case 5. Exponential or logarithmic curve
- Case 6. General parabolic curve
- Case 7. Sine-cosine curve.

Case 1

The construction consists essentially of three steps, which are fundamental not only to this case but to all the others except Case 7. For this reason they are enunciated at the start, and their importance cannot be overemphasized. They are:

1 Transfer of the x- and y-axes to an origin located at the average of the x- and y-values of the plotted points.

2 Summation of x'^2 and $x'y'$ with the planimeter, x' and y' being the coordinates of the plotted points, referred to the new axes.

3 Drawing of the empirical line through the new origin and the point whose coordinates are $\Sigma x'^2$ and $\Sigma x'y'$.

The detailed process of locating the new origin is best performed with the adjustable-arm planimeter, unless there are only a few points, in which case the draftsman may prefer to do his averaging by arithmetic. Mark off equal spaces—inches, half-inches, or some other convenient unit—on the x-axis of the chart, equal in number to the plotted points. In Fig. 1, a, b, c, to h are the plotted points, and j, k, l, to r mark the equal spaces. Set the adjustable points of the planimeter to the total distance jr; start the planimeter at j, go straight up to the abscissa of a, across one space, up to the abscissa of b, across one space, up (or down) to the abscissa of the next plotted point, and so on until the tracing point is directly above r. Then trace down to r and across to j. The reading of the planimeter will now be the average height of the figure traced, which obviously coincides with the average of the observed y-values. The average of the x-values may be measured in the same way, using the y-axis as a base line.

If an averaging planimeter is not available, one of the fixed-arm type may be used. It is a well-known proposition in the theory of this instrument that the area traced equals the length of the tracing arm multiplied by the true rolling distance of the drum.

Hence, if a figure is planimetered whose length equals the arm length times one-tenth the circumference of the contact flange of the drum,[3] the reading of the drum will coincide with the

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[2] "Statistical Methods," by F. C. Mills, p. 368.

Contribution by the Applied Mechanics Division and presented at the Annual Meeting, New York, December 3 to 7, 1928, of The American Society of Mechanical Engineers.

[3] Assuming that the customary graduation is engraved on the drum: ten major divisions, each of which registers 1 sq. in.

average height of the figure. Accordingly, if a fixed-arm instrument is to be used, the distance jr (Fig. 1) should be made equal to $\pi cda/10$, where c = a simple constant like 5 or 10, d = diameter in inches of contact flange of drum, and a = arm length in inches from pivot to tracing point. This distance, which is a constant for any planimeter, can then be subdivided into the requisite number of equal parts and the averaging process performed exactly as described in the preceding paragraph, except that the planimeter reading must be divided by c.

The point determined by these two averages is O, Fig. 1.

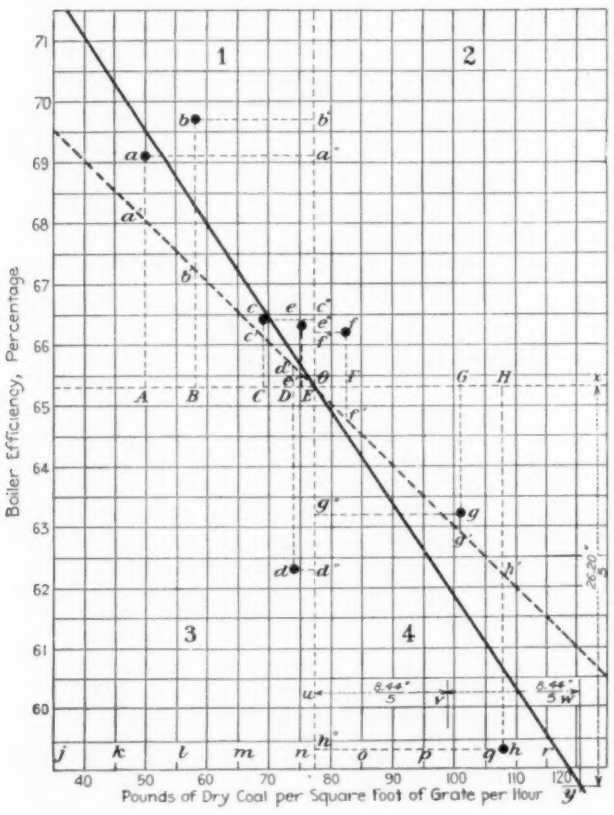


FIG. 1

Next, the planimeter is again brought into play, and the total area of the 45-deg. triangles OAa', OBb', OCc', etc. is obtained in the usual manner. It is not necessary to note down the area of each individual triangle or to pay any attention to the scale of the diagram; simply start at O and trace the lines OA, Aa', $a'O$; OB, Bb', $b'O$; etc. in turn, remembering always to go around in a clockwise direction, until all the triangles have been accounted for. The final result is then read off the dial or drum of the planimeter, and a numerically equal distance is laid off *twice* to the right (or left) of the vertical line through O. In Fig. 1, for instance, the triangles added up to 8.44 sq. in. on the author's original layout; so $\frac{8.44}{5}$ in. were laid off from u to v and again from v to w. Here, again, it should be explained that the particular scale that is used makes no difference, but for the sake of accuracy it is well to make it as large as possible, and the nearer w is to the edge of the chart, the better will be the final result. In this manner the x'^2-summation is obtained.

The $x'y'$-summation is very similar, the only difference being that instead of taking the areas of a number of triangles the rectangles $OAaa''$, $OBbb''$, $OCcc''$, etc. are measured and the total area obtained. In the original of Fig. 1 this was 26.20 sq. in.; accordingly, 26.20 units of the same size as in the preceding paragraph (fifth-inches) were laid off downward from a random point x on the horizontal through O, giving point y at the intersection with the previously drawn line through w. A straight line through y and O is, then, the "best" straight line that can be drawn to indicate the relation between boiler efficiency and rate of firing, for the particular set of tests recorded.

It will be noticed that the horizontal and vertical lines that were drawn through the central point O divide the diagram into four sections or quadrants that may be numbered 1, 2, 3, 4, as indicated. In this example, all the points a, b, c, etc., lie in the diagonally opposite quadrants 1 and 4, with the exception of d and f, which are in quadrants 3 and 2. The rectangles which these two points locate should therefore be traced *counterclockwise* by the planimeter, in distinction from the clockwise direction used for the points in the quadrants which contain the 45-deg. line. In all cases the triangles are traced in a clockwise direction.

The equation for the empirical line is $Y = aX + b$, where a is the ratio of $x'y'$ to x'^2, as found by the planimeter, and b is the intercept of the line on the original y axis.

CASE 2

If the x- and y-observations are both in error, a good approximate rule to follow is this: Draw an empirical line, by the

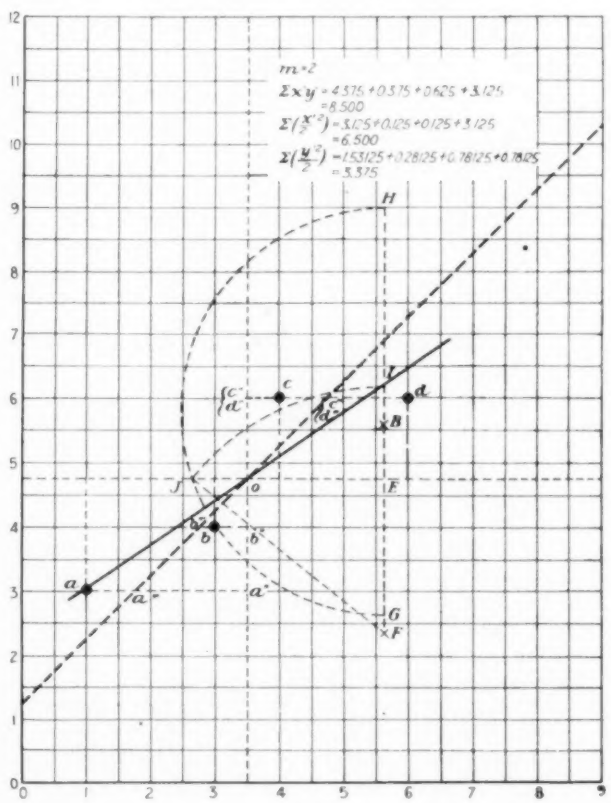


FIG. 2

method of Case 1, on the assumption that the y-values only are in error; draw another line by the same method, assuming that the x-values alone are in error; then draw the final line between these two, inclining it toward one or the other according as the x- or y-values have greater weight. In case the deviations appear to be fairly large, as in Fig. 2, it may be better to use the accurate,

though somewhat longer, process. First, locate the mean point O and transfer the axes as before. Lay off OE on the new x-axis equal to $\Sigma x'y'$, using any convenient unit of measurement; from E lay off EB vertically upward, making it equal to $\Sigma(y'^2/2)$, obtained by tracing the triangles $Oa''a''$, $Ob''b''$, etc.; and from B lay off BF vertically downward, making it equal to $m\Sigma(x'^2/2)$, m being the weight of the x-observations relative to the y-observations. The latter summation is obtained exactly as in Case 1. Prolong the line through F, E, and B and points G and H on it, such that $EG = OE$ and $EH = OE \times m$. Make GH the diameter of a semicircle cutting the new x-axis in J, and swing radius FJ onto line FH. The point I thus located, together with point O, determine the empirical line.

The equation of the line is of course $Y = aX + b$. Exact formulas for a and b are given in the Appendix.

Case 3

Frequently the x-observations increase while the y-observations decrease, suggesting a reciprocal relation and a curve of hyperbolic form. The general hyperbola does not lend itself readily to graphical treatment owing to the large number of constants to be evaluated; but by "conditioning," it is very easy to locate and plot an equilateral hyperbola, having one of the axes as asymptote, and satisfying the least-square criterion. For example, in the direct-current series-motor test plotted in Fig. 3 it may be assumed with reasonable truth that the starting torque is of a much greater order of magnitude than any of the values obtained under running conditions. That being the case, the curve is obtained as follows:

First, draw the ordinates through the plotted points a, b, c, to h, and with the upper left-hand corner of the diagram as a center, draw a quarter-circle large enough so that the tangent to it at the point where the a-ordinate cuts it will extend approximately to the upper right-hand corner of the chart. From the point where this tangent cuts the top line of the diagram, drop an ordinate to a' on the same horizontal with a. In the same manner draw tangents to the quarter-circle where it is intersected by the b-ordinate, c-ordinate, etc., and locate points b', c', etc.

For those ordinates that do not cut the quarter-circle the reverse construction is used; i.e., for the e-ordinate draw mn through its upper end and tangent to the quarter-circle, and then draw an ordinate through the point of tangency m, locating e' on this ordinate on the same horizontal with e. The entire construction of ordinates and tangents can be performed very rapidly with a sharp-cornered draftsman's triangle, as suggested by the dot-dash outline in Fig. 3.

If the hyperbola is a suitable curve for the problem in question, it will be found that the auxiliary points a', b', c', etc. are approximately in line, and this fact may be used as a test for the appropriateness of such a curve.

The next step consists in fitting a straight line rs to the auxiliary points. This is done in precisely the same manner as outlined for straight-line diagrams, and therefore need not be elaborated here. Having located rs, it only remains to set dividers to the respective vertical deviations—i.e., the vertical distances between the auxiliary points and the intersections of their ordinates with rs—and transfer them to the original a, b, c, etc. ordinates. This gives a series of points on the desired curve, which may then be sketched in and completed with a curved rule. This curve, shown by a heavy line in Fig. 3, is the "best" hyperbola with two arbitrary constants that can be fitted to the given data. Its equation is $Y = \dfrac{a}{X} + b$, where a is the slope of the auxiliary line rs multiplied by the square of the radius of the quarter-circle (measured in the same units as X), and b is the y-intercept of the auxiliary line; i.e., in Fig. 3, $a = \dfrac{820}{200} \times 800^2$ and $b = -2260$.

Case 4

The general parabolic or hyperbolic form of curve, with axis of symmetry passing through the origin, is one that frequently lends itself well to the representation of test data. The equation of such a curve is $Y = aX^b$, a and b being either positive or negative, integral, or fractional. If we try to fit a curve that will make the root-mean-square of the y-deviations a minimum, the normal equations are very unwieldy, and a graphical method is apparently out of the question. But by making the well-known transformation to logarithmic coordinates, the equation becomes $\log Y = \log a + b \log X$, and it is a simple enough matter to find a line such that the root-mean-square of the deviations of $\log y$ will be a minimum. This is a mathematical subterfuge that may seem unjustifiable according to the strictest canons of

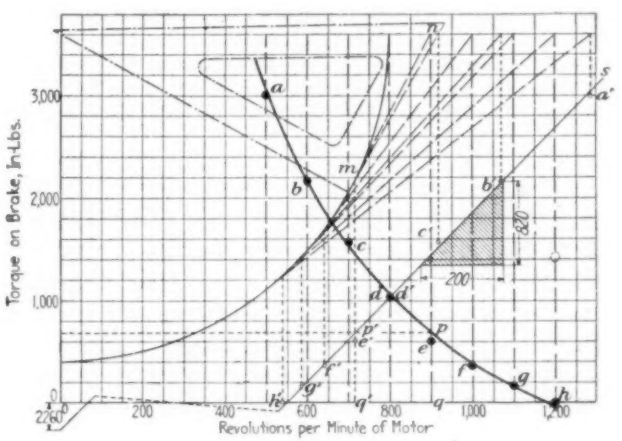


Fig. 3

logic, but it is recommended by statisticians[4] and is certainly acceptable on the ground of simplicity.

The use of logarithmic coordinates is so common in engineering practice that it is hardly necessary to work out a sample problem to illustrate the graphical method. Briefly, the process consists in transferring the observations to full-logarithmic paper, using any convenient combination of scales, fitting a straight line to the points by the method of Case 1 or Case 2, and then, if desired, retransferring to regular coordinates by picking off a sufficient number of points from the straight line to insure a smooth curve on the regular-coordinate paper.

Case 5

A type form very similar to the preceding is the exponential or logarithmic curve $Y = ae^{bX}$ or $Y = a \log X + b$. By plotting the points on semi-logarithmic paper and fitting a straight line by the basic graphical method, a curve is obtained which, for the exponential case, gives least-square values for the deviations from $\log y$, and for the logarithmic equation gives least-square values for the deviations from y.

Case 6

The observed points may frequently indicate a curve which is not symmetrical about the x- and y-axes, or does not pass through the origin, or have the form of a logarithmic or exponential curve. If it appears that it should be concave upward, and rising from left to right, try plotting values of x^2 against y and see if the points lie in a fairly straight line. If they still curve upward,

[4] "Statistical Methods," pp. 290–297.

a larger exponent may be tried. Conversely, if the curvature is downward, a fractional exponent is indicated, and if the curve is to slant downward from left to right, the exponent will be negative. Fig. 4 shows clearly the form of the curves belonging to this general category and the relation between the direction of curvature and the value of the exponent.

In any event the procedure is to choose arbitrarily the value of the exponent n in the equation $Y = aX^n + b$, plot the values of x^n versus y from the observation data, fit a straight line to this plot, read off on this straight line the values of Y that belong with the various x^n-values, and plot these against x to form the final curve. An example will make this clear. Fig. 5-*a* shows by means of solid dots seven observations ranging from $x = 2$, $y = 1$; to $x = 20$, $y = 11$. The general location of these points indicates a positive fractional value for the exponent n: say, $n = 1/2$. The observed values of x are accordingly raised to the one-half power and plotted against the observed values of y, giving the solid dots in Fig. 5-*b*, which lie in a flat S-curve that can readily be correlated with a straight line without excessive deviations. The straight line CD, obtained by the basic method, is drawn in Fig. 5-*b*, and the Y-values pertaining to the plotted $x^{1/2}$-values are shown by small circles. When these have been transferred to Fig. 5-*a*, the locus for the required curve AB

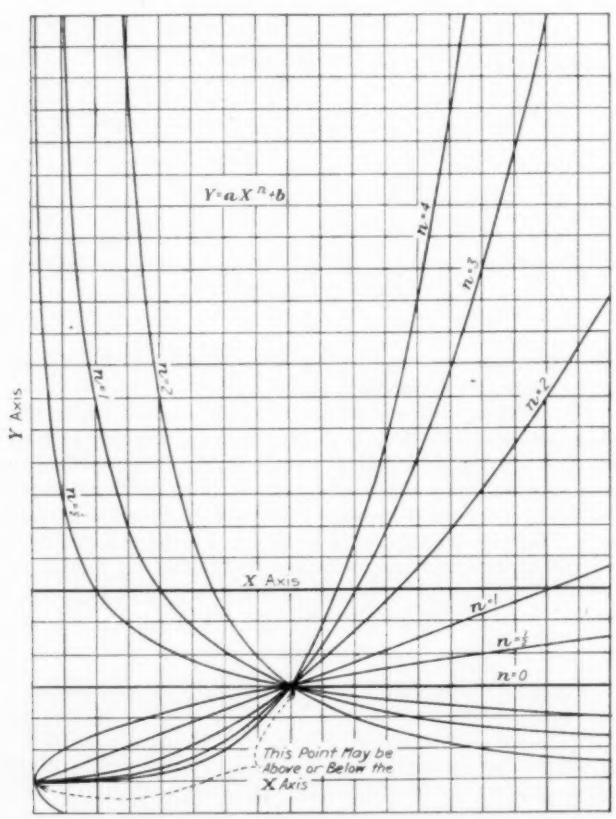


FIG. 4

has been duly obtained. The equation of the curve is $Y = aX^{1/2} + b$, or $a^2X = Y^2 - 2bY + b^2$, in which a is the slope of the straight line CD (with due regard to possible differences in the horizontal and vertical scales) and b is the intercept of CD on the y-axis.

Curve EF in Fig. 5-*a*, which fits the observed points more closely than AB but has certain offsetting disadvantages, will be referred to at the close of the paper.

CASE 7

This curve, the last to be discussed in the present paper, differs in several respects from those that precede it. Hitherto, all the curves have been derived from a basic straight line, fitted to a series of points by what may be termed a standard method. In the case of the sine-cosine curve the normal equations are

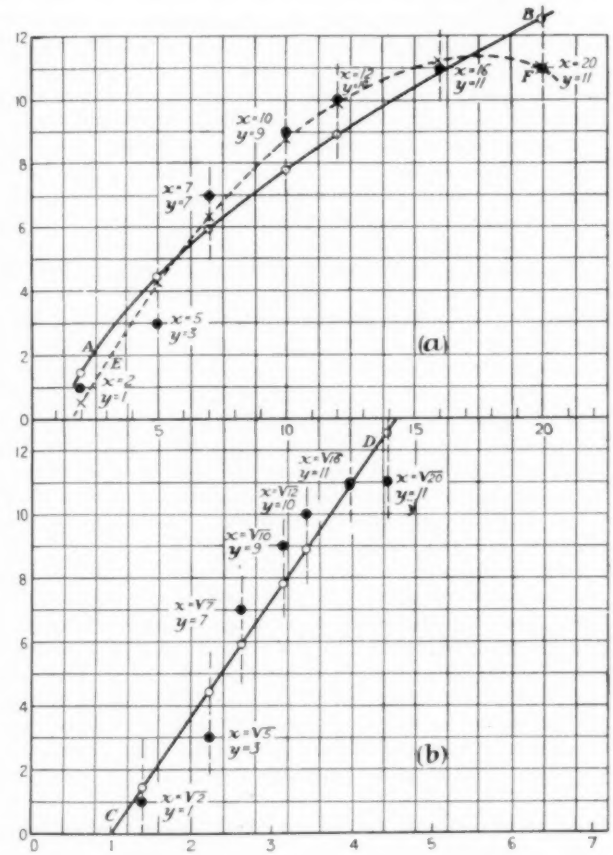


FIG. 5

markedly different; the only similarity lies in the device of shifting the x- and y-axes in order to simplify these equations and in limiting the number of constants in the empirical equation to two. In order to avoid the presence of three or more constants, it is assumed that (*a*) the period or frequency of the wave is known, and that (*b*) the x-axis is so located that the mean of all the y-observations is zero. The unknown constants to be determined are thereby reduced to the amplitude and the phase of the periodic curve.

To locate the new y-axis, the observed data[5] are plotted on transparent tracing vellum, and a double-frequency sine curve of any desired amplitude is laid out in ink or soft pencil on a separate sheet of paper (so that it will be visible through the vellum), using either the table of functions or the standard graphical method for its construction. It should extend for at least $2^1/_4$ wave lengths and have the first $^1/_4$ wave length accurately marked (AB, Fig. 6-*a*). This is then placed under the plot of observed values, making A coincide with the origin of

[5] Data taken from a forest and stream-flow experiment at Wagon Wheel Gap, Colo., by the U. S. Forest Service and Weather Bureau. To make the problem more typical, a limited set of readings were used, running from January, 1911, to September, 1912. Ordinates are monthly mean temperatures; abscissas are months.

coordinates (Fig. 6-*b*); the ordinate passing through each point is noted; and all the ordinates are added algebraically with dividers and the sum laid off vertically at *DE* (upward if positive and downward if negative). Then the tracing vellum is shifted to the left so that *B* coincides with the origin of coordinates (Fig. 6-*c*), and the same process of ordinate marking and summation is performed, giving the horizontal line *DF*. Sometimes, as in Fig. 6-*c*, $DF = 0$. This construction determines the angle *EFD*, the bisector of which gives the angular displacement of the new *y*-axis from the original axis. In Fig. 6-*c* angle *EFD* = 270 deg.; hence, the new *y*-axis, shown in Fig. 6-*c*, is located to the right of the original *y*-axis at a distance $c = \frac{135}{360} \times AC$.

The construction of the final empirical curve is accomplished in two steps: in the first, the amplitude of the sine component is obtained, and in the second, that of the cosine component. The resultant is found by plotting the components and simply adding them graphically, or by adding the two amplitudes vectorially, determining the phase of the resultant, and plotting a single sine curve to fit these conditions.

Fig. 6-*d* illustrates the first step. A sine curve of fundamental frequency and any convenient amplitude is drawn up on a separate sheet and slipped under the transparent plot, making its axis coincide with the new *x*-axis and its origin with the new origin *O* of the plot. A 45-deg. line *GH* is also drawn through *O*. Then ordinates are drawn from each of the plotted points 1, 2, 3, etc., to the sine curve, and from their intersections with this curve are drawn corresponding abscissas to the new *y*-axis. The intersections of these abscissas with *GH* are numbered 1′, 2′, 3′, etc., the primed numbers each being related to the corresponding original numbers. The planimeter is now used to measure and sum up a series of rectangular areas, one for each plotted point, thus: Start tracing at *O*, go down to the abscissa through 1′, over to *GH*, straight up to the new *x*-axis, and back to *O*; repeat for the other points, always starting out on the new *y*-axis and returning to *O* on the new *x*-axis. The final result is laid off vertically from the *x*-axis at a point where the underlying sine curve has its full amplitude, and any convenient scale is used (see *IJ*, Fig. 6-*d*). Then another series of rectangles is summed up, by starting the planimeter at *O*, going down to the abscissa through 1, across to the ordinate through 1′, up to the new *x*-axis, over to *O*, and so on for all the other points. Here again it is necessary to start tracing each rectangle on the new *y*-axis, as a result of which it will be found that some of the rectangles will be traced counterclockwise. It is therefore well to note the reading of the planimeter each time that the tracing point arrives at *O*, so that in case the total area is negative its true value may be found by subtracting the final reading from the maximum scale graduation of the planimeter. In Fig. 6-*d* this final reading is positive and is laid off horizontally as *IK*, to the same scale as *IJ*. Horizontal line *LM*, determined by the two legs of the triangle *IJK* and the point *L* where the sine curve intersects *IJ*, is then the amplitude of the sine component of the final empirical curve.

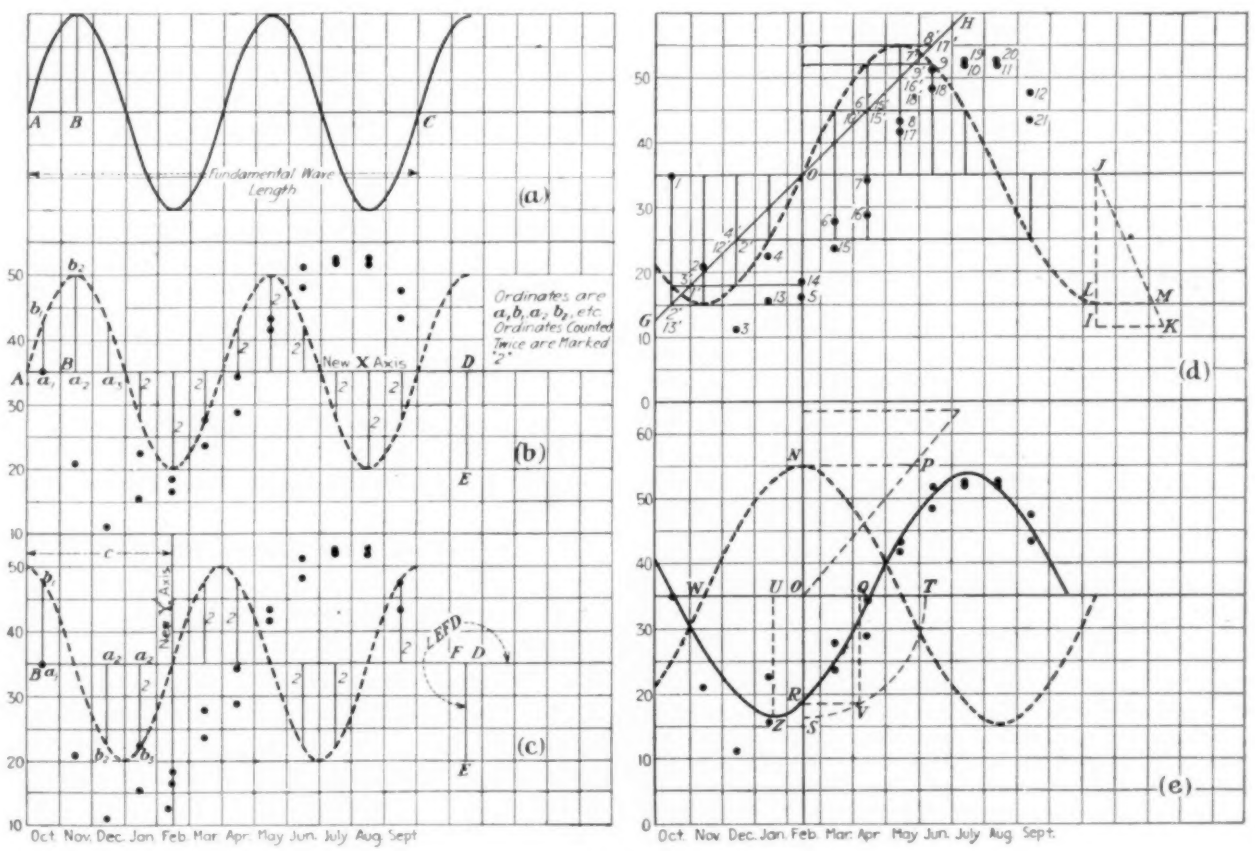


FIG. 6

The cosine component is determined in exactly the same way, except that the underlying curve is moved along so that its highest point coincides with the new *y*-axis (Fig. 6-*e*). The amplitude of this component for the illustrative problem is shown at *NP*; it is negative in sign.

To combine the two components graphically without plotting

either one, lay off $OQ = LM$ and $OR = NP$. Measurements up and to the right of O are positive; down and to the left are negative. Draw arc ST with O as center and OV as radius, measure arc SV with a protractor, lay off UO so that $\frac{UO}{WO} = \frac{\text{arc } SV}{90°}$, and locate Z at the intersection of horizontal and vertical lines through S and U; Z is then the minimum (or maximum) point on the required empirical curve, and since the wave length and axis of the curve are already known, it can be plotted by simple geometrical means.

In conclusion, a few general rules will be given that apply with equal force to all types of curve plotting:

1 Do not extend a curve by extrapolation more than 10 per cent at either end.

2 If observations are of unequal weight, multiply the weights by the least common factor that will make them all integral, and then consider that each point is replaced by several discrete but coincident points, equal in number to the weight of the point in question.

3 When an empirical relation between two sets of observations is derived by applying a known mathematical or physical law, it makes some difference whether the law is applied to each pair of observations individually or to a primary empirical relation between the observed quantities; e.g., the water rate of a steam turbine at different loads is not found by observing loads and corresponding pounds of steam per kilowatt-hour, but by observing loads and total steam consumptions per unit of time. Usually these latter observations are individually divided by the loads, and the resulting points of pounds per kilowatt-hour versus load are used as the basis of an empirical water-rate curve. However, it would be slightly more logical to find an empirical curve for the relation of load to total steam consumption, and then derive the water-rate curve from this primary curve by dividing its ordinates by the corresponding loads.

4 In all the cases described in this paper, except Case 2, it is assumed that the y-observations alone are subject to error. Hence, in plotting observations in which both sets are inexact, those which are most nearly correct should be plotted as abscissas.

5 The number of arbitrary constants in any empirical equation should be as small as possible consistent with good correlation. This somewhat begs the question, since there is no absolute standard of correlation. In the theory of statistics a relative measure of correlation is found by comparing the standard error of the assumed line with that of a horizontal line passing through the mean of the y-values. It may be stated as an unprovable but common-sense principle that the use of more than two constants in the equation is not justified unless they are needed in order to make the "index of correlation,"[6] as it is called, greater than 85 per cent, the maximum value with perfect correlation being 100 per cent. In Fig. 5-*a*, curve AB, representing an equation with two constants, has an index of correlation of 95.45 per cent. Curve EF, with three constants calculated by solving three simultaneous normal equations fits the points slightly better (i. c. = 98.74 per cent), but the peak near the right end suggests a downward trend which is not justified by the observed data. A somewhat rougher approximation, such as is furnished by curve AB, is in reality far more useful.

Appendix

Let

x_1, y_1, x_2, y_2, etc. = coordinates of plotted points

X_1, Y_1, X_2, Y_2, etc. = variables of empirical equations (in general X is independent and Y is dependent)

d_1, d_2, d_3, etc. = deviations of the dependent variable from the empirical curve; i.e., $d = y - Y$ when $x = X$

n = number of related pairs of observations or number of plotted points

x'_1, y'_1, x'_2, y'_2, etc. = coordinates of plotted points with reference to axes passing through the mean of the x and y observations; i.e., $x = x' + \frac{\Sigma x}{n}$, $y = y' + \frac{\Sigma y}{n}$, $\Sigma x' = 0$, $\Sigma y' = 0$

a, b, c, e, etc. = constants in the empirical equation

Σ = summation of a series of similar terms; it affects everything that follows it, up to the next + or − sign; i.e., $\Sigma xy^2 = x_1y_1^2 + x_2y_2^2 + \ldots$, and $(\Sigma xy)^2 = (x_1y_1 + x_2y_2 + \ldots)^2$.

Case 1

To determine a and b in the empirical equation $Y = aX + b$, assuming the x-observations perfect, $x_1 = X_1$, $x_2 = X_2$, etc.

$y_1 - d_1 = aX_1 + b$, hence $d_1 = y_1 - aX_1 - b$ and $d_1^2 = y_1^2 - 2ay_1X_1 + a^2X_1^2 - 2by_1 + 2abX_1 + b^2$

$y_2 - d_2 = aX_2 + b$, $d_2 = y_2 - aX_2 - b$, $d_2^2 = y_2^2 - 2ay_2X_2 + a^2X_2^2 - 2by_2 + 2abX_2 + b^2$

etc. etc. etc.

$$\Sigma d^2 = \Sigma y^2 - 2a\Sigma xy + a^2\Sigma x^2 - 2b\Sigma y + 2ab\Sigma x + nb^2$$

If $\sqrt{\frac{\Sigma d^2}{n}}$ is a minimum, Σd^2 must also be a minimum; hence

$$\frac{\partial}{\partial a}\Sigma d^2 = -2\Sigma xy + 2a\Sigma x^2 + 2b\Sigma x = 0$$

$$\frac{\partial}{\partial b}\Sigma d^2 = -2\Sigma y + 2a\Sigma x + 2nb = 0$$

$$a = \frac{n\Sigma xy - \Sigma x\Sigma y}{n\Sigma x^2 - (\Sigma x)^2} \qquad b = \frac{\Sigma x^2\Sigma y - \Sigma x\Sigma xy}{n\Sigma x^2 - (\Sigma x)^2}$$

$$a = \frac{\Sigma x'y'}{\Sigma x'^2}$$

When $X = \frac{\Sigma x}{n}$, these values of a and b give $Y = \frac{\Sigma y}{n}$.

The line constructed by the graphical method satisfies the foregoing condition for a and passes through the point $X = \frac{\Sigma x}{n}$, $Y = \frac{\Sigma x}{n}$.

Case 2

To determine a and b in the empirical equation $Y = aX + b$, when both x- and y-observations are in error and the x-observations have a weight of m times the y-observations.

Let d_x represent the deviation of an observed point from the empirical line assuming the y value as perfect, and d_y the deviation assuming the x-value as perfect (see Fig. 7). Then, if the x- and y-values are both in error but of equal weight, the total deviation d should equal $d_y \cos\theta$ and also $-d_x \sin\theta$, or $d^2 = -d_xd_y \sin\theta \cos\theta = \frac{d_x^2d_y^2}{d_x^2 + d_y^2}$. But the x-value is m times as accu-

[6] "Statistical Methods," p. 436.

rate as the y-value; hence the square of the deviation should be shifted toward d_y by substituting md_x^2 for dx^2. This makes

$$d^2 = \frac{md_x^2 d_y^2}{md_x^2 + d_y^2} = \frac{md_y^2}{m + d_y^2/d_x^2} = \frac{md_y^2}{m + a^2} = \frac{m(y - ax - b)^2}{m + a^2}$$

$$\Sigma d^2 = \frac{\Sigma y^2 - 2a\Sigma xy + a^2\Sigma x^2 - 2b\Sigma y + 2ab\Sigma x + nb^2}{1 + a^2/m}$$

The minimum value of Σd^2 with respect to a and b is obtained when

$$\frac{\partial}{\partial a}\Sigma d^2 = \frac{(\Sigma y^2 - 2a\Sigma xy + a^2\Sigma x^2 - 2b\Sigma y + 2ab\Sigma x + nb^2)\,2a/m - 2(a\Sigma x^2 - \Sigma xy + b\Sigma x)\,(1 + a^2/m)}{(1 + a^2/m)^2} = 0$$

and

$$\frac{\partial}{\partial b}\Sigma d^2 = \frac{2(a\Sigma x - \Sigma y + nb)}{1 + a^2/m} = 0$$

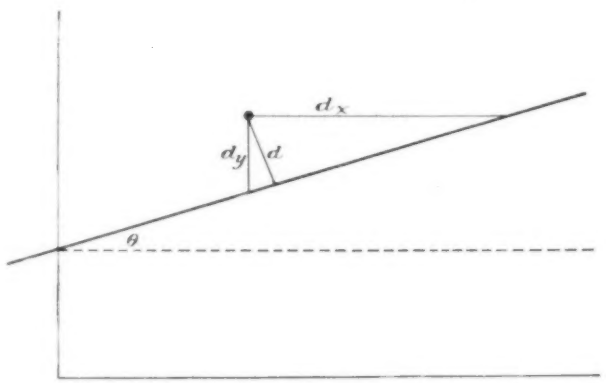


FIG. 7

This gives

$$\frac{a}{m}\Sigma y^2 - 2\frac{ab}{m}\Sigma y + \frac{nab^2}{m} + \Sigma xy - a\Sigma x^2 - b\Sigma x - \frac{a^2}{m}\Sigma xy + \frac{a^2 b}{m}\Sigma x = 0$$

and

$$a\Sigma x - \Sigma y + nb = 0.$$

Therefore

$$b = \frac{\Sigma y - a\Sigma x}{n}$$

and

$$na\Sigma y^2 - a(\Sigma y)^2 + n(m - a^2)\Sigma xy - (m - a^2)\Sigma x\Sigma y + am(\Sigma x)^2 - nam\Sigma x^2 = 0.$$

The last equation, when arranged in the type form of a quadratic, is

$$a^2 - \left[\frac{n\Sigma y^2 - (\Sigma y)^2}{n\Sigma xy - \Sigma x\Sigma y} - \frac{m[n\Sigma x^2 - (\Sigma x)^2]}{n\Sigma xy - \Sigma x\Sigma y}\right]a - m = 0,$$

or

$$a^2 - \frac{(\Sigma y'^2 - m\Sigma x'^2)}{\Sigma x'y'}a - m = 0$$

The solution of this is

$$a = \frac{\Sigma y'^2 - m\Sigma x'^2}{2\Sigma x'y'} \pm \frac{1}{2}\sqrt{\left(\frac{\Sigma y'^2 - m\Sigma x'^2}{\Sigma x'y'}\right)^2 + 4m}$$

Referring now to Fig. 2 and its construction, it will be remembered that

$$EF = \Sigma\left(\frac{y'^2}{2}\right) - m\Sigma\left(\frac{x'^2}{2}\right)$$

Also

$$EJ^2 = EG \times EH = OE \times EH = m(OE)^2 = m(\Sigma x'y')^2$$

$$\text{Therefore } JF = \sqrt{\left[\Sigma\left(\frac{y'^2}{2}\right) - m\Sigma\left(\frac{x'^2}{2}\right)\right]^2 + m(\Sigma x'y')^2}$$

and $EI = JF - EF$

$$= \frac{\Sigma x'y'}{2}\sqrt{\left[\frac{\Sigma y'^2 - m\Sigma x'^2}{\Sigma x'y'}\right]^2 + 4m} - \frac{\Sigma y'^2 - m\Sigma x'^2}{2}$$

The slope of the constructed line is

$$\frac{EI}{OE} = -\frac{\Sigma y'^2 - m\Sigma x'^2}{2\Sigma x'y'} + \frac{1}{2}\sqrt{\left[\frac{\Sigma y'^2 - m\Sigma x'^2}{\Sigma x'y'}\right]^2 + 4m}$$

but this is identical with the expression for a above; hence the line has the correct slope. Moreover, the value of b by construction is $\frac{\Sigma y}{n} - a\frac{\Sigma x}{n}$, which agrees with the required value of b given above.

CASE 3

To determine a and b in the empirical equation $Y = \frac{a}{X} + b$.

In Fig. 3 let x_1 = abscissa of c, u_1 = abscissa of c', and r = radius of quadrant. Then, from similar right triangles, $\frac{x_1}{r} = \frac{r}{u_1}$ and $x_1 = \frac{r^2}{u_1}$.

$$y_1 - d_1 = \frac{a}{x_1} + b = \frac{au_1}{r^2} + b$$

and similarly for y_2, y_3, etc.,

$$\Sigma d^2 = \Sigma y^2 - \frac{2a}{r^2}\Sigma uy + \frac{a^2}{r^4}\Sigma u^2 - 2b\Sigma y + \frac{2ab}{r^2}\Sigma u + nb^2$$

$$\frac{\partial}{\partial a}\Sigma d^2 = -\frac{2}{r^2}\Sigma uy + \frac{2a}{r^4}\Sigma u^2 + \frac{2b}{r^2}\Sigma u = 0$$

$$\frac{\partial}{\partial b}\Sigma d^2 = -2\Sigma y + \frac{2a}{r^2}\Sigma u + 2nb = 0$$

$$a = r^2\frac{\Sigma u'y'}{\Sigma u'^2} \qquad b = \frac{\Sigma u^2\Sigma y - \Sigma u\Sigma uy}{n\Sigma u^2 - (\Sigma u)^2}$$

To prove that the curve in Fig. 3 correctly represents the equation, select any point where the curve cuts an ordinate of one of the observed points, such as p, the corresponding point on the auxiliary line being p'. Then, by construction, $pq = p'q'$, and $op' = \frac{r^2}{op}$. Also, the auxiliary line was so constructed that $p'q' = \left(\frac{\Sigma u'y'}{\Sigma u'^2}\right)u' + \frac{\Sigma y}{n}$

$$= \left(\frac{\Sigma u'y'}{\Sigma u'^2}\right)\left(\frac{r^2}{op} - \frac{\Sigma u}{n}\right) + \frac{\Sigma y}{n}$$

$$= \frac{r^2 \dfrac{\Sigma u'y'}{\Sigma u'^2}}{op} + \frac{\Sigma u^2 \Sigma y - \Sigma u \Sigma uy}{n\Sigma u^2 - (\Sigma u)^2}$$

Substituting $Y = pq$ for $p'q'$, and X for op, and inserting a and b in place of their equivalents, this equation becomes identical with the type form at the head of the paragraph; hence the construction is correct.

Case 7

To determine a and b in the empirical equation $Y = a \sin \omega X + b \cos \omega X$, it being conditioned that $\Sigma y = 0$ and $\omega\left(\text{or } \frac{2\pi}{\omega}\right)$ is known.

Let x_1, x_2, x_3, etc., be redefined as the abscissas of the plotted points relative to the new y-axis, the abscissa of this axis relative to the original y-axis being c.

Let f = amplitude of double-frequency sine curve used in construction.

Let g = amplitude of fundamental frequency sine curve used in construction.

Then, by construction (Figs. 6-*b* and 6-*c*),

$$\frac{\dfrac{c}{2\pi}}{\omega} = \frac{1}{2} \times \frac{\angle EFD}{2\pi}$$

$$\omega c = \frac{\angle EFD}{2} = \frac{1}{2} \text{arc tan} \frac{DE}{DF} = \frac{1}{2} \text{arc tan} \frac{\Sigma f \sin 2\omega x}{\Sigma f \cos 2\omega x}$$

$$\tan 2\,\omega c = \frac{\Sigma \sin 2\omega x}{\Sigma \cos 2\omega x}, \quad \sin 2\,\omega c \Sigma \cos 2\,\omega x = \cos 2\,\omega c \Sigma \sin 2\omega x$$

$$\Sigma \sin 2\,\omega(x - c) = \Sigma \sin 2\,\omega x' = 0$$

According to the theory of least squares

$$d_1 = a \sin \omega x_1 + b \cos \omega x_1 - y_1, \text{ etc.}$$

$$d_n = a \sin \omega x_n + b \cos \omega x_n - y_n$$

$$d^2{}_n = a^2 \sin^2 \omega x_n + 2ab \sin \omega x_n \cos \omega x_n + b^2 \cos^2 \omega x_n - 2ay_n \sin \omega x_n - 2by_n \cos \omega x_n + y_n{}^2$$

$$\frac{\partial}{\partial a} \Sigma d^2 = 2a\Sigma \sin^2 \omega x + 2b\Sigma \sin \omega x \cos \omega x - 2\Sigma y \sin \omega x = 0$$

$$\frac{\partial}{\partial b} \Sigma d^2 = 2b\Sigma \cos^2 \omega x + 2a\Sigma \sin \omega x \cos \omega x - 2\Sigma y \cos \omega x = 0$$

$$a = \frac{\Sigma y \sin \omega x - b\Sigma \tfrac{1}{2} \sin 2\omega x}{\Sigma \sin^2 \omega x} \qquad b = \frac{\Sigma y \cos \omega x - a\Sigma \tfrac{1}{2} \sin 2\omega x}{\Sigma \cos^2 \omega x}$$

Since the empirical curve is determined solely by the y-deviations of the observation points and does not depend on the location of the axes of coordinates, the y-axis may be shifted arbitrarily to the right or left. Assume that it is moved to the right (Fig. 6-*d*) so that the new abscissas x' are related to the old ones x by the equation $x' = x - c$. Then the new values of a and b are given by the equations

$$a = \frac{\Sigma y \sin \omega x' - b\Sigma \tfrac{1}{2} \sin 2\omega x'}{\Sigma \sin^2 \omega x'} \qquad b = \frac{\Sigma y \cos \omega x' - a\Sigma \tfrac{1}{2} \sin 2\omega x'}{\Sigma \cos^2 \omega x'}$$

As proved in the foregoing, the second term in the numerator of each of these expressions equals zero. According to the construction used in Fig. 6-*d*, $LM = \frac{IK}{IJ} g = \frac{\Sigma y\,(g \sin \omega x')}{\Sigma\,(g \sin \omega x')^2} g = \frac{\Sigma y \sin \omega x'}{\Sigma \sin^2 \omega x'} = a$; likewise, $NP = b$.

Discussion

Carroll F. Merriam.[7] There is an important application for least squares in studies upon which are based important contracts. Slight differences in the location of the "best line" may make differences in the terms of these contracts amounting to considerable sums of money, and if the location be left to judgment it would give rise to disputes. Therefore any simplification of this problem and the enabling of it to be more commonly used should prove of assistance to engineers handling work of this kind.

Although analytical methods involve complicated computations, the development of the modern electric-driven computing machine makes possible a quick and easy solution, especially if the machine has a sufficient number of banks of keys to enable the products x^2 and $x'y'$ to be multiplied simultaneously and totaled at the same time. The graphical method has little if any advantage when mechanical means of calculating are available.

In all seven of the cases mentioned, it has been assumed that the best line would have a comparatively simple equation. Although this assumption may be of great aid in deriving formulas from experimental data, there are many engineering problems in which no attempt is made to derive such a formula, and, furthermore, this assumption may be misleading. We must not lose sight of the fact that we have information frequently, other than the mere coordinates of the points. Attention should be given to the location of the axes and the probable relation between the variables as evident from the nature of the problem. Failure to observe these may easily lead to false conclusions.

We may take as an example Fig. 1. To one unfamiliar with engineering, it would appear that the straight line is claimed to be the best representation of the relation between the two variables and that inasmuch as efficiency is increased with decreasing rate of firing from 120 lb. of dry coal per sq. ft. per hr. to one-third of this rate, it can be expected that maximum efficiency will be reached when the rate of firing is zero. It is perfectly evident from the nature of the case that this is not true.

We know that the curve must end at a value of x equal to the stand-by losses and that at this point y will equal zero. This fact is just as important as the location of any of the eight points, but it has been ignored necessarily by assuming the best line to be straight. Furthermore, no attempt has been made to locate the probable maximum value for efficiency, even though this may be of prime importance in such a series of tests.

These experimental results can be replotted by using the vertical scale for the product of the original x and y values. Inasmuch as the coal burned times efficiency equals the coal usefully burned, it will be apparent that the losses, which would then be represented by the distance that each point falls below a 45-deg. line, appear to be made up of certain constant losses plus others which are approximately proportional to some power of the rate of firing. It will be seen, furthermore, that five of the points lie very distinctly along such a curve, whereas point d is out of line with the others and therefore extremely doubtful. One would certainly be justified in at least heavily discounting the value of point d, if not in eliminating it altogether. This fact is by no means so clearly evident from Fig. 1. Points g and h may be considered to have about equal values in determining the trend of the curve for the high rates.

[7] Pennsylvania Water & Power Company, Baltimore, Md. Mem. A.S.M.E.

In many engineering problems it is not wise to trust the location of an important curve either to a draftsman or to mathematical analysis. There is need for engineering judgment and skill. For example, it is seldom justifiable to make sudden changes in the general trend of the curve to accommodate apparently wild points, yet there are cases where sudden turns have been shown to be inherent to the nature of the work. On the other hand, it is easy to become misled by opinions formed in advance regarding the shape to be taken by a curve, and great care should be taken to guard against this kind of mistake.

The method of least squares can be applied graphically to test the merits of several proposed locations for the most probable curve passing through a group of points. The plotted points should be connected to any curve to be tested by vertical lines. With the length of these lines as radii, circles are described using the points themselves as centers. The tracing point of the planimeter is started at a distinctly marked point on the curve, anywhere to the left of the left-hand point. The curve should be followed to the right to the foot of the vertical line drawn from the first point, and then around the circle clockwise, returning again to this point and proceeding along the curve to the second vertical line, and so on.

After following around the last circle, the starting point is reached by returning along the curve, and the total area of the circles are read on the dial of the planimeter. Assuming that all curves tested are consistent in shape with the nature of the data, the one which shows the smallest area included in the circles may be considered to be the best representation of the probable relation between the two variables.

The foregoing applies to the case where it is assumed that there is no error in x, but in case both x and y are assumed to be subject to error, slanting lines can be substituted for the vertical lines, these lines being inclined in proportion to the probable error in each of the two directions.

T. H. Gronwall.[8] In a paper of this kind, the question of greatest interest to the engineer is that of the speed and accuracy with which the methods proposed can be applied to practical problems. It may be stated that, at present, graphical methods are generally held inferior in both respects to the numerical ones, except possibly in some quite special cases of the application of graphical statics. Professor Whitaker, who established the first laboratory of applied mathematics in any English-speaking country at the University of Edinburgh in 1913, says in the preface to his "Calculus of Observations:" "When the Edinburgh laboratory was established in 1913, a trial was made, as far as possible, of every method which had been proposed for the solution of the problems under consideration, and many of these methods were graphical. During the ten years which have elapsed since then, the graphical methods have almost all been abandoned, as their inferiority has become evident, and at the present time the work of the laboratory is almost exclusively arithmetical."

The same kind of evidence of the superiority of the numerical methods may be gathered by a perusal of the volumes of the German "Zeitschrift for angewandse Mathematik und Mechanik," and the experience of the writer has been the same, with regard to both engineering and purely scientific problems. With the increasing use of calculating machines, this superiority of the numerical methods will be more and more pronounced. In the paper under discussion, the graphical construction of reciprocals in case 3, Fig. 3, for instance, is decidedly slower than reading them off from a table of reciprocals, and the construction in case 7, while quite interesting from a theoretical point of view, is exceedingly complicated.

Another consideration is that if one does not use these intricate graphical processes constantly, one generally finds that when the occasion to use them arises, one has forgotten how to do it and has to read up on them again. On the contrary, the simple mathematics underlying the numerical processes is easily retained in the memory.

Criticism may also be directed against the application of the method of least squares itself to problems of the type used as illustrations in the paper. It is not often that one can assert with any degree of plausibility that the Gaussian law holds for the errors of observation in engineering data, and hence the applicability of the method of least squares is very limited. Even where it is applicable, a rough and ready determination of the constants involved will frequently be as acceptable from a practical point of view as the least-square determination, and accomplished with much less labor. The writer has encountered many instances of this kind.

G. B. Karelitz.[9] The paper presents an opportunity to discuss in general the comparative value of graphical, mechanical, and numerical integration. A number of years ago the writer had an opportunity to investigate the accuracy of area determination by planimeters. It was found that in order to obtain an accuracy of 1 per cent the boundary of the area must be traced at least twice. An objection to mechanical computation is the ease with which an error can be made in tracing the triangles; one of these may be traced only once, while another three times instead of two. And when an error is made there is no means to locate it; the calculation must be repeated; it is therefore necessary to read the dial of the planimeter after each triangle is traced.

Now, in cases where a large number of determinations is to be made, such as the mentioned several hundred steam diagrams, or in shipbuilding computations where from 100 to 200 integrals of waterlines and cross-sections are to be found, the mechanical integrator is very serviceable. In mechanical engineering the accuracy generally needed is comparatively not high, and simple numerical computation proves to be the most reasonable method for evaluation of integrals. The method of least squares may be used advantageously only in separated occasions, the arithmetical average of values observed being a sufficient approximation in most cases.

Another general remark which the writer would like to make has no direct bearing on the paper itself. It has been quite customary to interpret a set of experimental data by an analytical formula. This procedure is rather dangerous; the formula may be reprinted several times, the range of the experiments not being mentioned, and unknowingly the results of the experiment in question will be extrapolated without any justification. Anybody can quote a number of examples to corroborate this statement. Only when a certain law governing the shape of the curve is known, the constants and coefficients being determined by experiment, is there reason to express test results by analytical formulas. In transactions of the last year or two one may find a number of papers giving such unjustified formulas having practically no meaning. Introducing a general practice to present experimental data by a curve or by a table, or to define clearly the range of validity of a formula, should one be used, would be of much value.

A. E. R. de Jonge.[10] I think that Mr. Waters is to be congratulated on drawing the attention of engineers in general to the method of least squares and on giving a simple and efficient method which will allow us to solve problems of this kind quickly on the drawing board.

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[10] New York Edison Co., New York, N. Y. Mem. A.S.M.E.

The question which Prof. Whitaker in England raises, that graphical methods are not as accurate as are analytical or computing methods, is of course quite true when it is necessary to apply them with great accuracy; but in practically all cases of average engineering problems this great accuracy is neither necessary nor wanted. I must say I disagree with Professor Whitaker completely in his statement that graphical methods are easily forgotten. I think it is very much easier to remember a process like the one given by the author than to remember a formula, and, as the speaker before outlined, there are limitations to formulas.

Formulas are so often used without accurate knowledge by the person who uses them, of how they have been derived and what limitations are involved; and I must say that I honestly believe that the method which Mr. Waters has outlined is as good as any that may be given in this particular case.

I was originally a naval architect, and in that science the question of integration enters as an everyday problem. We have to find integrals, but we never do it by computation. We always do it graphically, either purely graphically by a simple method of geometrical construction, or by the assistance of an instrument, either the integraph, the integrator, or at least the planimeter.

I think that the use of graphical methods can be recommended, as far as engineering problems are concerned.

The question of using analytical methods or graphical methods is, to my mind, largely a question of personal inclination and ability. Some people like the analytical way of dealing with problems and others the geometrical way, and it is largely a question of the person who uses the method to choose which one suits him best.

I have especially in mind the question of graphic statics. In graphic statics there is nowadays such a tremendous lot of work being done with analytical methods which could so profitably be replaced by a much simpler graphical method. I mention only the question of the elasticity ellipse, which is practically unknown in this country and practically unknown on the other side, too, except in Switzerland, where it originated, has been developed, and used to a very large extent. Professor Culmann is the originator and has developed the method, and after his death Dr. W. Ritter has perfected the method in such a way as to be incomparably simpler than all analytical methods. I say this unreservedly and it is a great pity that these methods are not known either in this country or in England.

I would like to advocate that whenever curves are being given the points from which the curves are derived should be marked on the diagram. It is unfortunately true that many, many curves have been published which do not represent values correctly. The omission of the points does not give one a chance of checking up the actual value of the curve or the actual average which the curve represents. It would be very much more valuable if all those curves and data diagrams which are being published would indicate definitely those points from which they are derived and if this would be made a general practice by members of this Society.

Author's Closure

Mr. Merriam has called attention to the comparatively simple equation for each of the seven cases illustrated in the paper. It was not the author's intention to emphasize equations, but rather the curves which are the real object in any graphical representation of data. However, since the point has been raised, he would state that the equations *are* simple, and for two reasons: (1) The planimeter is admirably suited for performing the operation $a_1b_1 + a_2b_2 + a_3b_3 + \ldots + a_nb_n$ at one setting, as required when two constants are to be determined; but when more than two constants are involved, the products are of higher degree, and consecutive multiplication and addition are not so easy. Incidentally, the standard computing machine would also present difficulties at this point. (2) Simple equations show the trend of the plotted points adequately, without forcing a proximity of the curve to doubtful points.

Fig. 1 illustrates this feature very well. The straight-line relationship between efficiency and rate of firing is of course unreasonable when extrapolated to the y-axis, yet it was quite properly used by the author of the book from which the plotted points were taken (the data of the figure are not original with the present paper) to derive further relations in locomotive-boiler operation of marked importance and value. It would have been quite possible to plot efficiency times rate of firing versus rate of firing as Mr. Merriam suggests, but neither this relationship nor the probable value of the maximum efficiency was considered to be essential.

It is absolutely certain that judgment and common sense must be applied in curve plotting. If points or regions of discontinuity are known to exist, from experience with the tests and observations, they must of course be recognized, the plotted points separated into groups, and individual curves drawn through each group. Skill must also be used in the discarding or weighting of "wild" points. The graphical method is only a technique—a kind of tool; and common sense is necessary in using it just as in applying any other technique to the solution of an engineering problem.

Mr. Merriam's method of testing a curve by tracing circles around the plotted points is interesting and theoretically correct, as far as it goes. Its evident drawback lies in the fact that it satisfies a necessary but not a sufficient condition for proof that a given curve is the best line, as judged by the least-square criterion.

Mr. Karelitz has presented some interesting observations on the relative accuracy of graphical and other methods of integration. It is not at all clear to the author why the sources of inaccuracy mentioned, such as tracing triangles the wrong number of times, are not equally possible when performing the equivalent operation by machine. Furthermore, it is stated that arithmetical averages of observed values are good enough in most cases. How, then, is a straight line, much less a curve, to be located by arithmetical averages? There is an infinite number of straight lines that may be drawn through a field of plotted points, any one of which will have the sum of the deviations equal to zero, but only one of these lines will be the right one.

The criticism that Mr. Gronwall made represents pretty generally the point of view of statisticians, both in this country and abroad. In all of the books on statistics that I have consulted I find that arithmetical methods are explained at great length, and many chapters are devoted to exposition of the best system for tabulating the figures, setting down the products and sums, and giving hints as to how to avoid errors, all of which points to the conclusion that the arithmetical method is not quite as simple as it is claimed to be.

It would seem that the preference of method is largely a matter of the point of view of the user. One designer, in obtaining the size of a machine part, may figure it out by formula; another may use a chart or nomogram of some sort; a third may use both methods, checking one against the other. Personally, I should not like to say dogmatically that any one of these designers uses the best method. My experience with engineering offices in industry has been that graphical systems are frequently preferred; the average engineer can grasp the real meaning of a set of observations by a glance at a graphical chart much better than from a table of figures, and, that being the case, he is more prone to favor graphical methods in any application than he is to favor arithmetical methods.

APM-51-19

Flow Characteristics of Submerged Jets

By M. J. Zucrow,[1] Lafayette, Ind.

In this paper, which deals with submerged square-edged and chamfered short tubes or jets, the author shows by dimensional analysis and corroboratory experiment that the coefficient of discharge for geometrically similar jets is a function of the rate of discharge, the fluidity of the liquid, and the jet diameter. The effect of changing the angle of chamfer was studied as well as the influence of the variations in the length-diameter ratio on the discharge characteristic.

THIS paper deals with the discharge characteristics of submerged square-edged and chamfered short tubes or jets which were calibrated with chemically pure benzol. The diameters of the jets varied from 0.020 to 0.088 in., and the ratio of the length L to the diameter D ranged from 0.318 to 10.57.

By means of dimensional reasoning the conditions for similarity in the motions of liquids through geometrically similar jets are established, and the results checked experimentally. It is shown that the coefficient of discharge C, for geometrically similar jets, is a function of the dimensionless product $w\phi r$, where w is the rate of discharge (gravimetric), ϕ the fluidity of the liquid, and r the reciprocal of the jet diameter in inches.

The effect of changing the angle of chamfer α at the entrance to jets having the same L/D ratio was studied; two chamfer depths were used, namely, 0.008 and 0.040 in. To determine the influence of variations in L/D on the discharge characteristics several square-edged tubes that had the same diameter (0.0326 in.) but different ratios of L/D were calibrated.

Dimensional Treatment of Flow Through a Jet. According to the ordinary laws of hydraulics the flow of liquids through jets and orifices is given by Torricelli's equation as

$$Q = Ca\sqrt{2gh} \quad \ldots\ldots [1]$$

where

Q = volumetric rate of discharge,
C = coefficient of discharge and
a = area of the orifice, or jet, and
h = head expressed in terms of the flowing liquid.

Experiments have demonstrated that C varies with the rate of discharge and with the liquid flowing, and for a particular jet it is affected by certain definite physical quantities, represented by the functional relationship

$$C = f(v,d,\rho,\mu) \quad \ldots\ldots [2]$$

where v is the actual mean velocity of the fluid, defined by the equation $Q = av$, d is the diameter of the jet, ρ is the density, and μ the viscosity. By means of dimensional reasoning it is possible to express dimensionally the foregoing relationship of physical quantities as the product of powers of the fundamental quantities, mass, length, and time.[2] Since C is a pure number, it has no dimensions, and [2] expressed dimensionally becomes

$$O = L^aT^{-a} \cdot L^b \cdot M^cL^{-3c} \cdot ML^{-1}T^{-1} \quad \ldots\ldots [3]$$

The values of the exponents are found to be

$$a = -1,\ b = -1,\ c = -1$$

and

$$C = f\frac{vd\rho}{\mu} \quad \ldots\ldots [4]$$

By a simple transformation it is possible to put [4] in the more convenient form

$$C = f(w\phi r) \quad \ldots\ldots [5]$$

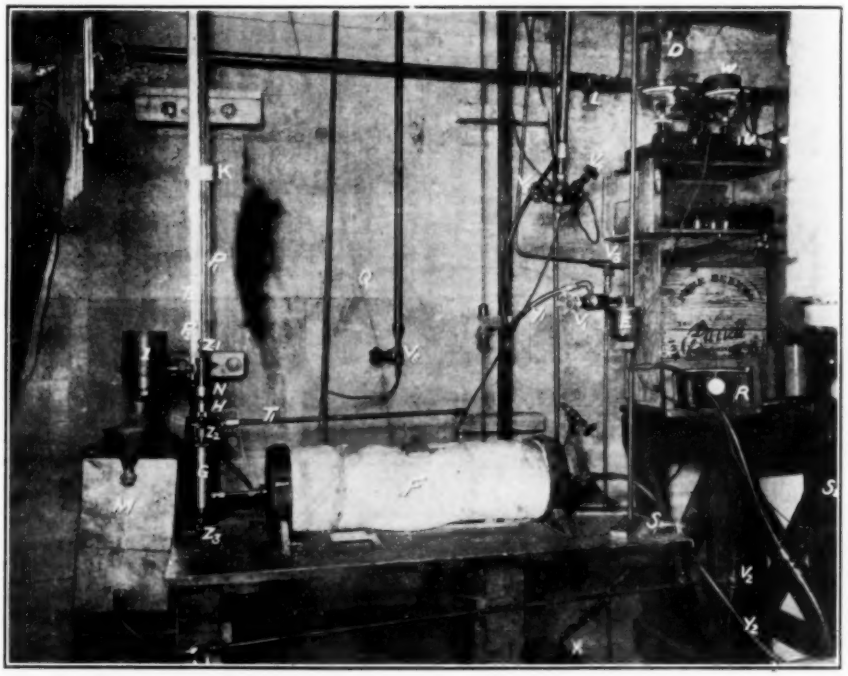


Fig. 1 View of Test Plant

where w = actual rate of discharge (gravimetric)
r = reciprocal of the diameter = $1/d$
ϕ = fluidity of the liquid = $1/\mu$

Since $w\phi r$ is non-dimensional, either English or metric units may be used in its evaluation.

The validity of [5] depends entirely upon the truth of the assumption that v, d, ρ, and μ are the only variables entering into the problem.

Equation [5] tells us that, if C is plotted as a function of $w\phi r$, the resulting curve gives information regarding the effects of temperature and fluid changes upon C from experiments using one liquid and an adequate range of heads. The curve so obtained for a given jet can be regarded as its characteristic curve, and should hold true for all geometrically similar jets.

[1] Research Associate, Engineering Experiment Station, Purdue University.

[2] Bridgman, P., "Dimensional Analysis," Yale Univ. Press.

Description of Apparatus

The test plant is shown in Fig. 1, and the arrangement of the parts is indicated schematically in Fig. 2. The plant embraces the following main components:

1 Jet chamber (*H*)
2 Weir (*I*)
3 Filter (*G*)
4 Heater (*F*)
5 Piezometers (P_1) and (P_2)
6 Float chamber (*E*)
7 Fuel-weighing and electrical apparatus.

Only the last item need be described in detail.

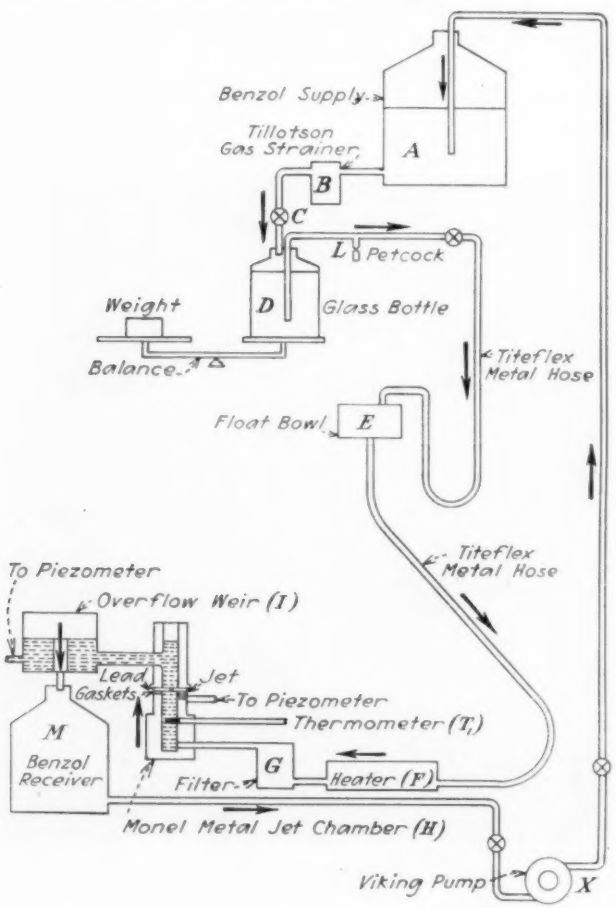


Fig. 2 Arrangement of Apparatus in Test Plant

Fuel-Weighing and Electrical Apparatus. The benzol is piped from the tank *A*, Fig. 2, into the glass bottle *D* located on the left-hand pan of the balance, and shown in the upper right-hand corner of Fig. 1. To give the balance freedom of motion the benzol is siphoned from the bottle to the carbureter float chamber *E*, used to maintain a constant static head upon the jet. The balance is sensitive to less than one-tenth of a gram, and is equipped with a contact wire dipping into the mercury cup *U*. The latter completes an electric circuit when a balance is reached between the scale pans. The completion of this circuit operates the relay *R*, which is used for starting and stopping the decimal-faced stop watch shown in the picture.

The connections from the siphon to the float chamber and from the latter to the heater are made of Titeflex flexible metal tubing. The petcock *L*, shown in the upper part of Fig. 1, is used for priming the siphon.

Course of Flow. Referring to Figs. 1 and 2, the benzol flows from the supply tank *A*, through the Tillotson gas strainer *B*, past the gate valve V_4, into the glass bottle *D*. From here the liquid is siphoned into the float chamber *E*; it then flows through the heater and filter to the jet chamber *H*, upward through the jet into the weir *I*, which maintains a constant submergence of approximately 4 in. upon the jet. The liquid discharged from the weir is caught by the receiver *M*, from where it is returned to the supply tank *A* by means of the motor-driven Viking pump *X*.

Outline of Investigation

The investigation subdivided itself into three subordinate series of experiments.

1—*Geometrically Similar Jets.* This part of the investigation consisted of the calibration of eight different jets having substantially the same *L/D* ratio. The jets were machined in pairs so that there were four different diameters; i.e., each pair had the same diameter. The dimensions and identification numbers of these jets are given in Table 1.

TABLE 1 GEOMETRICALLY SIMILAR JETS

Jet No.	Diameter (*D*) In.	Diameter (*D*) Cm.	Length (*L*) In.	*L/D*
1[1]	0.0882	0.224	0.0289	0.328
1[2]	0.0879	0.224	0.0290	0.330
3[1]	0.0593	0.151	0.0195	0.329
3[2]	0.0593	0.151	0.0195	0.329
4[1]	0.0318	0.0808	0.0101	0.318
4[2]	0.0317	0.0805	0.0106	0.334
5[1]	0.0205	0.0518	0.0069	0.338
5[2]	0.0204	0.0518	0.0069	0.338

The object in making and calibrating two like jets of each diameter was to determine if any desired jet could be reproduced if necessary; further to use the data obtained with one jet as a check on the results obtained with the other.

2—*Chamfered Jets.* To obtain the information pertaining to this phase of the research, ten different jets having substantially the same lengths and diameters were used. One of these, No. 27, was square-edged, the others were chamfered with different included angles. Four of these were chamfered to a depth of 0.008 in., and five to a depth of 0.040 in. The dimensions of these jets are given in Tables 2 and 3.

TABLE 2 CHAMFERED JETS (0.008 IN. DEEP)

Jet No.		Diameter (*D*) In.	Diameter (*D*) Cm.	Length (*L*) In.	*L/D*
20	20	0.0615	0.1562	0.2186	3.56
22	60	0.0615	0.1562	0.2186	3.56
24	90	0.0617	0.1567	0.2187	3.55
25	120	0.0618	0.1569	0.2185	3.54
27	180	0.0615	0.1562	0.2185	3.56

TABLE 3 CHAMFERED JETS (0.040 IN. DEEP)

Jet No.		Diameter (*D*) In.	Diameter (*D*) Cm.	Length (*L*) In.	*L/D*
28	20	0.0615	0.1562	0.2185	3.56
29	60	0.0615	0.1562	0.2180	3.56
30	90	0.0617	0.1567	0.2187	3.55
31	120	0.0618	0.1569	0.2185	3.54
32	150	0.0618	0.1569	0.2185	3.54
27	180	0.0615	0.1562	0.2185	3.56

3—*Square-Edged Jets With L/D Varying.* For this part of the investigation seven different square-edged jets having substantially the same diameter but different lengths were calibrated. Table 4 lists the dimensions of these jets.

TABLE 4 SQUARE-EDGED JETS

Jet No.	Diameter (*D*) In.	Diameter (*D*) Cm.	Length In.	*L/D*
7[1]	0.0323	0.0820	0.0300	0.929
7[2]	0.0323	0.0820	0.0310	0.960
8	0.0326	0.0828	0.0465	1.435
9	0.0326	0.0828	0.0622	1.907
10	0.0326	0.0828	0.0928	2.845
12	0.0328	0.0833	0.1550	4.730
14	0.0324	0.0823	0.3427	10.570

Properties of Liquid. Commercial chemically pure benzol was used throughout the investigation. The physical properties of this benzol were determined by the Bureau of Standards and are presented in Fig. 3.

Note on Temperature Control. Since plotting C as a function of $w\phi r$ takes into account the effects of temperature changes, no attempt was made to control the temperature of the liquid during the experiments; the temperature variation during any one set of experiments was small in every case, rarely exceeding more than 3 deg. cent.

Presentation and Discussion of Results

Geometrically Similar Jets. Figs. 4, 5, and 6 summarize the investigation with square-edged jets having the same L/D. In Fig. 4 the observed net head h is plotted as ordinate against the rate of discharge W as abscissa. The reason for using W as the independent variable instead of h is to enable the assembling of the complete data on one sheet. Each of the four curves presents the data obtained at substantially constant temperature, with a pair of jets having the same diameter and practically the same length. Of the eight jets, seven have substantially the same L/D ratio, about 0.333; the remaining jet (No. 4^1) has an L/D ratio of 0.318.

The plotted results for each jet of a pair having the same dimensions fall on one curve. This not only confirms the accuracy of the data, but also indicates that it is possible to produce duplicate jets.

Fig. 5 shows the coefficient of discharge C as a function of h. This is the usual method employed for presenting the results of hydraulic experiments.

It is apparent that four different curves, one corresponding to each jet diameter, are required to describe the results. It is seen that when h and L/D are both constant, the coefficient of discharge decreases as the jet diameter is increased. Further, in the case of such short jets, the value of C increases as h decreases. This result is true in a general way of all the jets, but the limits between which C varies narrow as the jet diameter increases. This is readily seen from the curve for jets Nos. 1^1 and 1^2. For these jets C varies from a minimum of 0.620 to a maximum of 0.690, the corresponding values of h being 19 and 3. C does not vary noticeably at the higher heads, the large changes in its value occurring at the lower heads. There appears to be a definite head below which C increases rapidly; this head is different for each pair of jets and decreases with the increase in diameter.

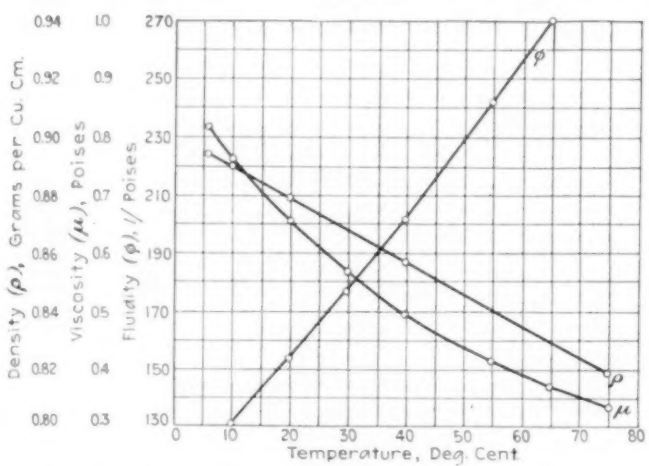


Fig. 3 Physical Properties of Chemically Pure Benzol

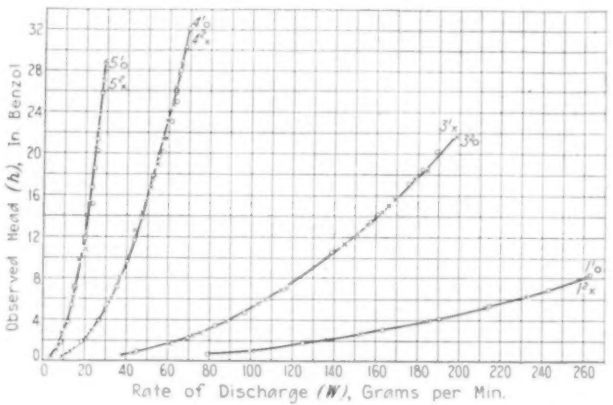


Fig. 4 Submerged Square-Edged Jets, Benzol Flowing. Observed Head Plotted Against Rate of Discharge

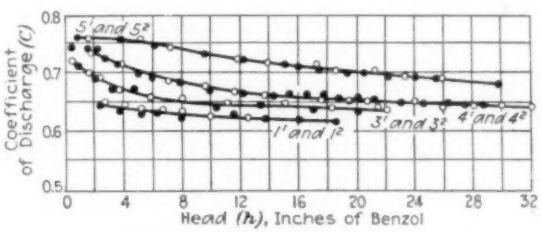


Fig. 5 Submerged Square-Edged Jets, Benzol Flowing. Coefficient of Discharge Plotted Against Head. L/D Substantially Constant

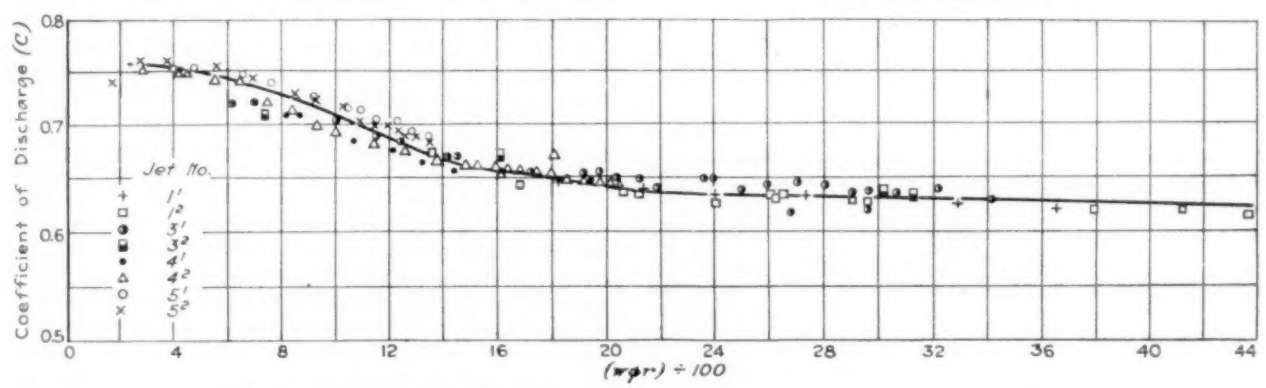


Fig. 6 Submerged Square-Edged Jets, Benzol Flowing. Coefficient of Discharge Plotted Against $w\phi r/100$

The plotting of C against h, as in Fig. 5, presents the results so that they can be readily applied in practice. But from an experimental point of view the mode of presentation has certain disadvantages. To obtain the necessary information a large number of jets must be calibrated, and the temperature must be maintained substantially constant through the tests. Furthermore, if the effect of changing the liquids discharged by the jets is to be studied, the investigation must be repeated for each liquid having a kinematic viscosity different from that used in the original calibration of the jets.

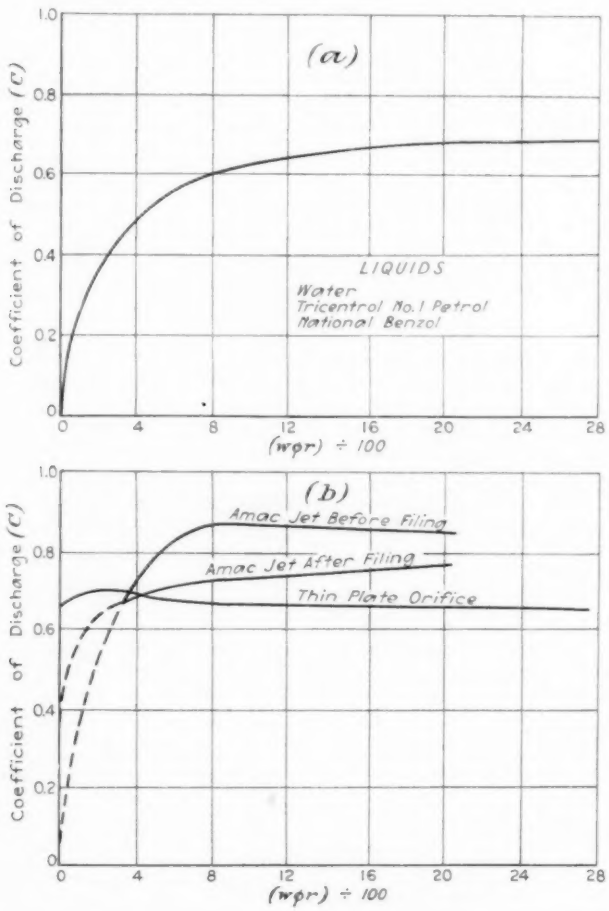


FIG. 7 CHARACTERISTIC CURVES OBTAINED BY BOND AND FINLAYSON

[(*a*) Characteristic curve of Zenith jet No. 110. (*b*) Characteristic curves of Amac jet No. 27 and orifice 1.463 mm. diam. (Orifice curve from results of W. N. Bond.)]

Fig. 6 shows C as a function of the non-dimensional group of variables $w\phi r$. It is apparent that, within the limits of experimental error, the points for all of the jets fall upon a single curve. The curve therefore applies to a single jet, or to any one of a group of square-edged jets with the same L/D.

Before discussing this curve in detail, it should be pointed out that one of the features of this method of presenting the results is the readiness with which it reveals small errors in measurement. For example, if an error of minus 1 per cent is made in measuring the diameter of a jet, two consequences result. First, the coefficient of discharge is increased about 2 per cent. Secondly, the value of $w\phi r$ is increased 1 per cent, which brings the plotted point into the region where the values of C are decreasing. Thus the point is moved upward and to the right, and plotted further off the curve than is the case where C is plotted against h. This may be the reason why the points for jets Nos. 5[1] and 5[2] lie slightly above all the others.

Referring to Fig. 6, we see that as $w\phi r$ is increased, C decreases, but reaches an almost steady value when $w\phi r$ is greater than 2000. The widest variations in C occur within the region containing value of $w\phi r$ less than 1800. If the curve is considered as applying to a single jet, the shape signifies that as the effect of viscosity diminishes (that is, as the fluidity increases) the condition of flow approaches that for a perfect liquid, and C tends to remain constant. The portion of the curve which includes the low values of $w\phi r$ corresponds to a condition of flow where the effect of viscosity is dominant, and probably represents a state of motion which is different from that prevailing where $w\phi r$ has large values. These results are in agreement with those obtained by Bond and by Finlayson (see Fig. 7). Their results, determined independently, were obtained by varying the viscosity of the liquid discharging through a single jet.

The results herein presented, and those of previous investigations, demonstrate conclusively that for similarity in the motions of different liquids through jets, the only condition necessary for geometric similarity is that L/D should be the same for each jet. Consequently, the curve shown in Fig. 6 is equally true for gasoline, water, or any liquid, and for all jets having $L/D = 0.333$. It should be remembered, however, that the viscosity of the liquid must be independent of the rate of shear; a condition which may not exist for heavy oils such as that used by Davidson[3] and oils at low temperatures.[4]

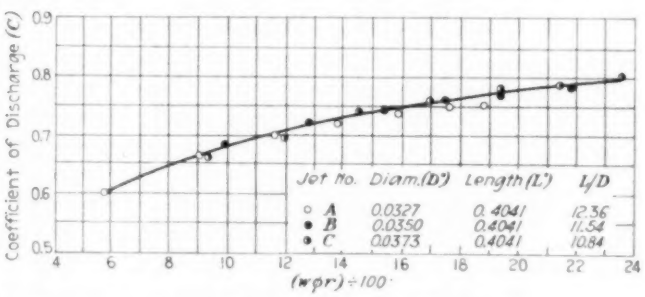


FIG. 8 SUBMERGED CHAMFERED JETS, WATER FLOWING. COEFFICIENT OF DISCHARGE PLOTTED AGAINST $w\phi r/100$

(Obtained by recomputing data of P. S. Tice, N.A.C.A. Report No. 49.)

The results just discussed certainly apply to short jets, but are they equally valid for longer jets? To obtain the answer to this question some of the experimental results, obtained by P. S. Tice[5] with chamfered jets having nearly the same L/D, and discharging distilled water, were recomputed. The recomputed data are shown plotted dimensionally in Fig. 8. Although these jets do not have exactly the same L/D and the depth of chamfer is not proportional to the jet dimensions, these small deviations in geometric similarity produce no material effect; the points for the three jets lie on one curve well within the limits of allowable error. The shape of the curve, however, is quite different from that obtained with the short jets of Fig. 6, for in the case of these chamfered jets C increases as $w\phi r$ is increased.

The work of Bond, Finlayson, Tice (recomputed), together with the experiments of the author, demonstrate that the characteristic curve obtained by plotting C against $w\phi r$ is valid for all liquids flowing through a jet, and for all jets which are geometrically similar to it, within allowable limits.

[3] Ibid.

[4] Wilkin, R. E., Oak, P. T., and Barnard, D. P., "Motor Oil Characteristics at Low Temperatures," *Jl. S. A. E.*, Feb., 1928.

[5] N.A.C.A. Report No. 49.

The foregoing discussion shows that a complete analysis of the flow of liquids through jets can be made with a minimum of experimental work if dimensional plotting is used. For all that is required is experiments with a single typical jet to determine its characteristic curve, one liquid, and an adequate range of heads. From the characteristic curve, the effect of changing the liquid, the rate of discharge, and the temperature can be calculated.

Effect of L/D—Square-Edged Jets. For this part of the investigation eight different square-edged jets were calibrated. The results are shown in Fig. 9. Of the eight, seven had substantially the same diameter, 0.0326 in., and one jet, No. 4², was slightly smaller, with a diameter of 0.0317 in. The dimensions of the jets are given in Table 4.

The two series of points for jet No. 14 refer to two different sets of experiments conducted with this jet to determine the quality of the edges at the entrance and exit of the jet. The

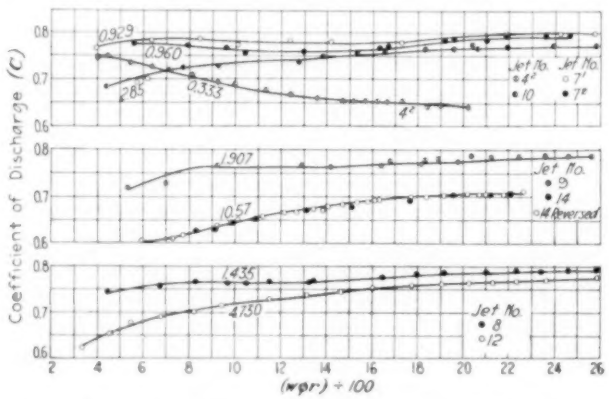


FIG. 9 CHARACTERISTIC CURVES OF SQUARE-EDGED JETS
(L/D varying from 0.333 to 10.57. Numbers refer to values of L/D.)

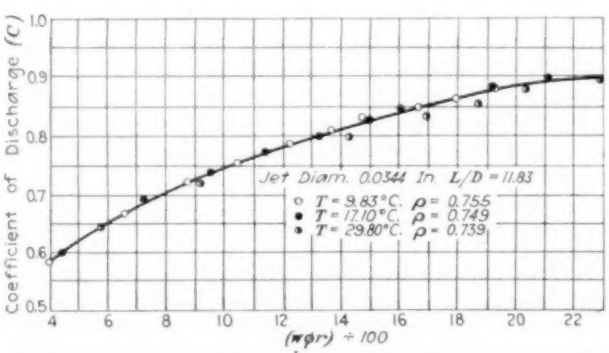


FIG. 10 SUBMERGED CHAMFERED JETS, GASOLINE FLOWING. COEFFICIENT OF DISCHARGE PLOTTED AGAINST $w\phi r/100$
(Obtained by recomputing data of P. S. Tice.)

side which pointed upstream during the first experiment was facing downstream in the second, so that the exit edge was used for the entrance to the passage. The points referring to this condition correspond to the points labeled "Jet No. 14 Reversed" in Fig. 9. The close agreement between the points indicates that the quality of both edges was about the same, and that if there was any taper or roughness in the passage, it was not of a serious nature.

An examination of Fig. 9 shows that varying L/D exerts a great influence upon the flow conditions within the jets. When L/D is 0.333, the high values of C occur at small values of $w\phi r$ between 200 and 500, and as $w\phi r$ is increased to about 1400 the value of C diminishes, but from this point on C remains practically constant. Increasing L/D to 0.929 completely changes the shape of the characteristic curve. The values of C corresponding to values of $w\phi r$ less than 350 are lower than those for $L/D = 0.333$; but in the region of high values of $w\phi r$ the values of C are much higher than they are for the thinner jet. Thus, when $w\phi r$ is 2000, $C = 0.642$ for $L/D = 0.333$,

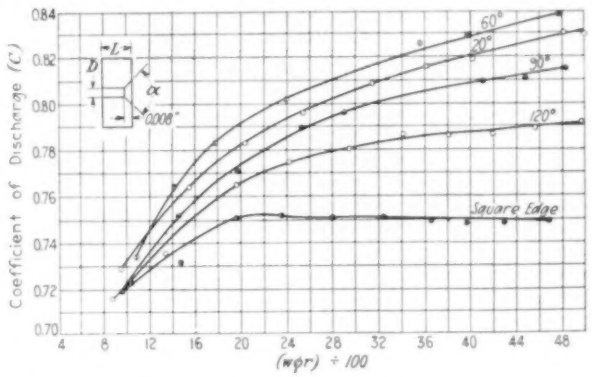


FIG. 11 EFFECT OF SMALL CHAMFER ON UPSTREAM EDGE
(Numbers refer to included angle of chamfer.)

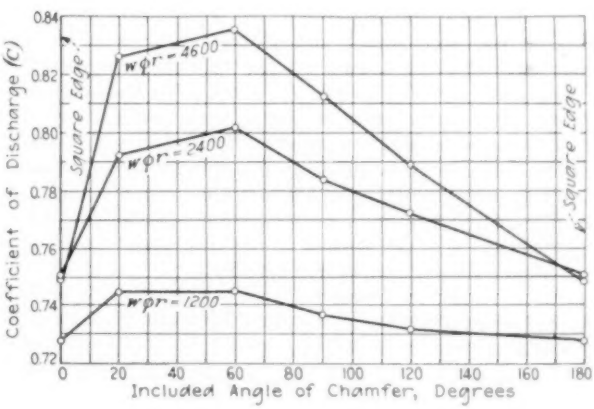


FIG. 12 EFFECT OF CHAMFER
(Depth = 0.008 in.; $L/D = 3.55$.)

and $C = 0.792$ for $L/D = 0.929$. It is also evident that, when $L/D = 0.929$, the variation in C for all values of $w\phi r$ between 700 and 2700 is slightly less than 1 per cent.

A small increase in L/D from 0.929 to 0.960 does not have an appreciable effect upon the shape of the characteristic curve, but the values of C are slightly diminished for the entire range of values for $w\phi r$. As L/D is further increased the values of C corresponding to low values of $w\phi r$ decrease more rapidly than those corresponding to the higher values. For example, consider an increase in L/D from 0.929 to 4.73. When $w\phi r$ is 400, C for the short jet is 0.767 and is only 0.643 for the longer jet, a decrease of 16.2 per cent. When $w\phi r$ is 2600, the corresponding values of C for the short and long jets are 0.789 and 0.770, or C is diminished only 3.5 per cent. At $w\phi r = 600$ the decrease in C is 11.7 per cent.

A comparison between the curves for $L/D = 0.929$ and $L/D = 10.57$ shows that, when $w\phi r = 600$, the decrease in C due to lengthening the jet is 22 per cent. Thus it appears that the increasing L/D decreases the values of C for all values of $w\phi r$, and that the greatest effects occur at the low values of $w\phi r$. It is also evident that the range of values for which C remains substantially constant is reduced, and is only reached with very high values of $w\phi r$.

Effect of Changing Temperatures. The effect of changing the temperature of the liquid is to change its viscosity and density. A rise in temperature increases ϕ, and decreases ρ.

It has been demonstrated that C is a function of v, d, ρ, and μ, and that no other physical properties need be considered. Consequently the effect of increasing or decreasing the temperature

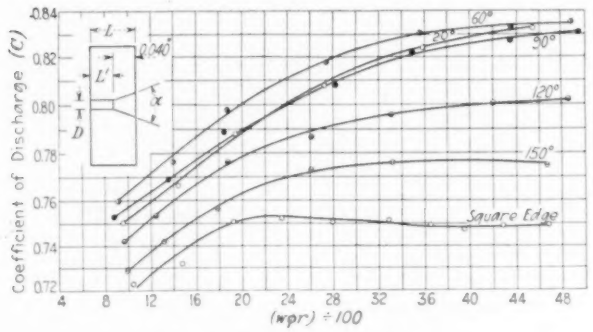


FIG. 13 EFFECT OF CHAMFER
(Depth = 0.04 in.; numbers refer to included angle of chamfer.)

is merely to change the values of $w\phi r$, but the change should have no influence upon the shape of the characteristic curve.

To investigate this proposition, the temperature data given by Tice was recomputed and plotted dimensionally. Tice obtained his data by flowing gasoline through a jet 0.0344 in. in diameter with L/D equal to 11.83. He obtained three different curves, one corresponding to each temperature. The results of the recomputation are shown in Fig. 10, and it is seen that the three curves combine into a single curve, demonstrating that, if an adequate range of heads is used in calibrating a jet, the effect of temperature upon its discharge coefficients can be predicted, and it is unnecessary to perform special experiments to determine the influence of that variable.

Effect of Angle of Chamfer. This investigation is divided into a study of the effect of changing the included angle when the depth of chamfer is 0.008 in., and when the depth is 0.04 in. For simplicity the first set of experiments are termed "small-chamfer" experiments and the second "deep-chamfer" experiments.

Small-Chamfer Experiments. The results of this investigation are presented in Figs. 11 and 12. Fig. 11 shows C as a function of $w\phi r$ for a square-edged jet, and also for four jets having the same total L/D but included chamfer angles α of 20, 60, 90, and 120 deg. The results, as previously shown, apply to all jets which are geometrically similar.

It is evident from Fig. 11 that a small chamfer on the upstream edge of the passage greatly increases the values of C. For example, for the square-edged jet $C = 0.79$ when $w\phi r = 4600$, and for the 60-deg.-chamfered jet, the corresponding value of C is 0.836.

To determine the angle of chamfer which consistently gives the highest values of C, C is plotted as a function of the chamfer angle α for three constant values of $w\phi r$ (see Fig. 12). It is apparent that the maximum values of C occur when α is between 20 and 60 deg. Owing to the number of points being insufficient for an exact determination of this particular angle, the plotted points are connected by straight lines and not by a smooth curve. The graphs show that for low values of $w\phi r$, that is, with viscous liquids or low rates of discharge, chamfering the entrance to the jet has but a small effect upon the value of C; apparently any angle of chamfer gives substantially the same results. In the case of less viscous liquids, or higher rates of flow, a decided gain in the value of C is possible if the angle of chamfer is properly selected.

Deep-Chamfer Experiments. The results are presented in Figs. 13 and 14. In a general way, they are similar to those obtained with the small chamfer. Comparing Figs. 11 and 13 we see that with a deep chamfer the values of C are not as sensitive to small variations in the values of α between 20 and 90 deg., when $w\phi r$ is large, but that C is more sensitive to changes in α when $w\phi r$ is small. The small difference in the results with the deep and small chamfer may in part be attributed to the lack of geometric similarity between the two sets of jets.

The experiments with chamfered jets demonstrate that a slight or deep chamfer on the upstream edge of the passage gives higher values of C than are possible with square-edged jets having the same L/D. By selecting suitable values of α a widely varying range of values of C can be obtained.

CONCLUSIONS

The following conclusions regarding the discharge characteristics of submerged jets are drawn from this investigation:

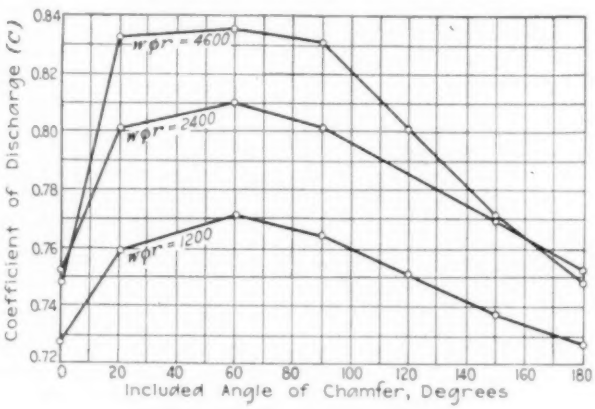


FIG. 14 EFFECT OF CHAMFER
(Depth = 0.04 in.; $L/D = 3.55$.)

1 If C is plotted against $w\phi r$ from data obtained with one liquid, one jet, and an adequate range of heads, the resulting characteristic curve is valid for all jets which are geometrically similar.

2 For square-edged jets and for chamfered jets geometric similarity is obtained when the ratio L/D is the same as that for the calibrated jet.

3 The characteristic curve is valid for any liquid flowing through the jet, and can be used to predict the effect of changing liquids.

4 The characteristic curve takes into account the effects which temperature produces upon the condition of flow.

5 The values of C for square-edged short tubes (L/D less than 1.5) are not very sensitive to changes in liquid, temperature, or rate of discharge, especially for values of $w\phi r$ greater than 600.

6 The length of the jet, the condition of the upstream edge of the passage, the angle of chamfer, and the depth of chamfer have marked influences upon the characteristic curves of jets.

7 Increasing the length of square-edged jets destroys the tendency to constancy of C, especially at low values of $w\phi r$.

8 Chamfering the upstream edge of the passage increases the values of C throughout the entire range of values of $w\phi r$, especially at the higher values.

9 Chamfered jets with an included angle of 20 to 60 deg. have the highest attainable values of C.

10 Increasing the depth of chamfer from 0.008 in. to 0.040 in. slightly increases the values of C.

11 The values of C for chamfered jets are more sensitive to changes in the liquid, temperature, and the rate of discharge, than is the case for square-edged jets with the same total L/D.

APM-51-20

A New High-Speed Fan

By CARL J. FECHHEIMER,[1] EAST PITTSBURGH, PA.

The author describes a centrifugal fan of novel construction for use on steam-turbine-driven generators. In the impeller, straight blades of sheet metal are bulged with respect to a plane passing through the longitudinal axis, and are riveted to the supporting rings. Further to reduce stresses, the blades are made of duralumin, having low density and good physical properties. A partition ring lowers the stresses still more. An analysis of the stresses set up in operation is given, as well as test data of an experimental fan built to deliver 50,000 cu. ft. of air per minute and having a peripheral velocity of 25,500 ft. per min. at 1800 r.p.m. The experimental fan was run up to 83 per cent overspeed.

AS THE sizes of steam-turbine-driven generators have increased, it has become difficult to make internal centrifugal fans wide enough to handle the large volumes of air and at the same time to keep the stresses low enough. The normal peripheral velocity is of the order of 25,000 ft. per min., and the rotors are guaranteed to operate at overspeeds of 20 per cent. The internal diameter cannot be decreased, because there must be sufficient space between the shaft and the fan to admit the air without prohibitively high loss at entrance.

In order that reasonably high fan efficiencies may be obtained, the blades should be inclined backward, and for many applications straight blades, set at about 25 deg. to a tangent at the inner periphery, give good performance.[2] In comparison with the centrifugal forces of the metal parts, the forces due to differences of air pressures and air velocities are negligible. The centrifugal forces acting upon flat blades are then as indicated in Fig. 1, all forces being along radial lines. The components normal to the plane of the blade are the ones requiring consideration.

Viewed from above, the blade and acting normal components are as shown in Fig. 2. These forces produce bending moments in the flat blade, as indicated in Fig. 3. Here $\frac{\text{C.F.}}{2}\cos\alpha$ is the summation of the components of centrifugal forces normal to the blade, for half the blade width; M is the bending moment; and f_s is the shearing force (usually on rivets) needed to balance the centrifugal components. Other things being equal, the bending moment increases as the square of the width, and the stress decreases only as the first power of the blade thickness. For wide fans the blade thickness for safe stresses becomes prohibitive. Several means for overcoming the difficulty suggest themselves: namely, use of partitions so as to reduce the width per blade;[3] use of light-weight material for the blades; and use of curved instead of flat blades. In this paper a description of a high-speed fan is given in which all three of these ideas are incorporated. The blades are curved, but not in a way that is usual in fan- or pump-impeller construction. Each blade is curved with respect to a plane passing through the longitudinal axis.

The forces C.F. cos α in Fig. 2 tend to bend the blade forward, and as soon as the deflection thereby produced becomes appreciable, end thrusts on the supporting rings ensue which tend to draw those rings toward each other. For lower-stressed blades the blade deflection is so small that what small end thrust is produced is counterbalanced by compression in parts of the blades themselves. But for the wide, high-speed fans the end thrust is too great, and increases the blade deflection. Consequently, the tow supporting rings are drawn toward each other and the fan is liable to collapse. The end thrust can obviously be balanced by compression struts, but they interfere with the air flow and complicate the construction.

Suppose that instead of making the blade flat, it be given an initial bulge. It is then possible to reduce the bending moment

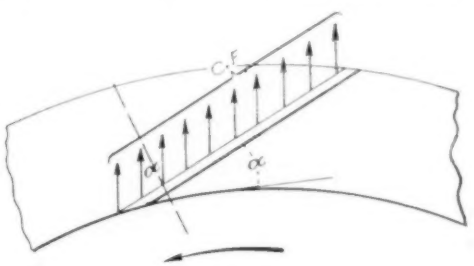


FIG. 1

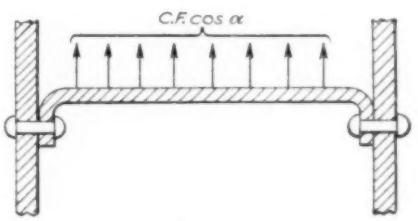


FIG. 2

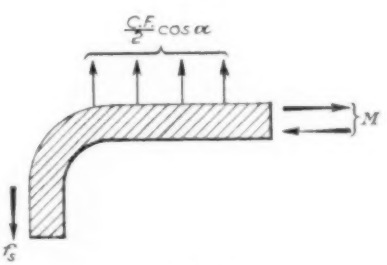


FIG. 3

to a negligible amount. This is the same idea that is embodied in many analogous applications, such as in boiler heads, arches, suspension-bridge cables, etc. If in Fig. 3 the bulge be forward, the resultant forces will be like those indicated in Fig. 4. The bending moment is now replaced by the tensile force f_t in the blade, which in turn is balanced by an end thrust F_t acting against the supporting ring. By properly designing the curvature of the blade, the bending moment is substantially eliminated.

If all the blades be curved forward as in Fig. 4, in order to balance the end thrusts it is necessary to supply axial struts, or their equivalent, as indicated in Fig. 5. However, it is easy to accomplish this in another way and without providing the compression struts.

If the blades are bent initially in the reverse direction, as in Fig. 6, the stress in the blade is changed from tension to compres-

[1] Electrical Development Section Engineer, Westinghouse Electric & Mfg. Co. Mem. A.S.M.E.

[2] See "Performance of Centrifugal Fans for Electrical Machinery," by Carl J. Fechheimer, Trans. A.S.M.E., vol. 46 (1924), p. 287.

[3] A partition is a ring placed equidistant between the two main supporting rings to which the blades are fastened. R_2 in Fig. 12 is such a partition, R_1 and R_3 being the main supporting rings. Obviously, more than one partition may be used to reduce the bending moment in the blades.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

sion, and the end thrust is the reverse of that shown in Fig. 4. It is then quite a simple matter so to construct a fan that the total number of blades is even, and to have the same number of forward-curved (tension) blades as backward-curved (compression) blades, alternating the tension and compression blades. The alternate outward and inward forces on the supporting rings (F_t and F_c in Figs. 4 and 6) balance each other, and no axial struts or bolts are needed.

As in the case of a suspended cord with a uniformly distributed load, the ideal curve should be a catenary. Or, as is well known,

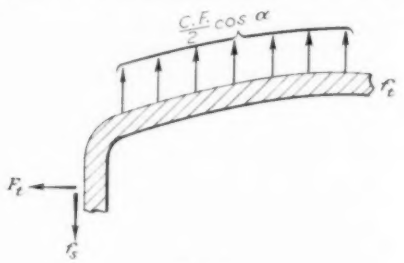


FIG. 4

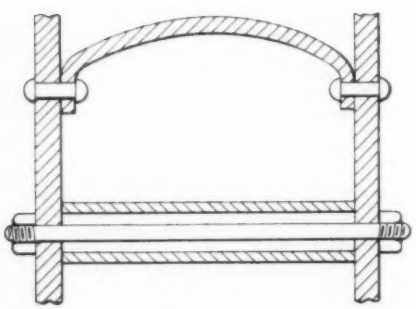


FIG. 5

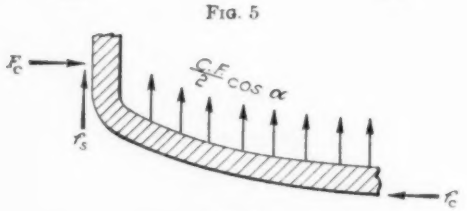


FIG. 6

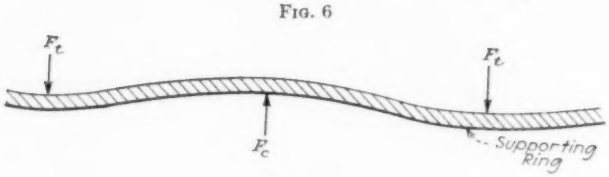


FIG. 7

if instead of uniform distribution along the blade the distribution of centrifugal forces normal to the axis be uniform, the ideal curve is a parabola. It may readily be shown that if the initial bulge is small, an arc of a circle is so close to either a catenary or a parabola that it may readily be used instead. The dies used in the manufacture of the blades are considerably simplified by bending them to an arc of a circle, and consequently that construction was adopted.

It will be noted that the alternate outward and inward forces on the supporting rings produce a bending moment therein. The conditions are like those in a continuous girder, as indicated in Fig. 7. There are various ways of securing an ample factor of safety from the standpoint of stresses due to this bending moment. These may be enumerated as follows:

1 Increase in number of blades, thereby reducing the distance between the points of application of the forces, which reduces the bending moment
2 Increase in section modulus of the supporting ring
3 Use of steel of great strength
4 Decreases in forces F_t and F_c. This may be accomplished in a number of ways:
   (*a*) By means of a partition which makes the axial width of the individual blade half as great
   (*b*) By reducing the thickness of the blades
   (*c*) By use of low-density material in the blades
   (*d*) By increase in initial bulge of the blades.

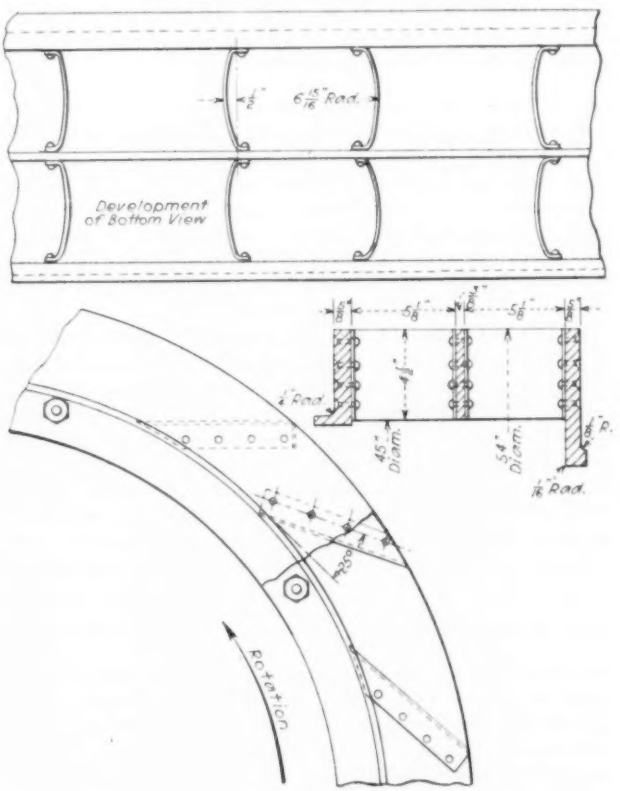


FIG. 8 PROPORTIONS OF EXPERIMENTAL FAN IMPELLER SHOWN IN FIG. 9

The manner in which these points were embodied in the design will now be elaborated upon. They refer to an experimental fan capable of delivering 50,000 cu. ft. per min., and having a peripheral velocity of 25,500 ft. per min., at 1800 r.p.m. Several generators are now operating with internal fans of this design.

1 The number of blades chosen was 18. In so far as performance from the air-flow standpoint is concerned, there is little difference for any number between 12 and 24. With 18 blades, and other proportions as chosen, the calculated stresses in the rings were satisfactory.

2 The supporting ring at the end adjacent to the rotor body is prevented from deflecting much by being backed by the rotor end plate, and by being of sufficient thickness. At the other end either an angle or channel section can be used, and then either one or both projecting parts at the inner or (and) outer diameter act, in conjunction with suitable stationary parts, effectively to seal against air leakage.

3 By using chrome-nickel steel in the supporting rings, the stresses are minimized, even with comparatively thin rings.

4 (*a*) Calculations indicated that for the width of fan needed it would still be advisable to put in one partition, although it might have been possible to employ one span only. (*b*) The blade thickness was made only 0.125 in., which was ample to secure a rigid structure. (*c*) All forces were greatly reduced by the substitution of duralumin for steel in the blades. This material, when suitably heat treated, has about the same physical properties as steel, but only about 36 per cent of its density. (*d*) The initial bulge of the blades was moderate, 0.5 in. for 5.125 in. span. The use of duralumin made for safe operation even with this small initial deflection.

At first glance it might appear that the stresses could readily be calculated with accuracy. But, as is frequently the case in composite structures, an analysis of the stress relations is not a simple problem, and some of the complications are due to incremental stresses arising from elastic deformations. Thus the side thrusts on the supporting rings permit of an incremental bulge of the tension blades, and a decrease of the bulge of the compression blades. Such incremental deflections decrease the tension-blade end thrusts, and increase those for the compression blades. Thus incremental bending gives unequal end thrusts, and the blade must have sufficient thickness to carry this as a bending stress. That is, there is a certain blade thickness below which buckling will occur in the compression blades. The deflections of the rings themselves admit of still further changes in the deflection of the blades, and so on. The resultant series is rapidly converging and the stresses are not much higher than those computed without allowing for incremental deflections, particularly with careful selection of proportions of the supporting rings, number of blades, etc. But from the calculation standpoint the problem becomes complicated, and it was therefore decided to construct an experimental fan. From the results of tests on the fan it will be seen that the construction is sound. In fact, the fan proved to be even better than was anticipated. The proportions of this fan are given in Fig. 8, and in Fig. 9 the completed fan is shown.

The backward-curved blades, as is usual in members under compression, are subject to buckling, and in the design it is important so to proportion those parts as to allow for an ample factor of safety. This was another reason why the partition was used, as without it the other proportions would have had to have been considerably modified to make the fan safe.

The results of calculation of some of the more important stresses are given in Table 1. They are given at 20 per cent overspeed (2160 r.p.m.) and at the maximum speed obtained in test (3300 r.p.m. = 0.83 per cent overspeed). The stresses at the higher speed were taken as $(3300/2160)^2 \times$ stresses at 2160 r.p.m. That is, no corrections were made for elastic nor for plastic deformations.

TABLE 1 CALCULATED STRESSES, POUNDS PER SQUARE INCH[1]

Location	Speed 2160 r.p.m.	Speed 3300 r.p.m.
Tension in blades (forward-curved blades)	1,780	4,150
Ring stress at inner radius, not allowing for blade thrust	25,700	60,000
Ring stress at outer radius, not allowing for blade thrust	27,000	63,000
Ring stress allowing for external loading	43,100	100,000
Bending in compression blades with incremental deflection and compression stress	6,010	14,000
Bending at corners near rivets of tension blades	24,000	56,000
Tension in rivet body	6,330	14,800
Shear of rivet heads	4,000	9,300
Shear of rivet body	4,500	10,400

[1] For calculating the bending in the rings due to the end thrusts, no allowance was made for the stiffening due to their L-shape. Consequently the calculated ring stresses which allow for end thrusts are high. They are about right where no end thrust is considered.

The factor of safety for a blade in compression against buckling is 10.2 at 2160 r.p.m.

The calculation of the bending at the corners of the tension blades requires some assumptions to be made, as the lever arm is uncertain. The figure given in Table 1 for 2160 r.p.m. is closer to the proportional limit or to the yield point of the material than any other, but if the yield point is passed, plastic deformation of the duralumin reduces the stresses.

In order to reduce the power required to drive the fan, a special set-up was made, in which the ingoing-air volume was reduced to a minimum by means of a wheel which revolved with the rest of the fan and cleared the rotating flange by only $^1/_8$ in. Even so, the power consumption was considerable, and the reason that higher speeds were not obtained was because the driving belt slipped, and the driving motor was overloaded. The set-up for the test is as shown in Figs. 10 and 11. The blower was placed in the frame of a turbine generator which was lined with heavy

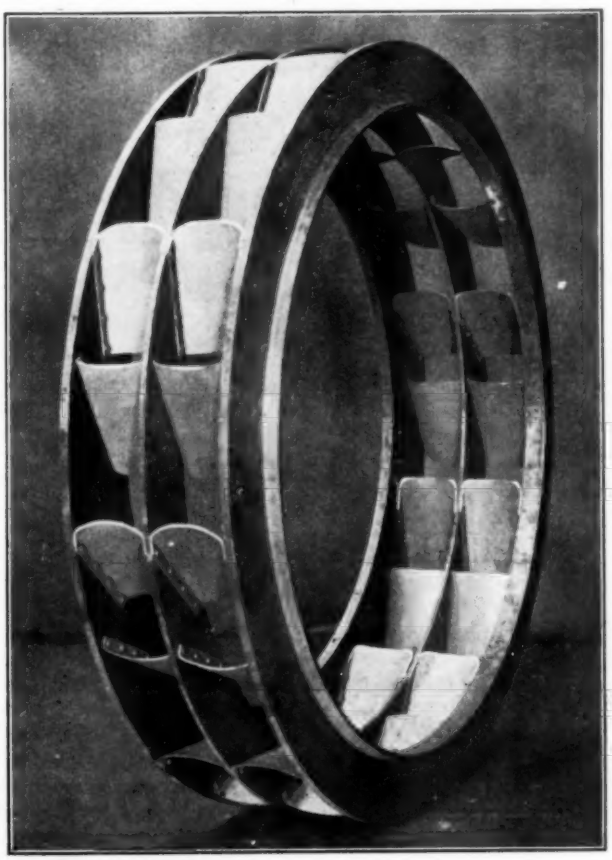


FIG. 9 VIEW OF EXPERIMENTAL FAN IMPELLER

timbers for safety in the event that the blower should fail at some high speed. The tests consisted of running at consecutively increasing speeds, starting with 1600 r.p.m., holding the speed for 10 min. at each speed, and shutting down between runs to take measurements. These measurements consisted in taking observations on a Berry strain gage[4] of (1) distances along the rings, (2) axial distances between rings, and (3) chordal distances between adjacent blades. As the air and fan got warm, and as the

[4] The unit for this gage is 0.0002 in., thereby detecting very small changes over a total Berry strain-gage length of $5^1/_8$ in. (the width of a blade). This corresponds to a strain of $(0.0002/5.125) = 39/10^6$ inches per inch. The stress for unit change then is: $(39/10^6)E$, where E is the modulus of elasticity. Taking E to be 30×10^6 as for steel, the stress for unit change is 1170 lb. per sq. in. For duralumin with a modulus of about 10×10^6, it is 390 lb. per sq. in., thus indicating the accuracy with which the measurements were made.

temperatures caused expansions, thereby affecting the strain-gage readings, the temperatures were also taken, and suitable corrections made therefore in the strain-gage readings. The blower was run at 2700 r.p.m. for one hour, to note the effect of longer time, and the final speed was 3300 r.p.m., for 10 min., after which, owing to the overload on the driving motor and belt, the tests were discontinued. There was no failure at the last-named speed, which corresponds to 83 per cent overspeed of the generator

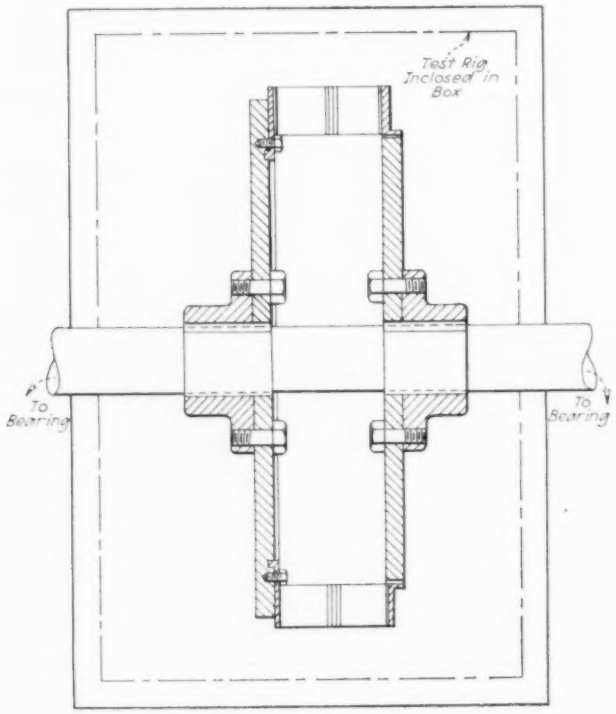


FIG. 10 SET-UP FOR TEST

and to a peripheral velocity of 46,700 ft. per min. (8.85 miles per min.).

The results of the tests indicated that there were no permanent sets in the rings such as was originally thought might take place (as indicated exaggerated in Fig. 7). What the strain-gage measurements did show was that the distance between the outside ring and the middle supporting ring increased slightly, and the distance between the inner ring and the one normally attached to the rotor end plate decreased a little. The measurements between adjacent blades corroborate these results. The first signs of permanent set occurred at 2300 r.p.m., i.e., above the overspeed usually specified.

An explanation of these deformations is offered herewith. Ring R_1 (Fig. 12) remains fixed. Ring R_2 is free to move, and it is substantially equally loaded on either side. R_3 is loaded on one side only, and largely owing to the non-uniformity of the section—the extension was at the outer diameter only—this ring was distorted as shown exaggerated by dotted lines. Had the section used been a channel instead of an angle, this distortion would have been nearly eliminated. It should be noted that the permanent sets were in the blades and were sufficient to hold the rings in the position shown, even after the fan was shut down. After having been run at high speed, the elongated outer parts of the blades meet opposing forces in the inner blades, causing the latter to compress a little, as was found from the Berry gage readings. It was noteworthy that at no speed was there a wavy deformation of the supporting rings, as the calculations had led the author to believe might take place. If there were such tendencies to deform, the elasticity of the ring caused it to resume its form before it was shut down.

In the last column in Table 1 are given the calculated stresses at the maximum speed attained. In Table 2 are given the physical properties of the materials as determined from measurements on suitable test bars taken from the ring forgings or blade sheet.

TABLE 2 PHYSICAL PROPERTIES OF MATERIALS USED IN RINGS AND BLADES

	Proportional limit[1]	Yield point	Ultimate tensile strength	Per cent elongation in 2 in.	Per cent reduction in area
Chrome-nickel steel in rings	...	88,000 to 91,000	122,000 to 124,500	22 to 25.2	61.8 to 63.8
Heat-treated duralumin in blades	17,500 to 18,000	34,270 to 34,800	54,100	18	26.4

[1] Proportional limit taken as the end of the straight-line part of the stress-strain diagram. Yield point taken as 0.2 per cent of the plastic deformation of duralumin.

If the calculations were correct, it will be seen that the bending stress at the corners of the tension blades at 2300 r.p.m. was 27,000 lb. per sq. in., or above the proportional limit but below the

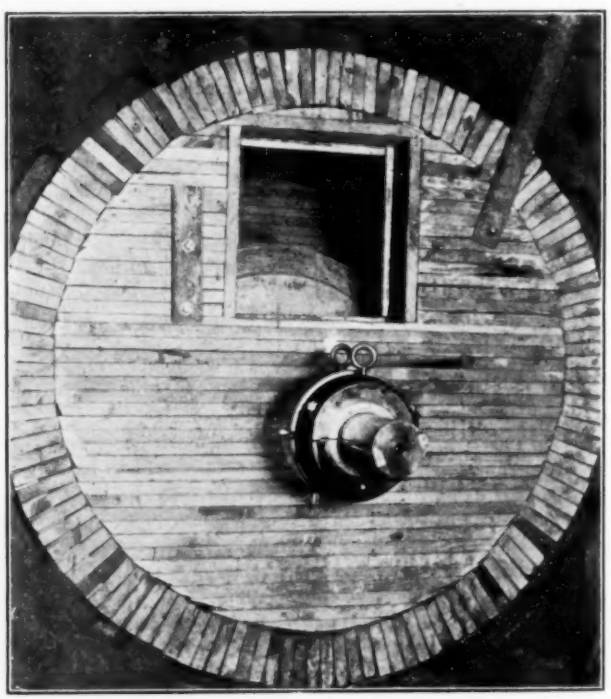


FIG. 11 SET-UP FOR TEST

yield point. Also, according to calculations the yield point of the chrome-nickel steel was passed shortly before the maximum speed of 3300 r.p.m. was reached.

Subsequent to making these tests, the duralumin blades were removed, and steel blades 0.156 in. thick, instead of 0.125 in. for duralumin, were substituted. The superiority of heat-treated duralumin was thereby proved. Permanent sets appeared at 1700 r.p.m., and at 2300 r.p.m. the vibration became so great, due to distortions, that tests at higher speeds were abandoned.

In Table 3 are given comparisons of bending stresses in flat blades for a fan 5.125 in. wide between rings and running at 2160 r.p.m. The blades were considered to be beams with fixed ends, giving two-thirds the stress that would obtain with a beam with free ends. With this assumption, the maximum stress occurs

near the corners of the blade, but the blade is highly stressed at the middle, too. At the corners, plastic deformation reduces the stresses beyond the elastic limit. This undoubtedly took place at the corners of the curved-blade fan, and yet failure was avoided, even though at the final speed at which the fan was run the calculated stress at the corners was more than double what it was at the speed corresponding to the yield-point stresses.

TABLE 3 BENDING STRESSES IN FLAT BLADES (LB. PER SQ. IN.)

Material	Thickness = 0.125 in.	Thickness = 0.1875 in.
Duralumin	28,400	19,000
Steel	79,500	53,000

As previously pointed out, the objection to the flat blades is that once they start to deform, the whole fan may collapse. With the curved blades, the highest blade stress occurs at the bend near the rivets, and plastic flow and permanent sets follow, but failure does not ensue.

Small models of the experimental fan with arched blades and of one with flat blades were run in the same test rig to determine their performances. The differences for the two in the pressure-volume and efficiency curves were negligible, being so small as to be within the range of experimental error.

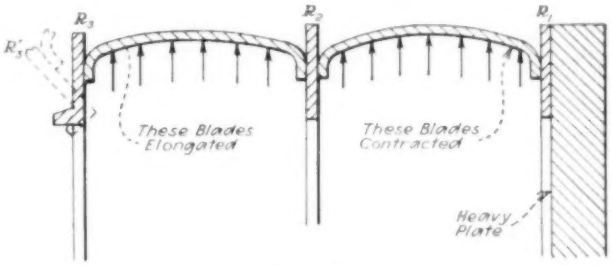


FIG. 12

During this investigation, Messrs. S. Timoshenko, G. W. Penney, and J. DenHartog advised regarding method of calculating stresses. M. Stone conducted the tests, and placed the results in report form. The work of these men was much appreciated, and the value of their work is hereby acknowledged.

APM-51-21

Vibration Damping, Including the Case of Solid Friction

By A. L. KIMBALL,[1] SCHENECTADY, N. Y.

The idea of logarithmic vibration decrement is briefly reviewed and several ways of expressing it and finding it are given.

The simple case of a vibrating reed is treated, three kinds of damping being considered—(1) an external liquid damping, (2) a liquid viscosity damping within the spring, (3) solid damping within the spring whereby the dissipation per cycle is independent of frequency but depends upon amplitude squared. For an experimental study and discussion of this type of damping see "Internal Friction in Solids," by A. L. Kimball and D. E. Lovell.[2]

These cases are discussed both for free vibrations and sustained vibrations.

A method is given for analyzing solid damping vibration problems which is of general application to all cases of vibration that can be treated on the assumption of an ideal viscosity damping. This method is used by Ormondroyd and Den Hartog in the closure to the discussion of their paper[3] *entitled "The Theory of the Dynamic Vibration Absorber."*

The discussion of vibration damping given herewith presents methods used by the writer for several years past in vibration work.

Logarithmic Decrement

By way of introduction the idea of logarithmic decrement will be briefly reviewed, as a clear understanding of it is of basic importance for the appreciation of what is to follow. Consider a damped vibration as given by the following equation and shown in Fig. 1:

$$y = y_0 e^{-\alpha t} \cos \omega t \qquad [1]$$

The logarithmic decrement δ is defined as the natural logarithm of the ratio of any two successive amplitudes of a vibration whose amplitude decreases exponentially, such as that given by Equation [1].

Expressing mathematically,

$$\delta = \log \frac{y_0}{y_1} = \log \frac{y}{ye^{-\alpha T}} = \log e^{\alpha T} = \alpha T \qquad [2]$$

where T is the period of a complete vibration.

Another useful way of expressing δ is derived by making use of the logarithmic series

$$\log x = x - 1 - \tfrac{1}{2}(x-1)^2 + \cdots \qquad [3]$$

For any amplitude y

$$\delta = \log \left(\frac{y + \Delta y}{y} \right)$$

(see Equation [2]), where Δy represents the decrease of y per cycle as shown in Fig. 1. Substituting in [3]

$$\delta = \frac{\Delta y}{y} \qquad [4]$$

where all terms of the series except the first are dropped since they are relatively small in most cases of vibration damping met with in practice.

The latter expression for δ is very useful because by means of it δ can be found at once by a simple measurement from a graph (or a photograph) of the damped vibration like that given in Fig. 1.

When δ is known, the amount of energy dissipated per cycle, ΔW, can be found as follows:

The total vibrational energy W, at some time t, equals ky^2

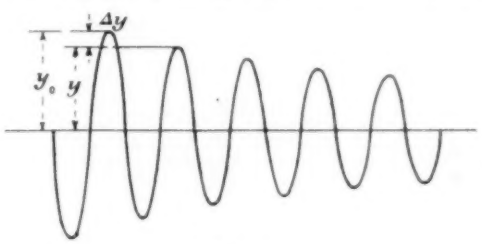


Fig. 1

since it equals the maximum potential energy of the system. The total vibrational energy one cycle later equals

$$k(y - \Delta y)^2 = W - \Delta W$$

Subtracting the latter from the former to find ΔW

$$\Delta W = k(y^2 - y^2 + 2y\,\Delta y - \Delta y^2) = 2ky\,\Delta y$$

where $-\Delta y^2$ is dropped as being negligible. Therefore,

$$\frac{\Delta W}{W} = \frac{2ky\,\Delta y}{ky^2} = 2\frac{\Delta y}{y} = 2\delta$$

$$\Delta W = 2W\delta \qquad [5]$$

In words, this equation says that the energy dissipation per cycle of a vibration whose amplitude decreases exponentially with time is equal to the total energy of the vibration at the time in question multiplied by twice the logarithmic decrement.

Any elastic vibration with any form of decrement has a definite instantaneous logarithmic decrement at each point. Expression [5] is expressed in terms of the logarithmic decrement as found from some point on a decrement curve of such a vibration. No matter what be the shape or the mode of vibration of the body, this relation is applicable. Although not mathematically exact, it is very useful in work involving vibration damping.

It is seen that three different expressions for the logarithmic decrement δ are given under expressions [2], [4], and [5].

The best way to appreciate the physical meaning of δ is through expression [4] as applied to the graph of Fig. 1. When once δ is known for any amplitude, for a vibrating system, the rate of energy dissipation through friction is at once obtained through [5], and if the decrement is constant, the resonance vibration amplitudes can also be found (see Table 2).

Three Cases of Vibration Damping

The vibratory motion of a reed carrying a mass m, such as that

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[1] Research Engineer, General Electric Co. Assoc. A.S.M.E.

[2] *Physical Review*, December, 1927, and Trans. A.S.M.E., vol. 48 (1926), p. 479.

[3] Trans. A.S.M.E., vol. 50 (1928), paper no. APM-50-7.

Contributed by the Applied Mechanics Division and presented at the Rochester Meeting, Rochester, N. Y., May 13 to 16, 1929, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

shown in Fig. 2, may be found from the well-known differential equation:

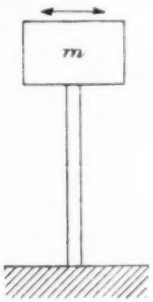


FIG. 2

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + cy = 0 \quad \ldots\ldots [6]$$

where m = mass carried by reed
b = a damping factor
c = elastic constant of reed
= elastic force exerted by reed when m is displaced unit distance from its rest position
y = displacement of m from its rest position at time t.

The solution of this equation is

$$y = y_0 e^{-\frac{bt}{2m}} \cos \omega_0 t \quad \ldots\ldots [7]$$

where

$$\omega_0 = \sqrt{\omega_c^2 - \frac{b^2}{4m^2}} \quad \ldots\ldots [8]$$

in which

$$\omega_c = \sqrt{\frac{c}{m}}$$

Equation [7] is seen to be the same as [1] when

$$\alpha = \frac{b}{2m}$$

Case I. *The damping such that b is independent of c and m.*

An example of such a system is one in which the damping is produced by the motion of the mass m in an ideal viscous liquid, neglecting any damping action within the reed itself. In this case b is assumed independent of c and m. However, the vibration frequency of the reed may be changed through the variation of c or m.

In this case from [1], [2], and [7]

$$\delta = \alpha T = \frac{bT}{2m} \quad \ldots\ldots [9]$$

From [8], the period of vibration

$$T = \frac{2\pi}{\omega_0} = \frac{2\pi}{\sqrt{\omega_c^2 - \frac{b^2}{4m^2}}} \quad \ldots\ldots [10]$$

Substituting [10] in [9]

$$\delta = \frac{\pi b}{m\sqrt{\omega_c^2 - \frac{b^2}{4m^2}}} \quad \ldots\ldots [11]$$

When α is relatively small, the second term under the radical is neglected and

$$\delta = \frac{\pi b}{m\omega_c} = \frac{\pi b}{\sqrt{mc}} \quad \ldots\ldots [12]$$

since $\omega_c = \sqrt{\frac{c}{m}}$

Case II. *Damping produced by friction within reed obeying the liquid viscosity law.*

In this case b is a parameter depending upon the form and properties of material of the elastic support, as expressed by Equation [18].

The calculation of an expression for δ in this case is best made from Equation [5] after evaluating ΔW and W.

Consider a unit volume of the solid as shown in Fig. 3.

Let the friction component of force per unit velocity of strain in the direction shown by the arrows be equal to η. This is, in

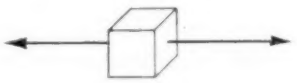


FIG. 3

fact, what is sometimes called the *coefficient of normal viscosity.*

Thus if ϵ = strain per unit length, the viscosity force produced upon the faces of the unit cube during elongation or compression is

$$\eta\frac{d\epsilon}{dt} \quad \ldots\ldots [13]$$

If

$$\epsilon = \epsilon_m \sin \omega t, \quad d\epsilon = \epsilon_m \cos \omega t d(\omega t)$$

where ϵ_m = half amplitude of the strain. Therefore, the damping force $= \eta\epsilon_m\omega \cos \omega t$

Now work per cycle per unit volume is

$$\Delta W_1 = \int \text{force} \times d\epsilon$$

taken over one cycle, therefore

$$\Delta W_1 = 2\int_0^{\pi} \eta\epsilon_m^2\omega \cos^2 \omega t d(\omega t) = \pi\eta\epsilon_m^2\omega \quad \ldots\ldots [14]$$

ΔW = total dissipation per cycle in entire volume of the material of the vibrating body

$$= \int\int\int \pi\eta\epsilon_m^2\omega dV \quad \ldots\ldots [15]$$

where V is the volume.

The total vibrational energy may be expressed in terms of the elastic potential energy of maximum strain

$$W = \int\int\int \tfrac{1}{2}\epsilon_m^2 E dV \quad \ldots\ldots [16]$$

Substituting [15] and [16] in Equation [5] and solving for δ

$$\delta = \frac{\pi\eta\omega_c \int\int\int \epsilon_m^2 dV}{E\int\int\int \epsilon_m^2 dV} = \frac{\pi\eta\omega_c}{E} \quad \ldots\ldots [17]$$

where ω is replaced by ω_c characteristic of a free vibration. Since the volume boundary is the same in the two cases, the integrals cancel. From [12] and [17],

$$b = \frac{\delta m\omega_c}{\pi} = \frac{\eta m\omega_c^2}{E} = \frac{\eta c}{E} \quad \ldots\ldots [18]$$

where in [17] ω_c is replaced by $\omega_c = \sqrt{\frac{c}{m}}$ as an approximation.

Case III. *Solid damping where dissipation per cycle is independent of the frequency.*

In accordance with previous work done by the writer[2] it will be assumed, as a good approximation, that loss per unit volume per

cycle $= \xi\sigma_m^2$, where σ_m is the maximum stress at the point considered,

$$= \xi\epsilon_m^2 E^2 \quad [19]$$

Then

$$\Delta W = \int\int\int \xi\epsilon_m^2 E^2 dV$$

and as before W is given by [16]. Substituting in [5] as before and canceling the integrals

$$\delta = \xi E \quad [20]$$

From [9],

$$b = \frac{m\omega_c \xi E}{\pi} = \frac{\xi E\sqrt{mc}}{\pi} \quad [21]$$

Comparison of the Three Cases

Table 1 shows the values of b and δ for the three cases arranged for comparison.

TABLE 1

		δ, log decrement	b, damping factor
I	Damping with b depending on external viscous friction only	$\frac{\pi b}{\sqrt{mc}}$	b independent of m and of properties of material of rod
II	Liquid damping (often wrongly applied to solids)	$\frac{\pi\eta\omega_c}{E}$	$\frac{\eta c}{E}$
III	Solid damping (correct law of solid damping)	ξE	$\frac{\xi E\sqrt{mc}}{\pi}$

In case I, although b is independent of the frequency of the vibration, the decrement δ decreases either with an increase of the mass m carried by the reed or its stiffness c.

In case II, b is proportional to the stiffness c but independent of m. The decrement δ increases with the frequency of the vibration.

In case III, the damping factor b depends upon both m and c. The decrement δ is *independent of frequency*, the loss per cycle being also independent of the frequency.

In this table, the values given for b apply, of course, only to a vibrating system with a concentrated mass m like that of Fig. 2. The value of δ under case I applies only to a system of one degree of freedom.

In cases II and III, however, the expression given for δ holds for any type of vibration or any vibrating system whatsoever which derives its elasticity and dissipation from one material such as a given type of steel. For instance, if a vibrating bar is substituted in place of the system shown in the figure, the expressions for δ under II and III still hold. The mathematical derivations given from Equations [13] to [17] are the same in this case as before.

The important difference between the δ of case II and that of case III (true solid damping) is that in the former it depends upon the vibration frequency, and is proportional to it; in the latter it is a constant for all frequencies.

Remarks on Method of Analysis

At first thought, it seems surprising that solid damping, where apparently the liquid viscosity law does not apply, should be handled by an equation of the form [6] whose damping term $b\frac{dy}{dt}$ gives a force depending upon velocity, like that produced by liquid viscosity.

The solution of this equation gives an exponential amplitude decrement curve from which it follows that a constant log amplitude decrement is obtained. Now the requirement for an exponential amplitude decrement of a vibration is simply that the dissipation per cycle be proportional to the amplitude squared. This condition is fulfilled in all three of the cases discussed, so that the amplitude decrement curve of Equation [6] applies in all three cases, provided the damping factor b is looked upon as a parameter and evaluated as previously illustrated for the special case considered.

The exact form of the vibration curve, however, aside from amplitude decrement, *does* depend upon the shape of the hysteresis loop. Equation [1] applies at all instants only when the hysteresis loop is of elliptical form. A vibration arising from some other form of hysteresis loop can be exactly represented by replacing the single cosine factor by the correct harmonic series.

Application to the Case of Sustained Vibrations

For a case of a sustained vibration, produced by a sinusoidal force $F\cos\omega t$, the differential equation of the system of Fig. 1 takes the form

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + cy = F\cos\omega t \quad [22]$$

This equation is the same as [6] except for the right-hand term. The solution of this equation is well known to be

$$y = y_o e^{-\frac{bt}{2m}}\cos\omega_o t + \frac{F\cos(\omega t + \beta)}{\sqrt{(c - m\omega^2)^2 + b^2\omega^2}} \quad [23]$$

where

$$\omega_o = \sqrt{\omega_c^2 - \frac{b^2}{4m^2}} \qquad \text{see Equation [8]}$$

and where

$$\tan\beta = \frac{b\omega}{c - m\omega^2} \quad [24]$$

Equation [23] contains two terms, the first the transient term which dies out with time, and the second a sustained-vibration term which remains. The first is the solution of Equation [6] which has been discussed already. In what follows, only the last term will be considered, and from it a study of the variation of amplitude with the frequency of the applied stimulus will be made for the three types of damping discussed previously for the system shown in Fig. 2.

Case I: b independent of m and of properties of the material of the rod.

$$y_m = \frac{F}{c\sqrt{\left(1 - \frac{\omega^2}{\omega_c^2}\right)^2 + \frac{b^2\omega^2}{c^2}}} \quad [25]$$

where y_m = maximum value of y for a given ω and $\omega_c = \sqrt{\frac{c}{m}}$ as in Equation [8].

When $\omega = \omega_c$, and when b is small, y_m has very nearly its maximum possible value, and the system is in resonance.

Case II. In this case for any unit volume the elastic stress is $E\epsilon$ and the frictional stress $\eta\frac{d\epsilon}{dt}$, Equation [13]. They are both sinusoidal and 90 deg. apart in phase. The ratio of the amplitude of the frictional stress to that of the elastic stress is seen to be independent of amplitude and thus equal to the same constant for every point in the vibrating elastic material. It follows that the ratio of the amplitude of the frictional term $b\frac{dy}{dt}$ to the elastic term cy for the entire vibrating body, as given by Equation [22], must also be equal to this constant. Expressing mathematically,

$$\frac{\eta\left(\frac{d\epsilon}{dt}\right)_m}{E\epsilon_m} = \frac{b\left(\frac{dy}{dt}\right)_m}{cy_m}$$

but since the vibration is sinusoidal and isochronous for every unit volume

$$\frac{\left(\frac{d\epsilon}{dt}\right)_m}{\epsilon_m} = \frac{\left(\frac{dy}{dt}\right)_m}{y_m} \text{ so that } b = \frac{\eta c}{E}$$

This is seen to agree with the value of b from Table 1 for this case. Substituting in Equation [25],

$$y_m = \frac{F}{c\sqrt{\left(1 - \frac{\omega^2}{\omega_c{}^2}\right)^2 + \frac{\eta^2\omega^2}{E}}} \quad \ldots\ldots [26]$$

This equation is seen to differ from [25] only by the const. $\frac{\eta^2}{E^2}$ which replaces $\frac{b^2}{c^2}$. Thus, the amplitude curve as a function of frequency of stimulus is of the same form in the two cases. This is what would be expected because in both case I and case II the damping is viscous in character, the friction being proportional to the displacement velocity.

Case III. In this case also the frictional force component per unit area can be expressed as $\eta \frac{d\epsilon}{dt}$ assuming, as before mentioned, that the actual hysteresis loop of solid friction is replaced by one of equal area but of elliptical form, as required by this relation for a sinusoidal vibration. The coefficient η will then be a parameter depending upon other physical quantities. It may be evaluated by equating the work per cycle arising from this frictional force (see Equation [14]) to that obtained by experiment for solid friction (see Equation [19]).

This gives

$$\eta\pi\epsilon^2{}_m\omega = \xi E^2\epsilon^2{}_m$$

therefore

$$\eta = \frac{\xi E^2}{\pi\omega} \quad \ldots\ldots [27]$$

Comparing the frictional and elastic terms for a unit volume with those for the entire body as under case II,

$$\frac{b}{\eta} = \frac{b\pi\omega}{\xi E^2} = \frac{c}{E}; \; b = \frac{\xi E c}{\pi\omega}$$

This also agrees with the value of b under case III of Table 1 except that in that case ω was given the value $\omega_c = \sqrt{\frac{c}{m}}$ characteristic of a free vibration.

Substituting in Equation [25],

$$y_m = \frac{F}{c\sqrt{\left(1 - \frac{\omega^2}{\omega_c{}^2}\right)^2 + \frac{\xi^2 E^2}{\pi^2}}} \quad \ldots\ldots [28]$$

The three values of y_m given by Equations [25], [26], and [28] corresponding to the three kinds of damping apply, of course, only to the particular case of the vibrating system of Fig. 1, whose motion is found from the Equations [6] and [22].

At resonance where $\omega = \omega_c$, at which frequency the amplitude is limited by the frictional resistance only, it is evident that for the same magnitude of the decrement δ, the amplitude must be the same in all three cases. This is shown to be the case in Table 2, where the values of δ are taken from Table 1.

TABLE 2

Case of damping	y for $\omega = \omega_c$	δ	y_m for $\omega = \omega_c$ in terms of δ
I	$\frac{F}{b\omega_c}$	$\frac{\pi b}{\sqrt{mc}}$	$\frac{\pi F}{c\delta}$
II	$\frac{FE}{c\eta\omega_c}$	$\frac{\pi\eta\omega_c}{E}$	$\frac{\pi F}{c\delta}$
III	$\frac{\pi F}{c\xi E}$	ξE	$\frac{\pi F}{c\delta}$

General Case of Solid Damping

Since the solid damping of case III is the type usually met with in actual cases where springs are used, this latter is of particular interest and importance. The method of solving the reed vibration problem for solid damping is general for this type of damping. Any such problem can be expressed mathematically by replacing η, the friction coefficient, as defined by Equation [13] for viscous friction, by its value for solid friction just derived under case III, Equation [27], namely

$$\eta = \frac{\xi E^2}{\pi\omega}$$

Thus all problems in elastic vibration which can be solved for cases involving an ideal viscous friction within the elastic material are also susceptible of solution for the case of solid friction, where dissipation per cycle is independent of frequency, but depends upon amplitude squared such that $\Delta W_1 = \xi\sigma_m{}^2$, where ΔW_1 is the frictional loss per unit volume per cycle.

The logarithmic decrement for all such vibrations is independent of the frequency, and is equal to ξE (see Equation [20]).

Discussion

R. H. Canfield.[4] In this paper we have a very useful comparison of different possible types of solid friction as regards the dying out of free vibrations. It should be emphasized that the application of the differential Equation [6] to the problem of Case III, Solid Damping Where Dissipation per Cycle is Independent of the Frequency, is, as he says, an approximation. The present writer has already[5] carried out the solution of this problem for the case of forced vibrations of constant amplitude in a rigorous form, but assuming a parallelogram form of hysteresis curve, without the use of a differential equation. The law of internal friction here specified is that which Mr. Kimball was instrumental in establishing as the general law for solid friction. Experiments by the present writer[5] indicate that the exact law is not quite as simple as the above. In fact, the loss per unit volume per stress cycle seems to be represented somewhat more accurately by the expression $\xi f(f - f_0)$ than by ξf^2, which is the author's original equation. In this case f_0 is a small "threshold stress" below which the internal friction is negligibly small. The threshold f_0 may be of the order of zero or range as high as 4000 lb. per sq. in., the larger values being found in freshly overstrained material.

J. P. Den Hartog.[6] The paper deals with the vibrations of a cantilever beam (Fig. 2) with different kinds of damping, of such a nature that the motion can be expressed by Equation [6] or [22]. In this connection the title of the paper might give rise to the misunderstanding that this method also covers cases where the damping is caused by solid friction between dry surfaces, like leaf springs, certain types of automobile snubbers,

[4] Associate Physicist, Naval Research Laboratory, Anacostia, D. C.

[5] *Physical Review*, vol. 32, no. 3, September, 1928, pp. 520–530.

[6] Research Department, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

etc. This is apparently not so, since the damping force in these cases is practically independent of the velocity dy/dt. Let us consider the three cases where the damping force is (*a*) independent of dy/dt, (*b*) proportional to it, and (*c*) proportional to its square, and let us assume that the shape of the vibration is approximately sinusoidal. Then for all three cases the relation:

$$\Delta W = 2\,ky\,\Delta y \quad \ldots\ldots\ldots\ldots\ldots\ldots [29]$$

appearing three lines above Equation [5] is true. For a constant damping force F the work dissipated per cycle is $\Delta W = 4\,Fy$, i.e., proportional to the amplitude. With a damping proportional to the amplitude the dissipation is proportional to the square of the amplitude and in case (*c*) it is proportional to the cube of the amplitude. Substituting in [29], it is seen that in the three cases we have:

(*a*) Δy is a constant
(*b*) Δy is proportional to y
(*c*) Δy is proportional to y^2.

Integrating these relations, the amplitude decrement curves of the three cases are found to be:

(*a*) A straight line
(*b*) A logarithmic curve
(*c*) A hyperbola.

The author has very clearly presented three manifestations of our second case. In the classical supposition that the factor b of Equation [22] is a constant of the system (as in the authors' first two cases), it is seen that the dissipated energy per cycle is proportional to the frequency ω. For if we have a sinusoidal vibration of a given amplitude the velocity will go up with the frequency; the damping force $b\,dy/dt$ is also proportional to the frequency; and since the path is constant, namely, the amplitude, the dissipation or work is proportional to the frequency.

Now it has been found experimentally by the author and others that for internal friction in shafts the dissipation is constant and independent of the frequency. This can obviously be incorporated in [22] by replacing the constant damping coefficient b by b_1/ω, where b_1 is an absolute constant for the system. The value of this constant has been calculated by an integration process by the author, giving the result $b_1 = \xi Ec/\pi$.

Therefore the differential Equation [22] for case III of the paper becomes:

$$m\frac{d^2y}{dt^2} + \frac{b_1}{\omega}\frac{dy}{dt} + cy = F\cos\omega t \quad \ldots\ldots\ldots\ldots [30]$$

As the author remarks in his closing paragraph, this result can be applied also to cases of more than one degree of freedom. In fact, in the closure to the discussion of a paper by J. Ormondroyd and myself, we have applied it to the particular two-degrees-of-freedom problem of the dynamic vibration absorber.[7]

C. Richard Soderberg.[8] It is possible to show in a simple manner that the effect of internal friction in solids is identical to that of viscous friction, regardless of the form of the hysteresis loop, as long as the strain variation can be considered sinusoidal. In other words, as long as the friction forces are not large enough to distort the form of the motion, their effect upon the amplitude of motion is equivalent to the effect of viscous friction.

Consider an elementary cube of the material subjected to a single component of strain ϵ, corresponding to a stress σ. If the strain is varied through complete cycles until a steady state is reached, the stress-strain curve will exhibit the ordinary hysteresis loop. When the strain is increasing there will exist, in addition to the elastic stress ϵE, a positive friction stress; when the strain is decreasing the friction stress will be negative. At the piont of reversal the friction stress will be zero. We may write, therefore, as an extension of Hooke's law

$$\sigma = \epsilon E + f(\epsilon) \quad \ldots\ldots\ldots\ldots\ldots [31]$$

where $f(\epsilon)$ is the friction stress.

Let Θ be a parameter varying directly with time, and let the strain variation be sinusoidal, so that

$$\epsilon = \epsilon_m \sin\Theta \quad \ldots\ldots\ldots\ldots\ldots [32]$$

where ϵ_m is the maximum strain. The friction stress $f(\epsilon)$ is obviously a periodic function in Θ, going through zero when $\Theta = \frac{\pi}{2}, 3\frac{\pi}{2}$, etc. Without restricting the form of $f(\epsilon)$ we may write it in a Fourier series

$$f(\epsilon) = a_1\sin\Theta + a_2\sin 2\Theta + \ldots + b_1\cos\Theta + b_2\cos 2\Theta + \ldots\ldots\ldots\ldots [33]$$

The work done in one cycle (the area of the hysteresis loop) is found to be

$$\Delta W = \int_c \sigma\,d\epsilon = \int_0^{2\pi} (\epsilon E + f(\epsilon))\,\epsilon_m\cos\Theta\,d\Theta = \pi\epsilon_m b_1 \ldots [34]$$

all integrals, except the first from the cosine series, vanishing. Thus, only the fundamental of the function $f(\epsilon_m \sin\Theta)$ influences the motion. Naturally this will hold only as long as the variation in Θ of ϵ is strictly sinusoidal.

Since the maximum potential energy in the element is

$$W = \frac{1}{2}E\epsilon_m^2 \quad \ldots\ldots\ldots\ldots\ldots [35]$$

we obtain for the logarithmic decrement of the motion

$$\delta = \frac{\Delta\epsilon_m}{\epsilon_m} = \frac{1}{2}\frac{\Delta W}{W} = \frac{\pi b_1}{\epsilon_m E} \quad \ldots\ldots\ldots\ldots [36]$$

It is obvious from this that the practical measurement of internal friction involves a measurement of the constant b_1. The experimental results available so far, of which those obtained by Mr. Kimball are the most important, indicate that b_1 is a function of the maximum strain ϵ_m, but is independent of the rate of stress reversal. The following cases may be considered of greatest interest.

(*a*) If b_1 is *independent of the maximum strain* ϵ_m, the logarithmic decrement will decrease with ϵ_m, and the actual decrement will be constant. Free vibrations will attenuate along a straight line. This may be characterized as a typical *coulomb friction*.

(*b*) If b_1 is *proportional to the maximum strain* ϵ_m, the logarithmic decrement will be constant. This case is equivalent to *viscous friction*. The area of the hysteresis loop is proportional to the square of the maximum strain.

In both of these cases Hooke's law can be written in the form

$$\sigma = E\epsilon + \frac{\Gamma}{\omega}\frac{d\epsilon}{dt} \quad \ldots\ldots\ldots\ldots\ldots [37]$$

if the parameter Θ is written as ωt. Γ is a *modulus of friction* expressed in the same units as E. In the first case Γ is inversely proportional to the maximum strain; in the second case it is a true constant. Here it is possible to treat the friction as a true

[7] Trans. A.S.M.E., vol. 50 (1928), paper no. APM-50-7.
[8] Västeras, Sweden, Assoc-Mem. A.S.M.E.

viscous friction, provided that the motion can be considered sinusoidal.

The same result is brought forward by the author by another method, and it is not claimed that the above treatment is more fundamental. It is felt, however, that the limitation with regard to the form of the strain variation needs special emphasis.

Another consequence of the above discussion is that any experiment on internal friction in solids, which measures the friction forces (or the logarithmic decrement) under sinusoidal stress variation, will convey no information with regard to the actual form of the hysteresis loop. The only element that will appear in the results is an elliptical hysteresis loop equivalent in effect to the actual one. Regardless of the value of the excellent work done on this subject, by the author of this paper as well as by others, we shall not penetrate into the true nature of this phenomenon until this deficiency in the methods of measurement has been eliminated.

M. STONE.[9] The author in his present paper has reviewed our ideas on vibration damping in a manner that makes the problem simple and of facile grasp. The realization that the results of his own experiments and those of others, yielding the fact that damping in solids is independent of frequency and proportional to the square of the amplitude, point to the conventional type of differential equation of motion

$$Mx'' + Cx' + kx = 0$$

is something that we should not be surprised to find. In fact, any type of vibration damping, in which the energy-loss per cycle is proportional to the square of the amplitude of vibration,

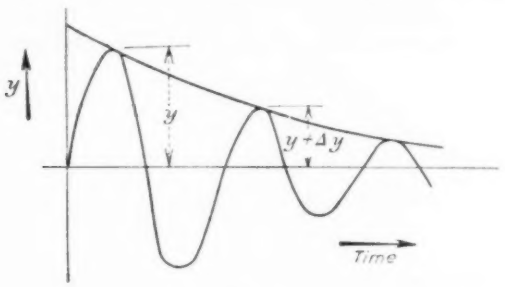


FIG. 4

can be expressed accurately in an equation of the above type. This proceeds from the discussion below.

First, let us consider the idea of logarithmic decrement more closely (Fig. 4). The author, in his second paragraph, defines it for vibrations whose amplitudes decrease exponentially; however, it should not be limited to only such motions, but to any type of decaying vibration. Thus, to use the author's notation,

$$\delta = \log\left(\frac{y}{y + \Delta y}\right) \quad \text{[38]}$$

Now, if we remain in the elastic region, i.e., where the energy at any time is proportional to the amplitude squared ($W = ky^2$), we may continue, writing

$$\psi = \frac{\Delta W}{W} = \frac{ky^2 - k(y + \Delta y)^2}{ky^2} = 1 - \left[\frac{(y + \Delta y)^2}{y}\right] \quad \text{[39]}$$

Now from [38]

$$e^{\delta} = e^{\log\left(\frac{y}{y + \Delta y}\right)} = \frac{y}{y + \Delta y}$$

or

$$\frac{y + \Delta y}{y} = e^{-\delta}$$

and $\left[\frac{(y + \Delta y)^2}{y}\right] = e^{-2\delta}$ and substituting in [39],

$$\psi = 1 - e^{-2\delta}$$

and expanding $e^{-2\delta}$ into its infinite series, we get

$$\psi = 1 - \left[1 - 2\delta + \frac{4\delta^2}{2!} - \frac{8\delta^3}{3!} + \cdots\right]$$

$$= 2\delta - 2\delta^2 + \frac{4}{3}\delta^3$$

and since δ is small, we neglect higher orders, arriving at $\psi = 2\delta$, which is the same as the author's Equation [5], and which he correctly states as being approximate, but which the writer wishes to point out is a relation holding for all types of vibration decay.

Hence, we can get the value of δ from a consideration of ψ, and quickly characterize the type of motion. In addition to the three types of damping discussed in the paper, the writer would like to add parallel developments of Coulomb damping, velocity-squared damping, and shear damping, all of which occur frequently in engineering applications. In each case we are interested in the energy loss per cycle.

Solid Damping: (experimental):

$$\Delta W = \xi x_{\max.}^2 = \xi \epsilon_m^2 E^2 \text{ (independent of } \omega)$$

Velocity Damping: $m\ddot{x} + b\dot{x} + kx = 0$

$$\Delta W = \oint (b\dot{x})dx \text{ where } x = x_m \sin \omega t$$

$$\dot{x} = x_m \omega \cos \omega t$$

$$dx = x_m \cos \omega t d(\omega t)$$

$$\text{Thus } \Delta W = 2\int_0^{\pi} b x_m^2 \omega \cos^2 \omega t d(\omega t) = \pi b \omega x_m^2$$

Liquid Damping: Viscosity force $= \eta \frac{d\epsilon}{dt}$ per unit volume

$$\Delta W = \oint \left(\eta \frac{d\epsilon}{dt}\right) d\epsilon \qquad \epsilon = \epsilon_m \sin \omega t$$

$$\dot{\epsilon} = \epsilon_m \omega \cos \omega t$$

$$d\epsilon = \epsilon_m \cos \omega t d(\omega t)$$

$$\Delta W = 2\int \eta \epsilon_m^2 \omega \cos^2 \omega t \, d(\omega t) = \pi \eta \omega \epsilon_m^2$$

Coulomb Damping: $m\ddot{x} + kx \pm F = 0$

$$\Delta W = \oint F \, dx \qquad x = x_m \sin \omega t$$

$$dx = x_m \cos \omega t d(\omega t)$$

$$\Delta W = 4\int^{\pi/2} F x_m \cos \omega t d(\omega t) = 4 F x_m$$

(Velocity)² Damping: $m\ddot{x} + kx \pm \beta(\dot{x})^2 = 0$

$$\Delta W = \oint \beta \left(\frac{dx}{dt}\right)^2 dx \qquad x = x_m \sin \omega t$$

$$\ddot{x} = x_m \omega \cos \omega t$$

$$dx = x_m \cos \omega t \, d(\omega t)$$

[9] Research Engineer, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

$$\Delta W = 4\int_0^{\pi/2} \beta x_m^3 \omega^2 \cos^3 \omega t d(\omega t) = \frac{8}{3}\beta x_m^3 \omega^2$$

Shear Damping (Fig. 5):

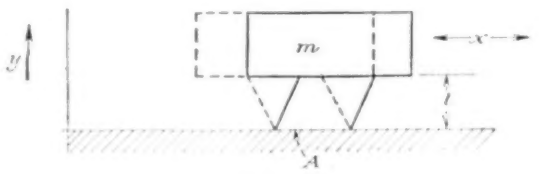


FIG. 5

$$m\frac{d^2x}{dt^2} + F + kx = 0$$

where F = frictional resistance due to shear action in elastic material, such as a rubber pad supporting an electric motor. This type of action is called "tangential viscosity," and is defined

$$F = \lambda \frac{dv}{dy}$$

where v is the velocity in x-direction of a particle under shear action, and λ is the coefficient of tangential viscosity. This type of damping occurs as the damping action of oil films in bearings, etc. Thus

$$F = \int\int \lambda \frac{dv}{dy}\, d(\text{area})$$

Now $v = cy$ and $\dot{x} = cl$ or $c = \dot{x}/l$

$$\text{and } v = \frac{x}{l}\, y$$

$$\text{and } \frac{dv}{dy} = \frac{x}{l}$$

$$\text{Thus} \qquad F = \frac{\lambda}{l}\frac{dx}{dt}\int\int d(\text{area}) = \frac{\lambda}{l}\frac{dx}{dt}A = \frac{\lambda A}{l}\frac{dx}{dt}$$

$$\text{and} \qquad \Delta W = \frac{\lambda A}{l}\oint\left(\frac{dx}{dt}\right)dx = \pi\frac{\lambda A}{l}\omega x_m^2$$

Now in order to get the δ's for each type of damping, we must write down the vibration energy in each case:

Solid Damping:

$$W = \frac{1}{2}kx_m^2 = \frac{1}{2}E\epsilon_m^2$$

and

$$\delta = \frac{\psi}{2} = \frac{\xi E^2 \epsilon_m^2}{E\epsilon_m^2} = \xi E$$

Velocity Damping:

$$\delta = \frac{\pi b\omega x_m^2}{kx_m^2} = \frac{\pi b\omega}{k} = \frac{\pi b}{\sqrt{mk}}$$

Liquid Damping:

$$\delta = \frac{\pi\eta\omega\epsilon_m^2}{E\epsilon_m^2} = \frac{\pi\eta\omega}{E}$$

These are given by the author, and the writer only repeats to preserve the continuity to the next.

Coulomb Damping:

$$\delta = \frac{4Fx_m}{kx_m^2} = \frac{4F}{kx_m}$$

Now since $\delta \approx -\dfrac{\Delta x_m}{x_m}$ for small damping as the author has shown, then

$$-\frac{\Delta x_m}{x_m} = \frac{4F}{kx_m}$$

and we get that

$$\Delta x_m = -\frac{4F}{k}$$

or see that the damping is linear (Fig. 6) and not exponential—an observed fact.

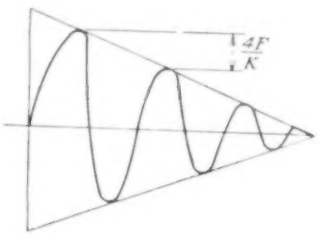


FIG. 6

Velocity² Damping:

$$\delta = \frac{8/3\beta x_m^3\omega^2}{kx_m^2} = \frac{8\beta\omega^2}{3k}x_m$$

Shear Damping:

$$\delta = \frac{\pi\dfrac{\lambda A}{l}\omega x_m^2}{kx_m^2} \text{ where } k = \frac{GA}{l}$$

$$= \frac{\pi\lambda\dfrac{A}{l}\omega x_m^2}{\dfrac{GA}{l}x_m^2} = \frac{\pi\lambda\omega}{G}$$

It will be seen from the above that in all the cases except Coulomb and velocity-squared damping the logarithmic decrement is independent of amplitude, which means an exponential decrease of amplitude with time—and thus the case of motion with solid damping can be represented by the conventional motion equations.

The case of Coulomb damping gives a logarithmic decrement inversely proportional to amplitude, resulting in a linear decrease in amplitude. This type of friction occurs in brake shoes, etc.—wherever solid dry surfaces rub together.

In the case of velocity-squared damping, we obtain δ proportional to the amplitude—and such a fact has been observed in the experiments of Honda and Konno[10] and Iokibe and Sakai.[11] They, however, wishing to adhere to the idea of constant δ, have attributed their linearity between δ and x_m due to too large amplitudes (although they do not go beyond the elastic range), while damping of the sort described above might well explain this consistent observation on many metals.

In the author's own experiments on copper δ,[12] there is a

[10] *Phil. Mag.*, vol. 42 (1921), p. 115.

[11] *Phil. Mag.*, vol. 42 (1921), p. 397.

[12] "Internal Friction in Solids." Trans. A.S.M.E., vol. 48 (1926), p. 450.

very decided increase in δ with amplitude—practically in a straight line. It seems that steels do not exhibit any appreciable change—the brasses, copper, nickel, lead, etc., showing the linear relation to a more or less definite degree, however.

This type of damping yields a decrease in amplitude according to a rectangular hyperbola, and is much faster for the larger amplitudes, for we have

$$\delta = \frac{\Delta x_m}{x_m} = \frac{8\beta\omega^2}{3k} x_m = Cx_m$$

or

$$-\Delta x_m = Cx_m^2$$

Now $-\Delta x_m$ is proportional to $\frac{dy}{dt}$, where y is taken as the envelope of the x_m's and

$$\frac{dy}{dt} = -\alpha y^2$$

or $$-\frac{1}{\alpha}\int\frac{dy}{y^2} = \int dt \text{ or } \frac{1}{\alpha y} = t + \gamma$$

or $$\alpha y t + \alpha\gamma y = 1$$

or $$y = \frac{1}{\alpha(\gamma + t)}$$

which is a displaced rectangular hyperbola (Fig. 12).

Now we must evaluate α and γ in terms of C and various observed characteristics of the observed curve of vibration decay. Now

$$y_t = \frac{1}{\alpha(\gamma + t)}$$

$$y_{(t+T)} = \frac{1}{\alpha(\gamma + t + T)}$$

$$\text{and } \frac{1}{y_t} - \frac{1}{y_{(t+T)}} = \alpha(\gamma + t) - \alpha(\gamma + t + T) = -\alpha T$$

or $$\frac{y_{(t+T)} - y_t}{(y_t)(y_t + T)} = -\frac{\Delta x_m}{x_m^2} = -\alpha T = C$$

and $$\frac{y(t)}{y(t+T)} = \frac{(\gamma + t + T)}{(\gamma + t)} = 1 + \frac{T}{\gamma + t}$$

from which two equations both α and γ may be determined. They are also given geometrically as shown in Fig. 7. The

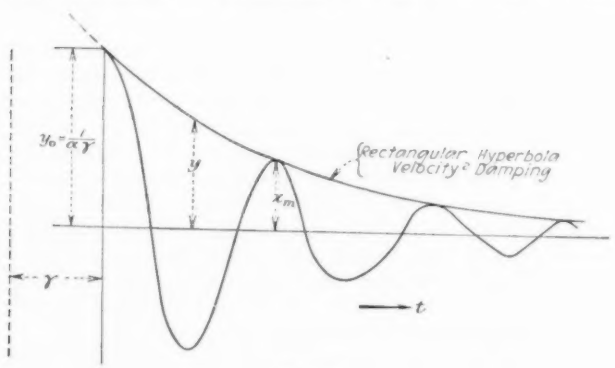


FIG. 7

best test as to the character of the curve, however, is to plot δ vs. x_m, when attempting to differentiate between exponential and hyperbolic damping.

Another observation, of an experimental nature, to discover such damping is to observe the curve of energy loss vs. x_m. For the case of velocity-squared damping, we should expect a cubic relation

$$\Delta W = kx^3$$

Many tests have been made, namely, by Rowett, etc., who have obtained exponents 2.3 and 2.6, showing that the actual damping is of the type

$$F = c\left(\frac{dx}{dt}\right)^k \text{ where } 1 < k < 2$$

The fact that many experimenters have found the energy loss to be practically independent of frequency can be explained by the mechanism of velocity damping, provided the specific resistance be considered proportional to $1/\omega$—and for velocity-squared damping, by considering the specific resistance proportional to $1/\omega^2$. These are hypotheses, but they have a considerable degree of substantiation in Quimby's tests,[13] where he found considerably lower values of specific resistance, working at very high frequencies.

From the consideration of normal viscosity damping, the value of δ arrived at is

$$\delta = \frac{\pi\eta\omega}{E} \text{ or } \frac{\eta}{E} = \frac{\delta}{\pi\omega}$$

It will be observed that the value of δ arrived at when considering tangential viscosity or shear damping is

$$\delta = \frac{\pi\lambda\omega}{G} \text{ or } \frac{\lambda}{G} = \frac{\delta}{\pi\omega}$$

From these two expressions, a close similarity is observed. From observed values of λ and η, we find no simple relation; however, if these two constants were in the ratio of G and E, then a general system-vibration decay need not be distinguished as to shear or direct stress damping. In general, however, we must recognize the difference in these actions, and this must needs modify the author's statement that his dampings II and III represent general system damping.

The general subject of vibration damping is one of increasing importance, for in many applications, instead of avoiding resonances, which we cannot always do, we introduce damping devices and count on existing damping (in a quantitative degree) to render the resonances of minimum violence.

The author has done much to clarify our ideas in this problem, in his present paper, and in preceding ones.

E. O. WATERS.[14] A number of A.S.M.E. papers in recent years have treated the subject of vibration from various points of view, and the present paper has a wholesome unifying effect in pointing out the different kinds of damping which might, at first thought, appear to have wholly dissimilar effects on the amplitude, but which actually, as it proves, produce a logarithmic decrement in every case.

It occurs to the writer that there is another kind of damping, which may not have evidenced itself in much of the author's experimental work, but which would seem to be present in a great deal of machinery, namely, vibrating systems in which one or more of the moving elements pass through poorly lubricated bearings or rub against stationary surfaces with solid contact. In such cases there is assumed to be a damping force

[13] S. L. Quimby, *Phys. Rev.*, vol. 25 (1925), p. 528.

[14] Associate Professor of Mechanical Engineering, Sheffield Scientific School, Yale University, New Haven, Conn. Assoc. Mem. A.S.M.E.

which is constant in magnitude, but which changes its sign with the direction of the velocity.

It would seem that such damping would decrease the amplitude at a constant rate, and that, in the absence of an external harmonic force to sustain the vibrations, it would result in zero motion at the end of a finite interval of time, as contrasted with the logarithmic decrement which theoretically does not terminate. Moreover, the final position of the system would not be one of zero displacement.

The equation of motion is

$$m\ddot{y} + cy = -d \text{ when } \dot{y} > 0$$
$$= +d \text{ when } \dot{y} < 0$$

and the solution, evaluating the integration constants by the condition that $y = y_0$ and $\dot{y} = 0$ when $t = 0$, is

$$y = \left[y_0 - (2n-1)\frac{d}{c} \right] \cos wt + \frac{d}{c} \text{ when } n \text{ is odd}$$
$$-\frac{d}{c} \text{ when } n \text{ is even}$$

n being the number of half-oscillations. The decrement per oscillation is $\frac{4d}{c}$; maximum number of half-oscillations is $\frac{cy_0}{2d}$, neglecting the remainder after performing the division; and the maximum duration of vibration is

$$\frac{\pi\sqrt{cm}\ y_0}{2d}$$

neglecting the time after the last point of zero velocity. The amplitude at the end of the nth half-oscillation is

$$\pm\left[y_0 - (2n-1)\frac{d}{c} \right]$$

and the energy dissipated during the nth half-oscillation is

$$2d\left[y_0 - (2n-1)\frac{d}{c} \right]$$

For the case of sustained vibrations, there are certain difficulties due to the discontinuities at each change of d from + to −. The hysteresis loop for this type of vibration is a parallelogram, and probably the best solution is obtained by substituting for it an ellipse of equal area, and using the standard equations presented by the author.

Author's Closure

Regarding the "threshold stress" observed by Dr. Canfield in his investigations, this is of so small a magnitude as to be almost out of the range of the revolving rod experimental method used by the author. This method as used in our experiments is not reliable for maximum fiber stresses much below 4000 lb. per sq. in., except for certain substances of low elastic modulus, like celluloid.

Mr. Stone has presented an interesting comparison of the decrements for different types of vibration damping as calculated from Equation [5]. In reference to Mr. Stone's remark about this relation "which the writer wishes to point out is a relation holding for all types of vibration decay," it was intended to convey this very idea in the first part of the second paragraph after Equation [5]. The author well appreciates that this relation holds for any law of vibration decay. It gives the logarithmic decrement δ at any instant during a vibration decay whether δ be a constant or not. Mr. Stone's comparison shows very well how δ increases with time when the dissipation per cycle is proportional to less than the second power of the amplitude, and how δ decreases with time when the dissipation per cycle is proportional to more than the second power of the amplitude.

As to the lack of equality between δ as obtained from torsional vibrations as compared with vibrations involving bending, it is probably appreciated that this must be due to a lack of isotropy of most materials as to their internal friction coefficients such that the coefficients vary with change of direction of the strain in the material.

Referring to the hyperbolic decrement curve derived by Mr. Stone, the author has used a similar method in deriving time decrement curves which may be of interest.

Let the energy of vibration $W = ay^2$

Let the time rate of energy dissipation $= \frac{dW}{dt} = -by^n$ where y represents vibration amplitude at some point and a and b are constants

From the first equation $\frac{dW}{dt} = 2ay \times \frac{dy}{dt}$

Equating the two values of $\frac{dW}{dt}$ and using the constant c in place of $\frac{a}{2b}$, the differential equation of amplitude decrement becomes

$$\frac{dy}{dt} = -cy^{n-1}$$

Integrating and evaluating the constant of integration we obtain

$$y^{n-2} = \frac{{y_0}^{n-2}}{(n-2)c{y_0}^{n-2}t + 1}$$

where y_0 = the amplitude y when $t = 0$.

For coulomb friction, $n = 1$ which gives

$$y = -ct + y_0$$

which is a straight line.

For $n = 3$

$$y = \frac{y_0}{cy_0t + 1}$$

which is a hyperbola, as obtained by Mr. Stone.

Turning now to sustained vibrations, an illustration will be considered of a case of an infinite number of degrees of freedom; namely, that of a uniform bar transmitting longitudinal waves. The differential equation for this case is

$$\rho\frac{\partial^2 u}{\partial t^2} - \eta\frac{\partial^3 u}{\partial x^2 \partial t} - E\frac{\partial^2 u}{\partial x^2} = 0$$

where ρ = mass per unit volume
η = friction coefficient
u = *total* strain at any point.

The friction term of this equation is derived on the assumption of frictional stresses in the bar given by $\eta\frac{d\epsilon}{dt}$, which is relation [13] of the author's paper. In the latter expression, the linear strain ϵ is identical with $\frac{\partial u}{\partial x}$ of the above equation of the vibrating bar.

The solution of this equation contains the constant η. Sub-

stituting its value as given by Equation [27], gives the solution for the case of "solid friction" where the dissipation is proportional to the square of the stress amplitude. When the dissipation is proportional to the nth power of the stress amplitude, Equation [27] becomes $\eta = \frac{\xi E^n \epsilon_m{}^{n-2}}{\pi\omega}$. If n has any other value than 2, the coefficient η contains ϵ_m which depends upon the variable x; thus it can be treated as a constant only when $n = 2$. For any other case the differential equation becomes non-linear and its solution more difficult.

It is important to note, however, that for a simple system like that given by Equation [22] the coefficient b can be treated as a constant and the solution for a sustained vibration thereby obtained for a frictional loss per cycle, ΔW, which is *any function*, $\phi(y_0)$, of the stress amplitude.

Using the method of Equation [14], the viscous frictional loss per cycle,

$$\Delta W = \pi b y_0{}^2 \omega$$

Equating this to the solid friction loss $\phi(y_0)$

$$b = \frac{\phi(y_0) y_0{}^{-2}}{\pi\omega}$$

where b is independent of either y or t. Solving [22] as it stands and substituting this value of b, y_0 can be obtained as a function of ω.

The discussion of Mr. Soderberg puts the fundamentals of the problem in a clear and interesting way. The exact mechanism of internal friction is still uncertain and much remains to be done on this subject.

AMP-51-22

Practical Determination of Torsional Vibration in an Engine Installation Which May Be Simplified to a Two-Mass System

By FREDERIC P. PORTER,[1] BELOIT, WIS.

This paper gives a practical solution for the torsional vibration in the shafting of a reciprocating-engine installation which may be reduced mathematically to a simple two-mass system. It presents methods for computing the damped vibrations in installations where the damping forces follow various laws. Particular attention is given to such damping forces as are caused by a marine propeller, elastic hysteresis, and a friction damper. The paper is illustrated by examples of computations for certain engine installations that have been investigated.

TORSIONAL vibrations have in the past caused many failures in the shafts and serious interferences in the operation of reciprocating-engine installations. Steam engines were probably the first type of reciprocating engines to experience these difficulties. As far back as 1900, engineers were beginning to recognize the existence of severe torsional vibrations in the shafting of steam-engine installations.

One of the first theoretical treatments on this subject was given in a paper by Frahm published in the *Zeitschrift des Vereines deutscher Ingenieure*, 1902, page 797, and translated in the *Journal of the American Society of Naval Engineers*, August, 1902, page 721. This paper was a very important contribution to the mathematical solution of torsional-vibration problems. It definitely established that torsional vibration was a problem that should not be neglected in marine propelling installations with engines of the reciprocating type.

The type of installation investigated by Frahm was one in which "the distance between the power-generating engine and the power-absorbing propeller is quite a considerable one." In this type of installation both the engine and propeller may be considered as inelastic flywheels connected to each other by the elastic shaft.

In his paper, Frahm gave a method for computing the damped torsional vibrations for the type of elastic systems just mentioned, damped by a single damping force proportional to the velocity. He considered the action of the propeller to be the principal source of damping, and gave a damping coefficient which he determined experimentally on a number of marine installations and model propellers.

Subsequent investigators have treated in various ways different phases of the subject of torsional vibration, notably that phase which deals with the determination of the synchronous frequency or the frequency of free vibration.

Recently the author presented a paper, "The Range and Severity of Torsional Vibration in Diesel Engines" (*Trans. A.S.M.E.*, Paper APM-50-8, 1928), in which he endeavored to give a comprehensive treatment of the subject of torsional vibration of irregular shafts. The material presented there is the result of a study to determine accurately the range of critical vibrations and the speeds at which they occur. The data also give the results of a study on the effect of elastic hysteresis and propeller damping forces in determining the maximum vibration at the various critical speeds. It is illustrated by examples of computations for the shafting of certain Diesel-engine installations that have been investigated. In the development of the mathematical theory given in the aforementioned paper it was the author's aim to present a general theory that could easily be adapted to any irregular shaft of a multicylinder installation.

Often actual engine installations approximate the mathematical system of two concentrated masses connected by an elastic shaft. In such a system the concentrated mass corresponding to the engine is receiving a succession of impulses stimulating vibration, while the other mass, together with the internal molecular friction, or elastic hysteresis of the material of the connecting shaft, is absorbing the energy given the shaft by the stimulating impulses.

It is the object of the present paper to give in detail the mathematical solution of the type of installation just described, with numerical applications of the theory developed, together with certain experimental verifications of the calculated results.

The forces stimulating torsional vibrations in any reciprocating engine installation are due in general to (1) the pressure forces in the working cylinders, (2) inertia forces of the reciprocating parts, (3) gravity forces of the reciprocating parts and unbalanced rotating parts, and (4) uneven absorption of power at the driven machinery.

The damping forces acting are due in general to (1) elastic hysteresis, (2) the driven machinery, such as a marine propeller, generator, hydraulic pump, dynamometer, vibration damper, etc., (3) slight slippage at the union of two parts of the shaft line-up, (4) surface friction between the moving and stationary parts, (5) energy absorbed by the oil film around the bearings of a crankshaft, (6) energy transmitted by side thrust of the crankshaft journals into the frame of the engine, and (7) air friction, etc.

Accuracy in calculating the amplitude of vibration of any vibrating system depends upon an accurate numerical statement of the stimulating forces and damping forces. It is convenient to handle these forces by expressing the turning moments due to each of the sources of stimulating impulses in terms of a Fourier series. The resulting Fourier series of the turning-effort curve acting at any particular section is then obtained by combining the various harmonic components of the same order acting at that section, where "order" means the number of complete cycles of the harmonic per revolution of the shaft.

A two-mass substitution is made for the connecting rod, one a rotating mass concentrated at the crank radius and the other a reciprocating mass acting at the wristpin. The values of these parts are taken inversely proportional to their distance from the center of mass of the rod.

The inertia of the reciprocating mass has approximately the same effect in vibration as a rotating mass concentrated at the crank radius equal to one-half of the reciprocating mass.

The numerical statement of the stimulating forces and damping forces was taken up in detail in the author's previous paper. An outline of the numerical values of these forces will be included at this time.

The harmonic stimulating impulses of the turning-effort curve

[1] Fairbanks, Morse & Co.

due to the usual pressure forces in a four-cycle Diesel-engine cylinder may be obtained from the coefficients given by Prof. Frank M. Lewis in a paper, "Torsional Vibration in the Diesel Engine," *Transactions of the Society of Naval Architects and Marine Engineers*, Vol. 33, 1925, plates 6 to 11, and "Vibration and Engine Balance in Diesel Ships," same publication, Vol. 35, 1927, plates 9 to 12. It is to be noted, however, that the signs of the coefficients of the cosine terms in the paper in Vol. 33 are opposite those given, i.e., the signs of the cosine terms for the orders $^1/_2$, 1, and $1^1/_2$ are positive and for the orders from 2 to 12 are negative.

The harmonic coefficients of the pressure forces of a triple-expansion, four-cylinder steam engine are given later in this paper. Similar coefficients may be obtained from the indicator cards of the pressure forces in any working cylinder by applying methods of harmonic analysis to the corresponding turning effort.

$$K = fS^{\lambda} \quad [2]$$

where S = one-half the stress range
f = hysteresis constant, and
λ = hysteresis exponent.

Lewis states that from experiments of Rowett made on thin-walled mild-steel tubes, stressed alternately in torsion to a point below the elastic limit, that $f = 1.37 \times 10^{-10}$ and $\lambda = 2.3$ in inch, pound, second, units for stresses less than 8000 lb. per sq. in., and for stresses greater than 8000 lb. per sq. in. the value of λ increases rapidly. The author has observed, however, that it is necessary to increase f to 3.29 times this value in order to obtain calculated results that agree with the recorded vibrations. This correction factor has been used in the investigation of a number of installations and has been found in all cases to be fairly reliable.

TABLE 1 HARMONIC COMPONENTS OF TORQUE DUE TO INERTIA FORCE OF RECIPROCATING PARTS

$$\text{Torque} = F_a \times \frac{\text{Weight}}{\text{Gravity const.}} \times (\text{Crank radius})^2 \times (\text{Angular velocity})^2$$
$$= 0.0000284\, F_a W_r r^2 (\text{r.p.m.})^2 \quad [\text{lb-in.}]$$

	Coefficients of harmonics in Fourier's expansion of Fa for										
Harmonics	$\frac{1}{k}$ = 4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0
sin α	0.06351	0.06192	0.06038	0.05894	0.05757	0.05626	0.05500	0.05380	0.05266	0.05156	0.05051
−sin 2α	0.50012	0.50011	0.50010	0.50009	0.50008	0.50008	0.50007	0.50006	0.50006	0.50006	0.50005
−sin 3α	0.19217	0.18718	0.18246	0.17804	0.17385	0.16973	0.16601	0.16234	0.15886	0.15551	0.15230
−sin 4α	0.01613	0.01532	0.01458	0.01389	0.01325	0.01265	0.01200	0.01159	0.01110	0.01063	0.01020
sin 5α	0.00258	0.00239	0.00222	0.00206	0.00193	0.00180	0.00167	0.00157	0.00148	0.00138	0.00130
sin 6α	0.00014	0.00013	0.00012	0.00010	0.00009	0.00009	0.00008	0.00007	0.00007	0.00006	0.00006
−sin 7α	0.00004	0.00004	0.00003	0.00003	0.00002	0.00002	0.00002	0.00002	0.00002	0.00002	0.00001
−sin 8α	0.00001	0.00001	0.00001	0.00001	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

TABLE 2 HARMONIC COMPONENTS OF TORQUE DUE TO GRAVITY FORCE OF RECIPROCATING PARTS FOR A VERTICAL ENGINE

$$\text{Torque} = F_b \times \text{Weight} \times \text{Crank radius} = F_b W_r r$$

	Coefficients of harmonics in Fourier's expansion of Fb for										
Harmonics	$\frac{1}{k}$ = 4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0
sin α	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
sin 2α	0.12701	0.12381	0.12077	0.11789	0.11515	0.11251	0.11001	0.10761	0.10531	0.10312	0.10102
−sin 4α	0.00103	0.00095	0.00088	0.00081	0.00076	0.00072	0.00066	0.00062	0.00059	0.00055	0.00052
sin 6α	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001	0.00000	0.00000	0.00000	0.00000

TABLE 3 HARMONIC COMPONENTS OF TORQUE DUE TO COUPLE FROM CONNECTING ROD

$$\text{Torque} = F_c \times \frac{\text{Weight of rod}}{\text{Gravity const.}} \times (h_1 h_2 - H^2) \times (\text{Angular velocity})^2$$

Where h_1 and h_2 are distances of the center of mass of the rod from the center lines of the crankpin and wristpin, and H is the radius of gyration of the rod. Also

$$\text{Torque} = 0.0000284\, F_c W_c (h_1 h_2 - H^2)(\text{r.p.m.})^2 \quad [\text{lb-in.}]$$

	Coefficients of harmonics in Fourier's expansion of Fc for										
Harmonics	$\frac{1}{k}$ = 4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0
−sin 2α	0.03124	0.02974	0.02834	0.02704	0.02582	0.02469	0.02362	0.02263	0.02170	0.02082	0.02000
sin 4α	0.00101	0.00091	0.00083	0.00075	0.00068	0.00063	0.00057	0.00052	0.00048	0.00044	0.00041
−sin 6α	0.00002	0.00002	0.00002	0.00002	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001

The harmonic impulses due to the inertia and gravity forces of the reciprocating parts are given in Tables 1 and 2. The inertia force of the connecting rod causes a torque on the shaft that is not given by the two-mass substitution for the rod. The harmonic impulses due to this source are given in Table 3. In these tables, α represents the crank angle from top center, k is the crank-to-connecting-rod ratio, and r is the crank radius.

The turning moment Q due to the unbalanced part of the connecting rod, crankpin, and crankwebs is given by

$$Q = W_u r \sin \alpha \quad [1]$$

where r is the radius to the center of gravity of the weight W_u of the unbalanced parts.

There are as yet very few experimental data on elastic hysteresis. Experiments on the energy absorbed by steel in reversed torsion show that for small ranges of stress it varies as a power of the stress range between the square and the cube. It may be stated per cubic unit by the equation

For a uniform circular shaft, the hysteresis energy loss per unit length will be

$$K = \int_{2r = d_1}^{2r = d_2} 2\pi r f \left(S \frac{2r}{d_2}\right)^{\lambda} dr = \frac{\pi f S^{\lambda} (d_2^{2 + \lambda} - d_1^{2 + \lambda})}{2(2 + \lambda) d_2^{\lambda}} \quad [3]$$

where d_2 and d_1 are the outer and inner diameters of the shaft and S is the maximum fiber stress. Putting the following values into Equation [3]: $S = 16 d_2 M/\pi(d_2^4 - d_1^4)$, $\lambda = 2.3$, and $f = 4.51 \times 10^{-10}$, where f is 3.29 times the value given by Lewis, the hysteresis loss for the length l of a uniform circular shaft is given by

$$K = \frac{69.7(d_2^{4.3} - d_1^{4.3})}{10^{10}(d_2^4 - d_1^4)^{2.3}} l M^{2.3} \quad [4]$$

where M is constant over the length l.

A marine propeller in vibration is affected in two ways by the surrounding water. The motion of the propeller is resisted

by a force proportional to the angular velocity, and the effective inertia of the propeller is increased by the water carried along with it. The factor for the increase in propeller inertia due to entrained water given by Frahm is between 20 to 32 per cent, with an average increase of about 25 per cent.

If, for a constant speed of advance, Q is the torque of the propeller and A its angular velocity in radians per second, the damping factor ρ may be obtained from the equation

$$\rho = \frac{dQ}{dA} \quad [5]$$

The damping factor given by Frahm[2] was $k = rW_m/v_m$. If R is the crank radius, the symbols used by Frahm in terms of the symbols used here are $k = \rho/R^2$, $W_m = Q/R$, $v_m = AR$, so that

$$\rho = r\frac{Q}{A} \quad [6]$$

The values of r given by Frahm fall between 3.6 and 4, with an average value of 3.8.

The method given by Lewis for adapting D. W. Taylor's model experiments to determine the propeller damping coefficient ρ is as follows:

Let s equal the nominal slip or slip in still water. Then the pitch of the propeller in feet times $(1 - s) \times$ r.p.m. equals the speed of advance in feet per minute. Thus it is evident that $(1 - s)A$ = constant. Differentiating this gives

$$\frac{ds}{dA} = \frac{1 - s}{A} \quad [7]$$

Therefore

$$\rho = \frac{\partial Q}{\partial A} = \frac{\partial Q}{\partial s}\frac{\partial s}{\partial A} = \frac{1 - s}{A}\frac{\partial Q}{\partial s} = \frac{Q}{A}(1 - s)\frac{\frac{\partial Q}{\partial s}}{Q} \quad [8]$$

The model experiments may then be used since

$$(1 - s)\frac{\frac{\partial Q}{\partial s}}{Q} = (1 - s)\frac{\frac{\partial Q_m}{\partial s}}{Q_m} = f_m \quad [9]$$

[2] *Journal of the American Society of Naval Engineers*, August, 1902, p. 759, Equation 7.

where the subscript m denotes the corresponding properties of the model propeller. Hence

$$\rho = f_m\frac{Q}{A} = \frac{60Q}{2\pi \times \text{r.p.m.}}f_m \quad [10]$$

Values of f_m at the nominal slip corresponding to maximum propeller efficiencies obtained from the model experiments of D. W. Taylor have been computed by the author and are given in Table 4. These nominal slips were from 12 to 17 per cent. The characteristics of the propellers are given in terms of pitch ratio, mean width ratio, projected area divided by the disk area, and blade-thickness ratio. These terms depend on the following definitions.

Diameter of the propeller is the diameter of the circle touching the blade tips.

Pitch ratio is the mean pitch divided by the diameter.

Developed area is the area of the helicoidal surface of the driving faces.

Projected area is the area of the projections of the blade faces upon a transverse plane perpendicular to the axis.

TABLE 4 PROPELLER DAMPING COEFFICIENTS

(Values of f_m at nominal slip corresponding to maximum propeller efficiency obtained from model experiments by D. W. Taylor. Models used were elliptically bladed with 3 blades, 16 in. diameter, 3.2 in. hub diameter, 1/64 in. tip thickness, towed at 5 knots speed of advance.)

Propeller number	Pitch in inches	Pitch ratio	Mean width ratio	Projected area ÷ disk area	Blade-thickness ratio and f_m, respectively							
					Original thickness		A — Cut		B — Cut		C — Cut	
1	9.6	0.6	0.15	0.2129	0.1033	1.7	0.0774	2.0	0.0516	2.3	0.0258	2.7
2	9.6	0.6	0.20	0.2838	0.0894	2.2	0.0672	2.5	0.0448	2.8	0.0224	3.2
3	9.6	0.6	0.25	0.3548	0.0800	2.5	0.0600	2.8	0.0400	3.2	0.0200	3.6
4	9.6	0.6	0.30	0.4257	0.0730	2.8	0.0544	3.2	0.0363	3.5	0.0181	4.0
5	9.6	0.6	0.35	0.4966	0.0676	3.1	0.0507	3.5	0.0338	3.9	0.0169	4.3
6	12.8	0.8	0.15	0.2023	0.1033	2.1	0.0774	2.5	0.0516	2.8	0.0258	3.3
7	12.8	0.8	0.20	0.2698	0.0894	2.5	0.0672	2.9	0.0448	3.3	0.0224	3.7
8	12.8	0.8	0.25	0.3373	0.0800	2.8	0.0600	3.2	0.0400	3.6	0.0200	4.1
9	12.8	0.8	0.30	0.4047	0.0730	3.1	0.0544	3.5	0.0363	3.9	0.0181	4.4
10	12.8	0.8	0.35	0.4721	0.0676	3.3	0.0507	3.8	0.0338	4.2	0.0169	4.6
11	16	1.0	0.15	0.1919	0.1033	2.4	0.0774	2.8	0.0516	3.3	0.0258	3.9
12	16	1.0	0.20	0.2557	0.0894	2.8	0.0672	3.2	0.0448	3.7	0.0224	4.2
13	16	1.0	0.25	0.3198	0.0800	3.1	0.0600	3.5	0.0400	4.0	0.0200	4.5
14	16	1.0	0.30	0.3837	0.0730	3.3	0.0544	3.8	0.0363	4.3	0.0181	4.7
15	16	1.0	0.35	0.4476	0.0676	3.5	0.0507	4.1	0.0338	4.6	0.0169	4.9
16	19.2	1.2	0.15	0.1814	0.1033	2.4	0.0774	2.7	0.0516	3.2	0.0258	3.7
17	19.2	1.2	0.20	0.2418	0.0894	2.8	0.0672	3.2	0.0448	3.6	0.0224	4.1
18	19.2	1.2	0.25	0.3023	0.0800	3.2	0.0600	3.5	0.0400	4.0	0.0200	4.4
19	19.2	1.2	0.30	0.3627	0.0730	3.5	0.0544	3.8	0.0363	4.3	0.0181	4.6
20	19.2	1.2	0.35	0.4231	0.0676	3.7	0.0507	4.1	0.0338	4.6	0.0169	4.9
21	24	1.5	0.15	0.1656	0.1033	2.7	0.0774	2.9	0.0516	3.3	0.0258	3.8
22	24	1.5	0.20	0.2209	0.0894	3.0	0.0672	3.3	0.0448	3.7	0.0224	4.1
23	24	1.5	0.25	0.2761	0.0800	3.3	0.0600	3.6	0.0400	4.0	0.0200	4.4
24	24	1.5	0.30	0.3313	0.0730	3.5	0.0544	3.8	0.0363	4.2	0.0181	4.6
25	24	1.5	0.35	0.3865	0.0676	3.7	0.0507	4.1	0.0338	4.5	0.0169	4.8
26	32	2.0	0.15	0.1394	0.1033	2.9	0.0774	3.2	0.0516	3.5	0.0258	4.0
27	32	2.0	0.20	0.1887	0.0894	3.2	0.0672	3.5	0.0448	3.9	0.0224	4.4
28	32	2.0	0.25	0.2323	0.0800	3.5	0.0600	3.8	0.0400	4.2	0.0200	4.7
29	32	2.0	0.30	0.2788	0.0730	3.7	0.0544	4.1	0.0363	4.5	0.0181	5.0
30	32	2.0	0.35	0.3253	0.0676	4.0	0.0507	4.3	0.0338	4.7	0.0169	5.2

Disk area is the area of the circle touching the blade tips.

Mean width ratio is h divided by the diameter, where h is the width of a rectangle whose length is the distance from the tip to the hub and whose area is the developed area of one blade. If D is the diameter of the propeller and d the diameter of the hub,

$$\text{Mean width ratio} = \frac{2 \times \text{developed area}}{3(D - d)D} \quad [11]$$

This formula is for a three-bladed propeller.

Blade-thickness ratio is AC divided by the diameter, where AC is obtained as follows: Let a plane through the axis of the propeller cut the center of a blade. Extend the lines of intersection of the plane with the blade faces to the axis. AC is the distance along the axis between these intersections.

It is noted that the factor f_m in Equation [10] is the same as the factor r in Equation [6] given by Frahm. For the purpose of comparing the numerical values of these factors, the value of f_m has been obtained for thirty-four different propellers given

by C. W. Dyson in "Screw Propellers" (1924), Vol. II, sheets 8 to 11. These values are listed below.

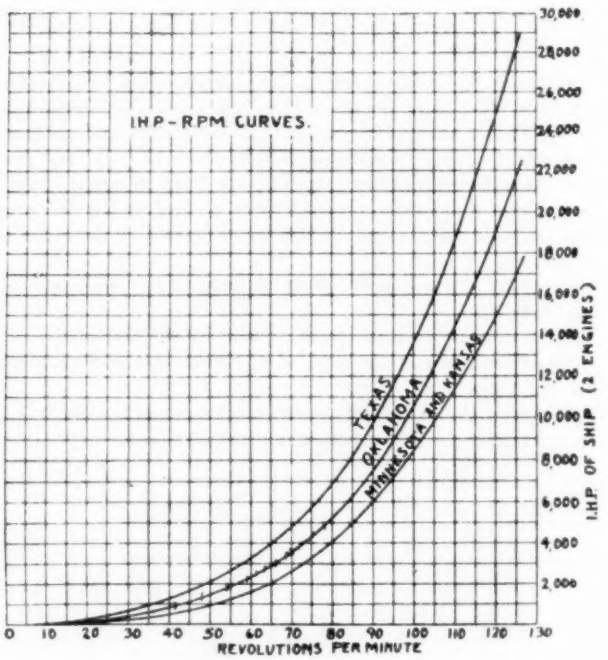


FIG. 1

Ship	f_m	Ship	f_m
Nebraska	3.6	Georgia	3.6
Wisconsin	3.9	Ohio (as adopted)	4.1
Denver	3.9	Kearsarge, Kentucky, and Illinois	3.4
New Orleans	3.9	Missouri	3.4
Albany	3.9	Atlanta	3.9
Montgomery and Detroit	3.7	Chicago	3.6
Marblehead	3.8	Maine (new)	3.6
Chattanooga	3.6	Texas (San Marcos)	3.8
Galveston	3.6	Baltimore and Philadelphia	3.8
Newark	3.5	Olympia	3.8
Alabama	3.9	Arkansas (Ozark)	3.5
Indiana and Massachusetts	3.5	Nevada (Tonopah)	3.5
Charleston (old)	3.6	Wyoming (Cheyenne)	4.1
Boston	3.6	Florida (Tallahassee)	4.1
Oregon	3.8	Connecticut	3.7
Iowa	3.6	Virginia	3.5
Minneapolis:			
Center screw	3.5		
Wing screws	3.4	Sum	125.7
		Average	3.70

The values of f_m for these thirty-four propellers fall between 3.4 and 4.1, and the average value is 3.70. The values given by Frahm for this same factor, i.e., his factor r, was between 3.6 and 4.0, with an average value of 3.8.

Other forms of damping that occur in the driven machinery of particular installations can often be calculated when the characteristics of this machinery are known.

A slight slippage between two of the members of the shaft line-up, such as between bolted flanged couplings, keyed couplings, friction clutch, etc., may be the source of great damping. This source of damping, however, is unreliable and in ordinary cases can be neglected.

The surface friction between the moving and stationary parts of a rotating shaft is nearly independent of the velocity, and therefore has practically no effect on damping the torsional vibrations. Such a friction is balanced by part of the mean torque of the engine.

Other energy losses such as the energy absorbed by the oil film around the bearing of a crankshaft, energy transmitted by side thrust of the crankshaft journals into the frame of the engine, air friction, etc., have a relatively small effect in damping vibrations. Without much error these sources of damping can be neglected.

For numerical illustration, the torsional-vibration investigations made on three steam-engine installations will be described in the following pages. These installations are the main propelling units of the (1) U. S. S. *Oklahoma*, (2) U. S. S. *Texas*, (3) U. S. S. *Minnesota*, and U. S. S. *Kansas*.

Each of these propelling engines is a triple-expansion, four-cylinder steam engine, operating with a maximum boiler pressure of 265 lb. per sq. in. gage. The full-power speed of these engines is 125 r.p.m. The power balance between the cylinders is: high-pressure cylinder, $^1/_3$; intermediate-pressure cylinder, $^1/_3$; and each low-pressure cylinder, $^1/_6$. From forward to aft the cylinders are arranged in the order of: forward low-pressure cylinder (F.L.P.), high-pressure cylinder (H.P.), intermediate-pressure cylinder (I.P.), and after low-pressure cylinder (A.L.P.). The working sequence of the cylinders for a port engine is F.L.P., A.L.P., H.P., I.P., and for a starboard engine

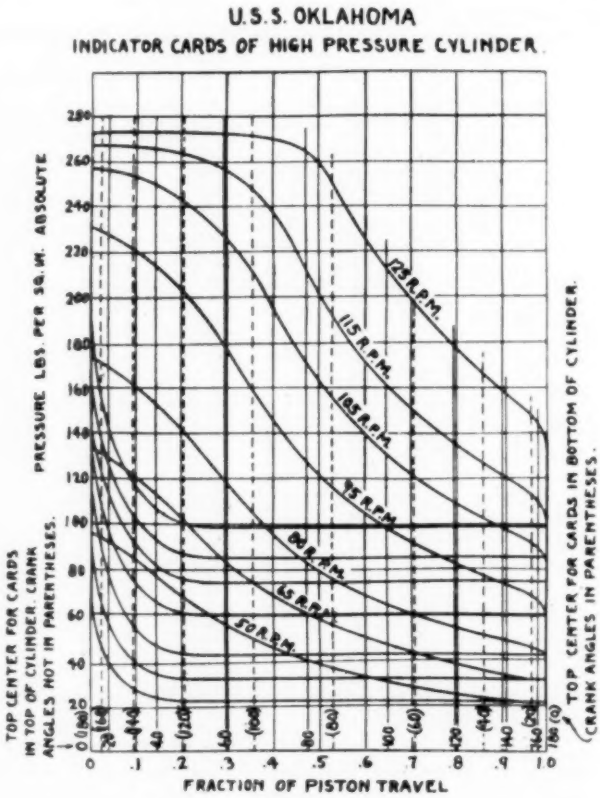


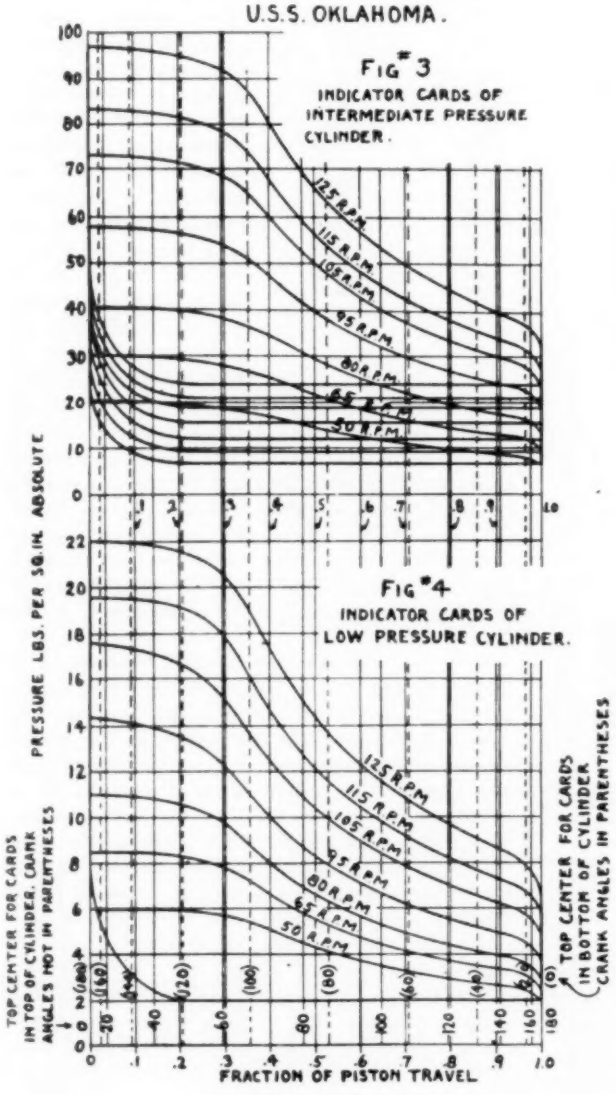
FIG. 2

is F.L.P., I.P., H.P., A.L.P. The crank radius of each is 24 in. and the connecting-rod length is 96 in. The indicated horsepower and revolutions per minute curves are given in Fig. 1. These curves give the total indicated horsepower for the ships, including both the port and starboard propelling units. Other data for these installations are given in Table 5.

I—U. S. S. "Oklahoma," Port Propelling Unit

The impulses causing vibration in this installation due to the turning effort of the pressure forces were obtained from the set of twenty-one indicator cards for the H.P., I.P., and L.P. cylinders shown in Figs. 2, 3, and 4. These cards are based on a set of actual indicator cards taken on the *Oklahoma's* engines. The turning effort corresponding to the pressure forces given

in these cards was analyzed into harmonic components by a schedule method of harmonic analysis. The results thus obtained for the combined effect of the pressures in both the top and the bottom of the various cylinders are given in Figs. 5, 6, and 7. The values (m_i) plotted in these figures give the magnitude of the first eight harmonic components per unit crank radius and unit mean piston area (i.e., mean of top and bottom area of piston). The phase angles of the first six harmonics are also plotted in these figures, and represent the lag of the maximum value of the harmonic behind top center of the piston. These phase angles are given in degrees and represent



FIGS. 3 AND 4

that part of one cycle of the harmonic component referred to, considering one cycle of the harmonic to contain 360 deg.

The resultant effect of all the impulses can now be obtained by assuming that all the impulses act at the center of the engine. This assumption is justified for the vibration in any installation where the amplitude of vibration over the section of the shaft receiving the impulses is nearly constant.

To illustrate the calculations necessary to obtain the resultant harmonic impulses, calculations for these impulses will be described for the first six harmonic components at the engine speed of 80 r.p.m. Let all phase angles be measured from the top center of the F.L.P. cylinder. The cylinders then follow in the order of F.L.P., A.L.P., H.P., I.P. The phase angles for the various orders at the different cylinders are thus increased by the angles given in Table 6.

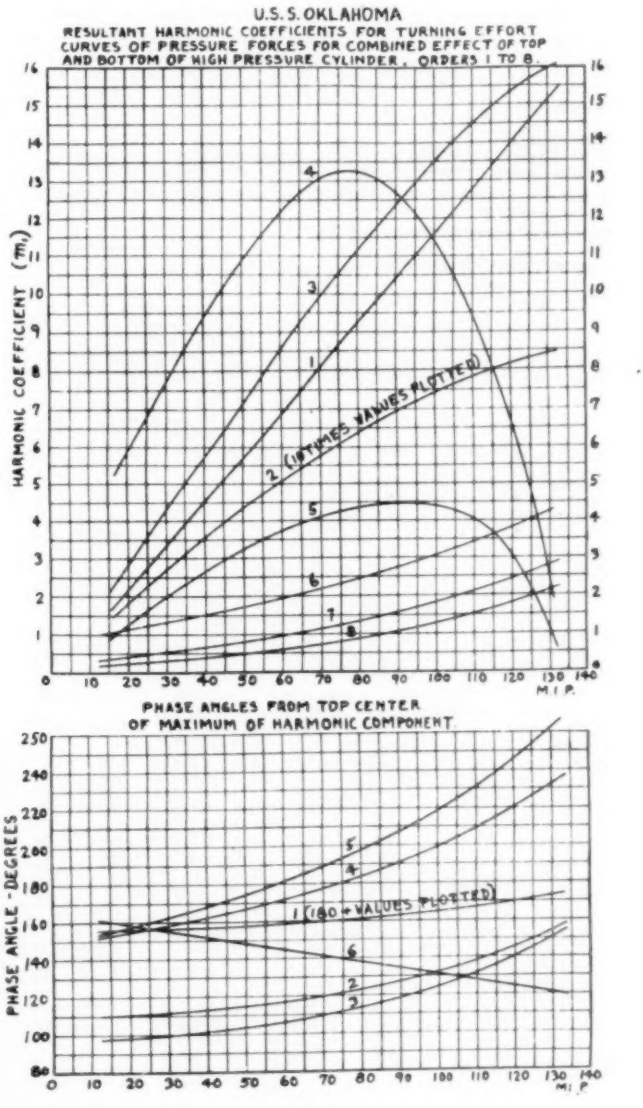


FIG. 5

Fig. 1 gives the i.hp. of the ship at 80 r.p.m. as 5200. Thus the i.hp. for each cylinder at this speed is: H.P., 867; I.P., 867; and L.P., 433; and the corresponding m.i.p. for each cylinder is: H.P., 47.6; I.P., 16.4; L.P., 4.68. The values of m_i and the phase angles for the pressure forces can now be obtained from Figs. 5, 6, and 7. Then multiplying the values of m_i by the mean piston area (mean of top and bottom area of each piston), the impulses due to the pressure forces are obtained. The values obtained in this way are given in Table 7.

The reciprocating part of the connecting rod is $0.4 \times 6470 = 2590$ lb. Thus the total reciprocating weight at the various cylinders is: H.P., 8800; I.P., 11,220; L.P., 11,320. The impulses due to the inertia of the reciprocating weights may be obtained from Table 1. For $r = 24$, r.p.m. $= 80$, these impulses are $104.8\ F_e W_r$.

Reading the values of F_a for the various orders, for a crank-to-connecting-rod ratio of $^1/_4$ from Table 1, the impulses due to this source may be computed. For 80 r.p.m. they are as given in Table 8. The harmonic components in F_a are all sine terms so that the phase angles of the impulses in Table 8 referred to the top center of the F.L.P. cylinder are 90 deg. greater than the phase angles for the various cylinders given in Table 6.

The impulses due to the couple from the connecting rod may be obtained from Table 3. For $h_1 = 38.4$, $h_2 = 57.6$, $H = 43$, $W = 6470$, r.p.m. = 80, these impulses are 424,000 F_c. Reading the values of F_c from Table 3, the impulses due to this source of stimulation at each cylinder are as given in Table 9. The phase angles for these impulses at the various cylinders are 90 deg. greater than those given in Table 6.

The impulses from the gravity forces of the reciprocating masses are given in Table 2. The evaluation of these impulses are given in Table 10. The phase angles for these impulses at the various cylinders are 90 deg. greater than those given in Table 6.

The unbalanced rotating part of the connecting rod is 0.6 × 6470 = 3880 lb. Thus the unbalanced rotating part of one connecting rod, crankpin, and two crankwebs is 3880 + 3760 = 7640 lb. The impulse due to the unbalanced rotating weight is given by Equation [1], so that its value at each cylinder is a 1-order impulse equal to 7640 × 24 = 183,500, having a phase angle at the various cylinders 90 deg. greater than those given for the 1-order in Table 6.

The resultant harmonic impulses acting at the center of the engine can now be obtained by combining the components of the various sources in their proper phase relation. This can be done graphically or analytically. If the latter method is used, the component impulses are multiplied by the sine and cosine of their phase angles. If A is the algebraic sum of all sine products for any order and B is the algebraic sum of all the cosine products, the resultant impulse for this order is $\sqrt{A^2 + B^2}$ and the corresponding phase angle is $\tan^{-1}(A/B)$. If this is done for the components computed above for the engine speed of 80 r.p.m., the resultants given in Table 11 are obtained.

In a similar manner the resultant harmonic impulses acting at the center of the engine can be obtained for any speed. Figs. 8 and 9 give the resultant impulses that have been computed for engine speeds between 40 and 125 r.p.m.

It is observed that the resultant impulse of the 1-order is due mainly to the pressure forces and the gravity forces of the reciprocating masses. The 2-order and 3-order are due mainly to the pressure forces and the inertia forces of the reciprocating masses. The 4-order is due mainly to the pressure forces, but is influenced to some extent by the inertia forces of the reciprocating masses. The effect of the couple from the connecting rod and the gravity forces of the reciprocating masses is seen to be small for the 4-order. It is also observed that the impulses of higher orders than the fourth are due principally to the pressure forces.

The principal damping forces occurring in this installation are due both to the action of the propeller and to elastic hysteresis of the shafting material. The energy absorbed by the elastic hysteresis may be obtained from Equation [4].

The data necessary to obtain the propeller damping coefficient for the *Oklahoma* are as follows:

Diameter	18 ft. 2 in.
Mean pitch	19 ft. 5$^1/_4$ in.
Number of blades	3
Developed area	92.4 sq. ft.
Projected area	77.7 sq. ft.
Pitch ratio	1.070
Mean width ratio	0.242
Projected area ÷ disk area	0.300
Blade-thickness ratio	0.0565

TABLE 5 DATA ON MAIN PROPELLING UNITS OF U. S. WAR VESSELS

Installation	*Minnesota* and *Kansas*	*Oklahoma*	*Texas*
WR^2 of engine	20,400,000	32,700,000	33,700,000
WR^2 of propeller plus 25 per cent for entrained water	60,000,000	66,900,000	74,200,000
WR^2 of shafting between center of engine and propeller	3,830,000	7,110,000	6,730,000
Equivalent length of shaft for stiffness between center of engine and propeller	1462 in. of 16.5 × 9.5 D. shaft	2020 in. of 17.75 × 8.75 D. shaft	1790 in. of 18.5 × 11 D. shaft
Full power rating for one engine, hp	8500	10,850	14,250
Diameter of cylinders, inches — H.P.	32.5	35	39
Diameter of cylinders, inches — I.P.	53	59	63
Diameter of cylinders, inches — L.P.	61	78	83
Diameter of piston rod, in	7.25	7.75	8.25
Unit weights of reciprocating parts, lb — H.P. piston	1160	2180	2540
Unit weights of reciprocating parts, lb — I.P. piston	1830	4600	3100
Unit weights of reciprocating parts, lb — L.P. piston	2280	4700	4970
Unit weights of reciprocating parts, lb — H.P. connecting rod	4640	6470	7260
Unit weights of reciprocating parts, lb — I.P. connecting rod	4490	6470	7260
Unit weights of reciprocating parts, lb — L.P. connecting rod	4290	6470	7100
Unit weights of reciprocating parts, lb — H.P. & I.P. piston rod	1153	1375	1694
Unit weights of reciprocating parts, lb — L.P. piston rod	1153	1375	1416
Unit weights of reciprocating parts, lb — Crosshead	1394	2660	2650
Distance of center of mass of connecting rod from crankpin center line ÷ center-line length	0.365	0.40	0.37
Radius of gyration of connecting rod, in	43.8	43	43
Weight of unbalanced rotating parts of one crank (2 webs and a crankpin) reduced to the center line of the crankpin, lb	2910	3760	3980

TABLE 6 PHASE ANGLES IN DEGREES

Cylinder	Order 1	2	3	4	5	6
F.L.P.	0	0	0	0	0	0
A.L.P.	90	180	270	0	90	180
H.P.	180	0	180	0	180	0
I.P.	270	180	90	0	270	180

TABLE 7 PRESSURE IMPULSES AT 80 R.P.M.

Cylinder	1-Order m_1	1-Order m	1-Order Phase	2-Order m_1	2-Order m	2-Order Phase	3-Order m_1	3-Order m	3-Order Phase
H.P.	− 5.4	−121,600	337	42.0	944,000	114	6.8	153,000	283
I.P.	− 1.9	−124,000	74	12.0	783,000	314	2.2	143,500	218
F.L.P.	− 0.54	− 61,800	165	3.3	377,000	133	0.58	66,300	146
A.L.P.	− 0.54	− 61,800	255	3.3	377,000	313	0.58	66,300	56
	4-Order m_1	4-Order m	4-Order Phase	5-Order m_1	5-Order m	5-Order Phase	6-Order m_1	6-Order m	6-Order Phase
H.P.	10.5	236,000	165	3.2	71,900	352	1.66	37,400	148
I.P.	1.5	97,800	184	0.63	41,100	128	0.64	41,800	274
F.L.P.	0.65	74,300	182	0.22	25,100	203	0.14	16,000	99
A.L.P.	0.65	74,300	182	0.22	25,100	293	0.14	16,000	279

TABLE 8 RECIPROCATING-WEIGHT IMPULSES DUE TO INERTIA FORCES AT 80 R.P.M.

Order	F_a	Impulses H.P.	Impulses I.P.	Impulses L.P.
1	0.0635	58,500	74,700	75,400
2	−0.500	−461,000	−588,000	−593,000
3	−0.1922	−177,000	−226,000	−228,000
4	−0.01613	− 14,850	− 18,980	− 19,000
5	0.00258	2,380	3,030	3,060
6	0.00014	130	160	170

TABLE 9 IMPULSES DUE TO COUPLE FROM CONNECTING ROD AT 80 R.P.M.

Order	F_c	Impulses
2	−0.0312	−13,250
4	0.00101	428
6	−0.00002	8

TABLE 10 RECIPROCATING-WEIGHT IMPULSES DUE TO GRAVITY FORCES

Order	F_b	Impulses H.P.	Impulses I.P.	Impulses L.P.
1	1.000	211,000	269,000	272,000
2	0.1271	26,800	34,200	34,500
3	−0.00103	−217	−277	−280
4	0.00001	2	3	3

TABLE 11 RESULTANT HARMONIC IMPULSES ACTING AT CENTER OF ENGINE AT 80 R.P.M.

Order	Impulses	Phase
1	88,700	146
2	448,000	69
3	220,000	243
4	480,000	183
5	33,900	343
6	36,300	217

Thus from Table 4, f_m is given as 3.6, making the damping coefficient from Equation [10] equal to

$$\rho = \frac{60 \times 3.6}{2\pi} \frac{Q}{\text{r.p.m.}} = 34.4 \frac{Q}{\text{r.p.m.}} \quad \ldots\ldots [12]$$

in which Q can be obtained from the i.hp. curve in Fig. 1 for a mechanical efficiency of 85 per cent from the equation

$$Q = \frac{33{,}000 \times 12 \times E \times \text{i.hp.}}{2\pi \text{ r.p.m.}} = \frac{63{,}000\, E \times \text{i.hp.}}{\text{r.p.m.}} \quad \ldots [13]$$

where E is mechanical efficiency.

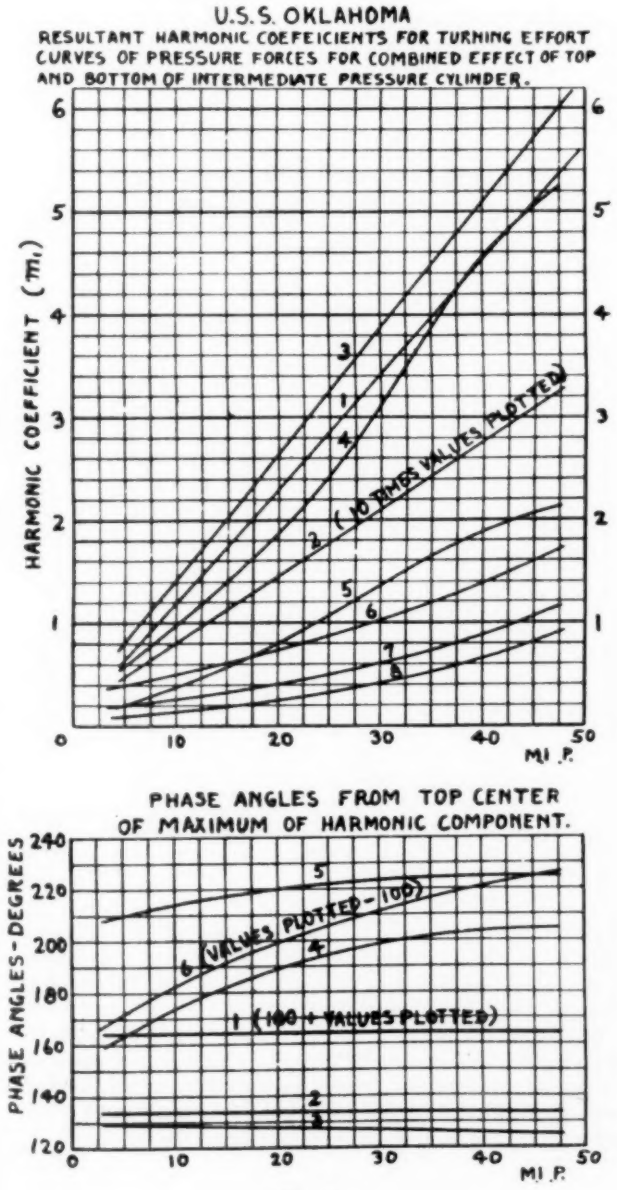


FIG. 6

As stated in Table 5, the stiffness of the shafting between the center of the engine and the center of the propeller is equivalent to 2020 in. of 17.75 in. outside and 8.75 in. inside diameter. This was obtained by calculating the deflection between these sections for a unit twisting moment, and finding the length of 17.75 × 8.75-in.-diameter shafting to give the same deflection. The crankshaft was assumed to have the same elasticity as a shaft of the same length having the journal section. For flanged couplings the shaft on either side was assumed to penetrate part way through the flanges.

The data given above for the *Oklahoma* are sufficient to calculate the amplitude of the first six orders of vibration over the speed range of 40 to 125 r.p.m. It is evident that the installation is approximately equivalent to the mathematical system of two concentrated masses connected by an elastic shaft having mass. If J_1, J_2, and J represent the WR^2 values, or the weight polar moment of inertias of the engine, propeller, and shafting, respectively, and if C represents the stiffness factor or proportional constant between the twisting moment M and the resulting angular deflection θ of the shaft between the center of the engine and the propeller such that $M = C\theta$, then

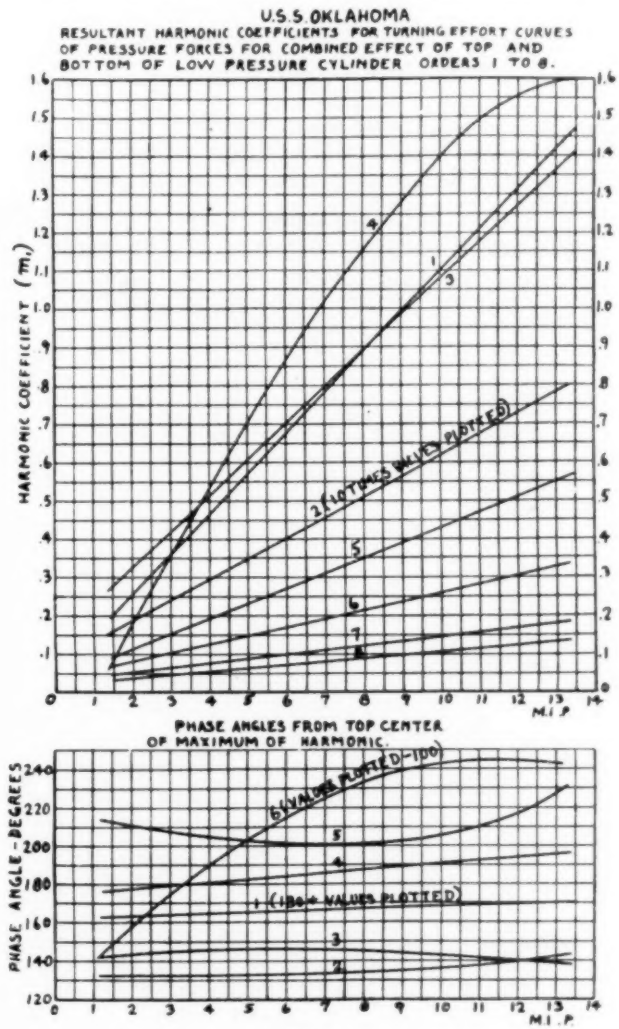


FIG. 7

$$J_1 = 32{,}700{,}000$$
$$J_2 = 66{,}900{,}000$$
$$J = 7{,}110{,}000$$
$$C = 53{,}500{,}000$$

Thus

$$\frac{J_1}{J} = 4.60, \frac{J_2}{J} = 9.41, \text{ and } \phi = \frac{360}{\sqrt{g}} \sqrt{\frac{J}{C}} = 18.33\sqrt{\frac{J}{C}} = 6.68$$

The synchronous frequency for any installation of this type may be obtained by the solution for n in Equations [24] and [39] of the author's previous paper.[3] These equations may be written in the following form:

$$\left.\begin{aligned} \tan\gamma &= \frac{\pi}{180}\frac{J_1}{J}\phi n \\ \tan(\phi n + \gamma) &= -\frac{\pi}{180}\frac{J_2}{J}\phi n \end{aligned}\right\} \quad \ldots\ldots\ldots [16]$$

where γ is a constant of integration in degrees.

It will be noted that there is an infinite number of roots for n as given by these equations. Considering the roots in an increasing order of magnitude, they are the synchronous frequencies of the 1, 2, 3, etc., noded modes of vibrations.

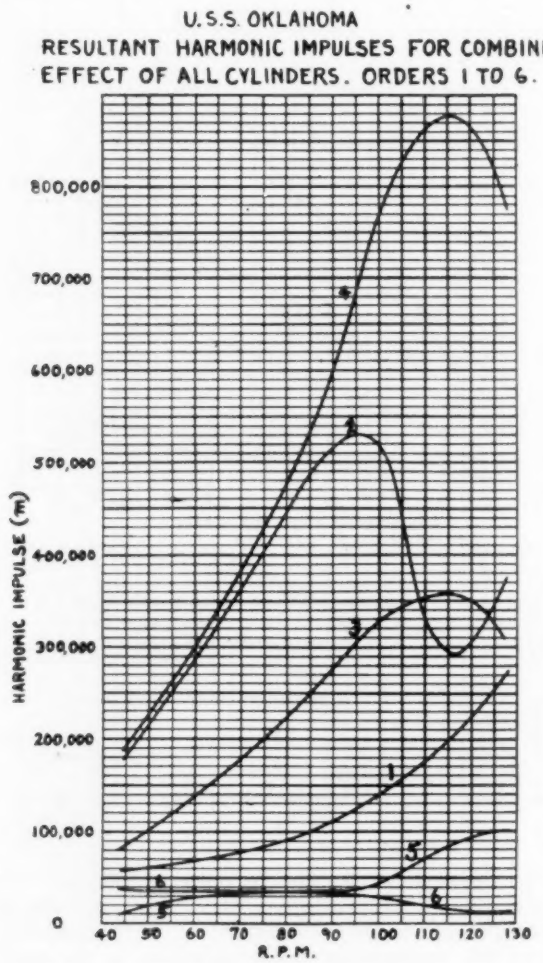


FIG. 8

From the solutions of these equations, a set of curves given in Fig. 10 has been constructed that give ϕn for the one-noded synchronous vibration for various values of J_1/J from 0 to 100 and J_2/J from 0 to ∞. From these curves ϕn can be obtained with a possible error of about one-half of one per cent, from which the value of n for the one-noded synchronous frequency can be computed by dividing by ϕ.

For the case of the installation under consideration, ϕn from the chart in Fig. 10, is 32.0, giving the value of n for the synchronous frequency of the one-noded vibration equal to 32.0/6.68 or 4.79 v.p.s.

When computing the amplitudes for the one-noded vibrations of such a vibrating system, however, where the inertia of the shaft is relatively small in comparison with the inertias of the concentrated masses, it is usually convenient to reduce the system to two concentrated masses connected by an elastic shaft having no mass. This may be done in such a way as to leave the frequency and the location of the node the same as before by using the curves in Figs. 11 and 12.

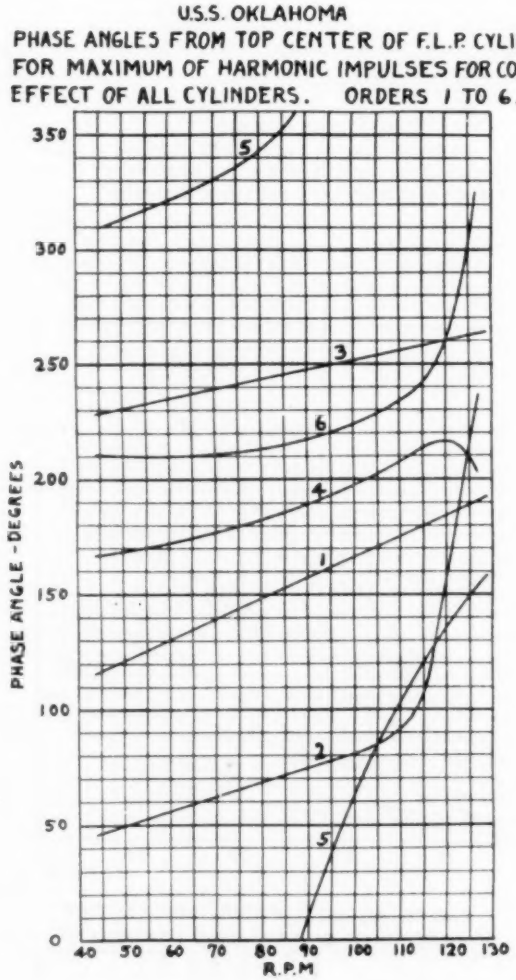


FIG. 9

Fig. 11 gives a factor to obtain the position of the node for the synchronous frequency of the one-noded torsional vibration for any two concentrated masses connected by an elastic shaft having mass from $J_1/J = 0$ to 20 and J_2/J from 0 to 12. The factor plotted in this figure is the absolute value of

$$\frac{x_{nd}}{l} - \frac{x_{cg}}{l}$$

where l = length of the shaft between the concentrated masses
x_{nd} = distance of the node from J_1
x_{cg} = distance of the center of gravity of the inertias from J_1; thus

[3] Loc. cit.

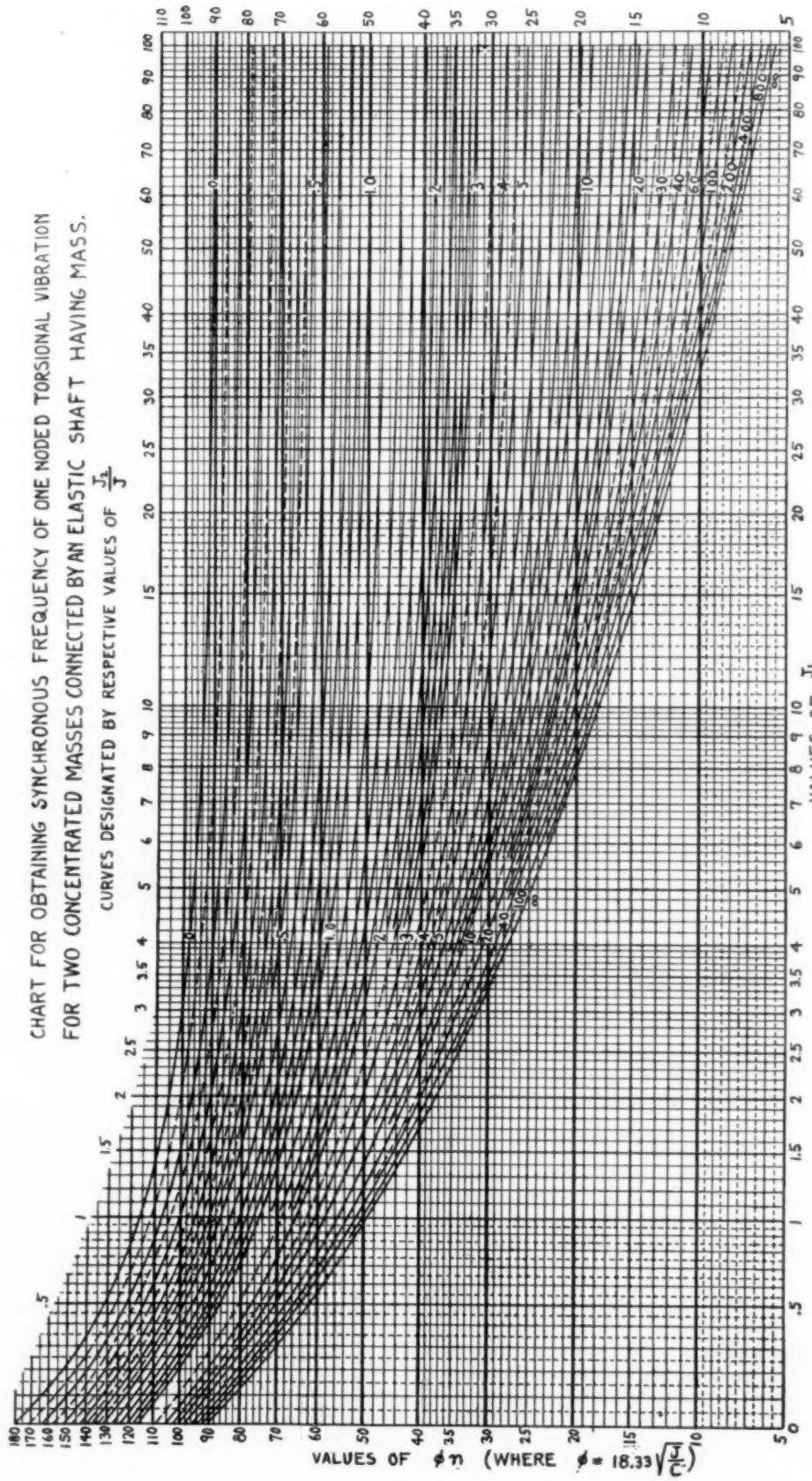

CHART FOR OBTAINING SYNCHRONOUS FREQUENCY OF ONE NODED TORSIONAL VIBRATION
FOR TWO CONCENTRATED MASSES CONNECTED BY AN ELASTIC SHAFT HAVING MASS.
CURVES DESIGNATED BY RESPECTIVE VALUES OF $\frac{J_1}{J}$
VALUES OF $\phi\eta$ (WHERE $\phi = 18.33\sqrt{\frac{K}{C_1}}$)
VALUES OF $\frac{J_1}{J}$

FIG. 10

$$\frac{x_{cg}}{l} = \frac{\frac{J_2}{J} + \frac{1}{2}}{\frac{J_1}{J} + \frac{J_2}{J} + 1} \quad \ldots\ldots [17]$$

The value of x_{nd} was computed from the relation $\frac{\phi n x_{nd}}{l} = 90 - \gamma$, where γ is the same constant as that stated in Equation [16]. This relation follows from Equation [22] of the author's previous paper where to obtain the node, θ is placed equal to zero. The position of the node is obtained from the factor given in Fig. 11 by computing x_{cg}/l from [17] and adding or subtracting $\frac{x_{nd}}{l} - \frac{x_{cg}}{l}$ in such a way as to place the node between the center of gravity of the inertias and the larger concentrated mass.

The synchronous frequency, as given by [18], for a shaft having mass that is fixed at one end and has a concentrated mass at the other can be obtained from Equation [19] by adding a certain fraction of the mass of the shaft to J_1. Let Δ be this fraction of J to be added to J_1. Then the synchronous frequency for the shaft fixed at one end will be given by

$$n = \frac{1}{2\pi}\sqrt{\frac{gC}{J_1 + \Delta J}}$$

Substituting this value of n in Equation [18] and changing the angle $\phi_a n$ to radians, there results

$$\cot \frac{1}{\sqrt{d + \Delta}} = \frac{d}{\sqrt{d + \Delta}} \quad \ldots\ldots [20]$$

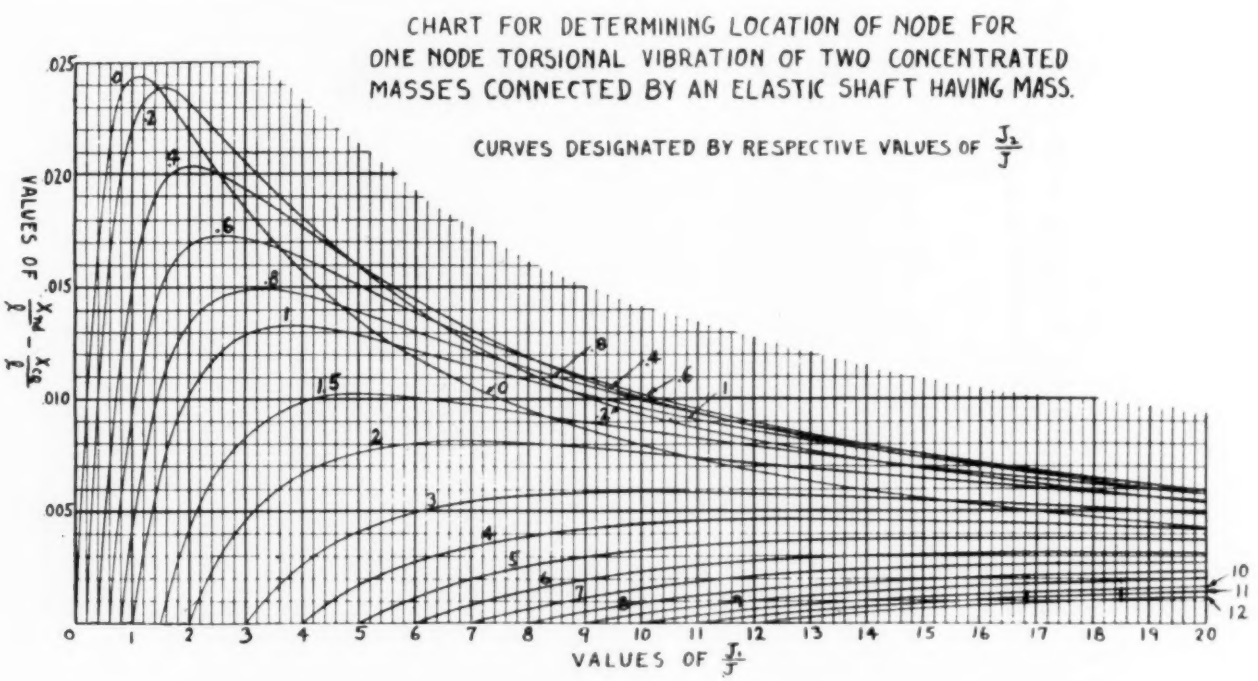


FIG. 11

After the node is determined for the synchronous frequency, the shaft can be considered fixed at the node. The concentrated mass and shaft on either side at this frequency can thus be considered as a vibrating system fixed at one end having a concentrated mass at the other. If ϕ_a is the value of $\frac{360}{\sqrt{g}}\sqrt{\frac{J}{C}}$ for a shaft fixed at one end, and if J_1 is the value of the inertia concentrated at the other, the synchronous frequency follows from [16] by putting $J_2 = \infty$. This gives $\phi_a n + \gamma = 90$, or, eliminating γ, the synchronous frequency follows from

$$\cot \phi_a n = \frac{\pi}{180}\frac{J_1}{J}\phi_a n \quad \ldots\ldots [18]$$

If an elastic shaft without mass is fixed at one end and has a concentrated mass J_1 at the other, the synchronous frequency is given by Equation [42] of Appendix No. 1, by putting $J_2 = \infty$. In this case

$$n = \frac{1}{2\pi}\sqrt{\frac{gC}{J_1}} \quad \ldots\ldots [19]$$

where $d = J_1/J$.

Where there is no concentrated mass at the end, $J_1 = 0$, $d = 0$, and

$$\cot \frac{1}{\sqrt{\Delta}} = 0, \text{ giving } \Delta = \frac{4}{\pi^2} = 0.40528$$

The limit of Δ when the inertia of the shaft is assumed indefinitely small may be obtained as follows:

Equation [20] may be written

$$1 = \frac{d}{\sqrt{d + \Delta}} \tan \frac{1}{\sqrt{d + \Delta}}$$

Expanding the tangent function into an infinite series of increasing powers of the angle, the above relation may be written in the following form:

$$1 = \frac{d}{\sqrt{d + \Delta}}\left(\frac{1}{\sqrt{d + \Delta}} + \frac{1}{3(d + \Delta)^{3/2}} + \frac{2}{15(d + \Delta)^{5/2}} + \ldots\right)$$

which may be changed to

$$\Delta = \frac{d}{3(d + \Delta)} + \frac{2d}{15(d + \Delta)^2} + \ldots$$

If J is taken indefinitely small, d becomes indefinitely large, and $\Delta = 1/3$.

The curves in Fig. 12 give the value of Δ for any value of $J_1/J(=d)$ from 0 to 12.

Hence two concentrated masses connected by a shaft having mass can be reduced to two concentrated masses connected by a shaft having the same elasticity but no mass in such a way as to leave the synchronous frequency and location of the node unchanged, as follows: Determine the location of the node from Fig. 11 and Equation [17]. Find Δ_1 from Fig. 12 for the part of the shaft that is on the same side of the node as J_1, and Δ_2 for the part that is on the same side of the node as J_2. The same frequency and location of the node will then exist for the synchronous vibration of the two masses

$$J_1 + \frac{x_{nd}}{l}\,\Delta_1 J \text{ and } J_2 + \left(1 - \frac{x_{nd}}{l}\right)\Delta_2 J$$

which are connected by a shaft having no mass and the same elasticity as before.

For the installation on the *Oklahoma* Equation [17] gives $\frac{x_{cg}}{l} = 0.660$ and the chart in Fig. 11 gives $\frac{x_{nd}}{l} - \frac{x_{cg}}{l} = 0.004$. Thus $\frac{x_{nd}}{l} = 0.660 + 0.004 = 0.664$. Therefore the inertia of the shaft from J_1 to the node is $0.664\,J$ and from J_2 to the node is $0.336\,J$. This gives $\frac{J_1}{0.664\,J} = 6.92$ and $\frac{J_2}{0.336\,J} = 28.0$ and from the chart in Fig. 12, $\Delta_1 = 0.336$ and $\Delta_2 = 0.334$. Hence $J_1 + \frac{x_{nd}}{l}\,\Delta_1 J = 34{,}300{,}000$ and $J_2 + \left(1 - \frac{x_{nd}}{l}\right)\Delta_2 J = 67{,}700{,}000$.

In this way it is seen that for the one-noded vibration of the installation under consideration, the shaft line-up in vibration is approximately equivalent to the mathematical system of two concentrated masses connected by an elastic shaft having no mass where

$$\begin{aligned} J_1 &= 34{,}300{,}000 \\ J_2 &= 67{,}700{,}000 \\ C &= 53{,}500{,}000 \end{aligned}$$

That this system has the same synchronous frequency as the previous one in which the shaft was considered to have mass may be seen by obtaining its synchronous frequency from Equation [42]; thus

$$n = \frac{1}{2\pi}\sqrt{\frac{386 \times 53.5 \times 102.0}{34.3 \times 67.7}} = \frac{1}{2\pi}\sqrt{907} = 4.79 \text{ v.p.s.}$$

The particular form of torsional vibration that may be expected to cause trouble in an installation of this type is the form that is continually maintained by the stimulating impulses at any constant speed of revolutions. The free or natural vibrations of the system that are caused by suddenly applied or nonperiodic forces rapidly disappear due to the action of the damping forces, if the installation is operated at a constant speed and load for a short time. For this reason the free vibrations may be neglected and consideration need be given only to the forced vibrations of a periodic nature.

The undamped resonance curves of the forced vibration in the installation under consideration may therefore be calculated from Equations [37] to [40]. For example, consider the undamped amplitude of the 4-order vibration at the frequency of $n = 4.65$ v.p.s. The r.p.m. of the shaft corresponding to a 4-order vibration of this frequency is $4.65 \times 60/4$ or 69.75 r.p.m. From expression [30] Appendix No. 1, $\alpha = -0.4178$, $\beta = -0.5677$. Thus $\alpha - \beta = 0.1499$. The value of m for the 4-order at this speed from Fig. 8 is 380,000 with a phase angle from Fig. 9 of 177 deg. Expression [30] then gives $m_0 = 0.4070$. Hence from Equation [39], $a = 0.4070/0.1499 = 2.715$, and from Equation [40], $b = -2.715 \times 0.5677 = -1.542$.

The stress in any uniform section of the shaft due to the deflection Ⓗ degrees of one mass relative to the other is

$$S = \frac{16Md_2}{\pi(d_2^4 - d_1^4)} = \frac{4d_2C\text{Ⓗ}}{45(d_2^4 - d_1^4)} \quad \ldots\ldots\ldots\ldots [21]$$

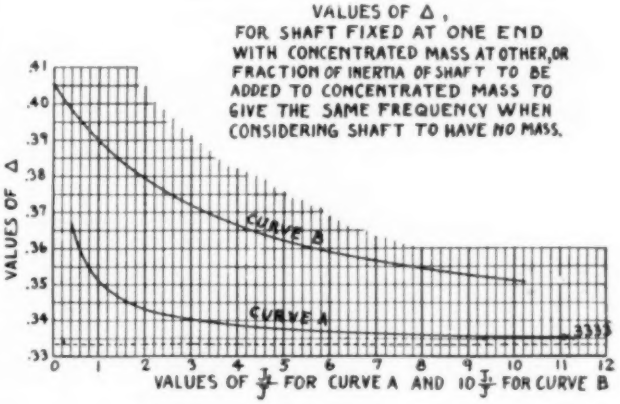


FIG. 12

where d_1 and d_2 are the inner and outer diameters of the section.

The total deflection over the shaft for the example considered is $2.715 + 1.542 = 4.257$, so that the stress in the small section of the shaft (17.5 in. outside and 8.75 in. inside diameter) from Equation [21] is 4050 lb. per sq. in.

The undamped resonance curve for the amplitude of the 4-order vibration at the engine and propeller computed as just described is shown in Fig. 13. The sign of the amplitude a in passing from a little below synchronism to a little above is considered to remain positive and the phase of the vibration to change by 180 deg. The calculated results are also tabulated in Table 12. It is observed that the amplitude of the undamped vibration at synchronism is infinite.

If the principal damping action comes from the propeller the damped resonance curves may be calculated from Equations [31] to [36]. Illustrating this, consider again the 4-order vibration of the installation on the *Oklahoma* at $n = 4.65$ or r.p.m. = 69.75. The value of ρ is given by Equation [12] which depends on Q. The i.hp. for this speed from Fig. 1 is 1808 per engine, so that from Equation [13], $Q = 1{,}390{,}000$. Then Equation [12] gives $\rho = 685{,}000$. From [30], $\rho_1 = 3740$. Therefore Equations [33] to [36] give, $a_N = 2.337$, $b_N = -1.326$, $a_d = 0.4960$, $\omega = 30.6$. These values are illustrated graphically in Fig. 14.

The resonance curve for the 4-order vibration damped only by the action of the propeller computed as just described is shown in Fig. 13 and also in Table 12. The manner in which the curves are plotted gives a picture of the amplitude of vibration at any point of the shaft. If at any speed the shaft is considered as represented by the ordinate in such a way that the engine and propeller are on their respective resonance curves, the amplitude at any intermediate section of the shaft is given at the corresponding point on the ordinate. It may be men-

tioned that although the maximum vibration does not usually occur precisely at synchronism, the difference can hardly be seen in ordinary cases.

The phase angles of the normal phase of the motion and of the resultant motion in degrees of the 4-order cycle behind the maximum value (m) of the impulse is shown in Fig. 13, as is also the phase angle of m behind top center of the F.L.P. cylinder. The phase angle for the vibration behind top center of the F.L.P. cylinder may be obtained by adding the phase angle of the vibration behind m to the phase angle of m behind the F.L.P. cylinder.

If the principal damping action comes both from the propeller

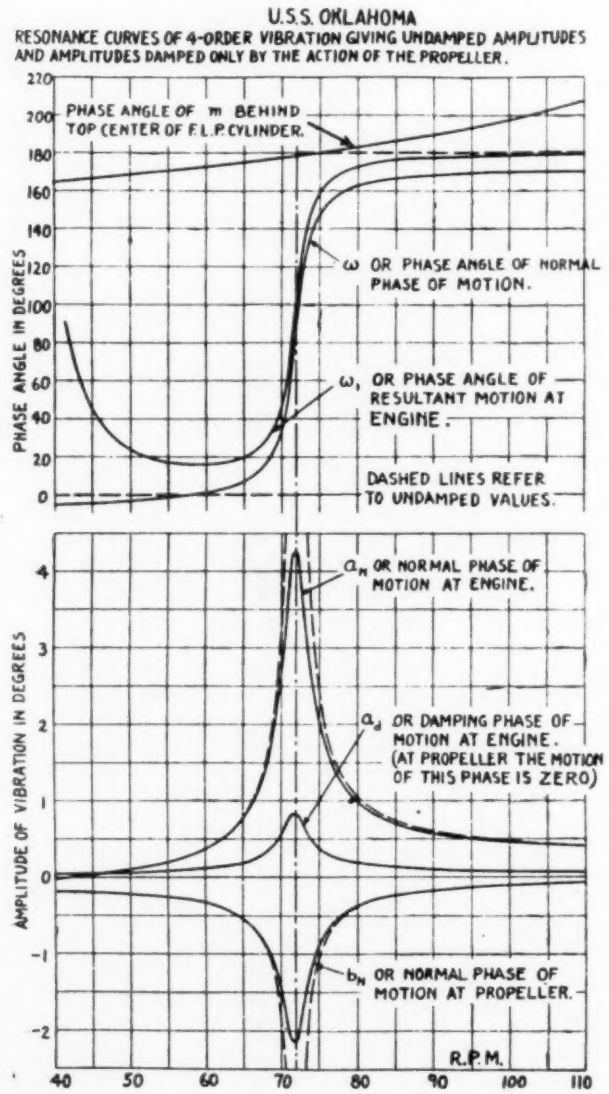


Fig. 13

and from the elastic hysteresis of the shafting, the damped resonance curves may be calculated from Equations [75] to [79]. Illustrating this, consider again the 4-order vibration of the installation on the *Oklahoma* at $n = 4.65$. The value of f_1 for the equivalent shaft whose length is 2020 in. and diameters are 17.75 in. outside and 8.75 in. inside may be computed from [70]. This gives $f_1 = 0.01230$. As before $m_0 = 0.4070$, $\alpha = -0.4178$, $\beta = -0.5677$, $\rho_1 = 0.3740$. Thus Equation [77] for the first approximation can be written

$$0.1657 - 0.00914\textcircled{H}^2 = [0.0566\textcircled{H} + 0.01744\textcircled{H}^{1.3}]^2$$

By substituting trial values in this equation, it will be found that $\textcircled{H} = 3.24$ deg. Thus the other values for the first approximation become $a_N = 2.067$, $b_N = -1.173$, $a_d = 0.4922$, etc.

For a second approximation, Equation [73] may be evaluated to give $f_2 = 0.01234$, and $\delta = 0.00259$. Thus Equation [75] can be written

$$0.1657 - 0.00989\textcircled{H}^2 = [0.05675\textcircled{H} + 0.01750\textcircled{H}^{1.3}]^2$$

From this equation it is found that $\textcircled{H} = 3.166$ deg., so that the other values of the second approximation become $a_N = 2.017$, $b_N = -1.149$, $a_d = 0.483$, etc.

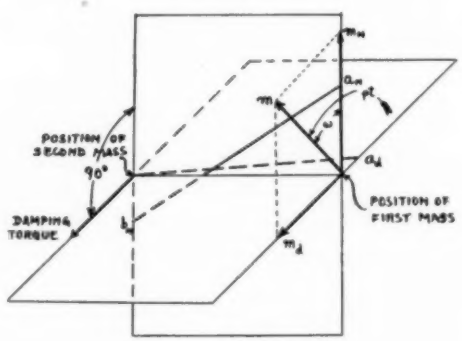


Fig. 14

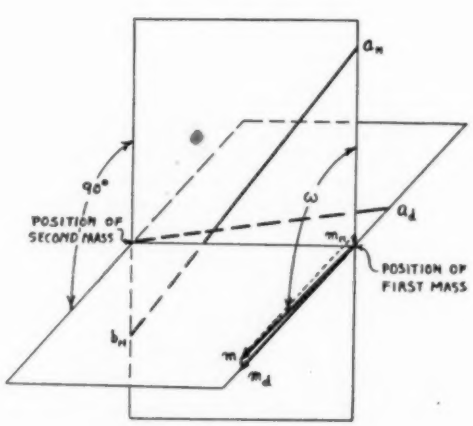


Fig. 15

A third approximation gives the same results as the second to the significant figures given. This shows that the results given by the second approximation are the ones sought. It is observed that the results of the first approximation are correct within about two per cent.

A graphical representation of the values at synchronism computed in a manner similar to that just given is illustrated in Fig. 15.

The resonance curve for the 4-order vibration damped both by the propeller and elastic hysteresis computed as just described is shown in Fig. 16, and the calculated results are given in Table 13. The phase angles of the vibration behind m, and m behind top center of the F.L.P. cylinder, are also shown in Fig. 16.

Several important characteristics of damped vibrations are

TABLE 12 CALCULATED AMPLITUDES OF 4-ORDER VIBRATION FOR INSTALLATION ON U. S. S. "OKLAHOMA"

		Undamped			Damped by propeller alone				
n	R.p.m.	a	b	Phase of a and b	a_N	a_d	b_N	ω	ω
2.780	41.70	0	−0.1875	0	0	0.0310	−0.1869	−4.8	85.2
3.20	48.0	0.0680	−0.209	0	0.0678	0.0434	−0.209	−3.5	29.1
3.60	54.0	0.1693	−0.250	0	0.1691	0.0614	−0.250	−1.9	18.0
3.905	58.58	0.302	−0.311	0	0.302	0.0856	−0.311	0	15.8
4.10	61.5	0.446	−0.380	0	0.446	0.1146	−0.379	2.0	16.4
4.25	63.75	0.628	−0.470	0	0.625	0.1511	−0.467	4.6	18.2
4.40	66.0	0.944	−0.627	0	0.932	0.212	−0.619	8.8	21.6
4.55	68.25	1.652	−0.984	0	1.570	0.342	−0.945	18.0	30.3
4.65	69.75	2.715	−1.542	0	2.34	0.496	−1.326	30.6	42.6
4.69	70.35	4.16	−2.25	0	3.07	0.630	−1.664	42.4	54.0
4.72	70.8	5.94	−3.16	0	3.56	0.723	−1.891	53.2	64.7
4.75	71.25	10.18	−5.20	0	4.01	0.807	−2.09	66.6	78.0
4.794*	71.90	∞	−∞	0	4.25	0.842	−2.15	90	101.2
4.83	72.45	12.55	−6.22	180	3.95	0.774	−1.954	108.3	119.4
4.86	72.90	6.98	−3.39	180	3.52	0.686	−1.712	120.3	131.3
4.90	73.5	4.43	−2.10	180	2.98	0.570	−1.415	132.0	142.9
4.95	74.25	3.06	−1.411	180	2.41	0.461	−1.112	141.7	152.5
5.05	75.75	1.962	−0.850	180	1.732	0.325	−0.752	152.0	162.8
5.20	78.00	1.304	−0.522	180	1.221	0.221	−0.488	159.6	169.8
5.50	82.5	0.844	−0.290	180	0.816	0.1408	−0.280	165.2	175.0
6.00	90.0	0.591	−0.1616	180	0.579	0.0950	−0.1583	168.4	177.7
7.00	105.0	0.436	−0.0817	180	0.430	0.0686	−0.0805	170.1	179.2
8.00	120.0	0.304	−0.0417	180	0.300	0.0471	−0.0412	170.3	179.2

* Synchronism.

illustrated by the results of the calculations shown in Figs. 13 to 16, and Tables 12 and 13. These may be summarized as follows:

1 The damping phase of the motion is small in comparison with the motion of the normal phase.

2 At frequencies a little removed from synchronism, the motion of the normal phase for damped vibrations is only slightly smaller than the undamped vibration.

3 The maximum value of the damped resonance curves usually occurs very near to synchronism, and its value is only slightly greater than the amplitude at synchronism.

These characteristics show that the range of severe vibration can be safely taken from the undamped resonance curve and the severity of the vibration can be judged by its amplitude and stress at synchronism.

The calculated amplitudes of the first six orders of vibration over the speed range of 40 to 125 r.p.m. are given in Fig. 17. It is evident from this figure that in addition to the large 4-order vibration at 72 r.p.m. there is also a large 3-order vibration at 96 r.p.m. Other vibrations in the speed range are relatively small.

In Fig. 18 is given the vibration stress in the smallest uniform section of the shafting (17.5 in. outside and 8.75 in. inside diameter) occurring in the thrust shaft. The stress due to the mean power torque is also shown in Fig. 18. By combining the vibration stresses at any speed in their proper phase relation, and adding the resultant to the stress due to the mean power torque, the maximum combined stress at any speed may be obtained. This value is represented in Fig. 18 by the dotted line.

Similar stress curves can be drawn for any section of the shafting. The stresses in the crankshaft are augmented by the bending stresses due to the reciprocating weights. In this paper, however, attention is confined to the stress in the small section of the shaft line-up remote from the crankshaft.

II—U. S. S. "Texas," Starboard Propelling Unit

Since the engines for the *Texas* are similar to those for the *Oklahoma*, the impulses due to the pressure forces may be obtained as described for the *Oklahoma* from Figs. 5, 6, and 7. Thus the combined impulses acting at the center of the engine may be computed.

The propeller damping coefficient for the *Texas* was found to be the same as that for the *Oklahoma* given by Equation [12].

Thus the principal vibrations for the *Texas* over the speed range from 40 to 125 r.p.m. may be calculated from the data given. The results of such calculations are given in Fig. 19, which gives the calculated vibration stresses in the smallest uniform section of the shafting (17.75 in. outside and 11.5 in. inside diameter) occurring in the thrust shaft. The stresses due to the mean power torque and the maximum combined stresses at any speed are also shown in this figure.

III—U. S. S. "Minnesota" and U. S. S. "Kansas," Port Propelling Installation

The engines for these ships are also similar to those for the *Oklahoma*. Hence the impulses due to the pressure forces may be obtained as described for the *Oklahoma* from Figs. 5, 6, and 7, and the combined impulses acting at the center of the engine may be computed.

The propeller damping coefficient for the original three-bladed propeller was found to be the same as that for the *Oklahoma* given by Equation [12].

Therefore the principal vibrations for the *Minnesota* and *Kansas* over the speed range from 40 to 125 r.p.m. may be calculated where the original three-bladed propeller is used.

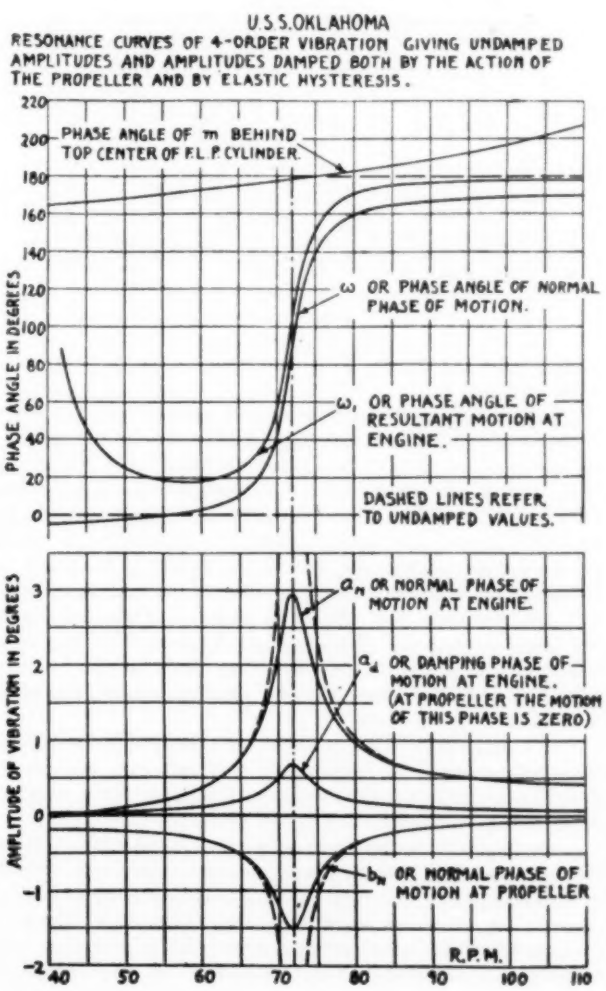


Fig. 16

The results of such calculations are given in Fig. 20, which gives the calculated vibration stresses in the smallest uniform section of the shafting (15.5 in. outside and 9.5 in. inside diameter) occurring in the thrust shaft. The stresses due to the mean

TABLE 13 CALCULATED AMPLITUDES OF 4-ORDER VIBRATION FOR INSTALLATION ON U. S. S. "OKLAHOMA" DAMPED BY ACTION OF PROPELLER AND HYSTERESIS

		First approximation					Second approximation				
n	r.p.m.	aN	ad	bN	ω	ω_1	aN	ad	bN	ω	ω_1
2.780	41.70	0	0.0335	−0.1869	−4.6	85.4	−0.0001	0.0335	−0.1866	−4.6	85.6
3.20	48.0	0.0678	0.0457	−0.209	−3.2	30.8	0.0675	0.0457	−0.208	−3.2	30.9
3.60	54.0	0.1691	0.0654	−0.250	−1.2	20.0	0.1684	0.0653	−0.249	−1.2	20.0
3.905	58.58	0.302	0.0921	−0.311	1.2	17.1	0.300	0.0918	−0.310	1.2	18.2
4.10	61.5	0.445	0.1239	−0.379	3.8	19.3	0.442	0.1231	−0.376	3.8	19.4
4.25	63.75	0.624	0.1645	−0.466	7.2	22.0	0.617	0.1633	−0.463	7.1	21.8
4.40	66.0	0.920	0.231	−0.611	13.0	27.1	0.907	0.229	−0.605	12.8	27.0
4.55	68.25	1.494	0.360	−0.890	25.1	38.7	1.464	0.354	−0.874	24.6	38.2
4.65	69.75	2.07	0.492	−1.173	40.4	53.8	2.02	0.483	−1.149	39.4	52.9
4.69	70.35	2.50	0.583	−1.353	53.0	66.2	2.44	0.572	−1.327	51.3	64.5
4.72	70.8	2.73	0.633	−1.448	62.7	75.8	2.67	0.620	−1.422	60.6	73.7
4.75	71.25	2.89	0.666	−1.507	73.5	86.5	2.87	0.663	−1.499	71.0	84.0
4.794*	71.90	2.96	0.672	−1.500	90	102.8	2.94	0.672	−1.498	87.4	100.3
4.83	72.45	2.86	0.644	−1.419	103.2	115.9	2.86	0.648	−1.428	100.7	113.4
4.86	72.90	2.69	0.595	−1.308	112.7	125.2	2.70	0.605	−1.321	110.6	123.2
4.90	73.5	2.44	0.532	−1.156	123.1	135.4	2.46	0.546	−1.190	121.6	134.1
4.95	74.25	2.10	0.454	−0.969	133.1	145.3	2.12	0.464	−0.990	131.7	144.0
5.05	75.75	1.614	0.340	−0.702	145.3	157.2	1.634	0.345	−0.712	144.7	156.6
5.20	78.00	1.181	0.237	−0.473	155.0	166.4	1.190	0.240	−0.478	154.7	166.1
5.50	82.5	0.805	0.1526	−0.276	162.5	173.2	0.809	0.1537	−0.278	163.4	174.2
6.00	90.0	0.575	0.1026	−0.1572	166.7	176.8	0.576	0.1033	−0.1582	166.6	176.8
7.00	105.0	0.428	0.0734	−0.0802	169.1	178.8	0.428	0.0735	−0.0803	168.9	178.6
8.00	120.0	0.299	0.0500	−0.0411	169.9	179.4	0.299	0.0500	−0.0411	169.9	179.4

* Synchronism.

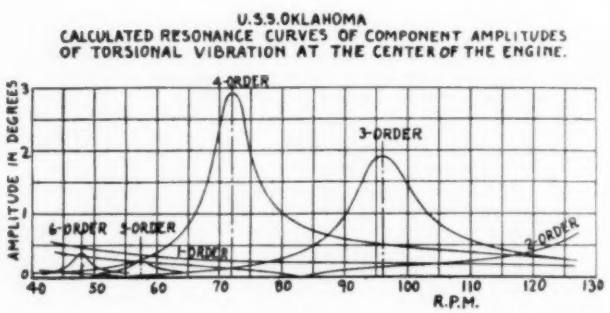


FIG. 17

power torque and the maximum combined stresses are also shown in this figure.

The agreement of the calculated and recorded magnitudes of vibration for the above installations will now be discussed.

Torsion-meter tests of the actual stresses in the shafting of the U. S. S. *Minnesota* and U. S. S. *Kansas* were made some time prior to 1917. The results of these tests were described by Ernest N. Janson and Lieut.-Comm. J. O. Richardson, U. S. N. ("Investigation of Shafting Failures and Engine Vibration on Vessels of the Louisiana Class," *Journal of the American Society of Naval Engineers*, Vol. 29, 1917, page 1). These tests were performed on both the *Minnesota* and *Kansas*. The *Minnesota* was tested with four-bladed propellers fitted in place of the former three-bladed ones, but the old three-bladed propellers were used on the *Kansas*.

The torsion meter used in these tests was made by Messrs. Denny-Edgecombe, of Dumbarton, Scotland, and was so designed as to obtain a continuous record on paper of the torsional stress during each revolution of the shaft.

A maximum stress was found to occur at 83 r.p.m. for the *Kansas* equal to 10,952 lb. per sq. in., and at 80 r.p.m. for the *Minnesota* equal to 11,380 lb. per sq. in. The calculated results, given in Fig. 20 above, show a maximum to occur at 85 r.p.m. equal to 12,400 lb. per sq. in. for the installations equipped with three-bladed propellers. The comparison of the calculated stresses and the observed values given by Janson and Richardson are shown graphically in Fig. 20. In regard to the observed difference in location of the maximum of the *Minnesota* and *Kansas*, it is believed that the WR^2 of the four-bladed propellers used on the *Minnesota* was greater than that of the three-bladed ones used on the *Kansas*, causing the synchronous frequency of the vibration on the *Minnesota* to be lower than that on the *Kansas*. The calculated and recorded stresses, however, are seen to be in fair agreement.

The instruments used in testing the installations on the *Texas* and *Oklahoma* were torsiographs. These instruments were designed by Dr. J. Geiger, and record on paper the amplitude of vibration at any section to which they are attached. Two torsiographs were used in testing each installation, one belted to the

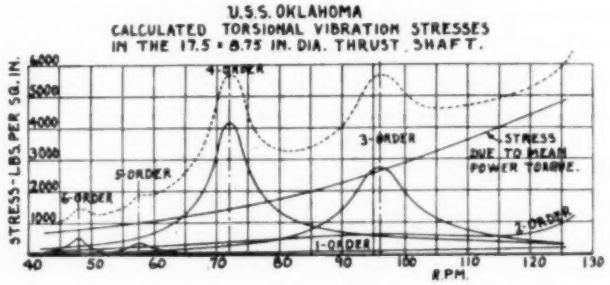


FIG. 18

shaft just aft of the engine and the other just forward of the after inboard coupling.

In the case of the *Texas*, records of the vibration in the shaft were obtained every five revolutions per minute apart from 40 r.p.m. to 105 r.p.m.

A large 4-order vibration was observed between the speeds of 70 r.p.m. and 85 r.p.m. Its maximum value of about 2.4 deg. at the forward instrument occurred at 77 r.p.m. A smaller 3-order vibration was observed at higher speeds having a maximum value at the forward instrument of about 1.3 deg. at 102 r.p.m. Several small vibrations of scarcely noticeable amplitude were observed at other speeds. The 5-order vibration was seen at 61 r.p.m. combined with a forced vibration of the 4-order of about the same amplitude. The 6-order vibration was seen at 51 r.p.m. and the 8-order at 39 r.p.m. The amplitudes obtained by the torsiographs and those obtained by calculations at the positions of the forward and after instruments are listed in Table 14. The calculated and recorded values are seen to be in fair agreement.

In the case of the *Oklahoma*, torsiographic records of the vibration in the shaft were obtained every five revolutions per minute apart from 40 r.p.m. to 80 r.p.m.

A large synchronous 4-order vibration was observed at 72 r.p.m. having a maximum value at the after instrument of about 1.25 deg. The calculated value of the 4-order synchronous vibration at this instrument was 1.36 deg. occurring at 72 r.p.m. These values are seen to be in fair agreement with the recorded values.

One of the main considerations in judging the severity of vibrations is the magnitude of the stress caused by the vibration. Furthermore, the seriousness of the vibration stresses is greatly magnified by stress concentrations at the various discontinuities in the shafting.

The results of fatigue tests gathered from various sources[4] show that the average endurance limit for steel in reversed torsion is about 25 per cent of the ultimate tensile strength, and that such discontinuities as square shoulders and V-notches reduce this endurance limit 50 to 60 per cent. The ordinary discontinuities in the shafting of a reciprocating engine installation probably have nearly the same effect in reducing the endurance limit of the shafting as square shoulders and V-notches. Thus the maximum allowable torsional vibration stress for an engine shafting is about 10 per cent of the ultimate tensile strength.

The ultimate tensile strength of the material used in the shafting of the installations described above was about 80,000 lb. per sq. in., which indicated a maximum allowable vibration stress of 8000 lb. per sq. in. Referring to Figs. 11, 12, and 13, it is seen that the 4-order vibration stress for the *Minnesota* and *Kansas* is above the allowable stress value, and failure of the

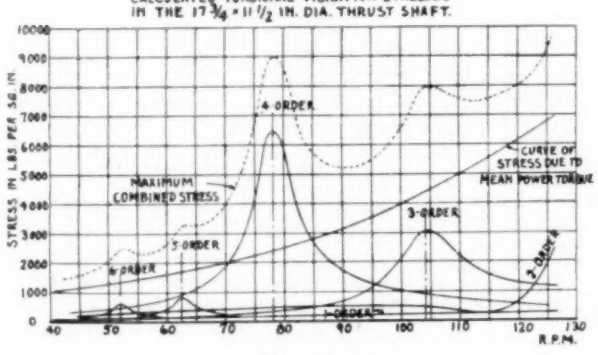


FIG. 19

shafting at some section of stress concentration could be expected eventually. This is what actually did occur several years ago on these ships.

The vibration stresses for the case of the *Oklahoma* and *Texas* are seen to be below the maximum allowable stress. In regard to these ships it may be stated that no failures have occurred in the ordinary discontinuities of the shafting, such as flanged couplings, changes in shaft diameter, oil holes, etc. Failures have occurred in these ships, however, in the outboard couplings. These couplings are the usual sleeve type of design, consisting of a sleeve into which taper portions of the line shaft and propeller shaft are inserted and drawn up by tapered wedges driven through large radial holes cut in the sleeve and shafts. The failures of the sleeves of these couplings show that they are a point of local weakness, which is probably due to high stress concentrations occurring around the large holes in the sleeves.

[4] See H. F. Moore and J. B. Kommers, "Fatigue of Metals," 1927.

In redesigning the above installations to avoid vibration troubles, the following remedies suggest themselves.

1 Redesign the outboard coupling to give it greater fatigue strength.

2 Install a damping device to absorb the energy of vibration before it reaches an excessive value.

3 Increase the mean diameter of the shafting to such a size that it can safely withstand the stress of the 4-order vibration.

4 Increase the mean diameter of the shafting to shift the 4-order vibration above the maximum operating speed.

5 Change the inertia of the engine and propeller to decrease the vibration stresses.

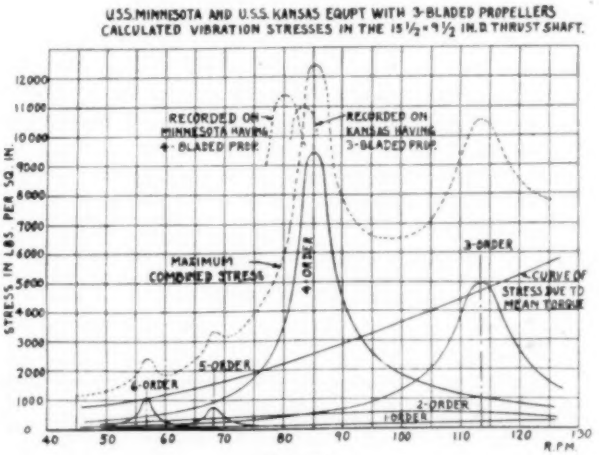


FIG. 20

The desirability of redesigning the outboard coupling to give it greater fatigue strength has been shown in the case of the *Oklahoma* and *Texas* by several failures that occurred to this part of the shaft line-up. This could easily be done by making, for example, the sleeves of the coupling out of some high fatigue strength material. It is probable that if this were done less trouble from torsional vibration would be experienced on these ships. Such a solution, however, would not correct the difficulty for the *Minnesota* and *Kansas*.

A well-known damping device is the Lanchester damper. This consists of a slipping flywheel whose only contact with the shaft is through friction surfaces. The equations whose solutions give the amplitude of vibration when the engine mass is acted on by a friction damper are given in Appendix No. 2.

To illustrate the effect of a friction damper, calculations have been made for the *Oklahoma* that are summarized in Fig. 21. Several different WR^2 values of the damper flywheel were considered for any friction torque that might act across the friction surfaces. The amplitude plotted in Fig. 21 gives the resulting amplitude of the normal phase at synchronism for the center of the engine. It is evident from this figure that the total friction torque for a friction damper is an important detail if the damper is to function in an efficient manner.

Fig. 21 shows that a flywheel having, for example, a WR^2 value of 6,000,000 lb-in.^{2} or 17.5 per cent of the engine WR^2 can be used to reduce the amplitude at the engine to less than one-half of its value without the damper if the friction surfaces between

TABLE 14 AGREEMENT OF CALCULATED AND RECORDED VIBRATION OBTAINED ON THE U. S. S. "TEXAS"

	Recorded values				Calculated values			
	4-Order		3-Order		4-Order		3-Order	
	R.p.m.	Amp.	R.p.m.	Amp.	R.p.m.	Amp.	R.p.m.	Amp.
For'd inst.	77	2.40°	102	1.46°	78.1	2.84°	104.2	1.35°
After inst.	77	1.10°	102	0.65°	78.1	1.30°	104.2	0.62°

the flywheel and engine are designed to hold about 180,000 in-lb. torque.

Calculations have also been made to show the effect of increasing the mean diameter of the shafting and changing the WR^2 values of the engine and propeller for the case of the *Minnesota* and *Kansas*. The results obtained will also apply qualitatively to the *Oklahoma* and *Texas*.

Fig. 22 shows the calculated results of the effect of changing the mean diameter of the shafting. It is noted that increasing the mean diameter of the shafting does not indicate necessarily a decrease in the vibration stress. The maximum vibration stress due to the 4-order occurs at 97.5 r.p.m. for 17.6 in. mean outside diameter. It is also observed that this stress is still undesirably large for a diameter of 20 in., which raises the synchronous speed for the 4-order to the maximum operating speed. This shows that the mean diameter of the shafting cannot be increased to such a value in the speed range of the installation that it can safely withstand the stress of the 4-order synchronous vibration.

It may be seen from Fig. 22 that if the mean outside diameter of the shafting were increased to 20.8 in., the 4-order synchronous vibration would be raised to 135 r.p.m. In this way the 4-order vibration could be removed from the operating range and the stress in the shafting could be kept to a moderate value.

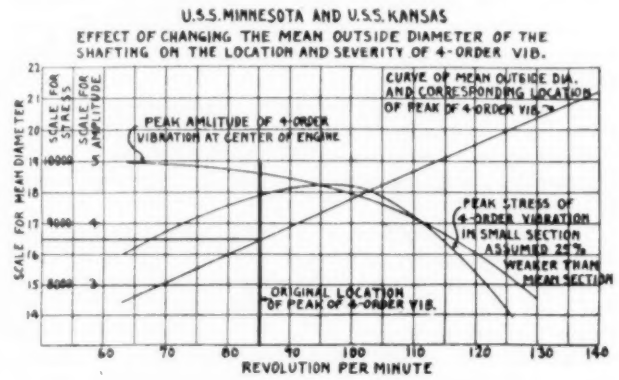


FIG. 22

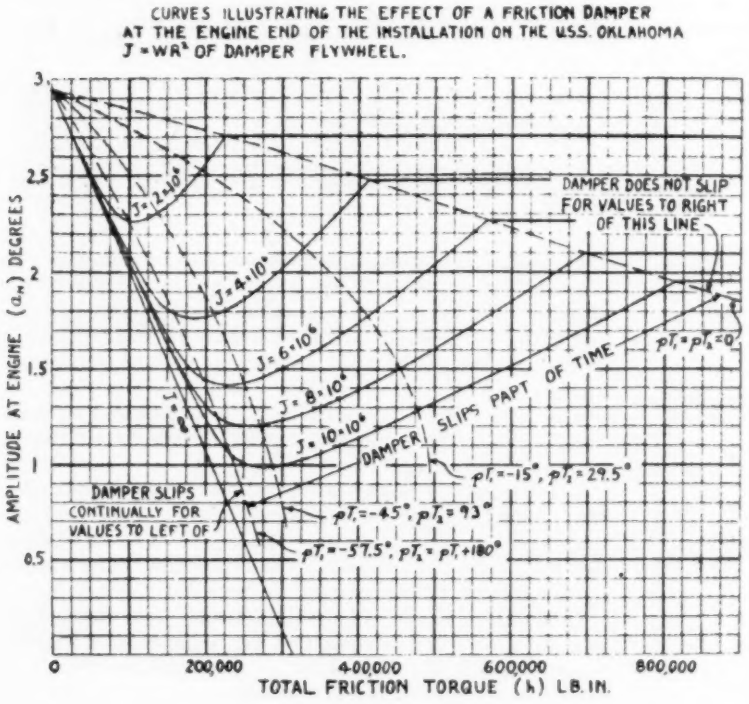


FIG. 21

Increasing the inertia of the engine and decreasing the inertia of the propeller has been found to have a large effect on decreasing the amplitude and stresses of vibration. This is due to the fact that such changes tend to decrease the amplitude at the engine and increase the amplitude at the propeller causing greater damping to occur at the propeller. The effect of changing these inertias in this manner is shown in Fig. 23. It is seen in this figure that if a flywheel is placed near the engine such as to increase the engine inertia 50 per cent, the vibration stress of the 4-order is decreased 39 per cent, while if the engine inertia is increased 100 per cent, the stress is decreased 58 per cent. Similarly if the propeller inertia is reduced 60 per cent, such as could be done by casting the propeller out of duralumin, the vibration stress is reduced 67 per cent. If the inertia of the engine is increased 100 per cent and at the same time the propeller inertia is decreased 60 per cent, the vibration stress of the 4-order is decreased 83 per cent corresponding to a stress of only 1600 lb. per sq. in. It is thus seen that such a solution offers rather wide possibilities.

Appendix No. 1

TORSIONAL VIBRATION OF TWO CONCENTRATED MASSES CONNECTED BY AN ELASTIC SHAFT HAVING NO MASS, WITH VARYING IMPRESSED TORQUES ACTING ON THE FIRST MASS AND A DAMPING TORQUE PROPORTIONAL TO THE VELOCITY ACTING ON THE SECOND

LET quantities with subscripts 1 and 2 not otherwise defined refer to the first and second masses, respectively. Consider that the first mass is acted upon by the impressed torques and the second by the damping torques.

If θ is the angular deflection in radians, the twisting moment acting on the shaft is $C(\theta_2 - \theta_1)$. If the weight polar moment of inertia of either mass is J, and its angular acceleration is $\frac{d^2\theta}{dt^2}$, the inertia torque of the mass is $\frac{J}{g}\frac{d^2\theta}{dt^2}$. Let the impressed torques acting on the first mass be given by M_t. Let the damping torque acting on the second mass be $\rho\ d\theta_2/dt$, where ρ is the constant of proportionality.

TABLE 15

Installation	Order of vibr.	R.p.m.	$-10^6\rho\beta/C$	$g\,10^6/J_2\rho$	$\rho 10^{-6}$	a_d/a_N	$-M_d/M_N$	$\sqrt{a_n^2+a_d^2}/a_N$	$\sqrt{M_n^2+M_d^2}/M_N$
Minnesota	3	113.6	0.242	0.1788	0.922	0.223	0.1650	1.025	1.014
	4	85.2	0.242	0.1788	0.664	0.1608	0.1188	1.013	1.007
	5	68.2	0.242	0.1788	0.487	0.1180	0.0872	1.007	1.004
	6	56.8	0.242	0.1788	0.409	0.0991	0.0728	1.005	1.003
Oklahoma	3	96.0	0.283	0.1885	0.916	0.259	0.1728	1.033	1.015
	4	72.0	0.283	0.1885	0.696	0.1970	0.1312	1.019	1.009
	5	57.5	0.283	0.1885	0.598	0.1695	0.1128	1.014	1.006
	6	48.0	0.283	0.1885	0.548	0.1550	0.1034	1.012	1.005
Texas	3	104.0	0.231	0.1575	1.318	0.305	0.208	1.046	1.022
	4	78.0	0.231	0.1575	0.968	0.224	0.1527	1.025	1.012
	5	62.5	0.231	0.1575	0.873	0.201	0.1378	1.020	1.009
	6	52.0	0.231	0.1575	0.817	0.1887	0.1288	1.018	1.008

Then the sum of the torques acting on one side of any section is equal to the torque in the shaft at that section. Thus it follows that just beyond the first mass

$$\frac{J_1}{g}\frac{d^2\theta_1}{dt^2} - M_i = C(\theta_2 - \theta_1) \quad \ldots\ldots [22]$$

and just beyond the second mass

$$\frac{J_2}{g}\frac{d^2\theta_2}{dt^2} + C(\theta_2 - \theta_1) + \rho\frac{d\theta_2}{dt} = 0 \quad \ldots\ldots [23]$$

Any form of torsional vibration of the two masses for the case considered will have torsional displacements that satisfy Equations [22] and [23]. The vibration may be periodic or changing depending on the impulses and the length of time of their application. If the impressed torques M_i are periodic and are applied to the first mass for a long time, the vibration will become periodic in nature. In the following solution only periodic or steady state vibrations will be considered.

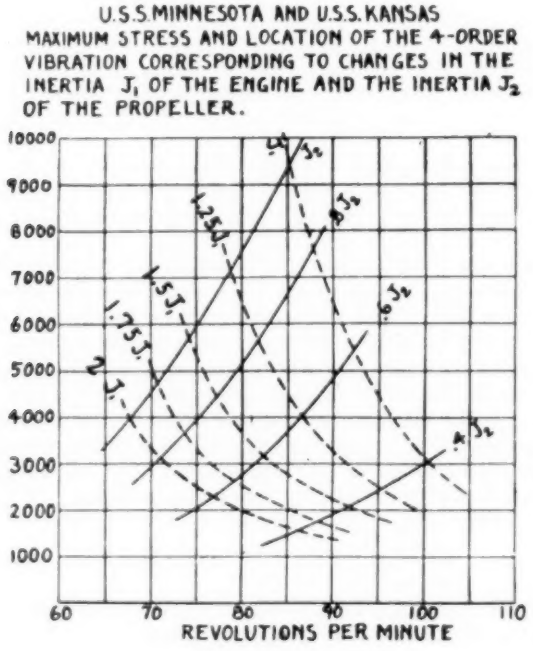


FIG. 23

Expand M_i by Fourier's theorem into a trigonometric series of the variable t. Consider that the vibration stimulated by each harmonic component of M_i is obtained as a solution of Equations [22] and [23] where M_i is replaced by the particular component of M_i considered. The algebraic sum of all such elements of vibration will satisfy Equations [22] and [23] and will be the resultant vibration stimulated by M_i.

Let the harmonic element of M_i to be considered be given by $m \cos pt$. Thus the reference point of time is taken as the instant that the component of M_i under consideration has its maximum value.

Replace the impressed torque with two components having a 90-deg. phase difference. Thus let

$$\left.\begin{aligned} m\cos pt &= m_N\cos(pt-\omega) - m_d\sin(pt-\omega) \\ \text{where } \omega &= \tan^{-1}\frac{m_d}{m_N},\ m^2 = m_N^2 + m_d^2 \end{aligned}\right\} .[24]$$

and $p = 2\pi n$, in which n is the frequency of the periodic force.

Any harmonic motion θ_N in the phase of the first component, i.e., in phase or 180 deg. out of phase with $m_N \cos(pt - \omega)$, is called motion of the normal phase. Similarly any harmonic motion θ_d in the phase of the second component or $-m_d \sin(pt - \omega)$ is called motion of the damping phase.

Since the damping torque at the second mass is given by $\rho\frac{d\theta_2}{dt}$, it is seen that any harmonic motion at the second mass causes a damping torque to occur 90 deg. out of phase with the motion. Thus any motion at the second mass in either the normal or damping phase causes a damping torque to occur in the other phase of the motion but in no other phase. It is evident, therefore, that for a steady-state vibration, the motion must be harmonic in nature and at each mass must be the sum of θ_N and θ_d.

As yet, however, the two components of $m \cos pt$ are only restricted by the condition that they have 90 deg. difference in phase. Let another condition to determine these components be that the amplitude of the damping phase of the motion at the second mass is zero. This makes the damping torque in the normal phase of the motion at the second mass equal to zero.

The forces of the normal phase, therefore, satisfy the following equations:

$$\frac{J_1}{g}\frac{d^2\theta_{N1}}{dt^2} - m_N\cos(pt-\omega) = C(\theta_{N2} - \theta_{N1}) \quad \ldots\ldots [25]$$

$$\frac{J_2}{g}\frac{d^2\theta_{N2}}{dt^2} + C(\theta_{N2} - \theta_{N1}) = 0 \quad \ldots\ldots [26]$$

And the forces of the damping phase satisfy the following equations:

$$\frac{J_1}{g}\frac{d^2\theta_{d1}}{dt^2} + m_d\sin(pt-\omega) = -C\theta_{d1} \quad \ldots\ldots [27]$$

$$-C\theta_{d1} + \rho\frac{d\theta_{N2}}{dt} = 0 \quad \ldots\ldots [28]$$

Adding the corresponding sides of Equations [25] and [27], [26], and [28], and putting $\theta_1 = \theta_{N1} + \theta_{d1}$, $\theta_2 = \theta_{N2}$, $M_i = m\cos pt$, it will be found that these equations reduce to Equations [22] and [23]. Hence a solution of Equations [25] to [28] will also be a solution of [22] and [23].

Writing

$$\left.\begin{aligned} \theta^\circ_{N1} &= a_N\cos(pt-\omega) \\ \theta^\circ_{N2} &= b_N\cos(pt-\omega) \\ \theta^\circ_{d1} &= a_d\sin(pt-\omega) \\ \theta^\circ_{d2} &= 0 \end{aligned}\right\} \quad \ldots\ldots [29]$$

where θ° is θ in degrees, the solution follows by evaluating a_N, b_N, a_d, m_N, m_d, and ω from Equations [24] to [28].

It is convenient in the solution to put

$$\left.\begin{aligned} &\alpha = 1 - \frac{J_1p^2}{gC},\ \beta = \left(1 - \frac{J_2p^2}{gC}\right)^{-1},\ \rho_1 = \frac{\rho p}{C} \\ &m_0 = \frac{180m}{\pi C},\ m_{N0} = \frac{180m_N}{\pi C},\ m_{d0} = \frac{180m_d}{\pi C} \end{aligned}\right\} \quad \ldots\ldots [30]$$

Hence putting [29] and [30] into Equations [25] to [28], they may be reduced to

$$\alpha a_N - b_N - m_{N0} = 0$$

$$\beta a_N - b_N = 0$$

$$\alpha\, a_d + m_{d0} = 0$$

$$a_d + \rho_1 b_N = 0$$

Therefore, solving these equations with [24] as simultaneous equations, the solution will be stated as follows:

$$\theta^\circ_1 = a_N \cos(pt - \omega) + a_d \sin(pt - \omega) \quad [31]$$

$$\theta^\circ_2 = b_N \cos(pt - \omega) \quad [32]$$

where

$$a_N = \frac{m_0}{\sqrt{[(\alpha - \beta)^2 + (\rho_1 \alpha\beta)^2]}} \quad [33]$$

$$b_N = \beta\, a_N \quad [34]$$

$$a_d = -\rho_1 b_N \quad [35]$$

$$\omega = \tan^{-1} \frac{\rho_1\, \alpha\beta}{\alpha - \beta} \quad [36]$$

The amplitude of the normal and damping phase of the motion at any point of the shaft and a statement of the resultant amplitude at any section of the shaft may be constructed from the above solution. Such equations are given in Appendix No. 2, Equations [65] to [68].

If there were no damping, i.e., if $\rho = 0$, the damping phase of the motion would be zero. In this case, omitting the subscript N,

$$\theta^\circ_1 = a \cos pt \quad [37]$$

$$\theta^\circ_2 = b \cos pt \quad [38]$$

where

$$a = \frac{m_0}{\alpha - \beta} \quad [39]$$

$$b = \beta a \quad [40]$$

It is seen from Equation [39] that if there is no damping the maximum values of θ_1 and θ_2 are infinite for values of n that make

$$\alpha - \beta \text{ or } 1 - \frac{J_1 p^2}{gC} - \left(1 - \frac{J_2 p^2}{gC}\right)^{-1} = 0 \quad [41]$$

unless m is zero. This means that vibrations of any amplitude, satisfying Equation [40], will exist indefinitely at this frequency with no impressed torque. The vibration at such a frequency is called "the normal mode of free vibration," and the frequency is called the "natural frequency," "synchronous frequency," or the "frequency of free vibration."

Solving Equation [41] for the value of p and putting $p = 2\pi n$, the synchronous frequency will be found to be

$$n = \frac{1}{2\pi} \sqrt{\frac{gC(J_1 + J_2)}{J_1 J_2}} \quad [42]$$

Substituting this value into Equation [40], it is seen that at synchronism

$$\frac{a}{b} = -\frac{J_2}{J_1} = \frac{1}{\alpha} = \frac{1}{\beta} \quad [43]$$

Hence the amplitude of undamped vibration at any point along the shaft at synchronism is given by

$$y = a \left\{1 - \frac{x}{l}\left(\frac{J_1 + J_2}{J_2}\right)\right\} \quad [44]$$

If x_{nd} is the distance of the node, or position of zero amplitude of vibration, from the first mass at synchronism for the case of no damping, Equation [44] gives

$$\frac{x_{nd}}{l} = \frac{J_2}{J_1 + J_2} \quad [45]$$

It is thus seen that the node for the undamped synchronous vibration of two concentrated masses connected by an elastic shaft having no mass is at the center of gravity of the inertia values considered as weights.

The characteristics shown by Equations [43], [44], and [45] to belong to the undamped synchronous vibration, are easily shown also to belong to the normal phase of the damped vibration at synchronism.

The sign of tan ω for the damped vibration depends on the sign of

$$\frac{\alpha - \beta}{\alpha\beta} \text{ or } 1 - \frac{J_2 p^2}{gC} - \left(1 - \frac{J_1 p^2}{gC}\right)^{-1} \quad [46]$$

Let $p^2 = p_s^2 - \delta$ where $p_s/2\pi$ is the synchronous frequency given by Equation [42] and δ is a small quantity. If δ is positive, $p/2\pi$ is less than the synchronous frequency, and if δ is negative, $p/2\pi$ is greater than the synchronous frequency. Thus

$$p^2 = \frac{gC(J_1 + J_2)}{J_1 J_2} - \delta$$

Substituting this value of p into [46], it can be written

$$\frac{\delta \dfrac{p^2 J_2}{gC}}{\dfrac{gC}{J_2} - \delta}$$

From this it follows that if δ is positive and approaches zero as a limit, tan ω is positive, and if δ is negative and approaches zero as a limit, tan ω is negative. Hence tan ω passes from $+\infty$ to $-\infty$ as n increases through the synchronous value. It will be assumed that near synchronism m_d is positive, therefore ω increases through 90 deg. at synchronism as n increases.

In a similar manner it can be shown that for the case of undamped vibration a is positive a little below synchronism and negative a little above synchronism.

Equations [34] and [35] can be combined to give

$$\frac{M_d}{M_N} = \frac{a_d}{b_N - a_N} = -\frac{g\rho}{J_2 p}$$

where M_d and M_N are the twisting moments in the shaft due to twisting the two masses through the angles a_d and $b_N - a_N$, respectively.

The significance of the last equation and [35] is best seen by numerical examples. In Table 15 these equations are evaluated at the synchronous frequency of several orders of vibration for the *Minnesota*, *Oklahoma*, and *Texas*, assuming that the only damping action comes from the propeller. As may be seen by a reference to Table 15, a_d and M_d are only a small fraction of a_N and M_N for the vibrations given, and the resultants $\sqrt{a_N^2 + a_d^2}$ and $\sqrt{M_N^2 + M_d^2}$ are only a small per cent greater than a_N and M_N. Hence for these vibrations, the damping phase of the motion can be neglected, and without much error the normal phase of the motion can be taken as the resulting vibration.

Appendix No. 2

GENERAL CASE OF DAMPED VIBRATIONS

THE solution for the steady-state vibrations stimulated by periodic torques acting at the first mass and damped by forces acting at either mass and at any point along the shaft will now be considered.

There are two types of damping forces that may act on a vibrating system. First, the damping forces may have their origin in some disturbance outside the system. In this case a damping force acting on the system coexists with an equal and opposite force acting on something outside of the system. Such a damping force, for example, acts on a marine propeller and will be called "outer damping." Second, the damping forces may exist between two members or particles of the system. For example, hysteresis damping forces may be considered as friction between the molecules of the shafting material. This type will be called "inner damping."

In general damping forces that act on a vibrating system have a certain characteristic in common. If the motion at the point of damping is harmonic such as

$$\theta^\circ = y_N \cos(pt - \omega) + y_d \sin(pt - \omega) \quad \ldots\ldots [47]$$

the function $F(pt - \omega)$ expressing the damping force at any instant obeys the relation $F(pt - \omega) = -F(pt - \omega + \pi)$, where $p = 2\pi n$ in which n is the frequency of the harmonic motion. Therefore the function F can be expanded into a Fourier series of odd multiples of $pt - \omega$. Thus

$$F(pt - \omega) = A \sin(pt - \omega) + A_3 \sin 3(pt - \omega) + \ldots$$
$$+ B \cos(pt - \omega) + B_3 \cos 3(pt - \omega) + \ldots$$

With very little error, the harmonics of F higher than the first can be neglected since their disturbing influence is small and their frequencies are much higher than that of the fundamental harmonic. Thus the expression for the damping force for the case of harmonic motion at the point of damping can be written as

$$\left.\begin{aligned} F(pt - \omega) &= A \sin(pt - \omega) + B \cos(pt - \omega) \\ A &= \frac{2}{\pi}\int_0^\pi F(\xi) \sin \xi \, d\xi \\ B &= \frac{2}{\pi}\int_0^\pi F(\xi) \cos \xi \, d\xi \end{aligned}\right\} \quad \ldots\ldots [48]$$

It is sometimes convenient to use Equation [48] in a slightly different form.

$$\text{Let } F(pt - \omega) = F_0(pT), \text{ where } pt - \omega - \omega_0 = pT \quad \ldots [49]$$

Then $F_0(pT) = A' \sin pT + B' \cos pT$

where

$$\left.\begin{aligned} A' &= \frac{2}{\pi}\int_0^\pi F_0(\xi) \sin \xi \, d\xi \\ B' &= \frac{2}{\pi}\int_0^\pi F_0(\xi) \cos \xi d \, \xi \end{aligned}\right\} \quad \ldots\ldots [50]$$

So that

$$F(pt - \omega) = A' \sin(pt - \omega - \omega_0) + B' \cos(pt - \omega - \omega_0)$$

Hence by expanding the trigonometric expressions, it is found that

$$\left.\begin{aligned} A &= B' \sin \omega_0 + A' \cos \omega_0 \\ B &= B' \cos \omega_0 - A' \sin \omega_0 \end{aligned}\right\} \quad \ldots\ldots [51]$$

Consider the action of a harmonic stimulating impulse given by $m \cos pt$. Resolve the impulse into two components having 90-deg. phase difference.

Thus let

$$\left.\begin{aligned} m \cos pt &= m_N \cos(pt - \omega) - m_d \sin(pt - \omega) \\ \text{where} \qquad \omega &= \tan^{-1}\frac{m_d}{m_N}, \text{ and } m^2 = m_N^2 + m_d^2 \end{aligned}\right\} \quad \ldots [52]$$

As in the solution of Appendix No. 1, any motion in the phase of the first or second component will be called motion of the normal or damping phase, respectively.

From Equations [47] and [48], it is evident that two harmonic motions having a 90-deg. phase difference, cause a damping force to occur in the phase of each of the vibrations. This characteristic is similar to that already observed for a damping force proportional to the velocity. Thus the motions produced by the components of the impulse in each of their phases will cause damping forces, and therefore motion in both of their phases, but (omitting the higher harmonics of the damping forces) in no other phase. For this reason the conditions for equilibrium of the forces acting in the vibrating system need only be written for the two phases of the components of the impulse.

The conditions for equilibrium of the forces may be stated as follows: The sum of all the outside forces on one side of any section in either phase of the motion is equal and opposite to the internal resisting forces in the shaft at that section. It should be noted that the inner damping forces are internal resistances in the material of the shaft.

As yet, however, the two components of $m \cos pt$ are only restricted by the condition that they have 90-deg. difference in phase. Let another condition to determine these components be that the amplitude of the damping phase of the motion at the second mass is zero.

Let θ_N and θ_d be the motion of the normal phase and damping phase, respectively. Consider that the function F is expressed as a torque. Let A_1 and B_1 be the amplitudes of the components of F for outer damping acting at the first mass. Similarly, let A_2 and B_2 be these amplitudes for outer damping at the second mass. Also, let A_i and B_i be the amplitudes of the components of F for inner damping along the shaft.

The forces of the normal phase of the motion, therefore, satisfy the following equations,

$$\left.\begin{aligned} \frac{J_1}{g}\frac{d^2\theta_{N1}}{dt^2} - m_N \cos(pt - \omega) + B_1 \cos(pt - \omega) &= Cl\frac{\partial \theta_N}{\partial x} \\ &\quad + B_i \cos(pt - \omega) \\ \frac{J_1}{g}\frac{d^2\theta_{N1}}{dt^2} - m_N \cos(pt - \omega) + B_1 \cos(pt - \omega) & \\ + B_2 \cos(pt - \omega) + \frac{J_2}{g}\frac{d^2\theta_{N2}}{dt^2} &= 0 \end{aligned}\right\} \quad [53]$$

The forces of the damping phase satisfy

$$\left.\begin{aligned} \frac{J_1}{g}\frac{d^2\theta_{d1}}{dt^2} + m_d \sin(pt - \omega) + A_1 \sin(pt - \omega) &= Cl\frac{\partial \theta_d}{\partial x} \\ &\quad + A_i \sin(pt - \omega) \\ \frac{J_1}{g}\frac{d^2\theta_{d1}}{dt^2} + m_d \sin(pt - \omega) + A_1 \sin(pt - \omega) & \\ + A_2 \sin(pt - \omega) &= 0 \end{aligned}\right\} \quad [54]$$

Writing

$$\left.\begin{aligned} \theta^\circ_{N1} &= a_N \cos(pt - \omega) \\ \theta^\circ_{N2} &= b_N \cos(pt - \omega) \\ \theta^\circ_{d1} &= a_d \sin(pt - \omega) \\ \theta^\circ_{d2} &= 0 \end{aligned}\right\} \quad \ldots\ldots [55]$$

the solution follows by evaluating a_N, b_N, a_d, m_N, m_d, and ω from Equations [52] to [54].

As in Appendix No. 1, it is convenient in the solution to use

the contants α, β, m_0, m_{N0}, m_{d0} as defined in [30]. It is also convenient to use $\textcircled{H} = a_N - b_N$.

Putting [55] into [53] and [54], they may be reduced to

$$\left.\begin{aligned} &\alpha a_N - b_N + \frac{180}{\pi C}(B_1 - B_i) - m_{N0} = 0 \\ &(\alpha - 1)a_N + \left(\frac{1}{\beta} - 1\right)b_N + \frac{180}{\pi C}(B_1 + B_2) - m_{N0} = 0 \\ &\alpha a_d + \frac{180}{\pi C}(A_1 - A_i) + m_{d0} = 0 \\ &(\alpha - 1)a_d + \frac{180}{\pi C}(A_1 + A_2) + m_{d0} = 0 \end{aligned}\right\} \quad \ldots\ldots [56]$$

Combining the first and second equations gives

$$b_N = \beta a_N - \frac{180}{\pi C}\beta(B_2 + Bi) \quad \ldots\ldots [57]$$

Combining the third and fourth equations gives

$$a_d = \frac{180}{\pi C}(A_2 + A_i) \quad \ldots\ldots [58]$$

Putting this into the third equation gives

$$m_d = -A_1 - \alpha A_2 + (1 - \alpha)A_i \quad \ldots\ldots [59]$$

Putting [57] into the first equation makes

$$m_N = \frac{\pi C}{180}(\alpha - \beta)a_N + B_1 + \beta B_2 - (1 - \beta)B_i \quad \ldots [60]$$

Now since $m^2 - m_N{}^2 = md^2$, Equations [59] and [60] give

$$m_0{}^2 - \left\{(\alpha - \beta)a_N + \frac{180}{\pi C}\left[B_1 + \beta B_2 - (1 - \beta)B_i\right]\right\}^2 = \left(\frac{180}{\pi C}\right)^2\left[-A_1 - \alpha A_2 + (1 - \alpha)A_i\right]^2 \quad \ldots\ldots [61]$$

Thus a_N can be obtained as a root of Equation [61]. At and near the frequency that makes $\beta = \infty$, however, Equation [61] becomes hard to evaluate. In this case it is more convenient to solve for $\textcircled{H} = a_N - b_N$. This may be done as follows: Subtracting βb_N from both sides of Equation [57], and dividing by $1 - \beta$ gives

$$b_N = \frac{\beta}{1 - \beta}\left[\textcircled{H} - \frac{180}{\pi C}\left(B_2 + B_i\right)\right]$$

Putting this into the first equation of [56] makes

$$m_N = \frac{\pi C}{180}\left(\frac{\alpha - \beta}{1 - \beta}\right)\textcircled{H} + B_1 - \frac{J_1}{J_2}B_2 - \left(1 + \frac{J_1}{J_2}\right)B_i \quad [62]$$

Thus since $m^2 - m_N{}^2 = m_d{}^2$, Equations [59] and [62] give

$$m_0{}^2 - \left\{\frac{\alpha - \beta}{1 - \beta}\textcircled{H} + \frac{180}{\pi C}\left[B_1 - \frac{J_1}{J_2}B_2 - \left(1 + \frac{J_1}{J_2}\right)B_i\right]\right\}^2 = \left(\frac{180}{\pi C}\right)^2\left[-A_1 - \alpha A_2 + (1 - \alpha)A_i\right]^2 \quad \ldots\ldots [63]$$

Therefore, summarizing, a_N may be obtained as a root of Equation [61], or $\textcircled{H}$ may be obtained as a root of Equation [63]. Then

$$\left.\begin{aligned} &a_N = \textcircled{H} + b_N = \frac{\textcircled{H}}{1 - \beta} - \frac{180\beta}{\pi C(1 - \beta)}(B_2 + B_i) \\ &b_N = \beta a_N - \frac{180}{\pi C}\beta(B_2 + B_i) = \frac{\beta}{1 - \beta}\left[\textcircled{H} - \frac{180}{\pi C}\left(B_2 + B_i\right)\right] \\ &a_d = \frac{180}{\pi C}(A_2 + A_i) \\ &\omega = \tan^{-1}\frac{m_d}{m_N} \end{aligned}\right\} \quad [64]$$

where m_d and m_N are given by Equations [59], [60], or [62], α and β are expressed in [30], and A and B with various subscripts are obtained from [48] or [51].

The vibrations at the first and second masses are therefore stated by Equations [55], the resultant vibration at either mass being $\theta_N + \theta_d$.

The amplitude of the normal phase at any point of the shaft will be

$$\left.\begin{aligned} &\theta^\circ{}_N = y_N \cos(pt - \omega) \\ &\text{where} \quad y_N = a_N - \frac{x}{l}\textcircled{H} \end{aligned}\right\} \quad \ldots\ldots [65]$$

The amplitude of the damping phase at any point of the shaft will be

$$\left.\begin{aligned} &\theta^\circ{}_d = y_d \sin(pt - \omega) \\ &\text{where} \quad y_d = a_d\left(1 - \frac{x}{l}\right) \end{aligned}\right\} \quad \ldots\ldots [66]$$

The statement of the amplitude of vibration at any section of the shaft may be stated in terms of a single harmonic by combining the normal and damping phase of the motion. Thus

$$\theta^\circ = \sqrt{y_N{}^2 + y_d{}^2}\cos\left(pt - \omega - \tan^{-1}\frac{y_d}{y_N}\right) \quad [67]$$

and at the first mass

$$\left.\begin{aligned} &\theta^\circ{}_1 = \sqrt{a_N{}^2 + a_d{}^2}\cos(pt - \omega_1) \\ &\text{where} \quad \omega_1 = \omega + \tan^{-1}\frac{a_d}{a_N} \end{aligned}\right\} \quad \ldots\ldots [68]$$

(a) Damping Proportional to the Velocity Together With Elastic Hysteresis Damping

Consider that an outer damping force acts at the second mass proportional to the velocity and that inner damping forces due to elastic hysteresis are acting along the shaft.

The energy absorbed per unit length by elastic hysteresis for one reversed cycle of torsional stress is given by Equation [4] as

$$K = \frac{69.7\,(d_2{}^{4.3} - d_1{}^{4.3})}{10^{10}(d_2{}^4 - d_1{}^4)^{2.3}}M^{2.3} \quad \ldots\ldots [69]$$

where M is the maximum positive twisting moment of the stress cycle.

Since the value of M in K is given by

$$M = \frac{\pi}{180}Cly'$$

where $y' = dy/dx$ is the maximum value of the angular deflection

in degrees per unit length, the hysteresis loss per unit length may be written as

$$K = \left(\frac{\pi}{180}\right)^{2.3} \frac{69.7\ (d_2^{4.3} - d_1^{4.3})}{10^{10}\ (d_2^4 - d_1^4)^{2.3}}\ (Cly')^{2.3}$$

The value of K is the same as the work done by the torque which opposes an increasing or decreasing stress given by

$$Q = \left(\frac{\pi}{180}\right)^{1.3} \frac{2.3 \times 69.7 (d_2^{4.3} - d_1^{4.3})^{2.3}}{4 \times 10^{10}\ (d_2^4 - d_1^4)^{2.3}}\ (Cl)^{2.3}(y')^{1.3}$$

for one reversed stress cycle where y' in this case is the absolute value of the deflection per unit length at any instant of the stress cycle. This may be seen by integrating Q over the stress

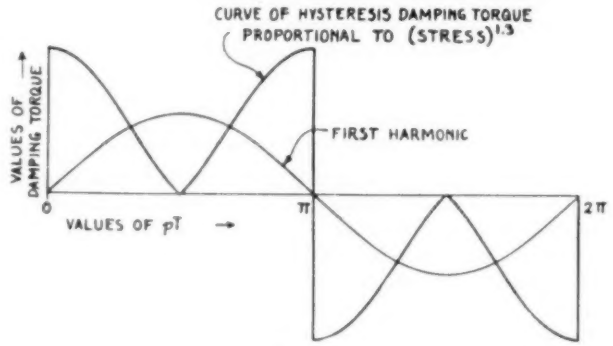


FIG. 24

cycle. Thus, since the work done by Q over the whole cycle is four times that done from $y' = 0$ to $y' =$ a maximum, the work done by Q is $\dfrac{\pi \times 4 \times y'}{180 \times 2.3}$ times Q, which evidently is equal to K.

For the purpose of simplifying Q, put

$$f_1 = \left(\frac{\pi}{180}\right)^{0.3} \frac{69.7 C^{1.3}\ (d_2^{4.3} - d_1^{4.3})}{\pi\ 10^{10}\ (d_2^4 - d_1^4)^{2.3}}$$

$$= \frac{6.58 l\ (d_2^{4.3} - d_1^{4.3})}{10^{10}\ C} \left(\frac{C}{d_2^4 - d_1^4}\right)^{2.3} \quad \ldots\ldots [70]$$

For $G = 11{,}800{,}000$ lb. per sq. in.

$$f_1 = \frac{0.0502\ d_2^{4.3} - d_1^{4.3}}{l^{0.3}(d_2^4 - d_1^4)}$$

and if $d_1 = 0$, $f_1 = 0.0502 \left(\dfrac{d}{l}\right)^{0.3}$

Hence Q may be written

$$Q = \frac{\pi \times 2.3\pi}{180 \times 4} f_1 C l^{1.3} (y')^{1.3} \quad \ldots\ldots [71]$$

Now, from [65] and [66]

$$\theta^\circ = \theta^\circ_N + \theta^\circ_d = y_N \cos(pt - \omega) + y_d \sin(pt - \omega)$$

Differentiating this and putting $\dfrac{dy_N}{dx} = -\dfrac{\text{Ⓗ}}{l}$ and $\dfrac{dy_d}{dx} = -\dfrac{a_d}{l}$, the value of y' given in [71] will be

$$y' = \frac{1}{l} \sqrt{\text{Ⓗ}^2 + a_d^2} \cos pT$$

where $pT = pt - \omega - \tan^{-1} \dfrac{a_d}{\text{Ⓗ}}$

Therefore substituting in [71],

$$F_0(pT) = Q = \frac{\pi \times 2.3\pi}{180 \times 4} f_1 C (\text{Ⓗ}^2 + a_d^2)^{1.3/2} \cos^{1.3} pT$$

where $F_0(pT)$ changes its sign when cos pT starts to decrease, and the value of cos pT is taken as always positive without regard to its actual sign. A graph of $F_0(pT)$ for one harmonic stress cycle is shown in Fig. 24.

The value of A_1' and B_1' may now be obtained from Equations [50]. Then from Equations [51] the values of A_i and B_i may be obtained. Thus

$$A_i = \frac{\pi C}{180} f_2 \text{Ⓗ}^{1.3} \text{ and } B_i = -\frac{\pi C}{180} \delta \text{Ⓗ} \quad \ldots\ldots [72]$$

where

$$f_2 = f_1 \left(1 + \frac{a_d^2}{\text{Ⓗ}^2}\right)^{0.15} \text{ and } \delta = f_2 \frac{a_d}{\text{Ⓗ}^{0.7}} \quad \ldots\ldots [73]$$

The outer damping torque at the second mass, due to the damping force proportional to the velocity is, $\rho \dfrac{d\theta_{N2}}{dt}$ which gives

$$A_2 = -\frac{\pi C}{180} \rho_1 b_N = -\frac{\pi C}{180} \frac{\rho_1 \beta}{1 - \beta} (1 + \delta)\ \text{Ⓗ}, \text{ and } B_2 = 0 \quad [74]$$

where $\rho_1 = \rho p / C$. The second relation for A_2 is obtained from the first by replacing b with its equal from [64], putting $B_2 = 0$, and $B_3 =$ its value in [72].

Since no outer damping forces are considered acting at the first mass $A_1 = B_1 = 0$.

Hence Equation [63] may be written

$$m_0^2 - \left[\frac{\alpha - \beta}{1 - \beta} + \left(1 + \frac{J_1}{J_2}\right)\delta\right]^2 \text{Ⓗ}^2 = \left[\frac{\rho_1 \alpha\beta}{1 - \beta}(1 + \delta)\ \text{Ⓗ} + (1 - \alpha) f_2 \text{Ⓗ}^{1.3}\right]^2 \quad \ldots\ldots [75]$$

where f_2 and δ are given by [73] and

$$\left.\begin{aligned} a_N &= \text{Ⓗ} + b_N = (1 + \beta\delta)\frac{\text{Ⓗ}}{1 - \beta} \\ b_N &= (1 + \delta)\frac{\beta\text{Ⓗ}}{1 - \beta} \\ a_d &= -\rho_1 b_N + f_2 \text{Ⓗ}^{1.3} = -\frac{\rho_1\beta}{1 - \beta}(1 + \delta)\text{Ⓗ} + f_2 \text{Ⓗ}^{1.3} \\ \tan\omega &= \frac{\frac{\rho_1\alpha\beta}{1 - \beta}(1 + \delta) + (1 - \alpha) f_2 \text{Ⓗ}^{0.3}}{\frac{\alpha - \beta}{1 - \beta} + \left(1 + \frac{J_1}{J_2}\right)\delta} \end{aligned}\right\} \quad [76]$$

As a first approximation δ may be taken equal to zero and $f_2 = f_1$. In this case the above equations become

$$m_0^2 - \left(\frac{\alpha - \beta}{1 - \beta}\right)^2 \text{Ⓗ}^2 = \left[\frac{\rho_1\alpha\beta}{1 - \beta}\text{Ⓗ} + (1 - \alpha) f_1 \text{Ⓗ}^{1.3}\right]^2 \quad [77]$$

or putting a_N for its equal $\text{Ⓗ}/(1 - \beta)$,

$$\left.\begin{aligned} m_0^2 - (\alpha - \beta)^2 a_N^2 &= [\rho_1\alpha\beta a_N + (1 - \alpha)(1 - \beta)^{1.3} f_1 a_N^{1.3}]^2 \\ b_N &= \beta a_N = \frac{\beta\text{Ⓗ}}{1 - \beta} \\ a_d &= -\rho_1\beta a_N + (1 - \beta)^{1.3} f_1 a_N^{1.3} = -\frac{\rho_1\beta}{1 - \beta}\text{Ⓗ} + f_1 \text{Ⓗ}^{1.3} \end{aligned}\right\} \quad [78]$$

At synchronism, this first approximation gives

$$\left.\begin{aligned} m_0 &= \rho_1\alpha^2 a_N + (1-\alpha)^{2.3} f_1 a_N^{1.3} \\ b_N &= \alpha a_N \\ a_d &= -\rho_1\alpha a_N + (1-\alpha)^{1.3} f_1 a_N^{1.3} \end{aligned}\right\} \quad \ldots\ldots [79]$$

where $\alpha = \beta = -J_1/J_2$.

Numerical evaluations of these equations have been given in the first part of this paper. An important characteristic that was illustrated by the numerical values was the accuracy of the first approximation. For this reason the assumption made for the first approximation, namely, that $f_2 = f_1$ and $\delta = 0$, will be used in the following pages.

(*b*) Friction Damper (Infinite Flywheel)

Consider the case of a friction damper at the first mass, damping proportional to the velocity at the second, and hysteresis damping along the shaft. Assume that the friction damper consists of a very heavy flywheel revolving at constant speed and having friction surfaces that press against the first mass. The value of the friction torque will be constant and independent of the velocity except that it reverses its sign whenever the velocity of the first mass reverses its sign. Let the absolute value of the total friction torque acting on the friction surfaces be h.

$$\theta_1^\circ = \theta_{N1}^\circ + \theta_{d1}^\circ = \sqrt{a_N^2 + a_d^2}\cos pT$$

where $pT = pt - \omega - \omega_0$, and $\omega_0 = \tan^{-1}\dfrac{a_d}{a_N}$

Therefore

$$F_0(pT) = -h \text{ from } pT = 0 \text{ to } pT = \pi$$
$$= h \text{ from } pT = \pi \text{ to } pT = 2\pi$$

Thus from [50] and [51]

$$A_1 = -\frac{4h}{\pi}\cos\omega_0$$

$$B_1 = \frac{4h}{\pi}\sin\omega_0$$

As obtained above (refer to [72] to [74] where $f_2 = f_1$ and $\delta = 0$),

$$A_2 = -\frac{\pi C}{180}\rho_1 b_N,\ B_2 = 0$$

$$A_i = \frac{\pi C}{180} f_1 ⓗ^{1.3},\ B_i = 0$$

Thus the equations for a_N, b_N, and a_d are as stated in [78], and Equation [63] becomes

$$m_0^2 - \left\{\frac{\alpha-\beta}{1-\beta}ⓗ + h_1\sin\omega_0\right\}^2 = \left\{h_1\cos\omega_0 + \frac{\rho_1\alpha\beta}{1-\beta}ⓗ + (1-\alpha)f_1ⓗ^{1.3}\right\}^2 \quad \ldots\ldots [80]$$

where

$$h_1 = \frac{720\,h}{\pi^2 C}$$

If there is no hysteresis damping acting, $f_1 = 0$ and $a_d = -\rho_1\beta a_N$ In this case if a_N is written for its equal $ⓗ/(1-\beta)$, Equation [80] becomes a quadratic in a_N. Its algebraic solution may be written thus:

$$a_N = \frac{(m_0^2 - h_1^2)\sqrt{1+\rho_1^2\beta^2}}{\rho_1\beta^2 h_1 + \sqrt{\{\rho_1^2\beta^4 h_1^2 + (m_0^2 - h_1^2)(1+\rho_1^2\beta^2)[(\alpha-\beta)^2 + \rho_1^2\alpha^2\beta^2]\}}} \quad \ldots\ldots [81]$$

At synchronism $\alpha = \beta$, and

$$a_N = \frac{(m_0^2 - h_1^2)\sqrt{1+\rho_1^2\alpha^2}}{\rho_1\alpha^2[h_1 + \sqrt{\{m_0^2(1+\rho_1^2\alpha^2) - \rho_1^2\alpha^2 h_1^2\}}]} \quad \ldots [82]$$

Approximately $1 + \rho_1^2\alpha^2 = 1$ and $\rho_1^2\alpha^2 h_1^2 = 0$ so that, as an approximation at synchronism,

$$a_N = \frac{m_0 - h_1}{\rho_1\alpha^2} = \frac{180\left(m - \dfrac{4h}{\pi}\right)}{\pi\rho p\alpha^2} \quad \ldots\ldots [83]$$

Hence, it is seen that for the case of a very heavy friction damper attached to the first mass, the stimulating impulse is effectively decreased by the amount of $4/\pi$ times the total friction force.

It is interesting to note that if the damping at the propeller were zero, i.e., if there were no damping forces other than that of the friction damper acting on the system, the amplitude at synchronism would be infinite in spite of the friction damper, if $m > 4h/\pi$.

(*c*) Friction Damper (Finite Flywheel)

Consider the case of a friction damper at the first mass, damping proportional to the velocity at the second mass, and hysteresis damping along the shaft. Consider the damper to be a slipping flywheel having an inertia J, whose only contact with the first mass is through friction surfaces. The total friction torque that can act on the friction surfaces will be constant and independent of the velocity except that it will reverse its sign whenever the relative velocity between the first mass and the damper flywheel reverses its sign. Let h be the absolute value of the total friction torque that can act on the friction surfaces.

The amplitude of vibration at the first mass is given by

$$\theta_1^\circ = \theta_{N1}^\circ + \theta_{d1}^\circ + \sqrt{a_N^2 + a_d^2}\cos(pt - \omega - \omega_0)$$

or

$$\theta_1 = a\cos pT$$

where

$$\omega_0 = \tan^{-1}\frac{a_d}{a_N},\quad a = \frac{\pi}{180}\sqrt{a_N^2 + a_d^2},\ \text{and } pT = pt - \omega - \omega_0$$

Thus

$$\frac{d\theta_1}{dt} = -ap\sin pT$$

$$\frac{d^2\theta_1}{dt^2} = -ap^2\cos pT$$

The torque that can act on the damper flywheel if it slips is

$$Q = \frac{J}{g}\frac{d^2\theta_0}{dt^2} = \pm h$$

where θ_2 is the amplitude of the vibration of the damper flywheel. Thus the possible acceleration of this flywheel is

$$\frac{d^2\theta_0}{dt^2} = \pm\frac{hg}{J}$$

There are three cases to be considered: First, when the damper flywheel does not slip; second, when the damper flywheel is slipping part of the time; and third, when the damper flywheel is slipping continually.

It will be convenient in the following development to use the following symbols:

$$h_1 = \frac{720h}{\pi^2 C}, \quad j = \frac{p^2 J}{gC}, \quad \omega_0 = \tan^{-1}\frac{a_d}{a_N} \ldots\ldots [84]$$

and α, β, ρ_1, m_0, m_{N0}, m_{d0} as stated in [30].

Case I.—When the Damper Flywheel Does Not Slip. If the possible acceleration of the damper flywheel is greater or equal to the greatest acceleration of the first mass, the flywheel will not slip.

This will occur if

$$ap^2 \leqq \frac{hg}{J} \text{ or if } 1 \leqq \frac{\pi h_1 \cos \omega_0}{4ja_N} \ldots\ldots [85]$$

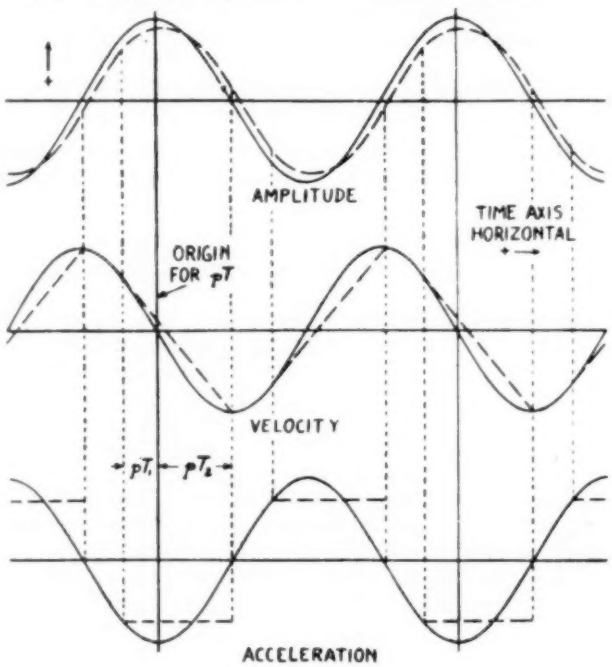


FIG. 25

In this case the solution is the same as the case of two concentrated masses, stimulated by $m \cos pt$, and damped by the propeller and hysteresis damping where the first mass is $J_1 + J$ and the second is J_2.

Case II.—When the Damper Flywheel Is Slipping Part of the Time. The damper flywheel will slip whenever the acceleration of the first mass becomes greater than the possible acceleration of the damper wheel. Thus slipping will occur if

$$1 > \frac{\pi h_1 \cos \omega_0}{4ja_N} \ldots\ldots [86]$$

The amplitude, velocity, and acceleration curves of the first mass and damper flywheel for the steady-state vibration of this case are illustrated in Fig. 25. Let T_1 be the negative value of T nearest zero where the velocity of the damper flywheel is equal to the velocity of the first mass. At this value the damper wheel will have reached its maximum negative acceleration. Its velocity will then continue decreasing at a uniform rate until the changing velocity of the first mass comes back to that attained by the damper wheel. Let T_2 be the value of T when this occurs. If the flywheel is to slip only part of the time $T_2 > 0$ and $< T_1 + \frac{\pi}{p}$. The velocity at $T = T_1$ is $-ap \sin pT_1$, and at $T = T_2$ is $-ap \sin pT_2$. Thus the slope of the velocity curve, or in other words the acceleration of the damper flywheel between T_1 and T_2, will be

$$\frac{ap}{T_2 - T_1}(-\sin pT_2 + \sin pT_1) = -\frac{hg}{J}$$

The acceleration of the first mass at T_1 is $-ap^2 \cos pT_1$. Thus the damper flywheel will begin to slip at this point provided

$$\cos pT_1 = \frac{1}{pT_1 - pT_2}(\sin pT_1 - \sin pT_2) = \frac{\pi h_1 \cos \omega_0}{ja_N} \quad [87]$$

It will slip only part of the time if

$$pT_1 < 0 < pT_2 < pT_1 + \pi \ldots\ldots [88]$$

The values of pT_2 corresponding to any value of pT_1 satisfying Equations [87] and [88] are plotted in Fig. [26].

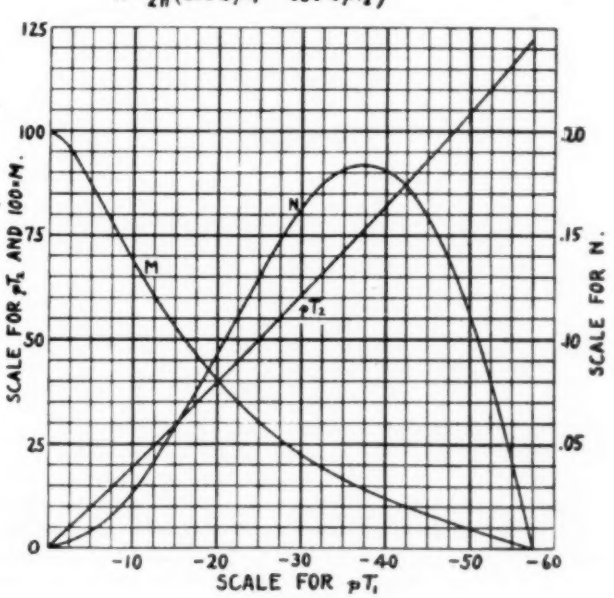


FIG. 26

The value of the friction torque $F_0(T)$ acting on the damper flywheel will therefore be $-h$ from $T = 0$ to T_2, $-\frac{J}{g}ap^2 \cos pT$ from $T = T_2$ to $\frac{\pi}{p} + T_1$, and h from $T = \frac{\pi}{p} + T_1$ to π. Hence from Equations [50] and [51],

$$A_1 = -\frac{2h}{\pi}\left[\cos(pT_1 + \omega_0) - \cos(pT_2 + \omega_0)\right] - \frac{\pi p^2 J}{180g}(a_d M - a_N N)$$

$$B_1 = \frac{2h}{\pi}\left[\sin(pT_1 + \omega_0) - \sin(pT_2 + \omega_0)\right] - \frac{\pi p^2 J}{180g}(a_N M + a_d N)$$

where

$$\left.\begin{aligned} 2\pi M &= 2\pi + 2pT_1 - 2pT_2 + \sin 2pT_1 - \sin 2pT_2 \\ 2\pi N &= \cos 2pT_1 - \cos 2pT_2 \end{aligned}\right\} \quad [89]$$

The values of M and N corresponding to any value of pT_1 and pT_2 which satisfy [87] and [88] as given by [89] are also plotted in Fig. [26].

As obtained above (refer to [72] to [74] where $f_2 = f_1$ and $\delta = 0$)

$$A_2 = -\frac{\pi C}{180}\rho_1 b_N, \quad B_2 = 0$$

$$A_4 = \frac{\pi C}{180} f_1 ⓗ^{1.3}, \quad B_4 = 0$$

Thus the equations for a_N, b_N, and a_d are as stated in [78], and Equation [61] or [63] may be written

$$m_0^2 - \{Ua_N - jNf_1(1-\beta)^{1.3}a_N^{1.3}\}^2 = \{Va_N + f_1(1-\alpha_1)(1-\beta)^{1.3}a_N^{1.3}\}^2 \quad [90]$$

CURVES ILLUSTRATING THE MOTION OF THE DAMPER FLYWHEEL WHEN IT IS SLIPPING CONTINUALLY. THE FULL LINES REFER TO THE FIRST MASS. THE DASHED LINES REFER TO THE DAMPER FLYWHEEL.

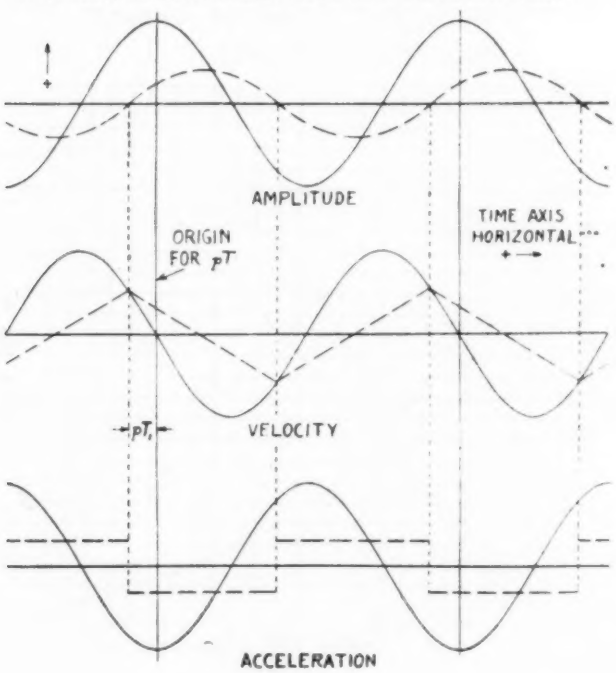


FIG. 27

where

$$\alpha_1 = \alpha - {}_jM = 1 - \frac{p^2}{gC}(J_1 - JM)$$

$$U = \alpha_1 - \beta + {}_jN\rho_1\beta + \frac{h_1}{2a_N}\left[\sin(pT_1 + \omega_0) - \sin(pT_2 + \omega_0)\right]$$

$$V = \rho_1\alpha_1\beta - {}_jN + \frac{h_1}{2a_N}\left[\cos(pT_1 + \omega_0) - \cos(pT_2 + \omega_0)\right]$$

$$\omega_0 = \tan^{-1}[-\rho_1\beta + f_1(1-\beta)^{1.3}a_N^{0.3}]$$

$$b_N = \beta a_N$$

$$a_d = -\rho_1 b_N + f_1(1-\beta)^{1.3}a_N^{1.3}$$

If there is no hysteresis damping the equations reduce to give

$$a_N = \frac{m_0}{\sqrt{U^2 + V^2}} \quad [91]$$

Case III.—When the Damper Flywheel Is Slipping Continually. The amplitude, velocity, and acceleration curves of the first mass and damper flywheel for the steady-state vibration of the case, where the damper flywheel is slipping continually are illustrated in Fig. 27. Let T_1 be the negative value of T nearest zero where the velocity of the damper flywheel equals the velocity of the first mass. The velocity at this point will be $-ap \sin pT_1$. The velocity at $T_1 + \pi/p$ will be $ap \sin pT_1$. Thus the slope of the velocity curve, or in other words the acceleration of the damper flywheel between T_1 and $T_1 + \pi/p$, will be $(2ap/\pi) \sin pT_1 = -hg/J$.

The acceleration of the first mass at T_1 is $-ap^2 \cos pT_1$. Thus the damper flywheel will slip continually provided

$$ap^2 \cos pT_1 \geqq -\frac{2ap^2}{\pi}\sin pT_1 = \frac{hg}{J}$$

or if $\tan pT_1 \geqq -\pi/2$. (This gives $pT_1 \geqq -57.5184$.) Therefore, for the flywheel to slip continually, the following relation must hold

$$\frac{hg}{J} \leqq \frac{2ap^2}{\sqrt{4+\pi^2}}$$

The value of T_1 for this case is given by

$$\sin pT_1 = -\frac{\pi^2 h_1 \cos\omega_0}{8ja_N}$$

where $$-\frac{\pi}{2} < \tan^{-1}-\frac{\pi}{2} \leqq pT_1 < 0$$

The solution follows from the several formulas given for Case II if $pT_2 = pT_1 + \pi$ subject to the conditions just given. Thus $M = N = 0$, and Equation [90] becomes

$$m_0^2 - U^2a_N^2 = \{Va_N + (1-\alpha)(1-\beta)^{1.3}f_1 a_N^{1.3}\}^2 \quad [92]$$

where

$$U = \alpha - \beta + \frac{h_1}{a_N}\sin(pT_1 + \omega_0)$$

$$V = \rho_1\alpha\beta + \frac{h_1}{a_N}\cos(pT_1 + \omega_0)$$

$$\sin pT_1 = -\frac{\pi^2 h_1 \cos\omega_0}{8ja_N} \geqq -\frac{\pi}{\sqrt{4+\pi^2}}$$

Appendix No. 3

LIST OF SYMBOLS

ALTHOUGH the symbols used in this paper were defined when they were first introduced, the following list of the more important ones is included for easy reference.

The dimensions of all values given are to be considered in the inch-pound-second system unless otherwise stated.

a Maximum value of undamped vibration at the first mass in degrees. Also in Appendix No. 2 (*c*) the value of $\frac{\pi}{180}\sqrt{a_N^2 + a_d^2}$

a_d, a_N Maximum values of θ°_{d1} and θ°_{N1}, respectively.

A Amplitude of the sine component of the fundamental harmonic of the function expressing the friction torque.

Subscripts 1 and 2 refer to outer damping at the first and second mass, and subscript i refers to inner damping along the shaft.

b Maximum value of undamped vibration at the second mass in degrees.

b_N Maximum value of θ°_{N2}.

B Same as A except that it applies to the cosine component.

C Proportional constant between the twisting moment M and the resulting angular deflection θ such that $M = C\theta$.

d Subscript denoting the damping phase of the motion.

f_1 Defined by Equation [70].

f_2 Defined by Equation [73].

f_m Defined by Equation [9]. Given in Table 4.

F Function representing the damping torque.

F_a, F_b, F_c Given in Tables 1, 2, 3.

g Gravitational constant.

G Shearing modulus of elasticity. For steel this is equal to 11,800,000.

h Absolute value of the total friction torque acting on the friction surfaces of the damper.

$h_1 = \dfrac{720\,h}{\pi^2 C}$

$j = \dfrac{p^2 J}{gC}$ [Appendix No. 2 (*c*)]

J Weight polar moment of inertia of shaft. Also in Appendix No. 2 (*c*) the polar moment of inertia of the damper flywheel.

J_1, J_2 Weight polar moment of inertia of the first and second mass, respectively.

k Crank-to-connecting-rod ratio.

K Energy absorbed by the damping forces.

l Length of shaft.

m Maximum value of the harmonically impressed torque. The following subscripts are attached: 0, $d0$, $N0$, 1 d, N. The subscript 0 is equivalent to multiplying by $180/\pi C$. The subscripts d and N refer to the components stimulating the damping and normal phase, and the subscript 1 refers to the value of m per unit mean piston area (i.e., mean of top and bottom area of piston) per unit crank radius.

M_i Impressed torque.

M Twisting moment. Also in Appendix No. 2, Case II of (*c*), M is defined by [89].

n Frequency.

N Subscript representing the normal phase of the motion. Also in Appendix No. 2 Case II of (*c*), N is defined by [89].

p Frequency constant $= 2\pi n$.

p_0 The value of p at synchronism.

Q Torque.

r Radius.

R Radius.

S Stress.

t Time variable.

T, T_1, T_2 Time variables referred to different initial values.

W Weight.

x Length variable from first mass.

y, y_d, y_N Maximum values of θ°, $\theta_d{}^\circ$, $\theta_N{}^\circ$, respectively.

$\alpha = 1 - \dfrac{J_1}{gC}\,p^2$

$\beta = \left(1 - \dfrac{J_2}{gC}\,p^2\right)^{-1}$

γ Constant of integration in degrees.

δ A small constant. Also in Appendix No. 2 (*a*), defined by Equation [73].

Δ Fraction of J for the shaft to be added to J_1 when the $J_2 = \infty$, to give the same frequency.

Δ_1 Fraction of the polar moment of inertia of the shaft from the first mass to the node to be added to the first mass.

Δ_2 Fraction of the polar moment of inertia of the shaft from the node to the second mass to be added to the second mass.

θ Amplitude of vibration in radians. This applies to θ with any subscript attached.

θ° Amplitude of vibration in degrees. This applies to θ with any subscript attached. The subscripts 1, 2, d, $d1$, $d2$, N, $N1$, $N2$ may be appended to θ or θ°. The subscripts 1 and 2 refer to the first and second mass, respectively. The subscripts d and N refer to the damping or normal phase of the motion, respectively.

$\textcircled{m} = a_N - b_N$

ρ Constant ratio for damping proportional to the velocity.

$\rho_1 = \dfrac{\rho p}{C}$

$\phi = \dfrac{360}{\sqrt{g}}\sqrt{\dfrac{J}{C}} = 18.33\sqrt{\dfrac{J}{C}}$ for $g = 385.9$ in. per sec. per sec.

ω Time-phase angle of normal phase of motion behind maximum value of impulse.

ω_0 Equations [49] to [51], any phase angle such that $pt - \omega - \omega_0 = pT$. Also elsewhere the special value of $\tan^{-1} \dfrac{a_d}{a_N}$.

ω_1 The time-phase angle at the first mass of the resultant of the normal and damping phase of the motion behind the maximum value of the impulse.

All positive time-phase angles represent the lag of the maximum positive value of the harmonic behind the origin of the time variable. They are given in degrees or radians and represent that part of one cycle of the harmonic where one cycle is considered to contain 360 deg. or 2π radians, respectively.

APM-51-23

Turbine Vibration and Balancing

By THOMAS C. RATHBONE,[1] LESTER, PA.

The limitations are discussed of the latest analytical methods employed in correcting the balance of turbine-generator rotors at speed and in operation. A strobo-vibroscope developed to study motion in a complex system is described, with the results of experiments made on a large rotor by means of this instrument. A method of balancing is then described, which takes into account the interaction of vibratory impulses communicated from one bearing to another, thus overcoming the chief difficulty with present methods. The proposed method also provides a means for evaluating the sensitivity of any installation, for comparative study of structures. There is also included a discussion of various factors other than simple unbalance which affect the vibration of large units, with references to some unusual cases encountered.

PART I FIELD BALANCING METHODS

Introductory

THE desirability of balancing large turbine-generator rotors in the shop at speed under conditions approaching those encountered in service, and also in actual operation in the field, has led to the development of various kinds of vibration instruments and analytical methods of balancing. Such methods are a great improvement over the old "hand balancing" process and yield fair results under favorable conditions. But being based on ideal assumptions which are never realized, particularly in the case of complex structures readily permitting interaction of vibration between bearings, these methods are sometimes found wanting.

It will be the purpose of this paper to discuss some of the more recent methods in use and their deficiencies, to describe apparatus for analyzing the vibratory motion at the bearings and the results of experiments carried out by their aid, and to propose a method of balancing on complex structures which compensates for the influences transmitted from one bearing to another. By this process the installation is evaluated in terms of the response from unit unbalance, and a means is thereby supplied for comparative study of the vibration characteristics of any unit particularly with regard to resonance, for purposes of design.

A brief discussion is also included of factors other than simple, fixed unbalance which affect the vibration of turbine-generator units.

Present Status of Field Balancing

As the sizes of turbine generators have increased and the operating conditions become more severe, it has been found that the balance obtained by means of the dynamic balancing machine, operating at relatively slow speed and room temperature, does not always insure quiet running in operation. The mass axis under operating conditions may not quite coincide with the initial axis. Because of this, it is now customary to check the balance of large rotors at speed and under temperature conditions simulating those in operation, in a special test set-up, in order to make any final adjustments of the running balance that prove necessary.

Changes in balance may occur in service due to seasoning, reblading, rubs, or other causes; therefore it becomes necessary at times to carry out balancing in the field under operating conditions. The problem of balancing at speed both in the shop and in service is therefore an important one and has received a great deal of attention in recent years.

Although some very creditable balancing has been accomplished by the familiar "hand balancing" methods, using the high-spot scribing on the shaft, this process is generally found tedious and unsatisfactory.

Various types of vibration instruments have been developed to aid in this work, and the quantitative information thus made available has been used as the basis for several new analytical methods of balancing. These methods are based on ideal assumptions, but have yielded fair results where the conditions have been favorable.



Fig. 1 Vibrometer

By means of the vibrometer,[2] a type of which is shown in

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Contributed by the Applied Mechanics Division and presented at the Akron Meeting, Akron, Ohio, October 21 to 23, 1929, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] This vibrometer, in common with most types, is based on the principle of the seismograph. For further description of vibrom-

Fig. 1, the amplitude of vibration in any direction at the bearings or any other part of the unit is determined. The balancing process consists generally in noting the changes in amplitudes resulting from applying known test weights at the end of the rotor nearest the bearing under observation. The amount and location of the unknown unbalance referred to the balancing holes are then deduced from the data obtained on three or more trials, by several methods.[3] It is assumed that the bearing

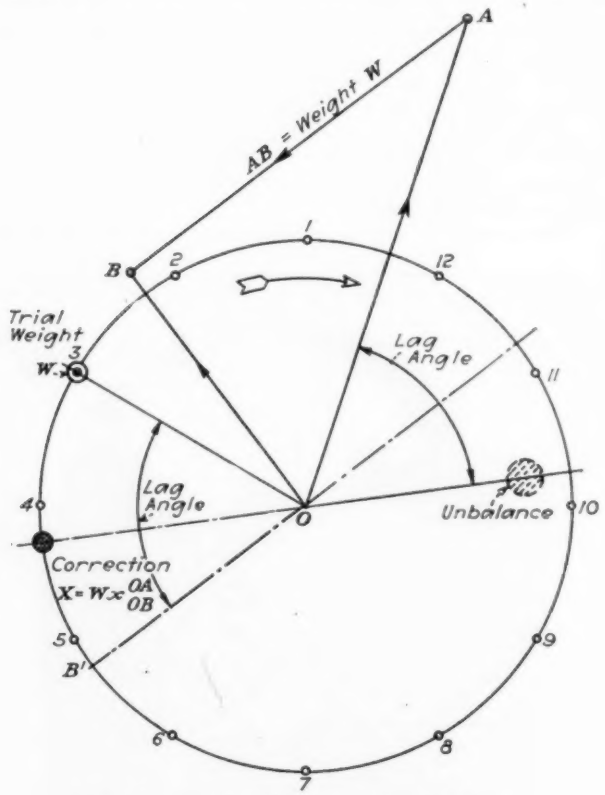


FIG. 2 SINGLE TRIAL BALANCING METHOD

under observation is not influenced by disturbances from other points and that amplitudes bear a linear ratio to unbalance.

Methods of balancing have also been devised, using the scribed high spot or employing the phase angle between a point in the vibratory motion and a reference datum on the rotor, by means of stroboscopic illumination of the vibrometer dial pointer. The change in the phase angle of the motion due to the initial unbalance on two successive trials with known weights in two positions affords sufficient data to determine the initial unbalance. The vector solution is simple.[4] It is assumed that the angle of lag remains constant for all amplitudes and under all conditions.

These ideal solutions have been reduced to the simplest possible terms by utilizing both the amplitude and phase relation of the vibratory motion. This solution has been termed the "single trial method" and has been used to advantage, especially on balancing machines. It is not believed to have appeared before in the literature,[5] and will be described here as use is made of this principle in developments to follow. By this method, the amplitude, with the phase angle between a point in the vibratory motion (for example, the vertical extremity) and the reference datum on the rotor, is observed for the initial unbalanced condition. In Fig. 2, representing the end of the rotor, the vector OA represents the phase and amplitude of this initial vibration. A known test weight is then inserted at any position, as at W. On this trial, a new amplitude and phase angle will result, representing the combination of the test weight and unknown initial unbalance. Let the new condition be shown by the vector OB. The closing vector AB now represents the known trial weight W in amount and phase. The initial vector OA, evaluated in terms of the weight scale of AB, determines the amount of the initial unbalance. The actual angle of lag is given by the difference in direction between OW and AB, which when laid off ahead of OA, fixes the location of the unbalance.

Under ideal conditions the high spot scribed on the shaft may be used instead of the phase relation of the vibration at the bearing.

The foregoing methods provide for balancing one end at a time. If the bearings can be considered as independent of each other, then the Marsland method[6] of compensating for the effect of the second end balance weight to prevent disturbing the first end already corrected, is applicable.

With the same assumptions as to independence of the bearings, and considering the system as a beam simply loaded, the two ends of the rotor can be balanced simultaneously by trial weights in both ends at once. In this case moments of forces about the bearings are used instead of direct weights at the balance planes.[7]

LIMITATIONS OF PRESENT METHODS

Some of these analytical methods have been used with fair success. They possess considerable advantage over "hand balancing." On many occasions, however, the disparity between the simple assumptions made and the actual conditions existing has been a source of trouble. Many additional trials then become necessary to obtain a satisfactory balance.

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eters, see: (*a*) "The Use of Vibration Instruments on Electrical Machinery," J. Ormondroyd, Trans. A.I.E.E., 1926. (*b*) "Measurements of Technical Vibration Phenomena," *The Electric Journal*, Feb., 1926. (*c*) "Vibration Recorder," A. V. Mershon, Trans. A.I.E.E., 1926. (*d*) "Vibration Problems in Engineering," Timoshenko, Van Nostrand Co., 1928, p. 327. (*e*) "The Prevention of Vibration and Noise," by Eason; Frowde, Hodder & Stoughton, London, 1923. (*f*) "Recent Applications of the Electric Micrometer," A. V. Mershon, *Gen. Elec. Review*, Oct., 1928, vol. 31, no. 10. (*g*) "Measuring Oscillations by the Vibrometer," Carl Schenck, *Machinery*, London, Feb. 25, 1926. (*h*) "The Davey Vibroscope," *Machinery*, vol. 35, no. 7, Mar., 1929. (*i*) "The Cambridge Vibrograph," *Engineering*, vol. 119, 1925. (*j*) "The Geiger Vibrograph," *V.D.I.*, vol. 60, 1916; also "Mechanische Schwingungen," Dr. Ing. J. Geiger, Julius Springer, 1927. (*k*) "Vibrometer," *Instruments*, vol. 2, no. 2, Feb., 1929, p. 71.

[3] For a description of some of the analytical methods employed, see: (*a*) "Field Balancing Rotors at Operating Speed," G. B. Karelitz, *Power*, Feb. 7 and 14, 1928. (*b*) "Balancing of Rotors in Factory and Installation," S. H. Weaver, *Gen. Elec. Review*, vol. 31, no. 10, Oct., 1928. (*c*) "Field Balancing of Rotors," J. J. Ryan, *Elec. Journal*, Dec., 1928, vol. 25, no. 2.

[4] This method, used by the author in 1923, is based on the principle that only one arrangement of the trial weight vectors is possible which satisfies the phase relations in the three trials.

[5] The method, based on combining amplitude and phase data, was suggested to the author by B. V. Anoschenko, Experimental Section, Large Turbine Div., Westinghouse So. Phila. Works.

[6] The correction for the first end balanced is allowed to remain. Then two weights are substituted for the second end corrector, one at each end of the rotor, by a simple simultaneous equation such that the moments about each bearing remain unaltered. This method avoids the labor of analyzing the static and dynamic components. See U. S. Patent No. 1,591,855, R. Marsland, 1923.

[7] The vector solution is similar to that described by M. W. Torbett in his discussion of the paper, "The Theory of the Dynamic Vibration Absorber," by Ormondroyd and Den Hartog, Trans. A.S.M.E., vol. 50, no. 17. A similar method was used by Akimoff. The determination of the center of gravity, however, is unnecessary.

The chief difficulty lies in the complex nature of the entire installation, regarded as a vibrating system. This is particularly true with relatively light structural steel foundations. The transmission of impulses from one end of the unit to the other through queer modes of vibration so confound the observed motions at a bearing that an analysis becomes difficult. The impulses from one bearing may augment or even neutralize the motion at another bearing. Observations on the shaft itself show the same influences. The author has often found the shaft "smooth" while the bearing pedestal indicated large vibration, and the reverse condition is frequently encountered.

It is perhaps a familiar experience to eliminate vibration at one bearing only to find it cropping up at another, or to suppress the vertical component at the expense of the lateral. It is difficult at times to determine which end of the rotor is responsible for the vibration. Often an intense vibration at the thrust end has later been found to originate at the coupling end. Again, much time may be consumed in attempting to balance the field when the spindle is the offender, and vice versa.

Because of the peculiarities of large units, wherein transient conditions are occasionally encountered, it is highly desirable to be able to make continuous observations from which the exact condition of the balance can be deduced over periods of time. This can only be partially accomplished by present methods.

Analysis of Motion

Regardless of the complexity of vibration characteristics encountered, it appeared probable that simple laws could be found which govern the make-up of the motion at any bearing or other point of the unit, thus making possible a prediction of the motion from a given set of conditions. In the following, simple harmonic motion is assumed throughout, which assumption is fairly well borne out in practice.

Fig. 3, upper drawing, represents a rotor on a non-rigid support, capable of communicating motion from one end to the other. Each bearing support may be considered as made up of two combined dynamic systems, one in the vertical and one in the lateral direction, each with its own inertia, elastic, and damping characteristics. The fundamental and higher natural frequencies in the vertical and lateral directions will therefore not be the same, unless by accident. The amplitude of motion in each direction occasioned by unbalance will depend on the ratio of the running speed to the natural frequency for the corresponding direction. Therefore, in going through the speed range, the vertical and lateral components will wax and wane independently of each other.

Assume a fixed speed and a given unbalance at A. The vertical component at the No. 1 bearing undergoes simple harmonic motion of an amplitude V_a, which may be represented as the diameter of the generating circle. This motion includes any effect coming from the No. 2 bearing due to A. Considering the balance-weight position as a reference point on the rotor, the angle of lag is the angle through which this point has turned beyond the vertical position at the instant the corresponding vertical component of motion has reached its upper extremity. In the diagram of Fig. 3, the point P_v in the path of motion V_a is shown at the instant when the unbalance is in the vertical position. Thus the corresponding points in the path of motion are laid off back from this reference.

Similarly the lateral motion L_a is shown as the diameter of the generating circle, with the phase of the motion referred to the reference A. The two simple harmonic motions compounded at right-angles generate the Lissajou ellipse P, and the point P on the ellipse is the instantaneous point in the path of motion when the rotor datum is passing through the upper vertical position. With this one point fixed, the elliptical path can be graduated in degrees or balance-hole positions to agree with the numbers on the rotor. It is important to note that if the unbalance is shifted through a given angle, the corresponding points on the ellipse will also move in agreement through the same angle, but the shape and inclination of axis will not change, nor the magnitude and direction of motion. It should also be noted that the phase difference between the vertical and lateral components may be such as to cause the bearing motion to rotate counter to the spindle. At first this appears to be a paradox. Any change in speed, inertia, or elastic scale will of course alter the figure.

Thus far, only the motion due to the unbalance A has been considered. This may be termed the "primary motion" for

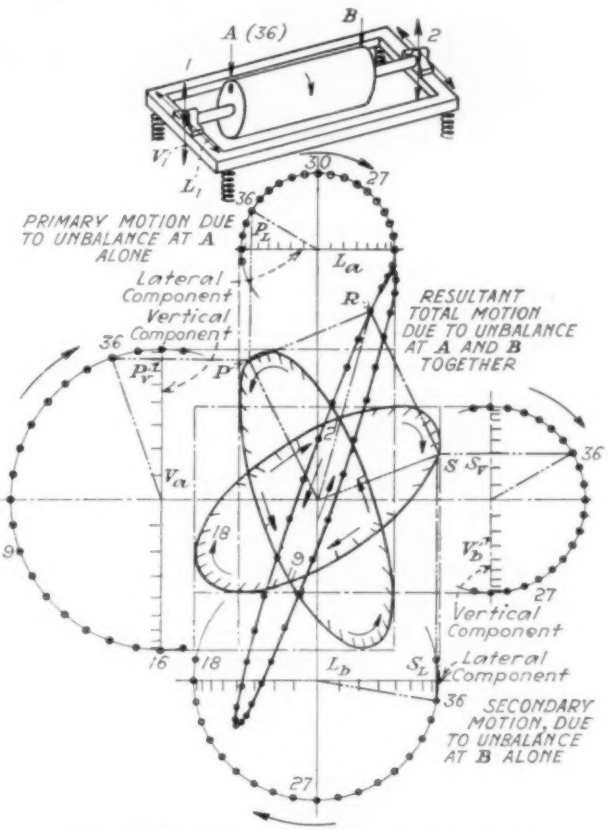


Fig. 3 Composition of Motion at a Bearing

the No. 1 bearing. Still considering this bearing, assume that the unbalance is removed from A and transferred to B. The centrifugal force developed influences the No. 1 bearing through the rotor, and also communicates a motion to it by way of the No. 2 pedestal and foundation. These two influences can be considered as a composite and need not be separated.

The generating circles for the vertical and lateral components of motion at the No. 1 bearing V_b and L_b due to the weight B are constructed with assumed phase relations, and compounding produces the ellipse of motion S. The point S in the path occurs when the reference on the spindle is at the top position and corresponds with the point P in ellipse P. This may be termed the "secondary motion" for the No. 1 bearing.

If the weight A is replaced, and the weight B left in, then the No. 1 bearing would be subjected to the two elliptic motions P and S. If the principle of superposition holds, then P and S can be compounded to give the resultant motion R, which

would obtain with both weights acting at the same time. The point R in this motion is the vectorial sum of OP and OS.

Similarly the same process holds for the No. 2 bearing, the data being obtained simultaneously. The primary motion for one bearing corresponds to the secondary motion of the other.

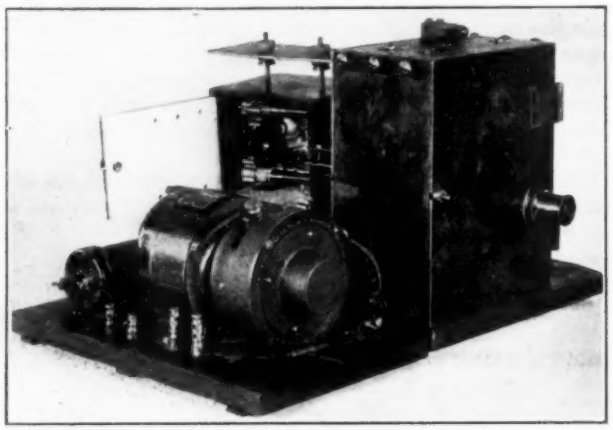


FIG. 4 VIBRATION PHASOMETER APPARATUS

These primary and secondary motions due to a unit unbalance may be termed "unit motions." They become a fixed characteristic or index for the system and completely determine the motion produced by all possible combinations of unbalance, with two provisos: every component magnitude of motion varies

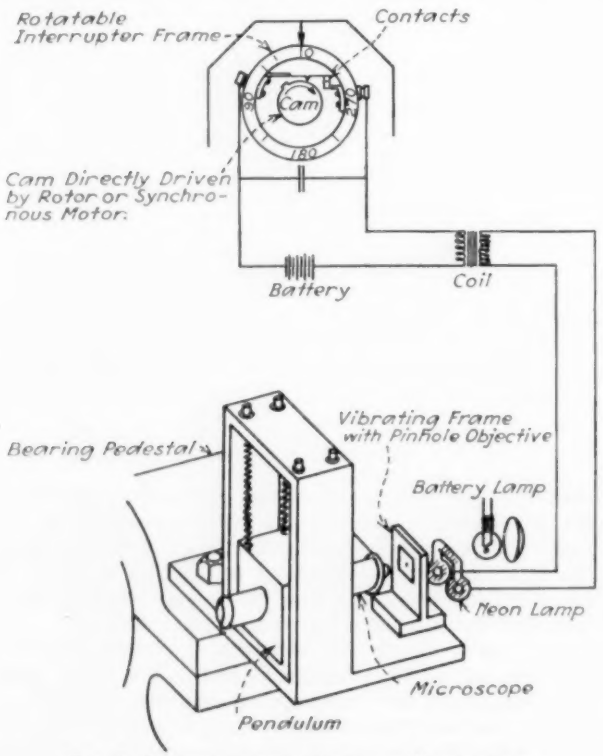


FIG. 5 SCHEMATIC ARRANGEMENT OF PHASOMETER

directly with the unbalance, and the several angles of lag remain constant.

THE STROBO-VIBROSCOPE

In order to demonstrate the principle outlined, experiments were carried out on a large rotor in its testing cylinder. Observations at the bearings were made with a special type of strobo-vibroscope, or vibration phasometer, shown in Fig. 4, developed at South Philadelphia for such studies.[8] The schematic arrangement is shown in Fig. 5. The microscope is mounted on a heavy pendulum suspended by light springs and is therefore practically stationary in space. The objective is an extremely small perforation in a sheet of foil, which is carried

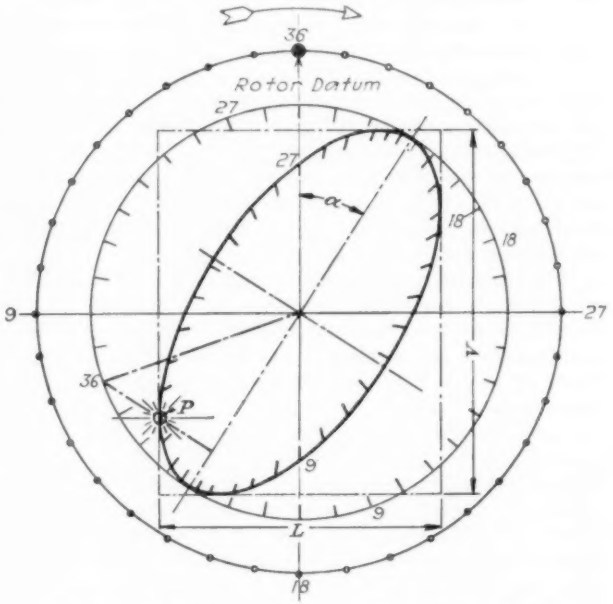


FIG. 6-*A* METHOD OF RECORDING ELLIPSE OF MOTION



FIG. 6-*B* DISTORTED MOTION

on a frame, the whole being rigidly mounted on the bearing and partaking of its motion.[9] Dual illumination is used; a steady light and a flashing neon lamp, both in the line of sight behind the pinhole.

When observing steady light through the pinhole the motion

[8] This is an improved design of the apparatus made by the author for investigations of obscure vibration phenomena in 1923. See "Unusual Vibration of a 25,000-Kw. Turbine Generator," by the author, *Elec. Journal*, vol. 25, no. 2, Feb., 1928.

[9] (*a*) See "Vibration Problem in Engineering," Timoshenko, p. 330, for description of method used by G. B. Karelitz, in observing motion figures through the microscope as reflected by points of steady light from emery cloth. The author has found the pinhole objective more satisfactory than reflected light or cross-hair intersection for stroboscopic work. (*b*) For an early attempt with stroboscopic methods see "L'Equilibramento delle masse rotanti a grande velocita," Dr. Ing. Carlo Luigi Ricci, Atti della Reale Accademia Delle Scienze di Torino, vol. 51, 1915–16, p. 188. A phase method using the Archimedes spiral is described. (*c*) See also a description of the Davey vibroscope, Electrocon Corp., loc. cit., for use with stroboscopic illumination. The Elverson oscilloscope and the stroborama are also examples of stroboscopic apparatus which may be adapted to study vibratory motion. See also "Optische Messungen an schnellaufenden Machinen," Dr. Ing. W. Kniehahn, in *Der Motorwagen*, Aug. 10, 1926, p. 2. (*d*) A phase-determining method is also described by Wheeler and Southwell in *Engineering*, vol. 99, p. 64, 1915.

appears as an illuminated figure. The size of the figure is determined by a graduated eyepiece, and the inclination of its axis is estimated by a graduated dial on the instrument.

A circuit interrupter, driven either by the main unit or a synchronous motor, is made to break a circuit including the primary of an induction coil once each revolution. The secondary coil operates the neon lamp, which flashes only when the circuit is broken. The flash is practically instantaneous. The interrupter frame can be rotated manually, its angular position being indicated by a graduated circle corresponding to the balance holes or graduations on the rotor.

The pink-colored neon flash observed through the pinhole appears as a pink spot somewhere in the illuminated path of motion. As indicated by *P* in Fig. 6-*A*, when the interrupter is rotated through 360 deg., the pink spot will make a complete circuit around the figure. A second neon lamp in the same circuit is provided to illuminate some exposed part of the shaft on which has been painted the balance hole or angular reference positions. These marks appear stationary in the stroboscopic illumination. By these means, the instantaneous position of the spindle corresponding to any point in the path of motion is directly determined. The interrupter dial may be adjusted to this setting and the spindle positions thereafter read directly from the dial position.

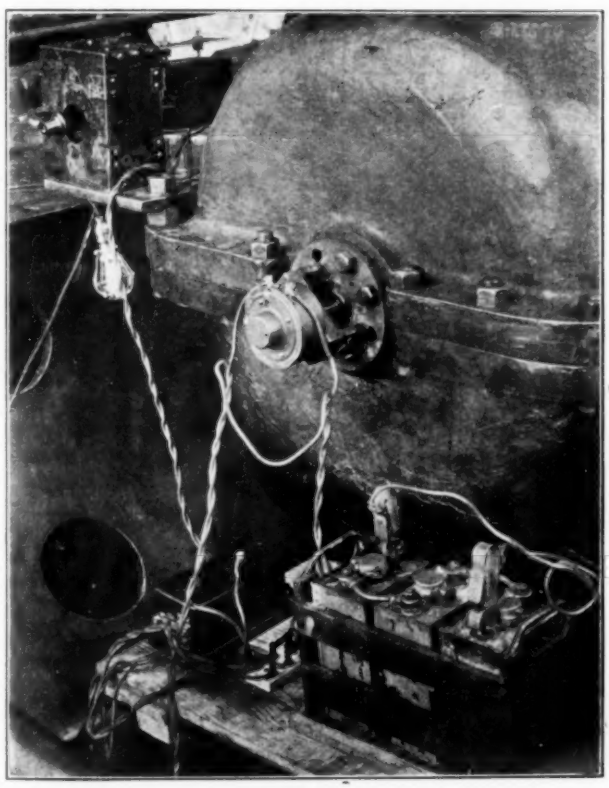


Fig. 7 Photograph of Apparatus on Testing Cylinder

Although the instrument can be arranged to record by photographic means, the one used was provided for visual observation only, in order to detect rapid changes. The method of recording an observation consisted in first sketching the shape of the ellipse and then determining the vertical and lateral dimensions of the enclosing rectangle, together with the inclination of the vertical axis. Then the phase angles for the important points in the path as shown were determined by the phasometer. Although only two points are needed (to locate all other points and show direction of motion), numerous points were observed for checking. The accuracy which can be obtained is quite remarkable. The phase angles repeatedly checked within three or four degrees with favorable conditions, and the amplitudes were estimated to a tenth of a mil. Two flashes per revolution 180 deg. apart were also used to obtain greater refinement.

Occasionally the figures observed are not true ellipses, and under unusual conditions they are greatly distorted, with a wide variation in the velocity of motion, as shown in Fig. 6-*B*. Such a condition is generally accompanied by a hard, sharp vibration, and is due to impact at some point. All sine motion at the same frequency lies in a plane. Axial components frequently occur, as this plane is not always perpendicular to the

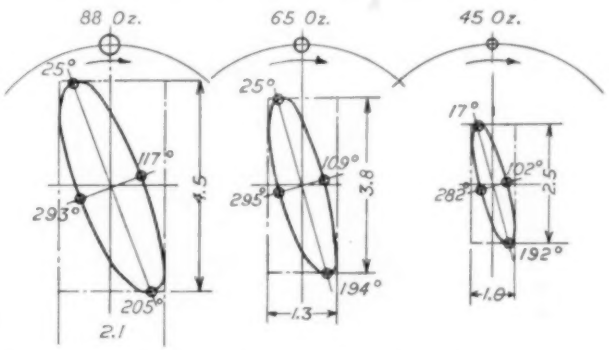


Fig. 8-*A* Preliminary Observations of Motion—Tests With Various Weights in Same Position (1400 R.P.M.)

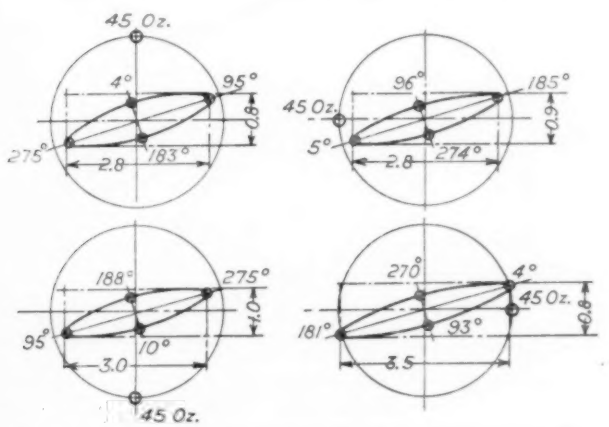


Fig. 8-*B* Preliminary Observations of Motion—Tests With Same Weight in Four Positions (1800 R.P.M.)

axis of the unit. Vibration figures simulating the figure 8 are produced by a curving of the plane of motion, and therefore one component must have a frequency of double the rotational speed. The true ellipse is the usual figure encountered.

Experimental Results

Fig. 7 is a view of the vibration phasometer mounted on a bearing housing of the overspeed and heat-testing cylinder for large spindles at the Westinghouse South Philadelphia Works. This set-up is very similar to an actual installation, as the bearings are both supported on the cylinder structure, and hence permit intercommunication of motion. A 50-ton spindle was used in the test, driven by an independently mounted turbine through a flexible coupling.

The spindle was first given a refined balancing at speed. No

vibration sufficient to interfere with the test was observed at any speed up to 2000 r.p.m.

First, a weight of 45 ounces was inserted at the 0 deg. reference at the thrust end of the rotor and observations made at the thrust-end bearing at various speeds up to 1900 r.p.m. Increasing the weight to 65 and 88 ounces resulted in a fairly proportionate increase in the size of the observed figure, as shown by

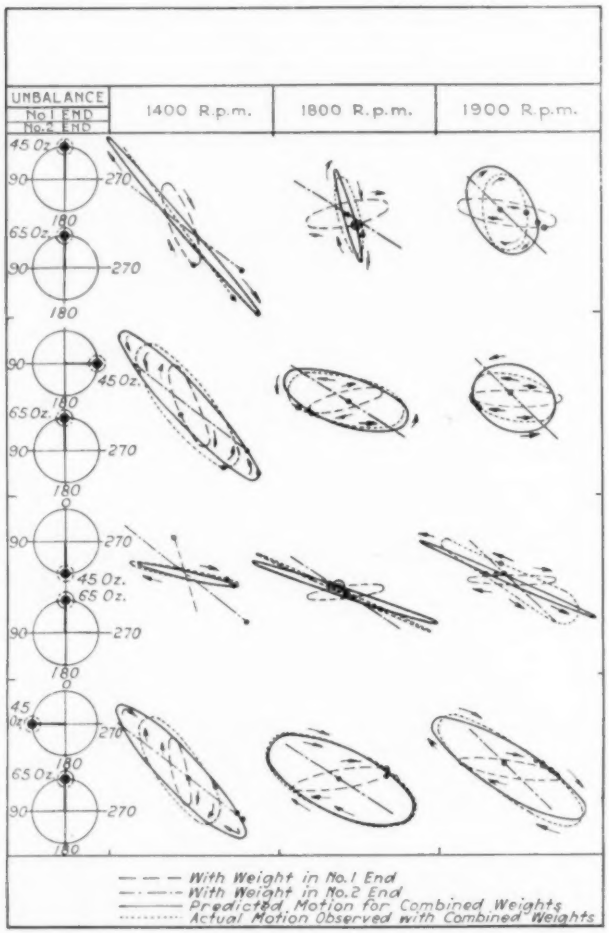


FIG. 9 ANALYSIS OF TEST RESULTS—VIBRATORY MOTION AT NO. 1 BEARING FOR VARIOUS SPEEDS

Fig. 8-*A*, but the shape and phase positions were not appreciably affected.

Observations were then made with the 45-ounce weight at 90, 180, and 270 deg. In each case the shape and size remained practically constant, but the phase positions shifted 90 deg. each time with remarkable fidelity, as shown by Fig. 8-*B*. The tests were repeated, removing the weight from the No. 1 end and trying 65 ounces in the No. 2 end at the 0-deg. and 180-deg. positions. The observed motions at the No. 1 bearing were the same for each case, and the phase positions reversed within about 20 deg. As this slight discrepancy held for all speeds, it was probably not due to error in maintaining a given speed, but to a small residual unbalance at the coupling end.

In the next series of tests, the No. 2 end weight of 65 ounces was kept at the 0-deg. position, and four trials were made with the No. 1 end weight of 45 ounces at the 0-, 90-, 180-, and 270-deg. positions on the rotor. The motions for each of these six cases separately having been observed, it remained to find if the weights in combination would produce motions agreeing with the motions predicted by superimposing the individual figures.

In Fig. 9 are shown the individual motions for each case, the predicted figure by superposition, and the actual motion observed by the weights in combination. The agreement is conclusive in all respects, in amplitude, shape or eccentricity of ellipse, inclination of axes, direction of motion (with or counter to rotor), and in the phase relation.

These results are particularly striking for the reason that the facilities for maintaining constant speed were not satisfactory. The phase relation of the motion to the rotor reference changes rapidly with speed. In a simple vibratory system, the angle of lag between the shaft deflection and the unbalance is expected to change swiftly from a few degrees to nearly 180 deg. going through the critical speed, the changes above and below being slight.

The experimental information obtained on these and other tests indicates, however, that the phase relation of the motion at a bearing of a large unit, which is really an index of the lag angle, changes continually with speed. In Fig. 10 is shown the relation between the phase angle and speed in tests on a large

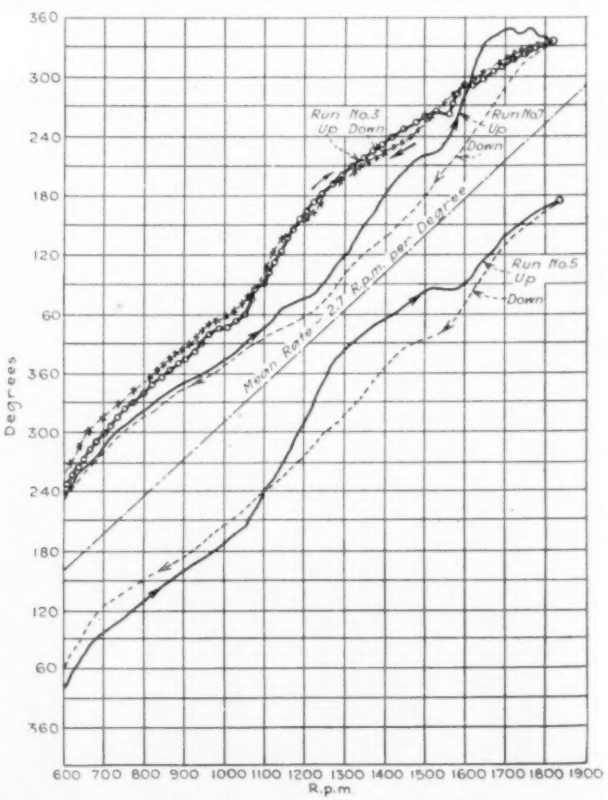


FIG. 10 CHANGE OF PHASE OF VIBRATION WITH SPEED—SPEED-PHASE CURVES OF LATERAL VIBRATION, NO. 1 BEARING, 50-TON SPINDLE, IN TEST CYLINDER

spindle in another testing cylinder made by Prof. Eric Therklesen[10] and the author in 1924. The remarkable fact developed that the angle of lag increased from 0 deg. to over 700 deg.—two complete revolutions—between the start and about 2000 r.p.m. The transit of vibration nodes across the point of observation and the influence of forces at other bearings must furnish the explanation. In the central station the slight variations in

[10] Dept. of Mech. Engrg., University of Montana, Bozeman.

frequency would not cause an appreciable change in the phase angle.

Other curious features observed in the present investigation deserve mention. The spindle shafts were carefully scribed for high spot on each test and at various speeds by A. R. Robertson.[11] There was little change in the mark between 1200 and 1900 r.p.m., and its position generally lagged about 90 deg. behind the weight. This ordinarily would have indicated resonance, but obviously resonance could not have existed throughout this entire range. Also noteworthy is the fact that when

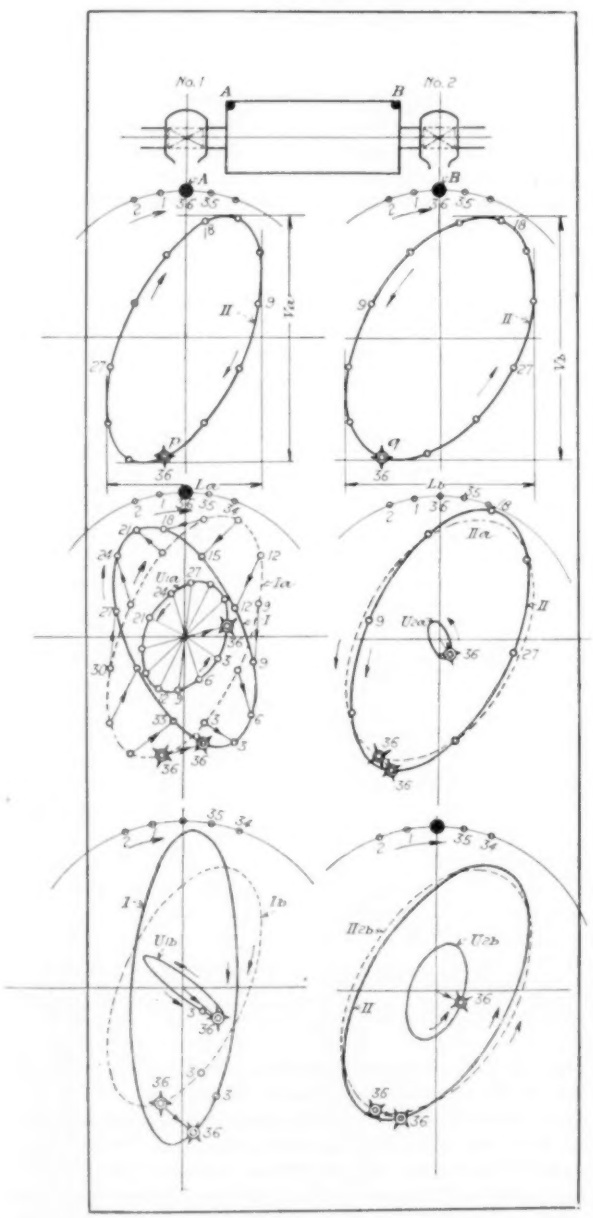


FIG. 11 METHOD OF DERIVING UNIT ELLIPSES OF MOTION FOR DETERMINING UNBALANCE

the 45-ounce unbalance at the No. 1 end led the 65-ounce weight in the No. 2 end by 90 deg., no high-spot location could be found. The shaft felt quite smooth, yet the vibration at the bearing was considerable. This may be explained by opposing impulses, and the occurrence emphasizes the unreliability of the scribed mark.

Another odd situation was developed by the experiment, but which might have been predicted by analysis. In the two

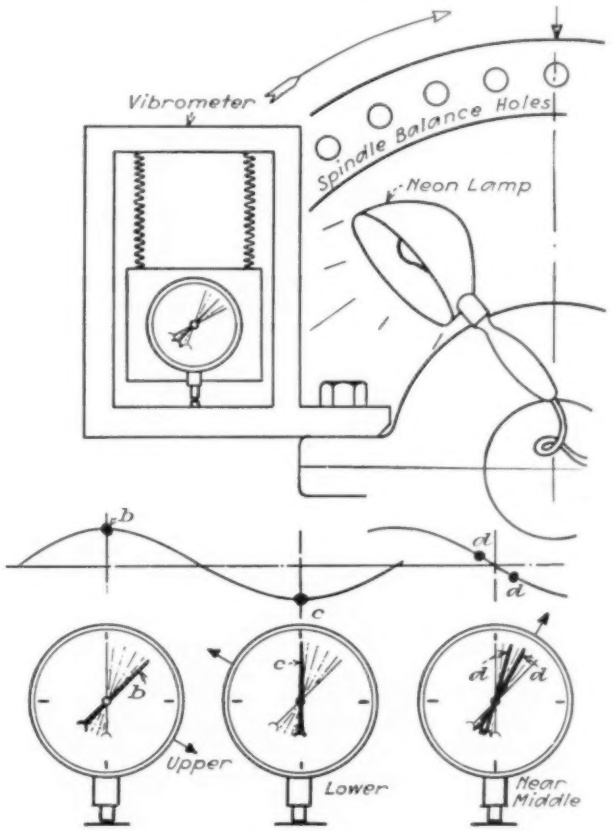


FIG. 12 VIBROMETER WITH STROBOSCOPIC ILLUMINATION FOR DETERMINING PHASE OF MOTION TO ROTOR

trials in which the unbalance of the No. 1 end first led the No. 2 weight by 90 deg. and then lagged 90 deg., it might have been expected from the usual academic treatment that the resultant vibration would be the same for both cases as the forces are identical. Actually, the results are totally different in every way. This indicates that a given combination of unbalance in a rotor mounted in intercommunicating supports will produce two sets of vibratory responses, one for each direction of rotation. On very large units the velocity of propagation of impulses through the structure from one bearing to another also enters the problem, as the rotor may turn through 10 or 15 deg. during this interval.

BALANCING SOLUTIONS BY UNIT MOTIONS

By utilizing the principles developed it is possible to determine any unknown unbalance in such a system. In addition to the two assumptions already made as to proportionality of amplitude for unbalance and constancy of lag angle, it must be assumed in the following that no change in the rotor or in the response takes place during observations. In Fig. 11 the rotor, operating at speed, is assumed to be unbalanced, requiring corrections at both balancing planes *A* and *B*. The first operation consists in determining the motion at each bearing as found and its phase relation to any reference datum on the

[11] Service Engineer, Westinghouse Elec. & Mfg. Co., So. Phila. Works.

rotor. Let these motions be represented by *I* and *II*, the points *p* and *q* representing the instantaneous positions in the paths of motion agreeing with the instant the reference on the rotor is passing through the upper position. Thus is the reference held throughout.

A known test weight, considered as a unit weight, is then inserted in balance plane *A*, for convenience at the reference point. The rotor is run to speed and a second set of observations made at each end represented by *Ia* and *IIa*, which will differ from the initial set. The weight is next transferred to the other end in the same angular position in plane *B*, and a third set of observations made, represented in the figure by *Ib* and *IIb*.

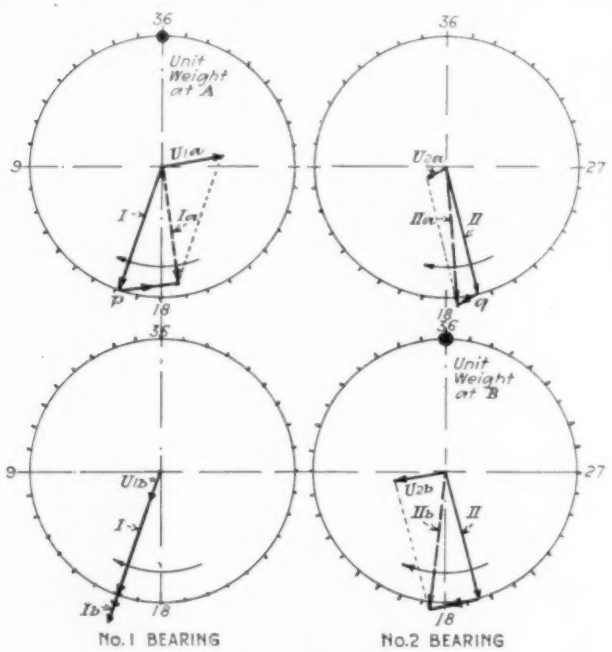


FIG. 13 DERIVATION OF UNIT MOTION VECTORS

(*I* and *II* represent motion at No. 1 and No. 2 bearing due to the initial unbalance. *Ia* and *IIa* result from adding unit weight at *A*. *Ib* and *IIb* result from adding unit weight at *B*. The closing vectors *U1a* and *U2a* are due to unit weight *A* alone, and *U1b* and *U2b* are due to *B*.)

No further information is necessary. The next step is the reverse of the procedure described in the experiment; that is, to find the ellipses of motion for the two trials which when added to the initial motions *I* and *II* would reproduce the composite motions *Ia*, *IIa*, *Ib*, and *IIb*. The process is indicated in detail for the figure *Ia* for the effect of the weight *A* on bearing No. 1. The ellipse *I* is the initial motion as before, and *Ia* is the motion with the weight added at *A*. The vectorial subtraction of *I* from *Ia* determines the motion *U1a*, which may be called the unit motion at the No. 1 bearing due to a unit weight at *A*. It is the motion at the No. 1 bearing which would have obtained had the rotor been in perfect balance before adding the weight *A*.

The process is repeated for deriving the unit motions for the other three observations, thus obtaining *U2a*, *U1b*, and *U2b*. With this information the resultant motions at both bearings for any combination of weights can be determined. Thus, by varying the weights at *A* and *B*, and shifting their location, the proper combination can be found by graphical or other means whose resultant motions will coincide with the initial motions observed, and the problem is completely solved.

The determination of the proper combination of weights can be made graphically, or by a mechanical device for combining two pairs of SHM's at right angles, or analytically. J. P. Den Hartog[12] has developed the complete mathematical analysis for the solution of the two-bearing unit by this method.

An analysis of the problem discloses that a superfluity of information is obtained from the ellipses; that two independent solutions exist which should check each other. This arises from the fact that the system considered is a dual one, having independent dynamic characteristics in both the vertical and lateral directions. Therefore, it should be possible to solve the unbalance by observing only the component of motion in either the vertical or lateral direction, without determining the shape of the ellipses. The method proposed takes advantage of this fact.

The necessary apparatus, therefore, can be greatly simplified, using the original arrangement with the vibrometer, observing the pointer with stroboscopic illumination as illustrated in Fig. 12.[13] If the vertical motion is chosen, then the phase reference is determined by the angle through which any datum on the rotor has turned from the vertical position, at the instant the motion reaches its vertical extremity. In the arrangement shown, the dial of the interrupter is turned until the needle stands out in the extreme clockwise position.

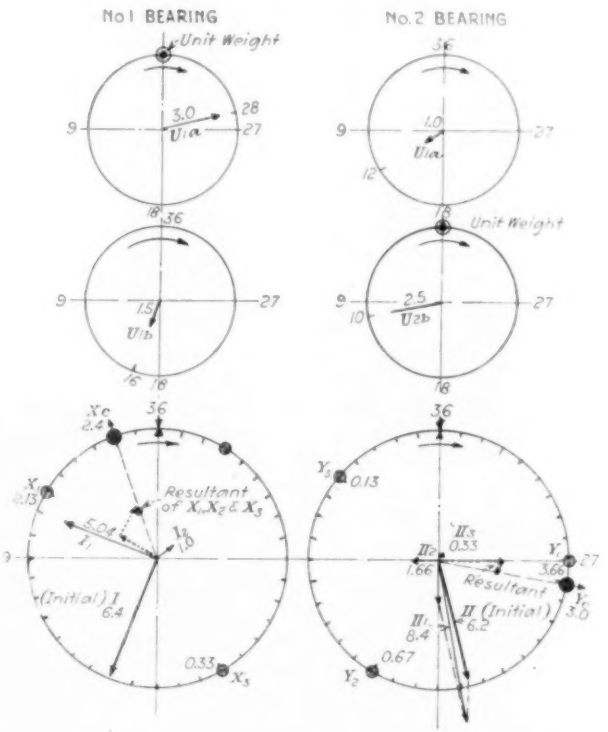


FIG. 14 GRAPHICAL METHOD OF DETERMINING INITIAL UNBALANCE BY MEANS OF UNIT MOTION VECTORS, FOR COMPONENT IN ONE DIRECTION

The conditions assumed in Fig. 11 are reproduced in Fig. 13 using only the vertical component of motion. The diagrams are again oriented by fixing the rotor datum in the upper position. *I* and *II* represent the initial vertical motion due to unknown unbalance, the points *p* and *q* being the instantaneous positions in the motion when the spindle datum is vertical. The motions can now be expressed as vectors representing amplitude and direction.

[12] Engineer, Mechanics Section, Research Dept., W. E. & M. Co., East Pittsburgh.

[13] This method was used to determine the phase data shown in Fig. 10.

With the known weight in A, the vectors are changed to Ia and IIa, and with the weight in B, vectors Ib and IIb result. The weights are assumed at the datum for convenience. The vectors representing the influence of the unit motions alone are derived, as shown, from the effect of the trial weights on the initial motion. In each case they are given as the closing vectors from the initial motion due to unbalance to the motion as affected by the weight. These unit vectors represent the motion which would have resulted had the rotor been in perfect balance before the trial weights were added.

As before, the solution consists in varying the amounts and locations of the two weights until the proper combination is found, producing vectors which duplicate the original vectors. This weight combination is then equivalent to the unknown unbalance. The mathematical solution, based on the linear component alone, is still quite involved.[14]

A simple graphical method for determining this combination will be described. In Fig. 14 are tabulated the unit vectors which are to be used as the basis for determining the unknown unbalance whose motion vectors are shown below. The graphical process consists in first assuming a weight at one end which cancels the initial vector at that end. Thus, in the example, the unit weight A would be multiplied by $I/U1a = X_1$ units and shifted to position X_1 to produce an equal and opposite vector nullifying vibration at this end. But in doing this, an additional vector is added to the motion at II, equal to $X_1 \times U2a$, in the corresponding direction. The resultant of this addition to the initial vector II is II_1.

For the next step, an assumed weight Y_1 at the B end is found whose vector cancels II_1. The only unbalance now remaining is that produced at the A end by the addition of Y_1 at the B end, and is equal to $Y_1 \times U1b/Ua$ and is canceled by X_2 at the corresponding location. As the continued process involves a rapidly converging series for the usual condition, one or two more adjustments should reduce the residual to an insignificant amount.

The several assumed weights, X, X_1, X_2 and Y, Y_1, Y_2, etc., are then added vectorially to give the resultant corrections Xc and Yc to apply to the rotor. If each weight produces a lesser motion at its own end than at the opposite end, then a motion at one end is corrected by inserting a weight at the opposite end, and in this way a converging series is obtained as before.

Four-Bearing System

The method outlined is applicable to the four-bearing system, as represented by the turbine-generator unit, but the solution is of course more involved. In addition to the initial observation of motion at all four bearings, four trial runs will be required to permit observations with the unit weight in each end of each rotor.

In Fig. 15 the four-bearing unit is represented with assumed initial motions at each bearing due to unbalances in each rotor. Below are tabulated assumed unit vectors, there being four sets of four simultaneous observations. The problem, as before, lies in adjusting the amounts and locations of the weight in A, B, C, and D until the combination is found which cancels all of the initial motions. Although the solution is much more tedious for the four-bearing unit than for the single rotor, an inspection of the unit vector table and comparison with the initial motions should yield clues as to about where the corrections belong, and thus indicate the sequence of trials, for the most rapid reduction of the initial motions.

Once having obtained the unit vectors, these methods then really represent balancing trials on a model of the unit, and are made on the drafting board, rather than on the actual unit itself, with obvious advantages. The solution by vectors based on the lateral component should check the vertical.

Evaluation or Vibratory Response of the Installation

The tabulation of unit weight vectors for each bearing in Fig. 15 supplies a criterion for evaluating the sensitivity of response in any installation at the operating speed due to unit unbalance. It has been a common experience to find that a given amount of unbalance produces a different vibratory response in two identically similar units on different types of foundations, or even on similar foundations. Types of units will also differ in their characteristics on similar foundations.

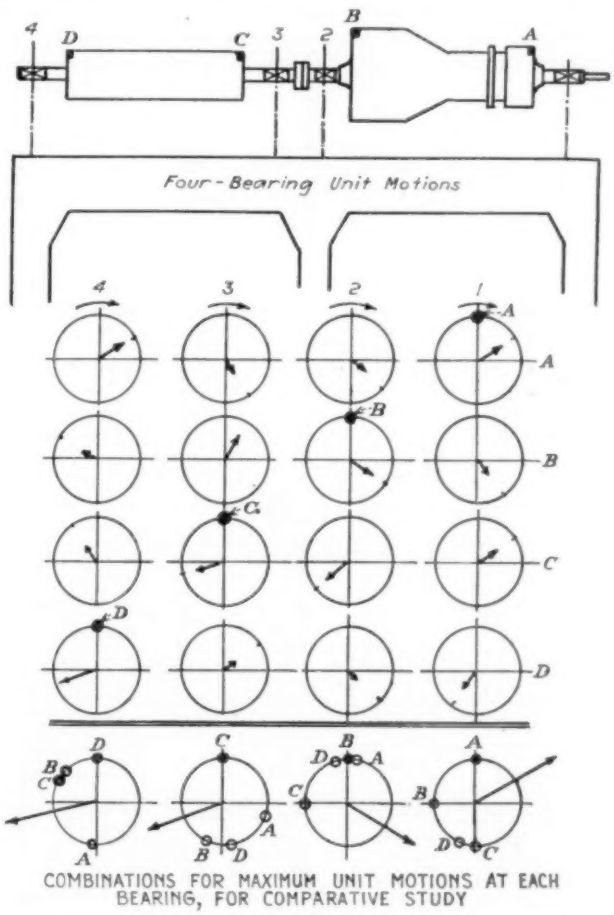


Fig. 15 Unit Motion for Four-Bearing Installation

The evaluation of sensitivity should therefore be valuable for comparative study of the characteristics of structures least conducive to vibration and in indicating desirable features of design. Such a criterion or index may be had by assuming a unit unbalance in both ends of both rotors at the same time, their angular relations being adjusted to produce a maximum vibration at each bearing in turn, as indicated in Fig. 15. The value of the largest response, and the average of the four, would furnish definite information as to sensitivity.

PART II FACTORS AFFECTING VIBRATION OTHER THAN SIMPLE, FIXED UNBALANCE

Vibration Due to Rotating Element

An analysis and solution of the vibration problem such as

[14] The solution, similar to Mr. Den Hartog's for the ellipses of motion, was worked out for the author by Nils O. Myklestad, undergraduate at the University of Copenhagen, Denmark.

outlined would not be complete without a discussion of its limitations and of complications encountered in practice which may on occasion make balancing difficult or even impossible. A recognition of these possible conditions must be had before any balancing process can be carried out intelligently and with assurance of results. The difficulties may have nothing to do with balancing.

One of the stipulations it is necessary to make in the method described is that the initial unbalance remains the same for each observation. It is obvious that any appreciable change in balance except that produced by the known trial weight will result in the derivation of a false unit motion. This is a most important point, as the usefulness of the method depends upon obtaining reliable and consistent data, and any small changes

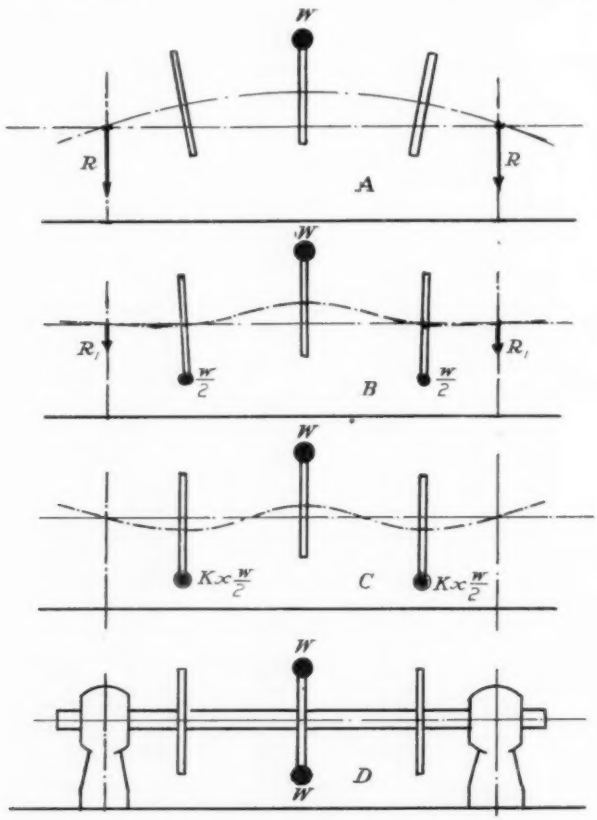


FIG. 16 EFFECT OF SHAFT DEFLECTION PRODUCED BY INTERNAL COUPLES AT SPEED

(*A*, deflection at speed due to unbalance at center. *B*, dynamic corrections $W/2$ at ends produce unbalance at speed because of greater displacement at center. *C*, additional corrector weights at ends to secure balance at a given running speed. *D*, ideal correction.)

which may occur throughout the range of operating conditions must be accounted for.

A number of purely mechanical faults may produce vibration, such as journal flats, rubbing contact at glands, dummy, spindle body, end bells, or loose windings. Vibrations set up by shaft whipping due to conditions in the oil film or to cramping fits have been described.[15] Other causes, such as pinched bearings and coupling misalignment, have been encountered. A slight wabble at the shaft has been suspected of setting up an oil-film condition conducive to local heating and hence increase of distortion. A bent shaft may set up axial vibration, particularly if the bearings are not properly self-aligning.

Rotors having considerable flexibility may undergo serious deflection at speed before the centrifugal and elastic forces balance. If the unbalance is chiefly near the middle, then the corrector weights at the ends produce double flexure at speed, and therefore must also compensate for an additional amount due to the preponderance of eccentric mass at the middle section. This

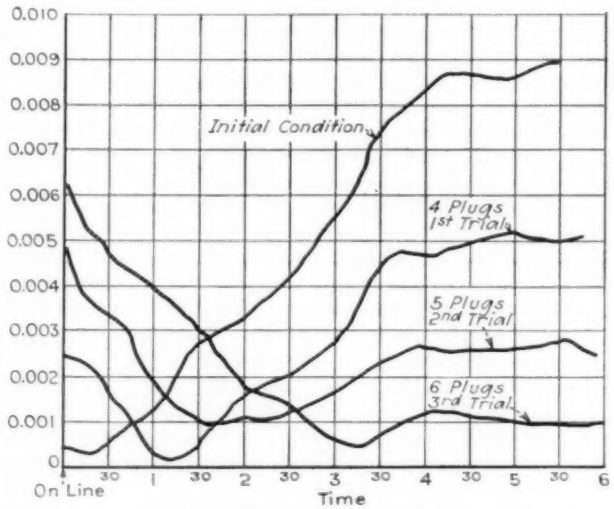


FIG. 17 VIBRATION-TIME RECORD, SHOWING EFFECT OF BALANCING OPPOSITE A DISTORTION REQUIRING ABOUT FOUR HOURS TO DEVELOP

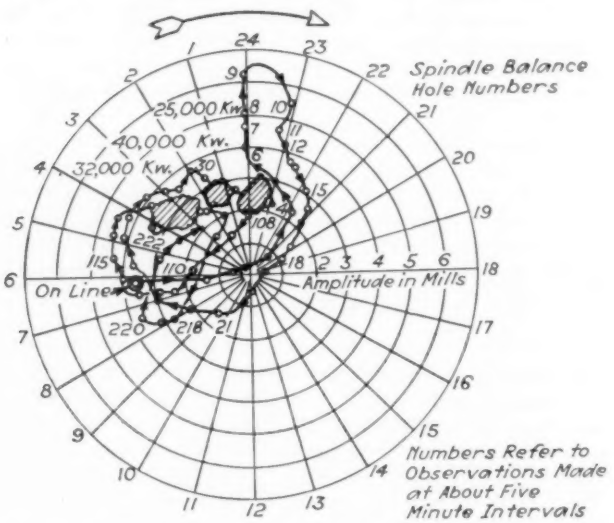


FIG. 18 CONTINUOUS RECORD OF VERTICAL VIBRATION VECTORS AT NO. 1 BEARING OF A 40,000-KW. UNIT, SHOWING CHANGES IN UNBALANCE WITH LOAD

(Numbers refer to observations made at about 5-min. intervals.)

is illustrated in Fig. 16. These excess corrector weights can be made to produce balance at a given speed, but only at that speed. In extreme cases axial vibration may be produced by the resulting wabble at the journals. In practice, the stiffness of most rotors permits satisfactory correction at the ends, but where flexibility must be taken into account, any large "static" unbalance correction is applied near the middle. So far as known, there is no device or method for determining the axial locations of unbalanced mass concentration, except by noting the effect on flexure developed at speed.

[15] (*a*) "Shaft Whipping." Dr. B. L. Newkirk, *Gen. Elec. Review*, vol. 27, no. 3, March, 1924. (*b*) "Shaft Whipping Due to Oil Action in Journal Bearings," Dr. B. L. Newkirk and H. D. Taylor, *Gen. Elec. Review*, vol. 28, no. 8, August, 1925.

Because of the practical impossibility of realizing ideal annealing, slight changes in the mass axis may occur under operating temperatures. These changes can often be detected with suitable instruments, and are sometimes sufficiently appreciable to warrant balance correction. On large units, from two to four hours of operation are required to develop fully the normal running condition. If the running condition is materially different from the cold condition, the direct procedure lies in lo-

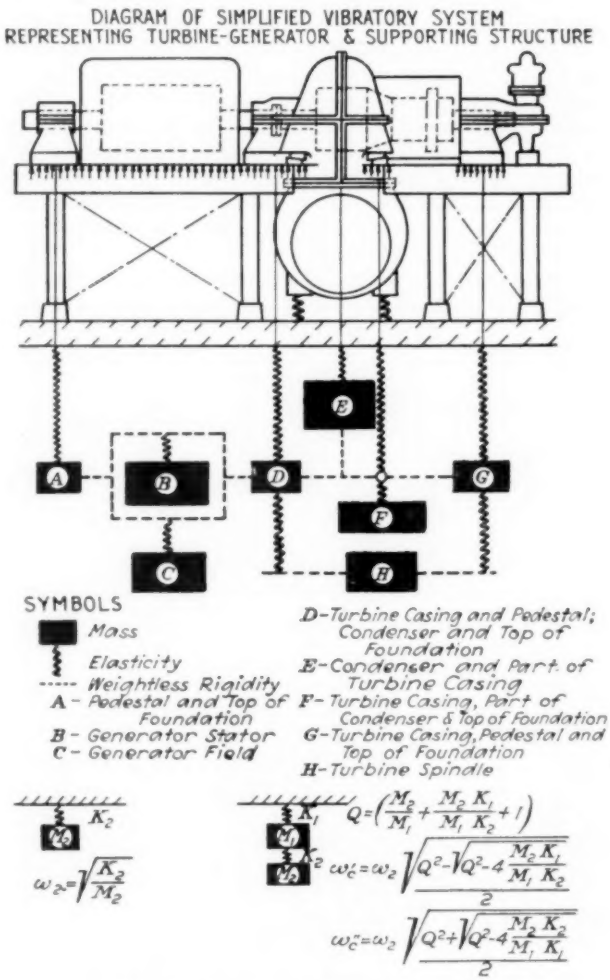


FIG. 19 SCHEMATIC FOUNDATION SYSTEM

cating the direction of the shaft distortion immediately after shutting down, and balancing opposite. Such a rotor will come to speed with some vibration, smoothing out after three or four hours. In Fig. 17 are shown vibration-time curves in an abnormal case, illustrating results of balancing opposite the bend. It is of course necessary to wait until conditions have stabilized before attempting observations for balancing by analysis of motion.[16]

Occasionally the balance is affected by changes in operating conditions, as by changes in load, temperature, etc., and in a few instances there was found a definite and consistent relation between distortion and certain loads. In such a case, the best average balance is desired. If the amplitudes at a bearing are plotted in their proper phase relation on a polar chart, and continued observations made throughout the range of operation at frequent intervals, a picture is had indicating the migration of the unbalance. Thus in Fig. 18, which is an actual record, the tendency for the points to group themselves for certain loads is apparent. The wide swings shown on this chart were due to other causes which will be described. As the best average balance is wanted, the center of the clusters is adopted for use in the balancing process.

Spindles are sometimes distorted in operation by outside agencies. Gland water at the high-pressure end getting to the hot spindle has often been the cause of trouble, especially of an intermittent nature, changing with the pressure behind the gland and simulating distortions supposedly due to load and temperature changes.

A spindle which has been standing while hot will "hump up,"

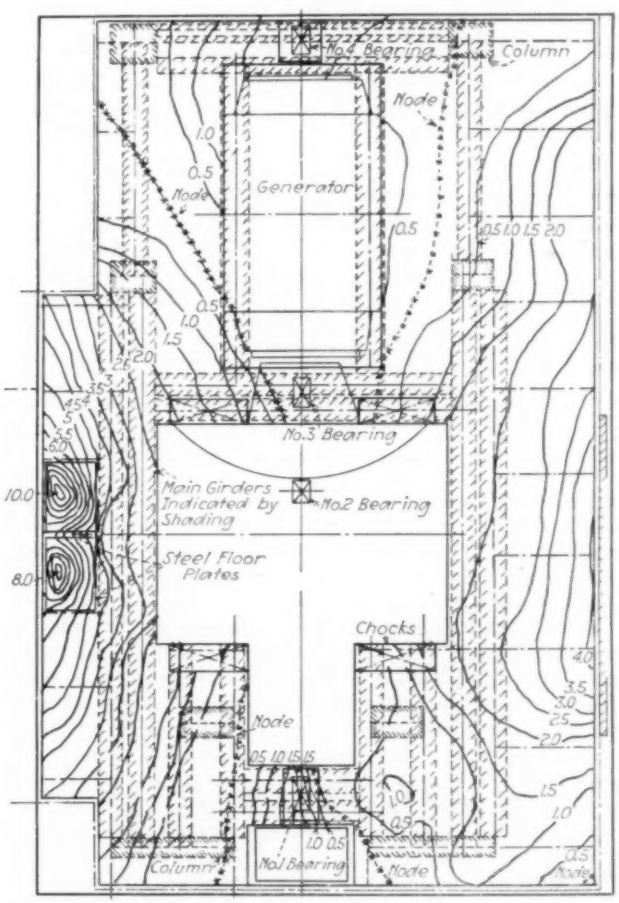


FIG. 20 FOUNDATION VIBRATION AT TURBINE FLOOR, IN MILS
(Load, 39,000 kw. Time, 10.15 to 10.45 P.M., June 24, 1927.)

due to the stratification of gases. This distortion reaches a maximum in from 8 to 12 hours on large solid rotors, and then slowly decreases. In starting up, most of this distortion must be "rolled out" before the unit is brought to speed. The spindle is usually rolled until the bend has straightened to within some prescribed amount, usually about 0.0015 in. (determined by experience from its effect on vibration), and then brought to speed. The location of this residual bend at speed is purely accidental, as the spindle may have stood in any position. The vibration which it produces usually disappears in from 30 to 45 minutes. It may either be such as to augment or reduce

[16] See also "Recent Applications of the Electric Micrometer," A. V. Mershon, *Gen. Elec. Review*, Oct., 1928, vol. 31, no. 10, p. 526, for examples of fluctuations after placing a unit in service, the records being made on the shaft itself by means of the electric micrometer.

other unbalances in the spindle, so that the vibration on going in service may vary greatly each time. These transitory conditions must be taken into account before attempting to balance.

Vibration may be caused by a fault in the rotor itself, as by a defective joint or bolting. Fortunately, such cases have been extremely rare. The wide swings shown in Fig. 18 were due to inadequate press fit in conjunction with a peculiar gland-water condition, which appeared to cause a working of the joint in one direction. On one other occasion of erratic vibration which could not be balanced out, an 8-lb. pipe plug was later found loose in the spindle cavity.

Vibration Augmented by Stationary Parts

It is now generally understood that the nominal critical speed

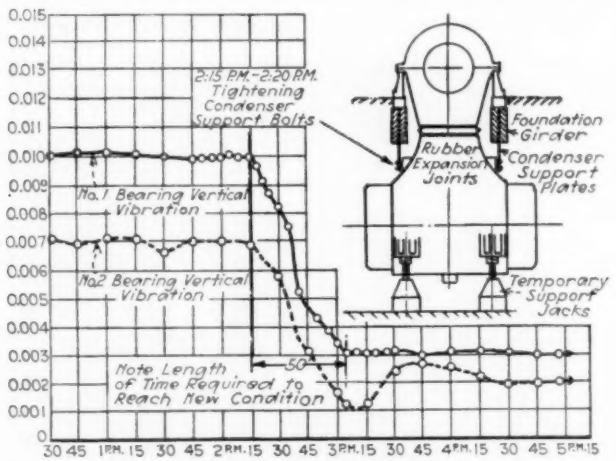


Fig. 21 Reduction in Vibration Brought About by Bracing Top of Foundation Through the Condenser

of the rotor (calculated with the assumption of rigidly supported bearings) loses its identity when the rotor is mounted on an elastic structure, and that the combination of rotor and foundation characteristics produces a number of different resonant speeds in various modes of vibration.

Eason[17] has expressed graphically the relations between the individual natural frequencies of rotor and support and the resulting critical speeds assembled, for the elementary compound system. Soderberg[18] suggests a diagrammatic method for solving the problem, with two or more degrees of freedom. These methods represent the nearest approach to the problem, but are for simple conditions only. Other treatments both here and abroad are based on such simple ideal assumptions, as mass concentration and elastic continuity, that their practical value is very limited.[19]

[17] Loc. cit.

[18] "On the Practical Application of the Theory of Vibrations to Systems With Several Degrees of Freedom," C. Richard Soderberg, *Philosophical Magazine*, vol. 5, Jan., 1928.

[19] See (*a*) Eason, loc. cit. (*b*) "Vibrations in Foundations," Dr. Ing. Ernst Schmidt, *Z.V.D.I.*, appendix, vol. 67, no. 2, Jan. 13, 1923. (*c*) "Von Bau der fundamente fur Turbogeneratoren," Von Obering. Dohme, *Elektrotechnische Zeitschrift*, vol. 46, no. 8, Feb. 19, 1925. (*d*) "Some Aspects of Structural Vibration," W. P. Digby, *Jour. of the Inst. of Structural Engrs.* (Eng.), Nov., 1926. (*e*) "Modern Steam-Turbine Foundations," Von Obering, Dohme, translated abstract in *Power*, vol. 66, no. 5, Aug. 2, 1927. (*f*) "Die Berechnung der Eigenschwingungen von Rahmenfundamenten," by Dr. Ing. Willy Prager, *Der Bauingenieur*, vol. 8, 1927. (*g*) "Berechnung der Schwingungserscheinungen and Turbodynamos," Dr. Ing. J. Geiger, *Z.V.D.I.*, Mar. 24, 1923. (*h*) "Rechnerisch Ermittelte und Gemessene Schwingungszahlen an einem Turbinen fundament," by Gèrhard Mensch, *Der Bauingenieur*, vol. 9, 1928. (*i*) "Stodola," Lowenstein, 1927, vol. II, p. 1138.

In Fig. 19 the author has attempted to reduce the dynamic characteristics of a large installation to the simplest representative system. Calculations of the many possible modes and critical speeds is practically out of the question. Yet the lateral vibration and the many diagonal and complex modes are also to be considered. To those who have made a study of the problem, it must appear remarkable that the operating speed of so many installations has not coincided with the natural frequency of any of these numerous modes.

Vertical vibration amplitudes on the top of the foundation of a large unit have been plotted and points of equal amplitudes connected in the diagram shown in Fig. 20. A symmetry of pattern is evident. This is more of a Chladni figure than a contour map, as the vibrations are opposite in phase on either side of the nodal lines. Vibration of this character is not at all unusual, and the figure indicates the complexity of motion usually encountered. These nodal lines of zero vibration could hardly have been predicted, and if the nodes are not determinable neither is the natural frequency.

The amplitude of vibration at any point in any direction depends not only on the amount of unbalanced forces, but also

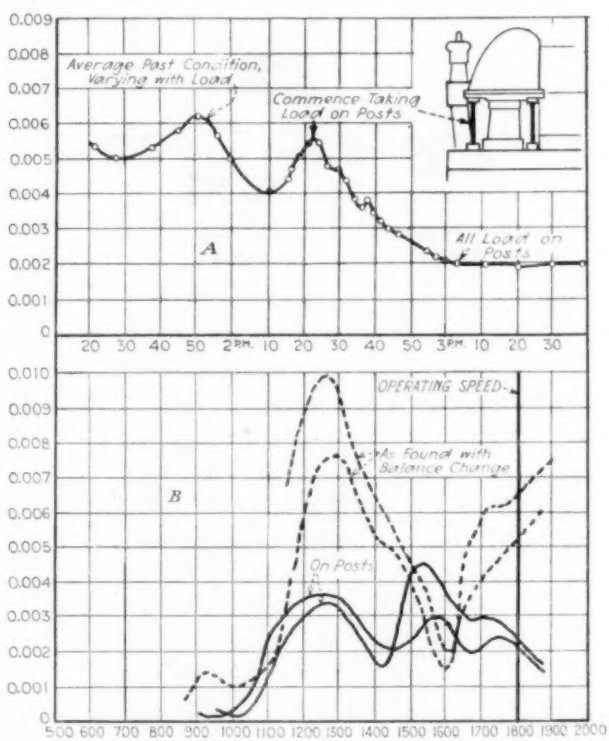


Fig. 22 Reduction in Vibration Effected on a 30,000-Kw. Unit by Changing Elastic Scale of the Support

(*A*, Vibration-time curve during alteration to scale of supports. *B*, Resonance curves before and after altering supports.)

on the proximity of the running speed to one of these numerous major or minor resonances. Herein lies at least a partial explanation of the dissimilar vibratory response of like units. Until more data are available governing the characteristics of an installation, the attempt to stipulate a certain fixed amplitude which the vibration must never exceed should be made with reservations.

Transient vibratory conditions can be set up through changes in the unit or foundation by temperature or other distortions Resonance characteristics depend among other factors on the distribution of loading. Distortions due to unequal heating

of the columns or warping of the structure can very materially shift the points of loading. Thus a natural frequency may be sufficiently changed to cause it to pass through the operating speed, producing a temporary increase in vibration. Such conditions are more apt to take place during the starting up period.

Several cases have been encountered where changes in the method of supporting have profoundly affected the vibration in operation. Fig. 21 is an actual record of the effect of altering the sling supports of the condenser of a 20,000-kw. unit and blocking up the bottom feet. In Fig. 22-*A* is shown the reduction in vibration secured by reducing the elastic scale of the cylinder supports at one end of a 30,000-kw. unit which had resisted all attempts at balancing until resonance was recognized as the cause. The condition encountered is readily explained by the Berger curves shown in Fig. 22-*B*. A resonance peak just below 1800 r.p.m. was converted into a valley by the substitution of the lighter supports. In this case it was easier

It has been a familiar experience to encounter localized vibration at various isolated points throughout the structure even when the units were known to have been in fair balance. On one occasion a rather severe vibration of the light flooring contiguous to the unit was practically eliminated by placing a weight on the antinode. Large vibrations frequently exist in one or more structural members which are nearly tuned to the running speed. On other occasions severe vibrations occur at lower speeds as they are passed through. Sometimes these conditions can be relieved; often, however, they are regarded as necessary evils which cannot be avoided except by drastic changes to the installation.[21]

In Fig. 24 is illustrated the manner in which structural members may vibrate. The motion of one column was parallel to the axis of the unit, while the other was transverse. Under certain conditions a violent and rhythmic energy exchange took place between the columns, one reaching maximum when the

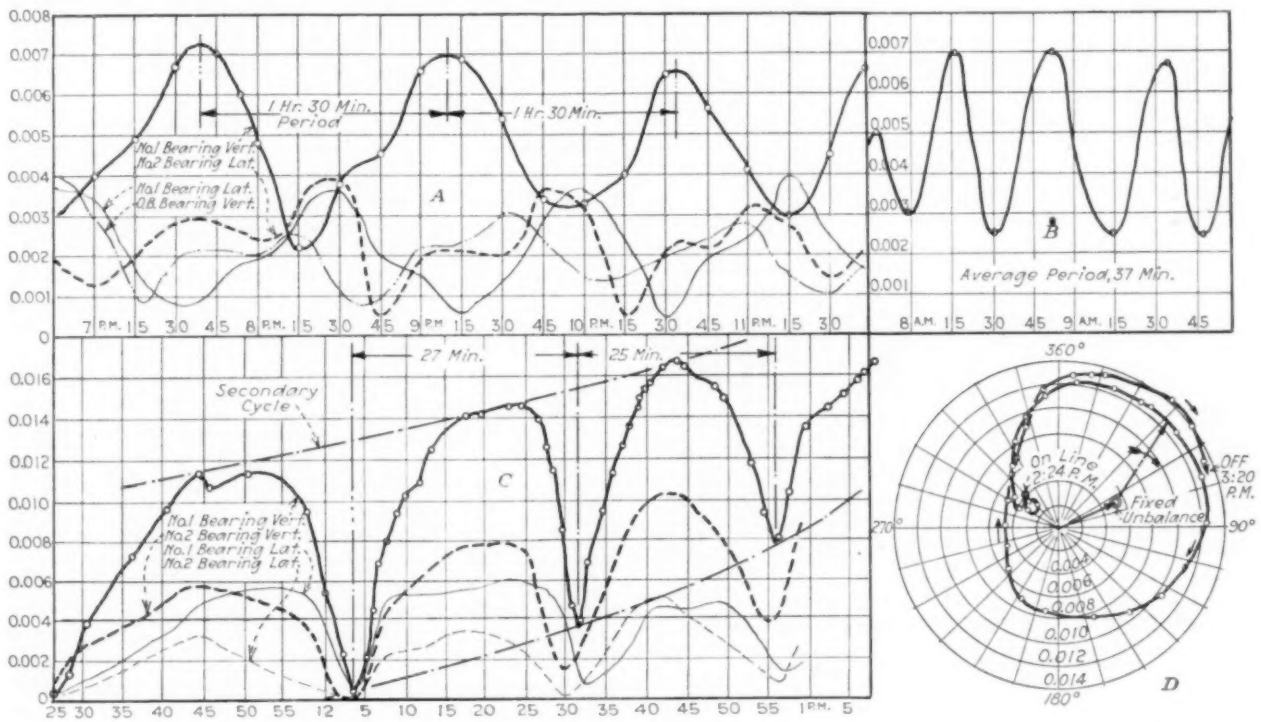


FIG. 23 CYCLIC VIBRATION

(*A* and *B*, cyclic vibration on a 20,000-kw. unit. *C*, cyclic vibration on a 25,000-kw. unit. *D*, vibration vectors plotted to show apparent combination of a fixed and rotating unbalance, causing cycles.)

to alter the turbine supports than the foundation. Neither was at fault; the combination simply was unfortunate.

A most remarkable phenomenon has been encountered in at least two installations, characterized by a rhythmic waxing and waning of the vibration, in regular periods ranging from fifteen minutes in one case to an hour and a half in another. The analysis of motion seemed to indicate a combination of a fixed unbalance and a steadily creeping unbalance, the maximum and minimum vibration occurring when the two were in conjunction and opposition, respectively. Fig. 23 is typical of several actual records made with the vibration phasometer, indicating the observed vibration was made up of a fixed and a rotating vector. The vibration disappeared when a peculiar resonance in the supports and a vibrational energy exchange between the cylinder and steam chest were eliminated.[20]

[20] See footnote 8.

other was at the minimum. It should be stated that this large vibration was due to excessive unbalance in the spindle, and that in this case additional bracing to reduce the column vibration actually increased the amplitude on the unit itself. In other words, the columns served to a slight extent as vibration absorbers at that running speed.[22] When the unbalance in the spindle was eliminated, the column vibration was reduced to a small amount.

While a comprehensive discussion of turbine-generator foun-

[21] See "Vibration of Turbine Generator Foundation" and "Curing Resonant Vibration in Turbine Units," by the author, *Power*, Apr. 3 and 10, 1928. For a simplified chart which is useful in determining readily the effect of mass and scale on the natural frequency of the unit, see "Vibration in Ships," by the author, in *Marine Engineering*, Sept., 1928, p. 507.

[22] See "The Theory of the Dynamic Vibration Absorber," footnote 7.

dations does not come within the scope of this paper, attention should be drawn to some of the important features which have a direct bearing on vibration characteristics.

The question of steel versus mass concrete is receiving more attention today.[23] The almost general tendency in Europe on the larger sizes has been toward the heavy concrete structure. In this country are to be found examples of both extremes, from the massive concrete foundation to the tall, light, steel structure.

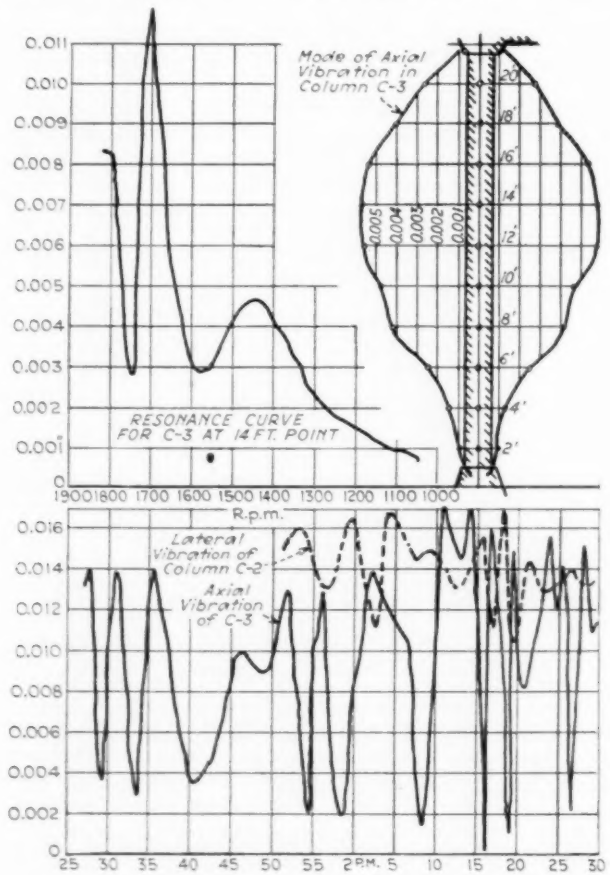


FIG. 24 FOUNDATION-COLUMN VIBRATION

Each has its advantages and disadvantages. The steel foundation is practically imperative where space is the paramount consideration. The problem of erection and the location of auxiliaries calls for this type of structure. Many large units are operating quite satisfactorily on relatively light steel foundations. But if insurance against vibration and allied operating difficulties is permitted to become the prime consideration, the indications are that the advantage lies with the massive concrete structure.

Upsetting the proper distribution of loading has been cited as affecting vibration. There have been cases found where steel columns expand 0.040 in. or more due to local heating, thus materially altering both the loading and the alignment. Local heating of longitudinal members, commonly encountered because of the location of the steam chest or piping, causes a lateral twisting and hence misalignment. Any attempt to relieve strains by postponing the final riveting at the joints until all the load is imposed can only be partially effective unless the structure has assumed the temperatures existing in operation. Some of these objections peculiar to steel because of its heat-absorbing propensity can be overcome by proper insulation. The welded structure, which is probably not so far off, will require especial attention with regard to prevention of internal stresses.

The unit stresses on even the lightest of structures from the static loading are commonly quite low, giving the impression that an ample factor has been allowed for the dynamic loading. Structural engineers, however, are prone to think of all dynamic loading in terms of the impact or "live" loading to which bridges and such structures are subjected. It is not always appreciated that dynamic stresses are a function of amplitude and that small harmonic forces may set up a much more severe condition than larger intermittent impacts. To take advantage of a time-honored illustration, a bridge designer would hardly attempt to compute the stresses caused by a body of soldiers marching in step unless he knew to a certainty the natural frequencies of his bridge.

Although the dynamic conditions at the operating speed are unquestionably of most importance, the operation at all speeds up to 10 per cent overspeed must also be considered. It would be very desirable to reduce the vibration which many units now go through at some lower "critical" speed. Stress calculations based on amplitudes under resonant conditions become so involved as to be impractical. But rather than assume an arbitrary minimum live-load factor, it would appear to be possible to adopt a procedure which would cover at least the fundamental modes of motion in the lateral and vertical directions. Such a method could be based on the centrifugal force due to an assumed eccentricity (say, 0.005 in.) and an amplification factor (say, 10) to take care of the ratio of the operating speed to any resonant speed, with the proviso that the stiffness should be such as to limit the resulting deflection to a given amount (say, 0.005 in.). The values given are tentative, by way of illustration, and neglect the mass of the stationary parts, but would insure only moderate vibration during abnormal conditions.

Because of the meager control now possible over the more complex resonance characteristics and the uncertainties which confront each new installation a recognition of the fundamental principles governing forced vibration in an elastic system, other than these resonance considerations, is desirable. The periodic forces of unbalance are absorbed in three ways: (1) In accelerating the masses involved. (2) In deflecting the elastic members. (3) In overcoming friction.

Other things being equal, the acceleration, and hence the resulting amplitude of vibration, is reduced by increasing the weight of the structure, increasing its stiffness, and employing material with the greatest internal damping. Simplicity of structure is also desirable, in that better control over the modes of vibration is secured.[24]

As the sizes of units become greater, with advances in design, and as other conditions such as are due to increases in temperature become more exacting, further attention may well be directed to the role which the foundation plays in influencing the vibratory characteristics of the unit as a whole.

In conclusion, the author wishes to express his appreciation of the valuable advice and assistance derived from his association with Prof. E. Therkelson, A. R. Robertson, J. Den Hartog, G. B. Karelitz, and B. V. Anoschenko, and to J. Burdette for aid in preparing this paper.

[23] See "Turbo-Generator Foundations," J. R. James, *Electrical World*, Dec. 1, 1928, vol. 92, no. 22, p. 1089, for discussion of comparison between steel and concrete. See also "Steel Construction in Power Plants," *Power Plant Engineering*, Oct. 1, 1927, p. 1028, and "The Design of Structural Supports for Turbo-Generators," E. H Cameron, presented at December, 1922, meeting of American Society of Civil Engineers.

[24] See "Foundations for Machinery," N. W. Akimoff, Trans. A.S.M.E., 1920, p. 441, for the extreme in simplicity.

Discussion

S. TIMOSHENKO.[25] The author is to be congratulated on a very interesting paper. Guided by the theory of vibration, he has developed a new method of balancing based on an application of the stroboscope together with the vibrometer. In this way he can obtain the amplitude and the phase of pedestal vibration with reference to a definite point on the rotor. Having these data for the initial condition of the rotor and for the case when a known unbalance is added, the position and the magnitude of the initial unbalance may be calculated on the basis of certain assumptions. The experiments with trial weights showed that in many cases the method is accurate enough and may be recommended for the practical solution of balancing problems. The author also described the limitations of the method. Calculations are based on the assumption that the phase relation between the vibration of a pedestal and a definite point on the rotor remains constant. There are cases, however, when this phase is continuously changing. The author mentions several possible causes for such changes in the phase relation, but it seems that this phenomenon is not yet completely explained and that further investigation is desirable.

The paper contains a description of the motion of pedestals. It will be very interesting to get records of the motion of the shaft itself. The usual theory of vibration is based on the assumption that there are no clearances between the shaft and the bearings. In practice such clearances always are present and, due to changes in the thickness of the oil film, a certain amount of lateral motion of the shaft with respect to the bearing becomes possible. Hence, vibration of the pedestal may not represent the actual lateral motion of the shaft. The measurement of lateral motion of the rotor may be interesting in investigating the above-mentioned phenomena of changing phase.

Another important point discussed in the paper is the investigation of the vibration of foundations and the establishment of certain methods for determining the sensitiveness of foundations to vibration. The author uses trial balance weights for this purpose. More satisfactory results may be obtained in a much simpler way by using a special d.c. motor with definite eccentric weights. By running such a motor at various speeds, the critical speeds for the foundation and the corresponding types of vibration can be investigated. Such a method has been used in several cases by Westinghouse engineers at the suggestion of the writer. Just recently an analogous method was used for studying foundation vibration in Germany.[26]

It would be well if the author should give additional explanations of the terminology used in certain cases. A reader interested in dynamics of machines but not working specially in practical balancing will find some difficulty in understanding such expressions as "balancing at speed," "balancing in testing cylinder," "Marsland method of compensating for the effect of the second end balance weight," etc. Additional footnotes would be very helpful in such cases.

A. NADAI.[27] A very interesting result of the author's elaborate tests seems to be the astonishing fact of certain fluctuations of the amplitudes he measured during longer periods of time on steam turbines. The amplitudes of certain vibrations seem to increase and to decrease periodically. The periods of these fluctuations of amplitudes are astonishingly long, as certain fluctuations of the amplitudes have periods of the magnitude of one-half hour or more. The writer is much interested in this and should like to ask the author if there can be found a certain mechanical or other explanation for such unusually long fluctuations in the elastic vibrations of machine parts.

B. A. ROSE.[28] The paper gives a very interesting discussion of the experimental work done by the author in connection with the vibration and balancing of turbine-generator units. There are a few points brought out, however, for which there are no satisfactory explanations. That concerning the very gradual increase in the lag angle of the vibratory motion of the pedestal as the speed of rotation increases is very remarkable. It would be very interesting in this connection to have a record of the change in the lag angle of the turbine spindle itself, or of a true spot on the shaft as close to it as possible, to compare with the above. If the turbine spindle and the pedestal bearing are considered to be separate, distinct masses of a vibratory system and the unbalanced force is on the spindle, then the motion of the spindle itself should be of considerable interest in the study of the vibration.

The point the author makes concerning the flexibility of extra long rotors having an influence on the axial location of the balance weights is well taken, but this effect is of no importance except on the very long rotors. This condition will not be encountered on the turbine in any case, and will be confined always to an exceptionally long generator rotor.

It is the general practice of the Westinghouse Electric & Manufacturing Company to remove the greatest possible amount of the unbalance from their turbo-rotors by drilling in the heavy side. This drilling is done in the body of the rotor itself and particular care is taken with the longer rotors to get the static unbalance weight removed uniformly along the rotor body. This eliminates any possibility of this "double flexure" effect causing trouble.

M. STONE.[29] The paper presents an excellent picture of the complexity of the vibration problem in the operation of large machinery. In the second part, the author has pointed the way toward matters which still remain unsolved, and that will necessarily guide the efforts of future workers. Experience of this kind can only be obtained by close work on such important problems.

In his first section, the author has reviewed the various methods that are commonly appealed to in making balance adjustments in the field—a procedure that is continually becoming more and more necessary as machines are getting larger, and particularly more flexible.

In addition to the various methods that the author has listed for obtaining quickly the amount and position of static unbalance, the writer would like to add the following, which presents a simple way of getting phase angle from a slowing-down curve.

If we let the motion of a rotating shaft in one plane be represented by the conventional differential equation

$$M\ddot{x} + c\dot{x} + Kx = Mw^2e \sin wt \qquad [1]$$

where w = rotational frequency, the steady-state solution for the motion is

$$x = \frac{Mew^2 \sin (wt - \epsilon)}{\sqrt{(K - Mw^2)^2 + c^2w^2}} \qquad [2]$$

where

$$\tan \epsilon = \frac{cw}{K - Mw^2} \qquad [3]$$

[25] Professor of Mechanical Engineering, University of Michigan, Ann Arbor, Mich. Mem. A. S. M. E.

[26] See paper by Prof. H. Kayser in *Z.V.D.I.*, vol. 73 (1929), p. 1305.

[27] Research Department, Mechanics Section, Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pa.

[28] Power Engineering Department, Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pa.

[29] Research Mechanical Engineer, Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pa.

ϵ is the lag angle between motion and force, and since w is rotational frequency of the shaft, ϵ is the physical angle as scribed on the shaft itself. Now from [3] it follows directly that

$$\sin \epsilon = \frac{cw}{\sqrt{(K - Mw^2)^2 + c^2w^2}} \quad [4]$$

Now, obtaining from [2] the maximum amplitudes X as a function of w, we have

$$X = \frac{Mew^2}{\sqrt{(K - Mw^2)^2 + c^2w^2}} \quad [5]$$

Combining [4] and [5],

$$\sin \epsilon = \frac{X}{w}\frac{c}{Me} \quad [6]$$

and at the critical frequency w_c, [6] becomes

$$\sin \epsilon_c = \frac{X_c}{w_c}\frac{c}{Me} \quad [7]$$

Now at $w = w_c$, $\sin \epsilon_c = 1$ from [4]. Then combining [6] and [7], we obtain

$$\sin \epsilon = \frac{X}{X_c}\frac{W_c}{c} \quad [8]$$

Thus from a slowing-down run, X_c and w_c are obtained, and at any w, X is measured on the vibrogram—this gives the phase angle ϵ of the unbalance ahead of the motion. Now if a mass m'

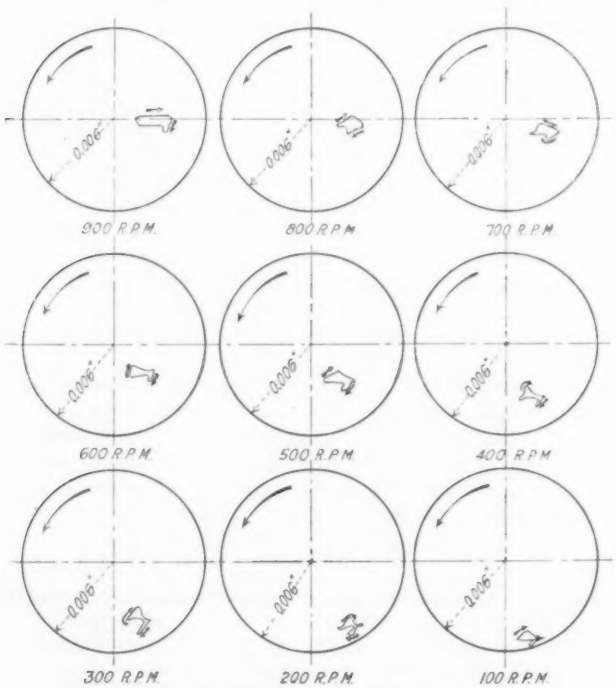


FIG. 25 EXPERIMENTAL STUDY OF SHAFT BEHAVIOR

is placed at a radius r, at the specified place, and a second run taken, we measure the amplitude at the speed w, X', we have from [5]

$$X = \frac{Mew^2}{\sqrt{(K - Mw^2)^2 + c^2w^2}} \quad [9]$$

and

$$X' = \frac{(Me - m'r)w^2}{\sqrt{(K - Mw^2)^2 + c^2w^2}} \quad [10]$$

Dividing [9] by [10] and solving for the necessary balance, as determined from the trial balance, (where $mr = Me$)

$$m = m'\left[\frac{1}{1 - \frac{X'}{X}}\right] \quad [11]$$

If m' is too large, then the correct value of weight at radius r is

$$m = m'\left[\frac{1}{1 + \frac{X}{X'}}\right] \quad [12]$$

The author's next important discussion has to do with the observation on bearing motion as affected by the vibratory condition of the rotor. In this connection, the writer made a series of very accurate experiments in 1927, studying the motion of the shaft of a two-bearing 7500-kva. 900-r.p.m. synchronous condenser. The motion was obtained to less than 0.00001 in. by very sensitive oscillographic recording of magnetic air-gap changes, and a series of pictures is shown in Fig. 25 giving the motion as a function of speed. The point the writer wishes to make in this discussion is the lack of simplicity of the motion—bearing little relation to the theoretical ellipse; the inability to determine phase relation; and further that vibrograph tests on the bearing pedestal showed quite different characteristics. This the author has also observed, but the important consequence is that balancing methods that depend on *accurately determined* values of lag angle and amplitude become difficult and often impossible.

The author's last discussion of the balancing of multiple-bearing sets presents the logical method of attack of the problem. However, the practicability of obtaining the correct combination of balances is difficult. In such combined sets, the simpler way to balance is to uncouple the parts, balance each unit separately, and then when vibration is observed in the coupled set, the cause is definitely attributable to incorrect alignment, which must then be corrected. In general, the author has emphasized the complexity of the balancing problem, when a rigorous solution is attempted.

AUTHOR'S CLOSURE

Dr. Timoshenko's suggestion that further investigation of the nature of the shaft motion itself would be desirable in the attempt to explain the continuous and progressive shifting of the phase of vibratory motion at the bearing with speed, as exhibited in Fig. 10, is quite in order. It is first necessary to differentiate between the absolute motion of the shaft in space and its net motion with respect to the bearings, as the apparent motion of the shaft is in part due to its own whirling from flexure, in part due to play in the oil film, and also very largely due to the bodily movement of the bearings. A. R. Robertson and the author have attempted to determine the amplitude and phase of the shaft motion coincident with a study of the bearing motion, and were able to derive the whirl motion of the shaft at a given point with respect to the bearing. If pendulum-type vibrometers are used, a vectorial subtraction of the shaft motion in space from the bearing motion yields the net shaft whirl as indicated in Fig. 26. Considering only the vertical (or lateral) component of motion, if s represents the amplitude and phase with respect to some datum on the rotor of the apparent shaft motion as determined by the pendulum vibrometer, and if b represents the motion at the center of

the bearing, then w, the vectorial difference, represents the net shaft motion or whirl neglecting the play in the oil film. The latter uncertainty can be eliminated by bringing the vibrometer

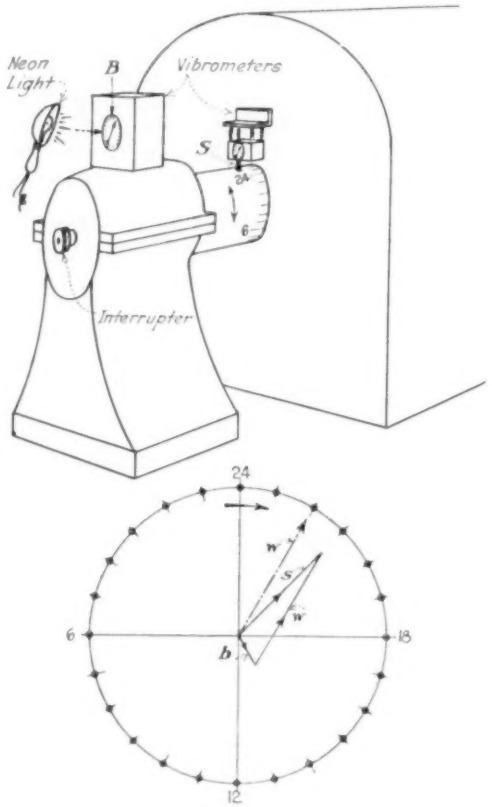


FIG. 26

b = amplitude and phase of vertical component at bearing B.
s = amplitude and phase of vertical component at shaft S.
w = vertical component of shaft whirl from flexure alone (neglecting position of other journal and play in oil film.)
NOTE: The complete paths of motion can be derived from the combination of simultaneous lateral and vertical observations at bearing and shaft.

indicator in contact with the journal itself. By a special arrangement the actual elliptical path of motion of the shaft at S can be determined, and from this the shape of the whirl due to shaft flexure alone. The data obtained by the author to date are not sufficiently complete to show just how the shaft motion follows the bearing motion throughout the speed range.

In one case the shifting of a node in the shaft itself along the axis was demonstrated for various conditions at the operating speed of 1800 r.p.m. It is proposed to extend the investigation to cover these and other points.

Some time ago, Dr. Timoshenko recommended that resonance characteristics of structures might be studied by means of an independent vibrating source, avoiding the necessity for operating the unit itself. This method appears to offer considerable advantage, as the vibrator may consist of an unbalanced motor which can be attached at various points on the structure or on the rotor itself. Although no important data have been obtained as yet, a motor-generator set driving a d.c. motor with an elaborate speed control has been secured to carry out such investigations.

With regard to the desirability of securing extreme flexibility in the turbine-generator supports, it is the author's belief that while such a system is feasible for smaller units which can be mounted on an integral and relatively stiff slab or base, with a flexible support, such an arrangement is impracticable for the larger sizes because of the difficulty in producing a continuous baseplate of sufficient rigidity. This baseplate itself in connection with the machinery masses might become resonant at the operating speed. If the various chock supports and pedestal bases are separately mounted on springs or the like, there would probably result so many simple, easy modes of vibration, each with its own critical speed, that it would be difficult to control these motions in bringing the unit to the operating speed without danger of rubs or coupling difficulties. In operation, there would exist little resistance to distortions or other disturbances, and the torque reactions would also present a problem in maintaining alignment.

To Dr. Nadai's question as to the cause of the cyclic vibration exhibited in Fig. 23, the author can state that every conceivable operating factor was investigated in the attempt to find something which could be correlated with the cycles. The only clue obtained was in the alternating nature of the vibration of the unit proper and the appended masses, consisting of the steam chest, throttle valve, and main steam line. These two systems reached their maximum and minimum vibration times about 180 deg. apart, suggesting an energy-exchange phenomenon. Such a system can be duplicated by means of a weight suspended by a spring, with means for altering the rotational moment of inertia of the weight, so that the torsional frequency can be made to agree very closely with the up-and-down frequency. If closely tuned, the bobbing motion will gradually be transformed into the purely torsional motion and so on indefinitely with a long period, as long as energy is supplied. The cyclic vibration described is very rare, and little opportunity has been available for a thorough investigation.

In reply to Mr. Donnell, the author believes that the progression of nodes and antinodes across a point of observation while running through the range of speed, accounts for the steady rotation of the phase of motion at the bearing with speed. During these tests the vibration waxed and waned in going continuously through the speed range apparently in some relation to the phase changes. These changes occur no matter where the vibrometers are located on the bearing, although the phase and rate may differ.

The question Mr. Rose asked concerning the motion of the shaft is answered in the reply to Dr. Timoshenko. Regarding the possibility of compound flexure in rotors, it was found necessary to balance out any appreciable static unbalance in the older long, double-flow spindles by applying the correction near the center, because of the deflections set up when concentrating weights at the ends of the spindle body. In fact, many of these spindles were provided with an extra balancing ring near the middle position. The later, solid types of spindles are sufficiently rigid to permit complete correction at the ends. Most of the static deflection occurs between the ends of the body and the journals. As the rotating elements become longer, however, the possibility of appreciable compound flexure must be kept in mind.

Mr. Stone's method of obtaining the value of phase angle from data obtained while the rotor is slowing down, is an interesting addition to the subject-matter, as it attacks the problem from a different angle and employs the fundamental concept of forced vibration of an ideal system. In 1923 the author carried out rather unsatisfactory experiments along these lines, in the attempt to determine the lag angle through observations at various speeds.

The equation of motion as stated specifies damping directly proportional to velocity. Although this assumption is known to be incorrect for mechanical vibratory systems, the discrepancy is probably small, and the solution for the equation is simple, leading to the following expression for the phase angle (using Mr. Stone's notations):

$$\tan \epsilon = \frac{C\omega}{K - M\omega^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots [13]$$

If it is assumed that the damping factor C, the elastic restoring factor K, and the inertia factor M remain constant, then the phase angle ϵ can be defined in terms of the speed ω. In the actual experiments, two things were determined: the speed and the corresponding "high spot" scribed on the shaft (or point in path of motion of bearing with relation to a datum on rotor).

In Fig. 27 the angular locations of the high spots found at three different speeds ω_0, ω_1 and ω_2 are indicated. The angles α and β are directly observed. The actual lag angle ϵ_0 between the initial high spot at speed ω_0 and the force P is unknown and is to be determined. As we are not interested in the absolute values of the damping, elastic, and inertia factors, the expressions for the phase angles at the three speeds may be set down:

$$\left\{\begin{aligned} \tan \epsilon_0 &= \frac{\omega_0}{\frac{K}{C} - \frac{M}{C}\omega_0^2} \\ \tan (\epsilon_0 + \alpha) &= \frac{\omega_1}{\frac{K}{C} - \frac{M}{C}\omega_1^2} \\ \tan (\epsilon_0 + \beta) &= \frac{\omega_2}{\frac{K}{C} - \frac{M}{C}\omega_2^2} \end{aligned}\right.$$

from which the ratios $\frac{K}{C}$ and $\frac{M}{C}$ may be eliminated and ϵ_0 determined.

Such methods, however, generally break down on any but elementary ideal systems. It was for this reason that the author was led to develop the unit vector method proposed in this paper which is applicable to the more complex systems encountered in practice.

Mr. Stone's experiments on the 7500-kva. condenser are most instructive in showing the gradual shift of the journal position and change in the oil film as the speed is varied. The shaft whirl indicated is between $1^1/_2$ and 2 mils, which may be considered insignificant for speed of 900 r.p.m. and below. Further improvement in balance would hardly appear necessary.

The superimposed erratic motions of a quarter of a mil or less are to be expected under the conditions described, and especially where the total motion is so small. When the practical difficulty of maintaining shaft roundness to within a quarter of a mil is appreciated, it becomes a question whether the ripples disclosed by apparatus recording to the hundredth of a mil are due to the surface irregularities or to actual shaft motion. It is also necessary to insure that such records do not indicate changes in the magnetic characteristics of the shaft periphery rather than changes in the air gap or displacement. The data observed in this isolated case may not be used as a basis for generalizing on conditions to be found in practice on all types of units.

The motion of the bearing pedestal is generally much more consistent and reliable than that of the shaft, and after all, the sum total of the disturbances to the installation must be transmitted via the bearings. In all but two cases, the author has encountered fairly uniform elliptical motion, sufficiently undistorted to permit repeated checks on phase points to within a few degrees. This is a statement of actual experience on eight large units, which may be considered representative. The distorted motions observed in the excepted cases were found to have been due to impactive disturbances between bearing and pedestal, or bearing housing and its seat, and actually furnished the clue to this condition. In such cases the mechanical difficulty should be corrected before any balancing process is attempted. This brings up the point that the proposed balancing process has an indirect value in disclosing conditions which are not due to simple unbalance, thereby indicating that investigation should be directed elsewhere.

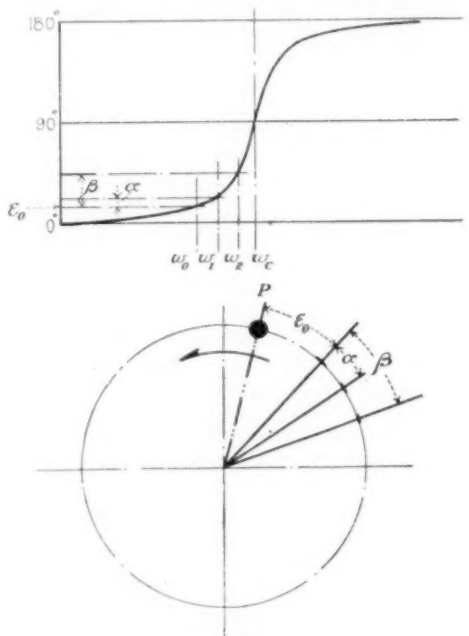


FIG. 27

The solution of the unbalance in the three- or four-bearing units by the proposed method is not so formidable as it may appear, as an inspection of the unit vector table should suggest at once the most advantageous sequence of graphic trials. On small, simple machines it is naturally in order to balance the two rotors individually. Unfortunately, this procedure is generally out of the question for large turbine-generator units in the central station, first, because it is rarely practical to take a second large unit out of service to bring up the generator field synchronously, and secondly, because of the desirability of balancing under normal load conditions at stabilized temperatures.

It has not been the author's intention to convey a too pessimistic impression of the complexities encountered, but rather to indicate that with improved methods of observation and better knowledge of the agencies at work, turbine operating difficulties are being more readily overcome.

APM-51-24

Stress Distribution and Hysteresis Losses in Springs

Progress Report No. 4 of the A.S.M.E. Special Research Committe on Mechanical Springs (Technological)

By M. F. SAYRE[1] and ANTHONY HOADLEY,[2] SCHENECTADY, N. Y.

The present report covers the first year of investigations carried on by the Committee at Union College, Schenectady, New York. The purpose of these investigations is to examine more closely than heretofore the fundamental characteristics of spring action in order that more reliable constants for a code of design on mechanical springs (see Bibliography, No. 18) might ultimately be established. The experiments were conducted by Anthony Hoadley under the direct supervision of M. F. Sayre. Certain of the results were covered in Progress Report No. 3, which report was not published. It should be considered as a progress report, in that, while a number of valuable results have been obtained, the field opened up is so wide that much of the work must be considered as preliminary only.

Particular points of interest are: (1) A study of distribution of stresses in bending in spring steel, at the very high unit stresses often used in practice. This has practical importance in furnishing a partial explanation of the large discrepancy between theory and practice as to allowable unit stresses, in furnishing a theoretical guide as to the amount of recoil to be expected when winding a spring about a mandrel of a given size, and in determining the effect of the initial stresses induced in a spring during the winding operation, upon the future behavior of the spring. (2) A comparison of proportional limit and of modulus of elasticity for the same material in bending and in tension at high and at low stresses, both in virgin condition and after a cyclical state has been reached. This has a possible importance in determining what weight should be attached to tensile tests in measuring the suitability of a proposed spring material. It also was done in order to investigate certain theoretical and practical discrepancies between properties in bending and tension. (3) A study of the comparative internal friction in spring material in bending and in tension, for varying stress ranges, up to very high loads. This has a bearing in its effect on the possible accuracy of spring-controlled measuring instruments, and a smaller bearing on the energy losses in power springs. It is also highly desirable to know accurately how this internal friction or hysteresis varies with the load range and the position of the load range in order to determine what relation may exist between it and both the proportional limit and the fatigue limit of the material, as governing allowable safe loads. An unexpected result of the investigation is that the hysteresis in tension was found to be far less in proportion to that in bending than was expected. This result should be considered as tentative only. Every effort is being made to discover any instrumental defects which may have contributed to this result, and lacking this, to find satisfactory theoretical reasons.

COMMON assumptions in engineering work are that all stresses dealt with are below the proportional limit of the material used, and that the distortions of the structure are of negligible magnitude. Both these assumptions are of doubtful validity in spring design. As a result, independently of the problems involved in the manufacture, selection, or protection of the spring material, many problems arise, which of course are of varying importance in different cases. Some of these problems are:

1 What is the effect upon true stress distribution in the metal of using working stresses which exceed the proportional limit of the material, and may approach the yield point?

2 During the manufacture of springs, internal stresses are introduced (*a*) during cold rolling or drawing, and (*b*) during cold shaping or coiling. How great are these initial stresses, and how much effect do they have upon the behavior of the spring after loading? Do these initial stresses make matters worse, or can they be so placed as to partly counterbalance the effect of the working stresses? How much effect, quantitatively, do they have?

3 Internal friction in the material operates to lower the accuracy of a spring as a measuring device, and to decrease its efficiency as a tool for storing up power for later use. What laws govern this hysteresis? Does the law of hysteresis variation in any way correlate with the fatigue limit of the material? Is this wasted energy in any way being used to gradually break the strength of the material?[3]

4 Springs distort sufficiently under loading so that the changes in shape are often not of inappreciable magnitude. What effect do these changes produce (*a*) directly, as through changes in load lever arms, and (*b*) indirectly, by causing secondary stresses? Can customary formulas be modified to care for these effects?

5 Natural frequency, particularly as it may cause resonance, often is an important factor. What is the natural frequency of springs of various types? What factors may affect natural frequency? What secondary vibrations exist?

Theoretical and practical studies have been made during the past year in an endeavor to open up an approach to the first three problems outlined. Attention has been concentrated on bending behavior rather than on torsional behavior, as a simpler problem and one present in practically all springs, even in helical springs under tension or compression, where the initial internal stresses due to the process of coiling are of importance. Some preliminary work on torsional theory has been done to lay a foundation for a future program. Tensile behavior has been studied as a necessary part of the program.

The experimental problems have not been entirely simple. The elastic changes in shape of a metal, particularly in tension or compression, are themselves small. Much of the work done has required the measurement of slight discrepancies in this elastic behavior of the order of magnitude of 0.00001 in., and so calling for the highest available degree of precision, with repetition of work to allow for elimination of erratic results and for use of average values. As a logical method of attack, one typical material was selected and its behavior studied in detail, with a view to later inclusion of other materials.

The material selected for the initial tensile and bend tests

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[2] Research Associate of the A.S.M.E. Special Committee on Mechanical Springs, and Assistant Professor, Union College.

Presented at the Annual Meeting, New York, N. Y., December 3 to 7, 1928, of The American Society of Mechanical Engineers.

[3] As a side issue also arises the question of external friction, both air friction and rubbing friction, in power springs and such springs as hair springs, which are constantly in motion. How great is this friction and how may it be reduced?

was a $^1/_2$-in. by $^1/_8$-in. oil-tempered wire. By using a fairly large size, the difficulties in determining the cross-section are made smaller. The flat shape was chosen so that as large a proportion of the material as possible should reach the proportional limit in bending at the same time.

The manufacturer states that the wire had gone through the following processes:

A tempered annealed rod $^1/_2$ in. × 0.162 in. is rolled and straightened and then lead-annealed at 0.160 in., rolled to 0.147 in., then to 0.140 in., lead-annealed at this stage, and then rolled successively to 0.135, 0.130, and 0.125 in.

In hardening and tempering this wire, the furnace for heating is at 1550 deg. fahr. and the temperature of the tempering furnace is 900 deg. fahr.

The analysis of this material is as follows:

Carbon	0.68 per cent
Manganese	0.67 per cent
Sulphur	0.046 per cent
Phosphorus	0.036 per cent
Silicon	0.201 per cent

The hardness numbers obtained were:

Shore	51.7
Brinell	374
Rockwell	C 39-C 40 (manufacturer's figures.)

The body of the report is divided into three sections: a statement of tension methods and results, followed by results of bend tests, and that in turn followed by a summary.

Physical Properties in Tension

This section of the work included the usual tensile tests,

TABLE 1 INITIAL TENSION STRESS-STRAIN CURVES

Bar	Max. stress —1000 lb.	Lower p.l. per sq. in.	Upper p.l.	E-1 Ascending —1,000,000 lb.	E-2	E-3 Descending per sq. in.—	E-4	Area of loop, in-lb. per cu. in.
75	79	...	...	30.1	30.1	30.1	30.1	0
71	80	...	...	29.9	29.9	29.8	29.8	0
72	80	...	...	30.1	30.1	29.9	29.9	0
81	139	104	...	29.7	28.3	..	29.5	10.5
82	139	103	...	29.8	..	..	..	7.0
77	150	67	117	30.5	29.1	29.7	29.7	27.7
73	160	83	133	29.5	28.2	29.2	30.1	29.9
83	159	100	135	29.3	26.4	28.1	28.9	28.4
95	170	106	150	29.7	26.3	28.0	29.7	58.8
121[1]	170	134	134	29.3	28.9	28.8	28.8	53.5
79[2]	170	105	150	30.3	28.8	29.4	30.0	10.7
109[2]	170	85	150	30.3	29.1	29.6	29.6	21.8
94	179	112	150	29.8	25.2	26.7	26.7	42.0

[1] Slow anneal from 200 deg. cent.
[2] Slow anneal from 335 deg. cent.

TABLE 2 CYCLICAL TENSION STRESS-STRAIN CURVES

Bar	Max. stress 1000 lb.	Lower p.l. per sq. in.	Upper p.l.	E-1 Ascending 1,000,000 lb.	E-2	E-3 Descending per sq. in.	E-4	Hysteresis, in-lb. per cu. in.	Median E	No. cycles
75	79	...	...	29.9	29.9	29.9	29.9	0	29.9	8
71	80	...	...	29.9	29.9	29.9	29.9	0	29.9	11
72	80	...	...	30.0	30.0	30.0	30.0	0	30.0	12
81	139	...	...	29.4	..	..	29.4	0	29.4	20
82	139	...	...	29.6	29.6	29.6	29.6	0	29.6	18
77	150	56	117	30.3	29.3	29.7	29.7	3.5	29.7	21
73	160	84	...	30.0	29.1	29.1	30.0	1.0	?	19
83	159	117	150	29.8	28.3	28.3	29.8	0.5	?	18
95	170	117	?	29.8	28.0	29.4	29.7	2.6	?	30
121[1]	170	100	150	28.9	28.2	29.2	29.2	2.8	29.2(?)	18
79[2]	170	134	150	30.7	29.5	29.6	29.6	2.0	29.6	24
109[2]	170	117	150	29.8	28.6	29.4	29.4	5.9	29.6	28
94	180	100	150	28.6	27.4	28.3	27.3	13.8	27.7(?)	75

[1] Slow anneal from 200 deg. cent.
[2] Slow anneal from 335 deg. cent.

carried to rupture. These were followed by an extended study of elastic behavior and of hysteresis for various maximum stresses, taking stress-strain curves with ascending and descending loads for a number of successive loads until cyclical conditions had been approached. The stress ranges studied and the results obtained are indicated in Tables 1 and 2.

Apparatus. The bars were tested in a machine of the hydraulic type which had been calibrated a short time before the tensile tests were made. The machine showed an error of not over one-sixth of one per cent over nearly its full range.

The extensometer used for most of the work was one of the mirror type developed by one of the authors.[4] The extensometer automatically averaged the elongations on both sides of the test bar. One division on the extensometer scale indicated a unit elongation of 0.00001 in. per in. An Olsen extensometer equipped with a "last word" dial on which one division corresponded to a unit strain of 0.00005 in. per in. was used in conjunction with a Riehlé electrical contact extensometer on the ultimate load tests and in conjunction with the mirror extensometer on some of the hysteresis loops.

The mirror extensometer was calibrated by taking a run on a previously calibrated bar. The mirror and Olsen extensometers were then both attached to a comparison bar with a 10-in. gage length. Simultaneous readings were taken on both extensometers during a load cycle so that their calibrations could be compared.

In the straight tension tests both the Olsen and Riehlé extensometers were attached to the bar at the same time. An initial load of 250 lb. (or 4200 lb. per sq. in.) was used, and readings were taken on both instruments or on only the Olsen up to the yield point. As its full range had been used in going to this point, the Olsen extensometer was then removed. Below the yield point the load was held constant while each reading was taken, but above this point the load was continually increased to the maximum, extensometer readings being taken "on the run."

The mirror extensometer was used in taking a majority of the hysteresis loops. The cycles were run between a stress of 4200 lb. per sq. in. and the maximum. The load increments were taken so that there would be 11 or 12 readings on each side of the loop. Readings were usually taken on the first, second, third, eighth, and eighteenth load cycles. Where the permanent set was likely to be large in the first few cycles, the Olsen extensometer would be used as it had a larger range than the mirror extensometer. When shifting from one instrument to the other, consecutive cycles, such as the third and fourth, or eighth and ninth, would be taken, using first the Olsen and then the mirror extensometer. The extensometers were left on the bar during the load cycles in which no readings were taken. It took about 9 min. to take readings on a complete cycle, and when readings were not taken, the load was applied at such a rate that a load cycle would be completed in 1.5 min.

Results. The tension tests showed characteristically, on first loading the specimen, an initial proportional limit of 80,000 to 126,000 lb. per sq. in., with a mean value of 103,000 lb. per sq. in. The modulus of elasticity up to this proportional limit ranged from 28.7 to 30.2 × 10^6 lb. per sq. in., with an average of 29.6 × 10^6 lb. per sq. in.

Above this initial proportional limit there was but a slight change in slope of the stress-strain curve, with usually a second straight portion of the curve with a slope indicating an apparent modulus of elasticity (for that portion of the curve only) averaging 26.8 × 10^6 and ranging from 25.2 to 29 × 10^6 lb. per sq. in. An "upper proportional limit" at the upper end of this second straight line came at 133,000 to 165,000 lb. per sq. in., averaging 144,000 lb.

This secondary straight portion of the stress-strain curve occurred so consistently in these tension tests that it seemed advisable to give throughout values for both the lower or true proportional limit, and of the upper pseudo-proportional limit.

[4] For a description of this extensometer see Proc. A.S.T.M., vol. 26, part 2, p. 660.

Extensometer readings were taken to a precision of 0.00001 in. per in. or better, so that the proportional limit as commercially determined would lie between the two apparent proportional limits as just given.

The yield point came at 175,000 lb. per sq. in., and varied but little in different bars. The ultimate strength ranged from 197,000 to 199,000 lb. per sq. in.

TABLE 3 TENSION-TEST SUMMARY

(Bars as received)

	Range	Average
Lower proportional limit (l.p.l.)	80–126,000	103,500
Upper proportional limit (u.p.l.)	133–165,000	144,000
Yield point		175,000
Modulus of elasticity below l.p.l.	28.7–30.2	29.6
Modulus of elasticity between l.p.l. and u.p.l.	25.2–29.0	26.8
Ultimate strength	197–199,000	198,700
Percentage of elongation in 2 in.:		
Including break	9.5–11	10.3
Adjacent to break	4–5	4.9

Table 3 summarizes the results of the tension tests. Yield-point, elongation, and ultimate-strength values are based on the tests of six bars. The other figures are based on tests of ten bars.

One erratic bar showed a lower virgin proportional limit of 67,000, with a second break in the curve at 117,000 lb. per sq. in.

Extensometer readings were taken at 175,000 and 180,000 lb. per sq. in. If a reading had been taken between these values it might have justified the reporting of a slightly higher yield point. The stress-strain diagrams were plotted to such a scale that below the proportional limit they make an angle of 55 deg. with the horizontal. The line between 175,000 and 180,000 lb. per sq. in. makes an angle of 5 deg. with the horizontal, so that there is a definite break in the line at 175,000 lb. per sq. in.

The fractures were about evenly divided between the cup and shearing types. When looking at the edge of the bar, the shearing fracture made an angle of about 45 deg. with the longitudinal axis of the bar. Both types of fracture showed comparatively smooth surfaces with a fine grain.

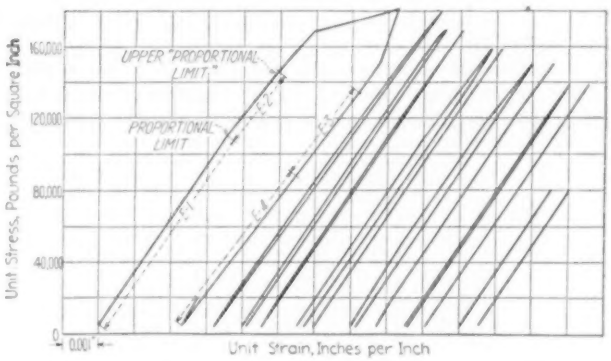


FIG. 3 INITIAL AND CYCLICAL LOOPS FOR BARS TESTED AT DIFFERENT MAXIMUM STRESSES

(The first loop of each pair is the initial one; the second cyclical.)

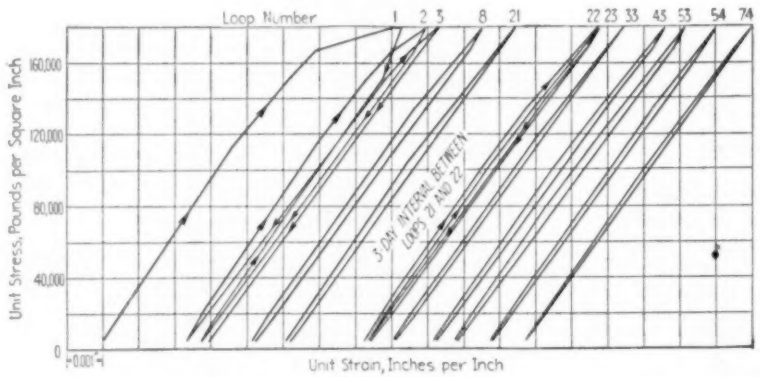


FIG. 1 SUCCESSIVE TENSILE HYSTERESIS LOOPS ON ONE BAR

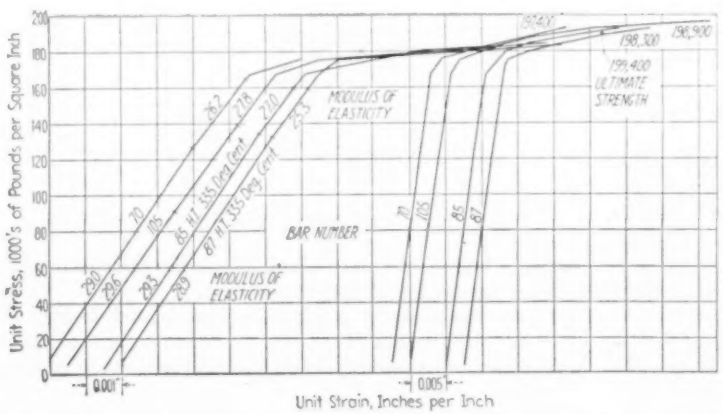


FIG. 2 TENSILE STRESS-STRAIN DIAGRAMS FOR SPRING STEEL. BARS NOS. 70 AND 105 AS RECEIVED; BARS NOS. 85 AND 87 AFTER SLOW ANNEAL FROM 335 DEG. CENT.

For the spring designer a knowledge of the condition of the material after it has reached a cyclical state, under varying loading, is probably at least as important as a knowledge of its behavior when first loaded. A particularly important problem in view of the high stresses which it is desirable to use in spring design is the effect upon elastic properties of repeated operation at stresses well above the proportional limit, approaching or exceeding the yield point. For this reason a large number of tests were made in tension, using varying maximum loads repeatedly applied, and taking accurate stress-strain curves for the initial loading and unloading cycle, and for successive cycles at intervals until a cyclical state had apparently been approached. Typical curves for successive runs on a single bar are shown in Fig. 1, and initial and final curves on different bars for various maximum stresses in Fig. 3. As indicated by these curves, the main part of the hysteresis loop consists of several straight lines, ordinarily two on the ascending and two on the descending segment, with shorter straight or curved lines connecting them at top and bottom. The two ascending segments in cyclical state are much more nearly parallel than for the initial loading, as would be expected, and it is somewhat uncertain whether the upper or lower break in the curve should be given as the true proportional limit. Both values are given in Tables 1 and 2.

It will be noticed that the lower proportional limit in the cyclical state is somewhat higher than for the initial loop, but that the upper proportional limit is usually the same. *E*-1, *E*-2, *E*-3, and *E*-4 are values of the modulus of elasticity corresponding to various parts of the loop, as indicated in Fig. 3.

The above values of the modulus of elasticity show the gradual changes that take place, but for purposes of computation or comparison a median value will ordinarily be more useful. For

this purpose a straight line has been drawn connecting the upper and lower ends of the loop, and its slope chosen as a median value of E. This median line is ordinarily closer to the descending than to the ascending side of the loop. The average value of the modulus of elasticity as given by this method is 29,700,000 lb. per sq. in.

Values of the hysteresis loss per cycle, in inch-pounds per cubic inch, have been plotted against maximum stress in Fig. 4. Below maximum unit stresses of 160,000 lb. per sq. in. the energy

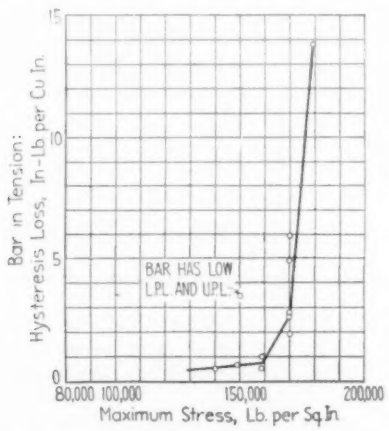


FIG. 4 LOSS OF ENERGY IN TENSILE HYSTERESIS

loss is extremely small, and corresponds to a mean width of loop of 0.000005 in. per in. or less. A sharp increase begins at 160,000 lb. per sq. in. This value lies between the upper proportional limit and the yield point of the material. These values are also plotted to logarithmic scale in Fig. 15. At 160,000 lb. per sq. in. the energy loss represents one-quarter of one per cent of the total elastic energy of the bar.

One particular bar gave erratic results. It showed a lower proportional limit of 67,000 on the initial loop and 56,000 after 21 runs at 150,000 lb. per sq. in. maximum load, both very much lower than for any other bar. Accompanying this low proportional limit was a relatively high hysteresis loss, reaching 3.5 in-lb. per cu. in., about six times the amount which would be expected based on the other bars tested.

The energy loss per cycle, for successive cycles on the same bar, is plotted in Fig. 6.

Most of the permanent set takes place in the first three cycles, so that after the third cycle the rate of energy loss greatly decreases. The first number after each curve is the serial number of the bar, and the second gives the maximum stress in thousands of pounds per square inch. After having partially attained a cyclical state, bar 94 had a three-day rest, and in that period it lost part of the effect of the cycles that had been run on it as shown by the large increase in loss of energy after the rest period. A loss of energy of 1 in-lb. per cu. in. represents a loop width of only 0.000006 in. when the maximum stress is 160,000 lb. per sq. in., so that the lower values on the curves cannot be expected to be very exact.

The tension-test bars were not run through as many cycles as the bend-test bars due to time limitations, and were probably not as near to the cyclical state. At the higher loads, where the maximum loop width would be 0.00005 in. or more, the set per inch in the last cycle might be 0.000005 in. to 0.00001 in. In the bend tests cycles were run until the last ten cycles showed a set in deflection of 0.0002 in., which corresponded to a set in the extreme fiber of 0.000001 in. per in., or from a fifth to a tenth of that obtained in the tension test.

One great difficulty in obtaining satisfactory hysteresis curves lies in the pseudo-hysteresis due to backlash in the extensometer used. This probably accounts for the sharp drop in energy loss in three of the bars after cycle 8, at which point a change in extensometers was made. All the other curves were made using the mirror extensometer. Calibration runs indicated that backlash with this extensometer was extremely small.[5]

BEHAVIOR IN BENDING

There are a number of factors entirely aside from the relatively simple features of the usual elastic theory of bending, which render the behavior of a material in bending much more complex than that of the same material in tension.

1 The elastic theory ceases to be valid as soon as stresses at any point pass the elastic limit. Furthermore, such initial overstressing, as in cold coiling a spring, introduces internal stresses in the metal and affects stress distribution for any future loading.

2 The side of the bar which is in compression tends to expand laterally, and the side in tension to contract, thus intro-

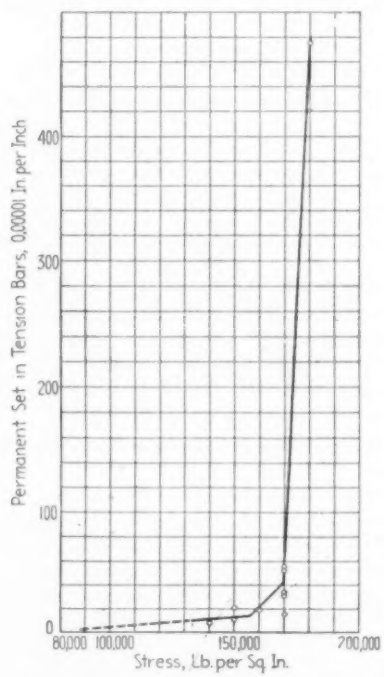


FIG. 5 PERMANENT SET AT CYCLICAL STATE IN TENSION

ducing a lateral curvature which affects the stresses in the bar and its apparent modulus of elasticity.

3 Bending moment is generally not uniform in magnitude over the entire length of the bar. As a result, stresses are also not uniform over the length and the yield point may be reached

[5] The amount of this backlash was estimated from several pairs of loops as follows:

Bar	Max. stress	Hysteresis areas, in-lb. per cu. in. Extensometer No. (1)	(2)	Difference between (1) and (2)	Equivalent backlash on 2-in. gage length
96	170,000	17	5.4	11.6	0.000136
95	170,000	14.6	3.8	10.8	0.000128
94	180,000	31.5	25.0	6.5	0.000076
121	170,000	16.7	3.9	12.8	0.000150

The two calibration runs which were subsequently made, with both instruments simultaneously on the same bar, indicated a backlash of 0.00018 and 0.00022 in. for extensometer (1) and zero for extensometer (2). The above table shows that the backlash can entirely hide the actual hysteresis value.

at certain places while others are still lightly stressed. Simultaneously, horizontal and vertical shearing stresses come into play, which modify the apparent stiffness of the bar.

These factors have of course been investigated in the past.[6] The results of much of the work done in England and Germany are not readily available, and consequently are not well known in this country; and apparently very little work has been done toward the direct application of these results in spring design. For these reasons both theoretical investigation and experimental work seemed necessary, and a start has accordingly been made.

The experimental work done to date has included studies of values of the proportional limit and of the modulus of elasticity on first loading, for comparison with corresponding tensile values; studies of the amount of recoil after overstressing, as an aid in determining the magnitude of internal stresses; of the rate at which the bars approach cyclical conditions when repeatedly loaded; and of values of hysteresis in cyclical conditions for ranges of stress from zero to a maximum value P, from zero to $^1/_2\,P$, from $^1/_4\,P$ to $^3/_4\,P$, and from $^1/_2\,P$ to P, for various values of P.

So far as could be discovered, the procedure employed in previous investigations of bending behavior had been to treat the test specimen either as a cantilever or else as a simply supported beam centrally loaded. In each case the bending moment varies over the length of the beam from zero to a maximum. Neither one even approximates the condition of uniform stressing present in most springs, whether laminated springs, power springs, or helical springs in bending. There are three other major objections: In the first place, when the test bar is loaded as a cantilever the bending moment varies along the length of the bar, and the material near the fixed end will be stressed to failure before the material near the load is even moderately stressed. In the second place, the presence of a shearing force leads to complications in computing the residual stresses in the bar after the outer fibers have been overstrained in bending. And the third reason is that the high local stresses at the jaw of the clamping device will have a considerable effect on the ultimate strength shown by the test specimen. Much better results can be obtained by so loading the bar that it is subjected to uniform bending over its entire length. This eliminates horizontal shearing stresses, and equalizes the tensile and compressive stresses so that the outside fibers over the entire length of the bar reach the proportional limit simultaneously. This condition of uniform bending moment can be obtained over the central section of a specimen by supporting it as a simple beam and loading it with two symmetrically placed equal loads. Except for unduly thick bars, however, the deflections will be very large before the proportional limit of the material is reached, and to allow them to take place the end supports will have to be on rollers. However, even if the details of this arrangement could be worked out satisfactorily, there would be a longitudinal tension in the beam due to its slope at the end supports.

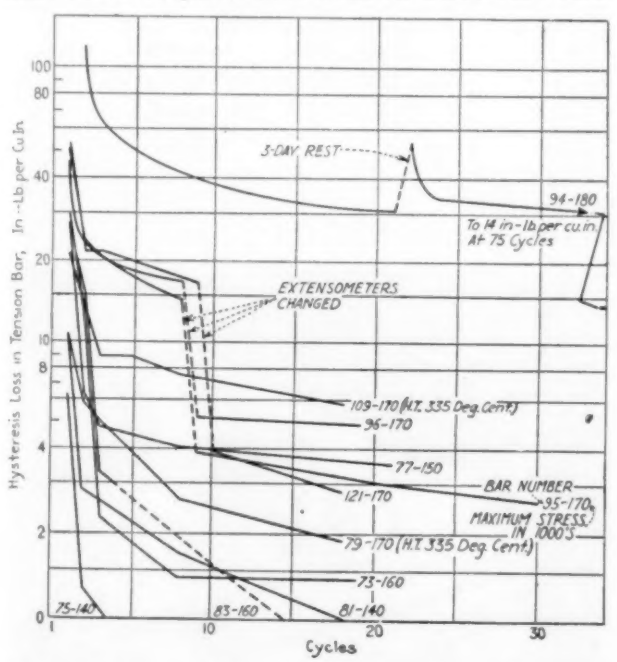


FIG. 6 INFLUENCE OF STRESS REPETITION ON TENSILE HYSTERESIS

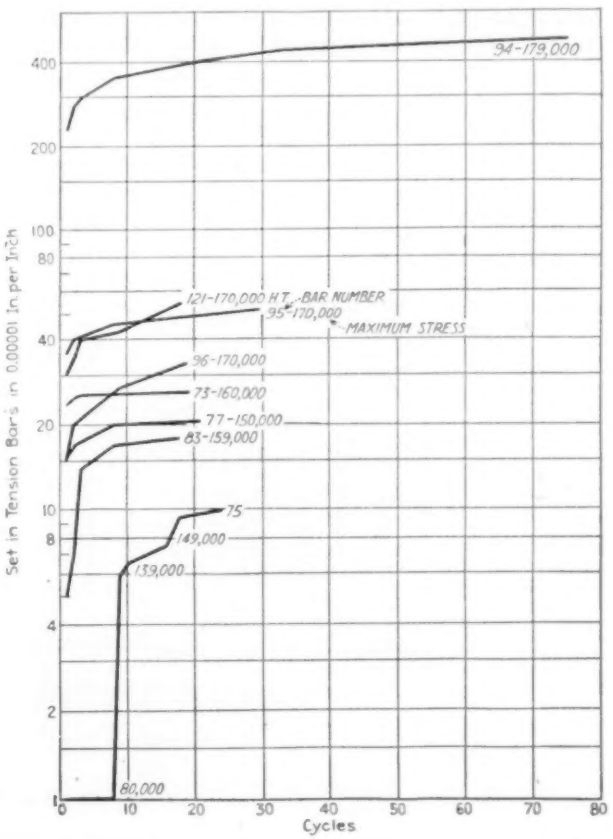


FIG. 7 INFLUENCE OF STRESS REPETITION ON TENSILE SET

It is believed that the difficulties already mentioned have been essentially avoided in the device actually used. In this the bar is clamped between hardened steel blocks at one end. At the other end, instead of applying a single load and compelling the bar to act as a cantilever, a torque or bending moment is applied. This is composed of two equal parallel forces, acting in opposite directions at points 180 deg. apart on the edge of a disk 6 in. in diameter. Fig. 8 gives details of the apparatus as originally built, while Figs. 9 and 10 show the apparatus with later modifications.

The method used is believed to be new, and naturally a number of details had to be carefully worked out. Parallelism of the cords which applied the loads was secured by carrying them over pulleys on a radius arm which could be adjusted by hand as desired. To reduce friction to a minimum, the pulleys used

[6] See Bibliography at end of report.

were supported on carefully made ball bearings. Difficulty was experienced at first in finding a cord which was at the same time sufficiently strong and sufficiently flexible. This was finally overcome by using two strands of 50-lb.-test braided silk line. Vertical load on the test bar was avoided by supporting the torque disk from the ceiling, using a carefully balanced counterweight.

The depth gages and micrometer heads by which the deflections were measured were attached to a section of $^5/_8$-in.-square

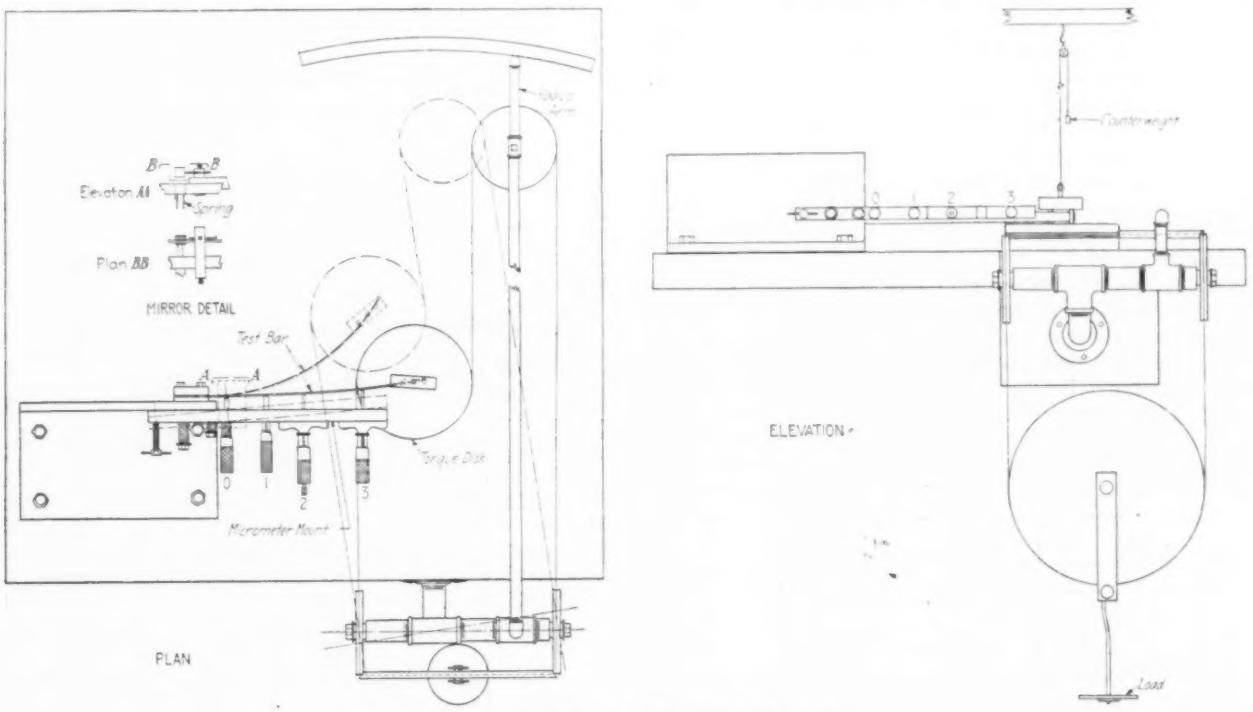


FIG. 8 DIAGRAM OF BEND-TEST APPARATUS

steel bar. These measuring instruments read directly to 0.001 in. and by estimation to 0.0001 in. The center line of the micrometer mount was in the same horizontal plane as the center line of the test bar. It can be seen in the photograph of the bend-test apparatus that the left end of the micrometer mount rests on top of the fixed block and is separated from the part of the mount carrying the gages by a spacer. The spacer also acts as an insulator between the two parts of the micrometer mount. The micrometer mount was pivoted about a $^1/_4$-in. vertical pin which was placed in the fixed block as near as possible to the point at which the test bar enters the clamping block. The micrometer mount can be rotated about this pin by means of a tangent screw located at the left end—the end farthest from the torque disk—of the mount, so that it is parallel to the test bar in its initial position.

Under ideal conditions the test bar, originally perfectly straight and free from irregularities in cross-section and from internal stresses, would be held in a perfectly fixed condition at point C of Fig. 11. The bar will be bent into a circular arc CA. The radius of curvature R can be determined from the deflection if the base distance B is known:

$$R = \frac{B^2}{2D} + \frac{D}{2}$$

Actually difficulty is caused by a slight yield at the clamping block (point B of Fig. 12). This shift does not affect the radius of curvature to which the bar is bent for a given load. It does, however, shift the location of the point at which the circular arc (prolonged if necessary) becomes tangent to a line parallel to the original position of the test bar. By measuring the deflections to three points on the test bar, the location of this point of tangency could be obtained, as explained more fully in Appendix No 2. Sixteen computations for the location of the point of tangency showed h (Fig. 12) to have a mean value of 0.146 in., varying within a maximum range of from 0.091 in. to 0.220

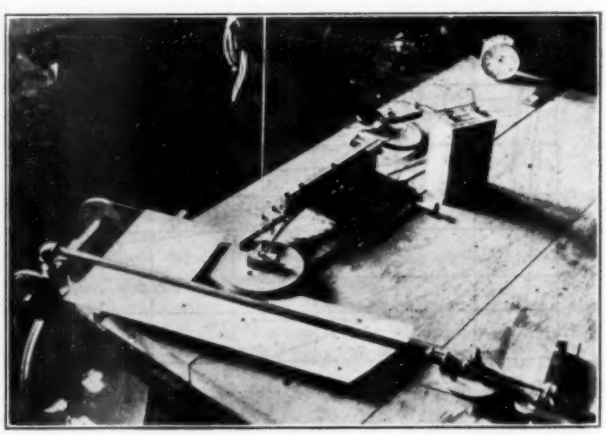


FIG. 9 PHOTOGRAPH OF BEND-TEST APPARATUS

in., and $R-k$ to have a mean value of 0.0014 in., within the range 0 to 0.003 in. In other words, the effective position of the point of tangency was not at the face of the hardened steel clamps, but was in from the face a distance averaging slightly greater than the thickness of the test specimen.

A study was made of various methods of overcoming errors due to this factor. One method used was to support a mirror

on the test specimen next to the clamp, and a second mirror on the micrometer mount. By using a telescope, and sighting on these mirrors, the micrometer mount was adjusted initially parallel to the test specimen. After each load increment on the specimen, the micrometer mount was so adjusted as to keep the mirrors parallel. It was found that settings could be duplicated so that micrometer readings checked within 0.0005 in. This method was used particularly during the preliminary runs, before the clamping blocks had been improved in form. A second method was to adjust the micrometer mount parallel to the test specimen initially, and thereafter keep it fixed in position, taking micrometer readings of the specimen at four points along the bar. These readings were used to compute the actual position of the point of effective tangency, this position in turn being used when computing the proportional limit and modulus of elasticity. After a time sufficient data were obtained to justify the use of mean values for the position of the point of tangency. This

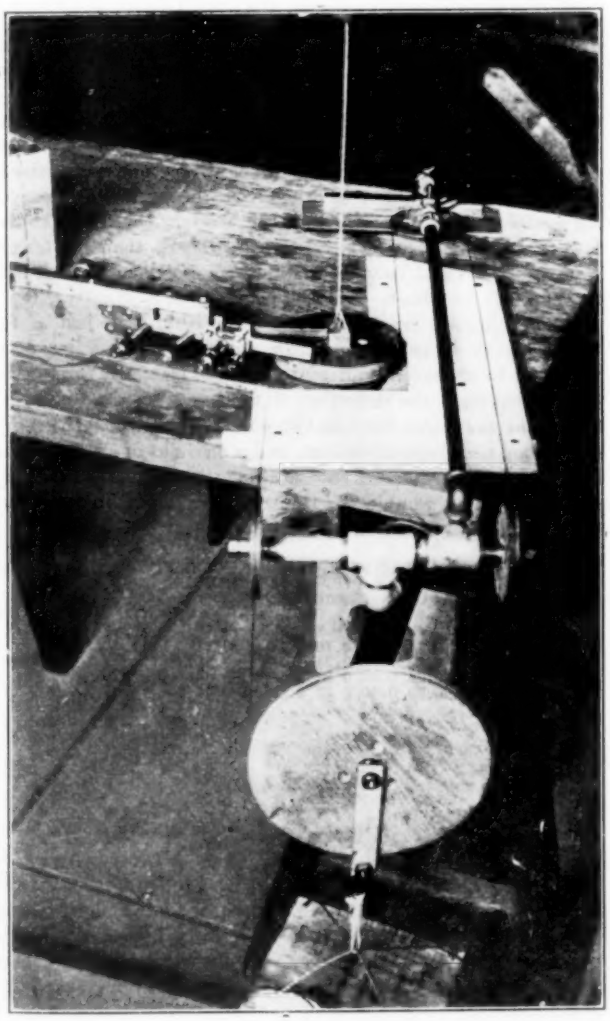


Fig. 10 Another View of Apparatus Shown in Fig. 9

accelerated the experimental work very much, as it was then necessary to take micrometer readings at only one point on the bar for each loading, and a large amount of laborious computation was avoided.

The resultant error in hysteresis loss and in proportional limit can be shown to be well within the range of other probable errors. Individual values of the modulus of elasticity may be affected by ± 2 per cent through uncertainty as to the exact position of this point of effective tangency, while the error of the mean value of several runs would be negligible. An effort was made

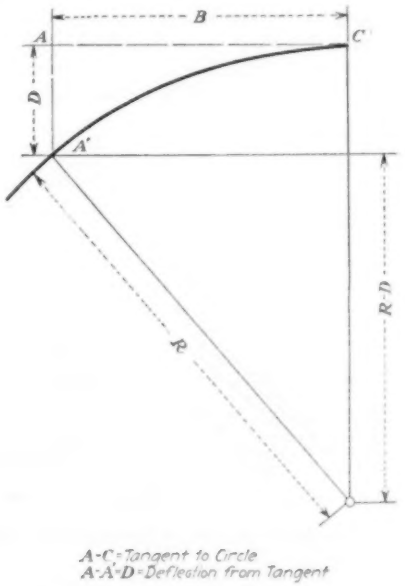


Fig. 11 Elements of Circular Arc

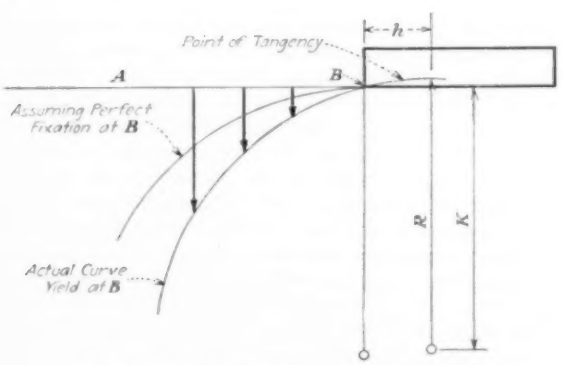


Fig. 12 Position of Point of Tangency

to correlate this variation in point of effective tangency with other factors, but it was apparently independent of the magnitude of load applied, being governed more by slight variations in the shape of the bar itself and in the way the clamping plates were tightened.

Operation of Bend-Test Apparatus

Before carrying out the bend-test experiments it was necessary to adjust the micrometer mount so that it was parallel to the face of the fixed block to which the test bar was clamped, and to measure the distances from the points from which the deflections were measured to the edge of the clamping block. This latter point, as has been previously pointed out, was very nearly the point of tangency.

A straight steel bar was supported by wooden blocks in the position to be later occupied by the test bars, and clamped against the vertical face of the fixed block. The leg of a 12-in. micrometer caliper was used for this purpose as it was straight and stiff enough to eliminate the possibility of any lateral displacement. The micrometer mount was adjusted by means of the

tangent screw until it was parallel to the straight bar, measuring the distance between the micrometer mount and straight bar with Johansson gage blocks. Micrometer readings were taken and these values later used as the "zero" readings.

The distance between the depth-gage rods and micrometer-head spindles and the distance between the first micrometer head and the clamping block were measured with gage blocks. From these dimensions and the diameters of the rods and spindles the distances between the edge of the clamping block and the

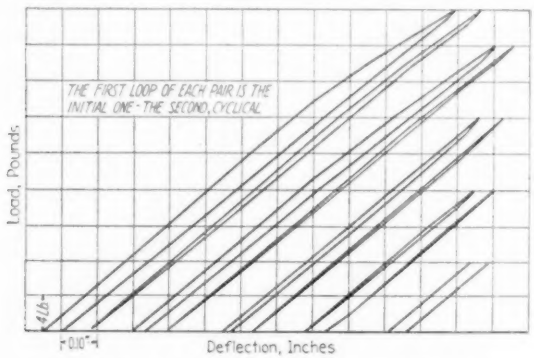


FIG. 13 FLEXURAL LOAD-DEFLECTION LOOPS

edge of the rod or spindle nearest to the block were determined. These distances were:

Measuring point.....	0	1	2
Distance in inches.....	0.337	2 535	4.483

The diameter of the torque disk was 6.02 in. and that of disk plus cord was 6.08 in. so that the effective diameter was 6.05 in.

The first step in making a bend test was to wash out any dust that might have collected in the pulley bearings and to make sure that they worked freely. The test bar was then clamped in place with the flat side vertical and the longitudinal axis parallel to the base plate. In tightening up the clamping-block bolts, care was taken to draw the block into as nearly uniform bearing as possible against the test bar. Micrometer readings were then taken, and a comparison with the readings taken on the straight bar indicated the amount by which the test bar deviated from a straight line. The torque disk was slipped into place on the free end of the bar and leveled up. It was necessary to see that both cords left the disk at the same elevation in order to insure that no torsion was applied to the test bar.

For most of the test runs the loads were applied in 4-lb. increments, this load causing an increase of 10,000 lb. per sq. in. in the stress in the outer fibers of the bar. The weights used were correct to the nearest 0.01 lb. or better. In putting on or removing the weights it was not feasible to do it so smoothly that the bar would gradually move from one load position to the next. Instead there was a fairly rapid movement of the bar, followed by a second or two of oscillation about the equilibrium position for the load. Thus when the bar was being loaded it might reach its equilibrium position from either side of that point. This would tend to prevent the friction in the pulleys from consistently making the deflections too small as the loads increased, and too large as they decreased. Duplicate readings of deflections, moving the load up or down, checked within 0.001 in.

After applying each load it was necessary to move the radius arm a small distance so that the cords leaving the torque disk would be parallel. Readings were taken at all of the micrometers for the first few bars tested, but later only the micrometer located 4.483 in. from the clamping block was used. While this did not give complete data as the first system, it saved a great deal of time. The point of contact was determined by the closing of an electrical circuit. The gap between the end of the micrometer and the test bar was connected in series with a 22.5-volt battery, a head phone, and a 100,000-ohm resistance. At first a condenser was used across the gap, but there was not much sparking and its use was discontinued. With this system the point of contact could be determined, as shown by duplicate readings, as closely as the micrometer could be read, i.e., to 0.0002 in. Readings were not duplicated unless an error was suspected at the time the reading was taken. Ten to fifteen seconds usually elapsed between the time the bar was loaded and the reading taken, this interval being taken up by the time required to adjust the radius arm and to set the micrometer. With two operators working it took 20 min. to run a cycle from 0 lb. to 72 lb. and back. This required 35 readings, so that the total time per reading was about 34 sec. When a single operator was working, a little more than twice this time was required.

RESULTS OF BEND TESTS

Typical load-deflection curves taken from bend tests are given in Fig. 13. The loops there shown are very similar to the hysteresis loops in the tensile stress-strain diagrams. The proportional-limit values are typically less definite than for the tension tests. A reasonable value for the cyclical condition seems to be 110,000 to 125,000 lb. per sq. in., as indicated in Table 4. Values of a lower proportional limit and an upper pseudo-proportional limit are given in Table 7 as taken from the stress-strain curves for initial loading, but generally the bend in the curve at the lower proportional limit was much less sharp than for tensile tests, and the exact value of the lower proportional limit is correspondingly uncertain.

For the load-deflection curves for the bend test, just as for the hysteresis loops in the tensile stress-strain diagrams, it is necessary to use several different moduli of elasticity in order to exactly represent the deflection of the bar. In place of doing this, a line was drawn between two points that bisected the width of the loop near top and bottom, and a modulus of elasticity was computed which would give deflections that would fall on this median line and would thus be reasonably close to the actual deflection.[7]

TABLE 4 CYCLICAL HYSTERESIS—SUCCESSIVE RUNS MADE ON DIFFERENT BARS

Maximum stress	Cyclical hysteresis, in-lb. per cu. in. — Experimental values	Mean	Reverse bends[1]	Median modulus of elasticity
40,000	0.11, 0.10, 0.10, 0.14	0.11	0.11	..
50,000	0.22, 0.20, 0.21. ..	0.21	..	..
60,000	0.22, 0.32, 0.32, ..	0.29	0.37	..
80,000	0.60, 0.60, 0.72, 0.66	0.65	0.74	30.9
100,000	.. 0.82	0.82	1.16	..
110,000	1.18, 1.18	1.18	..	31.0
120,000	1.40, 1.50, 1.68 ..	1.53	1.68	30.8
140,000	.. 2.24, 2.34 ..	2.29	..	30.4
160,000	3.76, 2.82, 4.45 ..	3.68	4.73	30.5
180,000	.. 6.56	6.56	5.40	30.8
201,000	.. 8.11	8.11	..	..

[1] Second set of runs made on the same bar, but with bar bending in opposite direction, to reverse the direction of stresses.

The values obtained for this median modulus of elasticity in bending averaged 30,300,000 lb. per sq. in. for 12 initial load-deflection loops, and 30,800,000 lb. per sq. in. for the corresponding cyclical loops. There was a maximum variation of 2.6 per cent from these mean values, with an average variation of 0.9 per cent.

The hysteresis values obtained for the bend-test bars are shown in Figs. 14 and 15. The maximum apparent stress (computed from $SI = CM$) in the outer fibers has been plotted on the x-axis, and the energy loss in inch-pounds per cubic inch on the y-axis. The latter value was obtained by dividing the total loss in a given length of the bar by the volume of the bar included in that length

[7] For method of computation, see Appendix No. 2.

The solid line in Fig. 14 represents the energy loss for bars in a cyclical condition, i.e., when no added set is noticed during the running of a load cycle. The individual points have been plotted to show the range in energy loss for several different bars tested at each of several maximum stresses. This curve apparently shows a slight break at somewhere between 130,000 and 150,000 lb. per sq. in., which is about the upper proportional limit in tension. For the lower portion of the curve $E_0 = 1.183 \times 10^{-13} \times S^{2.80}$, and for the upper portion $E_0 = 2.52 \times 10^{-17} \times S^{3.30}$, where E_0 is the energy loss in inch-pounds per cubic inch, and S the maximum fiber stress in the extreme fiber. Detailed values are given in Table 4.

The dotted line in Fig. 14 shows the energy loss in the first cycle and is really a measure of the set produced in that cycle more than of hysteresis loss. At a point between the virgin

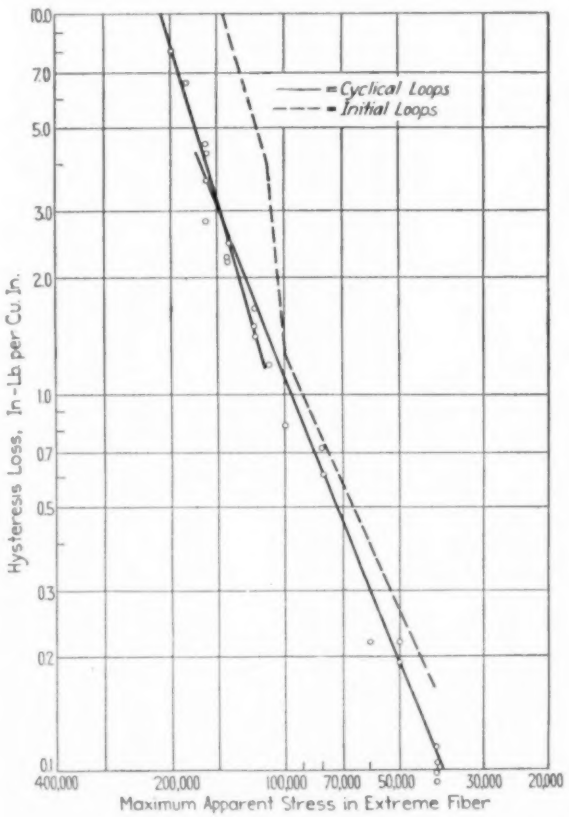


FIG. 14 FLEXURAL HYSTERESIS LOSS

proportional limit and the proportional limit as found in tension, it is seen to take a sudden increase, the initial loops showing naturally a greater proportionate loss above the proportional limit.

Certain of these bars were later reversed in position and retested in such a way that the center of curvature in bending was placed on the other side of the bar. In this way those fibers which had previously adjusted themselves to a cyclical condition of stress in compression were now compelled to adjust themselves to cyclical conditions in tension, and vice versa. The range of load in each case was from zero to the same maximum load as previously used. The resulting hysteresis values were but little, if any, greater for the reverse bend than for the initial conditions.

This was true even for bars loaded well beyond the proportional limit, with nominal maximum stresses of 200,000 lb. per sq. in.

Bar No. 33 was run through 10 cycles at 40,000 lb. per sq. in. maximum stress, then through 10 cycles at 60,000 lb., and by similar steps up to 120,000 lb. per sq. in. For the higher stresses more than 10 cycles were used to put the bar in a cyclical state. The results are shown in Fig. 15 and in Table 5.

Heat treating the bars at 335 deg. cent. by packing them in sand at that temperature and allowing them to cool very slowly

TABLE 5 CYCLICAL HYSTERESIS—SUCCESSIVE RUNS AT INCREASING LOADS MADE ON SAME BARS

Maximum stress	Cyclical hysteresis, in-lb. per cu. in. Bar as received	After low-temperature anneal at 335 deg. cent.
40,000	0.14	..
50,000	..	0.21
60,000	0.36	0.27
80,000	0.75	0.73
100,000	1.82	1.28
120,000	..	1.60
140,000	..	2.46
160,000	..	3.53

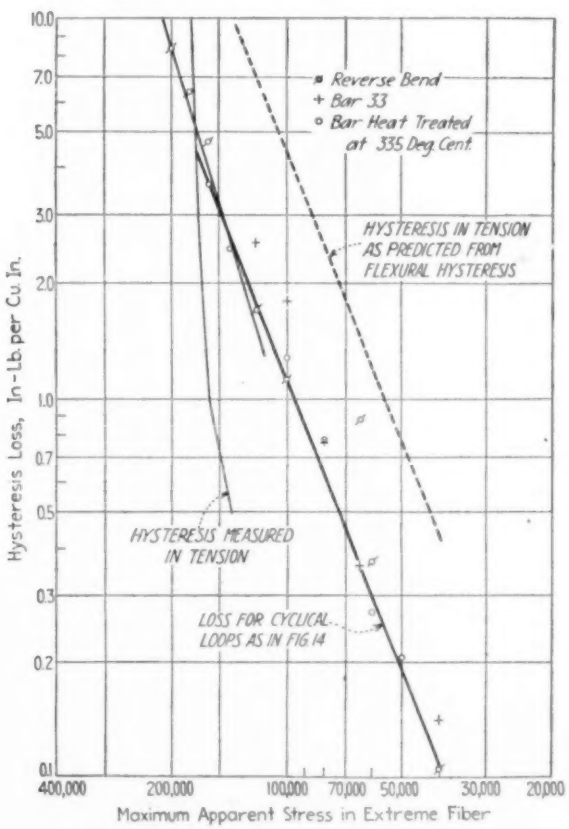


FIG. 15 FLEXURAL AND TENSILE HYSTERESIS LOSS

had no appreciable effect on the hysteresis loss. Fig. 15 shows the results obtained with a bar that was so treated.

An extended series of experiments were carried out to find out whether the hysteresis loss depended on the limits between which the cycle was run as well as on the stress range in the cycle.

Bars which had at first been tested between load limits of 0 and P were loaded and unloaded until they were in the cyclical state in the successive ranges 0 to $\frac{1}{2}P$, $\frac{1}{2}P$ to P, and $\frac{1}{4}P$ to $\frac{3}{4}P$. The hysteresis losses in these successive ranges are given in Table 6. There is evidently a definite progressive increase in hysteresis as the position of the center of the stress range is raised, the hysteresis for a load range $\frac{1}{2}P$ to P being approximately double that for a load range 0 to $\frac{1}{2}P$. The loss in the middle range lies between that in the other two.

Set and Elastic Afterworking in Bending

For ranges of stress above the virgin proportional limit of the material, the permanent set of the bar tends to increase under successive applications of loading, and to approach a certain maximum value corresponding to a cyclical condition in the bar. The relation between set and number of load cycles is given in Fig. 16, while Fig. 17 shows the relation between maximum stress and set. These values represent the permanent set in inches in the bar as measured by a micrometer at 4 53 in. from the edge of the clamping block.

At least 90 per cent of the total set is produced in the first 10 cycles, the rate of increase being comparatively slow after that. For the higher stresses the bar seems to lose its cyclical condition with a period of rest. Bar No. 38 (maximum apparent stress 200,000) had reached a state of nearly constant set at cycle 60, but after a 17-day rest it showed a sudden increase of 30 per cent in set. Bar No. 46 (maximum apparent stress 180,000) in the reverse-bend test showed the same effect, but to a less marked degree. Between cycles 50 and 64 it showed little increase in

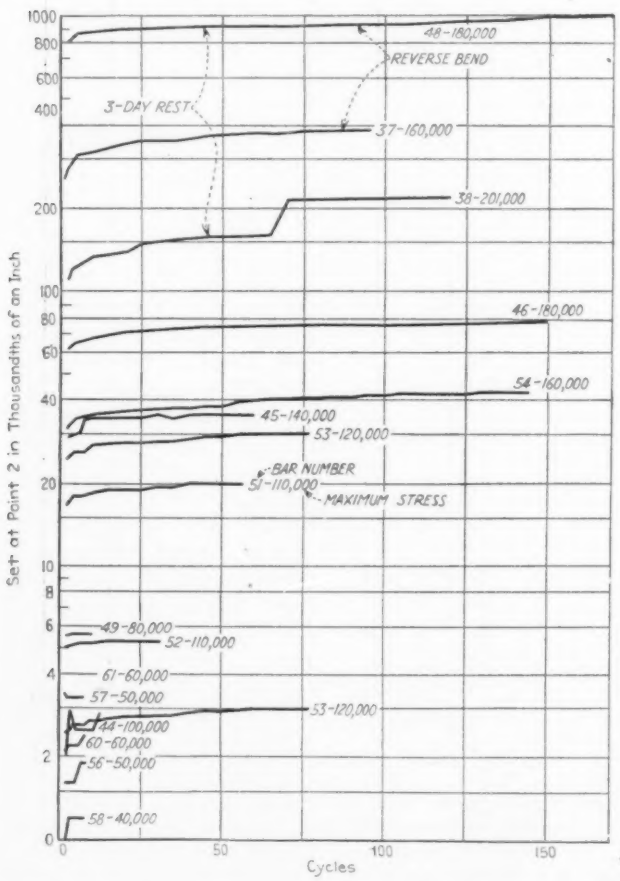


Fig. 16 Influence of Stress Repetition on Flexural Set

set, but after a three-day rest period it was necessary to put the bar through 100 more cycles before the set again became constant.

The set increases with the stress as shown in Fig. 17. At the lower loads the points are widely scattered. Some of these discrepancies may be due to the fact that the bars were not perfectly straight to begin with. There is a definite increase in the amount of set and in the rate of its increase above a stress of 155,000 lb. per sq. in.

The reverse-bend tests showed a more uniform relation between set and stress (Fig. 17). Below a stress of 155,000 the set increases as the 2.27 power of the stress and is 7 to 8 times as great as that in the direct bend test. The break in the curve takes place at 155,000 as it did in the direct-bend tests, but is a much sharper break. There is a little doubt as to the reliability

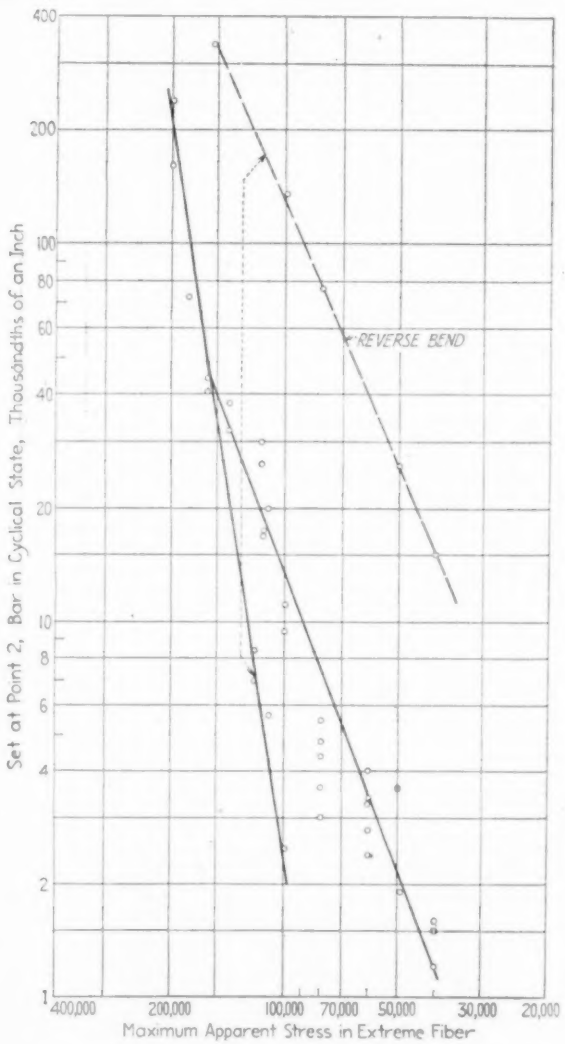


Fig. 17 Permanent Set at Cyclical State in Bending

of this part of the curve as it depends on one point only, but it would be expected that there would be more set above the yield point in the reverse bend than in the direct bend.

A study of the set and hysteresis results shows that if several bars are tested to the same maximum load the bars with the smaller amounts of set tend to show the lower cyclical hysteresis values, and that those with the larger sets will show the higher cyclical hysteresis values. This was true in the sets of bars tested to a maximum stress of 40,000, 80,000 and 120,000 lb. per sq. in. No conclusion could be drawn from those stressed to 60,000 and 100,000 lb. per sq. in., and this relation did not hold for the group of bars stressed at 50,000 lb. per sq. in.

The tendency of an overstressed bar to return to its original position during a period of rest at zero load has been called "elastic afterworking." For bars stressed beyond 150,000 lb. per sq. in. this elastic afterworking caused a reduction in set

varying from 0.0005 in. to 0.0014 in. during periods of rest varying from 1 to 16 hours.

Loads giving stresses as low as 40,000 or 50,000 lb. caused a yield or increase in set of 0.0002 in. and 0.0010 in., respectively, when applied continuously for a 15-hour period. The bars had previously undergone load cycles to maximum apparent stresses of 160,000 and 200,000 lb. per sq. in., respectively.

The increase in set was very noticeable when the bar was loaded for a period at stresses above the proportional limit. A bar loaded to 160,000 lb. per sq. in. for 15 hours showed an increase in set of 0.0460 in. Another bar loaded to 200,000 lb. per sq. in. yielded 0.0140 in. during a load period of 75 min.

Summary and Discussion

The real value of these experimental results of course can be most definitely brought out by making practical applications. The most immediate application lies probably in a study of true stress distributions in bending, as far as they affect the selection of proper unit stresses, and as they indicate the effect of internal stresses caused by various methods of manufacture.

Let us visualize what actually occurs in a round or rectangular bar as it is being bent. The fibers on the concave side are being shortened; on the convex side they are being lengthened; and the rate of shortening or lengthening is proportional to their distance from some central, unaffected, row of fibers called the neutral axis or neutral plane. The next step in the usual computation is to assume that unit stress is proportional to unit strain, and that therefore the unit stress is proportional to distance from the neutral axis, whence follows easily the well-known formula $S = Mc/I$. There also follow two less well-known but very convenient formulas:

$$M = EI\,(1/R_2 - 1/R_1)$$
$$S = Ec\,(1/R_2 - 1/R_1)$$

where R_1 is the radius of curvature of the center line of the bar at that particular point before applying load, and R_2 the radius of curvature at the same point after the load has been applied. E is the modulus of elasticity, usually assumed to have the same value as that obtained in a tensile or compressive test.

Any material placed under compression, and so caused to shorten longitudinally, tends to expand laterally in both directions. Consequently, as the compression fibers on the concave side of the bar expand laterally, while the tension fibers on the convex side contract laterally, the cross-section of the piece tends to warp, the face of a rectangular strip becoming concave in a direction at right angles to the main curvature of the piece. This effect is often spoken of as "anticlastic" curvature. Saint Venant in 1856 was the first to call attention to it. Love in his "Mathematical Theory of Elasticity," Prescott in his "Applied Elasticity," and Timoshenko and Lessells in their work of the same title all treat it in detail. Its presence can be forcibly called to one's attention by the simple experiment of gripping one end of a small rectangular steel bar in a vise, and then bending the bar into a small-radius circle.

There are three results from this tendency to lateral expansion:

(1) The distance from neutral axis to extreme fiber becomes greater on the compression side, and less on the tension side. A slight shift of the neutral axis may result, which has no particular importance.

(2) The lateral curvature, if allowed to occur freely, causes a change in the shape of the cross-section with a resultant gradual slight increase in the moment of inertia, and so in the apparent stiffness of the bar. The resulting lateral radius of curvature would equal the major radius of curvature divided by Poisson's ratio for the material.

(3) In wide, thin strips the lateral curvature is restrained or entirely prevented because of the arching action in the material which comes into play as soon as the lateral distortion begins. On the compression side of the material this is equivalent to introducing a hypothetical lateral compressive force which pushes the fibers back to their original width, simultaneously causing expansion at right angles to this (lateral) direction, forcing the fibers to increase in length parallel to the length of the bar, and so reducing the effective unit strain of the extreme fibers. As a result the bar deflects less under the applied loading, and its apparent modulus of elasticity is increased. In the limiting case, with a bar of infinite width in proportion to its thickness, the stiffness will go up in the ratio 1 to $1/(-\sigma^2)$, where σ = Poisson's ratio. For $\sigma = 0.27$, the resulting increase is 8 per cent. In the experimental work here described, the average value of the modulus of elasticity in tension was 29,700,000 lb. per sq. in., and in bending, 30,800,000 lb. per sq. in., an increase in stiffness of 3.7 per cent for bars $^1/_8$ in. thick by $^1/_2$ in. wide, with rounded corners. In Fig. 18 the discrepancy below the proportional limits, between the stress-strain curves in bending and in tension, is due to this cause.

This increased stiffness comes into play as soon as the bar begins to deflect, whereas the increase in stiffness due to the change in shape, noted under (2) above, comes into play only gradually as the bar deflects, and ordinarily factor (3) is much more important than factor (2). As a result, a wide, thin strip will show a greater stiffness in bending than a piece of square cross-section, and the latter a slightly higher stiffness than a round rod, all being made of identical material.

Returning to the main theme, imagine that the rod is still further bent, so that the extreme fibers pass beyond the proportional limit. The unit strain still remains proportional to the distance from the neutral axis, but the unit stress is no longer proportional to this distance. If, as in this experimental work, the radius of curvature of the bar is known, the true unit strain in the extreme fiber may be found, and then from a tensile stress-strain curve, or more correctly from such a curve corrected for the anticlastic effect, the true unit stress in the extreme fiber may be read off. The ordinarily used bending formulas are of course not valid for this case. For the particular case shown in Fig. 18, under the highest loading, the unit stress as computed by the formula $S = Ec\,(1/R_2 - 1/R_1)$ was 217,000 lb. per sq. in.; and by the formula $S = Mc/I$, it was 203,000 lb. per sq. in.; whereas the true maximum unit stress was only 176,000 lb. per sq. in.

Other methods of determining the true value of the unit stress in the extreme fiber may be used. One very engaging and theoretically exact method for bars of rectangular cross-section has been suggested by L. Prandtl. Take readings of load and deflection while the bar is being loaded. Compute the resulting unit stress and unit strain, using the formula $S = Mc/I$. Plot this stress-strain curve, and draw a straight line tangent to this curve, at the desired load point, as shown in Fig. 18. Read off the unit stress at the intersection of this line with the axis of zero strain, and subtract one-third of it from the nominal unit stress to obtain the true value.

The after-effects of an overstrain of this nature are more important than the immediate effect. These after-effects take the form of (*a*) a permanent set, or curvature, in the bar as a whole; (*b*) a permanent elevation of the effective proportional limit of the overstressed fibers; and (*c*) an introduction of trapped internal stresses throughout the entire width of the bar. The first effect is more or less obvious. The methods of prophesying its quantitative magnitude will be more evident after a study of the other two effects.

The character of the second effect may be visualized by a study of the typical stress-strain curves in tension and in bending for loads beyond the proportional limit, shown in Figs. 3 and 13.

The elastic return for loads beyond the virgin proportional limit is evidently not perfect. That is to say, the return line is not perfectly straight. Certain time-lag effects exist, and there are some other departures from perfect proportionality. Even after the load has been removed and reapplied a sufficient number of times so that cyclical conditions are approached, the stress-strain lines are still not perfectly straight, and a hysteresis loop remains. On the other hand, if the bar is held loaded for even a moderate length of time, so giving time for the deformation to occur, the stress-strain line on unloading will be so nearly straight that the bends may be averaged out, and a mean modulus-of-elasticity value used. In other words, although the extreme fibers did not behave elastically throughout the entire loading period, they will behave essentially elastically during the entire unloading period, and during future loading and unloading periods, particularly if future load values are kept moderately below the initial load.

The decrease in unit stress during unloading, measured at various points across the depth of the bar, will therefore be proportional to the distance of these points from the neutral axis, and will be accurately given by the usual elastic theory ($S = Mc/I$ formula). Thus in Fig. 19[8] the true stress in the extreme fiber, loaded, was 174,000 lb. per sq. in. tension; the decrease in stress by the formula given was 221,000 lb. per sq. in.; and the trapped stress in the extreme fiber with the load removed, was 47,000 lb. per sq. in. compression. If now this bar is again bent in the same direction, this trapped stress operates to relieve conditions, so that the true stress will be 47,000 lb. per sq. in. less than the computed value, while on bending in the opposite direction, the true stresses are 47,000 lb. greater than the computed values. This of course is the theoretical reason for the caution never to load a spring so as to cause it to uncoil under the load.

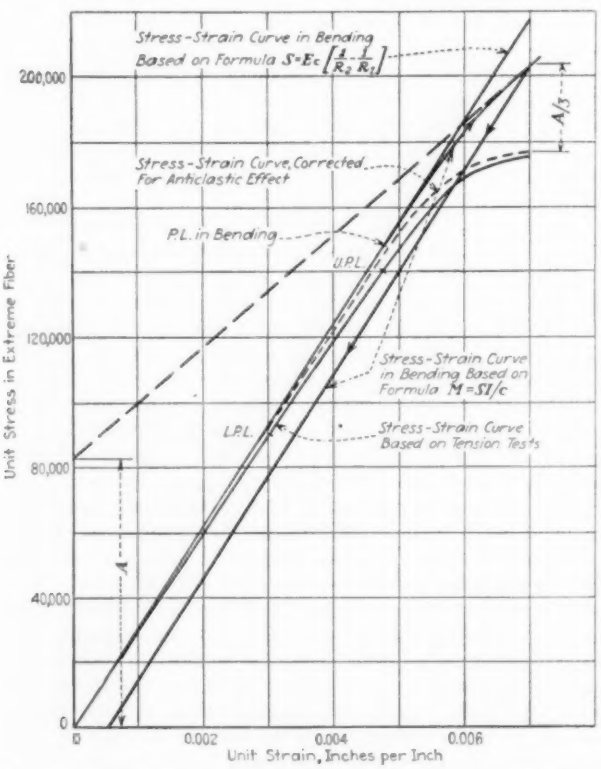


FIG. 18 · STRESS DISTRIBUTION IN BEAM LOADED BEYOND PROPORTIONAL LIMIT

Within the interior of the bar, conditions are analogous. At a point half-way between the neutral axis and the extreme fiber, the decrease in stress is 110,500 lb. As this is less than the true stress in the loaded condition at that point, the residual trapped stress at that point on unloading is one of tension. The true unloaded stress condition across the width of the bar is given by the dotted line in Fig. 19. The usual equations for equilibrium must be complied with by that dotted line: the total tension must just balance the total compression, and the moment of the forces acting clockwise about the neutral axis must balance the moment of the counterclockwise forces. This last condition determines the amount of permanent set remaining in the bar.

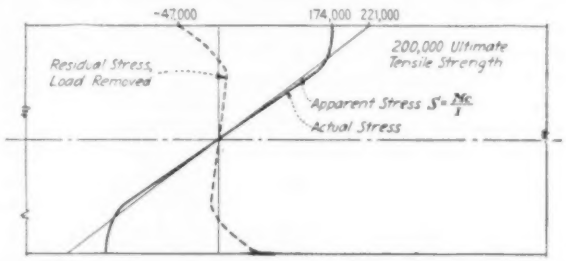


FIG. 19 TRAPPED STRESSES FROM FLEXURAL OVERSTRAIN

With modifications, therefore, it will furnish theoretical reasons for the amount of recoil of helical or power springs on winding, and should furnish an explanation for the occasional erratic behavior of different lots of wire which makes it difficult to comply with close specifications for size of coil. Some phases of these practical cases are more difficult to handle, as the wire is under tension as well as bending while being wound. The future program as outlined calls for a detailed study of this and of the simple case, applying the results directly to different types of wire. It also calls for an experimental study of certain approximations in this theory, particularly the effect of time lag and of elastic after-recovery.

The usual beam formula $S = Mc/I$ is properly limited to beams which are straight or nearly so. The error introduced through applying it to bars of appreciable curvature has so far been neglected. For the range of ratio between radius of curvature and thickness of piece which was used in this experimental work, this neglect is allowable, as the error will not exceed 0.5 per cent.

The magnitude of the internal friction or hysteresis loss in the material is best shown in Fig. 15. This figure is a logarithmic chart using as abscissa the hysteresis loss in inch-pounds per cubic inch of material corresponding to a single stress cycle starting at zero, rising to the maximum stress in the extreme fiber corresponding to the ordinate, then returning to zero. Before measuring these loop areas the material had been put through from fifty to one hundred or more stress cycles of this same magnitude, until further cycles caused no further increase in set, in order to avoid the effect of "initial set." The bending results show no signs of any break in the curve opposite the virgin proportional limit in tension, hysteresis being present even for light loads, and steadily and uniformly increasing as the load increases up to a stress approaching 150,000 lb. per sq. in. At this point there is a slight and not any too clearly indicated break in the line. The lower part of this line may be represented by the formula

$$\text{Hysteresis} = 1.183 \times 10^{-13} \times (\text{max. stress})^{3.6}$$

The upper part has a slope which indicates an exponent of 3.3. This compares fairly closely with the exponent of 3.0 given by

[8] This figure is based on a different bar and on greater loads than those used in Fig. 18, so that numerical values do not agree.

G. H. Keulegan of the U. S. Bureau of Standards (see Bibliography, Nos. 14 and 15). Mr. Keulegan worked with Armco iron, and with low unit stresses, a fraction only of the stresses indicated on Fig. 15. An extension of his curve upward, corrected for the fact that he worked with cantilever loading, whereas the bars here tested had moments of uniform magnitude over their length, came some distance above the author's experimental results, indicating that the hysteresis losses in the spring steel were approximately one-fourth of those in Armco iron. Not much reliance should be placed on this figure, as the stress ranges used in the experimental work do not at all overlap, and too much extrapolation of formulas is needed to permit of the comparison. Other experimenters have used coefficients ranging from 2 to 3, but very little work has been done at stresses approaching those used in the work here reported.

TABLE 6 RELATIVE HYSTERESIS IN DIFFERENT PARTS OF LOAD RANGE

Maximum stress	Range 0–P	0–$\frac{1}{2}P$	$\frac{1}{4}P$–$\frac{3}{4}P$	$\frac{1}{2}P$–P
		Hysteresis, in-lb. per cu. in.		
80,000	0.60	0.11	0.18	0.24
80,000	0.66	0.14	0.15	0.28
120,000	1.40	0.28	0.52	0.62
120,000	1.50	0.41	0.55	0.62
160,000	3.76	0.64	0.90	1.18
160,000	4.45	0.64	1.09	1.15
201,000	8.11	1.24	1.68	2.55

TABLE 7 PROPORTIONAL LIMITS IN BENDING

Bar No.	Maximum stress —1000 lb. per sq. in—	Initial loop L.p.l.	Initial loop U.p.l.	Cyclical loop Cycle No.	Cyclical loop P.l.
36	120	50	...	83	>120
41	120	80	110	97	>120
59	140	95	110	98	...
45	140	105	130	63	110
42	160	80	115	81	120
54	160	70	150	151	115
46	180	120	150	107	125

TABLE 8 COMPARISON BETWEEN HYSTERESIS IN TENSION AND IN FLEXURE

Apparent stress, 1000 lb.	Flexural hysteresis	Conversion factor, $n+1$	Tensile hysteresis based on bend test	Measured tensile hysteresis
180	5.7	4.3	24.5	14.0
160	4.0	4.3	17.2	1.0
140	2.55	3.6	9.2	0.5
80	0.66	3.6	2.4	0.0

From the experimental work (see Table 6) there seems very definite evidence that for a given total range of stress the magnitude of the hysteresis increases rapidly as the minimum stress in the range increases in value.

Attention should be drawn to the results for tensional hysteresis, shown by the upper line in Fig. 15. For stress ranges below 160,000 lb. per sq. in., the energy loss corresponded to a mean width of loop of not over 0.000005 in. per inch of length. Above this stress as the yield point of the material was approached the hysteresis loss increased sharply. No formula is given, but the slope of the line as drawn suggests that the hysteresis varies as the 20th power of the stress. These values represent conditions after the bars had been loaded and unloaded in tension a sufficient number of times so that the increase in set per cycle was appreciably below the width of the loop. Data are on file covering the progressive decrease in hysteresis loss as the bars approach cyclical conditions. A typical case is shown in Fig. 6.

Comparison cannot be made directly between hysteresis in tension and in flexure, as the whole volume of the bar is stressed equally in tension, whereas only the extreme fibers in bending receive the stated maximum stress. For an energy loss in bending varying as the 2.6 power of the maximum stress, the expected loss in tension, for the same maximum stress, would be $n + 1$ or 3.6 times as great. The actual measured values were much lower as indicated by Table 8. This suggests that some factor is present in flexure which causes hysteresis to be a more important factor in it than in tensile stress. Further study of this topic is planned.

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NOTE: For a general bibliography on mechanical springs see the Committee's Bibliography on Mechanical Springs published by The American Society of Mechanical Engineers.

Appendix No. 1

EFFECT OF PITCH ANGLE ON HELICAL-SPRING DEFLECTIONS AND STRESSES

AS A PRELIMINARY step to more detailed work on torsional conditions of stress, an investigation was made of the numerical effect of various approximations tacitly made in the ordinary design of helical springs, in order to be able to allow for them. The most prominent of these approximations is the assumption ordinarily made that the pitch angle of the spring is negligible, and that consequently in tension and compression springs the torsional stresses only need be allowed for, the bending stresses and deflections being ignored. A corresponding error occurs in formulas for helical springs used in torsion, where customarily bending stresses only are considered and torsional stresses and their effects are neglected.

Corrected formulas for these cases, making proper correction for the pitch angle, have long been known. They have been summarized here. Results are given in more easily available form in Fig. 20. These charts show for various values of the pitch angle the percentage correction which must be applied to the results of the usual formulas to make correction for the effect of the pitch angle. The pitch angle ϕ used in these formulas is the angle which the center line of the wire in the coil

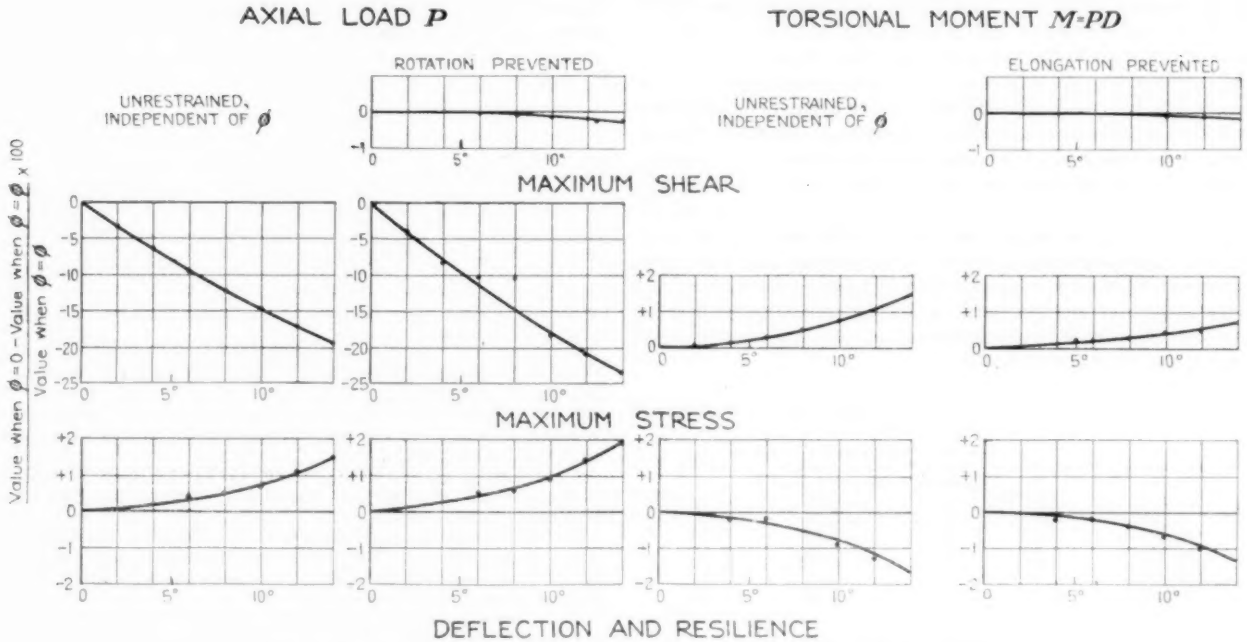


FIG. 20 EFFECT OF PITCH ANGLE ON HELICAL SPRINGS OF CIRCULAR SECTION

would make with a plane perpendicular to the axis of the coil. It is given numerically by the statement that $\tan \phi = p/\pi D$, where p = pitch or distance between center lines of consecutive wires, measured parallel to axis of coil, and D = diameter of coil, center line to center line of wires. Other items in the notation are as follows:

- P = axial load on springs, in pounds, considered + when it stretches spring
- M = torsional moment in inch-pounds applied from outside upon spring, considered + when it tends to bring coils closer together
- M_t = resulting torsional moment in wire of spring
- M_b = resulting bending moment in wire of spring
- F = linear deflection of spring, inches
- t = angular deflection in radians
- E = modulus of elasticity in bending, lb. per sq. in.
- G = modulus of elasticity in shear, lb. per sq. in.
- I = moment of inertia of wire
- I_p = polar moment of inertia of wire
- d = diameter of wire, inches
- L = total length of effective portion of the wire in the spring.

I—Springs Carrying Axial Load P[9]

1 *End Unrestrained* (Free to rotate as well as to move axially):

$$M_t = PD \cos \phi \qquad S_s = \frac{16\ PD \cos \phi}{\pi d^3}$$

$$M_b = -PD \sin \phi \qquad S_b = \frac{32\ PD \sin \phi}{\pi\ d^3}$$

$$\text{Maximum Stress} = \frac{S_b}{2} + \sqrt{S_s^2 + \left(\frac{S_b}{2}\right)^2} = \frac{16\ PD}{\pi\ d^3}\ (1 + \sin \phi)$$

$$\text{Minimum Stress} = \frac{S_b}{2} - \sqrt{S_s^2 + \left(\frac{S_b}{2}\right)^2}$$

$$\text{Maximum Shear} = \frac{\text{Max.} - \text{Min.}}{2} = \sqrt{S_s^2 + \left(\frac{S_b}{2}\right)^2} = \frac{16\ PD}{\pi\ d^3}$$

$$F = \frac{32}{\pi\ d^4} PD^2L \left(\frac{\cos^2 \phi}{G} + \frac{2 \sin^2 \phi}{E}\right)$$

$$t = \frac{32}{\pi\ d^4} PDL \sin \phi \cos \phi \left(\frac{1}{G} - \frac{2}{E}\right)$$

$$W = \frac{PF}{2}$$

2 *End Restrained* (so that relative rotation of ends is prevented):

$$\text{Moment caused by } P = \frac{-PD \sin \phi \cos \phi\ (E - 2G)}{E \sin^2 \phi + 2G \cos^2 \phi}$$

$$M_t = \frac{-PD \sin^2 \phi \cos \phi\ (E - 2G)}{E \sin^2 \phi + 2G \cos^2 \phi} + PD \cos \phi \qquad S_s = \frac{16\ M_t}{\pi\ d^3}$$

$$M_b = \frac{-PD \sin \phi \cos^2 \phi\ (E - 2G)}{E \sin^2 \phi + 2G \cos^2 \phi} - PD \sin \phi \qquad S_b = \frac{32\ M_b}{\pi\ d^3}$$

$$\text{Maximum Shear} = \frac{16}{\pi\ d^3} \sqrt{M_t^2 + M_b^2}$$

$$= \frac{16\ PD}{\pi\ d^3} \sqrt{1 + \left[\frac{(E - 2G)(\sin \phi \cos \phi)}{E \sin^2 \phi + 2G \cos^2 \phi}\right]^2}$$

$$\text{Maximum Stress} = \frac{16\ M_b}{\pi\ d^3} + \text{Maximum Shear}$$

$$F = \frac{32\ PD^2L}{\pi\ d^4} \left[\frac{\cos^2 \phi}{G} + \frac{2 \sin^2 \phi}{E} - \frac{(\sin \phi \cos \phi)^2 \left(\frac{E}{2} - G\right)^2}{\frac{E^2G}{4} \sin^2 \phi + \frac{EG^2}{2} \cos^2 \phi}\right]$$

$$W = \frac{PF}{2}$$

[9] In the formulas given on this page for springs carrying axial load, the term D should be taken as representing the center-line radius of the coil, rather than the center-line diameter.

II—Springs Transmitting Moment M

1 *End Unrestrained* (Free to elongate as well as to rotate):

$$M_t = M \sin \phi \qquad S_s = \frac{16\,M \sin \phi}{\pi\, d^3}$$

$$M_b = M \cos \phi \qquad S_b = \frac{32\,M \cos \phi}{\pi\, d^3}$$

$$\text{Maximum Stress} = \frac{16\,M}{\pi\, d^3}(\cos \phi + 1)$$

$$\text{Maximum Shear} = \frac{16\,M}{\pi\, d^3}$$

$$F = \frac{32}{\pi\, d^4} RLM \sin \phi \cos \phi \left(\frac{1}{G} - \frac{2}{E}\right)$$

$$t = \frac{32\,ML}{\pi\, d^4}\left(\frac{\sin^2 \phi}{G} + \frac{2 \cos^2 \phi}{E}\right) \qquad W = \frac{Mt}{2}$$

2 *End Restrained* (Change of length of coil prevented):

$$M_t = M \sin \phi - M\left[\frac{\sin \phi \cos^2 \phi\,(E - 2G)}{E \cos^2 \phi + 2G \sin^2 \phi}\right]; \qquad S_s = \frac{16\,M_t}{\pi\, d^3}$$

$$M_b = M \cos \phi + M\left[\frac{\sin^2 \phi \cos \phi\,(E - 2G)}{E \cos^2 \phi + 2G \sin^2 \phi}\right]; \qquad S_s = \frac{32\,M_b}{\pi\, d^3}$$

$$\text{Maximum Shear} = \frac{16}{\pi\, d^3}\sqrt{M_t^2 + M_b^2}$$

$$= \frac{16\,M}{\pi\, d^3}\left[1 + \left\{\frac{(E - 2G) \sin \phi \cos \phi}{E \cos^2 \phi + 2G \sin^2 \phi}\right\}^2\right]^{1/2}$$

$$\text{Maximum Stress} = \frac{16\,M_b}{\pi\, d^3} + \text{Maximum Shear}$$

$$t = \frac{32ML}{\pi\, d^4}\left[\frac{\sin^2 \phi}{G} + \frac{2 \cos^2 \phi}{E} - \frac{2(\sin \phi \cos \phi)^2\left(\frac{E}{2} - G\right)^2}{\frac{E^2 G \cos^2 \phi}{2} + EG^2 \sin^2 \phi}\right]$$

$$W = t\,\frac{Mt}{2}$$

Appendix No. 2

FORMULAS AND METHODS OF COMPUTATION

I—Method Used in Locating Effective Position of Point of Tangency of Strip

THE equation of a circle whose center is not at the origin of a system of rectangular coordinates is

$$(x - h)^2 + (y - k)^2 = R^2$$

where h and k are the coordinates of the center of the circle and R is its radius. If the coordinates of three points on the circle are measured, the x's being the distance from the micrometer to point B (see Fig. 12) and the y's the deflections measured by the micrometers, the values of h, k, and R can be found from the solution of the three simultaneous equations

$$(x_1 - h)^2 + (y_1 - k)^2 = R^2$$
$$(x_2 - h)^2 + (y_2 - k)^2 = R^2$$
$$(x_3 - h)^2 + (y_3 - k)^2 = R^2$$

Actually it was found to be more convenient to put the equations in the form

$$K + 2x_1h + 2y_1k = x_1^2 + y_1^2$$
$$K + 2x_2h + 2y_2k = x_2^2 + y_2^2$$
$$K + 2x_3h + 2y_3k = x_3^2 + y_3^2$$

where $K = R^2 - h^2 - k^2$, solve for h, k, and K, and then find R. The difference between k and R gave the y-coordinate of the point of tangency, so that it was necessary to compute values to five significant figures in order to obtain this distance to the nearest 0.001 in.

II—Method of Computing Flexural-Hysteresis Loss

The hysteresis loss during a given load cycle equals the difference between the work done on the bar while it is being loaded and the work returned by the bar during the unloading. If a gradually increasing moment M is applied to a bar the work done is $Md\theta$, where θ is the angular change at the point where the moment is applied. Plotting an M-θ curve the area between the curve and the axis will give the work done on the bar if θ represents the angular change for an increasing load, and will give the work returned by the bar when θ represents

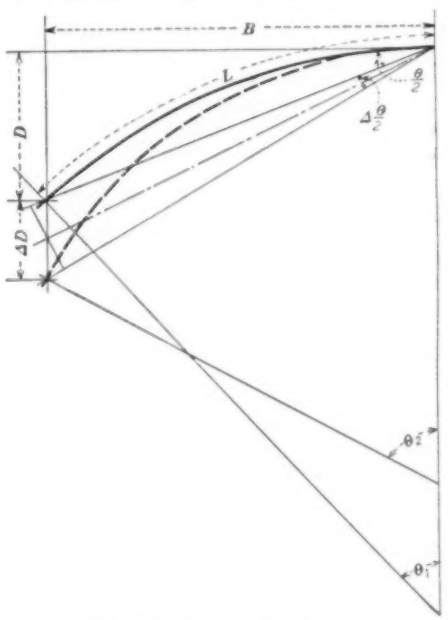


Fig. 21 Relation Between ΔD and $\Delta\theta$

the angular change for a decreasing load. The area of the loop formed between these two curves, which is analogous to the loop of a stress-strain curve as M is proportional to stress and θ to strain, gives the loss of energy during the load cycle. By plotting a M-$\Delta\theta$ curve, where $\Delta\theta$ is the difference in angle for increasing and decreasing loads, we have in effect greatly increased the scale of the plot. The area under the M-$\Delta\theta$ curve also represents the energy loss and can readily be obtained using a planimeter. Computations were greatly shortened by using a load-deflection difference curve in place of the M-$\Delta\theta$ curve.

The relation between $\Delta\theta$ and the deflection difference is seen from Fig. 21 to be

$$\Delta\theta = 2\Delta(\theta/2) = \frac{2\,\Delta D \cos(\theta/2)}{B/\cos(\theta/2)} = \Delta D\left[\frac{2 \cos^2(\theta/2)}{B}\right]$$

This figure also shows that as the bar is deflected the length between the point at which the deflection is measured and the fixed end increases. In other words, the deflection difference,

ΔD for $\theta = 30$ deg. is a measure of the loss of energy in a slightly greater length of bar than it is when $\theta = 10$ deg. In order to obtain the loss of energy in a constant length B, $\Delta\theta$ will have to be divided by L/B. The deflection difference for a given load, multiplied by $\frac{(2/B)(\cos^2[\theta/2])}{L/B}$, will give the value of $\Delta\theta$ for a length of bar B. A curve showing the variation of this factor with the magnitude of the deflection is shown in Fig. 22. The largest values of hysteresis, and consequently of $\Delta\theta$, are found at about half the maximum load in the cycle, so that the correction factor corresponding to the half-load deflection is relatively the most important. At 32 lb. load the deflection is

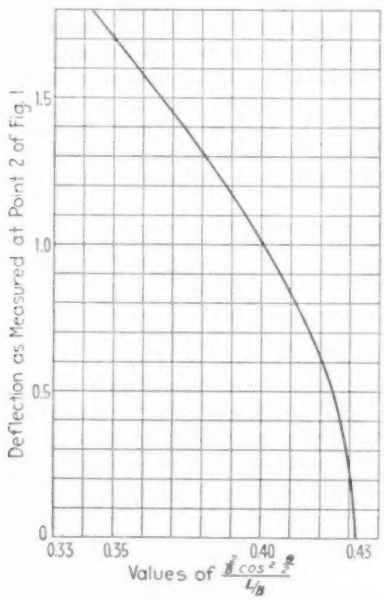


FIG. 22 CURVE GIVING FACTOR FOR OBTAINING $\Delta\theta$ FROM ΔD

0.50 in., so that up to 64 lb. the factor 0.424 is used. The deflection at half the maximum load of 80 lb. is 0.85 in., and it is seen from the curve that the corresponding factor is 0.408. Using these average factors for load up to 64 lb. and between 64 and 80 lb. rather than multiplying each value of ΔD by its exact factor to obtain $\Delta\theta$, introduces an error not greater than 4 per cent, which is less than the experimental differences and saves a great deal of time.

B_1 = 4.48 in. to edge of clamping block.
B_2 = 4.63 in. to point of tangency. Use $B = 4.55$ in.
Effective diameter of torque disk = 3.02 in.
Average area of test bar = 0.0596 sq. in.

Loss of energy in inch-pounds per cubic inch equals area of deflection-difference–load curve multiplied by 4.73 for loads to 64 lb. or 4.56 for loads above 64 lb.

$$\frac{3.02 \times 0.424}{4.55 \times 0.0596} = 4.73 \qquad \frac{3.02 \times 0.408}{4.55 \times 0.0596} = 4.56$$

III—MODULUS-OF-ELASTICITY COMPUTATIONS IN BENDING

It was much more convenient to plot load deflection curves in bending, and to study these directly, than to compute unit stress and unit strain values so that the usual stress strain curve could be plotted. This had the disadvantage that modulus of elasticity values could not be directly obtained from the graph.

As a result the median modulus of elasticity had to be computed from the expression $M_2 - M_1 = EI(1/R_2 - 1/R_1)$, where M_2 and M_1 were the bending moments applied to the bar under the upper and lower loads and R_2 and R_1 the resulting radii of curvature in the bar. These values of radius of curvature were obtained from the relation $R = B^2/2D + D/2$, where D is the measured deflection and B the perpendicular distance from the line along which the deflection was measured, to the effective point of tangency. This effective point was taken as being 0.15 in. inside of the edge of the clamping block, that being its average position, determined as already described.

Inserting numerical values in these equations,

$M = 3.02\,P$, where P = load applied to pulley system.
$I = 0.0000745$ (inches)^4 $\qquad B = 4.63$ in.

$$E = 40{,}540\,\frac{P_2 - P_1}{1/R_2 - 1/R_1}$$

Appendix No. 3

DETERMINATION OF SHAPE AND MOMENT OF INERTIA OF BAR CROSS-SECTION

THE material used throughout this investigation was the usual flat rolled wire, nominally $^1/_8$ in. by $^1/_2$ in. in cross-section. Due to the method of manufacture, the shape was roughly rectangular, with sides very nearly parallel, but with rounded ends. Consideration was given to the possibility of having the sides and edges of the wire carefully ground so as to make the shape truly rectangular.

This plan did not appear to be desirable, because of the possibility of drawing the temper of the surface skin of metal during the grinding operation, and because of the fact that the grinding operation was almost certain to draw these thin bars slightly out of true, so requiring later straightening.

As an alternative, the shape of the cross-section of the bar was carefully studied and the corresponding moment of inertia computed. In doing this, thin sections were cut from the bar and glued on a glass plate. Using a projector, the shadows of these sections were thrown on a screen and traced. The magnification was about 55 diameters. These tracings are shown in Figs. 23 and 24. It is seen that the curves traced lie a little outside of the true circular segment. The moment of inertia of a circular segment is 15 per cent greater than that for a parabolic segment, both having the same chord and rise. (This might not be true for a different ratio of chord to rise.) It was estimated that the moment of inertia of the actual segment would be larger than that of the circular segment, and the moment of inertia has been taken as 30 per cent larger than that of the parabolic segment. *This is slightly less than the moment of inertia of one-half of an ellipse. If t is the thickness of the specimen and a the ordinate of the vertex of the segment, the moments of inertia are as follows:

Parabola	$0.0334\,at^3$
Ellipse	$0.0490\,at^3$
Value used	$0.0435\,at^3$

On this basis the segments contributed 7 per cent to the total moment of inertia of the section.

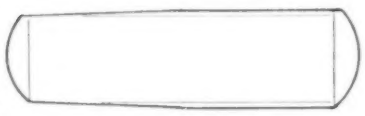
To check up the uniformity of cross-section of the bar, the thickness at each edge and at the center and the maximum width at inch intervals along the length were measured with a micrometer caliper. The width of the flat portion of the bar was measured with a micrometer microscope. The results of these measurements are given in Table 9.

The moment of inertia is very sensitive to any variation in the thickness of the bar, an error of 0.001 in. in thickness causing a 3 per cent error in moment of inertia, and for this reason the thickness had to be carefully measured.

TABLE 9 SUMMARY OF MEASUREMENTS OF 12-IN. BARS

(All dimensions in inches)

Bar	Mean dimensions: Max. width	Thickness: Edge 1	Thickness: Center	Thickness: Edge 2	Width across Flats: Top	Width across Flats: Bottom	$I \times 10^{-6}$
Bars as Received							
2	0.4984	0.1242	0.1242	0.1233	0.4355	0.4366	74.36
3	0.4987	0.1243	0.1244	0.1236	0.4387	0.4287	74.26
4	0.4991	0.1244	0.1246	0.1236	0.4244	0.4366	74.62
5	0.4991	0.1244	0.1248	0.1236	0.4268	0.4339	74.98
Bar With Edges Ground Off							
6	0.3999	0.1236	0.1248	0.1246	0.3999	0.3999	...
Average Variations in Dimensions of Bars 2–5, Inclusive							
(Max.):	0.0031	0.0004	0.0003	0.0006	0.0028	0.0028	...
(Mean):	0.0008	0.0002	0.0002	0.0002	0.0016	0.0016	...



SECTION AS ROLLED
THICKNESS VARIATIONS EXAGGERATED



SECTION ASSUMED IN COMPUTING
MOMENT OF INERTIA

FIG. 23 CROSS-SECTIONS OF WIRE

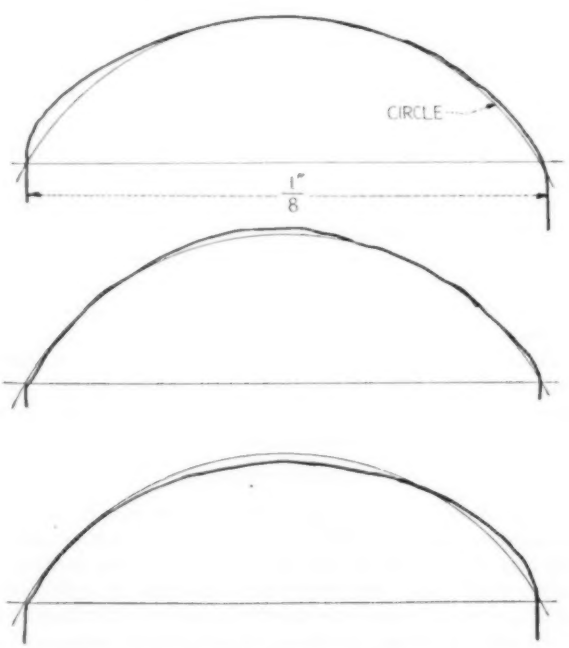


FIG. 24 TRACINGS OF PROJECTED SECTIONS OF WIRE

In computing the moments of inertia, the section was divided up into two rectangles and two segments as shown in the lower diagram of Fig. 23. The dimensions were determined from the mean thicknesses and widths previously mentioned. The greatest difference in the moments of inertia of the first five bars as computed from their mean dimensions was 0.7 per cent.

Taking bar No. 5, in which there was the greatest variation in section, it was found that the moment of inertia at the largest section was 1.5 per cent greater than the moment of inertia at the smallest section.

DETERMINATION OF AREA OF CROSS-SECTION

By using its weight and specific gravity, the average area of a bar can be found sufficiently, accurately and much more expeditiously than by using the mean of the dimensions taken at each inch along the length of the bar as was done in determining moment of inertia. The following values of specific gravity were determined by the usual methods of weighing in air and in water, making the necessary corrections:

Run	Specimen A	Specimen B
1	7.811	7.806
2	7.821	7.820
3	7.826	7.834
Mean	7.819	7.820
Max. deviation from mean, per cent	0.10	0.18

The bars were weighed on a laboratory balance, and the lengths measured to the nearest 0.01 in.

As a check, certain of the bars were ground down to a rectangular cross-section so that their areas could be accurately determined by linear measurements. The areas were then also determined by weighing as above described. The results of the two methods agreed within 0.2 per cent.

As illustrating the uniformity in size of the bar as received, the following measurements, taken on three groups of bars, may be of interest:

Group	No. of bars	Mean area	Maximum variation from mean, per cent
1 As rolled	9	0.05956	0.44
2 As rolled	5	0.05955	0.29
3 Ground to rectangular section	7	0.04956	0.22

Discussion

BENJAMIN LIEBOWITZ.[10] The preliminary results seem to justify more intensive investigation. Apparently, one of the greatest needs is for more sensitive and accurate extensometers. In spite of the excellence of Professor Sayre's instrument, it appears that perhaps nothing short of interferometer measurements will suffice for the extreme delicacy required in the direct-stress tests. The use of interferometers in engineering investigations is not new, but their application to strain measurements may involve considerable difficulties.

For the experiments in flexure no such extremes are necessary. Here the difficulty seems to be largely that of eliminating the effect of the clamped end. Any one who has ever tried to make built-up tuning forks knows how serious this influence can be. The nature of these experiments makes it all the more important to eliminate any possible doubt on this point. The clamped-end effect might perhaps be avoided by having a free instead of a fixed micrometer mount, and by using it after the fashion of a lensmaker's spherometer over the middle regions of the specimen.

If the result is substantiated that hysteresis in flexure is much greater than in direct stress, it may afford a partial insight into the fact that fatigue-stress limits are higher in flexure than in direct loading. For hysteresis, like any other frictional force, opposes internal motion of the specimen so that under a given cyclic load the cyclic yield or strain should be less. Thus hysteresis may exert a supporting influence which, under some circumstances, may offset its destructive effect. On the other

[10] New York, N. Y. Mem. A.S.M.E.

hand, for a given strain range, the fatigue limits in direct stress should be higher than in flexure, for the effect of hysteresis in that case is to increase the effective range of stress imposed on the specimen in flexure.

Another aspect of hysteresis may be mentioned, viz., its ability to function as a damping means. This can be shown strikingly by spreading the tines of an ordinary tuning fork, and then restoring it to its original shape. A marked reduction in tone volume results, but the effect is only temporary as the fork regains its original volume in a few hours or days. Hysteresis from this viewpoint may turn out to be a real ally in certain branches of engineering, e.g., crankshaft design.

One of the very interesting aspects of the paper is the emphasis which the authors have placed on the "cyclical state." It has always seemed to me curious and significant that there should be such a thing as a cyclical state, at least in the sense that in changing from one stress cycle to another, the specimen must in each instance develop a new cyclical state. In a report which the writer read before this committee in 1926, and which was published in the *Journal* of the Society of Automotive Engineers, a "hypothesis of minimum hysteresis" was suggested according to which the hysteresis in the cyclical state will in general be a minimum, and certain evidence from the literature in support thereof was adduced. The implication of this hypothesis is that in an acyclic fatigue test, the average hysteresis for a given maximum-stress range would be greater than in a corresponding cyclical test, and presumably therefore the fatigue life shorter or the non-fatiguing stress range lower. One experiment in particular which is mentioned in this report has a bearing on this point, viz., where the specimen was subjected in succession to the stress ranges O-$\frac{1}{2}P$, $\frac{1}{2}P$-P, and $\frac{1}{4}P$-$\frac{3}{4}P$. The writer would like to ask the authors, when the specimen was in the cyclical state $\frac{1}{2}P$-P and the range was then changed to $\frac{1}{4}P$-$\frac{3}{4}P$, did the hysteresis temporarily increase before the cyclical state in the new range was reached?

G. H. Keulegan[11] and W. G. Brombacher.[11] That portion of the report on elastic hysteresis and other elastic defects, such as elastic afterworking, are of particular interest. The experimental results obtained by Messrs. Sayre and Hoadley on the values of the elastic hysteresis when the bars are subjected to tension stresses are unexpected and, as they state, should be considered tentative until all possible sources of error are eliminated. In this connection the writers would like to suggest the possibility of the masking of the small values of the hysteresis which would be expected at the lower values of the stress, by errors in the measurement of the stress. No discussion is given in the report of the testing machine, the method of its calibration, or the method adopted for measuring the applied loads. Is the $\frac{1}{6}$ per cent error mentioned one of precision or accuracy? These features have very likely been given consideration by the authors, and it is hoped that they will be given a place in their final discussion.

The experiments appear to show that the tensile hysteresis is zero at a maximum stress of 80,000 lb. per sq. in. This result is in some measure contradicted by Hopkinson and Williams[12] who obtained a smooth curve for the energy loss per unit volume per cycle when plotted against the maximum stress range. The specimen was of low-carbon steel with a tensile strength of about 67,000 lb. per sq. in. The specimen was subjected to reversed stresses ranging from 23,000 to about 66,000 lb. per sq. in. Their results show the energy loss proportional to the fourth power of the fiber stress.

[11] Bureau of Standards, Washington, D. C.

[12] "The Elastic Hysteresis of Steel," Proc. Royal Soc. of London, 1912.

The relation between flexural hysteresis loss and the maximum apparent stress in the extreme fiber plotted in Fig. 14 shows a break or discontinuity in the curve at the yield point. As pointed out by the authors, the maximum stresses in bending are not likely to exceed the values of those at the yield point (see Fig. 19).

It therefore follows that there is no significance in plotting hysteresis loss against apparent stress for values of the stress above the yield point. It might be fairer to plot all of these values against the yield-point stress. It is evident that the static or cyclical hysteresis loss should increase as the stresses are increased above the yield point, since the average values of the stress also increase. The analysis of the complicated phenomena occurring at stresses above the yield point is a difficult matter. A prime necessity is an accurate knowledge of the stress distribution at each loading.

It would be of considerable interest to users of springs in precision instruments to know the ratio of the average or maximum value of the hysteresis to the total deflection.

G. M. Eaton.[13] The maximum working stress imposed on any part of the material of a spring is always at some place on the surface of the material. This is true of stress resulting either from bending or from torsion. There are three major reasons why the material at the surface of a bar of steel is inferior to material somewhat remote from the surface:

1 The surface of the bar is composed of metal at or adjacent to the longitudinal surface of the ingot, and this surface material is inferior to metal embedded a little further inside of the outer skin of the ingot
2 More or less mechanical imperfection of the surface may occur as a direct result of the rolling operation
3 Surface decarbonization probably occurs in some measure in every heat cycle to which spring steel is subjected.

This is a very insidious enemy to spring life, because the decarbonized material does not respond to heat treatment and yet the surface material is subjected to the maximum working stress.

Fatigue failures of steel springs almost invariably start at some surface imperfection, as a minute crack. Once this crack is started, its leading ends are the seat of heavy stress concentrations, so that the little incipient crack progresses into material which is amply strong to resist the stress which would exist if the concentration of stress mentioned did not exist.

This brief discussion has been made to lead up to the suggestion that the special research committee add to their agenda the investigation of the influence of the condition of the surface of the stock on the service performance and life of steel springs.

Authors' Closure

M. F. Sayre. I quite agree with Dr. Liebowitz that more sensitive and accurate measurements of elongation are needed, and I am hoping that we shall be able to obtain this result in our uture work. Any such increase in precision of extensometer measurements of course also involves added care in measurement of the applied loading, in control of temperature variations, and in the removal of frictional drag or lost motion at the points of attachment between the extensometer and the specimen and at joints in the extensometer. I am inclined to think that this last difficulty in particular has been the cause of errors in some hysteresis results in the past.

I have not found it easy to develop an extensometer which will use interferometer methods. The difficulty lies in the fact that it is desirable to provide for a total range of measurement of

[13] Engineer, Molybdenum Corporation of America, Pittsburgh, Pa. Mem. A.S.M.E.

some five thousandths of an inch. Each interference ring or band represents only ten to fifteen millionths of an inch, so that it would be necessary to keep count of possibly five hundred bands as they passed the field of view, in order to get results over the full range. This would mean indefinite delay of course in doing the work, and also a likelihood of inaccuracy in counting the rings.

The testing machine was an Amsler 30,000-lb.-capacity machine which had been calibrated about a month before the tests were made, under the supervision of a mechanic from the Amsler plant in Switzerland, using an Amsler calibration box. I was present at the time, and saw the calibration carried out. The $^1/_6$ per cent error mentioned represented the largest discrepancy found in this calibration. As a matter of fact, in that part of the work where precision was most needed (the measurement of hysteresis loss), accuracy depended upon being able to take extensometer readings at the same actual load on the ascending and descending branches of the stress-strain loop. The precision of this work consequently depended upon the sensitivity of the machine, and not upon the absolute value of the individual loads.

Internal friction is often a highly desirable feature in that it serves to dissipate energy, and so to damp out vibrations. Theoretically, as Dr. Liebowitz mentions, it should also operate to resist internal motion in such a way that for a given cyclic load, the cyclic yield or strain would be less. Quantitatively, however, the internal friction is ordinarily so small that I doubt whether this second factor would be of much numerical importance.

I should like to say a word about the question of stress concentrations on the surface of the specimen. There has been some foreign work done on studies of salt crystals under tension. The crystal was placed under load, the lower half of the crystal being in running water and the upper half in air. Under these circumstances, rupture occurred each time in the open air until the part in the water had been dissolved down to one-twentieth of the area of the part in air.

Apparently the unit strength of the crystal in water was twenty times the unit strength of the same crystal in air. The strength in water was not so far different from the high strength which theoretically, based on surface-tension values, it should attain. The low strength in air was interpreted as due to the effect of very minute surface cracks which had little time or opportunity to form in water, where the surface was constantly being dissolved away. This thoroughly agrees with Mr. Eaton's suggestion that the character of the surface metal is highly important, but also suggests that a thin layer of soft decarburized material on the surface of a spring may not be entirely undesirable, as this ductile metal would be less likely to be affected by these small cracks or flaws.

Errata

Paper APM-51-9: The Piston-Crosshead Motion of the Oilgear Pump, By Elek Benedek

Page 86, Fig. 3: For right angle y_1Ox read y_1Ox_1.

Page 86, 1st column, 3d line from bottom: For O_2 read O_1.

Page 89, middle of 1st column: In last line of paragraph beginning "Equation [10] shows," for $2\rho \cos \psi$ read $2\rho \sin \varphi$.

Page 89, 1st paragraph under heading "Analysis of the Motion, Etc.:" Begin second sentence in paragraph with "In Fig. 3."

Paper APM-51-17: Stresses in Heavy, Closely Coiled Helical Springs, by A. M. Wahl (Discussion)

W. M. Austin.[1] In the writer's discussion of Mr. Wahl's paper, there are certain errors that should be corrected. The writer also submits a further simplification of his formulas, and a table and curves of the correction factors to be applied to the old formulas.

Referring to page 197 of APM-51-17, Equations [43] and [44] should read:

$$W = \frac{2\pi Sa^2(1-v)}{v^3(1+uv)(1+u)}\left[\left(1-\sqrt{1-v^2}\right)\left(\frac{2}{3}+u^2v^2+2uv\right) - uv^3 - \frac{v^2}{3}\sqrt{1-v^2}\right] \quad [43]$$

$$W = \frac{\pi d^2 S(1-v)}{2v^3(1+uv)(1+u)}\left[\left(1-\sqrt{1-v^2}\right)\left(\frac{2}{3}+u^2v^2+2uv\right) - uv^3 - \frac{v^2}{3}\sqrt{1-v^2}\right] \quad [44]$$

On page 198, expression [45] should read:

$$\frac{\pi(1-v)}{2v^3(1+uv)(1+u)}\left[\left(1-\sqrt{1-v^2}\right)\left(\frac{2}{3}+u^2v^2+2uv\right) - uv^3 - \frac{v^2}{3}\sqrt{1-v^2}\right]$$

can be reduced to

$$\pi\left[\frac{1}{8}v - \frac{5}{32}v^2 + \frac{37}{384}v^3 - \frac{133}{1536}v^4 + \frac{127}{2048}v^5 - \frac{4079}{73728}v^6 + \frac{4061}{98{,}304}v^7 + \ldots\right] \quad [45]$$

Equation [46] should read:

$$F = \frac{2\pi S(1-v)(1+uv)r}{G(v+uv)} = \frac{2\pi S(1-v)(1+uv)a}{G(v+uv)v} = \frac{\pi(1-v)(1+uv)d}{G(1+u)v^2} = \frac{\pi dS}{G}\left[\frac{1}{v^2} - \frac{1\frac{1}{4}}{v} + \frac{9}{16} - \frac{83}{192}v + \frac{155}{768}v^2 - \frac{403}{3072}v^3 + \frac{3168}{36{,}864}v^4 - \ldots + \ldots\right] \quad [46]$$

[1] Engineer, Westinghouse Electric & Mfg. Co., East Pittsburgh, Pa. Mem. A.S.M.E.

For comparison with the formulas given in most of the handbooks, Equation [44] can be transformed to:

$$W = \frac{\pi d^2 S}{8D}\left[1 - 1\tfrac{1}{4}v + \frac{37}{48}v^2 - \frac{133}{192}v^3 + \frac{127}{256}v^4 - \frac{4079}{9216}v^5 + \frac{4061}{12{,}288}v^6 \ldots\ldots\right]$$

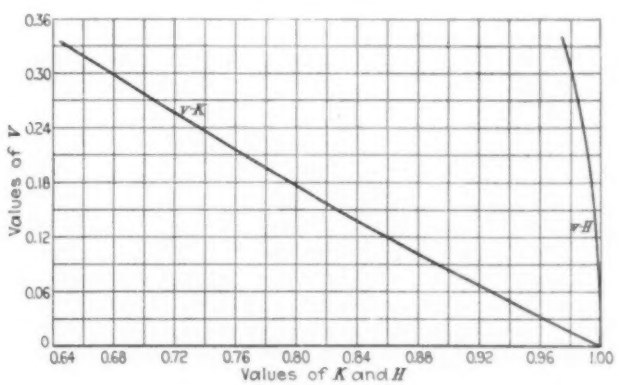
and Equation [46] can be changed to:

$$F = \frac{8WD^3}{Gd^4}\left[1 - \frac{5}{24}v^2 - \frac{77}{576}v^4 - \frac{4219}{110{,}592}v^6 \ldots\ldots\right]$$

If we denote by K the expression in the brackets in the formula for W, and by H the expression in the brackets in the formula for F, then we can calculate the values of K and H for various values of v. Table 1 shows the results of such a calculation for values of v from 0.335 to 0.000. The rate of change of K and H with respect to v varies so gradually that for intermediate values of v the values of K and H can be obtained by interpolation.

TABLE 1 VALUES OF K AND H FOR VARIOUS VALUES OF v

v	K	H	v	K	H	v	K	H
0.335	0.645	0.974	0.250	0.726	0.986	0.140	0.838	0.996
0.330	0.651	0.975	0.245	0.731	0.987	0.130	0.849	0.996
0.325	0.656	0.976	0.240	0.736	0.988	0.120	0.860	0.997
0.320	0.660	0.977	0.235	0.741	0.988	0.110	0.871	0.997
0.315	0.665	0.978	0.230	0.746	0.989	0.100	0.882	0.998
0.310	0.669	0.978	0.225	0.751	0.989	0.090	0.893	0.998
0.305	0.674	0.979	0.220	0.756	0.990	0.080	0.905	0.999
0.300	0.679	0.980	0.215	0.761	0.990	0.070	0.916	0.999
0.295	0.683	0.981	0.210	0.766	0.991	0.060	0.928	0.999
0.290	0.688	0.982	0.205	0.771	0.991	0.050	0.940	0.999
0.285	0.693	0.982	0.200	0.776	0.992	0.040	0.951	0.999
0.280	0.698	0.983	0.195	0.781	0.992	0.030	0.963	0.999
0.275	0.702	0.983	0.190	0.786	0.992	0.020	0.975	0.999
0.270	0.707	0.984	0.180	0.796	0.993	0.010	0.987	0.999
0.265	0.712	0.985	0.170	0.807	0.994	0.005	0.994	0.999
0.260	0.717	0.985	0.160	0.817	0.995	0.000	1.000	1.000
0.255	0.722	0.986	0.150	0.827	0.995	....	....	....



The accompanying curves v-K and v-H, Fig. 1, are plotted from the values of v, K, and H in the table. Values of K and H for larger values of v could easily have been calculated, but there is an old rule that springs having a wire diameter more than 1/3 of the mean diameter of the coil are not economical. This rule is no doubt the result of long experience. It also seems to be proved correct by the results of this and other analyses.

One thing is worthy of note, namely, that for the largest value of v in the table the variation in deflection from the "usual formula" is only 2.6 per cent, which corresponds to a variation in mean diameter of only 0.87 per cent. It must be remembered that this deflection formula holds good only for springs that have no internal stresses when free. Surging compression springs to a lower free height than that at which they were tempered

introduces other variables of which no account is taken in this formula.

While the correction factors in the brackets have been determined analytically to the sixth power of v, it will be observed that the sixth power is required only for the larger values of v in order to get K and H correct to the third place. In calculating the table, whenever a term became too small to affect the third place it was neglected. This reduced the time of calculation to less than half of what it would be if the sixth powers were used in each case.

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Foreword

THE Power division of the American Society of Mechanical Engineers is pleased to present the following papers which include all those presented at the Fourth Midwest Power Engineering Conference held in Chicago, Ill., February 12 to 15, 1929.

The meeting was sponsored and endorsed by the local sections, regional, and professional divisions of the American Society of Civil Engineers, American Institute of Mining Engineers, American Institute of Electrical Engineers, American Society of Mechanical Engineers, American Society of Refrigerating Engineers, American Society of Heating and Ventilating Engineers, National Electric Light Association, National Safety Council, and Western Society of Engineers.

During the last four years, the engineering societies of Chicago have been cooperating to provide a broader viewpoint of advances made in the power industry. Engineering conferences have been held, but there has been no ready means of publishing the papers. This year the A.S.M.E. decided to print the papers presented at the fourth conference as a part of its transactions. Circulation of these presentations through this publication should stimulate additional interest and provide discussion for future power conferences, that will serve further to clarify the issues involved.

The papers presented during the four days over which the conference extended, discussed power problems and ways in which the power resources of the country may be developed best for the benefit of the people and the industries. Separate sessions were devoted to power-plant substructure problems; metallurgical and chemical problems; electrical-engineering problems; heating, ventilating, and refrigeration problems; power-plant operation, and power-plant economies. Able authorities discussed various phases of the power situation, covering not only, the larger and broader viewpoint, but also specific and technical viewpoints as to what is being done in the power industry and probable future developments. The papers are arranged in the order in which they were presented and the table of contents shows the sessions into which the papers were grouped.

In reading the various papers, one is impressed with the attention being given to improved design and construction of power plants, to practices that result in added economies in power production and to better use of equipment now available. Metallurgical and chemical research has added much of value to power-plant maintenance and to provide higher grade materials for construction. Heating and ventilating problems are being solved to provide better operation of equipment and the refrigeration field is proving an added extent for power supply.

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Power Development

Engineering Advancement Depends on Progress Made in Many Sciences

By W. L. ABBOTT,[1] CHICAGO, ILL.

LIFE is change. When a city stops changing it stops growing, and when it ceases to grow it begins to die. Throughout the ages nature has been changing its animal forms and mankind has been continually changing its customs, costumes, religions, languages, laws, and governments; but the swiftest changes of all are now taking place and the practice of engineering is no exception.

It is true that not all changes are for the best. Nature makes something new only when she makes a mistake, and of those mistakes all except one in 10,000 are failures. Although she discards and disowns the 9999 mistakes, she cherishes the one success and says that her experiments were profitable.

Whether it was an accumulation of information resulting from a freer exchange of ideas that gradually solidified into a code of engineering, or whether engineering caused a freer exchange of information, we do not know and we cannot say which came first; but, egg or hen, we have learned the simple truth that the more information we give, the more knowledge we all have, and the more knowledge, the greater the progress and prosperity of the world.

It was not until after the freer exchange of information was recognized as good sense that the mechanic arts began their great strides and a profession of scholars was created expressly to form and develop original ideas and to make them known. These scholars, cutting paths in different directions through the wilderness of Nature's unknown laws and facts, developed a great diversity of engineering sciences, so great in number and so different in sphere that no one can follow all, and yet the whole are based on common fundamentals and each is so dependent on the others that many of those branches are represented here in our power program, and by their diversity in nature and their unity in purpose make this conference a success.

Such apparently unrelated subjects as substructure, Metallurgy, telemetering, heating and refrigeration, insurance, and sight-seeing among the molecules and atoms all find their appropriate places in our discussions of the great and comprehensive theme of power, whose development and utilization we are here to promote. Our friends farther down the avenue are displaying the latest styles in equipment for that purpose. We and they are here because of the great and increasing demands of civilization for some means to drive its wheels faster and faster, at ever lessening cost and with increasing benefit to man.

Power which once was obtainable in sufficient quantities from a slave or two or an ox or two is now demanded in amounts which would have appeared absurd to the users of a generation back—absurd because there was no demand for such quantities of power as are considered commonplace in this day, when nation vies with nation in its production and use. Excepting one or two small "hydro" countries, we lead all nations in the amounts of power used per capita. Therefore, we smugly assert that the use of power by a people in its industries and daily affairs is an index of that people's wealth, enterprise, industry, intelligence, mental alertness, good looks, etc.

While conceding all of that, to be agreeable, let us keep in mind that in this race we of America are favored with a great fundamental endowment of fuel, which accounts for much of our superiority in the use of power, and this fuel we are using with greater lavishness than efficiency. Great as that store of fuel is, it is limited in amount and cannot last as long as the human race will, and during this severe winter we shiver to think of the possible condition of the race with no coal left. Surely we can be rich in fuel and not spendthrifts of it, and if in using this wealth we do not employ all reasonable means to realize its last heat unit, we are squandering the heritage of our posterity.

It is true that immense strides have been made in efficiency since the day of the flueless cylindrical boiler, generating steam at or near atmospheric pressure, to be condensed by a jet of water in the cylinder of the slow-moving pumping engine. It is true that, the country over, scarcely half as much coal is now required to develop a unit of power as was needed twenty years ago, but today after 200 years of steam engineering, with the thermal efficiency of our best power plants under 25 per cent, we should blush rather than boast.

In this presidential address at the opening of the midwest Power Engineering Conference, it would be appropriate to list the more striking developments which have taken place during the past year, but upon consideration and advice I conclude that there have been no such striking developments during the past year nor during any preceding year. I might as properly take you into a great forest and ask which of those trees grew last year. None of them grew last year. All were growing. So with engineering. Progress made yesterday was but slight refinement of the development of last year, and that improvement was due to knowledge based on the accumulations of previous generations and centuries.

The airplane, whose continued roar is now heard over our flying fields, is a marvel of strength, power, and adaptation and a monument to the knowledge and ingenuity of its designer, but he could not have known how to start his design without the tragic experiences of flaming coffins and other wartime flying failures and successes. The building of those war planes was made possible by the Wright brothers at Kitty Hawk. They profited by the work of Langley on the Potomac, and he by the principles laid down by Chanute after his soaring flights in the Indiana dunes. All of this work were useless without the internal combustion engine of Dr. Otto. Back of Otto were electric ignition, engine design, thermodynamics, chemistry, metallurgy, and so on.

The steam turbine of today bears little resemblance to its immediate ancestors, which were exhibited by de Laval and Parsons at the Columbian Exposition, but the efficient, powerful giants that now drive most of the world cannot deny their rude parentage, and those toy turbines of thirty-five years ago have a direct lineage harking back twenty centuries to Hero of Alexandria. Neither the steam turbine nor the aeroplane were created; they "just grew up," with their roots in the deep past.

What is true of these modern marvels is true of all such marvels, now, here, or yet to be. None of them spring fully developed from any brain. They are dependent on too many different arts for any one discoverer or inventor to encompass, and each must wait for the work of others.

A marching column of soldiers advances by moving synchro-

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Address delivered at opening session of the Midwest Power Engineering Conference, Chicago, Ill., February 12 to 15, 1929.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

nously first its left feet, then its right. But not so with engineering. It is rather like the progress of a so-called 1000-legged worm that advances the front foot on one side, and before that is set down the foot next behind is being moved, and in turn the one behind that; the wave of activity moving back to the last leg, but by the time the fifth or sixth foot is lifted the first one has been put down and is again in motion. Meanwhile the same action is taking place with the legs on the other side and the body moves forward, but the final movement of the last leg was made possible by the movement of the first one some time earlier, and further advancement of the first leg must wait until the whole body moves, not due to the performance of any one foot, but because they all, as a part of the whole, have taken some mincing steps.

It is so with engineering advancement, which can only take place with progress in many sciences dependent on each other for light. Regarding engineering development during the past year, I can say only that the advancement of the arts has followed closely that of the sciences and waits further scientific progress. Today we are using our coal a little more efficiently, due to better furnaces and hotter fires; but we are moving slowly, waiting for better refractories; steam temperatures and pressures a little higher, but waiting for better boiler steel; transmission voltages a little higher, but waiting for better insulation. And so it is all along the line, each waiting for the one ahead to make further advance possible, and when one moves all move. Step by step, we are improving by adopting new ideas of value, and improving by discarding old ones that time has made uncouth; progress from year to year hardly perceptible, but from one decade to the next astounding.

These changes are going on in every country, for change is the order of the day, and comfort, prosperity, happiness, and pride of leadership are the stakes for which the various nationals contend.

The game is for scientists and engineers, but the winnings are for all.

Details of Design and Construction of the Stream-Flow Power Plant

By H. G. ROBY,[1] CHICAGO, ILL.

DESIGN requirements of any engineering structure are usually conflicting. The success of a design depends to a large extent on the good judgment used by the engineer in giving proper weight to the different requirements, and in working out a satisfactory compromise between them. This is particularly true in the design of low-head water-power plants.

A fundamental necessity is a low cost of construction. Unless the cost is low, the project will not be undertaken. In order to secure this, the designer attempts to hold as small as possible the quantities of excavation and of concrete called for by his plans. The tendency is to reduce the center-to-center spacing of the units, and to hold down the general dimensions, both in plan and in elevation.

The result of narrow and shallow water passages, however, is to speed up the water passing through the plant. A high velocity causes too great loss of head at the intake, due to turbulence and back eddies, and leaves too large a residuum of energy in the water as it is discharged from the power plant. For the best hydraulic conditions the plant should be ample in general dimensions, with deep and wide water passages. The good design finds the best middle ground between these extreme requirements.

Another conflict in requirements lies in the fact that the substructure is both a conduit for the water and a foundation for the machinery and building.

The passages carrying the water to and away from the wheel should be unobstructed. The discharge from the turbine in particular should be clear, as the water is being discharged at a high velocity, and in a whirl. But in order to carry down to the foundation the weight of the generating equipment, the weight of the building itself, and of the water in the scroll chamber, it is necessary to insert piers and columns which cut up the flow, causing turbulence, and loss of efficiency. It is quite remarkable that, with the diverse requirements to be met, the designers of water wheels and the designers of power stations have been able to work out arrangements which, even under low heads and while handling large volumes of water, give water wheels in place efficiencies up to 90 per cent.

There are limits beyond which experience has shown it is not desirable to go in speeding up the flow of the water through the power plant. A velocity through the entrance racks of 2 ft. per sec., or a little more, and a discharge velocity of about 4 ft. per sec. at the downstream line of the power plant has been quite generally adopted.

At Louisville, Kentucky, a new hydro station has recently been put into operation, which has a maximum head of 37 ft. The plant is equipped with eight units, each having a capacity of 10,000 kw. The water quantity required at full gate is nearly 4000 sec. ft. per unit or 31,000 sec. ft. for the entire installation. This is about four times the flow now being drawn from Lake Michigan through the Chicago River. The units are spaced 58 ft. centers except where there are expansion joints, where they are 64 ft. centers, and the total length of the power house is 508 ft.

The depth of the water at the entrance racks is 38 ft., and at the downstream line of the plant it is 24 ft. at lowest tailwater. It is seen from this that overall dimensions of low-head plants become large, in spite of the designer's constant effort to keep them down.

One of the important advances in water-power practice in recent years is the adoption of the propeller-type or high-speed runner. It is probable that for heads up to 50 or 60 ft. this type of runner will supplant entirely the Francis type which has been in use for 75 years. The large increase in speed of the propeller type makes possible the use of a much smaller generator, and this in turn permits the width of the power station to be cut down. In the first installations, this saving in cost was offset somewhat by a lower efficiency at part-gate or part-load operation. The latest improvement in the propeller design is an arrangement by which the pitch of the propeller blades may be changed to conform to the change in gate-opening or of head. By adjusting the wheel to the condition of flow at any given time, the efficiency is maintained at a high level, and the disadvantage of this type as compared with the Francis type is removed entirely.

Another advantage of the propeller type is its ability to keep up to speed under a much wider range of heads than the older model. At Louisville, this feature was particularly valuable. The tailwater rise at Louisville amounts to 65 ft. For only 75 or 80 days in the year does the water below the power house remain at the low-water stage. A Francis-type wheel, designed for 37 ft. head, would fail to maintain speed when the head was reduced to about 17 ft. With the propeller type, the plant at Louisville has been operated with heads varying from 37 ft. down to 7½ ft. The average head for the year is about 27 ft. It is estimated that the output of this plant is 10 per cent greater than it would have been if the Francis wheel had been adopted.

At Louisville, the dam was built for the United States Government, primarily for the improvement of navigation. There are two ways for vessels to pass the dam. Except for about 20 days in the year, they go through the canal and lock, paralleling the river at this point. At extreme flood the lock is drowned out, a pass is opened in the dam, and the vessels follow the natural channel of the river. Upbound vessels could not navigate the pass through the dam unless the drop from the upper to lower water level was kept small. Thirty-six hundred feet of removable dam, 10 ft. high, and 860 ft. of removable pass, 14 ft. high, make it possible to hold the upper pool at a fixed gage, while increasing the discharge through the dam and while the lower pool is rising to within a foot of the upper pool. After all of the removable dam and pass are taken out, the upper and lower pool continue to rise together. As I have stated, the maximum difference in level is 37 ft., and the tailwater at maximum flood rises 65 ft. The upper pool therefore rises 28 ft. at maximum flood.

This great variation in water levels necessitated a rather unusual design of the power station. Allowing for possible floods greater than recorded, the level of the entrances to the plant was placed 69 ft. above low tailwater. The head-works platform was placed a few feet above normal upper pool, but the gantry crane and all other features of the head works were designed so that the platform could be flooded to a depth of 29 ft.

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Presented at Midwest Power Engineering Conference, Chicago, Ill., February 12 to 15, 1929.

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In order to save concrete, the generators were placed on a floor 16 ft. below the entrance level. Even with the depression of the generator foundations, the main shafts, extending from the bottoms of the runners to the exciters above the generators, are 67 ft. long.

In addition to the unusual height of the substructure, several other features were injected into the design by the wide water-level variation. At maximum flood, the upward pressure on the turbine cover plate and on the roof of the scroll chamber become very large, amounting to 7,500,000 lb. for each 58 ft. unit. This uplift had to be counterbalanced by the weight of the building and machinery above the scroll-case roof, or there would have been a tendency for the upper part of the structure to float away. This limited the amount of coring out that could be practiced in the effort to keep down the concrete yardage.

For most of the year the turbines operate submerged. Leakage around the turbine shafts and around the pins operating the gate guides cannot be led directly to tailwater, as the tailwater is usually higher than the points of leakage. A sump had to be provided, to which the leakage is led by a drain running the full length of the power house and from which it is pumped over the high wall, the top of which is above possible flood level.

The preparation of the construction program required a careful study of the probable behavior of the river. The cofferdam around the power house site was built to a level 26 ft. above low water. It was designed so that it could be overtopped without damage, and unwatered quickly after having been flooded. It was expected that this high freeboard would give a working season from June 1 to November 1 for the excavation work, and for building the substructure up to an elevation high enough to permit the work to proceed during the winter and spring floods. There had not been a flood that would have interfered with this program since 1898, but as usual the season of 1926 was one of those unusual high-water seasons. The cofferdam was overtopped in August, which is the month of lowest average flow.

The construction schedule for the dam, which is 8650 ft. long, was arranged on the principle that it would be cheapest to build cofferdams to meet only moderate high water, and to work at a high rate of speed during the period when the comparatively low cofferdams would hold out the river. Following that scheme, a large part of the dam, over a mile in length and amounting to 50,000 cu. yd. of concrete, was built in 40 days.

FSP-51-3

Power-Plant Substructure Problems

By J. C. SANDERSON,[1] CHICAGO, ILL.

Varying conditions that are encountered in power-plant substructures are illustrated by discussing the problems that arose in the construction of four power plants at Mexico City, Pekin, Ill., Cincinnati, Ohio, and Chicago, Ill. The first was on soil of low bearing capacity, the second in water-bearing sand with a special cofferdam construction, the third required a deep foundation on water-bearing gravel, and the fourth was on a clay foundation.

SUBSTRUCTURE design is based almost entirely on the experience and judgment of the designer. There is no satisfactory method of determining the bearing capacity nor the lateral thrust of soil. At power-plant sites, which are of necessity near lakes or rivers, the character of the soil may be so variable in kind and quality that the information gained from borings or test pits may be worse than useless. In porous material water may be a difficult problem to handle, particularly where the variation from low to high water is considerable.

The difficulties of power-plant foundations do not consist so much of devising means for supporting heavy loads on unsatisfactory soils as in avoiding unequal settlements between the various parts of the building. Unequal settlement may mean a broken steam or water line or a high-speed machine out of level.

To illustrate some of the diverse conditions that may be encountered in power-plant foundations, we shall consider the foundations of four power plants built recently. The first is on a soil of low bearing capacity, the second in a water-bearing sand with a special cofferdam construction, the third a deep foundation on water-bearing gravel where the variation between low and high water is 64 ft., and the fourth on a clay foundation in Chicago.

The first plant is located in Mexico City which is near the center of an approximately circular level valley some fifty miles in diameter and encircled completely by high mountains. In the fifteenth century the Spaniards cut a tunnel through the mountains, lowering the water to its present level. The entire valley is supposed to have been a deep lake that has through the centuries been filled with alluvial mud, volcanic ash, and clay.

The thickness of the present top crust floating on this sea of mud cannot be determined as the water content increases with the depth to at least 50 per cent of the volume. This crust will stand a fair test load, but when a considerable area is loaded the settlement is as a basin, with the low point near the center of the load.

It is not practicable to use piles as the material gets softer with the depth. In most of the valley it is 1000 ft. or more to hard bottom. On account of the basin form of settlement, the heaviest bearing comes at the outer edges of the foundation and the lightest bearing at the center. After a careful study of existing buildings it was decided to adopt an average load of 850 lb., a maximum load at the edge of the footing of 1000 lb., and a minimum load of 520-lb. per sq. ft. at the center.

The foundation was designed as a light concrete barge with longitudinal and transverse steel trusses. These trusses took up the entire basement story. The bottom slab is 10 in., the deck or boiler room floor 6 in., and the sides 8 in.

The footing was extended at the west end of the building to bring the center of gravity of the foundation to the center of gravity of the load. The boiler loads and other concentrated loads were carried to the trusses by steel beams. On account of the unit bearing varying from the center of the load there was some difficulty in apportioning the load properly between the longitudinal and transverse trusses.

The old boiler room was torn down and the new boiler room constructed alongside the existing turbine room. The new foundation could not be carried below the bottom of the turbine room wall. Much of the interior of the turbine room was torn out and a new turbine foundation built on the same principle as the boiler room. Clearance was allowed around all pipes from the boiler room to the turbine room. There was no rigid connection made between the two buildings.

In preparing the site for the new boiler room some old boilers were dismantled. It was found that a turbine foundation nearby settled. When the new work was finished the turbine foundation had returned practically to the original elevation. The power station was completed about a year ago and has gone through a number of light earth shocks with no noticeable settlement.

The Powerton plant which is the second to be considered is on the Illinois river about three miles south of Pekin, Illinois. The plant site is about $^1/_2$ mile east of the river on ground 9 ft. above the flood plane. The top soil to a depth of 4 ft. to 6 ft. is a mixture of light clay and sand. Below this is a deep bed of sand and fine gravel. The plant is about 100 yd. from a deep sand and gravel pit in which the water level is nominally river level.

The lower point of the condenser pit is at extreme low water or 25 ft. below high water. The foundation is a concrete mat on the sand with a difference of 22.5 ft. between the bottom of the mat in the boiler room and condenser pit. The river level was nominally 20 ft. above the bottom of the condenser pit during the entire time the foundation was being installed. Inasmuch as the boiler-room foundations were in sand some 20 ft. above the pit bottom, we could not afford to lose sand during the installation of the deep footing.

The work was installed by excavating the entire site to the bottom of the boiler-room mat. Steel sheet piling was then driven around the entire area, approximately 120 ft. × 145 ft. on the neat line of the concrete. Inside the sheeting and about 1 ft. clear of the sheeting we drove 12-ft. to 18-ft. I-beams on 2 ft. 2-in. centers. These beams carried the earth thrust during the installation of the pit as well as after the pit was completed. The bracing was framed on the sand about 20 ft. above the bottom of the pit, and was spaced to give approximately 20-ft. dredging pockets. A concrete filler was cast between the I-beams and the sheeting for about 4 ft. from the top. When this concrete set, the bracing was tightened up with steel wedges to the extent of forcing the sheeting back into the bank. There was no bracing installed below this top set. When the excavation was completed there was nominally 20 ft. head room below the bracing. Wooden piles at intersections supported it.

As the excavation progressed the concrete filler between I-beams and sheeting was installed in 7-ft. sections. The I-beam had a 12-ft. plate riveted on the bottom to increase the bearing capacity. Two sumps about 5 ft. below the bottom kept the excavation dry. All reinforcing and concrete were placed in the dry. After the bottom was poured the I-beams were encased

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[1] Structural Engineer, Sargent & Lundy, Chicago, Ill.

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in concrete by pouring a solid wall around them. The beams were designed to carry the entire earth thrust. The concrete is used for the protection of the beams and for watertightness of the wall. Driving the sheeting on the neat line saved the outside form as well as the excavation of the three or four feet of space usually left between the concrete and sheeting. The ample head room, below the bracing, reduced materially, the cost of installing the reinforcing and concrete. The entire cofferdam, 572 lineal feet, was driven with a variation of less than 2 in. from the theoretical line.

After installing the basement floor the bracing was removed and the steel sheeting pulled. As the sheeting was pulled the space left was puddled with sand. We have had no settlement in the boiler-room floor that was cast next to, but free of, the pit walls.

During the progress of the work a check was made twice a day to be sure that the I-beams were not being forced in at the bottom. There never was any evidence of yielding. The sheeting was driven 10 ft. below the bottom of the pit and the I-beams 12 ft. The beams were probably driven farther than necessary, but as this was the first time the method had been used it was decided to take ample bearing area. There was some question as to the action of a saturated sand under lateral pressure. There was no evidence of the sand being displaced.

When the cofferdam was designed the groundwater was assumed at 15 ft. above the bottom. During construction the river level was actually 20 ft. above the bottom. Pipes were sunk at various distances from the pit and the elevation of groundwater was determined. The groundwater had a grade of practically 2 per cent and the elevation just outside the cofferdam was approximately half-way between the river level and the bottom of the excavation.

The intake crib on this job was built up partly on planking ways just above lake level and dredged about 30 ft. to position. The outside dimensions are 66.5 ft. × 67.25 ft. It was landed within 2 in. of the theoretical position and nominal level. Alternate screen partition walls were omitted to give larger dredging pockets. After the crib was sunk to proper depth the bottom was poured through the water with a bottom-dump bucket. When the sealing concrete had set, the water was pumped out and the inside finished in the dry.

The third power plant, at Columbia, is located on the west side of the Ohio River near North Bend, about twenty miles down river from Cincinnati, Ohio. The plant site was about 15 ft. below extreme high water. The variation between high and low water at the site is 64 ft. The soil is a light sandy clay on a thick bed of sand and gravel, about 45 ft. below grade. As an experiment we tried to install two piers by the Chicago well method. This was a failure on account of a water-bearing sand about 25 ft. below the surface. The pressure was so great at this depth that the lagging was pushed in until the hole was an ellipse. This same difficulty was encountered in putting in the intake and discharge tunnels. The earth thrust was much heavier at 25 ft. below the surface than at 40 ft. Several piers were installed with steel sheeting. The sheeting was braced as the excavation proceeded, then the pier cast dry. This method was slow and expensive. The sheeting was very hard to pull on account of the length and the very heavy pressure. Most of the piers were dredged down as was the condenser pit. The process was nominally the same for piers and pit.

The pier site was prepared by digging a hole 12 ft. to 15 ft. deep by open dredging. The steel cutting edge was leveled up on planks, forms were set up and about 12 ft. of pier poured. When this concrete was about 7 days old the pier was sunk by dredging the material from the inside. After the pier had been lowered to ground level another 12 ft. section was poured and the process repeated. When the pier reached gravel the bottom was leveled. A diver was sent down to inspect the bottom. The concrete seal was poured through the water and a cap put on the top, leaving the hollow pier partly filled with water. During the process of sinking it was necessary to keep a close check on the center line of the pier and see that it was vertical.

The condenser pit, on account of its size (77 ft. interior diameter) required close attention to assure that it was kept vertical and to exact position. Provision had been made for a maximum variation of 1 ft. in location. The pit was landed 2 in. out of level and 4 in. off center. The concrete seal of about 2000 yd. was made as a continuous pour.

The bottom was poured through some 20 ft. of water, starting at the center, pouring the full depth of the seal, and working toward the outside. The seal was figured without reinforcing for 2500 lb. per sq. ft. net when the pit was pumped out. The condenser pit was reinforced for the thrust of the sealing plug acting as a dome. The piers and condenser pit were also reinforced for shear and distortion.

Piers and condenser pit were made heavy enough to overcome a skin friction of 300 lb. per sq. ft. There were no material difficulties encountered as all piers were sunk without jetting or loading.

The fourth plant, Crawford Avenue of the Commonwealth Edison Company, is located on the drainage canal at Crawford Avenue, Chicago.

The soil is a blue clay, increasing in hardness with the depth. Hardpan is encountered at 25 ft. to 30 ft. below grade. The building walls and columns are carried on caissons founded on the hardpan. The caissons were sunk by the Chicago well method. There were no difficulties encountered in the installation except that the clay was quite hard toward the bottom. The turbine foundations, however, are mat footings on the clay. It would have been expensive to sink a number of lightly loaded caissons for turbine foundations. If fewer fully loaded caissons had been sunk, there would have been required as heavy a mat as for a spread footing. The mat foundation for the turbine was made strong enough to insure that the entire foundation for a machine would stay in one plane. If this is accomplished a slight settlement causes no trouble, even though the settlement is out of level. All of the piping connected to the machine was made sufficiently flexible to accommodate a slight change in elevation. There has been no noticeable settlement in any of the six machines now installed in this station.

Design and Construction of Substructures of Steam Power Stations

By P. E. STEVENS,[1] CHICAGO, ILL.

This paper discusses the substructures used under varying conditions in several localities. One located in an old river bed required the solution of many difficult problems. Another located 350 ft. from a river bank was constructed on a site which was overflowed to a depth of several feet during high-water stages. Still another developed an interesting method for solving the problem of wet excavation through pit and tunnel work.

NECESSITY for locating power stations adjacent to a plentiful supply of water makes wet excavation a usual condition, and the considerable differences in elevation at which foundation supports must be provided add complications to the task. These problems must be faced in order that there may be no limitation placed upon the design of those features of the plant having to do with the efficient production of commercial power, and the proper solution must be regarded as a contribution to that most desirable condition, a low operating cost.

The loads to be sustained by steam-power-plant foundations include very heavy concentrations from equipment in addition to the weight of the building. For example, a steel stack 18 ft. in diameter and 200 ft. high, will weigh, with its lining, about 900,000 lb. Such a stack will commonly rest upon a steel structure 70 to 90 ft. high above the foundations. In addition to the stack this steel structure will carry a boiler weighing 750,000 lb. with water walls, superheaters, insulating enclosure, etc., making the total weight of equipment on four columns a matter of 2,110,000 lb. In one case the columns carrying the stack deliver a concentration of 2,330,000 lb. to the foundation, while other boiler-room columns carry a total load of 840,000 lb. each. In the turbine room are found other heavy units of equipment. A very common size turbo-generator of 30,000 kw. capacity will weigh, with its steel support, about 600,000 lb. A condenser for such a unit will weigh about 400,000 lb. when filled with water. To take care of the concentrations of load found under the boiler room columns the desirability of developing a high intensity of reaction from the soil is apparent. Even where the soil is relatively dry and would afford a safe support for an ordinary manufacturing plant of similar dimensions, the concentrations from equipment and the necessity for substantial freedom from settlement make the use of piling advisable.

An excellent illustration of conditions characteristic of steam power plant substructures is found in the Hibridge station of the Northern States Power Company, St. Paul, Minnesota. The plant is located in the old river bed about 150 ft. from the present bank of the Mississippi river from which it derives its supply of cooling water. Low water in the river is approximately 20 ft. below the general surface of the ground and the maximum recorded high water was nearly 21 ft. above low water. The power plant occupies a building 168 by 205 ft. in plan and about 95 ft. in height. Additional areas are occupied by the coal receiving hoppers, a switching annex and an office. The building is surmounted by two self supporting steel stacks, 18 ft. by 200 ft. reaching a height of 330 ft. above the lowest footing. A general elevation for the basement floor was established at about 3 ft. above high water with a pit 40 ft. by 176 ft. in plan and 23 ft. deep for the condensers. The floor of the condenser pit is 9 in. above low water. The circulating water tunnels are below this level, making the deepest excavation about 13 ft. below low water or 31 ft. below the surface of the ground. The general elevation for column footings is approximately 19 ft. above low water so that the extreme variation in elevation of foundation levels is 32 ft. The coal pit, 40 ft. in width and extending 126 ft. along one end of the building, has its floor 1 ft. 3 in. below low water involving an excavation about 23 ft.

The subterranean survey indicated soft silt to a depth of 19 ft. or approximate low water, with groundwater at a depth of about 8 ft. Below this a 20-ft. stratum of soft clay and sand was found, beneath which a very firm sand was indicated. The footings were designed for McArthur pedestal piles to rest on the sand stratum at a depth of about 40 ft., or 20 ft. below low water. The small, shallow pits for walls and column footings were excavated by hand in open cut. For the deeper pits, steel sheet piling was driven and the material taken out by means of bucket and derrick.

Two steel stiff-leg derricks were placed to cover the heavy work and four pile drivers were used in driving the steel sheeting and the McArthur shells. The concrete plant comprised a ¾-cu. yd. mixer with overhead material hoppers and adjacent stock piles, with a stiff-leg derrick to handle the stock. Concrete was distributed by means of hopper cars running on narrow-gage industrial track.

Upon attempting to pull the casing from the piles under the condenser pit it was found impossible to do so without destroying it. It became necessary to change the design and to use a much larger number of wood piles instead of the concrete piles contemplated in the original plans. In some areas the depth of the soft ground was found to be greater than the limit of penetration of the McArthur shells, and in order to develop the desired bearing value a combination wood and concrete pile was resorted to. After sinking the steel shell to its full depth the plunger was withdrawn and a wood pile was dropped into the tube. The wood pile was then driven its full length below the tube, a concrete bulb was formed about the top of the wood pile, and the concrete pile completed in the usual manner.

The difficulties encountered at the station just described might well be contrasted with the relative simplicity of the construction problem at another plant about a mile farther up the river. The site is on what was once an island and the ground was only a few feet above low water. The relative elevation of the floor of the condenser pit is about the same, but the entire basement floor was placed a few feet above the condenser-pit floor. In the absence of a survey, a soft subsoil was anticipated and a contract let for driving wood piles with a penetration of not less than 20 ft. below the cut-off. Upon opening the excavation, a dense sand and gravel was found, into which piles could be jetted only about 12 ft. The entire excavation was kept dry with one 8-in. pump, and the only difficulty resulted from the engineer's insistence upon a real effort being made to meet the piling specification for a penetration of 20 ft.

In the Riverbank Station of the Oklahoma Gas and Electric

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

Company located about 350 ft. from the bank of the Arkansas River near Muskogee, we find a difference in level of various footings of 51 ft. The river has a high-water record approximately 37 ft. above its low-water stage, overflowing the site of the station to a depth of several feet. The building is 102 ft. by 175 ft. in plan and about 85 ft. in height above the basement floor. Two steel stacks, 14 ft. by 201 ft., give a total structural height above the lowest footing of 323 ft. The boiler-room basement and the turbine-room floor are 5 ft. above high water with the turbine-room basement 18 ft. lower and a condenser pit 34 ft. by 51 ft. in plan, extending to a depth of 23 ft. below the basement floor. Beneath the floor of the condenser pit which is about 1 ft. above low water, there is a valve pit 5 ft. 6 in. deep and circulating-water tunnels 6 ft. 0 in. in height, making the bottom of the excavation 13 ft. 6 in. below low water or about 35 ft. below the surface of the ground. A coal-receiving pit 24 by 37 ft. is also provided having a floor at a depth of 28 ft. 6 in. below the main floor involving an excavation about 20 ft. deep.

The soil underlying the site consists of 15 to 20 ft. of light sandy loam resting upon a stratum of wet sand and gravel about 17 to 25 ft. in depth, beneath which was found some brown shale and some oil-sand stone. This survey showed groundwater to be from 5 to 11 ft. above low water level.

The footings under the turbine-room columns are about 20 ft. above low water and those under the boiler-room columns about 18 ft. higher. These rest on McArthur pedestal piles designed to have a bearing value of 30 tons each. The coal-receiving hopper and pit and the condenser pit have no piles under them. The smallest footing carries 90 tons and the largest 600 tons.

Excavation was made largely in open cut, steel sheet piling being used only for the deeper pits. Material was shoveled into skips and buckets to be hoisted out by derricks. A steel guyed derrick, two steel stiff-leg derricks, and a wood stiff-leg derrick were placed to cover the work, and a revolving crane traveling on the railroad track was found valuable in handling cars and placing unloaded material to advantage. Concrete was distributed from a steel tower equipped with a boom for handling the spout.

Conditions of unusual difficulty were encountered in the substructure work for the Waterside station of the Louisville Gas and Electric Company on the banks of the Ohio River at Louisville. The erratic behavior of this river is well known. At this location its low-water shore line is about 580 ft. from the station and 37 ft. below the basement floor. Its rise to maximum high water is 45 ft., flooding the streets adjacent to the station. The oldest section of this plant was built in 1914 and it was extended in 1916 and 1924. It now occupies a building 182 by 322 ft. in plan and about 72 ft. high. From the bottom of the deepest concrete work to the top of the steel stacks is 322 ft. The general basement is about 8 ft. below high water. Pits are provided for the several condensers, as well as an ash pit 24 by 225 ft. in plan and 28 ft. deep. The bottom of the ash pit is about 16 ft. above low water. A screen house 35 by 51 ft. in plan is located on the station property about 480 ft. from the low-water shore line and connected with the plant, and with gratings in the river bottom by tunnels.

The condenser pit and the screen well were each constructed above ground as a caisson of reinforced concrete, open at top and bottom, and provided with a cutting edge of steel. As the reinforced-concrete walls and bracing were constructed the material within the cutting edge was excavated by means of a clam-shell bucket operated through the open wells of the caisson, thus permitting the structure to sink. The concrete was built up to its full vertical dimension before the cutting edge reached the desired depth. Additional weight to overcome the increasing friction of sinking was provided by loading with sand in large bins constructed on top of the concrete.

The footings of the 1916 extension are on McArthur pedestal piles resting on a stratum of firm gravel at approximately low-water level. For the 1924 extension it was decided to use a precast concrete pile of hexagonal cross-section tapering from 9 in. to 16 in. in diameter and 50 ft. long, reinforced with 6 bars $^3/_4$ in. square, and provided with a $2^1/_2$ in. jetting pipe in the center, reduced to $1^1/_2$ in. at the tip.

The discharge tunnel has its outlet in the river bed just below low water at a distance of about 480 ft. from the screen house. The intake tunnel extends beneath the river to a point about 590 ft. from the screen house. These two tunnels are in one concrete structure one above the other. They were constructed in a cofferdam of sheet piling, and the difficulties encountered would furnish material for a paper of many pages.

A relatively simple foundation layout was met with in the Horseshoe Lake station of the Oklahoma Gas and Electric Company near Harrah, Oklahoma. The site is about 120 ft. from the shore of Horseshoe Lake. This lake is an old river channel, now cut off from the new channel except at freshet stages, and is subject to a rise of about 12 ft. above present lake level, which has been raised 6 ft. by the construction of dams since the original plant was built in 1927. To this building which was 163 by 134 ft. in plan was added an extension 163 by 60 ft. in 1927. The turbine-room basement floor was placed 3 ft. below present lake level and the floor of the condenser pit 2 ft. 6 in. lower. The height from the basement to the roof is 114 ft., above which are two stacks, 11 ft. 8 in. by 73 ft. 6 in., the total height above the lowest footing being about 205 ft. Below the basement floor there is a sump, a clear water tank, and tunnels for cooling water, making the deepest excavation about 20 ft. below lake level.

A coal pit, 48 by 34 ft. in plan and 28 ft. deep, has its bearing on piles at a depth of 16 ft. below lake level. The floor of the screen well, located at the shore of the lake, is 16 ft. below lake level and the intake tunnel continues at this depth to the pump connection.

In the 1923 installation, the top of the piling under the turbine room was about at low water, and under the boiler room about 6 ft. higher. For these foundations precast concrete was used while the lower footings under the coal pits and cooling-water tunnels were placed on wood piles. In 1923, the lake level had been raised 6 ft. and wood piles were used for the entire addition.

Pit excavation was limited by means of steel sheet piling. A stream from a pipe nozzle was used to loosen the earth and sluice it to the suction pipe of a centrifugal pump, through which the water-suspended material was lifted to the surface and piped to the fill, by means of which the ground adjacent to the building was raised to a level about 1 ft. above high water.

The principal equipment included two steel, stiff-leg derricks, a pile driver, and a concrete plant consisting of a $^3/_4$ cu. yd. mixer served by wheeling in buggies from a stock pile to the batch hopper above the mixer. Concrete was distributed by spouting from a tower adjacent to the mixer, supplemented by the use of two-wheel buggies.

A very interesting method of solving the problem of wet excavation was employed in the pit and tunnel work for the steam power plant of the San Diego Consolidated Gas & Electric Company at San Diego, California. The station is located about 1100 ft. from the shore of San Diego Bay, an arm of the Pacific Ocean, from which it takes its cooling water. Extreme tidal range in the bay is $9^1/_2$ ft. A street leading from the water front and passing the plant has a grade of from 5.3 to 6.8 ft. above high water.

This station was built in 1910 and in 1921 the necessity for

additional generating capacity became manifest. Again in 1925 the growth in demand for power foreshadowed the need for a much larger circulating-water capacity, and in 1928 the station building was enlarged to its present dimensions of 179 by 167 ft. for the older portion, and 122 by 89 ft. for the new extension, which gives the building an L-shape. The basement of the turbine room in the old portion is about 9 ft. above low water and the earlier condenser pit about 2 ft. above low water. In 1922, additional tunnels were constructed under the boiler room at a depth of 27 ft. below the floor requiring excavation about 17 ft. below adjoining footings.

The soil is a fine sandy clay which is as hard as brick when dry and quite fluid when wet. A shallow trench was excavated through the boiler-room floor and in this were built the three longitudinal walls and roof of the tunnel, together with the end chambers for connection to the existing tunnels in the street alongside the station, and for the pump connections within the building. The tunnels were surmounted by longitudinal walls to prevent any possible caving from the sides. The entire structure 14 ft. 6 in. wide, about 200 ft. long, and 29 ft. high was sunk to its permanent elevation by excavating from the inside under compressed air. Excavated material was "blown" out through pipes and deposited on the roof of the tunnels between the extended side walls, providing the necessary weight for sinking. Wet concrete for the floor was introduced through a pipe having a hopper above ground giving sufficient head to overcome the air pressure.

When it became necessary in 1925 to increase the tunnel capacity, a new twin tunnel having two water ways, 4 ft. by 8 ft. 6 in., was sunk to a depth of 26 to 28 ft. below grade or 12 ft. below low water. This tunnel is about 1156 ft. in length and was sunk in four sections varying from about 200 to 373 ft. in length. The same method was used except that instead of retaining walls of concrete surmounting the tunnels, timber walls were used.

The new screen-house foundation which was a part of this work is 29 by 32 ft. in plan, and extends to a depth of 36 ft. below low water, or 8 ft. below the depth to which the harbor is to be dredged. From this depth up to about 10 ft. below low water is a concrete base constructed with 3 longitudinal and 4 cross-walls 3 ft. in thickness, forming 6 wells about 7 by 10 ft. in plan, and open at the bottom. Each wall is provided with a steel-protected cutting edge. Above the top of this base the middle longitudinal wall is omitted, thus providing three wells for the screens. The total height of this substructure from the cutting edge to the operating floor is 49 ft.

A cofferdam of steel sheet piling was driven, the bottom inside leveled off, and the concrete structure started in the dry, at a depth of about 10 ft. below low water. About 16 ft. of the structure was built up when dredging inside the wells was started. No important difficulty was experienced in sinking the foundation to its intended depth by this process of dredging, sinking, and building up. Upon completion of the sinking the 6 wells in the base were filled with sand to a depth of about 12 ft. below low water. An effort was made to pump out the structure in order to pour the 24-in. floor slab at the bottom of the screen wells. This was unsuccessful and the slab was poured under water by using a 4-in. tube or tremie pipe. The other interior construction below water level was then completed without further difficulty.

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Metals Used in Power Plants

By A. E. WHITE,[1] ANN ARBOR, MICH.

This paper deals with metallurgy as applied to power plants. It is divided into two major sections, one concerning the metals used in this field, together with special processes for producing certain desired results, while the other considers the tests applied to metals in order to determine their suitability for the various required applications.

The metals considered in the first section are divided into two groups, ferrous and non-ferrous. Under ferrous metals, cast iron, wrought iron, plain carbon steels, and certain selected alloy steels are considered. The metals in the non-ferrous group considered are copper, lead, brass, bronze, babbitt, and monel. Special emphasis is placed on the field of use of each of these materials in power-plant work rather than on their characteristics. The special processes considered in the first section are welding, plating, and nitriding.

In the second section, the tests applied to metals made to determine their suitability for power-plant use are discussed. Among the tests considered are surface examination, including both visual and macroscopic, tensile tests, impact tests, hardness tests, metallographic examination, magnetic tests, X-ray examination, strain determinations, heat tests, fatigue tests, corrosion tests, corrosion, fatigue tests, and high-temperature tests.

MATERIALS AND SPECIAL PROCESSES

THE chief ferrous metals used in power plants a few years ago were cast iron, wrought iron, and steel. Cast iron was used for many parts, particularly tuyeres, and as the casing for many power-plant units such as pumps, turbines, and various parts of condenser assemblies. As power plants went to higher operating pressures and temperatures, cast iron found less and less use. It is generally recognized that its chief outstanding asset is its cheapness, but in most instances it possesses no remarkable properties which would warrant its use over and above all other available materials. It is not intended to suggest that cast iron be stricken from the list of power-plant materials, as in cases of ordinary service it may have a perfectly proper field. It resists oxidation and corrosion to a greater degree than does ordinary steel. There are various grades of cast iron on the market for some of which outstanding claims are made by producers. Representations of this type should be taken with due caution.

One of the major characteristics of cast iron is growth under temperature. If an iron is made containing less than 0.50 per cent silicon and approximately one per cent of manganese, with around 3 per cent of carbon, growth will not take place. It should be pointed out that an iron of this composition is not machinable. Very marked improvements in the compositions of cast iron have been made through the addition of alloys, though these improvements have been largely in the matter of strength and machinability rather than a manifestation of any characteristics particularly applicable to power plants.

Wrought Iron. In the early days of power plants wrought iron was used extensively for boiler tubes. It now has little application in this field, however, it is being used to a degree as tube stock for economizers, in view of the fact that it is felt by some to possess greater resistance than does steel to the corrosive conditions which exist in economizers, as well as for other purposes.

Plain Carbon Steel. Plain carbon steel is used in power plants in cast, rolled, and forged conditions.

Cast Steel. Cast steel is used extensively in power plants, particularly in valve parts and fittings. Some of the fittings in modern stations weigh as much as 10,000 lb. and, in consequence, present real manufacturing difficulties. Most castings, in fact all of the important ones, used in power plants are made by the electric-furnace process. They are usually furnished in the annealed or normalized condition. Designers of power plants can be particularly helpful if they will observe a number of general precautions in the design of castings. Some of these are (1) the use of suitable fillets, (2) the maintenance, so far as it is possible, of the same thickness of metal throughout the cross-section of the casting, (3) the control of the total weight of the casting so as to keep it as much under 10,000 lb. as possible.

The foundryman can also help the casting situation very materially by observing certain practices: (1) He should control the density and porosity of the core closely and of the mould proper. (2) He should use risers of proper size and should have them properly located. In no case should a riser be less than 35 per cent of the weight of the casting, and it would be better practice to make it 50 per cent of the casting weight. (3) He should employ a proper pouring temperature and use a steel which has been killed thoroughly. (4) He should refrain from shaking out the casting until such time as there would be no possibility of the formation of surface cracks, and (5) he should bring the casting to heat slowly in the annealing and normalizing treatments and should refrain from cooling it under such rapid or uneven conditions as will produce strain.

Rolled Steel. Power plants are large users of rolled steel. The framework of most of the buildings and the reinforcing bars used in the concrete are all rolled material. Satisfactory standard specifications for the use of this material are available and, therefore, need not be discussed.

A very considerable amount of rolled material is used in the form of tubes which are of both the welded and seamless types. There is a general trend toward the use of seamless pipe since it is felt generally to be safer than welded pipe, though for many purposes welded pipe is acceptable. At the present time, a very high grade of welding is being done. Until recently, it was not possible to obtain seamless pipe beyond 6 in. diameter, except in relatively short lengths. With the advent of new types of piercing mills, however, one can obtain seamless pipe up to about 24 in. diameter in almost any desired length. A few years ago, plain carbon steel was about the only type of steel which could be obtained in a seamless condition. This situation is changed so that it is now possible to obtain seamless tubing, not only of plain carbon steel, but also of such alloys as stainless iron, Enduro (17 to 18 per cent chromium) and the high-chrome nickel alloys, though in all of these latter instances the carbon content in the metals is low, usually under 0.10 per cent.

This tubular material is used as boiler tubes, economizer tubes, superheater tubes, and fittings. In fact, it is used most generally in the boiler and turbine rooms, particularly in the former.

Forged Steel. The major uses of forged steel in power plants are in turbine shafts and turbine disks. Modern turbine shafts, because of their large sizes, call for particular care in manufacture. The ingots from which the forgings are to be made must be sup-

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plied with a riser of suitable size, and the forging operation must be performed to prevent the development of cracks in the metal. It is the author's opinion that, wherever the procedure is possible, an improvement in this metal is brought about by upsetting. Likewise, particularly in the case of large forgings, it is extremely

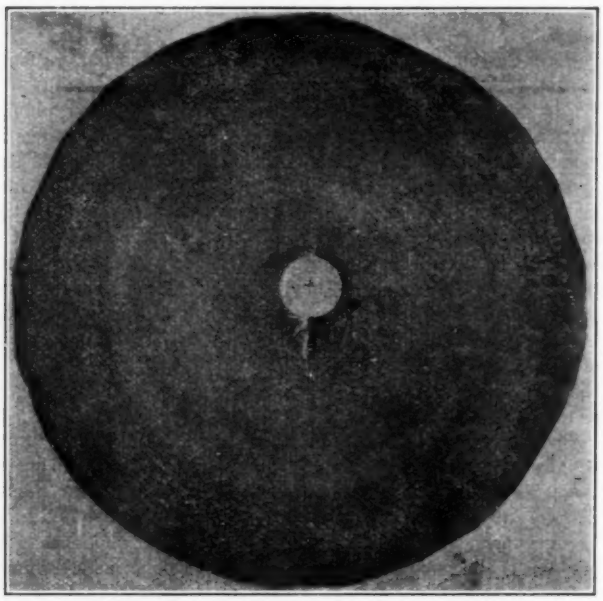




FIG. 1 SULPHUR PRINTS OF TWO CARBON STEELS

[(*a*) Above, an acceptable type as regards sulphide inclusions; (*b*) below, an unsatisfactory type regarding sulphide inclusions.]

important to bring the forging to heat slowly so that the metal may be the same temperature from center to surface.

Alloy Steel. The list of alloy steels which might be discussed in connection with power-plant uses would make several papers in itself. The author has, for the sake of brevity, arbitrarily selected seven steels. These are 3½ per cent nickel; a low-chromium, low-nickel; a low-chromium, low-tungsten; an 8 per cent chromium, 8 per cent tungsten; stainless iron, containing 12 to 14 per cent chromium; Enduro, containing about 18 per cent chromium; and a chromium-nickel alloy steel containing 19 per cent chromium and 8 per cent nickel.

The 3½ per cent nickel steel is used mainly for turbine disks, and in some cases for turbine shafts. It is preferred to plain carbon steel because of its higher strength and greater resistance to oxidation. The Chapman Valve Manufacturing Company has advocated chromium-nickel steel for castings for some time. Its uses give castings which have higher strength than is the case with plain carbon steel. Mr. Malcolm, metallurgist for the above company, was the first to bring to the attention of the public the value of a low-chrome, low-tungsten alloy; for high-temperature service.

Investigations made in the laboratories of the University of Michigan indicate that an alloy of 8 per cent chromium and 8 per cent tungsten has stability characteristics greater than those of the low-chromium, low-tungsten alloy, although it must be recognized that the greater the amount of chromium and tungsten present, the greater the difficulty with which forming and machining operations can be performed.

Stainless iron is used quite extensively for turbine blades. This is a metal which carries from 12 to 14 per cent chromium and under 0.10 per cent carbon. As turbine blades are one of the crucial parts in the power plant, considerable study has been given to the proper selection and design of blades. None of the manufacturers feel that the problem has been solved entirely, though, with the recent improvement in the design of the blades and the greater care which is being given to the selection of the stock, less difficulty should be encountered with this part than has been the case in the past.

With the trend toward higher operating temperatures, designing engineers have sought for a material acceptable for use in superheaters. It was soon recognized that plain carbon steel was not satisfactory. Stainless-iron tubes were then used, and this was followed by a stock made of Enduro. This material unfortunately appears to develop brittleness when held at around 1000 deg. fahr. (358 deg. cent.) for a long time and its use for this purpose, because of this weakness, has been discontinued.

An alloy made up of about 19 per cent chromium and 8 per cent nickel is now being used, for this particular composition. It manifests superior stability at elevated temperatures, as well as greater resistance to oxidation than does plain carbon steel.

Non-Ferrous Metals. Large quantities of non-ferrous metals are used in power plants. Chief among these are copper, lead, brass, bronze, babbitt, and monel. Copper, is employed mainly in the generator and transmission lines. It is a metal used in a high state of purity. Lead is used also in very large quantities as a covering for transmission cables. For this purpose it is used either as a pure metal or as an alloy carrying small quantities of antimony or possibly tin.

Brass is used in very large quantities in the condensers. There are three major types of this metal in use. One, which is known as Muntz metal is composed of about 62 per cent copper and about 38 per cent zinc; another type consists of 70 per cent copper and 30 per cent zinc; and the third, known as Admiralty, is made of 70 per cent copper, 29 per cent zinc, and one per cent tin. Consideration is also being given to nickel and Adnic tubes, although both of these latter types are in the process of development.

Bronzes are used in parts of impeller blades, and to a degree for heavy-duty bearings. Babbitts are used extensively for bearings. Monel, is made up of about two parts of nickel to one part of copper. One of its important uses was for turbine blades, and in this field it is still used, though a large proportion of the blades in modern turbines are made out of stainless iron.

Welding. The use of welding in power-plant construction is

receiving greater and greater attention daily. Engineers are yet fearful of poor welds and, unfortunately, no outstanding commercial method has been disclosed to date for the inspection of power-plant material after it has been welded. Yet improvements in the art of welding are being made constantly so that the welds of today are vastly superior to the welds that were made five years ago. With this improvement in welding, it is believed that all forward-looking engineers will give greater and greater consideration to the use of this method for joining together metal sections.

Plating. Very little plating has been done for power-plant parts. This is an operation which will in the future doubtless receive greater consideration than has been given it in the past. A recent use of plating has been made by the Detroit Edison Company at the Trenton Channel plant in nickel plating the valve stems of a number of important valves. A plating of $^3/_{16}$ in. thickness was put on the stems. As far as the author is aware, this is the thickest plate that has been used commercially. Over six months of work was necessary in order to work out methods for performing satisfactorily this operation.

Chromium plating also offers possibilities, though it should be recognized that it has not worked out satisfactorily from the standpoint of protecting the metals against oxidation at elevated temperatures. This failure is doubtless due to the fact that most chromium plate is exceedingly thin, and in consequence porous, and the gases, therefore, have an opportunity to get through the chromium to the base metal. It is possible to thicken chromium plate so as to obviate this condition. So far as the author is aware, however, this has not been done to date.

Nitriding. Nitriding is a process which aims to produce a very hard surface on steels, especially on certain alloy steels. It consists in subjecting steels containing special-alloying elements such as aluminum, chromium, molybdenum, and vanadium to an atmosphere of ammonia at a temperature in the neighborhood of 950 deg. fahr. (510 deg. cent.) for a period of from 12 to 90 hr.

One of the advantages of this process lies in the fact that the steel is heat treated previous to the nitriding process and no heat treatment, therefore, is necessary after the nitriding to give the case its hardness. This eliminates the warpage and scale often obtained in the other processes for producing hard surfaces. Another advantage of the process is the extremely hard surface obtained. Hardness values estimated at 1000 Brinell are secured. Application of this treatment has been made by the Chapman Valve Manufacturing Company, and possibly by others, for the hardening of their valve discs and seats.

METHODS OF TESTING

A very large number of types of tests are in use for the examination of materials used in power plants. Some of the most important ones will be discussed.. These include tests to determine surface characteristics, tensile properties, hardness, metallographic characteristics, the existence of blowholes, and sand pits (using X-ray), strain, and a few others.

Surface Characteristics

Practically all steel used in power plants is subjected to a surface inspection to determine the surface characteristics. This inspection consists of several supplementary processes including visual inspection, sulphur-print tests, and macroscopic examination.

Visual Inspection. Most surface inspection is of a visual type. No purchaser of metals will accept material which is badly scaled or pitted, or which is in a badly oxidized condition. The use of the periscope in this connection is gaining in popularity for examining tubular sections, especially in the examination of such materials as condenser tubes, boiler tubes, and the bore of turbine shafts. In this latter connection, the periscope test is a most important one, for if the material under examination discloses any cracks or other defects, rejection of the stock usually follows.

Sulphur Prints. The use of sulphur prints is also increasing since these give indications of segregations, particularly of sulphides. Fig. 1 shows two sulphur prints, one of steel of an acceptable type from this standpoint, and the other of a steel manifesting an undue number of sulphide inclusions. In the case of large forgings, sulphur prints are more and more coming into use for the purpose of determining the chemical center of the metal, for it is obvious that in a mass of metal, which is rotating at high speeds, less distortion and strain become manifest if the chemical and mechanical centers of the mass are in the same section.

Macroscopic Examination. The macroscopic test for steel is



Fig. 2 Macrograph of Half a Disk From Turbine Shaft

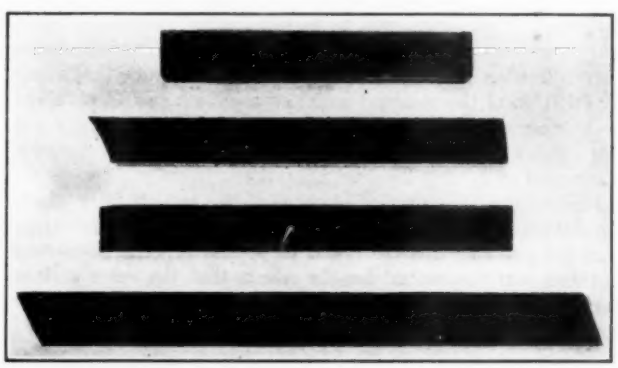


Fig. 3 Macrographs of Specimens From Seventh Stage of a Turbine Shaft

increasing in use in view of the fact that it too gives an indication of segregation, as well as flow lines, which result from forging or other mechanical forming operations. Figs. 2 and 3 show the results of a macroscopic etch on sections from a large turbine shaft. Fig. 2 is one-half of a cross-section of a turbine shaft. Fig. 3 is made up of four sections cut from a turbine shaft and illustrates the effect of the etch on the different faces. These examples are given, not as examples of good macroscopic characteristics, but rather, to indicate what a macroscopic test will disclose. Material to be acceptable from a macroscopic test should disclose a surface similar to that found on the outer face of Fig. 2. Yet it must be recognized, that the extent to which it is possible mechanically to work metal will affect, to a very considerable degree, the nature of the surface which will be obtained from a macroscopic etch. Material, which has not been appreciably worked will by its very nature disclose a more unsatisfactory surface from the standpoint of a macroscopic etch, than will material which has undergone an appreciable degree of mechanical reduction.

Tensile Tests

The method most commonly employed in the inspection of metals is the one involving the tensile test. This test gives values

for the proportional limit, elastic limit, yield point, maximum tensile strength, per cent elongation, and per cent reduction of area. The proportional limit, elastic limit, and yield point are greatly confused values, though within recent years, testing engineers have had a higher regard for the differences than was true with their predecessors. The proportional limit is usually accepted as that value at which the strain ceases to be proportional to the stress, as obtained by plotted results. The elastic limit is likewise looked upon as that value at which the strain ceases to be proportional to the stress, although it is obtained by a release of the load and is the value secured when a strain just begins to manifest itself upon the release of the load. As a rule the elastic limit is somewhat higher than the proportional limit. The modern tendency is to favor the use of the proportional limit rather than the elastic limit. The yield point is a rapid means of finding the load at which the strain ceases to be proportional to the stress. It is usually secured by means of the drop of the beam of the testing machine, or by means of the use of dividers employed for detecting a permanent set. It is the method most frequently used in commercial practice, though it gives values a few thousand pounds higher than does the proportional or elastic limit.

It is highly important in the case of rolled and forged stock, to take cognizance of the direction of the fibers; that is, whether the test sections are in line with, or perpendicular to, the direction of working. Sections taken parallel to the direction of work always show an equal, and usually a higher, reduction-of-area and elongation value, than do the sections taken at right angles to the direction of work. The author is of the opinion that a better criterion of the material under examination can be obtained by transverse-test sections than by longitudinal sections, for if there is any weakness in the stock it will be better disclosed by a transverse- than by a longitudinal-test specimen.

Furthermore, the proper selection of the section from which the test specimen is taken is highly important. Usually, sections near the outer surface are found to possess superior properties to those near the center, for the reason that the outer surface has been in a position to benefit to a greater degree, than has the center from mechanical work and the advantages to be derived from heating and cooling. As an illustration of these statements the results of four tensile tests made on a large forging are given in Table 1. One of the sections in Table 1 is transverse and the other three are longitudinal. The transverse section shows abnormally low elongation and reduction-of-area values, a part of which condition is attributed to the presence of snowflakes in the steel, though had the steel been free from this condition, it is doubtful if the elongation, as measured over two inches would have exceeded 20 per cent, and the reduction of area 3 per cent. It should also be noted that the elongation and reduction-of-area values for the longitudinal section, taken adjacent to the center are low. This is, without doubt, due to the unsatisfactory nature of the material. However, in comparison with these low values one should note the high values of the longitudinal sections, taken near and at the edge, where the metal had received all the possible benefits of work and the improvement due to heating and cooling. The elongation and reduction-of-area values for these two sections are normal and indicative of satisfactory material. Yet, with all the marked difference in the elongation and reduction-of-area values, there is not the same noticeable variation in the proportional limit, yield-point, or tensile-strength values, though, to be sure, the values for the transverse section are under those for the longitudinal. Such differences as are given in this table do not always exist, though it is seldom that transverse sections possess as good properties as do longitudinal sections, particularly with respect to elongation and reduction-of-area values.

TABLE 1 PHYSICAL TESTS ON SPECIMENS FROM LARGE FORGING

Specimen No.	Direction of specimen	Location	Treatment	Proportional limit, lb. per sq. in.	Yield point, lb. per sq. in.	Tensile strength, lb. per sq. in.	Elongation in 2 in., per cent	Reduction of area, per cent
1	Transverse	From center to outer edge	As received	30,000	40,000	63,825	7.0	8.52
2	Longitudinal	Near center	As received	40,000	55,000	76,450	11.5	18.82
3	Longitudinal	Between center and outer edge	As received	35,000	37,500	72,500	32.5	54.93
4	Longitudinal	Near outer edge	As received	35,000	40,000	72,500	32.5	59.59

Impact Tests

Impact tests of various types have been used for a number of years, although more as a research than as an inspection tool. In fact, it is generally conceded that the methods of preparing test specimens for impact tests play so great a part in the results obtained that the test, as such, has to date found little acceptance as a means of ascertaining the quality of material.

The impact tests are of two types. One is known as bending impact, and the other as tensile impact. The bending-impact type has been the one most frequently used, although there is now a tendency toward a greater use of the tensile impact.

Hardness Tests. A considerable number of methods have been devised for measuring the hardness of metals. The three which have to date found the greatest acceptance are the Scleroscope, Rockwell, and Brinell methods.

Scleroscope Tests. The scleroscope depends upon the principle of measuring the height of rebound of a hardened-steel plunger from the surface of the metal undergoing examination. It lends itself to shop inspection, and possibly that is one of the reasons for its rather general use.

Rockwell Tests. The Rockwell-hardness-testing machine has been on the market but a few years, though it is now finding a large field of usefulness. It operates on the principle of pressing a small ball or a diamond cone into the metal surface under a given load. The hardness of the metal being tested is determined by the depth of penetration. This method, due to its adaptability, is finding quite general use.

Brinell Tests. The Brinell test operates on the same principle as the Rockwell and, in fact, is the older of the two methods. The main objection to its extended use is that the stock has to be brought to it. Likewise, because of the size of the ball used in the test, it cannot be used for testing thin sections.

All of these methods, however, provide a means of testing without destroying. They are especially applicable for determining the quality of heat treatment done on parts such as bolts and nuts.

Metallographic Examination

Though metallographic methods have not been employed to any appreciable extent in specifications, yet they are used very extensively in investigations wherein difficulties are experienced with materials in either their manufacture or use. They are invaluable in determining the nature of constituents, grain size, segregation, inclusions, strains, cavities and blowholes, decarburization, and cracks.

Nature of Constituents. The nature of the constituents disclosed by metallographic examination is, of course, dependent upon the materials undergoing examination. In plain carbon steels, the constituents are usually free iron (ferrite) and carbide of iron, either in a free state or in a state of solution in the iron.

Grain Size. The extent to which the grain size of the above constituents may vary is shown in Fig. 4. In this photomicrograph, which was magnified 100 diameters, ferrite (white) is

shown in varying sizes and the carbide of iron, which in this case is present either as pearlite or sorbite (dark), is also of varying size.

Inclusions. This photomicrograph is also interesting in that it shows a sulphide inclusion (the dove-gray constituent) and a crack, as indicated by the dark streak through the center of the photomicrograph. It will be noted that adjacent to this crack the constituent is mainly ferrite and that the same is likewise fine-grained. Without doubt, the ferrite segregation adjacent to the crack is due to decarburization of the metal which has occurred at this particular spot. Therefore, in this photomicrograph the constituents in the metal are developed, grain size is shown, segregation is in evidence, and decarburization is apparent. Usually, however, most decarburization occurs at the surface of metal, though it is frequently adjacent, as in this case, to cracks.

Brass lends itself exceedingly well to metallographic examination. In Fig. 5 is a photomicrograph of Muntz metal in which the two constituents, the alpha and the beta, are shown. In this

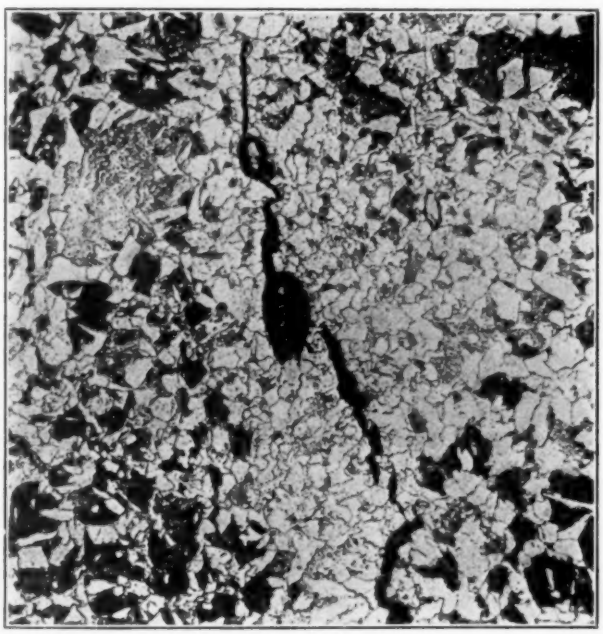


FIG. 4 PHOTOMICROGRAPH OF STEEL SHOWING INCLUSIONS, CRACKS, DECARBURIZATION, ETC.

particular photomicrograph strains are also evidenced by the parallel lines in the various grains.

An entire paper might be written on this subject, but it is believed that enough has been stated to demonstrate the usefulness of this research and inspection method for the examination of metals.

MAGNETIC TESTING

Magnetic testing, although not yet used to any great extent, offers possibilities of being reliable in detecting defects in metals. There are two forms of this test now in use. One consists in subjecting magnetized metals to a bath containing finely divided iron particles in suspension. If a flaw exists in the metal at or near the surface, then the iron particles will be deposited at this point and the defect thus shown.

The second method will discover flaws, not only at the surface, but also in the interior of the metal being tested. This test is not quantitative, but comparative. It consists in placing a perfect specimen in a magnetizing coil and comparing the readings obtained from it with those from exactly similar articles. If a marked variation in readings is obtained, then it is assumed that some foreign material or cavities exist in the section.

X-RAY EXAMINATION

The X-ray method of examining metals is one of rather recent origin. It has been used largely for the inspection of blowholes and sandspots in castings. In fact, the most notable use of the X-ray for this purpose is in connection with the examination of the high-pressure castings for the Weymouth station of the Edison Electric Illuminating Company of Boston. It is the author's understanding that every one of these castings has been examined by means of the X-ray, and that a considerable number of those initially presented for acceptance were rejected because

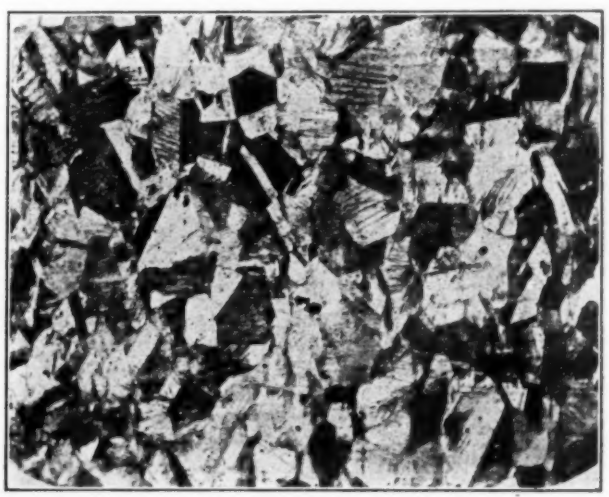


FIG. 5 PHOTOMICROGRAPH OF MUNTZ METAL IN STRAINED CONDITION

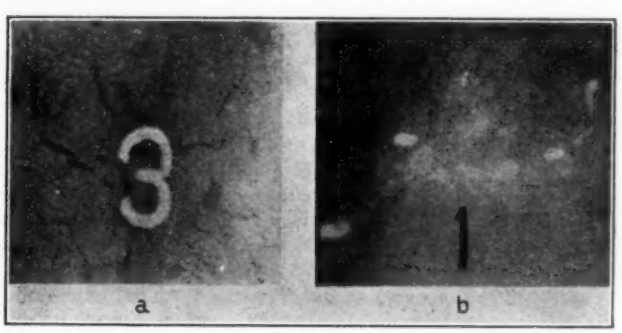


FIG. 6 RADIOGRAPHS OF CAST STEELS
[(*a*) At left, internal cracks; (*b*) at right, blowholes.]

of the presence of blowholes, sandspots and cracks in vital sections. A typical X-ray photograph disclosing some of the above conditions is given in Fig. 6.

In view of the fact that with present equipment it is not now possible to examine sections appreciably over two inches in thickness, it must be recognized that this method of examination is subject to limitations. However, with the development of tubes with higher voltage, it will be possible successfully to examine greater thicknesses of metal.

Another use of the X-ray which is of more recent origin relates to the detection of strains. When a section of metal is examined by the pinhole method, the resultant pattern is made up of con-

centric rings or of spots or striations brighter than the connecting segments. When the pattern is composed of concentric rings, the metal is strain-free [see Fig. 7 (*a*)]. When it is composed of spots or striations brighter than the connecting segments, the presence in the aggregate of large crystal grains or a preferred orientation of the crystal grains (indicative of strains) is manifest [see Figs. 7 (*b*) and 7 (*c*)].

This method of examination was employed on a turbine shaft, which was found to be in a strained condition, and patterns similar to those shown in Figs. 7(*b*) and 7(*c*) were disclosed.

Strain Determinations

It is not always simple to detect the presence of strains in metal, except in cases where the strains are excessive and produce actual rupturing of the crystals. Where such a rupture has taken place, the tensile tests, hardness tests, and metallographic examinations will disclose strains, as well as will the magnetic and X-ray methods. In some cases it is possible to get some criterion of the amount of strain existing in a metal by actual stress measurements. This method was employed in the case of a disk from a turbine shaft which was examined in the "as received" condition and a similar disk after it had been annealed at 1550 deg. fahr. (843 deg. cent.) for the purpose of relieving the stresses. The resultant changes in the amount of the stresses are given in Figs. 8 and 9. Fig. 8 shows the stresses in the disk of metal in the "as received" condition to be of much greater magnitude than those existing in the disk after annealing, as shown in Fig. 9.

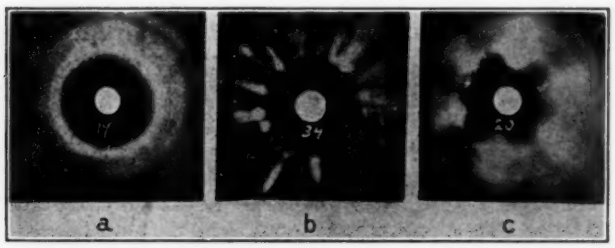
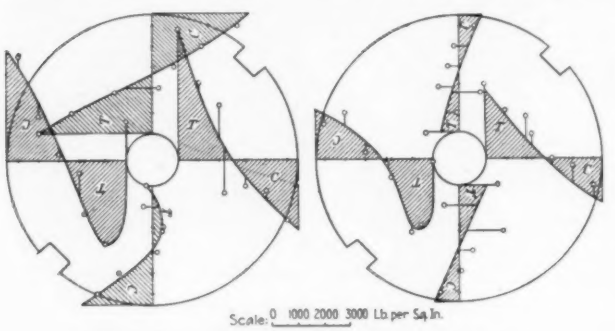


Fig. 7 X-Ray Diffraction Patterns of Steel
[(*a*) At left, strain-free iron; (*b*) in center, steel under extreme strain. (*c*) at right, cold-drawn steel perpendicular to direction of draw.]



Circumferential Stresses at Different Angles

Fig. 8 Disk as Received Fig. 9 Annealed Disk

Heat Testing

A recent development in the examination of turbine shafts is what is known as "heat testing." This test consists in rotating a turbine shaft slowly in a heated chamber. Various temperatures have been employed, although at present most turbine shafts being thus examined are heated to a temperature of approximately 400 deg. cent. (752 deg. fahr.). The deflection of the shaft throughout the heat cycle is noted but greater emphasis is placed on the first and second cold readings. If the deflection from the first hot to the second cold reading is, at any point, in excess of 0.002 in., the acceptability of the shaft is questionable. Most shafts during the heat-test cycle show a movement greater than 0.002 in., although the movement from cold to cold, as stated above, should not be in excess of 0.002 in. A typical turbine-shaft test report is given in Fig. 10, and the set-up for this test is given in Figs. 11 and 12.

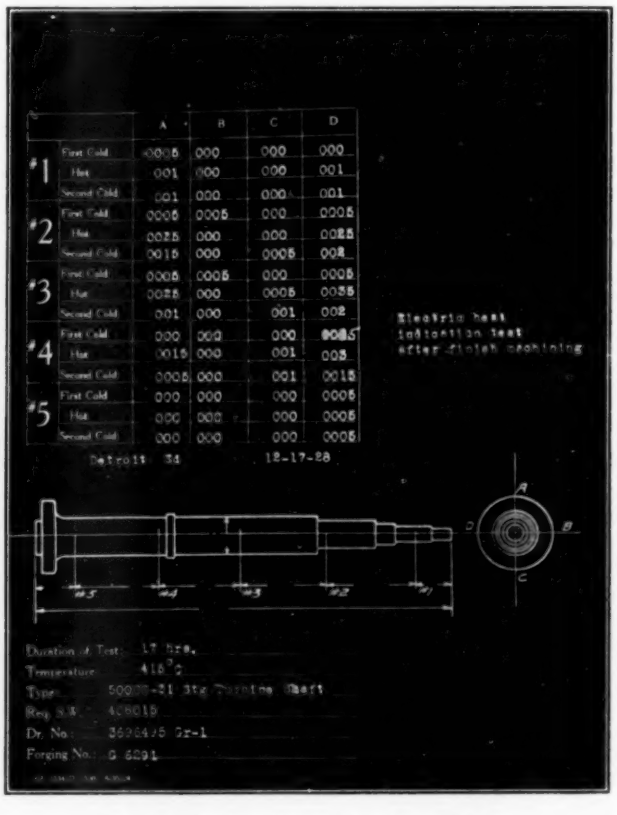


Fig. 10 Turbine-Shaft Heat-Test Report

Fatigue

Within recent years, a considerable amount of interest has been manifest in determining the fatigue characteristics of metals. The outstanding investigations in this country have been under the direction of Professor Moore at the University of Illinois. The larger builders of turbines in the United States have financed a considerable amount of this work, inasmuch as they desired a knowledge of fatigue characteristics of vital parts in prime movers. The results of Professor Moore's work are available through the University of Illinois' bulletins. These, summarized in the briefest of terms, show that the endurance limit for cast iron is about 35 per cent of the tensile strength, the endurance limit for cast steel is about 42 per cent, and the endurance limit for rolled and fabricated steel is about 50 per cent of this strength.

Corrosion

The engineering profession would enthusiastically welcome a short-time corrosion test which would disclose the respective abilities of the metals under consideration to resist corrosion. To date no single test has been developed, which adequately fulfills this purpose. Of the numerous methods which have been

employed for determining the relative resistances of various metals to corrosion, one finds both still and turbulent tests. It seems, however, that something is lacking in the case of all. The greater emphasis is, in consequence, placed on experience rather than on the results of any single test.

Corrosion-Fatigue

The corrosion-fatigue method of testing differs from the usual fatigue test in that the specimen is subjected to the influence of a corroding medium simultaneously with the repeated stress. This, test, although of rather recent origin, is finding wide use and is giving interesting results. Apparently the combined

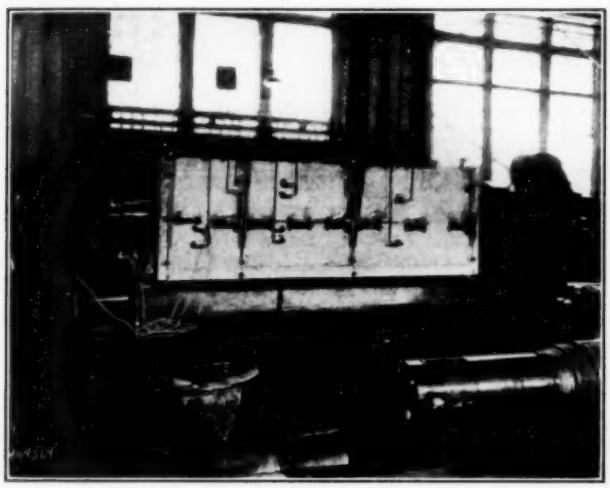


Fig. 11 Set-Up for Heat Tests on Turbine Shafts

action of stress and corrosion produce results entirely different than when these factors are applied separately.

If the stress-cycle graph obtained from this test be compared with one from the usual fatigue test, it will be found that the corrosion-fatigue limit is generally from 40 per cent to 90 per cent of the usual fatigue limit.

High-Temperature Tests

With the trend toward the use of higher operating temperatures in central stations, more information on the properties of metals at elevated temperatures than the central stations now have, is required. Not only should one have information with regard to the properties of metals at elevated temperatures as determined by short-time tests, but likewise information should be obtained with regard to the so-called "creep" characteristics.

Within the past few years a considerable amount of work has been done, both in this country and abroad, on this subject. The field, however, is so new and the number of alloys and the temperatures at which they should be examined are so great, and the time required for a single creep test is so long that the surface of this matter has hardly, as yet, even been scratched. These characteristics are being studied ably by a joint committee sponsored by The American Society of Mechanical Engineers, and the American Society for Testing Materials. Also a considerable number of laboratories have undertaken independent investigations, though many of these laboratories are carrying on activities in cooperation with the above mentioned joint research committee.

The properties of metals at elevated temperatures has been studied extensively in the laboratories of the University of Michigan for the past five years, as the result of grants, that have been made to the University by the Detroit Edison Company. The findings have been published from time to time and are consequently available to the public. Some selected data which have been obtained are given in Figs. 13 to 16.

Fig. 13 gives the data obtained by short-time tensile-strength tests on a 0.13 per cent carbon steel. These data are submitted for this steel, in view of the fact, that it is used so extensively in power plants. It does not possess outstanding properties at elevated temperatures, however, and for that reason it is being superseded by other grades of steel, when maximum stability at elevated temperatures and maximum resistance to oxidation are desired.

Fig. 14 shows the properties of a 19 per cent chromium, 8 per cent nickel alloy steel, when at elevated temperature, as determined by short-time tests. In comparing the data in Fig. 14 with that in Fig. 13, it will be noted that, within the ranges to which these steels are usually subjected, this high-chrome nickel steel has about twice the strength at elevated temperatures that plain carbon steel possesses, and in addition has a high resistance to oxidation.

Fig. 15 is of an eight per cent chromium, eight per cent tungsten

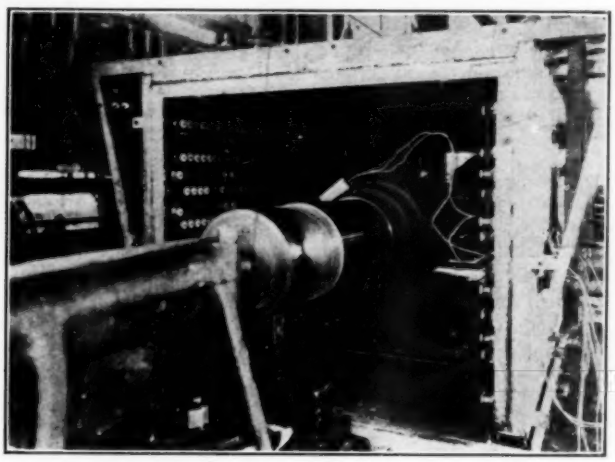


Fig. 12 Set-Up for Heat Tests on Turbine Shafts

steel, when subjected to short-time tensile tests at 1000 deg. fahr. (538 deg. cent.). This figure is included to illustrate the possibilities of heat treatment, since under the conditions of the test a range of proportional-limit values from about 15,000 to 100,000 lb. was secured. Though the 100,000 lb. value was accompanied by negligible ductility, a reasonable degree of ductility is possible by the use of a proper draw. This chart is an example of the possibilities of heat treatment when steels of suitable composition are employed. It is questionable, however, if many steels will show the same response to heat treatment that this particular steel showed. The properties of this particular steel over a range of temperature when tested in an annealed condition are not incorporated here, though they show characteristics, insofar as the proportional limit is concerned, twice those of the high-chrome nickel steel given in Fig. 14.

The laboratory is now engaged in a number of flow or creep tests on various grades of steel. The results obtained on a steel containing 1.43 per cent chromium and 2.19 per cent tungsten are given in Fig. 16. Various loads have been applied to this steel which on a short-time test at the temperature in question showed a proportional limit of 37,500 lb. This steel ruptured with a load of 45,000 lb. at 600 hr. It has been under a load of 40,000 lb. for 1800 hr. and, although to date it has not ruptured, the characteristics of the curve are such that it is expected to break in due time. A further sample was subjected to a load

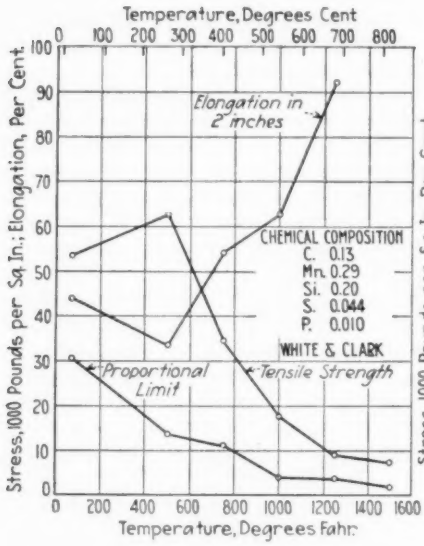


FIG. 13 PROPERTIES OF 0.13 CARBON STEEL AT ELEVATED TEMPERATURES AS DETERMINED BY SHORT-TIME TENSILE TESTS

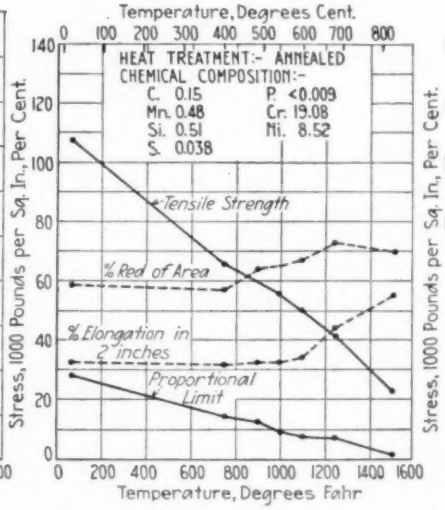


FIG. 14 TENSILE PROPERTIES OF A CHROMIUM-NICKEL STEEL AT ELEVATED TEMPERATURES

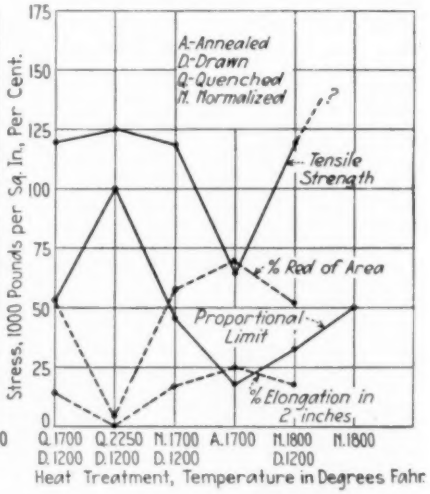


FIG. 15 EFFECT OF HEAT TREATMENT ON THE TENSILE PROPERTIES OF 0.41 PER CENT CARBON, 7.93 PER CENT CHROMIUM, 7.7 PER CENT TUNGSTEN STEEL AT 1000 DEGREES FAHRENHEIT

of 35,000 lb. but was discontinued after 1320 hr. At the time the test was discontinued the sample had not ruptured, although the curve characteristics indicated that it would break in due time. Further tests are in progress at 30,000 and 20,000 lb., respectively. Each of these tests has been in progress for over 1800 hr., and to date the maximum elongation is under two per cent in the case of the steel subjected to the 30,000 lb. and under one per cent in the case of the steel subjected to the 20,000 lb. load.

CONCLUSION

The foregoing tests and methods of examination are by no means the only ones which may be employed in the inspection of metals for imperfections, defects, and flaws. Likewise, it is recognized that the descriptive details of all of these methods are exceedingly brief. This course was purposely followed in order to keep the paper within reasonable limits, as it was the purpose to give a general birds'-eye view of this field rather than to take up any one item in detail.

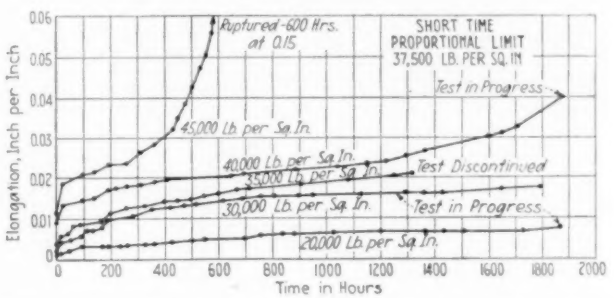


FIG. 16 FLOW TESTS ON 0.60 PER CENT CARBON, 1.43 PER CENT CHROMIUM, 2.19 PER CENT TUNGSTEN STEEL AT 1000 DEGREES FAHRENHEIT

The Corrosion Problem as Applied to Power Plants

By H. S. RAWDON[1] and K. H. LOGAN,[2] WASHINGTON, D. C.

WHILE the seriousness of the corrosion problem is generally recognized, it must be admitted that corrosion troubles are of secondary importance, as a rule, rather than primary so far as the main object of the plant is concerned. Nothing in the nature of a systematic study of this problem has been carried out by the Bureau of Standards. Advice on numerous phases of the corrosion problem has frequently been asked, however, and the following discussion relates to only a few of the more salient features of the corrosion problem as it relates both to the inside and the outside of the power plant, that is, to the generating of power and to its transmission.

Corrosion and Water Treatment

The most serious corrosion problems arise in connection with the generation and utilization of steam. It has been known by chemists for years and is now coming to be recognized more generally by engineers that the modern steam boiler is not merely an evaporator wherein takes place the conversion of water into steam, but is in effect an autoclave, the operation of which makes possible the concentration, to a relatively high degree, of impurities initially in solution in the water, together with chemical reactions involving these impurities such that there result very radical changes in the nature of the water within the boiler. The corrosion problem no longer remains the relatively simple one of the corrosion of iron by water, but rather we are confronted with a corrosive attack by substances, the presence of which is ordinarily considered as impossible. In order to ameliorate such conditions, the water is "treated," usually before it enters the boiler, and it may be questioned very seriously whether there is any natural water whatsoever which is entirely suitable for use as feedwater for a boiler without some sort of treatment. The story of boiler corrosion thus becomes inseparably bound up with that of "water treatment," in many respects the two being simply different aspects of the same problem.

Boiler-water treatment, in general, is for the purpose of bettering two conditions, scaling and corrosion. In the past, the former, probably because it gives very tangible evidence of its presence, has received most attention, and too many engineers apparently have been satisfied if the water treatment used controlled this phase of the problem. The other aspect, the control of corrosion, received but scant attention. Conditions have changed, however, and it is quite well recognized now that softening of the feedwater so as to prevent scaling may by no means constitute a complete and satisfactory treatment of the water.

Scaling, that is, the precipitation of insoluble substances resulting from the interaction of impurities present in the water, is usually not considered as a corrosion problem, although the consequences of scaling may at times be more serious than those of simple corrosion. The two problems are not entirely unrelated, however, and so far as corrosion alone is concerned, slight scaling giving rise to a thin, tightly adherent protective coating is to be considered as a distinct advantage.

For convenience of discussion, we may consider "boiler corrosion" as originating from two principal sources, both of which may be regarded as impurities in the water. Corrosion by steam is very limited in extent as compared to corrosion by water and such steam corrosion as does occur is usually reduced very materially by improving the water conditions. Under the influence of the conditions of temperature and pressure prevailing within a boiler many chemical reactions which are ordinarily regarded merely as "paper" reactions occur readily between some of the impurities in the boiler water, giving rise to conditions very favorable to corrosion. A number of natural substances, notably magnesium chloride, present either as such or produced by some preliminary boiler reaction, hydrolyze and actually produce acid in the water, the corrosive action of which upon the steel is accentuated by the prevailing conditions of temperature and pressure. Other salts and also organic impurities may act in like manner. Even traces of the lubricants used may do so. This form of attack may occur almost anywhere in the system but is most severe within the boilers and tubes. The form of the attack, that is, whether it occurs as a gradual and fairly uniform thinning of the tubes, for example, or as surface roughening in the form of pitting and streaking depends in large measure upon accompanying circumstances. The chemical character of the metal, as a rule, does not determine the form or the intensity of the corrosive attack. The only practical means of control of corrosion of this kind depends upon a preliminary chemical treatment of the water so as to insure the complete removal, so far as is practicable, of all those impurities which are either actual or potential corrodents. The subject of water treatment, though still somewhat controversial in a few of its aspects, is well understood and details need not be discussed here.

Of course, in any operating plant the amount of water added daily to the total amount in use within the closed system is only a very small percentage of the total. The greater part of the water is recovered after the steam has served its purpose and is used again. This, of course, serves to reduce enormously the corrosion within the system. Even with water of this character, however, corrosion is not eliminated and the character of the attack which occurs serves to introduce the second type of corrosive impurities in boiler water. These are the dissolved gases—oxygen and carbon dioxide. There is general agreement as to the serious consequences which may result from impurities of this kind. There is, however, decided difference of opinion as to the importance of the role played by oxygen and by carbon dioxide. Speller,[3] an outstanding American authority on corrosion states:

> "Regardless of the theory of corrosion, experience indicates that the oxygen dissolved in water is probably the greatest accelerator of corrosive action in boilers and accessories, and that it should be reduced to a certain minimum depending upon conditions."

On the other hand, Paul,[4] a British authority states: "Oxygen alone, even in the presence of water, has very little if any action

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not of the Society.

[3] F. N. Speller, "Corrosion, Causes and Prevention," McGraw-Hill Book Co., p. 400.

[4] J. H. Paul, "Boiler Chemistry and Feed Water Supplies," Longmans, Green & Co., p. 179.

on iron, but—carbonic acid has a very decided corrosive action upon iron." Paul admits, however, that oxygen may accelerate the attack of acids upon iron; he also considers that much of the attack of the carbonic acid is the result of some other carboxylic acid derived from and so closely related to carbonic acid that it is impossible to distinguish between them when they exist in such a dilute state. The reconciliation of such diametrically opposing statements upon such an important subject as this, even though they may seem to be of theoretical rather than immediate practical interest, would be very desirable, indeed. If, as suggested, oxygen in itself is of little or no account but is detrimental only to the extent to which it accelerates the action of some other corrodent which may be present, our concept of the problem may be quite materially modified. It is anticipated that some work now in progress at the Bureau of Standards dealing with the aeration factor in "under-water corrosion" will throw some light upon the relative importance of these two gases when dissolved in water upon the corrosive attack of iron such as occurs within a boiler. Be that as it may, however, it is quite generally admitted, at least in this country, that in addition to the regular water treatment, the elimination of dissolved air from all feedwater (which, of course, removes both oxygen and carbon dioxide) together with a close control of the oxygen content in the water throughout the various units of any steam-generating system is the most efficient and practical way of corrosion control.

Influence of the Character of the Metal

To what extent can the corrosion of boiler and allied equipment be controlled by a wise choice of the material entering into its construction is a question asked repeatedly.

The use of some of the recently developed corrosion-resisting steels is, of course, one means of reducing greatly, if not entirely eliminating, corrosion troubles. Tubes of such steels are now being made commercially, but up to the present their use as boiler tubes has been of decidedly limited extent and is to be regarded still as more or less experimental. Indeed, there seems to be no immediate prospect of any very great extension in this field, and even were such material put into wide use immediately for tubes and other parts, the corrosion problem in equipment already in use would still be a very active one for many years to come.

Variations in the chemical composition of the present-day material for boiler construction, provided, of course, that the material is of good commercial quality such as is required by current specifications, are not the determining factors so far as corrosion is concerned. Results have been published[5] calling attention to the inferior results obtained with low-quality steel. Such cases are exceptional, however. As a rule, serious corrosion is not primarily caused by the composition of the steel used.

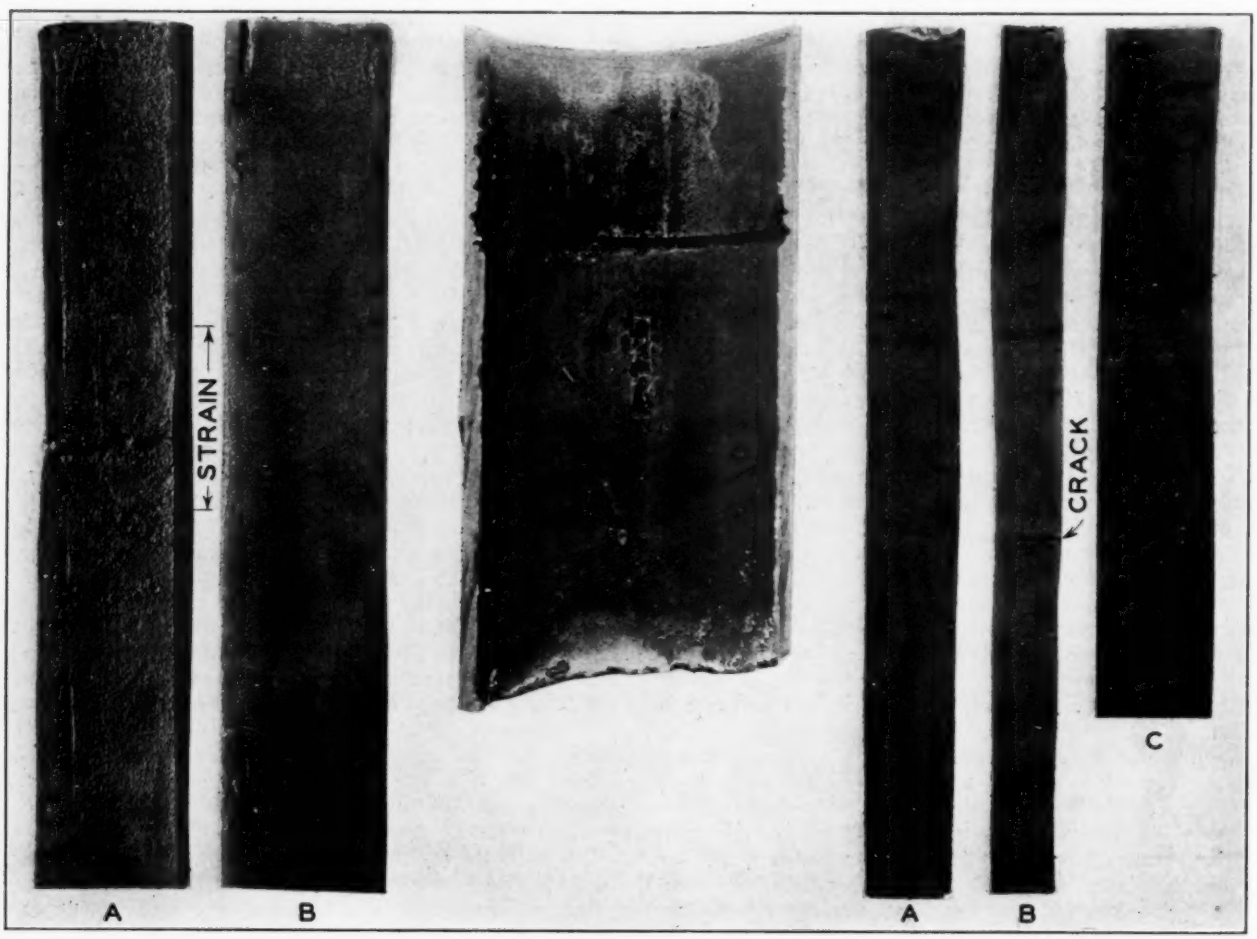


Fig. 1 At Left, Condition of Metal When Annealed End of Tube Was Cold Worked

Fig. 2 In Center, Tube Damaged as a Result of Mistreatment

Fig. 3 At Right, Non-Uniformity of Metal in Tube Resulting From Expansion

[5] G. R. Woodvine and A. L. Roberts, "The Influence of Segregations on Corrosion of Boiler Tubes and Superheaters," *Jl. Iron and Steel Inst.*, 113, p. 219, 1926.

On the other hand, steel, which according to the results of experience should be entirely satisfactory for the purpose, can be made worthless by the "brutal" treatment which it not infrequently receives in the erection of the installation. Corrosion of any metal is greatly accelerated and rendered more serious in its consequences, since it usually results in a localizing of the attack, by any condition which causes non-uniformity in the metal. Cold working by rolling, "expanding," shearing, punching, and the like is one of the most potent sources of trouble from this kind. While excessive cold working is forbidden in some cases, as in the punching of rivet holes, in other cases it is seemingly a necessity, as in the "expanding" of tubes into place. The following cases will illustrate the serious consequences which may ensue from excessive treatment of this kind.



FIG. 4 AT LEFT, CORROSION AT EDGES OF A 3/8-INCH STEEL PLATE



FIG. 5 IN CENTER, DAMAGED END OF A NIPPLE ON A DOWN-TAKE HEADER



FIG. 6 AT RIGHT, CORROSION IN CENTER PORTION OF 3/8-INCH STEEL PLATE

Fig. 1 illustrates the very serious damage resulting from mistreatment of a set of boiler tubes. The tubes failed by localized corrosion a few inches from the end, resulting in perforation within less than a month after installation. The greater part of the tube was in essentially as good condition as at the time of installation. In this particular case, the mechanic who installed the tubes insisted on annealing the ends in a blacksmith's forge evidently to facilitate the "expanding" of the tubes into the end plates. Fig. 2 shows the condition of the metal which resulted when the annealed end was cold worked by the "expanding" process. A zone of very severely deformed metal, as shown by the diagonal strain markings, was formed in that portion of the tube immediately adjacent to the annealed end. The conditions for electrolytic acceleration of the corrosive attack from any source whatever were excellent and the result was the perforation of the tube in this ring of localized corrosion within the course of a few weeks.

In Figs. 3 and 4 is shown the type of corrosion occurring near the edges of a 3/8-in. steel plate and at some distance away from the edge. The strain lines resulting from the shearing of the plate along the edge and extending for a distance of 2 in. into the plate have been revealed by a special etching method. The fact that the course of the corrosion attack has been directed by this strained condition of the metal cannot be disputed. This particular case of corrosion occurred outside of a boiler or heating system. Had it occurred within such a system, however, there is no reason for believing that the character of the attack would have been any different from that shown. In all probability it would have been much more serious than is shown in the figures on account of the favoring conditions of temperature and pressure.

That the foregoing cases are not far removed from actual conditions which may occur in the modern power plant is illustrated by Fig. 5, which depicts a case arising in one of the largest plants of the country in which the corrosion conditions are very carefully controlled and regulated. The illustration shows the damaged end of a nipple on a downtake header which was characteristic of a considerable number of such nipples. The circumferential crack shown, developed after the nipple had been "expanded" into place and the unit put into service. The decided non-uniformity in the working of the metal during the expansion of the tube is apparent from Fig. 6, and the fact that the circumferential crack developed in a zone in which the metal had been excessively rolled is, without doubt, significant as to the cause of the damage.

In the erection of new boilers and accessory units as well as in the replacement of parts in existing plants, the installation of tubes, nipples and the like by cold working the ends (expansion, flaring, and beading) is approved by present-day boiler codes.[6] The seriousness of the damage which may result if the operations are not carried out properly is not usually realized, however. The advisability of using methods which would eliminate such severe treatment of the metal is a matter for serious consideration. Although the damage produced by the cold working may not be immediately apparent and the job may be approved as satisfactory, the conditions may be exceedingly favorable for future trouble, and often all that is required is a corrosive attack

[6] A.S.M.E. Report of Boiler Code Committee, 1918.

of such parts. The only remedy that can be suggested is the old one of "avoiding extremes." Though the cold working may of necessity have to be quite severe, it should be as uniform as possible, both in its operation and in its effect on the metal. Any operation tending to produce abrupt changes in the structural condition of the steel either as changes in the visible structure or that indefinite condition usually referred to as "internal stresses" should be meticulously avoided as potential sources of trouble whenever corrosion is under consideration.

This is particularly true in cases of the so-called "caustic embrittlement" of boiler plate steel, a form of corrosive attack which manifests itself by the formation of cracks in the steel plate which suggest that the metal has undergone a complete change in its properties, from a ductile material to one almost entirely lacking in this characteristic. As a matter of fact, the metal is essentially as ductile as ever. The "embrittlement" has been the result of the action of a corrodent upon the metal while it was under stress. The combination of these two conditions gives rise to the cracking of the metal by the formation of intercrystalline fissuring—a phenomenon which has also been noted with a variety of metals when subjected to stress and corrosion simultaneously. The occurrence of a condition of internal stress such as results from excessive cold working as mentioned previously is very favorable to this type of corrosion.

Information recently published on this subject[7] has served to define quite clearly and unmistakably the conditions under which this form of corrosion may be expected to occur. In the laboratory tests, a stress above the yield point of the steel and a solution of sodium hydroxide of relatively high concentration (300 g. per liter) were necessary in order to produce the effect. An increase in the temperature, however, was found to have the same effect with more dilute solutions. In service, the necessary high stress in most cases is the resultant of the external or service stress and the internal stresses of cold working. The caustic soda results from improper treatment of the water. Although it would appear that this form of corrosion is restricted to certain geographic areas, depending upon the character of the groundwater, the seriousness of the problem is indicated by the recommendations concerning the precautions which should be taken, contained in the Boiler Construction Code for 1926.

Corrosion From Furnace Combustion Products

Corrosion from this source is confined practically to the relatively cooler portions of the furnace systems. The principal combustion products of coal and oil have no corrosive effect whatsoever upon iron or steel at the temperatures prevailing in service. It is the impurities in the fuel, particularly sulphur, to which the trouble can be attributed. Even these products, however, are seriously injurious only to the extent to which condensation occurs. The sulphur in the fuel, upon combustion, gives rise to sulphuric anhydride (SO_3) which may readily combine with other combustion products to form a neutral sulphate which is comparatively harmless. An acid sulphate is often formed and much of the uncombined gas escapes as such without corrosive effect. If the temperature falls to a point which will permit the condensation of moisture, acid is formed. Sooty deposits, if they be present on the inside of the smoke stack or elsewhere, form excellent absorbents for these corrosive gases and also serve to hold the condensed moisture in intimate contact with the metal. Corrosion of these parts, therefore, represents the idle periods of the plant rather than the opposite. Advice concerning the proper steel to use for withstanding this condition has been asked often. The use of special corrosion-resisting steel of the stainless type is, of course, beyond consideration. There is no simple and sure preventive for this type of corrosive attack. Anything of the nature of a paint coating is to be considered only as a palliative and of short-lived usefulness. While no recommendation based upon service can be made, it would seem that, inasmuch as the corrosive attack is essentially that of an acid, steels which have shown themselves to have superior acid-resisting properties under other conditions should, in general, be most suitable for construction work of this kind.

Corrosion Problems Outside the Plant

Distribution of power is attended by corrosion problems as varied and difficult as those encountered within the power plant. If overhead transmission lines are used, both the lines and the towers may be exposed to the attack of corrosive vapors.

Under most conditions copper exposed to the atmosphere develops a thin oxide coating which protects it from further corrosion. When the atmosphere contains ammonia the corrosion is accelerated. Corrosion of copper in an atmosphere containing salt, such as is encountered along the seacoast, is also greater than where the air contains moisture only. Serious corrosion of copper wires and cables occurs occasionally when these are exposed to atmospheres containing large quantities of smoke, as at railway crossings and in the neighborhood of industrial districts.

It seems probable that the presence of sulphur compounds, carbon dioxide, and carbon monoxide in the smoke is harmful. Hard copper is said to corrode more rapidly than soft copper, and stressed copper more rapidly than when not under stress. It is to be expected, therefore, that the corrosion of long-span transmission lines would be somewhat greater than on short-span lines, other conditions being equal.

The use of aluminum instead of copper for transmission wires appears to reduce corrosion problems to some extent. Under most conditions the oxide film which forms rapidly on the surface of aluminum protects the metal from further corrosion. Embrittlement of aluminum wire exposed near the seacoast has been reported. The relative positions of iron and aluminum in the electrochemical series would indicate that the use of an iron core for aluminum cables would set up in the presence of moisture a galvanic potential that would cause the corrosion of the aluminum. Experience indicates that such galvanic action does not occur to a serious extent, and the probable explanation lies in the high resistance of the oxide film on the aluminum.

During the World War, cables of aluminum were put into wide usage in Germany to replace copper and recent reports by a committee of the Deutsches Gesellschaft für Metallkunde on these installations constitute the most recent information on the subject. On the whole, aluminum overhead conditions appear to be satisfactory. In a great many cases no deterioration was found. In other cases serious sagging had occurred and restretching was necessary. Corrosion, when found, was confined almost entirely to the junctions with other metals, but by proper protection of these junctions, it can be eliminated. According to another report, no pronounced corrosive attack was found provided the aluminum was at least 99 to 99.3 per cent pure. It was concluded that the fear of electrolytic corrosion between the aluminum wires and the steel center is groundless.

Transmission towers and fittings are usually protected against corrosion by a coating of zinc. The protection thus obtained results from two causes, the mechanical protection of the zinc layer and the electrochemical action between the zinc and the underlying steel at points where the latter is exposed. Zinc, when exposed to the atmosphere, corrodes slowly on account of the film of zinc oxide that is usually formed. On the other hand, a zinc coating is seldom entirely free from pinholes and thin spots

[7] S. W. Parr and F. G. Straub, "Embrittlement of Boiler Plate," A.S.T.M. 27, Pt. II, p. 52, 1927.

and the potential of zinc with respect to steel is such that the latter is protected at the expense of the former.

Zinc corrodes rather rapidly when in contact with some of the alkali soils of the West and when in contact with wet cement. This results in the corrosion at the base of towers in some regions, but the trouble is not general. Guy wires used for the support of transmission poles are subject to corrosion especially just below the ground line. Heavily galvanized wires resist the action of acid soils considerably better than uncoated steel wires or rods, but in certain regions where the soil is high in salts corrosion of both zinc and steel is rapid. The Bureau of Standards is making a rather extensive study of the action of soils on galvanized and black steel buried underground. The data obtained apply more directly to completely buried materials than to metals such as culverts and guys which are more subject to atmospheric corrosion.

When the power line is put underground, quite a different set of corrosion troubles are encountered. Such lines are usually encased in lead sheaths and are placed in ducts, although lead is not always used and some cables are buried directly in the earth. The Bureau of Standards Soil-Corrosion Investigation includes specimens of parkway cable intended for burial in the earth for street-lighting service and specimens of chemical and antimony-lead cable sheaths. The specimens of parkway cable consist of a rubber-covered conductor encased in a lead sheath. This is wrapped with a layer of jute impregnated with a bituminous material, two layers of galvanized steel tape applied spirally in opposite directions and an outer layer of impregnated jute. The specimens examined most recently had been buried approximately six years. In many soils the steel tape was rusted seriously and a white corrosion product appeared on the lead sheath, but no indications of failure of the sheath have been discovered.

Specimens of lead-cable sheath buried for the same period were removed from 27 soils. In 26 of these locations the antimony lead, such as is now commonly used for telephone-cable sheaths, showed greater loss of weight than the specimens of chemical lead. The average rate of corrosion of the former material was 0.144 oz. per sq. ft. per year, while that of the latter was 0.085 oz. per sq. ft. per year, or but little more than half.

About half of the specimens showed pitting. In several soils the pits were quite deep although in most cases the depth could not be measured accurately because the specimens were not straight. In one or two soils the specimens were punctured.

The distribution of the pits seems to depend on the character of the soil. On some specimens the corrosion is nearly evenly distributed while on others only two or three deep pits will appear. This difference in types of corrosion is illustrated in Fig. 7.

These specimens were buried directly in the ground at depths of from two to four feet. Their conditions are such as to indicate that, unless the rate of corrosion due to soil action has been found to be negligible, it is unwise to bury lead-sheath cables directly in the ground. The results of the Bureau's study tend to confirm the results of a somewhat similar study made at Purdue University several years ago.

Sheaths placed in ducts are subject to less serious corrosion on account of soil conditions although it seems probable that some of the cable trouble attributed to stray current electrolysis has some other origin. The structure of lead is more homogeneous than that of steel and the differences of potential between different points on a lead sheath that can be attributed to non-homogeneity of the metal are small, as is the potential difference between oxidized and bright lead. On this account one might expect little corrosion of lead in a soil or a duct line. It has been shown, however, that differences of potential also are caused by differences in oxygen concentration at different points on the surface of a conductor and since a cable sheath is not in continuous contact with the bottom of a duct and since there is seldom contact except on the sides and bottom, excellent opportunities are afforded for corrosion due to oxygen concentration cells if the sheath is covered with a film of moisture as is frequently the case. Galvanic cells are formed also when the cable passes through different kinds of silt in the duct or from silt in the duct to water in a manhole.

Lead sheaths are destroyed easily by stray-current electrolysis on account of their thin walls. The usual method of preventing electrolytic corrosion is to maintain the sheaths negative to the earth by connecting them directly or indirectly to the negative bus of the source of the stray current. This method is effective in preventing stray-current electrolysis, but it occasionally results in another form of corrosion. In case the current flowing toward the cable passes through earth containing a considerable quantity of salt or lime, electrolysis of this material occurs and



FIG. 7 CORROSION OF LEAD IN SOIL

caustic alkali is formed on the surface of the cable sheath. Such a condition is established when salt water from ice-cream freezers is dumped near duct lines or when the concrete surrounding the duct contains lime and cases are known in which serious corrosion of cable sheaths by alkali transported by the drainage current has occurred. Such corrosion can be identified frequently by the colors of the corrosion product, for while the lead carbonate which usually forms on a sheath is white or gray the corrosion product formed by alkali in the presence of moist air is red or orange in color.

A rather peculiar case of corrosion was observed some time ago on the sheaths of a midwestern power company. Along the bottom of these sheaths was a ridge of soft lead, while on the sides was a thick crust of brick-red corrosion product. The best explanation for this condition that could be found was that since the sheaths were heavily drained electrically, alkali was deposited at the surface of the sheaths at points where the duct line contained water. This alkali dissolved some of the lead during the nightly periods of shutdown of the street-railway

system and at the surface of the water the red corrosion product was formed while on the bottom of the cable lead was deposited from the solution during the daytime by current flowing to the sheath.

A similar case of red alkali corrosion product was observed more recently in an eastern city where the cables were laid in a monolithic concrete duct line. It appears, therefore, that while it is necessary to drain cables electrically in order to protect them against electrolysis, it is wise to limit this drainage in order to minimize alkali corrosion as well as to avoid endangering neighboring structures.

Theoretically it is possible for reinforced concrete building structures to be injured by stray electric currents passing from the reinforcement to the concrete. Actually we know of no well-established case of the failure of reinforced concrete on this account. There are numerous cases of the failure of reinforced concrete on account of the rusting of the reinforcement, but such failures are usually accounted for by poor materials, the addition of salt to the concrete, or the placing of the reinforcement very near the surface of the concrete. There is very little probability that leakage of electricity will result in injury to power plants or other structures.

Use of Power in Chemical Industries

By CHARLES M. A. STINE,[1] WILMINGTON, DEL.

EXCLUDING the metallurgical industries, the total increase in value for material that passed through chemical industries was placed at $6,655,575,000 by the 1925 Bureau of Census Reports, and the value of the products of American industries which have a chemical interest, was $21,000,000,000. Power is conceived by Leonard H. Davis, as potential power existing in coal, oil, gas, or other fuels and in water capable of power development, as actual power such as mechanical or electric, and as utilized power which has been industrially transformed into commercial products. It is interesting to know that fertilizer can be shipped from Muscle Shoals to Saginaw, Mich., at a cost of 1.47 mills per kw-hr. whereas it would cost 1.5 mills per kw-hr. to transmit electricity 100 miles. As further examples of the consumption or transformation and of the distribution of power in the chemical industries, the explosives manufactured in 1925 contained 835,000,000,000 B.t.u., while the industries used over 4,930,632,000,000 B.t.u.

Because of lack of power development, in China 85 workers out of every 100 are needed for producing the necessities of life, whereas in the United States, due to the developed power per capita, 15 workers produce the necessities of life, while the other 85 are left for producing the luxuries and other items making life in the United States so desirable. During a ten-year interval in which progress was hindered by the World War, the power consumed per worker in the production of glass increased 50 per cent, of cement 38 per cent, of leather 25 per cent, and of lime 133 per cent. The importance of the use of power and of mechanical mass production in the economic life is shown by the fact that American industries, using $1^1/_2$ times as much horsepower per wage earner as British industries, turn out $2^1/_2$ to 3 times the production; and similar comparisons might be drawn between the chemical industries.

According to the 1925 Manufacturers' Census Report, those industries concerned with the changes in composition or properties of the raw materials, namely, the chemical industries, used 7,841,875 hp., while the mechanical industries used 6,525,016 hp. The 1927 report listed a total of 6710 chemical and closely allied manufacturing plants as reporting, while about 1400 plants did not report. The aggregate horsepower of the power-generating equipments installed in the 6710 establishments was 2,802,388 hp., and they burned 87,724,883 short tons of bituminous coal and 2,150,018 long tons of anthracite. Of the total primary horsepower, 706,022 consisted of steam engines and 793,784 of steam turbines. Figuring an average of 3.7 lb. of coal per hp-hr., the tonnage of coal burned in hp-hr. amounts to 5,528,600 hp-years for these 85 per cent of the entire establishments of the chemical and allied industries. Moreover, there are no figures at hand showing the amount of power purchased by these establishments. In 1925 the chemical industries spent $200,000,000 for coal, excluding thus the power purchased or the value of water power utilized.

In fifteen large chemical industries from two to seven times as much fuel is burned for process heating as for power generation, according to 1925 figures. One dye manufacturer uses 96 per cent of the steam generated at his plant for process work, 4 per cent for plant heating and purchases all electric energy. Another chemical plant, with an average daily capacity of 1,000,000 lb. of steam in 24 hours, 7 days per week operation, generates its full power load, uses 50 per cent of the live steam directly in processing, 25 per cent for power generation and the remaining 25 per cent for operating pumps, compressors, etc., and the exhaust steam is used for heating buildings and feed water. An explosive manufacturing plant uses 19,000 lb. of steam per day for power generation, 4000 lb. for refrigeration, 4000 lb. for compressed air, and 33,000 lb. for process heating. In a glass manufacturing plant 1,250,000 lb. of steam per day is generated, 30 per cent being for power generation, 25 per cent for process heating and 45 per cent for steam-driven equipment such as compressors and pumps.

The manufacture of synthetic ammonia for the year 1928 amounted to about 25,000 tons, and the fixed nitrogen production of the entire world to about 2000 tons a day or 1200 million lb. a year, which called, according to estimate, for a utilization of a minimum of two and one-half billion hp-hr., the Claude, Casale, Haber-Bosch, and less important ammonia processes being used. In the manufacture of synthetic ammonia, the direct synthesis of ammonia by the manufacture of hydrogen from water-gas consumes somewhere between 3.5 and 5.0 kw-hr. per kg. of fixed nitrogen; in the case of electrolytic hydrogen it is from 15 to 20 kw-hr. and in the case of the arc process as much as 75 to 80 kw-hr., so that the process of producing hydrogen from coke or coal is economically sound. The Linde hydrogen-nitrogen process for the production of fixed nitrogen by the Casale system yields nitrogen at a power cost of only 5.66 kw-hr. per 1000 cu. ft. There is also considerable requirement of power in the liquefaction of air to separate from it the nitrogen, oxygen, helium, argon, neon, krypton and xenon for commercial uses. The manufacture of argon, for example, requires a pressure of 4000 lb. per sq. in. and a temperature reduced to 220 deg. fahr. below zero, the temperature within the still being maintained at about 300 deg. fahr. below zero. In 1925, 1,693,460,000 cu. ft. of oxygen was produced by liquefaction, having a value of about $15,000,000, while the value of the other gases produced from the air was upward of $1,000,000. An interesting development is the production of a refrigerant, solid carbon dioxide snow, by the compression of carbon dioxide with subsequent cooling by expansion, using carbon dioxide from coke or by-product carbon dioxide.

The electrochemical and electrometallurgical industries are the industries most dependent on water power, which consume about 1,250,000 hp. and mostly from hydraulic turbines. An average of about 200,000 kw. capacity is probably a fair estimate of the world's annual purchase of direct current machines for electrolytic service. In the paper and pulp industries one plant generates 750,000 lb. of steam per day, uses 485,000 lb. of live steam for power generation, 175,000 lb. of live steam and 200,000 lb. of exhaust steam for heating buildings, etc. A large book-paper plant generates 5,300,000 lb. of steam per day, 400,000 lb. being for power, and 4,200,000 lb. for process heating. From 2500 to 40,000 gal. of water per ton of paper products is required in manufacturing paper and pulp, and 2 lb. of water must be evaporated for each pound of pulp or paper dried, or 2 tons of coal are needed for every ton of paper.

One petroleum plant uses 11,200,000 lb. of live steam per day and 1,300,000 lb. of exhaust steam for power generation, 9,800,000 lb. of live steam per day and 8,200,000 lb. of exhaust steam for

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process heating, and 450,000 lb. per day of live steam and 2,200,000 lb. of exhaust steam for heating the buildings. Another refinery uses 7,000,000 lb. of live steam per day for power generation, 89,000,000 lb. of live steam, and 13,000,000 lb. of exhaust steam for process work, including pumping, more than one gallon of water being converted into steam for every gallon of crude oil. Pumping, grinding, filtering, drying, mixing, centrifuging, combustion, and heat transfer are operations required in the paint, varnish, lacquer, and other chemical industries, the soap industry using one ton of coal to make one ton of soap. A valuable development in the chemical industry is the use of the by-product steam of power generation for process heating.

It is probably not too much to say that the American standard of living is dependent for its continuance largely upon our ability to further perfect our methods for the development of cheap power, after having made comparisons in cost of labor and cost of power in respect to competition in the field of chemical products between our country and foreign countries. The chemical engineer is bound to have an increasing part in the efficient utilization of the resources of fuels. He is interested in the methods of production of the power as well as in the ultimate costs of the finished, delivered power.

X-Ray Applications in Every-Day Engineering Problems

By GEORGE L. CLARK,[1] URBANA, ILL.

APPLICATIONS of X-rays are becoming so numerous and important in the daily life of every man that it seems of interest to take brief account of some of the achievements and possibilities of this great research agent.

What are these remarkably versatile X-rays? In every respect they are identical with ordinary light except that the wave lengths are only $^1/_{10,000}$ as great, on the average, as visible light. They are electromagnetic vibrations propagated as waves or as tiny bundles of radiant energy through space without transference of matter. X-rays, the cosmic rays which come from the far reaches of the universe and are able to penetrate 18 ft. of lead, the gamma rays from radium, the ultra-violet, infra-red, and radio waves are all like light except in wave length.

In all this great range of rays, from those only $^1/_{100,000,000}$ cm. long, to those hundreds of meters long used in radio broadcasting, the human eye is sensitive only to that extremely narrow band we call light. Because X-rays are shorter than light it follows that they are more penetrating, that they should pass through materials opaque to light, and that they should be associated with a finer subdivision of matter than is apparent by examination in visible radiation. As a matter of fact, we find that X-ray wave-lengths are of the same order of magnitude as the sizes of the ultimate atoms of all material things. By means of X-rays as the messenger, therefore, the knowledge of the true unit building blocks of matter and their interrelationships are gained.

Metal castings may be diagnosed for presence of internal gas cavities, sand and slag inclusions, pipe cavities, porosity, cracks, and metal segregation. The result is perfectly sound castings where the safety of human life is demanded such as in high-pressure power plants, oil stills, and aircraft motors. The greatest X-ray laboratory devoted to this subject is located at the Watertown Arsenal in Massachusetts. Some of the other radiographic applications in this country are the examination of welds for soundness, of coal for slate and ash, of minerals for classification, of golf balls for symmetry of the center, of cord tires for adhesion of rubber, of reclaimed rubber for nails and foreign matter, of shells and grenades for proper filling of explosive, of wood particularly when used in airplanes, for wormholes cracks and knots of trees and telephone poles for interior soundness, of logs in veneer manufacturing plants for imbedded nails liable to ruin cutting blades, of walls for hidden pipes and wires, of metal pipes and capillaries for measurement of internal diameters, of clogged gasoline lines, of radio tubes for proper position of electrodes, and of glass and pigments.

If a strip of iron is etched slightly with acid, the crystal grains are seen easily with the eye, each appearing perfectly homogeneous. Under the microscope, however, a homogeneous grain may show a heterogeneous fine structure. One of these microscopic units with X-rays indicates that it in turn is built up of ultimate crystal units. What is this unbelievably small unit crystal of iron—this block which is combined with an enormous number of others just like it to give the crystal grain which the human eye may see? We know it to be a tiny cube with an atom of iron at each corner and one in the center; the cube measures less than $^3/_{100,000,000}$ cm. on a side; it is the last thing that is still solid crystalline iron—its properties are those of the visible crystal grain of iron—it is the ultimate iron. Other metals such as tungsten and chromium have the same type of ultimate cubic architecture, but these have different sizes than that which characterizes iron. Still other metals such as the ductile aluminum, silver, and gold have a different type of crystal unit. The fact that this unit crystal is built in so orderly and perfect a fashion explains why X-rays yield such remarkably fundamental information.

X-rays lead us from information concerning the interior gross structures of material to knowledge of ultimate fine structures far beyond the power of any microscope. Nature has built all solid crystalline matter according to so orderly a plan that all the atoms and molecules are on parallel planes which are spaced at distances which compare with the short wave lengths of X-rays. Thus crystals, each one in its own definite characteristic fashion, diffracts X-rays just as finely ruled lines on glass diffract ordinary light. We are able to deduce the structure and size of the unit crystal cell serving as the architectural pattern which upon multiplication in all directions builds up the visible crystal. We determine the size, shape, and constitution of the single brick in an apparently homogeneous structure hundreds of millions of times larger. If this brick is deformed slightly by stress the X-rays patterns indicate it. If the material is a single crystal grain with all the tiny ultimate units perfectly aligned, or if it is a powder or aggregate of small grains chaotically heaped together, or if it is an aggregate with the grains oriented in a common direction as in asbestos or cotton fibers, or metal wires or rolled sheet, the X-ray patterns are perfectly characteristic. There are some 20 different types of fundamental information which may be ascertained from diffraction data, and these account in the realest sense for the actual behavior of materials.

So rapid has been the growth of this chemical science of X-rays that more than 500 kinds of crystals have been singularly analyzed ranging in complexity from common salt or aluminum to very complex silicate minerals and organic compounds. Prior to X-ray analysis the chemists knowledge of the solid state was decidedly limited and he strove to avoid this by melting or dissolving the substance before he attempted to do anything. Now he is trying to crystallize everything and determine the structure as our knowledge advances of the amazingly orderly forces which hold atoms and molecules in marshaled array in space. We are now thoroughly familiar with the crystalline architecture of most all the useful metals and alloys. We understand that the ductility and malleability of aluminum, copper, gold, silver, and the brittleness and hardness of chromium, tungsten, or molybdenum are properties directly related to the plan of building in the unit crystal. We can picture exactly from the X-ray data how zinc atoms elbow their way into the little unit cubes of copper and literally push the sides of the tiny structure apart to form brass. So, the whole problem of the constitutions and range of stability of metals and alloys is coming to be known and properties predicted definitely. In addition, the processes and results of fabrication and heat treatment, lead us to a new industrial science. When a metal is drawn into a wire or rolled

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into a sheet, remarkable changes occur in X-ray patterns. The mechanism of working may be quantitatively deduced; for example, in an aluminum wire every tiny ultimate cube is turned so that the cube diagonals are all parallel to the wire, axis, or in a copper sheet all the little cubes with one atom of copper at each corner and one in the center of each face lie with a cube face in the plane of rolling. All such worked metals are obviously characterized by strongly directional properties, and it is the task of heat treatment to remove this condition, particularly if the metal is to be formed into useful articles. There is no more remarkable achievement of X-rays than the commercial control of annealing which will assure a uniform product of highest quality upon the basis of absolute knowledge of recrystallization processes. With such a control a technique of manufacture inevitably results which involves no greater expense or difficulty.

It is a common belief that heat-treating metals for a long time at low temperatures in order to remove strains and directional properties achieves the same results as annealing for a short time at high temperatures. New X-ray studies show that this precept is far from true, particularly for silver and copper. In the presence of five-hundredths of one per cent of iron the recrystallization temperature of silver is brought down to room temperatures, and all silverware under these circumstances would soon be ruined. As a matter of fact this amount of iron is always present except in very specially refined silver, but its deleterious effects are always offset by the presence of the same slight amount of copper. This curious metallurgical fact, as ancient as silver itself, has come to light after hundreds of years through the agency of X-ray analysis.

Cast steel is characterized by an X-ray pattern which clearly shows a condition of great internal strain, even though it may be free of gross imperfections. The purpose of heat treatment is to relieve this condition, but until now there has been no control method sensitive enough to discover the best annealing technique so that the ideal structure may be obtained. From X-ray data alone it was possible in this laboratory recently to plot regions of equal strain in a large casting, and thus to predict where it would fail or how it should be annealed properly. As a result several large manufacturing plants have based their annealing methods on this scientific research basis instead of empiricism. Similarly, we might describe X-ray studies of the cause and prevention of transverse fissures in steel rails, which have caused so many wrecks, the proper manufacture of electric steel with lowest magnetic loss, the proof of the great superiority in terms of ultimate structure of welds made in a hydrogen atmosphere as compared with the ordinary arc method, the desired structure for aluminum alloys, the changes during metal fatigue, and many other problems. Every metal product which finds usefulness in our daily life is a potential subject for X-ray fine-structure research and then only a single possible field has been touched.

A few representative X-ray studies on non-metallic materials now being made in the laboratory at the University of Illinois and elsewhere include:

Asbestos. A clear differentiation between numerous varieties appearing outwardly the same but differing widely in practical behavior and a method of identifying the mines from various specimens of the same type.

Lime. Discovery of the cause of plasticity and methods to render non-plastic lime useful.

Enamels and Pigments. Composition, particle size, and crystallization as functions of covering power, tint, wear, etc.

Lubrication. A definite knowledge of mechanism in the lining of long grease molecules in successive layers like a pile of carpets, with the pile of each carpet representing the molecules which slide over each other instead of one metal surface on another.

Waxes, Soaps, Alcohols, and Other Organic Compounds. The analysis of crystalline structure, actual measurement of the sizes of molecules, the testing and verification of the structural theories of organic chemistry, as for example, absolutely definite proof of the benzene ring.

Ceramic Materials. Constitution transformations and internal strains as in spark plugs.

Rubber. The discovery that this remarkable substance develops a crystal-like fiber pattern upon stretching (as does also a muscle fiber), analysis of which removes much of the conjecture from our scanty knowledge; the development of a method of stretching rubber 10,000 per cent to threads insoluble in the solvents which usually dissolve rubber easily; and the establishment of these unique criteria never yet found in artificial or synthetic rubber.

Textiles. The behavior and constitution of the crystalline threads of cellulose and silk, and the deduction of a vastly improved type of rayon.

Catalysts. The deduction of optimum constitution and particle size for high-pressure synthesis of methanol and other products.

Gelatine, biological structures, road materials, paints, nitrocellulose explosives, lacquers, and transparent wrappers are studied by X-ray methods. The cause and prevention of cracking of patent leather, liquids of all kinds, carbon, cement, adhesives, paper, the tiny objectionable stones in pears, electroplating, food products—all of these and many more materials are yielding their secrets of constitution and of their good or bad practical behaviors in every-day use to the searchings of X-rays.

Synchronous Selector Supervisory Equipment and Telemetering

By C. E. STEWART,[1] PHILADELPHIA, PA.

AMONG the recent developments of centralized control of electrical energy are the supervisory and telemetering equipments which automatically give the dispatcher of a system load a continuous picture of the power-circuit conditions, and provide him with means for controlling them. The effect is equivalent to the actual removal of the substation switchboard to the power dispatcher's office, however, with only the control keys, indicating lamps and meters in view and concentrated on but a small panel, so that a large number of substations may be supervised. This paper is devoted to a description of one of the latest types of automatic supervisory equipments, its application to electrical power supervision and its functioning with automatic telemetering equipment. The supervisory equipment is designated as the synchronous selector type and is accurate in its selective features, highly speedy in selection, provided with an inherent "check-back" feature for checking the selection before performing the operation. It performs selection, operation, and indication over a maximum of four telephone or No. 19 B. & S. line wires between the dispatcher's office and each outlying station, requires no duplicate line wires for a moderate-sized substation, registers any change in the position of one or more supervised units promptly at the dispatcher's office by both visual and audible means, establishes a control circuit for operating any supervised device simultaneously with the closing of the dispatcher's control key, registers the correct indication simultaneously in the dispatcher's office when the dispatcher operates a substation unit, notifies the dispatcher automatically when the system is not operating due to open or shorted line wires, loss of operating potential and other major troubles. The individual devices are of standard design mounted on panels of the usual switchboard design.

The system employs very simple electrical circuits and very few common devices which is an aid in operation, inspection, and maintenance. A system for supervising 46 units contains as the common equipment the following electrically operated devices: at the dispatcher's office 3 selector switches, 11 small relays, and an alarm bell; the substation circuits connect to 3 selector switches and 7 relays, five of these relays being used for their convenience only, and four of the selector switches being used to supervise that group of substation units between 23 and 46 and not essential to units 1 to 23, so that basically the system consists of 2 selector switches and 13 relays, all other devices being added in accordance with the control and indication circuit requirements, or to facilitate circuit inspection, adjustments, and application. Circuit failure is guarded against by a minimum number of series contacts, two contacts in parallel on the smaller types of relays, increased operating voltage, and use of protective resistors in the battery circuits to selectors and relays.

The standard panels, completely wired with numbered terminal blocks for all external circuits, are 24 in. wide by 90 in. high for control of 46 substation units, 16 in. wide for 23 units, 32 in. wide for 69 units, and for 92 substation units there are two 24-in. panels. The dispatcher's supervisory panel contains the selector switches and common relays, control-power switches, fuses, line-wire test jacks, keys and indicating lamps, alarm bell, and meters for telemetering. The indication panel has one two-position type of polarized relay for each substation unit. Suitable panels are provided for any number of units for the selective and indicating equipment at the substation, including control power switch, selector synchronizing relay, line-wire jacks with the selector relays and auxiliary relays.

The polarized control relays operated over the control-line wire and generally mounted on the substation panel associated with the controlled unit may be of the two-position or three-position type. The synchronous-selector equipment is operated from two 60-cell storage batteries, one in the dispatcher's office and one in the substation. A continuous drain of 0.1 amp. is required of the dispatcher battery while the substation battery must deliver 0.1 amp. plus 0.02 amp. times the number of substation units indicated. Both batteries must also supply $^1/_2$ amp. intermittently. An independent 24-volt circuit supplies the dispatcher's indicating lamps with 0.04 amp. per lamp. All line wires are protected at each station with a self-restoring arrestor.

The selector switches in the dispatcher's office and the substation consists of 5 contact banks, each consisting of 25 contacts and an associated contact wiper which is stepped from point to point by the operation of a notching magnet and spring. The synchronizing bank and the notching banks at each station operate in conjunction with each other and their respective selector synchronizing relays in order to step the wiper arm of each slector ahead in exact synchronism. Individual select keys in the dispatcher's office feed potential to each of the contacts on the synchronizing bank, and the selectors can be stopped by opening the associated select key. The control circuit is connected directly from a two-position control key in the dispatcher's office to a two-position polarized control relay in the substation. The contacts of the indication relay establish potential across either red- or green-capped indicating lamps, depending on the position of the supervised unit. The immediate "check back" feature consists of checking the polarity as placed on the synchronizing line wires by the substation selector against the polarity as determined by the dispatcher's selector. If the polarities are correct, the control circuit is established for the dispatcher. An amber-capped lamp associated with the keys and lamps of the selected unit indicates the correct operation of the position-check relay.

The dispatcher's panel pilot lamp lights whenever a change occurs in the associated substation. The individual white-capped lamps indicate an automatic operation of a supervised device or incompleted operation started by the dispatcher. The equipment cannot start unless selectors are in synchronism on the normal starting position, a reset key being provided at the dispatcher's office for placing the selectors in synchronism at the normal rest position. The dispatcher is notified in case of operating-potential failure, if either selector steps from the normal at-rest position, or when there are line-wire troubles. The control wire closes only when the selectors stop on a contact, and they step from point to point at the rate of from ten to twelve impulses per second, so that the ninetieth breaker position can be selected within about 9 sec. Indication is in-

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stantaneous with the movement of the supervised unit. For manually operated stations, only about 11 sec. are required for the synchronous equipment to indicate the position of 275 units over four interconnecting line wires. From the dispatcher's office of the Cincinnati Street Railway, 18 railway substations are supervised.

The supervisory system provides also an easy means for selecting and obtaining intermittent meter readings over the 4 common line wires, the supervisory equipment being inoperative while the meter readings are being made; however, additional line wires are required for continuous or recorded meter readings. The telemetering equipment thus supplements the information given the dispatcher by the supervisory equipment, which may be used to inform him of division of load between generating stations, exchange of energy between power systems, load on machines, feeders, or transformer banks, and desirable voltage readings. A transformer and feeder station of the Bronx Gas and Electric Company, New York is provided with equipment for selectively reading a.c. amperes and a.c. voltage on 12 circuits by means of the rectified current-type of telemetering, the rectified-current type having no mechanical-motion parts except the indicator.

FSP-51-10

Trends in Development of Electricity-Supply Systems

By W. F. SIMS,[1] CHICAGO, ILL.

ALTHOUGH there is a growing use of automatic and remotely controlled equipment, the present trend seems to point toward simplification of this type of apparatus. On account of the complexities of the control wiring and equipment necessary, the initial cost was high, and an excessive amount of maintenance was required. The present trend is toward as simple and reliable a system as is possible.

The use of pulverized fuels has passed what might be called the hysterical stage and is now on a practical basis of consideration. This change in attitude toward the subject has in no way lessened its importance. The increase in the size of boilers has increased the difficulties of stoker firing, and this has stimulated the trend toward pulverized fuel. There has been some difference of opinion as to the relative merits of the bin and unit systems, with an apparent tendency toward the latter.

The improvements in combustion conditions have brought about a marked reduction in the size of furnaces. Water-cooled furnace walls are rapidly becoming standard practice, but it is not yet generallly agreed as to whether bare metal or a thin refractory covering should be used. The dust problem, which is yet unsolved, has become very troublesome, and it is important that definite progress be made in its solution. Automatic combustion-control systems are gaining in favor, particularly on large units where the initial expense can be justified.

In reference to higher steam pressures, although past developments in the 1200-pound class have been confined mainly to smaller installations, several installations of considerable size are going ahead. In these installations the steam temperatures are conservative.

For compound turbines in the 600-lb. class, reheating of steam seems to be more or less standard. Experience with live steam for reheating, however, has not yet established the fact that this is the most economical method. Although the mercury-vapor boiler installation at Hartford is being watched with a great deal of interest, the contemplation of no other such installation has yet been announced.

The subject of low-temperature coal distillation has attracted the attention of many engineers, and considerable research work is being done along this line. So far, it is only in the experimental stage. As the extent of interconnection and growth of individual systems increase, the trend is steadily toward larger units, which of course, is reflected into all the component parts which make up the generating station.

Definite progress has been made in the design of higher-voltage generators, and machines designed for 22 kv. have been built, and the possibility of even higher voltage is being given serious consideration.

The necessity for improvement in system stability in connection with hydroelectric plants feeding long transmission systems, has brought about the application of quick-response excitation equipment. Some application has been made, and a wider use may be expected, on steam turbo-generators connected to long transmission lines. There is also a field for its use on synchronous condensers used in connection with transmission systems. On the other hand, closely connected systems supplied from steam turbines, and with underground transmission such as is the case in most metropolitan areas, being much more stable, are not greatly concerned with this problem. As this method of stiffening the system under fault conditions is much less expensive than by changing generator characteristics to obtain the same result, it is likely that this feature will come into more general use. The use of the pilot exciter for exciting the main-exciter field, which has been developed in connection with quick-response excitation, in many installations also permits the elimination of the main-generator field rheostat.

More accurate frequency regulation has become a vital factor on interconnected systems, and even with hand regulation fairly uniform frequency is now being obtained. Considerable attention is being given to automatic frequency regulators, and several large companies have, on their own systems, made up regulating devices operated from the frequency recorder. It is interesting to note that although these various devices were worked out independently in different parts of the country, the fundamentals and some of the details are quite similar. Some of the manufacturers have brought out factory-made devices of this kind, and the attention being given to this class of equipment gives promise of much further development and application.

In the field of switching equipment, attention is being given to the further development and use of the truck type and of the metal-clad switch gear. Both types lend themselves to a maximum of factory assembly, and a minimum of field labor.

For some time manufacturers of circuit breakers have been investigating the possibilities of developing circuit interrupting devices that will open power circuits without the use of oil. Although none of these devices has yet been placed on the market, factory, and field tests show very encouraging results. A series of papers read before the midwinter convention of the American Institute of Electrical Engineers discussed the theory, design, and test performance of one such device. Although it is too early to predict the outcome, and there is some question as to the voltage limitations which may be reached, it is very possible that this form of current-interrupting equipment will come into use on power systems.

Under conditions where the attendance is a large factor in the cost of operation, there is a growing use of automatic and remotely controlled installations, which have demonstrated a high degree of reliability, and it is probable that a much greater use of them may be expected.

In synchronous-converter substations in densely built-up areas, and particularly where it is necessary to install the equipment in the basement or lower floors of commercial buildings, the problems of ventilation and elimination of noise often become quite serious. The use of enclosed converters offers many advantages under these conditions, one of which is the reduction of temperature in the general substation space. Much more positive ventilation can be secured as well as the practical elimination of noise from the machines. This arrangement also permits the use of air recirculation and air coolers.

In railway substations there is a growing use of mercury-arc rectifiers instead of synchronous converters. Although their

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first cost is at least as great as that of converters, their higher overall efficiency, and the advantage of static over rotating equipment, together with the practical elimination of the ventilation problem, and the great reduction of noise, makes their use decidedly advantageous. There is a demand for rectifiers of larger size, and it is expected that developments now in progress will be successful. If the size of rectifiers for potentials around 250 volts can be increased, and if the manufacturers are able to improve materially rectifier efficiency and regulation on the lower voltages, it is possible that some further installations may be made on direct-current network systems.

The development and application of the high-speed air circuit breaker marks the most important improvement in railway substation practice in recent years. Its ability to disconnect d-c. short circuits almost instantaneously eliminates the reflection of their effects back into the power system, and reduces greatly the number and severity of commutator flashovers. A much more extended use of this type of breaker is probable.

The increase in the development and use of household appliances, including refrigerators, oil burners, etc., has had a definite bearing on distribution conditions, making reliability of service to residence customers much more vital in the past. Regardless of the type of distribution system used, this situation emphasizes the need for reducing customer outages and generally improving service. Continued efforts in this direction may be expected.

The interconnection of systems is continuing, and it is reasonable to expect further developments of those already in operation, and the bringing together of many other systems now operating independently. There are still many places where the savings to be effected by the long-hour load of the more efficient plants, the reduction and better use of system reserve, and the improvement in reliability and quality of service are yet to be realized.

The problems of operating large interconnected systems are complicated by the lack of relay equipment that is adequate to meet all conditions. This situation is being studied by utility engineers and relay specialists in the factories, and continued improvement in relay methods and equipment is to be expected.

In general, transmission voltages are reasonably well standardized at 66 kv., and for the higher voltages from 110 to 140 kv., with 132 kv. as the most generally accepted figure. Recent developments, however, indicate that where the distances and amounts of power to be transmitted offer the necessary justification, there will be an increasing use of 220 kv. There is also a trend toward the use of higher voltage on short heavy lines because of better regulation and less liability of lightning interference.

The use of synchronous condensers for power-factor correction of transmission lines is increasing. The conditions under which their installation may be expected are where the length of the line is such that the value of the increased line capacity thus obtained, together with the capitalized value of the saving in line losses will offset the cost of the condenser installation. There are also systems where their installation may be justified by the improvement that their installation will make in regulation.

Notable improvements have been made in the design and construction of such machines. They are now being built for higher speeds, with a corresponding decrease in weight and floor space, and a considerable reduction has been made in operating losses. There have been several condensers built for outdoor operation, at least one of which is equipped with hydrogen cooling.

The active interest shown by transmission engineers in the study and discussion of the value and proper application of ground wires, arcing rings, and other protective measures, leads to the hope that the present differences of opinion on these subjects will be reconciled within a reasonable time. Field and laboratory tests now in progress should aid in an early settlement of these much debated questions.

Field investigations are now being made by the manufacturers on the transmission lines of some of the operating companies, looking toward data on the characteristics and wave shapes of lightning surges. Automatic cathode-ray oscillographs and other similar devices have been connected to the lines, and the use of the data which it is expected to secure should result in improvements is lightning-arrester design, and a better knowledge of how they should be applied.

Developments in high-voltage underground cable, and the successful installations at 132 kv. in both New York and Chicago, show the feasibility of its use. The field of application of such cable is primarily in extending from overhead lines to power houses in or adjacent to congested localities.

The large capacity of some of the high-voltage transformers manufactured during the past year, indicates a trend toward a greater use of larger units, and there is also a definite indication of a more widespread use of load-ratio control and other similar regulating equipment. Parallel with the development of the two-winding generator has come the split-winding transformer with similar reasons for its use and application.

As was the case several years ago on the lower-voltage systems, the need for reactors on high-voltage systems is beginning to become apparent, and the successful production of equipment shows that it can be procured when required. The necessity for a better knowledge of system conditions during disturbances has led to the installation of automatic-recording devices by several companies. The information already secured by this means has proved very valuable, and has resulted in a number of improvements in system design and operation.

Use of supervisory control has been extended, and in some instances includes the control from a central point of a system of hydroelectric stations and substations. The equipment developed for this purpose has proved to be extremely reliable, and its further application can be expected. The increased use of electric power in metallurgical and other industrial purposes, and also the progress of rural electrification are presenting new problems in power supply, some of which are not yet satisfactorily solved.

The size of the industry has now reached a point where on many systems the annual increase in load even of only a nominal percentage, in actual kilowatts is such that to meet the demand a very large increase in the capital accounts is required on all parts of the system. The magnitude of these additions is such that many companies have found it necessary to make organized studies of probable future development several years in advance, in order that probable capital required may be budgeted far enough ahead to permit of proper financing. This also permits the construction program to be so laid out that the new equipment may be provided and utilized in the most effective manner.

The notable developments in the industry which have taken place in the past, and which may be expected in the future, have been contributed to by men in every branch of the power industry, and also by the engineers of the various manufacturing organizations who are supplying the equipment required to build up the systems. The results obtained are an evidence of coordination of effort and cooperative action, of which the industry may well be proud.

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Refrigeration and Ventilation as Aids to National Efficiency

By S. C. BLOOM,[1] CHICAGO, ILL.

This paper describes developments in air cooling and air circulation that are being incorporated in modern office buildings. The amount of refrigeration required is large and current practice is to recirculate and recool 50 to 75 per cent of the air used. The volume of cooled air introduced is sufficient, usually, to change the air of a building 5 to 8 times per hour. The period during which cooling is required is but a small part of the year and current practice is to provide refrigerating equipment of capacity sufficient to handle the peak load. The power demand resulting is high and intermittent, and much expensive apparatus must remain idle a preponderant part of the time.

REFRIGERATION plays an important part in our national life, in the way of transportation, storage, and preservation of perishable food stuffs, and in its contribution to the comfort and health in the home. Of our great dependence upon this useful agent there cannot remain the slightest question; one need only to visualize the situation if it were withdrawn suddenly.

Similarly, ventilation in schools, office buildings, industrial plants, and the like is recognized as a vital necessity in the maintenance of decent and healthful conditions for those who must occupy them. In the past few years public attention has been focused upon the combination of refrigeration and ventilation as an aid to comfort in hot weather, as exemplified in an ever-increasing number of theaters, which have adopted cooling systems as a box-office attraction, with amazingly successful results.

This has, by no means, been confined to the metropolitan centers, with the result that the public has had a taste of this luxury, and there has been a spontaneous interest in the extension of its use to other fields. Thus we find department stores, restaurants, and hotels incorporating cooling apparatus into their ventilating systems to increase business by the provision of a comfortable environment for the patrons of their establishments.

In industry there are many manufacturing processes which cannot be carried on during the summer, when outdoor temperature and humidity exceed certain values. In such instances, the maintenance of production schedules or quality of the output has been successfully accomplished by the application of cooling systems, thus doing away with the expensive practice of seasonal shutdowns and throwing out of employment large numbers of people.

As a matter of fact, the industrial usage of air cooling was earliest, but knowledge of its success and possibilities spread very slowly, since many of the applications constituted trade secrets which were quite properly protected as long as they could be.

The direction which the development of the air-cooling field has taken is only natural, since it must be borne in mind that the use of cooling rarely springs from benevolent or philanthropic motives; it must pay dividends. In the applications previously referred to, it has been relatively simple to establish the worth of the investment.

There is a field of tremendous potential value yet to be exploited, where the investment yield is not so easily evaluated, yet none the less real. This lies in the cooling of office buildings, wherein a vast number of our people pursue their livelihood. Among them are the major portion of the executives directing the business of the nation; men whose minds must be concentrated upon the many and varied problems with which they are confronted.

One cannot be at his best when conscious of discomfort. We do not permit ourselves to be tortured by the rigors of cold weather, hence heating is considered an indispensable requirement of buildings. There is no more reason why we should allow ourselves to be uncomfortable in summer. That hot weather is accompanied by an extensive let-down in productive effort is apparent to every one. Thought of how to escape the heat is uppermost in mind, as is evidenced by the action of millions seeking the seashore, the mountains, the lakes, anywhere to get relief. The disturbance of business which this situation occasions is widespread and severe, reflecting itself in a reduction of national efficiency, the cost of which is incalculable.

We have become accustomed to accepting the situation as inevitable. Now, with the tools at hand to correct it, the confident prediction may be made that they will be utilized to best advantage. There have been a few outstanding examples of cooled office buildings in this country, operating for many years which afford a background of experience warranting the prediction just offered. It is the unanimous opinion of the managements that they could not afford to be without them.

Contrast the picture of the hot office with open windows admitting the dust and noises, the perspiring occupants and the disinclination to work, with that of one wherein the air is comfortably cool, dry and clean, where the windows may be closed to exclude noise and distraction. But little imagination is required to visualize the superior efficiency which will be obtained in the comfortable office.

Occupants will not only be happy to remain in such a place, but will feel like working while there. In one large office building where cooling has been used for a number of years, the refrigerating plant went out of commission in the middle of summer and was down for about two weeks. Although the ventilating system was operated in the interval, it was necessary to open windows which had not been opened for years in order to relieve the situation.

During this period the number of absentees went up decidedly and the office manager estimated that the efficiency of the office fell off at least 50 per cent. With the restoration of cooling its importance was appreciated as never before, a very common experience, when something essential to our comfort, health, or happiness is taken away temporarily.

At present there are several large office buildings planned and some under construction which will be cooled throughout, and they are but the forerunners of a practice which will become general. The cooling plant will take rank with the heating system as an essential item of equipment.

A cooling system, as applied under prevailing practice draws fresh air from outside into an air cooler where it comes into direct

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

contact with a veritable storm of cold water spray. The cold water simultaneously purifies the air, cools it, and condenses some of its moisture, thereby lowering the humidity. The cooled air is then forced by a fan or blower through a system of air ducts leading to the various rooms which are to be ventilated and cooled. There it absorbs the heat transmitted through the walls and windows, also the heat given off from the occupants and from lights and its temperature rises, with a corresponding drop in relative humidity.

The temperature of the incoming air, the amount of heat to be absorbed and the ultimate room temperature and humidity desired, govern the quantity or volume of air required, so that it is perfectly feasible to design a system for maintaining any condition wanted. The water which is sprayed in the air cooler is cooled and recooled, with refrigerating machinery, to the temperature required. There are some localities which are blessed with an abundant underground water supply of suitable temperature, which is easily available through the use of well pumping equipment. The upper limit of temperature at which well water is serviceable in this connection is about 58 deg. fahr.

With the contrariness of nature, we find that ground-water temperatures decrease the farther north we go, and at the same time the need for air cooling lessen or at least, the period during which cooling is needed is shorter. The amount of refrigeration required for air cooling is large. For example, to cool air at the rate of 1000 cu. ft. per min. requires the equivalent effect of melting ice at the rate of 2 to 3 tons per 24 hours depending upon the relative proportions of air taken from outside and that returned from the building for recooling.

Current practice is to recirculate from the building and recool 50 to 75 per cent of the air supplied, so that refrigeration at the rate of $2^1/_2$ tons per day per 1000 cu. ft. per minute of air supply is fairly representative of the amount provided. The volume of air introduced is sufficient usually completely to change the air of a building 5 to 8 times per hour, so that at a glance it is seen that the refrigeration requirement runs into big figures on a large building. There are some instances in which refrigerating machinery has been provided having a capacity in excess of 1000 tons equivalent ice-melting capacity for 24 hours.

There is at present a diversity of ideas as to the extent to which cooling should be done in order to maintain satisfactory conditions, and the tendency is to work toward higher room temperature and lower humidity. When cooling was first advocated, the prospective users almost invariably wanted a condition of weather 70 deg. fahr. and 50 per cent relative humidity indoors, with hottest summer outside. A very few such installations were made and experience demonstrated quickly that such a contrast was unbearable.

Today a condition of 80 deg. fahr., with a relative humidity of 55 per cent is considered quite satisfactory, when it is 95 deg. fahr. outdoors. The combination of experience and research promises to establish a standard which will not only assure a suitable degree of comfort, but which will take into consideration the factor of expense for producing such a condition, with a view to its reduction.

The period during which cooling is required is a relatively small part of the year and current practice is to provide refrigerating equipment of capacity to handle the peak load, with the result that power demand is high and intermittent, and much expensive apparatus must remain idle a preponderant portion of the time. These characteristics add greatly to the overall expense for cooling.

Development of means whereby the operation of the refrigerating machinery could be made more nearly continuous during the cooling seasons, even though the ventilating plant is inoperative when the building is unoccupied, would go far toward solving the expense problem and make a much more desirable power load. A few attempts at a solution along this line have been made, but without conspicuous success to date, although the possibilities in this direction have by no means been exhausted.

Multi-story office buildings present some interesting problems in the application of cooling, because of the necessity for conserving usable or rentable area, and of obtaining proper distribution of cooled air to the various stories. As stated previously, the current practice is to recirculate and recool a very substantial portion of the air. This requires valuable space, for the running of return air ducts in addition to those for the supply. Recirculation of air is done primarily to reduce the amount of refrigeration required, to temper the air supply and to obtain desirable humidity conditions within the building. In addition it is an expedient whereby existing standards as to the volume of air supplied to ventilated spaces are apparently complied with.

It is perfectly evident that if air recirculated from a building is recooled, it will require less refrigeration than if the same volume of air were taken from outdoors and cooled to the same temperature. Similarly, if a portion of the recirculated air is mixed with the remainder of the air which has been cooled, the resultant mixture will have a higher temperature and lower relative humidity than if all the air had been cooled. This device has been resorted to in order to avoid difficulties from drafts attendant upon the introduction of relatively large volumes of air at temperatures too far below the required room temperatures. It also facilitates obtaining satisfactory relative humidity at room temperatures which have been found desirable.

Existing ventilation standards differ widely throughout the country and many of them were formulated prior to the advent of air purification, cooling, proper usage of ozone, etc., as auxiliaries to ventilation. Strict adherence to them would preclude the use of cooling in many localities because of the excessive cost for refrigeration. Under these circumstances the air-conditioning engineer has boldly utilized the method of recirculating air, which in effect amounts to a corresponding reduction in volume of outdoor air taken into the system. To his credit it should be recorded that he has demonstrated in a practical way that quality of ventilation has not been diminished by the practice of recirculation, or to put it another way, that satisfactory ventilation may be had with greatly reduced volume of outdoor air.

The significance of this demonstration is very great in connection with the future of office building cooling and ventilation, for the reason that recirculation of air can be abandoned completely, and ventilation accomplished by the use of a relatively small quantity of outdoor air, purified and cooled to a low temperature. No special provision for its escape from the building would have to be provided, and thus a slight internal air pressure would prevail. The air would escape through cracks around the windows and elsewhere, and thus reduce or prevent inleakage of outdoor air, which under normal conditions greatly increases the air volume and refrigeration required for cooling. It will also be evident that the exclusion of inleakage will result in a greater air purity within the building.

The reduced volume of air supply, the omission of recirculation and of special provision for the exhaust or escape of air, greatly reduces the space requirement for air ducts and for the ventilating apparatus. At the same time there is a very desirable simplification of the system. Still further opportunity for lessening the space requirement lies in the direction of increasing air velocity in the ducts or conduits 5 to 6 times that used in conventional ventilating systems, and of operating blowers at pressures ranging upward to 1 lb. per sq. in. instead of the very low pressure customary. The power required to handle the greatly reduced air volume at the higher pressures will compare favorably with that needed in systems of current designs.

The use of higher air pressure will aid materially in the equal distribution of air throughout a high building, where many stories are interconnected by a common piping system, through offsetting the chimney effect prevailing in such a building. Furthermore, the maintenance of sufficient pressure at the branches will facilitate meeting the effects of windage which frequently disturbs distribution.

The form of conduit required to handle the air at the pressure suggested would perhaps require a change from the ordinary galvanized ducts which are not sufficiently tight. Light tubing or galvanized sheet-iron pipe of the type made to withstand low pressures would prove serviceable. The use of cold air would necessitate some insulation, but it would not have to be very heavy for the reasons that the prevention of condensation on the exterior is its primary object, and the low humidity of the air within the building would reduce the tendency for condensation to form.

Use of small quantities of cold air results in a low relative humidity, which is a desirable objective, for the common complaint in summer is that "it isn't the heat, but the humidity," which is the principal cause of discomfort. The lower the relative humidity, the higher the temperature can be for an equal degree of comfort. The higher the temperature indoors the less heat is transmitted through the walls and windows of a structure, thus reducing the amount of cold air and refrigeration required.

The tendency to carry room temperatures too low should be discouraged; it is expensive, unnecessary and even undesirable. Excessive cooling in summer is just as objectionable as overheating in winter. The provision of a reasonable contrast between indoor and outdoor conditions should be the objective. Avoidance of excessive contrasts is essential in order that readjustments of the body-temperature controls may not be too great, for it must be borne in mind that the passage of people from indoor to outdoor environment is sudden, and the effect is temporarily magnified. Furthermore, lighter clothing is worn in summer and street dress and indoor dress are the same, thus the body is more sensitive to changes in conditions.

So long as the answer to what is sufficient remains in the realm of opinion, there will be a diversity of viewpoints, however, there is some research work bearing upon this matter which has been underway for some time, and the results coupled with experience gained in existing installations should serve as a valuable guide in the determination of correct practice.

The prospect of providing for the comfort, health, happiness, and efficiency of people in summer, who are confined indoors by virtue of the necessity of gaining a livelihood, is most alluring. There are many well-qualified, forward-looking engineers who are concentrating their talents upon the problem of how to make this great boon to heat-ridden humanity available; men who are unprejudiced, who are not slaves to convention and who are boldly setting out on new and uncharted paths, confident of arrival at the goal of their endeavors. The spontaneity of the demand for cooling is most surprising and it appears to offer a most fruitful field for development. As an aid to national efficiency, it is difficult to conceive of anything which offers greater promise.

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Application of Refrigeration to the Preservation of Foods

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This paper tells how the application of refrigeration to perishables has brought about a very marked betterment in the standards of living throughout the country. It has placed the products of a nation of diversified soils and climates in the home of every one. The health of the whole people has been safeguarded in increasing measure as more and better products are being made available. The applications of refrigeration to perishables have made possible the agricultural development of sections of the country which would otherwise lay as wasted useless land.

REFRIGERATION in its application to perishable foods divides itself into three major departments. These are transportation, storage, and small commercial and home divisions. Refrigeration in transportation is mainly railroad car icing and steamship refrigeration. A number of different methods of cooling freight cars have been devised but none of these methods are commercially important at the present time.

There are a number of perishable food products which are peculiarly adapted to production in certain locations or climatic conditions. Among these are citrus fruits, hops, and figs. Other fruits and vegetables can be grown in nearly every locality in this country during suitable seasons but because of adaptability to certain combinations of climate and soil these in mass production have been confined to certain areas. These include apples, peaches, celery, grapes, pears, onions, and similar products. This localized production is carried on such a scale that no nearby market could absorb any appreciable quantity of product. For this reason refrigerated transportation has been adopted to carry these commodities to distant markets or to their points of distribution.

Refrigerated transportation combined with storage has had a part in the economic development of our country which is not even realized by a number of the men in the refrigeration industry, much less by the general public. None of the winter-garden areas and none of the western mass-production centers could have had more than a small percentage of their development without the aid of refrigeration in distributing and marketing their products. Climate and soil would have been valueless, had there been no way to transport their products to market.

Present Development in Transportation

The products of the packing industry form a large field for the use of refrigeration in transportation. From the center of this industry in Chicago shipments go to all parts of the country. Centralized packing has grown up in the face of great local competition and has now reached such a scale that 300,000 refrigerated carloads of packing-house products are shipped every year. These move to branch houses and wholesale storage houses in the larger towns. Smaller towns have been enabled by refrigerator-car shipments to secure fresh meat direct from the packer in cars which make periodical stops to deliver meat to the retailer. In this way markets have been widened to the greatest possible degree while people in very small towns have been enabled to supply their tables in the same manner as the large city dweller.

Refrigerated ships ply between this and every country which produces perishable products. From the ports of entry these products must be transported under refrigeration to their markets throughout the country. Bananas offer a good example of this type. A great portion of the bananas used in this country have in the past entered through the South and East ports. The new Western banana terminal of the United Fruit Company at San Francisco gives evidence of the present importance of this foreign product. Cargoes as high as 47,000 bunches are unloaded at this terminal for shipment to various points in the West. Present importance of this one commodity is shown by the figure of 64,000,000 bunches, which is given as the consumption during the past year. The refrigerator transportation of this fruit alone is quite a large item. Other tropical fruits are brought by refrigerated ships from Hawaii and the West Indies and we found some time ago that citrus fruit brought all the way from Italy could flood our market.

Approximately 1,550,000 cars of perishable food were transported in the United States during 1927. Of this amount 577,000 moved under ventilation while the remainder 973,000 were iced. The refrigeration of these cars consumed 12,500,000 tons of ice. In placing Imperial Valley lettuce on the table here in the Palmer House, the time in transit from San Antonio, Texas is the same as that from Tampa, Florida. The ice used per car on a given trip varies with the temperature maintained in the car. The latter depends upon the commodities being transported, some of which require the packing of ice in the load to keep them fresh. In coming from the Rio Grande Valley to Chicago in the summer, cars use 8 to 16 tons of ice, depending upon the commodity being carried.

Cold Storage

Refrigerated storage forms the next phase of our study of perishable food preservation. The central idea of all storage is to provide a reservoir to keep perishable products from the season of production, when there may be an oversupply, to the non-productive season or a season in which the demand exceeds the ability of the country to produce. Of course, there are certain commodities such as meats which are chilled and stored as a portion of their preparation for transportation and sale. This latter is more or less an activity of the packing-house storage, while the bulk of the business of the general cold-storage houses is the intro-seasonal preservation of perishables.

In the early days of refrigerated storage a lack of knowledge on storage methods in some cases brought about a reduction in the quality of the stored food. The public became conscious of this fact and developed a partial dislike for "cold storage" products. The cold-storage operator then in some cases tried to circumvent what he conceived to be an unfair prejudice by selling his products as freshly produced foods. This situation which based the sale of cold-storage products on the practice of deception was unfortunate and could not be maintained long.

Scientific study developed more effective methods in cold-storage practice. Producers of foods gave more attention to well-timed harvesting and proper lading and refrigeration in transit. These had their effect, cold-storage products were

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essentially as good as fresh products and a conversion of the public from its prejudice was begun. Cold-storage eggs were sold unblushingly as cold-storage eggs and the economic advantages of their use was stressed by the storekeepers. At the present time, the public is losing its disproval of the use of cold-storage products. This situation is very much more desirable from the viewpoint of the producer, warehouseman and consumer.

The refrigerated warehouse is now one of the most necessary units of the modern city. Without it we could not live in our accustomed comfort. Larger and larger storage houses are needed, and more and more of them are being built. Here are a few from recent press reports. The Pennsylvania Railroad completed a 2,000,000-cu. ft. storage and terminal at Philadelphia. A terminal and refrigerated storage of 4,600,000 cu. ft. capacity has been completed recently in Houston, Texas. A 1,500,000-cu. ft. terminal with cold storage is to be built at Walkerville, Ontario, Canada. The Chamber of Commerce of San Francisco has petitioned for the construction of a 2,000,000-cu. ft. harbor terminal and cold-storage house.

Cold Storage Available in All Parts of United States

Figures which include the latest installations are not now available, but the figures taken from the last biennial census (October, 1927) place the total cold-storage space in the United States in both private and public plants at 731,377,000 cu. ft. This indicates an expansion in the past 27 years of more than 400 per cent. Present building operations show no slackening in this expansion at the present time. It is illogical to assume that the saturation point is being approached. Relatively few people take marked advantage of the extension of the fresh-fruit and vegetable season with refrigerated storage. Then, too, the preservation of the highest quality of perishable foods is receiving increased interest and the application of refrigeration to a wider range of commodities can be expected during the next few years.

Refrigerated ships are enabling us to seek the markets of the world with a number of our products. American apples are finding favor in a number of countries. We see in the press notices that the largest shipment of Virginia apples ever made at one time, 40,000 bbl., went out recently from New York for Buenos Aires on a new refrigerated steamship. A recent Bureau of Commerce report carries the following "Barreled and boxed apples are arriving in the British markets in splendid condition, the stock having a good color and bright appearance." This export trade helps all along the line because it makes use of refrigeration in railroad transportation as well as in terminal storage.

Cold Storage as a Power Consumer

Looking at the power consumption side of the cold-storage business we see a gross consumption in that unit of the refrigeration application cycle is not inconsiderable. We must realize also that this is one of the integral parts of the system from producer to consumer. On the basis of 731,377,000 cu. ft. of cold-storage space in the United States and at a nominal rate of 60 kw-hr. consumed per 1000 cu. ft. per month, the power for cold storage amounts to 526,591,440 kw-hr. per year. It is not possible to determine what portion of this is electrified at the present time; but, if the same percentage obtains as in ice manufacturing it is 59 per cent and the year electrical consumption is 310,688,600 kw-hr.

The total production of ice manufactured and harvested in the United States during 1928 approximated 72,000,000 tons. Of this amount, 11,000,000 tons were harvested from lakes and streams and 61,000,000 tons were manufactured. When, as has been shown, 12,500,000 tons were used in icing refrigerator cars, the balance of 59,500,000 tons for domestic and commercial consumption seems small. It is only 1020 lb. per capita, yet per capita figures are usually small where a product not in universal demand is being considered. Ice is such a product. There are 5,000,000 less refrigerators in the country than wired homes. That is to say that there are 15,000,000 homes without refrigerators although a number of these are not available to ice and electric service. These statistics bring out the fact that refrigeration has touched little more than 50 per cent of its domestic market. The situation is one of challenging possibilities to the power industry as well as to the ice and refrigeration industry. In spite of past growth we yet have a large field for expansion before us.

The total number of ice plants in this country is somewhat in excess of 7000. Of this number 32 per cent are steam driven, while 59 per cent use electric drive and 9 per cent gas and oil drive. Assuming that the same percentage of the total production came from these plants, then 19,500,000 tons of ice were produced in steam-driven plants, 36,000,000 in electric-driven plants and 5,500,000 in gas- and oil-driven plants.

Using an average figure of 62 kw-hr. consumed per ton of ice made, the total of electric power used by ice plants last year was 2,232,000,000 kw-hr. This is a consumption which was exceeded by few industries in the United States and the future holds such a promise. Electrification is going forward rapidly in a number of locations. A new electric process of demineralizing water now makes it possible to electrify plants which were heretofore considered only from an angle of distilled water operation.

The ice industry, therefore, is one which uses more than 3 per cent of the total output of the electric-light and power industry. Added to this fact is the desirability of this ice-plant load which comes to its peak at a time far removed from the annual peak of the central-station load. The ice plant is also well suited to the use of synchronous motors, an aid in power factor correction.

Small Commercial Installations

There is another phase of this domestic and small commercial refrigeration field. This is in the field of the so-called electric refrigerator. The total of these units in homes and small stores is placed at 900,000 and the annual consumption at 700 kw-hr. per unit. These machines bring an increased load of 630,000,000 kw-hr. per year, a load which is very steady and reaches its peak demand during the summer months, when the demand on the central station is at its lightest point. The sale of mechanical refrigerators continues and this very desirable power load will increase in parallel with the sales volume.

Dairy and ice-cream plants have very extensive refrigeration possibilities and at the present time aggregate more than 80,000 tons of refrigeration. A portion of this is for use in manufacturing processes, but at the time the refrigeration preparation preserves the product in later handling. The refrigeration installations in meat markets and produce houses play a very important part in the national life. The influence of these installations has not made itself felt in the popular consciousness because a number of them have been in operation for twenty

YEARLY POWER CONSUMPTION IN APPLICATIONS OF REFRIGERATION IN THE UNITED STATES

	Kilowatt-hours
Cold storage	310,688,000
Ice making	2,232,000,000
Small electric refrigeration	630,000,000
Moderate size commercial installations	584,575,200
Total	3,757,263,200

years or more, yet it requires little study to see the effect they have upon our every-day life. From the point of power consumption these cooler and storage installations also have a real significance. Of all these there are 17,993 installations in the United States aggregating 159,720 tons refrigeration. On a 33⅓ per cent load factor these consume 584,575,200 kw-hr. per year.

On the score of power development we see that an industry has grown up which proves to be one of the best, if not the very best customer of the electric power industry. It consumes a great volume while it makes no exactions of extraordinary service. The accompanying tabulation is made up of the different figures arrived at in our discussion of the various units of the industry.

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Research as Applied to Heating and Ventilation

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This paper gives the results that are being accomplished, through various research programs, that are being conducted, in the field of heating and ventilation, by the research laboratory of the American Society of Heating and Ventilating Engineers. Cooperative arrangements are in effect with many universities, colleges, and departments interested in this research work and these enable the work to be carried on, at a cost, very much less than would be necessary otherwise.

RESEARCH as applied to heating and ventilation is worthy of attention and has unusual importance because it so profoundly affects the health and the comfort of the whole race, in contradistinction to those researches which constantly are under way in the laboratories of great manufacturers and which have usually as their object an ultimate financial reward. We can trust the manufacturers to study improved methods and can expect inventors constantly to search for and to experiment with improved processes and machines which will bring back dollars. The free-lance engineers of the country, however, know of a great many problems the solutions of which will benefit the whole race without immediate apparent monetary reward to any particular person or group of interested persons. These individual engineers do not have the means alone to pursue research.

The Research Laboratory of the American Society of Heating and Ventilating Engineers was organized for the purpose of securing contact between public-spirited individuals in order that research for the good of all, without any selfish angle whatever, might be conducted. The Laboratory entered into a contract with the Bureau of Mines under which the Laboratory was given space in the Bureau of Mines experiment station in Pittsburgh, for offices and for experimental apparatus, free of charge. A great deal of assistance and support is constantly being contributed by the Bureau of Mines in this friendly cooperation, in addition to the specific housing.

The Laboratory as first established was supported by gifts from members of the Society and from its friends. This method of financing being somewhat difficult when it came to establishing a budget, the constitution of the Society was eventually amended so that the dues paid by the members were raised 66 2/3 per cent, two-fifths of all receipts for dues from members thereafter being devoted to the Research Laboratory.

As is the case in most research situations, the need for work to be done soon expanded beyond the income, and in 1928 the budget of the Laboratory was double the income from member dues. The additional money is contributed by friends, who make such contributions without any restrictions as to the direction in which the funds shall be invested, and who contribute comparatively small sums each, year after year, rather than large sums such as might have been forthcoming from intensive money-raising campaigns. It is an inspiring situation that there is never any particular difficulty in raising the necessary money to finance the laboratory, and that in many instances, we are told by these friends—"If you find yourselves running short, let us know and we will send some more."

The activities of the Laboratory are directed particularly toward research in fields not likely to be covered by immediate commercial results, and which therefore tend to lag behind the developments and inventions emanating from the researches of the larger manufacturers. Thus our laboratory has studied intensively such things among many, as air infiltration through walls, heat conductivity of structural materials, effect of paint on radiating surfaces, carrying capacity of pipes, and the physiological and the psychological influence of different ventilating and temperature conditions on the human body.

There was so much work to be done by the Laboratory that the facilities of the Bureau of Mines at Pittsburgh available for our service soon were fully occupied. An opportunity came early in 1927 for making a cooperative arrangement with the University of Wisconsin, whereby research would be carried on at Madison under our general technical direction and approval, and under a joint expense account. This idea of cooperative research has proved very satisfactory to all concerned. It enables the University to make close contact, without cost, with the leaders in engineering thought in other Universities and in industry. It gives a practical aspect to the often otherwise routine work of the students. It trains research workers It lends authoritative prestige to the output.

This cooperation enables the American Society of Heating and Ventilating Engineers to carry on research at very much less cost than otherwise could be attained. It gives access to laboratories which have ready at hand most of the equipment needed. It brings educators of national reputation to collaborate with our own Director and technical advisory committees.

The contract developed and put into effect at Madison proved so satisfactory that the University cooperation idea has been extended, until today we have similar cooperative agreements with the University of Wisconsin, University of Minnesota, University of Kentucky, Carnegie Institute of Technology, United States Weather Bureau, health departments of thirteen representative cities, Purdue University, Texas Agricultural and Mechanical College, and Harvard University. Further extensions of the arrangement are imminent, and are delayed by our financial budget, rather than by any lack of good will or desire on the part of the cooperating agencies.

We have encouraged the two northern universities to study especially, the construction of buildings, so that our knowledge of the infiltration of air through walls and around windows and doors and our knowledge of the conductivity of materials used in building construction shall be increased. There are already in our literature exceedingly valuable papers from the University of Wisconsin on air infiltration, and we will soon publish the results of a years work on the leakage through various kinds of walls. We apprehend that these results will express among other things, to a startling extent, the effect of quality of workmanship in masonry walls.

We have devised heavy structural steel frames, at the Madison laboratory, in which any desired number of sample walls each about 8 ft. square and of any desired thickness can be built in the open, and allowed to dry out; then can be moved into the testing apparatus and have the leakage rate determined quickly. Prior to this arrangement, in other laboratories when the walls were built inside the testing maching it sometimes took six months or more for a wall to dry out enough to reach a stable condition as to air infiltration.

At the University of Minnesota we have what is probably the most authoritative facilities, that there are anywhere in the

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world, for testing the conductivity of materials. Both the hot-plate and the hot-box electrical methods are in use. This work, of course, has to do most intimately with the amount of heat-generating and transmitting apparatus the heating engineer shall provide for the building. In the development of the use of the comparatively recent building-insulation materials, so potent in their effect on operating costs with gas and oil fuels, our laboratory is giving an important service.

The tendency has been for the inventors and manufacturers of insulating materials to be rather more optimistic than otherwise. A given wall construction, for instance, may show a conductivity rate of some number of heat units per degree temperature difference between the two sides, and the manufacturer may so advertise in the best of faith. He may know that with an air temperature of 100 deg. near the ceiling of a normal uninsulated 70-deg.-breathing-line room, he has an attic-air temperature of 40 deg. He may not and usually does not know that after the insulation shall have been applied to that ceiling the air near the ceiling under the insulation may be still warmer than it was before, since it now has met with resistance to outgo; while the attic, no longer receiving a great flow of heat from below, due to the insulation, may drop from the former 40 deg. to zero or below, thus giving a considerably greater temperature difference than that on which the optimist based his calculations, and showing regretably, a much smaller saving due to insulation than that which he at first estimated.

There has been a vast increase in the number of oil- and gas-heated houses. The heaviest demand for gas boilers today is for the small homes having from five to six rooms, and a surprising number of these small installations are being made. Gas gradually is being sold at a lower price while other fuels are rising in price, and this militates in favor of homes insulated against rapid heat flow. The explanation of the reduction in price of gas seems to be that the gas companies have through research, discovered that their overall load factor can be improved vastly by acquiring a heating load.

Experience has demonstrated that it is easy to gain a return in cost of fuel saved, of from 10 per cent to 35 per cent, on money invested in house insulation. The cost of the heating apparatus can be reduced around 18 per cent by a reasonable and justifiable investment in better building construction. Research by the Gas Association, with which we maintain, through our own laboratory, very friendly and beneficial contacts, has shown that the domestic gas load for cooking, water heating, etc., follows the weather conditions, increasing noticeably as the weather gets colder and decreasing as it gets warmer. It is very clear that both domestic and heating demand for gas occur simultaneously, and that in intensity they are a function of the outside temperature.

At the University of Minnesota, we have shown that the efficiency of an air space in the wall of a building is increased rapidly as the thickness of the air space is increased, up to about $^3/_4$ in., after which the heat-resisting efficiency of the air space increases very slowly as the space is widened. This is with reference to small air spaces, in which interior convection air currents do not interpose their potent influence. It is well known, of course, that large air spaces in walls, especially if they have considerable height, allow interior air currents, up the warm side and down the cold side, which are not conducive to effective retention of heat within the building.

We are endeavoring to develop a code for the uniform testing of air-cleaning devices. The difficulty which has handicapped us in the past is that the measurement of the effectiveness of any air cleaner depends on some standard form of dust determinator. Years of study in many leading nations of the world have been given to the development of a dust-measuring machine, but we have today no universal acceptance of any particular type of measuring machine or method, and therefore we have no relative standards of efficiency of cleaning devices.

We have been studying a Hill machine, which is of the type which draws a sample of dusty air through an orifice, impinging the dust on a sticky, ruled object glass, where the particles can be counted under a microscope. We have devised a special apparatus in which several orifices and object glasses are arranged in series and have proved that with this machine from 80 to 85 per cent of the dust in a given sample sticks to the first object glass. We have discovered also the most effective distance from orifice to object glass and can foresee that any standard method of laboratory dust filter testing by this method must provide for a specific air velocity through the orifice of the dust determinator.

The smoke problem in cities gets worse year by year, as far as we can learn. It is more troublesome in some cities than in others. Some cities have a higher death rate from those diseases which we think have to do with air conditions than do other cities. In cooperation with the Weather Bureau, our laboratory is going into this matter with considerable elaboration. We know that results may be attained by this study which will be of undreamed value.

For instance, each Weather Bureau keeps a record of the hours of sunshine each day. The automatic sunshine-recording machine, however, must be favored with a twilight correction factor, being an allowance for the varying interval between true sunrise and true sunset, and the time when the sun is high enough above the horizon to penetrate the overhanging cloud of smoke and dust and so sensitize the instrument. We suspect very strongly that we have discovered that the "twilight correction" is a true measure to a degree, of the amount of suspended impurities in the air above cities. We hope soon to prove or to disprove this suspicion, and to check authoritatively over a period of years, the apparent truth that the "twilight correction" factor consistently is lengthening.

The Research laboratory has arranged with the Weather Bureau to have simultaneous daily observations made of the dust in the air, all with the same dust determinator in the following thirteen cities: Atlanta; Boston; Chicago; Cleveland; Denver; Detroit; Lincoln, Nebr.; Madison, Wis.; New York; Pittsburgh; St. Louis; Washington, D. C.; and Salt Lake City. We have arranged with the health departments of these thirteen cities to furnish us the statistics of the daily incidence and mortality of the infectious diseases in the standard international list of communicable diseases.

The results of all this cooperation will show for the first time the relative air cleanness of the cities, using identical measuring technique, and these data will be coordinated with temperature, humidity, wind velocity, sunshine, and twilight correction, as well as with health conditions. Illness may or may not be a function of dust. These determinations should help to settle the suspicion. The results should be of interest, at least, to the various smoke-prevention bureaus, if not to the various chambers of commerce. The technical advisory committee having the matter in charge, the various health officers, and the Government will all contribute to the authenticity of this study, and the results will, of course, be available in the publications of the Laboratory. When publicity to this research was first given, we were waited upon by one Chamber of Commerce which had gathered the impression that we were undertaking research to prove what city was the most healthy one in which to live. We have also been visited by laundry men and soap manufacturers.

At Harvard University, with the School of Public Health, we are carrying on cooperative research on Heating and Ventilation in its relation to Health and Comfort. Exceptional laboratory

facilities are available there, especially as evidenced in imported apparatus, for the most delicate psychological as well as physical observations.

At Purdue University, a radiator-testing laboratory has been built, to fulfil the most modern conception of what such a plant should comprise and work is under way intensively. There is a strong demand for data to check radiator ratings published by, sometimes, over-optimistic manufacturers and we hope to make Purdue, with our cooperation, an authoritative court of last resort on the subject. There are so many recent improvements in radiators, concealed and exposed, direct and indirect, copper and brass, that the test codes of former days may be obsolete.

At the University of Kentucky, we aided in the construction and inauguration of one of the most complete boiler testing laboratories in existence. Facilities are provided there for the rapid and authoritative testing of any kind of heating boiler, with up-to-the-minute instruments. An exceedingly interesting feature of this laboratory is the arrangement whereby every possible happening and variation during a test is recorded automatically, thus checking and fortifying the human element. Even the opening of a fire or draft door is recorded as to duration and time of incidence, by a machine.

At Carnegie Institute of Technology, Pittsburgh, cooperative research is conducted on the flow of steam and condensation water in pipes. This is fascinating work, profound in the possible economic results. These are published in the Guide, the annual handbook of the American Society of Heating and Ventilating Engineers, and the information gleaned has resulted in many corrections in pipe tables, the authenticity of which was clouded in hoary tradition. In general, it has proved possible to reduce the sizes of pipes for given duties, with tremendous savings in cost to building owners. This work will continue indefinitely.

At the Texas Agricultural and Mechanical College, Austin, Texas, similar work is being conducted directly under Professor Giesecke, a national authority on hot water heating pipe sizes and arrangements. New tables already are available in the Guide. Importance must be stressed on the reaming off of the pipe-cutter burrs in all piping and nipples, as these restrictions in area reduce pipe capacities greatly.

At Pittsburgh, in the Bureau of Mines, we have two psychrometric chambers, where controlled atmospheric conditions are available. We are equipped with the necessary instruments and facilities to study the effect on human beings of the varied atmospheric conditions encountered in industry, as in steel mills and in theaters or places of assembly. Our study in the psychrometric room covering heat production in and dissipation from the human body has extended over many years work. It has involved a study of the rate of metabolism and required the development of apparatus for measuring the heat outgo from the lungs as well as that from the skin.

The study which we are making in this connection is of profound value in all man-cooling projects. Many reports have been published covering the results of this research. We did not discover that there is a certain temperature and moisture condition above or below which the metabolism rate increases, but it is in any event an interesting thing to know. When the surrounding air is too cool, the heat production in the body speeds up to compensate. When the surrounding air is too warm, the heat production in the body speeds up to enable the body to get cool because of greater heat given off as by evaporation of perspiration. We are now publishing data dealing with the bodily heat-controlling mechanism, the results of a year's observation in these test rooms.

The results of this research, on which perhaps more time and resources have been spent than on any other of our studies, are classical, involving participation in the well known "Comfort Zone" chart of the American Society of Heating and Ventilating Engineers and covering fundamentals which are of almost inestimable value in all projects involving the promotion, by air conditioning, of human comfort and well being.

The results of the Research Laboratory appear in the monthly reports of the American Society of Heating and Ventilating Engineers and are the basis for most of the technical information given in the Guide of the society, an annual publication. This Guide is a combination text book and directory and contains the latest engineering data available. It is contributed to by all of the members of the society without compensation and is revised every year.

FSP-51-14

Commercial Problems in Steam-Turbine Design

By R. C. ALLEN,[1] PHILADELPHIA, PA.

This paper discusses developments in steam-turbine design. Concentration of power, in modern turbines, has led to an increase of operating stresses of turbines parts. The tendency now is toward higher blade speeds and reduced size and cost of machines. Manganese copper is used for blades of low stress and of small size; for the larger and more important blades, stainless steel is used. It can be forged and machined without difficulty.

IN THE CONSTRUCTION of large machinery, steam-turbine equipment, for example, the element of time is a dominating factor. We must invent, design, construct, install and place in service machines up to 6000 kw. in three months' time. This rapid procedure in the building of turbine equipment is not confined to simple types of units that are altered slightly to meet specific operating conditions. On the contrary, many extremely complicated industrial turbine combinations are required on a short delivery basis.

In the central station field, the turbine builder is called upon to design and construct units of the largest size in less than one year's time. Single-cylinder machines of the largest size have been designed, built, installed, and placed in service within ten months' time. With multiple-cylinder machines, there has been a corresponding record of achievement.

The nature of the large steam-turbine business makes it impossible to standardize designs, even in the turbine blades, at least not to any appreciable degree. Experiments are carried out with miniature turbines and the results applied, when predicting the performance of the full-sized units. The real test of engineering comes when the finished product is placed in service.

Turbines of today require larger and much more expensive blades than formerly as they are subjected to heavy stresses. For reasons of strength alone we would like to make them tapered rods or tubes. To secure a reasonable thermal efficiency the blade sections must be crescent shaped, with the horns of the crescents as the extreme fibers from the neutral axes of their least moments of inertia, when the blades are considered with regard to bending stresses. The inlet and outlet edges of blades must be reasonably thin to secure the desired efficiency. Formerly, the inlet and outlet edges were left ragged after the machining operations, so that fatigue failures were not uncommon. It required an unbelievably long time for the turbine builders to discover that the rounding of the inlet and outlet edges would give a substantial improvement in the fatigue-resisting properties of the finished blades. The fatigue resistance of turbine blades has been increased at least three times, by the introduction of the simple process of rounding the edges. The rounding of the upper edges of blade-packing pieces and the rounding of the edges of lashing-wire holes are other examples of the value of little details in blade design.

The study of blading materials has occupied the attention of metallurgists for many years. One important property brought to light has been the matter of solder cracking. If a $^3/_8$-in. rod of 30 per cent nickel steel is fluxed and coated with silver solder, it may be snapped off short with but little effort while at the soldering temperature. This is due to an attack by the molten solder upon the intercrystalline cement. Stress is required to bring the action into play. This obscure effect caused many troubles before it was discovered.

Long turbine blading requires a reinforcement in the form of shroud bands, or lashing wires, to keep down the bending stresses, whether they be caused by direct steam loads or by blade vibration. It is believed that silver soldering is the most-durable method of preventing relative motion between the blades and their lashing wires. It is therefore essential that blades and lashings be of material that can be silver soldered easily.

In the practice of the Westinghouse Company, manganese copper is used for blades of low stress and small size in turbines of the industrial class. For the longer and more important blades stainless steel is used. Stainless steel is a general term covering a large class of materials, most of which are not suitable for blading practice. The present standard material has a chromium content of $11^1/_2$ to 13 per cent and a carbon content of 0.07 to 0.12 per cent. It can be forged and machined without serious difficulty. It is believed to be the safest material now available for use in the extreme low-pressure zones. It is not a perfect material, as there may still be some erosion at high blade speeds. On the other hand, this steel is available in quantity and with sufficient uniformity so that the production of reliable blades is further assured. The physical properties are retained satisfactorily under high-temperature conditions.

There are many stainless steels that are less corrosive than the material now in common use, and while some of these are less susceptible to the erosion found in the last rows of large machines, there are definite objections to the use of these materials for blading; if not that of solder cracking, there is the question of low elastic properties. Some of these materials are extremely hard to fabricate.

Starting in about 1918, several large three-cylinder turbines of around 60,000-kw. capacity were designed and constructed by the Westinghouse Company. Among these units were two 60,000-kw. machines installed at the Colfax plant of the Duquesne Light Company. The turbine parts of one of these units weighed 1,652,000 lb. The low-pressure blades were 18 in. high with a tip speed of 607 ft. per sec. The rotative speeds were 1800 and 1200 r.p.m. for the high- and low-pressure elements, respectively.

In November of last year a 60,000-kw. 1800-r.p.m. Westinghouse single-cylinder turbine was placed in service at the Lakeside plant in Milwaukee. The turbine parts of this new machine weigh 635,000 lb., over a million pounds less than the corresponding parts of the older Colfax turbines of the same capacity. The generator parts of the new Milwaukee unit provide a further reduction of 350,000 lb. below the weight of previous designs. The low-pressure blades are 34 in. high with a tip speed of 1147 ft. per sec. The turbine at Lakeside occupies about one-third of the floor space required for the older Colfax machine, while, on the other hand, the engine efficiency of the new turbine is higher. The reason for the greatly reduced proportions of the new frame is found in the high blade speed and the great length of the last row of low-pressure blades.

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It must not be assumed that the new machine at Lakeside cost the turbine manufacturer the same average amount per pound as the older turbine at Colfax. Details of construction are more expensive in the smaller and higher-speed unit when considered on the pound basis. In the 60,000-kw. Colfax turbine, the spindle parts were designed so that they could be made from steel castings. In the Lakeside turbine the spindle parts are machined from steel forgings, while the coupling end is forged of $3^1/_2$ per cent nickel steel. In the Lakeside turbine, an improved form of blade fastening has been developed that permits a substantial shortening of the spindle, although adding greatly to the cost. Central-station people appear to be intensely interested in the largest sizes of single-cylinder turbines. It is believed that for steam pressures not exceeding 650 lb., the multiple-cylinder units will become increasingly hard to justify.

The great concentration of power in modern turbines has called for increases rather than reductions in the operating stresses of the turbine parts. Years ago a designer usually expressed pride in a machine design in which there were lower operating stresses than found in similar equipment of earlier types. Today the reverse condition exists, in that the engineer must work all material to the maximum possible stress consistent with good reliability. In many instances we have been asked, during the negotiation stages of large jobs, if the stresses in certain rows of blades were greater than those existing in previous designs. In some cases we were forced to reply in the affirmative, thereby creating doubts in the minds of the prospective purchases as to the sufficiency of the new designs. The author would like to bring out one important point in connection with the determination of operating stresses: namely, that most of the developments in the past few years have been made in the direction of discovering and eliminating high local stresses. In other words, in our modern designs we can operate with higher average stresses because our factor of ignorance is less and it has been made so by continued study and research on the problem of local stresses. It should also be borne in mind that little is known of the true stress distribution in a bar in simple tension. We still have factors of ignorance in machine construction that will be reduced gradually, permitting in the future still higher average stresses and securing at the same time an improved reliability.

The tendency toward higher blade speeds in modern turbine design has been the most important influence that has led to reductions in size and cost, and that has permitted the construction of the huge machines that have been placed in service the past year. How far we can go in further increasing blade speeds, we do not know. We have not yet reached the practical limits with the materials now available. The Westinghouse Company has in service, or on order, five turbines of the single-cylinder 1800-r.p.m. frame, designed for a normal rating of 65,000 kw. The General Electric Company has sold a single-cylinder machine for Cahokia with a 75,000-kw. rating. If the industry can use still larger single-cylinder machines, they can be constructed with good assurance as regards operating reliability and efficiency. Large multiple-cylinder turbines of 100,000 kw. capacity and greater, are, in most cases, tailor made to suit particular needs. The general trend of turbine design is naturally in the direction of reliability and simplicity. The Westinghouse Company has introduced the oil governor as a speed-controlling mechanism. The oil governor has been adopted because of the greater reliability that is to be secured through the elimination of the many wearing parts of the older flyball governor mechanisms.

In the general arrangement of Westinghouse turbines on their foundations, an attempt was made formerly to standardize the placing of steam chests, throttle valves, and other important parts. We tried to use standard assemblies of oil piping and similar auxiliary details. This was found to be impossible to carry out with the larger central-station turbines of recent designs because of the many types of foundations on which the equipment must be installed. It is therefore necessary for the members of our designing staff to cooperate with the engineers of the power-plant constructor and to work out with them the best arrangement of steam chests, oil tanks, oil piping, and other similar equipment.

It may seem anomalous that the reliability of central-station turbines is increasing in spite of the higher blade speeds, longer blades, and higher operating stresses. This is bound to follow from practical experience, as well as from advanced research. It is believed the future holds further possibilities in the way of greater concentrations of power and higher efficiencies, and that we shall at the same time be able to accomplish these things with increased reliability and with reduced labor on the part of our most loyal friends, the turbine operators.

FSP-51-15

Centrifugal-Pump Economics

By A. F. SCHERZER,[1] ANN ARBOR, MICH.

In this paper the author presents some developments in the theory of the centrifugal pump with which he has been connected and shows their application to pump practice. In reviewing the technical and theoretical aspects of the problem, he refers to the fact that the performance of the centrifugal pump is shown best graphically by the quantity-head curve. In the extensive centrifugal-pump literature existing there are to be found a number of theoretical analyses of the action of such pumps, all of which are based on somewhat similar assumptions. In each case the striking fact is the wide difference between the so-called theoretical curve and the one actually obtained by a test. To bring about any sort of an agreement it is necessary to assume in some cases large and disproportionate losses. To show this difference the author presents a chart showing the theoretical quantity-head curve and the actual curve obtained by a test of the pump. Around the point of zero discharge there was a discrepancy of about 100 per cent between so-called theory and practice. At or near the shut-off point the head actually obtained from the pump was nearly twice as much. A careful study of these matters resulted in the discovery of a surprising error in the derivation of the fundamental theory for the centrifugal pump.

Another interesting point brought out in the paper is that the use and the benefits claimed for guide vanes are founded on a misconception of theory. The same is true of the so-called volute or spiral casing into which the runner discharges the water.

The author states that the real function of the pump case is that of compelling the water to revolve in a more or less circular path. Thus the natural form of a case should not be volute or spiral, but rather a plain concentric circular case of constant cross-sectional area. Tests of a circular case pump and a good design of the volute type showed a remarkable increase in head development by the former. At its point of maximum efficiency the circular case pump developed an excess head of 50 per cent over a similar volute design.

PERFORMANCE of a centrifugal pump is shown best graphically, by the curve representing the relation between the quantity discharged and the height to which it is lifted, the speed of rotation being constant. This is known as the quantity head curve, or sometimes the pump characteristic. Such a curve is shown in Fig. 1. The shape of this curve is fixed by the design and proportions of the pump, and it can be determined exactly only by a test. It is not a question of impeller design or shell design, but rather the combination of the two. The same impeller in different shells will not give the same curve.

In our rather extensive centrifugal-pump literature, there will be found a number of theoretical analyses of the action of such pumps. Most of these are based on somewhat similar assumptions, and what is true of one is with some exceptions, common to all of them.

The striking fact in each case is the wide difference between the so-called theoretical curve and the one actually obtained by a test. In order to bring any sort of agreement, it is necessary to assume, in some cases, large and disproportionate losses. In view of the high efficiency of well-designed pumps, it seems impossible to account for the existing difference on any such theory of losses. When this problem first came to the author's attention fifteen years ago, he dismissed from his mind any idea that the commonly accepted theory of the centrifugal pump could be in any way open to question. As time went on, however, his confidence in it weakened gradually and the belief amounted almost to certainty, that some error had been made somewhere in its derivation. What this error could be, was the real question. The derivations found in texts were, in general, very simple and direct in their reasoning, and for some time no fault could be found with them. Without going into the details of existing pump theory, it will be sufficient to state, that the so-called

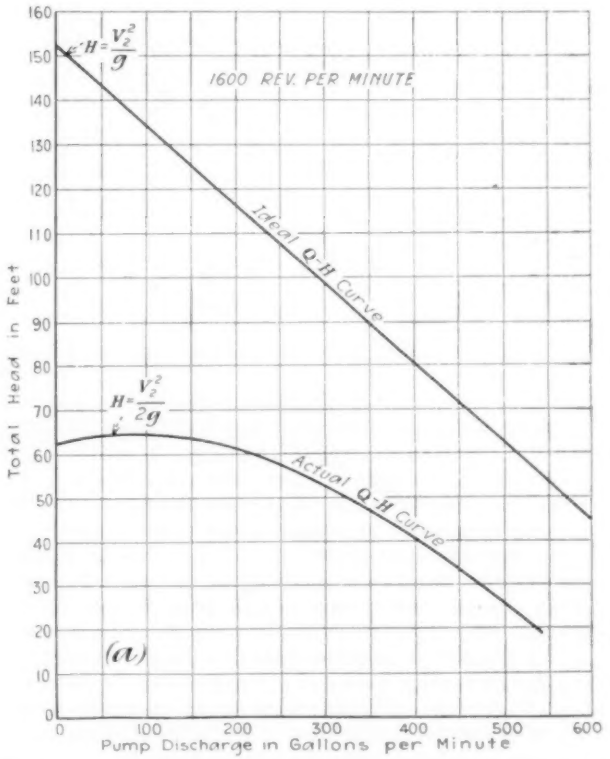


FIG. 1 COMPARISON OF ACTUAL AND THEORETICAL QUANTITY HEAD CURVES

theoretical relation between the quantity and head can be expressed by the relation:

$$H = \frac{V_2}{g} + \frac{V_2 \cos B_2 Q}{g A_2}$$

where H is the theoretical head developed, V_2 the peripheral speed of the impeller at its outer diameter, B_2 the vane angle as shown in Fig. 2, g the constant of gravity, and A_2 the area of the water passages in the impeller perpendicular to the relative velocity W_2. This equation leads to the straight-line relation between Q and H shown in Fig. 1 and marked ideal Q-H curve." Below this is shown the actual Q-H curve as obtained by a test. As can be seen around the point of zero discharge, there is a discrepancy of about 100 per cent between socalled theory and practice. At or near the shut-off point the head actually obtained from the pump is approximately $V_2^2/2g$, while the head called for by present theories is about V_2^2/g, or nearly twice as much. This

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difference is explained and accounted for in various ways, by different authorities, none of which appear to the author as either probable or convincing and further do not meet certain requirements of fact. A great deal may be found in current pump literature on this subject. A careful study of these matters resulted in the discovery of a rather surprising error in the derivation of the fundamental theory for the centrifugal pump, which will be explained in some detail.

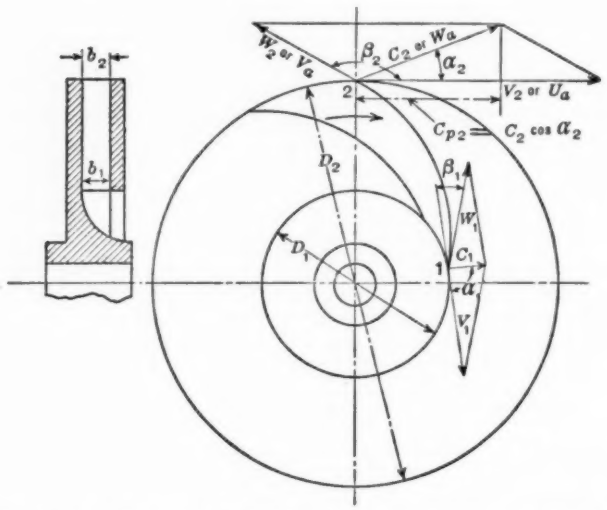


FIG. 2 PUMP DIAGRAMS AT ENTRANCE AND DISCHARGE OF IMPELLER

The usual explanation of this discrepancy indicates a serious misconception, not only of the action of the centrifugal pump, but of one of the fundamental laws of hydraulics. The error is due to a misapplication of the well-known Bernoulli theorem.

When water flows in a pipe, it is customary to say that the total head is obtained by measuring the pressure head by a piezometer at right angles to the flow, and adding the velocity head. This is a correct application of the theorem. The pressure and velocity exist as separate and distinct factors. However, when water is set in rotation, in a cylindrical vessel by the action of a paddle or impeller, there is a pressure created, by centrifugal force which is equal to $V_2^2/2g$ and which is part of the velocity itself. The total head may be regarded as either, but not both. The pressure exists merely because of the velocity in the circular path, but certainly not in addition to it.

It would be as logical to say that the "hammer," swung around by the athlete, has a total energy equal to the sum of the kinetic energy of the ball and the tension in the wire. Clearly the tension is not energy, but merely a force existing because of the tendency of the ball to fly off at a tangent. The energy is entirely kinetic. The act of letting go of the wire (destroying the tension) does not change any of the original energy, and the hammer flies off at a tangent with a velocity V which is the same as its former peripheral velocity. Therefore the total energy is $V_2^2/2g$.

It is clear that the pump gives a head of only $V_2^2/2g$ at "shut off," instead of V_2^2/g as required by all present theories, for the very simple reason that the energy imparted is only $V_2^2/2g$, and naturally the water can have no more energy than it is given by the impeller.

One of the reasons that might be advanced as to the cause of this error is the widespread failure to distinguish properly between pressure and energy. Pressure may measure potential energy and may serve to transmit energy, but it is not in itself energy any more than force is work.

Fig. 1 shows graphically the generally accepted theory of the centrifugal pump. Here the ideal quantity-head curve is shown beside the actual quantity-head curve for comparison. Similar curves may be found in numerous works of reference. There is an obvious inconsistency between the equations representing the head imparted to the water and their graphical representation as Fig. 1. For example, the ideal quantity-head curve is seen to pass through a point representing more than double the head actually developed at the point of zero discharge, an obvious impossibility, as the pressure is caused only by centrifugal force.

According to the present theory, the total head is considered to be composed of two items, pressure and velocity. A measurement of the pressure indicates an average value of about $V_2^2/2g$ at or near the point of zero discharge. Discharge diagrams of runners show that at small rates of flow the absolute velocity C_2 is nearly equal to V_2 and, hence, the energy represented by it is

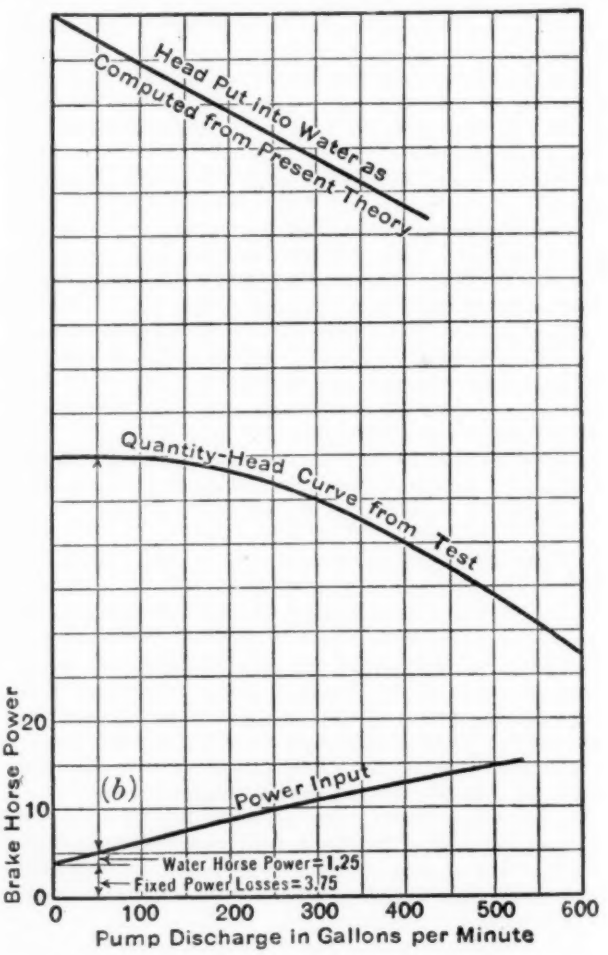


FIG. 3 DEMONSTRATION OF ERROR IN PRESENT PUMP THEORY BY A TEST

nearly $V^2/2g$. This would apparently mean that the total head is approximately $V_2^2/2g + C^2_2/2g$, or very nearly V_2^2/g. As a matter of actual experience, only a head of about one-half the ideal is developed by the pump. The difference is commonly said to be due to shock losses, and many elaborate formulas have been developed to account for it.

Fortunately, it is possible to settle this question beyond any doubt by a simple test in the laboratory. Referring to the characteristic curves obtained from a test of a centrifugal pump,

as in Fig. 3, it will be noticed that at the point of zero discharge the power required to drive the pump is the minimum (3.75 hp.). This power is consumed in doing the following things: (1) Overcoming the bearing friction; (2) overcoming the stuffing-box friction; (3) overcoming the disk friction of the impeller; and (4) pumping the internal leakage.

Since no water is being discharged by the pump, it follows that no useful work is done. If the discharge valve is opened and a little water allowed to flow, it will be noticed that the power required to drive the pump is increased. Assume that 50 gal. per min. is allowed to flow. The four losses mentioned will continue substantially as before, since there is nothing to change them at small rates of flow. To cause this flow requires an input of 5 hp.; the losses are 3.75 hp.; the difference, 1.25 hp., represents the amount of energy imparted to the water. Now, determine to what height 1.25 hp. would raise 50 gal. per min. A simple computation shows this to be 100 ft. Referring to Fig. 3, it will be seen that that is exactly the head developed by the pump.

According to the current pump theory, the head put into the water under the conditions of the test would be about 195 ft. If this were true the dynamometer would have to show this power input. As a matter of fact, it merely indicates the power required to lift the water to the head developed by the test and, at the same time, to overcome the various mechanical losses in the pump.

One of the questions debated for years by centrifugal-pump designers concerns the use of guide vanes. Some state that it is always possible to secure better efficiency by their use; others would never design a pump with guide vanes. Still a third class claims that it recognizes their advantages in some cases and uses them when by so doing the efficiency can be improved, thus giving the impression that they have some possible use.

In order to answer this question, consider what is supposed to be their function. It has been shown in the various theories that at discharge from the runner the water has a pressure head $V_2^2/2g$ as well as a velocity head $C_2^2/2g$. It is often stated that this velocity head $C_2^2/2g$ will be lost in shock, friction, etc., unless suitable provision is made for its recovery. The commonly accepted idea has been that at very small rates of flow this energy is about equally divided between pressure and velocity, and that the entire velocity head under these conditions was apt to be lost unless special provision was made in the form of guide vanes or diffuser vanes. Such vanes were designed so as to receive the absolute velocity of the water. By increasing the area of the water passages between them, the velocity was expected to be converted most efficiently into pressure.

Evidently, from the previous discussion, the use and benefits of guide vanes are founded on a misconception of the theory. In a centrifugal pump it is absolutely wrong to assume that the total energy is represented by the pressure in the casing plus the kinetic energy of the absolute velocity of discharge.

As the water is confined to a circular path, its kinetic energy is necessarily converted into pressure without any assistance from the guide vanes. In fact, they often serve to obstruct the natural flow of water, which entirely defeats their purpose and renders them useless. From the point of view of practice, equal or better efficiency can be obtained with a properly designed volute.

What has just been said with regard to the use of guide vanes may also be extended to include the so-called volute or spiral casing into which the runner discharges the water. The advantage of the volute casing is supposed to lie chiefly in permitting the efficient conversion of velocity into pressure through the recovery of a part of the energy represented by the absolute velocity of discharge from the runner.

While it is true that velocity is converted into pressure in the casing, yet it is not at all in the manner, or for the reasons usually given. If a body of water is set in rotation so that the peripheral speed of the outermost particle is V_2, then a pressure will be developed equal to $V_2^2/2g$, a case of perfect conversion of velocity into pressure. This is exactly what is taking place in the centrifugal pump.

Apparently, the only function of the casing is to restrain the water and compel it to move in a circular path. The development of pressure then follows naturally. The pressure obviously exists only because of the velocity, and one may be said to be equivalent to or measure the other. The very existence of pressure here indicates clearly that the velocity has been converted into pressure. It is difficult to see how it can be done a second time, yet this is exactly what is supposed to be accomplished by the guide vanes and spiral casings. This shows how hopeless it is to expect any gain from the use of guide vanes and spiral

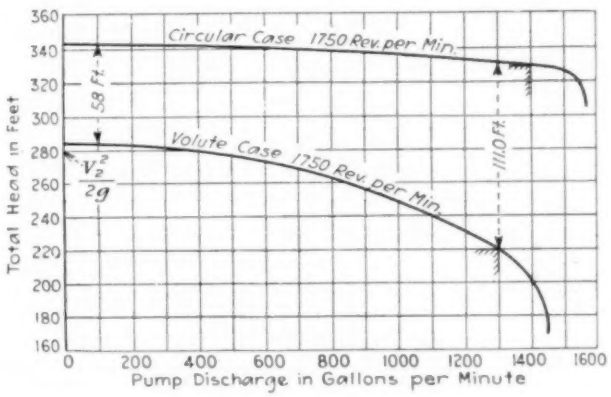


FIG. 4 COMPARISON OF A CIRCULAR-CASE PUMP AND A VOLUTE-CASE PUMP OF THE SAME DIAMETER, SHOWING EXCESS HEAD DEVELOPED BY CIRCULAR-CASE PUMP

casings, since their advantage is based on conditions, which can easily be proved never to have existed.

Up to this point we have been concerned chiefly with the technical and theoretical aspects of the problem. It is now proposed to show to what extent the conclusions which may be logically drawn from the foregoing discussion of the centrifugal pump theory may be applied toward improving performance.

The fallacy in the use of guide vanes and volutes or spiral casings has already been explained. The real functions of the pump case is seen to be, therefore, that of compelling the water to revolve in a more or less circular path, whereby the pressure is developed in much the same way that a stone whirled at the end of a string develops tension in it. From this it would appear that the natural form of case is not the well-known volute or spiral case, but rather a plain concentric circular case of constant cross-sectional area. This was such a radical departure from present accepted theories and practice that a pump was designed to test out the value of the new ideas.

The results of even the first design were quite remarkable and indicated in general two distinct points of advantage:

a Higher heads for a given peripheral speed
b Lower rotation losses for a given wheel diameter.

These two taken together made possible an increase of several points in the efficiency above that usually obtained in similar commercial designs.

Fig. 4 shows the very remarkable increase in head developed by a circular-case pump as compared with a very good design of the volute type. Each pump has its maximum efficiency at about the same capacity, so that they are fairly comparable. It is seen here, that the circular-case pump develops at its point of maxi-

mum efficiency an excess head of 111 ft. (50 per cent) over a similar volute design. The fact should be kept in mind, that both these impellers were of exactly the same diameter and both rotate at 1750 r.p.m. The circular-case design is clearly superior to the volute type in the matter of conversion of velocity into pressure within the case. This is accomplished by permitting a vortex of greater diameter than is possible in a volute pump.

Fig. 5 shows a similar comparison on another pump. Here a 17-in. impeller in the circular-case type is seen to produce at points of best efficiency about 60 ft. more head than a $17^1/_2$-in. impeller in a volute pump. If these two were compared on the basis of both $17^1/_2$-in. impellers the excess of the circular case over the volute design would be 77 ft. or 35 per cent.

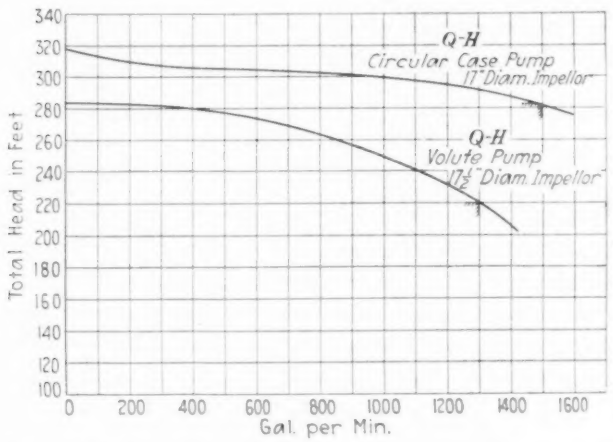


FIG. 5 COMPARISON OF HEAD DEVELOPED BY A CIRCULAR-CASE AND A VOLUTE-CASE PUMP AT 1750 REVOLUTIONS PER MINUTE

It is of special interest to note that the circular, concentric case is by no means new as the earliest pumps of this type were so constructed. Later on the circular case was replaced by the volute which was regarded as better hydraulically. It is now possible to demonstrate both theoretically and practically, that better results may be obtained with the circular case design, other things being equal.

The well-known failure of the various theories commonly accepted at present to agree with facts determined by tests, led to a very close study of both theoretical and practical phases of the problem. This has resulted in the development of an entirely new theory for the centrifugal pump. The author believes it to be theoretically sound, and its continued application to actual practice has resulted in exceptionally good agreement. The simplicity of the new theory should appeal both to those unfamiliar with present theories who desire to get some workable ideas as to pump performance, as well as those experienced in such work but hampered by the intricate and laborious calculations which now seem necessary.

The elementary action of a centrifugal pump will now be considered. First, imagine the pump revolving with the discharge valve closed. The water contained in the impeller revolves with it and through the action of centrifugal force has a tendency to move out into the case. As the case is filled with water, there is no place for it to go. Its desire to get out of the impeller is communicated, to the water outside in the case in the form of a static pressure. Thus the flow from the impeller is prevented by the equal and opposite back pressure of the water in the case, and no water is discharged. The pump is now said to be operating at shut-off, and the pressure head developed to approximately $V_2^2/2g$. Now the water in the case between the impeller and walls, also the water in the radial space beyond the impeller, is set in rotation by the dragging action of the revolving impeller. The angular velocity of this mass is less that that of the impeller, but due to its larger radius, in some cases, a pressure greater than $V_2^2/2g$ may be produced. The circular case is particularly favorable to this action. It is shown in Fig. 5 that a head 65 ft. in excess of $V_2^2/2g$ was developed by the circular case pump and only 7 ft. excess by the volute design.

Now assume the discharge valve to be opened a little. Water will flow through it. The back pressure in the case will be reduced and water will flow from the impeller into the case, due to the pressure difference existing. The water thrown off from the impeller has a large tangential component and mingles with the water revolving in the case. Some assistance may be received therefore from this source and a further increase in head may be obtained. This effect is especially noted in pumps having so-called rising characteristics.

The whole action might be compared in a rough way to water in a tub or barrel set in rotation by a revolving paddle, assisted at the same time by a jet of water directed more or less tangentially into it.

Consider a pump running at a certain speed, but with its discharge valve closed. The water in the impeller and the casing is set in rotation, creating a pressure in the pump, but flow from the runner is prevented owing to the pressure of the water in the casing. Now, assume that the discharge valve is opened a trifle to permit a slight amount of flow. The pressure in the casing falls, and the pressure opposing the flow from the impeller decreases, but the centrifugal force tending to throw the water out of the impeller remains the same, since the speed of the pump is constant. Consequently, water will flow in from the impeller into the casing, the quantity depending on the difference in pressure as well as on the area of the water passages. As soon as the water begins to move relative to the impeller, friction comes into play opposing the flow. The pump will adjust itself to these conditions so that the resultant of the pressure in the casing, the drop in pressure due to the flow, and the frictional losses in the impeller will equal the total pressure due to centrifugal force. From now on the pump will operate continuously at this discharge.

On further opening the discharge valve, similar readjustments continue until the entire pressure due to centrifugal force has been consumed in forcing the water through the runner, or else a point is reached where the atmospheric pressure is no longer able to force any greater quantity of water into the suction of the impeller. Thus, the flow from the runner is governed simply by the difference between the pressure due to centrifugal force and the pressure in the casing. Once this is understood clearly the problem is merely that of the flow through a very special form of orifice due to this pressure difference.

After the water has left the runner and has passed out into the casing, it must flow through the so-called throat of the pump before passing the discharge flange. This throat is merely the point of maximum cross-section of the volute. All the water discharged from the runner passes through this, a third orifice. Hence the entire problem of the centrifugal pump is that of flow through three orifices in series. The first is at the suction entrance to the impeller, and this determines the maximum capacity of the pump. The second is at the discharge from the runner, and the third is at the throat of the pump. Each of these orifices or points of restriction has its own characteristics and their combined effect determine the pump characteristic.

Consider that the pump is running at a constant speed. The pressure developed by centrifugal force, tending to force the water out of the impeller, is constant and is quite independent of the flow or the amount of it. Centrifugal force is equal to MV^2/R evidently, neither V nor R is changed greatly by the flow of

water and the mass M need not be considered in determining the energy input per pound of water. In other words, if there were no loss of pressure in the impeller and casing, the pump would deliver any quantity of water within the limit of the suction capacity at a constant pressure of approximately $V_2^2/2g$. A drop in pressure at the discharge flange is necessary if any water is to flow from the impeller.

The quantity that will flow depends on the area of the water passages; and its velocity, on the pressure difference established. The velocity is therefore quite independent of the quantity being discharged. The quantity is the product of the velocity and the area of the water passages measured perpendicularly to it.

The amount that the pressure at any point falls below the horizontal-line-tangent to the quantity-head curve at "shut-off" is very nearly that required to force this quantity of water through the pump. The total drop in pressure is (1) that required to cause the given quantity of water to flow, and (2) that required to overcome the hydraulic losses. These are fundamentally different in their nature and yet produce similar effects on the pump characteristic.

There is a rather close relation between the centrifugal pump and the electric generator. The drop in pressure required to produce flow is an example of a loss in pressure without any loss in power. A similar instance occurs in the field of electricity. An electric generator would deliver a constant voltage at its terminals, were it not for two principal losses. These are the drops in pressure due to armature reaction and to internal resistance. The drop in pressure due to armature reaction causes no loss of energy, whereas that due to resistance wastes energy in the form of heat. A loss of pressure that does not involve

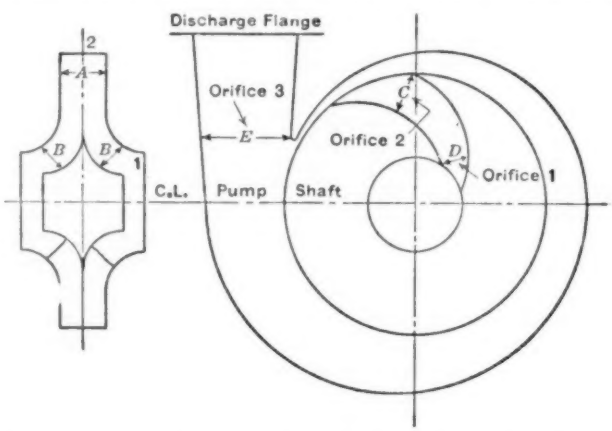


Fig. 6 Shell and Runner Showing Location of the Three Orifices

a loss of power is called a pressure drop, while a loss of power is called a pressure loss. These same terms might apply to the pump theory. In fact, many close analogies exist between the centrifugal pump and a separately excited shunt generator.

In the pump, the drop in pressure without loss of power merely means that a greater pressure drop is required to discharge a given quantity of water through a small orifice than for one of larger area; or that for a given pressure drop, more water will flow through a large orifice than a small one. This does not mean that the large orifice is necessarily more efficient than the smaller one for both may have a coefficient of nearly 100 per cent.

When water flows through the runner it has to overcome the frictional losses. These produce a drop in pressure, which is also a loss of power. Since the length of the path of the water in the impeller is short and the water ways are smooth, this loss of power due to friction is not large.

To make the new theory clear and indicate its application to practice, a characteristic curve, showing the relation between the head, and capacity at constant speed, has been worked out for a typical pump. The machine is of usual design and construction and of well-known make. Fig. 6 showing typical views, serves to illustrate the three points of control or orifices. The first is at the entrance to the impeller-vane passages, and its area is equal to the product of the dimensions, B and D, and the total number of vane passages. The second is at the discharge from the impeller-vane passages; its area is the product of the dimensions, A and C, and the total number of vane passages. The

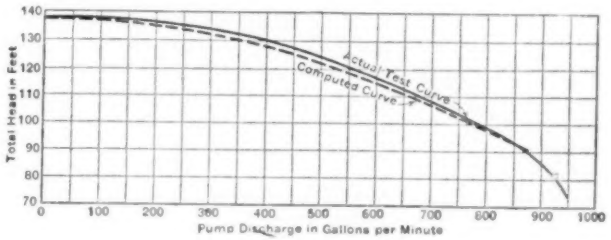


Fig. 7 Comparisons of the Quantity-Head Curves as Computed by the New Theory With That Obtained From a Test at 1750 Revolutions Per Minute

third is at the throat of the pump; its area is usually that of a circle of diameter, E (Fig. 6).

The following analysis of the quantity-head curve of a 4-in. double-suction pump will make the new theory clear. The areas given were determined from the pump design.

Area of orifice No. 1 equals $B \times D \times$ number of vanes = 0.0553 sq. ft.
Area of orifice No. 2 equals $A \times C \times$ number of vanes = 0.0625 sq. ft.
Area of orifice No. 3 equals $\pi E^2/4$ = 0.0694 sq. ft.

The velocity of flow through the orifice is equal to quantity ÷ area, hence the velocity through orifice No. 1 is $C = Q/0.0583$, through orifice No. 2 is $C_2 = Q/0.0625$, and through No. 3 is $C_3 = Q/0.0694$. The drop in head required to produce these velocities C_1, C_2, and C_3 can be compiled from the relation $H = C/2g$. Assuming Q = 200 gal. per min. $H_1 = 1.010$, $H_2 = 0.765$, and $H_3 = 0.645$. The total drop in pressure required to get 200 gal. per min. through the pump is the sum of the above three or 2.410.

Now assume Q = 400 gal. per min. and proceeding in the same way the total drop in pressure is 9.84.

For 600 gal. per min. the total drop is 22.0 ft.
For 800 gal. per min. the total drop is 39.0 ft.

The head developed at shut off was found by test to be 137 ft. The actual heads developed by the pump are therefore obtained by subtracting the total drop for various rates of flow from the head developed at shut off. Hence:

The head at 200 gal. per min. = 137.0 — 2.41 = 134.6 ft.
The head at 400 gal. per min. = 137.0 — 9.81 = 127.2 ft.
The head at 600 gal. per min. = 137.0 —22.0 = 115.0 ft.
The head at 800 gal. per min. = 137.0 —39.0 = 98.0 ft.

Fig. 7 shows graphically the agreement between the new theory and an actual test on this pump.

It will be noticed that in computing the drop in pressure to flow through orifices No. 1, No. 2, and No. 3 that no coefficient was used and also no account was taken of any possible gain in velocity head or of frictional losses. This was intentional. It

is evident that some coefficient must be used for each orifice also that some of the velocity produced at one orifice is available at the next.

The problem is so complex that any attempt to determine the coefficient or the regain of velocity head for each orifice separately would no doubt be beyond our present ability. It was therefore decided to neglect each as one tends to offset the other and then apply a blanket factor at the end to bring about agreement. It was surprising to note that even from the first, no great modification was needed and the results agreed pretty well as first calculated. This can mean only that neglect of one about cancels the other which is pure coincidence. It may be later on when further and more comprehensive data is available some factor may be used to advantage. If so it should be determined experimentally. Owing to lack of time the new theory[2] has been condensed to its essentials and some of the finer points have been omitted.

[2] The complete theory may be found in the Proceedings of the American Society of Civil Engineers, October, 1927, as well as in the author's closure Proc. A.S.C.E., January, 1929.

FSP-51-16

Coordination of Industrial-Plant and Central-Station Power

By P. J. GAUDY,[1] CHICAGO, ILL.

In this paper, the author discusses the generation of power for use of industrial plants. In general, he finds that central stations are able to deliver power as required by industrials, cheaper than the latter can produce it themselves. An industrial can generate electric energy for power purposes profitably only when the utility does not maintain proper service or charge a reasonable rate or when the industrial has energy available in some form, for conversion to power, which is a by-product, or natural result, of its major manufacturing process.

COORDINATION of industrial plant and central station power is, primarily, one of policy. It is unfortunate, in the past, that in many cases, policy has been a restricting element in a proper solution of this coordination, even when the economic situation is perfectly sound. The public-utility sales organization unquestionably demonstrates that it is earnest in its desire to cooperate with and serve the customers who supply the patronage which has allowed the utilities the prosperity which they have enjoyed. However, the sales effort and customer's cultivation have, in many cases, been handicapped by inability to surmount prejudice and rules which previous bad experiences have engendered.

The coordination of industrial-plant and central-station service primarily means the possibility of parallel operation or contribution by both the buyer and the central station in serving the buyer's block of load. The cases wherein the local industrial is so situated as to be a dominant or responsible factor in rendering service to the public utility are very few and are not to be omitted, although they represent so small a portion of the total relations as to make them negligible in this discussion.

If the generation of power by the industrial plant, for its own needs, is to be an economic move, it must necessarily be that the industrial has some facility whereby the combination of the capital cost of power equipment and the operating cost of generating energy is so substantially lower than the purchase price of energy as to make it an attractive venture from the profit-producing standpoint. This is, in general, a perfectly non-competitive proposition, as central stations, even at a profit, are able by their concentrated facilities and diversity to deliver the requisite power to industrials, cheaper than they can produce it in a similar manner themselves.

The exceptions are those cases where industrials can generate part or all of their power at a gross cost which is very much less than the purchase price for a corresponding portion of their power. An industrial can generate electric energy for power purposes profitably only when one of the following conditions is satisfied: The utility must be inadequate to maintain proper service or be operating under conditions which do not make possible a reasonable power rate, or the industrial must have energy in some form available for conversion to power which is a by-product or natural result of its major manufacturing processes.

There are singular instances where energy as fuel is available as a by-product to the operation wherein the failure to use this energy would be a total economic waste. For example, a steel mill having blast-furnace gas available, which is far in excess of the heat requirements as such, has a potential possibility as to power generation to the full extent of its own requirements, and, perhaps, in excess. A woodworking plant, where the creation of wood rubbish due to manufacture, delivers in combustible condition fuel as a direct waste, can with proper facilities generate power at a much less expense than the average purchase price.

Both of these instances may not represent a true absorption of the available fuel as the creation of wood rubbish or the production of gas must obviously concur with the power demands or facilities provided for storing, and later using these commodities, and in no case is the storage of a by-product fuel inexpensive.

There are also, and in much greater quantity, conditions where waste heat enters the picture to a dominant extent as referred to power generation. This waste heat may be either in the form of gases of sufficient temperature, to be economically diverted to the generation of power, or, as is more universally true, this heat may be in the form of steam at a higher pressure than that corresponding to its ultimate use, or be exhausted at a pressure higher than necessary if utilized directly for power.

One example of the foregoing conditions is a cement plant where the exhaust kiln gases will generate adequate power for the mechanical requirements. Here the matter of concurrence enters most forcibly, as the energy is represented only in the sensible heat of the products of combustion. The storage of this commodity is obviously impossible, and the motive-power load, if served, must be identical with the kiln loading.

Another example is a forge plant where, from the necessity of past practice, steam is exhausted at a pressure above atmosphere from its useful manufacturing equipment; this permits condensing the exhaust through a turbine and extracting a very substantial increment of power.

On the other end of the scale, where the economy and a portion of the usages justify high-pressure steam, there are many applications where the steam at the high pressure cannot be applied directly to manufacture, and in its reduction from the pressure of generation to the temperature of use, there is an energy increment which represents a very substantial power output.

In most instances, the ideal arrangement and one which can withstand any critical analysis is a coordination between the power purchased and the power generated. There are many difficulties inflicted upon the utility by any permission to an industrial to operate in parallel with the system. Most of these difficulties can be absolutely eliminated by a small and logical engineering outlay. However, the public utility has a responsibility in the continuity or service to other customers than the industrial in mind, and it should allow no laxness in the construction, or interconnecting arrangements, which will result in the imposition on the utility service of hazards or difficulties which were not existent before the parallel operation was permitted.

There is, of course, the feeling on the part of the industrial executive, if this great concession of parallel operation were extended, that his plant equipment either owned or to be installed under however improvident supervision would be perfectly good

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

apparatus when connected to the utility lines. This might be ridiculously untrue, and the author is not extending the hope that parallel operation will become seriously an element of the public-utility policy, and, parallel with that, expecting that any inadequately constructed or improperly applied equipment will be permitted for one second to encumber the public-utility service.

The possibilities of profitable generation by an industrial are not universal by any means, and unless the electric load exceeds 25 per cent load factor, and unless the demands involved are considerably in excess of 1000 kw., then the additional attention and skill, neglecting entirely the outlay, would almost disqualify such an industrial load as a possibility for parallel operation. If the steam load for all purposes other than power exceeds a demand of 100,000 lb. per hour, and the electric load is in excess of 5000 kw., then the elements of the problem will almost universally justify a thorough economic study.

There are analysts whose principal objective is to create a situation where the analysis will be productive of a construction problem. Nothing could be more destructive to progress and economy. It is necessary that the engineer who presumes to analyze a potential parallel-operating situation have the economic elements and the responsibility of the public utility to the beneficiaries thoroughly in mind, without which the true picture will not be reflected in the engineer's presentations to the owner.

The viewpoint of the owner, meaning the power purchaser, is susceptible to improvident recommendations, and the power which he purchases, as is often discussed, is a commodity which does not come in tonnage or visible quantity, and therefore he feels that, without true comparison, all his power bills are extraordinarily high. It is playing upon this angle that much improvident and destructive work is done in the development of industrial power-generating possibilities.

It is, to the author's belief, not the thought of the public-utility engineers to combat situations which are fundamentally and truly economical, where customer generation and even parallel operation are involved. Our experience shows that there are many utility organizations who are most willing to cooperate.

This paper is not intended as a plea for the discriminating engineer, except that it may open a field of record in which it must ultimately be agreed that both the industrial and the public utility must make concessions and look the economic factors in the face, before any intelligent interrelation as regards power supply can be established.

As a matter of interest, in a recent program which has accomplished all that could be expected in the line of fundamental economics, the author will describe a few features within the power rehabilitation program of the Packard motor plant at Detroit. This factory purchases its supply of power, and depends for adequacy and continuity entirely upon the serving utility. Economics, however, have been worked out both in the production of steam and in the utilization of steam which, although they are not dominant as referred to the power usage, are important as to the fundamental economy, which must not be overlooked.

Wood-mill rubbish and janitor refuse are burned in boilers of modern setting at a high evaporation, and consequently with a marked influence on the amount of coal required. High-pressure steam is generated for factory heating and passed through a turbine which operates against a vacuum in a surface condenser corresponding to the required water temperature for factory heating. The circulating water through the condenser is forced through the factory radiation, and in this way the heat is converted from the point of generation to the proper point of use, for every condition, utilizing the entire range of drop for the development of by-product power.

From an operating standpoint, an interesting application of evaporators is also involved wherein the steam used for forge-shop purposes is generated in an evaporator plant by the condensation of high-pressure steam through the evaporator and returning to the main power house, generating the steam from raw water, since its exhaust is to the atmosphere.

An isolated forge-plant boiler house is primarily one of 100 per cent makeup. While in this large factory where the total steam demand exceeds 300,000 lb. and where 100,000 lb. is used for forge-hammer purposes, the total boiler-plant makeup has been so worked out as to the accumulation of returns that the raw water is less than 2 per cent to the boiler-feed system. This installation is cited merely as exemplary of what can be done in the direction of power economy, in the average industrial.

The author wishes to visualize the electric and steam load over a year, taken from midsummer to midsummer. The industrial steam load, due to both processes, and heating will rise during the winter months. This rising peak in the industrial is also concurrent with other annual public-utility load conditions, exclusive of the connecting industrial. When this steam load rises, that portion, if the quantity is not negligibly small, which goes to building heating may be utilized as a means of power generation. With this power generation the industrial may be able entirely to cut off the rising peak represented by changing electric load in the wintertime. This not only improves the load factor so far as this customer is concerned, but relieves the utility of the necessity for preparation to meet both the rise on the part of this industrial and the natural increase in demand on all other increments of load.

This concurrence of elements influencing capital costs might develop to be very important from the standpoint of adequacy of utility service. In this direction, a rule of thumb may assist in visualizing what this by-product load from heating may be. Forty feet of building radiation will dissipate 10,000 B.t.u. per hour. With a bleeder turbine on the line, every additional 10,000 B.t.u. will develop 1 kw. Forty feet of radiation will handle slightly over 2000 cu. ft. of contained space. At an average factory-ceiling height, this 2000 cu. ft. of space will represent 130 sq. ft. or less of floor area.

The rule, therefore, for quantitative mental comparison is that for every block of floor area 10 ft. $\times$ 12 ft. or thereabouts, there is, in the conversion of heat from economical generation to economical use, the potential possibility of developing 1 kw. of load under maximum heating-season conditions, referring particularly to the suppression of the power required to be furnished by the utility at such times.

The value of power as a commodity is one factor which must be impressed thoroughly upon the industrial owner. We believe that a careful analysis will bring any owner to the admission that power, when necessary and in the quantity required, is a much different commodity than power dumped when available and in any quantity which may be excess.

We have worked on situations where the exchange of power between the public utility and the industrial is an element of the problem, and this situation is one which offers, particularly for the small utility, some distinct capital advantages. However, the occurrence of the period of available power as referred to the utility's needs is not apparent to the industrial without some study, but an analysis will show, and we believe that this thought should be introduced into every problem of coordination, that excess power turned into the utility's lines when, due to production conditions it happens to be available, is worth but a small fraction per unit of the power which is delivered by the utility when it is required and, more important, when it is demanded in utter disregard to the other beneficiaries, who may

be simultaneously in need of power at a rate which makes them more entertaining customers.

Industrial power cannot be generated under usual conditions with economy to the industrial unless:

1 The rates are inordinately high
2 The supply is inadequate or irresponsible
3 The by-products of manufacture contain an element of energy which is not usable
4 The manufacturing processes inherently contain energy uses which do not exhaust the power possibilities of the medium.

Fundamental economy demands more attention to interconnection. The author does not subscribe to wildcat engineering in this direction and also feels that the element of concurrence of responsibility and demand, together with the relative load factors, bears as great a weight on the justified interconnection as does the most attractive occurrence by-product fuel or energy.

There is one suggested solution for this problem of coordination that lies in the following direction:

Public utilities will not dispute the necessity of making proper use of waste fuel and by-product power where this can be produced in economic quantities. The author therefore suggests what could be termed an equilateral power-house proposition. The power plant at the industrial site could be built with the joint engineering work of the local public utility and the industrial's engineer. This plant would be tied into the utility system. The power would be taken in quantity sufficient for the industrial's needs from this plant; either generated therein, or furnished through it for the utility system. This plant would become a substantial support in capacity for the utility load.

Financing could be done jointly between the utility, the industrial, and public investment. The entire revenue problem could be based upon cost plus accounting, as has been successfully worked out in connection with power delivery.

In this way, full use would be made of the by-product possibilities. The construction and operation would have the advantage of the skill of the utility, and the engineer for the owner could act between the parties to see that the service to the industrial, and that the return to the generating company would be commensurate with the costs and requirements. Such a situation would enjoy best possible operation, and would attain that economy represented by the fullest consistent utilization of by-product power, and would support both the public utility and the industrial by a plant which is in every sense a joint benefit to the two, and would, in the last event, place the skill in the cheap production of steam from fuel for the industrial, in the hands of the operating staff of the utility, whose particular business it is in their other plants to produce like results.

There are several hundred situations in this country where hundreds of thousands of dollars are involved annually in the difference between the present setup and the ultimate economy, which could be worked out by such a coordination, and this equilateral power plant, so termed, is offered as a very substantial step in the satisfaction of the requirements of the industrial and the coordination of its motive—power problems with the capacity of the serving utility.

In analyzing such a situation, the author has found no problem which has not been separately worked out satisfactorily which, therefore, could be combined into such a serving unit.

FSP-51-17

Welding of Power-Plant Piping

By F. G. OUTCALT,[1] CHICAGO, ILL.

In this paper, the author discusses the possibilities offered for reducing costs of installation of power-plant piping, through welding. Erection problems are simplified when welded construction is used. Heavy fittings are eliminated and the piping is easier to handle and can be installed in less space. Many engineers are unwilling to use welded designs due to difficulty in telling when welds are not made properly. The author describes improved methods which insure that the work is of high quality and uniform character.

ECONOMIES through use of welding for power-plant piping begin in the drafting room, due to greater simplicity in designing and laying out systems for welded construction. Material costs are lowered through elimination of expensive cast or forged fittings. This is particularly important in the case of headers, elbows, bends, steam traps, anchorages, and supports, where all castings and flanges may be eliminated except at the valves. Through the use of welding, elbows and bends are made of material that is identical in strength and other physical properties with the rest of the system. Substitution of welded steel for large cast fittings results in decreased weight, thereby reducing the difficulty of supporting the line properly.

Erection problems are also simplified when welded construction is used. Due to the elimination of heavy fittings, the pipe is easier to handle and can be installed in less space. This means that the piping can be nested closer in pipe shafts or tunnels. The changes, that quite frequently occur during erection, can be taken care of quickly and easily. Even if the change may require a special fitting this can be welded from standard pipe right on the job, thus avoiding the delay that would result, if a special cast fitting had to be ordered to meet the emergency. Absence of flanges on a welded line reduces materially the cost of applying insulation. Welded construction eliminates leakage and thus results in higher operating efficiencies. To all intents and purposes there are no joints in a properly made welded line.

Where welding is done under procedure control, maintenance is eliminated. The weld is as good as the metal in the pipe itself, and consequently, the life of the line is limited only by the useful life of the pipe. In practice a welded-piping system usually becomes obsolete before any maintenance is necessary. Any changes that may be necessary in systems after they have been placed in service, can be made most readily on welded lines. When, for example, it becomes necessary to place an additional branch connection in the line, the necessary outlet can be cut quickly with the cutting blowpipe and the new connection welded in with minimum disturbance to the existing line and with minimum interruption of service.

It is evident that the full advantage of welded construction cannot be obtained unless it is specified by the engineers who design the system. Many engineers are already using this type of construction, but there are others who still hestitate to take full advantage of the welded joint in power-plant piping. They are willing to admit that all that has been said above is perfectly true of a properly made weld, but they want to know how they can make certain of obtaining high-quality welds throughout the entire system. Developments during recent years have reached the point where it is possible for the power-plant engineer to treat welded construction exactly as he does other types.

What are the steps taken by an engineer to secure high quality results in any type of installation? For convenience they may be grouped under three general heads: (1) Exact specification of materials and design; (2) Selection of competent organization to do the work; and (3) Provisions for inspection, to see that specifications have been carried out. The detailed information that the power-plant engineer requires in order to draw up specifications for welded construction is already available in large measure. Leading organizations in the welding industry have done a great deal of work along these lines and are prepared to cooperate with power-plant engineers in all phases of welded construction. In order to specify materials and design for welded piping, the designing engineer will require data on the strength of welded joints as well as on correct principles of design.

Strength of Welded Pipe Joints

Developments within the last three or four years make it possible to design and obtain welded joints that are consistently stronger than the metal joined. Two factors have had an important influence on this development. One is the introduction of high-strength welding rod, by means of which an average welder is able to obtain an immediate increase of about 10,000 to 15,000 lb. per sq. in. in the tensile strength of his welds. The other factor is the use of procedure control to insure uniform high quality. A procedure control is a detailed production specification covering the following six points:

a Check of welders' ability
b Selection and inspection of material
c Design and layout of welded joints
d Preparation for welding
e Welding technique
f Inspection and test.

Experience with procedure control in many fields where strength and reliability are important factors has shown conclusively that it does insure work of uniformly high quality. Indeed, there is less variation in welds made under procedure control than there is in the metal joined. Procedure control regulates the human element in welding by standardizing welding procedure, so that it is subject to specification and control exactly as are other methods of fabrication.

In considering strength of welded joints, power-plant engineers will be interested particularly in the recent tests conducted by the National Tube Company, to determine the strength of oxyacetylene welded joints in full-size pipe specimens. In these tests, welded specimens of 4-in., 8-in., 12-in., and 16-in. pipe were subjected to tensile tests in a 600-ton hydraulic machine capable of gripping the full-size specimens and pulling them to destruction. The tests also covered four different designs of weld. A complete report of the results will be found in a recent publication of the American Gas Association, "Report of Sub-Committee on Pipe Joints, Serial Report No. 1, Oxy-Acetylene Welding of Steel Pipe." From this investigation, the following conclusions were drawn:

1 A properly made oxyacetylene welded joint is as strong as the pipe, fully 100 per cent efficient.

2 The simple butt-welded joint is the most efficient and is recommended for use in welding pipe. This is in agreement with past experience as this design is practically universally used in pipe work.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

3 Very high strength and nearly equally satisfactory results were obtained with the normal socket and socket with groove joints. In joint design, however, other factors must be considered, as cost of preparation, stress distribution under strain, and ease of welding, which make these designs less satisfactory than the butt joint. The weakest joint was the socket with welded rivets. This design should not be used because of cost, and difficulties in welding, as well as lower joint strength.

4 Due regard must be given to penetration and to the shape of the joint, particularly the reinforcement of the weld. This is more important in the larger than in the smaller pipe sizes. Butt welds should be reinforced at least 25 per cent and should be built up so as to present a gradual increase in the reinforcement from the pipe wall to the center of the weld. Fillet welds, when used, should be built up well in excess of the wall thickness of the bell.

5 There is no detrimental heat effect or structural weakening of the pipe metal due to welding.

6 The uniformly high strength obtained in these numerous joint tests points out the dependability of properly made welds. With qualified welders under procedure-control methods, a pipe joint of maximum efficiency can be made consistently and uniformly by the oxyacetylene process.

Power-plant engineers will also be interested in the effect of temperature on strength of oxyacetylene welds. Elevated-temperature tests of welds have been conducted jointly by the San Francisco section, American Welding Society, and the Materials Testing Laboratory of Leland Stanford Junior University. Results were reported in the Journal of the American Welding Society for September, 1927, and September, 1928. The data indicate that strong welds at ordinary temperatures are also strong welds at the temperature range of these experiments, namely 900 to 1000 deg. fahr. That is, increase in temperature will have no more effect on a properly made oxyacetylene weld than it does on the base metal. Indeed, these tests indicate that weld metal made with high-test welding rod is superior to the base metal in this respect.

Design for Welded Power Piping

Consideration of fundamentals of design should begin logically with a discussion of the line joint since it is employed to far greater extent than all others. Although quite universally used in all other applications of pipe welding and proved from a strength and economy standpoint, both from years of service and comprehensive tests, the single-vee butt joint has not been accepted altogether for steam installations by power-plant engineers. In place of it, at times a mechanical joint provided with a lip for seal welding such as the Sargol joint, has been recommended by some engineers and others have employed the sleeve joint with fillet welds joining the sleeve to the pipe. The two reasons given by those who have employed either of these joints are that the single-vee butt joint cannot be relied upon for the strength required or that it is unsatisfactory because of the possibility of weld protrusions on the inside of the pipe which increase the friction and present a source of possible trouble, because sections of the weld protrusion may become broken off in operation and carry along the pipe system into the valves and other moving parts. With respect to the butt weld, we believe, sufficient evidence has been given in a previous section of this paper to prove that single-vee butt welds in all sizes of pipe can be made consistently which will have greater strength than the pipe itself. Through procedure control, furthermore, it can be assured definitely that such welds will be produced in any piping system.

The elimination of any protrusion of weld metal beyond the inside wall of the pipe can be controlled easily by an experienced operator. Where, furthermore, there is any question as to the skill of the operator in controlling this factor, a liner may be employed which will eliminate this possibility entirely. Recent developments by the National Tube Company have indicated, that exceedingly satisfactory results may be obtained by the use of such a liner with practically no additional cost.

It is unnecessary to go into any discussion of the merits or demerits of other types of welded joints for straight pipe line construction since the single-vee butt joint is the most economical and entirely satisfactory.

For a single-vee butt weld, the ends of the pipe should be beveled to 45 deg., with a shoulder, usually $^1/_{16}$ in., at the bottom to facilitate alignment and penetration. Specifications should give the space to be left between ends to aligning for welding, call for 100 per cent penetration to root of weld, and state the reinforcement required. Using high-test welding rod, the average reinforcement is 20 per cent, with a stated minimum of $12^1/_2$ per cent. For design purposes, the ultimate tensile strength of a butt weld made with high-test rod is taken as the strength of the pipe material up to 52,000 lb. per sq. in. This is used with the same factor of safety that is applied to the pipe itself. It is essential that the weld taper off gradually toward the toe.

For outlets and take-offs from the straight-line piping system, there are three designs which may be employed for varying conditions, namely, the straight-type nozzle, the flared-type nozzle, and the saddle-type nozzle. The straight-type nozzle is made by cutting an opening in the line pipe, shaping the end of the nozzle to fit it and welding the two together. This design is recommended for pressures which produce a line stress of less than 5000 lb. per sq. in. With this type, the weld is made at the point of maximum stress and, furthermore, a right-angle joint results which may not be entirely satisfactory from a standpoint of efficiency of flow.

The flared-type nozzle is made by pressing or forging out the pipe by means of dies so that a butt weld may be made to the nozzle. This design gives a rounded contour at the take-off, and furthermore removes the weld from the point of maximum stress. It is, however, more expensive than the straight-type nozzle and may not be practical to produce in all sizes of pipe.

The saddle-type nozzle employs a forging specially designed to reduce to normal the stresses which are ordinarily concentrated about any opening. This forging is butt welded into an opening in the header. Where the diameter of the nozzle permits, the welder may also reach in and weld from the inside. Strain-gage measurements made on welded headers using this design have shown that the stresses are normal throughout the nozzle.

The straight-type nozzle is used almost universally for low-pressure take-offs, but the latter two designs are receiving considerable attention especially for high-pressure header construction. The most satisfactory flange design for welding is one in which the hub is forged or cast integral with the flange and of sufficient length so that it may be butt welded to the pipe nozzle or pipe end for connection to flange valves. In response to a general demand for such flanges prepared for welding, one of the largest suppliers of pipe fittings has produced dies for forming forged-steel flange welding necks in all sizes of pipe to 16 in., and is prepared to supply them in any quantities desired.

Welded reducers are used in header construction and in line work. They are made by cutting vee-shaped pieces from the end of a piece of pipe, using suitable templets, so that the sections remaining will form the desired reducer after they have been heated, swedged into contact and then welded. For low-pressure service, bends may be made up from two or more pieces of pipe miter cut at the proper angle so that when welded together they will form the desired bend. The two-piece 90-deg. elbow made by welding together two lengths, each miter cut to 45 deg. is

the simplest example, but this design is undesirable in many cases due to the abrupt turn. Usually a 3-piece turn is to be preferred.

For high-pressure service, tube turns are recommended. These are designed especially for welded construction and are available in all standard sizes. They are shaped mechanically from standard pipe in such a way that the wall thickness remains uniform throughout. They have the same strength as the adjoining pipe. The end of a line may be closed by means of a welded bull plug. The orange-peel type of plug is made very much like a reducer, except that the sections when swedged down and welded, close the end of the pipe completely.

Welded construction offers special advantages in the design of anchors. An anchor sleeve carrying welded brackets, may be welded to the pipe, or preferably be shrunk on by taking advantage of the contraction during welding. The anchor sleeve is split and fitted tightly over the pipe. When the longitudinal weld is made in the sleeve, the contraction of this weld shrinks the sleeve tightly on the pipe.

Insuring Correct Fabrication and Installation

After completing the welded design, the engineer's next problem is to make sure that his designs are followed correctly during fabrication and installation. As with other types of construction this may be done by selecting competent organizations to do the work. There are a number of pipe-fabricating concerns which are qualified to fabricate and install welded high-pressure steam piping systems in strict accordance with the principles of procedure control.

Engineers may insure high-grade workmanship by specifying qualification tests for the welders who are to do the work. As an illustration, the following qualification test is taken from a specification prepared for welded construction of a 400-lb. pressure super-heated steam-piping system, maximum temperature 750 deg. fahr.

General.

1 Only competent gas welders, having experience in welding steel piping of the sizes involved herein, shall be employed for the welding of the piping covered by these specifications.

2 Any welder employed for this work shall be required to pass the following qualification tests, before starting work on any of the piping to be welded under this specification.

3 The contractor shall at all times, during the course of construction, maintain on file for the inspection of the engineer or his authorized representative, a record of the welders employed for this work, together with a copy of the report of the results of each welder's qualification test as required herein.

4 The qualifications of welders shall be arranged for by the contractor within —— days, after the formal award of contract and shall be done in the presence of engineer or his authorized representative. This may be done in the contractor's shop or on the premises of the work. The contractor shall notify the engineer —— days in advance of the date set for the qualification test.

5 The material for these tests shall conform to the material specifications herein for piping and welding rod.

Leak-Proof Test. This test requires the welding of the joints in one pipe bomb to be made up of 6 or 8 in. extra-heavy pipe containing two circumferential position butt welds approximately 8 in. apart, one of the joints to be welded with the bomb lying horizontally, the other to be welded with the bomb vertical. The bomb shall not be rotated during welding. The preparation of the joint shall coincide with the welded joint detail as shown on engineer's drawing ——. Weld reinforcement to be $^1/_8$ in. gradually tapered where it approaches the base metal. Penetration shall be to the inside wall of the pipe. The ends of the bomb are to be closed in a suitable manner to withstand the test pressure to which it shall be subjected. At completion of welding and after the bomb has cooled to room temperature, the bomb shall be subjected to an hydrostatic test of 600 lb. per sq. in. or $1^1/_2$ times working pressure, after all air has been expelled therefrom. While under this pressure, the welds shall be struck with a 4-lb. sledge at four equi-distant points, the blows being struck with a full-arm swing. Under these conditions the welds shall not leak or develop any visible imperfections.

Tensile Test. This test requires the position welding of a joint between two pieces of 8-in. extra-heavy pipe, 9 in. long, to form a single piece of 8 in. diam., 18 in. long. The joint edge, reinforcement and penetration shall be specified similarly to that for the leak-proof specification above. The pipe shall not be rotated during the welding operation. Upon completion of welding and the cooling of the pieces to room temperature, standard tensile-test coupons shall be taken therefrom, one from the bottom of the joint, one from the top of the joint and one from each side of the joint, four in all, and be tested by a recognized testing laboratory to rupture, at a testing speed not to exceed $^3/_8$ in. per minute within the yield point. All weld reinforcement shall be removed by machining or grinding. Under this test, the coupons shall break outside of the weld or otherwise develop an average tensile strength of not less than 52,000 lb. per sq. in. with a minimum tensile strength of not under 50,000 lb. per sq. in. The cross-section surfaces of the coupons at the weld shall show complete fusion and penetration to the bottom of the vee.

Ductility Test. When this test is required, two additional coupons 2 in. wide × 18 in. long are taken from the tensile test specimens between the quarter-points. This test requires that the coupons be clamped in a vise, with the edge of the weld in line with the top of the vise jaws, in which position the exposed section of the coupon shall be rotated in either direction gradually until the section is bent through an angle of not less than 90 deg. Under this test, the weld shall not develop any visible surface cracks or fractures.

Inspection and Test

For inspection of welding, if the organization does not have a qualified inspection staff, there are several commercial testing laboratories now fitted to render the service. Furthermore, several of the larger laboratories of companies interested in the promotion of welding will gladly aid in training employees of purchasers of equipment so as to obtain satisfactory shop inspectors.

In distinct contrast to other types of construction, welded construction can be tested up to the yield point of the material used. It is customary to specify that headers and specials fabricated in the shop be tested hydrostatically at three times the working pressure.

Installations may be tested under high pressure if desired by blanking off sections between valves. Where welding is done under procedure control, the line is frequently not tested above the safety blow-off pressure.

From the foregoing discussion, it is evident that through the application of procedure control, power-plant engineers can now specify welded construction with complete assurance of obtaining an installation that will reflect credit on their organizations.

FSP-51-18

High-Pressure Turbine Installation of Kansas City Power and Light Company

By EDWIN JOWETT,[1] KANSAS CITY, MO.

This paper describes the high-pressure turbine installation at the Northeast station of the Kansas City Power and Light Company. An operating pressure of 1200 lb. at the turbine throttle and a total temperature of 725 deg. are used. It is believed that the heat consumption of the station will be lowered 1830 B.t.u. by the operation of the high-pressure equipment. With an annual output of 530,201,400 kw-hr. and with fuel at 67,000 B.t.u. for one cent, the saving in fuel will result in an annual reduction in cost of $144,700. This is nearly three times the fixed charges on the difference in cost of the two installations.

FROM its two generating stations in Kansas City, the Kansas City Power and Light Company, furnishes electrical energy to Kansas City proper and 102 outlying communities in Missouri and Kansas. Its development has, of necessity, been very rapid in order to keep ahead of this fast growing industrial center. It has been the demand for cheap dependable power for industries that has been the stimulus, in working for higher economies in power production.

The Northeast Station which has for nearly eight years carried the entire system load was designed originally by Sargent & Lundy of Chicago, and the first 23,000-kw. unit was put into service in the fall of 1919. Since that time it has had a continual expansion, under the direction of the company's own engineers, until at the present time it contains 140,000-kw. generating capacity. While the initial steam pressure of 300 lb. has been maintained throughout all additions, many other changes and improvements have been made, both in new and old equipment, which along with improving load factor, has reduced the B.t.u. per kw-hr. of the station from about 21,000 to 16,700. By the addition of radiant-heat booster superheaters, the operating steam temperature has been raised from 600 deg. to about 650 deg. fahr. Original boilers were fired with natural-draft-type chain grates, later boilers with forced-draft chain grates, and within the last year one boiler has been converted to unit-type pulverized-coal firing.

It was quite evident that with the existing steam pressure and temperature, practically no further substantial improvements could be made in the economy of the station. The plant contains three 23,000-kw. turbines and two 30,000-kw. units served by twelve 1350-hp. boilers. With a limitation of approximately 250 per cent of boiler rating as the economical load with the fuel used, the present plant was lacking in boiler capacity. The existing building contained space for four additional boilers, at least two of which would have been required for the 1928–1929 peak. Accordingly, following up a line of study made by the company's engineers in 1922–1923, a decision was reached to install two high-pressure boilers and a turbine which would exhaust through reheaters to the station 300-lb. header. In this manner, the desired additional boiler capacity would be obtained along with some additional turbine capacity. When the original studies were made, it became evident that the installation of the equipment would have to be delayed until the minimum load on the system had grown to a sufficient amount to permit operation of the high-pressure equipment, and its compliment of low-pressure turbines at a very high daily load factor. This point was reached in the fall of 1928, shortly after which, the high-pressure equipment was ready for operation.

An operating pressure of 1200 lb. at the turbine throttle and a total temperature of 725 deg. was chosen for the turbine. The pressure was but little above that which had already been tried at Boston and Milwaukee, and the temperature such that the expansion to the present station header pressure through the turbine would give steam of a temperature but slightly above the saturation point. The size of the turbine was chosen such that its output, together with that of the present turbines using its exhaust steam, would practically meet the minimum-load requirements, and at the same time could normally be supplied with steam from two boilers of a size that could be fitted into existing vacant boiler bays. While two boilers will normally be in service to carry full load on the turbine, the furnaces and fuel-burning equipment is proportioned so that 3000-kw. load can be carried on one boiler.

The investment costs for this particular installation were considered more as the difference between the installation cost of two boilers of 300-lb. pressure and equal capacity, and two high-pressure boilers with reheaters. The turbine gave additional plant capacity at practically the same cost per kw. as a 300-lb. turbine, condenser and its auxiliaries. Furthermore, it was possible to locate this high-pressure turbine in the boiler room adjacent to the high-pressure boilers, where an addition of a like capacity of 300-lb. turbine would have necessitated an expensive addition to the turbine room. The proximity of the high-pressure turbine to its boilers and reheaters helped materially to reduce the additional cost of high-pressure piping over 300-lb. piping. It also served to hold down the pressure losses through the steam connections. The design of the turbine and generator was such, that no trouble is anticipated from dust of the boiler room. Fig. 1 shows the general arrangement of the equipment in the station.

From a close analysis of the probable system load and the proposed equipment, it was concluded that the following operating schedule could be followed in 1929:

Expected kw-hr. generated on 1200-lb. unit (82.5 per cent L.F.) (Based on expected turbine and boiler outage and load conditions)	=	72,200,000
Expected kw-hr. generated on present units with exhaust steam from hp. unit	=	245,600,000
Expected kw-hr. generated on balance of present units	=	212,401,400
		530,201,400
On the above basis, the consumption would be:		
Average B.t.u. per kw-hr. on 1200–300-lb. combination (Assuming full load on hp. unit).	=	13,810
Average B.t.u. per kw-hr. on balance of 300-lb. units (Assuming the same as for 1928 performance)	=	16,800
Composite average B.t.u. (net)	=	14,970

A careful analysis of the relative costs of a 1200-lb. installation

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

and a 300-lb. installation at Northeast station afforded a means of determining whether the saving in fuel would justify the investment. In order to put the two installations on a strictly comparable basis, both installations were figured on the most economical size of equipment for this station and reduced to unit costs.. The high-pressure job furnished 10,000 kw. electrical energy and steam for a total of 45,000 kw. The boilers for the 300-lb. unit would be capable of furnishing steam for 30,000 kw., and the turbine would be a 30,000-kw. unit. From the unit costs so figured, the comparison was worked out for 10,000 kw. electrical output and sufficient boiler capacity for 45,000 kw.

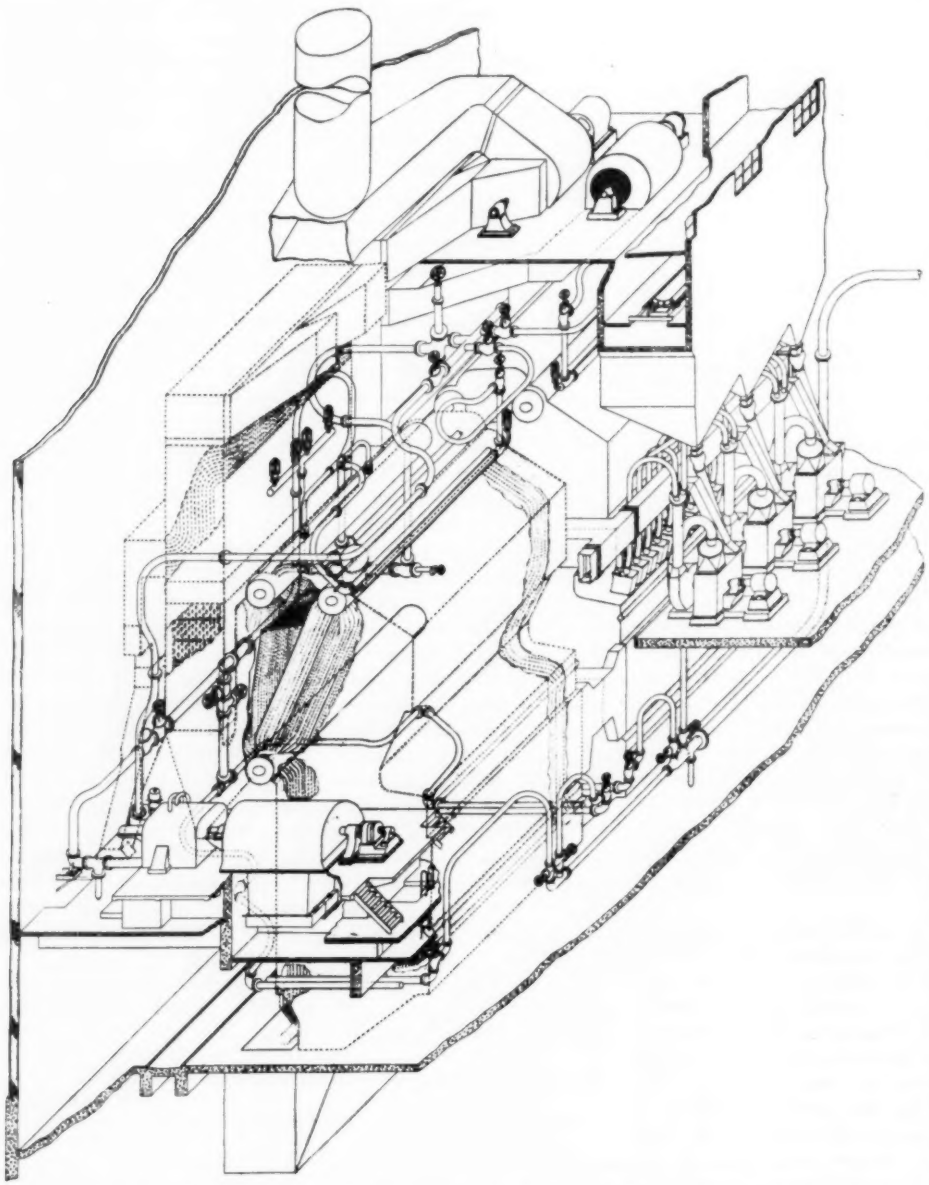


FIG. 1 BOILER AND TURBINE INSTALLATION, NORTHEAST STATION; KANSAS CITY POWER & LIGHT COMPANY

A summary of the costs for the 10,000-kw. turbine installation is as follows:

1400-lb. boilers (steam for a total of 45,000 kw.)	$1,342,138.00
1200-lb. turbine	307,246.00
	$1,649,384.00
300-lb. boilers (steam for 45,000 kw. total)	918,000.00
300-lb. turbine	335,000.00
	$1,253,000.00

From the above, the high-pressure installation costs $396,384 more than the 300-lb. installation of like capacity. This is an increase of 31.6 per cent. It must be borne in mind that these figures apply only to the particular job in question. If fixed charges at 13.25 per cent are figured on the difference between these two investments, the annual saving from the operation of the high-pressure equipment must at least equal 0.1325 × $396,384 or $52,500.

As previously shown, it is estimated that the heat consumption of the station will be lowered from 16,800 B.t.u. per kw-hr. to 14,970 B.t.u. or 1830 B.t.u. by the operation of the high-pressure equipment. For the annual output of 530,201,400 kw-hr. and with fuel at 67,000 B.t.u. for one cent, this saving in fuel will result in an annual reduction in cost of $144,700. This is nearly three times the fixed charges on the difference in cost of the two installations.

Whether or not the new equipment will involve higher maintenance is problematical and can only be determined by time. The practically completely water-walled furnaces of the two high-pressure boilers should result in lower furnace maintenance than the old masonary-lined furnaces.

A detailed description of the various parts of the equipment is given below:

GENERAL OPERATING CONDITIONS

Maximum pressure for which boilers are designed	1400-lb. gage
Normal operating drum pressure	1280-lb. gage
Pressure at turbine throttle	1200-lb. gage
Temperature of steam at turbine throttle	725 deg. fahr.
Back pressure at turbine exhaust at full load	315-lb. gage
Superheat in exhaust at full load	40 deg. fahr.
Pressure steam out of reheater	290-lb. gage
Temperature steam out of reheater	700–725 deg. fahr.

Many studies and investigations were made on the two general types of high-pressure boilers now in service in this country. While it appeared that both types were working out quite well, several points seemed to favor the bent-tube boiler in the minds of the company's engineers. It was felt that better circulation obtained in the steeply-inclined tubes and the number of high-

pressure joints were reduced to a minimum. It was also believed that the divided feed in the lower drum would aid circulation. As explained later, this feature had to be abandoned. Where head room was limited, a higher furnace could be obtained with the bent-tube boiler. Numerous arrangements of boiler, economizer, air heater, and furnace were made and in practically all respects the bent-tube boiler fitted best into the allowable space. Investigation showed remarkable success in maintaining the drum water level without priming with this type of boiler.

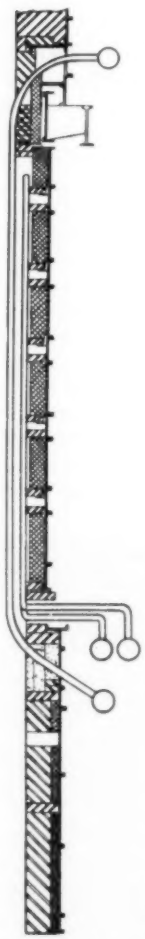


FIG. 2 SIDE WALL CONSTRUCTION

The largest boiler that could be installed in the existing bays contained about 17,000 sq. ft. Two boilers of this surface would be sufficient to carry full load on the turbine at an economical boiler rating. However, even for boilers of this size, many changes had to be made in the existing steel framework of the building to increase clearances, support new floors and to carry the greatly increased weight of equipment. Two hundred and fifty tons of additional structural steel were required to carry the boilers and turbine, and some new concrete footings had to be installed.

It was the original intention to fire the boilers with forced-draft chain grates. It was, however, found next to impossible to get sufficient grate area under the boilers to carry maximum load with the low grades of bituminous coals used at the station. Stokers of greater width than had yet been built would have been required, and manufacturers were very reluctant about building them.

An experimental installation of two unit pulverizers firing a 1350-hp. boiler had been in operation in the station about six months at that time. To date it had proved very satisfactory both from the dependability and efficiency standpoint. Largely on the performance of this installation, it was decided to follow the same general scheme for the two high-pressure boilers. The plant is situated away from the business and residence district, so that the problem of removal of fly ash did not influence the decision. It was possible to install a water-cooled furnace of sufficient size to keep the combustion rate down to 20,000 B.t.u. per cu. ft. of furnace at maximum rating and about 17,000 B.t.u. per cu. ft. at normal rating. This is based on a furnace volume of 15,670 cu. ft.

The furnace below the burner line is made up of fin-tube water walls on the front and back, and bare tubes on $7^1/_2$ in. centers on the two sidewalls; all tubes being 3 in. outside diameter by $^1/_{16}$ in. thick. Immediately back of the side-wall water tubes are the reheater elements made up of single loops of $1^3/_4$ in. outside diameter tubes about 17 ft. 6 in. long. Fig. 2 shows the construction of these side walls. Secondary air ports are provided in the front wall with independent control in horizontal rows. A typical form of water screen is provided at the bottom of the furnace with tubes arranged on 12-in. centers.

Nine 8-in. forced-blast Lopulco burners fire downward at a slight angle with the vertical through a tile-backed, water-cooled arch. These water-cooling tubes bend upward and form the front wall and also the top arch. These backing tiles are hung on independent cast-iron hangers and do not rest on the tubes. The upper side walls are made of firebrick and insulating material. The entire furnace, ash hopper and boiler is steel encased.

A very effective means of providing these water walls with water has been employed. A large header at the lowest part of the furnace at the rear is supplied with water from the lower drum by fifty-two 3-in. downcomer tubes outside of the furnace. This header is a forging 18-in. outside diameter by 3 in. thick and 36 ft. 8 in. long. The rear water-wall fin tubes rise directly from this header to the lower drum passing through connecting tubes to one row of the first-bank boiler tubes, thence to the front steam-liberating drum. The water-screen tubes come out of the back of this header, pass between the rear water-wall tubes and

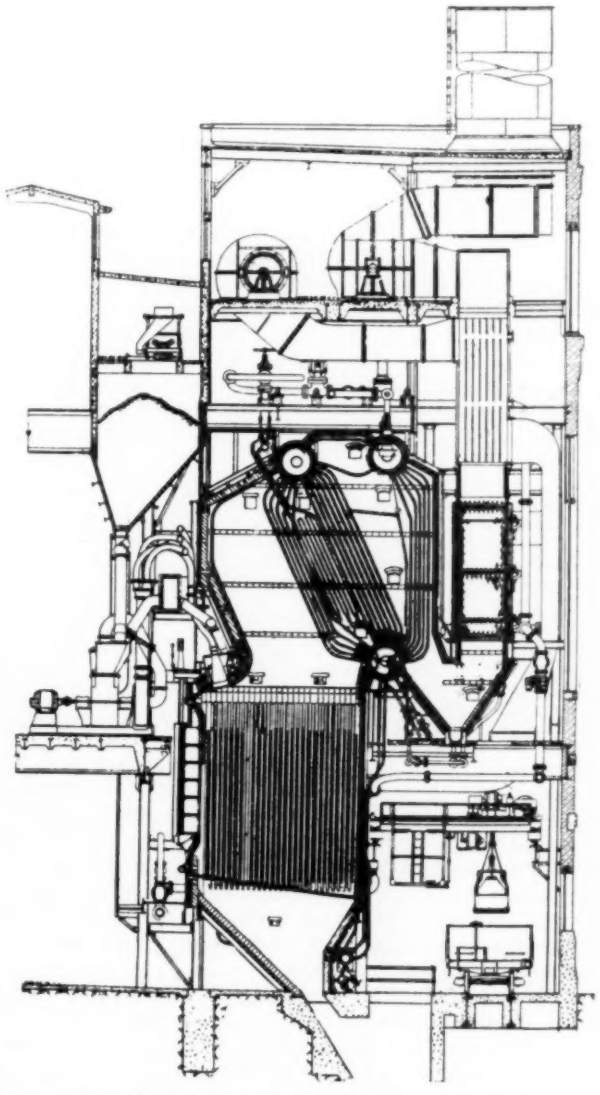


FIG. 3 CROSS SECTION OF PLANT SHOWING SLOPING-FURNACE BOTTOM FOR CONVENIENT ASH HANDLING

across the furnace to the lower front header, thence the circulation is through the front-wall fin tubes to an intermediate header. From this header, the arch and upper front-wall tubes carry the water on up to the front drum. Eight short tubes from each extended end of the lower rear header supply water to the lower side-wall headers. The water rises through the side-wall tubes to the upper side-wall headers, thence through twelve external-riser tubes on each side of the boiler to an extension of the front drum outside the furnace casing.

An ash-handling arrangement similar to that employed in the

other part of the plant will be used on these two boilers. A sloping furnace bottom as shown in the cross-section drawing, Fig. 3, empties into a more or less shielded water-sealed pit. From this pit the ash is removed by a clam-shell bucket and bridge crane also shown in the cross-sectional view.

The two high-pressure boilers are of the three-drum Stirling type, so baffled that the gas makes three passes in traversing the boiler. A convection-type primary superheater is located between the first and second passes. The drums are seamless steel forgings 40 in. inside diameter and $5^1/_4$ in. thick. The lengths of the three drums vary from 29 ft. 8 in. to 34 ft. 2 in. The heaviest drum weighs about 77,000 lb. All tubes in the boilers are 3 in. outside diameter by $^5/_{16}$ in. thick. Fig. 4 shows the manner in which the tube holes in the drum are counterbored and the tubes rolled therein. The two small grooves shown are about $^1/_{32}$ in. deep. The large counterboring is to facilitate tube renewals. These boilers do not employ the standard Ladd method of feed. Contrary to what was first planned, the water is fed direct to a feedbox in the upper rear drum. Steam is taken off of the rear drum through two 6 in. nozzles studded and step jointed to the drum. Each outlet is provided with a steam purifier drained externally. Water columns are of forged steel with flat glasses mica protected. General boiler data follows:



FIG. 4 ROLLED-TUBE JOINT

Normal boiler evaporation per hour, lb.	170,000
Maximum boiler evaporation per hour, lb.	200,000
Heating surface in boiler, sq. ft.	16,950
Wall surface: water screen (plain tubes), sq. ft.	330
Lower front finned tubes, sq. ft.	415
Upper front plain tubes, sq. ft.	670
Rear finned tubes, sq. ft.	765
Two side walls, plain tubes, sq. ft.	870
Total wall surface, sq. ft.	3,050
Reheater surface each boilers, sq. ft.	470
Primary superheater surface each boiler, sq. ft.	2,650
Safety valves each boiler	Four 4-in., one 2-in.

Each boiler is provided with three impact-type mills each with an hourly capacity of 15,000 lb. of bituminous coal containing 15 per cent moisture. The primary air entering the mills will be heated to any desired amount up to 400 deg. fahr. The mills are direct connected to 200-hp. motors. As previously mentioned, each boiler is equipped with nine forced-draft Lopulco burners. Heated secondary air is admitted at the burner, and tertiary-heated air through the front furnace wall below the burners.

Economizers. Each boiler is provided with a fin-tube economizer having 10,891 sq. ft. of effective heating surface. The tubes, which are $2^1/_2$ in. outside diameter No. 2 gage, are arranged in staggard rows. The inlet ends of the upper row of tubes as well as the outlet ends of the lower row are rolled into headers. The water passes through each vertical row of tubes from top to bottom, adjacent tube ends being rolled into forged-steel junction boxes provided with inside oval handhole plates. The economizer is designed for a working pressure of 1490 lb. per sq. in. The junction boxes are a unique type of forging which under test of 4750 lb. per sq. in. showed no serious deflection.

Water will be fed to the economizer at a temperature of 210 deg. fahr. and will leave at a temperature varying from 319 deg. to 324 deg. depending on the boiler rating.

Air Heaters. The gas after leaving the economizer on each unit, passes up through a plate-type air heater of 39,270 sq. ft. heating surface. The air heater is typical in every way of this type of equipment. The final air temperature of about 425 deg. is obtained, while the leaving gas temperature is about 330°.

Draft Fans. The induced-draft fan on each boiler is designed for a maximum draft of 10 in. water and is driven by a 450-hp. motor. The forced-draft fans are designed for $7^1/_2$ in. and are driven by 225-hp. motors. One short steel stack supported on the building steel serves both boilers.

Boiler-Feed Pumps. Two motor-driven boiler feed pumps each of sufficient capacity to handle both boilers have been provided. Each pump is of 800 gal. per min. capacity against a total discharge head of 1500 lb. The pumps are provided with water at their suction from the present station boiler-feed pumps at 350 lb. pressure. The pumps operate at 3550 r.p.m., and are direct connected to 900-hp. induction motors. The pump discharge pressure is controlled ahead of the economizer, by a differential pressure regulator which maintains a constant differential between the drum pressure and the water entering the economizer. Its chief purpose is to protect the economizer against the high shut-off pressure of the feed pumps, and incidentally to improve operation of the automatic feedwater regulators.

Turbo-Generator. The turbo-generator is of the combined impulse reaction type. The generator is rated at 10,000 kw. at 80 per cent power factor. Fig. 5 shows a general view of the unit. The turbine cylinder is machined from steel forgings as is also

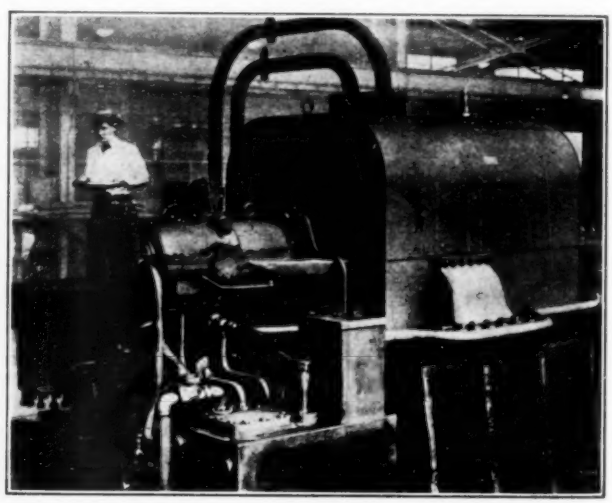


FIG. 5 TURBO-GENERATOR IN NORTHEAST STATION

the steam chest which sits along side the unit. The throttle valve and single governing valve is located in this chest. The steam chest is connected by four flexible pipes to the turbine cylinder. The cylinder is supported in a novel manner from the exhaust casting suspended by steam-heated steel links from stationary brackets attached to the foundation. Fig. 6 shows the essential elements of this support. This type of support allows free cylinder expansion and at the same time maintains a fixed center line regardless of what pipe strain might be imposed upon the exhaust casting by the exhaust pipe.

The impulse element of the turbine consists of two rows of revolving blading and one row of stationary blading made of stainless steel. This is followed by 14 rows of reaction blading, made of pure nickel. All blades are shrouded and of the end-tightening type. A stainless-steel built-up nozzle block expands the 1200-lb. steam down to 845 lb. before it enters the first impulse blades. The dummies are of a special design so that should any rubbing occur, either in the dummy packing or in the blading.

the area of the contact is exceedingly small and the intensity of rub very light.

The high-pressure gland has the same arrangement of end tightening strips as the high-pressure dummy. The gland at the exhaust end is fitted with radial contact strips because of their remoteness from the thrust bearing. A Kingsbury thrust bearing with ready handwheel adjustment of the axial position of the rotor is provided. Stops for limiting the movement of the thrust are provided so that starting and running positions are found readily.

When operating under the conditions hereinbefore described, the turbine will have a water rate varying from about 34 lb. per kw-hr. at full load to 45 lb. per kw-hr. half load. The superheat in the exhaust at the above loads varies from 22 deg. to about 60 deg., respectively.

Piping Connections. Fig. 7 is an isometric showing the piping arrangement. It will be noted how the proximity of the boilers and high-pressure turbine has permitted exceedingly short pipe connections. The scheme of the connections is more or less typical of installations of this sort except that some additional complications arose, due to the fact that it was a two-boiler installation, whereas all high-pressure jobs in operation at the time were single-boiler installations.

The problem of the correct distribution of steam from the ex-

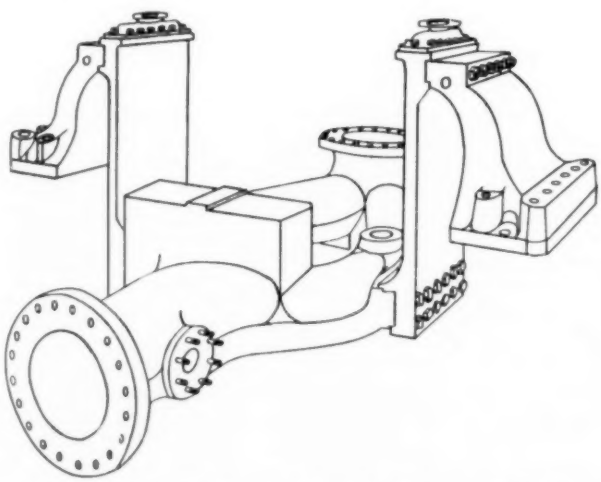


FIG. 6 CYLINDER SUPPORT FORMED BY EXHAUST CASTING

haust of the high-pressure turbine, to the four sections of reheaters (two in each boiler) presented a problem. The two reheaters in each boiler were piped together to a single connection with duplicate lengths of pipes, bends and fittings to give equal pressure drop from the single-branch connection to the reheater in each side of each boiler. The pair of reheaters in each boiler is then supplied with steam from a reheater supply header. An electrically operated regulating valve is placed in the connection between this header and the single connection to each pair of reheaters. These valves are arranged so that even in the closed position they will pass sufficient steam to protect the tubes of the reheater from burning. The control for these two valves is brought to the boiler-control board beside a recording thermometer indicating the steam temperature out of each reheater. The reheater regulating valves will be so regulated as to maintain equal reheater outlet temperatures. These two reheater regulating valves are synchronized mechanically, so that as one valve opens the other closes a proportionate amount. If manual control of this steam distribution becomes too much of a problem, equipment is arranged so that it can be made automatic, actuated either by the volume of steam out of the primary superheater of each boiler or by the temperature of the steam out of the reheater of each boiler.

A connection is provided to bypass automatically the high-pressure steam through a reducing valve and desuperheater, in the event of complete loss of load on the high-pressure turbine. This bypass can also be put into service should it be desired, to use the high-pressure boilers on the present 300-lb. system with-

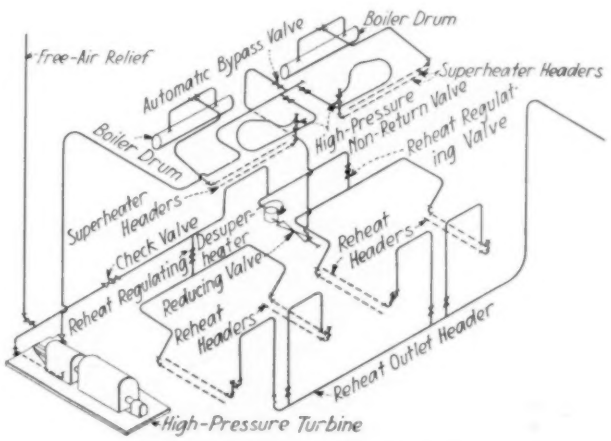


FIG. 7 DIAGRAM OF STEAM PIPING

out the use of the high-pressure turbine. The reducing valve is a motor-operated variable-orifice arrangement, actuated by an automatic control. The desuperheater is of the absorption type with automatic temperature control.

All high-pressure steam joints, 6 in. and above, are Sargol type except those connecting immediately to the boilers, superheaters and turbine. These particular joints as well as all steam joints between 6 in. and $1^1/_2$ in. are small tongue and groove. Steam joints below $1^1/_2$ in. are large male and female. Only lap joints are used, all tongue-and-groove or male-and-female facings being made in the lap of the pipe. All joints in the boiler feed pipes are small tongue and groove. In most cases, Norway iron gaskets are used in the tongue-and-groove and male-and-female joints.

Feedwater. It was realized that probably the most important factor in the successful operation of high-pressure boilers is pure feedwater. In the present part of the plant, no evaporators or feedwater treatment was used; complete deaeration of the feedwater has, however, been affected for several years and no trouble from oxygen pitting has been experienced. The undesirability of using untreated make-up in the high-pressure boilers even in as small a percentage as 2 to 3 per cent was realized. The general arrangement of the station piping and heat balance was such that an installation of evaporators would have been difficult and expensive. An arrangement has been worked out for feeding the high-pressure boilers entirely on condensate. A hot-process lime and soda ash treating plant has been installed for treating the make-up for the low-pressure boilers. It is possible that an after treatment, with trisodium phosphate may be added. Special precautions have been taken to reduce condenser leakage to an absolute minimum and condenser-leakage recorders have been installed on all condensers.

Boiler Control. Centralized manual control of the boilers is provided. It is arranged so that such parts of it as may prove desirable, after some experience with the operation of the equipment has been obtained, can be made automatic.

FSP-51-19

Power-Plant Maintenance and Engineering Insurance

By J. P. MORRISON,[1] CHICAGO, ILL.

EARLY boiler-insurance policies written in America, as do the policies written today, furnished a specified indemnity for loss resulting from explosion, rupture or collapse of the boiler, and further defined the explosion or rupture, or collapse as a sudden, substantial tearing asunder of the boiler caused by the pressure of steam, or as the sudden crushing inward of its furnace or flue. The expressions used are intended to define the occurrence insured against, as an accident of a disastrous or violent character. The boiler-insurance policy is not intended to furnish indemnity for the cost or expense of maintaining a boiler in serviceable condition, nor to guarantee it for any period of usefulness under normal operating conditions; nor does it include any damage from fire resulting from any cause whatsoever.

Of course, the steam-boiler policy does not provide indemnity for an accident which would have occurred if there had not been the timely discovery of the unsafe condition; nor to the loss which proceeds the occurrence of the accident insured against. Practically the same definitions and provisions apply to the unfired pressure vessel.

Our first policy was a valued form, providing specific indemnity in case of damage to each of the scheduled boilers and a specific indemnity for damage to the surrounding property. However, the policy as now written is a blanket form, in which the principal sum is applied to the payment of loss due to damage to any property of the assured, or to the property of others for which he may be liable.

The continued development of the steam prime mover resulted in many designs of a purely experimental nature which effected not only the dependability of the engine, but also the safety of the flywheel which all recognize as a magazine of stored energy. The numerous failures of engine flywheels, some of which were accompanied by loss of life and great property damage, created a market for protection against the loss which followed the accidental liberation of the energy stored up in the rapidly revolving mass. As there was no question but the explosion of a flywheel could rightly be termed a catastrophe, the accident insured against, in the flywheel policy, was termed an explosion and was defined as the sudden and substantial bursting and disrupting into two or more parts while revolving, whereby one of those parts was separated completely and dislodged from the other part of the wheel. The definition leaves the distinct impression that the accident insured against was to be a violent one.

It has been the underwriting practice to specify the limits of speed of the wheel, stated in terms of revolutions per minute, but really based upon the rim speed and the type of joint securing the parts of the rim together, just as we approve a maximum working pressure in the boiler policy. The presence of a flywheel, of course, implied the existence of an engine which likewise is subject to difficulties of an accidental nature as distinguished from those repairs due to wear and use. The framers of the engine policy broadened the protection afforded the owner of the flywheel, so instead of limiting the loss to those accidents having the characteristics of a catastrophe, the accidental breaking of any part of sufficient importance to require immediate repairs was defined as an accident and the loss was covered up to the face of the policy.

The engine insurance policy, instead of being referred to as explosion insurance, is known in insurance circles as "breakdown" insurance, for a modern engine may be damaged, and in fact its value destroyed entirely by the breaking which originated in a minor part.

The steam turbine did not require a very great length of time to refute the statement, that all it needs, is to be let alone, but on the other hand it displayed its ability to disrupt, explode, breakdown, burn out and do other things which contribute to the entertainment of the power-plant operator, and to the financial loss of its owner. So insurance protection has been provided to cover not only the driving steam rotor, but also the driven generator, driven compressor, driven pump, or other driven objects forming a part of the turbine unit. The most complete turbine protection afforded is termed "Breakdown Insurance" and includes, practically, all parts of the turbine above the foundation.

The owner may desire limited protection, and that can now be afforded under policy condition to suit the ideas of most any one. The protection can be restricted to the steam end of the machine and may be limited to the explosion of the rotor, or the steam casing exclusive of loss which results from damage to stationary or revolving plates or buckets.

The electrical generator operated by a steam turbine or by an engine, may be protected by insurance independent of the driving unit. The electrical policy contract defines the accident as including any disturbance which prevents the machine from functioning somewhat similar to the broad definition of an engine breakdown. The object insured may include also, motors, rotors, transformers, and types of revolving and stationary electrical equipment.

The engine, flywheel, turbine, and electrical policies have been combined under one contract form now known as the machinery insurance, which may also include pumps, compressors, and other objects having the engine operation peril, although the losses and inspection expense are classified separately.

It is doubtful if any other line of insurance has been broadened and modified within reasonable limits, to afford the protection desired by the owner of the equipment, to a greater extent than has the engineering-insurance policy, so at this time the power plant can obtain insurance protection against minor accidents, as well as those of a violent nature, in as small or as great an amount as the value of the exposure justifies.

The standard policy written by companies engaged in engineering insurance affords protection against loss due to direct damage to property of the owner, and of others for which the owner may be liable, and may be extended to include personal injuries and death to the public, as well as to employees in those states having no legislation to the contrary.

The loss of profit, loss of production, loss of sales, loss of rents, loss of tuition fees, and similar losses, are not included in the protection afforded by the direct-damage policy, which specifically excludes, any loss from delay or interruption of business or manufacturing process. However, complete protection against loss of that kind may be obtained under a "Use and Occupancy"

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

policy which affords a specific daily indemnity for each day that production is wholly prevented as the result of an accident insured against, under the terms of the direct damage policy, and affords pro rata protection when the production is but partially prevented. The use-and-occupancy policy may become effective at the time of the accident, or at any subsequent midnight, and may continue for any predetermined length of time, although 100 days is the average duration of the protection.

In connection with every business there are a great number of expenses which must continue with regularity regardless of whether or not the plant continues to operate. They must be paid, production or no production, and without use-and-occupancy coverage the owner will find taxes, guarantee dividends on stock, interest on bonds, and other indebtedness, insurance premiums, salaries to executives and other employees whose services cannot be dispensed with in case of shut down, auditor's fees, director's fees, advertising if under contract, penalties for failure to fill contracts, agency contract expense, expenses of branch and local offices, depreciation of plant machinery not damaged by the accident, so not to be considered when the direct damage claim is filed. Use-and-occupancy insurance is becoming a vital necessity, for regardless of how well the owner is protected against direct-damage loss from a power-plant disaster, without the additional protection afforded by the use-and-occupancy policy, he is exposed to heavy financial drains which might completely ruin his business.

Another probable source of loss is the freezing of growing plants, and the spoiling and fermenting of articles of food in storage or in process of preparation for use. Loss of this kind is covered under the "Consequential Damage" policy, and the protection becomes available immediately following the accident.

The direct-damage insurance premium, to an extent, depends upon the amount of financial protection afforded, but the premium increment is not proportionate to the increase in the face of the policy. This is due to the inspection expense being carried by the lower bracket, while the excess premium is based upon the cost of carrying an inspected risk. Too much insurance protection is a waste of money and too low protection is sure to lead to regret when the extent of the damage is determined following an accident. The exposure is difficult to estimate with any degree of certainty, for no one can tell the direction to be taken by the drum of an exploding boiler, which may damage property one-half mile or more from its original setting, while objects in the immediate vicinity escape injury.

The boiler of the modern power plant is of more value than the cost of the complete power plant in use when steam-boiler insurance was inaugurated, and the insurance protection should be proportionately greater, although local conditions must be the determining factor.

The protection carried under the steam-boiler explosion policy, should surely be sufficient to cover any loss which might result from the explosion of one of the boilers. In a large central station, this loss might not exceed the value of the boiler plant above the foundation, but in the average industrial plant, the exposure will include other buildings and property within a radius of possibly 150 ft. of the boiler plant, and, of course, if public liability is to be carried under the boiler policy, that hazard, too, should be taken into consideration when the total amount of coverage is determined. There have been numerous cases where the loss was total, that is, the amount of the loss exceeded the face of the policy, and the insurance protection was not sufficient, although a few dollars spent for additional protection would have been of great help toward placing the industry on its feet after the boiler accident had occurred.

The present level to which the insurance rates have decreased is due, not only, to the efficiency of the inspection service, but also to the ability of the plant operators, and the respectful promptness given the demand for repair. The dependability of operation varies directly as the maintenance varies, so the underwriters feel privileged to expect the owner and his employees to use reasonable care in operating, and due diligence in maintaining the power-plant equipment, for any other attitude must be reflected in the insurance rates even if the plant does not reach a condition where insurance protection is not obtainable.

FSP-51-20

Modern Stoker Equipment

By C. F. HIRSHFELD,[1] and G. U. MORAN,[2] DETROIT, MICH.

This paper describes improvements that have been made in stoker construction to adapt them to modern needs. Improved types are constructed carefully, are capable of accurate control and are used in large sizes. In choosing stoker equipment, the authors feel that generous allowance should be made above the apparent maximum requirements for fuel feed.

FORCED-DRAFT traveling grates were developed in two different sections of the country, for two different purposes. In the Chicago district this type of stoker was developed for burning the types of fuel mined in the neighborhood. These fuels are characterized by low softening temperature of the ash and, generally, by comparatively high water and ash contents and low heating value per unit of weight. They are also non-coking coals in the ordinary meaning of that expression.

In the eastern section of the country a somewhat similar type has been developed for burning the smaller sizes of anthracite. This coal is characterized by very low volatile content, high ash content, high ash softening temperature and, frequently, fairly high moisture content. The sizes are generally very small and are constantly getting smaller as the ability to burn small sizes becomes greater.

These stokers are definitely not well adapted to the burning of coals containing small amounts of ash, as a certain, rather large quantity of refuse is required to protect the grate itself. Also, if reasonably good efficiency is desired, the discharge end of the chain should be fairly well covered to prevent excessive air leakage into the furnace in that region. They were developed, as stated, for the burning of certain types of fuel which could not be satisfactorily dealt with by other means. When properly designed, installed, and operated they are capable of giving remarkably good results with such fuels.

The underfeed stoker of the modern multiple-retort type was developed primarily for burning the better grades of bituminous coals with rather low ash content of high softening temperature. During the past few years, however, models have been produced, which give very satisfactory performance with Illinois and similar varieties of coal. This type of stoker appears to be undergoing most rapid improvement at the present time, both with respect to increase of size, increase of fuel-burning capacity, improvement in thermal performance, and improvement with respect to maintenance.

It would not be correct to consider the evolution of stokers alone. Certain other factors have been active and they must all be considered together, if a true picture of the situation is to be obtained. Beginning about twenty years ago, there was a definite movement toward increased size of steam generating units and increased steam output per square foot of boiler surface. This in turn brought about both increase of stoker size, increased size, and particularly height of furnace. The weak point, at this stage of the development, was the refractory which was being subjected constantly to more severe usage and which definitely set a limit to possibilities.

Water cooling of furnace walls and even furnace arches was resorted to. There was doubt in the minds of many regarding the virtues of water protected furnace walls, in spite of the fact, that other and less perfect forms of such construction had been used for years in other places, such as in the railway and marine fields. Eventually, however, it was demonstrated that properly used water- and steam-cooled furnaces opened up possibilities for further improvement, increase of size, and increase of capacity which could not be ignored.

At about the same time that water- and steam-cooled surfaces came in for serious consideration, the use of preheated air began. The driving force behind this development came through a change in the type of steam cycle. At first, it was assumed that the use of preheated air would yield only an approximation to a certain calculated thermal saving, the calculation being based upon the degree to which the temperature of the flue gases could be lowered by this means. It was, however, discovered early, that the use of preheated air resulted in rather remarkable improvement of combustion, so that an unexpected gain was realized. The use of preheated air and the use of water-cooled surfaces in the furnace are really tied together to a certain extent. High preheat, with a great extent of refractory wall, is apt to cause trouble through too early failure of the refractories. Too great an extent of water-cooled furnace surface, with a low degree of preheat or no preheat, is apt to cause trouble through too low a furnace temperature. Great progress has been made in deriving satisfactory working rules with which to connect these factors, but much still remains to be done before we can be sure of obtaining the most economical arrangement for each case.

There is still much discussion as to the maximum permissible temperature of preheated air in stoker firing. Stoker manufacturers will say that their stokers are perfectly satisfactory for use with air up to 550 deg. or 600 deg. fahr. This may be true, but the authors confess that they are inclined to be more conservative as yet. At present, they prefer to limit the temperature of preheat to about 300 deg. or 350 deg. fahr. They do not mean that it is necessarily impossible to operate with higher values; this has in fact been done and is being done today. They mean simply that they believe that under what we may call normal conditions, much more certain and satisfactory results with minimum investment and minimum maintenance are expectable with the lower values.

Stoker design must be modified for different air temperatures. The stoker which is proportioned properly and designed for use with air at room temperature will, in general, not perform satisfactorily with highly-preheated air and vice versa. Use of highly preheated air sometimes radically changes the way in which fuel acts on the stoker. It is therefore not wise to take too great a step in the direction of increased air temperature, without preliminary experiments or observations under conditions similar to those contemplated.

Stokers have been made both wider and longer. Increase of width may be said to increase the fuel-burning capacity in direct proportion to the increase of stoker width. This is true, except for a slight difference due to the diminishing effect of side wall conditions as a per cent of the total, as the stoker gets wider. Increase of length, on the other hand, produces more than a proportionate increase in fuel-burning capacity, at least up to a certain limit not yet determined.

In the case of the chain grate, the increase of width and length bring in very serious design problems, both of a purely mechani-

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

cal character and also with respect to air distribution. In the case of the multiple-retort stoker, increase of width is a comparatively simple matter, but increase of length brings up problems of mechanical strength, problems of adequate coal feed, and of adequate coal forwarding.

The increase during the past few years in coal-burning capacity per stoker fired furnace, is well shown in Fig. 1 in which results obtained with both chain grates and underfeed stokers have been combined. The figures used are those given by the manufac-

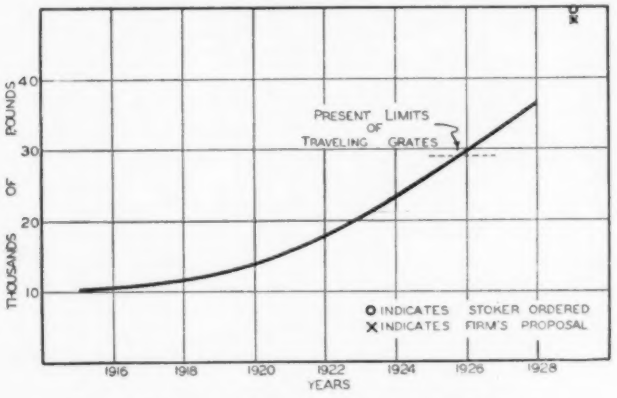


FIG. 1 MAXIMUM CONTINUOUS COAL CONSUMPTION PER FURNACE PER HOUR

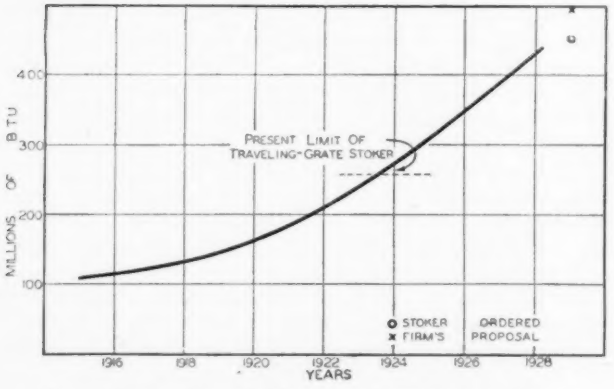


FIG. 2 TOTAL HEAT RELEASE PER FURNACE PER HOUR SHOWN GRAPHICALLY

turers of the stokers as maximum continuous possibilities. Even though they may represent a certain degree of optimism on the part of the manufacturers, it is reasonable to assume that the same degree of optimism prevailed throughout so that the relative values for different years may be accepted, even though the absolute values may be incorrect.

A more significant curve is given in Fig. 2 in which the total heat release per furnace is given, thus eliminating variations in heat value of coal. It will be observed that both figures have more than tripled in a period of thirteen years, a very remarkable performance for so short a period.

The data presented in Fig. 3, giving the increase in the rate of heat release per sq. ft. of grate surface over the same period, show a less rapid change. It must be inferred that a great part of the increase of output per furnace has resulted from increasing the dimensions, rather than from increasing the intensity of use.

It has long been known that stoker-fired equipment is not so limited with respect to rate of heat release per unit of furnace volume as is equipment fired with pulverized fuel. Cases are on record in which reasonably good results have been obtained with stoker-fired furnaces, in which the rate of heat release approached 100,000 B.t.u. per cu. ft. per hr. There is, however, a realization that furnace volumes have been increased greatly since the early years of this century and naturally there is some confusion as to the exact status of the art at the present time. It is impossible to give any simple rule for furnace volume. Too many variables are concerned. However, it may be said that as walls are protected better with air-cooled, water-cooled, or steam-cooled surfaces and as more highly preheated air is used, the rate of heat release per unit of volume may increase without affecting materially the operating results. The principal limit is set by the tendency to deposit fused ash on the boiler tubes nearest the furnace. This can be obviated by the use of an adequate slag-screen arrangement, that is, by the use of one or more rows of widely spaced tubes which cool the molten fly ash before it strikes closely spaced tubes.

It may be useful to show how improvements in furnace con-

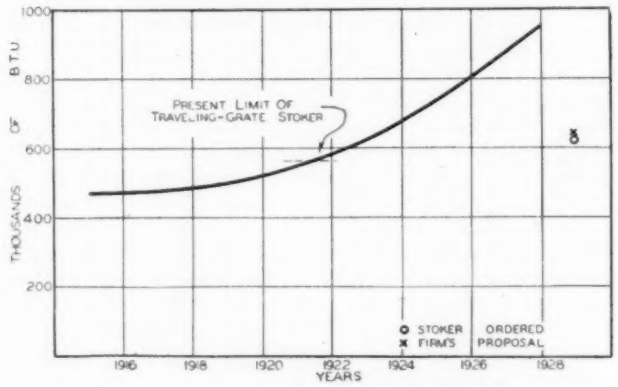


FIG. 3 HEAT RELEASE PER SQUARE FOOT OF GRATE AREA PER HOUR

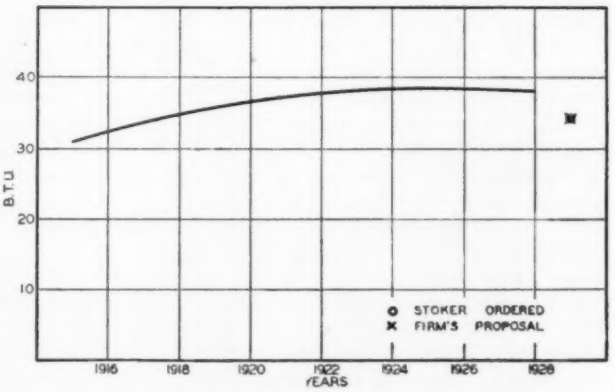


FIG. 4 HEAT RELEASE PER HOUR PER CUBIC FOOT OF FURNACE VOLUME

struction have affected the rate of heat release in recent years. The trend is shown in Fig. 4 and it may surprise many to discover that the general trend has been toward relatively smaller furnaces, with stoker firing during the past decade. This is explained partly by the facts stated above and partly by a certain sacrifice of the advantages of very large furnace volume in exchange for smaller investment.

The effect upon the boiler, of high rates of heat release per furnace and per unit of furnace volume, is exceedingly interesting. This is shown graphically in Fig. 5. Apparently we have not increased boiler area continuously at the same rate as we have the rate of heat release per furnace. This is equivalent to saying

that we have operated the boilers at successively higher rates of output in recent years. This fact is already well known, and it is also well known that the tendency toward lowered thermal efficiency has been balanced by the extensive use of economizers and air preheaters.

The most astounding picture results from a consideration of the increase in the quantity of steam produced per furnace. This is shown in Fig. 6. While it is true that the highest values shown do not as yet represent accomplished facts, it should be noted that they do represent actual orders placed. It is certainly a far cry from the "large Delray boilers" of twenty years ago with an output of about 140,000 lb. of steam per hour to the unit now available and capable of producing 500,000 lb. per hour.

A typical arrangement of forced-draft chain grate of the largest size as used with Illinois coal is shown in Fig. 7. This is a section through one of the Crawford Avenue boilers and furnaces. The small extent of the uncooled refractory arch and the extensive use of water-cooled surface are notable developments in this

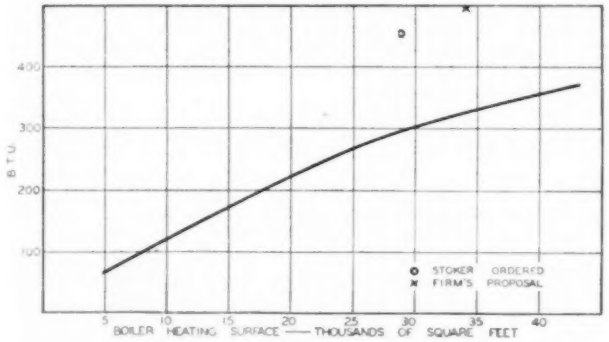


FIG. 5 HEAT RELEASE PER BOILER VS. BOILER SURFACE IN SQUARE FEET

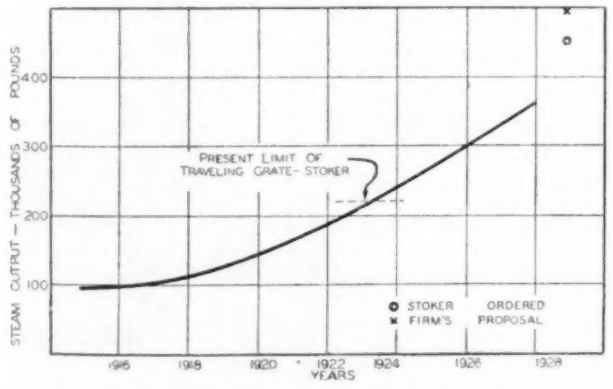


FIG. 6 MAXIMUM CONTINUOUS EVAPORATION PER BOILER PER HOUR

class of stoker. This represents one of the largest installations of its type being designed to produce 150,000 lb. of steam continuously and to carry peaks of 190,000 lb. The stoker pictured is 24 ft. wide by 20 ft., 6 in. long, giving a total area of 490 sq. ft.

An arrangement of the forced draft chain grate, as used with small sizes of anthracite, is shown in Fig. 8. The reversed arch over the discharge end of the stoker is a characteristic feature of this type of furnace. The construction shown in full lines was adopted after that shown in dotted lines was found to have undesirable operating characteristics and limitations. The original design had a throat of only 3 ft. 6 in. The later design gives a much wider throat with a very long rear arch and almost no front arch. It was found that the lean and rich gases mixed very satisfactorily and combustion was more nearly complete with the second arrangement. Combustion rates as high as 75 lb. per sq. ft. per hr. have been attained with this design but the

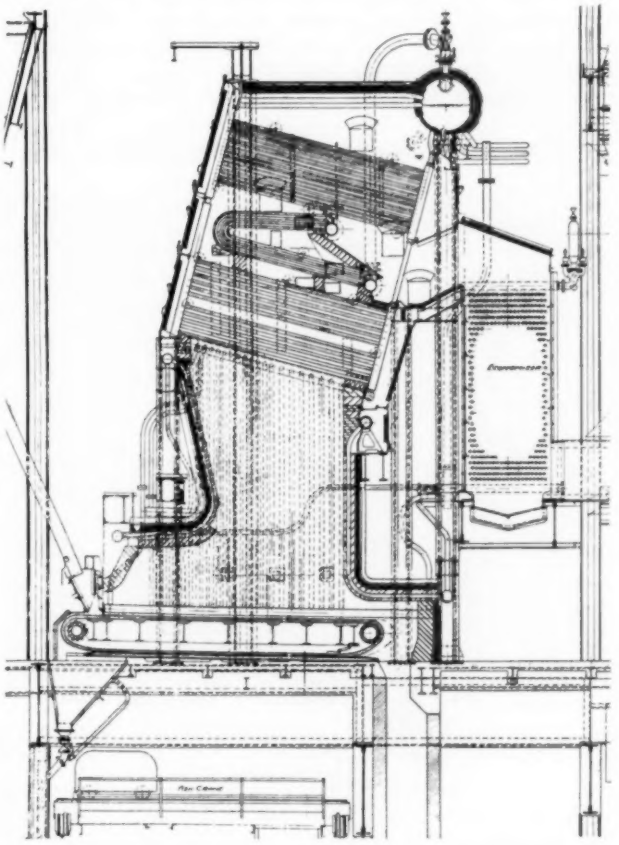


FIG. 7 ARRANGEMENT OF LARGE-SIZE FORCED-DRAFT CHAIN GRATE AS USED WITH ILLINOIS COAL

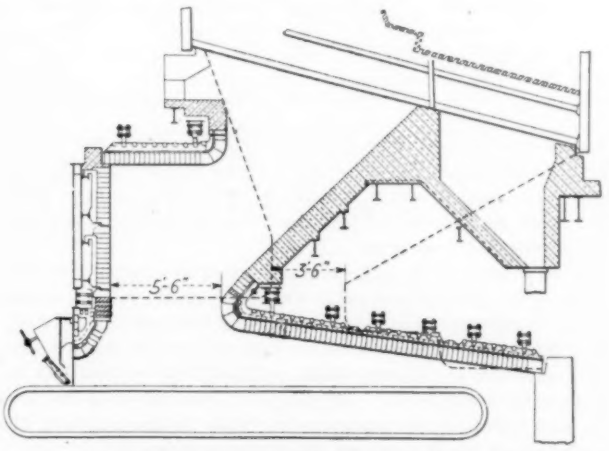


FIG. 8 ARRANGEMENT OF CHAIN GRATE USED WITH SMALL SIZES OF ANTHRACITE COAL

fuel bed must be thin and the stoker speed high (about 100 ft. per hr.) in order to maintain this rate.

A very modern arrangement of a multiple-retort stoker is shown in Fig. 9, as applied to bent-tube boilers in an English power plant known as Deptford East. This is an 11-retort, 37-tuyere stoker. A similar unit of 17 retorts and 49 tuyeres produces

350,000 lb. of steam per hour continuously. The cast-iron covered water tubes along the clinker line and those which form the bridge wall are particularly noticeable. In this way, the most critical wall sections are protected amply in a relatively inexpensive but effective manner, while cheaper masonry construction can be used for the remainder of the furnace enclosure. Incidentally the front wall construction shown is not to be recommended as it may be expected to have a comparatively short life.

A somewhat similar arrangement, but one which makes a

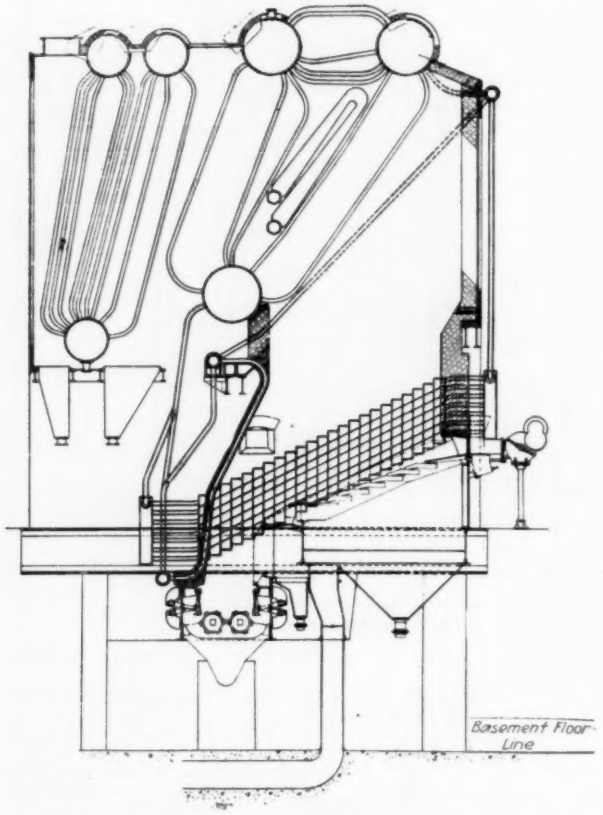


FIG. 9 MODERN ARRANGEMENT OF A MULTIPLE-RETORT STOKER

more extensive use of water-cooled walls as developed for the Richmond Plant in Philadelphia, is shown in Fig. 10. The preheated-air temperature in this case is about 330 deg. fahr. under maximum conditions and the maximum continuous steam output is 190,000 lb. per hour.

A very interesting, high-capacity installation with a double-ended firing arrangement under two single-ended boilers, is used by the Allegheny County Steam Heating Company, Pittsburgh, Pa. The stokers have 14 retorts each and are 36 tuyeres long. The maximum continuous rating is 300,000 lb. of steam per hr. with peaks of 400,000 lb. Fig. 11 gives a view of one of the stokers and parts of the water-cooled walls, looking toward the upper end of the stoker from the clinker-grinder pit.

A single-ended setting of very modern design is used in the Hudson Avenue Station, Brooklyn, N. Y. This unit has a maximum continuous output of 345,000 lb. of steam per hr. with a peak capacity of 410,000 lb. per hr. Water-cooled walls are used extensively. The stoker is 33 tuyeres long with approximately 6 ft. of overfeed rocking grate surface. A unit of similar capacity is now under construction for Delray No. 3, Detroit. The stoker in this case has 15 retorts and is 57 tuyeres long. The continuous steaming capacity is placed at 350,000 lb. per hr. and this is believed to be very conservative as stokers are in successful operation at much higher unit combustion rates. A design prepared in connection with a proposal for a unit capable of producing 500,000 lb. of steam continuously has 18 retorts and is 65 tuyeres long. It is intended to be used with highly preheated air. Although this unit has not been ordered, one very much like it and intended to produce substantially the same output is on order for a station near Berlin, Germany. This stoker has 20 retorts and is 49 tuyeres long, with an estimated maximum continuous capacity of 450,000 lb. per hr.

In the day when the stoker was itself a crude device and was used in small sizes under boilers of small output, it did not pay to attempt many refinements in associated equipment, or frequently even in operation. The modern types, constructed much more carefully, capable of much more accurate control and used in much larger sizes, make possible and even call for things not previously justifiable. The authors would like to call attention to some of the things that they think they have learned from experience in the use of stokers. What they have to say applies more to the underfeed stoker with which they have had the most experience but much of it is easily translated into terms of the forced-draft chain grate.

They have found that it pays to allow a very generous amount of leeway in all respects. Thus, while it may seem like throwing

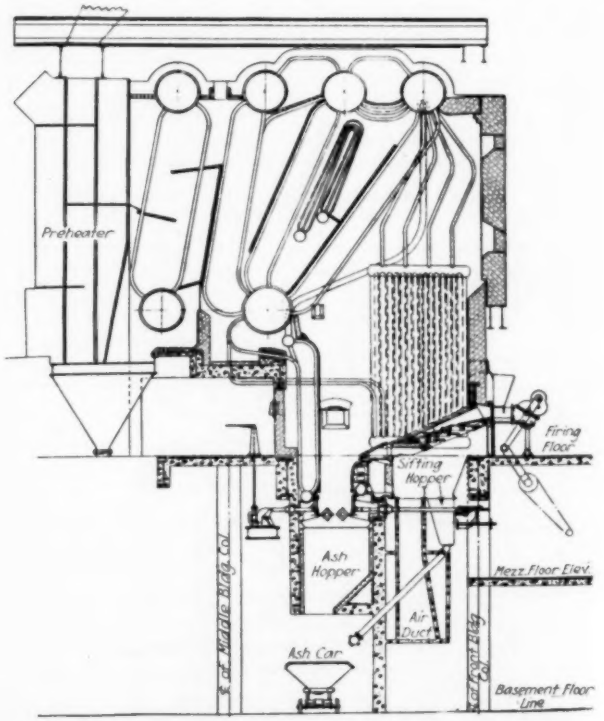


FIG. 10 EXTENSIVE USE OF WATER-COOLED WALLS USED IN THE RICHMOND PLANT, PHILADELPHIA

money away, it will be found usually that the purchase of stokers which are generously large for the job will pay handsomely as an operating proposition. Such action makes the user more independent of variation in the quality of fuel supply and much better able to cope with the various vicissitudes of daily operation.

Similarly, when choosing the motive power for stokers, one should not be satisfied with something that figures just large enough on the basis of calculated maxima. There should be a

very generous allowance above the apparent maximum requirements for fuel feed. It is frequently necessary to build up a fire rather hurriedly while at the same time burning at a rapid rate and this obviously calls for a great surplus over the calculated maximum speed. Too many installations are undesirably limited in this respect. At the present time most stokers, and particularly those of large size, are motor driven, and it pays to expend the few extra dollars required to obtain a generously large motor. Again, it generally pays to have a great many control points, or the equivalent, so that when the character of the load permits, the speed can be adjusted so as to prevent undesirable building up or thinning of the fire. This is particularly true at the lower speeds. It is also important to make certain that the motor, when operating at the lower speeds, will have sufficient torque to drive the stoker satisfactorily.

The authors have also found that it is usually not safe to base the choice of forced-draft fan characteristics upon too optimistic a conception of the performance of the stoker and furnace. Even though it is expected to maintain, say, 14 per cent CO_2, a fan should be big enough to deliver the air required for maximum combustion with a much lower CO_2 performance. As a general rule, something of the order of 11 per cent should be used for cases in which 14 per cent is to be expected. With such capacities, emergency conditions can be met with an assurance of safety. Similarly the maximum air pressure at maximum output should, in general, represent a generous margin of safety over the discharge pressure called for by the stoker manufacturer.

Here also proper speed regulation is important. This is a problem that many believe has not as yet been solved satisfactorily. Direct-current drive is beautifully flexible, but costly in all respects. Alternating-current drive becomes complicated and costly if the desirable flexibility is approximated. It seems as though the best present solution for general application is a mixture of damper and speed control, in which the power losses resulting from dampering, are offset by lower first cost, less complication, and less maintenance expense. Similar remarks apply, in general, to induced-draft fans although possibly they are not considered properly as part of the stoker installation.

It is also very important to distribute coal properly to a stoker. The stoker cannot be held responsible for results if coarse coal is fed to one or two retorts and much finer coal to the neighboring retorts. This fact has been known for a long time and appreciated by some, but ignored by most. There are now several different devices on the market which are capable of producing a very satisfactory distribution of coal in the stoker hopper and it is believed that they should be used more frequently than they now are.

The authors do not believe we need contemplate in the near future automatic combustion control, which will give results better automatically than, or even as good as, can be obtained by a conscientious, careful, and skilful operator assisted properly by instruments and other facilities. This is not to be interpreted as meaning that there is no field for automatic control in connection with stoker operation. In the first place, operators of the type indicated are rather rare and there are now available, automatic combustion-control systems capable of giving better results than are obtained by many of the stoker operators now employed. In the second place they believe it is wrong to expect the best results when a man is used for doing that which a machine

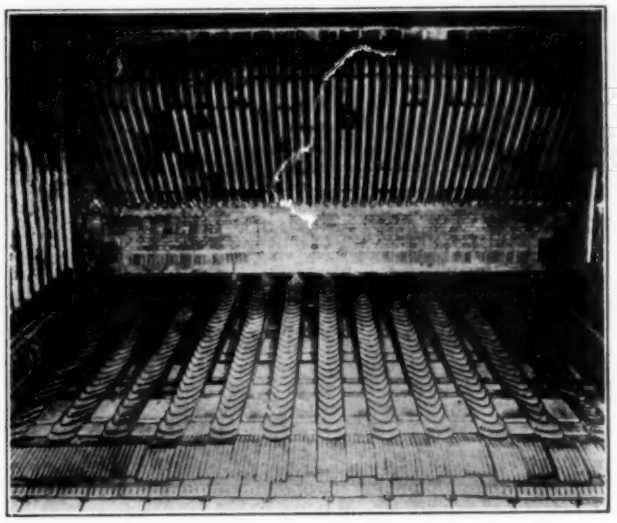


FIG. 11 VIEW OF A STOKER AND PARTS OF WATER-COOLED WALLS LOOKING TOWARD THE UPPER END OF THE STOKER FROM THE CLINKER-GRINDER PIT

can do as well or better, and thus use up a good part of his energies, instead of conserving them for doing only those things that he can do better than an available machine. For this reason they personally favor automatic control to a certain extent even when exceptionally good operators are available. The degree must vary with the economics of the particular situation as determined by cost of fuel, cost of labor, size of unit, etc. In the simplest case the automatic feature may be confined to boiler damper control, while at the other extreme will fall complete automatic control of all factors in an overall way and according to certain mean requirements, leaving the skilled operator free to make such secondary refinements in the relations between the different parts of the equipment as his experience may indicate to be necessary from time to time.

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Progress in Fuel Utilization in 1928

Contributed by the Fuels Division

Executive Committee: Victor J. Azbe, *Chairman*, Wm. G. Christy, *Secretary*, E. C. Schmidt, John Van Brunt, R. T. Haslam, and Geo. A. Orrok

IN ALMOST all phases of fuel utilization the past year has been characterized by consistent development of methods previously introduced and of equipment, the use of which had already been initiated. The greatest advances have been made in those branches of fuel utilization in which the largest quantities of fuels are employed. However, the predominant tendency to increase the effectiveness of fuel-utilization processes generally has caused an increasing amount of attention to be paid to combustion processes in those industries which have hitherto not shown active progress.

The report of this Committee is intended to summarize the more important advances which have been made during the past year. So many detailed improvements have been noted, however, that for a more complete statement reference must be made to the appended bibliography.

Specific attention must be drawn to the very valuable material reported at the three principal symposia on fuels conducted during 1928; namely, the fall meeting of the Fuels Division of the A.S.M.E. in Cleveland, the Fuels Conference of the World Power Conference held in London, and the more recent Second International Conference on Bituminous Coal in Pittsburgh. The transactions of these three meetings are replete with the most advanced material of interest to those concerned with fuel utilization.

Production, Preparation, and Storage of Fuels

Bituminous coal is the principal source of energy for the generation of power. Such an industry should be characterized by stability, constant output, and consistent earnings. Its present condition, on the contrary, is not far from one of distress. The maximum producing capacity of the bituminous-coal industry is variously estimated to be from 700,000,000 to 850,000,000 net tons, and shows no steady rate of increase. The production data for the period 1925–28 clearly indicate the present trend.

Year.....	1925	1926	1927	1928
Net tons.....	520,052,741	573,366,985	517,763,352	500,000,000 (estimated)

Comparative data indicating the relative use of coal are available for only a few industries. Those for central stations, however, show the present tendency.

	1927 Jan.–Aug.	1928 Jan.–Aug.
Energy generated from fuel, 1000 kw-hr..	29,556,431	30,941,071
Coal used, net tons.....	23,638,154	23,914,597
Fuel oil used, bbl.....	4,414,761	4,219,881
Gas used, 1000 cu. ft.....	38,454,403	47,280,154

The only material change noted is the increase in the use of gas. The figures for the last four months of this period show a decrease in coal used, and marked increases in oil and gas used for power generation in central stations. The increase in total energy generated in central stations noted elsewhere is due to a steady upward trend in the amount of power produced in the hydroelectric plants.

Coal-Producing Methods

In the mining of coal the efforts to reduce production costs and increase the percentage of recovery have been vigorous. This work has largely centered in the more extensive use of mechanical cutters, loaders, and conveyors, and in the adoption of improved mining systems. New types of mining equipment of practically all kinds have been introduced, and the older ones have been improved. Of particular significance is the work along these several lines by the larger coal producers. Much interest is being displayed in the development of a CO_2 cartridge for shooting the coal, a method which would eliminate flame. An outstanding activity is the increased attention paid to safety measures. The problem of supplying bituminous coal on a basis of quality and adaptability to specific purpose or type of combustion equipment has attained major importance, and hence producers are paying much attention to methods of cleaning and preparing coal. The labor situation in the bituminous industries is satisfactory, and there are no indications of potential strikes.

Anthracite

In the anthracite industry cumulative production for the first seven months of 1928, as compared with the corresponding period of 1927, shows a decrease of 4,500,000 net tons. Competition from oil and coke is severe. The decrease in production has caused the anthracite mining companies to make more intensive efforts to improve mining methods and to add labor-saving equipment. Old and expensive workings are being abandoned, and the larger mines are being equipped with automatic machinery in place of the old equipment. Extensive changes in auxiliary appliances are being made. The organization of the anthracite operators to carry on a well-planned research program, which has been effected, is undoubtedly an event of far-reaching importance in this industry. The anthracite industry has definitely started a series of investigations, the purpose of which is, first, to increase the efficiency of utilization of anthracite fuel generally, second, to develop means for applying steam sizes of anthracite to domestic use, and third, to carry on investigations which will enable the anthracite industry to turn some of its output into materials of greater value. The industry is having the close cooperation of domestic-heater manufacturers, and it is believed that this work will result in a distinct increase in the use of anthracite for domestic heating.

Coke

By-product coke for the first eight months of 1928 showed an increase of 2,000,000 tons over the same period for 1927. Beehive-coke production dropped 2,500,000 tons, and the net production is just under the figure for 1927 for the corresponding period.

Liquid and Gaseous Fuels

The increased use of natural gas for power generation has been referred to. During the past year at least three large gas lines have been completed which bring natural gas in increasing quantities into such centers as Denver, Omaha, and Kansas City. Considerable effort is being made by oil and gas producers to reduce the tremendous wastage of natural gas which has occurred and to divert it into domestic use, power generation, and to the repressuring of oil wells. This fuel is now being found in such

large quantities that the fears felt some years ago for its rapid extinction no longer exist. Production of crude petroleum for the first ten months of 1928 has been 4 to 5 per cent under that for the corresponding period during 1927. A very concerted effort was made to control petroleum output in the United States, both as a means of conserving this natural resource and as a necessary method of insuring the financial stability of the oil industry. At the present time the known oil reserves of the world are greater than at any other time. The petroleum industry is still in a critical state as far as the regulation of the supply of crude petroleum is concerned, and the efforts made this year to cooperatively hold down production in the U. S. must undoubtedly be extended to the principal foreign producing districts.

Furnaces

No radically new types appear to have been developed during the past year, but distinctive development along two lines has taken place, i.e., the further extension of water- and steam-cooled wall surface, and the decided increase in the use of automatic combustion-control equipment. Increased water cooling of the walls through water screens, which are a part of the boiler circulatory system, or by the use of radiant superheaters is taking place, because such development is found to reduce furnace maintenance costs and to facilitate improved combustion. Refractory surfaces continue to be used in the smaller installations, and particularly when the use of certain varieties of fuel is required. Furnaces with air-cooled walls are still being built, but operators believe that this type of wall will be gradually superseded by the water-cooled type, despite the higher cost of the latter.

Stokers

Reports from manufacturers indicate that the principal changes in stoker design which have taken place during the period covered by this report are: first, strengthening of those parts subject to strain by substitution of steel for cast iron; second, increase of the ruggedness of the ash-disposal mechanisms; and third, increases in both length and width. Last year this report mentioned the trend toward specific control of air admission to different parts of the fuel bed. This trend has continued uninterruptedly, and the newest stokers are being supplied with air-control systems intended to permit highly selective admission of air to the different portions of the stoker. Moreover the advantages of controlled supplies of air over the fuel bed are now thoroughly appreciated, and many such installations are being made. The burning of low-grade and clinkering coals at high percentages of rating on stokers has been successfully carried out by means of stokers specifically designed for this type of operation. The limit for air preheat in ordinary installations continues to be 400 to 500 deg. fahr., although the presence or absence of an air preheater in a given installation is dependent on the particular heat-balance conditions of a given installation. Corrosion of air preheaters which has been experienced has been eliminated by increasing the exit gas temperature, and the burning of stoker tuyeres has been corrected by using greater clearances and in some cases using coal with a higher ash-fusion point. A great deal of attention is being paid to tuyere and retort design, notably in altering the tuyere-to-retort area ratio, the overfeed-underfeed ratio, and the use of large rams with variable-width tuyeres. Sufficient data are now available so that stoker manufacturers are designing equipment with constantly increasing exactitude.

This year has seen a real tendency toward the installation of mechanical stokers in marine practice, and this type of equipment is being tried out in a sufficient number of ships to indicate that progress in this direction from now on will be rapid. The trial installations seem to be proving successful.

Pulverized Fuel

The increased use of pulverized fuel both in converting old equipment and as regards new installations continues at about the same rate as for the preceding year. In public-utility plants the central storage system predominates over the unit system in the ratio of 2.4 to 1, whereas in the industrial plants the installed horsepower supplied by the unit system is greater than that of the central system in the ratio of 1.7 to 1. The rate of increase of the unit system has been greater than that experienced by the central system, and a few installations of the unit system have been made in connection with large boilers. The use of preheated air in the mill for drying coal seems to be prevalent in new installations, except for those fuels which have a low ignition point. In these cases the use of steam for drying reduces the tendency to take fire in the drier. The elimination of dust in the stack gases from pulverized coal is giving concern in a great number of cases. This problem is being attacked in a variety of ways, e.g., by the use of cyclone collectors in series, water sprays, electrical precipitation, etc. As may be expected, no one method seems to be uniformly applicable, and the particular means used have to be specifically selected for each installation. Installations have been made in which the regular bank of boiler tubes is protected by an extra series of tubes on extended centers below the regular bank as a means of chilling the ash in such a way that it does not cause slagging on the boiler tubes. This method may prove to be of particular advantage in small furnace chambers where short-flame burners are used. No new significant points in the design of furnaces for use with pulverized fuel are noticed, the trend being the same as for stoker-fired furnaces in that the use of water- and steam-cooled walls practically enclosing the combustion space is almost universal.

The more extensive use of the turbulent-type burner referred to last year continues to be noted, the principal advantages being increased combustion rates per unit of furnace volume, reduction of excess air, and facilitation of ash separation.

The turbulent-type burner has been successfully applied to Scotch marine boilers. While the extension of the use of pulverized fuel to marine practice in the United States has been slowed up by the low prices of bunker fuel oil, it is certain that interest abroad in this development is constantly becoming more intense. On at least three British ships at the present time a part of the boilers are being fired with pulverized fuel. The bunker-fuel situation in England will encourage intensive work to perfect the use of pulverized fuel for cargo carriers. The British government is expected to appropriate £100,000 for such development during the coming year.

Steam Pressures

Aside from the fact that additional 1400-lb. units have continued to be built, no radical change has taken place in steam pressures and temperatures, although in the larger stations the trend seems to be from the 400-lb. to the 600-lb. range. A consistent realization of steam temperatures of about 750 deg. fahr. has not yet taken place.

Two-Fluid Systems

In last year's report the possibility of the bi-fluid system, using organic compounds of the type of diphenyl oxide as a substitute for mercury, were indicated. Research work carried out during the past year has indicated that whereas the thermal stability of such compounds at operating temperatures can be assured, the peculiar thermodynamic properties of such organic

liquids requires the use of such large heat-exchange equipment that the necessary capital cost of the equipment is so high as to render the use of an organic-boiler-fluid system non-competitive with the 1400-lb. steam system at present.

Locomotive Practice

In this field the selection of fuel has been one of the major means of conserving fuel. The pounds of coal per thousand gross ton-miles continues to decrease at the rate of about four per cent per annum, and the corresponding figure per passenger car-mile is decreasing nearly two per cent. The number of locomotives in service which are equipped with mechanical stokers has increased until about one-fourth of the total number have stokers installed. Over one-half of the total number of locomotives employ superheaters, and the use of automatic fire doors on locomotives is now general practice. Railroads are paying particular attention to the proper selection of fuel and to supplying divisions constantly with the same kind of fuel. This permits effective training of firemen.

A great deal of interest is displayed abroad in the Schmidt high-pressure locomotive which employs two steam pressures, 850 lb. and 205 lb. The 850-lb. steam is expanded in a center cylinder, the exhaust from which is mixed with the 205-lb. steam, which is then expanded in two outside cylinders. This development has presaged the building of additional locomotives to operate on a similar principle, in that the high-pressure portion will be further enlarged at the expense of the low-pressure system. In addition a locomotive working at 1700 lb. pressure according to the Loeffler principle is now under construction. The advantages for this particular design are fuel economy, with only slightly higher first cost than that of existing locomotives. In Russia particularly, as well as in other European countries, the application of Diesel engines to railway work continues. This is being facilitated by a solution of some of the difficulties of gear transmission previously experienced.

Gas Manufacture

This field of fuel utilization is becoming distinctly interested in two auxiliary processes, namely, the dry quenching of coke and the dehydration of gas. The economical advantages incident to using the sensible heat of the coke for producing steam are beginning to be appreciated as the success of existing installations becomes better known. This type of plant is adaptable to a large variety of carbonizing units and conditions, and the facts indicate that the dry-quenched coke produces less breeze and has some advantages as water-gas fuel over water-quenched coke.

Meter repairs have always been an appreciable item in the cost of manufactured gas. Data gathered both in the United States and England indicate that substantial savings on this item may be made by dehydrating the gas before distribution. Certain processes for dehydration are being installed in the United States, and the forthcoming year should produce data really indicative of the value of these processes. Research in England has indicated that specific constituents of the ash in coal may have significant effects on the properties of the coke in so far as its ability to decompose water and CO_2 are concerned. The practical aspects of this point remain to be investigated.

Low-Temperature Carbonization

The Second International Conference on Bituminous Coal at Pittsburgh brought forth many papers and considerable discussion of this topic, and still there do not seem to be definite data establishing the economic utility of this process. Until the two large-scale plants in the United States have operated for a sufficient time to demonstrate the earnings possible, little real progress can be reported. The theoretical as well as the engineering aspects of low-temperature carbonization are doubtless well worked out, but it is believed that even in England where the market for low-temperature coke should permit of a higher premium than in the United States, definite assurance as to commercial practicability seems to be lacking. The value of the tar seems to be a doubtful point. It is possible that low-temperature tar could be hydrogenated to produce such a large proportion of saturated hydrocarbons that its value could be greatly enhanced with a consequent improvement in the economic situation of the process as a whole. Continued experimentation both in the United States and abroad proceeds, as well as continued discussion of the general scheme of combined gas and electric plants. A paper by Rosin at the World Power Conference summarizes the possibilities and draws the conclusion that the combination has the best chance of economic success when low-grade fuels such as lignite are used.

Experimental work on low-temperature carbonization processes both in the United States and abroad is being actively continued on a wide variety of processes, most of which have been described fully in the literature. Taken as a whole this work is being supported financially to an extent almost unequaled in any other branch of the fuel-utilization field.

Smoke Abatement

The necessity for smoke abatement in the large cities is constantly becoming more acute. Data as to the economical losses and health hazards involved are becoming more available, and a few of the more progressive municipalities are adopting smoke-abatement ordinances and instituting control organizations to the end that only combustion equipment designed for minimum production of smoke will be permitted and that constant effort will be made to modify existing installations. Notwithstanding this interest there is as yet no effective or uniform means available for controlling or abating the smoke nuisance.

Research

Despite the interest in this phase of fuel utilization displayed at such meetings as the Second International Coal Conference and the fact that The American Society of Mechanical Engineers, the American Gas Association, National Coal Association, and other similar organizations have research committees and tentative research programs, no concrete organization fully organized to secure the necessary funds for carrying on a well-considered program of fuels research has yet been developed. This is probably the greatest single need in fuel utilization at the present time.

The papers presented at the conferences noted above constitute quite a comprehensive statement of the work completed and under way.

Mention of even the subjects discussed at the Second International Conference on Bituminous Coal would be too voluminous for this report. The impression gained by those attending the sessions was that throughout the whole world an enormous amount of well-chosen and well-conducted fuels research is being carried out. Abroad the greatest importance is attached to transformations of coal and its derivatives into materials of enhanced value such as liquid fuels.

Without minimizing in any way the importance of the other papers recently presented, attention should be called to two papers vitally affecting furnace design, namely, those by Wohlenberg and Anthony, soon to appear in the publications of the Society; and by Rosin at the Second International Conference on Bituminous Coal.

Some of the more important work being carried on in Europe on which definite progress seems to be assured includes:

Development of Internal-Combustion Engines Using Pulverized Coal as Fuel

Application of Pulverized-Coal Firing to Locomotives

Long-Distance Transmission of Coke-Oven Gas

Formation of Benzol and Other Hydrocarbons from Methane.

Details of the work done on these projects will be found in the transactions of the Second International Conference on Bituminous Coal.

J. T. WARD,[1] *Chairman*,
Progress Report Committee.

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Discussion

O. P. HOOD.[2] If one seeks a reason for the popularity of these national and international fuels meetings, he will find it in the breadth of subject that is brought up.

Mr. Ward's review of the year's situation has a number of points of contact between fuel and our modern civilization. This paper is not so much one for discussion, but it is certainly one for amplification or further development.

JOHN VAN BRUNT.[3] Professor Ward mentions the amount of steam fuel produced in the mining of anthracite as 30 per cent. Just what is included as steam fuel?

PROFESSOR WARD. I included everything below nut size, but others might restrict such a figure to those sizes below pea coal.

MR. VAN BRUNT. You mentioned that in the future that will increase to 50 per cent.

PROFESSOR WARD. That figure of 50 per cent must not be taken too literally. It is doubtless more correct to state that under present conditions the proportion of anthracite coal mined that is not salable as domestic fuel is approximately 30 per cent. A fair estimate of the rate of depletion of present anthracite reserves may be made, and this indicates that inside of fifty years we will be mining varieties of anthracite that upon being crushed will yield a larger proportion of sizes not salable as domestic fuel. It is estimated that these softer anthracite deposits will yield not over 50 per cent of salable domestic sizes.

Authorities agree that as time goes on the percentage of high-revenue anthracites will decrease, and the anthracite operators are in the uncertain position of any industry that foresees a decline in revenue. The remedy is doubtless research leading to the use of anthracite fines to produce products of greater value.

MORGAN B. SMITH.[4] For some time past the coal situation in Detroit and surrounding cities has been in a thoroughly demoralized state, as is evidenced by the wide price range asked for nut and slack coal of so-called 2 in. size; namely, 25 cents to $1.10 per ton f.o.b. mine. Over-production is largely responsible for this condition.

In addition bankers tell us that there has been approximately 41 per cent decrease in demand for coal because of:

(*a*) Increased efficiency of locomotives
(*b*) Increased efficiency of public-utility steam power and heating plants
(*c*) By development of hydroelectric power plants
(*d*) By the increased use of substitutes such as oil and gas.

We have in this Society men capable of studying the financial and general economic status of the coal industry and well able to assist in stabilizing the situation. With this thought in mind I wish to suggest that the Fuels Division include this phase of the topic in their work and give us a report of progress a year hence.

VICTOR J. AZBE.[5] Mr. Ward's report is not as voluminous as it will be next year. He is getting together a very able committee, and I am sure that this committee will aid greatly in stimulating the proper use of fuels in this country, and we certainly do need such stimulation.

Professor Ward's report indicates progress; you get a picture that things are all fine, everything is lovely, and that we burn coal with very high efficiency, which really on the average is far from so.

We should have still another committee, the exact opposite of Professor Ward's committee. Professor Ward's committee tells us about the progress, and the other committee should tell us about the things that are happening that are the exact opposite of progress.

Another thing that we ought to have is a committee that will evaluate the efficiency of the average engineer. I will say that his efficiency is very low. If you attempt to measure progress in some industries you actually have to use a micrometer to find it. It is really outrageous in this twentieth century to find that certain industries are about in the same state as they were some fifty years ago.

MASON A. STONE.[6] I wish to take exception to the remarks of the speaker divorcing economics and engineering. I think that a speaker before him illustrated the fact that a fuel engineer is primarily an economist and that our profession is one of those which applies political economy to the problems of industry.

The political economist theorizes about economics, but the engineer puts the theory into practice, and no engineering de-

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[2] Chief Mechanical Engineer, U. S. Bureau of Mines, Washington, D. C. Mem. A.S.M.E. (Chairman of the meeting at which the report was presented.)

[3] Combustion Engineering Corporation, New York, N. Y. Mem. A.S.M.E.

[4] Engineer, General Motors Corporation, Detroit, Mich. Mem. A.S.M.E.

[5] Consulting Engineer, St. Louis, Mo. Mem. A.S.M.E.

[6] Structural Designing Engineer, E. L. Phillips & Co., New York, N. Y. Mem. A.S.M.E.

signer is a good designer unless he takes into consideration the economic features. He may not have studied political economy, but he certainly applies it every day of his life.

MR. HOOD. I brought one major impression from the London meeting. The question of fuel is of interest to the government in England. It has been shown in many ways. It is not only of interest to the government as an organization; it is of interest to each political party. Just imagine our Republican or our Democratic platform with some really meaningful reference in it about the efficient use of fuel.

The American Government was one of the first, under President Roosevelt, to make appropriations to do something in the matter of efficient use of fuel resources. Something has been done, a little each year, since 1910. But the rate at which the Government is interesting itself in American fuel is comparatively small as compared with the interest shown in England.

The Government Research Laboratories at Greenwich under Dr. Lander propose to spend for the coming year £90,000, or $450,000, on one line of research. Our American appropriation for similar work is about $150,000, of which something more than $50,000 is taken up with routine testing for the Government itself.

I think that shows the difference in the two countries as to their interest in the fuel problem.

As has been said here, Lord Redding was quite willing to come out with a flat statement that English prosperity rested on the fuel industry. Our own people are not ready to say that as yet. We are interested in too many things. We are too rich in fuel. Perhaps it is just as well that we get along as we do, but the time will doubtless come when we, too, must take an increased interest in this basic resource that we are exploiting.

A. E. R. DE JONGE.[7] American engineers are indeed very fortunate, for the coal they use has such a small content of ashes. The coal used in power stations here has hardly more than 10 per cent of ashes. They can therefore hardly visualize the difficulties that other engineers have.

I have been associated for over nine years with the Shanghai Municipal Electricity Department and its large power station. There we had considerable difficulty with ashes and grit. The coal we were burning contained up to 25 and even over 30 per cent of ashes, and one of the most difficult problems we had to deal with was the question of reducing the amount of grit that passed out of the stacks.

I did not gather from Mr. Ward's report whether he had any data on the progress that has been made in this country on the elimination of grit and the losses of ashes from the stacks.

As I said, we had considerable difficulty and have tried various equipment to eliminate these cinders. We also installed some equipment of our own from my ideas, as we found that all the equipment we had used would not work to our satisfaction. We have, however, not been able to get rid of more than 50 per cent of what passes out of the stack, although the manufacturers of some of the equipment claimed that it would eliminate between 85 and 90 per cent.

The reason why I mention this is that we have been rather unfortunately located. The wind direction prevailing in Shanghai for practically three-quarters of the year is southeast, and this means that there is a very narrow patch of country lying to the northwest of the station on which all the grit passing from the stacks was deposited. This land is unfortunately agricultural land.

Naturally we had for some time tremendous opposition from the Chinese farmers. Tons and tons of black grit were deposited on the vegetables they were growing. They complained to the Government, and the Chinese Bureau for Foreign Affairs complained to the Municipal Council. The situation became desperate, and we had to do something. But, as I stated, we have not been able to eliminate these stack losses to any great extent.

The best results, and I think this is worth stating, were obtained through the installation of air preheaters which acted more or less as cinder catchers and eliminated about 6 per cent.

May I add one more thing to the subject? That is the domestic application of coal. A large amount of coal is burned in small boilers and in small furnaces in a domestic way, but the efficiency is frightfully low, and I think most of the pollution of the air is due to this source. In England they have attacked the problem of air pollution from this source, and as far as I know there is a definite movement on foot to reduce smoke and cinder losses, or rather soot, issuing from these small furnaces and thereby to reduce the pollution of the air. We are human beings who are living in the air, and I think that everything that can be done to do away with pollution of the air should be done.

A MEMBER. What differences are there between German brown coal and American lignite? In reading about English coal the price of patent fuel is mentioned—is that the briquet coal?

MR. HOOD. "Patent" is a term used for briquetted fuel, and it varies considerably in quality. The Englishman has not ruined his briquet market quite as much as we have by using high-ash material in the briquets.

In answer to the question about German brown coal, if one thinks of it as a fuel that has not been metamorphosed as much as lignite, you will not be far wrong. Its heating value is, in round numbers, about one-half of a fair American coal. It varies all the way from 40 to 60 per cent in inherent moisture—not wetness on the surface, but inherent moisture—and that is its main characteristic. Our own American lignites in Texas and Dakota run about 30 to 35 per cent inherent moisture. Our lignites must be mined by blasting, just the same as our bituminous coal. That is, they are hard. German brown coal is so soft, at least in some places, after the bed is stripped, that with a little exertion one can thrust a spade into it.

It varies in color all the way from deep cream color to brown-black. In Central Germany there are bands of brown coal that are deep cream in color. Even the amber that is used for jewelry, I believe, comes from a vein of brown coal. It varies widely in color, but the greater quantity of it is brown.

Occasionally one will see a pile of this brown coal exposed to the sun that shows its woody structure in a peculiar manner. One can pick out pieces where the annual layers of wood can be separated one from the other. It will curl up like drying chips. I know of no better illustration than that of the occasional hen you see with its feathers turned the wrong way, and that is the way a pile of this material looks.

According to one way of classifying coal, a German brown coal is included under the general term of lignite. But we in this country have no commercial brown coal. I am told there is a spot in Texas that has a little of it. We have read what Germany has done with its lignites and that because we have lignites therefore we can do what Germany has done. The logic is bad, since the two materials are unlike.

FREDERICK A. SCHEFFLER.[8] I would like to suggest that the Fuels Division undertake to induce the Department of the Interior at Washington to separate in their monthly report of fuels used in public utility plants the amount of coal used in pulverized form from the total raw coal used.

[7] Designer, New York Edison Co., New York, N. Y. Mem. A.S.M.E.

[8] Special Representative, Fuller-Lehigh Co., New York, N. Y. Mem. A.S.M.E.

Pulverized coal is a new form of fuel and is used now to such a large extent that it is just as important to have it tabulated as it is for oil, gas, and hydroelectric power to be tabulated.

There are installed and under construction in public-utility plants in this country over 4,000,000 kw. capacity which will use pulverized coal to the extent of approximately 13,000,000 tons net annually, which is 31 per cent of the total cost reported in 1928 bulletins.

C. Harold Berry.[9] With respect to brown coal, it might be of interest to call attention to the fact that the brown coal of Australia differs from European brown coal and from our own lignite. I understand that it has about the mechanical properties of brown sugar. A typical analysis shows fixed carbon, 14 per cent; volatile matter, 18.8 per cent; ash, 1.2 per cent; moisture, 66 per cent; heating value, 3000 B.t.u. per lb. It is burned successfully in several plants, but requires a special procedure in handling and firing.

[9] Professor of Mechanical Engineering, Pierce Hall, Harvard University, Cambridge, Mass. Mem. A.S.M.E.

Professor Ward. We will place before the Executive Committee Mr. Scheffler's suggestion that the statistics of the Department of the Interior be changed to show the amount of pulverized coal used in public-utility central stations.

Mr. Smith's suggestion that the Fuels Committee include a discussion of the economic status of the fuels industry is probably too great an increase in activities for the Fuels Progress Committee to undertake, but there is no question about the desirability of such a survey. Doubtless the Executive Committee will be glad to discuss Mr. Smith's suggestion.

In conclusion, it is doubtless sufficient to reiterate that while there is an abundance of fuel research being carried on and consistent progress is being made in fuel utilization, the past year has seen no developments so radical as to be productive of results that will change the present trend of fuel utilization either here or abroad.

FSP-51-22

Joint Research Committee on Boiler-Feedwater Studies

THROUGH the Joint Research Committee on Boiler-Feedwater Studies, six national technical organizations are cooperating actively in a study of the various processes employed in the purification of feedwater in steam power plants and on railroads. This research is being carried on to determine the fundamental principles underlying certain phenomena which take place in steam boilers. The Committee is sponsored by the American Boiler Manufacturers' Association, the American Railway Engineering Association, the American Water Works Association, the National Electric Light Association, the American Society for Testing Materials, and The American Society of Mechanical Engineers, and has been subdivided into nine sub-committees and an executive committee. Each sub-committee is collecting data relating to the subject assigned to it, and has been requested to compile and submit yearly reports. After these reports are reviewed by the Executive Committee, they are presented for publication in the journals of the organizations participating in these studies.

Progress Report of the Boiler-Feedwater Studies Committee

Report of the Executive Committee[1]

By SHEPPARD T. POWELL, BALTIMORE, MD., CHAIRMAN

[1] The personnel of the Executive Committee of the Joint Research Committee on Boiler-Feedwater Studies is as follows:

S. T. Powell, *Chairman*, 213 St. Paul Place, Baltimore, Md.

W. L. Abbott, Chief Engineer, Commonwealth Edison Company, Chicago, Ill.

A. J. Authenreith, Vice-President, Middle West Utilities Company, Chicago, Ill.

R. C. Bardwell, Superintendent of Water Supply, The C. & O. Railway Company, Richmond, Va.

Prof. Edward Bartow, The State University of Iowa, Iowa City, Iowa.

James H. Buell, Middle West Utilities Company, Suite 1500, Edison Building, 72 West Adams Street, Chicago, Ill.

Prof. A. G. Christie, Department of Mechanical Engineering, The Johns Hopkins University, Baltimore, Md.

Wellington Donaldson, Consulting Engineer, Fuller and McClintock, 170 Broadway, New York, N. Y.

Harold Farmer, Chief Chemist, Philadelphia Electric Co., Central Service Building, 23rd and Market Streets, Philadelphia, Pa.

Prof. C. W. Foulk, Analytical Chemistry, Chemistry Department, Ohio State University, Columbus, O.

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Abbott L. Penniman, Jr., Superintendent Steam Station, Consolidated Gas, Electric Light & Power Co. of Baltimore, Lexington and Liberty Streets, Baltimore, Md.

V. Bernard Siems, Vice-President and General Manager, North American Water Works Corpn., 11 Broadway, New York, N. Y.

Frank N. Speller, Metallurgical Engineer, National Tube Co., 1810 Frick Building, Pittsburgh, Pa.

George A. Stetson, Associate Editor, The American Society of Mechanical Engineers, 29 West 39th Street, New York, N. Y.

C. P. Van Gundy, Engineer of Tests, Baltimore & Ohio Railroad, Baltimore, Md.

Prof. Albert E. White, Director of Department of Engineering Research, University of Michigan, Ann Arbor, Mich.

Abel Wolman, Chief Engineer, Maryland State Department of Health, Editor-in-Chief, Journal American Water Works Association, 16 West Saratoga Street, Baltimore, Md.

Progress reports presented at a joint session of the A.S.M.E. Power Division and the Joint Research Committee on Boiler-Feedwater Studies at the Annual Meeting, New York, N. Y., Dec. 3 to 7, 1928, of The American Society of Mechanical Engineers.

THE functioning of the Committee during the past year has been along the lines originally laid down when these studies were inaugurated. The aim has been to collect information and data on various phases of feedwater treatment and allied subjects with the ultimate view of perfecting the art. The importance of such work must be apparent to all engineers who are faced with the necessity of producing an adequate and satisfactory supply of water for power-plant uses. This rapidly changing art requires continued study since new factors are constantly being injected into this phase of engineering which requires readjustment of operating methods. Under the existing conditions the art of water treatment for industrial uses has become an exact science and water is now generally classified as an important engineering material.

Although considerable apathy exists in the engineering profession on certain phases of this problem there has been manifested in general a keen interest in the work of these committees, not only in this country but among engineers in Great Britain, Europe, and the far East.

Considerable effort has been given during the year to the accumulation of a fund for carrying on research work necessary for more exact knowledge of fundamentals of the physics and chemistry of water treatment and allied problems. There has been contributed to this fund by various persons or companies interested in the work $3000. In addition to this sum, $20,000 a year for a five-year period will be available when the full budget has been raised. The release of this contribution is contingent on the availability of the full budget required for the entire research program of $60,000 annually for a five-year period. One of the larger engineering associations has under consideration at the present a contribution of $30,000 a year for five years.

No active research work is planned until the entire fund is available since the successful promotion of the work must depend upon an uninterrupted program which cannot be assured without assured financial assistance. It is the earnest desire that the program may be undertaken early in 1929 so that a full year's work may be reported next year.

Committees on filtration, corrosion, water softening by chemicals, evaporators and condensers, and embrittlement have

reported progress, but no formal reports are available from these groups this year. Committee No. 8 on water analysis has prepared a tentative method for analyses for the determination of dissolved oxygen. This method is now being employed in a large number of steam stations as a result of cooperative effort between this committee and the Chemists' Committee of the Prime Movers Committee of the National Electric Light Association. A similar coordination is maintained with the American Water Works Association which is responsible jointly with the American Public Health Association for the last edition of the Standard Methods of Water and Sewage Analysis. It is the desire of Committee No. 8 to develop a complete set of standard methods of analysis application for power-plant use.

The successful fulfilment of this program will be an invaluable contribution to the art since there is now no uniformity in methods of analysis or in reporting of results. It is the purpose of the committee to utilize the present standard of water analysis as a basis and to modify or develop new methods as may be desirable to fulfil the specific requirements in power-plant practice.

Attention is directed to the report of the committee on trade-waste disposal as it affects the quality of water used in steam stations. The importance of this report may not be fully realized except by those who have experienced heavy financial losses as a result of exaggerated cases of gross pollution of their water source. The apparent failure to recognize the importance of this matter is accountable only by lack of knowledge of the degree of pollution which may be translated into specific financial losses. Complete solution of these problems is not always apparent and frequently requires extensive research which may be relatively costly. In most instances, as has been shown by the committee's reports, this procedure is warranted. The recommendations of the committee concerning these problems are important and if adopted by the profession should prove effective in improving the quality of water used in many sections of the country which are now grossly contaminated.

Constructive work has been done by Committee No. 9. This committee has prepared for publication during the year eight series of bibliographies including 244 articles or papers on water treatment for industrial uses or allied problems. These abstracts have been published in the *Journal* of the American Water Works Association, the abstract journal of the American Chemical Society, and elsewhere. This work is a valuable contribution to the profession since it has made available for ready reference current literature on the subject from widely diversified sources.

There were presented at the San Francisco convention of the American Water Works Association in June of this year six papers on problems relating to boiler-feedwater treatment and surface-condenser operation. The papers were not committee reports but were secured as contributions by the committee's activity. These papers have been published or will be published in the journals or proceedings of the associations affiliated in this work. The previous reports of this committee have directed attention to the necessity for constructive research on numerous problems encountered in steam-station practice in the maintenance of an adequate and satisfactory water supply. The past year has completely confirmed this opinion. There is an increasing tendency toward higher steam pressures and temperatures and the introduction of many new problems, the solution of which may not be solved without fundamental knowledge on the subject.

Although progress in these studies has not been as rapid as desired, constructive work has been done. The most important accomplishment has been the lessening of duplication of effort which in the past has been a gross waste of time and energy. There has resulted a unified purpose and a more intelligent interpretation of the various phases of feedwater treatment than was formerly possible. In addition to these conditions much good has been accomplished by merely directing attention to the economic importance of these problems in the operation and maintenance of steam generation and equipment.

Zeolite Softeners, Internal Treatment, Priming and Foaming

Progress Report of Sub-Committee No. 3 on Zeolite Softeners, Internal Treatment, Priming, and Foaming[2]

By C. W. FOULK, COLUMBUS, OHIO, CHAIRMAN

[2] The personnel of Sub-Committee No. 3 on Zeolite Softeners, Internal Treatment, Priming, and Foaming, of the Joint Research Committee on Boiler-Feedwater Studies is as follows:

Prof. C. W. Foulk, *Chairman*, Analytical Chemistry, The Ohio State University, Columbus, Ohio.

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Dr. A. M. Buswell, Director, Water Survey, Urbana, Ill.

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Prof. H. L. Olin, Department of Chemistry, The State University of Iowa, Iowa City, Iowa.

Prof. O. R. Sweeney, Chemical Engineering Department, Iowa State College, Ames, Iowa.

C. P. Van Gundy, Engineer of Tests, Baltimore & Ohio Railroad, Baltimore, Md.

THE report of this Committee published in 1926 outlined the present state of our knowledge of foaming and priming and indicated the general lines of research that should be followed in the investigation of this subject. Last year's report[3] reviewed two important papers on foaming and priming and advanced a working hypothesis of foam formation to serve at least as a tentative guide in investigation. During the last year this hypothesis has been used at the Ohio State University in an extended experimental study of film formation[4] in water solutions. This study was completed too late to be written up at this time. It can be said, however, that the hypothesis presented in last year's report has proved of value in research and may now perhaps be raised to the dignity of a general theory of

[3] Trans., A.S.M.E., vol. 50, no. 15, May–August, 1928, Paper No. FSP-50-46.

[4] Doctor's dissertation by V. L. Hansley.

film formation. It should nevertheless be emphasized at this point that laboratory experiments in glass vessels cannot solve the industrial problems of boiler-water chemistry, however important they may be in suggesting general principles. Experimental and full-size boilers must be used before decisive results are reached.

The Committee has as yet made no reports on those topics other than foaming and priming that have been assigned to it, namely, the internal treatment of boiler water and the use of zeolite-softened water. These topics have nevertheless been under consideration and it is hoped that during the next year reports on the present state of the art of these two subjects can be prepared. It should also be stated that a topic closely allied to boiler-water subjects has been taken on by Committee No. 3, namely, the art of steam purification.

Future Work of the Committee

The Committee has two possible futures, one to continue as best it may without funds for research and the other, the happy contingency of having such funds at its disposal. In the first situation little or nothing can be expected beyond the preparation of reports setting forth the present-day knowledge of the topics assigned to it. This is a slavish and somewhat thankless task for busy men to perform but nevertheless we shall do the best we can. When, however, funds become available for the building of experimental boilers and for experimental work with commercial boilers, we suggest the following as important points of attack:

1 Comprehensive boiler experiments on the causes and prevention of wet steam

2 Comprehensive boiler experiments on ways of preventing the adherence of scale.

Topic No. 1 is seen to be a general statement of the problem of foaming and priming. Topic No. 2 is, however, but one of the subdivisions of the broader subject of internal treatment. It is suggested as the most important one for the first attack on the problem.

Committee No. 3 has frankly been waiting the completion of the financial program because there is little or nothing that can be done without money for equipment and technical assistance in carrying on experimental work.

Municipal Water Supplies and the Effect of Trade Wastes in Relation to the Use of Water in Power-Plant Practice

Progress Report of Sub-Committee No. 7 on Municipal Water Supply in Relation to Boiler Use[5]

By V. BERNARD SIEMS, NEW YORK, N. Y., CHAIRMAN

IN THE two former reports prepared by this Committee attention has been directed to the important effects of trade-waste pollution upon the quality of water employed for boiler-feed use and for cooling water in surface condensers. During the past year data have been accumulated on this subject which clearly indicate the growing importance of these problems. There are a large number of contributing factors to pollution of this kind but all of them may be classified under the following general headings:

(*a*) Acid wastes and salts which are converted into acid under the influence of heat

(*b*) Strong alkaline wastes

(*c*) Waste liquors high in organic solids, especially volatile organic material

(*d*) Liquids containing relatively large amounts of suspended solids, either mineral or organic.

The detrimental effect of these by-products is resulting annually in heavy financial losses due principally to corrosion and to increased maintenance and operation of boilers, condensers, and appurtenances.

A number of state and governmental agencies are actively engaged in attempting to reduce the pollution now discharged into surface streams, but such activity is confined almost exclusively to the health aspect of the problem with little or no consideration to the suitability of the supplies from the viewpoint of the industrial user. In the majority of cases, control of trade-waste pollution for sanitary reasons will improve the quality of the water for industrial use. The problems are not, however, strictly analogous and the desired result from the viewpoint of the industrial user may not be reached unless special consideration is given to these requirements.

The following specific feedwater problem and condenser operation difficulties encountered in steam-station practice have been brought to the Committee's attention:

(*a*) Active corrosion of superheater tubes resulting from high CO_2 and volatile corrosive organic solids due to direct trade-waste pollution of surface streams above the intakes of power houses

(*b*) Operating troubles in keeping surface-condenser tubes clean due to oil deposits, especially in the presence of suspended solids

(*c*) Trouble from prolific growths of algae and similar microorganisms resulting from organic trade waste, seriously affecting the efficiency of water purification apparatus and condensers.

[5] The personnel of Sub-Committee No. 7 on Municipal Water Supply in Relation to Boiler Use, of the Joint Research Committee on Boiler-Feedwater Studies is as follows:

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H. B. Lawrence, Henry L. Doherty & Company, 60 Wall Street, New York, N. Y.

S. T. Powell, Consulting Engineer, 213 St. Paul Place, Baltimore, Md.

D. N. Randolph, Water Purifying Department, Wm. B. Scaife & Sons Company, Oakmont, Pa.

Caleb M. Saville, Manager and Chief Engineer, Water Works, 53 N. Beacon Street, Hartford, Conn.

Isaac S. Walker, New Chester Water Company, Drexel Building, Philadelphia, Pa.

A number of important studies have been inaugurated during the past year which give promise of success in controlling the contamination of many of the more grossly polluted surface streams. Cooperation of the industries responsible for the pollution with state and Federal authorities appears to have accomplished the best results This plan of attack has a marked advantage over dogmatic legislative enactment of remedial laws.

At the present time the states of Ohio, Pennsylvania, West Virginia, Kentucky, Maryland, and New York are bound together in an agreement of uniform policies concerning interstate stream conservation. A number of other states may shortly join this group. Exhaustive studies carried on at the Wisconsin and Maryland State Departments of Health during the year have offered a number of solutions to specific trade-waste problems, including wastes from paper mills and canneries, acid wastes from steel mills, starch wastes, and other industrial by-products.

The states of Ohio, Pennsylvania, New York, Maryland, Massachusetts, Illinois, and Wisconsin have been particularly active during the past year in attempting to control the number of trade-waste nuisances. The nature and magnitude of the undertaking preclude any hope of complete success for many years to come but continued gross pollution may be eliminated by such cooperative effort.

The Fox Valley Federation is an interesting organization formed in 1923 with the ultimate object to provide pure water, to end organic-waste pollution, to regulate and control stream flow, and to restore recreation areas. This federation put through the general legislative statute providing for the organization of a conservancy district. Similar organizations, if properly set up for the specific purpose of eliminating pollution detrimental to industrial use of water, offer promise of a practical solution for the protection of surface water courses.

Much attention has been given to oil pollution of streams in the Middle West oil fields and oil refineries discharging wastes into the streams on the Atlantic seaboard. This problem is a perplexing one because of the difficulty of adequately treating these liquors and the large volume of the wastes to be purified.

There are no general methods of trade-waste treatment owing to the diversity of the problem and the variety and complexities of the constituents in the polluting liquid. Each case must be considered as an independent research study. Considered broadly, the most difficult problems to be encountered are those which involve the disposal of organic solids which are volatile under the influence of heat. Although adequate treatment of many of these trade wastes is possible, the cost of treatment is prohibitive or at least is so considered by those who have not given sufficient study to effective methods of disposal.

The most outstanding fact that has been brought to light in the Committee's work is the revelation that in practically all trade-waste problems that have been intelligently studied there has resulted a marked financial saving due either to the recovery of valuable by-products or to reduction in the cost of plant operation from better control of the processes contributing to the stream pollution. As a graphic example of this condition two cases may be quoted which have come to the Committee's attention. There has resulted in one small steel mill a net annual saving of $25,000 in pickling acid, resulting directly from better control of the pickling process in an attempt to reduce stream pollution by pickling-liquor wastes into a small surface stream. Still further reductions are expected at this plant. The company has now decided to carry on a constructive research program to eliminate completely these by-products by conversion into valuable materials.

A more impressive example is the success obtained in a cooperative study of wastes from a group of mills. This has resulted in an annual saving of over three million dollars in the treatment of their liquors. This particular problem has in the past been considered an unsolvable one which for many years has caused gross pollution of many surface water streams.

A number of tannery-waste testing stations have been established throughout the country and a number of treatment plants have been built and operated with a fair degree of success. Tannery wastes are objectionable when present in boiler-feedwater supplies and in water which is to be employed as circulating water in surface condensers. Gross pollution of streams from such wastes should be controlled, especially since methods for control are now available.

Packing-plant wastes, although not an important problem over wide areas, is a highly important one in restricted sections of the country. These by-products are highly objectionable from the standpoint of boiler-feedwater use. The grease and other organic solids are removed with difficulty and small amounts of grease remaining in the water after treatment cause serious operating difficulties, especially from the effect of these products in promoting priming and foaming and from corrosion resulting from their decomposition.

Much study has been given recently to the control of pollution from milk-product wastes. These wastes may be adequately treated by chemical precipitation followed by tank treatment and filtration. Cannery wastes such as these contain a high organic content highly objectionable when discharging into streams which are used as a source of supply for power-house use. The treatment of these wastes, although somewhat difficult, is feasible, and pollution of this kind should be given careful consideration by power-house operators, since the products, especially when decomposed, are highly objectionable from the organic matter and acid produced.

Gas-house wastes and wastes of a similar kind are difficult to control but the liquids when present in feedwater supplies are highly objectionable and make adequate purification difficult. It is the hope of the committee to collect and correlate data on trade-waste treatment of this and other by-products during the ensuing year.

The Committee has been impressed with the fact that heavy financial losses are being sustained in power-plant operation due to diversity of trade-waste pollution of surface-water courses, and there is little hope for immediate relief without the concerted interest of the industry in the intelligent and cooperative study of these problems.

The Committee presents the following specific recommendations for control of the problem:

1 Cooperation with state and Federal bureaus in intelligent study, leading to constructive legislative enactment for general stream-pollution control
2 Collection and interpretation of steam-station records to determine the effect of trade-waste pollution on the cost and operation of boilers, condensers, and allied equipment
3 Cooperative study by steam-station operators of surface water in areas grossly contaminated, to allocate the source and degree of trade-waste pollution
4 Distribution of scientific literature on trade-waste control to industries responsible for such pollution
5 Enlisting the interest and cooperation of industries contributing to the pollution of surface water.

Bibliography

A selected bibliography is included as part of the Committee's report, listing the important trade-waste studies reported during the current year.

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Standard Methods of Water Analysis

Progress Report of Sub-Committee No. 8 on Standardization of Water Analysis[6]

By HAROLD FARMER, PHILADELPHIA, PA., CHAIRMAN

DURING the past year the Committee on Standard Methods of Water Analysis has devoted its efforts to a satisfactory test for dissolved oxygen in boiler feedwater.

The Committee presents the following method as a *tentative method* for trial and criticism by the various industries represented by the joint societies.

The method as it is here presented has been accepted by the N.E.L.A. Chemists' Committee as a tentative method of that group, and at the present time is in daily use in the majority of the large power plants.

Reports from the power plants indicate that the tentative method is very satisfactory. This is of considerable interest because the set-up in a modern power plant is such that any erroneous results would be readily apparent. The Committee, therefore, feels that the use of this method by the N.E.L.A. group will furnish some valuable information.

It was the early object of this Committee to indicate or develop a test that would be satisfactory for water-works practice, railroads, power stations, and other uses. It may be that a universal test is not possible; however, only general use of the tentative test by the interested groups will indicate the limits and possibilities of the proposed test.

The Committee has received some criticisms of the proposed method. These are chiefly relative to the reagents employed and the reactions involved. One objection to the method is here cited because it illustrates the need for further research on this subject:

"Objection to the use of manganese chloride and sulphuric acid or to hydrochloric acid, on account of the danger of introducing *free chlorine* which liberates *iodine*."

Such being the case it would appear to be more desirable to employ manganese sulphate and sulphuric acid. On the other hand, we have considerable evidence to show that operators using hydrochloric acid and manganese sulphate, or manganese chloride and sulphuric acid have no difficulty in obtaining *zero oxygen*.

The objection here raised is of considerable interest and shows the necessity for some extensive research work before any *standard* for dissolved oxygen in water can be acceptable.

In the absence of any standard test for dissolved oxygen in boiler feedwater we recommend that the tentative method herein described in detail be given general use wherever possible.

CHEMICAL REACTIONS INVOLVED IN THE WINKLER METHOD

(1)	$MnCl_2 + 2\ KOH$	=	$Mn(OH)_2 + 2\ KCl$
	manganous chloride + potassium hydroxide		manganous hydroxide + potassium chloride
(2)	$2\ Mn(OH)_2 + O_2$	=	$2\ H_2MnO_3$
	manganous hydroxide + oxygen		manganous acid
(3)	$2\ KCl + H_2SO_4$	=	$K_2SO_4 + 2\ HCl$
	potassium chloride + sulphuric acid		potassium sulphate + hydrochloric acid
(4)	$Mn(OH)_2 + H_2SO_4$	=	$MnSO_4 + 2\ H_2O$
	manganous hydroxide + sulphuric acid		manganous sulphate + water
(5)	$H_2MnO_3 + 4\ HCl$	=	$MnCl_2 + 3\ H_2O + Cl_2$
	manganous acid + hydrochloric acid		manganous chloride + water + chlorine
(6)	$2\ KI + Cl_2$	=	$2\ KCl + I_2$
	potassium iodide + chlorine		potassium chloride + iodine
(7)	$2\ Na_2S_2O_3 + I_2$	=	$Na_2S_4O_6 + 2\ NaI$
	sodium thiosulphate + iodine		sodium tetrathionate + sodium iodide

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Tentative Method for Determination of Dissolved Oxygen in Boiler Feedwater

SCOPE

The presence of dissolved oxygen in water used for boiler feed is objectionable, since it increases the corrosive properties of the water. It is essential in steam-station operation that the dissolved oxygen content of the feedwater be kept as low as possible. In order to control the dissolved oxygen, tests should be made at frequent intervals.

APPLICATION OF TEST

The application of the Winkler method, herein described in detail, is intended for use on all boiler feedwater supplies except raw water.

If organic matter, nitrites, or iron salts are present in appre-

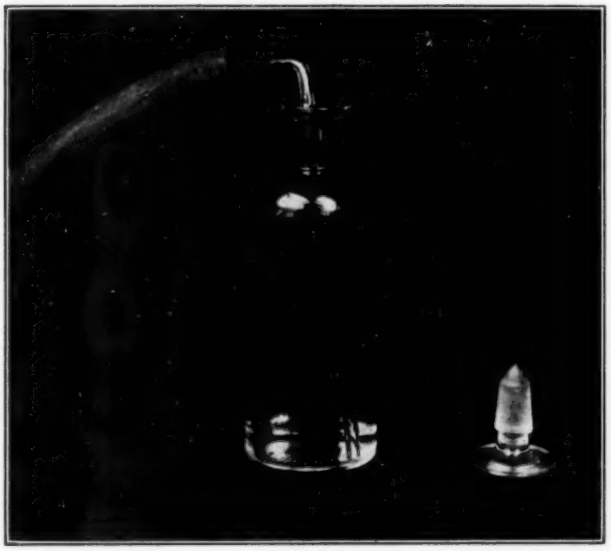


FIG. 1 SET-UP OF BOTTLE FOR COLLECTING WATER SAMPLES FOR DISSOLVED OXYGEN SHOWING STOPPER GROUND TO POINT



FIG. 2 SAMPLE OF WATER BEFORE TREATMENT WITH REAGENTS TO FIX DISSOLVED OXYGEN SHOWING PIPETTES FOR REAGENT ADDITIONS

ciable amounts, the Rideal-Stewart method (appearing in "Standard Methods of Water Analysis," A.P.H.A., 6th edition, 1925) should be applied.

After further investigation it may be possible to indicate the permissible amounts of organic matter, nitrites, or iron salts under which the method outlined may be employed.

WINKLER METHOD FOR DETERMINING DISSOLVED OXYGEN

The Winkler iodimetric method depends upon the liberation in the solution of an amount of iodine exactly proportional to the amount of oxygen dissolved. Chemically, it amounts to the oxidation of the manganous-chloride solution to manganic acid by the oxygen present, and a subsequent reduction of iodide by the manganic acid. The starch indicator indicates the presence of iodine, and the standardized solution of sodium thiosulphate has a definite iodine absorbing value. The calculations by the formula are in terms of oxygen, but the intermediate steps are computed in terms of iodine.

The chemical reactions are given on page 6.

REAGENTS

1 *Manganous Chloride.* Dissolve 425.5 grams of crystalline manganous chloride c.p. ($4H_2O$) in 1000 cc. of distilled water.

2 *Alkaline Potassium Iodide.* 700 grams of potassium hydroxide c.p. and 150 grams of potassium iodide c.p. are dissolved separately in distilled water and the mixture diluted to 1000 cc.

3 *Sulphuric Acid.* Equal volumes of c.p. concentrated acid (sp. gr. 1.84) and distilled water.

4 *Starch Solution.* Dissolve 5 grams of Litners soluble starch

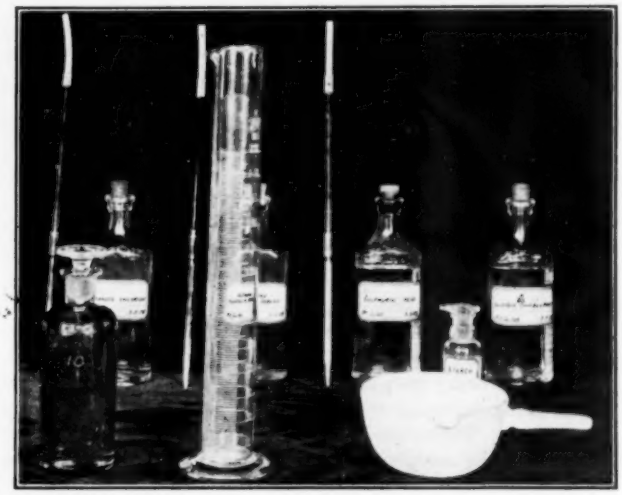


FIG. 3 SHOWING SAMPLE OF WATER AFTER REAGENTS HAVE BEEN ADDED TO FIX DISSOLVED OXYGEN AND APPARATUS AFTER MEASURING OUT NECESSARY VOLUME FOR THE TEST



FIG. 4 SAMPLE TRANSFERRED TO CASSEROLE SHOWING APPARATUS AFTER ADDITION OF REAGENT TO ESTIMATE DISSOLVED OXYGEN

in 50 cc. of distilled water to make a thin paste and then pour into 200 cc. of boiling distilled water and continue boiling for 4 or 5 min. When cold add a few drops of chloroform and keep this solution in a dark glass-stoppered bottle. Starch solution does not keep well and should be made up fresh every week.

5 *Sodium Thiosulphate: 0.025 N Solution.* Dissolve 6.205 grams of chemically pure recrystallized sodium thiosulphate in distilled water and dilute the solution to 1000 cc. with freshly boiled and cooled distilled water. Each cc. is equivalent to 0.2 mg. of oxygen or to 0.1395 cc. of oxygen at 0 deg. cent. and 760 mm. pressure. Inasmuch as this solution is not permanent it should be standardized weekly against a 0.025 N solution of potassium dichromate or 0.025 N solution of potassium permanganate. If special investigations are being made the thiosulphate should be calibrated daily.

Note: When facilities for the preparation of the above reagents are not available they can be purchased from any reliable chemical supply house.

Collection of Sample

Water supplied in excess of 125 deg. fahr. and above atmospheric pressure shall be cooled through a copper coil condenser, throttled at the discharge end, to room temperature.

A narrow-necked glass-stoppered bottle of 250 to 300 cc. capacity is satisfactory. To the end of the sample connection, attach by means of heavy-walled rubber hose, a glass tube, which will permit filling the bottle by displacement of the air and water by overflowing. The minimum rate of filling should be 250 cc. per 30 sec. and the sample bottle should be permitted to overflow not less than 5 min. This procedure insures the complete removal of air in the bottle and subsequent solution in the water of such air. Make certain that the sample line from the source of sampling to the bottle has been freed of air bubbles and deposits by a strong flushing of the line. After the water has been displaced in the specified time, the glass tube admitting the water is removed and the glass stopper quickly inserted in place to prevent exposure of the sample to air. Make certain that no air space is left between the stopper and water. The glass stopper should be ground to a point or cone to eliminate trapping of air.

The next step of adding the reagents should proceed immediately.

Adding the Reagents

Remove the stopper from the sample bottle and add immediately 2 cc. of manganous chloride and 2 cc. of alkaline potassium iodide in the order mentioned and quickly replace the stopper. The pipettes used to deliver these reagents should extend to the bottom of the bottle and the additions made in sequence with individual pipettes should be rapid. A short length of rubber tubing should be attached to the upper end of the pipettes to reduce the possibility of drawing the solution into the mouth. Do not expel solution by breath.

After the addition of the alkaline potassium iodide, replace the stopper, mix by shaking, and *permit the precipitate to settle.* Then add 2 cc. of sulphuric acid and mix by shaking.

The procedure to this point must be carried out in the field, but after the acid has been added and the stopper replaced the rest of the test shall be performed as soon as convenient, preferably within one hour.

Titrating the Sample

Measure 250 cc. of the sample and transfer to a 300-cc. porcelain casserole. (The entire sample may be conveniently transferred and titrated if the volumes of the sample bottles are previously determined.)

Titrate the sample with 0.025 N sodium thiosulphate until the solution becomes a pale yellow color, then add 3 cc. of the starch solution and titrate until the first disappearance of the blue color.

If the sample contains oxygen in excess of 0.5 cc. per liter the water will have a decided yellow color.

If zero or only traces of oxygen, the water will not appear yellow. The starch solution should then be added and if any blue color appears thiosulphate should be added until the blue color disappears. The difference between the initial and final

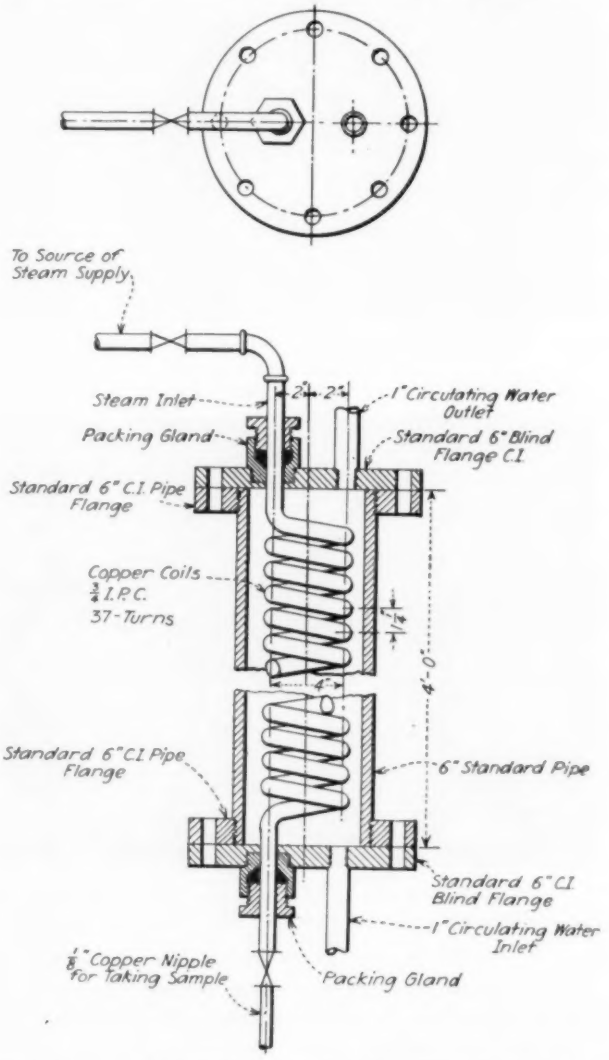


Fig. 5 Sketch of Cooling Coil for Dissolved Oxygen Test

readings of the burette will be the number of cubic centimeters of thiosulphate used, which is proportional to the amount of oxygen present.

Calculations

If 250 cc. of water were titrated the calculations will be as follows:

$$\frac{1000}{250} \times (\text{cc. of 0.025 N Thio.} \times 0.1395)$$

Apparatus

= cubic centimeters of oxygen per liter standard condition (0 deg. cent. and 760 mm.). Express individual results to the second decimal place. Report values below 0.03 cc. as "trace."

NOTES WHEN MAKING THIS DETERMINATION

1 When the manganous chloride and alkaline potassium iodide are added a white precipitate will be noticed (manganese hydroxide).

2 If oxygen is present a brown-colored precipitate will be noticed which is manganous acid depending upon the amount of oxygen present.

3 The addition of the sulphuric acid dissolves the brown precipitate and sets free an amount of iodine equivalent to the quantity of oxygen in the sample.

4 The solution at this time will be yellow in color; the depth of color depending upon the amount of oxygen present.

5 If oxygen is present and starch is added to the solution at this stage the color will change to a deep green or blue color depending upon the amount of iodine (oxygen) present.

TOLERANCE

With care, the above procedure will give reproducible results as low as 0.03 cc. of oxygen per liter.

The illustrations in Figs. 1 to 4 show the apparatus and steps in the operation. An itemized specification of the apparatus follows. Sufficient construction details for making the cooling coil are shown in Fig. 5.

1 Copper cooling coil.

2 Rubber tubing, heavy wall, 1/4-in. bore, 1/8-in. wall.

3 Flint-glass bottles, narrow mouth with mushroom glass stopper ground to a point. Capacity 8 ounces for collecting samples.

4 Flint-glass bottles, narrow mouth for rubber stoppers, capacity 8 ounces for reagents. (The number of sample bottles required is optional depending on the number of sampling points in the station.)

5 One bottle dropping TK type, flat glass stopper, 125 ml. capacity, for starch solution.

6 One glass cylinder, heavy base, and lip for pouring, capacity 250 ml., subdivisions 2 ml.

7 One casserole, porcelain, glazed both sides, flat bottom pour out, and porcelain handle, capacity 375 ml. Size No. 4.

8 One burette with straight glass stopcock, capacity 10 ml., subdivision 0.05 ml.

9 One apparatus support, rectangular base 4 × 6 in., length of rod 18 in.

10 One universal burette clamp, rubber-covered jaws.

11 Four pipettes, transfer type, bulb in center, graduated to deliver 2 ml. fitted with 2 in. of pure gum-rubber tubing.

Bibliography

Report of Sub-Committee No. 9 on Bibliography[7]

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THE extension of the Engineering Index Service has made possible more frequent publication of the bibliographies issued by the Committee. The chairman receives daily from the Engineering Index Service every indexed item pertaining to subjects of interest to the technical committees and from time to time items from *Chemical Abstracts* which are furnished by Dr. A. M. Buswell.

Up to October 1, 1928, there had been issued 13 of these bibliographies containing a total of 368 items. Since last year's report 8 bibliographies containing 244 items have been issued.

The Committee expects to continue the service during the coming year and will be grateful for any suggestions as to means by which its usefulness can be increased.

Discussion

SAMUEL B. APPLEBAUM.[8] One of the subjects that Committee No. 3 handles is zeolites. A sub-committee has been formed for that purpose, and as chairman of that sub-committee I wrote a report which was confined to a statement of what is known about zeolite water softening, as used in the industry today. But in addition to the work of designing, installing, and operating water-softening plants, there is a great deal of work that may be done of a research nature to find the theoretical explanations for some of the practical results that have been empirically obtained. This work belongs, more or less, in the field of pure research which the committee is anxious to do with the assistance of university men when the financial facilities are at hand.

For example, from a practical point of view we know what causes a boiler to foam and what steps should be taken to stop a boiler from foaming, but theoretically the explanation as to the fundamental cause of foaming has not yet been worked out, and research work on this subject may result in an entirely new point of view which may be of great help to the practical boiler operator. A boiler operator knows that to prevent foaming he must keep oil out of his boiler, he must keep the concentration of soluble solids down, and he must keep the water in the boiler free from suspended matter. But the influence that various salts may have, such as sodium sulphate, sodium carbonate, or calcium salts, and the influence of other similar factors can best be determined in a research laboratory.

Frequently pure scientific research results in disclosing factors of a very novel nature which open up a new field of application. As an illustration of this, the origin of zeolites for water softening may be mentioned. A professor in a college was engaged in research work in agricultural chemistry and developed a synthetic zeolite to prove his theory as to the reasons for the different behavior of various soils. From this purely scientific work the application of zeolites to water treatment developed and resulted in an entirely new method of softening water. Thus, though one starts far away from a practical result, pure scientific research may result in a very practical suggestion.

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However, in the experience of the average commercial company manufacturing water-treatment apparatus, new problems do arise from time to time, which makes this field of water treatment so intensely interesting. As an illustration, the use of natural impure brine for the regeneration of zeolites may be mentioned. This problem was presented to us not long ago by an oil company out West which had such impure brine available, because oil wells do contain brine in various stages of their development. The problem presented was whether such brine could be used to regenerate zeolites. It has been the practice, in offering zeolite apparatus, to specify pure salt; that it, containing not less than 98 per cent sodium chloride. The brine from the oil wells contained considerable amounts of calcium and magnesium hardness, and the effect of such hardness on the regeneration had to be investigated. Large samples of brine were obtained and regeneration experiments conducted, and the design of the zeolite softener, together with the regeneration equipment, was based on these experiments. A large zeolite plant has been in use at this refinery since then, successfully operating with this impure brine for regeneration purposes. This experience has suggested a new field for the use of natural sea water for regenerating zeolites, because sea water has about the same amount of calcium and magnesium salts as the natural brine from the oil wells. We ran experiments with sea water for regeneration purposes, and on the basis of those experiments, we installed four large plants using sea water for that purpose. Wherever industrial plants are located along the Atlantic or Pacific coasts and where these plants can obtain strong sea water, not diluted by fresh river water, there is the possibility of using such sea water for regenerating zeolite plants, and thus the zeolite plants can function without any cost for chemicals.

Another problem which has been presented to manufacturers of water-treating equipment recently is the suggestion to increase the sulphate-carbonate ratio in waters used in boilers. This has required considerable experimentation to develop reliable acid feeds. Boiler operators are averse to using acid unless they can be shown that it can be fed with impunity. To do this, feeds of a dependable and accurate nature had to be designed and tested out, and the materials out of which these feeds were built had to be chosen to withstand the corrosive effects of acid, and means also had to be provided for insuring non-corrosion of the water system using the acid-treated water.

A successful method of accomplishing the latter result was worked out, and this method consists of returning a portion of the alkaline boiler blow-off water into the acid-treated feedwater stream. This is particularly practicable where the boilers are being blown off continuously and where a portion of the continuous blow-off stream may thus be conveniently diverted and pumped back into the outlet of the feedwater heater. Since the boiler blow-off water is concentrated and alkaline, a mixture of only a small amount of this water with the feedwater raises the pH value of the feedwater, so that considerable assurance is thus provided of the water being non-acid.

With the usual acid feed, the precaution should be taken of thorough mixture of the water with the acid so as to avoid streaks of acid passing through the feedwater system, and in addition means should be provided for aerating the water after acid treatment to rid it of the CO_2 which has formed by the reaction between the acid and the bicarbonates in the natural feedwater. Then in most boiler plants the aerated, acid-treated water is passed into an open heater where all the oxygen and free CO_2 is released by heating. At that point the small stream of alkaline blow-off water can be added, which raises the pH value and thus protects the feedwater system and economizers. The boilers themselves usually would not need such protection because there would be a storage of alkaline water in them.

With regard to the illustration showing a sketch of a cooling coil to cool hot water in which oxygen tests must be made, we have used a similar coil, but with a slight modification. Instead of using two stuffing glands, one at the top flange and the other at the lower flange, we have used an open piece of 8-in. pipe with a cap only at the bottom, and the inlet and outlet of the coil both pass out of the open top of the pipe. This has a further advantage in that the coil acts as a trap, so that if one is trying to condense steam to determine oxygen in the steam, a more complete dissolving of the gases as the steam condenses is insured.

While on the subject of oxygen testing I have a suggestion to make for a rapid qualitative method, which our laboratory has devised, of determining the absence of oxygen in feedwater. In this method the reagents are a mixture of pyrogallic and sulphuric acid and some potassium hydrate solution. These two reagents are added in succession, and if a brown coloration is obtained it indicates the presence of dissolved oxygen. If the water remains either colorless or with a slight violet or blue coloration, then the water is free of dissolved oxygen.

We had occasion to use this method in several plants where it was desired to remove oxygen by chemical means after the feedwater left the open heater. The ordinary open heater may leave a small amount of oxygen in the feedwater, ranging from several tenths of a cubic centimeter of oxygen per liter of water up to about 1 cubic centimeter per liter. Where steel tube economizers are employed these small amounts of oxygen may cause corrosion of the tubes. One of our clients asked whether they could remove this small amount of oxygen chemically, rather than install the apparatus on their open feedwater heater to permit the heater itself to remove this oxygen more completely. We accomplished this by feeding small amounts of sodium sulphite or sodium bisulphite, which, in the presence of suitable catalyzers, rapidly reacted with the oxygen and thus rendered the water oxygen-free.

The foregoing qualitative test was worked out to help the operators in these plants to determine quickly whether their chemical feeds are working properly, so as to be certain that the oxygen was completely removed from the water.

JOSEPH D. YODER.[9] The illustration given in the progress report of the Boiler-Feedwater Studies Committee shows a method of obtaining and condensing a sample of steam with a view of determining the oxygen in the steam, although the test refers only to determining the oxygen in water. The inference is that if steam is condensed and a sample collected in the manner shown by the diagram, the amount of oxygen originally in the steam can be accurately determined by analyzing the condensate for oxygen by the Winkler test.

Observations that our engineers have made in the field where such attempts were made to determine oxygen show that this method is not dependable, for the reason that the oxygen in the steam is disposed to bubble up through the condensate without being dissolved. The oxygen is not absorbed by the condensate nearly as rapidly as might be expected by the person not having actually made the attempt to determine the oxygen in this way. My opinion is that the method illustrated is worthless for the purpose of determining oxygen in steam.

HAROLD FARMER.[10] Mr. Yoder's statement that the sketch of the condensing coil shown in the report is presumably for steam whereas the method refers only to water needs explanation. The cooling coil is intended for use with hot water (above 125

[9] Manager, Water Purification Department, Cochrane Corporation, Philadelphia, Pa. Mem. A.S.M.E.

[10] Chief Chemist, Philadelphia Electric Company, Philadelphia, Pa.

deg. fahr.), and no mention is made of its use for sampling *steam* in the report.

Unfortunately the sketch, employed to illustrate only the principle of the cooling coil, was one which we had employed for steam, and there appears a note on the sketch "steam inlet." This should have been changed to read "source of hotwater supply."

JOSEPH D. YODER. In discussing the report of Committee No. 3 on zeolite softeners, I would say that it is unfortunate that Professor Foulk is not present to read his paper, for I have heard that his recent experiments are very interesting and valuable. A great deal should be accomplished by attacking the problem of foaming in a scientific and unprejudiced manner. The author has nothing to guide him as to the way in which he should go except a desire to find the truth. With this in mind I predict that his accomplishment will be well worth while.

The foresight of the chairman of the Executive Committee of the Joint Research Committee on Boiler-Feedwater Studies, Mr. Sheppard T. Powell, who is also absent, is commendable in that he has succeeded in holding together a large committee to give consideration to a number of subjects related to feedwater treatment. The broad scope of the work and the variety of subjects considered by the committee is an active example of the fact that feedwater treatment is not merely the simple problem of preventing scale in boilers. It covers much more.

It involves a genuine, complete, and adequate protection of the boiler metal. This demands the prevention of deposits that would interfere with the free exchange of heat from the metal to the water. It demands that the metal be protected from pitting and corrosion and calls for the maintenance of boiler-water conditions that guard against embrittlement of boiler plate. All of these problems must be properly solved and at the same time a condition of boiler water must be maintained that avoids priming and foaming and insures steam as dry as possible.

The subject of foaming and priming of boilers demands a healthy respect from every engineer in water purification. Its solution is difficult and complicated. There is a great variety of possible causes of foaming and priming, of which several are likely to be active at every plant.

In practice it is impossible to isolate the various factors which may cause foaming and priming, and therefore to properly evaluate each one. Possibly Professor Foulk in his laboratory will be able to do that by working with the individual factors, and then in learning the value and characteristics of each, he may be able to reconstruct the problem as a whole, and thus be able to predict with a fair degree of certainty whether under any given set of conditions the boilers will or will not prime, and if priming does occur, just what can be done to prevent it.

Some of the factors which have an important bearing on foaming and priming and to which Professor Foulk is undoubtedly giving his consideration are sodium salts, the suspended solids, organic matter, the rate of liberation of steam per given unit of water surface, uniformity of steam load, design of boiler, etc. Many of these factors come beyond the province of the water-purification engineer, as he can only be concerned with the improvement of water supply.

It must be assumed that the demand for steam will be made as uniform as possible, and likewise the firing. The factors with which the water-purification engineer is therefore chiefly concerned are the sodium salts, the suspended matter, and the organic solids, the effects of which must be observed in connection with the rates of steaming the boilers.

Among water-purification engineers there seem to be two schools; one places the emphasis chiefly on the suspended solids as the cause of foaming and priming and advocates that the sodium salts make relatively little difference providing the suspended solids are kept as low as is possible of attainment by various methods of water purification, and the other school takes the position that the foaming is caused almost entirely by the soluble salts and that the suspended matter is of comparative insignificance.

My opinion, based upon the observation of boilers operating at steaming rates varying from 30 lb. of steam per hour per 10 sq. ft. of heating surface or nominal boiler horsepower to more than 200 lb. per hour per 10 sq. ft. of heating surface, is that both the suspended solids and the solids in solution are important contributing factors to priming. Whether a given amount of suspended matter will cause more or less priming than the same quantity of sodium salts is a difficult question to decide.

It is obviously true that the goal for the water-purification engineer should be to prepare a water that will give the smallest quantity of sodium salts and suspended matter within the boiler, if consideration alone is to be given to the foaming and priming. The problem may be stated in another way, i.e., that if water is to be treated so as to protect the boilers from scale and corrosion, that method of treatment should be selected which will introduce the smallest quantity of mineral solids into the boiler in order that the boiler may deliver as dry steam as possible.

Many of the boilers now installed and operated at high ratings are supplied with distilled water, although there are also boilers operating at high rating evaporating more than 200 lb. of water per hour per 10 sq. ft. of heating surface which are fed with 100 per cent make-up water.

To my knowledge all of these boilers, which are necessarily equipped with water-cooled walls and screens, are fed with a water properly softened to reduce the amount of mineral solids to a minimum. If the water concentrates in the boiler to the extent that causes foaming or priming for the given rate at which it is desired to operate the boiler, it becomes necessary to blow it down and thus correspondingly dilute the water in the boiler with fresh water.

Whereas the greatly increased pressures and ratings placed upon boilers have made the task of feedwater treatment far more severe, the evidence is that the water-purification engineers have kept pace with the development of boiler design, so that if any industrial plant which is compelled to use chiefly make-up water finds it economical to install high-pressure boilers to operate at high ratings, it is entirely practical to treat the water so that a satisfactory quality of steam will be delivered at these ratings and at the same time to protect the tubes from scale.

To insure a satisfactory quality of steam it is commonly advisable to place a good-sized receiver separator or steam purifier between the steam drum and superheater so that any mineral solids which concentrate in the boiler and may be carried off as moisture in the steam will be removed before the steam enters the superheaters. Very efficient steam purifiers are now available for this work. Accordingly if the power-plant engineer takes the proper precautions for treatment of the feedwater and the purification of steam by the installation of such equipment as is now available, he can feel assured that his high-pressure high-ratings boilers may be successfully operated even though he has 100 per cent make-up water relatively high in mineral and scale-forming solids.

In discussing the report of Committee No. 8 on "Standard Methods of Water Analysis," the illustration given in this report shows a method of obtaining and condensing a sample of steam with a view of determining the oxygen in the steam, although the test refers only to determining the oxygen in water. The inference is that if steam is condensed and a sample collected in the manner shown by the diagram, the amount of oxygen origi-

nally in the steam can be accurately determined by analyzing the condensate for oxygen by the Winkler test.

Observations that our engineers have made in the field where such attempts were made to determine oxygen show that this method is not dependable, for the reason that the oxygen in the steam is disposed to bubble up through the condensate without being dissolved. The oxygen is not absorbed by the condensate nearly as rapidly as might be expected by the person not having actually made the attempt to determine the oxygen in this way. My opinion is that the method illustrated is worthless for the purpose of determining oxygen in steam.

R. E. Hall.[11] In discussing Professor Foulk's paper, permit me to present a few facts that pertain to boiling conditions in boilers.

First, I wish to emphasize the importance of the work Professor Foulk is doing in obtaining exact data in order to build a comprehensive theory. He speaks of the accumulation of existing data as a slavish task, and from personal experience I fully agree with him. But, also, the further work of choosing and inventing apparatus for obtaining new data and, with methods established, the routine of careful data taking are fully as slavish. If Professor Foulk can realize, however, how keenly we appreciate the results of his work, and how definitely they are needed as a guide in the problems confronting us, he will feel repaid, in part at least, for his effort.

I have read the thesis referred to in the footnote, and advise all of you to watch for its publication. I believe Professor Foulk has struck at the very root of this problem and has presented supporting data to prove his points. Even though funds are not available, let us hope that he will continue with the glass-boiler experiments and laboratory data which are largely directly applicable to operating practice.

Mr. Yoder has spoken of the influence of suspended matter on boiling conditions. Doubtless suspended matter is of influence, but let us consider two factors relating thereto, namely, its amount and its character.

In any well-operated boiler the suspended solids in the boiler water will not amount to more than 500 p.p.m., or as an extreme maximum, 1000 p.p.m., and usually very much less than the lower figure. On the other hand, concentrations of dissolved solids vary from a few to several thousand parts per million. As dissolved solids in general exercise greater influence on boiling conditions than suspended solids, the conclusion is unavoidable that, in an uncontaminated water, the effects of dissolved solids far outweigh those of suspended solids.

If however, contamination is present in the form of soaps, or of oil, adsorption thereof by the suspended material may and practically always does occur. This destroys homogeneity of distribution of the suspended material in the boiler water and gives rise to segregation thereof at the steam-separating surface because the particles are no longer easily and uniformly wetted by the boiler water. Under these conditions, with but a few parts per million of dissolved solids in the boiler water, very bad boiling conditions may obtain. Of course, the answer is stoppage of the contamination before its entry into the boiler water.

Two materials common to boiler water are of interest in connection with foaming qualities of the water. Hydroxide plays a multiplicity of roles. In low concentration, it is protective to the boiler metal; in higher concentration, it may deteriorate the metal. It saponifies any organic fat—for instance, that used in compounding a cylinder oil—thus forms soap, and thereby works havoc with boiling conditions.

In conjunction with calcium, however, hydroxide makes the water boil very much more smoothly than the same water would boil without the hydroxide. Likewise, if the calcium is removed, boiling conditions become much worse. These observations probably are an explanation why sodium carbonate has been considered a primary cause of foaming. It is doubtless the removal of calcium thereby that increases foaming rather than any intrinsic property of the sodium carbonate itself. Undertreated waters give better boiling conditions than fully treated waters, doubtless because of the calcium content of the former.

If to the components calcium and hydroxide in the boiler water we add one of the tannates, as cutch, or any one of the multitudinous forms in which it occurs, boiling conditions are still better. In the absence of saponified material, tremendous concentrations of dissolved and suspended materials may be present, and yet boiling conditions be practically equivalent to those of pure distilled water. If the calcium is removed, however, as by addition of sodium carbonate, tremendous foaming occurs.

The reason why these effects occur must be answered by work such as that of Professor Foulk. Unless that work can point the way, there is a definite limit to what can be done chemically. This is especially true in view of the trend to higher operating pressures, and the consequent more exact limitations on the boiler water. At those pressures, calcium cannot be tolerated, concentration of hydroxide must be extremely slight, and any organic substance as tannate more readily undergoes decomposition. Thus the chemist is limited to obtaining cleanliness of the boiler water, and to maintaining those concentrations best suited to protect the boiler metal. Other measures, protective to the quality of the steam, must be mechanical.

K. Von Eltz.[12] We all hope that Professor Foulk will be able to carry on the experiments regarding foaming and priming. It would seem very important that not only each salt is tested individually in these experiments, but also a number of salts together, because only tests made thus will represent the actual boiler-water conditions. Occasionally one finds a statement such as this: The boiler is able to carry so many thousand parts of sodium chloride, or sodium carbonate, or sodium sulphate. But it is not stated how high the concentration could be if any two or all three of these salts were together in the water.

In a recent publication in Germany there was given a number of allowable concentrations for different salts. It was also stated there that the correct thing to do would be to judge, not only each salt individually, but to judge a mixture of the different salts. The allowable concentrations were greater for the lower pressures. With increasing pressures the concentrations had to be reduced.

This is contrary to the general belief that with higher pressures the danger of foaming and priming was lessened. That is not true according to these figures, which show that the concentrations had to be kept lower with the higher pressures.

This seems very important, since nowadays the tendency is to go to extremely high boiler pressure, up to 1800 lb.

Dr. Hall's observation regarding calcium seems to be borne out by our own experience. In most of the water softeners built by my company, barium carbonate is used instead of soda ash, therefore no sodium salts are added to the water by the treatment. I can say that foaming is practically unknown in boilers using barium carbonate treated water, even in spite of the fact that in some cases the raw water contains over 22 grains per gallon of sodium chloride and a considerable amount of sodium sulphate, as much as 8 grains per gallon.

Here I should state that the sodium sulphate present in the raw water is not changed into soda ash, as it is generally believed. The reason therefore is that the reaction between barium car-

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[12] Manager, Reisert Automatic Water Purifying Company, New York, N. Y. Mem. A.S.M.E.

bonate and sodium carbonate requires much time and agitation. Actually in our softeners the raw water is in contact with the barium carbonate only a short time, so that the before-mentioned reaction does not take place.

In not one of the cases mentioned, where a large amount of sodium sulphate is present in the raw water, has any sodium carbonate been found in the treated water.

Since with the barium carbonate treatment no sodium sulphate is formed, nor any free soda ash present in the treated water, boilers fed with such a water do not have the least tendency to foam even if worked at very high ratings.

Not long ago I had occasion to look over a boiler plant worked at 400-lb. pressure and having ratings up to 480 per cent. These boilers have no steam purifiers. At the time the boiler manufacturer made the test it was found that the maximum moisture in the steam going to the superheater was $^{3}/_{10}$ of 1 per cent.

Mr. Yoder mentioned that it should be the endeavor of water purification to have as little as possible of dissolved as well as solid matter in the treated water. Both requirements are met by the lime barium carbonate treatment.

DR. A. M. BUSWELL.[13] The method for the determination of dissolved oxygen as described by Mr. Farmer is essentially the same as the method adopted as a standard jointly by the American Public Health Association and the American Water Works Association. However, the American Public Health Association standard uses manganous sulphate, whereas the one described by the previous speaker uses manganous chloride. If it were possible to adopt the use of manganous sulphate in the determination of dissolved oxygen in boiler feedwater, we would then have essentially one standard method.

Unfortunately it is not possible for the American Public Health Association and American Water Works Association to change to the chloride, since we have found that the interference of certain substances, notably iron, is more serious when the chloride is used instead of the sulphate. (See Buswell and Gallaher, *Industrial and Engineering Chemistry*, 15, p. 1127; and Theriault, U. S. Public Health Bulletin 151.)

In writing the reactions for classes of engineering students the speaker has followed the plan of writing *KI* instead of *KCl* in Equation [3], thereby producing *HI*, which then reacts with the manganous acid in Equation [5], producing iodine instead of chlorine. This makes Equation [6] unnecessary. It would also seem possible to eliminate Equation [4], since it does not enter into the actual determination and merely represents the re-solution of any excess manganous hydroxide above that which is oxidized by the dissolved oxygen. This reduces the number of equations from seven to five.

HAROLD FARMER. Might I answer Dr. Buswell? So far as the use of manganous sulphate is concerned, I see no objection to that. The method as we have it here is a method which by common usage has become very nearly a standard of the utilities, and a survey showed that they were practically all using manganous chloride. We saw no reason why we should ask them to change their method until we had definite information that the same results could be obtained by using manganous sulphate. It would be very desirable if manganous sulphate could be employed, because then we would be right in line with the method of the American Public Health Association.

DR. FRANK E. HALE.[14] The following discussion is given at the request of Mr. Powell. The statements made are the result of a very wide experience in determination of dissolved oxygen, in salt water, in the polluted harbor waters about New York City, in thermocline studies in reservoirs in which the oxygen reduces to practically zero in the summer period at the bottom in contact with organic matter and also just below the thermocline where dead microscopic organisms collect, in condensed steam in heating system returns, and in corrosion tests with iron pipes, etc. Under these conditions oxygen has varied from complete absence to supersaturation, organic matter from little to large quantities, chlorides to that of ocean water, iron to many parts per million. Under all these conditions the method used has been that described in the 1912 edition of Standard Methods of Water Analysis of the American Public Health Association, including the reagents manganous sulphate and potassium hydrate and iodide, but concentrated hydrochloric acid has been used instead of 1 : 1 sulphuric acid. The reason for the use of hydrochloric acid is that sulphuric acid does not dissolve the manganese hydrate sufficiently quickly, and any undissolved oxide introduces an error.

It is my opinion that hydrochloric acid or chlorides do not interfere with the oxygen determination. Neither does ferric iron. How can iron interfere when it is possible to obtain oxygen values as low as 0.1 p.p.m. in water which has corroded iron pipe and contains 30 or 40 p.p.m. iron? The interfering effect of many substances has been overemphasized including organic matter, nitrite, etc. In the ordinary amounts found in water an error of 0.2 p.p.m. oxygen would not be exceeded, and yet it is recommended in present Standards Methods that correction is unnecessary for displacement of liquid by added reagents, which causes an error of 0.2 p.p.m. oxygen on 16 p.p.m. oxygen content, or 0.1 on 8 p.p.m. The ordinary Winkler method may be used on the great majority of waters without resorting to the permanganate–oxalate modification.

The proposed method specifies 700 grams of potassium hydroxide and 150 grams of potassium iodide, whereas the regular quantities, 360 and 100, respectively, have been found sufficient in all of our experience.

The proposed long pipettes with rubber bulbs seem to be an unnecessary and cumbersome refinement. In rapid work it is simpler to use short tubes, closing the top with the finger, and with two marks near the lower end measuring 2 cc. between them. It is only necessary to insert about 1 inch (up to lower mark) as the heavy solutions drop immediately to the bottom of the bottle. Also one precaution is not mentioned. These tubes should be rinsed on the outside under running water each time they are introduced into a bottle of water before inserting again into the reagent bottle, otherwise contamination and precipitation of manganese hydrate will result in the reagent bottles. The addition of this precipitate to the bottle will cause error.

With regard to starch solution one must guard against amidulin, a form of soluble starch giving blue with iodine, but readily oxidized by iodine to erythrodextrin. Such red end-points mean a loss of iodine. The formula we use and recommend is that of Gaston. Starting with as pure potato starch as possible to procure, rub to a thin paste with a small amount of cold water 5 grams of the starch mixed with 10 milligrams of mercuric iodide, pour into a liter of boiling water and boil for one-half hour. Allow to settle. The foregoing produces a stable starch emulsion for a considerable time.

Regarding the thiosulphate, dilute solutions do not keep well. We prefer to keep the stock as N/10 solution, which does keep well. N/100 is then made up as required. Correcting for displacement by the first two reagents added, a factor is used ranging from 0.811 to 0.813, depending on the size of bottle used.

Titrating 100 cc. instead of the entire bottle gives a chance for check in case of necessity.

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[14] Director of Laboratories, New York Water Department, Bureau of Water Supply, Brooklyn, N. Y.

In the collection of the sample, the minimum rate of filling is given as 30 seconds and to overflow for 5 minutes. It would seem more important that the flow be quiet so as not to mix air and water and therefore not too rapid. Overflowing about one-half bottle in our experience has been found ample to remove the first water in contact with air and get results as low as 0.1 p.p.m. oxygen. Oxygen does not absorb in water very rapidly.

The question of replacing the stopper in the bottle was brought up in the discussion. It should be replaced after each reagent is added, as oxygen is absorbed from the air in the alkaline condition, i.e., after adding the second reagent. Also catalytic increase of iodine takes place in contact with air after the addition of the acid.

Also if the bottle has been filled with slightly warm water, on cooling, air may be sucked into the bottle and a bubble of air will cause error.

I should like to correct the statement that a grain per gallon of soda ash was used. The dosage was half a grain. There was no excess of soda ash in the water. It reacted with the permanent hardness and merely increased the alkalinity from 8 to 15 parts per million. The total hardness of the water was 17 parts per million.

DUDLEY K. FRENCH.[15] I am very glad to be able to support the comments of Mr. Hall with considerable confirmation, both from research and practical results.

I think I may say, literally, with regard to the opportunity of observing the results in using various chemicals with waters, that there are few people who have had the opportunity to observe such conditions with as many kinds of water over as long a period of time as I have.

This period covers between 20 and 25 years and involves results on all sorts of water from Canada to South America.

With regard to foaming, I think that not enough attention has been paid to the tannin extracts. They have properties which we do not yet understand thoroughly. It is perfectly true, from a standpoint of the number of foaming waters that we have treated, that probably less than one-half of the treatment that we supply is treatment containing saponifiable oils. There are a great many cases where tannins with or without coagulants meet the situation more satisfactorily; in fact, when using saponifiable oils and castor oil is the principal in this class, we think that there must be enough lime in the water in the boiler to make insoluble soaps out of the fatty acids before the other elements upon which we depend are made effective, and we have been able to demonstrate from the analysis of sludges blown from such boilers that calcium soap is formed.

Castor oil has eeveral substances in it that other vegetable oils do not have, and it is apparently an effect of the solution of some of these in the interfacial film between the water and the steam space that permits the dropping out of some of the suspended matter and increases the ease with which the steam bubbles go through.

Frankly, I believe that the tannins do more to the surface film than the ingredients from the oil. Furthermore, I think that we are apt to overlook other effects which can be brought about by these vegetable materials. There has been a great amount of data printed about tannins. Go to almost any text on water treatment and it states that tannins are objectionable because they act as acids upon the metal. Well, the tannins are rarely present in a boiler water where there is enough acid to permit such a condition. They are almost invariably present in an alkaline condition.

In so far as their action in controlling corrosion is concerned, I might state that our experience with reference to oxygen-absorbing properties of the oxygen tannates shows that certain of the extracts are as effective as alkaline tannic acid or sodium tannic, and that the amount of oxygen that can be absorbed by as small a quantity as 1 grain exceeds 200 cc. It is really a very large amount when one considers the smallness of the material itself.

We have had opportunity to analyze waters going to boilers and waters coming from boilers, under all sorts of conditions, and it is a hope of mine that some time some condition may be worked out whereby certain types of water treated under certain conditions can be brought to the attention of some group in this committee whereby it can send out an observer and collect samples as it wishes, and analyze them as it wishes, in an effort to indicate in some unbiased, scientific way what is actually being accomplished.

From eight to ten thousand waters pass through our laboratory annually, and there are few days in which we do not make at least forty complete analyses, but many of these waters come to us with a fair background of operating and plant information, and the amount of information of a practical but unscientific nature that we collect is enormous. It is our hope that in some way means can be developed whereby this information that we accumulate can be converted into some scientific form or can be utilized so as to be of value to this committee in its investigations.

R. C. BARDWELL.[16] In stressing the report presented by Dr. Foulk, it is desired to call particular attention to the necessity for some accurate determination of the fundamental factors involved. Practically all the men and laboratories engaged in the correction of water supplies are so busily occupied in the actual operation of plants and handling matters pertaining to troubles with these supplies that they do not have the time or research facilities necessary for investigation of the fundamental facts.

In railroad practice we do know that boilers will foam at different points, probably due to three different conditions, such as (1) the amount and type of suspended matter present, (2) the concentration of dissolved solids, and (3) the working conditions. Not knowing the exact cause or the influencing ratio of the fundamental factors, it is necessary to determine the particular foaming point for the locomotives on the individual districts and apply the well-known rule-of-thumb methods for correction, which consists of either blowing down the boiler and removing the objectionable solids, preferably in accordance with predetermined program, or using a method of internal treatment which, as Mr. French says, is usually a compound of castor oil.

It is rather surprising to hear the interpretation which Mr. Hall has placed on the presence of calcium in quieting foaming in water. This is practically the opposite of our experience. We have found that, where waters were treated with soda ash alone, considerable foaming complaints resulted and this has been expected. However, on one division where this was particularly aggravated, the soda-ash treatment was changed to sodium aluminate and caustic soda. The scaling matter composed of calcium and magnesium salts was removed to a greater extent than with the soda-ash treatment. With less calcium in the water the foaming was greatly reduced.

This same treatment was extended to other divisions with similar satisfactory results. Therefore, we cannot help feeling that there is a controlling factor radically different from the presence of calcium which influences foaming, in locomotive boilers at least.

It is believed that there is a large number of fundamental facts which will require detailed investigation and impartial study to

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[16] Chesapeake and Ohio Railway Company, Richmond, Va.

determine the relative importance. I fully believe that Dr. Foulk is working along the right line and sincerely hope that funds will be available which will permit him to continue the valuable work which he has started.

It is believed that most of us have had experience with trouble caused by stream pollution. Damage to power-plant boilers along the Ohio River caused by contamination by coal mine drainage is an example.

About six years ago a conference was held in Philadelphia to consider means for offsetting the bad results from stream pollution. At that time I mentioned as an estimate an annual expense of $10,000,000 either in damage or in cost, to offset the trouble caused by sulphuric acid drainage from coal mines in the Ohio River basin. It is believed that this figure is reasonably conservative.

This is a very big problem which may require settlement by the National Government or a conference of state authorities. The information and data that are being accumulated by this sub-committee should be of considerable assistance when the time comes for definite action to be taken.

FSP-51-23

Orifice-Steam-Meter Coefficients

By ROBERT W. ANGUS,[1] TORONTO, ONT.

THE value of the thin-plate orifice as a fluid meter has been established beyond all question, and many very successful commercial applications of it are on the market. There are, however, not many tables of actual orifice coefficients, these being apparently retained by the manufacturers. Recently, the author was conducting some tests on a steam turbine of about 100 hp. which was not equipped with surface condenser, and as it was connected to the main header from a large battery of boilers, the feedwater method of measuring the steam consumption was not available. An orifice meter was then decided on, and after making the turbine tests, the orifice was set up in the steam laboratory of the Department of Mechanical Engineering of the University of Toronto and calibrated.

So far as the test was concerned, the coefficients under the test conditions were all that were necessary, but it was decided to enlarge the scope of the work and to determine the coefficients under a wide range of conditions, and this paper is the result of such work.

The orifice used in the test had a diameter of 1.838 in. and thickness of 0.015 in. It was set concentric with a 4-in. extra-heavy pipe having an internal diameter of 3.826 in. The set-up in the laboratory was therefore based on these dimensions, and a drawing of the experimental equipment is shown in Fig. 1. The 6-in. main steam pipe from the experimental boilers is shown at *S*, and by means of a 6-in. by $1^1/_2$-in. tee, connection was made to the experimental piping shown in Fig. 1. The 4-in. extra-heavy pipe started 1 ft. $10^3/_4$ in. from the center of the 6-in. main, and the 4-in. section had a total length of 7 ft. $10^3/_{16}$ in., with a pair of companion flanges in the center for insertion of the experimental measuring orifice. The pipe was again reduced to $1^1/_2$ in., which in turn delivered steam to a small steam turbine in the laboratory.

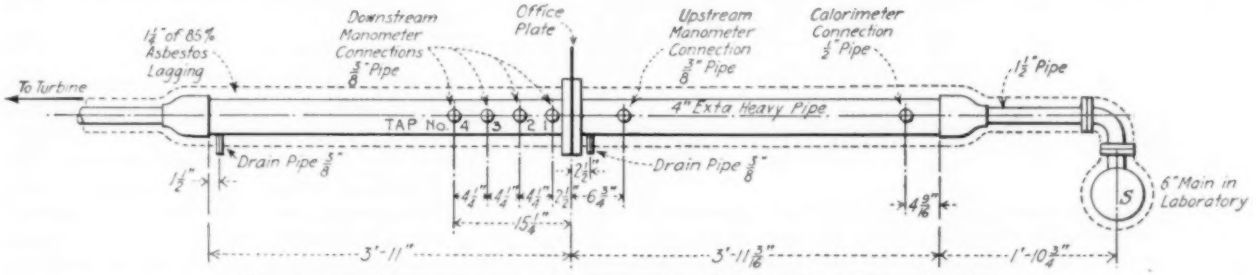


FIG. 1 PIPE CONNECTIONS FOR CALIBRATION OF ORIFICES (ALL PIPING LAGGED)

The entire piping was lagged with $1^1/_4$ in. of 85 per cent magnesia, and $^3/_8$-in. drains were placed at the two points shown, to prevent the collection of water. A throttling calorimeter was connected at the point indicated, and the orifice plates were inserted between the two companion flanges, and very carefully centered. Only one pressure tap was used on the upstream side for all experiments, this being placed $6^3/_4$ in., or 1.76 times the internal pipe diameter, above the orifice. Four taps were used on the downstream side, being placed in the positions shown in Fig. 1, and all of the pressure taps were made of $^3/_8$-in. pipe, screwed into the side of the 4-in. pipe so that they just came flush with the inside of the latter.

The differential pressures across the orifices were measured by a differential mercury manometer, reading to 0.05 in. mercury, and very great care was taken in every case to see that the connecting pipes were free of air and contained water up to the point of connection with the 4-in. pipe. Frequent readings were taken on this manometer for each test.

The orifice plates were made of galvanized iron 0.016 in. thick, the orifice being very carefully bored out in the lathe and afterward calipered, when hot, to thousandths of an inch. Each plate contained a $^9/_{32}$-in. hole as well as the orifice, this hole being placed so that its lower side was flush with the bottom of the pipe. Three orifice plates were used, with orifices A, B, and C, respectively 2.66 in., 1.838 in., and 1.33 in. in diameter, as indicated in Fig. 2.

A large number of experiments were made with the four downstream taps and the three orifices, each experiment lasting for from thirty to forty minutes. In all cases the steam pressure at the orifice was kept as close to 125 lb. per sq. in. gage as possible, and was very steady during any given test. The moisture of the steam entering the 4-in. pipe was also maintained very uniform by keeping the drip on the 12-in. main partly open; the quality remained uniformly constant at 98.5 per cent, and this quality was used in all calculations. Before each test the $^3/_8$-in. drains on the bottom of the 4-in. pipe were opened, and after they were closed the brake load on the turbine at the end of the $1^1/_2$-in. pipe was adjusted, and this load was kept constant, within a fraction of 1 per cent, during a test, thus producing a constant differential across the orifice. The steam passing through the orifice and turbine was then condensed and weighed, the average of at least eight weighings being taken in each case, and no test was used where the several weighings showed any material variation.

The maximum weight of steam that could be put through the equipment was 2423 lb. per hour, and the discharge varied from this amount down to a minimum of 510 lb. per hour; these ranges corresponded to differentials of from 0.13 in. to nearly 5 in. of mercury, and to orifice velocities of from 32.8 ft. per sec. to 161.2 ft. per sec. Unfortunately the steam available did not permit the use of these velocity ranges with each orifice.

Table 1 gives a summary of the results of the tests, which were made in February, March, and April, 1928. The calculations are based on Goodenough's Steam Tables, and the formula employed in getting the coefficient c was—

$$V = \frac{c\sqrt{2gH}}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}}$$

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[1] Professor of Mechanical Engineering, University of Toronto. Mem. A.S.M.E.

Contributed by the Research Committee on Fluid Meters and presented at the Annual Meeting, New York, December 3 to 7, 1928, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

TABLE 1 ORIFICE COEFFICIENTS FOR STEAM METER

(1) Test No.	(2) Steam discharged, lb. per sec.	(3) Absolute steam pressure, lb. per sq. in.	(4) Volume of steam per lb., cu. ft.	(5) Orifice area, sq. ft.	(6) Velocity through orifice, V ft. per sec.	(7) Differential pressure, in. mercury	(8) Head H, ft.	(9) Coefficient c
			No. 1 Downstream Pressure Tap					
1	0.1454	139.6	3.187	0.0100	46.3	0.49	102.3	0.567
2	0.2106	142.8	3.117	0.0100	65.6	1.03	210.4	0.560
3	0.2996	141.3	3.149	0.0100	94.3	2.06	425.1	0.566
4	0.3545	141.0	3.156	0.0100	111.9	2.80	579.0	0.575
5	0.4147	139.6	3.186	0.0100	132.1	3.85	803.5	0.576
6	0.4615	139.0	3.199	0.0100	147.6	4.80	1006.0	0.576
7	0.4186	138.8	3.203	0.0188	71.3	1.08	227.5	0.573
8	0.5135	139.8	3.182	0.0188	86.9	1.57	327.3	0.582
9	0.5756	141.3	3.133	0.0188	95.9	1.96	403.3	0.579
10	0.6010	139.3	3.192	0.0188	102.0	2.12	443.4	0.587
11	0.6571	141.3	3.133	0.0188	109.5	2.58	529.2	0.577
12	0.6730	138.3	3.214	0.0188	115.0	2.55	538.0	0.601
			No. 2 Downstream Pressure Tap					
13	0.1417	143.0	3.121	0.0100	44.2	0.48	98.2	0.552
14	0.2237	142.9	3.115	0.0100	69.7	1.06	215.9	0.587
15	0.3017	140.9	3.158	0.0100	95.3	1.95	403.8	0.586
16	0.4037	140.4	3.169	0.0100	127.9	3.05	633.2	0.629
17	0.4323	140.5	3.167	0.0100	136.9	3.69	765.6	0.612
18	0.5039	140.3	3.185	0.0100	160.5	4.95	1033.0	0.618
19	0.2798	140.8	3.160	0.0188	47.0	0.39	80.7	0.635
20	0.4258	140.2	3.174	0.0188	71.9	0.93	193.4	0.626
21	0.5313	140.2	3.174	0.0188	89.7	1.45	301.6	0.626
22	0.6180	138.8	3.203	0.0188	105.3	2.0	419.7	0.623
23	0.4343	141.0	3.156	0.0390	35.1	0.13	26.9	0.738
24	0.6307	138.9	3.197	0.0390	51.7	0.28	58.7	0.735
			No. 3 Downstream Pressure Tap					
25	0.1552	142.3	3.128	0.0100	48.6	0.50	102.5	0.593
26	0.2313	140.3	3.171	0.0100	73.3	1.03	214.0	0.620
27	0.3328	140.2	3.174	0.0100	105.6	2.08	432.6	0.628
28	0.4122	139.6	3.186	0.0100	131.3	3.23	674.2	0.626
29	0.4652	136.8	3.248	0.0100	151.1	4.21	895.8	0.624
30	0.4968	136.9	3.246	0.0100	161.2	4.75	1010.0	0.627
31	0.3213	140.3	3.171	0.0188	54.2	0.49	101.8	0.651
32	0.4620	138.6	3.208	0.0188	78.8	0.99	208.1	0.662
33	0.5833	139.6	3.186	0.0188	95.5	1.45	302.6	0.665
34	0.6543	139.0	3.199	0.0188	111.3	1.95	408.7	0.667
35	0.4757	137.6	3.230	0.0390	39.4	0.15	31.7	0.761
			No. 4 Downstream Pressure Tap					
36	0.3122	141.5	3.145	0.0188	52.2	0.47	96.8	0.643
37	0.4610	140.2	3.174	0.0188	77.8	0.93	193.4	0.678
38	0.5400	137.4	3.235	0.0188	92.9	1.33	282.7	0.668

where V denotes the velocity through the orifice in feet per second; A_1 and A_2 are the areas, in square feet, of the inside of the 4-in. pipe and of the orifice, respectively, the area A_2 including that of the small hole in the plate; H is the head, in feet of steam, corresponding to the differential mercury-gage reading, and c is the coefficient sought. The velocity V was found by taking the weight of steam discharged per second, multiplying it by the volume of the actual steam per pound, and then dividing the product by the orifice area, while the head H was given by the formula

$$H = \text{inches of mercury} \times \left(\frac{13.6 - 1}{12}\right) \times 62.4 \times \text{volume of 1 lb. of steam}$$

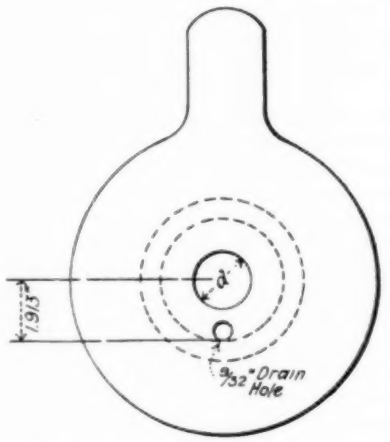


FIG. 2 DETAIL OF ORIFICE PLATE

Orifice	A	B	C
Diameter, in.	2.66	1.838	1.33
Inside diameter of pipe, in.			3.826

For the lower differentials the expansion of the steam through the orifice may be neglected, and the above formula treats the fluid as incompressible. The higher differentials, however, were nearly 5 in. of mercury, which was slightly under 2 per cent of the steam pressure, and in this case the expansion of the steam would have a slight effect. In Table 1, column 9 gives the value of the coefficient without making any allowance for expansion, as the latter should not affect the coefficients over about one-half of 1 per cent in the worst cases, and the effect would be negligible in the greater part of the work.

The results of the tests are plotted on the curve sheets Figs. 3 to 7, inclusive. In Fig. 3 the base is velocity V through the orifice, in feet per second, and three sets of curves have been plotted, one for each orifice. The three curves of the upper set all belong to the smallest orifice, of diameter 1.33 in., and show how the coefficient varies with the position of the downstream tap, the numbers referring to the position of the tap as indicated in Fig. 1. Unfortunately, it was not possible in the time available to get a curve for the fourth tap, but the middle group of curves for the 1.838-in. orifice gives the complete results for the four taps. The lower group, for the 2.66-in. orifice, was very limited owing to the steam capacity available.

The curves in Fig. 3 are very useful in showing how the position of the downstream tap affects the coefficients for a given orifice. All of these curves slope upward at first, indicating that the coefficient for a given orifice, and a fixed pair of pressure taps, varies with the velocity through the orifice, that is, with the differential manometer reading. The curves clearly flatten off and tend to become horizontal beyond a certain velocity; for example, above a velocity of 110 to 120 ft. per sec. the coefficients for the 1.33-in. orifice are constant.

The curves for the 1.838-in. orifice are not as evenly spaced as one would desire, and there may be some errors, even with the extreme care used in the work.

The curves of Fig. 4 are copies of those on Fig. 3 but grouped so as to show the effect of the size of the orifice on the coefficients,

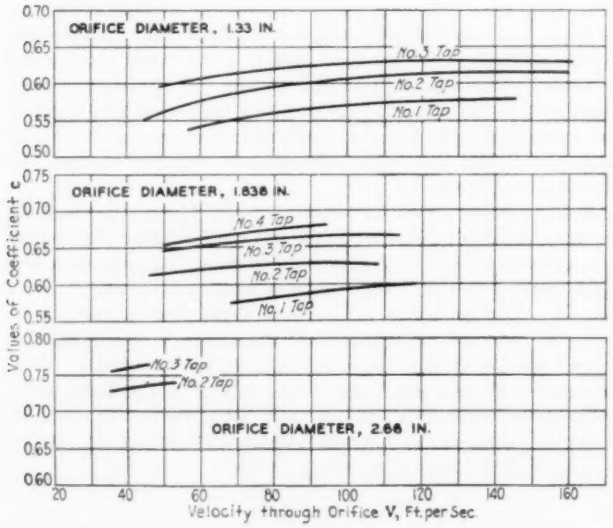


FIG. 3 VALUES OF COEFFICIENT FOR VARIOUS VELOCITIES OF FLOW THROUGH ORIFICES, SHOWING HOW POSITION OF DOWNSTREAM TAP AFFECTS THEM

for each given pair of taps. In this case the experimental points used in plotting the curves are shown. The upper group of curves belongs to No. 1 downstream tap and shows a marked change in the coefficient with the size of the orifice, for a given velocity in the orifice; and the same thing is true of each of the other three groups, which were taken with the second, third, and fourth downstream taps, respectively.

The curves in Fig. 5 show the relation between the coefficient c and the ratio A_2/A_1 of the area of the orifice to the area of the

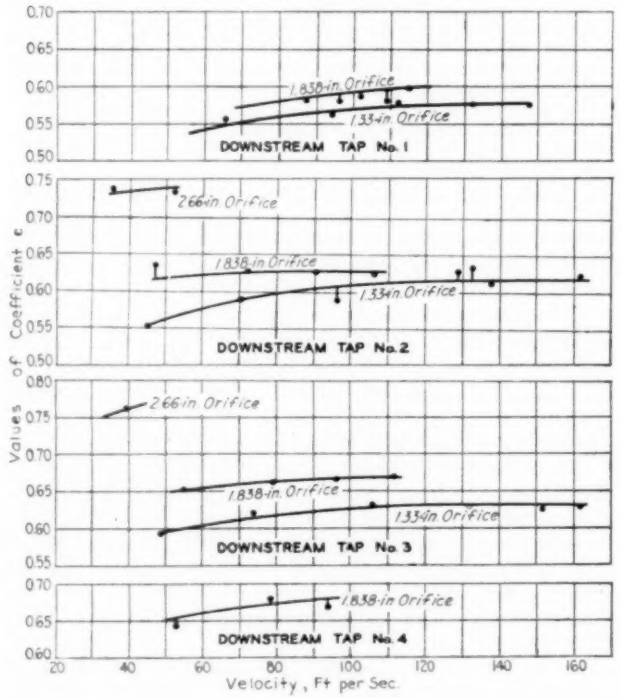


FIG. 4 VALUES OF COEFFICIENT FOR VARIOUS VELOCITIES OF FLOW, SHOWING FOR EACH PAIR OF TAPS THE EFFECT OF SIZE OF ORIFICE

pipe, these coefficients varying, as in the other curves, with the position of the downstream pressure tap.

The curves in Fig. 6 give the same information but referred to a base showing the ratio d_2/d_1 of the diameters of the orifice and of the pipe, respectively. It should be noted that the ratio A_2/A_1 takes account of the small $^1/_{32}$-in. hole in the orifice plate, whereas the ratio d_2/d_1 does not. In plotting these curves approximate average values of the coefficients were used as obtained from Figs. 3 and 4.

The final curves in Fig. 7 give the coefficients for the three orifices in relation to the position of the downstream pressure tap. As in the case of Figs. 5 and 6, the shape and position of the curves in Fig. 7 will vary with the coefficient adopted, as obtained from Figs. 3 and 4. The curves shown are based on approximately average values for each of the curves, although it might have been better to have used the coefficients corresponding to the horizontal parts of the curves.

A fairly complete discussion of the orifice meter is found in "Fluid Meters, Their Theory and Application," Part I, Report of The American Society of Mechanical Engineers' Special Research Committee on Fluid Meters.

This report was published by the A.S.M.E. in 1927 in a revised edition, and gives a complete bibliography on fluid meters, so that references are not necessary in the present paper.

The author's coefficients differ from those shown in this report, as none of the experiments discussed there takes account of the variation of the coefficient with the orifice velocity—and the author's experiments apparently establish such a variation. The author's upstream pressure tap was 1.76 pipe diameters above the orifice, whereas the curves of Fig. 14 in the report referred to were obtained with the tap one pipe diameter up-

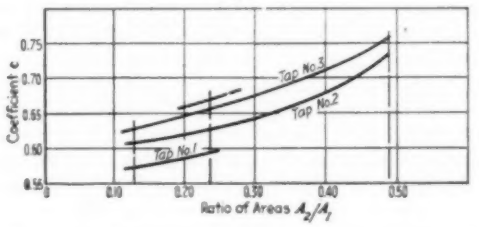


FIG. 5 RELATION BETWEEN COEFFICIENT c AND RATIO A_2/A_1 OF AREA OF ORIFICE TO AREA OF PIPE

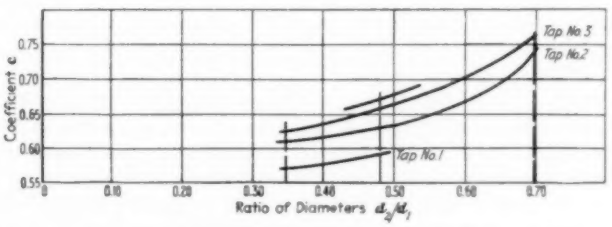


FIG. 6 RELATION BETWEEN COEFFICIENT c AND RATIO d_2/d_1 OF DIAMETER OF ORIFICE TO DIAMETER OF PIPE

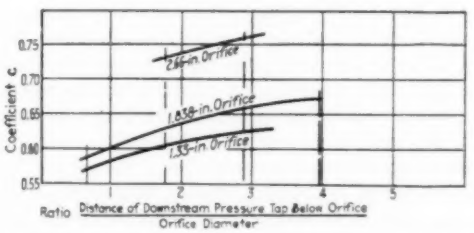


FIG. 7 COEFFICIENTS FOR THE THREE ORIFICES IN RELATION TO POSITION OF DOWNSTREAM PRESSURE TAP

stream; but tap No. 1 used by the author is very close to the vena contracta, and hence the author's coefficients should agree closely with those in the report.

In Fig. 14 the report gives a value $c = 0.608$ for a ratio of diameters $d_2/d_1 = 0.48$, a ratio corresponding to the 1.838-in. orifice used by the author with tap No. 1. For this case the author's values varied from 0.57 to 0.60, although the curve suggests that the value would be about 0.61 at the higher orifice velocities.

The author's smaller orifice had a ratio $d_2/d_1 = 0.348$, and with tap No. 1 the coefficients vary from 0.54 to 0.575, the latter value corresponding to the horizontal part of the curve. For the same ratio d_2/d_1 Fig. 14 of the report shows a value of about 0.602, although for this case the author's pressure tap was probably not at the vena contracta.

The experimental work described herein was carried out in the Steam Laboratory of the Department of Mechanical Engineering, University of Toronto, Canada, and the author acknowledges the help of Messrs. DeLaplante and Saunders, demonstrators in the laboratory, who did the actual experimental work.

Discussion

HORACE JUDD.[2] In looking over the paper it seems to the writer that the coefficients thus obtained would be applicable only to a layout similar to the one used in the tests, and could not be used, with assurance of accuracy, with other layouts differing in pipe size and in other proportions from the 4-in. test line used. With only twelve diameters' length of straight pipe above the position of the orifice meter, there would be danger of error due to the distorting effect of the small elbow near the entrance to the 4-in. pipe.

The differential head across the orifice for any given flow will be greater at the vena contracta than if read at any other downstream point of attachment of the manometer. It follows, then, that tap No. 1 would be the preferable point of attachment for this orifice meter in order to reduce the error of observation as much as possible. The number of tests for each orifice and for each tap position and the range of orifice diameters used appear to be too limited for more than general inferences to be made.

However, the writer agrees fully with the author's opinion that the thin-plate orifice is a valuable fluid-meter element and also that very few reliable data, especially for steam flow, are available for the general user. It is probable, also, that the manufacturers themselves have but a limited fund of reliable data at their command on steam flow under low differential heads. In these days of extended and increasing use of orifice meters for steam-flow measurement it is important and highly commendable that those who make such use of the orifice meter should be willing to share their experiences with others so that the all too limited field of practical knowledge may be enlarged and that much uncertainty now existing may be cleared away. This paper points out a great need for an exhaustive research of steam flow under varying flow conditions, using the orifice meter operating under relatively low differential heads. The ease with which orifice-meter coefficients for water and air have been and can be determined and the assumed adaptability of these coefficients to steam-flow measurement, together with the difficulty and increased expense encountered in conducting tests of steam-flow measurement, are probably the reasons why so few reliable data are available for orifice-meter coefficients for steam flow.

The Engineering Experiment Station at Ohio State University, recognizing this great need of more accurate information concerning orifice coefficients for steam flow, is now engaged in such a research in collaboration with the Bailey Meter Company. The limited number of tests made thus far has shown that for steam at 100 lb. gage pressure, 100 deg. fahr. superheat, and with differential pressures ranging from 2 in. to 10 in. of mercury, the coefficients are in fairly close agreement with those given in Fig. 14, p. 39, Fluid Meters Report, part 1, second edition.

C. D. NORTHAM.[3] The writer has endeavored to compare the coefficients given in the paper with those shown in Fig. 14 of the paper "Orifice Coefficients—Data and Results," by J. M. Spitzglass, presented at the 1922 annual meeting. In order to do this the tabulation given herewith was made.

It is at once evident that on the two orifices whose conditions approximate those of the Spitzglass paper the coefficients shown by the two papers disagree to the extent of from 6 to 7.5 per cent. The coefficients used by other authorities and shown in other papers and reports fall between or near the curves of the two papers compared. From the standpoint of the user this evident disagreement of the authorities does not add much to the confidence one can place in the accuracy of the orifice-type meter.

[2] Professor, Hydraulic Engineering, Ohio State University, Columbus, Ohio. Mem. A.S.M.E.

[3] Mechanical Engineer, Chicago, Ill. Jun. A.S.M.E.

	Orifice A	Orifice B	Orifice C
Diameter, d_2	2.660	1.836	1.330
Inside diameter of pipe, d_1	3.826	3.826	3.826
Ratio, d_2/d_1	0.722	0.480	0.348
Ratio, $(d_2)^2/(d_1)^2$	0.492	0.230	0.121
Ratio, A_2/A_1	0.492	0.230	0.121
Location of downstream tap No. 1, in percentage of pipe diameter (Angus paper)	0.654	0.654	0.654
Location of downstream tap for vena contracta spacing, in percentage of pipe diameter (Spitzglass paper)	0.44	0.68	0.72
Orifice coefficient (Angus paper), No. 1 tap, Fig. 5	...	0.595	0.575
Orifice coefficient (Spitzglass paper), vena contracta spacing, Fig. 14, 4-in. pipe	...	0.634	0.618
Ratio of Angus coefficient to Spitzglass coefficient	...	0.94	0.93
Ratio Spitzglass coefficient to Angus coefficient	...	1.065	1.075

In other words, in order to know how accurate any orifice meter is, the user must know the basis of the coefficients employed and judge the probable accuracy of the coefficient on the basis of the extent and accuracy of experimentation involved in the finding of these coefficients. The writer believes there should be agreement on some set of coefficients applicable to certain conditions of flow, pipe, and orifice. However, before such a standard could be formulated an investigation similar in the scale to the A.S.M.E. Steam Table Research should be undertaken.

In considering probable sources of difference in the two papers compared it must be remembered that the Spitzglass experiments were carried out using air at low pressures whereas the Angus experiments were made using steam partially saturated and at comparatively low pressures. Most new installations of steam meters today operate under conditions of high pressure and temperature, and it would be interesting to know how these affect the coefficients.

The writer was at a loss to understand the necessity for the small $^9/_{32}$ in. hole in the orifice used. Presumably this is a drain hole, but with the pipe drained as shown in Fig. 1 there should be no water on either side of the orifice; and if there is, it would probably fill this hole up, thereby reducing the effective area of the orifice. However, recalculation of the coefficient, using data in test 4, Table 1, shows that the effect of neglecting the area of this $^9/_{32}$ in. hole would change the coefficient as found on this test from 0.575 to 0.576, so it is apparent this has little effect on results except that flow through the small hole might set up eddy currents affecting the main stream.

The important point brought out by the author's experiments is the variation of coefficient with change in velocity. The writer believes this to be due to change in position of the vena contracta, with change in velocity but without change in position of the pressure taps. The curves also show that the coefficient becomes fairly constant at increased velocities.

With these points in mind it would seem important that when designing an orifice the probable flow through it be known and also that the orifice be designed for as high velocity as possible above 120 ft. per sec., and still within the limits of the manometer used, to determine the differential created.

R. E. SPRENKLE.[4] The paper presents an interesting addition to the literature already published on flow measurement work, particularly in reference to coefficient values for different sized orifices when used for steam measurement.

It is deeply regretted, however, that this work is not more

[4] Mechanical Engineer, Bailey Meter Co., Cleveland, Ohio. Assoc-Mem. A.S.M.E.

comprehensive in scope. To attempt to establish coefficient curves on the basis of tests of only three different sizes of orifices, as the author has done in Figs. 5 and 6, is both hazardous and too often misleading, especially when the slope of such curves is rather steep. For example, should the 2.66-in. orifice tests be in error, the slope of the entire curve between the 48 and 69½ per cent ratio points would be in error. Coefficient curves or tables lack authority unless supported by test data from a large number of different-sized orifices; intervals of 4 or 5 per cent in the ratio d_1/d_2 are not too large for work of this exactitude. It is presumed, however, that there was not sufficient time available for testing more sizes. In this event it would have been better to omit the final curves in Figs. 5 and 6 until they could have been substantiated by definite tests on the intermediate sizes.

No explanation is advanced in this paper as to the large increase in coefficient with throat velocity. Such a characteristic has not been found in similar orifice tests using water as the flowing medium, although with venturi meters and flow nozzles an increase of ¼ to 1½ per cent has been found for the same range of velocities. Some steam tests made at Ohio State University recently on a 6-in. line using a 49 per cent ratio orifice show a comparatively flat curve throughout a throat velocity range of 50 to 120 ft. per sec. It will be noted that the coefficient of the 48 per cent ratio orifice as used by the author of this paper varied 5 per cent, through approximately the same range of throat velocity.

If, however, we take the average coefficient for each of these three different-sized orifices tested by the author, we find for tap 1 in the case of the two smaller orifices that the coefficient is about 6 per cent lower on the flow basis than the coefficients obtained from similar-sized orifices in similar-sized pipes by other experimenters. For example, Mr. J. M. Spitzglass in his paper in 1922 gave a value of 0.618 as the coefficient for a 50 per cent ratio orifice measuring air in a 4-in. pipe; and the Bailey Meter Company value of a 48 per cent ratio, 4-in. pipe size, is 0.615 measuring water, as against the author's average of 0.583 for steam. Similarly, Mr. Spitzglass's value for a 31½ per cent ratio orifice, 4-in. pipe, is 0.614, and the Bailey Meter Company average for this same size of orifice is 0.615, as against the author's average of 0.57 for a 34.8 per cent ratio. It would have been interesting to compare the coefficient for the 69½ per cent ratio orifice, tap 1, with other experimenters' data if the author had chosen to obtain these additional data. Comparison between the coefficients obtained by taps 2, 3, and 4 and those obtained by other experimenters using the same tap locations also shows results from 8 to 10 per cent lower. However, since these taps are not at or close to the vena contracta, the results obtained therefrom are of no value whatsoever from a standpoint of commercial application. In passing, it should be pointed out that since there is such a close agreement between Mr. Spitzglass's results obtained on air measurement and the Bailey Meter Company's results obtained on water measurement, the possibility that the steam coefficient would be lower than water is very remote indeed.

Orifices of no greater thickness than 0.015 in. as used in these tests should not be used either for research or for commercial application. It is difficult to see how such thin orifices maintain the perfect vertical plane for any period of time without bulging or cracking and thereby destroying the accuracy of the machined hole. It is entirely possible for these orifices to bulge out appreciably on the high throat velocities, thus increasing the effective orifice opening and increasing the apparent coefficient of discharge. Such an action would help explain why the author's coefficients increased at high velocities, although it might be but a contributory cause.

The orifice flange should have been located about one foot from the exit end of the 4-in. pipe instead of in the middle. This would help materially in eliminating the jet effect which exists in the system due to the sudden increase in pipe size from the 1½ in. to the 4 in., and which might easily carry through in the higher velocities to the orifice flange as shown in Fig. 1. Such an effect would also tend to raise the coefficient, since the static pressure on the inlet side of the orifice would not register its full value.

It is also regretted that a high-pressure water manometer could not have been used in reading the differentials across the orifices. A description by the author would be appreciated as to the method used in reading a mercury manometer with any degree of accuracy when the differentials shown were no more than ½ in. or even 1 in. Magnification of such small differentials by water column would have reduced the possibility of error by a ratio of over 12 to 1 and thus would have made the results obtained more dependable.

J. M. Spitzglass.[5] In his introductory paragraph the author stated, "There are, however, not many tables of actual orifice coefficients, these being apparently retained by the manufacturers." As one of the manufacturers and as a member of the Fluid Meter Committee the writer wishes to take exception to that statement, because the fluid-meter manufacturers have actually, it may be said, forced themselves upon the Fluid Meter Committee by asking the members to utilize their data for publication. Some of the orifice coefficient data, as submitted by the manufacturers, are published in the Fluid Meter Committee report, to which the author refers. Other data are published in the numerous papers which are available in the literature of the Society. The writer would request that the author retract this statement, especially in view of the fact that the paper is supposed to be contributed by the Research Committee on Fluid Meters.

The field of orifice application is tremendously large, and every additional experiment adds another point to the composite curve or diagram. It is imperative to analyze the conditions of the test before the given results are registered on the list. The writer does not believe that reliable results could be obtained with a layout as shown in Fig. 1 and an orifice with a drain hole as shown in Fig. 2. The layout shows a 4-in. pipe trapped between 1½-in. extended inlets and outlets. The author has probably noted the statement in the Fluid Meter Committee report that a jet issuing from an orifice into an enlarged section extends to the walls of the pipe at 4 or 4½ diameters distance from the orifice. This property, however, does not apply to the case of a jet issuing from a long tube. In that case the restricted area of the jet may carry far beyond that distance, and it may have an effect on the differential pressure even at a distance of 10 pipe diameters, which the author allowed in his layout.

The orifice plate had a 9/32-in. drain hole at the bottom. The area of this hole in some cases formed 4½ per cent of the area of the orifice. The author allowed for that area, assuming that it will pass the steam in the same proportion as the opening of the orifice. He could not be certain of that, however, because the condensate lodging in the lower part of the 4-in. pipe in front of the orifice may at times prevent the full discharge through that opening, and thereby affect the coefficient of discharge. This opening is so close to the wall of the pipe that it may at times be partially closed by the packing.

In Table 1, of the thirty-eight tests shown, the only points that could be of value are the first twelve, which were made on the No. 1 downstream pressure tap. If the author had referred to Fig. 12 of the Fluid Meter report, he would have seen beforehand the results of locating the pressure tap farther away from the

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vena contracta. The region between the vena contracta and the point of restoration is uncertain at best and is never recommended for practical use in connection with an orifice plate.

Figs. 3 and 4 are intended to indicate that the coefficient of the orifice was reduced at low velocities. If we restrict ourselves to the data on tap No. 1 and if we plot the curves more accurately, this variation with velocity will be less apparent. At the top of Fig. 4 there was one point omitted on the first velocity of the smallest orifice. The result would show three almost identical points on the first three velocities and three identical points on the second three velocities of the small orifice.

In referring to the Fluid Meter Committee, the author is wondering why his results do not agree with the Fluid Meter Committee coefficients. The writer believes that the author's tests have demonstrated the value of the orifice as a measuring device in that it agreed so closely (within a few per cent) even though the installation was not what it should be. It may be added that recent experiments on orifices have proved beyond doubt that the orifice coefficient varies with velocity only when approaching the region of laminar flow, and in that region the coefficient is considerably increased, which is contrary to the author's indications.

Sanford A. Moss.[6] The paper shows evidence of a great deal of careful work, and the author presents very interesting data showing variation of orifice coefficient with different velocities, with different positions of the pressure holes with the different area ratios. The paper is a very valuable contribution to our knowledge of flow measuring apparatus. However, instead of assisting in the use of thin-plate orifices, as the author intends, the paper should have exactly the opposite effect and should discourage the use of thin-plate orifices. The author starts out by saying that "The value of the thin-plate orifice as a fluid meter has been established beyond all question," and he then goes on to demonstrate by a great many carefully conducted experiments that the thin-plate orifice is not a good fluid meter. The reason is of course that the coefficient of a thin-plate orifice is very uncertain and depends on all of the variables which the author takes into account. As the writer understands the paper, the author takes full theoretical account of all the theoretical differences, such as variations in density, moisture content, initial velocity head, and properties of steam. Therefore, the variations in the final coefficient of discharge c are purely empirical differences which cannot be taken account of by theory and which can be found only by experiments, such as those given in the paper. Table 1 shows that c has a minimum value of 0.552 and a maximum value of 0.761. This gives an extreme variation of 38 per cent. Of course, a large part of the differences are given by the different downstream taps. However, the author shows that the position of the venturi contraction varies with the different circumstances, so that there is no theoretically sound way for deciding just which downstream tap to use.

The author's work covers a great deal of territory, but nevertheless there is a vast amount of additional territory to be covered for different pressures, different velocities, different sizes of orifice, and different area ratios. A number of other papers have been published giving data for some of this other territory, and many systems for using thin-plate orifices have been proposed.

The author implies that the manufacturers of meters for thin-plate orifices have exact data which they are retaining. The possession by manufacturers of exact data covering all the variables is to be questioned.

There is to be contrasted with the thin-plate orifice, with such an uncertainty in the coefficient discharge, the nozzle with well-rounded approach with a very definite coefficient. This has been used for many years either as a venturi meter or with insertion in a pipe line in the same way as a thin-plate orifice. A detailed discussion of the use of a nozzle with well-rounded approach was given by the writer in a paper presented at the December, 1927, meeting, and its advantages as compared with the thin-plate orifice were pointed out. The data given in the paper confirm all of the statements made in the paper regarding difficulties of use of a thin-plate orifice.

The author has presented a very valuable paper as a result of a great deal of careful work, but his paper demonstrates the exact opposite of the point he intended to make.

[6] Engineer, Mechanical Research Department, General Electric Company, West Lynn, Mass. Mem. A.S.M.E.

Author's Closure

The author is very grateful to those who have discussed the paper and offered criticism that will be helpful in future work. It is a matter of regret that lack of time and opportunity prevented more extensive work, but it is hoped that the research will be continued.

Criticism of the use of taps 2, 3, and 4 is not justified, as these points may be used quite as well as No. 1, provided the coefficient is known. The desirable point of connection is one which gives a good differential and where the coefficient varies little for a slight change in location of the point. Tap No. 4 is a place where the jet in all probability fills the pipe, being four diameters downstream, and is therefore a good point; one meter builder places the downstream tap eight diameters below the orifice. Information about taps 2 and 3 is also very desirable if for no other purpose than helping to locate the vena contracta.

Objection to the thickness of the plate by Mr. Sprenkle is interesting in view of the fact that orifice B was an exact duplicate in dimensions, including the $^{9}/_{32}$ in. hole, of that supplied by the meter maker on the 100-hp. plant referred to except that orifice B was 0.016 in. thick, while that of the meter maker was 0.015 in. The small hole was presumably for drainage, and care was taken that it was not closed up by the packing. There seemed to be no alternative to including the area of the small hole in that of the orifice, although it had little effect in the case of the larger orifices. The $^{3}/_{8}$-in. drain pipes were closed during each test.

Both Professor Judd and Mr. Spitzglass criticize the distance between the $1^{1}/_{2}$ in. pipe and the orifice, but as no definite suggestion is offered it only proves that more information is needed as to the setting of orifices and the effect of changes in pipe section on them. In the tests the $^{1}/_{2}$-in. calorimeter pipe, which extended across the 4-in. pipe, must have largely broken up the jet.

There is considerable force in Mr. Moss' contention that meters with nearly constant coefficients, especially when these are close to unity, as in the nozzle and venturi tube, have much to commend them, but the orifice has very much to its credit also. The paper does not try to establish a case for the orifice, but it most clearly shows that where it is used great care must be exercised in the setting and in the coefficients used, and I have seen some unfortunate troubles due to neglect of these precautions. More information is needed about the device, and it is to be hoped that it will be forthcoming.

There was no intention in the paper to reflect on the meter manufacturers, and it may have been an error to assume that they had obtained actual coefficients for the meters they sold, but a very careful and fruitless search for coefficients for orifice B was the cause of the investigation being started, and its scope was afterward enlarged.

The question of the variation of the coefficient with the velocity has been raised by Mr. Northam, Mr. Sprenkle, and Mr. Spitzglass and is a most important one. Recently when in Berlin the author discussed this paper with a well-known authority, who then showed some curves which he had obtained with the

same general characteristics, and it is to be hoped that he will soon make these public so that a comparison may be made. It is a point that should be definitely settled, although a similar condition is well known to exist in the case of the venturi meter.

Unfortunately the author is unable to furnish further details regarding the work, as he is at this time abroad and cannot refer to his notes or to any details of the tests or papers mentioned in the discussion.

FSP-51-24

The Peak-Load Problems in Steam Power Stations

By A. G. CHRISTIE,[1] BALTIMORE, MD.

This paper deals with the problem of the annual peak load in steam power plants. The use of the load-duration curve for the purpose of studying annual peak loads is analyzed and the conclusions are drawn that (1) plant intended for annual-peak-load service should only be installed at the lowest possible first cost, and that (2) economy of peak-load equipment can be sacrificed to secure low initial cost.

Various schemes that have been proposed to provide equipment for carrying the annual peak loads are analyzed. The three plans that appear most promising are those involving the use of steam accumulators, of peak-load units, and of present plant after new base-load equipment has been added.

In order to indicate the possible annual savings through the application of these three methods, the author assumes an electric system with a maximum annual peak load of 200,000 kw., together with certain data for annual load factors of 65, 45, and 25 per cent on this system. With certain assumed plant costs, fuel costs, and operating performances, an analysis is made of the annual operating costs of the various methods, and the possible annual savings through the use of steam-accumulator plant or peak-load units are shown in the form of a series of curves. These savings in some cases may be of a large order.

MUCH thought has been devoted to methods of improving the daily load factors of steam-electric power stations. Night loads have been added, power demands for morning and early afternoon hours have been developed, and attention has been given to reducing peak demands which usually occur at about six o'clock in the afternoon. This paper will discuss the problem of carrying the maximum peak loads of the year.

The Load-Duration Curve

The load-duration curve represents the cumulative duration of all loads throughout the year as shown in Fig. 1. Each point on the curve indicates the hours per year in which the load has not been less than the kilowatts represented by the point. To plot this curve, the daily load curves are first taken and the average output for each hour of the twenty-four is noted. A record is made of the duration in hours of each load throughout the year. These loads are then plotted in decreasing order from the peak load with cumulative time as a base, until the loads for the whole 8760 hours of the year have been recorded. A smooth curve, or a stepped curve as preferred, drawn through the plotted points, represents the load-duration curve of the particular system.

The area under the curve represents kilowatt-hours output to a given scale, and may be used to check the yearly output computed by other means.

The annual load factor as used in this paper is the ratio of the kilowatt-hours generated to the maximum demand during the year multiplied by 8760, the total hours per year. This ratio is thus the area under the load-duration curve divided by the area represented by the product of the maximum load of the year and 8760. The annual load factor of that portion of the demand above any given load may be found by taking the ratio of the area under the curve above the given load, to the product of 8760 and the difference between the given load and the maximum peak load. The annual load factor of that portion of the load below any given load is the ratio of the area under the curve between zero load and the given load, to the product of 8760 and the given load. The steeper the load-duration curve, the lower will be the annual load factor, as is indicated by Fig. 1, which has an annual load factor of 30.4 per cent. On the other hand, the flatter the load-duration curve and the less it diverges from the flat curve at maximum and minimum load, the higher will be the annual load factor, as shown in Fig. 2, which has an annual load factor of 51.4 per cent.

Loads in excess of 50 per cent of the maximum peak in Fig. 1 occur for only 1050 total hours during the year, while loads in excess of 75 per cent of the maximum peak exist for only 200 hours in the year. The kilowatt-hours generated by the equipment employed to carry load in excess of 50 per cent of the maximum peak and shown by the cross-hatched area on the diagram represent only 4.8 per cent of the total annual output. The equipment carrying load above 75 per cent of the maximum peak in Fig. 1 produced only about 0.5 per cent of the total kilowatt-hours generated. Load in excess of 80 per cent of the maximum peak in Fig. 2 lasts for 320 total hours and develops about 0.62 per cent of the total annual output.

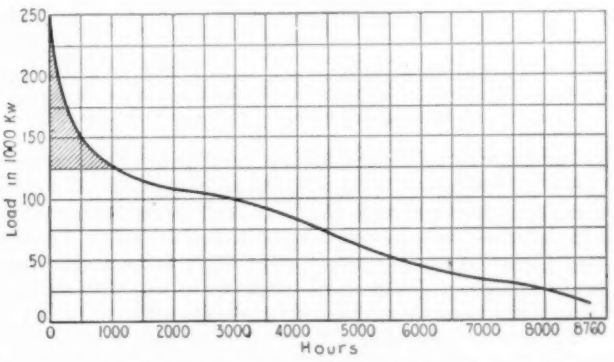


Fig. 1 Load-Duration Curve for System With 30.4 Per Cent Annual Load Factor

The annual load factor has been plotted in Fig. 2 for that portion of the load above each load. For instance, the annual load factor for that portion of the load in excess of 75 per cent of the maximum peak is 2 per cent, as shown on the upper scale. Also, the annual load factor for that portion of the load in excess of 50 per cent of the maximum peak is 18.6 per cent. This method of drawing the annual load factor for various portions of the maximum peak assists in analyzing the peak-load problem.

A study of load-duration curves leads to the formulation of two statements regarding peak-load equipment in a generating system: (*a*) Plant intended for peak-load service only should be installed at the lowest possible first cost; (*b*) Economy of peak-load equipment can be sacrificed to secure low initial cost. The first statement is evident from a consideration of the few hours of service of this portion of the system during the year.

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Contributed by the Power Division and presented at the Annual Meeting, New York, December 3 to 7, 1928, of The American Society of Mechanical Engineers.

Only equipment with a low first cost can be justified. The second follows from a consideration of the small amount of power generated by such peak-load equipment. The cost of each kilowatt-hour thus generated consists predominantly of fixed charges, with relatively smaller charges for fuel, labor and maintenance.

Proposed Schemes to Carry Peak Load

Electrical systems have had such rapid growth that additional generating capacity is needed every few years. The design of

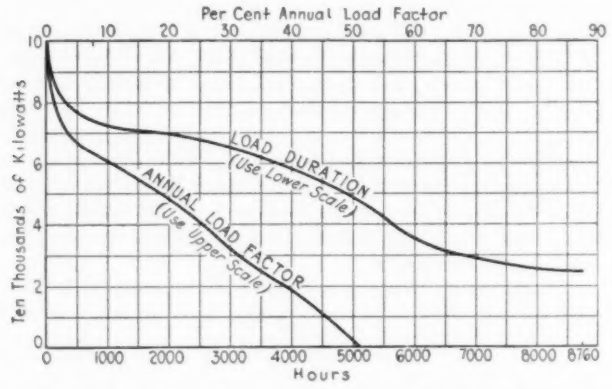


Fig. 2 Load-Duration Curve for System With 51.4 Per Cent Annual Load Factor and Annual-Load-Factor Curve for That Portion of Output Above Given Loads

power plant and equipment has also developed so that fuel economies are available by the addition of new units. Each new installation has been made the "base-load" plant of the system, although there have been exceptions to this practice. The high load factors of many systems which have well-sustained high loads throughout the greater portion of the day with no short, sharp peaks favor the addition of base-load units.

Methods of carrying peak loads were discussed at the convention of the Verband Deutscher Electrotechniker at Kiel, Germany, in June, 1927, and reported in *Elektrotechnische Zeitschrift*, June 30, 1927. High-level water storage, Diesel engines, electric storage batteries, and Ruths steam accumulators were considered in this connection.

Few sites are available in this country where the combination of lack of pondage on the river, low load factor, and nearby available natural storage basins at high head would make an economic possibility of hydraulic storage during off-peak hours for use on peak load.

Electric storage batteries for peak-load service are usually limited to those systems which still have Edison direct-current distribution. Few large batteries will be installed except for emergency reserve in order to preserve continuity of the direct-current service. Storage batteries are still of great value for such service in important districts of many large cities. They are costly and are not generally considered economical equipment to carry system peak loads.

Large Diesel engines are being installed in some European systems for peak-load service. Much can be said in favor of Diesel engines for this purpose on account of their quick availability and low stand-by and operating costs. Their future development should be closely watched by power engineers with this possible use in mind. The development of large Diesel engines for central-station service has been slow in America due to high first cost, and on this account it is not likely that Diesels will be widely used for peak-load service in the immediate future.

Steam Accumulators

The steam accumulator has been used for many years both here and abroad for service with exhaust steam from reciprocating engines, and its operation is well understood by engineers. A Swedish engineer, Dr. J. Ruths, applied the principle of steam storage to accumulators for large quantities of steam by making the pressure vary between wide limits—for instance, from 250 to 50 lb. per sq. in. gage. In the following discussion the use of the term "steam accumulator" will refer to one of this high-pressure type.

The steam accumulator finds economic justification in power plants to take violent load fluctuation of short duration, to enable the plant to carry sharp daily peak loads economically, and to carry a portion of the annual peak load.

The application of steam accumulators in industrial power plants with wide, frequent, and rapid fluctuations in steam demand has been described in R. A. Langworthy's paper "The Ruths Steam Accumulator."[2] Central stations which serve street railways, underground railways, and electrified trunk lines have power demands which vary quite as rapidly and between as great extremes of boiler capacity as in many industrial plants, and to which steam accumulators would seem desirable additions.

The cost of an accumulator will depend upon the extent and nature of the fluctuations in steam demand that it will have to meet. If the fluctuations are of short duration, the accumulator installation need not be large. If the fluctuations are of long

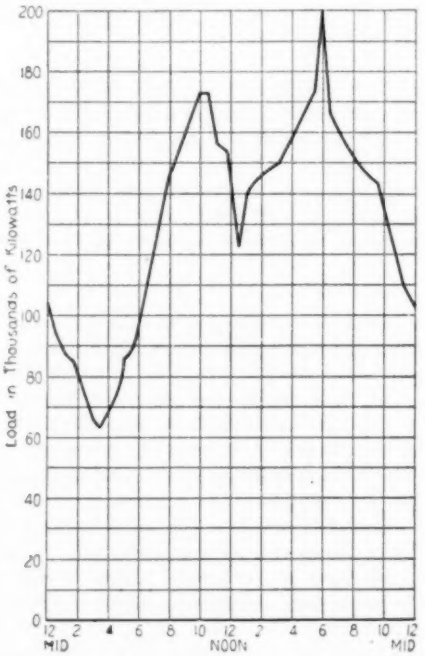


Fig. 3 Average Daily Load Curve for System With 65 Per Cent Annual Load Factor

duration and very irregular, more accumulator capacity will be required and the cost will increase. Where no increase in total station output is anticipated subsequent to the accumulator installation, the fixed charges on the new equipment, including costs of changes on turbines and piping, must be balanced against the savings resulting from better operation of the boiler plant.

A point is soon reached in electrical systems with growing loads when additional generating capacity must be installed.

[2] Trans. A.S.M.E., vol. 50, no. 15, 1928, paper FSP-50-33.

The choice may lie between additional boiler-house equipment and another standard high-pressure turbo-generator; steam accumulators and a special turbo-generator to carry fluctuations and peak loads; or a special peak-load turbine and boilers. A boiler plant which has been subject to wide fluctuations in load can generally carry a steady load of much higher average capacity with less difficulty and at better efficiency. Hence the addition of steam accumulators would improve the performance of older boilers, and would also save the cost of additional boilers, boiler house, and coal handling and other equipment. Accumulators are generally placed out in the open with no housing or any construction other than the supporting concrete foundations and the necessary piping and valves.

Steam turbines for accumulators are usually operated on saturated steam at moderate pressures, which permits the use of cast-iron casings. The low annual load factors for peak-load service will not warrant refinements in the design of turbine or condenser plant, and these for use with accumulators should be little, if any, more expensive than similar plant for peak-load service at standard pressures. Some operating advantages may be secured at slight additional cost by making the accumulator turbine a mixed-pressure turbine capable of operating steadily in emergencies on steam direct from the boilers, or desuperheated.

Steam accumulators are suitable for peak-load service when the peaks are sharp and of short duration. The Siemens-Schuckert Werke of Berlin, Germany, have recently received an order from B.E.W.A.G., the electricity works of that city, to install 16 Ruths accumulators in connection with the Charlottenburg Station, one of the older plants of the system. Vertical accumulators each 69 ft. high, 14 ft. 9 in. in diameter, and with a capacity of about 11,000 cu. ft. will be added. These will stand in the open with no housing, and in appearance will resemble a group of



FIG. 4 LOAD-DURATION CURVE AND ANNUAL-LOAD-FACTOR CURVE FOR SYSTEM WITH 65 PER CENT ANNUAL LOAD FACTOR

Cowper stoves. They will have a steam capacity sufficient to carry a 40,000-kw. peak with a three-hour base on two turbines each of 20,000-kw. capacity, and to generate 67,000 kw-hr. during this period. The boilers of the plant will supply steam to these accumulators at night and during drops in the day demand. This service will increase not only the load factors on the boilers, but also, as the accumulators float on the line, the rate of driving the boilers will become more constant, resulting in an increase in boiler-plant efficiency. The accumulator equipment will take the top portion of the Berlin daily peak, which has become very sharp, and thus permit other stations to operate on higher load factors with improved performance.

OTHER STEAM PLANT FOR PEAK LOAD

Old stations on a power system often offer possibilities of increased capacity for peak-load service. Many have stokers with low settings and old equipment. Some of these may be changed to powdered-coal firing with unit mills, water-cooled furnace walls, evaporators, air preheaters, and radiant-type superheaters, and the output of the boiler plant can thus be greatly increased during peak loads. If adequate cooling water is available, the old turbines may be rebuilt for greater steam capacity at peak load.

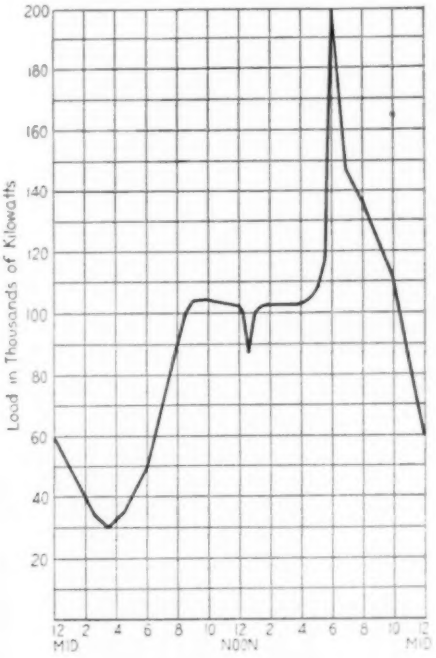


FIG. 5 AVERAGE DAILY LOAD CURVE FOR SYSTEM WITH 45 PER CENT ANNUAL LOAD FACTOR

Small turbines may be replaced by new, larger, peak-load units. Generators may also be rewound for added capacity, or may be replaced by units of larger output. Such changes will materially increase the capacity of an old plant as a peak-load station, and the resultant gain in system capacity may be secured at a lower total cost than by any other plant which will provide the same additional capacity.

When new and highly economical boilers and turbines are added to a power system and are operated as base-load units during the early years of their use, the capacity factors of the older equipment are decreased. The question now arises, whether the installation of cheap, rugged units of large capacity for peak-load service only, in place of highly efficient base-load units, would be economically justified. It is also desirable to determine whether such peak-load units would provide lower costs than steam-accumulator plant.

These peak-load turbines are of simple design and moderate efficiency so that as great an output as possible may be secured from a given frame, thus lowering the first cost per kilowatt of capacity. Condenser equipment for moderate vacuum may be installed. Peak-load boilers could be either oil fired or provided with water-cooled walls, large furnaces, and unit pulverizers for quick starting, low stand-by costs, and maximum ratings during their short daily periods of operation.

ECONOMICS OF METHODS OF CARRYING PEAK LOADS

In order to make some concrete comparisons of the relative economy of steam accumulators, base-load units, and peak-load units, the average daily load curves for a year (for 65 per cent

annual load factor and for 45 per cent annual load factor) of a plant with a total capacity of 200,000 kw. have been assumed from data in A. H. Markwart's paper "Power in California."[3] The corresponding load-duration and annual-load-factor curves

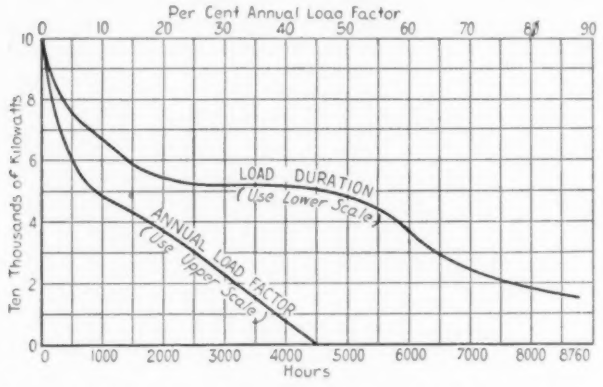


FIG. 6 LOAD-DURATION CURVE AND ANNUAL-LOAD-FACTOR CURVE FOR SYSTEM WITH 45 PER CENT ANNUAL LOAD FACTOR

lator was taken as $55 per kw. of capacity. Accumulators with their foundations and piping were assumed to cost $17 per kw-hr. of maximum charging capacity. The cost of accumulator plant in the following calculation was based on an arbitrary assumption that the maximum daily peaks of the year to be carried by the accumulators would have a time base twice as great as that shown by Figs. 3 and 5, which are average daily load curves. Hence for a given portion of the peak load, the daily storage capacity of the accumulator was assumed to be twice the kilowatt-hours shown by that portion of the average daily load curve. Similar assumptions were made in the calculations for 25 per cent load factor. These assumptions were made for the purpose of this paper only. In an actual case the daily load curve for that day on which the maximum yearly peak occurs should be used to calculate the necessary kilowatt-hour capacity of the accumulator.

The influence of variations in these prices will be discussed later. Fixed charges were assumed for all systems to be as follows: interest, 6 per cent; taxes, insurance, etc., 3 per cent; depreciation, 6 per cent; total, 15 per cent. Coal of 12,500 B.t.u. per lb. as fired was assumed at a cost of $5 per ton of 2000 lb.

The standard plant equipment to which additions are to be

TABLE 1 DATA ON ANNUAL LOAD FACTORS AND KILOWATT-HOURS OUTPUT OF A 200,000-KW. PLANT

	65 per cent annual load factor as in Figs. 3 and 4				45 per cent annual load factor as in Figs. 5 and 6				25 per cent annual load factor as in Fig. 7			
	Load below given load		Load above given load		Load below given load		Load above given load		Load below given load		Load above given load	
Load in kilowatts	Annual load factor	Per cent total kw-hr.	Annual load factor	Per cent total kw-hr.	Annual load factor	Per cent total kw-hr.	Annual load factor	Per cent total kw-hr.	Annual load factor	Per cent total kw-hr.	Annual load factor	Per cent total kw-hr.
200,000	65.0	100.00	0.0	0.00	45.0	100.00	0.0	0.00	25.0	100.00	0.00	0.00
190,000	68.5	99.95	0.7	0.05	47.3	99.96	0.4	0.04	26.6	99.95	0.23	0.05
180,000	72.2	99.77	1.5	0.23	49.8	99.81	0.8	0.19	27.7	99.72	0.68	0.28
170,000	76.2	99.39	2.7	0.61	52.6	99.59	1.3	0.41	29.2	99.36	1.07	0.64
160,000	80.0	98.41	5.2	1.59	55.7	99.26	1.7	0.74	30.9	98.95	1.3	1.05
150,000	83.8	96.47	9.2	3.53	59.2	98.77	2.2	1.23	32.8	98.40	1.6	1.60
140,000	86.5	92.96	15.3	7.04	62.8	98.07	2.9	1.93	34.9	97.67	1.9	2.33
130,000	88.3	88.20	21.9	11.80	67.0	96.96	3.9	3.04	37.2	96.80	2.3	3.20
120,000	90.8	83.63	26.7	16.37	71.3	95.43	5.1	4.57	39.9	95.80	2.6	4.20
110,000	92.8	78.37	31.3	21.63	76.3	93.39	6.6	6.61	43.0	94.52	3.0	5.48
100,000	94.8	72.82	35.4	27.18	80.9	90.09	8.9	9.91	46.4	92.78	3.6	7.22
90,000	96.8	66.93	39.1	33.07	83.7	83.70	13.3	16.30	50.3	90.59	4.3	9.41
80,000	98.3	60.46	42.8	39.54	86.0	76.61	17.5	23.39	54.8	87.67	5.1	12.33
70,000	99.5	53.52	46.6	46.48	88.7	69.12	21.3	30.88	59.9	83.88	6.2	16.12
60,000	100	46.07	50.2	53.93	91.7	61.21	24.9	38.79	65.8	79.00	7.5	21.00
50,000	100	38.40	53.5	61.60	94.7	52.75	28.3	47.25	72.6	72.60	9.1	27.40
40,000	100	30.71	56.4	69.29	98.2	43.73	31.6	56.27	80.5	64.38	11.1	35.62
30,000	100	23.04	59.0	76.96	100	33.41	35.2	66.59	89.0	53.42	13.7	46.58
20,000	100	15.36	61.2	84.64	100	22.27	38.8	77.73	97.0	38.81	17.0	61.19
10,000	100	7.68	63.2	92.32	100	11.15	42.0	88.85	100.0	20.00	21.3	80.00
0	0	0.00	65.0	100.00	0	0.00	45.0	100.00	0.0	0.00	25.0	100.00

are shown in Figs. 4 and 6. The data on load factors and kilowatt-hours output above and below given loads as shown in Table 1 were taken directly from Mr. Markwart's paper. The second and third columns in the table indicate the annual load factor and the percentage of the total kilowatt-hours output, respectively, for those portions of the load curves in Figs. 3 and 4 below the given load shown in the first column. The fourth and fifth columns show similar data for those portions of the load curves above the given load. The sixth to ninth columns show the corresponding data for Figs. 5 and 6. A load-duration curve, Fig. 7, was drawn for 25 per cent annual load factor from other available data. Table 1 also gives data on this load-duration curve in the tenth to thirteenth colums.

The following plant costs were assumed for the purposes of this discussion:

Peak-load plant complete.....$85 per kw. capacity
Standard plant complete.....$100 per kw. capacity
Base-load plant complete with latest heat-saving equipment.....$115 per kw. capacity

The steam-accumulator plant can be as simple in the turbine room as the peak-load plant. Its cost exclusive of the accumu-

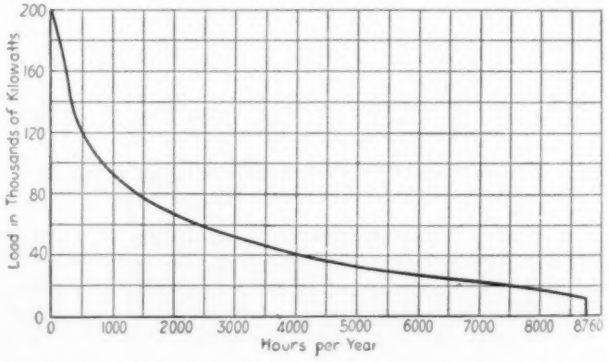


FIG. 7 LOAD-DURATION CURVE FOR 25 PER CENT ANNUAL LOAD FACTOR

made was assumed to approximate recent practice. The variation in heat rate with load factor of such standard plant, as expressed in B.t.u. per kw-hr., is given by curve A, Fig. 8, and is expressed by the formula

$$\text{B.t.u. per kw-hr.} = 13{,}600 + \frac{100{,}000}{\text{Annual load factor in per cent}}$$

[3] *Jl. Franklin Inst.*, Aug., 1927.

The steam leaving the accumulator will not be superheated. For the purpose of this paper, gage-pressure limits of 200 lb. to 20 lb. will be assumed on the accumulators, and an average steam consumption of 20 lb. per kw-hr. on the turbine. An allowance of 12.5 per cent will be added to take care of auxiliary power, radiation losses from the accumulators, and steam required to start up the turbine. The accumulator plant would therefore require 22.5 lb. of steam per kw-hr. net output. Assume that at night the boilers add 1000 B.t.u. per lb. to generate this steam at an efficiency of only 75 per cent. The average heat requirements of the accumulator plant will be $(22.5 \times 1000)/0.75 = 30{,}000$ B.t.u. per net kw-hr., which seems a liberal figure.

A special turbine and boiler plant, for peak-load service only, would be added to the standard plant where the steam conditions may be assumed as 400 lb. gage, 700 deg. fahr. The condenser could be designed for 28 in. of vacuum. The average steam consumption under these conditions was assumed to be 11 lb. per kw-hr. An allowance of 12.5 per cent additional steam at average load was made for auxiliaries and for losses in starting up. If 1200 B.t.u. are added per pound of steam in a boiler built for large capacity rather than for economy, and which operates at an average efficiency of only 65 per cent, the average B.t.u. per net kw-hr. output will be $11 \times 1.125 \times (1200/0.65) = 22{,}850$ B.t.u.

The base-load units will be assumed to operate with high pressure, high superheat, bleeders, low vacuum, and air preheaters or economizers to insure high plant economy. The performance of such a base-load unit is given in curve B, Fig. 8, and is expressed by the formula:

$$\text{B.t.u. per kw-hr.} = 12{,}900 + \frac{100{,}000}{\text{Annual load factor in per cent}}$$

Additional plant operators for steam-accumulator, peak-load, and base-load units are provided at the rate of 60 cents per hour. The steam accumulator is entirely automatic and requires little

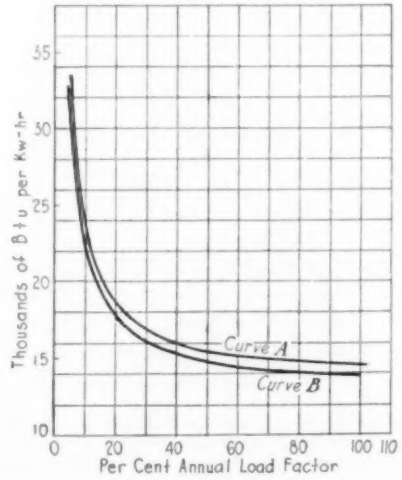


FIG. 8 RELATION OF B.T.U. PER KW-HR. TO ANNUAL LOAD FACTOR

attention. Its turbine will require an operator for a single shift of 8 hours per day during the peak load. If used for annual peak only, this operator need not be employed for the whole year. The peak-load plant is assumed to operate only during a single 8-hour shift with two additional operators, one in the boiler room and one in the turbine room. The base-load unit operates the full 24 hours and two additional operators per 8-hour shift are provided, one for the boilers and one for the turbine. When the added unit exceeds 40,000 kw., it has been assumed that adjustments will be made in the labor force of the original station so that no additional operators other than those just mentioned will be required in any new plant.

The assumption has also been made that no plant operates at a higher annual load factor than 80 per cent. This provision

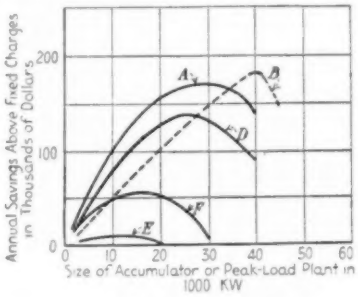


FIG. 9 ANNUAL SAVINGS ABOVE FIXED CHARGES THROUGH THE USE OF STEAM-ACCUMULATOR PLANT OR PEAK-LOAD UNIT ON A 200,000-KW. SYSTEM WITH 65 PER CENT ANNUAL LOAD FACTOR AND WITH COAL COSTING $5 PER TON

Curve A—Annual savings, steam accumulator over addition of base-load plant to standard plant.
Curve B—Annual savings, peak-load unit over addition of base-load plant to standard plant.
Curve C—Annual savings, combined accumulator and peak-load plant over addition of base-load plant to standard plant.
Curve D—Annual savings, steam accumulator over addition of standard unit to standard plant.
Curve E—Annual savings, steam accumulator over addition of base-load unit to system with standard plant and old low-efficiency plant.
Curve F—Annual savings, steam accumulator over addition of base-load unit to system with standard plant together with older plant of moderate efficiency and still older plant of low efficiency.
Curve G—Annual savings, steam accumulator over addition of base-load unit to system with a series of units with efficiencies decreasing from the standard plant to the very old plant.

NOTE: The differential between curves A and B at any size of accumulator or peak-load unit remains constant for that particular size. Curves have not been drawn for annual savings with peak-load units for conditions represented by curves D, E, F, and G, as this would confuse the diagrams. The savings by the use of peak-load units with these combinations can be found by deducting the differential between curves A and B from the curve of accumulator savings for the particular system in question.

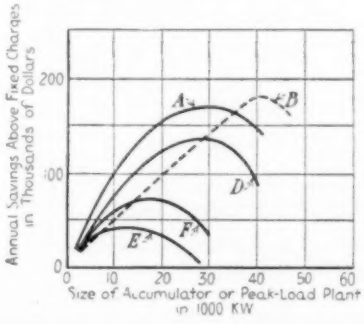


FIG. 10 ANNUAL SAVINGS THROUGH THE USE OF STEAM-ACCUMULATOR PLANT OR PEAK-LOAD UNIT ON A 200,000-KW. SYSTEM WITH 65 PER CENT ANNUAL LOAD FACTOR WITH COAL COSTING $4 PER TON
(See Fig. 9 for definition of curves.)

allows ample time for condenser cleaning, boiler and turbine maintenance, general inspection, and overhauling.

The following comparisons will be based only on fixed charges and fuel costs with the additional labor. It is assumed that repairs and maintenance, attendance, supervision, and other charges will remain substantially constant in all cases, and they are therefore not included in the comparisons, although it would appear that these charges should be less for accumulators than for boilers.

STEAM ACCUMULATORS VS. BASE-LOAD UNITS

To show the method by which the following analyses were made, a 20,000-kw. accumulator plant is assumed in Case 1 to

take the top 20,000 kw. of the maximum peak of 200,000 kw. of the 65 per cent annual-load-factor curve shown in Figs. 3 and 4. This will be compared with a base-load unit of 20,000 kw. capacity also applied to the load curves of Figs. 3 and 4 which will operate with an 80 per cent annual load factor. The standard plant is assumed to carry the remaining portion of the load in each case. The comparison of yearly costs is as follows:

CASE 1—STEAM ACCUMULATORS VS. BASE-LOAD UNITS, COMPARISON OF YEARLY COSTS

Steam Accumulator Plant (20,000 kw., cost $67 per kw.):		
Fixed charges, $0.15 \times 20{,}000 \times \$67$	=	$201,000
Fuel at 1.5 per cent annual load factor (30,000 B.t.u. per kw-hr.) $= 0.015 \times 8760 \times 20{,}000 \times \frac{30{,}000}{12{,}500} \times \frac{\$5.00}{2000}$	=	15,768
Standard Plant (180,000 kw.):		
Fixed charges, $0.15 \times 180{,}000 \times \$100$	=	2,700,000
Fuel at 72.2 per cent annual load factor (14,985 B.t.u. per kw-hr.) $= 0.722 \times 8760 \times 180{,}000 \times \frac{14{,}985}{12{,}500} \times \frac{\$5.00}{2000}$	=	3,411,933
1 extra operator, $\$0.60 \times 2920$	=	1,752
Total annual cost		$6,330,453
Base-Load Unit (20,000 kw.):		
Fixed charges, $0.15 \times 20{,}000 \times \$115$	=	$345,000
Fuel at 80 per cent annual load factor (14,150 B.t.u. per kw-hr.) $= 0.80 \times 8760 \times 20{,}000 \times \frac{14{,}150}{12{,}500} \times \frac{\$5.00}{2000}$	=	396,653
Total kw-hr. generated $(a) = 0.65 \times 8760 \times 200{,}000 = 1{,}138{,}800{,}000$		
Total kw-hr. on base-load unit $(b) = 0.80 \times 8760 \times 20{,}000 = 140{,}160{,}000$		
Total kw-hr. on standard plant $= (a) - (b) = 998{,}640{,}000$		
Annual load factor, standard plant $= \frac{998{,}640{,}000}{180{,}000 \times 8760} = 63.33$ per cent		
Standard Plant (180,000 kw.):		
Fixed charges, $0.15 \times 180{,}000 \times \$100$	=	2,700,000
Fuel at 63.33 per cent annual load factor (15,180 B.t.u. per kw-hr.) $= 0.6333 \times 8760 \times 180{,}000 \times \frac{15{,}180}{12{,}500} \times \frac{\$5.00}{2000}$	=	3,031,711
6 extra operators, $6 \times \$0.60 \times 2920$	=	10,512
Total annual cost with base-load unit		$6,483,876
Total annual cost, using steam accumulator		6,330,453
Annual saving by the use of steam-accumulator plant above fixed charges		$153,423

Table 2 presents Case 1 in tabular form together with Cases 2 and 3 for sizes of accumulator plants and base-load plants up to 40,000 kw. calculated for 65 per cent annual load factor as per Figs. 3 and 4. Two operators are provided for the accumulator plants in Cases 2 and 3, since these would have to operate for a portion of two shifts during the annual peak loads. The annual savings above fixed charges with coal at $5 per ton as shown in Table 2 are shown graphically by curve A, Fig. 9. The annual savings with coal at $4 per ton are shown by curve A, Fig. 10. Calculations were made in the same manner for the addition of steam accumulators or base-load plant to standard plant in systems with annual load factors of 45 and 25 per cent as shown by Figs. 6 and 7. The annual savings under these conditions with both $5 and $4 coal are shown by curves A in Figs. 11, 12, 13, and 14.

The annual savings above fixed charges that appear possible by the use of steam-accumulator plant, as shown by these tables and curves, are of such magnitude that an analysis will be made of the effect on the final results of changes in the assumptions on which the calculations are based.

In Case 1, if the first cost of the complete accumulator plant were increased as much as 25 per cent, the annual saving would still be $103,173. Doubling the fuel consumption for the steam

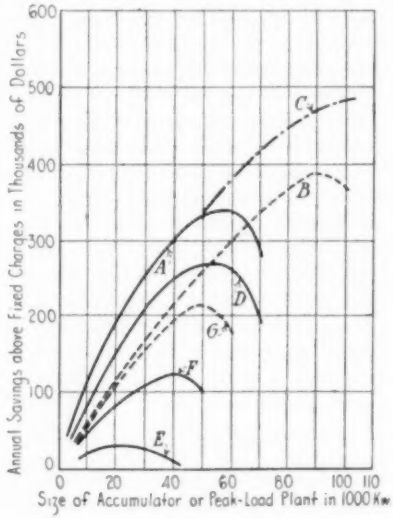


FIG. 11 ANNUAL SAVINGS THROUGH THE USE OF STEAM-ACCUMULATOR PLANT OR PEAK-LOAD UNIT ON A 200,000-KW. SYSTEM WITH 45 PER CENT ANNUAL LOAD FACTOR AND WITH COAL COSTING $5 PER TON
(See Fig. 9 for definition of curves.)

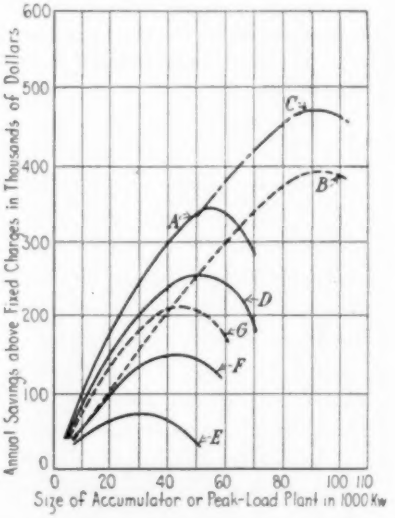


FIG. 12 ANNUAL SAVINGS THROUGH THE USE OF STEAM-ACCUMULATOR PLANT OR PEAK-LOAD UNIT ON A 200,000-KW. SYSTEM WITH 45 PER CENT ANNUAL LOAD FACTOR AND WITH COAL COSTING $4 PER TON
(See Fig. 9 for definition of curves.)

accumulator would decrease the saving only $15,768. Hence considerable variations in the initial cost and fuel costs of the accumulator plant will not wipe out its net savings. The effect of a change in the price of coal is variable, as can be seen by comparing points on curves A in Figs. 9, 11, and 13 for $5-per-ton coal with similar points on curves A in Figs. 10, 12, and 14 for $4-per-ton coal. An increase in coal price of $1 per ton will cause similar variations in savings in the opposite direction. It is interesting to note that in the case of standard plants the

TABLE 2 STEAM-ACCUMULATOR PLANT VS. BASE-LOAD PLANT FOR 65 PER CENT ANNUAL LOAD FACTOR AS PER FIGS. 3 AND 4 WITH COAL AT $5 PER TON

	Case 1, 20,000 kw.		Case 2, 30,000 kw.		Case 3, 40,000 kw.	
Steam-Accumulator Plant:						
Cost per kw	$67		$77		$87.50	
Fixed charges		$201,000		$346,500		$525,000
B.t.u. per kw-hr	30,000		30,000		30,000	
Annual load factor, per cent	1.5		2.7		5.2	
Fuel costs		15,768		42,574		109,325
Standard Plant:						
Capacity, kw	180,000		170,000		160,000	
Fixed charges		2,700,000		2,550,000		2,400,000
Annual load factor, per cent	72.2		76.2		80.0	
B.t.u. per kw-hr	14,985		14,910		14,850	
Fuel costs		3,411,933		3,383,885		3,330,202
Extra operators	1	1,752	2	3,504	2	3,504
Total		$6,330,453		$6,326,463		$6,368,031
Base-Load Plant:						
Capacity, kw	20,000		30,000		40,000	
Fixed charges		345,000		517,500		690,000
Annual load factor, per cent	80		80		80	
B.t.u. per kw-hr	14,150		14,150		14,150	
Fuel costs		396,653		594,979		793,306
Standard Plant:						
Capacity, kw	180,000		170,000		160,000	
Fixed charges		2,700,000		2,550,000		2,400,000
Annual load factor, per cent	63.33		62.35		61.25	
B.t.u. per kw-hr	15,180		15,200		15,230	
Fuel costs		3,031,711		2,822,689		2,614,930
Extra operators	6	10,512	6	10,512	6	10,512
Total		$6,483,876		$6,495,680		$6,508,748
Annual saving by use of accumulator plant in place of base-load plant with coal at $5 per ton		$153,423		$169,217		$140,717
Annual saving with coal at $4 per ton		153,290		170,975		146,975

effect of variations in coal cost is small over the ranges of size of accumulator plant that would probably be installed.

Considering the base-load unit in Case 1, a decrease in first cost to $100 per kw. will reduce its total costs $45,000. Even a 10 per cent reduction in heat consumption, resulting from higher boiler pressure and reheating, would only lessen total costs by $39,665. Normal variations in the costs of the base-load unit will not lower these costs to those of the accumulator.

Another factor is the variable element in the assumed heat rates. In the case of the base-load plant it is obvious that if the variable element is reduced, the coal cost of the base-load plant will also be reduced and the annual savings by accumulator or peak-load plant would be correspondingly reduced. For instance, if the variable factor is changed so that the heat rate of the base-load plant becomes

$$\text{B.t.u. per kw-hr.} = \frac{12{,}900 + }{\text{Annual load factor in per cent}}$$

$$\frac{80{,}000}{}$$

the base-load station's performance with 80 per cent annual load factor becomes 13,900 B.t.u. per kw-hr., a saving of (14,150 — 13,900)/14,150 = 1.77 per cent of its fuel cost. Except for the larger base-load plants this will have little effect on the final results shown by curve A in the various figures. If the variable factor of the heat rate of the standard plant is changed to

$$\text{B.t.u. per kw-hr.} = \frac{13{,}600 + }{\text{Annual load factor in per cent}}$$

$$\frac{80{,}000}{}$$

the annual saving in Case 1 would decrease $267. Calcu-

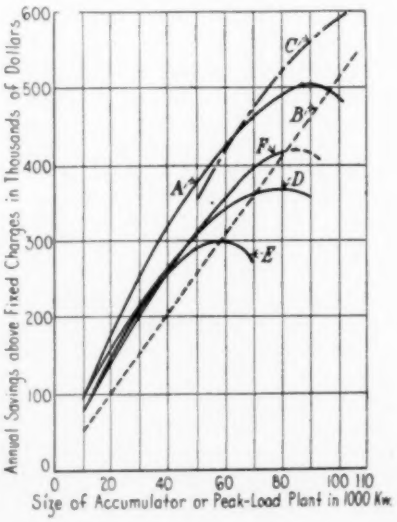


FIG. 13 ANNUAL SAVINGS THROUGH THE USE OF STEAM-ACCUMULATOR PLANT OR PEAK-LOAD UNIT ON A 200,000-KW. SYSTEM WITH 25 PER CENT ANNUAL LOAD FACTOR AND WITH COAL COSTING $5 PER TON

(See Fig. 9 for definition of curves.)

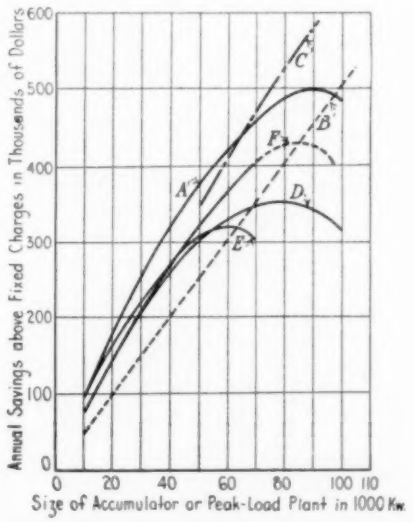


FIG. 14 ANNUAL SAVINGS THROUGH THE USE OF STEAM-ACCUMULATOR PLANT OR PEAK-LOAD UNIT ON A 200,000-KW. SYSTEM WITH 25 PER CENT ANNUAL LOAD FACTOR AND WITH COAL COSTING $4 PER TON

(See Fig. 9 for definition of curves.)

lations on the assumed plants indicate that reasonable changes in the assumed variable factor of the heat-rate equations for the standard plant, have no important effect on the annual savings of steam-accumulator plants.

PEAK-LOAD UNITS VS. BASE-LOAD UNITS

Calculations were made to determine the annual savings above fixed charges resulting from the addition of various sizes of peak-load plant in place of base-load plants. These savings are shown by the dotted curves B in Figs. 9 to 14. The difference between curve A and curve B in each case represents the difference in annual savings between the accumulator plant and the peak-load plant. This difference remains constant for a given size of accumulator and peak-load plant at a given load factor.

Curves A and B indicate the possibility of further savings with

25 and 45 per cent annual load factors from a combination of accumulator and peak-load plants. The annual savings from a few of these combinations are indicated by curves C in Figs. 11 to 14. These data indicate that the addition of peak-load plant effects no additional saving until the economic capacity of the accumulator plant is exceeded by the combined capacity of accumulator and peak-load equipment.

Steam Accumulators vs. Additional Standard Plant

Instead of adding a base-load unit to the system another unit may be added at the same cost and efficiency as those of the assumed standard plant. The annual savings that may be effected by the installation of a similar capacity of steam-accumulator plant to carry peak loads over the new standard unit are shown by curves D in Figs. 9 to 14.

Steam Accumulators on Systems With Standard and Old Plants

The figures having the greatest influence on the savings are the differences in the fixed charges of the accumulator or peak-load plant and the equivalent capacity of base-load plant. These differences will vary with the assumed plant costs. The next important influence is the saving in coal costs when load is transferred from one plant to another after the installation of the base-load plant. In the plants assumed in curves A, B, C, and D the differences in B.t.u. per kw-hr. between base-load plant and standard plant are not very great. Many electrical systems, however, are supplied from both modern and older stations which may have wide differences in heat rates. Consequently the shifts of loads between such stations when a base-load unit is added may result in a large saving in coal. If the relative sizes of old and modern standard plants in relation to the load-duration curve are such that this shift of load represents a large number of kilowatt-hours, the saving in coal costs by the installation of a base-load unit may offset, in a large measure, the savings in fixed charges that so greatly favor accumulator and peak-load plants.

An assumption was made that a system consisted of a certain capacity of standard plant whose station heat rate as before was represented by the formula:

$$\text{B.t.u. per kw-hr.} = 13{,}600 + \frac{100{,}000}{\text{Annual load factor in per cent}}$$

together with certain capacity of old plant with a heat rate as follows:

$$\text{B.t.u. per kw-hr.} = 19{,}000 + \frac{105{,}000}{\text{Annual load factor in per cent}}$$

A transfer of a kilowatt-hour of load from the old station to the standard plant, resulting from the installation of base-load plant, produces a coal saving of at least 0.4 lb. The annual savings that would result from the installation of an accumulator plant in place of a base-load plant, in a system with various combinations of standard and old plant with heat rates as above, are shown by curves E in Figs. 9 to 14. The value of the fuel saved by the shifts of load resulting from the installation of a base-load plant in such systems with high load factors offsets the advantage of lower fixed charges with accumulator plant, and leads to little or no net saving. With low load factors, however, this fuel saving is of less value, for fewer kilowatt-hours are involved in the shift of load, and the annual savings above fixed charges by the use of accumulator plant in place of base-load plant are large, as shown by curves E in Figs. 13 and 14.

Steam Accumulators on Systems With Mixed Plants

Many electrical systems do not have such wide variations between the heat rates of the old and the standard plant as in the preceding paragraph. Others may contain some plant with a heat rate intermediate between that assumed for the old and standard plant. Calculations were made to show the effect of the introduction of such plant, known as "medium plant," with a heat rate as follows:

$$\text{B.t.u. per kw-hr.} = 16{,}300 + \frac{100{,}000}{\text{Annual load factor in per cent}}$$

The annual savings above fixed charges resulting from the installation of accumulator plant in place of base-load plant in such a system are plotted in curves F, Figs. 9 to 14, and show a substantial increase over the savings shown in curves E. If the variations in heat rate were less extreme than those assumed and more intermediate plants employed, calculations indicate that the savings by the use of accumulator or peak-load plant in place of base-load plant will exceed those shown by curves F. This condition is approximated in many electrical systems where the operating equipment ranges from small, low-efficiency turbines and boilers, progressively up to the most modern highly efficient units. There are so many possible combinations of such plant with varying heat rates that many different costs may be obtained. Annual savings by the accumulator plant in one of these combinations are shown by curves G, Figs. 11 and 12. Calculations indicated that there could be a considerable variation in the proportions of the standard and older equipment for a given size of accumulator or base-load plant, without appreciably affecting the savings shown by curves E, F, and G, Figs. 9 to 14.

Attention has already been called to the differential between the annual savings of the accumulator and peak-load plant of a given capacity, which for given load factors, coal costs, and capacities of units is shown by the distance between the curves A and B. This differential at the same accumulator or peak-load unit capacity can be applied to curves D, E, F, and G in the respective figures if the annual savings by the use of peak-load units with these various combinations are desired.

Conclusions

With the assumed cost data and a triangular peak, the steam accumulator loses its advantage if the base of the peak that it is to carry exceeds four hours. These studies have been made on load curves with sharp annual peak loads such as occur in some cities with large lighting loads. Calculations indicate that peak-load units are less economical than an accumulator plant except in the large sizes. On the other hand, the availability of peak-load units for emergency service, particularly in the case of long-sustained peak loads on dark days, is greater than that of the accumulator. The same degree of availability may be secured on the accumulator turbine by making it a mixed-pressure unit.

The load-duration curves of systems with broad, flat daily peaks may show a certain portion of the annual peak which has a low annual load factor. The long duration of such daily peaks will favor the installation of a peak-load plant in preference to steam accumulators.

A steam-accumulator plant could carry peak loads other than during the annual peak, and this would lead to station economies due to the operation of fewer boilers and standard turbines. The charged accumulator is also available for emergencies during off-peak hours such as thunderstorm peaks, fog, etc. The charging of the accumulator with steam during the night would improve the load on the boilers, and this should lead to more efficient boiler-house operation during that period. Accumulators require little maintenance and repairs, and this cost will be less than for the equivalent capacity of the boiler plant.

Accumulators are subject to little wear and tear, hence their rate of depreciation should be less than that of a similar capacity of boiler plant. No financial credit has been given in this paper to the accumulator plant for any of these savings, although it is reasonable to expect that these may be of a substantial size. The annual savings shown by the various curves refer only to the annual peak load of the system.

The accumulator, with its lower first costs, saves in fixed charges. The base-load plant saves fuel. The net savings depend on the relative values of these two savings, and as shown by the various curves given in the paper, may amount to a large sum of money.

An important point shown by curves A, B, C, and D in the various figures is that steam accumulators and peak-load units effect their largest savings when added to a system consisting of standard plant with modern equipment only. Accumulators have usually been considered for addition to systems with older plants, whereas this study indicates that they can effect the largest savings when applied to systems with the most modern plants, provided these carry sharp peak loads.

A second striking fact is the large margin of saving that may be secured by the use of accumulators in place of base-load plant to carry annual peaks with any plant combination when the annual load factors are low.

A third point of interest is that the lower the price of coal, the larger the accumulator savings in systems with old and mixed plants, as shown by comparing the curves for different coal prices. This results from the fact that the value of the fuel saving effected by the addition of base-load units is less with low-priced coal than with expensive coal.

It is well to bear in mind in studying the figures presented in this paper that the results refer specifically to the assumed data. In conclusion, although these studies do not consider all possible cases of annual peak loads, they will direct attention to the importance of considering methods of carrying such peak loads and the possible place of steam accumulators and peak-load units in modern central stations. Since each system is a problem in itself, and should be considered separately, this paper may suggest ways of studying the peak-load problems of various power systems.

Discussion

GEORGE C. USHER.[4] It will probably be conceded that the peak-load problem in British power stations will undergo a very considerable change as the Electricity Act comes into operation and the linking up of selected stations takes place. It is reasonable to suppose that the larger selected stations will operate as base-load stations, while smaller stations in each area will be used as peak-load stations and brought into operation only to take the peak loads in each group. In the immediate future, however, each station will have its own particular peak-load problem to deal with, and whatever method is adopted there are two main considerations which must be generally satisfied. These are, first, that the response of the plant to peak-load demand shall be immediate and, second, that the response shall be capable of indefinite prolongation.

In the boiler house both of these important points can be met with pulverized fuel. Pulverized-fuel-fired boilers are capable of wide and immediate variation of load, together with a high standard of generating efficiency at all ratings. The steam generator, which is the logical development of such boilers, fitted with water screens and walls, has enabled a plant of high evaporative capacity to be installed at comparatively low first cost.

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For a station where the peak loads are frequent and of short duration, it is possible to take these peaks by driving the generators beyond their normal continuous rating. It is not possible to advocate this practice with stoker-fired boilers, however modern the design, since the maximum rating is definitely limited by the grate performance. It may seem an exaggeration to say that the evaporation from a steam generator is limited only by the rate by which it can be fed with water; but, provided intense circulation can be maintained and the burners are designed for short turbulent-flame production, the evaporation per square foot can be brought to exceedingly high figures, and in emergency can be pushed well above the normal maximum running.

Stations having a well-defined peak of several hours' duration are best served by the installation of peak-load generators. These can be brought up to pressure in the minimum of time and at minimum cost, and can with perfect ease be banked ready to go on the line; they are available, therefore, not only for peak-load service but for emergency service.

By far the greater part of the paper is taken up with a cost analysis of various methods of carrying peak load on a 200,000-kw. system with varying peak load and varying load factor. The disadvantage of such a method is that the results of such an investigation are only of special application, and for other reasonable cases could be assumed to give entirely dissimilar results, such as a station having high annual load factor with a fairly high percentage peak of steady duration. Further, the basic figures of plant cost compare very unfavorably with such as can be obtained in this country for a station of such large capacity. The coal cost figure also seems extremely high.

(1) A large increase in size is necessary with increasing duration of demand
(2) The heat consumption rate per kilowatt hour is extremely high, and often more than twice that of an efficient generating plant
(3) The accumulators are not necessarily available in sudden emergency such as the breakdown of plant.

M. D. ENGLE[5] AND G. R. DAVISON.[6] The paper is timely, and it brings up for discussion the question of the most economical equipment for generating the low-load-factor portion of the system load, or, in other words, those kilowatts which result in the sale of very few kilowatt-hours of electrical energy. To supplement the paper we would like to point out several factors which have not been brought out and which have a very important bearing on the subject.

The first of these is the factor of turbine-generator inspection and maintenance. This factor is not very important on systems which have a summer load that is very small compared with the peak load of the previous winter, but let us consider a system with a lighting and power load such as shown by Fig. 3 in this paper.

Fig. 15 shows the daily load curves of the Edison Electric Illuminating Company of Boston expressed, for the convenience of the discussion, as a percentage of the 1927 annual peak. The load factor of this system has been increasing year by year and is approximately 40 per cent at present. The peak load is growing at the rate of approximately 9 per cent per year, and so we believe can be taken as representative of many systems in this country. The peak load during the summer of 1928 was approximately 80 per cent of the peak load of the previous winter,

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[6] Generating Department, The Edison Electric Illuminating Company of Boston, Boston, Mass.

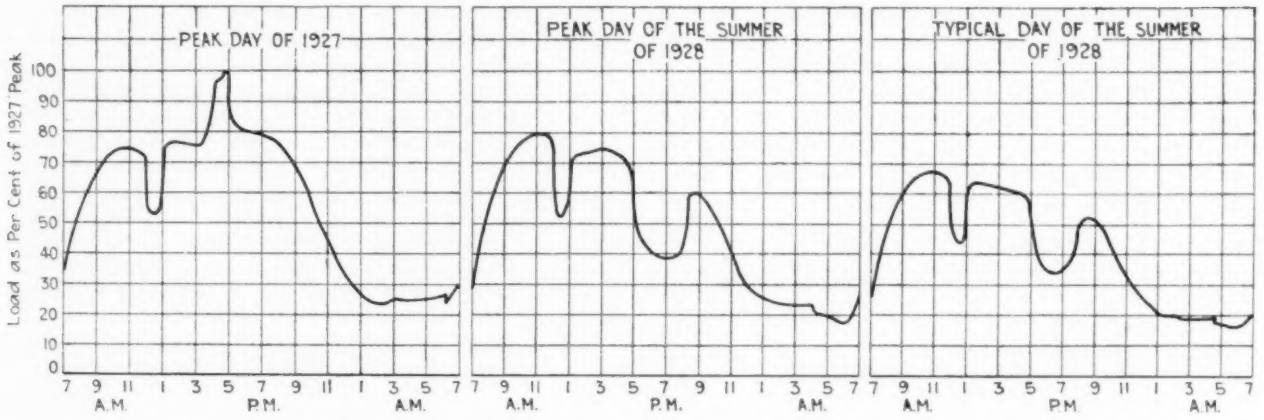


FIG. 15 DAILY LOAD CURVES OF THE EDISON ELECTRIC ILLUMINATING COMPANY OF BOSTON

and the peak load on a typical week day during the summer of 1928 was approximately 67 per cent of the peak load of 1927.

Let us consider a system which has daily load curves such as shown by this chart and the annual peak of which is 200,000 kw., as was assumed in the studies given in the paper. From this chart it is seen that such a system would have a peak load of 160,000 kw. during the summer following the 200,000-kw. annual peak. The peak load on a typical day of the same summer would be approximately 135,000 kw.

It is good operating practice to open up all turbine generators once a year for inspection and maintenance. If the number of machines on the system is large and the inspection and maintenance work is carried on during the whole year, to serve the system load will require an installed generating capacity between

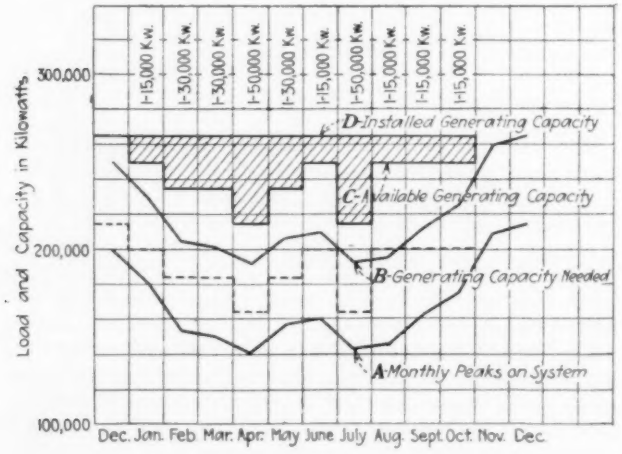


FIG. 16 MONTHLY PEAKS OF SYSTEM

20 and 30 per cent greater than would be required if all machines were available for carrying load during the few months of the year when the annual peak load might be expected to occur. We believe that all will agree that the economical method is to arrange the maintenance schedule so that all machines are available during the peak period.

The 200,000-kw. system that we are considering might be served by five 55,000-kw. turbine-generator units. In actual practice, however, such a system would probably be served by units varying in size from 15,000 kw. to 50,000 kw. Let us assume that there are five 15,000-kw. units, three 30,000-kw. and two 50,000-kw. units, having a total capacity of 265,000 kw.

Fig. 16 shows the monthly peaks on the system for the spring and summer following the 200,000-kw. annual peak and also the generating capacity needed and the generating capacity available.

The maintenance schedule for this system would require that ten turbine-generator units be opened up during the year. It has been our experience that this inspection and maintenance require an average of one month per machine. This time would probably be less for the smaller and longer for the larger machines, but for simplicity we have shown each unit out for one month.

Curve *A* shows the monthly peak loads on the system for the year following the peak load of 200,000 kw. in December. Curve *B* shows the generating capacity needed on the line to supply the load and protect the continuity of the service by having at all times sufficient capacity on the line so that the load can be carried if the largest unit trips out for any reason. Curve *D* shows the installed generating capacity of 265,000 kw., and the cross-hatched area shows the capacity of the units out of service for inspection and maintenance. The curve *C* at the bottom of the cross-hatched area shows the generating capacity available during the time when the various units are out of service.

The dotted curve shows that at no time during this period would there be sufficient generating capacity if a 50,000-kw. unit was unexpectedly lost. These curves show that in this setup we are not attempting to protect against the contingency of having a unit out of service for maintenance when a 50,000-kw. unit fails; and yet with the number of units involved that is really a possibility.

There are no units on such a system that are standing idle all year waiting for the annual peak. In fact, practically all of the available units will be operating for a considerable number of hours, practically every month of the year. This same general statement is true for steam-generating equipment.

We therefore do not see where the strictly peak-load equipment is justified unless for some special reason the system has an abnormal peak during a very short period of the year.

H. HOBSON.[7] The paper resolves itself into a comparison of the merits of installing Ruths steam accumulators together with suitable generating plant for dealing with the peak load with the benefits from installing new and efficient boilers and generating plant for dealing with a corresponding amount of base load. It is assumed, in either case, that the load on the balance of the system would be dealt with by plant referred to as "standard plant."

The argument in favor of the installation of steam accumulators

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and equipment is really based on the claim that the increased heat consumption per unit generated by the steam-accumulator plant is more than offset by the reduced capital charges as compared with new and efficient boiler and turbine equipment assumed to be installed for dealing with the base load.

On the basic data assumed in the paper, it would, however, appear to be impossible to make a case for the installation of base-load plant at all. This is due to the fact that, as compared with the standard plant, the reduced heat consumption of the base-load plant would not effect a sufficient saving in coal cost to cover the increased capital charges incurred.

Even if the base-load plant were operated at 100 per cent load factor, the saving in coal cost would be less than one-half of the extra capital charges incurred. Having regard to both capital charges and efficiency, the base-load plant is thus very much worse than the standard plant, and in no circumstances, therefore, could a case be made for its installation.

The foregoing anomaly has a direct bearing on Table 2, which gives the annual costs incurred by the installation of a steam-accumulator plant for dealing with the peak load as compared with the installation of a corresponding amount of base-load plant. It will be seen that the case for the installation of a steam-accumulator plant is seriously impaired, if the comparison is made with the costs which would be incurred by the installation of further standard plant instead of by the installation of base-load plant.

Apart from these considerations it is doubtful whether it is sound economics under present-day conditions to install plant for peak-load operation. If no further improvements in power station design could be foreseen, it would certainly pay to construct cheaper and less efficient plant for dealing with the peak load. Under present conditions, however, the improvements in power station design are so rapid as to warrant, in many cases, the scrapping of existing plant before it is actually worn out. In these circumstances, it is evident that the tendency should be toward running the older existing plant at peak load, and not toward spending capital on new plant to deal with this load.

So far as the conditions in Great Britain are concerned, a further aspect presents itself as a result of the transmission system which is now being constructed by the Central Electricity Board.

This transmission system, commonly known as the "grid," will interconnect all the stations required for generating the electricity supply of the country. The base load will be generated by efficient stations which will be in continuous operation, but the peak load will be generated by the less efficient stations which will be brought into operation when required.

From this point of view, it does not appear that the steam-accumulator plant would have any advantages to offer, as it necessarily involves the boilers generating steam for the purpose of charging the accumulators at times of light load on the station. In general, however, when the "grid" system is complete, it is not proposed to operate stations under these conditions. The peak-load stations will be brought into operation when required, during which time they will be generating nearly their full output. After the peak load has been dealt with, the stations will be closed down, thus enabling substantial savings in standby coal and labor costs to be effected, and this clearly would not be possible if the boilers had to be kept under steam for the purpose of charging steam accumulators.

LEE J. LEVERT.[8] The paper outlines an ideal system—a system where there are no troubles; a system where all machines, boilers, and feeders stay in; a system where all the overhauling is done on schedule time; a system where the customers are drilled to turn on and turn off the lights on certain schedules assigned to them by the public utility management.

The system outlined is a dream, however—a dream of every public utility executive; it is a dream of every operator. Our friends from Boston sound the first pessimistic note. You know the tea party which started the Revolution took place in Boston. They tell their brethren in Maryland that it is quite possible that the machine might trip out while under load. They also say that sometimes it is impossible or inadvisable to overhaul the machines on the off-peak time and there is a likelihood of the necessity of taking the machine out when the load is very heavy.

I am still more pessimistic than our friends from Boston, and I say that not only turbines but boilers and feeders are quite likely to have their tantrums. There is no relationship between the turbine outage and that of the boilers and feeders. It is quite likely that we lose turbines in one place, feeders in another, and boilers in a third. It reminds me of the case of a person having a blue coat and gray vest in one place and a gray coat and blue vest in the other; similarly, it is quite possible that we might have an excess boiler capacity due to turbine outage at a station in one place and an excess generator capacity in another. It does not do any good for the East River superintendent to know that Hell Gate has an excess steam capacity to run East River turbines, because that steam is just as unavailable as if Hell Gate station was in Shanghai, China. I take pleasure in paraphrasing Lincoln's saying as "You can run some of the equipment all of the time, and you can run all of the equipment some of the time, but you cannot run all of the equipment all of the time."

We can obtain the base load to be carried by the system from the load-duration curve. The economical capacity required, however, to carry the base load must be at least 30 per cent greater than the base load itself, because we must provide for an outage for repairs and an outage on the load at any point of the system. Inasmuch as the turbine represents the greatest concentration of capacity, which might not be true a few years from now when some of the 80,000-kw. boilers will be in operation, only turbines need to be considered. We have to assume that any station might have one of its largest units out for repairs and another unit in the same station might trip out during the peak load of the system.

I believe that all the peak capacity can be taken care of by the overload valves of the turbogenerators, forcing the boilers, and old generating stations. Why in the world should we spend more money for peak-load capacity when we have all generating capacity which can take care of the extreme peaks and be used as standbys for emergency?

Of course, there are cases when the dismantling of old equipment might be dictated by economical situations such as land values or change of the nature of the locality, or the advance of the art might be so great as to make it economical to replace this equipment by more efficient machines. I would suggest adding another curve to the multitude of curves which are already used for power-plant work. The curve I suggest is a curve which would show the capacity required to carry the loads shown by the load-duration curve, and for the want of a better name I would call this curve the "capacity duration curve." This capacity duration curve would give us the capacity which the system must have available in order to maintain certain reliability standards at all times when handling the load indicated by the load-duration curve.

I. E. MOULTROP.[9] There is no such thing as a base load station. Any station may be operated on base load for the first few

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[9] Chief Engineer and Assistant Superintendent, Construction Bureau, Edison Electric Illuminating Company of Boston, Boston, Mass. Mem. A.S.M.E.

years of its life or any station may be operated on the base load during certain parts of the year, but there is no such thing as a 100 per cent load factor station, and we should stop talking about such dreams.

From some of the discussions presented, the writer is inclined to believe that some of the discussors have read this paper very casually. If they have, they have made a mistake. Assuming the conditions that the author has set up, his figures are approximately correct. The only problem is to find a situation where those assumptions apply.

The power industry is growing so fast that the electric utilities find they have a large amount of obsolete generating equipment on their hands. This obsolete equipment generally proves to be the cheapest peak equipment that they can use.

There are a number of fields where the steam accumulator may be justified. The central steam-heating business is growing rapidly. It may be that the steam accumulator will play an important part in the development of this industry. It also appears logical that the steam accumulator may be economically justified in industrial installations which have rapidly fluctuating demands for hot water and steam.

Warren Viessman.[10] The author has presented a very interesting paper on the present practice of applying and using the steam accumulator, principally on the Continent. The manner in which he has covered the procedure for analyzing and selecting an accumulator is very useful. It is the kind of analysis that we need to consider in our problems, and although, as a number of speakers have said, there are certain cases where we would not want to consider an accumulator, there are also many others where it might be advisable.

My experience with large central stations has been in accord with that of the previous speakers; that is, there are many factors which enter into such a determination, and in order to be certain that the plant can carry the load when machines are out for repair, an accumulator or even a peak-load installation is not usually advisable, base-load machines being much preferred.

However, on central-station systems which have a peak load similar to that of the city of Berlin it is doubtless advisable to have a steam-accumulator installation to take care of the highest peak load, after standby equipment has been put on the line. In the small isolated central stations the peak loads are rather severe at times and of short duration, and in a central station of this type an accumulator unit might be most desirable.

I appreciate what the previous speaker said in regard to the use of an accumulator on a central district heating system. On such a system when heat is put on on cold mornings, the peak is very heavy, and for half an hour or perhaps an hour the use of an accumulator would be most beneficial.

One of the broadest fields for the application of such equipment is the industrial field. In that field there are many opportunities of application in industries where the steam consumption for process work is short but heavy, and application in industries of this type does not require a special turbine, but merely the accumulator installation itself to meet the process steam requirements, relieving the boilers which are usually operated nearly to capacity all the time. With an accumulator the process steam could be generated in the boilers and stored in the accumulator during the periods of light power loads and withdrawn from storage during the periods when the power and process loads are very heavy.

A. E. R. de Jonge.[11] The very able paper has interested me very much. There is one point which has not been brought out by any of the previous speakers, and this is that the steam accumulator can have application only in power systems where conditions are fairly stable; i.e., where large increases in capacity are not expected. In any rapidly growing system I do not think that the steam accumulator will stand any chance whatsoever.

I have been associated for over nine years with one of the most rapidly growing plants in the world, the power station of the Shanghai Municipal Electricity Department in China. You may picture to yourself power stations in China as some small and unimportant affairs, not very modern, erected in some out-of-the-way place, and probably kept up badly. But that is one of the most modern plants in the world.

The Shanghai station is only one plant, not a system of interconnected plants such as in this country, and yet the load factor is as high as 60 per cent (see *Engineering*, June 1, 1928, pages 685 and 805); and I believe that this is one of the highest, if not the highest, load factor for any single station. These remarks refer to conditions as they were in February, 1928.

This system has had a most remarkable growth, having grown during the nine years of my association with it from a capacity of 39,000 kw. to 161,000 kw. From my intimate experience with that system, the steam accumulator has no chance whatsoever in such a rapidly growing system.

R. A. Langworthy.[12] The author has emphasized a phase of power-plant development that is well worth the attention of those interested in the design, construction, and financing of such plants.

In the last ten years great strides have been made in the matter of increased efficiency, and the performance of the large modern power plant is remarkable when the results of ten or fifteen years ago are considered.

It seems to the writer, however, that in the endeavor to gain the last fraction of a per cent in efficiency some of our engineers and designers perhaps have lost sight of the very important factor of fixed charges on the investment. The fixed charges are an important part of the cost of electric power, and when the station is finished they are a permanent burden and cannot be affected in any way by the future operation of the plant, no matter how well organized it may be.

It is a matter of serious question sometimes whether, in our effort to reach the last notch in efficiency, our investment has not been increased by an amount which would actually show a net loss on the overall cost of the power, when both fixed charges and operating expenses are considered.

The modern power plant is a complex institution and contains a great deal of apparatus upon which the designers have expended their best thought. They have, undoubtedly, produced very efficient machinery, but as is often the case efficiency beyond a certain point is attained at the expense of a proportionately larger increased cost of the equipment involved.

In stations having an average load factor, much of this expensive equipment, representing a considerable part of the investment, is installed to meet the peak loads and is idle for a great part of the time, although the fixed charges continue 365 days in the year.

It is manifest that any design which will reduce the investment necessary to carry the peak load and thereby reduce the fixed charges should have most serious consideration.

The author has pointed out a method of meeting the situation of peak loads by means of the steam accumulator. So far as the writer knows, this piece of equipment has never been seriously considered in the United States in conjunction with the central power stations. Its application, however, to large power stations

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[11] Designer, New York Edison Company, New York, N. Y. Mem. A.S.M.E.

[12] United Engineers and Constructors, Inc., New York, N. Y. Mem. A.S.M.E.

for the purpose of carrying the peak loads might be termed unique in the history of such plants on this side of the water. It would seem that in the future design of large central stations this piece of equipment should be considered on its merits, the same as any other part of the power plant, and a decision on whether to install it should be based upon its effect on the total overall cost of generating electric power.

LOUIS ELLIOTT.[13] The author has done the profession and the utility industry a service by presenting in clear fashion a specific problem in providing for peak loads on a power system, with a solution comparing alternative schemes. In this country it has been the usual procedure, in building up a steam-power system, to install a plant unit to carry base load— at least for the first portion of its life. Such units have almost always been designed to give an efficiency at least equal to that of the best existing facilities, with the assumption that the low-load-factor portion of the annual requirements could be carried advantageously by the less economical equipment already in service.

Although this course of action has in general been wise, especially considering the radical improvement in the art, the rapid growth of power systems, and the frequent consolidations during the past decade, it is timely that attention should now be drawn to other methods of system extension. Basic conditions in not a few regions are becoming more nearly stabilized, load growth is going on at a lower rate, and station economies may not improve as fast as heretofore. In some cases the base load can be taken care of economically by existing capacity, and it is the increasing peak that must be provided for.

Whereas in the past steam accumulators have appeared to American utility engineers to be suitable chiefly for industrial work, Diesel engines have been used in relatively small sizes only, and plant units designed specifically to carry peak loads have been rare, a careful comparison of different methods for carrying peaks should now be made, with no prepossession in favor of past practice. As has not infrequently been the case in other developments, European engineers have led the way in the adoption of certain of the newer methods of handling peaks on steam-power systems—such as the aforementioned use of steam accumulators and the installation of Diesel engines. Although European conditions are frequently quite different from those obtaining in this country, and although solutions of problems on a moderate scale cannot always be applied on a large scale, certain combinations of circumstances may justify the adoption of methods heretofore little used by utilities in this country.

It is evident that a great deal of careful analysis was required in the preparation of the paper, and the resulting curves and conclusions, based on the assumed conditions, are most interesting. As is stated in the paper, however, it would be unwise to apply to any new problem the principles laid down, without a careful collection and analysis of pertinent data; a moderate variation in sharpness or duration of peak may change entirely the economics of the case.

ALPHONSE A. ADLER.[14] The writer sees a great many applications of steam accumulators in industrial plant work. He has in mind dyehouse work in particular. Here the peak loads come twice and sometimes three times a day, depending upon the type of goods in process. There are additional advantages in plants of that character because they can not only use the steam for process work but can also use the hot water to start the tubs at a higher temperature so as to release some of the boiler capacity required in warming up the water beforehand. That is an excellent application for steam accumulators. However, a great many local plants are not sufficiently large to warrant the installation of complicated equipment. Engineers are compelled to design these small plants for operators who know just enough to run a plant like a great many plants they have seen. If you give them a new machine, you sometimes have to train a new man for the job.

N. E. FUNK.[15] The author has presented in a clear manner the thought of the possibility of installing peak-load rather than base-load plants, and in his conclusions has pointed out the fact that suggestions in this paper are not a cure-all for every power problem.

The writer believes it may be well to emphasize some of the factors involved in this problem. If steam accumulators are installed instead of base-load units, there is a decrease in installation costs, but the old plant must produce much more energy than would be the case were base-load units installed. Therefore, basically, the savings in annual charges, due to lesser installation costs, are reduced by an amount equal to the product of the energy that would have been produced by the base-load plant had it been installed and the difference in the increment operating costs of the old plant and base-load plant. Decreases in the difference between the cost of installing the steam accumulators and the base-load units, or increases in the increment difference, reduce the net saving. These changes may even produce a negative result in respect to saving, in which case it would be more economical to install the base-load units.

A very rough comparison is given in Table 3, using some of the figures in the author's Case 1, but changing the increment energy cost difference between the old plant and the new base-load plant.

TABLE 3

Difference in mills per kw-hr. between increment energy cost of old plant and base-load plant	Annual savings by installing accumulators	
	With 15% annual carrying charges	With 12% annual carrying charges
1.50	−$ 35,000	−$ 64,000
1.30	− 7,000	− 36,000
1.00	+ 34,000	+ 5,000
0.7	+ 75,000	+ 46,000
0.3	+ 130,000	+ 101,000
0.1	+ 158,000	+ 129,000

The table distinctly shows that with large differences between the increment energy costs of the new base-load plant and the old plant the installation of steam accumulators would result in a loss as compared with the installation of new base-load units. The same situation would hold as the difference in cost between the installation of base-load units and steam accumulators decreases.

In combination hydroelectric and steam-electric generating systems, where a variable river flow occurs, the hydroelectric plant must be utilized during the high-flow period as a base-load plant and during the low-flow period as a peak-load plant. To obtain the maximum capacity value from the hydroelectric plant at extreme low flows it is necessary to utilize it on the extreme peak of the load. This would force the steam-accumulator section of the steam plant into a section of the load where the duration would be considerably longer than four hours and would immediately put it beyond practical consideration from the point of direct increased carrying capacity, although it still might have a use as an emergency reserve.

The steam accumulator has been used in the past for smoothing out very rapid power fluctuations such as in rolling-mill engines

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and for like services. An installation of this kind might be feasible and show a considerable economic advantage in a station supplying railway service only, with extremely high peaks of very short duration, in which case the accumulator would alternately store and release energy without very great change in pressure. It would thus permit the boilers to operate at a much better efficiency.

The steam accumulator has an additional disadvantage in its application in a large power system in that it is an inflexible tool and is incapable of as wide use in the load curve as a standard plant.

In determining the usefulness of this type of plant it is therefore necessary to predict the type of system load over the useful life of the projected accumulator plant, the projected base-load plant, or the projected peak-load plant in order to decide which would give the maximum economy for the entire life of each type of installation.

The author in his conclusions has definitely pointed out that the results presented in his paper do not indicate that steam accumulators should be applied to all systems having very sharp load curves, and distinctly warns against this assumption. Even though it is a repetition, the writer desires again to call attention to the fact that this application of steam accumulators must never be considered as a sure economic possibility and that each case must be carefully considered, giving due weight to all the variable factors in each specific problem, before a sound decision can be made.

H. L. H. Smith.[16] It seems to the writer that one of the most fundamental aspects of the whole question is whether a power company should spend money on handling its peak load or whether it should spend money only on handling the base load. It seems that the question can be considered from two sharply divergent aspects, one of which has a basis in reality and the other has not, although it has significance in clearing our ideas on the matter. If, for instance, any electric power company were confronted by the extremely unreal situation of building up a large power system in a community which had only started, but which it was known in a matter of half a dozen years would attain a large size, the electric company might reflect that in half a dozen years it would have to handle a peak load of a certain high value and a base load of much lower value, and it hence might consider actively whether it should scheme to handle the peak load by the use of accumulators. Actually, of course, no such situation ever confronts any company.

What we always have is a growing system, so that in considering the illustrations of the paper, the horizontal axis always remains the same, whereas the vertical axis is always expanding, so that if a company installed an accumulator system at any stage of its growth, as the years went on that accumulator system would be able to operate a lower and lower number of hours each year. Of course, the answer of American systems as to whether they should spend money to handle peak loads has unmistakably been in the negative. It is an open question as to whether the answer will always remain so. The writer thinks that all will agree that considerable light has been shed on the question by the paper.

Among the somewhat radical schemes that have been suggested to handle the peak load of a constantly enlarging system is one that occurred to an executive of this company, which is situated on the Atlantic Coast. He said that he would like to have a study made which would consider if there would be anything in the idea of constructing a steam station on a barge and hauling it up and down the coast line from one year to another in order to handle the peak load in the most efficient manner, as it might be acute in one season in one locality and more acute in another season in another locality; the idea being that the turbine and boiler plant constituting the floating peak-load station should be especially designed to represent low cost of installation, economics of operation of course working the other way. Such an investigation has been started, but no answer as yet has been obtained.

F. T. Leilich.[17] The author has presented a very interesting set-up of the economics of accumulators for hypothetical plants, but it is not possible to draw other than very general conclusions from it.

His assumed B.t.u. per kilowatt-hour and unit costs are somewhat better than for the average modern plant. Taking Case 1 as being representative, the saving of $153,000 for the accumulator is less than 12 per cent on the investment. This saving would be reduced to about $100,000, or 8 per cent net, if the slightly improved annual load factor was not credited with 1.3 per cent economy betterment. Estimating on the very elusive duration curve for future years does not provide a very secure basis upon which to make large expenditures. With the growth of power systems and interconnection the whole becomes more elastic for the load dispatcher, and as the most economic point of operation is generally less than full load, the reserve capacity in machines operating on the line is large, and short-duration peaks may be easily handled. In a small system an accumulator and special turbine installation must be backed up by reserve capacity; and for a large network the elasticity of the system will enable it to absorb swings and peaks. The accumulator and its turbine are not readily adaptable as a universal reserve.

The annual load factor is of little use in preparing an operating and cost schedule, because service outage, and the base, peak, and hydro power available must be considered in preparing load schedules during the year.

The study seems to overlook certain important practical considerations essential to insuring continuity of service. To fulfil its mission of carrying load over the peak hours as shown by the load-duration curve, the accumulator and its machine would have to be charged and on the line for more than the entire period over which the peaks extend because the amount and time of these are not exactly known nor can they be predicted much in advance. To carry the accumulator equipment as capacity reserve during times of reduction of available capacity for overhaul and inspection would involve operation of the equipment over greater periods, with resultant increase in the kilowatt-hour output. Based on total operating and fixed costs of the plants as assumed, the savings are less than $2^1/_2$ per cent and the fuel costs are roughly 50 per cent of the total; therefore the margin is only 5 per cent to cover unforeseen conditions of load, coal costs, etc.

The accumulator undoubtedly has a place in power generation, but each system must be considered as a special case and detailed studies must be made accordingly. Considering the uncertainties of future load and interconnections, it seems reasonable to insist that an installation of special equipment should clearly show a net return sufficient to cover a reasonable divergence from estimated conditions and at the same time assure the investors a substantial profit.

Author's Closure

We have heard most of the objections that may be raised to the use of steam accumulators in electrical power plants. These have been presented principally by men who are primarily interested in the operation of such plants. Their opinions have quite evidently been based on operating rather than financial considera-

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tions. Operators wish to have only standardized equipment in their plants in order to simplify operating problems, although, as the paper itself emphasizes, this may not result in the production of power at the lowest total cost for a given set of load conditions.

It has been pointed out repeatedly in other discussions that whereas the production cost of power has decreased through the use of large high efficiency units, the fixed charges per unit of output have changed but little for several years. Public-utility executives are asking engineers to consider first cost more carefully, and will soon demand plants of lower first cost in order to further reduce total power costs. This will mean a reconsideration of plant design, of unit sizes, and of means for carrying peak loads in the cheapest possible manner. When engineers study the problem from this viewpoint, the advantages of steam storage will be obvious where high peak loads of short duration must be carried. The author believes with Mr. Moultrop that many engineers have only read this paper in a casual way.

Several speakers referred to the advantages of steam accumulators in industrial plants and on heating systems. It is now quite generally recognized that steam storage has distinct advantages in these services and that such uses will be extended very rapidly as opportunities are presented to add such new equipment. In view of the unanimous agreement in regard to this use of steam accumulators discussion of these applications was omitted from the paper, which is confined solely to demonstrating the economics of adding accumulator plant to steam power stations to carry peak loads.

The possible use of older equipment for stand-by and peak-load service is fully recognized and discussed in the paper. There is little question concerning the suitability of this equipment for such service. The question raised in the paper is whether it may not be cheaper to add low-cost accumulator plant to carry peak loads and to maintain the present plant in continuous service carrying regular loads for more years, than to buy further new standard equipment and to relegate present equipment to peak-load service only. This is a financial as well as an operating problem, and as shown in the paper, there are conditions where the addition of accumulator plant may save large sums of money annually.

There is another point in regard to the growth of load in this country that must not be overlooked. Power salesmen have been very active in our utilities until almost all the present available power loads have been added to the utility's service. These additions have materially improved load factors and have lessened the sharpness of peak loads. Are we assured that the power demands will continue to grow at an equal rate as lighting peak demands? This is questionable, and in some cases there are indications of the reappearance of sharp lighting peaks. As one of the discussors pointed out, some customers have already been notified to limit their peak loads. The author knows of a large system which can force certain consumers to discontinue their demands at the time of peak load and furthermore pays them rebates for so doing. Are such policies right? Would it be better economics to utilize steam-accumulator plant to carry these peak loads and to allow the customers to continue manufacturing at their maximum output? After all, it is the extra marginal production over a certain definite output that fixes profit in an industrial plant. The necessity to curtail this extra production must diminish the manufacturer's profits to a greater extent than he is recompensed by any possible rebate that the utility may pay. The utility should therefore be prepared to meet any possible peak demands that may occur in serving the consumer.

Referring specifically to some of the points raised in the discussion, Mr. de Jonge objects to the use of steam accumulators on a rapidly growing load. That is exactly the type of load in the city of Berlin on which the German engineers decided that the accumulator had its best possible field of application. Why should one set aside, only for peak service, large units that have been installed only a few years, when they are still serviceable and economical and when the peak load can be carried by cheaper accumulator sets? Dr. F. Munzinger has stated that the large Klingenberg Station in Berlin, with its 80,000-kw. units, cost $60 per kilowatt to build, while the accumulator plant now being installed by the same company at its Charlottenburg station cost only $45 per kilowatt. Character of load, cost of coal, and first costs of equipment must determine whether steam accumulators should be added to any system regardless of rate of growth.

Mr. Smith's interesting discussion indicates that utility executives are already giving consideration to the economics of handling peak loads. He calls attention to the difficulty of predicting the character of future loads, which difficulty is quite generally recognized.

Mr. Langworthy has emphasized the importance of watching investment costs in striving for high efficiency, and particularly when plant equipment has few hours use per year.

The discussion by Messrs. Engle and Davison attempts to show that the outage schedule of turbines for overhaul and maintenance necessitates the operation of every unit for a considerable number of hours each month and that steam-accumulator plant would not be suitable for this service. An analysis of their data indicates that this condition is largely due to their local combination of unit sizes and that quite other conditions might prevail had other turbines been used. Fig. 15 indicates that 20 per cent of the maximum peak, or about 40,000 kw., is of the type best handled by accumulators. The writers state that load growth is 9 per cent per year, or about 20,000 kw. If 30,000-kw. units had been added in place of 50,000-kw. sets, the flexibility for reserve and maintenance would have been increased, and a possible further addition of two 20,000-kw. accumulator units for the peak might have resulted in a cheaper and more economical system from a financial standpoint than the present one. Also their outage requirements would have been less pressing if new equipment for the next winter's peak had been available in the preceding fall. Furthermore, the use of an accumulator set would lessen the peak demands on the boilers and furnaces and would lower stoker maintenance.

Mr. Usher calls attention to the need of immediate response at the time of peak load. Steam-accumulator plant fills this need. As the accumulator turbines operate on saturated steam, they can be started easily and in less time than when superheated steam is used. Once on the line they can take full load immediately. These turbogenerators could spin on the line as synchronous condensers, when their response to load demands would be instantaneous. For instance, a steam-accumulator plant at Malmo, Sweden, in a demonstration in 1927 picked up the complete load of a hydroelectric system when the line breakers were tripped, with not more than a momentary fluctuation in voltage. Peak-load units do not need to be capable of carrying load indefinitely, for other reserve is available for use at all loads under the peak. Attention is called in the paper to the use of special peak-load units for long-sustained peaks due to fog, etc., as Mr. Usher urges. The accumulator is best fitted for short, sharp peaks. Regarding the heat-consumption rate of the accumulator set, it can be shown easily that this is an insignificant item in total costs for peak-load units and can be virtually neglected. Contrary to Mr. Usher's opinion, accumulators are available for sudden breakdown emergencies and can carry load until other boilers or turbines can be put into service.

Mr. Funk's table showing the effect of the difference in cost between energy from base load and old plant, on savings, illustrates very effectively the influence of this factor on possible

savings. It is apparent that steam-accumulator plant is not particularly fitted to serve with hydroelectric developments on rivers with variable flow, as indicated by Mr. Funk.

Mr. Levert insists that the system with which he is associated has all the generating capacity needed to take care of extreme peaks and to serve as stand-by. He asks why they should find more money for peak-load capacity. It is quite possible that a careful analysis of the total cost of providing and maintaining this excess capacity is greater than would have been the case if cheap steam-accumulator plant had been installed to carry peak loads and to serve as emergency reserve at other times. Too little regard has been paid to the total investment in certain cases.

Mr. Hobson has called attention to the unfortunate choice of figures giving the B.t.u. per kilowatt-hour on standard and base-load plants. These figures were chosen as representative of what one might expect by adding fuel-saving improvements to a plan without changing pressure and temperature conditions. Mr. Leilich believes these assumed performances are better than could be obtained in practice. The paper, in discussing Case 1, shows that even if a heat consumption were taken 10 per cent below that assumed for the base-load plant, the gain through the use of the accumulator would still be significant. This also applies to the other cases shown. A curve for a plant of such remarkable efficiency would fall somewhat below curve D in the various figures.

The situation in Great Britain as influenced by the "grid" is an exceptional one, as Mr. Hobson points out. The large number of less efficient stations made available through the grid interconnection can provide for peak-load demands for several years, particularly since this equipment is already bought and paid for. This excessive capacity is partly due to the large stand-by reserve capacity carried in most British stations.

One speaker criticized the use of a heat rate of 22,850 B.t.u. per kw-hr. for the peak-load turbine. A triangular peak load was assumed, and the average heat consumption over the whole of such a peak was figured as 22,850 B.t.u. per kw-hr., based on some turbine guarantees. The performance of the accumulator turbine at its point of highest economy should be considerably less than this figure.

Mr. Leilich has calculated net savings in per cent of first cost. It must be remembered that these net savings are over and above fixed charges. In effect they represent possible dividends and as such provide liberal marginal earnings on the investment. Accumulator charging can generally be done during off-peak hours at night, when the increase in boiler load is welcomed, and also during the noon-hour dip.

No claim has been made in the paper that the proposed accumulator plant can be universally applied. As many discussors have pointed out, its field is special, and each case of possible application must be studied as a separate problem. There may be places where base-load and special peak-load units should be installed, just as there are other places where the installation of accumulators would result in the lowest total power costs.

As pointed out in the paper, savings of several hundred thousand dollars a year may be possible through the use of steam accumulators in certain plants. In such cases, both operating and designing engineers will have difficulty in convincing their utility executives that steam storage should not be used. The steam accumulator is not a cure-all for all peak-load problems, but it may find a useful place in American central stations for the reduction of the total cost of power.

FSP-51-25

Condition Curves and Reheat Factors for Steam Turbines

By CLARENCE C. FRANCK,[1] PHILADELPHIA, PA.

This paper offers a theory for "condition curves and reheat factors" developed by a study of the basic thermodynamic principles involved. A method is developed which makes possible the construction of "condition curves" for any turbine, when certain operating or guarantee characteristics are known. The question of "condition curves" immediately presents the closely related "reheat factor," and the theory developed for the "condition curve" has been applied to determine values for "reheat factors."

I—CONDITION CURVES

A CONDITION curve may be defined as a line on a Mollier diagram which connects points representing the condition of the steam at the entrance of each successive stage, the end point of the curve representing the condition of the steam leaving the last row of blades. This curve is of assistance to engineers in the design of turbines, and particularly in heat-balance calculations involving bled steam. A method that will facilitate the drawing of correct condition curves should prove helpful and useful to engineers. The construction of condition curves has been much discussed, and this work will be briefly reviewed.

Previous Data

Brown and Drewry[2] assume that the internal efficiency of the turbine is 10 per cent greater in the superheated than in the saturated region. This assumption gives a means for estimating the division of Rankine heat drop in the two regions, and by a cut-and-try method, the point at which the condition curve crosses the saturation line may be approximated.

Baumann in a recent paper[3] states that the stage efficiency in the saturated region is a function of the moisture content of the steam and that this stage efficiency decreases 1.0 per cent for every 1.0 per cent increase in the moisture content of the steam.

Warren and Blowney, in a paper[4] based on more recent test data, find that the stage efficiency in the saturated region should be decreased 1.15 per cent for every 1 per cent increase in the moisture content of the steam.

Goudie[5] gives a method of locating the condition curve by solving for the pressure at which the condition curve crosses the saturation line. This is done by combining an assumed equation of the condition curve with an equation of the saturation line to find their point of intersection.

The condition curve is assumed to be capable of expression by the relation

$$p_1 v_1^{\,n} = p_s v_s^{\,n}$$

and that of the saturation line by

$$v_s = \left(\frac{\text{Constant}}{p_s}\right)^{\frac{1}{\lambda}}$$

which may also be written in the form

$$p_s v_s^{\,\lambda} = 490$$

where p_1 = pressure, lb. per sq. in. abs., at initial condition
p_s = pressure, lb. per sq. in. abs., at saturation condition
v_1 = volume, cu. ft. per lb., at initial condition
v_s = volume, cu. ft. per lb., at saturation condition
n = coefficient of expansion and is a function of the stage efficiency
λ = 1.067.

Combining the equations of these two curves we have

$$p_s^{\,1-\frac{n}{\lambda}} = \frac{p_1 v_1^{\,n}}{490^{\lambda}}$$

Substituting for λ

$$p_s = \left(\frac{333.4}{p_1^{\,y} v_1}\right)^{z} \quad \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

where $z = \dfrac{n\lambda}{n-\lambda}$, and $y = \dfrac{1}{n}$.

If proper values are substituted in Equation [1], p_s, the pressure at which the condition curve crosses the saturation line, may be calculated.

Equation [1] is based on the assumption that

$$v_s = \left(\frac{\text{Constant}}{p_s}\right)^{\frac{1}{\lambda}}$$

is the equation of the saturation line.

Goodenough[6] gives the following equation to express the conditions at the saturation line:

$$\log p = A - B/T - c \log T - DT + ET^2 - \Delta \quad \ldots. [2]$$

$$v - c = BT/p - (1 - 3ap^{1/2}) \times m/Tn \quad \ldots\ldots\ldots\ldots [3]$$

These equations might be combined to give an equation for the saturation line. It is seen, however, that such an equation would be complicated and would not take the simple form of the Goudie equation.

Callendar[7] gives the following equations to express the conditions at the saturation line:

$$\log p = 22.2787 - \frac{5226.1}{T} - 4.71734 \log T + 0.7302\,(c-b)\,\frac{p}{T} \quad \ldots\ldots\ldots [4]$$

$$v = \frac{RT}{Ap} - c + b$$

These equations might also be solved to find an equation for the saturation line, but again it would be in a complicated form. Goudie[8] also states that his simple equation for the saturation

[1] Westinghouse Electric & Manufacturing Co., South Philadelphia, Pa. Submitted in competition for the 1928 A.S.M.E. Student Prize.

[2] "Economy Characteristics of Stage Feedwater Heating by Extraction." Trans. A.S.M.E., vol. 45 (1923), p. 713.

[3] "Some Recent Developments in Large Steam Turbine Practice." *Jl. I.E.E.*, vol. 59, p. 565.

[4] "The Increase in Thermal Efficiency Due to Resuperheating in Steam Turbines," Blowney and Warren. Trans. A.S.M.E., vol. 46 (1924), p. 563.

[5] "Steam Turbines," Goudie, 2nd edition, p. 577.

[6] "Properties of Steam and Ammonia," Goodenough, pp. 4 and 6.

[7] "Abridged Callendar Steam Tables."

[8] "Steam Turbines," Goudie, 2nd edition, art. 97, p. 171.

line, $pv^{\lambda} = 490$ is approximate, although the error involved by its use is small.

Goudie's method of considering the equation of the condition curve to be of the form pv^n = constant, where n is a function of the stage efficiency, appears to be rational. In fact, all of the condition curves which have been drawn in this investigation appear to follow this exponential law in the superheated region. Goudie's values of n are calculated by the following expression:

$$n = \frac{1}{1 - \eta_s\left(1 - \frac{1}{\gamma}\right)}$$

where n = coefficient of expansion for a given stage efficiency
η_s = stage efficiency, and
γ = adiabatic coefficient of expansion.

The results are based on the assumption that γ is a constant equal to 1.3. Callendar[9] and Mollier give the value of γ as 1.3. Zeuner assumes γ as 1.33. Schüle[10] finds γ equal to 1.33 but states that the value may vary between the limits of 1.3 and 1.35. Prof. G. A. Goodenough, in a personal communication, stated that there is no particular virtue in the value 1.3 or 1.33, and that different tables of superheated steam would probably give different values of γ.

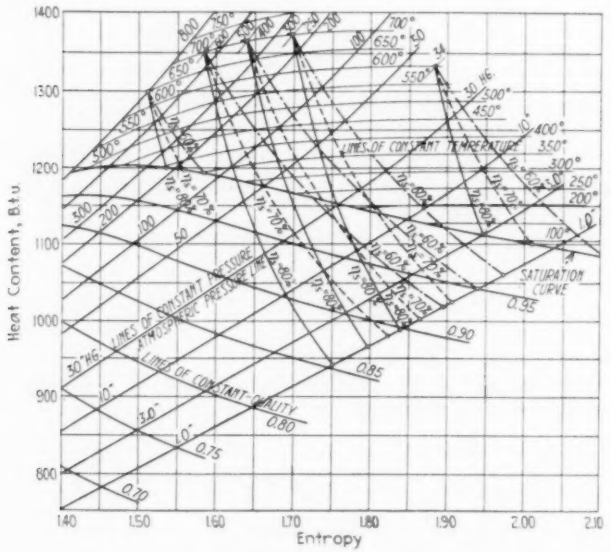


FIG. 1 CONDITION CURVES FOR STEAM TURBINES
(Goodenough Mollier diagram.)

The value of γ was found to vary slightly over any one Mollier diagram. Goodenough's Mollier diagram was used to plot the condition curves in this paper. It was found that γ for superheated steam varies from 1.32 in the high-pressure region of low entropy to 1.31 in the low-pressure region of high entropy. The value of γ may be calculated by taking an adiabatic pressure drop from a given initial pressure in the superheat region to some lower pressure above the saturation line, and by applying the characteristic equation pv^{γ} = constant, to these initial and final conditions. The average value of γ between the low and high entropy limits of the Goodenough Mollier chart was found to be approximately 1.315.

The value of γ was also calculated for the Ellenwood chart[11] which is based on the Marks and Davis.[12] It was found that values of γ over the entire superheated region varied from 1.34 to 1.30.

A slight difference in the value of γ shifts the position of the condition curve to the right or left of its true position. For example, consider that the true value of γ for a particular section of the Mollier diagram is 1.31, and the assumed value of γ used in calculating the coefficient of expansion n for a certain stage efficiency is 1.30. Then the value of n resulting from the use of the incorrect γ is approximately 0.5 per cent less than the true value. The overall effect of the use of 1.3 instead of the true value, 1.31, would be to decrease the stage efficiency approximately 2 per cent for the entire expansion in the superheated region.

Reference has already been made to the fact that an increase in the moisture content of the steam in the saturated region is accompanied by a decrease in the stage efficiency. This assumption was used in this investigation of the saturated portion of the condition curves. Warren and Blowney's figure of 1.15 per cent decrease in stage efficiency for every 1 per cent increase in the moisture content of the steam, which appears to be the latest available data on the subject, was assumed.

Condition curves may be accurately drawn by the step-by-step method when the stage efficiency is known. This method is very tedious, and considerable time is required to draw the complete condition curve in this manner.

Many designers approximate the condition curve by plotting the three controlling points, viz., the initial condition, the point where the condition curve crosses the saturation line, and the end point at exhaust. These fixed points may be connected by straight lines which have been assumed to be sufficiently accurate to serve for ordinary bleeder calculations. A smooth curve may also be drawn through the points, with a steeper slope in the superheated than in the saturated regions, to provide for the difference in stage efficiencies in the two regions. While either of these two methods may be satisfactory for ordinary purposes, careful bleeder calculations make greater accuracy in plotting the condition curve desirable as the condition of the bled steam is figured directly from the pressure given at the bleeder points on this curve.

There have been recent cases in which the predicted bleeder conditions were considerably different from those actually obtained later on completed turbines. This deviation from the expected conditions frequently has esulted in a decrease in the overall station economy and was therefore undesirable. The increases in heat consumption may have been small in certain cases, but these persist throughout the whole useful life of the machine. Such losses might have been prevented by a more accurate knowledge of the turbine stage efficiencies and by the use of a more carefully drawn condition curve to represent the true steam conditions at all points during expansion in the turbine.

Our increasing knowledge of steam-turbine design and power-plant heat balance will in the future require more accurate data on the conditions of all steam bled from the turbines. This can only be predicted with assurance when correct condition curves have been drawn on the basis of known stage efficiencies.

INVESTIGATION

SUPERHEATED REGION

The first step was to investigate whether condition curves in the superheated region could be expressed by an equation in the

[9] "Properties of Steam," Callendar.
[10] "Technische Thermodynamik," W. Schüle, 3rd edition, art. 9, p. 48.
[11] "Steam Charts," F. O. Ellenwood.
[12] "Thermal Properties of Superheated and Saturated Steam," Marks and Davis.

form pv^n = constant. Curves for different stage efficiencies were plotted by the step-by-step method for this purpose.

A second object was to determine whether there was any fixed relation between the value of the coefficient n in this equation for the condition curve, pv^n = constant, and the stage efficiency η_s.

Initial points for the condition curves were taken at 800 lb. per sq. in. abs., 650 deg. fahr.; 600 lb., 400 lb., and 250 lb. per sq. in. abs., 700 deg. fahr.; and 34 lb. per sq. in. abs., 600 deg. fahr. This choice gave the full range of conditions on the Goodenough Mollier diagram. Other initial conditions could be selected at intermediate points on the plotted curves.

The step-by-step method was used in plotting the condition curves for stage efficiencies of 80 per cent, 70 per cent, and 60 per cent, respectively, as shown in Fig. 1, giving a group of three condition curves from each initial point.

Each curve was plotted by assuming a small constant value of Rankine heat drop for each step, so that the number of steps between the initial conditions and the saturation line was in no case less than twenty. This was done in order that the curves obtained would approach expansion with an infinite number of stages. However, it was found on replotting that the form of the curve did not change substantially when half the number of steps was used. A smooth curve is drawn through the loci of the points representing the conditions of the steam at the entrance of each step, and this is taken as the condition curve for the given stage efficiency.

The corresponding pressures and volumes at different points along the condition curves were plotted on logarithmic-coordinate paper. These are shown in Fig. 2. The various points fell along the same straight line for the same stage efficiency and indicated that the equation of the condition curves was of the form, pv^n = constant.

The values of n can be taken as the slope of the lines from Fig. 2 or calculated by the following formula:

$$n = \frac{\log p_1 - \log p_2}{\log v_2 - \log v_1}$$

where p_1, v_1 = pressure and volume, respectively, at initial point
p_2, v_2 = pressure and volume, respectively, at any point on the condition curve down to the saturation line.

The values of n thus obtained are given in Table 1.

TABLE 1 VALUES OF n

η_s, per cent	800 lb., 650 deg.	600 lb., 700 deg.	400 lb., 700 deg.	250 lb., 700 deg.	34 lb., 600 deg.	Average
60	1.170	1.169	1.169	1.169	1.169	1.169
70	1.203	1.205	1.203	1.204	1.204	1.204
80	1.241	1.241	1.239	1.240	1.239	1.240

As the average of these values shows a maximum error of $\pm$ $^1/_{1200}$ or $\pm$ 0.08 per cent when compared to the individual values and as these are within the limits of accuracy of the chart and personal errors, it is reasonably safe to assume that the average values may be used.

The values of n were plotted against corresponding values of η_s taken from Table 1, and it was observed that the resulting curve was of parabolic form. In order to determine the values of n at intermediate-stage efficiencies the equation of the curve must be set up with n as a function of η_s.

Representing $n = f(\eta_s)$ by a general equation of the parabola,

$$(n - b)^2 = 4p(\eta_s - a)$$

where n corresponds to the abscissa and η_s to the ordinate.

The values of the constants a, b, and p may be determined by substituting the average values of η_s and n from Table 1 in the general equation. This gives a set of three equations and three unknowns, from which the following values of the constants were found:

$$a = 2.47428$$
$$b = 2.469$$
$$p = -0.22385$$

Substituting these in the general equation, we have

$$n = 2.469 - \sqrt{2.22536 - 0.8954\,\eta_s}$$

The values of n for any assumed stage efficiency calculated by this general equation are as shown in Table 2.

TABLE 2 VALUES OF n

η_s	n	η_s	n	η_s	n	η_s	n
60	1.169	68	1.198	76	1.224	84	1.256
61	1.174	69	1.201	77	1.229	85	1.259
62	1.176	70	1.204	78	1.232	86	1.263
63	1.179	71	1.208	79	1.236	87	1.267
64	1.183	72	1.211	80	1.240	88	1.271
65	1.186	73	1.214	81	1.244	89	1.275
66	1.190	74	1.218	82	1.248	90	1.278
67	1.194	75	1.221	83	1.252		

These values of n enable one to readily plot a condition curve

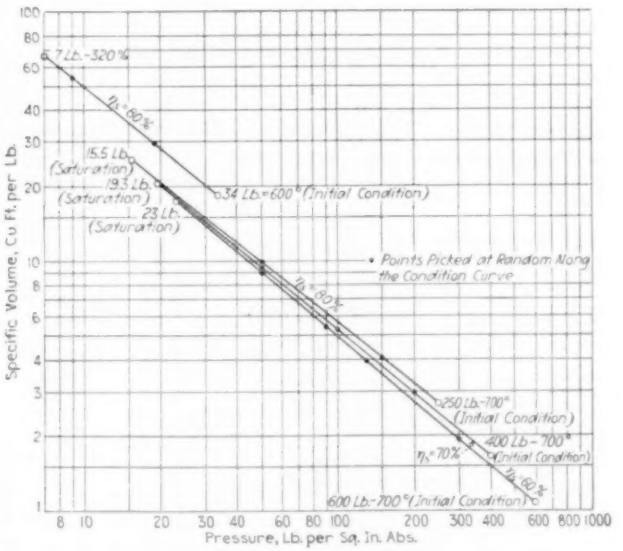


FIG. 2 LOGARITHMIC PLOT OF CONDITION CURVES IN THE SUPERHEATED REGION

in the superheated region for any assumed value of the stage efficiency on the Goodenough Mollier diagram.

SATURATED REGION

The saturated portion of the condition curve extends from the point where this curve crosses the saturation line to the exhaust pressure.

This portion of the condition curve is more difficult to draw by the step-by-step method as the stage efficiency is decreasing with increasing moisture contents of the steam. However, a cut-and-try method of assuming such a Rankine heat drop that the end point of the step is at a given quality eliminates intermediate calculations. For example, the first step will be the continuation of the condition curve of the superheated region from the saturation line to the 99 per cent quality line. The average stage efficiency for this step is:

$$\eta_a = \eta_{s1} - \left[\frac{1.15\,(1 - x)}{2}\right]$$

where η_a = average stage efficiency for the step, in per cent (from 100 per cent to 99 per cent quality)

η_{s1} = stage efficiency of the beginning of the step, in per cent (i.e., at 100 per cent quality)

x = quality of steam at the end point of the step, in per cent (i.e., 99 per cent quality).

Then

$$\eta_a = \eta_{s1} - 0.575$$

Assume some value of h_r, a Rankine heat drop to a pressure p_x (Fig. 3). The corresponding value of the actual heat drop converted to work per stage is $h_s = \eta_a h_r$.

Next scale from the point a the value h_s. The end point b of the step is where this constant-total-heat line intersects the pressure line p_x. If the point b does not fall on the assumed quality line of 99 per cent, a new value of h_r should be assumed giving a new value of p_x, and the end point recalculated. The point b on the 99 per cent quality line is the beginning of another step which can be calculated and plotted in the same manner. The condition curve can be completed in this way to any desired final back pressure, the stage efficiency decreasing in proportion to the progressive increase in the moisture content of the steam. In this investigation the condition curves were carried to the absolute-pressure line for one inch of mercury (see Fig. 1).

The same procedure as in the superheated section was used to determine whether the portion of the condition curves in the saturated region was of the form pv^n = constant. It was found, however, that the curves when plotted on logarithmic-coordinate paper had decided curvatures as shown, for example, in Fig. 4. This indicated that the condition curve in the saturated region

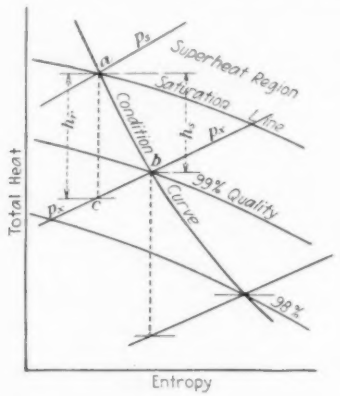


FIG. 3 CONSTRUCTION OF CONDITION CURVE
(Section from Goodenough Mollier diagram.)

did not follow the law pv^n = constant, probably owing to the use of decreasing stage efficiencies with added moisture in the steam.

However, upon examination it was found that the ratio H_s/H_r in the saturated region gave a constant coefficient C for each of the condition curves, where H_r = Rankine heat drop from the point where the given condition curve crosses the saturation line to the 1-in. of mercury pressure line; and H_s = heat available for work between the saturation line and the end point of the condition curve on the 1-in. of mercury absolute-pressure line.

The values of C found in this investigation for various values of η_s where the condition curves cross the saturation line are given in Table 3.

TABLE 3 VALUES OF C

η_s per cent	600 lb., 700 deg.	400 lb., 700 deg.	250 lb., 700 deg.	Average
80	0.780	0.782	0.781	0.781
70	0.689	0.692	0.690	0.690
60	0.594	0.597	0.597	0.596

The product of C for a given value of η_s at the saturation line, and H_r, the total Rankine heat drop from the point where the given condition curve crosses the saturation line to the exhaust pressure, gives the value of H_s which fixes the end point of this condition curve on the exhaust-pressure line. Also when the coefficient C is multiplied by the Rankine heat drop from the

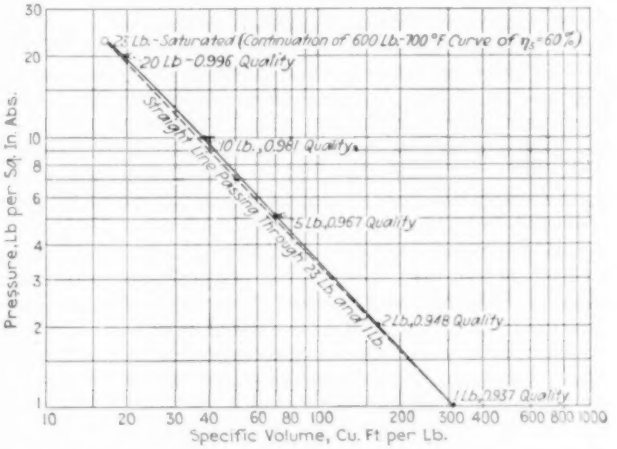


FIG. 4 LOGARITHMIC PLOT OF CONDITION CURVES IN THE SATURATED REGION

saturation line to any intermediate pressure above the final exhaust pressure, the corresponding value of H_{s1} to that pressure is found and the point at which the condition curve intersects this intermediate pressure line is determined. The point where the condition curve crosses the saturation line is fixed by the condition curve of the superheat region, and the remainder of the condition curve may be drawn by plotting points at various intermediate pressures between the saturation line and exhaust pressure by means of this coefficient C. The values of C for intermediate stage efficiencies are given in Table 4.

TABLE 4 VALUES OF C

η_s per cent	C	η_s per cent	C	η_s per cent	C	η_s per cent	C
60	0.596	68	0.671	76	0.746	84	0.816
61	0.605	69	0.680	77	0.755	85	0.825
62	0.614	70	0.690	78	0.764	86	0.833
63	0.624	71	0.700	79	0.773	87	0.842
64	0.633	72	0.709	80	0.781	88	0.851
65	0.642	73	0.718	81	0.789	89	0.859
66	0.653	74	0.727	82	0.798	90	0.868
67	0.661	75	0.737	83	0.807		

DISCUSSION OF PRECEDING METHODS

Trials were made to determine whether the data obtained by the preceding methods would hold for other cases.

A curve with initial conditions of 400 lb. abs., 600 deg. fahr., η_s = 70 per cent, was drawn by the step-by-step method.

Intermediate points on the condition curve between the initial conditions and exhaust pressure were computed by the methods developed in the preceding paragraphs. These points did not show any appreciable variation from the curve plotted by the step-by-step method. Another trial with initial conditions of 250 lb. abs., 600 deg. fahr., η_s = 70 per cent, gave the same comparative results by the two methods. Another condition curve was drawn with initial conditions of 800 lb. abs., 750 deg. fahr., η_s = 80 per cent, and again the points were substantially the same by both methods.

To test the flexibility of the tables a curve with initial conditions of 400 lb. abs., 700 deg. fahr., η_s = 85 per cent, was drawn by the step-by-step method. It was found that the predicted condition curve cut the exhaust line at 0.6 B.t.u. below the end point of the curve drawn by the step-by-step method. This was

the greatest deviation between the two values throughout the entire length of the curve, and may have been partly due to personal errors in reading small heat drops.

Checks were also made between the two methods with initial conditions picked at random as follows:

(1) 160 lb. abs., 520 deg. fahr., and η_s = 70 per cent
(2) 240 lb. abs., 490 deg. fahr., and η_s = 80 per cent
(3) 220 lb. abs., 530 deg. fahr., and η_s = 60 per cent
(4) 260 lb. abs., 600 deg. fahr., and η_s = 85 per cent
(5) 350 lb. abs., 700 deg. fahr., and η_s = 67.4 per cent.

Substantial agreement was found in all cases.

In the preceding investigation no consideration was given to the effect of supersaturation. If this had been considered, the condition curve would have continued beyond the saturation line according to the law pv^n = constant, for the supersaturated region, until a point was reached representing the limit of supersaturation. Beyond this point it would probably be necessary to use a larger moisture correction than 1.15 per cent decrease in the stage efficiency for an increase of 1 per cent in the moisture content of the steam.

Methods of Drawing Condition Curves

Condition curves used for practical purposes are drawn by assuming that the stage efficiency throughout the expansion is either constant or variable. Some designers assume that the stage efficiency increases down to the saturation line and decreases in the saturated region. Constant stage efficiency is sometimes assumed for the whole expansion.

In all cases the stage efficiency should be corrected for the moisture contents of the steam. As the data given in Table 4 take this into account, no further mention wil be made to the effect that *the stage efficiency decreases 1.15 per cent for every average 1 per cent increase in the moisture content of the steam.*

Constant Stage Efficiency

The following example shows the steps that must be taken to construct the condition curve on the assumption that the stage efficiency is constant.

Assume a turbine with a constant stage efficiency of 80 per cent operating under the initial conditions of 400 lb. abs., 700 deg. fahr. at the first-stage nozzles and exhausting at an absolute pressure of 1 in. of mercury.

(1) Locate the initial point on the Mollier diagram.
400 lb. absolute
700 deg. fahr.
1.6387 = E = entropy
1.669 = v_1 = specific volume, cu. ft. per lb.
1362.4 B.t.u. per lb. heat content.
(2) Table 5 shows the method of tabulating and computation for the superheated portion of the condition curve.

TABLE 5 COMPUTATIONS FOR SUPERHEATED PORTION OF CONDITION CURVE

(The value of n for η_s = 80 per cent is 1.240.)

Points	Pressures, lb. per sq. in. Initial p_a	Final p_b	Ratio of pressures, i.e., $\left(\frac{p_a}{p_b}\right)^{\frac{1}{1.24}}$	Specific volumes — Initial v_a	Final $v_b = \left(\frac{p_a}{p_b}\right)^{\frac{1}{1.24}} \times v_a$	B.t.u. per lb. at p_b and v_b[1]
Initial	400	...	1.000	1.669	1.669	1362.4
2nd	400	300	1.260	1.669	2.10	1334.0
3rd	400	200	1.753	1.669	2.925	1297.6
4th	400	100	3.059	1.669	5.12	1242.6
5th	400	50	5.364	1.669	8.99	1193.7
6th[2]	400	33	7.871	1.669	12.50	1167.0

[1] Entropy could be used with the same results.
[2] The sixth point taken falls just over the saturation line into the saturated region and no further points are necessary.

(3) Draw a curve through these points terminating at the saturation line.
(4) Table 6 shows the method of tabulating the result for the saturated portion of the curve.
(5) Connect these points by a smooth curve to form the complete condition curve.

The curves shown in Fig. 5 may be used for values of n and C in place of the tables.

TABLE 6 TABULATION OF RESULT FOR SATURATED PORTION OF CURVE

(The value of C for η_s = 80 per cent is 0.781.)

Points	Pressures, lb. per sq. in. Initial p_a	Final p_y	B.t.u. per lb. at sat. line $H_{sat.}$	B.t.u. per lb. at Rankine drop to p_y	Rankine drop, B.t.u. H_r	$0.781H_r = H_s$	B.t.u. per lb. at end, $H_{sat.} - H_s$
Saturation	33.5	33.5	1167.5	1167.5	0	0	1167.5
2nd	33.5	20	1167.5	1129.5	38.0	29.7	1137.8
3rd	33.5	14.7	1167.5	1108.2	59.3	46.3	1121.2
4th	33.5	20 (in. Hg)	1167.5	1079.5	88.0	69.3	1098.2
5th	33.5	10 (in. Hg)	1167.5	1035.8	131.7	102.8	1064.7
6th	33.5	3 (in. Hg)	1167.5	968.0	199.5	155.9	1011.6
Final[1]	33.5	1 (in. Hg)	1167.5	909.0	258.5	202.0	965.5

[1] Exhaust condition, 1 in. mercury absolute.

Variable Stage Efficiency

The condition curve for variable stage efficiency may be computed as in the previous case, with the exception that different values of n and C must be taken for every change in stage efficiency as shown in the following example.

Assume a turbine of ten stages, five in the superheated region and five in the saturated region. The stage efficiencies of the five stages in the superheated region will be assumed as 76, 77, 78, 79, and 80 per cent, respectively. The stage efficiencies, not corrected for moisture contents, of the five stages in the saturated region will be assumed as 80, 78, 76, 74, and 72 per cent, respectively. The operating conditions of the turbine will be 500 lb. abs., 700 deg. fahr., at the nozzles of the first stage and exhausting at 1 in. mercury absolute pressure.

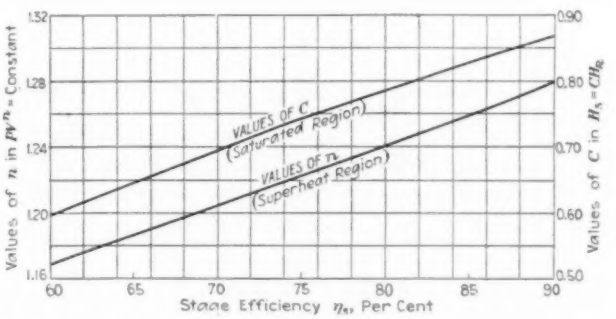


Fig. 5 Values of Constants Used in Construction of Condition Curves

Superheated Region.

(1) Let the first stage operate between the pressure limits of 500 lb. and 300 lb. From Fig. 5 or Table 2 the value of n at η_s = 76 per cent is 1.224. The initial steam volume, v_{s1-1}, at 500 lb. abs., and 700 deg. fahr. is 1.32 cu. ft. per lb.

The end point of stage No. 1 will be fixed by the relation

$$v_{s1-2} = v_{1-1}\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}$$

or

$$v_{s1-2} = 1.32\left(\frac{500}{300}\right)^{\frac{1}{1.224}}$$

$$= 2.003 \text{ cu. ft. per lb.}$$

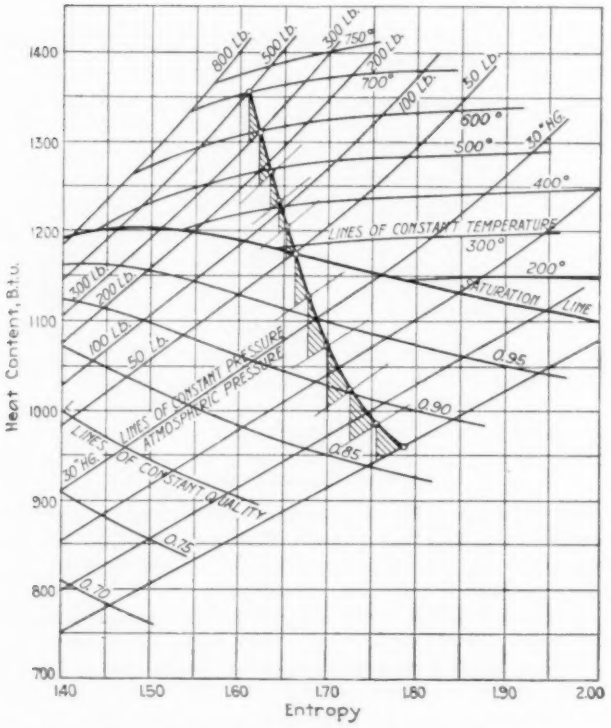


FIG. 6 CONDITION CURVE FOR A TEN-STAGE TURBINE OF VARIABLE STAGE EFFICIENCY
(Goodenough Mollier diagram.)

Therefore the end point of stage No. 1 is located at 300 lb. and 2.003 cu. ft. specific volume or 1310.0 B.t.u.

In the case of the first stage as well as in the succeeding ones, a point could be located at some intermediate pressure between the stage limits in order to fix three points through which to draw the condition curve for each separate stage.

(2) Let the second stage operate between 300 lb. and 180 lb.

n at η_s = 77 per cent is 1.229.

$$v_{s2-2} = 2.003\left(\frac{300}{180}\right)^{\frac{1}{1.229}}$$

$$= 3.035 \text{ cu. ft. per lb.}$$

Therefore the end point of stage No. 2 is located at 180 lb. and 3.035 cu. ft. specific volume or 1266.5 B.t.u.

(3) Let the third stage operate between 180 lb. and 120 lb.

n at η_s = 78 per cent is 1.232.

$$v_{s3-2} = 3.035\left(\frac{180}{120}\right)^{\frac{1}{1.232}}$$

$$= 4.218 \text{ cu. ft. per lb.}$$

Therefore the end point of stage No. 3 is located at 120 lb. and 4.218 cu. ft. specific volume or 1236.1 B.t.u.

(4) Let the fourth stage operate between 120 lb. and 75 lb.

n at η_s = 79 per cent is 1.236.

$$v_{s4-2} = 4.218\left(\frac{120}{75}\right)^{\frac{1}{1.236}}$$

$$= 6.1705 \text{ cu. ft. per lb.}$$

Therefore the end point of stage No. 4 is located at 75 lb. and 6.1705 cu. ft. specific volume or 1203.3 B.t.u.

(5) Let the fifth stage operate between 75 lb. and 49 lb.

n at η_s = 80 per cent is 1.240

$$v_{s5-2} = 6.1705\left(\frac{75}{49}\right)^{\frac{1}{1.24}}$$

$$= 8.696 \text{ cu. ft. per lb.}$$

Therefore the end point of stage No. 5 is located at 49 lb. and 8.696 cu. ft. specific volume or at the saturation line.

In the above computations for stage No. 5 the end point fell exactly on the saturation line. In case the point does not fall on the saturation line, another trial with a different lower pressure limit may be carried out. However, in most cases it is not necessary to terminate a stage directly at the saturation line.

Saturated Region.

(6) Let the sixth stage operate between 49 lb. and 22 lb.

C from Fig. 5 or Table 4 at η_s = 80 per cent is 0.781.

This figure allows for the correction to stage efficiency caused by the moisture content.

$$\begin{aligned} H_{6-1} &= 1159.6 \\ H_{6-1R} &= 1114.6 \\ H_{R_6} &= 45.0 \times 0.781 = H_{S_6} = 35.1 \end{aligned} \qquad \begin{aligned} H_{6-1} &= 1159.6 \\ H_{6-2} &= 1124.5 \text{ B.t.u.} \end{aligned}$$

Therefore the end point of stage No. 6 is located at 22 lb. and 1124.5 B.t.u.

(7) Let the seventh stage operate between 22 lb. abs., and 17 in. of mercury.

C at η_s = 78 per cent is 0.764

$$\begin{aligned} H_{7-1} &= 1124.5 \\ H_{7-R} &= 1058.4 \\ H_{R7} &= 66.1 \times 0.764 = H_{S7} = 50.5 \end{aligned} \qquad \begin{aligned} H_{7-1} &= 1124.5 \\ H_{7-2} &= 1074.0 \text{ B.t.u.} \end{aligned}$$

Therefore the end point of No. 7 stage is located at 17 in. of mercury and 1074 B.t.u.

(8) Let the eighth stage operate between 17 in. of mercury and 6 in. of mercury.

C at η_s = 76 per cent is 0.746

$$\begin{aligned} H_{8-1} &= 1074.0 \\ H_{8-1R} &= 1010.2 \\ H_{R8} &= 63.8 \times 0.746 = H_{S8} = 47.5 \end{aligned} \qquad \begin{aligned} H_{8-1} &= 1074.0 \\ H_{8-2} &= 1026.5 \text{ B.t.u.} \end{aligned}$$

Therefore the end point of No. 8 stage is located at 6 in. of mercury and 1026.5 B.t.u.

(9) Let the ninth stage operate between 6 in. of mercury and 2.5 in. of mercury.

C at η_s = 74 per cent is 0.727

$H_{9-1} = 1026.5$

$H_{9-1R} = 976.5$

$H_{R9} = 50.0 \times 0.727 = 36.3$

$H_{9-1} = 1026.5$

$H_{S9} = 36.3$

$H_{9-2} = 990.2$ B.t.u.

(10) Let the tenth and final stage operate between 2.5 in. of mercury and 1.0 in. of mercury.

C at $\eta_s = 72$ per cent is 0.709

$H_{10-1} = 990.2$

$H_{10-1R} = 941.5$

$H_{R10} = 48.7 \times 0.709 = 34.5$

$H_{10-1} = 990.2$

$H_{S10} = 34.5$

$H_e = 955.7$ B.t.u.

The end point of the expansion is 955.7 B.t.u. at 1 in. of mercury. This figure does not contain the leaving loss, and the true condition of the steam entering the condenser is found by adding that loss.

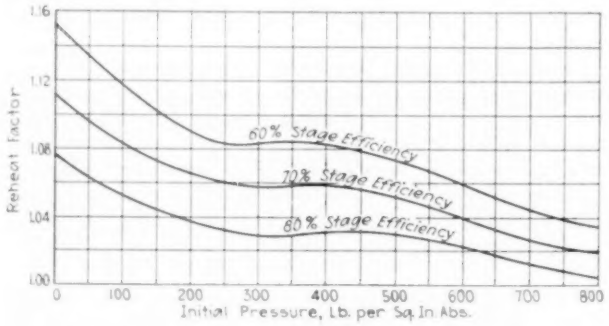


FIG. 7 REHEAT FACTORS, SUPERHEATED REGION
(Based on an initial temperature of 600 deg. fahr. Expansion from initial condition to saturation line.)

The condition curve is formed by passing a smooth curve through the points. The completed curve is shown in Fig. 6.

II—REHEAT FACTORS

Reheat factors are of great importance to the turbine designer, as they are indications of the ratio of the actual heat energy available for work in the stages to the total Rankine heat drop. Reheat occurs when a portion of the stage losses are reconverted into heat energy, some of which is available for work in succeeding stages. The reheat factor is always greater than unity and depends upon stage efficiency and steam conditions.

PREVIOUS WORK

The value of the reheat factor R for any definite expansion is expressed by the ratio of the summation of Rankine heat drops Σhr for the various stages to the total Rankine heat drop Hr from initial conditions to final pressure, i.e.,

$$R = \frac{\Sigma h_r}{H_r}$$

Also the ratio of the heat H_s converted to work to the total Rankine heat drop H_r is a measure of the energy conversion in the turbine, and is known as the *Internal Efficiency*, η_i; i.e.,

$$\eta_i = \frac{H_s}{H_r}$$

The total heat converted to work, H_s, is equal to the summation of the heat converted to work, h_s, in each stage of the turbines, i.e., $H_s = \Sigma h_s$. The values of h_r and h_s denote, respectively, the Rankine heat drop and the heat actually converted into work per stage, and the ratio h_s/h_r expresses the stage efficiency, η_s; i.e.,

$$\eta_s = \frac{h_s}{h_r}$$

If the stage efficiency is assumed to be constant for all stages of the expansion considered, then

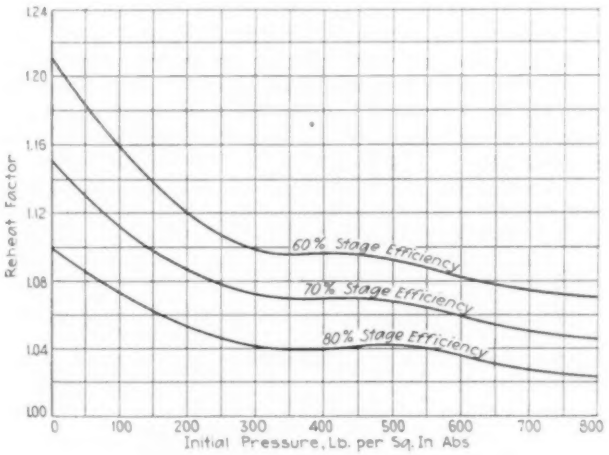


FIG. 8 REHEAT FACTORS, SUPERHEATED REGION
(Based on an initial temperature of 650 deg. fahr. Expansion from initial condition to saturation line.)

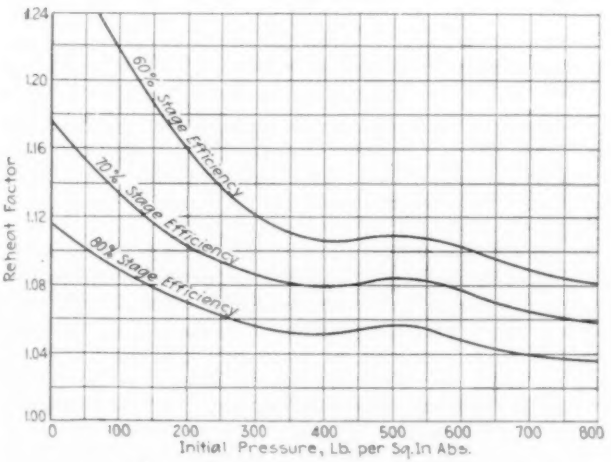


FIG. 9 REHEAT FACTORS, SUPERHEATED REGION
(Based on an initial temperature of 700 deg. fahr. Expansion from initial condition to saturation line.)

$$H_s = \eta_s \Sigma h_r$$

and $$\eta_i = \frac{\eta_s \Sigma h_r}{H_r}$$

or $$\eta_i = \eta_s R$$

Hence $$R = \frac{\eta_i}{\eta_s}$$

Several mathematical expressions have been derived to calculate reheat factors, but these derivations depend upon a constant value of the coefficient of adiabatic expansion γ for superheated steam. However, as shown in the preceding discussion of condition curves, γ varies for various steam tables and even for different steam conditions on the same Mollier diagram.

It is therefore apparent that these theoretically derived reheat factors may not truly represent actual values.

Investigation

The value of the reheat factor for the complete expansion from initial condition to final pressure as determined by the relation $R = \eta_i/\eta_s$ holds true when the value of the stage efficiency η_s is constant. The construction of the condition curves explained in previous paragraphs has been based on expansion with constant stage efficiency in the superheated region, and uniformly decreasing stage efficiency in the saturated region. This fact suggested that the investigation of reheat factors be separated into two parts: namely, the superheated region with constant stage efficiency, and the saturated region with variable stage efficiency.

Superheated Region. In plotting the condition curves shown in Fig. 1 the step-by-step Rankine heat drops h_r were determined. The relation $R = \eta_i/\eta_s$ can be checked by means of

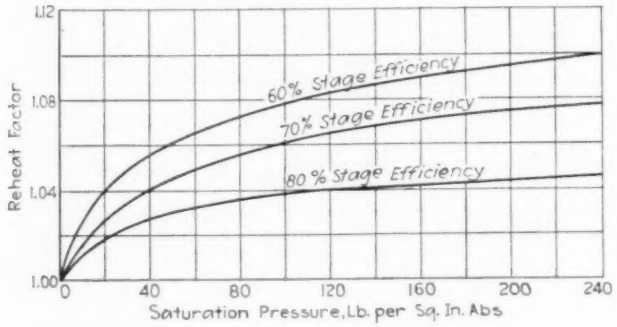


Fig. 10 Reheat Factors, Saturated Region
(Expansion from saturation line to 1 in. Hg absolute back pressure.)

the equation $R = \Sigma h_r/H_r$, as the summation of the individual heat drops Σh_r may be made from the initial conditions. This procedure gave exact checks of reheat factors in all cases. These calculations provided reheat factors for fifteen different initial conditions.

In order to determine values of reheat factors for the entire superheated region, the previously outlined method of constructing condition curves for the superheated region was applied for other steam ranges. Initial conditions were selected and different stage efficiencies assumed, so as to give a number of points through which curves showing the variation of reheat factors with initial conditions could be constructed. In these determinations of η_i for use in the relation $R = \eta_i/\eta_s$ it was only necessary to locate the point at which the condition curve crossed the saturation line, so that the values of H_r and H_s could be calculated.

Figs. 7, 8, and 9 show values of reheat factors in the superheated region. These values are for expansion with constant stage efficiency, with an infinite number of stages, from the initial superheat conditions to the saturation line.

Saturated Region. The problem is decidedly different in the case of the saturated region. The effect of the decreasing stage efficiency on the construction of the saturated portion of the condition curves was shown in the previous discussion of condition curves. This variable stage efficiency, combined with certain properties of steam, make it difficult to determine a true average stage efficiency for the entire expansion in the saturated region. Hence for the saturated region the relation $R = \eta_i/\eta_s$ cannot be used, and the laborious method of summing up the individual Rankine heat drops Σh_r must be followed.

The previous work on condition curves gave data to calculate reheat factors for nine different conditions in the saturated region. The method previously outlined was used to construct condition curves of different stage efficiencies from various saturation pressures to the 1 in. of mercury back-pressure line. The summation of the individual Rankine heat drops h_r was obtained from the plotted condition curves by stepping off along the curve with a pair of dividers.

Fig. 10 shows values of reheat factors in the saturated region. The stage efficiencies shown on the curves of Fig. 10 are the values assumed for the beginning of expansion at the saturation line. These stage efficiencies are decreased with increasing wetness of the steam as explained in the earlier section. The values of the reheat factors are for expansion with decreasing stage efficiency for an infinite number of stages, from a given saturation pressure to the 1 in. of mercury absolute exhaust pressure.

Discussion. Knowing the initial condition and the stage efficiency, the reheat factor in the superheated region may be found from Figs. 7, 8, and 9 without previous knowledge of the shape or location of the condition curve. However, to determine the reheat factors in the saturated region, the pressure where the condition curve crosses the saturation line must be predetermined, which can easily be done by the methods developed in this study.

III—CONCLUSIONS

This investigation has been made by the use of Goodenough's Mollier diagram, and the constants used in constructing the condition curves and the reheat factors can only be used in connection with that diagram.

The author has applied the constants determined by this investigation to plot condition curves on different Mollier diagrams. It was found that these condition curves did not check with similar ones drawn by the step-by-step method. This showed quite conclusively that different diagrams based on various steam tables have different characteristics. To plot the condition curves correctly on such different diagrams, it is only necessary to determine the characteristics of the particular steam tables that are used.

The author considered investigations of the characteristics of different steam tables. However, it was felt that the research on the properties of steam, sponsored by the A.S.M.E., and carried on jointly at the Bureau of Standards, Harvard University, and the Massachusetts Institute of Technology, will fix certain relations from which standard properties of steam can be calculated. With these properties thus permanently established, the basic principles of this investigation can be applied to determine values for the constants, and to calculate reheat factors which could be universally used.

The author wishes to acknowledge his indebtedness to Prof. A. G. Christie, of the Johns Hopkins University; Prof. G. A. Goodenough, of the University of Illinois; W. B. Flanders, of the Westinghouse Company; Prof. J. H. Keenan, of Stevens Institute of Technology; and G. B. Warren of the General Electric Company for valuable information and suggestions.

FSP-51-26

Power Supply for New England Industry

By F. M. GUNBY,[1] BOSTON, MASS.

IN CONSIDERING the broad subject of the power supply for New England industry, it should be of interest to know how much power is required, whether or not the amount is increasing, the sources from which power is obtained, and its cost compared to that in other parts of the country. One of our largest industries is composed of the electric power and lighting companies, referred to generally as the central stations. It is of interest to know how much of the total energy used is supplied by these central stations, and how much is supplied and used by the manufacturing plants, referred to as the industries.

The principal objects of this paper are therefore:

a To show how much power New England is using and the rate of increase, including power generated by the industries and by the central stations.

b To analyze the growth and sources of power in the industries to see if any definite trend is indicated as to procurement of power.

c To compare figures for the approximate cost of power in various sections of the country.

The facts brought out in attempting to attain these objects will show what is going on in connection with our power supply, and may give the answer to some of the points on which there has been much discussion in the past.

New England attained her industrial development through the inventive and industrial capacity of her people, and because Nature had helped her in some respects although handicapping her in others. One of her many advantages in the early days of manufacturing was that there was an abundant supply of water power which could be easily harnessed.

As the manufacturing units grew larger, the water powers were supplemented by steam, and later by purchased electricity.

Throughout this development we can see the abandonment of small factories and water powers as being no longer able to meet the increasing competition, first, from more efficient plants, and, later, from plants located where some conditions made production costs lower. It appears that we must always keep the element of comparative cost clearly before us. We can see today examples of small water powers, or ones which would be excessively expensive to redevelop, being abandoned when worn out.

From these small beginnings the power requirements have developed until it was estimated by the Power Investigating Committee of the Associated Industries of Massachusetts that the energy required for all of New England in 1928 would be in excess of 8¾ billion kilowatt-hours, not including railroad locomotives.

This Power Investigating Committee was appointed in 1923 by the Associated Industries of Massachusetts to "ascertain the facts with reference to the power situation in New England." The members were Charles T. Main, Chairman, Henry I. Harriman, Dugald C. Jackson, George L. Finch, and B. Preston Clark. It served without pay, but the work of its staff cost several thousand dollars, and it spent about a year in preparing its report, which was published in April, 1924.

The report of this committee is a most comprehensive study of the power situation in New England, and has been used in this paper as the basis for much of the data as to power requirements, and the author wishes to make suitable acknowledgments here for use of the data contained therein.

It has been possible to get figures in regard to Massachusetts industries for 1925, and for her public utilities for 1927, with which to test the accuracy of the estimates made in that report. The results of these checks indicate that the actual rate of increase in the amount of power required has closely approximated the rates predicted by the Committee. It is probable that what is true of Massachusetts will hold for the rest of New England, so that the data given in the Committee's report may be used with confidence today.

These statements refer to the total power, which shows a consistent increase. Individual elements, of course, vary from the average. The textile industry, and particularly the cotton portion of it, shows a decrease in power used. Some other industries have grown to offset this. Some large public-utility systems, partly by absorbing the load of others, have grown at rates far above the average, five of the larger ones showing, in the five-year period from 1922 to 1927, an average increase of about 75 per cent. The interchange of energy between companies shows a large increase for this five-year period.

Amount of Power Required

Fig. 1 shows the total energy requirements for New England, not including railroad locomotives, from 1915 to 1923, and the Committee's estimated rate of increase. The power shown here includes that for the industries, for public and domestic lighting, power for homes, and for all of the miscellaneous purposes for which public-utility energy is used. It shows that for 1928 about 8¾ billion kilowatt-hours will be required, of which about 5¾ billion would be generated by central stations and about 3 billion by the industries. The Power Investigating Committee predicted an annual increase of total energy, after eliminating duplication, etc., of from 1.5 to 2 per cent of the 1923 output. It also estimated that the central-station output would increase faster than this, or at the rate of about 9 per cent of the 1922 total. The actual average rate of increase from 1922 through 1927 was just over 9 per cent.

It was further shown that about two-thirds of this power is used in an area of Massachusetts, Rhode Island, and New Hampshire centering about Eastern Massachusetts. Massachusetts uses approximately one-half the energy required for all the New England industries.

In 1922 about 70 per cent of the total energy was generated by fuel and 30 per cent by water, and the same proportions held for the energy generated by central stations and industrial plants. In 1927 about 75 per cent of the energy generated by the central stations was from fuel, and it is probable that the same figure would hold for the industries.

Taking the six public-service electric companies reporting over 100 million kilowatt-hours generated in 1927 and adding certain other companies closely associated with them, we find that in 1922 they accounted for about 85 per cent of the total energy generated by all the electric companies, and in 1927 for about 89 per cent.

Another important point is that in 1927 all the public utilities and municipal plants, excluding street railways, reported a total of about 3,500,000,000 kw-hr. on their switchboards. Of this, however, only about 2,400,000,000 kw-hr. was generated or purchased from sources which would not be duplicated in the re-

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[1] Associate, Chas. T. Main, Inc., Consulting Engineers. Mem. A.S.M.E.

Contributed by the Power Division and presented at the New England Industries Meeting, Boston, Mass., October 1 to 3, 1928, of The American Society of Mechanical Engineers.

ports, showing that about 1,100,000,000 kw-hr. was sold or interchanged between public utilities. This was about 2½ times the amount of energy so interchanged in 1922.

This shows that the companies did a large volume of business on interchange of power, and undoubtedly a great conservation of fuel and other costs resulted. During this year there was purchased from plants not a part of the public-service companies, about 40 million kilowatt-hours. This was purchased mostly from a few industrial plants having surplus water power, and from the Metropolitan District Commission, generated by its water-supply works.

Another interesting feature is that in Massachusetts the increase of load from 1922 to 1927 for the industries only, was about 130,000,000 kw-hr., while the increase in load of the central stations was about 930,000,000 kw-hr. for the same period, showing that the industries used only about one-seventh of the increase in the central-station loads, the remainder probably being used for public and domestic purposes, including lighting, washing, ironing, cooking, and other domestic purposes, and "sitting up to listen in."

TABLE 1

Industry	1909	1914	1919	1922	1925
Cotton goods (incl. smallwares)	362,000	438,100	526,400	536,500	572,200
Woolen and worsted goods (incl. felts)	127,200	154,100	190,100	199,000	230,000
Paper and wood pulp	115,800	133,900	145,300	139,100	153,200
Iron and steel works, foundry and machine-shop products	.....	.....	.....	100,000[a]	133,000[a]
Dyeing and finishing textiles	.....	30,900	39,100	53,000	72,300
Boots and shoes (incl. cut stock and findings)	.....	39,700	48,400	50,700	43,700
Rubber goods	.....	17,500	34,400	43,000	62,000
Electric machinery, apparatus and supplies	.....	29,900	36,400	40,000	57,500
Leather, tanned, curried, and finished	.....	26,500	34,200	37,900	40,500
Cordage, twine and jute goods	.....	28,600	32,400	33,000	.....
Wire and wire working	.....	15,100	24,800	23,900	31,400
Gas	.....	11,500	18,800	22,200	25,700
Printing and publishing	.....	.....	21,100	20,500	24,200
Cutlery and misc. tools	.....	.....	20,900	19,900	.....
		925,800	1,172,300	1,318,700	1,445,700
Misc. other industries	?	471,000	557,200	471,400	567,300
Total	1,175,100	1,396,800	1,729,500	1,790,100	2,013,000

[a] These figures are probably not strictly comparable.
NOTE.—The total for 1926 was 2,016,000 hp.

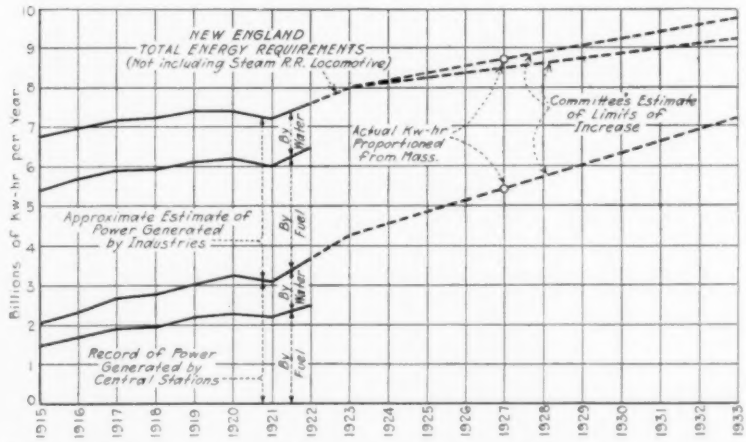


FIG. 1 GROWTH OF NEW ENGLAND POWER REQUIREMENTS IN EQUIVALENT KILOWATT-HOURS
(Taken from report of Power Investigating Committee of Associated Industries of Massachusetts, January, 1924.)

From 1922 to 1927 the generation by street-railway plants decreased by about 38,000,000 kw-hr., or about 10 per cent.

POWER REQUIRED IN VARIOUS INDUSTRIES

Table 1 shows the installed horsepower in the principal power-using industries in Massachusetts for the past several years. (All figures given to nearest 100.) These are arranged in order of size for 1922.

The figures given are for the installed capacities as reported to the Massachusetts Department of Labor and Industries. The actual load requirements will be less, and have been estimated.

The boot and shoe industry is the only important one in Massachusetts which shows less installed horsepower capacity in 1925 than it had in 1922, the decrease being from 50,700 to 43,700 hp.

Fig. 2 shows the growth of power capacity in all the industries in Massachusetts, both in installed capacity and in probable actual load.

This chart shows that there has been practically no change in the amount of installed water-power capacity, a slight decrease in the installed fuel-burning power-generating equipment, and a rapid increase in the amount and proportion of purchased power.

In 1922 the purchased power for all industries was about 38 per cent of the total, and in 1925 it was 43 per cent of the total, showing that purchased power accounted for much of the growth of the total power in the industries. In fact, the increase in purchased power accounted for about 90 per cent of the total increase in the installed horsepower in the industries.

The fuel-burning capacity installed in the generating plants of the industries in 1925 was almost exactly the same as it was in 1914, while the water-power capacity was the same as it was in 1900. This indicates clearly the trend toward the purchase rather than the generation of power in the industries, at least as far as growth is concerned. It also means that as some generating plants wear out, their places are taken by purchased current.

Some industries require large amounts of steam for their process work, and for which fuel must be burned, whether power is generated or not. Among these are the woolen, paper, dyeing and finishing, rubber, soap making, sugar refining, laundry, and similar industries. The installed capacity in these industries in Massachusetts in 1925 was about 570,000 hp., or about 28 per cent of the total of the state's industries. The purchased current represents about 28 per cent of the power uses of these industries, and this ratio has not changed much in recent years.

The remainder of the industries have about 1,440,000 hp. installed, and of this about 50 per cent in 1925 was for purchased current. The industries in this group have relatively little use for steam in their processes, the principal uses being for warming the buildings.

The proportion of purchased power in all the industries of the state has increased from 16 per cent in 1914 to 43 per cent in 1925, and it will probably increase materially in the future.

PRINCIPAL POWER-USING INDUSTRIES

The three largest users of power among the industries of Massachusetts, and probably of New England as a whole, are the cotton manufacturing, woolen and worsted manufacturing with their several subdivisions, and the paper and wood-pulp industries.

Table 2 shows the installed horsepower in 1922 and 1925 as reported to the Massachusetts Department of Labor and Industries.

These figures represent installed generating capacity and motor capacity for purchased current, and surplus water-power capacity.

TABLE 2

Industry	Hp. installed, including secondary water and other surplus capacity 1922	1925	Increase, Per cent
Cotton manufacturing	536,500	572,200	7
Manufacturing of wool, including felts	199,000	230,000	15
Paper and wood pulp	139,100	153,200	10
All other industries	915,500	1,057,600	15½
Total	1,790,100	2,013,000	12½

In the textile industry there has been a considerable increase in the installed motor capacity for the same machinery, due to the use of individual drives, etc.

For this and other reasons it is felt that the Bureau of the Census data as to the number of spindle-hours operated during the year are a better guide to the total power used by the cotton textile industry than is given by the installed horsepower capacity. In 1922 the cotton mills of Massachusetts operated 25,233,000,000 spindle-hours. In 1927 they operated 19,662,000,000 spindle-hours, or 78 per cent as much as in 1922. Assuming that a kilowatt-hour will operate an average of 37 spindles for an hour, the power required in 1922 was 682,000,000 kw-hr., and in 1927 it was 530,000,000 kw-hr., showing a decrease of 152,000,000 kw-hr. per year in the power actually used by the cotton mills in Massachusetts. The decrease for all New England cotton mills was 227,000,000 kw-hr.

This loss is accounted for by plants running at part capacity, or that have liquidated, or have moved to locations where manufacturing costs would be lower, or by plants that have scrapped some of their older equipment. This shrinkage is still going on.

The statistics for the woolen and worsted mills and allied plants do not give the spindle- or loom-hours operated except for the country as a whole. It seems probable that the New England mills may have run fewer hours in 1927 than in 1922. However, the installed horsepower capacity in Massachusetts mills increased about 15 per cent between 1922 and 1925 and the use of power probably increased in about the same ratio. The same figures may be assumed to hold for all of New England. The actual load for this industry was estimated at 125,000 hp. for 1922, and it was probably 145,000 hp. in 1927.

The installed horsepower in the paper and wood-pulp industry of the state increased from about 139,100 to 153,200 hp. or in the neighborhood of 10 per cent. It is probable that the actual load increased in about the same proportion.

From the above it appears that between 1922 and 1925 the textile industry's actual power load decreased by about 11 per cent while in the same period the actual load of the remainder of the industries in Massachusetts increased about 15 per cent. The total actual load increased from 1,250,000 to 1,302,000 hp., which is just about the increase predicted by the Power Investigating Committee.

Data for the textile mills for 1927 show only a small decrease from 1925. The figures as to installed capacity in the other industries are not yet available.

These checks show that the estimates for the future requirements for power for New England industries as given by the Power Investigating Committeee of the Associated Industries of Massachusetts have proved accurate.

The figures also give a clear indication that while New England's largest power-using industry has suffered severely in recent years so that the amount of power used by it has shown a substantial decrease, the growth of the other industries has been sufficient to more than offset this loss.

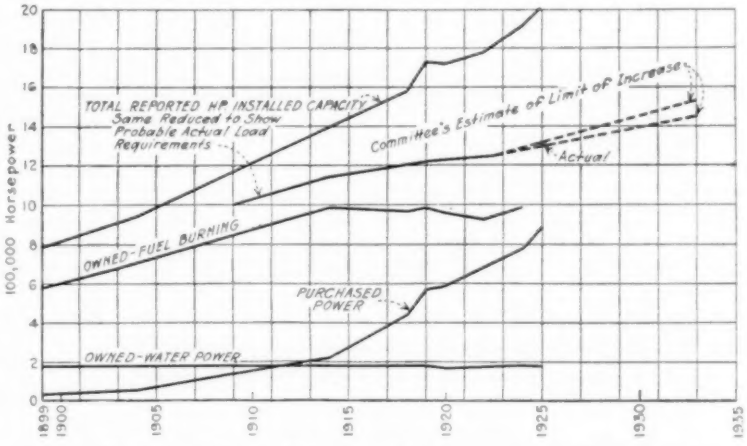


FIG. 2 GROWTH OF POWER CAPACITY—ALL INDUSTRIES IN MASSACHUSETTS
(Taken from report of Power Investigating Committee of Associated Industries of Massachusetts, January, 1924.)

COMPARATIVE COSTS OF PURCHASED POWER IN VARIOUS SECTIONS OF THE COUNTRY

Since the products of our industries must compete with those from other sections of the country, a comparison of the cost of production in various sections is of interest. A comparison of the costs of purchased power in various parts of the country has been made. It may be assumed that the same features which affect the cost and price of power of the public utilities will have a similar effect on the cost of power if made by the industries, so the relative cost of purchased current should give a reliable picture of power costs in those sections.

The cost per kilowatt-hour delivered at the mill voltage has been estimated from the rates of large power companies in the Central West, Middle Atlantic, and some of the Southeastern states. In calculating these costs, the same loads and hours of use were assumed in all cases, so that the results would be as fairly comparable as possible, and for each locality the rate applying which would result in the lowest cost has been used.

These figures must not be taken too literally as applying to any particular load because, with the various companies, the form of the rate, the method of determining the demand, power-factor clauses, etc., may apply in actual practice differently from the assumptions made in calculating the cost from the published rates.

In Fig. 3, the upper curves show the cost per kilowatt-hour for various loads or demands when the plant runs 200 hours per month, and assuming that the average load is 80 per cent of the demand, or, in other words, that the demand is used 160 hours per month. This would correspond to a fairly steady load, like a textile plant running one shift.

The lower curves show the cost based on using the demand 368 hours per month, corresponding to a steady load running two 55-hour shifts. The prices for all current were reduced to the basis of low tension delivery at the industry's plant.

The graphs for the cost in New England were made up by averaging the rates charged by the

Central Maine Power Company
Edison Electric Illuminating Company of Boston
Hartford Electric Light Company
Narragansett Electric Company of Providence
England Power Company, and the
Turners Falls Power & Electric Company.

Similarly, that marked "Central West" was made up from the rates of the

Cleveland Electric Illuminating Company
Commonwealth Edison Company of Chicago
Union Gas and Electric Company of Cincinnati, and the
Union Light and Power Company of St. Louis;

that marked "Middle Atlantic" was made up from the rates of the

Consolidated Gas, Electric Light & Power Co. of Baltimore
Philadelphia Electric Company, and the
Public Service Company of New Jersey;

and that marked "Southeast" was from the rates of the

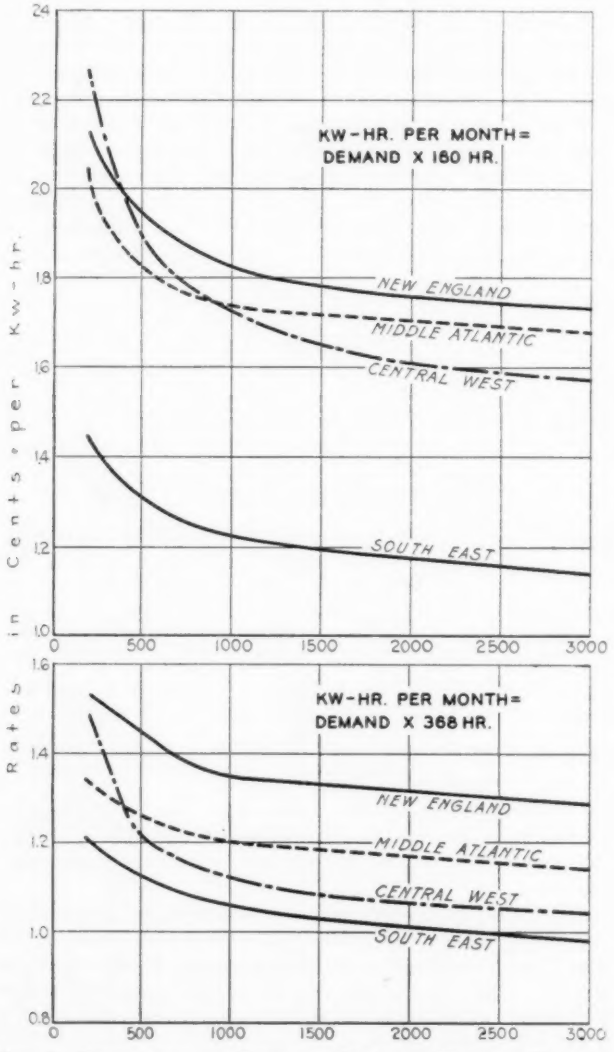


FIG. 3 COMPARISON OF COST PER KILOWATT-HOUR IN VARIOUS SECTIONS OF THE UNITED STATES

(Rates shown are averages of prices for purchases current from several important companies in each section of the country. Rates are reduced to low-tension basis.)

Alabama Power Company
Georgia Power Company, and the
Southern Power Company.

DIVERSITY FACTOR AND INTERCONNECTIONS

The power required to drive a single machine will follow all the variations in the cycle of operation of that machine, showing the maximum and the minimum requirements at the appropriate periods. The power to operate 100 such machines will not show 100 times the maximum, nor 100 times the minimum, but will show variations much less than these, and only the average power will increase in even approximate proportion to the number of machines making up the load.

In a similar way, the load characteristics for different factories and of different industries are different, and if many of these are supplied from the same power source, its capacity does not need to be so great as the sums of the peak loads of each unit connected. In the same way, the lighting of homes and streets would not usually reach its peak during the period of the industrial day shift.

An individual plant or a small utility will get little benefit from these conditions, while a large power system will be able to realize great savings in capacity required and from improvement in load factor from being able to combine the loads of many plants and industries with one power system. The result has been larger and more efficient generating stations and the shutting down or holding in reserve of the less efficient fuel-using stations. Most of the large companies are now interconnected so that they may exchange power with each other. Such arrangements make it possible to use the generating equipment to the best advantage, to make use, at times, of water power which would otherwise be wasted, and to give greater assurance of reliable service.

With these modern arrangements, a considerable portion of the energy required in New England is bought and sold between power companies from day to day almost on a commodity basis. What one has available but has no immediate use for, another may be able to use—at a price. One example of this, in which several large companies in Massachusetts and Connecticut are interested, is the Connecticut Valley Power Exchange.

Another of our large water-power companies has for years had interchange arrangements with companies having large steam power plants, and also with industries and others having steam and water powers, and with large water-power companies operating in other states.

INDUSTRIES REQUIRING PROCESS STEAM

There are certain industries which must burn large quantities of fuel to get steam for use in their processes regardless of how their power is obtained. In this group are the wool manufacturing, dyeing and finishing of textiles, paper manufacturing, rubber industry, sugar refining, soap manufacturing, and a good deal of the chemical industry.

Where this steam can be used in the processes at low pressure, as much of it usually can be, it is simple and efficient to put high-pressure steam through a prime mover and generate power with a small portion of its heat and use the exhaust in the processes. This is common practice in most of these industries. Much has been written on the subject, including a paper by W. F. Ryan[2] on the use of unusual high pressures for this work, and by M. K. Bryan on the importance of selecting the proper apparatus for the conditions, and of guarding against losses due to fluctuating conditions of power and steam loads.

One important matter in the use of non-condensing turbines in connection with process steam is to avoid waste where the requirements for steam and power do not coincide. Where the steam requirements are far in excess of the amount produced in making power, there will be no waste.

There are many cases, however, where there is an excess of power requirement at one time of day and an excess of steam requirement at another. In cases where warm water is used in the process in large quantity, it has been possible to balance these conditions by making and storing warm water.

It would seem that these cases should give grounds for the

[2] Trans. A.S.M.E., vol. 47 (1925), p. 779.

power companies to come in and act as the balance wheel and sell all the kilowatt-hours the industry did not make from its steam needs. Special rates would generally be desirable, probably working on a buy-and-sell arrangement. There is about a half-million horsepower in industries having these conditions, and in this field the rate of growth of purchased current is small and may be expected to remain so unless some such arrangement as indicated can be worked out.

There are now cases of waste where, because an industry cannot afford not to generate some of its power with process steam, it is forced to generate it all, and at times to waste steam. It would seem that there should be some solution which would meet these cases and permit the power companies and the industries to share what must now be wasted. In the case of industries using, say, less than 500 kw., there will probably not be sufficient savings possible to justify doing anything special.

Each of these cases would require individual consideration, but if the problem is approached by both sides in the same spirit which has led to the great interchange of power between the utility companies, a constructive solution should be found. Something may be accomplished in localities where manufacturing is concentrated by the use of central stations using very high initial pressures and exhausting at a pressure high enough to permit the economical transmission of steam for considerable distances.

General Summary

In a paper of this kind it is possible to draw only conclusions applying to the situation as a whole. Such conclusions cannot be applied to particular cases, as each case may have special features which would materially affect the answer.

There is no hard and fast formula by which we may say that all users of a given amount of power should purchase it or should not purchase it, and there will not be so long as the rates of power companies differ and conditions in plants vary, and the individual retains his independence.

The total amount of power used by the industries is increasing, and part of this increase has been provided for by an increase in the generating capacities in the industries, while a greater part has been provided in the form of purchased current. This condition demonstrates that the users have found that, for some conditions, it is better to generate the power required, and, for other conditions, to purchase it, and, for still others, to generate some and purchase some. That the proportion of power purchased is increasing shows that there are more conditions which make that course advisable than there are which make generation advisable.

There are probably cases where it would have been better to adopt a different solution than the one chosen, due to lack of sufficient engineering information or data, or to financial conditions, or salesmanship, or prejudice; but these cases form only a small portion in what the industry has done as a whole.

The investment in plant will nearly always be less when power is purchased than when it is generated by the industry. The cost of power is a matter which must be determined when considering how to obtain it, and will play an important part in arriving at the decision.

The power supply where the installations are well made has been satisfactory, whether generated by the industry or purchased. Poor installations have given unsatisfactory results in both cases.

In general, plants served by long transmission lines are subject to more frequent interruptions and annoyance than those with good installations of their own. On the other hand, there is less likelihood of a long shutdown, due to accident, when served by a modern, well-managed utility than where the industry has its own plant and does not have sufficient spare capacity.

The following points are of particular interest. In these, 1922 is used as a base.

1 The total power load of New England, excluding railroad locomotives, is increasing at an average rate of about 3 per cent per year.

2 The load of the industrial plants of Massachusetts is increasing at the rate of about $1^1/_2$ per cent per year, and this figure probably holds for all New England.

3 The load on the central stations is increasing at an average rate of about 9 per cent per year.

4 The increase of load in the industrial plants accounts for only about one-seventh of the increase in load on the central stations, the balance being used for public lighting, domestic purposes, and the many new uses which are constantly developing.

5 The bulk (about 90 per cent) of the increase of installed power capacity in the industries is in the form of purchased-power facilities. This is not true to the same extent for those industries which use large amounts of steam in their manufacturing processes.

6 As gaged by the relative rates for purchased current, the cost of power in New England, while considerably higher than in the Southeast, is, on the average, only about 5 to 10 per cent higher for single-shift work than in the Central West and Middle Atlantic states. For two-shift work the excess cost in New England is more marked. This is natural in view of the distance from the coal supply.

7 There are some who hesitate to face the fact that our textile industry is in a deplorable condition, and there are others who seem convinced that all New England industrial development is on the decline. Such attitudes do not change facts, and the facts are that what we have lost and are losing in some lines, notably textiles and shoes, we have made up for by growth in other lines; so that the total use of power, and the prosperity which goes hand in hand with it, has maintained its normal rate of increase. As long as this is so, it seems that New England must be fundamentally sound.

Discussion

Charles L. Ware.[3] In the woolen industry the trend has been toward increased use of heat in processing, particularly in the finishing process. As this is almost wholly at comparatively low temperatures the opportunity for using the heat rejected from power units is especially favorable. It would therefore seem that the use of purchased power in this industry should properly show a downward trend if full advantage is taken of the possibilities.

In certain branches of the woolen industry it is found that as much as two-thirds of all steam generated is used directly in process work besides utilization of the waste heat from power units. In other branches a very satisfactory balance is had when nearly all steam first goes through the engine or turbine. It is only in cases where the dyeing and finishing processes are not incorporated with the preparation and weaving plants that any excess of waste heat is to be anticipated, provided that a proper selection and application of driving units is made.

In one large plant a change from hydroelectric to steam power has been made in which normally all steam generated goes to the power units and all process steam is bled at suitable pressures and the remainder passed to a condenser which furnishes the requisite warm water. By means of a suitable control system the temperature is kept constant and the fluctuation in quantity equalized to process by storage tanks. In this instance the fuel consumption

[3] Engineer, Steam and Water Power, American Woolen Co., Andover, Mass. Mem. A.S.M.E.

was not increased by the addition of the power units, as experience developed that there had previously been a material waste in the use of steam when the supply was taken direct from the boilers and that the interposition of turbines as reducing valves aided in checking this waste.

There are notable examples in this industry of private plants which could supply to the central station considerable power for a part of the time at a low cost by reason of their need of the process heat obtainable by steam extraction or from condenser water and which would also be benefited by having a supply of cheap power available for small overtime uses or when process requirements were smaller than the supply available when generating all of the power demand. Under such an arrangement the individual plant could depend on the central station for breakdown service, thus avoiding the cost of spare units.

Such arrangements would be more feasible when the connection was with a large power system so that a single plant either supplying or drawing its rated capacity would have no serious effect on the system as a whole.

A plan of cooperative connections of this sort would probably require the formulation of special rates, methods of protection, and rules and regulations of a comprehensive nature if the practice were to become at all general among this class of power users.

Study and suggestion along these lines would seem to be quite within the province of this organization.

S. L. Elden.[4] Mr. Gunby's paper has utilized statistics which give a complete review of the existing situation as it has been developed in actual practice in New England and elsewhere.

In pointing out the increasing tendency toward the utilization of purchased power for industry in general, it is of course apparent that favorable results follow from such a course, otherwise it would not continue to develop in an ever-increasing ratio.

Observation of industrial operations utilizing purchased power from the larger companies has always brought into view the relief which the power user seems to have secured from having an inexhaustible source of power at his command as compared with the anxieties which frequently exist where the operation of a private plant is not all that could be desired.

It is undoubtedly true that the cost of power in most processes is today a very small element in the total cost of any finished product, hence the choice between purchased and privately made power must usually be made from a consideration of other points of view than the relative cost of power from different sources.

We have found in many situations that the ease with which power requirements can be extended to meet a growing business may overcome any unfavorable cost factors which under various methods of computation may possibly appear to make private operation more economical.

As to comparative costs for purchased energy in various portions of the country, there would undoubtedly have been different results obtained by Mr. Gunby by the selection of other companies in obtaining the average costs in various locations.

In the New England group where the highest costs were shown it would have seemed more reasonable to include only strictly hydroelectric companies when comparing costs with that group designated as "Southeast" where only hydroelectric companies are involved. By readjustment to provide a comparable basis, it appears upon a rough check that New England might well be placed in a more favorable position than appears from the computations as shown in the paper.

There appears to have been no recognition taken of the rate schedules in existence in New England whereby the central stations supply large blocks of service to industrial customers under so-called "off-peak" rates. These rates average lower than any rate shown in the graphs up to approximately 2500 hours of use.

Some large power users have effected substantial savings in power costs through the purchase of energy under these rates whereby energy which they are required to develop from their own plants is but a small fraction of their yearly requirements.

In referring to New England conditions and costs for purchased energy, it seems important to consider the entire rate question and to give credit to New England ingenuity in the development of rate schedules which result in an actual economy to the user and at the same time are productive of a large amount of business to the utilities.

In the conclusions which Mr. Gunby has drawn, the old contention of unreliable service derived from long transmission lines is again brought to the front, and it is desirable to point out that under conditions existing in New England most long transmission lines are parts of systems that are backed up by both hydroelectric and heavy steam reserves; where the steam reserves are not actually a part of the system owning the transmission lines other companies supply the necessary steam reserves through the extensive interconnections which now exist.

In the largest system in New England the multiplicity of sources of power and the subdivision of the generating plants and transmission lines into units limiting the capacity involved in the case of the failure of a single line or unit have gone far toward relieving the system as a whole of the troubles which were long considered inherent in systems employing long transmission lines.

It is believed that this factor will shortly become of even less importance, when duplications of lines, both over the same and different routes, are completely developed from all important sources of power as is now contemplated in a number of important systems. Further, the development of protective equipment capable of contending with lightning, now the greatest source of trouble, will undoubtedly be such in the future that even less trouble will result from lightning than occurs at present.

With respect to the future power supply for New England, there is no evidence to show that any of our public utilities are neglecting to provide for the future in an ample and careful manner. Extensions to steam generating plants are contemplated to meet forecasted requirements where dependence is placed upon steam generation. Hydroelectric developments of the first magnitude are either under way or in contemplation, to utilize important sites which can be properly developed and coordinated into a single operating system, thus combining the output of both hydroelectric and steam sources of supply throughout New England.

The extensive interconnections which have been and are being developed will unify practically the entire power sources throughout New England with the exception of the state of Maine. New England can rest assured of continuous power developments adequate in every respect to meet the requirements of its industries in the future as in the past.

An important innovation in the operation of private power installations appears to rest on the possibility of some of our utilities utilizing the power by-product of certain important industries along lines which would result in the conservation of fuel and a reduction in the cost of power to the industry.

A number of these possibilities appear in sight which may be proved profitable to both interests and go far to place an even greater approval upon the use of purchased energy.

D. S. Boyden.[5] Mr. Gunby's paper gives many valuable data that will be useful for reference purposes. It shows a definite trend toward the use of purchased power by the industries. I would like to point out a few facts in reference to the use of steam for process and heating which materially affect the eco-

[4] The Edison Electric Illuminating Co. of Boston, Boston, Mass.

[5] The Edison Electric Illuminating Co. of Boston, Boston, Mass.

nomics of the problem of purchased power compared with generated power.

To a considerable extent the economical use of steam in the industries has not always received the careful consideration and study which it deserves. Where the exhaust of prime movers is used for process and heating it is sometimes treated as a by-product of little value, not justifying much effort to conserve it. This is due to the difficulty of determining the real value of the by-product steam. In fact, few industries know the demand or consumption of steam by their various processes and services. Furthermore, steam metering equipment has been high in cost and has not been altogether satisfactory because of the varying flow and pressure. It is often difficult, therefore, to make an intelligent comparison of the cost of purchased power and of generated power.

The general tendency to overestimate the steam requirements for heating and process under the guise of "playing safe" reduces the apparent cost of generated power. Quite often careful study will show that much steam is being wasted through inefficient utilization. Many cases have come to my attention where the manufacturer has continued the operation of his plant at a considerable loss, still believing that he was operating under the most economical plan. A careful study of the situation based on accurate records of steam utilization will often pay handsome dividends.

Fortunately these conditions are being gradually corrected due largely to the results obtained in plants where power is purchased and steam for process and heating is supplied independently. In these plants the cost of the steam service is accurately known and focuses the attention of the management upon its efficient utilization.

Proper metering equipment is usually justified because it can be made to show where the steam is being wasted and steps can then be taken to stop the waste. Meters are also a constant check on the operation of the various processes in the plant and often indicate improper routine of production. This last factor alone often justifies the expense of the metering equipment.

In the last few years the engineers of the company with which I am connected have cooperated with many industries in an effort to determine the most economical balance between generated and purchased power. In many cases which at first seemed to be especially suited for generation of their own power we found that when all factors were taken into consideration the industry would save money by purchasing at least a considerable portion of their power requirements.

Quite often it has been found economical to use some other heating medium than steam. The change of heating medium has often showed not only higher economy but also an improvement in the product being manufactured.

Where these savings have been effected production has not suffered but has usually been speeded up.

I have thought it worth while to point out these facts because more attention to the efficient use of steam will result in a proper division between purchased and generated power. This, in turn, will result in lower power rates to the industries and will bring the rates in New England as low as the rates in the Middle Atlantic and Central West.

PHILIP CABOT.[6] There are two points on which I should like to comment briefly.

The first is in regard to the probable future growth of the use of industrial power. The author shows that during the last few years the rate of increase has been between 1.5 and 2 per cent in spite of the fact that during that period the textile and shoe industries have been so depressed that their consumption of power showed an actual decrease and the situation has been saved only by the growing use of power by other industries. This is a condition which in my judgment is temporary. The boot and shoe industry already shows signs of improvement. The cotton textile industry of New England has been brought to its present condition by unskillful management. This may have been because the managers followed the teaching of the theoretical economists as to the operation of "the law of supply and demand." This so-called law is very misleading—in fact, the most recent studies on the subject seem to indicate that there is no one law of supply and demand but that there are several and, moreover, that the problem of dominant importance has long since ceased to be that of regulating supply and has become that of stimulating demand. This important fact our textile manufacturers seem to have overlooked. In the early days of the industry it was possible to run a mill successfully with almost a single eye on manufacture, the objective being to reduce the cost of production to a minimum by running the mill full for long periods of time on some standard line of goods without much attention to the demand or market side of the business. The mills have found to their cost that this cannot now be done. The stimulation and regulation of demand now are the major problem; not manufacturing but selling is the chief duty of the manufacturer; herein the New England manufacturers seem to have been weak.

This situation is temporary, however. New men will be found who understand modern conditions and it will not be long before our textile mills will resume their place as consumers of power upon an increasing scale. The result will be that the market for electric power, which during the last few years has appeared to be saturated, because it showed a rate of growth about equal to the growth of population, will increase at a normal rate—at least double that of recent years.

My other comment on Mr. Gunby's paper is this: The new men in the textile industry will not tie up their quick assets in power plants unless they are forced to do so. If power of reliable quality can be bought as cheaply as it can be produced they will buy power and will keep their money in quick assets where by rapid turnover it can be made most profitable. This means that there will be an opportunity for the central stations of New England to build up a much larger power load if they are alert and skillful merchants. Their major problem will be to ascertain first the prices at which a maximum volume of business can be obtained, and second, whether they can produce this quantity at a cost that will show a profit. The community has a right to demand that they exert themselves to the utmost to attain maximum volume, which means a low price; and in order to make any profit the highest possible efficiency in production is essential. But to reduce production costs much below their present level is not a simple problem; in fact, it seems improbable that the cost at the switchboard of steam generated power *at a given load factor* can be reduced by any substantial amount from the figure now reached by the more efficient stations. So far as hydroelectric power is concerned the same holds true because the cheapest and best located sites have already been developed. Those that remain are either so far from the market that the transmission costs will be high or involve a high development cost. A method is in sight, however, which if skilfully used may enable the companies to make a substantial reduction in price without an undue sacrifice of profit. I refer to the possibilities of improving load factor by interconnection.

If all the principal producing stations in New England are tied together by a well-designed transmission network and if the operation of them is coordinated with skill, great improvements in the load factor and the capacity factor of the whole group can be made. This will result in reducing operating expenses by produc-

[6] Professor of Public Utilities, School of Business Administration, Harvard University, Cambridge, Mass.

ing the power at the most efficient point in the system taken as a whole and also in reducing fixed charges because the same total system load can be carried with a smaller investment in generating equipment.

One good way to accomplish this result would be to combine all the operating units in New England by means of a holding company. There have always been too many small independent producers in New England for the good of the customers; and the area, population, and market are not too large to be handled successfully by a single organization. The efforts of the New England Power Association seem to look in this direction and it is earnestly to be hoped that no narrow or selfish interest will be allowed to hamper this scheme of coordination.

It must be admitted that the provincialism of New England people and politicians is a somewhat serious obstacle in the road of progress in this direction. It is still illegal in Massachusetts to combine the operating companies of the state in the way which would be most useful to their customers and the state of Maine is still inclined to prevent the development of its water powers by prohibiting the sale of the output in the only markets now available, which are outside the state.

Fortunately, so far as all the other New England states are concerned, there is a method by which the advantages of combination can be secured without sacrificing the independence of the present operating units. The objective is such coordination of all efficient generating stations that the diversity factor of all the loads in New England may be used to give the maximum system load factor while at the same time the power is generated at the most economical point. This can be done by a well-designed transmission network centering on a load dispatcher who has complete control. Experiments with this method have already been made in the Connecticut Valley Power Exchange through which the operations of the stations at Turners Falls, Springfield, and Hartford have been coordinated with profitable results. Similar arrangements for interchange have been employed by the New England Power Company. These are only beginnings and if important economies are to be achieved the thing must go much further. The economy of quantity production in all industries depends upon a large and constant market. As already noted, demand determines supply, and large-scale or quantity production must rest upon a large market. Big units of production and small or fickle markets cannot live together.

The power market of New England taken as a whole is large enough to sustain large units of production; broken up, as it now is, such production is not profitable. Before really low production costs can be reached the whole New England market must be combined at some central point in the office of a load dispatcher, or rather a power broker, where all the power produced in the area will be bought and sold. Such a broker would act as the agent of all the producers, who would call on him daily, or even hourly, to know the prices and quantities of firm or secondary power which his clients had for sale. This market would be open not only to all the central stations but to those manufacturers for whom, as noted by Mr. Gunby, power was a by-product and who at times might have power to dispose of at a very low price. Such an arrangement operating like a wheat or cotton market might introduce a form of competition into the electric power market which would keep the producers on their toes and prove to be a factor of major importance in the industrial development of New England.

H. E. Thompson.[7] Both conditions of interconnections of companies and exchange of power exist at the Amoskeag, and a description of the manner in which they are handled may be of interest.

Enough steam is used in process work to justify the installation of an exhaust turbine generating 4000 kw. with an initial pressure of 140 lb. gage pressure, and exhausting at 25 lb. back-pressure. This steam is delivered directly from the turbine to the dyehouses, wet finishing department, slashers and tenters, and is used for heating in the winter.

The regulation of the turbine is controlled entirely by the back-pressure, the steam demand in the mills determining the load generated by the turbine. This load varies tremendously during the day, sometimes as much as 1500 kw. within two or three minutes. This change in load is automatically taken care of at our hydroelectric plant a mile away, which has two 5000-kw. and one 6000-kw. units, synchronized with the steam plant.

During the middle of the day, especially in winter, the exhaust turbine generates as much as 4200 kw. about 4 p.m., but just when our lighting load is coming on, the turbine will have dropped its entire load, so we must be prepared to produce about 5000 kw. by some other means for the remainder of the afternoon.

Part of this power we produce by starting a 3000-kw. condensing turbine located in the same station. The boilers are going, the men are in the boiler house, and the same engineer can run it, so practically the only cost is the fuel used. The balance of 2000 kw. can be made up from water if available; and if not, it can be purchased from the Public Service Company of New Hampshire, with which we are interconnected.

This interconnection is of great benefit to both parties as we often have a surplus of water power when the Public Service Company can use it to advantage, and during the low-water period it is often a great advantage for us to be able to take power from them and not be obliged to keep an auxiliary steam plant in readiness.

William H. Larkin, Jr.[8] Mr. Gunby's interesting paper brings to the front once more the need of some standardized method of power cost accounting for use in the industrial plants of New England and the country at large. Until such a method is generally used, it will be difficult to make intelligent comparison between individual plants, between privately generated power, and purchased power, or from year to year at the same plant.

Such a standard power report should be compiled at regular intervals, preferably monthly, and should be largely financial, with enough supporting data of technical and engineering character to show underlying causes of fluctuations in power production costs. Such a report should show a certain amount of general data such as amount of factory production for period; amount, characteristics, and cost of fuel; total productive man-hours, cost of all power, etc.; a certain amount of engineering data sufficient to check the main facts of the report, such as total evaporation, boiler efficiency, engine, generator, water power and purchased power loads, with total horsepower-hours or kilowatt-hours produced or consumed; a section containing operating costs, labor, supplies, repairs, and charges covering the various departments of the power plant, that is, boilers, engines, generators, water power and purchased power; and, finally, a section containing the important unit costs, such as steam per thousand pounds, engine power per horsepower-hour, generated or purchased electric power per kilowatt-hour.

Such a report should be quite different from the usual engine-room log and should not concern itself with numerous engine-room details usually found on such operating records. Boiler and engine-room logs or records are useful and are needed but not generally outside the power plant itself. From them are drawn

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the engineering facts which, with other facts and figures from the accounting department of the factory, go to make up the monthly report.

The "monthly power report" is for the use of the manager of the industry rather than for the chief engineer. Its function is to let the manager know how well he is being served in his power department—whether his costs for steam and power are greater or less than in previous years; whether or not the considerable expenditure made at the chief engineer's recommendation resulted in the expected reduction in unit cost; whether or not his engine or generated power unit costs compare favorably with the unit cost of purchased public service power offered. Such a report has essentially the nature of a continuous plant test made by the chief engineer and his assistants over long periods, which, if properly done, is more valuable than a short test, however carefully made, by some outside engineer.

One or two of our large industries operating several power plants have such power reports which have been in continuous use for many years. The writer believes that one long step in the direction of fuel conservation and elimination of economic loss in the power field would be the adoption by the industrial power users of some standard method of power cost accounting and report.

SAMUEL FERGUSON.[9] Mr. Gunby mentions the fact that our New England prices are higher than those paid by competitive industries in the Middle West and properly attributes this to our distance from the coal fields.

I wish to take this opportunity to state that public utility executives in New England very generally recognize their responsibility so to conduct their business that their customers shall not labor under this handicap longer than is absolutely necessary. The relief will come from the dilution of our fuel costs with our water power resources, but, as the author intimates, this will be fully effective only when interconnections are more general and more complete than at present.

Without interconnection the steam companies' costs are high due to cost of fuel; and water power companies' costs are high due to the impossibility of using the seasonal full flow of our streams during every hour of the month including Sundays and holidays.

The start has been made, however, but as with every new undertaking at first the fixed charges on the very large investment are consuming a large percentage of the benefits. It is my belief that a few years will see us all well beyond this stage as every month sees an improvement over the preceding one.

The use of averages by the author is possibly a little misleading in the case of his comparison of prices because the spread in prices between companies in the same region is quite as great as the spread indicated between New England and the Middle West.

To illustrate, I have compiled a graph, two years old, which explains the point. The spread between the high and low city at both the one-shift and the two-shift points is quite comparable with the spread in averages which the author mentions. Superimposed upon these curves are those of Hartford and Waterbury, showing, as of 1926, a reasonable accord with the prices in cities located much nearer the coal fields. In the case of the companies in both these cities, a reasonable start has been made toward an amount of interconnection which will maintain a proper balance of water- and steam-generated power.

Another point on which Mr. Gunby touched is bound to become of importance, namely, the use of the central station load to absorb all of the power which the factory process and heating steam is able to generate.

The reason that this has not already been done to a greater extent is because the industries have not been ready to recognize the fact that the extent of the saving to the power company is the increment fuel cost only. I expect to see the power companies bearing the cost of the installation of turbines to extract this power which will be used in part on the premises and in part on the system; and to see the savings effected equitably divided after the carrying charges on the turbine are absorbed.

We have recently installed a 500-kw. turbine in a customer's factory for the dual purpose of (1) insuring a 10-mile single-circuit transmission line to his plant and (2) effecting fuel economies. In this case, since the cost of the turbine was less than the cost of an otherwise necessary second circuit, the estimated fuel saving of 6 mills per kw-hr. is divided on a 50–50 basis. All the power generated is therefore fed into the system on the system side of the customer's meters. Consequently, customer pays his regular power bill and is paid 50 per cent of the savings effected by running the turbine to the extent of the steam necessary for other purposes; or 3 mills per kw-hr. generated.

Obviously it would not be equitable to credit the customer with more than the savings effected as would be the case if he reduced the amount of purchased power by feeding into his load inside of the meters.

C. W. CONRAD.[10] Colonel Gunby's excellent paper on the power situation in New England is not encouraging. Our higher cost of purchased power and probable higher cost of power generated by the industrial is a serious menace in that more industries may be attracted with the textile and shoe industries to other sections of the country. In order that this factor may not contribute to further exodus of industry it is essential that the power cost per manufactured unit be reduced. The engineer, public utility operator, manufacturer, and other business men must cooperate to secure the most economical generation and utilization of power.

To secure cheaper power, popular clamor appeals for development of water power. Projects like those proposed on the St. Lawrence River and the Bay of Fundy have encouraged hope. Unfortunately these developments have serious limitations from the standpoint of investment required and feasibility of transmission of output to market that make then economically unattractive. Water power development may help somewhat in reducing power cost, but it is certain we can never reach the limit necessary until we utilize more economically the heat in the fuel consumed.

The generation of power from fuel and the utilization of heat are problems that present enormous possibilities for economy if they are properly analyzed on a thermal efficiency basis. The operations of the central station are compared on the B.t.u. per kilowatt-hour output. Analysis of an industrial plant on the basis of useful and total B.t.u. per manufactured unit will bring out some amazing conditions, particularly where large quantities of process steam are required. The 13,000-B.t.u. per kw-hr. central station indicates something quite definite. The 6000-B.t.u. per kw-hr. industrial power plant, which utilizes all exhaust from back-pressure generating equipment for process, may mean little because the heat in the process steam may not be utilized efficiently. As some one has well expressed it, we must become "B.t.u.-minded" in order that heat will be as efficiently used as other manufacturing materials.

I am going to express some opinions on generation of steam and power, part of which may not be attainable now, but I am confident that the time will come when they will be common practice.

1 In localities where manufacturing is concentrated, central steam-generating plants should be established and the industrials

[9] Hartford, Conn.

[10] Plant Engineer, Bird & Son, Inc., East Walpole, Mass. Mem. A.S.M.E.

should purchase steam. As it is necessary to use medium pressures for steam transmission each industrial should have sufficient equipment to generate the maximum power economically attainable to supply its process heat requirements. Any additional power needed should be purchased and if there is a surplus it should be sold. The central steam plant would operate at higher pressure and utilize back-pressure generating equipment to reduce steam to proper distributing pressure. This steam plant would be owned by an industrial, groups of industrials, the power utility operating in the district, or by a corporation which would have the viewpoint that its principal business is to generate and distribute steam and that the power obtained is in the nature of a by-product. If the power company did not operate the steam plant it should purchase the power available.

2 In localities where manufacturing is not concentrated the industrial plants should generate only steam required for process, using efficient back-pressure equipment to generate the maximum power economically attainable in reducing steam pressures for process requirements. Additional power necessary should be purchased.

3 The utility should have efficient equipment to generate and distribute a reliable power supply to the manufacturing plant at a cost cheaper than the industrial can generate power with condensing equipment.

In New England we have a large interconnected power system which can be expanded and utilized for distribution. Its operators are cooperating in making it more reliable and useful. However, we need the cooperation of the engineer, the manufacturer, the utility operator, and other business men to study the economical use of heat to try to weld these forces into an organization which will solve the problem and result in cheaper industrial power.

W. W. Gaylord.[11] Mr. Gunby's paper gives a very interesting survey of the power supply situation in New England and especially that part of New England within seventy-five miles of Boston.

In western Connecticut, although we notice the same general trends, there are several factors which modify conditions considerably.

First, our largest power users are the metal-working industries rather than any of the three mentioned by Mr. Gunby.

Second, our water power resources are of relatively small importance, although one of the power companies is making the interesting experiment of using a storage reservoir located on a small watershed as a giant storage battery. (See paper by E. J. Amberg on Rocky River Hydroelectric Development, presented at meeting of A.I.E.E., New Haven, Conn., May 11, 1928.)

Third, the policy of certain public service electric companies of "educating the public to service interruptions" has compelled some industries to generate at least that part of their power which cannot be subject to interruption without loss of a large amount of work in process.

The brass mills of the Naugatuck Valley are large power users not only for metal working but also for electric melting and annealing. As they have relatively small steam demands for process and heating they make a very desirable load for the central station. But because an interruption of over half an hour in the power supply to an electric melting furnace results not only in the loss of the charge but usually of the furnace lining as well, it has been found desirable in most cases to install at least enough generating capacity to take care of this portion of the load.

The industries using large amounts of process steam are becoming increasingly alert to the need of balancing steam and power demands, and some interesting plants of this type are now under construction.

Mr. Gunby suggests the possibility of the power company's coming in to act as the balance wheel by supplying power when the process steam demands were lower than the power requirements and by buying power when an excess was available. This looks like an interesting possibility, but from several studies we have made of such cases we are inclined to believe its application will be decidedly limited as there are several serious obstacles which must be overcome before such an arrangement can be made successful. Among these are:

1 The difficulties of running non-condensing turbine-driven generators of relatively small size (unless of the induction type) in parallel with the power companies' generators.

2 The fact that as the industries' excess power seldom comes at the time of the power companies' peak loads the saving to the power companies, and therefore the amount they can afford to pay, is less than their fuel cost of the power received.

3 The additional cost of installing such a system will increase the fixed charges and further reduce the value of any fuel saving.

4 The rates a power company will have to charge for intermittent service will be almost, if not quite, as high as the cost of generating the power non-condensing and wasting the exhaust steam.

5 The added cost of dispatching and accounting to both the industrial plant and power company.

Mr. Gunby states that industries using less than 500 kw. probably cannot show sufficient savings to justify such an arrangement, but from studies I have made I doubt if industries using less than 2500 kw. can show any attractive savings.

The suggestion of central stations' using very high initial pressures and supplying both power and steam to industrial plants appears to have interesting possibilities in places where there is a sufficient concentration of industrial plants to warrant the building of a fair-sized station.

There is another possible way of reducing the cost of industrial power, which, although it has some objections, we believe should be given more consideration than it has had. That is the common power plant for groups of industrial plants located adjacent to each other, this common power plant to supply both steam and power as needed by the various plants. This scheme would certainly reduce the labor cost and should reduce the fixed charges by eliminating the duplication of spare equipment.

In cases where plants using large quantities of process steam are located near plants having large power loads a considerable saving in fuel would be possible. A study of the savings possible was given by the writer in an article in *Power Plant Engineering* for May 15, 1928.

Of course, as already stated, each case must be studied on its merits, but the fact that there is such a widespread interest in power problems by industrial executives indicates that substantial progress has been made.

Harry W. Benton.[12] I am not enthusiastic on the subject of central station power for industries located where the buildings require heating for seven months out of the twelve. In plants the size of Pratt & Whitney it is an absolute paying proposition to produce their own power because of the fact that for the winter period they produce all necessary electric current as a by-product of the heating system and at no extra cost for fuel.

Irving E. Moultrop.[13] I have read the paper several times

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[12] Plant Engineer, Pratt & Whitney Company, Hartford, Conn.

[13] Chief Engineer, The Edison Electric Illuminating Co. of Boston, Boston, Mass. Mem. A.S.M.E.

and I get something new out of it each time I read it. The author's summation in reference to the question of purchased power versus installed plant is very good. There are undoubtedly cases where an industry can afford to install its own power-generating plant, but such cases are few and far between.

Professor Cabot in his discussion has said that the cost of central-station power is not going to come down very much. He has stated that many central stations are overbuilt. I am not sure that either of these statements is correct.

It is true that the thermal efficiency of steam-electric generating stations has been raised to a point where material improvement in the near future is probably not economically justified; but do not forget that fuel costs are only a part of the cost of generation. The fixed charges on the generating station are much greater than the fuel costs, and I expect that in the near future we are going to be able to reduce these costs the same as we have reduced the fuel costs. We are at this time more interested in dollar efficiency than in thermal efficiency. I believe that by better design we are going to be able to obtain a material improvement in dollar efficiency.

In comparing the cost of power in New England with the cost of power in other parts of the country, you should not forget the fact that practically all of the coal burned in New England for power-generating purposes is hauled approximately 400 miles by railroad and 600 miles by water. When you consider this fact, I think you will admit that we are doing mighty well in generating power as cheaply as we are.

In reference to the central stations being overbuilt, I would say that when all factors are considered I do not believe we have installed any more generating capacity than is economically justified. There are many factors that govern this situation that are not obvious unless you are thoroughly familiar with the requirements of the service.

Another factor that materially affects the cost of generating power is the load factor of the system being served. We have made great strides in this direction during the past few years, and I believe that we will make further progress in the future. The development of high load factor business, the increase in the reliability of equipment, and extensive interconnections are all helping to increase the load factor of our generating stations and will no doubt help materially in our efforts to supply cheap and reliable power to the New England industries.

Charles H. Bigelow.[14] In relation to unit costs I have worked out these costs from month to month for different plants from the actual expenses and find that it is not possible to get a constant figure as is given in estimated costs, to be expected with any particular arrangement of apparatus. The unit cost per kilowatt will vary over a considerable range due to various reasons, and this applies whether the power is purchased from a public utility or is manufactured in an isolated plant.

The management and accounting departments think that they should have a constant figure, so they can say that their power costs one and a half cents, two cents, or whatever it may be per kilowatt-hour, but the actual results do not give this.

In one company, located in Pennsylvania, where the power was purchased, the net cost per kilowatt without any change in the rates varied from 1.78 cents per kilowatt to 2.33 cents per kilowatt, an increase of 32 per cent within a few months. This was due to changes in load, the increased unit cost coming when the load had decreased about 51.5 per cent, due to slack business, and was mainly caused by the fact that the demand charge was based on the maximum demand in the year and did not change for a year unless the demand increased, even although the total load had decreased over 50 per cent, the demand factor being 25.5 per cent of the total with the heavy load and 40.5 per cent with the small load.

In another company the cost varied from 1.60 cents to 2.31 cents from the same reason.

In New Jersey the Public Service Commission requires the public utilities to base their maximum demand charges on the average maximum pulls of the working days for each month, which is much fairer to the customer, making the necessary changes each month.

If the company makes its own power, the same variable results will be obtained, depending on the various factors involved, such as load, heating in cold weather, maintenance, repairs, etc.

In general the charges are made up of "fixed charges," such as depreciation (in one plant I was in that was taken at 10 per cent, which is high), taxes, insurance, etc., and over this factor, which in this case varied from 20 to 30 per cent of the monthly expenses, the operating department had no control. Of course the larger the output the less the unit cost.

"Maintenance" was due to repairs, breakdowns, etc., but the amount charged depended on the bills paid during the past month irrespective of the time when the work was done, although the labor would come into the proper month.

"Operating labor" had been reduced to a practical minimum of one man on watch in the turbine room and one in the boiler room, plus some extra help in the daytime during the heavy load. Here also the larger the output the lower the unit cost.

"Fuel" is the only item that varies in any kind of ratio to the power produced, and even that would not be in exact ratio.

Therefore, it can easily be seen that the unit cost in this case will not be constant, but will vary over a large ratio.

Where process steam is used together with the heating, etc., particularly in a plant that is already built and the investment made, it would not pay to buy power, because the fuel cost would be the only variable, and if the pro rata of fixed charges, maintenance, and operating labor was taken off of the electric power, it would greatly increase the charges for heat, process steam, etc.

In one case, where the prorated charge for electric power was a little more than the power could apparently be bought for, if the power was bought the charges for the other services would be so increased that the gross cost of both services would be increased about $24,000 per year, besides an extra investment of $16,000 to $20,000 for transformers, switchboards, etc., and they would still have the old equipment.

In conclusion, I would say there is no real constant unit cost, whether power is bought or made, and the question of buying or making must be determined in each case on its own merits.

A. W. Milliken.[15] One of the reasons for early New England's prominence in manufacturing was the availability of power. If now, as shown in Colonel Gunby's paper, New England power costs slightly more on the average than elsewhere, other factors must be more important than cheap power, since manufacturing in New England is steadily advancing as shown by the increasing power use. This leads to the conclusion that cheap available power is no longer of the primary importance it once was, as compared to many other economic factors affecting industry generally. Furthermore, I would call attention to the fact that if there were some method of weighting the average of the utility rates studied, according to population, number of industrials served, value of products manufactured, or some other measure of the importance of the individual rate structure as applied to industry within the territory for which the average is being computed, a more nearly accurate average would be obtained.

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[15] Assistant to Manager, Electric Department, New Bedford Gas and Edison Co., New Bedford, Mass.

Author's Closure

I will take up the points in the order in which I made a few notes. No one would be more delighted than I to see the tendency in the textile work of New England reversed promptly and enthusiastically. Mr. Cabot said that he thought it was going to be. I hope that he is as good a predictor on that as he has been on other things. It would be a very desirable result. He also mentioned interconnections, and put in a thought that has been in the back of my mind for a long time. That was that there ought to be interconnection not only between the central-station people, but also between them and many industrial generating plants. As you noticed very roughly from the curve showing our total load, a little over one-half of our total requirements are now generated by the public-service people, and a little less than one-half is generated by the industries, and if we can get a closer cooperation between those two groups, we are undoubtedly going to get some actual savings. Never mind how they divide; there will be an actual saving to the pocketbook of New England from whatever economy can come about. And interconnection has some large possibilities of economy.

Mr. Elden and Mr. Milliken bring up the question of how the rates for which I gave you some graphs were made up. Perhaps I not only "muffed the ball," but also dropped it in the mud, in not making it entirely plain that it is not the rates of power companies that I am trying to compare. I am trying to compare the cost of power to industrial users. Perhaps if we had, let us say, some cotton mills located in each one of the sections mentioned, and I had the costs from them, they would serve my purpose admirably. We did not have them, so I used the things we did have in all communities; namely, the power rates as published by power companies.

Now, as a matter of fact, there are some fundamental things that affect the cost of power. If you have a central station, one of your costs is going to be fuel, and roughly speaking, the relative cost of coal is not very different in those communities from what the charts show about the relative cost of power, except that in New England the cost of coal bears a greater disparity than the cost of power, which can only mean one thing, that we have been more efficient in the use of coal in our central stations up here than some other people have. Here we pay nearly $7 for coal; in some other sections of the country coal costs $3 to $4; so under similar load conditions power can be made for less. The companies selected for the South get most of their energy from water power. They have the same problem we do—will their large customers buy power or generate it? The economics of the situation are the same in all sections.

Remember, the curves I showed are just an arithmetical average and are for companies in cities picked out as fairly representing general conditions. They might be illogically selected, using the wrong companies. I was not trying to prove anything. I wanted to find out what might be the facts as regards average conditions. They do not present a finished picture of the situation, but they do, I believe, in general, show that our power costs a little more than it does in some other sections. I think if we figured it out more closely we would get a greater disparity. In fact, I am surprised that the graphs show as little difference as they do. I believe, for instance, that we in New England require a higher degree of reliability of service than has been the custom in some other sections. This would tend to make our costs higher.

Now, another thing. I wonder if Mr. Milliken was entirely correct when he said New England grew up on cheap power. I am not a native New Englander, so I do not know as to the price. Some of the information I have is only hearsay or from documents that we have come across once in a while in connection with some old plant. I doubt if it was cheap power that made New England. I think it was power. I doubt if those old manufacturers did know what power cost them. I doubt very much if they cared. Up in Lawrence a while ago we got hold of an old document showing where they paid the people every three months if the plant ran, and the mill ran when they had water; they always shut down on a cloudy day because they did not have enough light, as they had no method of artificial lighting. The operation was entirely dependent on getting the combination of circumstances under which they could run the factory. I ran across another funny case, interesting only in showing some of the crudities they had back in those days, as they appear now. This was the letter to the treasurer by an agent of a mill about one hundred miles from here, in which he said that the help were kicking on account of being paid once a year, and they wanted to be paid once in six months. He said that of course it was going to increase the amount of clerical work, but he thought it probably would be a good thing. And he further thought that they would kick next to be paid more often, as often as once in three months. I do not believe those fellows knew much about the cost of power. They were trying to get the power, and I believe that is still our problem. We can keep better track of the costs today than we used to, but I think the power, getting the power, is the important thing.

I saved Mr. Cunningham's comment until last, because he said that he would like to be shown that New England was growing. I think the facts show that she is growing. I will talk now entirely about the industries that use power, and will leave out the central stations, which undoubtedly are growing very fast. The arithmetic shows that there is no question of that, but the thing that Mr. Cunningham has on his mind is the industrial situation. Admitting that we have had 150,000,000 kilowatt-hours drop in the amount of power that is used by cotton textiles alone in a five-year period, the fact remains that applying the same hours of use to the rest of the industry, you get an increase in the amount of power used. Another way of looking at the same thing is to take that same cotton industry, the worst thing we have as far as decreased activity is concerned. In 1922 it had installed 535,000 horsepower, and by 1925 it had 572,000 horsepower, and in 1926 it had dropped back to 550,000 horsepower. These were installed capacities. It actually decreased about 30,000 horsepower in the amount of power. In other words, that much horsepower had gone out of business in this industry. This decrease is easy to account for in liquidation and things like that. But despite this, the total power installed in all industry had not decreased at all, but had gone up a little bit. Some other industries, of necessity, had to make up for that 30,000 horsepower dropped by the cotton mills. I think what is happening is that new industries are coming up to take the place of those which are going down. Many of these new industries are in small units. We saw the same thing happen to our farming people a couple of generations ago. The farms were doubtless quite valuable when they built all those stone walls on them. That value is very largely gone now. Our cotton mills were very valuable a few years ago. Even we young fellows can remember that. Now you can buy many of them fairly cheaply, just like farms. The farming largely disappeared, and something else took its place. The cotton mills may go, but something else will come along and take their places. All that is going to happen is a perfectly natural development, and when somebody beats us at our game, we will get into some other game.

FSP-51-27

Balancing Heat and Power in Industrial Plants

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The purpose of this paper is to indicate the importance of various factors in increasing the amounts of by-product power that can be generated in industrial plants using process heat. It has been shown that there are opportunities for economies in increased boiler pressures, higher superheats, and particularly by reducing the back pressure on turbines and engines to the lowest possible point. Utilization of waste heat from furnace gases offers an opportunity for additional power generation in mixed-pressure turbines or in low-pressure condensing turbines. It is shown that hot air can be most economically used by returning it to the boiler furnace and heating air for process work with exhaust steam.

Two curves are presented which give the results of a detailed analysis of the problem of water heating, one showing the advantages of two-stage water heating as compared with heating by exhaust steam only, and the other giving the best condenser vacuum for various initial and final temperatures of water in a two-stage process. Attention is called to the importance of an adequate study, both of the heat and power requirements of the process and of possible improvements or alterations of the process as a basis for the improvement of the heat-power balance.

INDUSTRIAL plants use large amounts of heat in such processes as drying and evaporating, water heating, cooking and digesting, and in kilns, furnaces, and ovens. Practically all plants in the Northern States have a substantial space-heating load during six months or more of the year. Moreover, all industrial plants have power and electric-lighting loads. In plants like paper mills, textile finishing mills, and chemical factories, where there is a substantial heating load, the possibility of generating power as a by-product of the process steam presents an opportunity for possible economy.

When all of the power used in the plant is thus generated without rejection of heat in condenser water, the power and heat loads may be said to be balanced.

This paper discusses the most important factors that determine or affect this heat-power balance. These factors are:

1 Boiler steam pressure and superheat
2 The pressures and temperatures required by the process
3 Type and characteristics of power-generating equipment
4 Utilization of waste heat.

Basic Principles

Before discussing these factors in detail, it will be desirable to recall the fundamental laws of the interconversion of heat and power. These may be simply stated as follows:

1 Power can be completely converted into a definite amount of heat at any desired temperature. A kilowatt-hour of electric power is convertible into 3412 B.t.u., whether in producing an electric arc or in melting ice.

2 Heat can never be completely converted into power. Only a definite maximum percentage is theoretically capable of conversion, and this only by a proper prime mover. (Fig. 1.) The maximum percentage depends on (*a*) the temperature at which heat is supplied to the prime mover, and (*b*) the temperature at which the remaining heat can be rejected from the prime mover.

3 For every kilowatt-hour of power generated, a definite quantity of heat (3412 B.t.u.) is actually used up and disappears as such.

4 The conversion of this portion of heat into power is accompanied by a definite drop in temperature of the remaining heat not so converted.

In more familiar terms, the maximum theoretical power that can be developed by an engine per pound of steam handled depends fundamentally on the temperature of the steam entering and leaving the engine, and therefore on pressures and superheats.

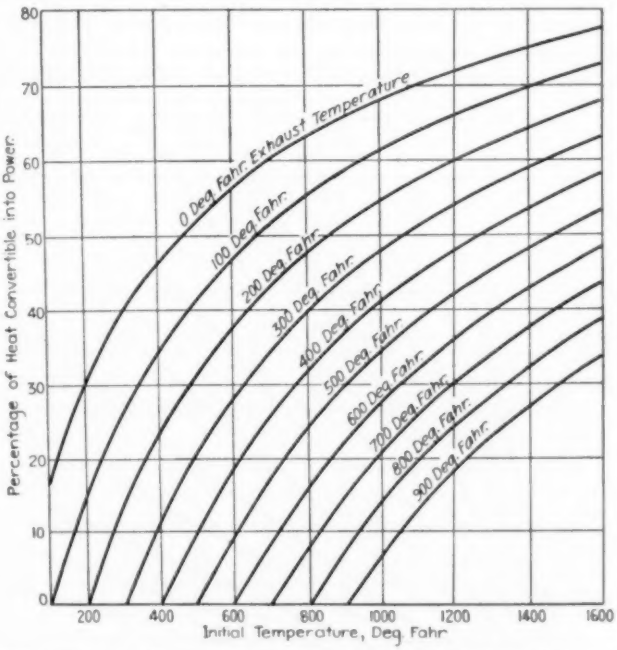


Fig. 1 Heat Convertible Into Power Between Various Initial and Final Temperatures

The portion of this maximum theoretical power which the engine will actually deliver depends on how closely it follows the theoretical cycle and on mechanical losses.

By-Product Power

Since the combustion of coal liberates heat at a temperature far above that required in most industrial processes, this heat can generate power before being used in process. The power so generated will be referred to in this discussion as "by-product power." In the near future power-plant engineers will regard this by-product power as too valuable to be neglected. We are frequently told that it is a crime to burn raw coal because of the waste of tar and gaseous by-products that are of value as basic chemicals. The value of these material by-products from a pound of coal is roughly $^1/_4$ cent, and the remaining coke has $^3/_4$ of the heating value of the original coal. In generating by-product power, a pound of coal may easily yield $^1/_2$ kw-hr., worth at least $^1/_2$ cent, and $^7/_8$ of the heating value remains for use in such processes as drying and water heating. The burning of coal to

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furnish low-pressure steam for heating only is therefore a greater crime.

Most industrial plants can find a use for an almost unlimited amount of power if it can be obtained cheaply enough. Mechanical power is readily convertible into electrical, chemical, and luminous forms of energy as well as into heat, and in proper form can be readily transmitted, controlled, and applied to an endless variety of processes. In Northern paper mills, where water power is cheap and coal is difficult to obtain, electric boilers are economically used to furnish steam for cooking pulp and drying paper. A textile finishing plant was able to save $2000 per month by replacing gas singers with electric-plate singers operated on by-product power. The problem of balancing heat and power therefore resolves itself into the problem of generating from the process heat the maximum by-product power that can be economically used.

The Heat-Power Survey

The first step in attempting to balance the heat and power loads of an industrial plant should be a careful survey, involving accurate measurements, of the heat and power required by the various processes. In making this survey, careful attention should be paid to points where slight changes in operating procedure may make substantial improvement in the balance of heat and power. For this work, intimate knowledge of the process concerned and cooperation with the plant superintendent are necessary. While it is not usually regarded as within the province of power-plant engineers to consider changes in process, experience has shown that intelligent study of conditions frequently brings about changes that are desirable not only from the power standpoint, but from the point of view of decreased labor and improved product. In one paperboard mill, a rearrangement of the steam supply to the driers not only decreased the back pressure on the turbine from 40 lb. to 10 lb., but also increased the daily production beyond all previous records. The advantages of a survey, quite aside from any saving of fuel, are further illustrated by the case of a large Southern sawmill, where redistribution of steam saved a contemplated investment in additional boilers. Table 1 summarizes the results of a heat-power survey of a large mill making coated book paper. This plant is of particular interest on account of the great variety of its heat and power requirements. The table is very much condensed.

TABLE 1 HEAT-POWER SURVEY OF BOOK-PAPER MILL

Unit	Live steam, lb.	Power developed, kw.	Exhaust, lb.	Exhaust steam used, lb.	Pressure, lb. per sq. in.
Beater engine	20,000	520	17,700	....	175
Variable-speed engines	4,400	100	4,000	....	175
Boiler-room auxiliaries	1,000	10	900	5,000	175 and 10
Paper-machine driers	8,000	...	....	4,000	40
Coating-mill driers	400	...	....	13,600	10
Evaporator	5,000	...	....	....	40
Sulphite digesters	5,000	...	....	....	80
Soda digesters	2,000	...	....	....	110
Bleaching	4,200	...	....	....	10
Chemical plant	2,000	...	....	....	40
Total summer steam	52,000		22,600	22,600	
Winter space heating	10,000				
Total winter steam	62,000				
Total engine power		630			
Water power, owned rights		350			
Water power, purchased water		750			
Conversion losses		120			
Total load if all electric		1850			

Distribution of Process-Steam Requirements

Pressure, lb. per sq. in.	10	40	80	110	
Exhaust steam, lb.	18,600	4,000	...	...	
Live steam reduced, lb.	4,600	15,000	5000	2000	
Total, lb.	23,200	19,000	5000	2000	Totals
Available kw-hr. from 175 lb. per sq. in. sat.	740	355	54	11	1160
(Based on 60% eff.) from 300 lb. per sq. in. sat.	865	523	92	28	1508

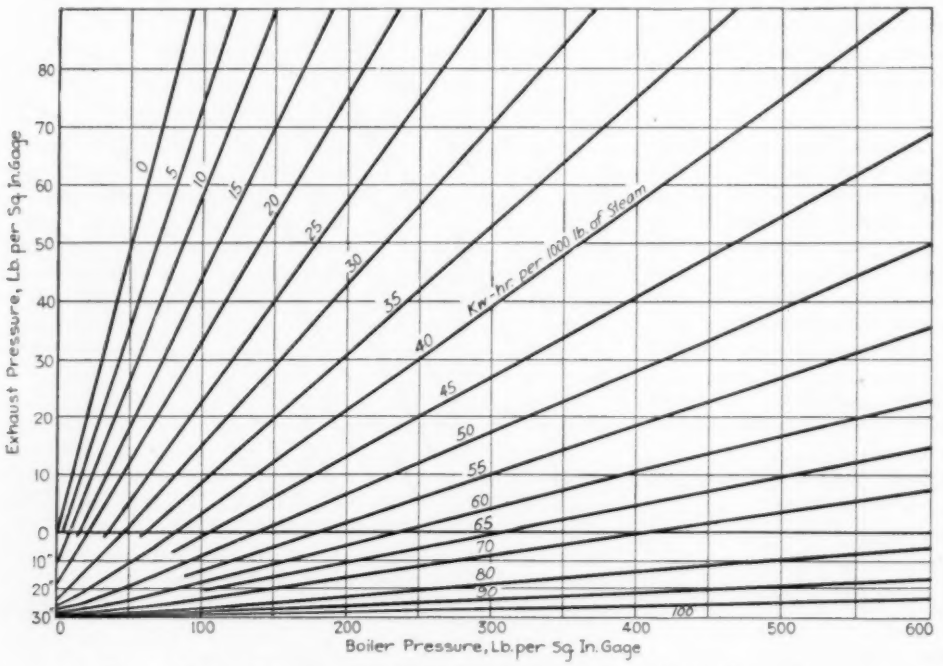


Fig. 2 Power Developed by Ideal Engine for Various Steam Conditions

Heat-Power Ratio

In order to determine at the outset whether a balance of heat and power can probably be worked out, and along what lines, it is desirable to determine the overall heat-power ratio for the plant. This ratio is the total process-heat requirement divided by the total power requirements, and is expressed in B.t.u. per kw-hr. By comparing this ratio for a given plant with the values given in Table 2, the probability of a balance of heat and power will be readily determined.

TABLE 2 TYPICAL HEAT-POWER RATIOS

Plant	Exhaust temperature, deg. fahr.	Heat-power ratio B.t.u. per kw-hr.
Non-condensing plant	212	30,000 up
High-pressure central station, condensing	80	15,000
Mercury-steam plant	80	10,000
Diesel-engine plant	700	10,000
Diesel-steam plant	80	8,000

Effect of Pressures

The accurate determination of the necessary steam and power conditions to produce a proper balance can be greatly simplified by certain charts that have been used with success for this purpose by the author. The object of these charts is to show at a glance what steam conditions are necessary in order to get a given

amount of power from a definite quantity of process steam. Fig. 2 shows one such chart which is very useful in considering the effect of decreasing exhaust pressure or increasing boiler pressure. This chart is based on an ideal engine, but it is a simple matter to allow for any actual efficiency with regard to the ideal Rankine engine. The type of information that can be obtained from this chart can best be illustrated by an actual example. The present boiler plant of a paper mill operates with saturated steam at 160 lb. pressure. The plant uses 40,000 lb. of steam per hour at 30 lb. pressure in driers. It is necessary to generate 1150 kw. from this process steam in order to balance the heat and power loads. By making certain changes in the paper machines, it will be possible to reduce the back pressure to 10 lb. in the driers.

The problem to be considered is: Are the changes in the paper machines justified, or shall a new high-pressure boiler plant be considered? By referring to Fig. 2, we see that 160-lb. steam in expanding to 30 lb. back pressure will theoretically generate 31 kw-hr. per thousand pounds of steam. Assuming 70 per cent turbine efficiency, the 40,000 lb. would generate $31 \times 0.70 \times 40$ or 828 kw. The required 1150 kw. represents 139 per cent of this 828 kw. power which is available under present conditions. Assuming the same turbine efficiency under higher boiler pressure or lower back pressure, we shall require theoretically 31×1.39 or 43 kw-hr. per thousand pounds of steam. From the chart (Fig. 2), we find that this can be obtained by raising the boiler pressure to 290 lb. with the present 30-lb. back pressure, or by reducing the back pressure to 10 lb. with the present 160-lb. boilers. A detailed consideration of costs showed the second to be the more desirable solution. This illustration serves to bring out very clearly the great importance of keeping down back pressures, since a decrease in back pressure of 20 lb. (from 30 lb. to 10 lb.) was equivalent to raising the boiler pressure by 130 lb. (from 160 lb. to 290 lb.).

Fig. 4 shows the importance of back pressure in the case of the actual performance of a 1500-kw. turbine.

Although decreased back pressure is usually the simplest method of increasing the power from process steam, it is frequently necessary to use high boiler pressure as well. It is possible to deduce, from Fig. 2, the fact that the percentage increase of available power due to raising the boiler pressure a given amount is greater for high exhaust pressures than for condensing plants. For example, raising the boiler pressure from 200 to 400 lb. with 42 lb. back pressure will increase the theoretical power per thousand pounds of steam from 30 to 45 kw-hr., an increase of 50 per cent, while at 26-in. vacuum the same increases in boiler pressure will raise the available energy from 90 to 102 kw-hr., or an increase of only 13 per cent. The industrial plant that is using exhaust steam under pressure may well consider the advantages of boiler pressures as high as are used in the best condensing central-station plants.

Superheat

Although Fig. 2 is by far the most convenient to use when pressures alone are to be considered, as is the case with saturated steam, it is not convenient to take account of superheat in this particular type of chart. For problems involving superheat, which includes practically all power-plant problems today, it is necessary to use some such chart as Fig. 3. This chart shows the theoretical power obtainable in going along the theoretical expansion line from any pressure and superheat indicated in the lower right-hand quadrant to any final pressure. The saturated-steam condition is taken as the horizontal axis, and the total power

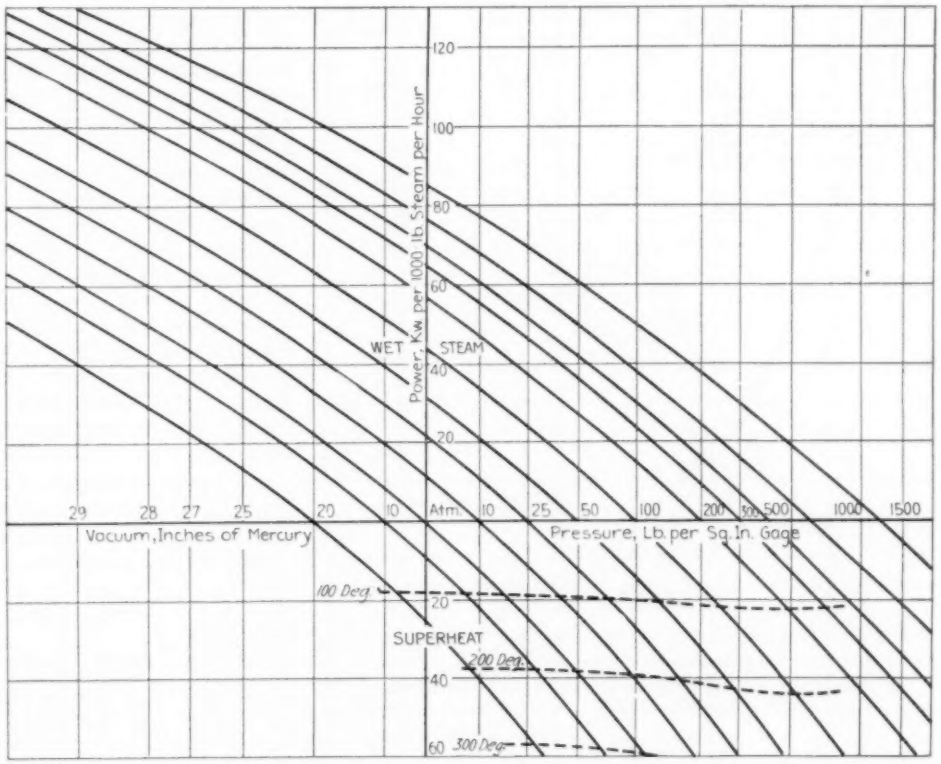


Fig. 3 Power Developed From Superheated Steam by Ideal Engine

developed by 1000 lb. of steam is the sum of the powers indicated in the superheated and wet-steam regions. Suppose that it is desired to use dry saturated steam for bleaching at 50 lb. pressure, bled from a turbine operating under 300 lb. boiler pressure. Fig. 3 shows that the expansion line intersecting the saturated-steam line at 50 lb. intersects the 300-lb. line at 44 kw. on the power scale. If the turbine efficiency is 75 per cent, the actual power developed will be 75 per cent of 44 kw., or 33 kw., and the 33-kw. point on the 300-lb. line represents about 160 deg. of superheat. It is therefore necessary to use 160-deg. superheated steam at 300 lb. in order to bleed dry steam at 50 lb. pressure.

While considering the effect of superheat, it may be possible to clear up some mystery which seems to surround the effect of superheat on steam economy. It is quite obvious that higher pressures increase the indicator-card areas of a reciprocating engine and hence develop more power. The effect of superheat is less obvious. Its action can be indicated by noting that the volume occupied by a pound of steam at a given pressure increases very considerably with superheat. Thus a pound of saturated

steam at 215 lb. pressure occupies 2 cu. ft., while at the same pressure and 150 deg. superheat, it occupies $2^1/_2$ cu. ft. A given cylinder volume filled with the superheated steam has, therefore, only 80 per cent as many pounds of steam in it as when it is filled with saturated steam, although the pressure exerted on the piston is in both cases the same. Many other factors modify this crude conception slightly, but it is illuminating to note that the addition of 8 per cent of heat in the form of superheat increases the volume of the steam by 25 per cent.

One other factor must be borne in mind in considering the effect of superheat on power developed by process steam. When saturated steam develops power in an engine of reasonable efficiency, a portion of it condenses into moisture which is ordinarily removed in a separator before the steam is used in process. In

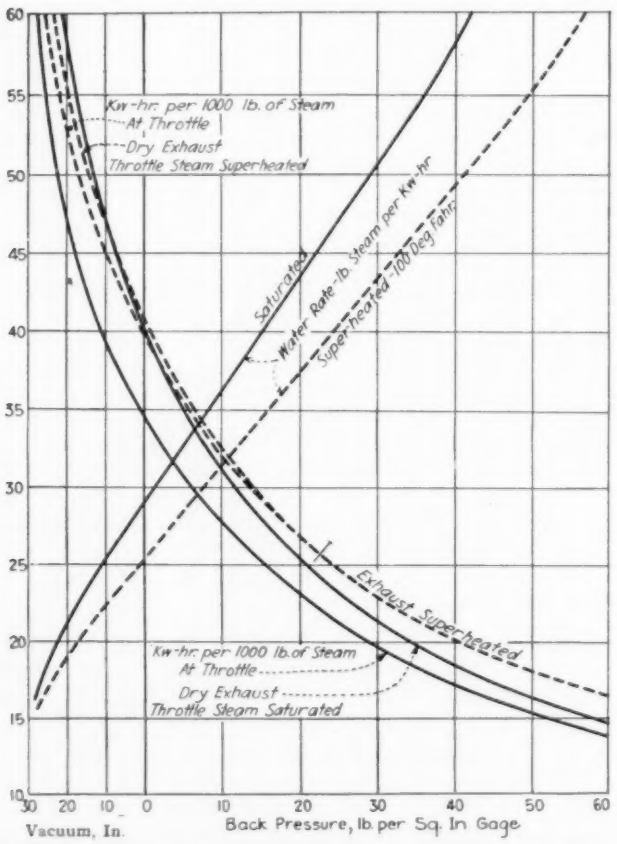


FIG. 4 EFFECT OF BACK PRESSURE AND SUPERHEAT ON HEAT-POWER RATIO OF 1500-KW. TURBINE, THROTTLE PRESSURE 160 LB. PER SQ. IN. GAGE

this way, the dry steam sent to process is less than the steam supplied to the engine throttle by an amount which may easily be as high as 10 per cent. When any large amount of superheat is present in the throttle steam a correspondingly smaller amount of steam is condensed, so that the dry exhaust steam will in this case usually be very nearly equal to the steam supplied to the throttle. It follows that, although superheat decreases the steam supplied to the engine throttle, there is not a proportional decrease in the pounds of dry process steam discharged. Fig. 4 illustrates this point very well. These curves show the power generated both per thousand pounds of throttle steam and per thousand pounds of dry exhaust steam which later would be used in process. In the case of superheated steam the exhaust would be dry at 22.5 lb. back pressure and superheated at all higher back pressures. At vacuums below about eight inches of mercury, the superheated steam actually generates less power per thousand pounds of exhaust process steam than does the saturated steam. This will usually not be the case, however.

Although the effect of superheat in increasing the power developed from a given amount of steam is not large, the addition of superheaters is usually not a difficult problem in an existing boiler plant, and it frequently happens that the resulting economies are very substantial. Quite aside from any question of heat and power balances, superheat is always essential in high-pressure boiler installations in order to insure dry steam to as many as possible of the turbine stages. It is probable that the advantages of superheat even in low-pressure boiler plants have received far too little attention in the past.

CHARACTERISTICS OF POWER-GENERATING EQUIPMENT

The theoretical curves given in Figs. 2 and 3 must, of course, be modified in any actual case by the efficiencies and characteristics of the equipment in which the steam is to be used. This subject is one that might well take up an entire paper and an extensive one at that. A brief résumé of the facts is, however, pertinent here.

The two main types of steam power-generating equipment—turbines and engines—have very different characteristics. The efficiencies of turbines do not vary greatly over various pressure ranges. There is some decrease in stage efficiency at very low pressures. But factors of equal or greater importance are the size of unit and the degree of efficiency that is justified in view of increased costs of high-efficiency designs. Turbines below 1000 kw. decrease rapidly in efficiency with decreasing size. Such small turbines range from 40 to 65 per cent in efficiency. The sizes above 1000 kw. range from 50 to 85 per cent in special cases. Special high-efficiency turbines are expensive, but are frequently justified in such plants as paper mills, where the loads are steady and continuous. Such mills occasionally operate with overall

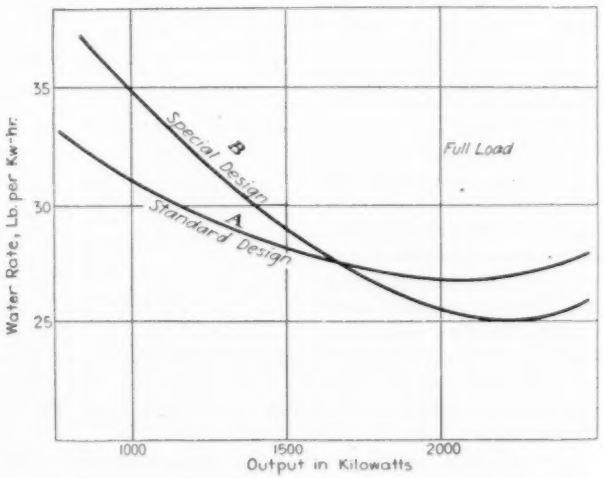


FIG. 5 COMPARATIVE WATER RATES OF STANDARD AND HIGH-EFFICIENCY TURBINES

annual load factors of over 75 per cent, and under such conditions specially designed turbines are in order. However, attention should be called to the fact that special high-efficiency turbines are normally designed for very closely limited conditions as to pressures, steam flows, and power loads. If plant conditions are subject to considerable fluctuation, overall efficiencies may not be improved by such design. Fig. 5 shows the water-rate curves for two turbines; *A*, a stock machine, and *B*, a special design. Machine *B* is essentially a base-load machine and as such gives high economy, but on fluctuating loads machine *A* is as good in overall performance.

The efficiencies of reciprocating engines vary much more than do those of turbines. The efficiencies in small sizes are noticeably higher than for small turbines when the engines are in good condition. Engines do not utilize low or very high pressures as well as turbines. The steam consumption of a reciprocating engine is not affected nearly so much by changes in back pressure as is that of a turbine. If a small amount of steam at high back pressure is needed, it can frequently be obtained from engines having throttling governors without a measurable increase in their steam consumption.

The maximum-efficiency point of an engine usually occurs between $^1/_2$ and $^7/_8$ load, while for a turbine it is generally at or near full load.

Utilization of Waste Heat

We have so far considered only steam because it is by far the most frequently used medium for distributing and utilizing heat. There are several sources of heat for power generation which are usually referred to under the general heading of "waste heat." When process heat is used at high temperatures as in cement kilns, furnaces, retorts, etc., a large amount of heat is lost in the high-temperature exhaust gases. The development of waste-heat boilers has made it practicable to utilize this heat. Since this heat is essentially at a lower temperature than that usually found in boiler furnaces, an analysis is required to indicate the most suitable steam pressure at which to operate a waste-heat boiler, both from the point of view of percentage of heat recovered and the power that can be developed from this recovered heat. Even small amounts of waste heat can frequently be utilized by the use of a mixed-pressure turbine. Reference to Fig. 3 will indicate the very considerable amount of power that can be developed by relatively low-temperature heat in condensing engines running under high vacuum.

In connection with the utilization of waste heat, the use of air heaters as developed for power-plant work is of special interest. It is frequently suggested that, where air is used in large quantities for drying, the most suitable source of heat is an air heater in the

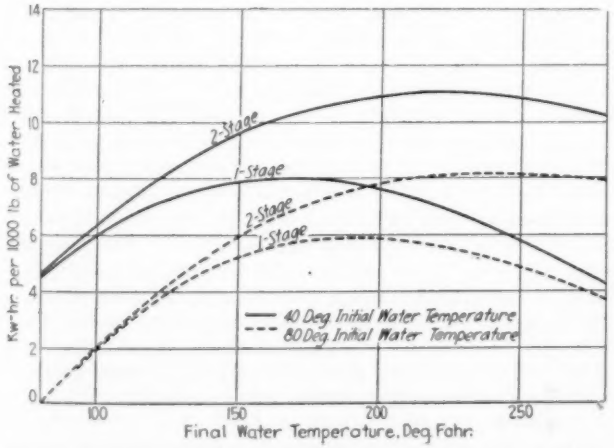


Fig. 6 Comparison of Single-Stage and Two-Stage Water Heating

boiler flue. A more careful analysis shows, however, that from the point of view of obtaining maximum power from a given consumption of coal, it is more desirable to use this heated air as far as permissible in the boiler furnaces, thereby increasing the liberation of high-temperature heat and the output of high-pressure steam. This steam can then develop by-product power, and the low-temperature exhaust can be used for air heating. There is a definite theoretical advantage as well as the added convenience of steam piping, as compared with air ducts, for transferring large quantities of heat.

In some industrial plants there are certain other sources of waste heat that deserve more attention as sources of heated air, either for combustion or for process work. In one interesting re-

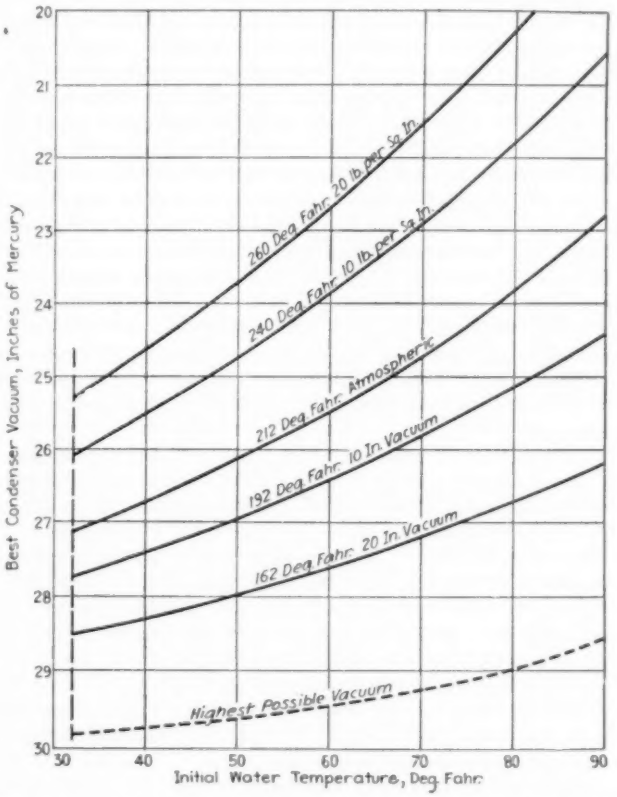


Fig. 7 Best Condenser Vacuum for Two-Stage Water Heating

arrangement, an economizer was removed from the boiler flues and put into the exhaust stack of a black-ash burner and its place was taken by air heaters. In this way, the apparent output of the boiler plant, neglecting the heat from the black-ash burner, was close to 90 per cent.

Internal-Combustion Engines

A development which gives promise of materially widening the field of balanced heat and power loads in industrial plants is in connection with high-efficiency internal-combustion engines. The power developed per heat unit by such engines can, under proper conditions, exceed that of any of the other commercial types of power-generating equipment. While there are a number of factors in the economic utilization of such engines which have not been worked out with complete satisfaction, it is equally true that analyses of performance of these engines have usually not been given proper credit for the waste heat that can be utilized. Hot water from the cylinder jackets of these engines, heated further, if desired, by the exhaust gases, can be used in many industrial operations. There is also a possibility for the commercial development of a heat and power unit based on a Diesel-type engine which will combine an efficient and compact small-size generating unit with a boiler heated by exhaust gases to furnish small amounts of process steam.

Water Heating

Although most process-heat requirements are too specialized to

permit a general analysis, the problem of water heating is one of great importance, and offers an interesting field for improvement of the heat-power balance. The old idea that water heating with live steam is 100 per cent efficient because all the heat units in the steam appear in the hot water is, in view of the concept of by-product power, entirely obsolete. Under certain conditions, the use of an injector operating on live steam for combined pumping and heating of water, especially at fairly high temperature, is permissible. In the near future, however, the condensing bleeder turbine will become the standard equipment for water heating. The simplest method of heating water, which we shall refer to as "one-stage heating," involves the operation of a condensing engine or turbine at a suitable vacuum so that the condenser cooling water is discharged at the temperature required by the process. The idea that it is economical to operate a condensing engine at 28 or 29 in. vacuum using large amounts of cooling water and then using either atmospheric pressure or live steam for water heating is one which cannot be tolerated in the modern industrial plant. Any heat discharged from the plant in condenser water is to be regarded as a serious loss. If all the water that can be heated by the steam from the engine can be used, the heat and power loads will be balanced, even though the engine is running on a condensing cycle.

A more efficient arrangement for heating water has been designated as two-stage water heating. In this arrangement the condenser is run at a somewhat higher vacuum and steam is bled at a pressure corresponding approximately to the final temperature required in the water. A comparison of the power theoretically developed in one-stage and two-stage water heating is given in Fig. 6. It will be noted that as the final water temperature increases, a point is reached at which the power generated per 1000 lb. of water heated is a maximum. If temperatures beyond this point are required it is more economical to use live steam or an additional stage of bled steam for heating. Fig. 7 shows the best relation between condenser vacuum and final water temperature for various initial water temperatures. These analyses are similar to those that have been made in connection with multistage feedwater heating in central stations.

Discussion

F. M. Gibson.[2] To secure that balance of heat and power in an industrial plant which will give the best dollar efficiency requires the consideration of a great number of factors. To secure a satisfactory balance it is necessary to study the hourly change in the ratio of heat demand to power demand under all conditions of operation. Various processes, types, and sizes of equipment and the cost of each must be considered and be compared with the various systems, sizes of equipment, and costs of generating heat and power. Not only is it necessary to determine the most economical balance of present conditions, but the possibilities of the next 15 or 20 years must be considered as well in order to provide the best economy over the life of the equipment. Such consideration requires patient study and time, factors that are not always appealing to the management.

The large plants could well afford to spend a period of time, up to two years, purchasing their excess power and correcting methods and equipment employed in the utilization of heat and power in order to reduce their demands to an economical minimum before determining the system of generation. Recognition must be given to the fact that heat and power are service commodities, and therefore the primary consideration is service. Too zealous an effort to reduce the amount of installed horsepower unit of manufactured product or the cost per installed horsepower may interfere with profit-making production. The important factor is the effect of the cost of heat and power upon production rather than the basic cost of heat and power.

Furthermore, the time is rapidly approaching when the major consideration will be the balancing of heat and power in a group of plants rather than that of the isolated plant. In a few cases some firms are seeking locations for small plants near larger industrial plants for the purpose of purchasing heat. Large industrial firms and central-station companies have combined in joint ownership of a power station. It is probable that the next major development in industrial heat and power will be in the direction of central systems and exchange of heat and power in groups of plants.

In contemplating the use of high-pressure steam, sufficient consideration is not given to the advantages to be secured in its utilization in process equipment. Particularly is this true in those industries using a moderate steam pressure (60 to 80 lb.) for process work and a low-pressure exhaust system (5 to 10 lb.). In plants of this class at least 30 per cent of the steam generated passes through the prime movers into the exhaust-steam system, and when the high-temperature head on the heating surfaces in the boilers is compared with the low-temperature head on the surfaces in the process equipment, it is evident that the total heating-surface area in the process equipment is several times the area of the heating surface in the boilers, an item large enough to be given major consideration. With material processed at 160 deg. fahr., an exhaust system of 10 lb., and a process system of 75 lb., the temperature head on the heating surface is doubled if process steam is used instead of exhaust steam. This means much greater heat transfer per square foot of heating surface, or much less area to accomplish the same amount of heating.

The existence of two steam systems throughout the manufacturing portion of a plant involves a large amount of piping, many valves, regulation, interconnections, and duplicate condensate system. If all heating could be done with steam from the process system, the economy and simplicity of a single system could be obtained. This can be obtained by high steam-pressure and high back-pressure turbines, and in some plants at least the advantages in the utilization of the heat would more than offset the additional cost of generation.

A few months ago Messrs. Jacobi and Shoudy demonstrated the advantage of a term expressing 1000 B.t.u. Those who calculate industrial heat balances find the need of a term that will express 1,000,000 B.t.u., or approximately the heat content of 1000 lb. of steam. In determining total heat demands in process work the B.t.u. must be used, and it involves unwieldy figures. In calculating the supply of heat it is more convenient to use the unit of 1000 lb. of steam to express heat content. This is a generality, and when it is necessary to be accurate, long expressions must be used in explaining the pressure, superheat, or moisture of the particular pound of steam under discussion. In these calculations there must be considered steam generated at high pressure with superheat, process steam at a moderate pressure, and exhaust steam at low pressure with a percentage of moisture, a different heat content for each condition.

The heat unit is the basic factor and would be used to a greater extent in discussions if there were short terms to express large quantities of heat units. The item itself is not of great importance. Fate might just as well have made the Watt a Smith or a Jones, or have made it a unit of heat instead of power. It is not reasonable that the advantage of such terms should be lost because of the difficulty in supplying a term. If it is necessary that the construction of the terms should indicate the definition, I would suggest the words "betu" to express the B.t.u. "thoutu" to express 1000 heat units, and "milltu" to express 1,000,000 heat units. These terms are short, easily pronounced, indicate their

[2] Plant Engineer, American Sugar Refining Co., Brooklyn, N. Y. Mem. A.S.M.E.

meaning, and are at least consistent in that they do not apply to the English system of measurement quantitative prefixes of the metric system. This is not an effort to coin trick words, but an endeavor to hasten the consummation of the thought of Messrs. Jacobi and Shoudy and to carry it one step farther.

THOMAS J. SULLIVAN.[3] Dr. Kleinschmidt's interesting paper lays down a general strategy for the intelligent guidance of industrial power engineers in their study of heat-power balance. Fitting the general principles to specific cases is "a job for the cobbler," who in this case is the power engineer.

The point of departure or datum in the study of economic heat-power balance in any industrial plant is the process itself. What is the heat requirement of the process, in what form is it released, and what is now done with the excess either before or after the process? With these basic data, carefully assembled, we are ready for the next step. What are the power requirements, is this power purchased or generated in the plant, and what does it cost per unit? If the power is purchased, its cost is quickly and accurately found; if generated in the plant, the engineer must not permit either accounting or engineering myths to deter him from nailing it down. With these tools supplemented by his own engineering skill, the power engineer is ready to sharpen his pencils and prove his case. Let the pencil labeled "economics" be the sharpest in his kit, for—assuming the soundness of his plan—the convincing presentation of the dollars-and-cents angle determines his success or failure.

In a copper-refinery heat balance it was found that 14 per cent of the total heat input, including all combustibles, fuel oil, poles, brands, and coke, went to the copper and 42 per cent to the generation of steam in waste-heat boilers. In another department where the practice persists of installing waste-heat boilers directly over reverberatory furnaces, the boiler-heating surface seeing the furnace hearth, the heat to steam was 54 per cent. The installation of well-designed recuperators rather than waste-heat boilers with waste gases around 2000 deg. fahr. would return to the furnace 40 to 45 per cent of the total heat generated, speed up the refining operation, and reduce furnace outage for refractory repairs. Let me interpose here that the practice of installing waste-heat boilers to "see" the furnace results in the theft of radiant heat from its primary purpose, the melting of the product, and diverts it to the secondary operation of steam production. This practice is uneconomical and unwise, since a leak in the boiler shuts down the furnace, sometimes disastrously.

The elimination of the waste-heat boiler unties the hands of the power engineer, who may now go unfettered to the higher pressures and superheats indicated by current practice. Steam required for process heating and drying may then be bled from two or even three stages of the prime mover.

The reduction of back pressure on prime movers is another important method of improving the heat-power balance. Where large quantities of solution are to be heated, stage bleeding may be employed very economically with heaters of the jet or surface condenser type. The corrosiveness of the solution need not stop the power engineer if resistant metals are available. In some cases the use of a small prime mover exhausting directly into such a heater, the solution to be heated used as the cooling medium, may be indicated.

The function of the power engineer in relation to process changes to improve heat-power balance or general plant operation may be illustrated by two concrete examples:

1 In a copper refinery where the power load-factor was nearly 100 per cent, study of the waste heat and direct-fired boiler steam-production curves superposed showed spots on the same time ordinate, 35,000 lb. apart. The condition was corrected by la-

[3] Consulting Engineer, New York, N. Y. Mem. A.S.M.E.

dling two of the day furnaces at night. This was done without increase in personnel, the regular night shift having to extend itself, however. A beneficial effect also was the reduced congestion on the production floor during the day.

2 In a heavy chemical plant where the product involved a reaction in a closed vessel at high pressure and temperature, the non-interference of the power engineer brought an unhappy result. The process, imported from abroad during the war, had proved economical when the reaction occurred under a pressure of 20 atmospheres. The steam pressures available to start the reaction in our plants were such that the operation could not be carried above 15 atmospheres. This greatly increased the time increment. The existing vessels, in which the reaction occurs, are rated at a working pressure of 24 atmospheres. The new high-pressure boiler plant is designed for 250-lb. operation. Obviously the reaction vessels cannot be worked up to their designed rating, and very little if any speeding up of the process may be expected.

The aggressive engineer in an industrial plant who dares to peer into the process in an endeavor to improve the power and heat balance is frequently disliked but never despised. To discharge his function fully the power engineer must literally chase every B.t.u. released and every watt produced to its point of useful employment.

ANDREW A. BATO.[4] The question of balancing power and heat in industrial plants with the aim of utilizing all the forms of off-fall heat is neglected to such an extent in small, medium, and even large industrial plants that every endeavor to promote and to spread the solution of this question is very welcome to all those interested. As the author begins with the statement that it is desirable to generate as much power as possible in connection with the various forms of heat figuring in industrial processes, first of all another method of utilizing waste heat should be mentioned to complete the list of the possibilities in this field.

One of the greatest industries in Germany is the manufacturing of briquets out of lignitic coal. This coal contains about 45 per cent of moisture and is dried before further processing. The water vapor driven out from the coal is absorbed with the aid of water screens, and warm water at a temperature of 160 deg. fahr. is thus obtained. This water is brought to boiling under vacuum, and the steam formed is used for the driving of turbines.

An example of utilizing heat contained in hot waste liquids is that of heat exchangers recently installed in power laundries, distilleries, etc. These exchangers, in the case of laundries, transmit part of the heat contained in the soapy, hot discharge of the washing machines to the cold, clean water that is to be heated for use in the washing. On one hand, coal savings up to 30 per cent have been thus obtained; on the other hand, the heat balances of such plants were revolutionized. Before the installation of the heat exchangers there was a great demand for exhaust steam to heat the water; consequently, engines with a low economy were used. The preheating of the washing water with the aid of these heat exchangers, however, cut down the demand for exhaust steam to such an extent that now in most cases even the most economical engines can hardly fit into the heat and power balance of the plant.

Another example demonstrating the value of medium-pressure steam as compared with lower is the method of compressing low-pressure steam to higher pressures with the aid of a compressor called sometimes a "heat-pump." This is being done, for example, in sugar plants and refineries, where there is a great amount of steam available at pressures too low for the process. If the initial pressure of the steam used in the engines or turbines is increased, additional power can be obtained to compress this

[4] Consulting Engineer, East Orange, N. J. Mem. A.S.M.E.

low-pressure steam to such pressures as are sufficiently high for the process in question. A modified application of this method is represented in the case of a steam laundry in Detroit installed in a building in which only low-pressure steam at 15 lb. was available. For use in steam mangles this steam is now compressed to 75 lb. by using power from the public-utility station.

The utilization of exhaust steam for heating is neglected in many cases on account of the time factor; that is, because the exhaust steam cannot be utilized at the time it is generated. In cases in which the waste steam should be used to heat water in tubs, vats, etc., it is not done, because the men in charge do not realize that the heat could be stored in a central hot-water-supply tank even better and cheaper than in a steam accumulator.

The way the author presents his material is in itself a proof of the fact that the personnel of plants he had to deal with, while highly specialized in the manufacturing of their respective products, are very ignorant in the matters of efficient and economical steam engineering. For such people the effect of superheat might be, as he called it, a "mystery;" but it is hardly a mystery to those reading or listening to the presentation of his paper, making the explanations referring to it superfluous. On the other hand, some other points might need more elucidation. Thus, in connection with the chart shown in Fig. 4, the statement that, when using a vacuum of more than 8 in. of mercury, 1000 lb. of exhaust steam will correspond to more power in the case of saturated steam than in the case of superheated steam is quite surprising. The explanation may be that the chart refers to one turbine designed for conditions which are very different from the superheat used and the 8-in. vacuum, and does not refer to turbines especially designed for every case shown on the chart. Explanations in connection with this point will possibly be of a far greater interest.

While the paper as a whole renders a very good service to the cause of heat and power economy by giving a general view of the question in the text and especially in the charts, the most valuable part of it is, it is believed, the chapter on single-stage and two-stage water heating, in the charts of Figs. 6 and 7, through their clear presentation as well as their originality.

The National Laundry Owners Association estimates the cost of power generated in a way so that the exhaust steam is utilized at from $^1/_2$ cent up to 1 cent per kilowatt hour, including all costs, such as depreciation.

H. D. Fisher.[5] The writer agrees with Dr. Kleinschmidt's engineering data, but is inclined to view the problem from the business angle; that is, what return will the expenditure on higher pressure boilers, etc., produce? It is only recently that power equipment has been commercially available for 400 or 600 lb. pressure with 150 to 200 deg. fahr. superheat, and already a number of plants are under construction and some of them in operation. In most locations they show very satisfactory returns, but one instance that the writer saw investigated by a well-known firm of engineers, where coal and hydroelectric power were available at low prices, showed too low a return to warrant tying up capital in the venture.

One feature of the greatest importance in the economical operation of installations of this sort has been rather slighted in the paper. Process heat and power requirements can be approximately balanced in the design, but no process is self-balancing in this respect, and some means of supplying the deficiency or taking care of an excess of steam economically must be provided. A process in which the steam required would just generate the necessary power at full capacity would be badly out of balance at partial capacity, since every plant has certain auxiliaries whose steam or power requirements will be approximately constant regardless of mill output, and also the steam consumption of prime movers is higher per unit of power at the lower loads; so, as in most processes the heat required per unit of output varies to a much less extent with varying output, there would be a considerable surplus of exhaust steam to be wasted. Where process steam and power requirements are reasonably uniform and the cycle of change is long—for example, as between operating one of two or more paper machines or the whole mill or as between summer and winter load conditions—the most desirable solution is to design for a balance with the maximum power development per unit of output occurring in any regular operating period and supply the excess process steam required at other times through reducing valves. Where the variations are extreme and their period comparatively short, thermal storage of one sort or another may be used, steam accumulators being available for higher temperatures and comparatively short cycles, and water heating providing an excellent means of balancing in processes requiring considerable quantities of hot water. One woolen mill with which the writer is acquainted heated the morning dyehouse supply with the exhaust of the turbine on the night load, which was chiefly lighting and power-plant auxiliaries.

Dr. Kleinschmidt's remarks on possibilities of savings by changes in process requirements deserve the careful attention of factory superintendents and managers, as in many cases back pressures are increased much beyond the point where proportional increases in output are secured. Temperatures, with which process work is chiefly concerned, increase much more slowly with increase of pressure as the higher pressures are reached, thus:

Gage pressure, above 14.7 lb. abs.	Increase in temperature per pound increase in pressure, deg. fahr.
0	3.3
10	2.1
20	1.7
30	1.3
40	1.1
50	1.0

But apparently the increase in steam consumption of prime movers is more nearly proportional to the increase in pressure. This factor of diminishing returns with increase of back pressure is shown in drying paper, as beyond 20 lb. gage the increase in drying is very slow and beyond 30 lb. negligible.

The company with which the writer is connected manufactures paper-box board and is at present installing a boiler and turbine for 400 lb. gage with 160 deg. fahr. superheat. From available information, companies making the higher grades of box board but not operating converting plants can secure a satisfactory balance at this pressure, but those which must generate additional power for box shops, etc., must operate at about 600 lb. gage.

TABLE 3 OUTPUT 110 TONS PER 24 HOURS; AVERAGE TURBINE LOAD, 1360 KW.

	Summer	Winter
High-Pressure Steam		
Steam to turbine throttle, lb. per hr.	35,500	35,500
Steam to boiler-feed pump, lb. per hr.	3,000	3,600
Supplied through reducing valve, lb. per hr.	4,500	19,100
Condensation, etc., lb. per hr.	3,000	3,500
Totals, lb. per hr.	46,000	61,700
Exhaust Steam		
Paper-machine dryers, lb. per hr.	30,000	35,000
Heating beater water, lb. per hr.	3,000	6,500
Fan coils, lb. per hr.	4,000	10,000
Feedwater heater, lb. per hr.	1,000	1,200
Condensation, etc., lb. per hr.	2,000	2,500
Totals, lb. per hr.	40,000	55,200

[5] Plant Engineer, New Haven Pulp and Board Company, New Haven, Conn. Mem. A.S.M.E.

These require, however, very economical prime movers and, it will usually be found, complete electrification so that all the power may be generated at the very low water rate of the main turbine.

A comparison of our design figures with those for the book-paper mill cited by Dr. Kleinschmidt may be of interest (see Table 3).

In order to provide a safe margin for operating one paper machine, a 2000-kw. 37-stage turbine is being installed with steam at the throttle 370 lb. gage, 160 deg. fahr. superheat, to exhaust against 20 lb. gage back pressure, the water rates being:

Kilowatt load	Power factor	Pounds of steam per kw-hr.
1000	0.80	28.75
1500	0.80	25.65
2000	0.80	24.20

The twin-slide-valve variable-speed engines are being replaced by single-motor variable-speed drives. One boiler to carry the full load is being installed, having integral economizer section, partially water-cooled furnace, underfeed stoker with clinker grinders and air preheater; stoker, forced- and induced-draft fans, and one boiler-feed pump being motor driven, the other boiler-feed pump being turbine driven. We expect to get this plant into operation early in the coming year and to be able to report operating data later.

R. J. Gordon.[6] Dr. Kleinschmidt has made a substantial contribution to a question that is becoming more important every year in the industrial field. Necessarily he confined his discussion to the bare essentials, but this subject is of such importance that it would be of much value if at least two additional papers could be submitted—one of which would treat the application of the high-pressure steam engine and the other of the application of the steam turbine in the industrial field. If similar conditions could be assumed in considering the advantages and disadvantages of each type of prime mover, the papers would be of greater value.

It was noted that the use of load curves was not included in the paper. Load curves plotted over the same intervals for both steam and power will readily indicate the limitations of the use of back-pressure-type steam-generating equipment. These load curves used in conjunction with the charts shown by Dr. Kleinschmidt would make it possible to determine quickly in a general way the applicability of back-pressure-type steam-generating equipment.

Dr. Kleinschmidt mentions that the efficiency of reciprocating engines varies much more than does that of turbines. Is it to be understood that this variation extends to the high-pressure type of engines now being built abroad? Recent developments in Europe indicate that high-pressure reciprocating engines are being received with considerable favor, and it would be interesting to have Dr. Kleinschmidt furnish any information that he has regarding the performances of the latest type of high-pressure engines.

I. E. Moultrop.[7] There have been so many papers of late years on the subject of the heat balance of steam-electric generating stations that it is indeed refreshing to remember that the same subject is also receiving the attention of mechanical engineers who have to deal with the operation and construction of industrial plants. There is the opportunity for as great an improvement in the heat balance of industrial plants as there was in the central stations of ten years ago.

The most important facts brought out in Dr. Kleinschmidt's paper are that to obtain an economical heat balance in an industrial plant requires a careful engineering study and that quite often the best solution of such problems lies in the utilization of existing boiler plants rather than in the construction of new high-pressure boiler plants.

Supplementing the comments of the author, I would like to point out that surprisingly often an engineering study of the processes of an industrial plant for the purpose of heat conservation will produce much greater savings at less capital cost than an engineering study to determine the maximum amount of by-product electrical energy that can be generated from the steam being used.

In the majority of cases, industrial plants are started on a small scale with limited financial backing that precludes the installation of the most efficient apparatus for the processes of the industry, let alone the most efficient apparatus for the production of the by-product, electrical energy. As the industry grows, plant extensions are made with low first cost as the primary requisite, with the result that often a large property is brought into being which is a great waster of heat both in the processes of the industry and in the factory-building heating. A careful study followed by a moderate expenditure of new capital for rearrangement of the equipment which utilizes heat will quite often, in fact almost always, pay a handsome dividend on the money and effort expended. The most common excuse for deferring such changes is the lack of money. Many factory managers will admit that such changes are worth while, but also will declare that the same money invested in new manufacturing equipment will pay a greater dividend.

If the steam requirements of the industry are such as to warrant the installation and operation of power-generating equipment, it is very seldom economical to install equipment for utilizing the space-heating steam for power generation. A well-designed space-heating system very seldom has an annual load factor in excess of 25 per cent. With such a handicap to start with, the possibilities of showing a profit with equipment for utilizing such steam for power generation are remote.

Even large steam-distribution companies, serving large central-heating systems, find it difficult to justify such equipment.

The statement in the paper that most industrial plants can find a use for an almost unlimited amount of power, if it can be obtained cheaply enough, is dangerous. Many are of the opinion that the only expense chargeable to power generated by pressure-reducing turbines is about one-half pound of coal per kilowatt-hour. To those persons it should be pointed out that the fuel costs are only a part of their power costs. Fixed charges are the big item; and operating labor, maintenance, supplies, and supervision are probably greater than the coal charges. If the total costs of generation are compared with the costs of purchased power, it will be found that purchased power is generally the cheaper. This is logically true, for the electric utilities specialize in the generation and distribution of electrical energy, and it would be expected that they can do this at a lower price than someone else who is specializing in some other business. The utilization of purchased power also releases capital for use elsewhere in the industrial plant.

When an industrial plant is faced with the necessity of increasing its steam supply it will, in the large majority of cases, find it more economical to revamp its existing boiler plants for operation at higher ratings rather than to rush into the construction of high-pressure boiler plants with the idea that the plant will be able to make a handsome profit by the generation of its own power requirements.

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[6] Power Engineer, Procter & Gamble Co., Cincinnati, Ohio. Mem. A.S.M.E.

[7] Chief Engineer, Edison Electric Illuminating Company of Boston, Boston, Mass. Mem. A.S.M.E.

The writer cannot agree with the author in his thought that industrials should consider boiler pressures as high as are used in the best condensing central-station plants. Very few industrials can show a profit by the installation of such equipment, and even where they can break even under normal conditions a business depression will very quickly turn their expected profit into a loss.

This condition is inherent in the economics of the situation. The financial structure of most successful industrials is based on a large gross income per dollar invested in plant, whereas the very nature of the electrical generation and distribution business requires such heavy investments in plant that the gross income per dollar invested is relatively small. The industrial must keep its fixed charges low to prevent a serious loss at times of curtailed output due to business depression or to loss of business to competitors. The electric utilities very seldom show a reduction in output from year to year and so can afford to operate with a financial structure wherein the fixed charges on the money invested in plant are a large part of their total annual expenses. These differences are inherent in the economics of the public-utility business as compared with industrial business.

High-pressure equipment has proved economical and profitable in steam-electric generating stations where the expense of high-grade operating labor and supervision is distributed over a large installation. Very few industrials, however, can afford to operate and maintain a high-pressure boiler plant unless the high-pressure steam is essential in their processes.

Warren B. Lewis.[8] Dr. Kleinschmidt's paper is a scholarly presentation of a fundamental principle which unfortunately is not understood by mill executives who make final decisions. It would be well if such a paper with full discussion could be presented to a group of such executives and repeated again and again.

Certain paper mills and many bleacheries and dyehouses present opportunities for an almost complete balance between heat and power, but there are many mills all over the country where the power requirements are very much in excess of the heat requirements, and in many of these cases water is not available for condensing purposes. The writer presents a solution of such cases which has proved satisfactory from every angle, and it has been made possible through the altered attitude of the public utilities.

It is safe to say that with few exceptions mechanical power cannot be produced with a non-condensing prime mover exhausting to the atmosphere in competition with purchased power and that exhaust steam flowing to the atmosphere is an indication of economic waste. Where a limited amount of process heat is required, the solution is the combination of a non-condensing unit exhausting to a process line and synchronizing with public-utility power, with automatic control of the prime mover to maintain a uniform back pressure in the process line.

As an illustration the writer refers to a paper mill where the demand for low-pressure steam is 15,000 lb. per hour in the coldest weather and 5000 lb. per hour in warm weather, with daily fluctuations of considerable extent at any season. In this mill a low-speed Corliss engine drives a generator, the engine taking steam at 125 lb. pressure and 100 deg. of superheat and exhausting at 7 lb. gage. The generator is synchronized with the incoming power from public utilities, and the total mechanical load is 800 kw. The load on the engine is governed entirely by the amount of exhaust steam taken from the process line. The load on the engine varies from 250 kw. as a maximum, which is a limit of safe load, to as low as 75 kw., under certain conditions. With this arrangement full advantage is taken of the principle laid down in Dr. Kleinschmidt's paper, and in this particular case the over-all cost is a minimum.

[8] Consulting Engineer, Providence, R. I. Mem. A.S.M.E.

In a new installation it is probable that high steam pressures and superheats with a different type of prime mover would develop a larger amount of mechanical power with the same amount of steam passing through. Many plants have equipment which with slight changes could be adapted at small cost to operate as outlined. This same plan has been carried out in a chemical manufacturing plant with equally satisfactory results.

A few years ago public utilities did not favor this method of operation, but at present there seems to be a recognition of advantages to both parties, and whatever objections have existed heretofore have been removed through the installation of protective devices such as reverse power relays and current limit relays.

Great emphasis should be placed on the use of process steam at the lowest possible pressure and with the least waste, as in too many instances the mere fact that exhaust steam is available has brought about an uneconomical use of it and the real principle of economy has been more or less violated.

R. A. Langworthy.[9] Dr. Kleinschmidt has made a timely contribution to the literature of heat and power in industrial plants. This subject is one that has been discussed for many years, but definite action has been lacking. Far too many plants having heavy demands for process steam are generating this steam at boiler pressure and taking it through reducing valves to the lower process pressure or pressures required. This in most cases is an economical waste and should be eliminated from any modern manufacturing establishment.

In most plants having any appreciable demand for process steam, the entire electric-power requirement can be generated from this steam before it is used in the process work and at a very low cost. This not only reduces the operating expense of the plant, but is a conservation of our natural fuel resources.

It has been the experience of the writer that in many plants the changes in process pressures can be made without having any serious effect on the quality or quantity of the output. It is often difficult to bring this home to the men in charge of production if they have become accustomed to certain process pressures and have formed the opinion that any lower pressure will not properly accomplish the purpose.

Manifestly, the lower the pressure on the process the greater the amount of power which can be taken out of the steam before it is so used. It is therefore necessary to make a thorough study of the process work and arrange to carry it on with the lowest permissible pressure. This requires the cooperation not only of the power engineer but of the factory executives and the men in responsible charge of the operations.

In studies of this kind it should be borne in mind that the steam accumulator has a very good application in many plants. The electrical-load peaks and the steam-load process peaks seldom coincide, with the result that the amount of power that can be obtained from the process steam is a compromise between these two conditions. If a steam accumulator is placed in the system, it will balance the two loads and allow the generation of much more power from the process steam than would be the case were the accumulator omitted.

The tendency for the last few years has been toward large networks of the electric power lines covering various parts of the country. It is to be hoped that some day when a full realization of the economies of the situation is attained many industrial plants located along these lines will not only generate the power required for their own use but will feed their excess of very cheap power into the system.

[9] United Engineers and Constructors, New York, N. Y. Mem. A.S.M.E.

CROSBY FIELD.[10] One fact has become impressed upon my mind in listening to the very able discussions of this most excellent paper. That fact is that the engineers studying this problem of the utilization of steam are most woefully limited by the fact that they have become self-conscious. They remind themselves constantly that they are power engineers and that there are many chemical engineering features involved. Some day when we realize that engineering is fundamentally engineering and most of these classifications of power engineer and chemical engineer are passing fads, then we will only have the natural limits of the problem to confront us and not the additional handicaps we self-impose by these arbitrary distinctions.

Because our engineers studying this problem are power engineers and not chemical engineers, they go right up to and then stop before one of the most important points of the whole proposition, the time factor. We then apologize for not seeing it through to its ultimate, laying stress on the fact that it has its basis in chemical or other engineering problems, and therefore we are not competent in ourselves to study and must therefore have cooperation with other branches which cooperation is mainly important because of its lack. Since 1914 the writer has been confronted with the problems of high back-pressure steam and knows of several cases where the taking away of the title "power engineer" and renaming the individual "engineer" and letting him loose in a plant has resulted in a large number of economies, not only in the power situation, but also in the chemical process itself. For example, in one plant where over 4,000 boiler hp. was recommended by the power engineer, because of the constantly increasing demand of steam of the units of a large chemical plant, this engineer, by a simple management study including the ordinary handling of materials, use of power, and scheduling of operations, found that by a simple rescheduling of several steps of the process he was able to eliminate all thought of the 4000 boiler hp. additional. It was simply a matter of not letting everybody use steam at the same time, but working out the several steps so that they could use it intermittently and in a dove-tailing fashion without in any way deleteriously affecting production.

The author's example, of course, was taken to illustrate his various recommended steps, but the average reader may fail to realize the very complicated condition in the average plant. In this example the process shown is fairly well balanced and the machines more or less run continuously, so that the character and sudden changes with a wide diversity of connected steam loads ordinarily found in such a survey are missing. For an example, one is summarized herewith, in which there was finally worked out one line-steam pressure and two back-pressure systems and to provide for one duplication of back pressure in any one locality. In order to obtain this there had to be several novel installations, such as the use of steam compressors in certain isolated spots.

In this survey, 379 vats consuming steam in open boiling widely varied their requirements, from 110 lb. in 5 minutes for one to 33,500 lb. in 18 hours for another. The pressure also varied from 5 lb. per sq. in. to 100; with very slight changes to some they were all made to operate satisfactorily on the final back pressure selected. The total steam consumed in this way each day amounted to 634,000 lb. There were 39,000 sq. ft. of room heaters (radiators), requiring 13,000 lb. of steam per hour; 42 pieces of chemical equipment, using steam at 5 lb. per sq. in. pressure or less, requiring maximum of 10,000 lb. per hour; 80 pieces using from 5 to 10 lb. per sq. in., requiring maximum of 33,000 lb. per hour; 120 pieces using from 10 to 20 lb. per sq. in., requiring maximum of 33,000 lb. per hour; 170 pieces using from 20 to 25 lb. per sq. in., requiring maximum of 32,000 lb. per hour; 537 pieces using from 50 to 100 lb. per sq. in., requiring maximum of 13,000 lb. per hour, of which 239 could be changed to 25 lb. or lower, with a maximum of 90,000 lb.; 80 pieces using from 100 to 150 lb. per sq. in., requiring maximum of 15,000 lb. per hour; 12 pieces using from 150 to 250 lb. per sq. in., requiring maximum of 7000 lb. per hour; 5 pieces using from 250 to 300 lb. per sq. in., requiring maximum of 2000 lb. per hour.

There has been quite some discussion regarding the 75 per cent steam load factor on a 24-hour basis, 360 days in the year. The writer has seen this duplicated in seven plants under one management having a wide diversity of chemical products, so it is by no means out of the question to obtain.

J. D. ROBERTSON.[11] In considering the specific application of the principles of the paper to a particular case, the cost of the by-product power and the investment required to make it available are the governing factors. The writer wonders if the author had average figures as to the cost of by-product power per kilowatt hour, also has he formulated procedure for predicting the cost of by-product power if its consumption were increased in any given plant. Such a standardized procedure would be of value when considering the advisability of changing processes so as to use more current.

In the case of a plant whose demand for process steam at exhaust pressure is in excess of the exhaust from its power units, the question of changing processes so as to use more electricity depends on an accurate comparison of the present cost of the process using exhaust-pressure steam from a reducing valve with the cost of the process using electricity.

A carefully worked out procedure for finding the cost of by-product power including all items of direct cost and overhead would give added standing in the eyes of some treasurers to an engineer's comparison between the cost of a process using such by-product power and an existing process using purchased central-station power. Possibly at some future meeting it may be considered worth while to present such a paper.

WARREN VIESSMAN.[12] This paper has opened a field of discussion which has been gratifying because the field of industrial power work has been considerably neglected and now is quite ready for extensive development. The writer has in mind two large meat-packing plants on the opposite sides of the same street. One concern continually made every effort to improve its power plant. The other one neglected it. So far as one could see, the two establishments were otherwise about the same. Due to the recent depression in that industry one plant has gone into the hands of receivers, but the other one is still in existence and now doing an extensive business. The concern with the improved power facilities has won out, and it is entitled to the profits that will come to it principally because of its reliable and economical power plant.

Engineers in the past have been somewhat inclined to decide that a certain type of power installation should go into a particular plant because some other plant has one of that type. But with the wide field in the selection of bleeder turbines, low-pressure turbines, accumulators, and the use of purchased power detailed studies must be made of every heat requirement and every power requirement.

The author cited the case of a paper mill where large economies were effected by the rearrangement of a steam plant. In a paper mill which was operating its own electric power and steam

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heating plant five years ago, the electric requirements were about 2000 kw. It operated 24 hours a day at practically full load. This plant was about to increase its size to twice the capacity, and it made a study of how to improve its power plant, which had been very much neglected in regard to heat balance. A decision was made to purchase power, but still generate its own heat, and immediately after doubling its plant, business fell off to such an extent that it was practically at a standstill. If it had doubled its steam installation, the concern would have been out of existence now. But as it was it was able to get through with a low plant investment and it is now doing very well.

The fluctuations in the industrial load certainly are an important item in determining the advisability of a heavy private plant investment, and purchased power should always be taken into consideration as well as any other sources of supply for heat and power.

This one item of investment is very much neglected in the comparison of various plant operations. In making studies of certain types of plants, pumping stations in particular, the writer recently went to one of the largest cities in this country and asked for their costs on various stations; they had eight or ten steam and electric plants. They produced a nicely tabulated blueprint made out month by month showing what was supposed to be the total cost of pumping water at these plants. These figures were used as a criterion on which to build new plants. In every one of those tabulations, when analyzed, the cost of construction was entirely eliminated, and that is a very big factor.

In a water-pumping station, with an output at present of about 30,000,000 gal. per day, the new station being built will require new units of about 4500 hp. The present station is steam. It was decided to build a new station, an electric station, and abandon the old steam one because of economies. This would not always apply and could not be done in every city. In this particular case the cost of coal was $5 a long ton. The cost of electricity was 0.62 of a cent per kilowatt hour. But when operation costs were considered, labor, maintenance, and investment, in this particular case no heat being required, an electrical installation was much to be preferred. Every case must be decided on its own merits and full consideration given to every heat and power requirement.

Author's Closure

The author heartily agrees with Mr. Moultrop's remarks on the amount of room there is for improvement in the average industrial power plant; also, that the place to make your original saving is in cutting down the use of steam to the minimum at the very start. Money is saved only by not buying coal. After you have bought as little coal as possible, then get as much power from it as possible. We usually estimate 4 lb. of steam lost per kilowatt-hour of by-product power generated.

One feature that Mr. Moultrop did not mention which should perhaps be taken into account is the interruption of service on the central station company's lines. In the Middle West this has been a deciding factor in some cases, where an interruption of service of even as much as 2 or 3 hours would so seriously disrupt a necessarily continuous process as to mean the loss of many thousands of dollars.

Mr. Gibson brought out an exceptionally good point in regard to looking into the future of the plant. That is the most difficult of all the problems of the engineer who is attempting to make a power survey. In the case of a paper mill which we surveyed about three years ago we were informed shortly after submitting our report that they expected within two years to entirely change their production schedule so that it would be necessary for us to make another complete survey when that was done. Some people do not realize that a complete change in the production schedule necessitates a new survey, and they attempt to fit a recommendation to a very much altered situation.

Regarding the relative heating surface in boilers and in steam-using equipment, the discussion failed to take account of the fact that heat transfer in a boiler is from gases to boiling liquid with a relatively low heat transfer coefficient, which is further cut down by scale and soot. Therefore the boiler requires a relatively large surface, whereas equipment using steam very often transfers the heat from condensing steam to a liquid, or to wet paper passing over the surface of a drier where the heat transfer coefficient is perhaps 100 times as great as it would be in the boiler. The boiler will ordinarily have surfaces comparable with if not greater than the steam-using equipment.

It is a point well taken that if we can raise the boiler pressure and then raise the pressure at which the steam is used we will increase the flexibility of the process very considerably.

The use of a heat pump is an interesting possibility in certain cases, particularly where there is an excess of power such as water power and where evaporation is one of the major problems of the plant.

High-pressure engines are not as yet in very extensive use in industrial plants. No doubt it is possible to produce a reciprocating engine which will have a high economy over almost any particular pressure range except the very low pressure ranges. But, in general, a single turbine will operate (with reheating) from even 1200 lb. down to any vacuum required, in one or several cylinders operating as a unit, and get high efficiency throughout the range, and I think no one would attempt to make a single reciprocating engine that would cover any such pressure range with high efficiency.

In connection with the use of water power let me mention the case of a mill having a high load factor—the condition under which by-product power will always be most interesting—which had recently completed a 1000-kw. hydroelectric plant. They were buying the water for this plant from a local water company, at a cost of 7 mills per kilowatt hour. By putting in high-pressure boilers in place of their old hand-fired, horizontal return-tubular boilers, the increased efficiency alone would pay for the new boilers, and charging the excess cost of high pressure over low pressure boilers to power, the actual cost of by-product power was between 3 and 4 mills. It would therefore pay to shut down the new hydroelectric plant, in so far as they could generate by-product power. The hydroelectric plant was a great comfort for standby, but not an economic proposition.

The question of hot water for heat storage is one that deserves a great deal of attention. Heating a large supply of water from off-peak steam to be used throughout the day should be of interest in a number of industries.

In connection with Mr. Sullivan's discussion, one must of course be sure that waste heat is really going to waste and that it cannot be otherwise utilized. The idea of exposing boiler surfaces to direct radiant heat from a furnace is not what could be called waste heat practice, although similar conditions are often encountered.

Whenever clouds of steam are rising into the atmosphere, it is pretty certain that a plant is wasting money. A small exhaust to atmosphere will waste more money than is necessary to pay the investment charges on a higher pressure boiler or other equipment to conserve that steam.

Cooperation with public utility plants should receive more attention. Rate schedules of public utility plants should be so arranged as to take over those portions of the industrial power load which cannot be generated economically by by-product power. It has in the past been largely the custom of public

utilities to make rates in such a way that the plant which has two or three hundred kilowatts of load that it cannot generate as by-product power is charged such a high rate, 2 or 3 cents, that it can afford to put in a condensing bleeder turbine rather than the cheaper back-pressure turbine. When the public utilities realize that they are losing a field in which they could be of real benefit to the community, perhaps they will arrange to take over some of this excess power load.

Mr. Field's example of a really complex survey strikes a sympathetic chord. The complexity of the apparent uses for steam startles one at first, but as a rule we find that steam at 10, 30, 40, 60, 80, 110, and 150 lb. is not necessary.

In one strawboard mill the superintendent solemnly informed me that it was impossible to cook straw at any pressure but 60 lb. Below that too much water got into the straw, and above it the straw was burned. With the approval of the plant manager, we arranged a connection which would turn the 10-lb. bleeder line from the turbine into the straw-cooking line, and turned it on secretly one Monday morning. The straw came out as good as ever, and they have continued it ever since. I do not know whether they ever broke the news to the man.

Meat packing plants are interesting because they have a large refrigeration load, as well as some heating and power load. Refrigeration load can usually be best treated as a power load, although occasionally the absorption system using low pressure steam is indicated.

As to the average cost figure of by-product power, the figure of 5 mills to 1 cent strikes me as being a little high for most conditions. In paper mills we can usually figure by-product power at around $3^1/_2$ to 4 mills, $^1/_2$ to 1 mill for the heat (that is, the excess coal consumption) and about 3 mills for equipment cost. Extra operating labor is in many instances negligible.

FSP-51-28

Coal Pulverizers

By W. J. A. LONDON,[1] WALDEN, N. Y.

Commercial coal pulverizers vary greatly in their performance. While there are two principles of attrition and impact on which pulverizing mills are based, there are such radical differences of design as to raise the question of correct fundamentals. Guarantees are given on performance when the engineering problems are difficult of determination with any degree of accuracy. Analysis of what actually takes place in a mill leads to a suggested design.

THE serial report on pulverized fuel issued by the National Electric Light Association, dated August, 1927, contains much information for all those interested in this subject. This publication, containing reports from the operating engineers of nearly all stations using powdered coal, is of value to all prospective users of this system and of still greater value to the designers of pulverizers. It gives data on actual performances. In Fig. 1 there are plotted all of the complete tests published in this report, with a few odd additions taken from other publications to date. It is generally accepted that the power consumption per ton bears a direct relationship to the fineness of the fuel, so that in this chart only those tests are plotted that give the fineness performances. Leaving out a few of the tests that are "off the map," it will be seen that approximate curves can be plotted showing that the power consumption at 60 per cent through 200 mesh varies from 13 kw. per ton to 22 kw. per ton. Taking the 13 as a basis, we get a difference of no less than 70 per cent between one figure and the other.

Looking at the subject from a strictly engineering standpoint, this cannot be said to be a very flattering commentary on the state of the art today. There are few examples in engineering practice where we see such a large discrepancy in the performance of commercial machines.

It can hardly be said that the pulverized-fuel problem is in a state of infancy, inasmuch as powdered coal has been used for 50 years in the cement industry, and there has been a development going on for many years in the application of this fuel in our largest and most up-to-date power plants.

It is generally accepted that there are only two actual principles employed today in the different mills on the market, attrition and impact. Attrition mills comprise those of the ball and roller types and the impact mills those of the hammer type. An analysis of the design of the different mills will show that while all attrition mills operate on substantially the same principle, there seems to be a difference of opinion among the designers of impact mills as to the proper or most efficient way of attacking the problem. A brief review will be given of some of the more generally used machines with the object of pointing out that there are sufficient radical differences in design to ask ourselves if all of them are correct, and if so, why the art of pulverizing coal should differ so greatly from the development of other engineering devices, where only one or two fundamental principles are finally accepted as the correct ones on which to base commercial design.

A considerable part of this paper is devoted to the subject of guarantees and their relation to subsequent performances in the field.

Many unknown factors enter into the problem confronting the manufacturer when requested by a purchaser to make specific guarantees. There is no question that the builder today will guarantee performances that he should not be expected to; the customer, on the other hand, is demanding performance guarantees that reputable manufacturers should refuse to give. There should be an appreciation on the part of the purchaser of the difficulties encountered in making guarantees, and these difficulties should be emphasized by the manufacturer. Guarantees have never been known to run a plant. A better basic understanding would lead to a more rational handling of the guarantee and contract situation.

Types of Impact Mills

In the following descriptions of various mills only those of the impact or hammer type are considered. While accepting as true that all mills pulverize coal by the combined actions of impact and attrition, the impact forces in a ball or roller mill must be relatively very small—so small in fact, that a mill of this type would operate just as satisfactorily if no impact forces were introduced at all, except perhaps for the breaking up of the larger lumps. That there is a difference between the breaking up of lumps of coal and pulverizing is obvious.

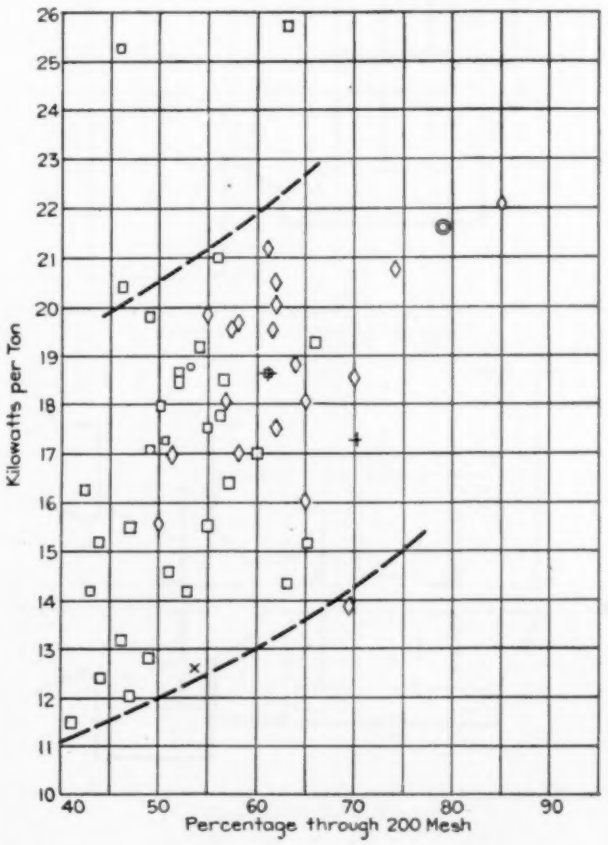


Fig. 1 Chart of Tests, Giving Fineness Performances
(Shape of symbols indicates types of mills.)

In support of this contention it is necessary only to state that designers of impact mills consider paddle or hammer velocities of 200 or 250 ft. per sec. necessary for efficient pulverization. If

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we consider the effect of impact forces varying as the square of the velocity, and then take a roller velocity of 20 ft. per sec. as compared with the 200 ft. mentioned, the deduction is obvious.

In any attempt, therefore, to analyze the internal performance of mills, we find the ball or roller mill conforming to the general principles of grinding, while mills of the hammer type contain one or two interesting problems as to how the work is actually performed in the mill. Designers attack the problem in so many different ways that it is quite interesting to compare these, and then follow this with an analysis as to what is taking place at any one time in the mill and see if we can satisfy ourselves as

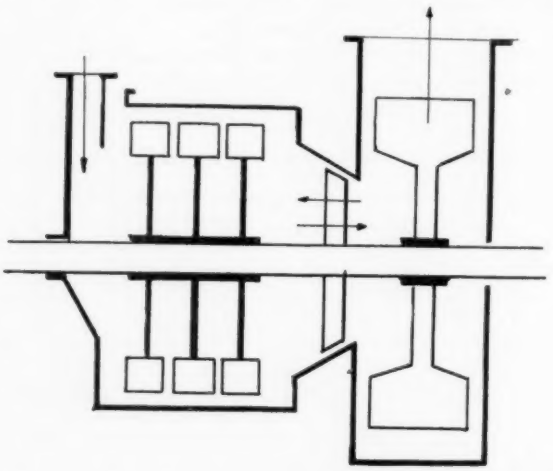


FIG. 2 MILL WITH COAL ADMITTED AT CENTER AND WHIRLED BEFORE CONTACT WITH HAMMERS

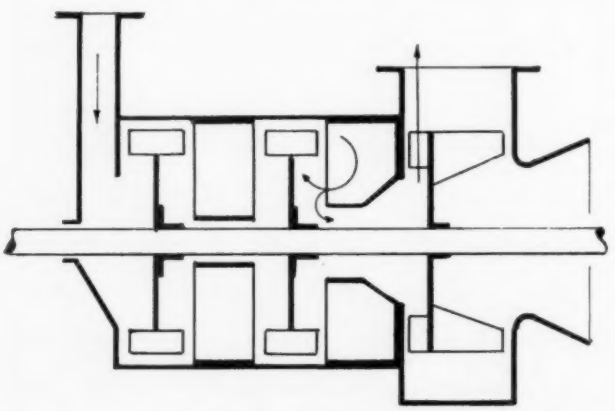


FIG. 3 MILL WITH COAL ADMITTED AT CENTER AND WITH TWO SETS OF HAMMERS

to how much is done by impact in the so-called impact mills, how much is really done by straight attrition, and how much by shear.

It has been asserted that the prevailing idea that all hammer mills operate on substantially the same general principles is erroneous, and this will be substantiated by Figs. 2 to 6, which show diagrammatically the general designs of some of the more important mills.

Fig. 2 represents a mill with coal admitted at the center and which receives a certain whirling motion before its contact with the hammers. The coal flows through the mill in a horizontal direction, and at the outlet it is met by a whizzer which picks up the unground particles and returns them to the mill for further treatment.

In the mill shown in Fig. 3 the coal is again admitted at the center. After passing through the first set of hammers it is brought substantially to rest by a series of stationary ribs, and then submitted to a second treatment by another series of hammers similar to those in the first stage. On leaving these hammers it is led to a large pocket fitted with radial ribs. The object of this pocket is to whirl the heavy particles back to the wheel for further grinding while allowing the fines to escape along the small opening around the shaft.

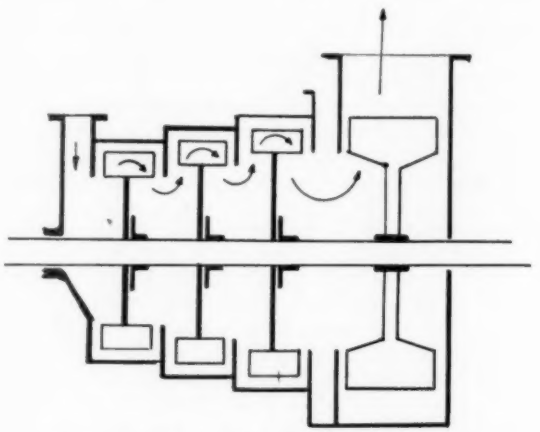


FIG. 4 MILL WITH SEVERAL SETS OF HAMMERS

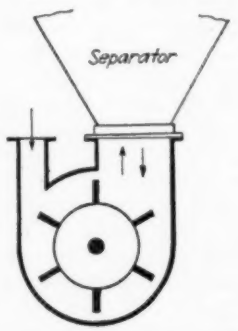


FIG. 5 MILL WITH COAL ADMITTED DIRECTLY IN PLANE OF HAMMERS

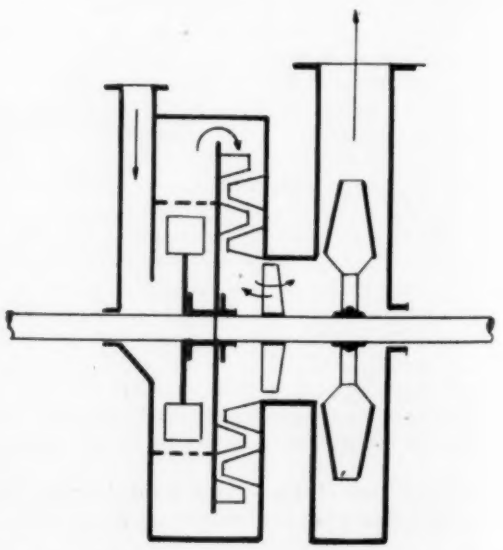


FIG. 6 MILL DESIGNED TO INCLUDE BOTH IMPACT AND ATTRITION

Mills of the type shown in Fig. 4 are built with a varying number of stages. Each stage has the hammers surrounded by a pocket in which it churns the coal to a certain fineness before it passes on to the next stage. In this mill the coal is again fed to the center of the mill.

Fig. 5 shows a mill employing an entirely different principle to those mentioned. Here the coal is admitted directly in the plane of the hammers, insuring maximum impact. The coal is discharged in an upward direction, as distinguished from the mills previously described, and enters a separator, which liberates the fines and returns the coarse particles to the mill.

Among the builders of this type of mill a great difference of opinion exists as to the most efficient speed that should be used. One maker at least uses a relatively high rotative speed in order to obtain the maximum amount of work from impact forces, whereas other makers use a relatively low speed, getting what impact they can, and relying on attrition for the balance of the work.

Of particular interest is the mill shown purely diagrammatically in Fig. 6. The designer of this mill took cognizance of the fact that there were two separate and distinct actions taking place in a so-called impact mill. He appreciated that impact could go only so far and that attrition must do the rest. This mill is divided into two separate stages. Coal is subjected to the regular hammer action in the first stage, and from there it is immediately discharged in a natural direction, outwardly radial. After passing over a partition it has to find its way in again toward the center against restrictions and eddying counter-currents. In other words, everything has been done to create attrition, either by friction against projecting pins or by collisions and rubbing of the particles against each other created by counter-currents of air in the attrition chamber.



Fig. 7 Diagram to Show Ratio of Circumferential Flow of Air to Horizontal Flow

How Fine Pulverization Is Produced in Impact Mills

Having described the general principle of operation of these various mills, let us try to analyze just what takes place inside of them during operation.

How much work is done by impact and how much by attrition? It can, the author thinks, be shown that no mill using impact forces alone can pulverize coal to a useable fineness. There are several ways in proving that this contention is correct. Let us first take the analogy of a ball suspended by a string, this ball in itself having a certain mass. We can take a bat and hit that ball with sufficient force to break it into small pieces. On the other hand, if we suspend a feather by a string and attempt to break this feather with a bat we will not be successful. The feather will begin to fly away from the bat before the bat hits it because of the cushion of air between the bat and the feather; in other words, the bat will just catch up with the feather, and when it does catch up with it, will not have sufficient relative velocity to do the feather any damage.

In a hammer mill having a tip velocity of 250 ft. per sec. the ratio of circumferential flow of air to horizontal flow is extremely high, as will be seen by the diagram in Fig. 7. This diagram is plotted in the following way: A mill of standard design has been used and the velocity of the mixture of air and coal has been computed in a horizontal direction, through the annulus A. This horizontal component will be seen to be almost negligible compared with the circumferential velocity created by the paddles. We therefore have air pockets between the paddles, with the air in these pockets traveling at substantially the same speed as the hammers.

Now if we could design a mill to give maximum impact, it would be in the form of that shown in Fig. 8, and the hammers would rotate in the direction indicated by the arrow.

Neglecting the slight downward velocity due to gravity and assuming a particle in the position shown, a certain upward

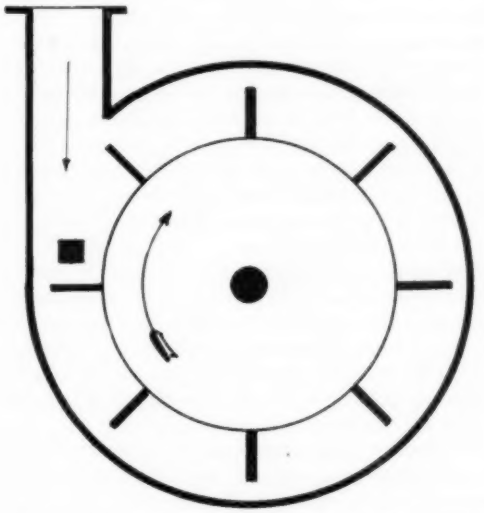


Fig. 8 Suggested Design of Mill to Give Maximum Impact

velocity of air will be sufficient to retain a particle of given weight in constant suspension.

Assuming the weight of coal to be 0.047 lb. per cu. in., then 1 cu. in. of coal would exert a downward pressure at rest of 0.047 lb. per sq. in.

The theoretical velocity of air to create this static pressure will be found from the approximate formula

$$V = 350\sqrt{p}$$

Where V = velocity in feet per second
p = pressure in pounds per square inch

Therefore assuming the particle in question in the diagram to be 1 cu. in., the upward velocity to keep this in suspension is only 75 ft. per sec. Any velocity less than this will permit the hammer to hit it. Any velocity in excess of this will drive it away. Now where do we stand with mean hammer velocities of 200 ft. per sec.? The size of coal in suspension would be very much larger.

Commercial mills, for obvious reasons, cannot operate in the direction of rotation shown in the diagram, so that in practice the impact forces are considerably less in consequence.

Theoretically, therefore, unless the hammer strikes the coal immediately upon its entrance to the mill, before the air currents have time to affect it, little impact can actually take place, provided the coal particle remains in the zone of the rotating hammers and air pockets.

But we have every reason to believe that the hammers in the mill are doing quite a little work in breaking up the coal. Let us go a step further. The high velocity of the hammers creates centrifugal forces sufficient to throw all solid particles out against the casing. Their contact with the casing will tend to bring these

particles to rest, with the result that positive impact can then be obtained. Judging from the character of the wear that takes place in the hammer of these mills that have fine radial clearances, it is fair to assume that a great percentage of the work done is confined to a very narrow zone extending from the casing to a very short distance down the hammers.

We are not considering pulverization at this time. We are only considering the first process of breaking up, and this brings in another factor. With lump coal concentrated at the periphery of the casing, and hammers rotating at high velocities close to it, granting that there is breaking up due to impact, is there not a possibility that there is quite an appreciable amount of work being done by direct shear?

In support of this argument we might again refer to the wearing of the hammers at the tips, and we could go a step further and argue that these shearing forces might account for the big differences that occur in fineness when the clearances are small, with new paddles, and when they are large, due to wear. The author, however, does not feel that this shearing effect has anything to do with fineness and that the change in the production of fines with small or large clearances can be traced to contributory conditions attendant upon said clearances.

We have been considering the work done by impact, with the possible assistance of shear. If we consider the analogy of the ball and the feather, and if we calculate the effect of windage in the casing, there are strong arguments to support the theory that in the so-called impact mills once the coal is broken up into small lumps (with a certain amount of attendant fines), impact forces cease so far as pulverization is concerned.

As a contributory argument it might be mentioned that pure impact mills have been experimentally built with velocities of impingement of 640 ft. per sec., the effect of which is 10 times as great as that of the average mill with a hammer velocity of 200 ft. per sec., but without satisfactory results so far as pulverization is concerned.

Once the coal is broken up into small lumps, the centrifugal force created by the rotating paddles is sufficient to throw all the particles of coal against the casing. Referring again to Fig. 7, the horizontal velocity of the mixture (air and coal) was computed to be 680 ft. per min. when passing 8000 lb. of coal through the mill per hour and assuming 18 cu. ft. of air per pound of coal. The coal in this mixture will hug the casing in its passage through the mill. This being the case, given a certain discharge from the mill, the thickness of this film of coal can be approximately calculated.

Assuming the coal to weigh, in round numbers, 80 lb. per cu. ft., then the cubic feet of coal per hour will be 100, or 1.66 cu. ft. per min. Assuming the velocity of coal through the mill to be the same as the mixture, or 680 ft. per min., then the film of coal will be less than one-tenth of 1 in. thick.

In commercial mills the minimum clearance usually allowed between the tip of the hammers and the casing is about $^1/_2$ in. If the material is thrown out against the casing, as we have every reason to believe, then all particles less than $^1/_2$ in. will be immune to impact or even shearing.

Let us now consider the question of pulverization as distinguished from breaking up. It has been shown that when the coal is broken up into particles less than the clearances in the mill it is out of the way of impact or shearing forces, because the film of coal at any one time in the mill is less than the clearance. In the grinding down of our coal from small lumps to 100 and 200 mesh, attrition and only attrition must be doing this all-important work. And this attrition will naturally be a function of the whirling velocities of the particles and the time element.

Regarding the marked effect of the wear of hammers on fineness, we cannot look for the answer to this because of any great loss in impact or in shearing. We can look for the answer in the effect that increased clearances have on the velocities of rubbing.

Take for instance the case of a mill having zero clearance: the particles will be whirled around the casing at the speed of the hammers. We can now add sufficient clearance between the hammers and the casing so that the induced windage created by the hammers will be insufficient to overcome the frictional resistance of the coal rubbing against the casing, in which case the particles would remain stationary.

Conclusions based on the foregoing arguments would be:

1 In any so-called impact mill, the actual work done by impact forces is relatively small.

2 In hammer mills, where the no-load power or windage losses vary somewhere between the square and the cube of the speed, are high-speed hammer losses warranted by the fact that they are probably doing little hammer work, but the velocity energy created by the hammers rotating at high speed is only producing velocities necessary for attrition that might be accomplished in a more efficient and scientific way?

Guarantees

It is hard to conceive a more difficult engineering problem than the determination, with any degree of accuracy, of the exact performance of a coal pulverizer.

Steam engines, turbines, pumps, and blowers can all be tested accurately by short runs in a manufacturer's plant, and these can be made without prohibitive expense. Readings can be made with sufficient precision to establish reliable guarantees and performance curves.

This condition unfortunately does not hold good in pulverizer work. Tests are extremely expensive, and any attempt toward an accurate determination of true performances in a manufacturer's plant involves an expense entirely out of proportion to any return that can now be expected.

Friability, moisture, load, and fineness all very materially affect the horsepower per ton. With the difficulties and expense involved in carrying out shop tests, the manufacturer has been obliged to rely on station tests for a great percentage of the data he uses in determining subsequent guarantees on future installations.

We are again confronted with the question as to the accuracy of the best station tests. The manufacturers owe a great deal to the cooperation that has been afforded them by the engineers of large power plants, not alone in allowing the manufacturer to inconvenience him a great deal by conducting experiments in his plant, but by whole-hearted cooperation in arranging his loads so that the manufacturer could get data as reliable as possible. Under the best of all these conditions, however, it is difficult to obtain data that will enable the manufacturer to state with any degree of accuracy what he will get on the same mill in another station. Referring to the difficulties involved in getting accurate data from station tests, one specific example is sufficient to explain this point. A series of tests were run under the best possible conditions, extending over a period of five weeks, tests being conducted practically every day. During this period the moisture content of the coal varied from 4 per cent to 14.5 per cent. There was no record as to whether the friability remained constant.

With the foregoing difficulties confronting us, therefore, is it plain common sense for the purchaser to demand guarantees on something that he himself knows must only be a good (or bad) guess on the part of the guarantor? Is it good engineering for any manufacturer to make guarantees with no concrete data to back them up? Are such demands and acceptances conducive to the development of an art, the prime object of which should be the mutual interest of all concerned in the conservation of natural resources?

In asking the builder to make guarantees, let us ask ourselves just what the purchaser is really interested in. The answer will be that he is interested in the following factors and in this order of importance:

1 Satisfactory continuous operation

2 Efficiency.

Regarding item 1, the engineer appreciates that he cannot get continued satisfactory operation unless the machine fills the following specifications:

(*a*) The design must be such that it will operate without legitimate mechanical troubles

(*b*) Over a legitimate period of time between shutdowns for replacements of worn parts, he must not experience excessive "snowstorms;"[2] in other words, during this period he is interested from an operating standpoint in his 40 mesh performance and he is not interested whether his 200 mesh is 60 per cent or 90 per cent

(*c*) The design must be such as to facilitate quick replacement of all worn parts, including bearings.

Item 2 is considerably involved. It contains many factors that must be taken into account if we are to make an absolute analysis as we would in a heat balance in a station over a period of one to two months.

Major items necessary for the tabulation of absolute data would include the following:

(*a*) Reliable information relative to the power consumption over the whole range of operating conditions

(*b*) A close approximation as to the mean fineness performance over a satisfactory period of operation

(*c*) Maintenance in cents per ton.

Regarding the factors mentioned in item 1, the purchaser is entitled to expect and the builder should be willing to give protective guarantees to cover all these points.

When we come to item 2, however, it is a different story. If we make calculations to determine the answer to any problem, we must know all the factors involved. If there is a variable x in the problem, the answer is not correct until we find the absolute value of this x for the particular case we are considering. In coal pulverizer calculations there are many x's.

As *a*, *b*, and *c* of item 2 are all more or less affected by the same variables, we must, to consider the situation properly, make a new classification. This classification is confined to only two subjects, but it will be seen that in each one treated the variable x must be so multiplied as to include a great many other letters of the alphabet.

The first subject to be considered will be the nature of the material to be pulverized. By this is meant the complete nature of the material, including friability, moisture, and chemical characteristics. All these factors affect power per ton for pulverizing and materially affect the cost of maintenance.

With all these variables, how can a purchaser expect to get any reputable manufacturer to give guarantees unless he himself is in a position to furnish all the data necessary for the making of such guarantees?

If this same purchaser is buying a turbine he will state that he will furnish to the turbine steam at a certain pressure and quality and will guarantee to take the exhaust steam from this machine at a certain pressure. Under these conditions the turbine builder can make concrete guarantees that should always be met. In the case of a pulverizer, however, until the purchaser can give the same definite particulars with regard to the coal that is to be pulverized—i.e., complete physical and chemical characteristics of the coal—he has no more right to demand or even to expect the builder to make binding guarantees either on kilowatts per ton or maintenance than he would to expect the turbine builder to make guarantees with unknown steam conditions.

Again, looking at the matter from the builder's standpoint, if the prospective customer could give all of the information outlined, has the builder sufficient data today to make computations from standard performances so as to make positive guarantees? If not, he has no right to commit himself to guarantees.

The second subject to be considered under the general classification referred to is that of the meaning and understanding of the term "fineness."

All questions pertaining to the performance of mills, and in fact all discussions on the subject of pulverized coal, center on the performance through a 200-mesh screen. This is well illustrated by the fact that so many test reports refer only to this screening. Now there are many reasons why performances and comparisons based on a 200-mesh standard are neither logical, reliable, nor of any great value.

In support of this argument it is not necessary to go into any great detail. Some of the points to be enumerated will be self-evident to all, while others will be appreciated only by those having had first-hand experience in the collecting, screening, and weighing of samples.

1 What is a 200-mesh screen? Mr. Peabody pointed out in his paper before the Society of Naval Architects and Marine Engineers that so long as manufacturers of screens disregard any idea of adhering to recognized standards and use different sizes of wires (as unfortunately they do) various screens having different sizes of openings may be used, any of which might comply with the literal rule of so many "mesh to the inch," but no two of which would give the same results on screening a sample of pulverized coal. Incidentally, Mr. Peabody recommends the general adoption of the A.S.T.M. standard, which specifies the size of the screen by the size of the opening. Until we all learn to talk the same language with regard to screens, and especially with regard to the finer screens, all questions relating to reported performances are absolutely worthless.

2 What is the shape of the particle when it reaches the 200-mesh screen? Supposing it is slaty in formation and supposing that it just passed the 100 or 150 screen because its widest measurements permitted it to, but that its thickness measured $^1/_{500}$th of an inch, it might not pass the 200-mesh screen, but it would possess much better firing qualities than a cube or a sphere that did pass the 200-mesh screen.

3 The actual taking of representative samples is acknowledged to be a problem of no mean proportions to those who have attempted it. Assuming that a correct or representative sample has been obtained, difficulties are encountered in getting a proper screening. Where there is moisture in the sample, it is still more difficult, and the results are often unreliable.

When all is said and done, and the manufacturer has guaranteed 80 per cent through this ambiguous 200 mesh, and the sample only shows 70 per cent, the manufacturer has fallen down on his guarantee, probably owing to his ignorance of the coal used. Controversy arises over payments, but the boiler goes on operating with the same efficiency at 70 per cent as it would show with 80 per cent.

Now, if on the other hand, all guarantees were based on a flat and definite performance of 100 per cent with an allowance of, say, a minimum of 99.5 per cent through the 40 per cent screen, let us see what would happen. In the first instance the screening through a 40-mesh sieve is simple and it takes a very short time to get an absolutely accurate screening. That which does not go through a 40-mesh screen can be seen. It is in the form of easily visible lumps. On the firing end, the particles larger than 40 mesh can be seen in the furnace in the form of "snowstorms," which is a serious condition and liable to cause shutdowns. This

[2] A term used to describe large particles in the furnace.

constitutes one proposition relative to the importance of the 40-mesh screening.

Granted that the 40-mesh sample is what it should be and that snowstorms are not experienced, what guarantee have we that we are getting enough fines and superfines to give us the best economy in the boiler? The answer to this question, the author thinks, is given in Figs. 9 and 10. In Fig. 9 there are plotted over a hundred tests from seven different types of commercial mills.

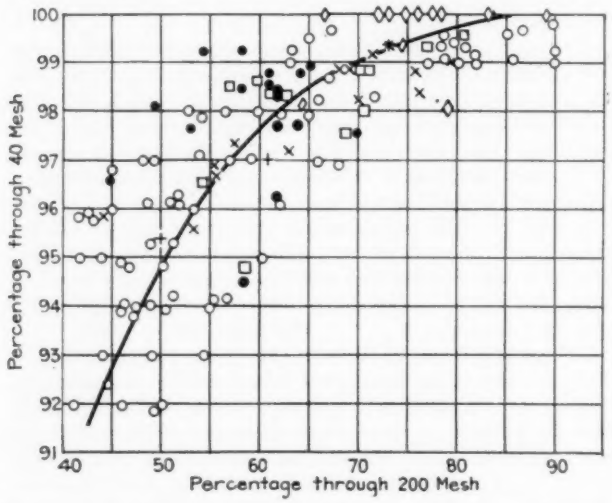


FIG. 9 TESTS PLOTTED ON BASIS OF PERCENTAGE THROUGH SCREENS OF DIFFERENT MESHES
(Shape of symbols indicates the seven types of mills included.)

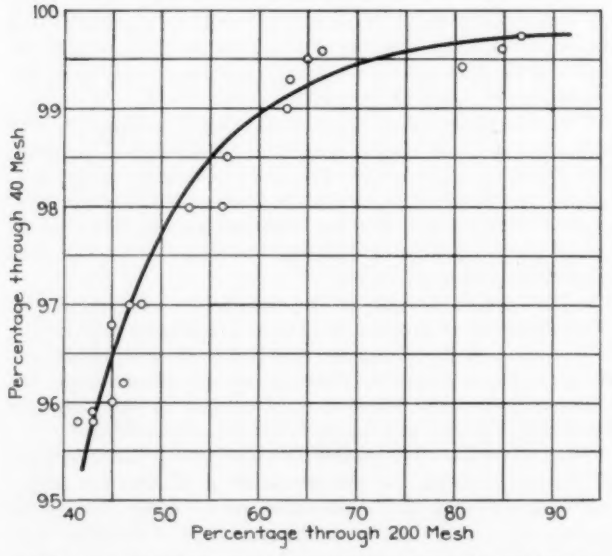


FIG. 10 CHART PLOTTED FROM ASHLEY STEEL PLANT TESTS

They are plotted on the basis of the percentage that went through 200 mesh and what the same sample did at 40 mesh. In taking the worst example it will be seen that 100 per cent through 40 mesh was not reached until the fineness at 200 reached 67 per cent. Taking a mean curve of all the tests, the average is about 85 per cent through 200-, with 100 per cent through 40-mesh screen.

In order that there can be no misunderstanding as to whether this characteristic holds good with various loads and conditions, Fig. 10 has been plotted from the Ashley Street tests given in the N.E.L.A. report referred to. In this series of tests we have loads varying from one-half to 20 per cent overload; we have the moisture content at 6 per cent, 9 per cent, and 12 per cent; and we have tests with new paddles and with paddles badly worn. Throughout all these tests the general characteristics are remarkably consistent.

If, then, the actual performance of all mills is such that with a given performance of, say, 100 per cent through a 40-mesh screen—and we cannot help but accept the plain fact that the 40 mesh is an all-important screening from the standpoint of operation and efficiency—what are the logical arguments against accepting this 40-mesh screen as the standard upon which to base all guarantees?

The average engineer in charge of a power plant is no fool. On the contrary, we find the best engineers in the country today in charge of our large generating stations. There never was a time when the manufacturer needed the cooperation of the station engineer as he does today for the proper development of his apparatus. The builder will not get this cooperation until he is honest with himself, and he cannot make ten or fifteen guarantees on a proposition with so many unknown quantities involved and be honest with himself.

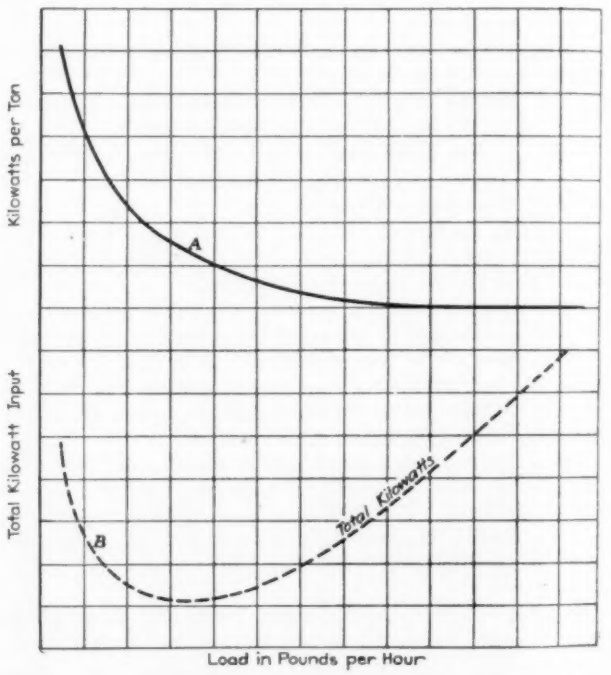


FIG. 11 CURVE OF AVERAGE POWER CONSUMPTION PER TON OF ONE MILL

On the other hand, this same engineer when properly approached with sufficient sane guarantees that protect him in the operation of his plant and with enough supplementary data backed by sound engineering judgment to enable him to estimate with a reasonable degree of accuracy what his general performance will be, will be found to be the most receptive of purchasers.

Apart from the question of giving impossible guarantees, the builder can be severely criticised for the publishing of unauthentic data in his catalogs. In Fig. 11 there is reproduced to scale a catalog curve of the average power consumption per ton of a certain mill. This is curve *A* and looks as though it might have the right shape. If we replot this curve, however, on the basis of totol kilowatts against load, as shown in curve *B*, we find curve *A* valueless and misleading.

Conclusion

Until we know more about the art of pulverizing coal than we do today, the builder should be honest with the purchaser in any guarantees he makes. Engineering judgment prompts us to confine all of our guarantees to the following simple code, which is presented here for discussion:

1 Guarantees for power per ton and maintenance to be based on one accepted standard grade of coal.

2 Moisture content under guarantee conditions must not exceed 5 per cent.

3 Fineness guarantees to be based on 99.5 per cent through a standard 40-mesh screen.

Multitudinous guarantees, based on guesses, are worthless to the purchaser and harmful to the builder. Simple guarantees can be made accurate and reliable and thus are worthwhile and mean something to the buyer.

Discussion

O. P. Hood.[3] There are other arts equally interested in fine pulverizers. The metallurgists are greatly interested in this subject. The metallurgist has wanted for some time a means of determining the efficiency of crushing machinery. It would appear that we have the same problem here. If it were possible to have a crusher with 100 per cent efficiency, what would be the form of the power curve as fineness increased? It has been questioned as to whether the amount of power required was in direct proportion to the amount of surface produced or not.

Work that has been done during the last year or two by the Bureau of Mines has shown that Rittinger's law is correct. If the amount of power required is in direct proportion to the amount of surface produced, why is it that in coal pulverization we get such scattering results?

I think it must be admitted that the absolute efficiency of any of these pulverizing machines is relatively low, so that the big factor is a variable waste factor which masks a great many of the real facts of the case. Also, it has been pointed out that pulverization varies greatly with the moisture of the coal.

The standard method of determining moisture is empirical. It is entirely possible with some coals, by using a different method of determining moisture, to find anywhere from 3 to 4 per cent more moisture than the standard method reveals.

Moisture in coal exists in several forms. Which form affects coal pulverization and which does not? Here again we have factors which when plotted mask the facts of the case.

It is not at all surprising to learn that different coals, even when dry or at the same moisture content, take a different amount of power to reduce them to equivalent sizes. If one will look at the shape of the particle that is produced, he will find a great deal of variation in the different coals. It is obvious that if spheres could be produced, you would have a minimum surface for a given size produced. The nearest thing you can get to that in coal would probably be small cubes. But if one looks at powdered lignite, or even some bituminous coals, the characteristic shape of the small piece only a few microns in average dimension may be plate-like or rod-like, having a considerably larger surface than either the cube or the sphere. There is, therefore, a characteristic form or surface area for each particular coal.

Not only that, but each different ingredient of the same coal presents this same group of moisture and breaking problems. The so-called mother of coal very prominent in black coal from Brazil, Ind., is made up of very fine friable particles to begin with. It is like fine charcoal. In that same coal will be a dark band named by our English friends vitrain because it breaks like a vitreous substance with a conchoidal fracture.

These factors which mask the facts of crushing should be borne in mind when plotting the results of experiments.

C. J. Jefferson.[4] The author emphasizes the important fact that manufacturers of pulverizing equipment should know exactly what their equipment can produce and their guarantees should be based on facts rather than on guesswork.

He states in his conclusion that guarantees for power per ton on maintenance should be based on one accepted standard grade of coal. This is quite in order, but it should be supplemented by coefficients indicating what the performance will be when using the various fuels that will be available at the plant where the pulverizer is to be installed. It is appreciated that at present this is most difficult, if not impossible, as there is not sufficient knowledge available as to just what are the "grindability" characteristics of the various fuels of the world. This quality of coal should be made a matter of research. As it is so vital to the whole coal industry the research should be sponsored by the mining interests, coal transportation interests, manufacturers of equipment, and the operators of the power plants themselves.

The second item in the conclusion states that moisture content under guarantee conditions must not exceed 5 per cent. This is ideal for the manufacturers of pulverizing equipment, but does not meet operating conditions. Our experience with actual bunkerings of the steamship "Mercer" has shown that out of seven different bunkerings the moisture contents in five were in excess of 7 per cent. This high moisture content is due to the exposure of barges and railroad cars to the weather during transportation of coal from the mines to the point of bunkering. Our experience therefore has forced us to adopt a policy that all pulverizing equipment submitted for test at the Philadelphia fuel-oil testing plant (where we are now carrying on our laboratory developing work for the adaptation of pulverized fuel to marine boilers) must function with a moisture content as high as 7 per cent; it is also understood that the fuel used is the ordinary bituminous coal found along the Atlantic seaboard.

If I were purchasing pulverized-fuel equipment for a plant using mid-West coal or Southwest lignite, I would insist on a still higher moisture content, as 5 per cent moisture content in Illinois coal would be practically dry coal.

The fineness guarantee to be based on 99½ per cent through standard 40-mesh screen may be suitable for boilers fitted with enormous refractory-lined combustion chambers, but where the combustion space is limited and a heat release of 30,000, 40,000, and 60,000 B.t.u. per cu. ft. per hr. is required, this qualification is meaningless, because in the case of these boilers it is necessary to have a great number of superfines in order to speed up flame propagation. In the case of such furnaces the guarantee should be based on the finer coal rather than on the heavy granulars. Personally I do not think it is practicable to set a definite standard on the degree of fineness required which will cover all boilers. The type of furnace relative to size and to whether or not it is refractory-lined or water-cooled affects this characteristic so much that a blanket specification as to the degree of fineness is not only ridiculous but dangerous. On the one hand, excessive power may be called for in obtaining the specified fineness, and on the other hand, failure to call for the required fineness will result in an impossible combustion condition.

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[4] Head, Fuel Conservation Section, U. S. Shipping Board, New York, N. Y.

A. H. KRAUSS.[5] The author has discussed a subject of great importance to those concerned with the burning of fuel. The question of guarantees is a vital one, and there are several points upon which the writer does not agree with the author. It is necessary to obtain specific guarantees from the manufacturer in respect to both fineness and power consumption. It is true that guarantees do not run a plant, but the choice and selection of a given type of plant must necessarily be predicated upon guarantees. Also, the writer's experience has shown that by having guarantees, changes and adjustments have been made to mills in order to correct faults therein to meet guarantees, and such a procedure is conductive to the development of the art because it forces the manufacturer to work with the purchaser in obtaining the maximum efficiency from the plant.

It is admitted that there is bound to be a large variance in power consumption for pulverizing different coals. The author has plotted in Fig. 1 a chart of tests on fineness performances, with abscissas giving percentage through 200 mesh and ordinates giving kilowatts per ton. From this chart he shows a difference in power consumption of no less than 70 per cent between the upper and lower limits for any given percentage through 200 mesh. The author states farther on in his paper that friability, moisture, load, and fineness all very materially affect the horsepower per ton. In plotting Fig. 1, are the points plotted for maximum rating of the mills or any other specific rating? This factor has a great bearing upon the power consumption, and unless it is taken into account, the figure is misleading. The writer has found that there is an increase in power consumption per kilowatt of coal pulverized of 60 to 70 per cent between full rating and one-third rating on a type of impact mill with which he has had experience. This may account for a great part of the variation that the author shows in his Fig. 1.

Moisture in the coal also has effect upon the power consumption, as stated by the author, and this should be taken into account in plotting such a curve; in other words, those points in which the moisture is in great variance with the average should be discarded from such a plot.

The writer believes that if these various factors were taken into account it would be found that there is not nearly such a wide variation in actual performance as the author has stated. It is necessary for the purchaser to obtain power consumption guarantees, although it is admitted that there are various factors such as pointed out by the author which affect the results. These factors should be taken into account by the manufacturer in making his power consumption guarantees, and the writer does not agree with the author that there are too many variables and that it is unfair to ask the manufacturer to make such guarantees.

With regard to fineness, it is true that we are mainly interested from an operating standpoint in the 40-mesh performance but we are also interested in the 200-mesh performance. The writer has had experience with mills where the ratio, as shown in the author's Fig. 9, of 40- and 200-mesh fineness was not obtained until considerable adjustments and changes had been made to the mill and its classifiers. It is not seen how the maximum efficiency and best performance of the mill could have been obtained unless there had been guarantees to work to. Neither is there any apparent objection in making a guarantee on performance through a 200-mesh screen. Qualification, however, may be made as to what standard 200-mesh screen the guarantees are based upon.

Unless guarantees are made, there will be no incentive to obtain the best performance from the equipment furnished, and the writer believes that all facts necessary can be given to the manufacturer and that he should be in position to make guarantees which he can honestly give.

[5] Assistant Mechanical Engineer, Electric Bond & Share Company, New York, N. Y. Assoc-Mem. A.S.M.E.

AUTHOR'S CLOSURE

Mr. Krauss, Mr. Van Blunt, Professor Work, and others have made particular reference to that point in the paper where I recommended guarantees based on 40-mesh screen.

I know this is a very disputable question, but I think the gentlemen have rather missed the point I wanted to bring out, and that is that if we confine ourselves to a minimum of 99.5 or 100 per cent through the 40-mesh screen, we cannot get this until we have reached a minimum of 60 to 70 per cent through the 200-mesh. That is the main point, and the actual characteristics of the curves are immaterial. Now 60 to 70 per cent through a 200-mesh is considered in most power plants as very satisfactory performance.

Mr. Hood brought out the point that there is quite a difference in the methods of measuring the actual moisture in coal which has to be taken into account. This only emphasizes what I brought out in the paper, that until we have all of these variables boiled down to concrete factors and know just what we are talking about, only then will we be in a position to talk absolute guarantees.

I tried to point out in the paper that I think we ought to begin by the simplest kind of code possible, confining our guarantees for the time being to such factors as we have concrete data upon.

Mr. Jefferson has brought out that he considers it impracticable to confine guarantees to a maximum of 5 per cent. I agree with Mr. Jefferson that the moisture content in practice is generally very much higher than this, but the effect of moisture in different types of mills is quite different. We have very little data to tell us just what the difference would be in any mill with 5, 10, or 15 per cent moisture. Until we have this data, the performance with higher moisture content is purely conjectural, so I still feel that it would be much more sensible to confine guarantees to substantially dry coal and then to make what we consider approximate corrections for high moisture rather than to give definite guarantees on high-moisture coal without any data whatever to work with.

I was glad to note that Professor Work agrees with me relative to standardizing on some fixed grade of coal. The question has been raised by several speakers relative to the accuracy given in Fig. 1. I appreciate that this is a very approximate curve. It was introduced to show the immensely wide variation in commercial work. The tests plotted were at the manufacturer's rated full load. I think it was Mr. Van Blunt who brought out the point that the load has quite a marked effect on the performance. This is granted; but if a manufacturer, as he does in some cases, sells a mill for one-half the rated load that the mill will carry in some other cases, in order to improve fineness performances, then the full rated load automatically becomes one-half the normal, with a consequently higher power consumption.

I am sorry I could not raise a better discussion on the design of mill. Mr. Van Blunt pointed out that in his opinion the hammer mill is not a good mill for fine pulverization. Why not? I think I brought out clearly in the paper that in a hammer mill very little work is done by impact and the real pulverizing is done by attrition, and I therefore feel that a scientific study of this type of mill will demonstrate that it is quite possible to get fine pulverization, provided that proper attrition is provided for.

Mr. Hood brought out the fact that the efficiency of pulverizers is probably very low. It is in the full appreciation of this fact that I have endeavored to point out that very little scientific research work has been done in an attempt to improve this.

FSP-51-29

Steam-Generating Capacities of Boilers

By FAIRFIELD E. RAYMOND,[1] CAMBRIDGE, MASS.

In this paper the author gives reasons why the maximum steam-generating capacity of any complete boiler unit should be calculated entirely on a heat-liberation basis, except in so far as the features of design are reflected in the value used for the weight or volume of fuel consumed per hour in the furnace and in the overall efficiency. He presents formulas which represent the fundamental conditions, and gives several examples of their use.

THE selection of suitable boiler equipment to meet the steam requirements of a given power plant is becoming more and more complex, and recent developments and innovations in the design of boilers and furnace equipment present an increasing number of obstacles to the direct determination of the maximum steaming capacity of any unit. At present more consideration than ever before is being given to the amount of heating surface exposed to radiant heat, such was water walls, water screens, and the lower banks of boiler tubes, as such surfaces have a much greater capacity for steam generation than those which are more remote and obtain heat by convection only. Besides, the amount of fuel that can be burned in the furnace under any boiler has been greatly increased by the use of pulverized coal, as well as oil. Likewise the effect of air preheaters, heat exchangers, feedwater heaters, air-cooled walls, economizers, etc., must be considered in determining the overall efficiency of the proposed boiler installation and the ultimate generating capacity of the boiler. Eventually all this must be reduced to terms of pounds of steam generated per hour in order to compare it with the requirements of the power plant. This problem assumes serious proportions when it comes to the determination of the number of safety valves required for each boiler unit, as the methods available for solving this problem follow in most cases arbitrary rules which do not provide for the conditions encountered in modern practice. It therefore seems desirable to establish a direct method of obtaining the maximum steaming capacity of any complete boiler unit without involving indeterminate factors.

Existing Methods of Determination

In the past the accepted method of determining the steaming capacity of a given boiler depended upon the assumption that the rate of heat liberation in the furnace, or the rate of heat absorption by the boiler tubes, was practically a constant for most installations. In general this was true as long as hand firing existed and boiler pressures remained moderate. Evidence of this fact is visible in the various boiler codes in use up to this time.

Certain codes refer to the unit of grate area (lb. per sq. ft.) as a measure of the fuel burned per hour, having assumed that a uniform amount of coal is supplied during the hour to each square foot of grate surface and that the heat content of that coal is the same for all conditions; and following this it is assumed that the rate of evaporation is also uniform for all conditions, and thereby an expression is arrived at which represents the probable steam-generating capacity of the boiler. The conditions here mentioned are naturally those existing when the boiler is being forced to its maximum. Examples of this type of code are to be found in those published by the United States Board of Inspection Service of Steam Vessels, and in the Massachusetts Steam Boiler Rules, and the Canadian Interprovincial Regulations—for Boilers.

Other codes refer to the unit of heating surface (lb. per sq. ft.) as a measurement of the steam generated per hour in the boiler proper, omitting the conditions existing in the furnace and the amount of fuel fired. This method therefore implies that the rate of firing is constant for all installations, and assumes that the rate of heat transfer from the products of combustion to the water in the boiler is constant for all cases except where corrective factors are given for special types of boilers or various pressure ranges. Here, too, the boiler is considered as being forced to its maximum. Examples of this type of code are to be found in those published by the Committee of Safety Valves of the American Railway Master Mechanics' Association, in the statements given in Par. 274 of the A.S.M.E. Boiler Code, and in the codes applying to the regulation of boilers throughout the British Empire.

Improvements in Firing Rate

With the improvement of firing methods and the use of mechanical stokers it was found that boilers could be forced to higher steaming rates than had ever been possible by hand firing. As a result a greater amount of fuel could be burned under any given boiler, with a consequent greater rate of heat liberation. Therefore the original assumption of a constant rate of heat liberation, as found in the codes referred to, no longer holds true. To account for this it became customary to state that a boiler was operating at 200 per cent or 300 per cent of the nominal boiler rating given by the manufacturer and as calculated on the original assumptions. This method proved satisfactory in most instances until the advent of oil and pulverized coal, by means of which it is possible to force a boiler with a properly designed furnace to 500 per cent or 600 per cent of rating, and finally, with the development of boilers similar to the "Combustion" steam generator, it is possible to obtain 1500 per cent of the nominal rating. It seems that with ratings increasing so rapidly, any method of calculating the maximum steam-generating capacity of a boiler which depends on such arbitrary assumptions and corrective factors is entirely unreliable and fails to represent the actual conditions.

Improvements in Boiler Design

Likewise, with the introduction of water walls and water screens it was found that such surfaces, as well as the lower banks of tubes which are exposed to the direct effect of radiant heat, absorb more heat in proportion than more remote sections of the boiler, and that the rate of heat transfer from the products of combustion to the water in the tubes is no longer a constant for all sections of the boiler. Accordingly, in order to calculate correctly the actual steam generated on the basis of total heating surface, it would be necessary to segregate each section of the boiler and calculate the steam generated therein by applying the appropriate rate of heat transfer for that section. The combined results would give the total generating capacity of the boiler. Practically this would be impossible as it would require tedious calculations involving details of design the data for which can rarely be found outside the possession of the boilermaker, and therefore it would seem desirable to have some simpler and more direct method established.

Present Practice in Purchasing Equipment

Moreover, it is becoming common for engineers and designers

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

of boiler plants to select the most efficient type of firing system and grade of fuel for a given plant, and design the furnace and equipment to suit; whereupon a boiler which will most satisfactorily fulfil the conditions is superimposed upon the furnace design. Auxiliary equipment such as air preheaters or economizers, etc., have become a definite part of the complete boiler installation and therefore must also be considered, as the effect of these units upon the overall efficiency of the boiler will influence the final choice of equipment for the power plant. Therefore, it is becoming impossible to purchase a boiler as a unit, complete in all respects, as was customary in the past, and likewise to expect arbitrary rules to hold for determining the generating capacity when the component parts are obtained from unrelated sources.

Use of Fundamental Principles

At this point it becomes apparent that the methods provided in the codes mentioned are inadequate for modern conditions and that in the future the actual maximum possible steaming capacity must be calculated from facts derived from the actual layout of each installation. To avoid complex calculations involving details of design it is proposed to utilize the fundamental principle that the heat input must equal the heat output, by basing the calculation upon the heat liberated in the furnace, and upon the heat content of the steam as delivered by the boiler to the main steam line, with due consideration of the losses, expressed as a factor of overall efficiency. The data for such calculations are readily obtained from the original determination of the heat balance for the power plant and from the guarantees of the manufacturers whose equipment has been selected.

Recognition of Fundamental Principles

The soundness of this principle was definitely recognized by the A.S.M.E. Boiler Code Committee as early as 1914 and was incorporated in the code published that year, being continued in subsequent editions, and may be found in Par. A-12 of the Appendix. Other codes have come to recognize to a small degree, as already mentioned, that the heat liberation per unit of grate surface or the heat transfer per unit of heating surface is not constant for all fuels or pressures, and have provided, in a superficial way, arbitrary tables or factors for corrective purposes. These other methods, however, do not approach the problem from the right angle, and show that the authorities responsible for them have not grasped the fundamental conditions involved or else have not applied them to the fullest extent possible.

Proposed Method of Calculation

The maximum steam-generating capacity of any complete boiler unit, therefore, should be calculated entirely on the basis of heat liberation without regard for the details of design, except in so far as the features of design are reflected in the value used for the weight or volume of fuel consumed per hour in the furnace and in the overall boiler efficiency. The following formulas represent the fundamental conditions:

$$W = \frac{C \times H \times E_0}{q + xr - q_f} \quad \text{(for use when steam is saturated or wet)} \quad [1]$$

$$W = \frac{C \times H \times E_0}{q + r + C_p\,(t - t_{sat}) - q_f} \quad \text{(for use when steam is superheated)} \quad [2]$$

where W = maximum steam-generating capacity of any complete boiler unit (lb. per hr.)
C = total weight or volume of fuel burned per hour at time of maximum forcing, lb. or cu. ft.
H = heat of combustion of fuel, B.t.u. per lb. or per cu. ft.
q = heat of the liquid, B.t.u.
r = latent heat of vaporization, B.t.u.
x = dryness factor (for saturated or superheated steam $x = 1.0$)
C_p = specific heat of superheated steam at constant pressure ($C_p = 0$ if steam is saturated or wet), B.t.u. per deg. fahr.
t = temperature of superheated steam, deg. fahr.
t_{sat} = temperature of saturated steam, deg. fahr.
q_f = heat content of the feedwater, B.t.u.
E_0 = overall boiler efficiency = per cent ÷ 100.

The derivation of these formulas depends entirely upon simple thermodynamic principles and the data required can be obtained from the calculations for the individual plant. The value of C is obtained from the manufacturer of the firing equipment; the value of H is obtained from the vendor of the fuel as his average analysis for the grade required; the values of q, x, r, C_p, t, t_{sat}, and q_f can be found from any steam table of the proper pressure-temperature range. The efficiency factor E_0 includes the effect of all primary and auxiliary equipment composing the boiler unit, and accounts for all heat losses such as stack temperature, incomplete combustion, radiation, and moisture. Heat that is reclaimed from other sources outside the boiler is represented in the term for the heat content of the feedwater, and is subtracted from the total heat in the live steam. The denominator thereupon represents the heat required to generate one pound of steam at the maximum pressure and superheat from feedwater having the highest obtainable temperature. It may be well to note here that the data required are consistent with the present practice of purchasing such equipment.

The A.S.M.E. Evaporation Factor

Another important step in the same direction was the adoption by The American Society of Mechanical Engineers of a new unit of boiler rating, the "evaporation factor" (equal to 1000 B.t.u. of heat transferred), in place of the commonly used "boiler horsepower" (equal to 34.5 lb. of steam per hour from and at 212 deg. fahr.). Engineering practice has so changed in the past few decades that the term "boiler horsepower" has become purely nominal, as its use involves obscure factors and tends to avoid the underlying conditions of heat transfer. The new term, "evaporation factor," states exactly the proposed number of B.t.u. a boiler can absorb for the purpose of generating steam, and lends itself readily to the simple determination of the style of boiler to be superimposed upon the furnace, depending entirely upon the heat liberated in that furnace. This also conforms to the practice of purchasing the boiler equipment in separate units.

Approximate Method of Calculation

Where only general or approximate results are required, the formula as given in Par. A-12 of the A.S.M.E. Boiler Code is most applicable and sufficiently accurate. It is derived from Formula [1], wherein it has been assumed that the difference between the total heat in the steam and the heat returned in the feedwater is constant, so that the denominator of the first equation becomes equal to 1100 B.t.u., as follows:

$$W = \frac{C \times H \times E_0}{1100}$$

where the various terms have the same meaning as already given.

Overall Boiler Efficiency

For general purposes the overall boiler efficiency can be assumed as 75 per cent. However, if test data are available or the

manufacturers' guarantees indicate a definite efficiency, whether of a higher or lower value, that should be used in preference. If any uncertainty exists as to the proper efficiency to be used, one should choose a higher value rather than a lower one, especially if a safety factor is concerned. Such doubt implies extraordinary conditions of operation and new methods or designs, which might give exceedingly high ratings of steam generation and a resulting increase in overall efficiency. As efficiencies of 90 per cent have been attained, it is wholly within reason to use in such cases the figure of 100 per cent until a more accurate figure can be determined from actual operating conditions.

Application to Safety-Valve Problems

All these facts apply particularly to the determination of the proper number of safety valves. Many times in the experience of the author, as well as of others richer in such knowledge, has the number of safety valves provided proved insufficient to relieve the boiler due to the discrepancy between Boiler Code requirements as represented by existing formulas and the actual maximum steam generation possible. The number of safety valves required to protect a given boiler can be found simply by dividing the maximum amount of steam generated (lb. per hr.) in a boiler (W from the formulas) by the stated discharge capacities (lb. per hr.) of any make of safety valve as guaranteed by the manufacturer and stamped upon the casing or tag permanently affixed to the valve. If no such data are provided, safety valves should be rated as specified in the various codes with minimum capacities until the correct data for them can be obtained. Here again, if the actual overall boiler efficiency is unknown or it seems inadvisable to use an efficiency as low as 75 per cent owing to radical features of design, an efficiency of 100 per cent can be safely used without involving too great an expenditure for extra valves; the 25 per cent added efficiency above the average efficiency will only require one additional safety valve per boiler at most, and often this extra valve must be supplied as it is practically impossible to provide an aggregate safety-valve relieving capacity exactly equal to the steam-generating capacity of the boiler, and the resulting reserve capacity will cover a possible increase in efficiency even up to 100 per cent. Again this is evidence that a more exact method of determining the maximum steam-generating capacity is needed similar to that proposed in this paper.

Conclusions

In conclusion, the necessity for rating boilers on a heat-liberation basis in B.t.u. units for the English system cannot be overemphasized. The steam-generating capacity should be stated in one of three ways:

(*a*) Pounds of steam evaporated per hour
(*b*) Heat liberation in the furnace; together with an overall efficiency and the operating or limiting steam pressures and temperatures and feedwater temperatures
(*c*) A.S.M.E. evaporation factor (1000 B.t.u.) together with the operating or limiting steam pressure and temperatures.

Similarly, specifications for safety valves should give the steaming capacity of the boiler in one of the three ways expressed, and if the second way (*b*) is preferred and the total B.t.u. input is not at hand, the data should include the maximum weight or volume of fuel that can be consumed in the furnace and the heat content of the fuel as also stated, but *never* the grate area or the heating surface alone. Engineers and responsible authorities should give this subject much consideration, as progress demands a change in existing practice, and sufficient influence should be brought to bear upon them so that formulas for the determination of the maximum steam-generating capacity of a complete boiler installation, based on fundamental facts, will be adopted by them and incorporated in all codes regulating boilers.

Examples

Specific examples may well illustrate the suggested methods:

CALCULATIONS FOR STEAM-GENERATING CAPACITIES OF BOILERS

(Using Formula [1])

Example 1 Original Condition.

Data: Maximum coal fired = $6\frac{1}{2}$ tons per hr.
= 13,000 lb. per hr.
Heat content of coal = 14,500 B.t.u.
Overall boiler efficiency = 80 per cent
Maximum steam pressure = 385 lb. gage
Maximum superheat. = 150 deg. fahr.
Feedwater temperature = 445 deg. fahr.

W = pounds of steam generated per hour

$$= \frac{13{,}000 \times 14{,}500 \times 0.80}{422 + 780 + 0.5(150) - 422} = \frac{150{,}800{,}000}{855} = 176{,}300 \text{ lb. per hr.}$$

Example 2 Firing Rate Increased.

Data: Same as for Example 1 except
Maximum coal fired = 8 tons per hr.
= 16,000 lb. per hr.

$$W = \frac{16{,}000 \times 14{,}500 \times 0.80}{422 + 780 + 0.5(150) - 422} = \frac{185{,}600{,}000}{855} = 217{,}000 \text{ lb. per hr.}$$

Example 3 Assuming 100 Per Cent Efficiency.

Data: Same as for Example 1 except
Overall boiler efficiency = 100 per cent.

$$W = \frac{13{,}000 \times 14{,}500 \times 1.00}{422 + 780 + 0.5(150) - 422} = \frac{188{,}500{,}000}{855} = 220{,}400 \text{ lb. per hr.}$$

CALCULATIONS FOR SAFETY VALVES

(For conditions given in Examples 1, 2, and 3)

Example 4 Original Conditions.

Data: Steam-generating capacity (W) from Example 1.
Discharge capacity of safety valve at 385 lb. = 56,000 lb. per hr.

N = number of safety valves required for boiler.

$$N = \frac{176{,}300}{56{,}000} = 3.15. \qquad \textit{Use 4 valves.}$$

Example 5 Firing Rate Increased.

Data: Steam-generating capacity (W) from Example 2.
Discharge capacity of safety valve at 385 lb. = 56,000 lb. per hr.

$$N = \frac{217{,}000}{56{,}000} = 3.88. \qquad \textit{Use 4 valves.}$$

Example 6 Assuming 100 Per Cent Efficiency.

Data: Steam-generating capacity (W) from Example 3.
Discharge capacity of safety valve at 385 lb. = 56,000 lb. per hr.

$$N = \frac{220{,}400}{56{,}000} = 3.95. \qquad \textit{Use 4 valves.}$$

In the foregoing examples it was assumed, first, that the original design data were used; second, that a change in the firing equipment was made, the increase in rating being known; and third, that a change in the firing equipment was made, the increase in rating being unknown. However, the resulting number of safety valves required for each case, as calculated in Examples 4, 5, and 6, is the same, that is, four, justifying the suggested use of 100 per cent efficiency in such calculations when the firing rate of a given boiler installation is unknown, as there is no penalty imposed in the form of the cost of additional valve equipment.

FSP-51-30a

Analysis of Costs of Variable-Load and Multiple Power Supply

By EDWARD H. SCOFIELD,[1] MINNEAPOLIS, MINN.

This study is based upon the actual performance of a plant in operation and includes a classification of costs designed to facilitate the calculation of the influence made thereon by a load of variable demand. Also it attempts to simplify the consideration of load characteristics and to determine the value of constants relating to certain factors of operation. Reference is made to the application of the results to a study for determining the economics of a power supply for the local electric railway system.

IN CONNECTION with an electric power load having a variable demand, the problem of costs as influenced by such variable demand becomes important. When such a power load is supplied from several sources, the problem becomes involved and must include a consideration of the various factors of manufacturing costs, the practical and economical distribution of the load to the sources of supply, and the apportionment of charges to the various operations to which this power is applied.

The problem in some cases also extends to a consideration of transmission, conversion, and distribution economies. The manufacturing costs may be listed under four general classes:

Fixed costs, including investment, overhead, and management costs, which are determined mainly by capacity of the plant, the character of its design and the amount of reserve units provided. These costs in the aggregate for an established plant are not influenced by the volume of output.

Service costs, including radiation losses, an element of attendance labor, idling or floating operation of power units, operation of house service and certain auxiliary equipment, emergency provision and other ready-to-serve elements, which are determined by the used capacity of plant during any period, and the policy of management regarding reliability of service, and not influenced by the output of the plant.

Demand costs, including the starting, stopping, and banking of power units, a part of fuel, labor, supplies, and maintenance cost, which costs are governed by the maximum power demand over a limited period. This maximum demand is usually considered as the average of daily maximum one-hour peak loads, occurring during an operating period of one month. The value of the constants determined for this factor will of course depend upon the manner in which the maximum demand is determined.

Output costs, including all operating costs for fuel, labor, supplies, and maintenance, which vary directly with the volume of net output of the plant, measured in kilowatt-hours, or average kilowatts of continuous output over a definite period.

The determination of values of coefficients for these various factors of cost is a matter of analysis. In predetermining the operating cost for a new plant, this analysis must be based upon equipment manufacturers' guaranteed performance and shop-test data as compiled and modified by the experience of the engineer. Some margin should be added to the summation of individual unit performance.

After a plant has been put into operation, observations may indicate the necessity of modifying the value of some of the items assumed in the preliminary determination. After a plant has been in operation over an extended period, and particularly where the conditions are such as to give experience over a wide range of power demand, detailed data may become available, from which an analysis may be made and values obtained which may differ materially from the predetermined value. This is true particularly because of the unknown and emergency conditions of operation, the divergence of variable-load performances from fixed-load performance of power units, the depreciation of performance due to efficiency of upkeep, lack of knowledge covering the character of load demand, changes in quality and price of fuel, supplies and wages, and operating conditions of labor.

Study to Determine Power Efficiency

With such data now available from the operation of the Twin Cities Electric Railway plant, a brief description of which appears later, over a period of 25 years, a study and analysis has been made and certain values for the factors and relations of costs have been determined. This study was undertaken for the purpose both of determining the economies resulting from the water powers now in use and for estimating the possible cost for additional power purchased, or the replacement of the present supply by a new steam power plant. As this study is still in progress it will be impossible to consider it in all its details or to give results or conclusions. All that is attempted is to indicate the general method employed in arriving at certain details of costs.

For properly evaluating the various items and factors of operation and expense, a number of charts are presented. As these charts are more or less general in character, the scale of ordinates in some cases is decimal. For specific values a proper constant is to be applied. For example, fuel costs as indicated on the charts will be modified by quality and price, also by efficiency of performance due to temperature and selection of generating units; labor costs will be modified by wage rates and conditions of employment; and supply costs by the variation of commodity prices.

In the operation of a steam plant the chief item of expense in this locality is fuel. The cost of fuel is made up largely of transportation and handling charges, which amount to from 50 to 70 per cent of the total cost. This makes the factor of fuel performance of first importance. The fuel performance depends upon the efficiency of firing, the adaptability of the furnace and boiler equipment to the coal used, the efficiency of generating and auxiliary units, the management of the plant, the seasonal weather conditions, and the character of load demand. On account of the number of variables involved and of the great divergence of their values, it seems impracticable to solve for values of all such variables by a mathematical process. However, it seems that by selecting a period over which the general conditions are fairly constant or vary over a limited range, plant performance may be grouped with relation to certain major factors and the data from such grouping introduced into equations, from which the relative values of these variables can be solved.

An attempt at a solution was made from data covering a period of thirty-six months' operation of the plant described at the end of the article. The method of grouping the data of major

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Contributed by the Power Division and presented at the Summer Meeting, St. Paul-Minneapolis, Minn., August 27 to 30, 1928, of The American Society of Mechanical Engineers.

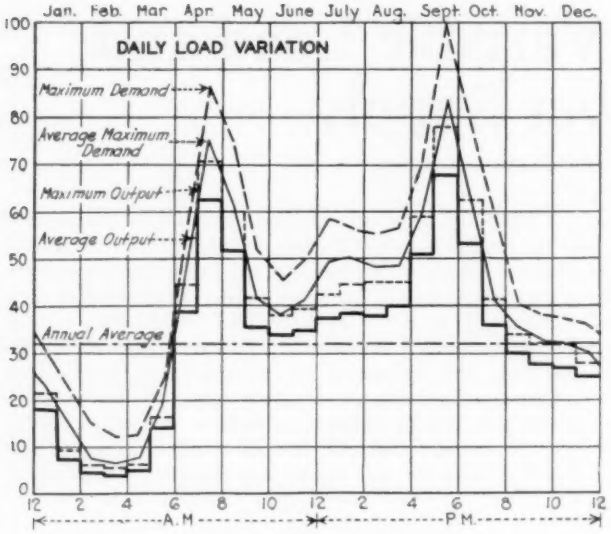


FIG. 1

(Shows the essential data of a typical urban electric-railway load in its relation to the hourly demand and the departure from normal of the maximum individual hours of the year, the average maximum hours during the months, and the maximum monthly average.)

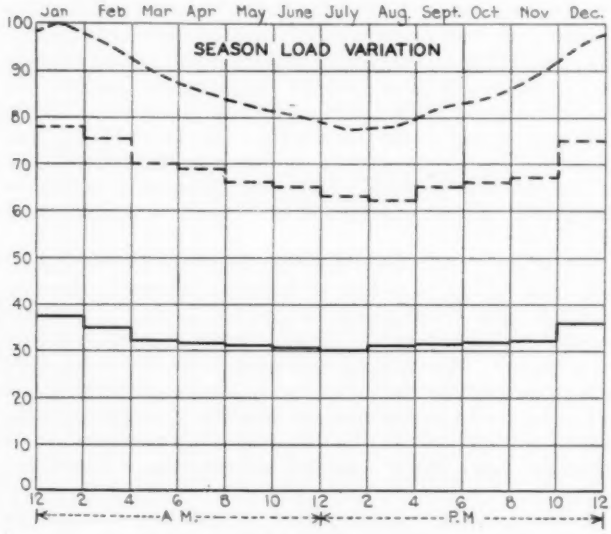


FIG. 2

(Shows the seasonal variation of the same data as in Fig. 1 for the total system load.)

importance and also the solution for value of constants are given in Table 1.

Fig. 5 was plotted to determine the relation of monthly average maximum one-hour demand to average output. This relation is described by the formula

Maximum kw. demand

= (1.5 average kw. output + 11,000 kw.) ± 10 per cent

The individual daily one-hour peak may, however, have a deviation as high as 50 per cent.

Fig. 6 was also plotted to show the relation of coal used to average output (the coal quantities being reduced to an equivalent heat value). This chart indicated a definite general relation with a deviation corresponding in character with the demand deviation, suggesting that the general form of equation should include a factor for maximum demand in addition to average output.

LOADING AND TEMPERATURE THE MAIN INFLUENCES

A controlling influence in the fuel rate of this plant is the loading of the preferential electric-generating unit. Also in plants of this character, the temperature, as relating to circulating water, air for combustion, condensate, and radiation losses, has a material influence on fuel rate. To compensate for these items, percentage factors are introduced. In the problem worked out these factors are made to modify only the average output factor. From some considerations it might be thought advisable to apply the temperature coefficient to both power factors.

The formula for fuel performance based on the foregoing influences may take the form

$$\text{Coal, lb.} = RX + Sy\left(\frac{100 \pm Ta \pm Ub}{100}\right)$$

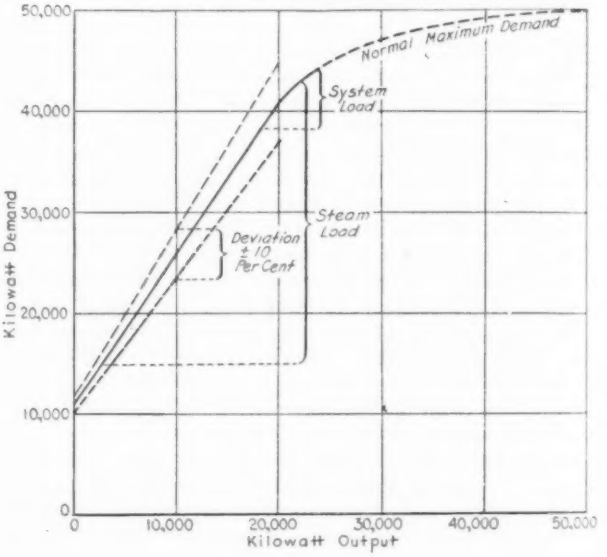


FIG. 3

(See description under Fig. 4.)

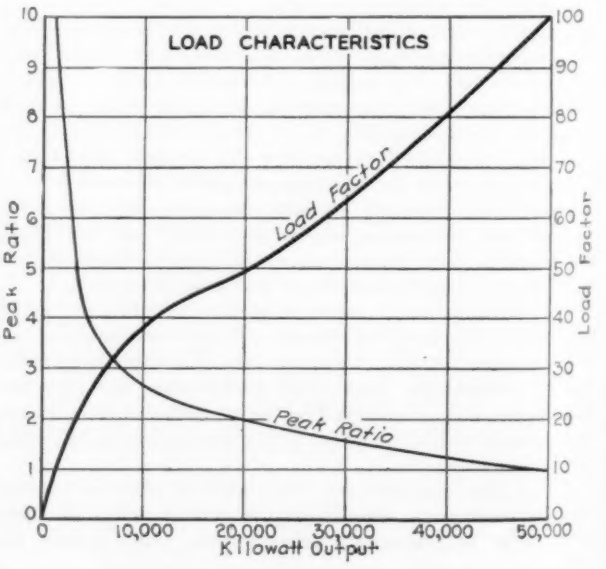


FIG. 4

(This, with Fig. 3, shows the relation of maximum demand to output were the loading of a plant carried to the ultimate of 100 per cent plant factor, and the corresponding load factor and peak ratio resulting therefrom.)

Where

R = average of one-hour peaks for each day of the month, kilowatts

S = average continuous output for the month, kilowatts

T = departure of temperature from normal, 45 deg. fahr.

U = departure of loading of No. 4 turbine unit from normal, 50 per cent of total turbine hours.

A solution gives values for constants as follows:

Maximum kw. factor	X	= 6.0
Average kw. factor	y	= 56.4
Temperature factor	a	= ±0.18
Unit hours factor	b	= —0.2

Substituting the values of these coefficients in the formula,

$$\text{Coal, lb. per day} = 6.0R + 56.4S\left(\frac{100 \pm 0.18T \pm 0.2U}{100}\right)$$

TABLE 1 MONTHLY AVERAGE PERFORMANCE

Maximum kw.	Average kw.	Temp., deg. fahr.	Unit No. 4, per cent	Coal, 1000 lb.
37,232	17,130	14	51	1184
37,861	16,770	22	52.5	1084
35,048	15,330	17	54.3	1036
34,990	16,070	17	55.6	1072
35,511	16,120	24	55.2	1064
33,410	14,430	15	51.8	954
32,656	14,290	14	58	944
32,573	13,950	23	60	930
33,687	16,590	9	52.8	1074
312,968	140,680	155	491.2	9342
34,774	15,631	17.2	54.6	1038
31,304	12,150	35	64.3	806
30,474	12,540	38	65.5	878
31,027	13,520	33	53.7	922
32,112	12,820	26	56.3	908
29,042	12,500	64	67	846
27,686	12,280	73	55.8	902
23,640	6,820	36	59	514
21,747	6,640	67	71.5	490
27,071	10,560	30	67	730
254,103	99,830	402	560.1	6996
28,234	11,092	44.7	62.2	777.3
25,791	9,000	53	70	700
27,874	11,800	72	68	874
27,213	10,800	66	41	810
23,352	11,650	62	61.4	834
27,037	11,710	70	17	740
26,096	9,825	28	69.7	684
19,754	5,950	70	60	430
20,236	6,050	63	53	492
20,098	6,180	52	40.4	500
217,451	82,965	536	480.5	6064
24,161	9,218	59.6	53.4	673.8
24,278	9,070	55	70	650
23,784	7,450	67	71	572
24,265	8,570	71	70	670
25,797	8,510	44	58.8	618
17,723	4,725	57	51.2	400
17,981	4,220	47	58	350
17,607	3,525	46	18	314
15,535	2,900	54	0	266
16,523	3,230	65	28.5	282
183,493	52,200	506	425.5	4122
20,388	5,800	56.2	47.3	458

$$\begin{aligned}
X + 0.450y\,(1 - 0.278a + 0.046b) &= 29.85\\
X + 0.393y\,(1 - 0.003a + 0.122b) &= 27.55\\
X + 0.382y\,(1 + 0.146a + 0.034b) &= 27.9\\
X + 0.285y\,(1 + 0.112a - 0.027b) &= 22.46\\
X + (0.450 - 0.1250a + 0.0207b)y &= 29.85\\
X + (0.393 - 0.0012a + 0.0480b)y &= 27.55\\
X + (0.382 + 0.0558a + 0.0130b)y &= 27.9\\
X + (0.285 + 0.0319 - 0.0077b)y &= 22.46\\
(0.057 - 0.1238a - 0.0273b)y &= 2.30\\
+ 0.0249 - 0.0539a - 0.0119b &= +1/y\\
(0.011 - 0.057a + 0.035b)y &= -0.35\\
- 0.0314 + 0.163a - 0.100b &= +1/y\\
(0.097 + 0.0239a + 0.0207b)y &= 5.44\\
+ 0.0178 + 0.0044a + 0.0039b &= +1/y\\
+ 0.0563 - 0.2169a + 0.0881b &= 0\\
+ 0.0411 - 0.1586a + 0.0644b &= 0\\
- 0.0492 + 0.1586a - 0.1039b &= 0\\
- 0.0081 - 0.0395b &= 0\\
b &= -0.204\\
a &= +0.1765\\
y &= +56.35\\
X &= +6
\end{aligned}$$

As a check upon these values of coefficients a calculation was made by this formula for the coal demand by months for the three-year period considered in the problem. The results of this calculation varied from the actual amounts charged to operation less than one-half of 1 per cent for the total period, 2 per cent for a one-year period, and monthly within 6 per cent,

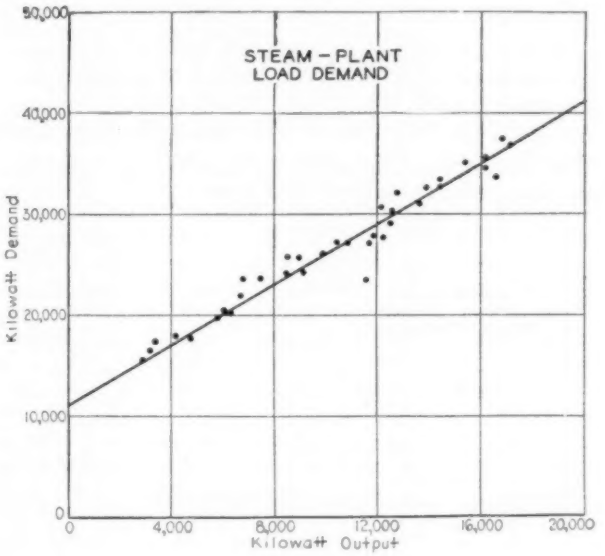


FIG. 5

(Shows the actual relation of maximum demand to output of the load imposed on a steam plant where a variable base load of the power system is diverted to a water power having no storage capacity. This chart and that shown in Fig. 9 coordinate when the proper constant factor is applied.)

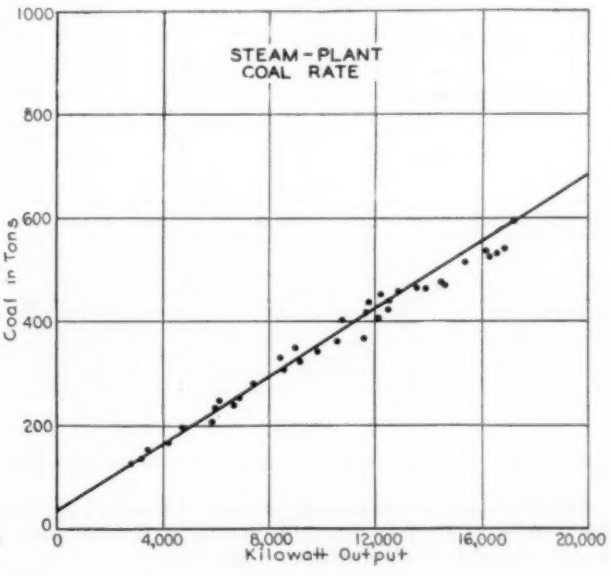


FIG. 6

(Shows the relation of coal demand to output, and in conjunction with chart shown in Fig. 5, indicates the influence of maximum demand upon coal demand.)

with the exception of two months when arbitrary adjustments had been made of the amount of fuel charged to the plant.

These satisfactory results were obtained notwithstanding the fact that the amount of coal charged to the plant each month is arrived at by methods which are subject to error as high as 5 per cent.

From these results the following deductions may be made:

The basic coal rate for this plant under normal conditions—i.e., 45 deg. fahr. temperature and 50 per cent turbine hours loading on No. 4 unit—is 56 lb. of coal per day average kilowatt, equal to 2.33 lb. of coal per kw-hr.

The basic rate is increased by 6 lb. of coal per day per kilowatt of 1-hr. maximum demand. This amount added to the basic rate gives the total coal rate as follows:

Load factor 100%	0.25	+ 2.33	=	2.58 lb. per kw-hr.
Load factor 50%	0.50	+ 2.33	=	2.83 lb. per kw-hr.
Load factor 33⅓%	0.75	+ 2.33	=	3.08 lb. per kw-hr.
Load factor 25%	1.00	+ 2.33	=	3.33 lb. per kw-hr.

The basic rate is increased or decreased directly 1 per cent for a change in temperature of 5 deg. fahr.

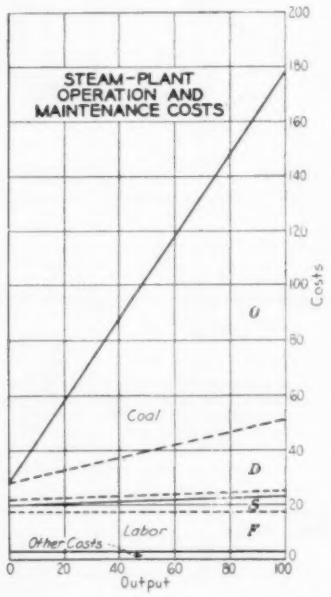


Fig. 7

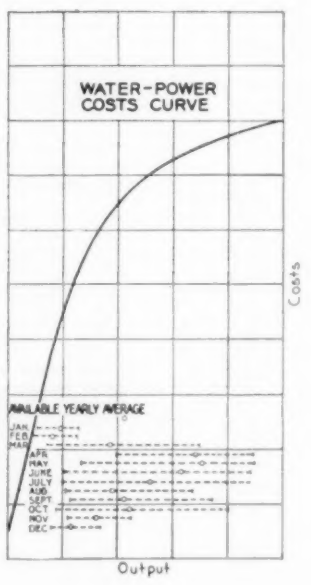


Fig. 8

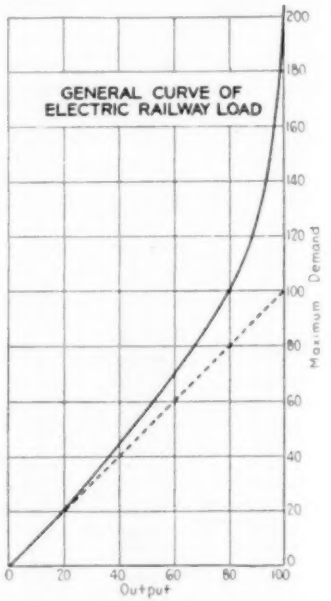


Fig. 9

FIG. 7

(Shows the normal relation, throughout the range of ouput, of the general division of operation and maintenance expense—i.e., fuel, labor, and other costs. These are indicated by the full-line division of the chart. For any output referred to the base scale the expense of each item is indicated by the distance between division lines referred to the vertical scale, and the total expense by the upper line. This chart is plotted from the actual operating data of a plant in which the monthly output varied from point 20 to point 80 of the chart. The secondary subdivision of this chart by the broken lines into parts—F, fixed; S, service; D, demand; and O, output—is for the purpose of illustrating the discussion, at the beginning of the article, on manufacturing costs. The determination of values of subdivisions O and D is the basis of the discussion of the items of expense in the operation of a steam plant.)

FIG. 8

(Shows the variable availability and trend of costs of the water power used on Twin City Electric Railway system. The circles indicate for a period of 10 years the annual and monthly averages of output available. The broken lines indicate the variation of the monthly averages during this period. The vertical ordinates of the full-line curve show the cost of any output.)

FIG. 9

(Shows the general characteristic of an electric railway load. This chart is plotted in terms of the base-load outputs expressed in percentage of total average output for any period to the base scale and the maximum demand of such base load in percentage of total average output to the vertical scale. That is, this chart indicates the peak ratio or inverse-load factor of any base-load portion of the total load, or by differential the load factor of any portion of the total load. The extended broken line indicates a condition of 100 per cent load factor for the total output. From this chart may be determined approximately the characteristics of the loads which may result from a division of load to two or more sources of power.)

The basic rate is increased or decreased inversely 1 per cent for a change of 5 per cent in the percentage of turbine hours of service carried by the preferential turbine unit.

Arriving at the Fuel Costs

A summation of these four factors gives the total coal rate for any character of load under any condition of operation, and this rate multiplied by the current price of coal delivered to the plant gives the fuel costs per kilowatt-hour.

As the rate per kilowatt of maximum demand includes such losses as are entailed by banking of boiler equipment, warming up, and starting and stopping of generating units and main auxiliary units, this item would in great part be eliminated if the plant ran continuously on a 100 per cent load. In this case the continuous coal rate would approach 2.33 lb. per kw-hr. Were the plant equipped fully by generating units such as the No. 4 unit, this rate would be reduced by 10 per cent to 2.1 lb. per kw-hr., which corresponds closely with the estimated performance based on test data of the plant equipment.

Having determined the normal coal rate and costs for any period by the foregoing method, the labor and supplies costs may be added and the total costs for operation and maintenance with relation to output determined as indicated by Fig. 7. The amount and trend of these added costs are based on performance of the plant as modified by variation in wages and prices.

By coordinating the costs thus determined for a varying output of the steam plant with costs as shown on Fig. 8 for a varying water-power supply, the aggregate cost of any load demand is determined, the characteristics of loads on each plant being indicated by reference to Fig. 9.

Other diversion of load to or from the steam plant may be made and the characteristics of load and its influence on cost of output determined by these charts and formulas.

The results of changes in equipment and methods of operating or of the price and quality of fuel may be satisfactorily analyzed and compared by this method.

Description of Power System

This study is based on data collected from the operation and development of the power supply for an electric-railway system comprising approximately 400 miles of single trackage

and operating 1000 cars with an average weight of 24 tons. The development covers a period of 40 years, during which it has passed successively through every phase of electric-railway improvement.

The original power supply consisted of hand-fired boilers and triple-expansion engines belted to small high-speed direct-current generators. About the year 1897 this was supplemented by 10,000 hp. of water power generating alternating-current power which was distributed through rotary converter substations. This water power was leased on a 40-year contract. The payments thereof were on a sliding scale based upon the amount of available power. These powers becoming inadequate, in 1902 a considerable capacity of storage batteries was added as a temporary relief. About 1904 plans for an adequate steam-plant auxiliary were made, and an initial capacity of equipment was installed, consisting of overfeed stokers, water-tube boilers, vertical compound engines direct-connected to 13,200-volt three-phase alternating-current generators. Units of this description were added as required, and the original steam plant and the storage batteries were abandoned. About 1906 two vertical Curtis-type steam turbines of 5000-kw. capacity were added. Following this, overfeed stokers were replaced by chain grates, and additional smoke-stack capacity was provided.

In 1908 an added block of 12,000 hp. of water power was added on a lease, to terminate at the same time as the original water-power lease. The payments on this lease are at a flat rate per year. Neither of these water powers have appreciable pondage for seasonal or daily storage.

From 1909 to 1917 the four 3500-kw. engine-driven units were replaced by three 15,000-kw. vertical Curtis-type turbo-generators, and in 1917 by a 20,000-kw. horizontal Curtis-type turbo-generator. About 1922 a number of chain grates were replaced by underfeed stokers, and increased superheater surface was added in the boilers.

The present continuous rated capacity of this steam plant is 50,000 kw. The combined capacity of the water power has a seasonal variation from 1000 kw. in winter to 16,000 kw. during favorable water supply. The aggregate output of water power available for use on the railway load averages over a period of years approximately 40 per cent of the load demand. This output has a seasonal variation from 12 per cent to 62 per cent and an annual variation from 30 per cent to 60 per cent of the entire load demand. As a result of this variable water power, the auxiliary steam plant is called upon for a very unfavorable load as to economy of operation. The total load demand approximates an average daily load factor of 50 per cent. The load on the steam plant at times is below 20 per cent load factor.

This condition led to a study of the economy of the use of these combined powers, also of the possibilities of replacement of a part or all of these powers by a supply of power from the local light and power company, whose distribution covers the same territory as that covered by the railway system.

A formidable obstacle to such a plan was the condition that the railway power system had been built up on a frequency of 35 cycles, while the power system was at 60 cycles. On the other hand an advantage for the plan existed in the condition that the existing transmission and substation system of the railway was overreached. Car lines were extended beyond the economic distance for trolley-current distribution, and the installation of added substations in such localities would be attended with high costs for extending transmission lines.

The problem therefore includes a study of transmission, conversion, and distribution economy, in addition to costs of generation and supply.

FSP-51-30b

Operation of a Combined Steam and Hydro System

A Discussion of Operating Problems of the Northern States Power Company's System Located in Minnesota and Wisconsin

By JAMES A. COLVIN,[1] MINNEAPOLIS, MINN.

WITH THE growth of the public-utility industry, the responsibility of meeting increased power requirements and providing higher standards of service has presented problems in every system. In an interconnected steam and hydro system which extends over a wide area and which contains a main load center located at some distance from almost one-half of the available energy, and with other scattered load centers, the task of providing continuous and adequate service is quite an undertaking. The maintenance of a continuous supply of power and the regulation of the voltage at substation and point of use constitute the most important of these problems.

To obtain the maximum overall economy consistent with good service is the aim of the operating organization of the system. To do this, full advantage of the diversity of stream flow and of possible reservoir and pondage manipulation must be realized and these operations must be coordinated with the establishment on the steam plants of the most economical load factor that is consistent with the seasonal variations of energy available from hydraulic sources. In order fully to accomplish this aim for high overall economy it is necessary to have centralized control of the system and also communication systems through which the operation of all plants of the system may be properly coordinated.

The general problems of all combined steam and hydro systems are similar, but each system also has certain peculiarities which apply to it alone. These peculiarities result from the method in which the particular system has been developed, the area over which it is spread, the climatic conditions, and the ratio of steam to hydro capacity. The author will endeavor to present some of the problems of operation which obtain on the interconnected system of the Northern States Power Company to which his direct experience has been limited.

Description of Interconnected System

The following brief description, combined with the sketch shown in Fig. 1 in which a small portion of the primary transmission system is shown and certain plants are located, will serve to present a picture of the system under consideration.

The system extends over an irregular area approximately 250 miles east and west and 150 miles north and south. Approximately 75 per cent of the load is concentrated in the small area of Minneapolis and St. Paul.

The heart of the transmission system consists of a 110-kv. loop which encircles the Twin Cities. This loop encloses an area of approximately 200 square miles, and to it are connected two major steam plants (which have a total capacity of 152,000 kw. and which are located inside the loop), and two 110-kv. lines interconnecting 64,000 kw. of hydro capacity from outside the loop. This hydro capacity is obtained from three plants located in middle-western Wisconsin approximately 100 miles east of the loop. The above-mentioned steam and hydro plants, together with one other hydro plant of 21,400 kw. capacity, which feeds through one of the major substations over two 50-kv. lines approximately 40 miles long, constitute the major sources of power of this system. There are, in addition, some eighteen other smaller hydro and steam plants which are connected at various points on the system.

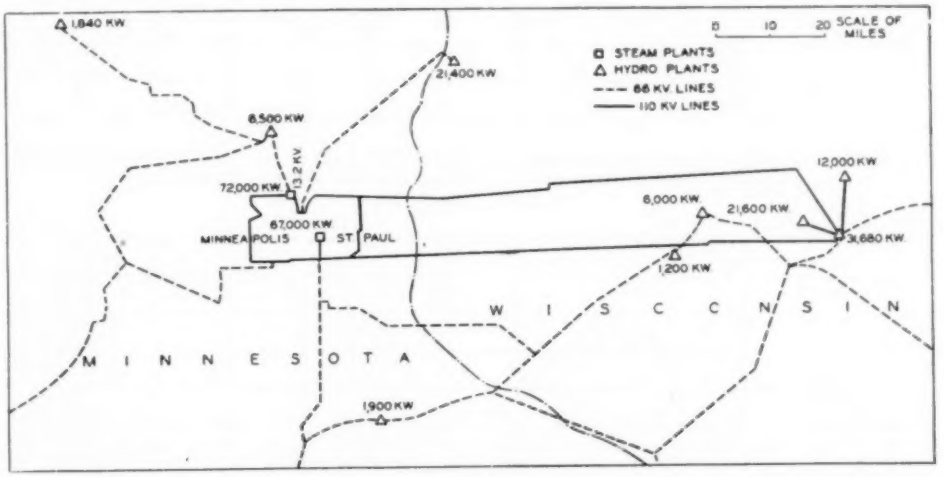


FIG. 1 PRIMARY TRANSMISSION SYSTEM OF THE NORTHERN STATES POWER CO. SHOWING PRINCIPAL TRANSMISSION LINES AND GENERATING STATIONS

Major substations as well as the steam plants, which, in addition to supplying electrical energy, act as major substations, are tapped on the 110-kv. loop. These major substations take care of the entire load of the 110-kv. loop which as mentioned before is approximately 75 per cent of the system load. A little less than one-half of the remainder of the load has for its main source of power a substation connected to the largest of the three hydro plants in Wisconsin, referred to above.

Development of the System

The development and growth of a combination steam and hydro system, where the annual output from the two sources is fairly well balanced, requires a more involved analysis than that

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required by systems which are mainly hydro or solely steam.

Several years are sometimes required from the time a decision is reached to construct a certain hydro development until the plant is actually complete, on the line, and ready for service. It therefore follows that studies on which decisions are based to develop either additional steam- or hydro-plant capacity must be projected many years in advance, and the predicted outputs of these plants coordinated with reference to the anticipated system load curve. Factors must be applied to adjust for line losses and to compensate for the potential capacity of existing hydro plants, the full utilization of which may not have been realized under existing load conditions because of lack of system absorbing capacity during the off-peak period.

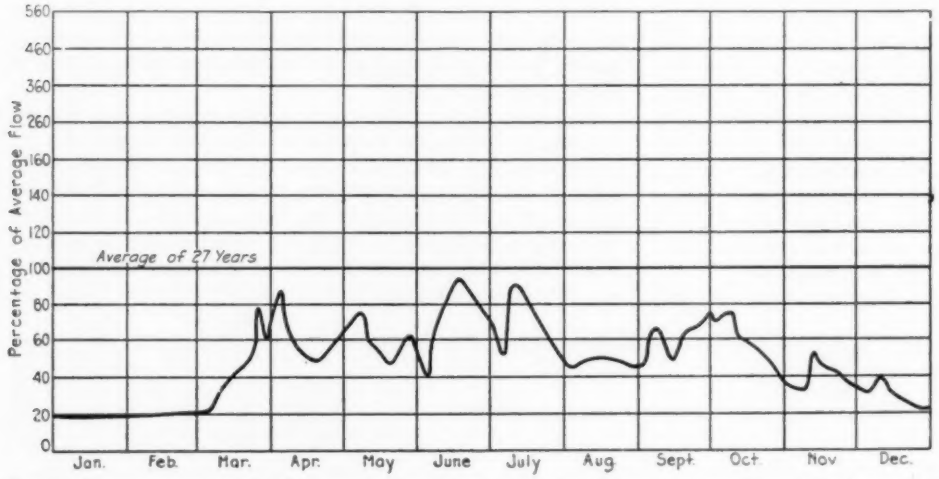


FIG. 2 HYDROGRAPH OF THE MISSISSIPPI RIVER AT COON RAPIDS, MINN., FOR THE YEAR 1925 SHOWING STREAM FLOW IN PERCENTAGE OF AVERAGE FLOW FOR PAST 27 YEARS
(Shows low stream flow.)

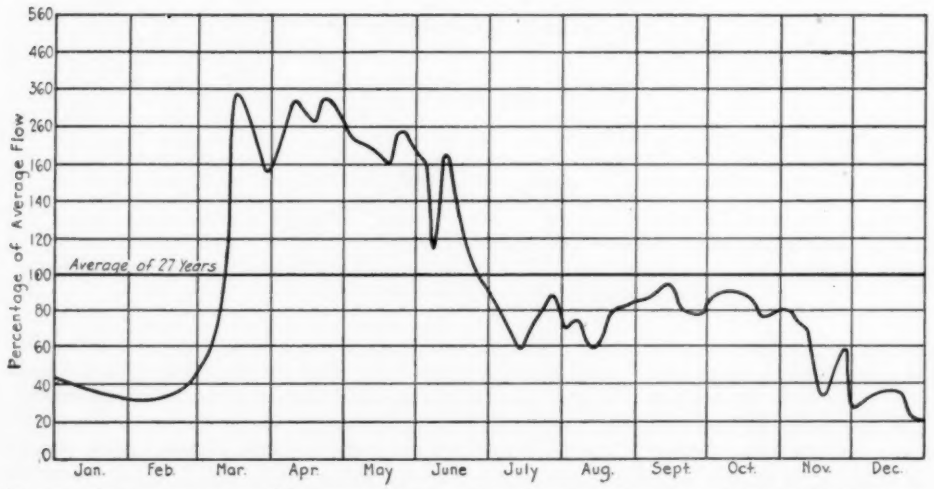


FIG. 3 HYDROGRAPH OF THE MISSISSIPPI RIVER AT COON RAPIDS, MINN., FOR THE YEAR 1927 SHOWING STREAM FLOW IN PERCENTAGE OF AVERAGE FLOW FOR PAST 27 YEARS
(Shows high stream flow.)

The useful capacity of the maximum system requirements must be considered on the basis of anticipated minimum flow conditions as they exist naturally, or when supplemented by discharges from storage reservoirs. Usually when there is a comparatively high proportion of hydro-plant capacity with good pondage facilities and control, the maximum energy output of a projected hydro plant that can be assured during the fourteen to sixteen hours of heavy load of the annual peak day or days will control its economical justification from a capacity standpoint. Under other conditions or stages of development, for instance, where the proportion of steam-plant capacity is greater, the maximum assured capacity of the projected hydro plant during the maximum annual demand period, i.e., the two- or three-hour period of peak demand, will control its justification from a capacity standpoint. Thus, the same plant in a combined steam and hydro system may have entirely different importance, depending upon the period of system development.

The minimum stream flow and the corresponding minimum output of a hydro plant may be fairly accurately predicted from a study of the records over a large number of years, but no accurate forecast can be made of the exact year in which this minimum may occur. In the operation of the system the operating organization must be fully aware of the possible variations in stream flow and also have a good knowledge of the plan on which the system was developed in order to be prepared to meet anticipated load requirements with the equipment available.

In the regulation of reservoirs that control seasonal fluctuations of stream flow, care must be taken so that sufficient storage is retained for maximum system demands—even though losses in power may occur by non-utilization of storage which sometimes might be refilled by subsequent floods. The extent to which stream-flow variations occur, even with limited reservoir control, is best shown in Figs. 2, 3, 4, and 5, which show actual stream flows of two streams for two different years. The average flow over a large number of years is shown by the heavy horizontal line.

In this section of the country maximum flood conditions occur during the months of April and May, and they may occur at any time for short intervals between June 1 and November 15. This period of possible floods is a period when the system load requirements are a little less than average. The maximum system demand period, however, occurs in the early part of December, at which time natural stream flows are usually at very nearly minimum stages. Seasonal variations in system loads are shown in Figs. 6, 7, 8, 9, and 10, which also indicate typical steam and hydro outputs which are obtained during these periods.

VOLTAGE REGULATION

Probably the most important feature of a combined steam and hydro system is the problem of maintaining satisfactory voltage

throughout the system in the face of extreme variations in power flow. In Fig. 1 the two 110-kv. lines, approximately 100 miles long, may at times carry as much as 65,000 kw. to the largest load center. At other times such as periods of low hydro output, even during normal load periods, the power transmitted over the lines is very little. Under night-load conditions the power supply may be actually reversed. In order properly to regulate the transmission voltage under these varying conditions it has been found necessary to carry on the line approximately 75 per cent of the steam capacity normally required for power generation during the seasonal peak period. This is done in order to absorb reactive kva. and maintain voltage within the range of regulation and compensation provided on distribution feeders.

During the excess hydro period it is necessary to generate a comparatively small amount of energy on the steam plants, and at times there is sufficient capacity at the hydro plants to carry the entire system load. Unless special provisions are made for keeping the turbine blading and wheels from overheating, the manufacturers of large turbine units generally recommend carrying a minimum load of not less than 10 per cent of the rated capacity of the unit. On 30,000-kw. units this would require between two and three tons of coal per hour, the majority of which might be saved by arranging a proper supply of cooling steam to the turbine and carefully checking the turbine temperatures. With these precautions the turbo-generator units may be "motored" as synchronous condensers.

The possible number of hours that it would be desirable to "motor" the units would, of course, depend upon the ratio of hydro-plant capacity to the system load. A study of load curves for a system which has a hydro capacity during flood periods of 85 per cent of the system peak at those times indicates that it would be possible to "motor" normal-sized steam units for a total of 1000 to 1500 hours a year. If units are "motored" it is possible to regulate the system voltage as well as to have "on the line" an immediate source of reserve capacity for protection against the failure of transmission lines carrying energy developed at distant hydro plants, and at the same time save 2000 to 3000 tons of coal per year.

Similarly, hydraulic units located at subsidiary load centers are "motored" with gates closed and vented draft tubes, thereby reducing the amount of power required to carry the unit on the line by 50 per cent or more, and thus conserving water for generation of power during peak periods. Steam units are "motored" during periods of maximum hydro output whereas hydro units are normally "motored" during periods of minimum hydro output. In this particular system the use of hydro and steam capacity, combined with certain motor-generator equipment used in the conversion of alternating current for direct-current systems, has provided sufficient synchronous condenser capacity to satisfactorily regulate the voltage of the system within the limits of regulators on distribution feeders, even though there are wide variations in the amount of load carried by the transmission lines. With the continued growth of substation loads and corresponding higher utilization of generating capacity, it is anticipated, however, that the installation of corrective equipment will be necessary.

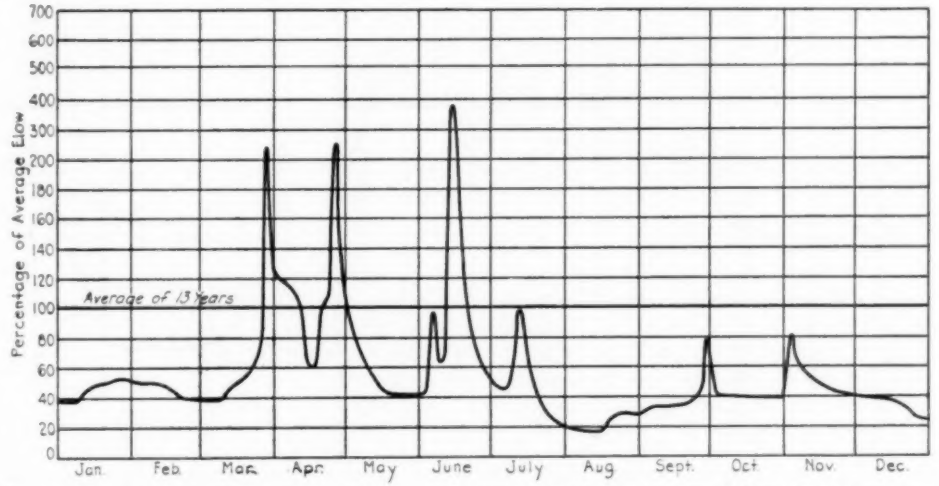


FIG. 4 HYDROGRAPH OF CHIPPEWA RIVER AT CHIPPEWA FALLS, WIS., FOR YEAR 1925 SHOWING STREAM FLOW IN PERCENTAGES OF AVERAGE FLOW FOR PAST 13 YEARS
(Shows low stream flow.)

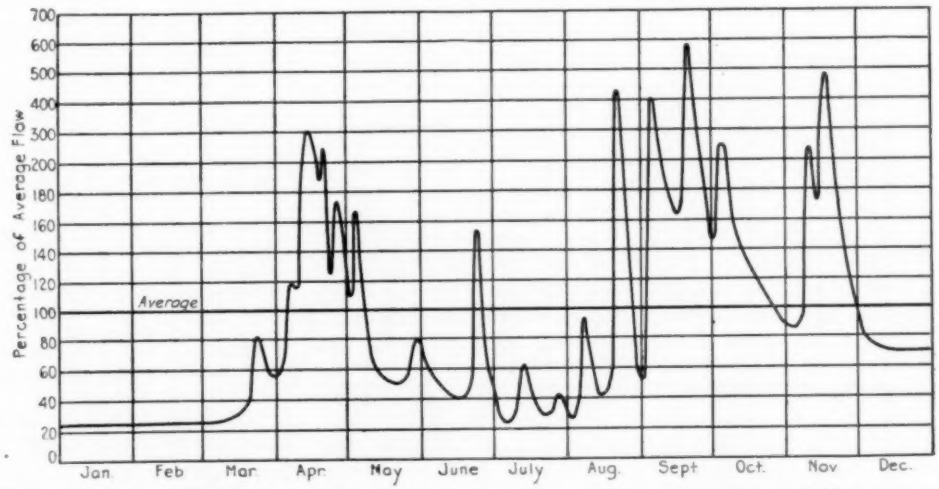


FIG. 5 HYDROGRAPH OF CHIPPEWA RIVER AT CHIPPEWA FALLS, WIS., FOR THE YEAR 1926 SHOWING STREAM FLOW IN PERCENTAGE OF AVERAGE FLOW FOR PAST 13 YEARS
(Shows relatively high flow.)

CHARACTERISTICS OF STEAM-PLANT OPERATION

Wide ranges of loading, seasonal variations of loading, and low-load operation are the outstanding characteristics of the steam-plant operation in a combined steam and hydro system. Generally, the comparatively low annual load factor at which steam plants are operated in such a system will not justify the instal-

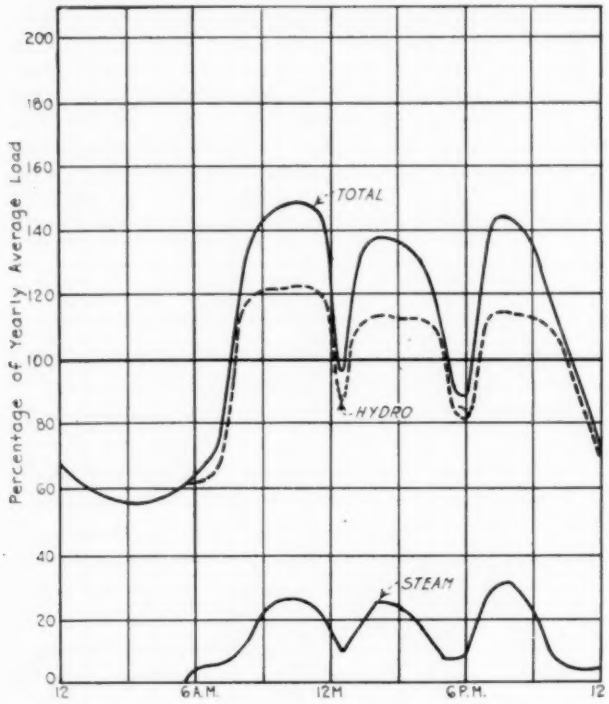


FIG. 6 TYPICAL DAILY SYSTEM LOAD DIAGRAM OF THE NORTHERN STATES POWER CO.'S INTERCONNECTED SYSTEM FOR A WEEK DAY IN MARCH SHOWING HYDRO, STEAM, AND TOTAL LOADS IN PERCENTAGE OF YEARLY AVERAGE LOAD

(Shows maximum hydro-minimum steam condition.)

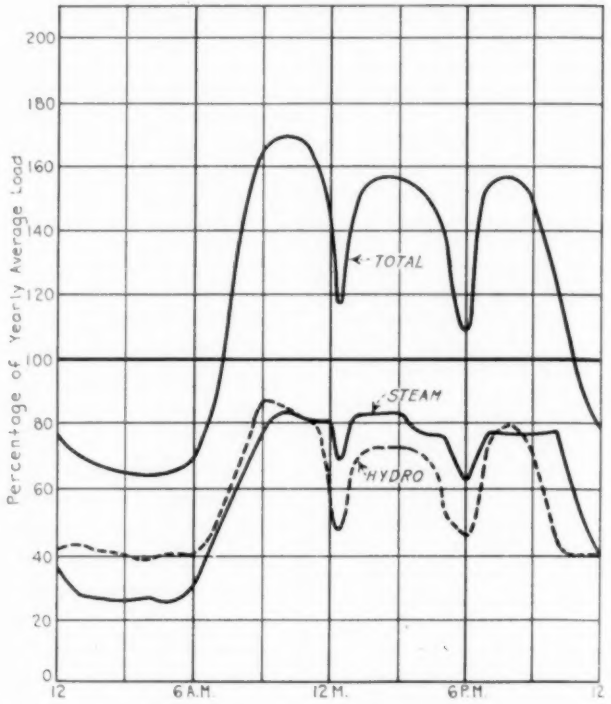


FIG. 8 TYPICAL DAILY SYSTEM LOAD DIAGRAM OF THE NORTHERN STATES POWER CO.'S INTERCONNECTED SYSTEM FOR A WEEK DAY IN SEPTEMBER SHOWING HYDRO, STEAM, AND TOTAL LOADS IN PERCENTAGE OF YEARLY AVERAGE LOADS

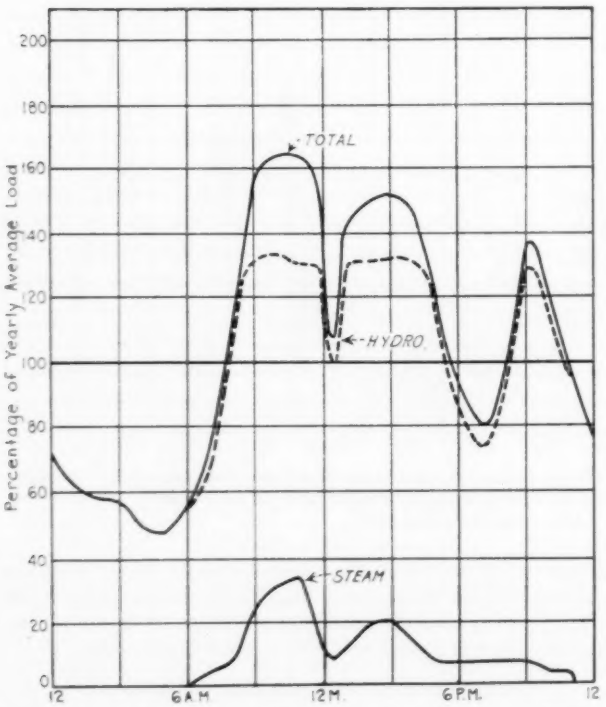


FIG. 7 DAILY SYSTEM LOAD DIAGRAM OF THE NORTHERN STATES POWER CO.'S INTERCONNECTED SYSTEM FOR A WEEK DAY IN JUNE SHOWING HYDRO, STEAM, AND TOTAL LOADS IN PERCENTAGE OF YEARLY AVERAGE LOAD

(Shows maximum hydro-minimum steam condition.)

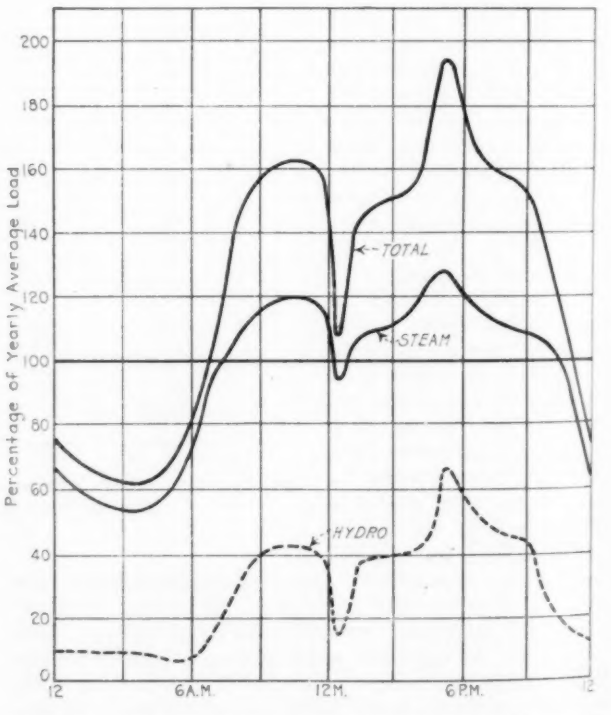


FIG. 9 TYPICAL DAILY SYSTEM LOAD DIAGRAM OF THE NORTHERN STATES POWER CO.'S INTERCONNECTED SYSTEMS FOR A WEEK DAY IN DECEMBER, SHOWING HYDRO, STEAM, AND TOTAL LOADS IN PERCENTAGE OF YEARLY AVERAGE LOADS

(Shows peak load and minimum hydro-maximum steam condition.)

lation of equipment producing high thermal efficiencies. More attention must be given to ruggedness of equipment, to its ability to withstand rapid starting and sudden load fluctuations and to provision of equipment that is capable of carrying wide ranges of load with the highest overall economy. Flexibility of operation is more important than high thermal economy.

As stated, no accurate prediction can be made of the hydro output which may be anticipated for any individual year; and this output from a group of hydro plants may exceed the minimum output by 50 per cent during an exceptionally good water year. This deviation from normal stream flow results in a corresponding variation in requirements for energy generated at steam plants. Coal contracts must be based upon minimum hydro conditions, or with sufficient tolerances to take care of the maximum anticipated steam production. In order to obtain favorable contracts, reasonable tolerances are permitted; but with the wide variations in hydro output, the amount of coal contracted for to meet the minimum hydro conditions may exceed actual requirements by as much as 50 per cent. This makes desirable the establishment of substantial storage facilities to eliminate unreasonable tolerances in the coal contracts.

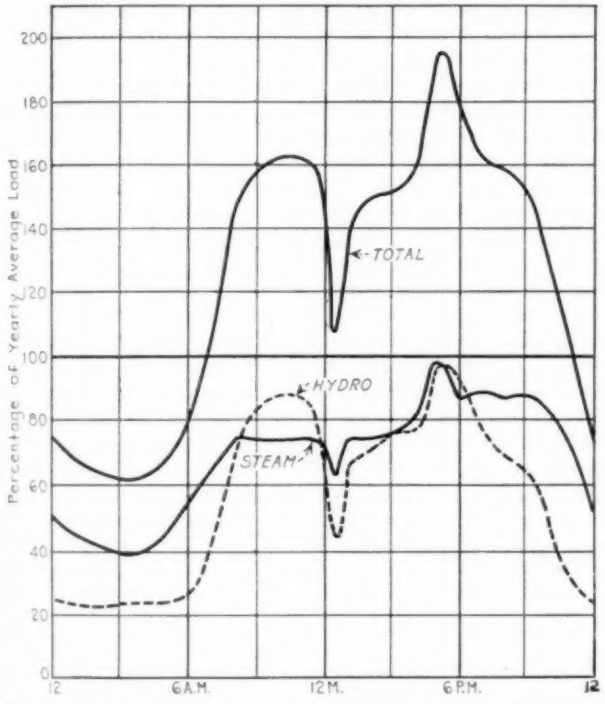


FIG. 10 TYPICAL DAILY SYSTEM LOAD DIAGRAM OF THE NORTHERN STATES POWER CO.'S INTERCONNECTED SYSTEM FOR A WEEK DAY IN DECEMBER SHOWING HYDRO, STEAM, AND TOTAL LOADS IN PERCENTAGE OF YEARLY AVERAGE LOADS
(Shows peak load with hydro and steam approximately equally divided.)

During the summer months changes of scheduled operations will vary on extremely short notice following sudden rains, and make it absolutely essential for the steam supervisory organization to be kept in constant touch with the system operating conditions in order to establish economical operating schedules to meet the different requirements.

During the normal excess-water period it is necessary to carry sufficient capacity in service to protect against transmission-line failures, and to regulate properly the voltage of the system. The operation of the boilers during this period was analyzed, and it was found that by applying the same careful study to this low-load operation as was applied to heavy load conditions, startling economies could be obtained. Not many years ago it was considered that a plant must be resigned to poor boiler efficiency when operating with banked fires or at very low ratings. It was found, however, that efficiencies above 75 per cent could be maintained for long periods when operating with "live-banked" boilers at about 25 to 30 per cent of rating. Efficiencies as high as 78.0 per cent have been maintained at a station over a period of four months, during which time the average rating of boiler capacity in service was approximately 25 per cent. Very substantial savings in fuel resulted from this system of "live-bank" operation and discontinuance of the former procedure of carrying whatever small load there was on the station on one or two boilers and "dead" banking the balance of the boilers.

Probably the most decided advantage that a steam plant enjoys in a combined system is the period available for maintenance and overhaul of large units during the flood period in the spring. Two months can be definitely relied upon for this period. In emergencies, at almost any time of the year, the power supplied normally by a large steam unit may be taken care of for very short periods by pondage draw-downs, thus making it possible to take care of minor repairs of an emergency nature at once before they develop to interfere seriously with service. During the winter months when the bulk of the power is carried on the steam plants, the hydro plants can take the swing of the system load, thus making it possible to establish extremely economical load factors on the steam plants at a time when circulating-water temperatures are low. This partially offsets the low thermal economy which accompanies low load and poor load factor at which the steam plants have had to operate during periods of excess hydro output.

GENERAL SYSTEM OPERATING CONDITIONS

The responsibility for properly operating the system and establishing load schedules is placed in the hands of a centralized dispatching organization and a production engineer. The active work of dispatching requires a centralized office from which the loading of all plants on the system may be controlled by means of suitable communication facilities between dispatchers and plants. For this purpose line telephones, carrier current, and radio are used, and properly trained crews are provided for inspection and maintenance of these communication facilities. The dispatching organization is guided in the loading of the system by fairly well established instructions for operation of plants under normal conditions. These instructions are prepared by the production engineer, whose duty it is to study carefully the operation of the plants and their relation to the system, analyze test curves of both hydro and steam plants, and from this information determine the best schedule for overall system economy. These instructions must be arrived at with the cooperation and complete understanding of the steam and hydro supervising crews. Such instruction must be provided not only for daily but also for seasonal operations, and during normal conditions set up for as long a period in advance as can safely be predicted for the convenience and clear understanding of plants which are located some distance away and which are controlled through subsidiary dispatching centers. Successful results depend largely upon the teamwork of these organizations.

It is true that the operation of a combined steam and hydro system requires a great amount of detailed analysis, complicated study, and careful planning, but since stream-flow and system-load conditions are seldom similar to those previously encountered, the variety which is introduced into the work, and the prompt action which is required when uncontrollable changes in conditions spring up, add ample spice to the drab routine of daily operation and make this work one of the most interesting phases of the public-utility business.

Discussion

H. G. Roby.[2] There is one point that Mr. Colvin's paper did not bring out that might be touched upon, and that is the different ways in which the water powers are operated in the system at different periods of the year. When there is plenty of water, the water powers are operated 24 hours per day, up to plant capacity. The steam plants come in on the short daily peak loads. This happens in the spring and early summer and occasionally in the fall. Ordinarily in the fall and early winter the reverse takes place, and the water-power plants are then operated on the peak and the steam plants carry the base load. The water powers are then deficient in the ability to turn out kilowatt hours, but have considerable plant capacity available. The water is ponded during the off-peak periods and the capacity of the hydro plants is brought into use at the time when the additional capacity is required. By doing this the water-power plants get their maximum use. During the spring of the year the water-power plants are turning out kilowatt hours at their full installed capacity, and in the fall and winter they are turning out kilowatt hours to the full value of the stream flow. It is only by combining steam and water-power plants that such economy is possible.

R. L. Goetzenberger.[3] I have read Mr. Colvin's paper with considerable interest. One question I would like to put is regarding frequency control. No reference appears to have been made to it. However, I would suppose, with the diversified power requirements of a system such as the Northern States Power, that frequency control would be an item of great importance. It would be interesting to know what weight is given to frequency control, as well as something about its accomplishment.

[2] Byllesby Engineering & Management Corp., Chicago, Ill.

[3] Minneapolis-Honeywell Regulator Co., Philadelphia, Pa. Assoc. Mem. A.S.M.E.

Author's Closure

I presume that what Mr. Goetzenberger would like to know is what we think about the problem of frequency control. Frequency control, in a combined steam and hydro system such as ours, is a matter which must be taken care of in a different way each day, each hour, and sometimes each minute. Thus far we have not felt it practicable to consider automatic control, for the simple reason that there are so many plants which have the responsibility of maintaining speed regulation that it would require a large number of control equipments to do this.

We endeavor to maintain a reasonably economic load factor on our steam plants, and during the loading-up period the system frequency control is transferred to the hydro plants after the steam plants have been loaded up. This is done in successive order as we desire the hydro plants to take on the load. As soon as each plant is loaded, the dispatcher is notified and the frequency control is transferred to the next plant. Finally a stage is reached where the balance of the load of the system is within the capacity of the last plant concerned, and this plant watches speed for the two or three hours in the morning, afternoon, and evening. The time of these periods varies with the seasons of the year.

You can see from the system load diagrams for the month of January there is a long period of flat load excepting for a time during the noon hour. In the spring and fall there are three peaks, in the morning, afternoon, and evening. The frequency control must be watched by each individual plant, and it is transferred so frequently we cannot see at the present time any method of automatically regulating frequency control. On our system we have relatively complicated operation, but we endeavor to maintain a time control within a few seconds on standard Western Union or observatory time. So far, we have been successful in doing this.

It is doubtful whether any automatic frequency control could be developed which would be fully applicable to our system.

FSP-51-31

Refractories Service Conditions in Furnaces Burning Anthracite on Traveling-Grate Stokers[1]

Progress Report of the A.S.M.E. Special Research Committee on Boiler Furnace Refractories

By R. A. SHERMAN,[2] EDMUND TAYLOR,[3] and H. S. KARCH,[3] PITTSBURGH, PA.

THIS report presents data which pertain to furnace conditions that affect the life of refractories in furnaces burning anthracite on traveling-grate stokers. The investigation, which was made at the Nanticoke, Pennsylvania, power plant of the Glen Alden Coal Co., is the last of a series of nine in the study of service conditions that govern the life of refractories in boiler furnaces conducted by the Bureau of Mines in cooperation with the Special Research Committee on Boiler-Furnace Refractories of The American Society of Mechanical Engineers. The study of the properties of refractories which affect their life is being continued by the Bureau of Standards and the University of Illinois. A list of the previously published reports of the field investigation is appended to this report.

The size of anthracite now burned in power plants is largely buckwheat and smaller, or that passing a $^{9}/_{16}$-in. round-hole screen. It is generally burned on forced-draft, traveling-grate stokers. A front arch is placed over the stoker to secure and maintain ignition of the coal, and a rear arch is provided in modern design to bring the excess air, which comes through the back of the fuel bed, forward to mix with and burn the combustible gases from the front of the fuel bed. In order to obtain complete combustion of the small particles of fuel it is desirable to maintain such pressures in the air compartments under the rear arch that will lift the coal from the grate; it is then carried forward in the current of gas passing toward the front of the furnace under the arch and again deposited on the grate, so that it is kept in the furnace for a longer time and more complete combustion is obtained.

Although the ash content of the small sizes of anthracite may be as high as 20 or 25 per cent, its softening temperature is also generally high, and normally little difficulty is occasioned by the formation of slags which erode the refractories or adhere to the boiler tubes. The life of the refractories used to line these furnaces is therefore relatively long normally, and failure results more from general weakening of the wall structure and, to some extent, from spalling rather than from slag erosion. However, the operators of this plant reported and inspection showed that erosion by slag was a cause of failure of refractories of equal importance to spalling or structural weakness, and that the accumulation of slag on the exposed boiler tubes was a source of considerable difficulty in the operation of the furnace and boilers.

OBJECT AND SCOPE OF INVESTIGATION

The general object of this investigation was to obtain data on the service conditions which govern the life of refractories in these furnaces, and in particular to study the nature of the slag action. The conditions investigated were:

1 Temperature of furnace gas
2 Composition of furnace gas
3 Velocity of furnace gas
4 Temperatures of refractories
5 Amount and composition of the solids carried in the furnace gas.

Data were obtained on these conditions over the full range of boiler load with the furnace as originally designed and built.

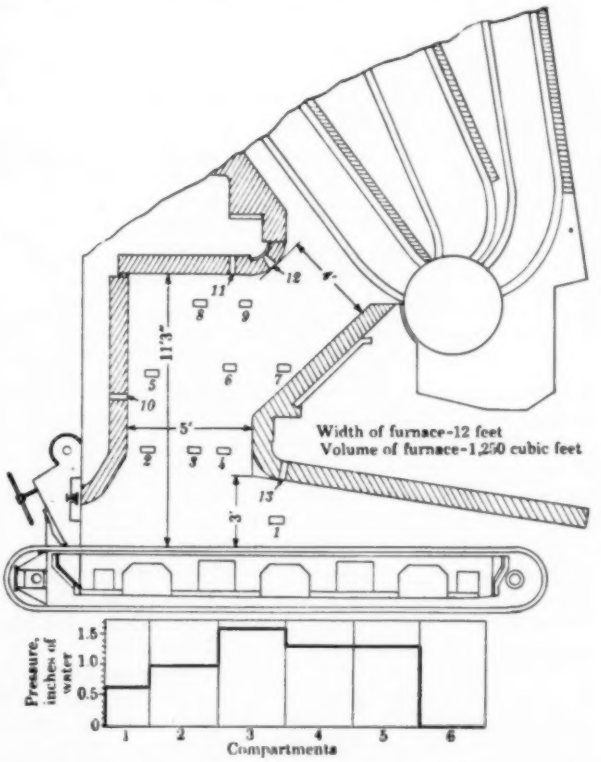


FIG. 1 SECTION OF FURNACE SHOWING PRINCIPAL DIMENSIONS, LOCATION OF BRICKS CONTAINING THERMOCOUPLES, AND TYPICAL DISTRIBUTION OF AIR PRESSURES. FURNACE BURNING ANTHRACITE ON TRAVELING-GRATE STOKER, NANTICOKE, PA.

The company engineers then decided to make a change in the front arch; after this change was made data were taken at one boiler load.

PARTICULARS OF FURNACE

The steam demands of this plant, which furnished power to several collieries of the Glen Alden Coal Co., were supplied by 25 boilers; 20 were horizontal, straight-tube boilers, each with 3030 sq. ft. of heating surface; 5 were inclined, bent-tube boilers, each with 6680 sq. ft. of heating surface.

This investigation was carried on in the furnace of one of the

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[2] Fuel Engineer, Pittsburgh Experiment Station, Bureau of Mines.

[3] Assistant Fuel Engineer, Pittsburgh Experiment Station, Bureau of Mines.

larger boilers, known in the station as No. 25. Fig. 1 shows a side sectional elevation of the furnace and gives the principal dimensions. The numbered rectangles on the figure mark the position of the bricks in which temperatures were measured.

The design of this furnace differed from others burning small size of anthracite in that the front arch was at a considerable height above the grate. The front and rear arches were of the

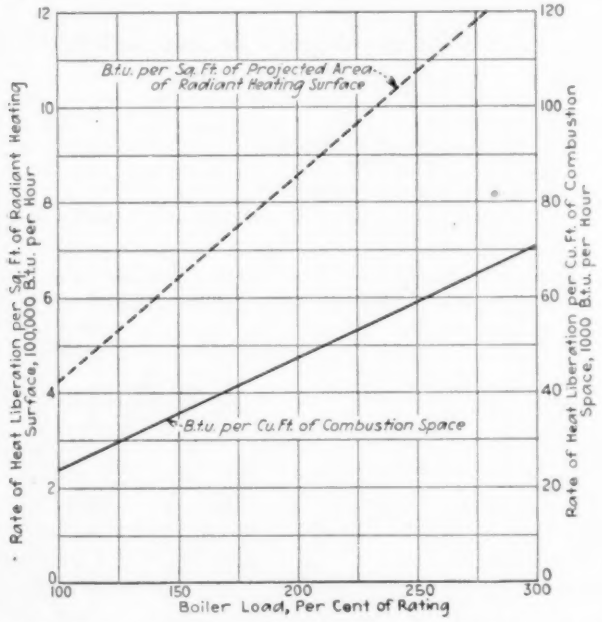


FIG. 2 RELATION OF BOILER LOAD IN PERCENTAGE OF RATING TO RATE OF HEAT LIBERATION, FURNACE BURNING ANTHRACITE ON TRAVELING-GRATE STOKER, NANTICOKE, PA.

flat suspended type; the front wall was of the sectional supported construction in which each section, $10^1/_2$ by 30 in., was independently supported on vertical iron hangers. The tiles in the arches and front wall were 10 in. in thickness, with no covering or insulation on the outside. The total thickness of the side walls was 27 in., of which 18 in. were of firebrick and the remainder face brick. The lining for the first three courses above the grate line was of silicon carbide bricks. There was no air or water cooling in the furnace other than the exposed boiler tubes whose projected area, computed as all that which could "see" any part of the furnace or fuel bed, was 68 sq. ft.

The stoker was fitted with six air compartments in which the air pressure could be separately controlled. The graph in the lower part of Fig. 1 shows a typical distribution of air pressure in the compartments when the boiler was operating at 200 per cent of rating. The area of air space in the grate was about 5 per cent of the total grate area. The normal depth of fuel bed carried was 4 in.

THE COAL BURNED

The coal burned at this plant was from the several collieries of the Glen Alden Coal Co. in Luzerne County, Pennsylvania. As more coal was received from the Truesdale than from the other collieries, this coal was burned during the investigation in the furnace tested except on a few days when none was available.

The size of the coal burned was known as buckwheat No. 4, or barley No. 2, which is that passing through a $^3/_{32}$-in. round-hole screen. Table 1 gives the proximate and ultimate analyses, and calorific value of a composite of a number of samples of this coal on the moisture-free basis, and the range of ash softening temperatures.

TABLE 1 ANALYSES OF COAL USED IN TESTS

Source	
Mine	Truesdale
County	Luzerne
State	Pennsylvania
Proximate Analysis	Per cent
Volatile matter	7.0
Fixed carbon	72.9
Ash	20.1
	100.0
Ultimate Analysis	
Hydrogen	2.3
Carbon	72.6
Nitrogen	0.8
Oxygen	3.0
Sulphur	1.2
Ash	20.1
	100.0
Calorific value, B.t.u. per lb.	11,820
Softening temperature range, deg.(a)	2380–2570
Softening-interval range, deg.	100–300
Fluid-interval range, deg.	80–260

(a) Fahrenheit scale used throughout this paper.

PRESENTATION OF RESULTS

The plan for this investigation was to obtain data at three specified boiler loads. It was found difficult, however, to fix the boiler output at a given value and maintain it constant long enough to secure the desired data. The inability to maintain a constant given load was due to rapid changes in the fuel bed which were beyond the control of the fireman. Therefore data were obtained at a number of different boiler loads, and no attempt has been made to present the results individually, but they are arranged and presented in the curves and tables according to three rating bands, 120–150, 175–200, and 210–225 per cent of rating. The corresponding range of rate of heat liberation per cubic foot of combustion space is also indicated on figures. As it may be desired to compare conditions on the basis of the rate of heat liberation per square foot of exposed radiant heating surface, Fig. 2 shows the relation of the boiler load in percentage of rating to this rate and also to the rate of heat liberation per cubic foot of combustion space.

TEMPERATURE OF FURNACE GAS

Fig. 3 shows the temperature of the furnace gas at 6 in. and 36 in. from the side wall at each of the 11 positions of sampling and for the three ranges of boiler load. The positions are indicated by the numbered circles on the outline of the furnace. The temperatures for positions 2 to 10 are presented graphically; those for 1 and 11 are indicated numerically near the circles which mark the positions. From two to five determinations were made in each range of rating, and all are presented.

There was considerable variation among separate determinations at the same position and similar rating, which was to be expected because of the frequent changes in the condition of the fuel bed. Although the coal gate was set to give a 4-in. depth of coal, accumulation of slag on the gate caused variation of the depth; the coal which was carried from the grate at the rear and toward the front by the air blast built up in hummocks on the grate in no consistent or regular order, and holes were sometimes temporarily blown in the fuel bed. Because of these changes in the fuel bed the distribution of the air pressure in the various compartments was sometimes adjusted by the fireman. An attempt was made to correlate the compartment pressures with the temperatures and other data, but it appeared that the variations in other conditions were so great as to mask the effect of the difference in the air distribution under the grate.

The lowest temperature measured in the furnace was 1280 deg. at 6 in. from the wall at position 1, under the rear arch, at the lowest rating, although another determination at the same position and similar rating gave 2330 deg. The highest temperature

measured in the furnace was 2900 deg., which occurred at 36 in. from the wall at position 10, at the rear of the front arch, at both the medium and high ratings, and at position 11, at the nose of the front arch, at the high rating.

The temperatures increased, in general, with increasing height above the grates, but the difference decreased with increase in rating, and the temperatures were quite uniform at the 7 upper positions at the highest range of rating.

The average temperature at position 4, at the nose of the arch, was lower at 6 than at 36 inches from the wall by from 150 to 250 deg., and for most of the other positions the temperatures were somewhat lower at 6 in. from the wall.

Composition of Furnace Gas

Fig. 4 presents the composition of the furnace gas at the 11 positions of sampling for distances of 6 in. and 36 in. from the side wall and for the three ranges of boiler load; they are shown in the same form as were the temperatures in Fig. 3. The nonuniformity of the fuel bed caused a similar lack of uniformity in separate determinations of the composition for one position and rating as of the temperature of the gas.

At the front wall, the CO_2 content was normally high, up to 16 per cent; CO and H_2 were found at all positions at both 6 in. and 36 in. from the side wall and the amounts were greater at ratings above 175 per cent than at 120 to 150 per cent. Toward the rear of the furnace, positions 4, 7, and 10, at 6 in. from the wall the CO_2 was normally somewhat lower than at the front wall and no CO or H_2 was found; at the same positions at 36 in. from the side wall, although the CO_2 in most samples was higher than at 6 in., CO and H_2 were found in only one sample, which was from position 7 at the highest range of rating. Between the front and rear of the furnace, positions 3, 6, and 9, CO and H_2 were found in most of the samples at both 6 in. and 36 in. from the side wall, but the content was, in general, less than at the front wall. Under the rear arch, position 1, appreciable amounts of CO and H_2 were found in only one of the samples taken at 6 in. from the sidewall, but were found in all samples at 36 in. from the wall. At the nose of the front arch where the gas was leaving the furnace, position 11, CO and H_2 were found in but 4 of the 13 samples taken; the maximum total content of the two gases in one sample was 1.6 per cent with 16.1 per cent CO_2. The maximum CO_2 content found without CO or H_2 was 14.6 per cent.

These results indicate that the gases directly above the fuel bed from the front wall to at least the rear arch contained considerable amounts of CO and H_2. Air for their combustion came

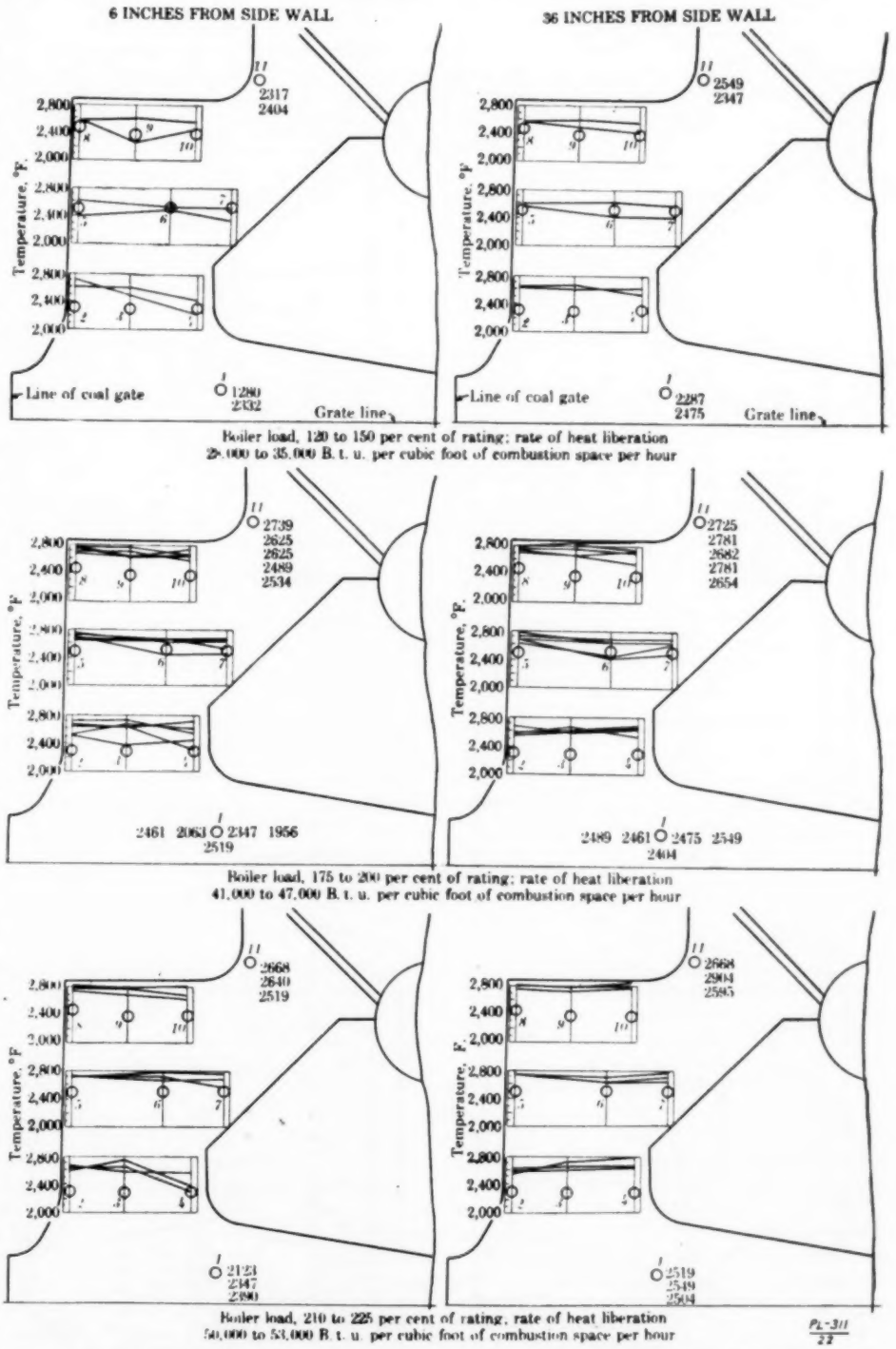


Fig. 3 Temperature of Furnace Gas at 6 In. and 36 In. From the Side Wall at Varying Boiler Loads, Furnace Burning Anthracite on Traveling-Grate Stoker

principally from the rear of the grate, and at the nose of the rear arch no CO or H_2 was found. The air reached to the middle of the throat as the combustible content of the gas was less there than at the front wall, but some combustible remained unburned up to the arch at the front wall; the amount of combustible increased with the rating. Hence the front wall and part of the side walls were in a reducing atmosphere most of the time.

Sulphur Content of Furnace Gas

The sulphur content of the furnace gas, calculated to SO_2, was determined at 6 in. from the side wall at four positions, 3, 6, 10, and 11, at boiler loads of from 140 to 235 per cent of rating. A total of 26 determinations was made. The results varied from 0.024 to 0.088 gram SO_2 per cubic foot or from 0.03 to 0.12 per cent SO_2 by volume, with no relation to position or rating.

The ratio of SO_2 to the total carbon in the gas as computed from its analysis varied from 0.012 to 0.031 and the average ratio for the 26 determinations was 0.022. The ratio of SO_2 to carbon, calculated from the coal analysis, is 0.033. Therefore 67 per cent of the sulphur of the coal was found in the gas at the points of measurement.

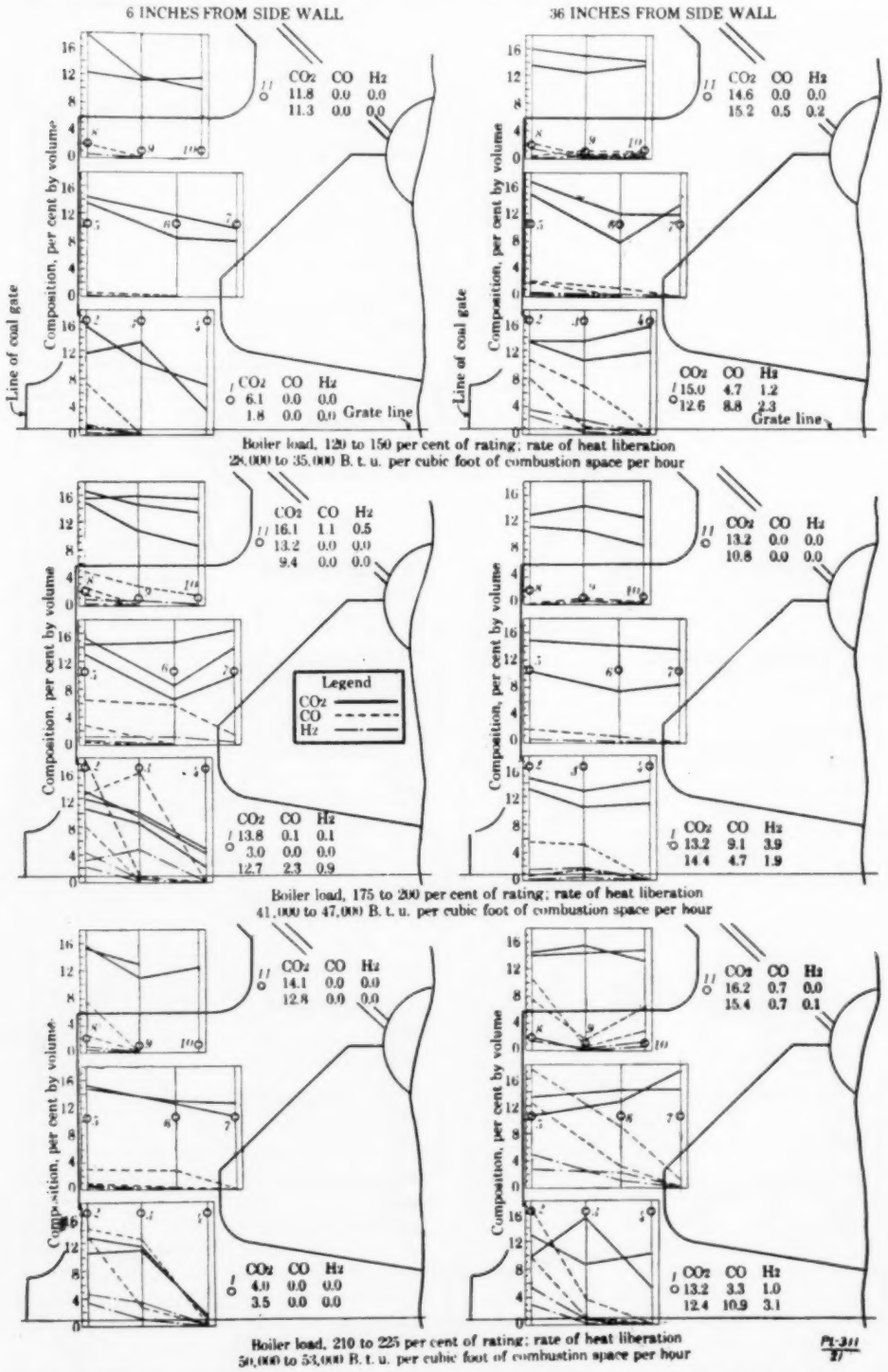


Fig. 4 Composition of Furnace Gas at 6 In. and 36 In. From the Side Wall at Varying Boiler Loads, Furnace Burning Anthracite on Traveling-Grate Stoker

Velocity of Furnace Gas

The velocity at which the gas passes through the furnace determines the amount of ash which will be carried to be deposited on the walls. The velocity was determined at each of the 11 positions in the furnace and at several different boiler loads. Fig. 5 shows typical values obtained at 6 in. and 36 in. from the side wall at a boiler load of 200 per cent of rating. The velocities in feet per second are indicated near the circles which mark the positions. The shading is introduced with its density varied approximately as the velocity in order to assist in the visualization of the path and relative velocity of the gas.

Considering first the velocities at 36 in. from the side wall, that at position 1, under the rear arch, was relatively high, 45 ft. per sec., because this was over the compartment in which the highest air pressure was carried. The velocity was low at the nose of the rear arch; it was approximately the same at the middle of the throat, position 3, as under the arch. The velocities at positions 6, 10, and 11 were of similar order and were higher than at the front or rear of the furnace, which indicates that the main body of gas passed to the furnace outlet in a relatively narrow path at high velocity. The maximum velocity of 50 ft. per sec. was found at the nose of the front arch.

The velocities at 6 in. were of similar order to those at 36 in. from the wall. The principal differences were in a lower velocity

under the rear arch and a higher velocity at the nose of the rear arch. The velocity at both these positions may have been affected by the accumulation of slag on the wall. It is also probable that the measurements made at positions 2, 5, and 8 were affected by their proximity to the front wall; eddy currents were set up, and the direction of flow indicated by the pitot tube was often variable.

Temperature of Refractories

The temperatures of the refractories were measured by means of thermocouples placed in bricks in various parts of the walls and arches. The numbered rectangles in Fig. 1 mark the position of the bricks, of which there was a total of 13; 9 in the side wall, 1 in the front wall, 2 in the front arch, and 1 in the rear arch.

Figs. 6, 7, and 8 show the temperatures measured in six of the bricks over several days of operation; hourly readings are plotted.

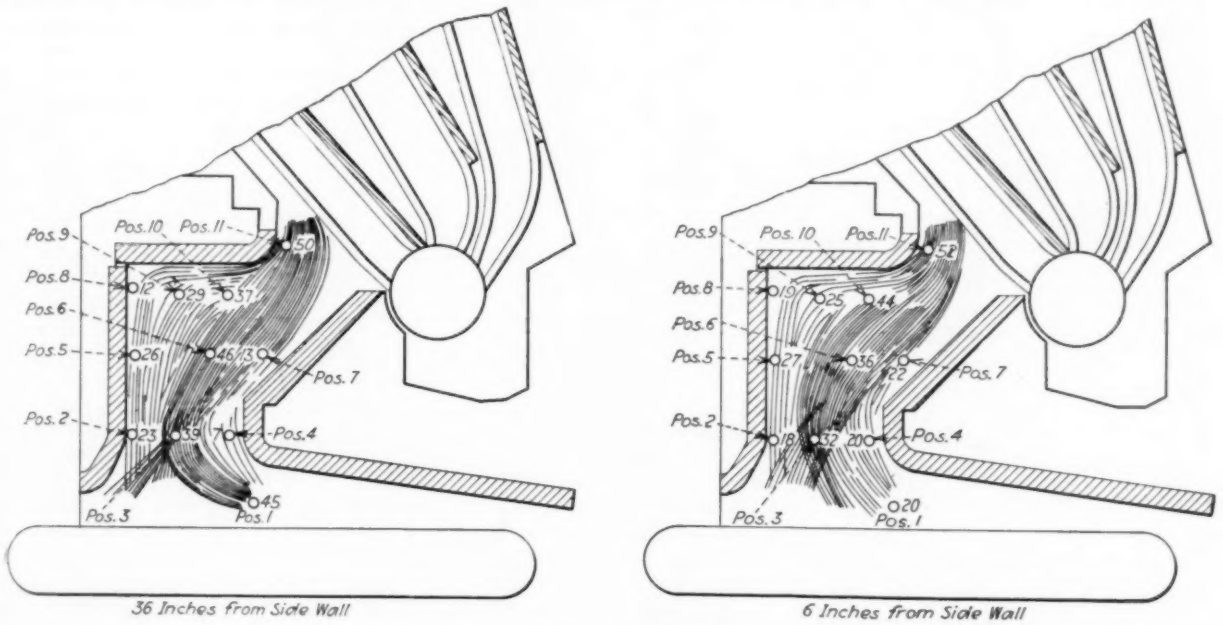


Fig. 5 Velocity of Furnace Gas in Feet per Second, Boiler Load 200 Per Cent of Rating, Furnace Burning Anthracite on Traveling-Grate Stoker

The curve in the upper part of the figures shows the boiler load in percentage of rating, as read on the scale at the left, or the rate of heat liberation in B.t.u. per hour per cubic foot of combustion space as read on the scale at the right.

A low fire, the boiler not steaming, was maintained for 45 hours because there was a relatively large area of new brickwork in the furnace. During this period the temperatures fluctuated widely owing to changes in the low fire carried; the maximum temperature attained at $\frac{1}{2}$ in. from the hot face of any of the bricks was less than 1100 deg.

Following the heating-up period the temperatures were measured over five days of normal operation of the boiler during which it steamed at about 200 per cent of rating and was banked for five hours on one night. The rate of steaming was subject to frequent changes of from 10 to 20 per cent and these changes were reflected in the variations of the temperature of the refractories.

The temperature at 1 in. from the hot face of brick No. 1, in the side wall under the rear arch, which is shown as curve 1-B in Fig. 6, was subject to the most frequent and widest variations. It has been shown that the two determinations of the temperature of the furnace gas at this position were widely different. The temperatures at 3 in. and 6 in. from the surface of this brick, curves 1-C and 1-D, respectively, decreased after the period of banking, although the rate of steaming was, on the average, higher than before. This was undoubtedly due to the insulating effect of an accumulation of slag upon the surface.

After the banking period the temperatures at all points shown in the curves, except 1-C and 1-D, gradually increased through the remaining period of observation. As the average load carried on the boiler did not increase, it would be expected that the temperatures at $\frac{1}{2}$ in. from the face of the bricks would have reached a steady state in less than this time. It is probable that this increase in the temperature of the refractories was due to decrease in the rate of heat absorption from the furnace by the exposed boiler tubes because of the accumulation of slag upon them.

The temperatures at $\frac{1}{2}$ in. from the surface of brick No. 6, in the side wall above the throat of the furnace, and of brick No. 11, in the front arch, shown in Fig. 7, and of brick No. 5, in the side wall, and brick No. 10, in the front wall, shown in Fig. 8, were quite similar during the entire period of observation. The highest temperature measured in any of these bricks was approximately 2600 deg. at $\frac{1}{2}$ in. from the hot face.

Fig. 9 shows the temperature gradients through four of the bricks at 88 hours after lighting the fires when the boiler was operating at 200 per cent of rating or at a rate of heat liberation of 47,000 B.t.u. per cubic foot of combustion space per hour. The temperatures at the surface were obtained by extrapolation of the curves as indicated by the dotted lines. At this time the surface temperatures of the four bricks lay between 2530 and 2600 deg. This uniformity of temperature of the refractories in different parts of the walls above the throat of the furnace was as to be expected from the uniformity of the gas temperatures in this zone.

The gradient through brick No. 10, which was in the front wall, was steeper than through the other bricks which were in the side wall because the front wall was but 10 in., whereas the side wall was 27 in. in thickness.

Quantity of Solids Carried in the Gases

The weight of the solids carried per cubic foot of gas was de-

termined at 6 in. from the side wall at four positions, 3, 6, 10, and 11. Table 2 presents the average results for from 3 to 6 determinations at each of the positions at 200 per cent of rating. The combustible in the sample was assumed to be and is reported as carbon.

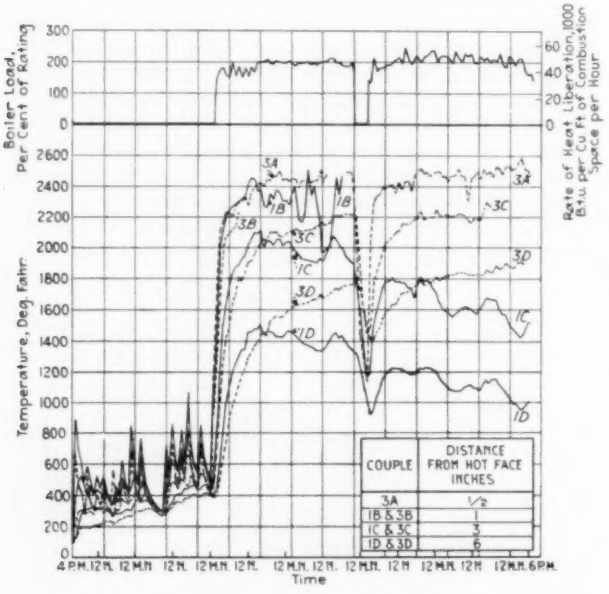


FIG. 6 TIME-TEMPERATURE CURVES IN BRICKS IN SIDE WALL, ANTHRACITE FURNACE, NANTICOKE, PA.

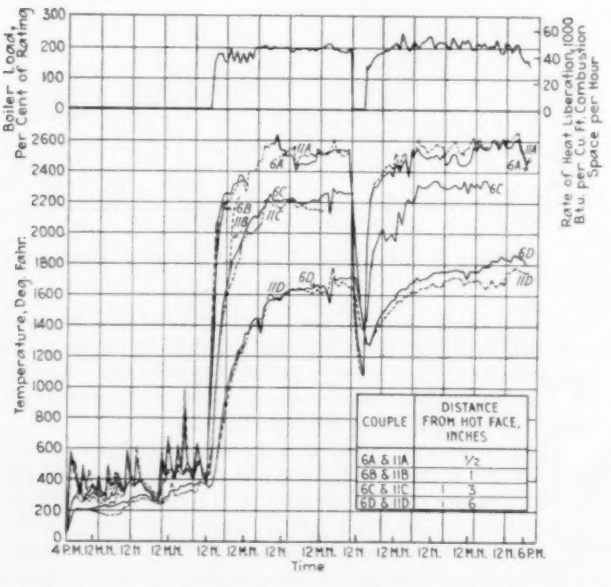


FIG. 7 TIME-TEMPERATURE CURVES IN BRICKS IN SIDE WALL AND FRONT ARCH, ANTHRACITE FURNACE, NANTICOKE, PA.

The weight of solids carried did not decrease with increasing distance from the grate, as might be expected because of particles settling out of the gas stream. Calculation from the equation developed by Blizard[4] from an analysis of Audibert's[5] experimental data on the carrying power of gases for solids shows that gas moving at a velocity of more than 15 ft. per sec. will carry upward particles of anthracite of density 1.5 and of a mean diameter of $^1/_8$ in. or less. As the velocity of the gas at these positions and this rating was not less than 30 ft. per sec., and as all the coal particles were less than $^1/_8$ in. in diameter, any particles which rose from the grate into the stream of gas would be carried and would not be deposited except as thrown out on change of direction of the gas flow. Because some of the ash sintered together on the grates to form large pieces, the amount of ash actually carried out was not as great as would be indicated by this calculation.

It is interesting to note that the percentage of carbon in the sample at the furnace outlet, position 11, 12 ft. from the grates, was not less than at position 3, 4 ft. from the grates, as would be expected if they were in the same stream of gas. There was evidently little combustion of solid carbon in this distance of travel. These values were confirmed by the composition of the dusts taken from the bottom of the second and third pass of the boiler, both of which contained 87 to 88 per cent of combustible.

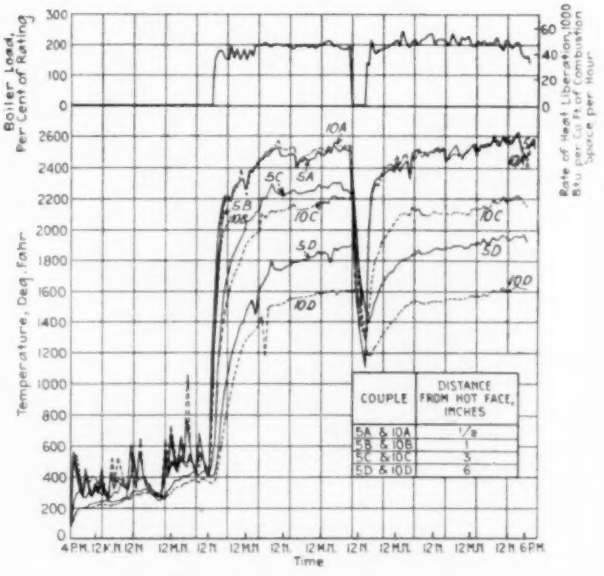


FIG. 8 TIME-TEMPERATURE CURVES IN BRICKS IN SIDE AND FRONT WALLS, ANTHRACITE FURNACE, NANTICOKE, PA.

It is also worthy of note that the dust which passed out of the furnace contained more carbon and less ash than the coal fired to the furnace. The action of the air blast on the coal on the grate was in the nature of a washing process, in which the lighter coal particles were carried by the gas and the heavier slate particles and those fused or sintered together remained on the grate.

The last column but one of Table 2 gives the solid carbon which was carried in the gas stream as a percentage of the total

TABLE 2 AMOUNT AND COMPOSITION OF SOLIDS CARRIED IN THE GAS AT 6 IN. FROM THE SIDE WALL AT 200 PER CENT OF RATING, FURNACE BURNING ANTHRACITE ON TRAVELING-GRATE STOKER

Position	Wt. per cu. ft. of gas, gram		Percentage composition of sample		Carbon, percentage of total carbon(a)	Ash, percentage of total ash(b)
	Carbon	Ash	Carbon	Ash		
3	0.424	0.072	85.3	14.7	16	10
6	0.725	0.106	87.3	12.7	30	16
10	0.228	0.045	83.9	16.1	12	9
11	0.481	0.069	86.8	13.2	21	11

(a) Total carbon is sum of gaseous and solid carbon.
(b) Total ash is that ash which should accompany carbon based on ratio of carbon to ash in coal.

[4] "The Terminal Velocity of Particles of Powdered Coal Falling in Air or Other Viscous Fluid." *Journal of the Franklin Institute*, vol. 197, 1924, p. 199.

[5] "Study of the Entrainment of Powder by an Air Current." *Annales des Mines*, 12th Series, vol. 1, 1922, p. 153.

carbon in the gas, that is, the solid carbon plus that as CO_2 and CO. This ranged from 12 to 30 per cent; at the furnace outlet, position 11, it was 21 per cent.

The last column gives the ash as a percentage of the ash which should accompany the total carbon in the gas on the basis of the ratio of ash to carbon in the coal. From 9 to 16 per cent of the ash fired to the furnace was found in the gas at the points of sampling, and the remainder stayed on the grates.

These samples were taken at only 6 in. from the wall and cannot therefore be definitely taken as representative, but they are indicative of conditions over the entire furnace, and calculations based on these results are of interest. The ash in the gas at position 11 was 11 per cent of the total ash fired to the furnace; the content of ash in the coal on the dry basis was 20.1 per cent. Therefore, the ash carried in the gas was $11 \times 20.1 = 2.2$ per cent of the coal fired. The percentages of carbon and ash at this position were 86.8 and 13.2, respectively. Therefore the carbon was $2.2 \times 86.8/13.2 = 14.5$ per cent of the coal fired. The total material carried in the gas was $14.5 + 2.2 = 16.7$ per cent of the coal fired to the furnace. Because of the high content of combustible this constituted a considerable loss, but a part of this was recovered in these furnaces by returning the dust which collected in the bottom of the second pass to the furnace by gravity through metal chutes.

Composition of Coal Ash and Slag

Table 3 gives the composition and softening temperatures of samples of coal ash, dusts from the furnace gases, and slags from various parts of the furnace.

The first analysis is that of a composite sample of the ash of Truesdale coal which was burned during the tests. The total flux content, the constituents other than silica and alumina, was 17.4 per cent, and the softening temperatures of the ash, as determined on separate samples, varied from 2380 to 2570 deg. Samples 2 and 3 were coals from the Pettibone and Hallstead collieries which were also burned in the plant. Their flux content was 15.1 and 11 per cent, and the softening temperatures of the ash were from 2610 to 2700 and from 2750 to 2810, respectively. The principal difference between the flux content of these two coals and of the Truesdale coal was in the lime content. Although the Hallstead coal would normally be judged as the best of the three coals because of the low flux content and high softening temperature of its ash, it gave the most trouble in the fuel bed, apparently because of clinker formation. As it was not reported to give either more or less trouble from slagging of the refractories, no further analysis of the reason for the action of this coal was attempted.

Sample 4 is the ash of coal burned at the Hunlock Creek station of the Luzerne County Gas and Electric Co., where very little difficulty with slagging or other failure of refractories was reported. The total flux content and the percentage of the various fluxes were very similar to those of the Pettibone coal ash; the content of alumina was somewhat higher than that of any of the three coals from the Nanticoke plant.

Samples 5 to 8, inclusive, are composites of the dust collected from the furnace gases, at positions 3, 6, 10, and 11. Their flux content varied from 18.8 to 20 per cent, which was greater than that of the coal ash; the principal difference was in the somewhat greater content of Fe_2O_3; however, the content of CaO was lower than in the coal ash.

Sample 9, taken from the exposed boiler tubes, was slag whose composition would be expected to be similar to that of the dust carried in the gases. However, its composition was more nearly like that of the Truesdale coal ash, although it was not from the burning of this coal only because, as previously stated, it was occasionally necessary to burn coal from other collieries in this furnace.

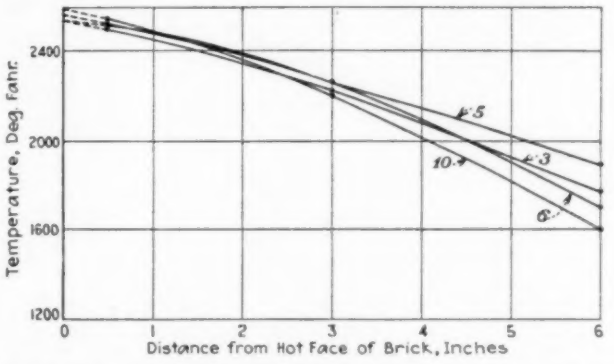


Fig. 9 Temperature Gradient Through Bricks in Walls of Anthracite Furnace at Nanticoke, Pa., 88 Hours After Lighting Fires, Boiler Load 200 Per Cent of Rating, Rate of Heat Liberation 47,000 B.t.u. per Hr. per Cu. Ft. of Combustion Space

Samples 10 and 11 were dusts from the second and third pass; these represented the burning of Truesdale coal only. The content of both the Fe_2O_3 and CaO in the dusts was greater, and the total flux content was about 5 per cent greater, than in the dusts collected from the furnace gases. The material at the bottom of the passes was probably the heavier particles of the dust which were deposited from the gas stream as it changed direction on passing around the baffles.

The content of SiO_2 and Al_2O_3 in the ash from the ashpit, sample 12, differed slightly from that of the coal ash, but the content of the several fluxes and the total flux content was practically the same as of the ash of the coal fired to the furnace.

Sample 13 is that of the dust from the rear door of the furnace which consisted of the small, light particles which rose from the ashpit and grate as the ash fell from the grate; it was largely the unfused or slightly sintered portions. The content of SiO_2 was 7 per cent less, the Al_2O_3 content 7 per cent more, the Fe_2O_3

TABLE 3 COMPOSITION AND SOFTENING TEMPERATURE OF COAL ASH, DUST, AND SLAG SAMPLES, FURNACE BURNING ANTHRACITE ON TRAVELING-GRATE STOKER

Sample number	Description of sample	Composition, per cent: SiO_2	Al_2O_3	Fe_2O_3	CaO	MgO	SO_3	Na_2O(a) K_2O	$\frac{SiO_2}{Al_2O_3}$ ratio	Softening temp., deg.	Softening interval, deg.	Fluid interval, deg.
1	Truesdale coal ash	54.6	28.0	8.8	3.7	0.6	2.0	2.3	1.95	2380–2570	100–300	80–260
2	Pettibone coal ash	56.4	28.5	7.4	1.8	1.0	1.2	3.7	1.98	2610–2700	210–290	110–120
3	Hallstead coal ash	59.5	29.5	6.8	0.3	0.7	0.3	2.9	2.02	2750–2810	110–140	40–130
4	Hunlock Creek coal ash	55.4	30.5	7.8	1.1	0.9	0.9	3.4	1.82	2700	200	60
5	Dust from gases, Position 3	50.5	29.5	11.8	2.1	0.8	0.7	4.6	1.71			
6	Dust from gases, Position 6	49.1	31.2	12.0	2.1	0.8	0.5	4.3	1.57			
7	Dust from gases, Position 10	48.6	31.6	13.7	1.5	0.8	0.3	3.5	1.54			
8	Dust from gases, Position 11	48.3	32.9	11.3	1.5	0.8	0.4	4.8	1.47			
9	Slag from boiler tubes	52.6	30.5	9.9	3.8	0.9	0.2	2.1	1.72	2360	80	270
10	Dust from second pass	45.6	27.1	19.1	5.2	0.8	1.5	0.7	1.68	2210	110	220
11	Dust from third pass	45.4	29.3	17.6	3.3	0.6	1.6	2.2	1.55	2350	200	180
12	Ash from ashpit	56.6	25.7	9.3	4.0	0.9	0.7	2.8	2.20			
13	Dust from rear door of furnace	49.5	32.8	10.3	2.5	0.7	1.3	2.9	1.51	2700	190	200+

(a) Alkalis by difference.

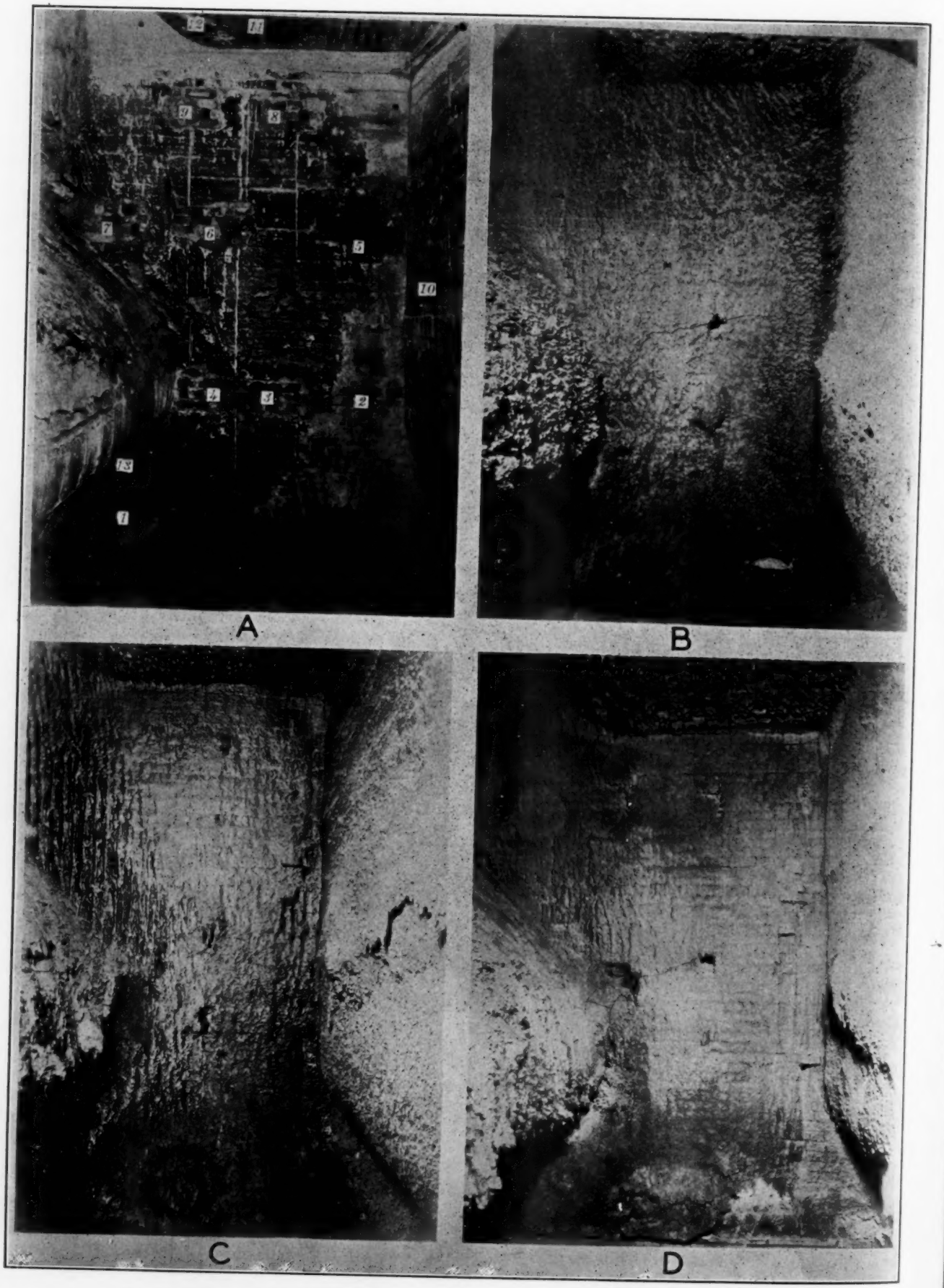


FIG. 10 VIEWS OF INTERIOR OF FURNACE BURNING ANTHRACITE ON TRAVELING-GRATE STOKER

content 1 per cent more, and the lime content 1.5 per cent less than in the ash from the ashpit. The softening temperature of this dust was 2700 deg., 130 deg. higher than the highest for a sample of the Truesdale coal ash.

The softening temperature of the dust from the passes and the slag from the boiler tubes was lower than that of the coal ash. The softening temperatures of eight samples of slag taken from the side walls, arches, coal gate, and front wall of the furnace ranged from 2190 to 2330 deg., all lower than the lowest softening temperature of the coal ash; this is as has been found in other furnaces investigated.

The SiO_2/Al_2O_3 ratio in the Truesdale coal ash was 1.95; it was lower in the dust and slag samples, ranging from 1.47 to 1.71; and it was 2.2, somewhat greater, in the ash from the ashpit. Therefore a somewhat greater percentage of alumina than silica left the grates.

Refractories Service

View A in Fig. 10 shows the side wall and portions of the front wall, front arch, and rear arch at the time the tests were started. The front arch and the nose of the rear arch had just been rebuilt for the second time since the furnace went into service 12 months before. The side wall had been rebuilt once and was still in good condition except for the arch of the observation door which was rebuilt. The patches of new brickwork in the side wall surround the bricks in which thermocouples were placed to measure the temperature of the wall. These bricks are numbered in the photograph as in Fig. 1. The holes in the side wall for the sampling of gases and the measurement of temperature may also be seen.

The tiles in the arches and front wall were hand made of Pennsylvania clay; the inner 9-in. lining of the side walls was of Georgia kaolin, the next 8 in. were of Pennsylvania bricks, and the remainder of buff face bricks. The bricks in which thermocouples were placed and those surrounding them were of Pennsylvania clay. The first three courses above the grate were of silicon carbide bricks. Several silicon-carbide bricks were also placed in other parts of the furnace for this run.

Views B, C, and D in Fig. 10 show the furnace at the end of two, four, and ten weeks of service, respectively, during which the boiler steamed at an average rating of approximately 190 per cent. After the first two weeks of service, view B, the front wall, and the nose of the rear arch were covered with a layer of friable, granular ash. This layer had a maximum thickness of 7 in. on the front wall at about 4 ft. above the grate. This was evidently the coal and ash which were thrown from the stream of gas from under the rear arch as it changed direction to pass up toward the tubes.

The heavy accumulation of slag on the lower part of the side wall and on the front wall had been removed before the photograph of view C was taken at the end of four weeks of service. The slag on the side wall was harder and more glassy than before, and congealed streams of slag may be seen on the upper part of the wall above the rear arch.

At the end of a total of 10 weeks of service, view D, the slag, except in the thick deposit on the lower part of the front wall, was relatively thin. The outlines of the individual bricks and tiles may be seen in the photograph. The differences in the appearance of the walls in these three photographs do not necessarily indicate different conditions of service over the entire period of service. The condition of the slag of the walls when the boiler is taken out of service is governed by the conditions of operation the last day, or even the last few hours of steaming.

The most severe erosion of the refractories by slag was in the corners of the side and front walls; the maximum depth of erosion was 2 in. at position 2. Considerable erosion had occurred on the upper part of the front wall, but the lower part had not eroded because of the protection of the accumulation of slag.

A course of bricks may be seen to project from the rest a little more than half-way up the front wall and others project on the side wall; these were the silicon carbide bricks. The maximum loss from these bricks in the 10 weeks of service was $^1/_8$ in. from a brick near position 5.

Considerable loss from spalling had taken place on the front arch; the greatest loss was in the nose blocks. Reference to Fig. 9 shows that the rate or frequency of change of temperature at $^1/_2$ in. from the face of brick No. 11, which was in the front arch, was little if any greater than at the same distance in brick

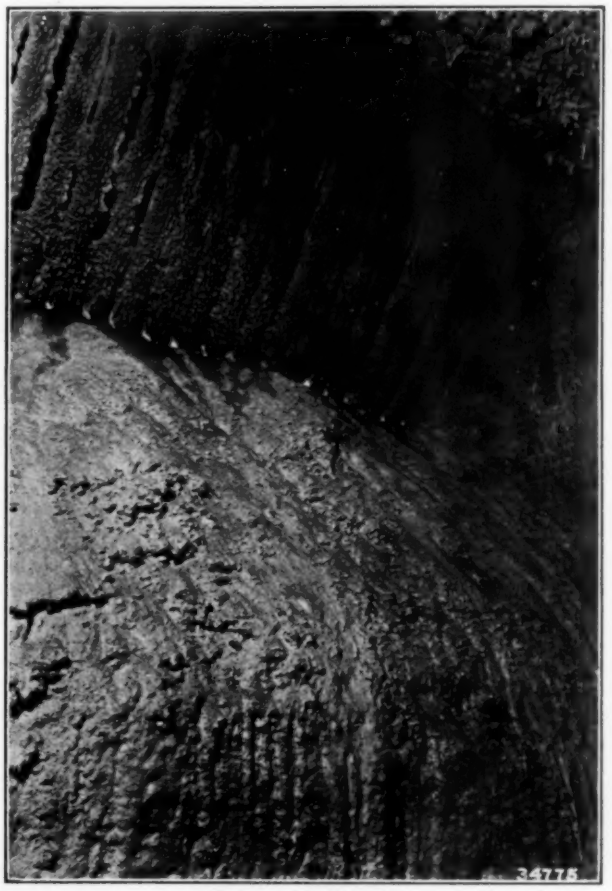


Fig. 11 View of Boiler Tubes With Slag Accumulation, Furnace Burning Anthracite on Traveling-Grate Stoker, Nanticoke, Pa.

No. 6 which was in the side wall where spalling was not a cause of failure. It is probable that pinching was a major cause of the spalling, as many of the blocks were oversize and there was not enough room for expansion.

Fig. 11 shows the boiler tubes at the end of the ten-week period. The tubes were partly cleaned of slag by air pressure through a hand lance once every eight hours and were thoroughly cleaned each time the boiler was down for inspection, which was approximately every two weeks. All of the exposed surface of the tubes was covered with slag and a large portion of the area between adjacent tubes was covered.

Summary of Service Conditions

Although, as has been pointed out, the temperature, composition, and velocity of the furnace gases at any given position and

rating were subject to wide variations due to changes in the fuel bed, the conditions in this furnace may be generally summarized as follows:

The gases arising from the front part of the fuel bed had a high content of CO and H_2. The air for the combustion of these gases came principally from the back of the fuel bed. This air did not penetrate to the front wall, so that this wall and a considerable portion of the side walls were maintained in a reducing atmosphere. Combustion continued throughout the furnace above the throat, and because the area of surface of the boiler tubes exposed for heat absorption was small the heat was absorbed at a lower rate than it was released by combustion; hence the highest temperatures were found in the upper part of the furnace.

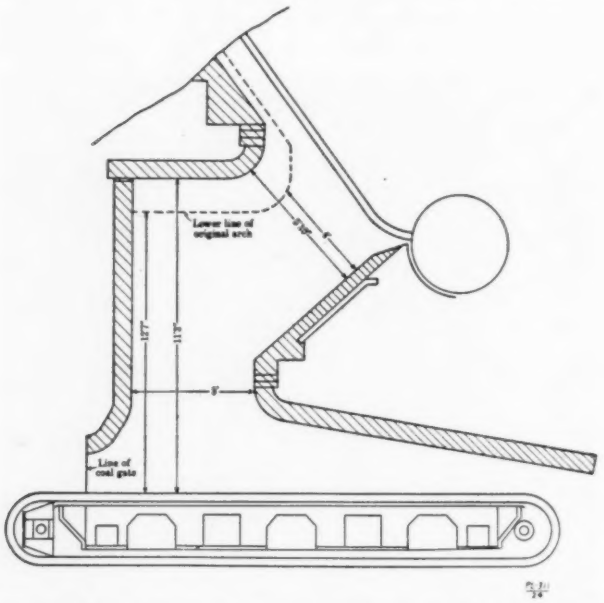


FIG. 12 SIDE SECTIONAL ELEVATION OF FURNACE AFTER FRONT ARCH WAS RAISED, SHOWING LOWER LINE OF ORIGINAL ARCH WITH PRINCIPAL COMPARATIVE DIMENSIONS, FURNACE BURNING ANTHRACITE ON TRAVELING-GRATE STOKER, NANTICOKE, PA.

As the particles of coal were small in size and the gases moved at a high velocity a large amount of coal and ash was carried by the gases to be deposited on the walls, on the tubes, and carried out of the furnace.

As the area of the outlet of the furnace at the boiler tubes was the smallest area in the furnace for the passage of gases, and because the main stream of gases passed upward toward the nose of the arch in the shortest path, the velocity of the gases reached a maximum at the furnace outlet. The high velocity with which the softened particles of ash and slag struck the boiler tubes probably increased their tendency to adhere to the tubes. As the tubes became covered by slag the rate of heat absorption decreased, the temperature in the upper part of the furnace increased, and the slagging of both the refractories and the tubes increased.

The bridging of the slag over the spaces between the tubes increased the pressure drop through the boiler and thus decreased the rate at which the boiler could be operated with the available draft.

COMPARISON WITH CONDITIONS IN OTHER FURNACES BURNING ANTHRACITE

Refractories service conditions have been observed in five other power plants burning the small sizes of anthracite on forced-draft, traveling-grate stokers, although no data were taken as in this furnace. The failure of the refractories in the furnaces of these plants was due to general weakening of the structure after a prolonged period because of repeated expansion and contraction of the walls, and to spalling. Ash was deposited on the refractories in conditions varying from a deposit which could be brushed from the wall with the hand, toi a sintered spongy deposit which adhered rather firmly but whech had not apparently harmed the refractories. Congealed strbams of slag or thin glassy coatings with evidences of erosion by slag were not seen.

The two major differences between the Nanticoke and the other furnaces were the size of the fuel and the design of the furnace.

Whereas the coal burned at the other plants was largely buckwheat No. 3 or barley No. 1, that burned at Nanticoke was almost entirely buckwheat No. 4 and smaller. This difference in the size of the coal may have been an important factor in the difference in the slagging in that a greater amount of material might have been carried in the gases with the smaller size. It was probably not a major factor, however, as there was enough ash on the walls in all furnaces to have caused serious erosion had it been fused.

The designs of the furnaces differed from those at the Nanticoke plant, in that there were front arches over the grates at about the same height as the rear arch. Mixing of the gases and complete combustion was probably accomplished at a greater distance from the boiler tubes, and the furnaces above the throat were exposed to a larger area of boiler tubes so that in the passage of the gases from the throat to the tubes heat was being absorbed faster than it was liberated. Hence it is considered probable that a smaller area of the walls was in a reducing atmosphere, and that the temperatures of the gases and the refractories were not as high as in the Nanticoke furnace.

The softening temperature of the ash of the Truesdale coal which was used in largest amounts at Nanticoke was lower than that of the coal used at the other plants and also than of the other coals at the same plant by from 200 to 300 deg. The operators at Nanticoke did not report, however, that the coals whose ash had high softening temperatures gave less slagging than the Truesdale coal with the ash of low softening temperature. This was probably because the temperatures reached in the furnace at normal boiler loads were well above the softening temperature of the ash of any of the coals.

POSSIBILITIES OF CHANGES

The slagging of the refractories in this furnace might be decreased by decreasing the velocity of the gases, the temperatures, and the content of reducing gases.

It has been stated that calculation, based on available experimental data, shows that a gas velocity of above 15 ft. per sec. will carry upward anthracite particles $^1/_8$ in. in diameter. As all the coal was less than this size and the ash particles still smaller, and as the velocities were well above 15 ft., it would not appear feasible to reduce the velocity by change in furnace design enough to decrease appreciably the amount of solids carried in the gases.

The temperature of the gases in the upper part of the furnace could be reduced by increasing the area of tubes exposed for heat absorption, or by obtaining complete combustion earlier in the path of the gases so that they would have more time for radiation to the absorbing surface.

The area of tubes exposed to radiation could be increased by raising the front arch. This would also serve to decrease the pressure drop through the furnace and boiler, for the area of this section was the least in the furnace. It would also decrease the

mean velocity of the gases at the outlet, but the gases would tend to take the shortest path, and therefore the main stream of gas would pass directly under the nose of the arch as before. Inclination of the arch by raising it more at the nose than at the front wall should also aid in decreasing the temperature of the arch as it would expose the surface at a more favorable angle for radiation to the tubes.

If more complete combustion could be obtained earlier in the path of the gases, the temperature in the upper part of the furnace would be decreased because of the greater time for radiation and also a lesser area of walls would be maintained in a reducing atmosphere. Such mixing as was effected by the high front arch took place so late that little space remained for combustion. Mixing might be effected earlier by a front arch nearer the grate or by the introduction of secondary air in jets at high velocity along the front wall at the height of the nose of the rear arch.

Change Made in the Furnace

It would have been impossible to introduce a front arch near the grate without extensive changes in the furnace. The operators of the plant had previously considered raising the present front arch, and, as the desirability was confirmed by the data taken in this investigation, the change was approved.

Fig. 12 is a side sectional elevation of the furnace after the arch

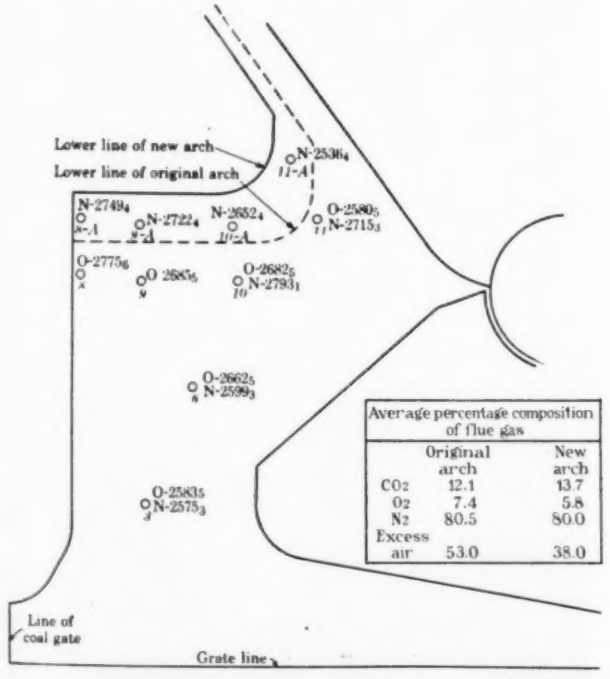


Fig. 13 Temperature of Furnace Gases at 6 In. From Side Wall and After Front Arch Was Raised, Boiler Load 200 Per Cent of Rating, Furnace Burning Anthracite on Traveling-Grate Stoker, Nanticoke, Pa.

(Letter "O" before temperature denotes original arch; letter "N" before temperature denotes new arch; subscript following temperature shows number of determinations averaged.)

was raised and shows the lower line of the original arch in dashed lines and the principal comparative dimensions. The arch was raised 16 in., which made the area of the furnace outlet greater than that of the throat between the front wall and rear arch instead of less as it had been before. The volume of the furnace was now 1380 cu. ft., an increase of 130 cu. ft., and the projected area of boiler tubes was 104 sq. ft., an increase of 36 sq. ft.

Four sampling holes were placed in the side wall at the same position relative to the new arch as the four positions 8 to 11 had to the original arch; these were numbered 8A to 11A. The temperature, composition, and velocity of the gas and the quantity of solids carried in the gas were determined at these and other positions at a boiler load of approximately 200 per cent of rating.

Figs. 13 and 14 show the average temperatures measured at 6 and 36 in., respectively, from the side wall after the arch was

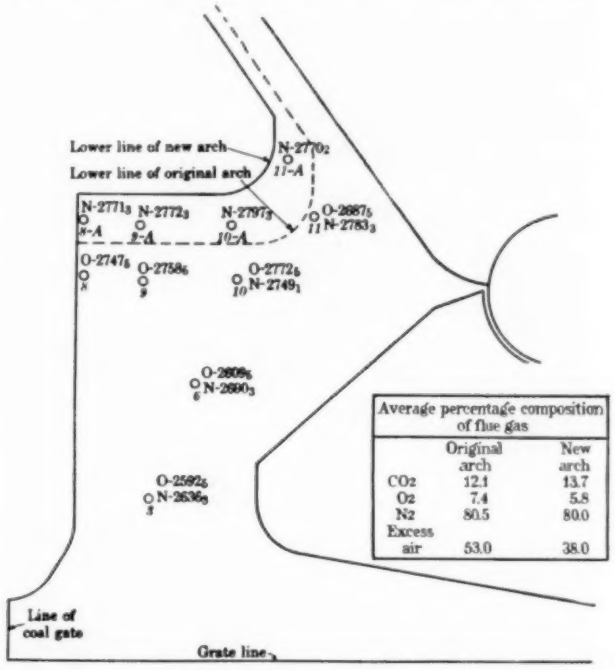


Fig. 14 Temperature of Furnace Gases at 36 In. From Side Wall Before and After Front Arch Was Raised, Boiler Load 200 Per Cent of Rating, Furnace Burning Anthracite on Traveling-Grate Stoker, Nanticoke, Pa.

(Letter "O" before temperature denotes original arch; letter "N" before temperature denotes new arch; subscript following temperature shows number of determinations averaged.)

raised, together with those at similar positions before it was raised. The values for the original arch are prefixed by the letter O, and those for the new arch by the letter N; the subscript denotes the number of determinations averaged.

At 6 in. from the side wall the average temperatures with the new arch were from 63 deg. lower to 135 deg. higher than with the original arch. At 36 in. from the wall, however, the average temperatures at all positions, except position 10 where only one determination was made with the new arch, were from 14 to 101 deg. higher than with the original arch. This was contrary to what was to be expected with the increased area of radiant-heat-absorbing surface. It will be noted, however, that the CO_2 in the stack gas averaged 13.7 per cent, equivalent to 37.5 per cent of excess air, after the arch was raised, whereas it was 12.1 per cent, equivalent to 51 per cent of excess air, before the arch was raised. This difference in excess air was probably enough to account for the differences in temperature found, but enough data are not available to allow an accurate calculation of the temperatures for equal percentages of excess air.

The composition of the gas at the various positions is not shown as it was relatively similar to that before the arch was raised except that the CO_2 content, and likewise the CO content, was generally higher in the furnace as it was in the stack gas. This relatively small change in the height of the arch could scarcely account for the lower excess air; it was more probably due to difference in the manipulation of the fire.

Fig. 15 gives the velocities at the positions under the front arch and at positions 3 and 6 in the lower part of the furnace; the values for the original and new arch are prefixed by the letters O and N, respectively. The velocities in the lower part of the furnace were appreciably lower with the new than with the original arch, which was probably due to a change in the position of the main gas stream with the new arch so that it did not pass the points of measurement. At positions 10 and 11 the velocities were lower with the new arch, but at positions 10A and 11A they were practically the same as at the corresponding positions under the original arch.

These data confirm the expectation that, although the increased height of the arch would decrease the mean velocity of the gases at the outlet of the furnace, it would probably not decrease the velocity at the nose of the arch due to the tendency of the gas to flow out of the furnace in the shortest path.

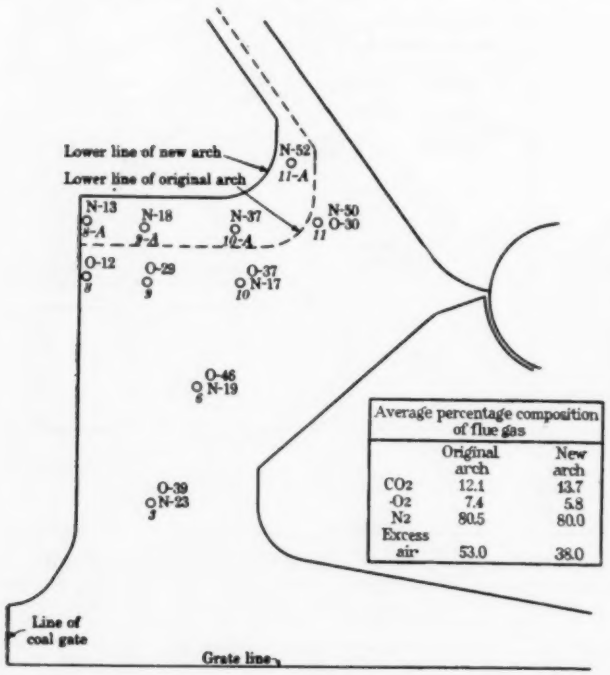


FIG. 15 VELOCITY OF FURNACE GASES IN FEET PER SECOND AT 36 IN. FROM SIDE WALL BEFORE AND AFTER FRONT ARCH WAS RAISED, BOILER LOAD 200 PER CENT OF RATING, FURNACE BURNING ANTHRACITE ON TRAVELING-GRATE STOKER, NANTICOKE, PA.
(Letter "O" before velocity denotes original arch; letter "N" before velocity denotes new arch.)

Table 4 presents the average results of the determination of the solids carried in the gases at positions 11 and 11A with the new arch together with the results at position 11 with the original arch as taken from Table 2; from 4 to 6 determinations at each position were averaged. At position 11 with the new arch, which was at the nose of the original arch, the weight of carbon per cubic foot of gas was greater and the weight of ash was nearly the same as at this position with the original arch; however, the solid carbon as a percentage of the total carbon, and the ash as a percentage of the total ash, were approximately the same with the two arches.

At the nose of the new arch, position 11A, although the percentages of carbon and ash in the sample were not greatly different from those at position 11 with either arch, the weight of both the carbon and ash per cubic foot of gas was about one-third, and the solid carbon and the ash as percentages of the total carbon and ash were about one-half of the values found at position 11. This was not to be expected as the velocity of the gas at 11A was higher than at 11. The probable explanation is that the flow of gas around the nose of the arch after it had been raised was not the same as before, and that the gas passing position 11A was not of the same stream as that passing position 11, but was that which turned upward after passing horizontally under the arch from which a large part of the solids had been deposited either on the arch or by having been thrown out as the stream changed direction.

TABLE 4 AMOUNT AND COMPOSITION OF SOLIDS CARRIED IN THE GAS AT 6 IN. FROM THE SIDE WALL BEFORE AND AFTER THE ARCH WAS RAISED, BOILER LOAD 200 PER CENT OF RATING. FURNACE BURNING ANTHRACITE ON TRAVELING-GRATE STOKER

Position	Arch	Wt. per cu. ft. of gas, gram Carbon	Wt. per cu. ft. of gas, gram Ash	Percentage composition Carbon	Percentage composition Ash	Carbon, percentage of total carbon(a)	Ash, percentage of total ash(b)
11	Original	0.481	0.069	86.8	13.2	21	11
11	New	0.557	0.062	89.8	10.2	26	10
11A	New	0.161	0.023	84.7	15.3	9	5

(a) Total carbon is the sum of gaseous and solid carbon.
(b) Total ash is that which should accompany the total carbon, based on ratio of carbon to ash in the coal.



FIG. 16 VIEW OF SIDE WALL OF FURNACE AFTER 12 DAYS OF OPERATION WITH ARCH RAISED, FURNACE BURNING ANTHRACITE ON TRAVELING-GRATE STOKER, NANTICOKE, PA.

Fig. 16 shows the side wall of the furnace after the boiler had operated for 12 days at approximately 200 per cent of rating with the raised arch. The entire front arch and the upper 16 in. of the side and front walls were new at the beginning of this period, but the remainder of the walls were the same as those in the views of Fig. 10 two months before. The walls have the same general appearance as in the other photographs. The

accumulation of slag on the walls was, perhaps, somewhat heavier in parts, but there were similar evidences of fluid slag and erosion.

Fig. 17 shows the boiler tubes at the same time. Comparison of this view with Fig. 11, which was of the tubes at the end of a similar period before the arch was raised, shows that the accumulation was somewhat lighter, the spaces between the tubes were not as completely covered, and the slag had fallen from some of the tubes. It cannot be definitely concluded that this improvement in the condition of the tubes was entirely due to the increased height of the arch. The amount of slag on the tubes when the boiler comes out of service must vary with the thoroughness of the cleaning by hand lancing during operation, particularly of the last time before taking the boiler out of service.

The increase in the height of the arch did, however, result in marked improvement in the operation of the boiler in that higher loads could be maintained. With the original arch the limit of maintained output of the boiler was about 50,000 lb. of steam per hour, equivalent to about 225 per cent of rating; after the arch was raised an output of 56,000 lb. of steam per hour, equivalent to about 255 per cent of rating, could be easily maintained. This was largely because of the lessened resistance of the larger outlet of the furnace. The average pressure drop from the furnace to the stack at 200 per cent of rating was 0.74 in. of water before and 0.66 in. after it was raised.

The possibility of increased boiler output on these boilers was of a particular advantage in this plant for two reasons: (1) The older and smaller boilers operated at much lower efficiency than the newer and larger boilers; therefore increased output from the latter would result in a lesser number of the former in service and a great overall plant efficiency. (2) As the coal cost was low the investment costs were a large percentage of the total cost of current; therefore increased output per boiler reduces the investment cost per unit of current generated.

GENERAL SUMMARY OF THE INVESTIGATION

The range of values of the refractories service conditions, namely, the temperature, composition, and velocity of the furnace gases, the temperatures of the refractories, and the amount and composition of the solids carried in the gases when burning the small sizes of anthracite on a traveling-grate stoker has been determined and recorded.

The slagging of the refractories and the boiler tubes was more severe than is normal for furnaces burning anthracite. The factors which caused the slagging were the small size of the fuel, the high velocity of the gases, the reducing atmosphere, and the high temperatures.

The velocity of the gases was high because of the restricted area for gas passage and because the gases passed upward in a narrow stream toward the furnace outlet. Because of the high velocity and the fine size of the fuel a large amount of coal as well as ash was carried and deposited on the walls, arches, and boiler tubes.

A reducing atmosphere was maintained over the front wall and a part of the side walls because complete mixing of the combustible gases with air for their combustion was not effected until near the boiler tubes.

The temperatures were high because of the delayed combustion, because the area of exposed boiler tubes was small, and because the heat-absorbing surface became less effective as it was covered with slag.

The data taken after the arch had been raised did not show conclusively that the service conditions were less severe because a lower excess of air was maintained during the period of observation. At the end of a 12-day period, however,[6] it appeared that the boiler tubes were not as badly slagged as before, and it was definitely shown that higher loads could be carried on the boiler because of the lower pressure drop from the furnace to the stack.

This investigation disclosed no reason why, with the proper design of furnace to obviate the extremely high temperatures and high content of reducing gas, the burning of anthracite of the sizes smaller than buckwheat No. 3 should be accompanied by more difficulty with refractories than the burning of the larger steam sizes.

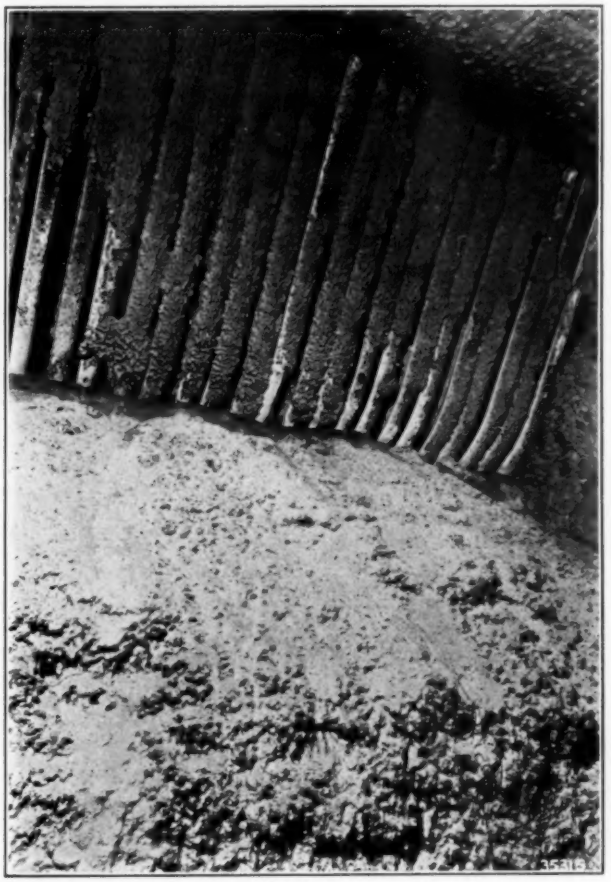


FIG. 17 VIEW OF BOILER TUBES WITH SLAG ACCUMULATION AFTER 12 DAYS OF OPERATION WITH ARCH RAISED, FURNACE BURNING ANTHRACITE ON TRAVELING-GRATE STOKER, NANTICOKE, PA.

ACKNOWLEDGMENTS

This investigation has been carried on under the general direction of the Special Research Committee on Boiler Furnace Refractories of the A.S.M.E., of which C. F. Hirshfeld is chairman, and under the direct supervision of P. Nicholls, supervising fuel engineer, Bureau of Mines.

The study covered in this report was made possible by the cooperation of H. M. Warren, consulting engineer, C. R. Seem, electrical engineer, and R. B. Allbee, chief engineer, Nanticoke power plant of the Glen Alden Coal Co., Scranton, Pa., and the operating force of the plant.

The analyses of coal ash and slag were made in the general analytical laboratory of the Bureau of Mines at Pittsburgh under the direction of W. A. Selvig, and the coal analyses and softening-temperature determinations in the coal-analysis laboratory under the direction of H. M. Cooper.

[6] The engineers of the plant reported after five months' operation with the arch raised that the slagging of the tubes was less than before.

Other published reports of this investigation are as follows:

1 "Preliminary Findings in Refractory Investigations," by R. A. Sherman, P. D. Helser, H. W. Brooks, and G. A. Bole. *Power*, vol. 62, Aug. 18 and 25, 1925, pp. 234–237, 277–280.

2 "Refractories Service Conditions in Boiler Furnaces," by R. A. Sherman. *Power*, vol. 63, Jan. 19, 1926, pp. 113–115.

3 "Refractories Service Conditions in Furnaces Burning Pittsburgh Coal on Chain Grates," by R. A. Sherman and W. E. Rice. MECHANICAL ENGINEERING, vol. 48, November, 1926, pp. 1115–1122.

4 "Refractories Service Conditions in Furnaces Burning Illinois Coal on Chain Grates and Eastern Bituminous Coal on Underfeed Stokers," by R. A. Sherman, W. E. Rice, and L. B. Berger. MECHANICAL ENGINEERING, vol. 48, December, 1926, pp. 1389–1396.

5 "Temperatures in Powdered-Coal Furnaces Having Extended Radiant Heating Surface," by R. A. Sherman. MECHANICAL ENGINEERING, vol. 49, April, 1927, pp. 335–338.

6 "Service Factors Governing the Slagging of Boiler-Furnace Refractories," by R. A. Sherman and Edmund Taylor. MECHANICAL ENGINEERING, vol. 49, 1927, pp. 735–741. Also printed in the *Journal of the American Ceramic Society*, vol. 10, August, 1927, pp. 629–643.

7 "Refractories Service Conditions in a Furnace Burning Pittsburgh Coal on Underfeed Stokers," by R. A. Sherman and W. E. Rice. MECHANICAL ENGINEERING, vol. 49, October, 1927, pp. 1085–1092.

8 "Refractories in Power Plants," by R. A. Sherman. *American Refractories Institute Technical Bulletin* No. 18, December, 1927. Also *Power*, vol. 67, May 8, 1928, pp. 829–830.

9 "Refractories Service Conditions in Furnaces Burning Powdered Illinois Coal with Long-Flame Burners," by R. A. Sherman and Edmund Taylor. Trans. A.S.M.E. Fuels and Steam Power Section, vol. 50, no. 15, May-August, 1928, pp. 125–140.

10 "Refractories Service Conditions in Furnaces Burning Fuel Oil," by R. A. Sherman, Edmund Taylor, and H. S. Karch. Trans. A.S.M.E., Fuels and Steam Power Section, vol. 50, no. 33, Sept.-Dec. 1928, pp. 313-322.

FSP-51-32

Design and Test of a Venturi Stack

Result of Tests on a Stack Operating on the Venturi or Ejector Principle, Through Which the Gaseous Products of Combustion Are Discharged for the Purpose of Inducing the Proper Draft

By ALEXEY J. STEPANOFF,[1] SAN FRANCISCO, CALIF.

IN ONE form of induced-draft system the gaseous products of combustion are discharged through a venturi-shaped stack, using the ejector principle to obtain the proper draft. The venturi stack was originated in France about 1913 and was adopted in European countries during the war, but is not widely used in America. Herewith is presented a résumé of the results of a study and tests on a stack operating upon the venturi or ejector principle.

The stack tested was designed for use in connection with a vertical fire-tube boiler (Main Iron Works) 42 in. in diameter and 9 ft. high, with 358 sq. ft. of heating surface. The boiler is located in the Mechanical Laboratories of the University of California at Berkeley. Because of a structural change in the original stack involving the addition of two 45-deg. bends, the natural draft was reduced from 0.5 in. to 0.2 in. of water. The venturi stack was installed to provide additional draft. The boiler is equipped with a Ray mechanical oil burner, and since about one-third to one-half of the air required for combustion at full load is furnished by the burner blower, the draft system of the boiler is a combination forced- and induced-draft one, tending toward the balanced-draft type.

A study of the literature available on venturi stacks revealed the lack of design data for the proper proportioning and arrangement of this stack. The two arrangements possible are the "in circuit" and "out circuit;" in the former, hot gases are drawn from the breeching by a blower and discharged through a nozzle into the venturi-shaped stack (see Fig. 1), and in the latter, cold air is discharged through the nozzle and into the stack by the blower.

Contradictory information with regard to venturi stacks may be found in several articles appearing in American technical journals during the period from 1913 to 1926.[2]

The advantages which were claimed for these stacks are that the capacity of the blower need only be 20 per cent of that for the direct induced draft (i.e., a system of draft where all of the stack gases are handled by the fan), and a saving of power of 4 per cent. These statements were confirmed by long theoretical deductions. But later these figures were considerably reduced and practically nullified in the published account of an actual test (*Power*, Mar. 19, 1925). The results of this test showed an increased power consumption of about 40 per cent as compared with the direct induced-draft system operating under the same conditions.

The dimensions of the stack are shown in Fig. 1. The proportioning of different parts of the stack was a matter of judgment with respect to best practice as exemplified in the design of existing apparatus for fluid flow. The final arrangement of parts, the choice of type and size of nozzle and fan, and the position of the nozzle were made after preliminary tests of the stack as an air-jet pump. The stack was placed in a horizontal position and cold air used as the working fluid.

The investigation with cold air covered the following problems: (1) Performance of the stack for different locations of the jet and the determination of the best position; (2) the performance characteristics for different back pressures (corresponding to different stack resistances) and for different driving air (nozzle) pressures, involving the calculation of the ratio of induced to driving air; (3) the performance of the jet pump with different forms of nozzles: namely, (*a*) single-jet type (Fig. 2); (*b*) annular type; (*c*) circular single jet with spiral guide vanes; (*d*) single jet with a diffuser; and (*e*) multi-jet type.

The quantity of air discharged was measured by means of a standard pitot tube. The differential pressures, in some instances as low as 0.0005 in. of water, were measured by a special inclined manometer developed at the University of California and shown in Fig. 3.[3]

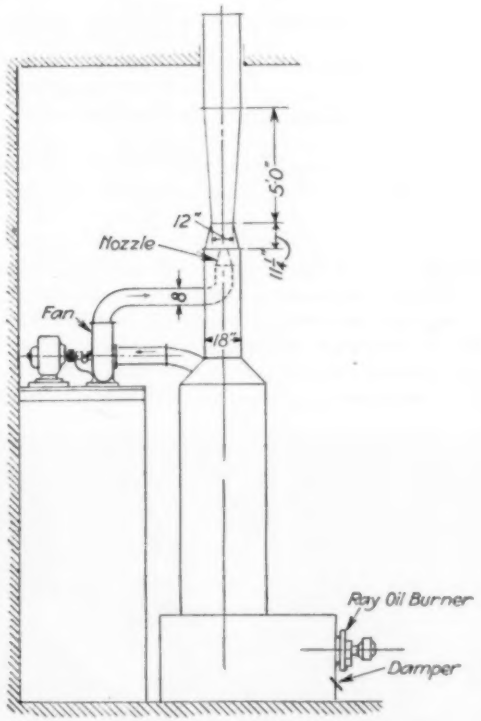


FIG. 1 ARRANGEMENT OF VENTURI STACK

The general conclusions from these tests are as follows:

1 The best position for the nozzle in the stack is found to be near the base of the converging cone; the change in position in close neighborhood to this point has very little effect upon the efficiency of the stack, while as the nozzle approaches the throat, the effect of the change in position becomes more pronounced. The reason for this lies in the fact that near the throat of the

[1] Engineer, Byron Jackson Pump Manufacturing Company. Assoc-Mem. A.S.M.E.

[2] A. M. Bellis, *Engineering Magazine*, July, 1915; *Power*, November, 1913, p. 371; A. W. H. Griepl, *Combustion*, April 22, 1922.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] A detailed description of this manometer is given in "Ventilation of Mines," by W. S. Weeks, McGraw-Hill, 1926.

stack the nozzle presents more obstruction to the flow of induced air than when it is near the base of the cone.

2 The nozzles may be listed in the following order when considered from the point of view of efficiency—that is, the ratio of induced to driving air—the first being the best, after which come the circular nozzle with spiral vanes, multi-jet nozzles, plain circular, annular, and circular with diffuser. The best performance was obtained by using the nozzle equipped with spiral vanes, because the rotation of the entering stream caused a uniform velocity distribution throughout the cross-section of the stack, while with the other nozzles higher velocities were observed in the upper half of the stack. This condition was probably due

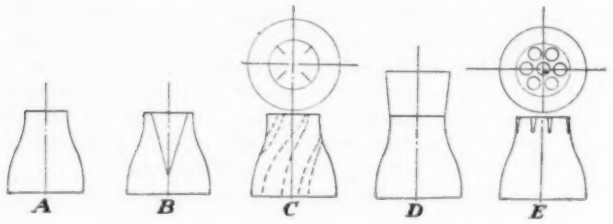


FIG. 2 AIR-JET PUMP WITH DIFFERENT FORMS OF NOZZLES

to the position of the stack near the floor, with obstructed flow of air into the intake. When used in a vertical position the spiral-vane nozzle has no advantage over the circular nozzle. The low efficiency of the annular nozzle is probably due to the large angle of the central cone. The addition of the diffuser to a nozzle approximately doubles the discharging capacity for the same nozzle pressure.

For all nozzles it was observed that the low driving heads were more efficient than the higher ones. The same was found to be true for hydraulic jet pumps recently tested by the author. The draft produced increased with an increase in pressure of the driving air or with an increase of the area of the nozzles; i.e., with an increase in the quantity of driving air in both cases. But it was found to be more economical to increase the quantity of driving air by



FIG. 3 SPECIAL INCLINED TYPE OF MANOMETER USED

increasing the nozzle area rather than by increasing the pressure back of the nozzle.

A Sturtevant fan (Monogram No. 3) was chosen for the installation of the stack on the boiler. This unit is rated to deliver 700 cu. ft. of gases per minute against a static pressure of 3.6 in. of water at 65 deg. fahr. at 1750 r.p.m., or 700 cu. ft. of gases against a static pressure of 2 in. of water at 500 deg. fahr. at the same speed. The ratio of the fan pressure to the required draft in this particular case is about 4, and the ratio of the fan capacity to the total quantity of boiler gases is about 0.7. A direct-current motor was used to drive this fan. The general arrangement is shown in Fig. 1. The furnace was equipped with a Ray burner, size No. 5, capacity 20 gal. of oil per hour. Three dampers were in the path of the gases through the stack: one in the pit below the burner, which was closed during these tests; the second just above the hood of the boiler; and the third on the top of the stack. The last two dampers were wide open for all runs. The

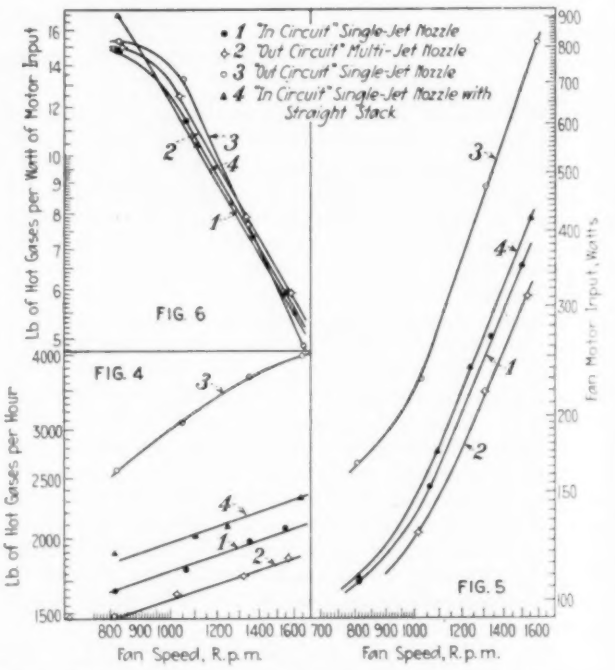


FIG. 4 WEIGHT OF HOT GASES DISCHARGED PER HOUR AT VARIOUS SPEEDS

FIG. 5 FAN-MOTOR INPUTS AT VARIOUS SPEEDS

FIG. 6 WEIGHT OF HOT GASES PER WATT OF MOTOR INPUT AT VARIOUS SPEEDS

necessary air for combustion was delivered by the fan of the burner, and through a clearance between the burner and the firebox door. The lower damper was kept closed because the quantity of air produced in this way was sufficient to support a good combustion of about 12 per cent of CO_2 content, and to obtain a boiler output up to 25 per cent overload. The rated capacity of the boiler is 25 boiler hp. From the preliminary runs it was found that with the lower damper open, double the rated capacity of the boiler could be obtained.

The following arrangements were tested on hot gases at different fan speeds: (1) "In circuit" with a single-jet nozzle $4^1/_2$ in. in diameter, area 15.9 sq. in.; (2) "in circuit" with a nozzle having seven jets each $1^1/_2$ in. in diameter, combined area 12.36 sq. in.; (3) "out circuit" with the first nozzle (for these runs the suction of the fan was disconnected from the boiler hood); (4) the same arrangement as (1) but with the venturi-shaped part of the stack replaced by a cylindrical pipe; (5) the same arrangement as (1) but with the fan not running, the draft being produced by steam jets (for these tests a $^3/_8$-in. pipe ring with 20 orifices each 0.052 in. in diameter was placed around the tip of the nozzle and connected to the steam line); (6) the arrangement (1) with the fan not running, the draft being due to the natural draft only. The quantity of hot gases was measured by means of a standard pitot tube, the readings being taken at twenty points.

The results of these tests are plotted in Figs. 4 to 9, and given in Tables 1 and 2. The logarithmic scale has been used on these

TABLE 1 GAS TEMPERATURES AT FAN DISCHARGE AND STACK TOP FOR VARIOUS NOZZLE ARRANGEMENTS

	In-circuit single-jet nozzle				In-circuit multi-jet nozzle			
Fan speed, r.p.m.	820	1055	1350	1530	820	1020	1315	1560
Temperature of gases at fan discharge, deg. fahr.	582	614	621	614	618	618	627	627
Temperature of hot gases at the top of stack, deg. fahr.	432	466	450	476	436	424	428	424
	Out-circuit single-jet nozzle				In-circuit single-jet nozzle, straight stack			
Fan speed, r.p.m.	820	1140	1330	1630	800	1090	1240	1580
Temperature of gases at fan discharge, deg. fahr.	109	109	109	109	659	666	649	613
Temperature of hot gases at top of stack, deg. fahr.	359	382	386	372	436	424	428	424

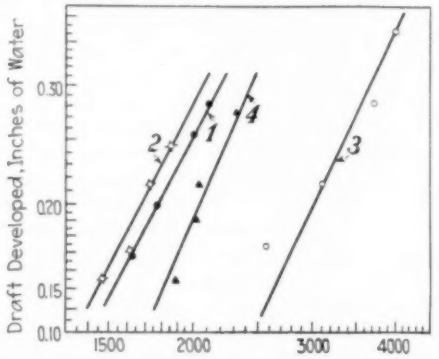


FIG. 7 DRAFT DEVELOPED AT BASE OF STACK FOR VARIOUS WEIGHTS OF HOT GASES PASSING PER HOUR
(For notation see Fig. 5.)

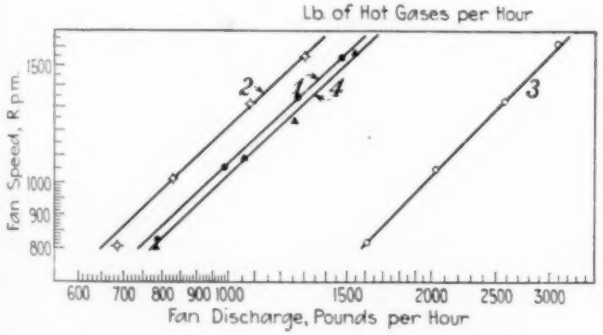


FIG. 8 FAN DISCHARGES AT VARIOUS SPEEDS OF FAN

graphs. Fig. 4 shows that the quantity of boiler gases varied considerably for different arrangements, but the efficiency is approximately the same for all of them, as is seen from Fig. 6, where the quantities of hot exhaust gases per watt of the fan-motor input are plotted against the motor speed. From these curves it is seen that the quantity of gases moved per watt decreases with higher fan speeds and hence higher fan discharge pressures. The same result was obtained from the runs on cold air. The steam consumption for steam-jet draft was on the average 8 per cent of the boiler output. It is evident that this is the least economical system tested and should be used only in the

TABLE 2 STEAM-JET INDUCED DRAFT

Run number	1	2	3
Steam consumed per hour, lb.	83.5	77.6	57.3
Per cent of boiler output	8.97	9.6	7.7
Weight of hot boiler gases, lb. per hr.	2095	1960	1770
Pounds of hot gases per pound of steam used	25.1	25.2	30.8

case of emergency. The substitution of the cylindrical part instead of the venturi-shaped portion of the stack decreased slightly the quantity of gases moved per watt (see Fig. 6). The quantity of hot gases moved by natural draft was found to be 1290 lb. per hour, the draft being 0.125 in. of water.

It should be noted that the out-circuit arrangement (3) is the most effective. The quantity of hot gases handled in this arrangement is approximately twice that for the in-circuit system. The reasons for this are the greater density of cold air, and hence the greater quantity of air discharged by the fan.

It has been mentioned that the venturi stack using the ejector principle for producing draft is less economical than the direct induced-draft system. The same conclusion may be based upon experimental data available on water-jet pumps. It is evident that it is more economical to use one low-pressure centrifugal pump to raise a certain quantity of water to a definite level than to use a smaller high-pressure pump for a portion of water to be raised and use the discharge of this pump to drive a jet pump to raise the remainder of the water. This is true because the efficiency of the jet pump is about one-half that of the centrifugal pump.

But the venturi stack with the jet method of producing draft has the following advantages: (1) To move the same quantity of boiler gases the size of the fan is considerably smaller; (2) a high-speed motor can be used, as the fan discharge pressures are higher in this system; and (3) the space occupied is smaller and the support can be lighter than for a large fan with a slow-speed motor. All three items result in a lower first cost of installation. For intermittent service, such as in the case described, and also to meet high peak loads for a short time, this system of draft will be found more convenient and economical than any other. It is possible to install it with an existing cylindrical stack, and to start the fan for peak loads and run on natural draft for normal loads. The "out circuit" arrangement is simpler to connect, though no trouble was experienced in running the fan on hot gases at temperatures up to 650 deg. fahr., the fan being equipped with plain bearings without water cooling

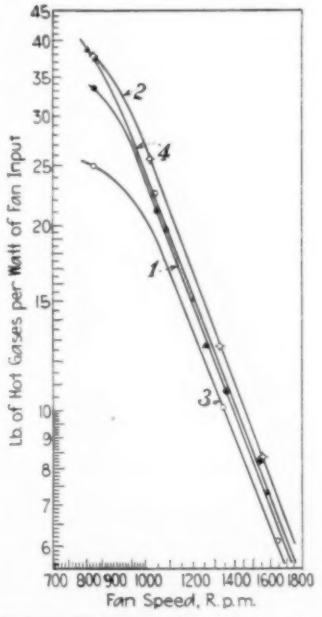


FIG. 9 WEIGHT OF HOT GASES PER WATT OF FAN INPUT AT VARIOUS FAN SPEEDS
(For notation see Fig. 5.)

FSP-51-33

A Graphical Treatment of Heat-Exchange Problems

By JOS. S. STEPANOV,[1] HARRISON, N. J.

In this paper the author points out that the graphical method has been an important tool in engineering calculations and investigations. Applying this method to the investigation of certain heat-transfer problems, he finds it convenient to introduce some new relations between the variables involved in heat transfer and to give them corresponding mathematical expression, thus simplifying the graphical presentation of the heat-transfer equations. An arrangement of interconnected diagrams around the origin of a coordinate system furnishes the numerical values of the variables pertaining to the given set of heat-transfer equations and simultaneously satisfying them. Picturing the change of variables for different conditions, these diagrams can be used for calculations as well as for investigations and for analysis of experimental data.

In the paper are given and explained equations and diagrams for handling the heat-transfer problems pertaining to surface heat exchangers and direct-contact heat exchangers, using countercurrent and parallel flow of substances, and their applications in general calculations and investigations, and in the analysis of experimental data.

THE problems confronting the modern engineering art are becoming more and more complicated. The number of factors involved in certain engineering problems and affecting their final solution is so great that it becomes impossible to visualize the character and value of the influences they exercise upon the sought-for solution.

In the experience of the author, the graphical method of handling such problems has proved to be a helpful tool in the hands of designers or investigators.

Heat-transfer problems no doubt belong to this class of problems, and the author believes it worth while to develop for their treatment a graphical method which he devised many years ago,[2] for the calculation and investigation of ship propellers and which was found useful. Its general idea may be applied also in the solution of other engineering problems.

For convenience in locating desired material, the author has adopted the following sequence treatment of the subject:

I Transformation of Heat-Exchange Equations
  (*a*) Surface Heat Exchangers
  (*b*) Direct-Contact Heat Exchangers
II Graphical Representation of Equations
III Logarithmic Curves
IV General Properties of the Diagrams
V Applications of the Diagrams

I—TRANSFORMATION OF HEAT-EXCHANGE EQUATIONS

(*a*) Surface Heat Exchangers

Fundamental Equations. The amount of heat transferred per hour in surface heat exchangers from one substance to another at a lower temperature is usually expressed in the following ways:

1 In terms of the properties of the hot substance:

$$H = G_h c_h \Delta T \quad \ldots\ldots [1]$$

2 In terms of the properties of the cold substance:

$$H = G_c c_c \Delta t \quad \ldots\ldots [2]$$

3 In terms of the heat-transmitting surface:

$$H = SU\theta_m \quad \ldots\ldots [3]$$

where

$$\theta_m = \frac{\theta_1 - \theta_2}{\log_e \frac{\theta_1}{\theta_2}} \quad \ldots\ldots [4]$$

in which H = B.t.u. per hr. transmitted from the hot substance to the cold one through the total heat transmitting surface under consideration

G_h, G_c = weight of the substance, in lb. per hr. (Subscript h for hot substance, c for cold substance.)

c_h, c_c = mean specific heats at constant pressure, in B.t.u. per lb. per deg. fahr.

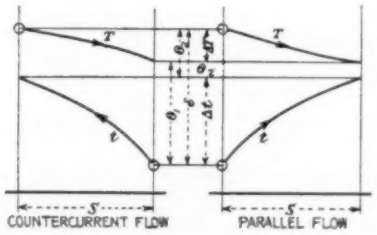


FIG. 1 SCHEME OF TEMPERATURE NOTATION EMPLOYED

T = temperature of the hot substance, deg. fahr.

t = temperature of the cold substance, deg. fahr.

ΔT, Δt = corresponding temperature drops, deg. fahr.

θ_1, θ_2 = temperature differences at the ends of the heat-transmitting surface, deg. fahr.

θ_m = mean temperature difference along the total heat-transmitting surface, deg. fahr.

U = heat-transfer coefficient, in B.t.u. per hr. per sq. ft. per deg. fahr.

S = heat-transmitting surface, in sq. ft.

δ = available temperature drop, in deg. fahr., i.e., the difference between the highest temperature of the hot substance and the lowest temperature of the cold substance.

The temperature notations are given also in Fig. 1.

The Specific Heats c_h and c_c for any substance are beyond the control of designers, and therefore can be conveniently considered together with the weight of the substance as single variables $W_h = G_h c_h$ and $W_c = G_c c_c$, which represent the heat capacities of the interacting substances and which, according to German investigators,[3] may be called "water values" of the substances. These express the amount of heat given out or absorbed by a temperature change of the substance of 1 deg. fahr., and are obviously equal to such an amount of heat in W lb. of water, hence the name "water values."

[1] Worthington Pump & Machinery Corporation. Mem. A.S.M.E.

[2] Published in *Viestnik Ingenerov*, 1915, Petrograd.

Contributed by the Power Division and presented at the Annual Meeting, New York, December 3 to 7, 1928, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

[3] Grober, Nusselt, ten Bosch and others.

The Temperature Drops ΔT and Δt as well as the *temperature differences* θ_1, θ_2, and θ_m, can be expressed in common terms of the available temperature drop δ as follows:

$\frac{\Delta T}{\delta} = e_c$ shows to what extent the hot substance is cooled, and may be called the "cooling efficiency" of the heat exchanger.

$\frac{\Delta t}{\delta} = e_h$ shows to what extent the cold substance is heated, and may be called the "heating efficiency" of the heat exchanger.

$\frac{\theta_m}{\delta} = e_m$ shows to what extent the available temperature drop δ is used to provide the thermomotive force causing the heat transmission, and may be called "thermomotive coefficient" of the heat exchanger.

Hence

$$\Delta T = e_c\delta$$
$$\Delta t = e_h\delta$$
$$\theta_m = e_m\delta$$

For countercurrent flow, Fig. 1,

$$\theta_1 = \delta - \Delta T = \delta - e_c\delta = \delta(1 - e_c)$$
$$\theta_2 = \delta - \Delta t = \delta - e_h\delta = \delta(1 - e_h)$$

and for parallel flow,

$$\theta_1 = \delta$$
$$\theta_2 = \delta - (\Delta T + \Delta t) = \delta - \delta(e_c + e_h) = \delta[1 - (e_c + e_h)]$$

The Transformation of Equations [1]–[4] may be effected by substituting the heat capacities W_h and W_c and the values e_h and e_c for the corresponding variables of these equations. This gives

$$H = W_h e_c \delta \quad [1a]$$
$$H = W_c e_h \delta \quad [2a]$$
$$H = SUe_m\delta \quad [3a]$$
$$\theta_m = e_m\delta \quad [4a]$$

or, dividing all members by δ,

$$\frac{H}{\delta} = W_h e_c \quad [1b]$$
$$\frac{H}{\delta} = W_c e_h \quad [2b]$$
$$\frac{H}{\delta} = SUe_m \quad [3b]$$
$$\frac{\theta_m}{\delta} = e_m \quad [4b]$$

where Equation [4b] is given in general form and may be developed by using [4] and the foregoing assumptions as follows:

For countercurrent flow,

$$e_m = \frac{e_c - e_h}{\log_e \frac{1 - e_h}{1 - e_c}} \quad [5]$$

for parallel flow,

$$e_m = \frac{e_c + e_h}{\log_e \frac{1}{1 - (e_h + e_c)}} \quad [6]$$

or in general form for both flows,

$$e_m = \phi\,(e_c, e_h) \quad [7]$$

The fraction H/δ in Equations [1b], [2b], and [3b] expresses the mean amount of heat transferred per hour and per degree fahrenheit of the available temperature drop δ, i.e., the intensity of the utilization of this drop.

To avoid having more than three variables in Equation [3], a new one is introduced, namely, $h_m = Ue_m = H/\delta S$, which expresses the thermal load of heat-transmitting surface in B.t.u. per hr. per sq. ft. per deg. fahr.

Now the heat-transfer equations appear in the following form:

$$\frac{H}{\delta} = W_h e_c = W_c e_h \quad [8]$$
$$\frac{H}{\delta} = SUh_m \quad [9]$$
$$h_m = Ue_m \quad [10]$$
$$e_m = \phi\,(e_h, e_c) \quad [11]$$

(*b*) Direct-Contact Heat Exchangers

Assumptions. Such heat exchangers as are represented by dry coolers of different kinds of granular material (coke, lime, cement clinker, etc.) or gas coolers by direct contact with water, have no measurable heat-transmitting surface. Therefore the heat-transfer equations should be changed to take care of these cases.

It is assumed that heat transmission takes place on the surface of the particles (whatever size they may be), and some heat-transmitting surface exists, though not readily accessible for measurement and calculations. This surface depends upon the size of the particles, and for a given size it depends only upon the space the material occupies.

Instead of a heat-transmitting surface we may consider a heat-transmitting volume V expressed by the factors cross-sectional area A, perpendicular to the flow of the substance, multiplied by L, the length of the contact; i.e., $V = AL$, for each of these factors has its own function in the heat exchange.

Formation of the Equations. Arranging Equations [8], [9], and [10] as follows:

$$\frac{H}{\delta} = W_h e_c = W_c e_h = SUe_m$$

and replacing S by $V = AL$,

$$\frac{H}{\delta} = W_h e_c = W_c e_h = ALUe_m$$

dividing all members by AL,

$$\frac{H}{\delta AL} = \frac{W_h}{A} \times \frac{e_c}{L} = \frac{W_c}{A} \times \frac{e_h}{L} = Ue_m = h_m$$

In different ways, all members express the same thing, namely, h_m, or the amount of B.t.u. transferred per hour per degree fahrenheit, per cubic foot of heat-transmitting volume ($V = AL$).

U, the heat-transfer coefficient in this case, means the number

of B.t.u. transferred per hour per degree fahrenheit per cubic foot of the heat-transmitting volume.

$\frac{W_h}{A}$ and $\frac{W_c}{A}$ express the heat capacities of interacting substances per hour corresponding to the square foot of the cross-sectional area, and therefore they may be correspondingly indicated by W_h and W_c. Hence

$$h_m = \frac{W_h e_c}{L} = \frac{W_c e_h}{L} = U e_m$$

The products $W_h e_c = W_c e_h$ express the portions of the heat capacities exchanged in the process, in B.t.u. per hr. per deg. fahr. per sq. ft. of cross-sectional area, and may be indicated as one variable by the symbol K.

The following set of equations expressing the heat exchange in the direct-contact heat exchanger may now be offered:

$$K = W_h e_c = W_c e_h \quad [12]$$

$$K = L h_m \quad [13]$$

$$h_m = U e_m \quad [14]$$

$$e_m = \phi\,(e_c, e_h) \quad [15]$$

II—GRAPHICAL REPRESENTATION OF EQUATIONS

Inspection of Equations [8]–[11] and [12]–[15] shows that—

1 Each pair of equations, [8] and [12], are mathematically identical, for the variables have the same range of numerical values.

2 In Equations [11] and [15], developed in [5] and [6], the variables e_c and e_h are interchangeable: the value of e_m does not change if e_c is replaced by e_h, and vice versa. The combined effect of the foregoing permits the determination of both variables, although [8] and [12] have been plotted as one equation.

3 Each equation involves three variables, and any two consecutive equations in arrangement given above have a common variable.

These properties make it possible:

(*a*) To plot each equation in the rectangular-coordinate system as a family of curves, each of them corresponding to the definite numerical value of the variable selected as the variable parameter. Using only the positive values of the variables, each equation will occupy only one quadrant of the coordinate system.

(*b*) To locate all four equations in four quadrants around the origin, plotting any pair of equations with common variables in two adjacent quadrants, so that the common variable may be measured in the same coordinate direction for both equations. The arrangement of both sets of equations is shown in Fig. 2.

After the selection of the variables measured in the directions of the coordinate system, the remaining variables in each equation should be treated as variable parameters.

In this arrangement Equations [8], [9], and [10], and [12], [13], and [14] represent the families of straight lines passing through the origin: each straight line for a definite numerical value of the variable parameter.

Equation [11] or [15] expresses the family of curves, each curve corresponding to a definite value of the variable parameter (e_c or e_h, ad lib.).

The general arrangement of the diagrams is shown in Fig. 3.

Any rectangle around the origin like that shown in Fig. 3, with sides parallel to the coordinate axis, gives, in its intersections with the axes and curves or straight lines in the quadrants, the numerical values of eight variables simultaneously satisfying all four equations plotted in the quadrants.

In addition, taking the given or the first selected value of e_h (or e_c, ad lib.) along the axis, the value of e of another substance is found at the point of intersection with the e-curves (e_c or e_h, correspondingly); taking this value along the e_h-axis and dropping a perpendicular to its intersection with the horizontal line SW_c projected, the heat capacity W_h of the other substance is determined.

The same diagrams can be used for both sets of equations, i.e., for investigations of both surface and direct-contact heat exchangers, bearing in mind that some of the variables have different meanings and ranges of numerical values, and therefore that the scale for such variables on the same diagrams should be correspondingly different for each case.

For convenience in further development, the quadrants may be distinguished according to the variable parameters as follows:

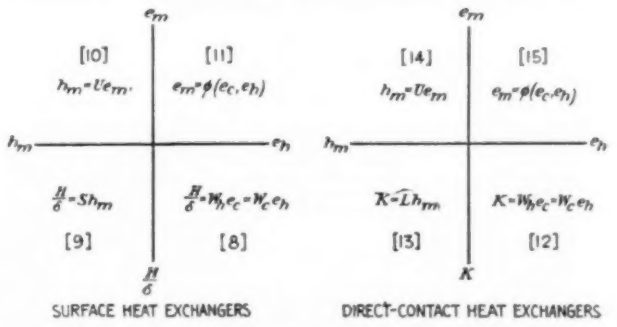


FIG. 2 ARRANGEMENT OF EQUATIONS AROUND ORIGIN OF COORDINATE SYSTEM

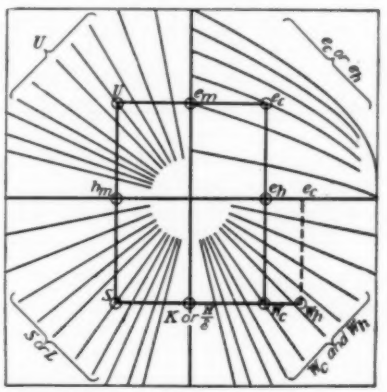


FIG. 3 GENERAL ARRANGEMENT OF FOUR INTERCONNECTED DIAGRAMS AROUND THE ORIGIN OF THE COORDINATE SYSTEM

1 Temperature quadrant, parameter e_c or e_h
2 Heat-transfer-coefficient quadrant, parameter U
3 Heat-transmitting surface (S) or length (L) quadrant, parameter S or L, and
4 Heat-capacity quadrant, parameter W.

III—LOGARITHMIC CURVES

In the temperature quadrant the equation $e_m = \phi\,(e_c, e_h)$ is plotted. The explicit forms of this equation differ for the countercurrent flow and parallel flow as shown by [5] and [6].

The computation, the plotting, and even the use of the logarithmic curves expressed by these equations are much easier if their peculiarities are taken in consideration.

(*a*) *Countercurrent Flow* (Fig. 4)

$$e_m = \frac{e_c - e_h}{\log_e \frac{1 - e_h}{1 - e_c}} \quad [5]$$

1 The numerical range of e_m, e_c, and e_h, as follows from their definitions, is from 0 to +1.

2 All values of e_m are positive for the values of e_c and e_h in the given range.

3 For all values of the parameter e_c in the given range, $e_m = 0$ if $e_h = 1$ (limit value). This means that all curves have a *common end point* on the e_h-axis, at a distance from the origin $e_h = 1$.

4 For all values $e_h = e_c$, values of e_m become indeterminate (0/0). The evaluation of this expression shows that the real values of the function in these points are $e_m = 1 - e_h = 1 - e_c$. All these points are located on the straight line connecting the points of coordinate axis: ($e_h = 1$, $e_m = 0$) and ($e_h = 0$, $e_m = 1$). It is the locus of equal cooling and heating efficiencies ($e_h = e_c$) which correspond to the equal heat capacities

$$W_h = W_c \text{ or } \frac{W_h}{W_c} = \frac{e_h}{e_c} = 1$$

connected with the different values of this ratio. In such cases the heat capacities may change, keeping the same ratio $W_c/W_h = \alpha$. Introducing this condition in [5] and [6], new equations involving the variable parameter α are obtained as follows:

For countercurrent flow,

$$e_m = \frac{e_h(\alpha - 1)}{\log_e \dfrac{1 - e_h}{1 - \alpha e_h}} \quad \ldots\ldots\ldots\ldots\ldots\ldots [16]$$

For parallel flow,

$$e_m = \frac{e_h(\alpha + 1)}{\log_e \dfrac{1}{1 - e_h(\alpha + 1)}} \quad \ldots\ldots\ldots\ldots\ldots\ldots [17]$$

(*c*) *Peculiarities of Curves* expressed by [16] and shown in Fig. 5.

1 If $\alpha = 0$, [16] becomes equivalent to [5]; for $e_c = 0$, curves $\alpha = 0$ and $e_c = 0$ coincide. This means that $W_c/W_h = 0$ only if $W_h = \infty$. Since e_h is finite, W_c is finite, therefore W_h should be ∞ in order that $W_c/W_h = 0$; that is, the heat capacity of the substance changing the state of aggregation is infinite.

2 If $\alpha = 1$, $e_m = 0/0$, that is, indeterminate, and the real values coincide with those of [5] and are graphically represented by the same straight line [case (*a*) par. 4].

3 If $e_h = 0$, $e_m = 0/0$, also indeterminate. The evaluation of the indeterminate form shows that the real value of $e_m = 1$. This means that all curves have a common end point on the e_m-axis at a distance from the origin $e_m = 1$.

4 If $e_h = 1/\alpha$, $e_m = 0$ for any value of $\alpha > 1$. This means that these curves have their end points on the e_h-axis, but each at a different distance from the origin and equal to $e_h = 1/\alpha$.

5 If $e_h = 1$ and $\alpha < 1$, $e_m = 0$ for all values of $\alpha < 1$. This means that part of the α-curves, for $\alpha < 1$, have a common end point on the e_h-axis at the distance $e_h = 1$ from the origin.

(*d*) *Peculiarities of Curves* expressed by [17] and shown in Fig. 6.

1 If $\alpha = 0$, curves $\alpha = 0$ and $e_c = 0$ coincide as in case (*c*), par. 1.

2 If $\alpha = 1$, $e_m = \dfrac{2e_h}{\log_e \dfrac{1}{1 - 2e_h}}$, the curve of equal efficiencies and heat capacities has its end point on the e_h-axis at the distance from the origin $e_h = 1/(\alpha + 1) = 1/2$.

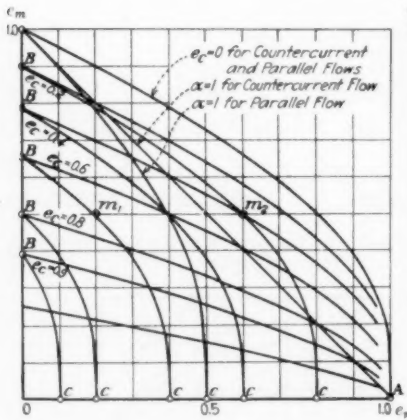


FIG. 4 PECULIARITIES OF THE e_c-CURVES
A = Common end point for countercurrent flow.
B = Common end points of pairs of the e_c-curves, presenting the same value of e_c for both flows.
C = End points of e_c-curves for parallel flow.

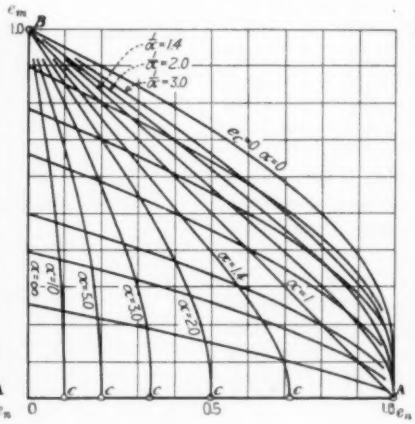


FIG. 5 PECULIARITIES OF THE α_c-CURVES FOR COUNTERCURRENT FLOW
A = Common end point for the α-curves $\alpha < 1$.
B = Common end points for all the α-curves.
C = End points for the α-curves $\alpha \geq 1$.

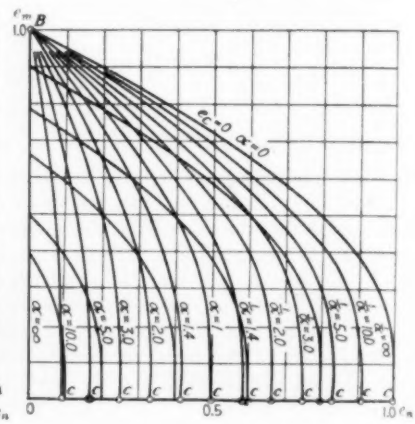


FIG. 6 PECULIARITIES OF THE α-CURVES FOR PARALLEL FLOW
B = Common end point for all the α-curves.
C = End points for different α-curves.

(*b*) *Parallel Flow* (Fig. 4)

$$e_m = \frac{e_c + e_h}{\log_e \dfrac{1}{1 - (e_c + e_h)}} \quad \ldots\ldots\ldots\ldots\ldots\ldots [6]$$

1 The numerical values e_m, e_c, and e_m are as in (*a*) above. In addition, $e_c + e_h$ lies in the same range, because $\Delta T + \Delta t \leq \delta$.

2 For all values of e_c, $e_m = 0$ if $e_c + e_m = 1$ (limit value). This means that all curves have end points on the e_h-axis, but that each curve starts at a different distance from the origin; that is, the values of e_c and e_h at the starting points should satisfy the condition $e_c + e_h = 1$.

(*c*) *Common Points and Curves* (Fig. 4). Both families of curves [(*a*) and (*b*)] have the following in common:

1 Both Equations [5] and [6] become equivalent and represent the same curve when $e_c = 0$. This means that if one of the interacting substances does not change its temperature as by a change in the state of aggregation, both principles of the heat exchange—countercurrent and parallel flow—are equivalent.

2 Similarly, both equations are equivalent if $e_h = 0$; i.e., for the same values of e_c both curves [for cases (*a*) and (*b*)] have a common end point on the e_m-axis.

Sometimes a definite relation (W_c/W_h) between the heat capacities may be desirable on account of characteristic properties

3 If $e_h = 0$, the properties are the same as in (*c*), par. 3.

4 If $e_h = 1/(\alpha + 1)$, $e_m = 0$ for any value of α.

This means that *all* curves have their end points on the e_h-axis at *different* distances from the origin, i.e., at $e_h = 1/(\alpha + 1)$.

All the foregoing statements concerning the peculiarities of the curves in cases (*a*), (*b*), (*c*), and (*d*) can be readily proved by applying the usual methods of the calculus.

Dividing Lines. All families of curves in the temperature quadrant are divided by the lines $\alpha = 1$ into two fields each possessing equivalent properties, but this afforded an additional convenience to the investigator.

1—Fig. 4, Families of Curves *a* and *b*.

On account of the interchangeability of e_c and e_h the final temperature conditions of the interacting substances may be expressed in two ways: If the investigation is started by the condition $W_c/W_h > 1$, i.e., $e_c/e_h > 1$, the point representing this case will be at the left side of the dividing line on the vertical from the point of *greater* heat capacity W_c (point m_1). But by interchanging e_c and e_h, the same temperature condition will be represented by the point at the right side of the dividing line, on the perpendicular from the *smaller* heat capacity W_h (point m_2), $W_h/W_c < 1$.

Which of these equivalent points is to be used depends upon which heat capacity is given or selected first, the greater or the smaller.

2—Figs. 5 and 6, Families of Curves *a* and *c*, and *b* and *d*.

From the foregoing another conclusion may be drawn: By studying the cases where α = constant, for $\alpha > 1$ the vertexes of the rectangles will be on the α-curves at the left side of the dividing lines.

For $\alpha < 1$ the equivalent rectangles will have their vertexes on the α-curves with reciprocal values $= 1/\alpha$ at the right side of the dividing lines.

When possible, it is more convenient to use the left field of curves, because they are located at greater distances from each other.

IV—GENERAL PROPERTIES OF THE DIAGRAMS

(*a*) *The Coordinate Axes*, in addition to their usual function, represent also the loci of the limiting values of the variable parameters in corresponding quadrants. Thus,

The h_m-axis is the locus for S or $L = 0$ and for $U = \infty$

The H/δ- or K-axis is the locus for S or $L = \infty$ and for $W = \infty$

The e_h-axis is the locus for $W = 0$ and for $e_c = 1$ in countercurrent flow, and

The e_m-axis is the locus for $U = 0$ and for $\alpha = \infty$

(*b*) *The Limiting Rectangles.*

1 $S = 0$ or $L = 0$

Such a rectangle is shown in Fig. 7 as No. 1. Here $e_m = 1$, S or $L = 0$, H/δ or $K = 0$, $e_c = 0$, $e_h = 0$, W_c and W_h may have any value (starting points of all W straight lines are at the origin), U = some real value, and h_m a corresponding one. Both countercurrent and parallel flow have similar rectangles—or identical if the same U is selected for both cases.

2 $S = \infty$ or $L = \infty$ (countercurrent flow).

Such a rectangle is shown in Fig. 7 as No. 2. Here $e_h = e_c = 1$, $e_m = 0$, W_c and W_h = any real value, U = any value (all U straight lines start from the origin), and H/δ or K = real values corresponding to heat capacities.

Note the following inconsistency in the algebraic analysis:

$H/\delta = USe_m = U \times \infty \times 0$ (indeterminate)

$H/\delta = W_c e_h = W_h e_c = W \times 1$ (real value)

This can be explained by assuming that the total ∞-surface cannot be fully utilized for transmitting the heat, because during a part of the time of their contact the substances already have equal temperatures.

3 $S = \infty$ or $L = \infty$ (parallel flow).

Such a rectangle is shown in Fig. 7 as No. 3. Here $e_c + e_h = 1$ and $e_m = 0$, the other variables being as in the preceding case.

(*c*) *Typical Rectangles.*

4 The heat capacities of both substances are equal: $W_c = W_h$, and

$$\frac{W_c}{W_h} = \frac{e_c}{e_h} = 1$$

Such a rectangle is shown in Fig. 7 as No. 4. Its vertex in the temperature quadrant will be always on the dividing line $\alpha = 1$, for countercurrent flow as well as for parallel flow.

5 $W_h = \infty$, W_c = finite value, $e_c = 0$, e_h = finite value, and

$$\alpha = \frac{W_c}{W_h} = \frac{e_c}{e_h} = 0$$

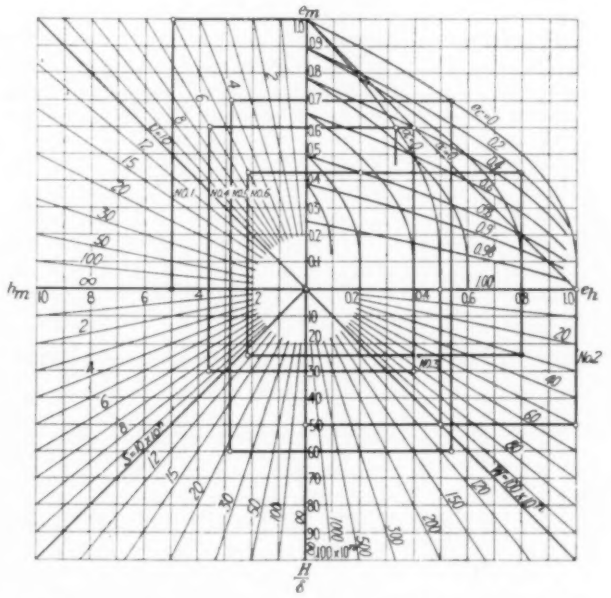


FIG. 7 SHOWING GENERAL PROPERTIES OF DIAGRAMS—LIMITING AND TYPICAL RECTANGLES

One of the interacting substances changes the state of aggregation. Such a rectangle is shown in Fig. 7 as No. 5. Its vertex in the temperature quadrant for any other conditions will be always on the ($e_c = 0$)-curve. The curves $e_c = 0$ and $\alpha = 0$ coincide, therefore for the heat-capacity ratio $\alpha = 0$, the temperature change will be expressed by the same curve ($\alpha = 0$), but for the reciprocal ratio $1/\alpha$, $\alpha = \infty$ and the α-curve coincides with the e_m-axis, thus showing that no temperature change is occurring ($e_h = 0$).

6 $W_h \neq W_c$, e_h and e_c have finite values,

$$\frac{W_c}{W_h} = \frac{e_c}{e_h} = \alpha, \quad \infty > \alpha > 1, \text{ and its reciprocal}$$

$$1 > 1/\alpha > 0$$

The rectangle No. 6 in Fig. 7 is typical for this case. This rectangle shows two vertexes in the temperature quadrant: one for greater heat capacity ($\alpha > 1$), on the left side of the dividing line, and another, for smaller heat capacity, on the right side, on the α-curve having a reciprocal value. This rectangle is characteristic for both countercurrent and parallel flow.

(*d*) *The Scale of Working Diagrams.*

The limiting values of e_m, e_c, and e_A are unity; the range of $h_m = Ue_m$ is narrow, but the variables S, H/δ, and W may vary within an extremely wide range depending on the size of the heat exchanger.

The author used the following scales:

For U-values, the locus of $U = 10$ B.t.u. (for gases) is represented by the diagonal at this quadrant, hence the largest value of h_m on the h_m-axis is $10 \times 1 = 10$.

In the S-quadrant the diagonal expresses $S = 10$ sq. ft., hence max. $H/\delta = 10 \times 10 = 100$.

In the W-quadrant, max. $e_A = 1$; the diagonal of the quadrant expresses 100, hence max. $H/\delta = 100 \times 1 = 100$, corresponding to the S-quadrant.

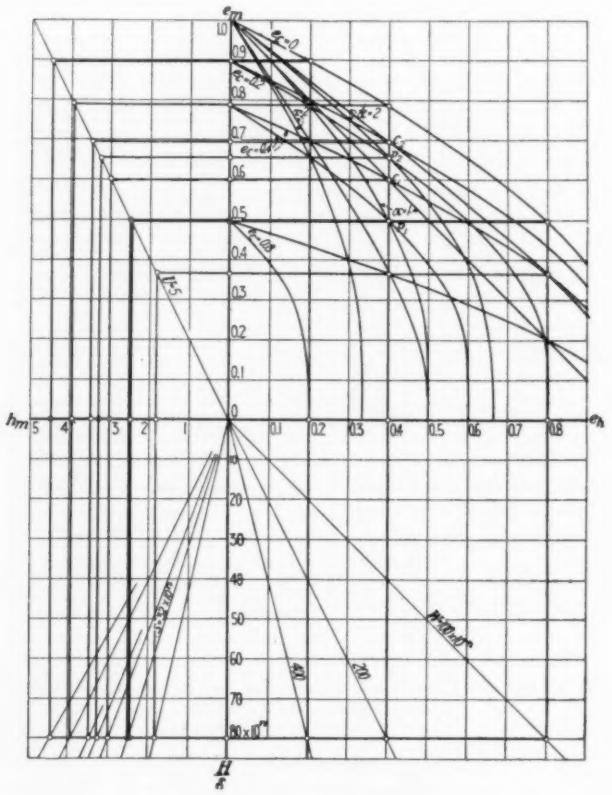


FIG. 8 APPLICATIONS OF THE DIAGRAMS
(Only the lines and curves involved in the examples given are shown.)

To take care of values for S, H/δ, and W greater than these, the figures given on the diagram are multiplied by 10^n, i.e., the figures on the diagram express units, tens, hundreds, thousands, etc., depending on the size of the plant.

In similar way the scales for L, K, and W are chosen (direct-contact heat exchangers).

V—APPLICATIONS OF THE DIAGRAMS

The use of the diagrams for investigations and calculations will be shown by a few examples.

Consider a surface water heater for raising the temperature of 2000 lb. of water from 50 deg. fahr. to 110 deg. fahr. by forcing along the heating surface a gas at 200 deg. fahr. inlet temperature and where $c_p = 0.24$ B.t.u. per lb. per deg. fahr. For these conditions

$W_c = 2000 \times 1 = 2000$ B.t.u. per hr.

$\delta = 200 - 50 = 150$ deg. fahr.

$e_A = 60/150 = 0.4$ where $60 = 110 - 50$

$H/\delta = W_c e_A = 2000 \times 0.4 = 800$ B.t.u. per hr. per deg. fahr.

$U = 5$ B.t.u. per hr. per deg. fahr. per sq. ft. (assumed).

CASE 1: $W_c/W_A = e_c/e_A = 1$, i.e., the heat capacities and efficiencies are *equal*.

The rectangles representing this case for countercurrent flow (No. 1) and for parallel flow (No. 2) are shown in Fig. 8.

The numerical values of the remaining variables as taken from the diagrams are:

	Countercurrent	Parallel
e_m	0.60	0.50
h_m	3	2.5
S	267	320

To transfer the same amount of heat, maintaining the same temperature conditions, the parallel flow requires (320 — 267)/267 = 0.2, or 20 per cent more heating surface than the countercurrent flow.

This difference is caused by the different values of e_m and is expressed on the diagram by the distance c_1-p_1 between the intersections of the rectangle side with the ($e_e = 0.40$)-curve for countercurrent flow (point c_1) and with the same curve for parallel flow (p_1).

The diagram shows that this distance is decreasing with e_A and becomes 0 if $e_A = 0$. This means that the difference between the countercurrent and the parallel flow is negligible if small temperature changes are involved.

CASE 2: $W_c/W_A \neq 1$, $e_c/e_A \neq 1$, i.e., the heat capacities and efficiencies are unequal.

Assume that $W_c/W_A = 1/2$. As before, for $W_c = 2006$ B.t.u. per hr. and $e_A = 0.40$, the values of $W_A = 4000$ and $e_c = 0.20$ result. The rectangles for these conditions are also shown in Fig. 8—for countercurrent flow as No. 3, and for parallel flow as No. 4.

The numerical values of the remaining variables are as follows:

	Countercurrent	Parallel flow
e_m	0.695	0.654
h_m	3.47	3.27
S	230.5	244.2

Again the parallel flow calls for more heating surface, but not to the same extent as at $\alpha = 1$ (case 1), i.e., (244.2 — 230.5)/230.5 = 0.06 or only 6 per cent difference.

Similarly to case 1, this difference is expressed by the distance $c_2 — p_2$. The points c_2 and p_2 are also on the intersections of the e-curves with the α-curves of the corresponding directions of the flow. Therefore the distance $c_2 — p_2$ depends upon the shape of these curves for the same value of α, but for different flows.

All α-curves, for both flows, start from the same point $e_m = 1$, $e_A = 0$ [III, (*c*) and (*d*), Figs. 5 and 6], and each pair of curves for the same α are divergent from the starting point, the divergence being maximum at $\alpha = 1$ and zero at $\alpha = \infty$. The distance $c_2 — p_2$ depends on this divergence, and therefore the conclusion follows that the greater the difference between the heat capacities ($W_c/W_A = \alpha$), the smaller the difference in using countercurrent or parallel flow, which disappears completely if $\alpha = \infty$, i.e., if one of the substances is changing its state of aggregation ($W_c = \infty\beta$).

Assume now that $W_c/W_A = 2$ and that as before $W_c = 2000$ and $e_A = 0.40$; then $W_A = 1000$ and $e_c = 0.80$.

In this case parallel flow is impossible. The rectangle side passing through $e_A = 0.40$ does not intersect the ($e_c = 0.80$)-curve

for parallel flow. The limit for the use of parallel flow is $e_c + e_h = 1$, whereas in this case $e_c + e_h = 0.40 + 0.80 = 1.20$.

For countercurrent flow the case is represented by rectangle No. 5 in Fig. 8.

The numerical values of the remaining variables are as follows:

$$e_m = 3.63;\ h_m = 1.82;\ s = 440.$$

CASE 3: One of the substances (water) is changing its state of aggregation (boiling): $W_c = \infty$, $e_h = 0$, $W_c/W_h = e_c/e_h = \alpha = \infty$.

To make this comparable with the preceding, let us keep the amount of heat transferred H and the available temperature drop δ as before, i.e., $H/\delta = 800$ B.t.u.

The heat capacity W_h and the corresponding temperature change e_c of another substance (gas) may have any values in the limits of given conditions $W_c = \infty$ and $e_h = 0$, but it is interesting to investigate this case with the same values of W_h as in cases 1 and 2, i.e., with $W_h = 2000$, 4000, and 1000.

In this case the interchangeability of e_c and e_h on the diagrams should be made of use, because for the given $W_h = 2000$, 4000, and 1000 corresponding values of $e_c = 0.40$, 0.20, and 0.80 are to be taken on the e_h-axis.

The rectangles representing these conditions are shown in Fig. 8 as Nos. 6, 7, and 8, and express the countercurrent and parallel flow in the following order:

For $W_h = 2000$	No. 6
$W_h = 4000$	No. 7
$W_h = 1000$	No. 8

The values of the remaining variables are as follows:

	No. 6	No. 7	No. 8
e_m	0.783	0.897	0.497
h_m	3.91	4.48	2.48
S	205.5	178.5	322.5

The size of heating surface depends on the location of the rectangle vertex on the ($e_c = 0$)-curve: the nearer the vertex to the e_m-axis, i.e., the smaller the temperature change of the gas (or the greater its heat capacity), the less the surface required to transfer the same amount of heat.

Now compare rectangles Nos. 1, 3, 5, 6, 7, and 8, all representing countercurrent flow (the last three parallel flow as well).

	No. 3	No. 1	No. 5	No. 7	No. 6	No. 8
W_c	2000	2000	2000	∞	∞	∞
W_h	4000	2000	1000	4000	2000	1000
W_c/W_h	1/2	1	2	∞	∞	∞
S	230.5	267	440	178.5	205	322.5

To transfer the same amount of heat to the same weight of a given substance, the surface required will be the less the greater the weight of the second substance, whereby the case of $W_c = \infty$; the effect of the W_h is *smaller* than in other cases because the ($e_c = 0$)-curve has a greater slope than the other e_c-curves.

Making the same comparison for parallel flow it can be seen that the same effect is *greater* for $W_c = \infty$ than in other cases because the others e_c-curves have a greater slope than the ($e_c = 0$)-curve.

In a similar way the influence of other variables can be investigated.

The same method is applicable in the investigation of direct-contact heat exchangers if the process of heat exchange is continuous, but it will be necessary to use a different scale for the diagrams and to bear in mind the different meanings of the variables.

From the application of the diagrams in investigations of heat exchangers in design work, it is clear that the same method can be applied in the investigation of the experimental data taken from existing plants.

In developing these diagrams the author is fully aware that their value is restricted by the same conditions which are involved in the heat-transfer equations: the process of heat exchange is considered continuous and stationary, the heat-transfer coefficient U as well as the heat capacities W being constant along the whole heat-transmitting surface.

The cross-current flow and intermittent process of heat exchange, on account of their two-dimensional change of temperature, are not covered by these diagrams.

The intermediate values of the temperatures of the interacting substances, as well as the proper values of the heat-transfer coefficients to be used in given conditions are highly important for the intelligent handling of heat-transfer problems.

The functional relations of the heat-transfer coefficient U to the heat capacities W, and of the temperature conditions and properties of the substances to the heat-transmitting surface and pressure drop to be expected for the corresponding value of U, as well as the temperature change along the selected heat transmitting surface, permit a similar graphical representation in the diagrams around the origin of a coordinate system, but space limitations prevent the author from dealing with it in the present paper.

Discussion

M. A. STONE.[4] In the diagram on the third page of the paper, the author has given a tool which is of universal application to the solution of all heat-transfer problems.

The writer has had a varied experience with heat-transfer problems in the past, both in connection with the design of apparatus for the transfer of heat and the insulation of furnace walls and of power-plant piping, and he has found a diagram of this same form, which was published by A. Lanstaff Johnson in *The Engineering Magazine* in 1915, of very great assistance in the latter case.

This particular diagram was prepared for the determination of the economic size of steam pipe and thickness of insulation, taking into account the cost of pipe, insulation and steam, the rate of heat transfer, flow of steam, loss of heat, and pressure drop. A glance at the expressions in the text of the paper is sufficient to show for such a problem the superiority of a graphical solution, in which the relation of the many variables can be grasped at a glance, to the tedious solution involved in setting up the algebraic equations with their many variables and exponential functions and then arriving at the correct arithmetical solution by trial and error.

ARTHUR J. WOOD.[5] Has the author used alignment charts to meet the same requirements as indicated in his paper? The quadrant method of plotting the results of certain equations may have advantages, but since the alignment chart has come into such universal use and has greater possibilities in its application, it is suggested that this method be tried out to meet the requirements of the author in presenting to the eye some of the complex relations in order that they may be more clearly interpreted by the average reader.

CROSBY FIELD.[6] This paper requires a great deal of study in order to fully comprehend its many fields of application and to fully realize its great merit. The writer hopes that this splen-

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[6] Engineer and Vice-President, Brillo Mfg. Co., Inc., Brooklyn, N. Y. Life Member, A.S.M.E.

did paper is only a starting point for further papers by Mr. Stepanov on the same subject and would like to see applications of this paper to those many problems arising in the art of refrigeration.

Author's Closure

The advantage of the diagrams developed in this paper consists of not only eliminating the tedious figuring, but chiefly in visualizing how the different variables are affecting each other. The latter property is not represented in alignment charts which chiefly replace the slide rule.

As to the quadrants, with straight lines, representing mere multiplications, they are of value only in connection with other quadrants showing the influence of variables on each other. This property is fully secured only by functional interconnection of all the equations represented around the origin; there are several forms of the diagrams around the origin, with equations functionally disconnected. Such diagrams are deprived of most valuable property for investigations.

In the last page of the paper the author mentioned that the diagrams involve the properties and restrictions hidden in the fundamental equations. Therefore, being represented here in most general form, these equations may be plotted differently according to any special case, as for instance: (1) Only for countercurrent flow, if no problem of using parallel flow exists; (2) only α-curves, if the relationship of heat capacities be of first interest; (3) only ϵ-curves, if the temperature differences are to be specially investigated. Scale and size of the diagrams depend upon the heat exchanger under consideration.

Sometimes the use of logarithmic paper may be of advantage, if a large range should be investigated, whereas the precision of interpolation is not important. From this point of view the author hardly would recommend the reproduction of the full-sized diagrams as they are demonstrated in this paper; they will not conveniently cover the whole variety of the heat exchangers.

As the author mentioned in the last paragraph of his paper, the similar diagrams can be constructed for the investigations of the temperature change along the heat-transmitting surface as well as for selection of the heat-transfer coefficient in function of the pressure drop in the substance forced through the heat exchanger.

The author still hopes to publish these diagrams and to combine them with the diagrams of this paper adding detailed figures and several full-sized diagrams for the most widespread cases.

FSP-51-34

Low-Temperature Distillation of Low-Grade Fuels, Especially Lignites

By MAX TOLTZ,[1] ST. PAUL, MINN.

This paper is not intended to be a description of the various types of apparatus which are used in low-temperature distillation of low-grade fuels. On the contrary, its main purpose is to point out the advantages which inhere in these processes, and which result in greater usefulness and economies, particularly with reference to the lignite as found in the northwestern portion of the United States.

ABOUT 9 billion tons of lignite are deposited in North and South Dakota and Montana, but very little use is made of it except for domestic purposes at the rate of about 2½ millions of tons per year.

During the past 30 years attempts have been made to briquet this fuel, but these have not been successful because it has been necessary to transport the binding material over a long distance to the supply fields.

The lignites, as found in the above states, differ considerably in quality. This is evident from the facts revealed by the following analysis:

Site	Moisture, per cent	Ash, per cent	Volatile, per cent	Fixed carbon, per cent	B.t.u. as received	B.t.u. dry
Rosebud (Montana)	23.65	9.51	30.00	36.86	8750	11450
North Dakota	36.18	8.70	29.17	25.35	6700	...
North Dakota	32.64	11.42	29.10	26.25	6970	...
North Dakota	43.87	5.82	24.88	25.43	5938	...

Low-Temperature Distillation

On account of the high water content, this lignite cannot be shipped long distances. It has another disadvantage in that it slacks readily when stored, and crumbles into powder. Therefore low-temperature distillation seems to be the only method whereby this fuel may be utilized advantageously and conveyed to the people of the Northwest within a radius of about 600 miles of its source.

The following question arises: What is the nature of low-temperature distillation?

Low-temperature distillation may be defined as a heat treatment, not exceeding 1000 deg. fahr., whereby the decomposition of the tar is prevented, thereby releasing the maximum amount of liquid by-products and at the same time producing a smokeless fuel called semi-coke or char. The yield of tar of course is eliminated when the use of a gas of high thermal value is preferred. It should here be mentioned that tar water constitutes another by-product. The reason why low- instead of high-temperature distillation is preferable may be found principally in the larger yield of liquid fuels.

Although there are many different processes of low-temperature distillation, nevertheless all these types may be broadly segregated into two main classes, viz.:

1 The type in which heat is applied directly to coal in suitable retorts, or, in other words, in which the coal comes in direct contact with the hot gases or superheated steam

2 The type in which the heat required for distillation is applied externally, that is, through the walls of the retorts which contain the coal.

The retorts themselves may be placed in horizontal or vertical positions, they may be stationary or rotating, and the operation may be either continuous or intermittent.

European Progress and Requirements

It may be of interest to discuss briefly what Europe has accomplished and is now doing in the field of low-temperature distillation of fuels. In England the purpose is to obtain a smokeless solid fuel for domestic use. This aim has been achieved. But in Germany the principal needs consist in the production of a motor fuel called benzol which is similar to our gasoline, also other oil substitutes. The coke or char is used either in powdered form in the various power plants or in the form of briquets. Because Germany has no oil wells, oil must be purchased from other countries. Hence the securing of a substitute for crude or natural oils is a vital problem in Germany. Eventually all of the coal used in that country will be treated by this process. Then a sufficient supply of liquid fuel and oil substitutes will become available. In this connection, it should be noted that the benzol which is used for automobile purposes costs, in Germany, from 60 to 85 cents per 4 liters (1.2 U. S. gallons).

The term "braun Kohle" (brown coal) is applied in Germany to this coal. It is less valuable than North Dakota lignite, but it is superior to peat. It contains up to 60 per cent moisture, and has a heating value of from about 3700 to 5000 B.t.u. per pound in the dry state. At present Germany produces about 1,000,000 tons of semi-coke or char from about 3,000,000 tons of brown coal. The yield of tar is about 7 per cent.

In the northwestern part of the United States the problem is entirely different. Here a commercial fuel for domestic and industrial purposes is needed. The char, which is a product of the low-temperature distillation process, consists of distinct small granules or particles which range in size from 1/16 in. to nearly ¾ in. Special grates are required in order to use char of this kind under boilers. In Europe the char is used on chain grates, but the tendency is toward the employment of this fuel in powdered form. In several European plants inspected by the author, pulverized char was used. In one case which came under the author's observation pulverization was temporarily abandoned on account of the danger of producing explosions in the mill. It was hoped, however, that this danger would be overcome in the near future.

The conditions which prevail here require the briqueting of the char. This is easily accomplished by the addition of about 6 per cent of by-product tar, as a binder. It is noteworthy that in one plant located in central Germany experiments have been conducted during the past two years for the purpose of evolving a method of making char briquets without a binder. These efforts proved successful, but the author was unable to learn the details because the process was being protected by pending patents. He was informed that the manager of this plant had been instructed to preserve strict secrecy concerning the nature of this process, but he (the manager) was confident that in the near future complete details would be revealed.

Solution of Northwest Fuel Problem

It is evident that in this discovery lies the solution of the fuel problem of the Northwest. When its use is available here, the

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problem of the handling of lignite when in the form of char briquets as a domestic, industrial, and railroad fuel will have been solved. Moreover, economical transportation throughout the tributary territory will then be possible. Furthermore, char briquets could then compete with coke or any eastern, Illinois, or other bituminous coal which is now sold in this territory.

Tar and Its By-Products

During the process of distillation, tar as a by-product is obtained through the condensation of the gases. The average yield of tar from lignite is about 9 per cent of the total weight. Its value for chemical purposes increases in proportion to its freedom from dust and other substances.

As pointed out already, in Europe light oils (benzol) and heavy oils (for lubrication) are extracted from this tar. The most valuable product, however, is paraffin. Its consumption is increasing every year, and it has an extensive field of application in the manufacture of electrical appliances and accessories. This tar also contains a small amount of asphaltum which is too small, however, to warrant extraction.

Tar Water and Its By-Products

Ammonium sulphates are distilled from the tar water as by-products. It is well known that ammonium sulphates, $(NH_4)_2SO_4$, are used in the manufacture of fertilizers, smokeless powder, and ice. It is not necessary to give particular consideration to the two last-mentioned uses, but the employment of ammonium sulphates in the manufacture of fertilizers is of high importance, not only in Europe but also in America. In order to bring ammonium sulphate acid, NH_4HSO_4, into the use which its properties so highly warrant, a well-organized educational campaign among the farmers of the Northwest is required. Economical farming will be greatly augmented by the scientific use of suitable fertilizers which will give back to the soil that which has been removed in the growing of crops. At present, however, in United States the market is swamped with ammonium sulphates.

Acetate of lime, in the form of brown or gray lime, is another by-product of tar water. This is used in the distillation of acid salts and pure vinegar.

Indirect Heat Treatment in Germany

Of all the plants which the writer visited, one located in central Germany proved the most interesting. In this plant brown coal is carbonized by the indirect method. Experiments extending over a period of four years showed that the best results were obtained from coal which had been crushed to a size of about $^1/_2$ to $^3/_4$ in. The moisture content of this crushed product is about 55 per cent before drying in a rotary dryer to about 15 per cent. After the moisture content has been thus reduced by the drying process, the brown coal is deposited in retorts, similar in design to the Koppers coke ovens, the top of which is 4 in., while the bottom is $5^1/_2$ in. wide. The open spaces between the retorts are available to the hot gases of combustion, whose heat is transmitted through the retort shells into the coal mass within. This manner of transmitting and conducting the heat to the coal is a well-known essential feature of the indirect method. A retort unit consists of twelve retorts which are structurally connected in order to facilitate handling. It is of course evident that units may consist of other suitable group arrangements. Each retort unit is supported upon and transported by a steel carriage or truck.

In this method the ordinary retort is not used. Instead, an oven about 150 ft. long is employed as a chamber for both the retort units and their supporting trucks. A track is provided which permits the direct transportation of the retort unit by means of their supporting trucks into the oven. A receiving chamber about 10 ft. long is provided at one end of the oven for the reception of one truck and its retort unit. This receiving chamber can be shut off by a sliding partition from the main portion of the oven. After the receiving chamber has been loaded with one truck containing one retort unit, the outer door of the chamber is closed, and the sliding partition between the chamber and the main oven is opened. The car or truck is then connected with the train of cars which is already located in the oven proper. The temperature which is maintained in the oven proper does not, at any time, exceed 932 deg. fahr. The retort units and their contents are subjected to the continuous temperature of the oven proper. The progress of the train of cars through the oven is slow because a time period of from 10 to 12 hr. is required to complete the process of distillation. From the front end of the train, a car is discharged into a discharge chamber as soon as the work of distillation has been completed. The operation—the forward movement of cars, and the discharge of cars from the discharge chamber—is continuous.

Upon leaving the discharge chamber, each retort unit is lifted from its supporting truck and the char contents of the retorts are automatically deposited upon the truck platforms. Steel brushes are used to sweep the char from the truck platform into a hopper which is situated below the truck.

The char is conveyed upon a belt from the hopper to the storage bin. During its forward progress on the belt, the char is cooled. The storage bin is located either above the pulverizer or the briqueting plant. The pulverized char is used under the boilers of a comparatively large power plant (6500 kw.). When the briquets are manufactured, they are shipped to dealers.

In the plant just described attempts were made to use pulverized coal in the retorts, but it was found that much trouble ensued because of the adhesion of fluid coal to the sides of the retorts during the process.

Reports of the tests made, of course, show that the yield of tar was higher when finer material was used. Nevertheless, it was determined that coal ranging in size from $^1/_4$ to $^1/_2$ in. insured the most economical handling.

Another experiment was tried which deviated from the usual procedure of condensing the gases for their tar and tar-water products. Instead, the gas, after cleaning and scrubbing, and with a heating value of from 550 to 800 B.t.u. per cu. ft., was piped to the local gas works for distribution to consumers. In this case the gas of combustion in the retort oven was obtained from separate gas producers. This gas had a heating value of about 110 to 125 B.t.u. per cu. ft.

Conclusion

The writer is of the opinion that the fuel economy of a power plant will be raised by distilling coal, using the low-temperature process, and producing gas of a high heating value whenever such a power plant can be connected with a gas works. The char should be used in pulverized form under the boilers. Where superheated steam instead of gas from gas producers is used in the distillation of the coal, a great benefit can be derived from the heat of the char by the dry-quenching of this material, the heat to be utilized under a separate boiler for raising low-pressure steam or for superheating the steam used for the process of distillation.

It is not within the province of this paper to compare the different systems of distillation, except in so far as to point out that, for the lignite problem, low-temperature distillation, with externally heated retorts, seems to be the most advantageous. The reasons for this are that it yields the highest quantity of char, containing about 10 to 15 per cent of volatiles, and produces

the greatest amount of tar. But compared with the system of internally heated retorts or high-temperature distillation of fuels, the yield of gas of high heating value will be low.

Because extensive experience with distillation of low-grade fuels is lacking in this country, safe conclusions can be drawn from the results achieved in Europe, particularly in Germany. From this evidence it is inferred that the success of low-temperature distillation lies along the line of briqueting the char, thus facilitating its handling and shipping to the tributary country lying within the radius of low cost of transportation, within which it is possible to meet competition with other fuels. It is certain that housewives will approve the use of this fuel for cooking and heating purposes. For industrial purposes the briquet may be used, but if the distance from the carbonizing plant is not too great, the char could be shipped and used as fuel after pulverization at the boiler plant. Even the railroads may use the briquets with excellent results and with the assurance of obtaining smokeless combustion.

If gases of high heating value can be used, the whole process will be simplified, but if it is impossible to use them directly, they should be condensed and the tar and tar water separated. The tar, if clean and practically dust free, will find a ready market in the East.

The separation of light and heavy oils from the tar is not recommended, because they are comparatively plentiful and inexpensive in the United States. This condition, very probably, will prevail in America for many decades.

Since the farmers in the lignite states can make excellent use of ammonium sulphates, it is advisable to separate this pr ct from the tar water.

A low-temperature distillation plant of the Lurgi system is under construction at Lehigh, North Dakota, and is expected to be set in operation during September, 1928. All interested in the development of the use of lignite in the form of char briquets and its by-products hope that the process of low-temperature distillation has solved the problem of fuel supply for the Northwest in a feasible and practical manner crowned with commercial success.

[The above-mentioned Lurgi plant has been in operation since the middle of January, 1929, and it is reported that the operation of the mechanical apparatus as well as the economical results are exceeding expectations. Its capacity is only 100 tons char briquets per 24 hours.—AUTHOR'S NOTE.]

Discussion

EUGENE MCAULIFFE.[2] There are two things that I think the low-temperature carbonization of lignites will bring about. In the first place, any process that is not economical and financially successful is doomed to ultimate failure. If we could take out a material portion of the moisture content of lignite and other low-grade coals at the mine, reducing the transportation expense, a material saving would be created, and that would represent a substantial contribution toward financial success. Then comes that other problem—perhaps it is a collateral problem, but still directly related—the matter of attempting to obtain a better load factor for the mines that produce this low-grade material. Perhaps the worst neglected situation that exists in the coal-mining industry is the fact our load factor runs from 17 to 20 per cent. A tremendous capital investment is standing idle the major portion of the time, and that situation is accentuated by the fact that it costs more to maintain a coal mine idle than it does any other sort of factory. The forces of nature, mine water, the decay of mine timber, and things of that sort all make for heavy expense. If we could increase the return in tons of coal from our untreated timber, our pumpage, and ventilation expense, and things of that sort, we would effect a material economy in the cost of production. Where it takes seven or eight years to work out a certain area, the place must be retimbered several times, making the cost much greater. So those two things—the matter of reduction of transportation expense and the better load factor—that could be brought to the low-grade producing mines by making use of this process, thus reducing the idle period, together with storage facilities, and that would admit of putting back this temporary unmarketable situation to a time when the coal was marketable, seem to me two very compelling factors leading toward the success of the process.

I would like to ask the author what is the present situation with respect to low carbonization. We have a bituminous mine in the State of Washington, and the coal that we have is quite fit for locomotive fuel although it runs about 26 per cent moisture. There is a wide seasonal spread between our minimum and maximum demand as we are dependent wholly on the railroad fuel demand, which is very low during the summer, and then in the fall when the business picks up we have to reach out and assemble casual labor; and so I have thought that perhaps there was a possibility of carbonizing our coal during the off-peak demand period and storing it so that we might take it up in the fall and burn it during the heavy demand. The Northern Pacific and Santa Fe railroads have almost overturned all our past theories of locomotive fuel use by developing a new method of burning coal, and that is by using a small grate opening; they are using table grates with half-inch openings, providing from 13 to 17 per cent air openings, where they used to consider it an absolute necessity to provide not less than 35 per cent. That makes it possible to burn very fine coal successfully in a locomotive and with a very high rate of combustion. Could the author give me any light on that point?

AUTHOR'S CLOSURE

The difficulties of which Mr. McAuliffe speaks can be overcome by the use of low-temperature distillation, in which low-grade fuels of high moisture content are carbonized economically. The char, the result of this process, having the size of pea coal, with a moisture of about 5 per cent and a B.t.u. value of from 12,500 to 13,500, can be easily stored. It can be used on chain grates under stationary boilers or on locomotives with a special grate, but it is preferable to use this fuel in pulverized form. If this process is adapted for the mine mentioned, the load factor can be increased wonderfully, because the economical feature of carbonizing is a continuous process. Of course in this case the char has to be stored in summer time, to be shipped and used for the winter loads. The shipping expenses are also reduced because no freight has to be paid for water as in the case of lignite with 35 to 40 per cent moisture content.

Another method of use of low-grade fuel, if consumed quickly and freshly as it comes from the mine, is the adaptation of a special grate as has been developed by the Northern Pacific Railway on the locomotives of the Montana district. The fuel is semi-bituminous, almost a lignite, with a mixture content of about 23 per cent and a heating value of about 7600 B.t.u. The grates have openings of $^3/_8$ to $^7/_{16}$ in. and are from 16 to 18 per cent of the total grate area. This has been carried out successfully as compared with former methods. In Europe and in one stationary plant in North Dakota step grates are used. The low-grade fuel is partly gasified, and the gas combines in the combustion chamber with the fuel. Although a great improvement over the old methods, it is only a step forward. The only economical way of using low-grade fuels with high moisture content is by reduction of the carbonization process, whereby also valuable by-products are obtained.

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FSP-51-35

Effect of Alloying Elements Upon the Stability of Steel at Elevated Temperatures

By A. E. WHITE[1] and C. L. CLARK,[2] ANN ARBOR, MICH.

ALLOY STEELS during the past few years have made rapid strides in an entirely new field—in the construction of metallic parts which are to withstand stresses at elevated temperatures. It is true that plain carbon steels have been used because of their relative cheapness and ease of fabrication. Yet because of their recognized limitations their place will be taken in an ever-increasing degree by alloy steels. Owing, however, to the large number of alloy steels available there is often considerable confusion as to just what alloy steel is best suited for the problem at hand. It was for the purpose of furnishing information regarding the effect of various alloying elements upon the steels to which they were added that the work reported in this paper was undertaken.

Stability Characteristics

In order for a metal to be serviceable at elevated temperatures it must be stable at the temperatures to be employed. When a metal is stable, two distinct characteristics are implied:

1 That the metal or alloy is capable of resisting oxidation at the temperatures being considered
2 That the metal or alloy is capable of resisting indefinitely, or at least for the economic life of the piece of which it forms a part, the stresses to which it is to be subjected.

Since this paper is primarily concerned with the second of these two characteristics, the first will be dismissed with a few general statements.

Resistance to Oxidation. A generalization well set forth by Stoughton[3] regarding the effect of alloy additions upon the corrosion-resisting properties of the metals to which they are added is: "When a metal forms a solid solution with other metals, it gives to the alloy some of its own corrosion-resisting properties." Since oxidation may be considered as high-temperature corrosion, the same statement applies to it.

From this, if the resistance to oxidation is to be increased, then some alloying element must be added which is itself resistant under the given conditions and which when added to the steel dissolves in the ferrite. Two corrosion-resistant elements which may be considered in this connection are nickel and chromium. Nickel when added to steel dissolves in the ferrite and so increases the ability of the metal to resist either oxidation or corrosion. Chromium, on the other hand, has a greater affinity for the carbon than the ferrite. Thus if added in small amounts it will combine entirely with the carbon forming carbides, and the resistance to oxidation is not increased unless the steel is subjected to a heat treatment to throw the carbides, at least partially, into solution. If, however, the carbon is sufficiently low or the chromium is added in large enough amounts so that there is more than enough chromium to combine with the carbon, then the remaining chromium will dissolve in the ferrite and the resistance to oxidation is increased without any heat treatment.

As an example of the latter two cases, chromium stainless steels and chromium stainless irons may be considered. The stainless steels, containing approximately 12 per cent chromium and 0.30 per cent carbon, require a heat treatment to give them their oxidation-resisting properties, while the stainless irons, containing approximately 12 per cent chromium and 0.10 per cent carbon, resist oxidation without any previous heat treatment.

Load-Carrying Ability. The experimental work reported in this paper is concerned with the effect of temperature upon the tensile properties of metals and alloys. Before presenting the test results, however, it will be best to consider the entire field from a theoretical standpoint.

Ever since the presentation in 1912 of a paper by Rosenhain[4] concerning the conditions existing at the crystal boundaries, metallurgists have generally accepted the belief that metals consist of two states, a crystalline and an amorphous state. Also, as expressed by Beilby,[5] the crystalline phase or state is "thermally stable but mechanically unstable" and the amorphous phase is "thermally unstable but mechanically stable." This implies that the relative strength of these two phases is not fixed, but may vary according to given conditions.

If it is desired to increase the strength of a metal consisting of two phases of unequal strength, the following ways are open:

1 The addition or formation of some substance which by its presence increases the strength of the weaker phase
2 The addition or formation of some substance which tends to remove the weaker of the two phases.

From this it follows that in order to increase the load-carrying ability of a metal at any temperature it is first necessary to determine which of the two phases is the weaker and then take steps to strengthen or remove this weaker phase.

It is a well-established fact that at atmospheric temperature, fracture of ordinary steels proceeds through the grains and not around them. This is owing to the greater strength of the amorphous boundaries. As the temperature of testing is increased, however, the amorphous phase weakens at a more rapid rate than that of the crystal proper, and a temperature is finally reached where fracture proceeds around the grains and not through them. This temperature Jeffries[6] calls the "equicohesive temperature." In general, under very slow rates of loading, this corresponds to the lowest recrystallization temperature of the alloy when in a severely cold-worked state. Values for recrystallization temperatures reported in the literature vary considerably, but that generally accepted for commercial iron[7] is in the neighborhood of 850 deg. fahr. (454 deg. cent.).

From the foregoing it is evident that before the effect of alloying elements upon the load-carrying ability of metals can be de-

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[3] B. Stoughton, "Alloy Steels," Trans. A.I.M.E., vol. 75 (1927), pp. 497–525.

Contributed by the Power Division and presented at the Annual Meeting, New York, December 3 to 7, 1928, of The American Society of Mechanical Engineers.

[4] W. Rosenhain and D. Ewen, "Intercrystalline Cohesion in Metals," *Jl. Inst. Metals*, vol. viii, no. 2, p. 149, 1912.

[5] G. T. Beilby, "The Hard and Soft State in Metals," *Philosophical Magazine*, August, 1904.

[6] Z. Jeffries, "Effect of Temperature, Deformation, and Grain Size on the Mechanical Properties of Metals," Trans. A.I.M.E., vol. 60 (1919), pp. 474–577.

[7] Z. Jeffries and R. S. Archer, "The Science of Metals," first edition, 1924.

termined it will be necessary to know over what temperature range and for what length of time this stability is desired. That is, for plain carbon steels those elements, the addition of which may cause increased stability up to temperatures in the neighborhood of 850 deg. fahr. (454 deg. cent.), are not necessarily the ones that will have the greatest effect in increasing the load-carrying ability at the higher temperatures.

If stability is desired below the equicohesive temperature, then we advance the following hypothesis:

1 The addition or formation of any element, compound, or constituent which does not enter into solid solution with the matrix, but which by the very nature of its presence interferes with the crystal slippage, will tend to increase the load-carrying ability of the material of which it forms a part

2 The addition or formation of any element, compound, or constituent which tends to increase the amount of amorphous material will increase the load-carrying ability of the material of which it forms a part.

For stability above the equicohesive temperature, our hypothesis follows:

1 The addition or formation of any element, compound, or constituent which tends to decrease the relative proportion of amorphous material to crystalline material will increase the load-carrying ability of the material of which it forms a part

2 The addition or formation of any element, compound, or constituent which strengthens the grain boundaries by interfering with plastic flow will increase the load-carrying ability of the material of which it forms a part.

steels were subjected to short-time tensile tests at temperatures as high as 1500 deg. fahr. (816 deg. cent.) in some cases and to 1250 deg. fahr. (677 deg. cent.) in others. The last five, together with steels G and P′ were tested after various heat treatments only at 1000 deg. fahr. (538 deg. cent.).

It is realized that proportional-limit values as obtained in short-time tensile tests may not be, at temperatures above the equicohesive temperatures at least, a true criterion of a metal's load-carrying ability. Yet it is believed that for temperatures not too far above this point carefully determined proportional limits can be used as a good measure of the metal's ability to resist rupture for periods up to several thousand hours.

For this reason proportional-limit values rather than creep-test results are here presented, because in this way it was possible to investigate a much larger group of alloys. In two cases creep tests were run as a check on the proportional-limit values, and these results also are presented.

The method used for determining the proportional limit values was the same as described in previous papers by the authors.[8,9] On this one property the literature is full of conflicting data. Great care and attention to detail are necessary to get correct values. For example, on one of the manganese steels on which we have had occasion to do considerable work certain investigators have reported proportional-limit values at 1000 deg. fahr. (538 deg. cent.) of approximately 40,000 lb. per sq. in. The best value we have been able to obtain after various heat treatments, one of which is that which is supposed to bring out the best properties, is 15,000 lb. per sq. in. We make no claim that our values on this steel and on the others that will be reported are beyond question. We do contend that the work was carefully performed and that close adherence to those factors which give accuracy were followed. We have confidence, therefore, in our results.

TABLE 1 CHEMICAL COMPOSITION OF ALLOY STEELS INVESTIGATED

Symbol	Class	Composition										
		C	Mn	Si	S	P	Cr	Ni	W	Va	Mo	Al
D	Carbon	0.37	0.69	...	...	...	...	..	..	...	...	..
E	Manganese	0.35	1.17	0.115	0.014	0.019	...	..	..	...	...	..
F	Manganese	0.18	1.31	0.031	...	...	...	..	..	...	...	..
G	Manganese	0.44	1.55	0.171	0.028	0.021	...	..	..	...	...	..
H	Manganese	0.44	2.06	0.256	0.023	0.022	...	..	..	...	...	..
I	Nickel	0.10	0.43	0.160	0.013	0.024	0.020	4.95	..	...	...	..
K	Nickel	0.23	0.57	0.230	0.028	0.040	...	4.95	..	...	...	..
L	Chrome-nickel	0.38	0.60	0.180	0.023	0.010	0.580	1.46	..	...	...	..
M	Chrome-nickel	0.41	0.56	0.050	0.022	0.011	1.240	1.79	..	...	...	..
N	Chrome-nickel	0.15	0.48	0.510	0.038	0.005	19.080	8.52	..	...	...	..
O	Nickel-chromium	0.31	0.65	0.910	0.024	0.005	7.960	20.65	..	...	...	..
P′	Chrome-tungsten-vanadium	0.40	0.59	0.760	0.019	0.005	7.930	..	7.70	0.120	...	..
Q	Silcrome	0.48	0.59	3.540	0.009	0.024	9.160	0.03	..	...	...	..
R	Chrome-tungsten-vanadium	0.60	0.26	0.360	0.011	0.024	1.430	0.03	2.19	0.170	...	...
Ra	Chrome-tungsten-vanadium	0.55	0.24	0.310	0.007	0.012	1.490	..	1.87	0.400	none	trace
S	Chrome-aluminum	0.49	0.50	0.260	0.012	0.025	1.620	0.01	..	...	0.230	1.00
T	Chrome-manganese	0.49	1.21	0.210	0.014	0.021	0.332	0.06	..	0.084	0.031	..
U	Nickel-molybdenum	0.24	0.57	0.380	0.012	0.026	0.103	1.79	..	0.024	0.172	..
V	Chrome-tungsten-vanadium	0.52	0.42	0.160	0.009	0.023	0.610	0.13	1.79	0.141	none	0.05

Considering the foregoing principles and the effect of certain alloy additions upon the metallographic structure of the metals to which they are added, it should be possible to predict and explain the influence of the various alloy additions upon the load-carrying ability of the resulting alloy. Also the matter of heat treatment should be given careful consideration, for often the selection of proper heat treatment is equally as important as the selection of the proper alloying element. The alloying elements that will be considered in this paper are nickel, chromium, tungsten, manganese, molybdenum, vanadium, and various combinations of these elements.

Methods of Present Investigation

For the purpose of obtaining the data from which the factors producing stability at elevated temperatures could be determined, short-time tensile tests were conducted on a large number of alloy steels. The chemical composition of these steels is given in Table 1.

With the exception of the last five listed in Table 1, the

Some question arises in high-temperature testing as to just what heat treatment should be applied to the metal before subjecting it to the testing process. It has been shown several times that in all tempering or drawing operations at least two factors must be considered, (1) the temperature employed and (2) the time during which the metal is maintained at that temperature. Therefore, if it is desired to subject the specimens to any sort of a quenching operation, it is necessary to draw them at a sufficiently high temperature so that the time effect at the temperature at which they are to be used will not cause any marked change in structure with corresponding changes in physical properties.

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[8] A. E. White and C. L. Clark, "Properties of Boiler Tubing at Elevated Temperatures Determined by Expansion Tests," Trans. A.S.M.E., vol. 48 (1926), p. 1075.

[9] A. E. White and C. L. Clark, "Properties of Ferrous Metals at Elevated Temperatures as Determined by Short-Time Tensile and Expansion Tests," Dept. Eng. Res., Univ. of Mich., reprint No. 3, 1927.

Because of the necessity of following any quenching treatment with such a high degree of tempering, all of the metals reported in this paper on which tests were conducted at temperatures above 1000 deg. fahr. (538 deg. cent.) were subjected to either a normalizing or an annealing treatment. It is felt that these two treatments more nearly represent the working conditions encountered in service.

If, however, the metals are to be used at only relatively high temperatures, such as 1000 deg. fahr. (538 deg. cent.) or possibly

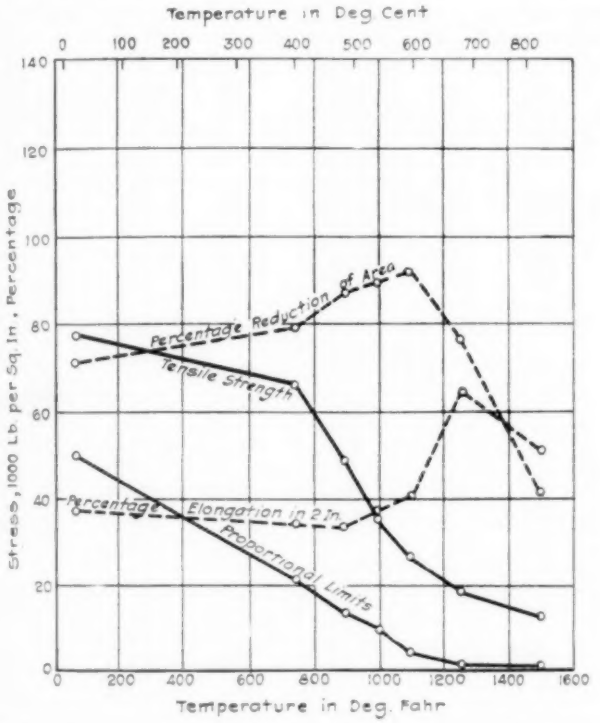


FIG. 1 TENSILE PROPERTIES OF A NICKEL STEEL AT ELEVATED TEMPERATURES

(Chemical composition: C 0.1, Mn 0.43, Si 0.16, S 0.013, P 0.024, Ni 4.95, Cr 0.02. Heat treatment: Normalized at 1500° F., 815°C.)

1100 deg. fahr. (593 deg. cent.), then quenching operations followed by a suitable draw will often result in greatly improved load-carrying properties at these moderate elevated temperatures. With this in view, six selected steels were subjected to various heat treatments and then tested at 1000 deg. fahr. (538 deg. cent.)

RESULTS OF TESTS

Nickel Steels. Nickel dissolves in steel at atmospheric temperature, especially in the ferrite crystals. Since nickel additions produce a marked lowering of the thermal critical range of iron, slowly cooled nickel steels can have various structures, as pearlitic, martensitic, or austenitic, depending on the relative amounts of nickel and carbon present.

With low-carbon nickel steels in the pearlitic zone, one of the effects of the addition of nickel is to reduce grain size. At atmospheric and slightly elevated temperatures this reduced grain size improves the strength of the alloy due to the presence of larger amounts of amorphous material. At elevated temperatures above the equicohesive temperature this reduced grain size results in a weakening of the alloy if the rate of application of the load is very small, due to increasing the amount of the weakest phase (amorphous) present.

Considerable percentages of nickel will produce an alloy with a structure entirely austenitic. Austenite, being a solid solution of carbon or iron carbide in gamma iron, fails to satisfy any of the four principles set forth for producing good load-carrying ability. For this reason one would not expect these steels to exhibit high tensile properties in any part of the temperature range. That this is the case will be shown in connection with the chromium-nickel steels.

The tensile properties at elevated temperatures of two nickel steels in the normalized state are given in Figs. 1 and 2. Tables 2 to 13 give the results for these steels, as well as for all the other steels to be considered. These two steels differ only in carbon content, one containing twice the amount of the other. In both cases the tensile-strength and proportional-limit values fall rapidly after 750 deg. fahr. (399 deg. cent.), which is probably close to the equicohesive temperature range, showing that the metal possesses little load-carrying ability above this temperature.

From the foregoing it can be concluded that plain nickel steels in the normalized state do not offer possibilities of having high load-carrying ability at temperatures much above 750 deg. fahr. (399 deg. cent.). Neither should various heat treatments be

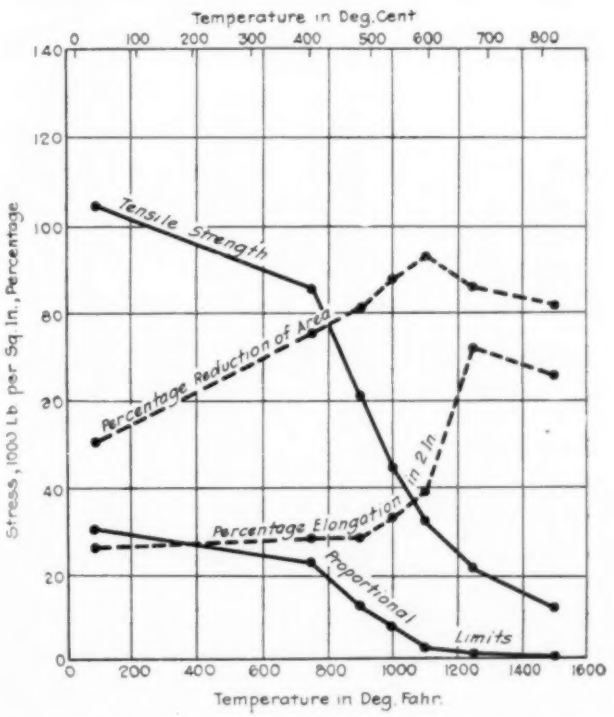


FIG. 2 TENSILE PROPERTIES OF A NICKEL STEEL AT ELEVATED TEMPERATURES

(Chemical composition: C 0.23, Mn 0.57, Si 0.23, S 0.028, P 0.04, Ni 4.95. Heat treatment: Normalized at 1500°F., 815° C.)

expected to greatly increase the stability of low-nickel steels at temperatures in the neighborhood of 1000 deg. fahr. (538 deg. cent.). That such is the case is shown in Figs. 3 to 5. It is true that the steel tested also contains some molybdenum, but the nickel is the predominating alloying element. As the quenching temperature is raised from 1450 deg. to 1650 deg. fahr. (788 deg. to 899 deg. cent.) a slight increase in hardness is obtained, but, as shown in Photomicrographs 1 to 6, this can be acounted for by refinement in grain size. The hardness, as determined by the Rockwell machine, shows a decrease as the quenching temperature is raised from 1650 deg. to 1750 deg. fahr. (899 deg. to 954 deg. cent.). From the photomicrographs this is again what should be expected, as the grain size increases as the temperature is raised above 1650 deg. fahr. (899 deg. cent.).

Considering the proportional-limit values obtained at 1000 deg. fahr. (538 deg. cent.) on this steel after various heat treatments, they are found to vary from 7500 lb. to 15,000 lb. per sq. in. As should be expected, the treatment producing the finest structure, quenching from 1650 deg. fahr. (899 deg. cent.), gives the lowest proportional-limit value. The highest values were obtained after quenching from 1750 deg. fahr. (954 deg. cent.) and also after normalizing from this same temperature. It should be observed that the specimens were in all cases drawn at 1200 deg. fahr. (649 deg. cent.) before being subjected to the tensile tests.

TABLE 2 TENSILE PROPERTIES OF TWO NORMALIZED NICKEL STEELS AT ELEVATED TEMPERATURES

Temperature of Test °F.	°C.	Tensile strength, lb. per sq. in.	Proportional limit, lb. per sq. in.	Elongation in 2 in., %	Reduction of area, %
Steel I—0.10% Carbon, 4.95% Nickel					
70	21	76,475	50,000	37.50	71.00
750	399	65,275	20,625	33.25	78.83
900	482	47,750	12,500	32.50	87.50
1000	538	34,375	10,000	36.00	88.65
1100	593	25,300	4,000	40.25	91.28
1250	677	17,540	625	63.00	75.11
1500	816	11,815	250	50.00	40.90
Steel K—0.23% Carbon, 4.95% Nickel					
70	21	104,525	30,000	26.25	50.71
750	399	85,500	22,500	28.00	75.48
900	482	61,400	12,500	27.75	81.45
1000	538	43,725	7,750	32.25	87.55
1100	593	31,515	2,375	38.25	92.75
1250	677	21,000	1,000	70.50	84.02
1500	816	11,550	500	65.00	81.57

TABLE 3 EFFECT OF HEAT TREATMENT UPON THE TENSILE PROPERTIES OF A NICKEL-MOLYBDENUM STEEL AT 1000°F. (538°C.)

Steel U—0.24% Carbon, 1.79% Nickel, 0.17% Molybdenum

Heat treatment[1]	Tensile strength, lb. per sq. in.	Proportional limit, lb. per sq. in.	Elongation in 2 in., %	Reduction of area, %
Q. 1450°F., D. 1200°F.	54,300	12,500	25.0	81.36
Q. 1550°F., D. 1200°F.	66,350	12,500	21.5	83.20
Q. 1650°F., D. 1200°F.	64,700	7,500	22.5	82.70
Q. 1750°F., D. 1200°F.	60,000	15,000	23.5	81.02
N. 1550°F., D. 1200°F.	46,250	12,500	30.0	81.02
N. 1750°F., D. 1200°F.	49,000	15,000	30.0	77.85
D. 1200°F., A. 1750°F.	47,000	10,000	27.0	70.00

[1] Q., quenched; D., drawn; N., normalized; A., annealed.

TABLE 4 TENSILE PROPERTIES OF AN ANNEALED CHROMIUM-TUNGSTEN STEEL AT ELEVATED TEMPERATURES

Steel P—0.40% Carbon, 7.93% Chromium, 7.70% Tungsten

Temperature of Test °F.	°C.	Tensile strength, lb. per sq. in.	Proportional limit, lb. per sq. in.	Elongation in 2 in., %	Reduction of area, %
70	21	110,140	50,000	24.5	54.38
750	399	88,700	28,750	20.0	51.93
900	482	74,963	20,625	22.5	60.16
1000	538	60,475	17,500	24.0	68.25
1100	593	47,563	15,000	27.0	73.23
1250	677	27,625	7,250	50.0	93.46
1500	816	12,350	2,000	62.5	97.30

TABLE 5 TENSILE PROPERTIES OF TWO ANNEALED CHROMIUM-NICKEL STEELS AT ELEVATED TEMPERATURES

Temperature of Test °F.	°C.	Tensile strength, lb. per sq. in.	Proportional limit, lb. per sq. in.	Elongation in 2 in., %	Reduction of area, %
Steel N—0.15% Carbon, 19.08% Chromium, 8.52% Nickel					
70	21	108,050	18,125	38.5	59.19
750	399	66,400	14,375	31.5	56.60
900	482	59,900	12,750	32.5	64.22
1000	538	55,650	9,500	32.5	64.79
1100	593	50,390	7,500	34.0	67.25
1250	677	41,575	7,000	44.0	73.03
1500	816	22,750	2,000	55.0	70.73
Steel O—0.31% Carbon, 7.93% Chromium, 20.65% Nickel					
70	21	93,415	22,500	35.00	55.95
750	399	74,865	19,375	30.00	51.43
900	482	68,425	15,000	26.50	52.24
1000	538	62,350	11,500	30.25	56.35
1100	593	57,525	11,000	30.50	62.29
1250	677	43,825	5,750	42.50	71.53
1500	816	25,300	2,000	57.50	82.70

The stress-strain curves from which the proportional-limit values, as shown in Fig. 4, were obtained are given in Fig. 5. The nearly perfect parallelism of these lines shows that the various treatments did not noticeably affect the modulus of elasticity of this steel at the temperature considered.

Chromium Steels. Chromium dissolves in iron in all percentages, forming solid solutions with the ferrite. In the presence of carbon, however, it forms double carbides with the iron. The

TABLE 6 TENSILE PROPERTIES OF TWO NORMALIZED CHROMIUM-NICKEL STEELS AT ELEVATED TEMPERATURES

Temperature of Test °F.	°C.	Tensile strength, lb. per sq. in.	Proportional limit, lb. per sq. in.	Elongation in 2 in., %	Reduction of area, %
Steel L—0.38% Carbon, 0.58% Chromium, 1.46% Nickel					
70	21	106,160	58,750	24.50	55.90
750	399	91,575	17,500	23.75	67.00
900	482	71,075	7,500	21.00	59.60
1000	538	57,750	6,000	21.75	60.85
1100	593	43,400	5,000	24.00	55.75
1250	677	24,500	3,250	26.95	50.85
1500	816	11,600	1,000	71.00	91.20
Steel M—0.41% Carbon, 1.24% Chromium, 1.79% Nickel					
70	21	133,500	51,250	16.0	43.40
750	399	117,425	45,000	18.0	60.90
900	482	93,625	31,250	14.5	54.95
1000	538	73,250	10,500	17.0	60.45
1100	593	52,725	5,250	27.5	79.40
1250	677	26,800	875	34.5	88.25

TABLE 7 TENSILE PROPERTIES OF ANNEALED SILCROME AT ELEVATED TEMPERATURES

Steel Q—0.48% Carbon, 9.16% Chromium, 3.54% Silicon

Temperature of Test °F.	°C.	Tensile strength, lb. per sq. in.	Proportional limit, lb. per sq. in.	Elongation in 2 in., %	Reduction of area, %
70	21	135,250	80,000	22.25	46.00
750	399	109,400	40,000	17.50	43.53
900	482	85,825	25,625	23.75	59.21
1000	538	68,850	15,190	26.75	69.91
1100	593	44,900	5,250	35.00	86.58
1250	677	23,025	1,925	45.75	96.35
1500	816	9,550	375	65.25	97.86

TABLE 8 EFFECT OF HEAT TREATMENT UPON THE TENSILE PROPERTIES OF LOW-CHROMIUM-TUNGSTEN STEELS AT 1000°F. (538°C.)

Steel R—0.60% Carbon, 1.43% Chromium, 2.19% Tungsten

Heat treatment[1]	Tensile strength, lb. per sq. in.	Proportional limit, lb. per sq. in.	Elongation in 2 in., %	Reduction of area, %
Q. 1400°F., D. 1200°F.	82,500	13,750	29.0	62.32
Q. 1500°F., D. 1200°F.	95,000	17,500	18.5	61.09
Q. 1600°F., D. 1200°F.	105,500	23,750	20.5	65.64
Q. 1700°F., D. 1200°F.	124,350	32,500	18.0	64.00
Q. 1800°F., D. 1200°F.	(Specimen cracked when quenched)			
N. 1750°F., D. 1200°F.	134,750	35,000	17.5	53.05
A. 1650°F.	71,250	17,500	31.0	80.15
O. Q. 1650°F.	136,500	22,500	18.5	59.34
O. Q. 1650°F., D. 1200°F.	142,200	37,500	15.0	56.50
Steel Ra—0.55% Carbon, 1.49% Chromium, 1.87% Tungsten				
N. 1575°F., D. 1250°F.	95,825	22,500	20.5	61.08
Q. 1575°F., D. 1250°F.	91,750	18,750	20.0	66.75
Steel V—0.52% Carbon, 0.61% Chromium, 1.79% Tungsten				
N. 1575°F., D. 1250°F.	101,650	37,500	24.5	59.09
Q. 1575°F., D. 1250°F.	93,000	35,000	24.5	75.68

[1] Q., quenched; D., drawn; N., normalized; A., annealed; O. Q. oil-quenched.

TABLE 9 EFFECT OF HEAT TREATMENT UPON THE TENSILE PROPERTIES OF HIGH-CHROMIUM-TUNGSTEN STEEL AT 1000° F. (538° C.)

Steel P'—0.40% Carbon, 7.93% Chromium, 7.70% Tungsten

Heat treatment[1]	Tensile strength, lb. per sq. in.	Proportional limit, lb. per sq. in.	Elongation in 2 in., %	Reduction of area, %
Q. 1700°F., D. 1200°F.	119,150	52,500	14.5	52.75
Q. 2250°F., D. 1200°F.	125,000	100,000	2.0	3.92
N. 1700°F., D. 1200°F.	118,100	45,000	17.0	57.04
N. 1800°F., D. 1200°F.	119,000	32,500	17.5	51.69
N. 1800°F.	.....[2]	50,000	...[3]	...[3]
A. 1700°F.	61,583	17,500	24.0	67.67

[1] Q., quenched; D., drawn; N., normalized; A., annealed.
[2] Tensile strength above 140,000 lb. per sq. in.
[3] These values not obtained as specimen was not broken.

carbides that are deposited within the crystal will interfere with the process of crystalline slippage, and for that reason materially increase the stability of the alloy at all temperatures below the equicohesive temperature. The carbides which are precipitated at the grain boundaries will interfere with the plastic flow of this material, and so increase the load-carrying ability of the metal above the equicohesive temperature. Therefore steels which contain chromium in sufficient amounts manifest a much greater load-carrying stability at all temperatures than plain carbon steels or nickel steels.

Likewise, compositions which produce the free carbides show greater strength in the alloys in which they are found than compositions which throw the carbides into solid solution, as evidenced by the superior results of 8 per cent chromium, 8 per cent tungsten steel (Fig. 6) over and above those of two alloys, one containing 20 per cent chromium and 8 per cent nickel (Fig. 7) and the other 8 per cent of chromium and 20 per cent of nickel (Fig. 8). To be sure, in the first case 8 per cent tungsten is present and in the other two relatively large percentages of nickel, but the structure of the first steel shows free carbides while the last two are austenitic—solid-solution alloys.

TABLE 10 TENSILE PROPERTIES OF NORMALIZED CARBON-MANGANESE STEELS AT ELEVATED TEMPERATURES

Temperature °F.	Temperature °C.	Tensile strength, lb. per sq. in.	Proportional limit, lb. per sq. in.	Elongation in 2 in., %	Reduction of area, %
Steel D—0.37% Carbon, 0.69% Manganese					
70	21	93,150	50,000	29.5	55.20
750	399	86,350	15,625	27.5	62.80
900	482	63,450	12,250	24.5	69.47
1000	538	55,650	10,500	25.5	75.29
1100	593	41,000	5,000	30.0	85.68
1250	677	23,050	4,500	26.0	88.53
Steel E—0.35% Carbon, 1.17% Manganese					
70	21	91,825	47,500	31.25	63.00
750	399	89,275	24,125	25.00	68.00
900	482	65,700	20,000	26.50	77.80
1000	538	53,250	17,500	28.50	87.30
1100	593	40,025	10,375	31.30	92.00
1250	677	23,425	2,500	38.00	96.70
1500	816	9,500	1,500	70.00	98.50
Steel F—0.18% Carbon, 1.31% Manganese					
70	21	66,175	37,500	38.75	66.78
750	399	69,500	16,750	28.90	62.80
900	482	52,725	9,750	26.50	76.35
1000	538	45,825	9,125	28.00	81.66
1100	593	33,825	7,375	30.00	85.39
1250	677	19,150	3,875	35.50	91.29
1500	816	10,200	....	62.50	79.08
Steel, G—0.44% Carbon, 1.55% Manganese					
70	21	100,550	67,500	32.25	69.50
750	399	88,450	31,250	26.70	80.15
900	482	68,100	24,375	29.60	81.65
1000	538	55,975	18,000	30.25	85.85
1100	593	39,900	10,000	34.50	90.80
1250	677	26,000	6,750	39.25	92.85
1500	816	13,650	1,500	65.80	96.00
Steel H—0.44% Carbon, 2.06% Manganese					
70	21	135,375	41,750	19.00	36.79
750	399	111,925	18,750	22.00	72.67
900	482	87,825	18,750	22.00	77.70
1000	538	71,250	14,875	24.00	83.80
1100	593	48,000	6,250	30.75	86.50
1250	677	31,490	3,625	39.75	90.40
1500	816	13,675	1,500	63.50	97.60

Chromium-Nickel Steels. The addition of both chromium and nickel to a steel should give an alloy having properties which are a combination of those mentioned in the foregoing two cases. Nickel dissolving mainly in the iron will increase the resistance to oxidation of the alloy, while the chromium, being a former of carbides, will tend to increase the load-carrying ability of the alloy. Since nickel additions will produce a marked lowering of the thermal critical range of the iron, the same varieties of structure obtained in the case of straight nickel steels are again possible.

Short-time tensile tests were conducted on four chromium-nickel steels, the results of which are shown graphically in Figs. 7 to 10. The steels presented in Figs. 9 and 10 contain small enough amounts of the alloying elements so that in the annealed condition their structures are pearlitic. Those represented by Figs. 7 and 8, however, contain such large amounts of both chromium and nickel that even in the annealed state they are austenitic.

TABLE 11 EFFECT OF HEAT TREATMENT UPON THE TENSILE PROPERTIES OF A CARBON-MANGANESE STEEL AT 1000°F. (538°C.)

Steel G—0.44% Carbon, 1.55% Manganese

Heat treatment[1]	Tensile strength, lb. per sq. in.	Proportional limit, lb. per sq. in.	Elongation in 2 in., %	Reduction of area, %
Q. 1450°F., D. 1200°F.	53,150	5,000	35.0	87.58
Q. 1550°F., D. 1200°F.	58,100	7,500	29.5	86.14
Q. 1650°F., D. 1200°F.	58,000	6,250	30.5	87.85
Q. 1750°F., D. 1200°F.	57,750	6,250	31.5	87.58
N. 1550°F., D. 1200°F.	47,700	12,500	38.0	85.09
N. 1750°F., D. 1200°F.	48,250	8,750	35.0	86.58
Q. 1800°F., Q. 1600°F., D. 1000°F.	64,250	....	30.0	91.03
Q. 1800°F., Q. 1600°F. D. 1200°F.	60,000	....	27.0	90.42
Q. 1800°F., Q. 1600°F., D. 1300°F.	56,350	12,500	29.0	91.18
Q 1800°F., Q. 1600°F., D. 1500°F.	52,000	12,000	31.5	88.80

[1] Q., quenched; D., drawn; N., normalized.

TABLE 12 EFFECT OF HEAT TREATMENT UPON THE TENSILE PROPERTIES OF A CHROMIUM-MANGANESE STEEL AT 1000°F. (538° C.)

Steel T—0.49% Carbon, 0.33% Chromium, 1.21% Manganese

Heat treatment[1]	Tensile strength, lb. per sq. in.	Proportional limit, lb. per sq. in.	Elongation in 2 in., %	Reduction of area, %
Q. 1400°F., D. 1200°F.	65,375	7,500	24.5	73.08
Q. 1500°F., D. 1200°F.	74,000	10,000	21.0	77.60
Q. 1600°F., D. 1200°F.	Specimen cracked in quenching			
Q. 1700°F., D. 1200°F.	Specimen cracked in quenching			
Q. 1800°F., D. 1200°F.	Specimen cracked in quenching			
N. 1550°F., D. 1200°F.	67,650	13,750	24.5	72.46
N. 1750°F., D. 1200°F.	70,750	11,250	21.0	75.49
A. 1700°F.	68,000	11,250	22.0	80.50

[1] Q., quenched; D., drawn; N., normalized; A., annealed.

TABLE 13 EFFECT OF HEAT TREATMENT UPON THE TENSILE PROPERTIES OF A CHROMIUM-ALUMINUM STEEL AT 1000°F. (538° C.)

Steel S—0.49% Carbon, 1.62% Chromium, 1.00% (?) Aluminum

Heat treatment[1]	Tensile strength, lb. per sq. in.	Proportional limit, lb. per sq. in.	Elongation in 2 in., %	Reduction of area, %
O. Q. 1450°F., D. 1200°F.	66,350	17,500	22.5	75.09
O. Q. 1550°F., D. 1200°F.	81,300	22,500	19.5	75.09
O. Q., 1650°F., D. 1200°F.	77,600	15,000	22.5	81.02
Q. 1450°F., D. 1200°F.	65,500	20,000	25.5	69.40
Q. 1550°F., D. 1200°F.	88,050	22,500	21.0	76.27
Q. 1650°F., D. 1200°F.	91,600	20,000	23.0	82.70
Q. 1750°F., D. 1200°F.	92,850	22,500	17.5	77.60
N. 1600°F., D. 1200°F.	94,750	17,500	19.5	70.98
A. 1650°F.	67,300	17,500	27.0	77.85

[1] O. Q., oil-quenched; D., drawn; Q., quenched; N., normalized; A., annealed.

In a later section it will be shown that not only is the presence of free carbides necessary for the securing of the best load-carrying ability, but the size of these particles as regulated by heat treatment is equally important.

First considering the two lower alloy-containing steels of this group (Figs. 9 and 10), the carbon and nickel contents are nearly identical, while the chromium content of the first is approximately one-half that of the second. From what has been said, the second steel should show the highest proportional-limit and tensile-strength values throughout the temperature range considered. An examination of the figures will show that such is the case.

The results obtained with the two austenitic steels of this group show that the proportional-limit values for both steels throughout the entire temperature range are low. Neither are the tensile-strength values in most cases high. From the conditions presented when discussing the nickel steels, this is what was

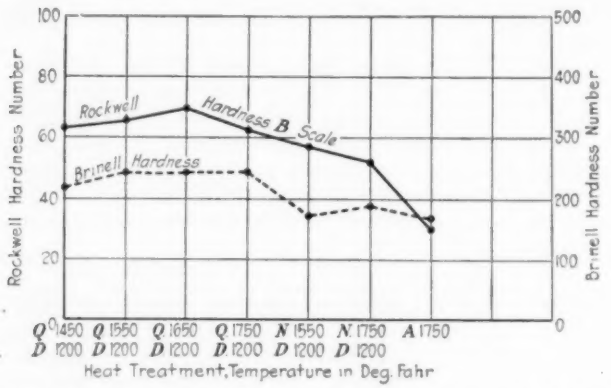


FIG. 3 EFFECT OF HEAT TREATMENT ON THE HARDNESS OF 0.24% C, 1.79% NI, 0.17% MO STEEL

(Q., quenched in water; D., drawn; A., annealed; N., normalized.)

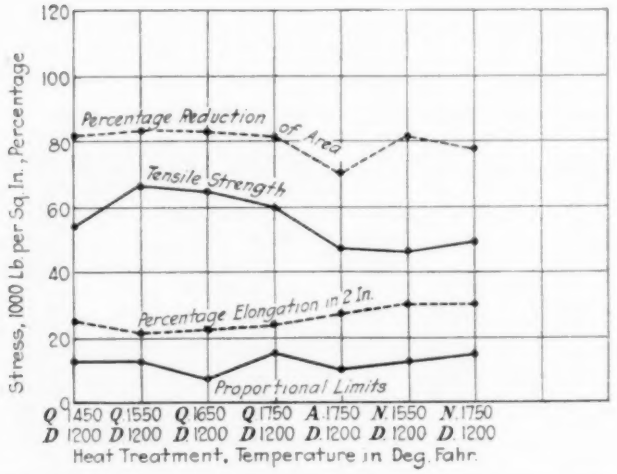


FIG. 4 EFFECT OF HEAT TREATMENT ON THE TENSILE PROPERTIES OF 0.24% C, 1.79% NI, 0.17% MO STEEL AT 1000°F.

(Q., quenched in water; D., drawn; A., annealed; N., normalized.)

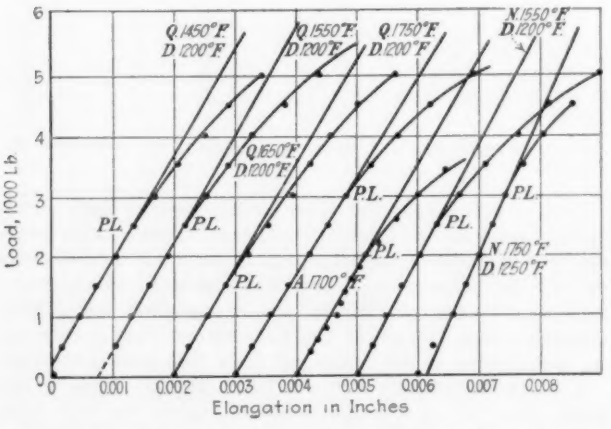


FIG. 5 STRESS-STRAIN CURVES OF 0.25% C, 1.78% NI, 0.21% MO STEEL AT 1000°F., 538°C.

(Q., quenched in water; D., drawn; A., annealed; N., normalized; P. L., proportional limits.)

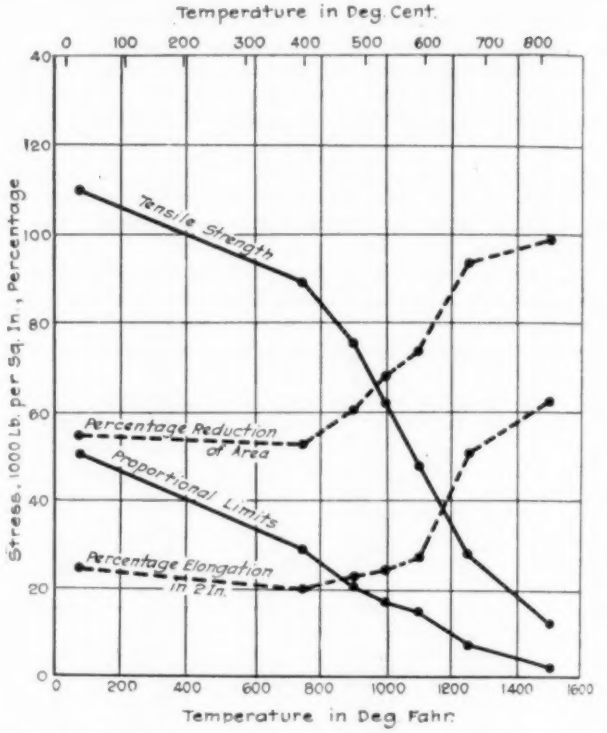


FIG. 6 TENSILE PROPERTIES OF CHROMIUM-TUNGSTEN STEEL AT ELEVATED TEMPERATURES

(Chemical composition: C 0.4, Mn 0.59, Si 0.76, S 0.019, P —0.005, Cr 7.93, Ni 0.26, W 7.7. Heat treatment: Annealed.)

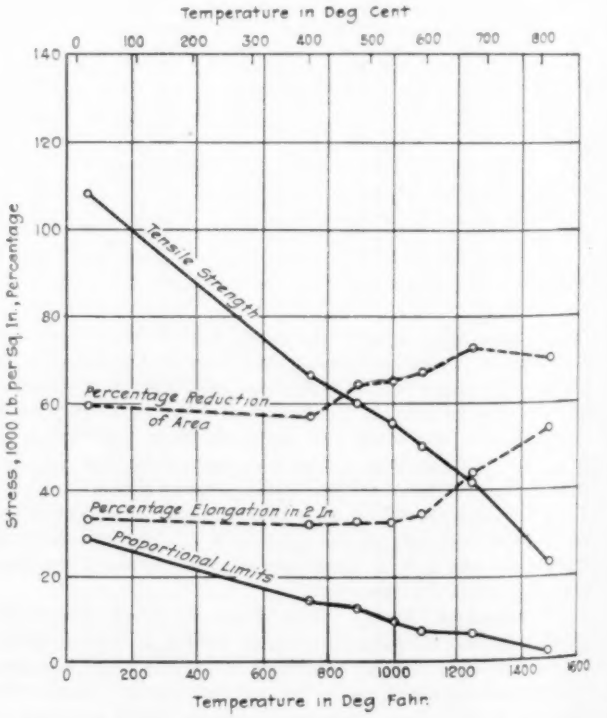


FIG. 7 TENSILE PROPERTIES OF CHROMIUM-NICKEL STEEL AT ELEVATED TEMPERATURES

(Chemical composition: C 0.15, Mn 0.48, Si 0.51, S 0.038, P —0.005, Cr 19.08, Ni 8.52. Heat treatment: Annealed.)

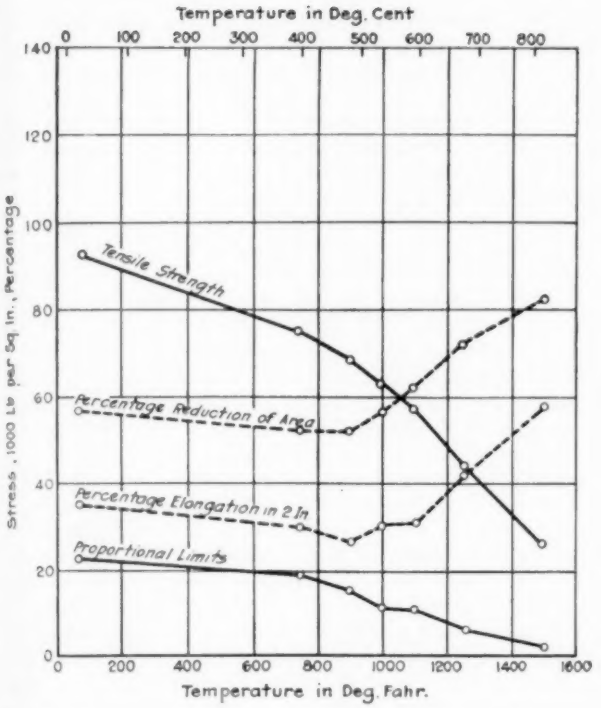


FIG. 8 TENSILE PROPERTIES OF CHROMIUM-NICKEL STEEL AT ELEVATED TEMPERATURES

(Chemical composition: C 0.31, Mn 0.65, Si 0.91, S 0.024, P —0.005, Cr 7.96, Ni 20.65. Heat treatment: Annealed.)

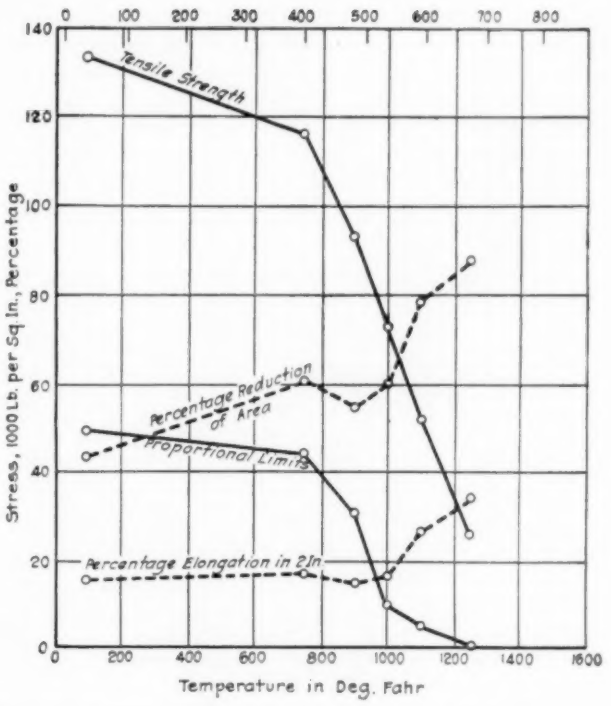


FIG. 10 TENSILE PROPERTIES OF CHROMIUM-NICKEL STEEL AT ELEVATED TEMPERATURES

(Chemical composition: C 0.41, Mn 0.58, P 0.011, S 0.022, Si 0.05, Cr 1.24, Ni 1.79.)

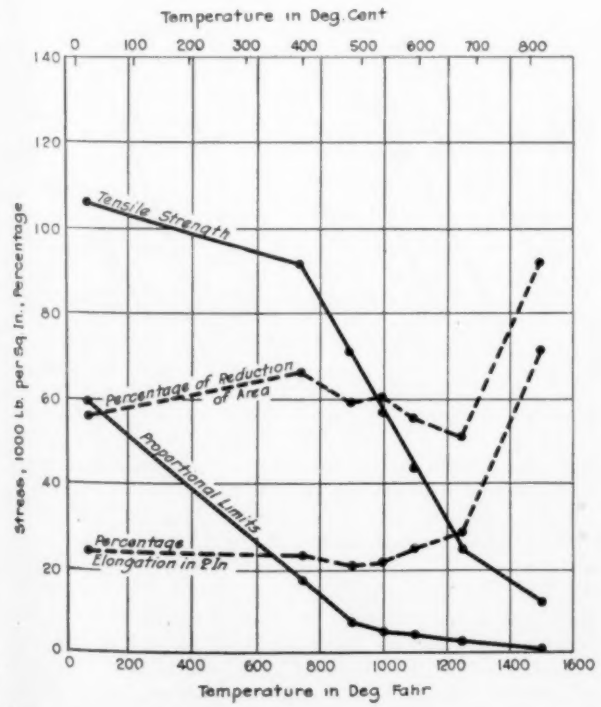


FIG. 9 TENSILE PROPERTIES OF CHROMIUM-NICKEL STEEL AT ELEVATED TEMPERATURES

(Chemical composition: C 0.38, Mn 0.6, P 0.01, S 0.023, Si 0.18, Cr 0.58, Ni 1.46. Heat treatment: Normalized at 1650°F., 899°C.)

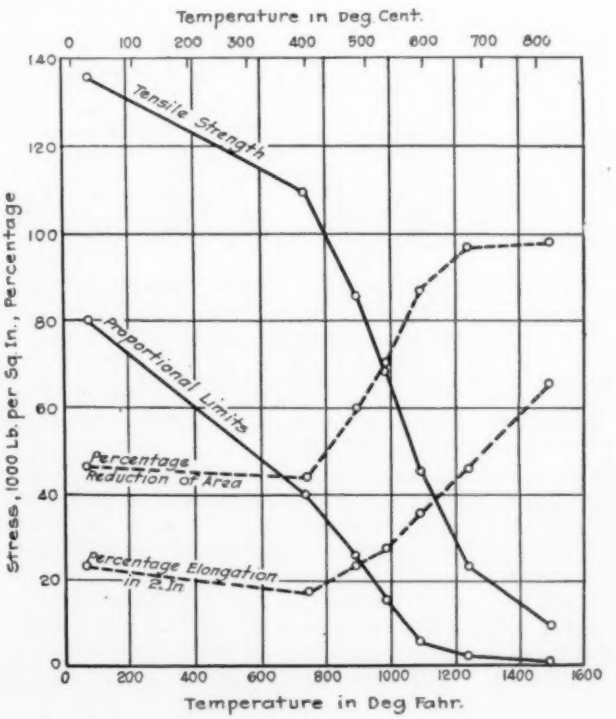


FIG. 11 TENSILE PROPERTIES OF SILCROME AT ELEVATED TEMPERATURES

(Chemical composition: C 0.48, Mn 0.59, P 0.024, S 0.009, Si 3.54, Cr 9.16, Ni 0.03. Heat treatment: Annealed at 1550°F., 843°C.)

expected; that is, low load-carrying ability throughout the entire temperature range. An interesting result in connection with these two steels is that even though the carbon, nickel, and chromium contents were varied considerably, the test results obtained in the two cases were nearly identical.

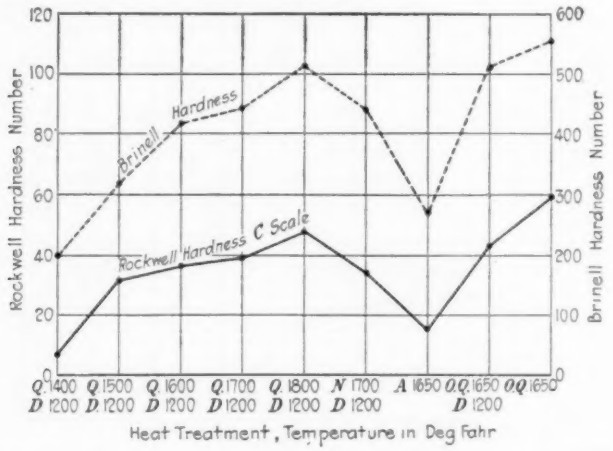


FIG. 12 EFFECT OF HEAT TREATMENT ON THE HARDNESS OF 0.6% C, 1.43% CR, 2.19% W STEEL

Q., quenched in water; O. Q., quenched in oil; D., drawn; A., annealed; N., normalized.)

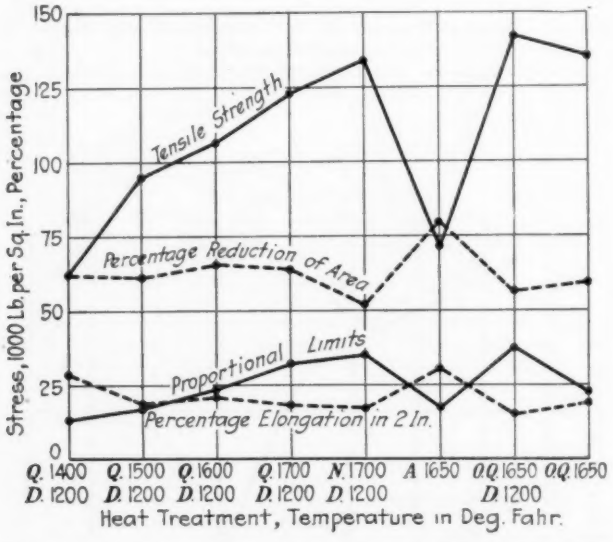


FIG. 13 EFFECT OF HEAT TREATMENT ON THE TENSILE PROPERTIES OF 0.6% C, 1.43% CR, 2.19% W STEEL AT 1000°F.

(Q., quenched in water; O. Q., quenched in oil; D., drawn; A., annealed; N., normalized.)

A comparison of the properties obtained with these four steels shows that if the best results, in regard to load-carrying ability at moderately elevated temperatures, are to be obtained, then the amounts of alloying elements added must not be sufficiently large to cause the steels to become austenitic.

Tungsten Steels. Tungsten combines with the carbon rather than with the iron when added to steel. Since it is similar in this respect to chromium, the tungsten steels should be expected to have load-carrying properties comparable to those of the chromium steels.

The effect of tungsten upon the properties of steel at high temperatures is shown in Fig. 6. It is true that this particular alloy also contains about 8 per cent chromium, but if these results be compared to those from Silcrome (Fig. 11), an alloy containing about the same amount of carbon and from 8 to 9 per cent chromium, then the effect of tungsten can be determined. It is true that Silcrome also contains from 3 to 4 per cent silicon, but this element, being similar to nickel in that it dissolves in the ferrite, does not greatly affect the load-carrying properties. A comparison of the results obtained with these two steels will show the tungsten-bearing steel to have the superior properties, especially at the higher temperatures.

Chromium-Tungsten Steels.[10] Chromium tungsten steels, owing to the presence of two carbide-forming elements, should exhibit good load-carrying properties at temperatures both above and below the equicohesive temperature. One steel of this class (Fig. 6), the results of which have already been discussed, was subjected to tests at temperatures up to 1500 deg. fahr. (816 deg. cent.). This same steel, in addition to two other steels of this group but differing from the first in that they contain much lower percentages of the alloying elements, were tested at 1000 deg. fahr. (538 deg. cent.) in various heat-treated conditions. The results obtained are presented in Figs. 12 to 19.

The steels containing the lower percentages of chromium and tungsten will first be considered. The variation in hardness with heat treatment of steel R is given in Fig. 12. As would be expected for steels of this class, the hardness can be varied considerably by proper heat treatment. In this case the hardness, as determined on both the Rockwell and Brinell machines, increases steadily as the quenching temperature is raised from 1400 deg. to 1800 deg. fahr. (760 deg. to 982 deg. cent.). This can readily be explained by the partial solution of the excess carbides and perhaps also by some refinement of grain size. Both normalizing and annealing treatments produce lower hardness values, a fact to be expected. The structures of this steel after the different heat treatments are shown in Photomicrographs 7 to 22, while

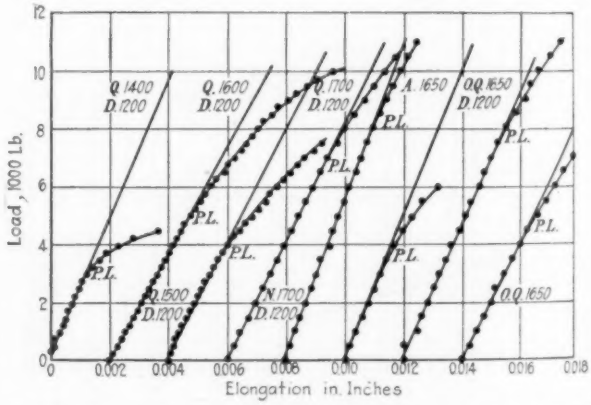


FIG. 14 STRESS-STRAIN CURVES OF 0.6% C, 1.43% CR, 2.19% W STEEL AT 1000°F., 538°C.

(Q., quenched in water; O. Q., quenched in oil; D., drawn; A., annealed; N., normalized.)

those of steels Ra and V (both low-chromium-tungsten steels) are shown in Photomicrographs 23 to 28.

As was the case with the hardness values, the proportional limit and tensile strength for steel R (Fig. 13) also increase rapidly as the quenching temperature is raised. Normalizing from 1200 deg. fahr. (927 deg. cent.) and quenching in oil from 1650 deg. fahr. (899 deg. cent.) gave the highest proportional-limit and tensile-strength values. There are two possible explanations as to why these treatments should give higher values than those obtained by water quenching, say, from 1700 deg. fahr. (927 deg. cent.). Either quenching in water from this high temperature

[10] The authors are indebted to Mr. Malcolm for calling their attention to the good load-carrying properties at elevated temperatures of chrome-tungsten steels of the "V" type.

produced quenching cracks invisible to the eye, an assumption supported by the fact that the specimen quenched in water from 1800 deg. fahr. (982 deg. cent.) was badly cracked, or else there is a critical size of both carbide particle and crystal giving the maximum strength values.

The proportional-limit results presented in Fig. 13 show that a rather large variation can be obtained depending upon the heat treatment. For example, the lowest value shown is approximately 15,000 lb. per sq. in., while with proper heat treatment this can be raised to 37,500 lb. per sq. in.

It will again be observed that all the quenched and normalized specimens with one exception were first drawn at 1200 deg. fahr. (649 deg. cent.) before being subjected to the high-temperature tensile test. The one exception was oil-quenched from 1650 deg. fahr. (899 deg. cent.) and then tested without a previous draw except that obtained in the testing furnace at 1000 deg. fahr. (538 deg. cent.). Both the proportional-limit and tensile-strength values in this particular case were lower than those obtained with the steel subjected to the same treatment but drawn at 1200 deg. fahr. (649 deg. cent.). This may be explained on the assumption that the residual stresses in the undrawn sample are of sufficient magnitude to lower the strength values.

Steels Ra and V (see Table 1) were tested at 1000 deg. fahr. (538 deg. cent.) in two heat-treated states. These steels were both subjected to the same treatments, and it is interesting to note that the steel with the lower chromium content gave the highest proportional-limit values, as shown in Fig. 15. This does not necessarily imply, however, that the same relationship would hold for all heat treatments, and it may be that this particular treatment is more favorable to the lower-chromium-carrying steel in that it produces the size of grain and carbide particle best suited to give maximum tensile properties. That this is probably the case is shown by the fact that when the quenching or normalizing temperature of the steel R, very similar to steel Ra, is raised, then the tensile properties approach very near to those shown by steel V at the lower quenching temperature.

Next considering the alloy containing higher proportions of both chromium and tungsten, Fig. 17 shows that even a greater increase in hardness values can be obtained than was the case with the first steels of this group. The hardness increases rapidly with quenching temperature, attaining a maximum value of 57 Rockwell C hardness for the specimen quenched from 2250 deg. fahr. (1232 deg. cent.). Again, this increased hardness can be

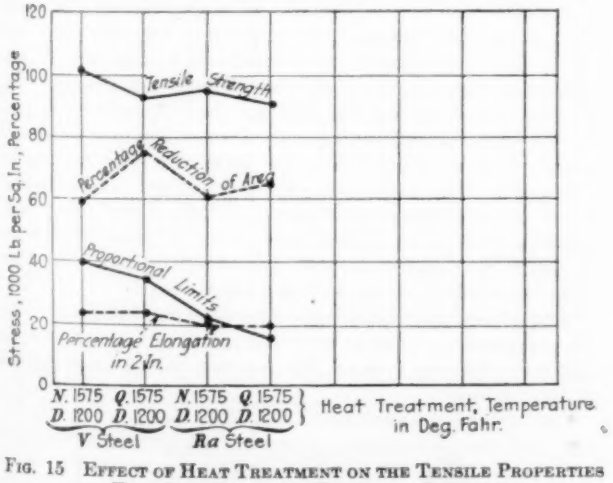


FIG. 15 EFFECT OF HEAT TREATMENT ON THE TENSILE PROPERTIES OF TWO LOW-CHROMIUM-TUNGSTEN STEELS AT 1000°F., 538°C.

(Q., quenched in water; D., drawn; N., normalized. Chemical composition of steels: Type V—0.52% C, 0.61% Cr, 1.79% W; type Ra—0.55% C, 1.49% Cr, 1.87% W.)

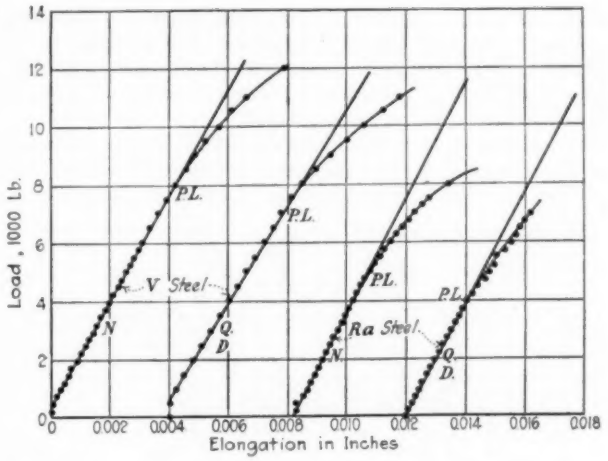


FIG. 16 STRESS-STRAIN CURVES OF CHROMIUM-TUNGSTEN STEELS AT 1000°F., 538°C.

(Q., quenched in water; D., drawn; N., normalized; P. L., proportional limits.)

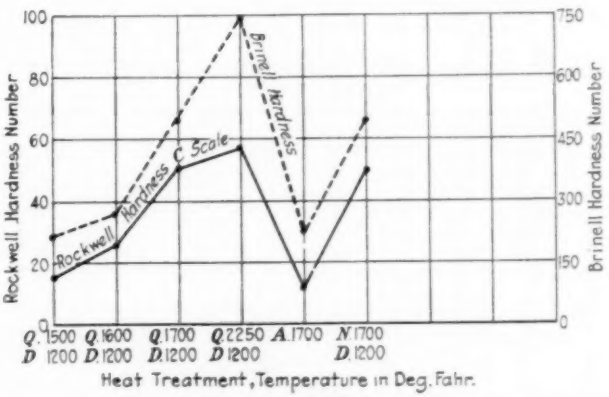


FIG. 17 EFFECT OF HEAT TREATMENT ON THE HARDNESS OF 0.4% C, 7.93% CR, 7.7% W STEEL

(Q., quenched in water; D., drawn; A., annealed; N., normalized.)

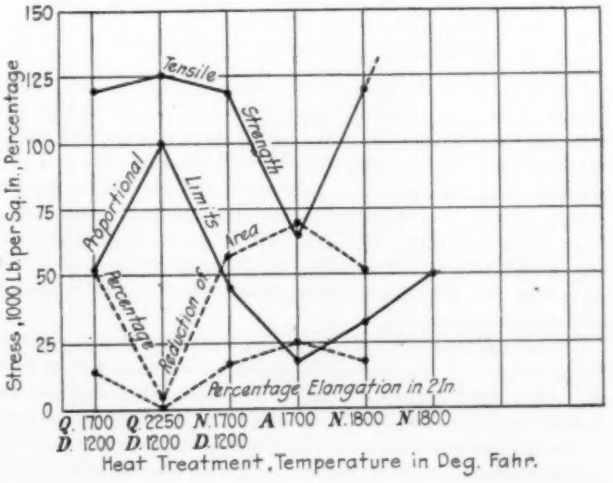


FIG. 18 EFFECT OF HEAT TREATMENT ON THE TENSILE PROPERTIES OF 0.41% C, 7.93% CR, 7.7% W STEEL AT 1000°F.

(Q., quenched in water; D., drawn; A., annealed; N., normalized.)

accounted for on the basis of the partial solution of the excess carbides. The metallographic structures corresponding to the various heat treatments are shown in Photomicrographs 29 to 33. These pictures show a decrease in the size of the carbide particles as the quenching temperature is raised.

The variation in tensile properties with heat treatment is even more marked than in the case of the hardness values. The proportional-limit values vary from 17,500 lb. to 100,000 lb. per

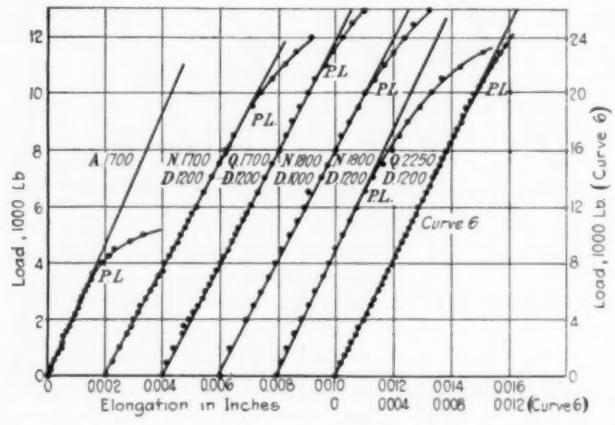


FIG. 19 STRESS-STRAIN CURVES OF 0.41% C, 7.93% CR, 7.7% W STEEL AT 1000°F.
(Q., quenched in water; D., drawn; A., annealed; N., normalized.)

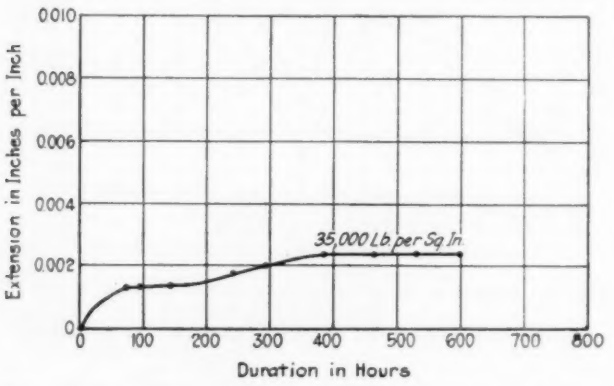


FIG. 20 FLOW OF NORMALIZED 0.4% C, 7.93% CR, 7.7% W STEEL AT 1000°F., 538°C.
(Heat treatment: Normalized at 1700°F., 927°C.)

sq. in. As far as the authors are aware, this latter value is one of the highest proportional-limit values ever reported at this temperature. With this same treatment, however, the ductibility of the metal, as shown by the reduction-of-area and percentage-elongation values, is very slight. Some of the other less drastic treatments, such as either quenching or normalizing from 1700 deg. fahr. (927 deg. cent.), give proportional-limit values almost as startling, being in the neighborhood of 50,000 lb. per sq. in. This is also a higher value than the authors have as yet seen reported.

The tensile-strength values are also remarkably high for this temperature. In the case of the specimen normalized from 1800 deg. fahr. (982 deg. cent.) and in the undrawn condition, the tensile strength could not be obtained as the proportional limit of the holders was reached before the load corresponding to the tensile strength of the specimen. All that can be said for the tensile strength in this case is that it lies above 140,000 lb. per sq. in., as it was at this stress that the holders began to give noticeably.

As was the case with one of the first steels of this group, two specimens with the same previous treatment (except that one was drawn and the other was not) were tested. In this case the specimen which had not been drawn except at 1000 deg. fahr. (538 deg. cent.) gave a higher proportional-limit value than the one previously drawn at 1200 deg. fahr. (649 deg. cent.). Although this appears contrary to the results obtained with the first steel, such may not be the case. In the first instance steels in the quenched condition were being considered, while in the latter the specimens were in the normalized state. If the same type of steel is considered in these two states, that in the normalized condition must be more stable than the one in the quenched condition, owing to the less drastic way in which its structure was obtained. Thus, even though the normalized steel was tested at the same temperatures at which it was drawn, its structure is possibly sufficiently stable to resist the action of the stress applied and the time.

In order to determine whether or not these proportional-limit values would be used as a criterion of the metal's load-carrying ability, a specimen after being normalized from a temperature of 1700 deg. fahr. (927 deg. cent.) was subjected to a load of 35,000 lb. per sq. in., and this load was maintained constant for 600 hr. The results of this test are shown in Fig. 20.

The results show that even after 600 hr. the elongation was only slightly over 0.002 in. If the test had been maintained longer, the metal might possibly have continued to have crept, but with no doubt at probably a slower rate than shown during the first 600 hr. Since the rate of creep is so small, it is evident that the metal would be able to withstand this stress for a considerable time.

Manganese Steels. Manganese is similar to both chromium and nickel in that when added to steel it unites with the carbon, forming double carbides, and also dissolves in the ferrite. The separation of free carbides is not obtained unless both the carbon and manganese are present in relatively large amounts. For this reason the addition of this element should increase the load-carrying ability of metals at elevated temperatures, but not to such a degree as chromium or tungsten.

The short-time tensile properties of various carbon-manganese steels at elevated temperatures are given in Figs. 21 to 24. For the purpose of comparison, the properties of a plain carbon steel containing the usual percentage of manganese is included. On the basis of proportional-limit findings, the 1.55 per cent manganese, 0.44 per cent carbon steel is the outstanding member of this group. This steel also shows the maximum reduction-of-area values over a large part of the temperature range. The maximum tensile-strength values were obtained with the 2.06 per cent manganese, 0.44 per cent carbon steel. None of these steels shows as high values, however, as were obtained with some of the other steels, especially in the higher temperature ranges.

The 1.55 per cent manganese, 0.44 per cent carbon steel was also subjected to different heat treatments and tested at 1000 deg. fahr. (538 deg. cent.). The results obtained are presented in Figs. 25 to 28.

The hardness values shown in Fig. 25 do not show much variation with heat treatment. Increasing the quenching temperature from 1450 deg. to 1550 deg. fahr. (788 deg. to 843 deg. cent.) slightly raises the hardness as shown by both the Rockwell and Brinell hardness numbers. This is undoubtedly due to grain refinement and some solution of the carbides. As the quenching temperature is further increased up to 1750 deg. fahr. (954 deg. cent.), the hardness, especially as obtained on the Brinell machine, falls off due to increase in grain size. The specimen normalized from 1750 deg. fahr. (954 deg. cent.) gives lower hardness values than the one subjected to this same treatment from 1550 deg. fahr. (843 deg. cent.). Again, this can be explained on the

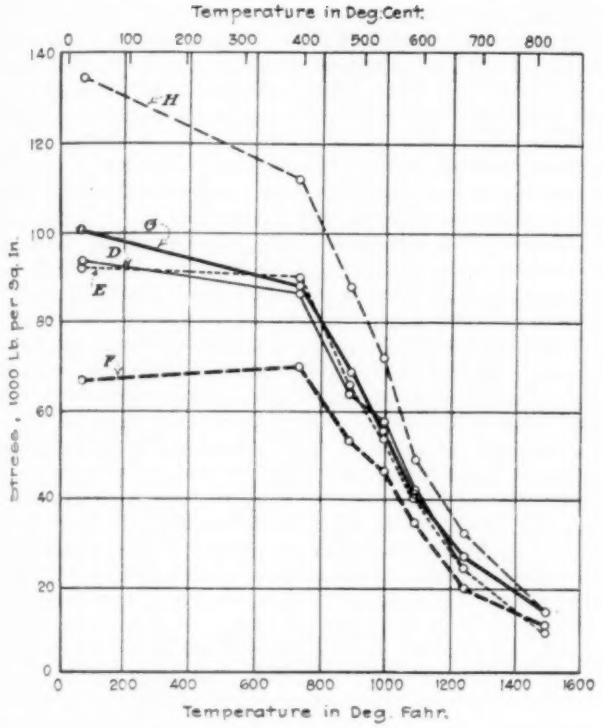


FIG. 21 TENSILE-STRENGTH CURVES OF VARIOUS CARBON-MANGANESE STEELS AT ELEVATED TEMPERATURES
(D, 0.69% Mn; E, 1.17% Mn; F, 1.31% Mn; G, 1.55% Mn; H, 2.06% Mn.)

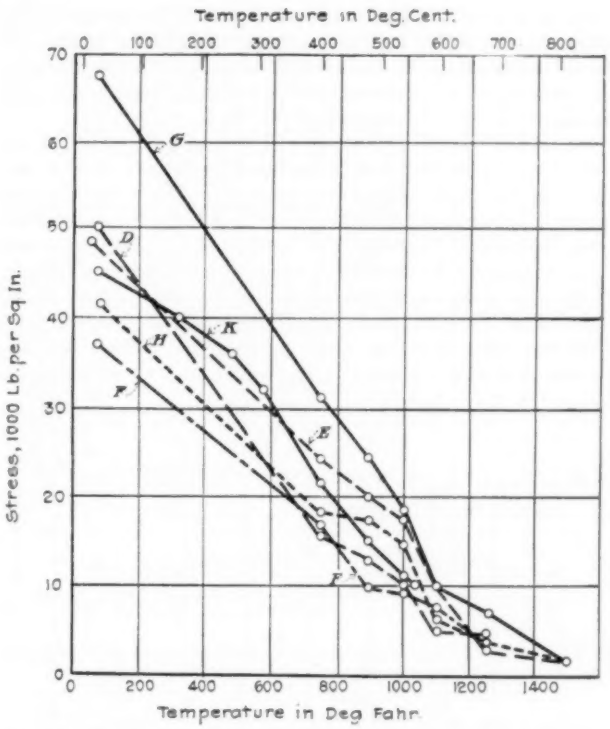


FIG. 23 ELONGATION CURVES OF VARIOUS CARBON-MANGANESE STEELS AT ELEVATED TEMPERATURES
(D, 0.69% Mn; E, 1.17% Mn; F, 1.31% Mn; G, 1.55% Mn; H, 2.06% Mn.)

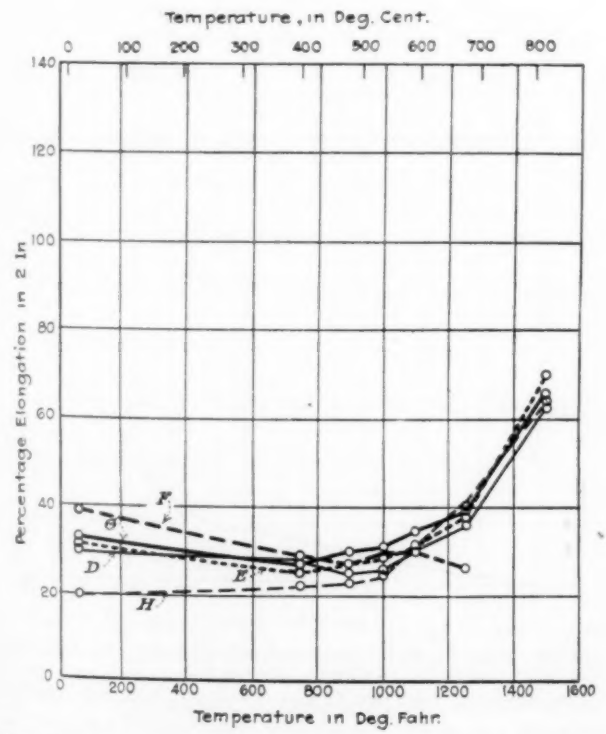


FIG. 22 PROPORTIONAL-LIMIT VALUES OF MANGANESE STEELS AT ELEVATED TEMPERATURES
D, 0.69% Mn; E, 1.17% Mn; F, 1.31% Mn; G, 1.55% Mn; H, 2.06% Mn; K, 1.26% Mn.)

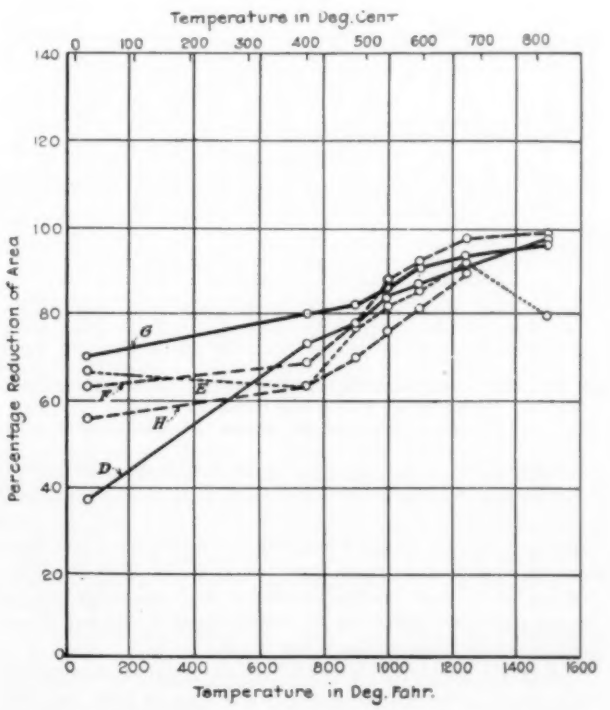


FIG. 24 REDUCTION-OF-AREA CURVES OF VARIOUS CARBON-MANGANESE STEELS AT ELEVATED TEMPERATURES
(D, 0.69% Mn; E, 1.17% Mn; F, 1.31% Mn; G, 1.55% Mn; H, 2.06% Mn.)

basis of grain size. The lower values shown by the specimens subjected to the double-quenching treatment are evidently due to the higher drawing temperatures employed. The metallographic structures obtained with the various heat treatments are shown in Photomicrographs 34 to 40.

Neither do the tensile properties of this steel, shown in Fig. 26, vary to any great degree with heat treatment. As was the case with the hardness values, the proportional-limit and tensile-strength values increase as the quenching temperature is raised from 1450 deg. to 1550 deg. fahr. (788 deg. to 843 deg. cent.) and then decrease as the quenching temperature is increased further. In the case of the tensile tests, however, the highest proportional-limit values were obtained from the specimen normalized from 1550 deg. fahr. (843 deg. cent.) and the one subjected to the double-quenching treatment. The higher drawing temperatures

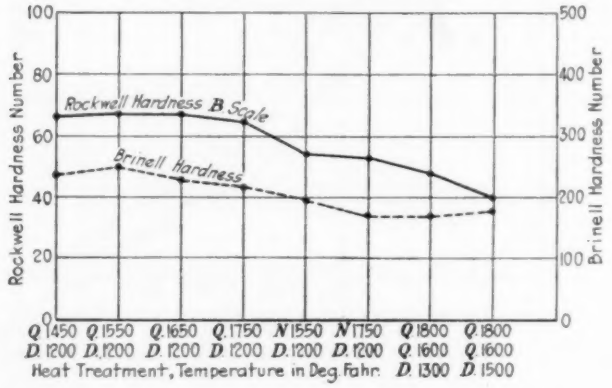


FIG. 25 EFFECT OF HEAT TREATMENT ON THE HARDNESS OF 0.41% C, 1.43% MN STEEL
(Q., quenched in water; D., drawn; N., normalized.)

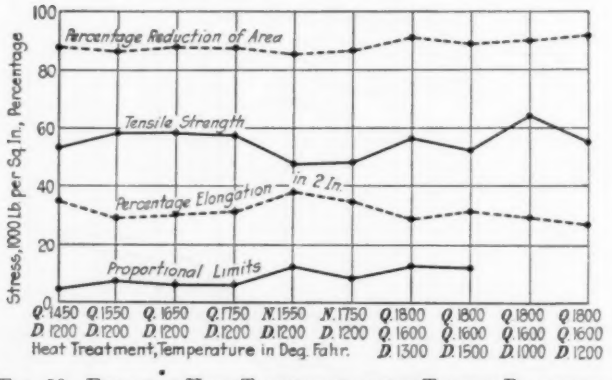


FIG. 26 EFFECT OF HEAT TREATMENT ON THE TENSILE PROPERTIES OF 0.43% C, 1.43% MN STEEL AT 1000°F.
(Q., quenched in water; D., drawn; N., normalized.)

were used in the steels subjected to the double-quenching treatments because for the lower draws no definite proportional-limit values could be obtained. In view of the fact that the other measurable factors obtained from the specimens with the lower draws were similar to those obtained with the 1300 deg. fahr. (704 deg. cent.) draw, we feel justified in the conclusions that the proportional-limit values for the lower draws would not be appreciably above those for the higher draws.

Certain investigators[11] have claimed much higher proportional-limit values for manganese steels of this type at 1000 deg. fahr. (538 deg. cent.) or temperatures in that neighborhood after being subjected to the double-quenching treatment shown in the figures. Since the highest values the authors were able to obtain are only in the neighborhood of 12,500 lb. per sq. in., it was decided to submit this steel to a creep test in order to determine which values were the more nearly correct. Accordingly, the specimen, after being given the preferred heat treatment recommended by those who reported the higher proportional-limit values, was subjected to a load of 10,000 lb. per sq. in., a value of the same order of magnitude as the proportional-limit value of the authors, but only about 25 per cent of the one reported by other investigators, and tested at 1000 deg. fahr. (538 deg. cent.).

The results of this long-time test are given in Fig. 28. It will be observed that the creep proceeds uniformly and rather rapidly for nearly 300 hr. and then the rate increases, the increased rate continuing up to rupture, which occurred at 435 hr. One of the outstanding facts about this test is the high degree of elongation obtained, 59 per cent. This is considerably more than is usually obtained in long-time tests.

The effect of heat treatment upon the tensile properties at 1000 deg. fahr. (538 deg. cent.) of another manganese steel was also investigated. This steel in addition to the manganese also contained some chromium. The results are given in Figs. 29 to 31.

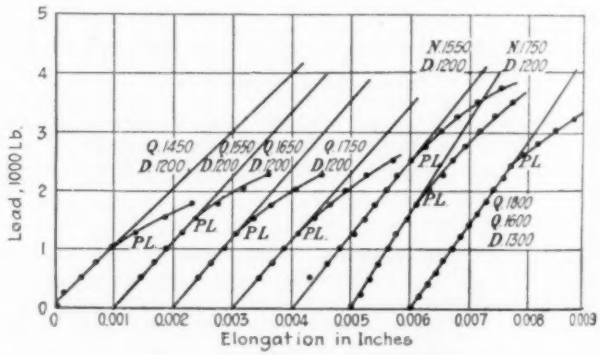


FIG. 27 STRESS-STRAIN CURVES OF 0.41% C, 1.43% MN STEEL AT 1000°F., 538°C.
(Q., quenched in water; D., drawn; N., normalized; P. L., proportional limits.)

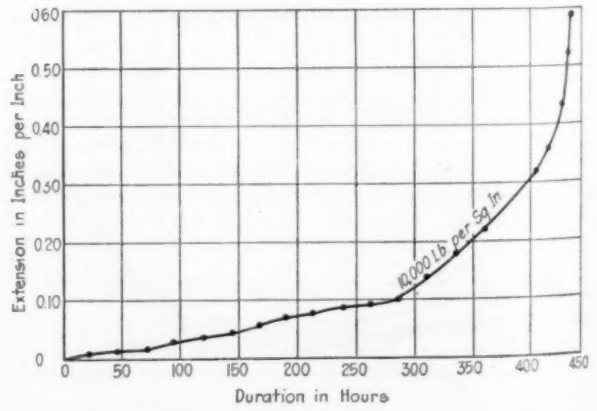


FIG. 28 FLOW OF HEAT-TREATED 0.44% C, 1.55% MN STEEL AT 1000°F., 538°C.
(Heat treatment: 8 hr. 1800°F., water-quenched; 1/2-hr. 1600°, water-quenched; drawn at 1000°F.)

The hardness values obtained with this steel differ from those of the straight-carbon manganese steel in that after increasing with quenching temperature up to 1600 deg. fahr. (871 deg. cent.)

[11] V. Krivobok, B. Larsens, W. Skinkle, and W. Masters, "Some Characteristics of Low-Carbon-Manganese Steel," Tech. Publ. No. 24, A.I.M.E., 30 pp., 1927.

they do not decrease but remain constant as the quenching temperature is further raised to 1800 deg. fahr. (982 deg. cent.). This is no doubt due to the partial solution of the chromium carbides, offsetting the increased grain size. This also explains why the specimen normalized from 1550 deg. fahr. (843 deg. cent.) exhibited lower values than the specimens normalized from 1750 deg. fahr. (954 deg. cent.). The structures corresponding to the various heat treatments are shown in Photomicrographs 41 to 45.

Heat treatment failed to materially increase the proportional-limit values of this steel. The specimens quenched at all temperatures above 1500 deg. fahr. (816 deg. cent.) were badly cracked in the quenching operation, and accordingly were useless for tensile-testing purposes. The maximum proportional-limit value obtained was approximately 14,000 lb. per sq. in., and this was obtained with the specimen normalized from 1550 deg. fahr. (843 deg. cent.).

Molybdenum and Vanadium Steels. Molybdenum and vanadium in that they combine with the carbon to form carbides when added to steel are both similar to chromium and so should, like chromium, increase the load-carrying ability of the alloys to which they are added. They both differ from chromium in that smaller percentages of these elements are required to produce the same change as in the case of the chromium alloys.

Vanadium in that it is a grain refiner is similar to nickel. In this respect it will, like nickel, increase the load-carrying ability at temperatures below the equicohesive temperature, but for temperatures above will have the opposite effect. It is also claimed that vanadium is beneficial in that it reduces intercrystalline brittleness, a condition often encountered in heat-resisting alloys.

No tests were conducted on steels containing only vanadium as the alloying element, but the low-chromium-tungsten steel considered in a previous section contained an appreciable amount of this element.

Likewise, no tests were conducted on steels containing only molybdenum as the alloying element. The effect of heat treatment on the tensile properties at 1000 deg. fahr. (538 deg. cent.) of a nickel-molybdenum steel was investigated, and these results were also given in a previous section.

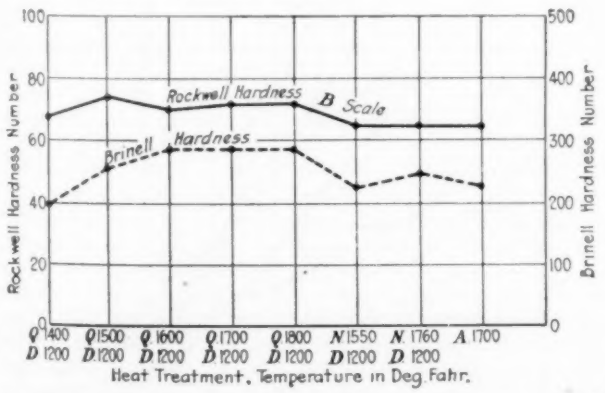


FIG. 29 EFFECT OF HEAT TREATMENT ON THE HARDNESS OF 0.49% C, 1.21% MN, 0.33% CR STEEL

(Q., quenched in water; D., drawn; A., annealed; N., normalized.)

Chromium-Aluminum Steel. The effect of heat treatment upon the tensile properties at 1000 deg. fahr. (538 deg. cent.) of a chromium-aluminum steel was also investigated. The aluminum, dissolving mainly in the iron, was not expected to greatly affect the load-carrying ability, but it was desired to determine the joint action of both chromium and aluminum on the high-temperature properties of steel. The results obtained are shown in Figs. 32 to 34.

The structures corresponding to the various heat treatments are given in Photomicrographs 46 to 54. The pictures portray some interesting conditions. For instance, the samples quenched either in oil or water from 1650 deg. fahr. (899 deg. cent.) are

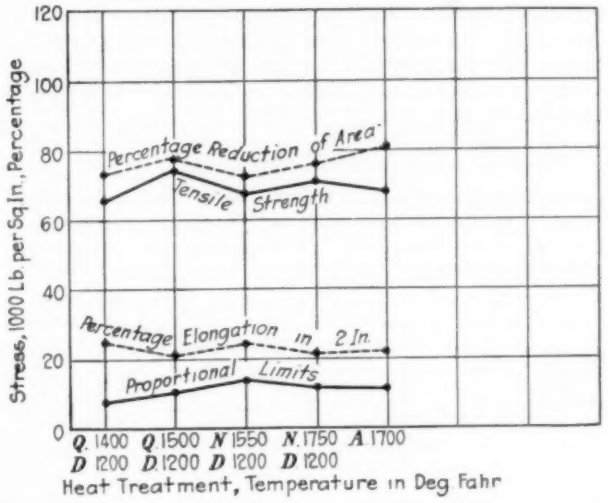


FIG. 30 EFFECT OF HEAT TREATMENT ON THE TENSILE PROPERTIES OF 0.49% C, 1.21% MN, 0.33% CR STEEL AT 1000°F.

(Q., quenched in water; D., drawn; A., annealed; N., normalized.)

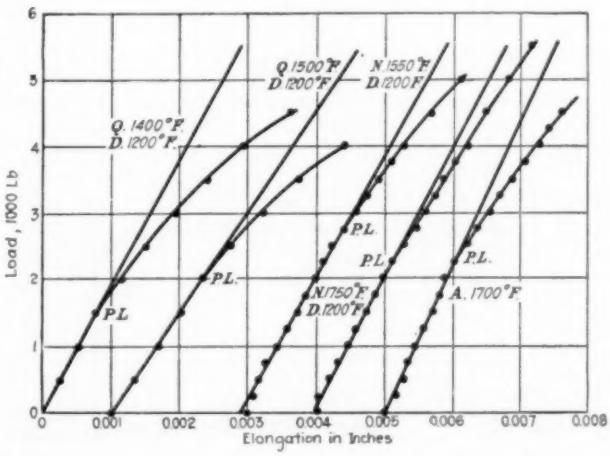


FIG. 31 STRESS-STRAIN CURVES OF 0.49% C, 1.21% MN, 0.33% CR STEEL AT 1000°F., 538°C.

(Q., quenched in water; D., drawn; A., annealed; N., normalized; P. L., proportional limits.)

completely surrounded with a ring of polyhedral grains of metal resembling either ferrite or austenite. The specimen water-quenched from 1700 deg. fahr. (927 deg. cent.), as well as the one annealed from the same temperature, contains a large amount of this same constituent, while the specimen normalized from this temperature consists entirely of it. That this constituent is relatively hard is shown by the hardness value obtained from the normalized specimen.

As would be expected from the photomicrographs, the hardness of this alloy can be varied considerably, depending upon the heat treatment. The increased hardness can be accounted for on the assumption that either the chromium carbides or else complex compounds of chromium, aluminum, and carbon are either partially or wholly dissolved in the ferrite as the quenching temperature is raised. In the case of the normalized specimen all

the carbon and chromium seem to have disappeared, and must be in solution in the polyhedral crystals shown.

As in the case of the hardness values, the tensile strength varies considerably depending upon the treatment, but the proportional limits remain nearly constant regardless of the treatment. The maximum proportional-limit values were obtained by oil-quenching from 1550 deg. fahr. (843 deg. cent.) or water-quenching from 1550 deg. fahr. (843 deg. cent.), or 1750 deg. fahr. (954 deg. cent.). The minimum value was obtained by oil-quenching from 165 deg. fahr. (899 deg. cent.).

Conclusions

This paper has endeavored to throw additional light on, and to furnish further data with respect to, the properties of metals at

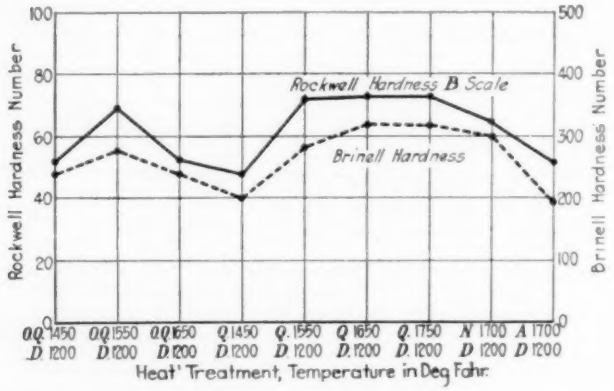


Fig. 32 Effect of Heat Treatment on the Hardness of 0.5% C, 1.6% Cr, 1% Al Steel

(Q., quenched in water; O. Q., quenched in oil; D., drawn; A., annealed; N., normalized.)

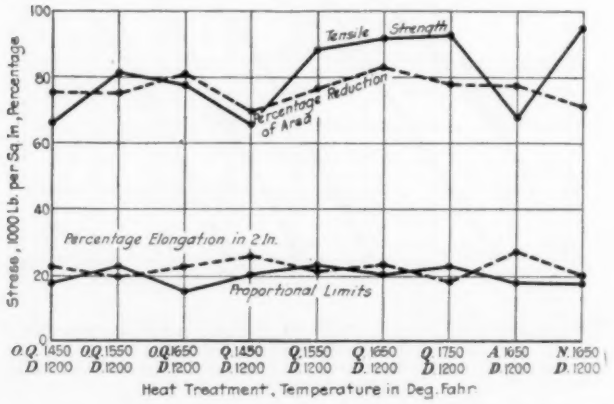


Fig. 33 Effect of Heat Treatment on the Tensile Properties of 0.5% C, 1.6% Cr, 1% Al Steel at 1000°F.

(Q., quenched in water; O. Q., quenched in oil; D., drawn; A., annealed; N., normalized.)

elevated temperatures. It has done this by (1) advancing a theory or hypothesis accounting for the stability of the various types of steels at elevated temperatures, and (2) by giving specific data on various types of alloy steels, showing (*a*) the effect of elevated temperatures on various types of steels when in an annealed or normalized state, and (*b*) the effect of heat treatments which would presumably bring out the best possible proportional-limit values on various types of steels when tested at a temperature of 1000 deg. fahr. (538 deg. cent.).

The hypothesis advanced to account for the influence of the different alloying elements on steels when at elevated temperatures is set forth in two parts, one dealing with the conditions below the equicohesive temperature and the other dealing with conditions when above this temperature. The hypothesis follows.

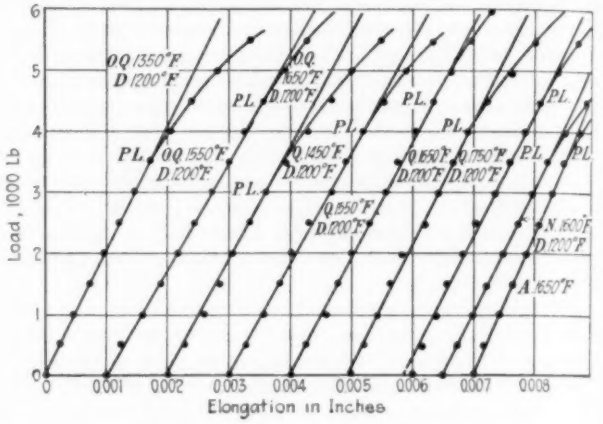


Fig. 34 Stress-Strain Curves of 0.5% C, 1.6% Cr, 1% Al Steel at 1000°F., 538°C.

(Q., quenched in water; O. Q., quenched in oil; D., drawn; A., annealed; N., normalized; P., proportional limits.)

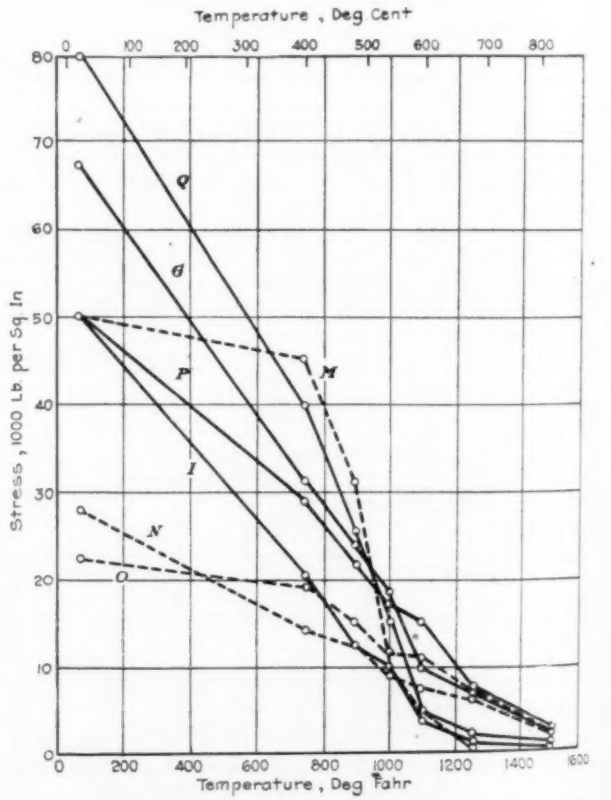


Fig. 35 Proportional-Limit Curves for Various Selected Steels at Elevated Temperatures

(Chemical composition of steels: Type G—0.44% C, 1.55% Mn; type I—0.1% C, 4.95% Ni; type M—0.41% C, 1.24% Cr, 1.79% Ni; type N—0.15% C, 19.08% Cr, 8.52% Ni; type O—0.31% C, 7.96% Cr, 20.65% Ni; type P—0.4% C, 7.96% Cr, 7.7% W; type Q—0.48% C, 9.16% Cr, 3.54% Si. Heat treatment: All steels normalized or annealed.)

Below the Equicohesive Temperature.

1 The addition or formation of any element, compound, or constituent which does not enter into solid solution with the

matrix, but which by the very nature of its presence interferes with the crystal slippage, will tend to increase the load-carrying ability of the material of which it forms a part.

2 The addition or formation of any element, compound, or constituent which tends to increase the amount of amorphous material will increase the load-carrying ability of the material of which it forms a part.

Above the Equicohesive Temperature.

1 The addition or formation of any element, compound, or constituent which tends to decrease the relative proportion of amorphous material to crystalline material will increase the load-carrying ability of the material of which it forms a part.

2 The addition or formation of any element, compound, or constituent which strengthens the grain boundaries by interfering with plastic flow will increase the load-carrying ability of the material of which it forms a part.

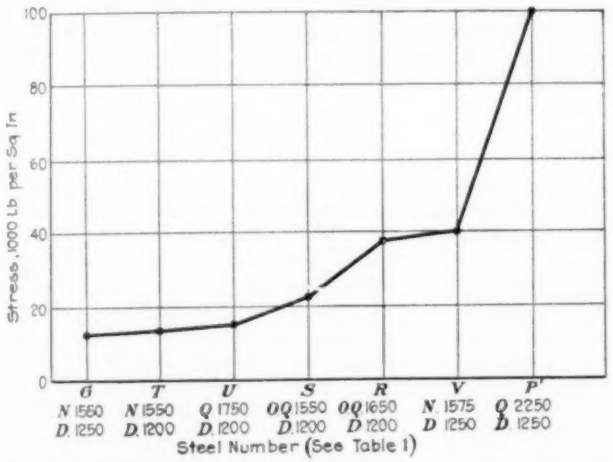


FIG. 36 MAXIMUM PROPORTIONAL-LIMIT VALUES OBTAINED AT 1000°F., 538°C, WITH VARIOUS SELECTED STEELS

(Chemical composition of steels: Type G—0.44% C, 1.55% Mn; type P'—0.40% C, 7.93% Cr, 7.7% W; type R—0.6% C, 1.43% Cr, 2.19% W; type S—0.49% C, 1.62% Cr, 1% Al; type T—0.49% C, 0.33% Cr, 1.21% Mn; type U—0.24% C, 1.79% Ni, 0.17% Mo; type V—0.52% C, 0.61% Cr, 1.79% W. Q., quenched in water; O. Q., quenched in oil; D., drawn; N., normalized.)

Two views are held by those using materials at elevated temperature. One is that all such materials should be used in the annealed or normalized state. The other is that, under certain conditions and with certain types of uses, materials may be employed which have been subjected to heat treatments that bring out the highest possible short-time characteristics at elevated temperatures. All will agree that the greatest stability is in material in the normalized or annealed condition. For that reason a very considerable amount of work done in this laboratory has been on materials in one or the other of these two conditions. In Fig. 35 are given the proportional-limit values at various elevated temperatures from steels which were initially in a normalized or annealed condition. Though the values at atmospheric temperatures are given, for the purposes considered only those values at 750 deg. fahr. (399 deg. cent.) or higher should be considered. As a matter of fact, the best criterion for the stability characteristics of these alloy steels will be secured by studying the values at 1000 deg. fahr. (538 deg. cent.) and higher. With such an analysis, the steel designated as P' and containing 7.93 per cent chromium and 7.70 per cent tungsten stands out above all of the other steels.

In Fig. 36 are given the highest proportional limits at 1000 deg. fahr. (538 deg. cent.) that it is possible to secure on various alloy steels which had been subjected to such a treatment as would bring out the highest possible proportional-limit value. The manganese steel of which the authors made favorable mention in a previous paper does not possess as high proportional-limit values as any of the other steels which were subjected to this study. Every possible effort was made to obtain as high a proportional limit as possible for this steel, but the authors were not able to obtain values in excess of 12,500 lb. per sq. in. Particular attention is directed to the steel designated as P', which contains 7.93 per cent chromium and 7.70 per cent tungsten. By quenching this steel from 2250 deg. fahr. (1232 deg. cent.) a proportional limit of 100,000 lb. was obtained. So far as the authors are aware, this is the highest proportional limit which has as yet been recorded at this temperature. On the steel designated as R, which is similar to the V steel (a type recommended by one of the largest valve-manufacturing companies), 1.43 per cent chromium and 2.19 per cent tungsten, a proportional limit at 1000 deg. fahr. (538 deg. cent.) of 37,500 lb. was obtained. Because of the lower chromium and tungsten values this should prove to be both a cheaper and a more readily workable steel. Therefore steel of this type is recommended when moderately high-temperature values are desired, although for the higher temperatures a steel of the P' type should manifest the maximum stability characteristics.

Discussion

L. W. SPRING.[12] With reference to the 8 per cent tungsten-8 per cent chromium steel mentioned, we have flow tests under way which show it to stand high among the alloys in the matter of flow resistance. Previous tests to determine retention of hardness when heated for a long time showed that this chrome-tungsten alloy with 0.57 per cent carbon, quenching from 2100 deg. fahr. and held continuously at 900 deg. temperature for 300 hours, advanced in hardness from 590 Brinell to 630 Brinell. Brinelling this piece every week, we found that after a total of 2000 hours it had retained its hardness of 630 Brinell, after which it gradually fell off, until after a total of 2900 hours at 900 deg. the hardness was 510 Brinell, at which time the test was discontinued. The 0.30 per cent carbon alloy with this tungsten and chromium had much the same general characteristics, but, as expected, it does not hold its hardness so long. The foregoing results were far better than most of the alloys tested, the 0.57 per cent carbon alloy described being second only to quenched high-speed steel of 17 per cent tungsten, 4 per cent chromium, and 0.60 per cent carbon. Other alloys tested were 13 per cent chromium–0.12 per cent carbon stainless iron, 13 per cent chromium–2 per cent tungsten steel containing 1 per cent molybdenum and 0.54 per cent carbon, 13 per cent chromium–4 per cent tungsten, 17 per cent chromium–1 per cent carbon alloy, 8 per cent chromium–2 per cent tungsten and 3 per cent silicon, none of which held their hardness as well as the 8 per cent tungsten–8 per cent chromium or the high-speed steel. Duplicate tests holding the temperature continuously at 750 deg. instead of 900 deg. showed all of these alloys capable of retaining their hardness more or less indefinitely.

From some of our flow tests we feel that the authors have somewhat underrated the high nickel-high chromium austenitic steels. Our tests of these steels at temperatures of 900 deg. and over have shown that they resist considerably higher stresses than the non-austenitic alloys. Two such alloys which we have tested are 23 per cent nickel–7 per cent chromium and 23 per cent nickel–17 per cent chromium. These austenitic steels, while having very good physical properties, are difficult to fabricate, a serious factor and an additional problem for the manufacturer, the cost of which

[12] Crane Company, Chicago, Ill.

eventually, of course, is passed on to the customer in one way or the other.

The authors of the paper remark on the effect of grain size. In our A.S.T.M. paper of June, 1928, on "Long Time or Flow Tests of Carbon Steels at Various Temperatures, With Particular Reference to Stresses Below the Porportional Limit," we called attention to the fact that our tests showed that steels of larger grain size resisted flow at higher stresses than the same steels in which smaller grain size was produced by heat treatment or other method such as forging. We already had evidence from our flow tests that well-known alloy steels behaved similarly. This of course brings up the question whether forgings or castings or other products for high-temperature service where highest resistance to flow is desired should have the fine grain which the engineer for many years has considered so desirable. He long has considered ductility, as measured by high elongation before breaking, the end to be sought for. The old standard of ductility probably was something like 25 per cent elongation in 2 inches, and if that was discounted and a stronger steel given by the manufacturer, he objected, preferring the lower strength with high ductility. As a case in point, something over ten years ago the alloy-steel stud was brought out for high-temperature bolting, and such alloy-steel studs have been used on 600 lb. and higher steam pressures at 700 deg. without failure. These studs were the basis of A.S.T.M. Specification A-96-28 for alloy-steel bolting for high-temperature service, of which Class C requirements are 105,000 lb. p.s.i. minimum yield point, tested at normal temperatures.

Naturally, such steel gives less than 25 per cent elongation in 2 inches, and the specification, based on actual experience was drawn calling for a minimum of 16 per cent. This Class C steel is being used very largely today, and apparently engineers now agree that it is better to have a high yield point with somewhat lower ductility than to have the less strong steel of low yield point with higher ductility. In other words, it seems to have come to this: So long as one gets reasonable ductility to avoid likelihood of extreme stiffness in the steel, it seems better to have a high enough yield point to cover operating stresses adequately so that flow or creep will be held to a minimum. With high enough yield point that the steel does not stretch under stresses and temperatures used, of what value is potential higher elongation which will never need be used? An elongation of 16 per cent is the minimum limit given in the specification. Probably no one would care to advocate lower than 12 per cent, as we would get rather more rigidity and perhaps increase the danger of failure by shock. However, the 16 per cent minimum elongation with 105,000 lb. p.s.i. yield point does seem to be well justified, and now has the sanction of approximately ten years' satisfactory use.

It was formerly thought that ductility has something to do with the endurance value. Recent and very thorough tests have shown that the endurance value is not connected either with the ductility or with the yield point, but is higher as the ultimate strength is higher. These high-strength steel studs therefore should have increased endurance limit under vibration, which is one of the things desired in many installations.

Authors' Closure

Professor White. I quite agree with what Mr. Spring said, to the effect that there is a place for the high chrome-high nickel alloys. Those alloys without doubt will be found to have quite acceptable properties from the standpoint of resisting oxidation at these high temperatures, particularly corrosion where there is a possibility of wet steam from time to time.

The paper itself of course was dealing primarily with the stability of these metals at elevated temperatures and was confining most of its attention to a discussion of the alloys which will give the highest physical properties. It was from that standpoint, therefore, that the authors felt there were other alloys than these austenitic ones which had high properties.

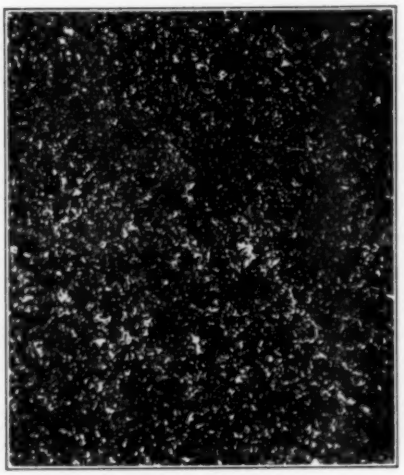
We are glad to have Mr. Spring say what he did with regard to large grain size, although we feel that a small grain size is preferable with conditions below the equicohesive temperatures. When we are above we are inclined to believe in improvements brought about by a large grain size, and we are glad to get the confirmation with respect to that matter.

Photomicrographs of Alloy Steels



No. 1 Nickel-Molybdenum Steel

(Steel R—Quenched 1450°F., drawn 1200°F. Nitric acid etch. X 100 D.)



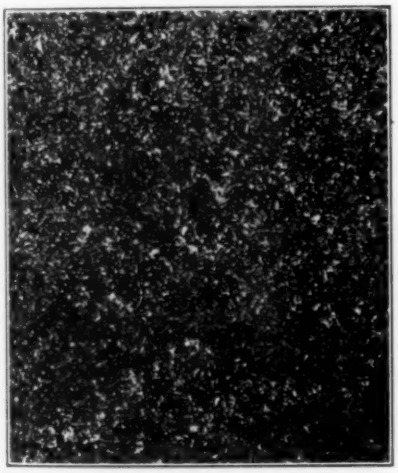
No. 2 Nickel-Molybdenum Steel

(Steel R—Quenched 1550°F., drawn 1200°F. Nitric acid etch. X 100 D.)



No. 3 Nickel-Molybdenum Steel

(Steel R—Quenched 1650°F., drawn 1200°F. Nitric acid etch. X 100 D.)



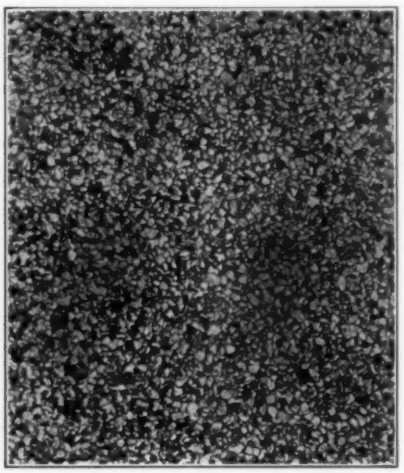
No. 4 Nickel-Molybdenum Steel

(Steel R—Quenched 1750°F., drawn 1200°F. Nitric acid etch. X 100 D.)



No. 5 Nickel-Molybdenum Steel

(Steel R—Normalized 1550°F., drawn 1200°F. Nitric acid etch. X 100 D.)



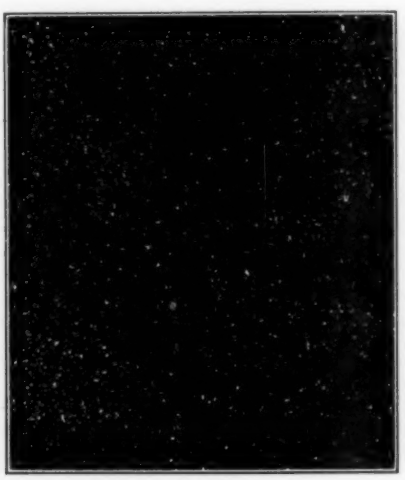
No. 6 Nickel-Molybdenum Steel

(Steel R—Normalized 1750°F., drawn 1200°F. Nitric acid etch. X 100 D.)



No. 7 Chromium-Tungsten Steel

(Steel R—Quenched 1400°F., drawn 1200°F. Nitric acid etch. X 100 D.)



No. 8 Chromium-Tungsten Steel

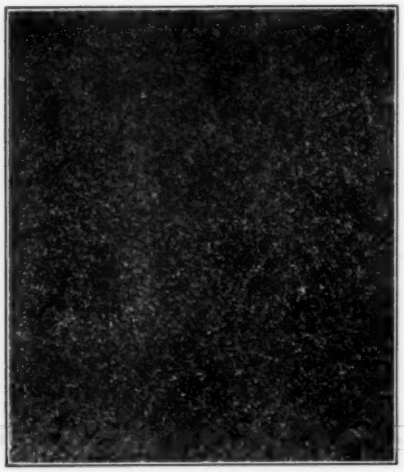
(Steel R—Quenched 1500°F., drawn 1200°F. Nitric acid etch. X 100 D.)



No. 9 Chromium-Tungsten Steel

(Steel R—Quenched 1600°F., drawn 1200°F. Nitric acid etch. X 100 D.)

Photomicrographs of Alloy Steels



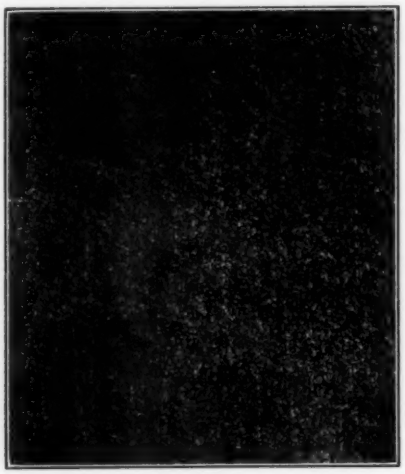
No. 10 Chromium-Tungsten Steel
(Steel R—Quenched 1700°F., drawn 1200°F. Nitric acid etch. X 100 D.)



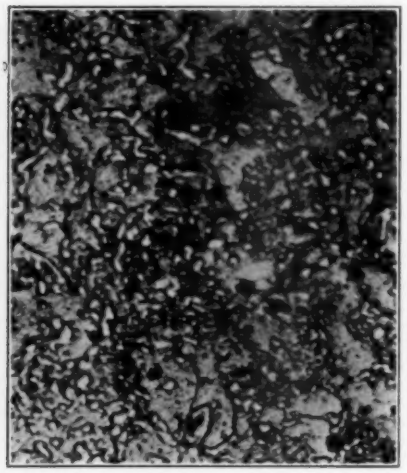
No. 11 Chromium-Tungsten Steel
(Steel R—Quenched 1800°F., drawn 1200°F. Nitric acid etch. X 100 D.)



No. 12 Chromium-Tungsten Steel
(Steel R—Quenched 1800°F., drawn 1200°F Nitric acid etch. X 100 D.)



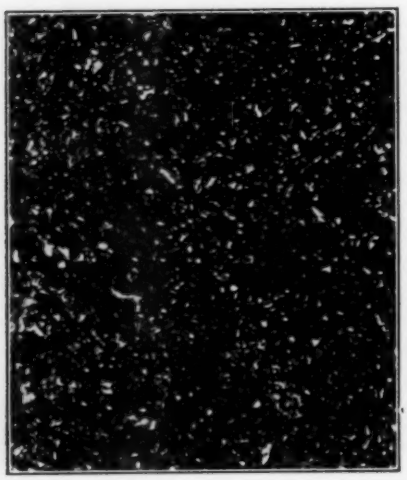
No. 13 Chromium-Tungsten Steel
(Steel R—Quenched 1650°F., drawn 1200°F. Nitric acid etch. X 100 D.)



No. 14 Chromium-Tungsten Steel
(Steel R—Quenched 1400°F., drawn 1200°F. Nitric acid etch. X 1000 D.)



No. 15 Chromium-Tungsten Steel
(Steel R—Quenched 1500°F., drawn 1200°F. Nitric acid etch. X 1000 D.)



No. 16 Chromium-Tungsten Steel
(Steel R—Quenched 1600°F., drawn 1200°F. Nitric acid etch. X 1000 D.)



No. 17 Chromium-Tungsten Steel
(Steel R—Quenched 1700°F., drawn 1200°F. Nitric acid etch. X 1000 D.



No. 18 Chromium-Tungsten Steel
(Steel R—Quenched 1800°F., drawn 1200°F. Nitric acid etch. X 1000 D.)

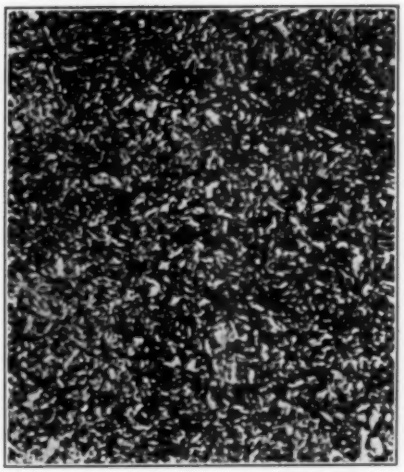
Photomicrographs of Alloy Steels



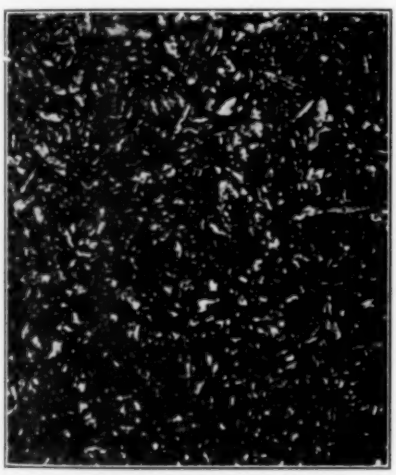
No. 19 Chromium-Tungsten Steel
(Steel R—Normalized 1750°F., drawn 1200°F. Nitric acid etch. X 1000 D.)



No. 20 Chromium Tungsten Steel
(Steel R—Annealed 1650°F., Nitric acid etch. X 1000 D.)



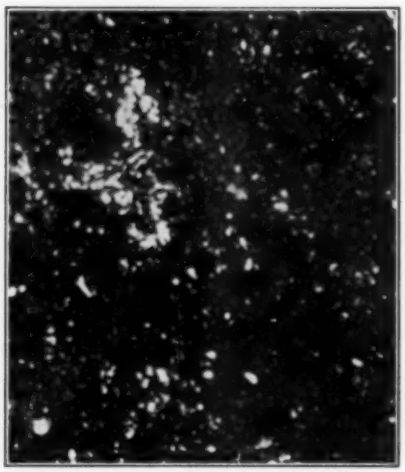
No. 21 Chromium-Tungsten Steel
(Steel R—Oil-quenched 1650°F., drawn 1000°F. Nitric acid etch. X 1000 D.)



No. 22 Chromium-Tungsten Steel
(Steel R—Oil-quenched 1650°F., drawn 1200°F. Nitric acid etch. X 1000 D.)



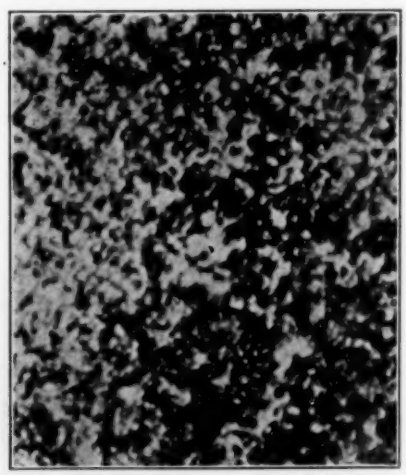
No. 23 Chromium-Tungsten Steel
(Steel Ra—Quenched 1575°F., drawn 1250°F. Nitric acid etch. X 1000 D.)



No. 24 Chromium-Tungsten Steel
(Steel V—Quenched 1575°F., drawn 1250°F. Nitric acid etch. X 1000 D.)



No. 25 Chromium-Tungsten Steel
(Steel Ra—Quenched 1575°F., drawn 1250°F. Sodium picrate etch. X 1500 D.)



No. 26 Chromium-Tungsten Steel
(Steel V—Quenched 1575°F., drawn 1250°F. Sodium picrate etch. X 1500 D.)

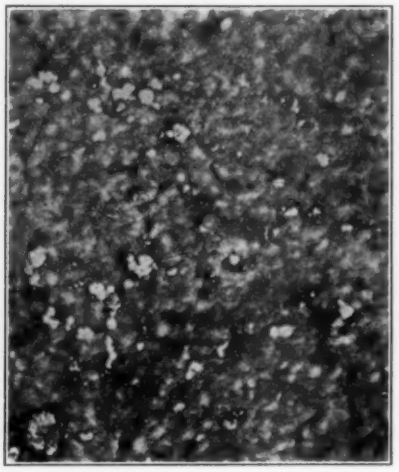


No. 27 Chromium-Tungsten Steel
(Steel Ra—Normalized 1575°F., drawn 1250°F. Nitric acid etch. X 1000 D.)

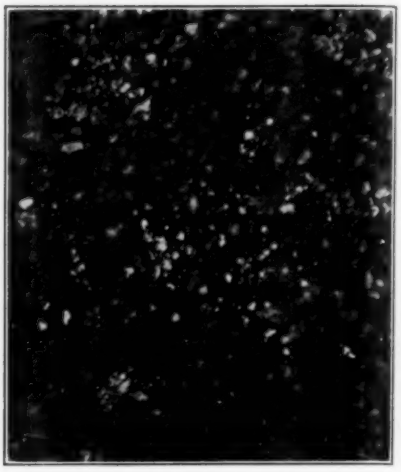
Photomicrographs of Alloy Steels



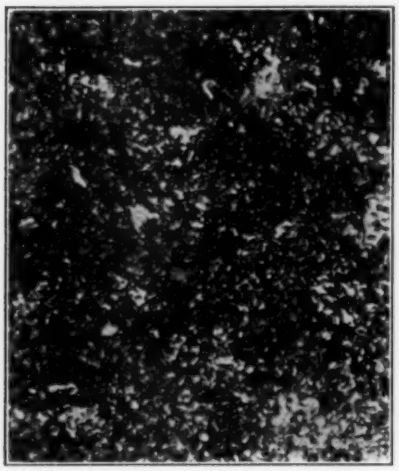
No. 28 Chromium-Tungsten Steel
(Steel V—Normalized 1575°F. drawn 1250°F. Nitric acid etch. X 1000 D.)



No. 29 Chromium-Tungsten Steel
(Steel P—Quenched 1500°F., drawn 1200°F. Nitric acid etch. X 1000 D.)



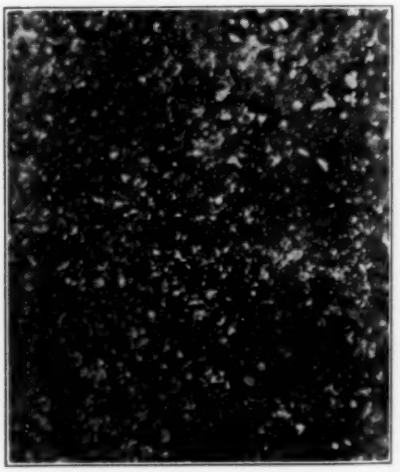
No. 30 Chromium-Tungsten Steel
(Steel P—Quenched 1500°F., drawn 1200°F. Nitric acid etch. X 1000 D.)



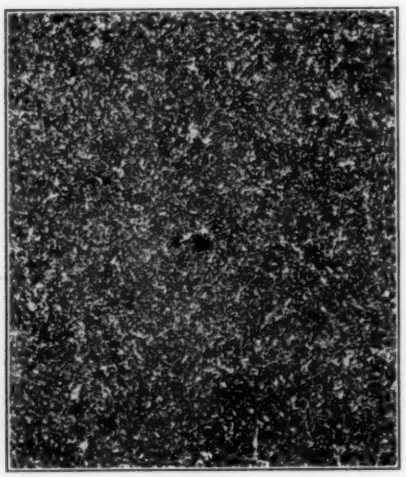
No. 31 Chromium-Tungsten Steel
(Steel P—Quenched 1700°F., drawn 1200°F. Nitric acid etch. X 1000 D.)



No. 32 Chromium-Tungsten Steel
(Steel P—Normalized 1700°F., drawn 1200°F. Nitric acid etch. X 1000 D.)



No. 33 Chromium-Tungsten Steel
(Steel P—Quenched 2250°F., drawn 1200°F. Nitric acid etch. X 1000 D.)



No. 34 Carbon-Manganese Steel
(Steel G—Quenched 1450°F., drawn 1200°F. Nitric acid etch. X 100 D.)

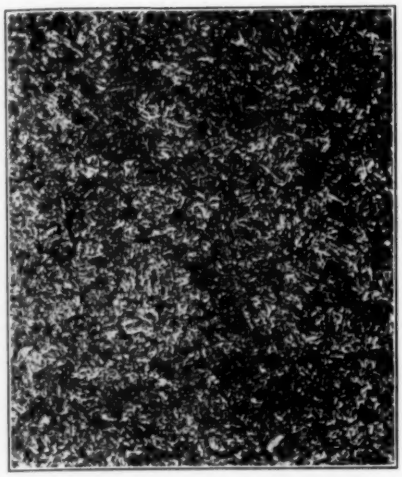


No. 35 Carbon-Manganese Steel
(Steel G—Quenched 1550°F., drawn 1200°F. Nitric acid etch. X 100 D.)

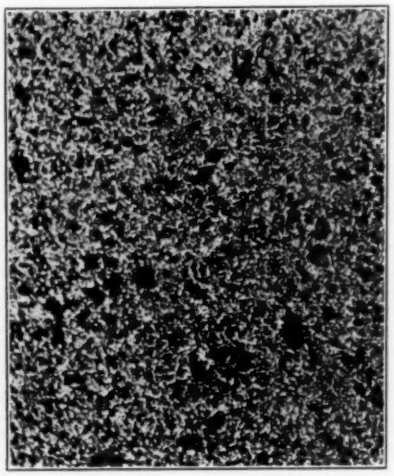


No. 36 Carbon-Manganese Steel
(Steel G—Quenched 1650°F., drawn 1200°F. Nitric acid etch. X 100 D.)

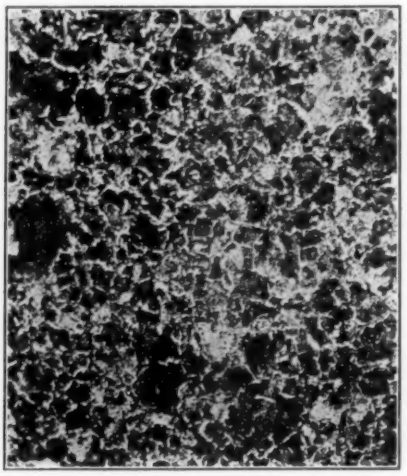
Photomicrographs of Alloy Steels



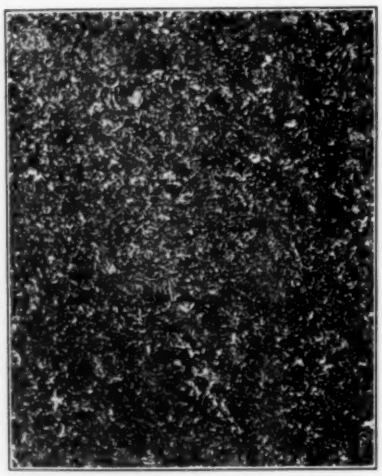
No. 37 Carbon-Manganese Steel
(Steel G—Quenched 1750°F., drawn 1200°F. Nitric acid etch. X 100 D.)



No. 38 Carbon-Manganese Steel
(Steel G—Normalized 1550°F., drawn 1200°F. Nitric acid etch. X 100 D.)



No. 39 Carbon-Manganese Steel
(Steel G—Normalized 1750°F., drawn 1200°F. Nitric acid etch. X 100 D.)



No. 40 Carbon-Manganese Steel
(Steel G—Quenched 1800°F., Quenched 1600°F. drawn 1200°F. X 100 D.)



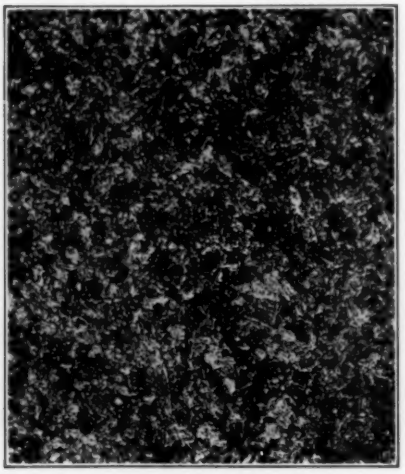
No. 41 Manganese-Chromium Steel
(Steel T—Quenched 1400°F., drawn 1200°F. Nitric acid etch. X 100 D.)



No. 42 Manganese-Chromium Steel
(Steel T—Quenched 1500°F., drawn 1200°F. Nitric acid etch. X 100 D.)



No. 43 Manganese-Chromium Steel
(Steel T—Normalized 1550°F., drawn 1200°F. Nitric acid etch. X 100 D.)

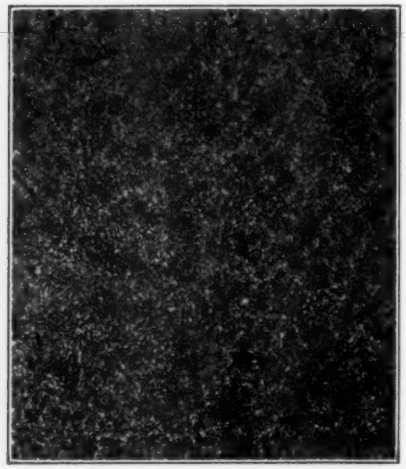


No. 44 Manganese-Chromium Steel
(Steel T—Normalized 1750°F., drawn 1200°F. Nitric acid etch. X 100 D.)



No. 45 Manganese-Chromium Steel
(Steel T—Annealed 1700°F. Nitric acid etch. X 100 D.)

Photomicrographs of Alloy Steels



No. 46 Chrome-Aluminum Steel
(Steel S—Oil-quenched 1450°F., drawn 1200°F. Nitric acid etch. X 100 D.)



No. 47 Chrome-Aluminum Steel
(Steel S—Oil-quenched 1550°F., drawn 1200°F. Nitric acid etch. X 100 D.)



No. 48 Chrome-Aluminum Steel
(Steel S—Oil-quenched 1650°F., drawn 1200°F. Nitric acid etch. X 100 D.)



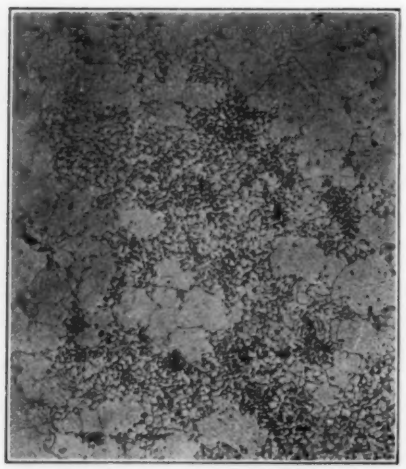
No. 49 Chrome-Aluminum Steel
(Steel S—Quenched 1450°F., drawn 1200°F. Nitric acid etch. X 100 D.)



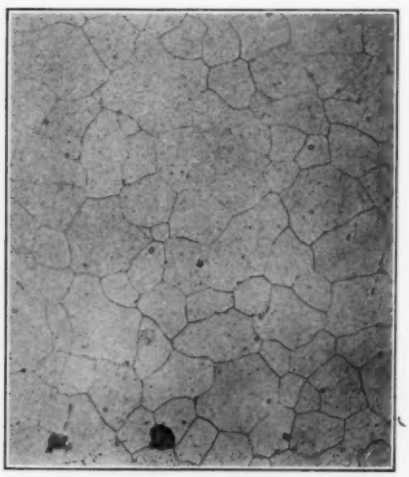
No. 50 Chrome-Aluminum Steel
(Steel S—Quenched 1550°F., drawn 1200°F. Nitric acid etch. X 100 D.)



No. 51 Chrome-Aluminum Steel
(Steel S—Quenched 1650°F., drawn 1200°F. Nitric acid etch. X 100 D.)



No. 52 Chrome-Aluminum Steel
(Steel S—Quenched 1700°F., drawn 1200°F. Nitric acid etch. X 100 D.)



No. 53 Chrome-Aluminum Steel
(Steel S—Normalized 1700°F., drawn 1200°F. Nitric acid etch. X 100 D.)



No. 54 Chrome-Aluminum Steel
(Steel S—Annealed 1700°F. Nitric acid etch. X 100 D.)

FSP-51-36

Influence of Coal Type on Radiation in Boiler Furnaces

By W. J. WOHLENBERG[1] and R. L. ANTHONY,[2] NEW HAVEN, CONN.

This paper includes the results of an application of the methods and data presented in a previous A.S.M.E. paper ("The Influence of Radiation in Coal-Fired Furnaces on Boiler-Surface Requirements, and a Simplified Method for Its Calculation," by W. J. Wohlenberg and E. L. Lindseth) to pulverized coal for a number of coals of widely varying composition. After an evaluation of radiation quantities for each of these coals, further investigation was made for the purpose of ascertaining what quantitatively measurable property of coals might be most significant in differentiating one from another during combustion with respect to the quantity of energy which is transferred as heat to the walls of a boiler furnace. It was found, of the properties investigated, that the calorific value serves as the most consistent and useful indicator of the radiation powers of a coal as burned in the pulverized form in boiler furnaces.

THE information contained in this paper is derived from the results of a large amount of painstaking calculation carried out by the men composing the graduate class in power engineering at Yale during the school year 1927–1928. At the outset, therefore, the authors wish to express their indebtedness to this group.[3]

The paper includes the results of an application of the methods and data presented by Wohlenberg and Lindseth[4] for pulverized coal to a number of coals of widely varying composition. After an evaluation of radiation quantities for each of these coals, further investigation was made for the purpose of ascertaining what quantitatively measurable property of coals might be most significant in differentiating one from another during combustion with respect to the quantity of energy which is transferred as heat to the walls of a boiler furnace. As will be shown later, it was found, of the properties investigated, that that one known as the calorific value serves as the most consistent and useful indicator of the radiation powers of a coal as burned in the pulverized form in boiler furnaces.

Haslam and Hottel include in their paper[5] a somewhat modified form of certain of the equations worked out by Wohlenberg and Morrow[6] on which the development of Wohlenberg and Lindseth is based. These modified formulas of Haslam and Hottel have a better mathematical basis than do those which they replaced in the Wohlenberg and Morrow analysis of this subject. Thus in the limits for exceedingly fine coal, the Haslam and Hottel formulas yield radiation magnitudes more nearly in accord with the facts than would be the case should the Wohlenberg-Morrow formulas be applied without correction to the same conditions. However, the inadequacy of the latter formulas for such conditions was recognized by the authors, so that the method of correction adopted resulted in radiation curves closely approaching the Haslam-Hottel curves.

Application of the two sets of equations to a particular set or sets of conditions leads to two sets of results which differ only slightly in magnitude. For instance, the computed radiation mean values of the flame temperature differ by only about 30 deg. for assumed conditions within the usual operating range.

In view of such small differences as these it appears that no material advantage could have been realized in this investigation by adopting the Haslam-Hottel system of nomenclature and constants. On the other hand, it will undoubtedly be advantageous to any who are familiar with the earlier papers[7] leading up to this subject to have the whole in harmony; i.e., to have a consistency throughout so that the relativity of things is not disturbed by a change in value of constants which in themselves lead to corrections so small as to be within the unavoidable error of necessary assumptions.

TABLE 1 PERCENTAGE ANALYSES OF COALS

	North Dakota lignite (4)	Washington sub-bituminous coal (5)	Colorado sub-bituminous coal (6)	Illinois bituminous coal (7)	Virginia semi-bituminous coal (8)	Kansas bituminous coal (9)	Pennsylvania bituminous coal (10)	Alabama bituminous coal (11)
Proximate Analysis								
Moisture	42.3	29.8	24.6	13.0	1.9	5.1	2.9	3.2
Volatile	25.0	32.9	29.8	37.0	14.0	34.5	33.8	31.0
Fixed carbon	28.7	29.5	42.0	39.3	68.9	52.2	53.1	59.6
Ash	3.9	7.8	3.6	10.7	15.2	8.3	10.2	6.2
Total	99.9	100.0	100.0	100.0	100.0	100.1	100.0	100.0
Ultimate Analysis								
Carbon	37.8	43.9	54.3	59.8	75.3	71.8	73.8	78.3
Hydrogen	7.3	6.6	6.3	5.6	3.6	5.2	5.3	5.3
Oxygen	50.0	40.2	33.8	19.1	4.8	10.2	8.1	7.6
Nitrogen	0.7	0.8	1.2	1.1	0.9	1.2	1.5	1.4
Sulphur	0.4	0.6	0.3	3.7	0.5	3.3	1.1	1.2
Ash	3.9	7.8	3.6	10.7	15.2	8.3	10.2	6.2
Total	100.1	99.9	99.5	100.0	100.3	100.0	100.0	100.0
Calorific value, B.t.u. per lb.	6,331	7,547	9,517	10,856	12,739	13,082	13,217	14,141

Individual Coal Types

Analyses of the coals investigated are shown in Table 1. The coals are numbered beginning with 4, because coals included in the Wohlenberg and Lindseth paper were designated by numbers 1, 2, and 3. It is to be noted that the eight coals included vary in composition from that of lignite to semi-bituminous coal, and the calorific value ranges from 6,300 to more than 14,000 B.t.u. per lb.

For each of these coals there are included flame and refractory isotherms similar to those given by Wohlenberg and Brooks,[8] and

[1] Professor of Mechanical Engineering, Sheffield Scientific School, Yale University. Mem. A.S.M.E.

[2] Laboratory Assistant, Yale University. Jun. A.S.M.E.

[3] R. L. Anthony, Yale, 1925; H. H. Dedo, University of Kansas, 1927; R. B. Dodds, Georgia School of Technology, 1925; E. D. Grimison, Lafayette, 1927; C. S. Hammer, University of Colorado, 1927; H. C. Longfellow, New York University, 1927; G. P. Vye, University of Minnesota, 1927; and T. H. Smith, Case, 1927.

[4] Trans. A.S.M.E., vol. 48 (1926), p. 849.

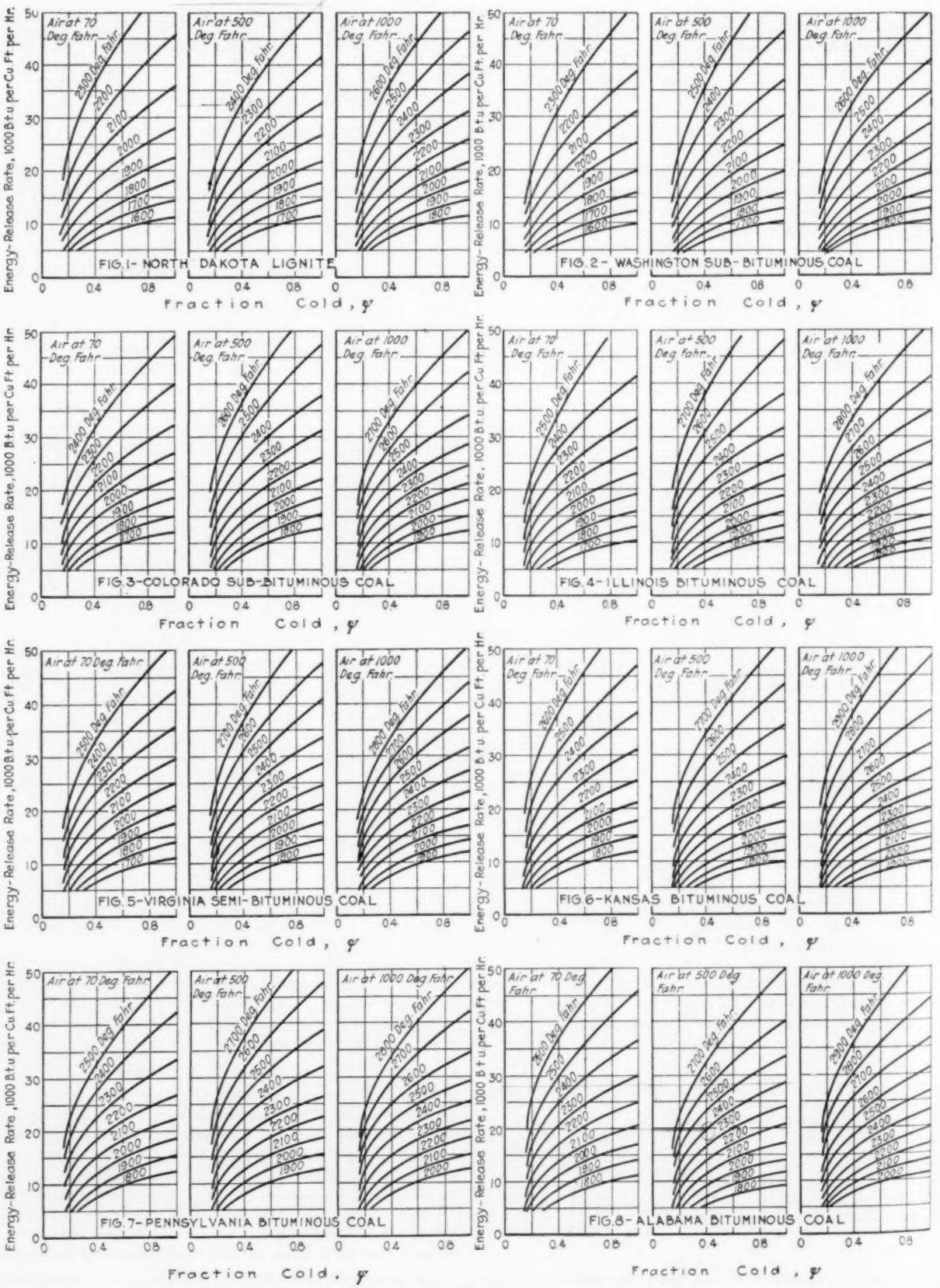
[5] "Combustion and Heat Transfer," Trans. A.S.M.E., vol. 50 (1928), no. 8, paper FSP-50-3.

[6] Trans. A.S.M.E., vol. 47 (1925), p. 127.

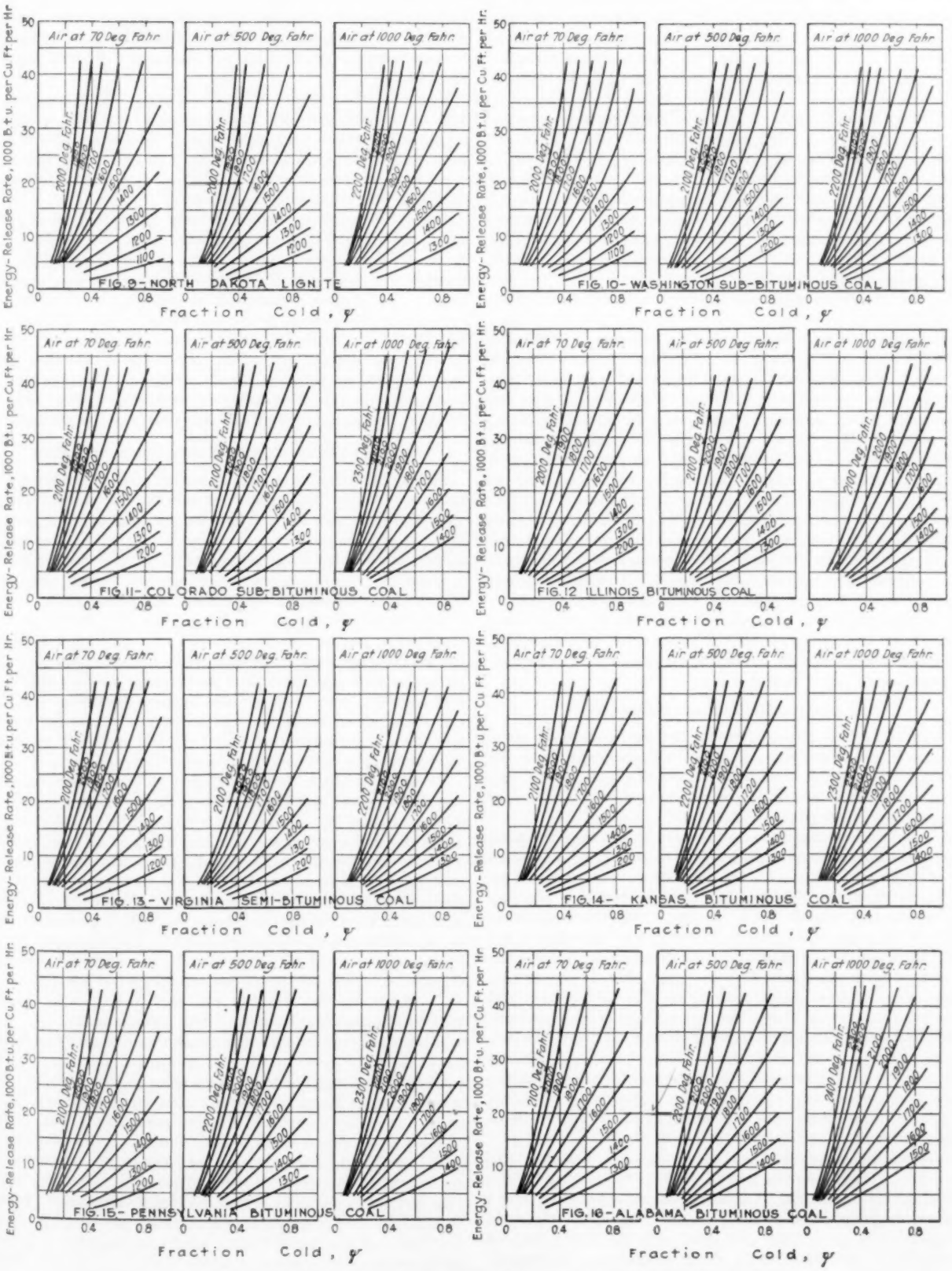
Contributed by the Power Division and presented at the Annual Meeting, New York, December 3 to 7, 1928, of The American Society of Mechanical Engineers.

[7] Wohlenberg-Morrow, Wohlenberg-Lindseth, Wohlenberg-Brooks.

[8] Wohlenberg and Brooks, Trans. A.S.M.E., vol. 50 (1928), no. 15, paper FSP-50-39.



FIGS. 1 TO 8 FURNACE-FLAME ISOTHERMS FOR VARIOUS COALS
(20 per cent excess air.)



FIGS. 9 TO 16 FURNACE-REFRACTORY ISOTHERMS FOR VARIOUS COALS
(20 per cent excess air.)

also heat-transfer curves showing the rate of heat transfer per square foot of projected cold, bare furnace-wall surface. It is to be noted that in all cases the air excess has been taken at 20 per cent and the furnace cavity as a cube 20 × 20 × 20 ft. containing a flame 19 × 19 × 19 ft. This is furnace No. 2 of Wohlenberg and Lindseth. The initial mean particle diameter entering the burner is taken as equal to 0.002 in. for all coals, which according to the analysis by Wohlenberg and Morrow is about that for 75 per cent through a 200-mesh screen. It should be noted in particular that the energy-release figures are all based on the volume of the flame and not that of the furnace. The total energy released is thus energy-release rate × 6859 (the assumed flame volume), and not energy release × 8000 (the furnace volume).

Results are shown for furnaces of several radiation symmetries. In all cases Furnace A, as in the former papers,[9] denotes a cube, the top face of which is covered with bare-iron water-cooled surface. Furnace B has both the top and bottom faces occupied by bare water-cooled surfaces, and Furnace E has all faces occupied by bare water-cooled surfaces. Walls not covered by bare water-cooled surfaces present uncooled refractory faces. The term "fraction cold ψ" denotes the fractional part of the furnace envelope which is occupied by bare-iron water-cooled surface. Thus for Furnace A the fraction cold ψ is equal to one-sixth, for Furnace B to one-third, and for Furnace E to unity.

As here described, the furnace envelope is comprised of bare water-cooled iron faces and more or less solid refractory faces. Water-cooled refractory faces should of course also be considered in a more complete analysis of the problem. An analysis including such furnace walls is under way. For the present, however, let it suffice to say that whatever influences are due to bare-iron water-cooled surfaces would also be present, but to a somewhat less degree for a refractory-covered water-cooled wall, other conditions being the same in both cases.

Flame and Refractory Isotherms

Flame and refractory isotherms for the several coals are given in Figs. 1 to 16, inclusive. Figs. 1 to 8, inclusive, have the radiation mean of the flame temperature as a constant on each curve, and in Figs. 9 to 16 the radiation mean of the refractory temperatures is a constant on each curve.

It is to be noted that the same kinds of curves have the same general shape regardless of fuel type, but that they are shifted up or down with respect to the axes, depending on fuel type. As an example, at 500 deg. fahr. air temperature Alabama bituminous coal, Fig. 8 (B.t.u. per lb. = 14,141), requires an energy-release rate of very nearly 20,000 for fraction cold $\psi = 0.4$ when the mean flame temperature is 2400 deg. fahr. Under the same conditions the North Dakota lignite, Fig. 1, requires an energy-release rate of 33,000. If the fraction cold is unity and other conditions are as above, the required rate of energy release is 28,500 for the Alabama coal and 52,000 for the North Dakota lignite. Obviously the combustion of the *lower-grade* coals is accompanied by a higher rate of energy release for the same mean flame temperature, and vice versa, for a given rate of energy release the lower-grade coals are accompanied by a lower flame temperature, all other conditions being the same.

Such conditions should be reflected in the mean refractory temperatures. Thus, referring to Fig. 16, it is found that for the Alabama coal at a 20,000-B.t.u. energy-release rate with $\psi = 0.4$ and an air temperature of 500 deg. fahr., the mean refractory temperature is about 1930 deg. fahr. For the North Dakota lignite, Fig. 9, at a 33,000-B.t.u. energy-release rate, corresponding to a 2400-deg. fahr. flame temperature, the mean refractory temperature is just above 1900 deg. fahr. For a 20,000-B.t.u. energy-release rate with the North Dakota lignite and $\psi = 0.4$ with the air temperature at 500 deg. fahr., the mean refractory temperature is about 1520 deg., as compared with 1930 deg. for the Alabama coal.

Neglecting the influence of impurities in the ash, it thus appears that the lower-grade coals may in general be burned at higher energy release than may the higher-B.t.u. coals. This is fortunate when it is considered that often the low-grade coals contain ash which is composed of low-fusing constituents and which may also otherwise attack the refractory.

Obviously the data included in these curves may be of some use in setting out by itself, as shown above, the expected influence of the radiation factors of the coal on the possible permissible rate at which the combustion process may be maintained. In all cases, however, it is essential that other factors such as slagging, erosion, proper conditions for good combustion, etc., be given thorough consideration. In this study these factors have been ignored, which simply means that they have been assumed as constant for all coals, and this assumption is made for the sake of isolating the radiation factor so that its variation in magnitude with different coals may be ascertained.

Heat Transfer at the Water-Cooled Surface

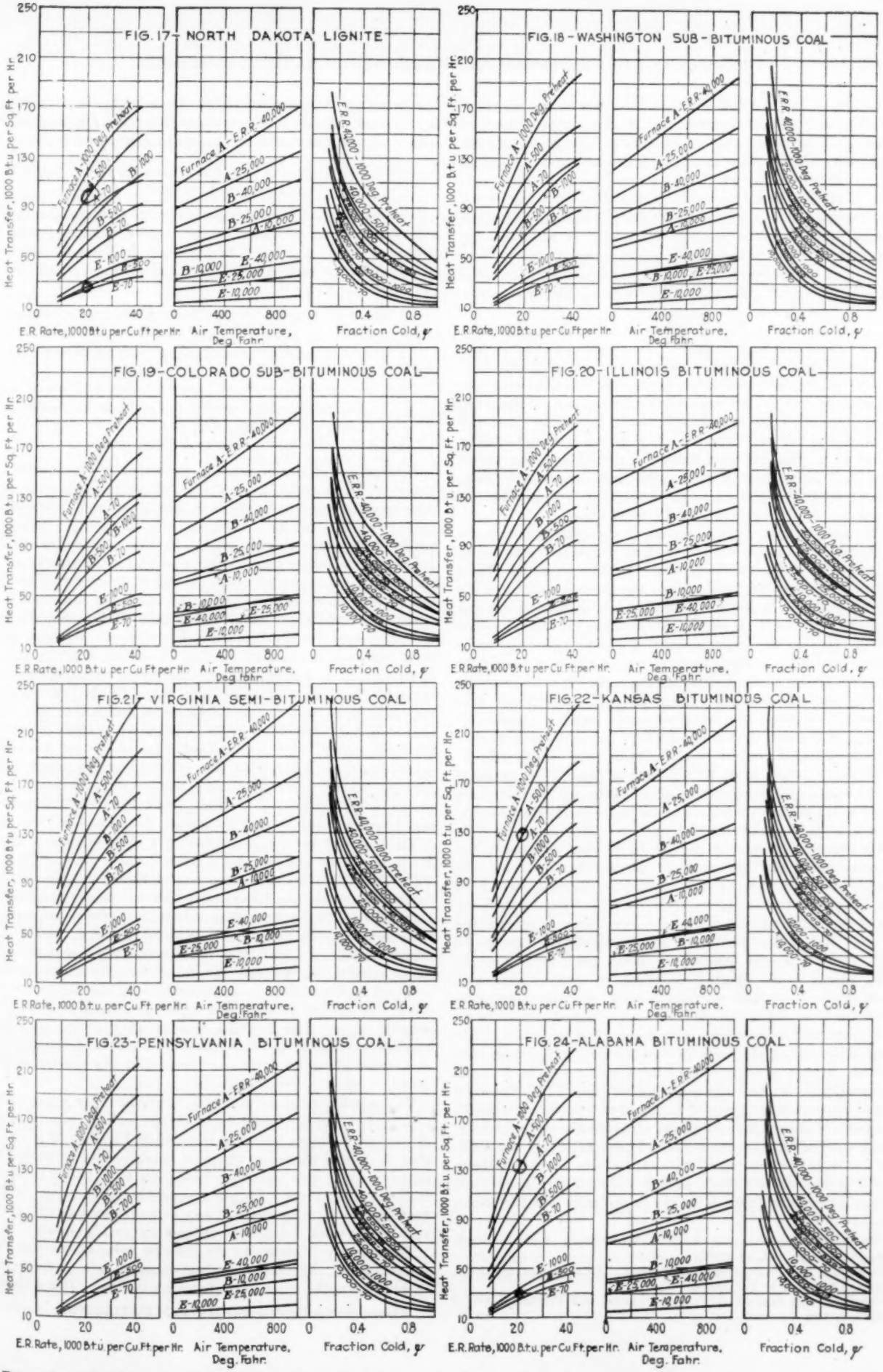
Figs. 17 to 24, inclusive, contain curves plotted in three different ways to show the variation in rate of heat transfer at the water-cooled furnace wall. The ordinates in all figures represent the rate of heat transfer per square foot of bare water-cooled surface as projected into the plane of the furnace wall at which it is located. The abscissas, taken from left to right, represent rate of energy release, air temperature, and fraction cold.

The groups of curves on the left-hand side of each figure show a steep rise in rate of heat transfer with rate of energy release for any given furnace, air temperature, etc. The groups of curves in the center show that the rates of heat transfer vary practically in a straight-line ratio with increase in air temperature, and the right-hand group shows how the heat-transfer rate per square foot of surface decreases as more bare cold surface is placed in the furnace, other conditions remaining the same.

To illustrate the influence of coal type, let us again consider the Alabama coal and North Dakota lignite shown respectively in Figs. 21 and 17. Taking first Furnace A ($\psi = 0.167$) with a 20,000-B.t.u. energy-release rate and 500 deg. air temperature, we find that for the Alabama coal the heat-transfer rate as above defined is equal to 132,000. For the North Dakota lignite, under the same conditions otherwise, the heat-transfer rate is 97,000. Thus for the lower-grade coals the heat-transfer rate is, as seen, considerably lower, and this persists regardless of furnace symmetry if for both coals the same furnaces, etc. are considered. Thus for the above conditions but for Furnace E ($\psi = 1.00$), the heat-transfer rate with the Alabama coal is 29,000 as against 25,000 for the North Dakota lignite.

It is to be noted that the above differences in rates of heat transfer to the water-cooled walls exist in spite of the fact that the total rate of energy liberation is the same in both cases for the same excess-air figure and for the same air temperature. Examination of the analyses of the two coals shows that for the North Dakota lignite the ratio of fixed carbon to volatile matter is 28.7 to 25, and for the Alabama coal, 59.6 to 31. The moisture contents are respectively 42.3 and 3.2 per cent. This high moisture content is partially responsible for a considerably higher thermal capacity in products of combustion per 10,000 B.t.u. of heat value of the North Dakota lignite as compared with that of the products of combustion of the Alabama coal on the same basis. Consequently a larger fraction of the liberated energy is absorbed by the products of combustion of the North Dakota lignite, which in itself leaves a smaller fraction to be transferred

[9] Wohlenberg and Morrow, Wohlenberg and Lindseth.



FIGS. 17 TO 24 HEAT TRANSFER AT COLD SURFACE IN FURNACE, B.T.U. PER SQ. FT. OF PROJECTED COLD SURFACE PER HOUR
(Plotted respectively against energy-release rate, temperature of air supply, and fraction cold. Excess air, 20 per cent.)

to the furnace walls. This accounts for a part of the difference in the two rates of heat transfer. Another part of the difference is accounted for by the above differences in ratio of fixed carbon to volatile matter. Since this ratio is greater for the Alabama coal, it follows that by weight the solids, or coked carbon particles, in suspension within the furnace cavity, will be larger for the Alabama coal than for the North Dakota lignite, resulting in a larger extent of carbon radiating surfaces for the Alabama coal. Thus from this source more heat is transferred to the walls in the case of the Alabama coal than is so transferred from the North Dakota lignite, and the total difference in the two cases is, as shown, of a considerable magnitude.

In computing the radiating surface of the solids in suspension the coefficient of volume change[10] during volatile distillation has been based on the same mean coke density for all of the coals and the validity of the above argument is based, to some extent at least, on the correctness of this assumption. Should the lignite particle swell considerably more in coking than the Alabama-coal particle, then, relatively speaking, more radiating surface from solid particles might be available in the lignite flame than it has been credited with, and consequently more heat would be transferred to the walls than is shown by the curves.

The rate of swelling of the particle in the ignition zone is dependent on the volatile content per unit weight of fixed carbon, the extent to which sticky or tarlike substances are formed as the volatile passes from the coal to the gaseous state, and the rate of heating. It appears that the rate of heating for the lignite particle must be lower than it is for the Alabama-coal particle, because the larger amount of moisture for the former will hold down the particle temperature as it is introduced into a furnace cavity in which, in turn, the radiation to the particle is itself less because of lower flame and wall temperatures, and probably because of less radiation from a smaller quantity of solids in suspension than would be so if the same furnace were fired by Alabama coal to yield the same total energy release. Consequently there must be considerably more time available for the volatile constituents to be dispelled through the interstices of the lignite particle than there would be for the discharge of the volatile from the Alabama-coal particle. This increased time factor probably operates to reduce the internal pressure in the particle during the gas-discharge period, thus reducing the tendency toward blowing up. Obviously the final answer to this question in quantitative form may be obtained only by actual experimental work of a most involved and painstaking nature. In a paper[11] before the International Coal Conference held at the Carnegie Institute of Technology, Pittsburgh, in 1926, Alfred White reported the results of some experimental work on the instantaneous carbonization of crushed and pulverized coal in which fine particles were dropped through a retort containing a reducing atmosphere. The tendency to abnormal swelling of the particles was observed under these conditions. It was noted, however, that the "cenospheres," as the abnormally swelled particles are called by Newall and Sinnatt,[12] had an outer shell composed of more or less transparent window-like patches of clarain tied together with ridges of carbon or coke. These window-like patches no doubt have a relatively low coefficient of radiation, so that in spite of the large increase in diameter it is doubtful if a single particle will emit very much more energy by radiation than it would in the more compact state. Furthermore the proportion of the clarain matter is very small in any coal, whence it appears that the foregoing method of evaluating the radiation from solids is reasonably correct.

[10] See "Coking-Diameter Ratio," Wohlenberg and Lindseth, Trans. A.S.M.E., vol. 48 (1926), p. 922.

[11] White: See Proceedings, International Coal Conference 1926, Carnegie Institute of Technology.

[12] Newall and Sinnatt: "Fuel in Science and Practice," vol. 3 (1924), p. 424.

Relation Between the Calorific Value of a Coal and Its Radiating Powers

In attempting to find a determining factor which, within reasonable limits of error, relates a coal to its radiating powers, George Vye, in a thesis,[13] investigated the influence of the following conditions on the radiating powers of a coal as fired in the pulverized form:

1 The total moisture content in the products of combustion yielded by a pound of coal
2 The total moisture content in the products of combustion yielded per 10,000 B.t.u. of heating value
3 The thermal capacity between 70 deg. and some high temperature of the products of combustion yielded per 10,000 B.t.u. of heating value
4 The calorific value of the coal, B.t.u. per lb.

Without going into a detailed discussion of the results at this time, let it suffice to state that, of the above factors, the calorific value was found to be most satisfactory as the determining factor. In Fig. 25 it is shown how the actual computed values of the flame temperatures fall for the conditions specified. It is seen that curves may be drawn so that with one exception the plotted values for all coals fall either on or very close to the curves. With this exception the maximum difference between curve and point values is less than 40 deg., but for the exception (coal No. 8, points shown in circles) the difference is in some instances as much as 100 deg. Coal No. 8 is very much like Coal No. 1, points for which are shown as crosses on the curve for the Type E furnace. The values for the latter points are taken from Wohlenberg and Lindseth, and it is to be noted that they fall much closer to the curve than do those in circles for Coal No. 8. It appears, then, that there may be some doubt about the accuracy of the computed temperature results for Coal No. 8. It is to be noted also that values for Coal No. 2, shown by crosses for type E furnace and taken from Wohlenberg and Lindseth, fall very near the curve.

The curves show that a coal having a calorific value of 9000 B.t.u. as against one that has a calorific value of 14,000 B.t.u. will result, for an energy-release rate of 10,000, in a flame temperature about 100 deg. lower for the same conditions otherwise in both cases. If the energy-release rate is 25,000 the corresponding reduction in flame temperature is about 150 deg., and if the energy-release rate is 40,000 the corresponding reduction in flame temperature is nearly 200 deg. Furnace symmetry has relatively a small effect on the magnitude of the differential in flame temperature caused by a variation in magnitude of the calorific value. This becomes obvious when noting that the curves for the A, B, and E furnaces are nearly equidistant from each other along the ordinates throughout the abscissa range shown.

The relation between rate of heat transfer per square foot of projected bare-iron water-cooled surface and the calorific value of the coal is shown for given furnaces in Fig. 26. It is noted that the points plotted to represent the calculated values fall so that in no case does the point value differ more than 6 per cent from the curve value. Thus it appears that within these limits, heat-transfer rates for any coal may be ascertained directly from such curves when the calorific value of the coal is known.

For a given difference in the calorific value of the coal the corresponding differential in heat-transfer rate increases as there is less cold surface in the furnace envelope. This is shown

[13] Presented for the degree of M.S. in M.E., Yale, 1928.

by the greater slope of the curves for types A and B furnaces when compared with the curve for the type E furnace. It is noted that this differential in heat-transfer rate increases also for increasing energy-release rates. Thus for an energy-release rate of 25,000 a shift from 9000 to 14,000 in the calorific value changes the heat-transfer rate from 28,000 to 30,000 for the type E furnace, whereas for the type A furnace the rates increase from 69,000 to 81,000 B.t.u. per sq. ft. per hour. If the energy-release rate is increased to 40,000 the heat-transfer rates increase respectively from 44,000 to 48,000 and from 161,000 to 193,000

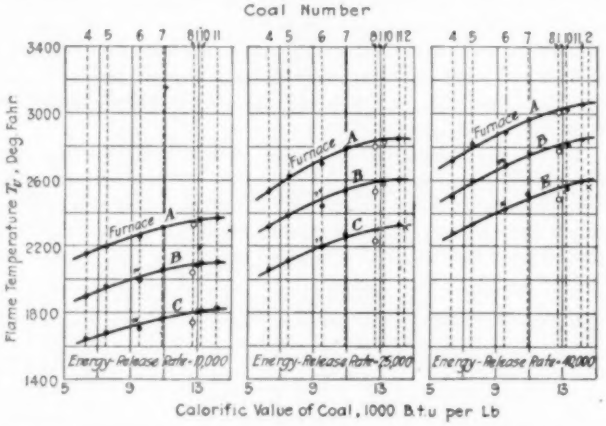


FIG. 25 INFLUENCE OF CALORIFIC VALUE OF COAL ON MEAN FLAME TEMPERATURE
(Air excess, 20 per cent; air temperature, 500 deg. fahr.)

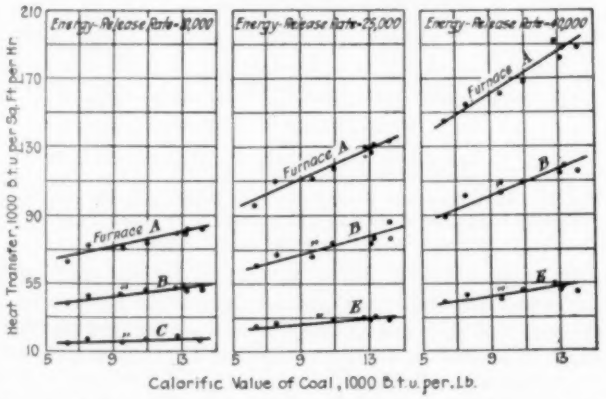


FIG. 26 INFLUENCE OF CALORIFIC VALUE OF COAL ON HEAT-TRANSFER RATE AT BARE COLD SURFACE IN FURNACE WALLS
(Air excess, 20 per cent; air temperature, 500 deg. fahr.)

B.t.u. per sq. ft. per hour for a shift in calorific value from 9000 to 14,000.

ISOTHERMAL AND ISO-TRANSFER CURVES

Curves in Figs. 27, 28, and 29 show the relation between the calorific value of a coal and the required energy-release rate for a given mean flame temperature. The curves are flame isotherms for given furnace conditions when the coal is a variable. Similarly, the curves in Figs. 30, 31, and 32 are "iso-transfer" curves for given furnace conditions with the coal as a variable.

It is seen that the flame isotherms have more curvature toward the right at the lower ends than the iso-transfer curves have. On first thought it might appear that the iso-transfer curves should follow very closely the trend in curvature of the above flame isotherms. The relatively sharp curvature of the flame isotherms at the lower ends is largely accounted for by the increase in thermal capacity, per 10,000 B.t.u. of heat liberated from the coal, for coals of lower and lower calorific values. Thus for a given flame temperature a larger fraction of the liberated energy is retained in the gases when low-heating-value coals are

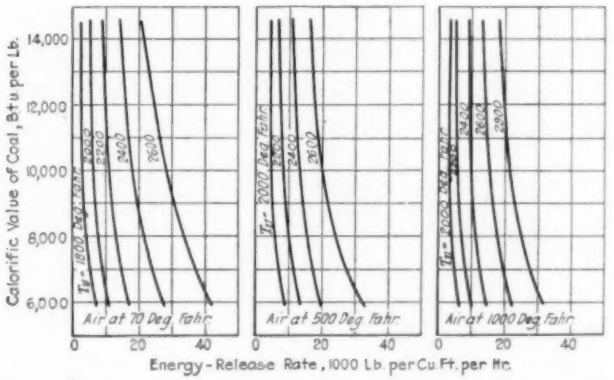


FIG. 27 INFLUENCE OF CALORIFIC VALUE OF COAL ON PERMISSIBLE ENERGY-RELEASE RATE FOR A GIVEN FLAME TEMPERATURE
(Furnace A, 20 per cent excess air.)

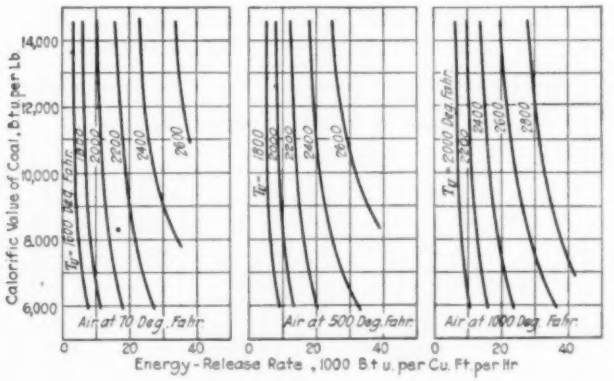


FIG. 28 INFLUENCE OF CALORIFIC VALUE OF COAL ON PERMISSIBLE ENERGY-RELEASE RATE FOR A GIVEN FLAME TEMPERATURE
(Furnace B, 20 per cent excess air.)

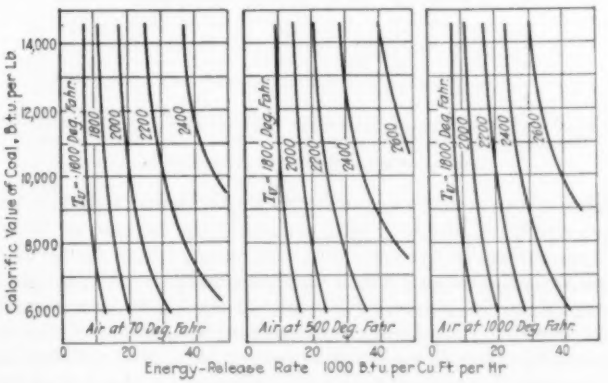


FIG. 29 INFLUENCE OF CALORIFIC VALUE OF COAL ON PERMISSILBE ENERGY-RELEASE RATE FOR A GIVEN FLAME TEMPERATURE
(Furnace E, 20 per cent excess air.)

burned than when high-heating-value coals are burned. It follows that for the lower-grade coals a larger amount of energy must be released to maintain a given furnace temperature.

At a given flame temperature, any increase in the radiation discharge from the flame of the lower-grade as compared with

that of the higher-grade coals will also be a factor in determining the trend of the curves toward the right. The trend of the flame isotherms is thus in general the result of two factors. The relative magnitude of these may be shown by an example. Referring to the left-hand curve groups in Figs. 27 and 30, it is seen, first from Fig. 27, that for a 2600-deg. flame temperature, a 10,000-B.t.u. coal will require an energy-release rate of 28,000. For a 6000-B.t.u. coal this must be increased to 42,000. The iso-transfer curves in Fig. 30 show that for these conditions the

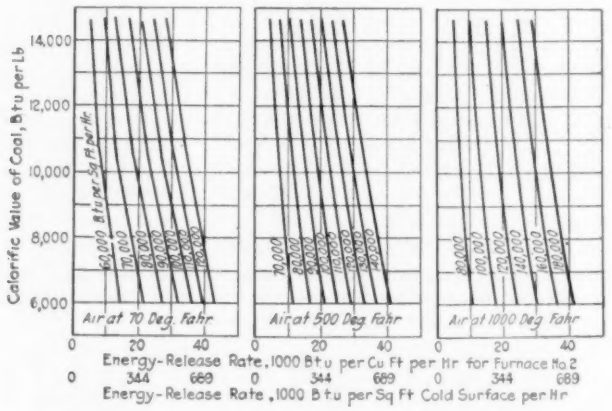


FIG. 30 INFLUENCE OF CALORIFIC VALUE OF COAL ON REQUIRED ENERGY-RELEASE RATE FOR A GIVEN RATE OF HEAT TRANSFER
(Furnace No. 2, Type A; 20 per cent excess air.)

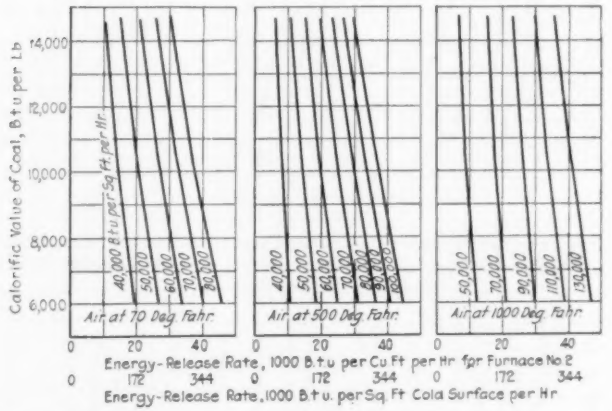


FIG. 31 INFLUENCE OF CALORIFIC VALUE OF COAL ON REQUIRED ENERGY-RELEASE RATE FOR A GIVEN RATE OF HEAT TRANSFER
(Furnace No. 2, Type B; 20 per cent excess air.)

rates of heat transfer are respectively 102,000 and 118,000 B.t.u. per sq. ft. of projected cold surface. This increase of 16,000 B.t.u. per sq. ft. per hour in the heat-transfer rate for the low-grade coal as compared with the high-grade coal, when the same mean flame temperature exists in both cases, is to be accounted for largely by the increased amount of radiation from the increased amount of water vapor, etc., in the products of combustion of the lower-grade coal. This difference, however, does not account for the total difference in energy quantities released in the furnace, as is obvious by comparing the two required energy-release rates: i.e., 28,000 as against 42,000. The remaining, and by far larger, part of the difference in total energy quantities released is thus absorbed in supplying heat to compensate for the differences in thermal capacities of the products of combustion of the lower-grade coal as compared with those of the higher-grade coal.

It may be said of the curves shown in Figs. 27 to 32 that they graphically portray the variation in radiation intensities to be expected in burning coals with respect to variation of calorific value of the coals, provided the actual composition of the range of coals under investigation does not depart too far from those considered here. It is apparent from an examination of Table 1 that the moisture and oxygen contents have been one of the chief variable factors. Should these constituents have remained very small in amount throughout the range and the B.t.u. content decreased almost wholly by an increase in the ash content of the coal, then unquestionably the trend of these curves would be somewhat different in the low-B.t.u. range. For instance, a 6000-B.t.u. coal might conceivably be composed of 0.41 lb. pure carbon and 0.59 lb. ash, in which case the products of combustion would be quite different from what they are for the 6000-B.t.u. coal here considered.

However, the composition of coal does not vary in this way. David White[14] made a study of more than 300 coals mined in the United States and has plotted several curves. One of his curves slopes from a high oxygen content and a low B.t.u. value to a low oxygen content and a high B.t.u. value. All points are

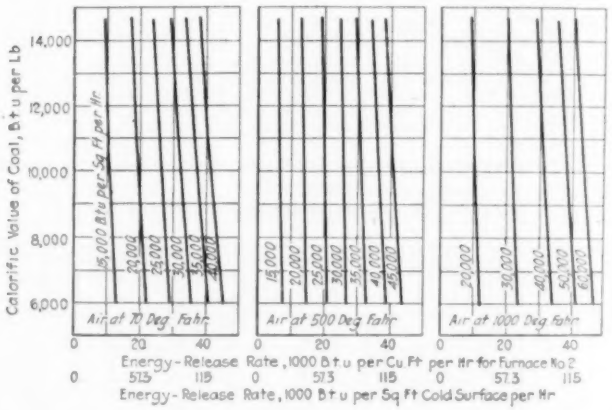


FIG. 32 INFLUENCE OF CALORIFIC VALUE OF COAL ON REQUIRED ENERGY-RELEASE RATE FOR A GIVEN RATE OF HEAT TRANSFER
(Furnace No. 2, Type E; 20 per cent excess air.)

within 2 per cent of falling on the curve. Another curve shows the carbon over the oxygen plus ash ratio to the B.t.u. value of the coal, and all points again fall within a 2 per cent accuracy. Again in general the moisture content of a coal bears a smooth-curve relation to the B.t.u. value of the coal. Thus it appears that the calorific value of a coal may be taken as a reasonably reliable determining factor for indicating what radiation intensities may be expected during combustion if, in evaluating these intensities throughout a range, representative coals in the several grades ranked according to their B.t.u. values are used as a basis for the calculations.

This concludes the authors' discussion of this subject. There have been no arguments for or against particular furnace arrangements or combustion methods. It is hoped that the information presented in curve form will be of some use to the engineer and may possibly aid him in determining what is and what is not a proper proportion. The curves are, it is thought, all fully titled, so that the uses to which they may be put are obvious. The most important information included appears in the form of the relativity of things. The absolute value of any result taken from the curves may be relied on only in so far as the conditions on which the analysis is based are realized in practice. The combustion reaction is a succession of energy transformations

[14] U. S. Bureau of Mines Bulletin No. 29 (1911).

which change so rapidly that they defy any attempt to follow its progress in minute detail, even though every change has its compensating reaction somewhere in the system. The theorist may only set up what he thinks from his knowledge and experience are the means or averages of the process, and, with this as a basis, work for average results. This is exactly what these data are, and no more.

Appendix No. 1

RELATION BETWEEN HEAT-TRANSFER RATE PER SQUARE FOOT OF SURFACE AND RADIATION ENERGY EQUATIONS

In Wohlenberg and Lindseth's paper the energy equation is given in the following form:[15]

$$dQ_U/dt + dQ_A/dt = dQ_B/dt + dQ_F/dt$$

Also

$$dQ_U/dt + dQ_A/dt = G_U U + G_A \Delta h_A$$

in which

$dQ_U/dt = G_U U$ = lb. of fuel per hr. $G_U \times$ B.t.u. per lb. fuel

$dQ_A/dt = G_A \Delta h_A$ = B.t.u. per hr. input above atmospheric temperature in G_A lb. air per hr. entering at temperature T_A

$dQ_B/dt = \Sigma G_B \Delta h_B$ = B.t.u. per hr. absorbed in raising ΣG_B lb. of gas per hr. from atmospheric temperature to mean flame temperature T_U, the symbol Σ denoting the several constituents

dQ_F/dt = B.t.u. per hr. absorbed at bare water-cooled walls in furnace envelope.

Let dq_F/dt = heat transfer B.t.u. per sq. ft. of projected cold surface in furnace envelope

S_C = square feet of projected cold surface in furnace envelope

μ_6 = fractional part of total input to furnace, B.t.u. per hr., which is transferred to cold furnace walls;

then

$$dq_F/dt = \frac{G_U U + G_A \Delta h_A}{S_C} \times \mu_6$$

and conversely

$$\mu_6 = \frac{S_C \times dq_F/dt}{G_U U + G_A \Delta h_A}$$

The quantity dq_F/dt is that shown as the ordinate scale in Figs. 17 to 24, and as a constant on the curves in Figs. 30, 31, and 32.

In arriving at values for the heat-transfer rate dq_F/dt, it is necessary to set up the energy equation for the conditions under investigation. This may be done in accordance with the method explained in Wohlenberg and Lindseth's paper. The flame temperature is arrived at by finding that value of it which makes the right- and left-hand sides of the energy equation equal to each other. This value of T_U determines the total heat dq_F/dt transferred to the cold walls, and hence also the unit heat-transfer rate dq_F/dt when the surface area S_C is known.

From the curves in Figs. 17 to 24 and 30 to 32 it is easy, by reversing the procedure, to determine both dQ_F/dt and μ_6. Thus

$$dQ_F/dt = dq_F/dt \times S_C$$

and

$$\mu_6 = \frac{dQ_U/dt}{\text{Flame volume} \times \text{energy-release rate} + \text{B.t.u. per hr. input above 70 deg. by preheated air}}$$

It should be remembered that the energy-release rate must be multiplied by the flame and not by the furnace volume in arriving at μ_6, because all figures on energy-release rate in this as well as the foregoing papers are based on the flame volume. For the furnace considered in this analysis the flame volume is 19 × 19 × 19 = 6859 cu. ft.

To facilitate heat-transfer computations for the zone beyond the furnace the thermal-capacity curves shown in Fig. 33 are included. Having determined the radiation mean of the flame temperature, which of necessity is equal to the mean temperature of the gases escaping from the furnace, reference to these curves at once shows how much heat is left in the products per pound of coal above a temperature of 70 deg. If the lower temperature is above 70 deg., as is generally the case, it is of course necessary to subtract the thermal capacity of the gases at this temperature as taken from the chart, from that for the flame temperature.

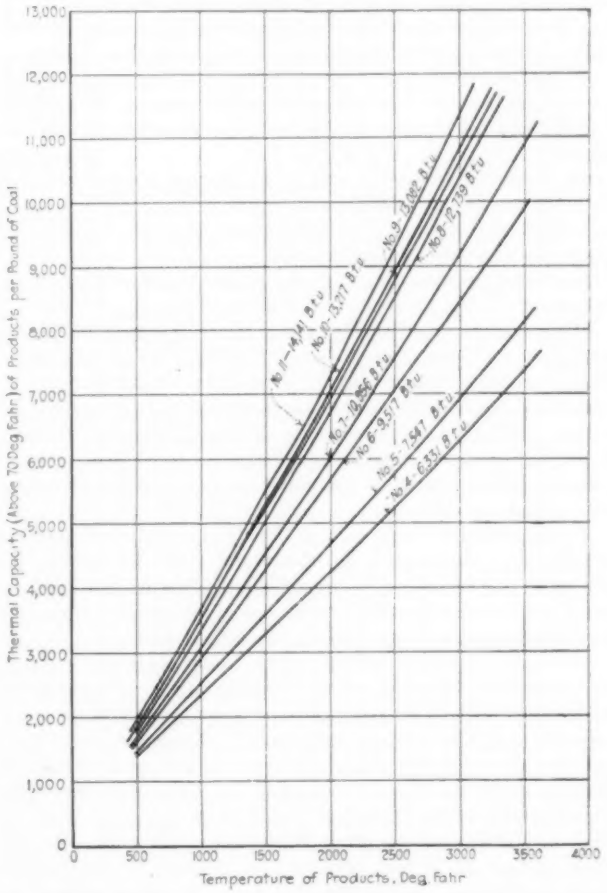


FIG. 33 THERMAL CAPACITY OF PRODUCTS OF COMBUSTION PER POUND OF COAL

(Excess air = 20 per cent for all curves.)

This difference represents the amount of heat to be withdrawn from the products of combustion per pound of coal in cooling them from flame temperature to the temperature in question.

[15] Trans. A.S.M.E., vol. 48 (1926), p. 884, Equation [11].

Discussion

HENRY KREISINGER.[16] This paper contains the results of a large amount of theoretical calculations involving the rate of heat absorption by bare water-cooled surfaces exposed to radiation in furnaces burning various kinds of coal in pulverized form. Such calculations require much labor and patience, especially when so little experimental data are available and consequently many assumptions have to be made. The paper would have been more complete and more easily followed if the appendix had contained more detailed explanations of the factors used in computing the rate of heat absorption. That is, the last member of the energy equation DQF/DT should have been more fully explained. Without this explanation in the paper, one has to study other papers in order to get an understanding of what factors this member contains. On consulting the papers referred to one learns that this member contains the following factors:

(*a*) Radiation from the surface of the coal particles, corrected for the heat reflected from the water-cooled surfaces. This factor varies as the difference of the 4th powers of the absolute temperatures.

(*b*) Radiation from CO_2 and H_2O corrected for the heat reflected from the heat-absorbing surfaces. This factor is a function of the concentration, the thickness of the layer of these gases, and the absolute temperature.

(*c*) A convection factor, consisting of heat transferred by direct contact of the hot gases with the water-cooled surfaces.

The inclusion of all the foregoing factors requires many assumptions and complicates the calculations. Usually in problems of furnace design only the radiation from the flame is considered and a coefficient is assumed that will give approximately the total heat received by the heat-absorbing surfaces. In all such problems a great deal of judgment is required, and the calculated results must be compared with the few experimental results that are available.

We greatly need experimental data to check our theoretical calculations. At present the theoretical study is running far ahead of the experimental investigation. We do not need to check the theoretical study, but we should push the experimental investigation to catch up with the theoretical study.

The paper presents only the mean values such as the flame temperature and furnace-refractory temperature. This is about all one can calculate with any reasonable certainty. These mean temperatures must be studied along with the size of the furnace and the factor of fraction cold in order that one can arrive at the possible extremes of such temperatures. For example, with Pennsylvania bituminous coal and furnace one-third fraction cold, rate of energy release of 30,000 B.t.u. per cubic foot per hour, and air at 70 deg. fahr., the temperature of the furnace flame as shown in Fig. 7 is 2500 deg. fahr. In some parts of the furnace the temperature of the flame may be above 3000 deg., in other parts of the furnace below 2000 deg. In other words, the extremes may be well 1000 deg. apart. Similarly, the mean temperature for the furnace refractory for the same coal and the same conditions of combustion is given in Fig. 15 as 2100 deg. fahr. Assuming that the furnace is water-cooled at the top and bottom and is a perfect cube with a side of 20 ft., this furnace gives the fraction cold factor as one-third. The side walls, the front wall and the rear wall are made of refractory. The mean refractory temperature as computed is 2100 deg. fahr. Although this mean temperature may be nearly correct, the extremes are very far apart. The temperature half-way up the furnace along the side walls will be very nearly the temperature of the flame. The temperature near the water-cooled bottom and top will be approaching that of the water-cooled surfaces. That is, the temperature may vary from 1500 to 2500 deg. fahr. The part of the walls near the water-cooled top and bottom will be safe from abrasion, but half-way up the furnace the temperature may be too high for the refractory to stand it.

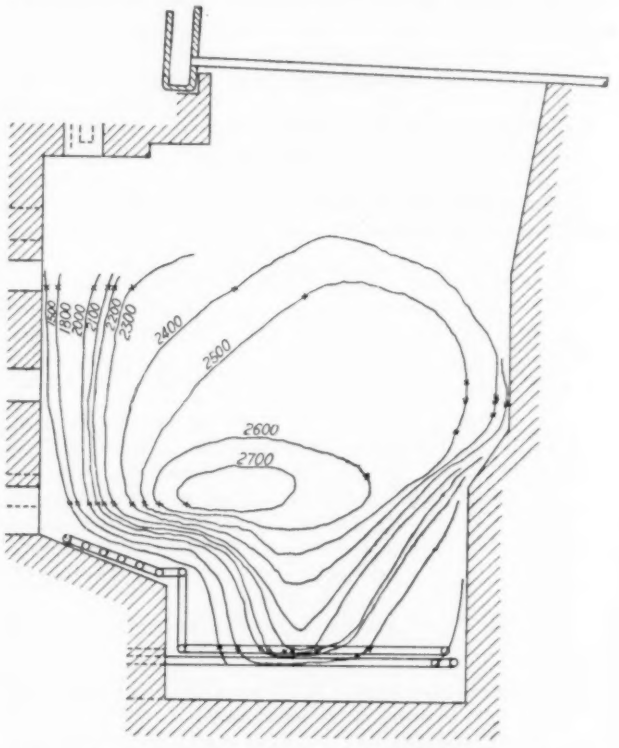


FIG. 34 FLAME TEMPERATURE IN POWDERED-COAL FURNACE WITH WATER-COOLED TOP AND BOTTOM

Fig. 34 shows the flame temperature in a powdered-coal furnace with water-cooled top and bottom. The fraction cold is 25 to 30 per cent. The rate of energy release was about 20,000 B.t.u. per hour. The temperature of the flame is shown to vary from about 1800 to 2700 deg. fahr. The mean temperature was probably 2100 deg. fahr. The temperatures were measured with a platinum couple consisting of a thin wire exposed to the furnace gases. The measurements are probably correct within 100 deg. fahr. The refractory of the side walls near the top and bottom were safe, but near the center of the furnace there was a bad abrasion from molten slag.

In the calculations of the furnace-flame temperatures the authors assumed that the flame filled the furnace to the same degree with all the rates of heat release. Actually this is never the case. At lower rates of combustion the flames fill only part of the combustion space nearest to the burners. Under such conditions the surface of the mass of flame is much smaller than it is when the flame fills the furnace completely. Since it is the surface of the flame which radiates most of the heat, the quantity of heat radiated from the flame at the low rates of combustion is smaller, not so much because of the lower flame temperature but because of the smaller flame surface radiating the heat. Thus the temperature of the flame at the low rates of combustion may not be much lower than it is at the high rates of combustion.

For example, a burning candle or a match placed in any part of a large completely water-cooled furnace will burn with a flame temperature of about 2200 deg. fahr., although the average rate of heat liberation based on the entire furnace volume may be

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less than 1 B.t.u. per cubic foot of combustion space per hour. Actually the combustion takes place in only a very small fraction of the furnace volume. The flame temperature of the candle or the match is high because the heat-radiating surface of the flame is very small and the heat radiated to the water-cooled walls from the flame surface is also very small. It is the extent of the flame surface and not the extent of the heat-absorbing water-cooled surface that determines the amount of heat radiated.

The average rate of heat liberation in combustion of pulverized coal can be reduced to 2000 B.t.u. per cubic foot per hour or even less in completely water-cooled furnace, provided the burners are of such a design that they produce proper mixing near the burner at such low rates of combustion. Usually at such low rates practically all of the air needed for combustion is supplied with the coal as primary air, and very little or no secondary air is used. Very little mixing takes place near the burner, and the coal is blown far into the furnace before ignition is started. In such cases it is the burner design that is responsible for the poor combustion results. If such burners produced as good mixing at the low rates of combustion as they do at the high rates, no difficulty would be experienced in good combustion results at the low rate of average heat liberation.

Much heat is radiated from a luminous and to a large extent opaque flame. In pulverized-coal furnaces such flame is produced by the combustion of the volatile matter of the coal. The volatile matter is driven from the coal as heavy hydrocarbons largely in the form of tar vapors. These heavy hydrocarbons are partly broken down by the heat into soot and light hydrocarbons and other light combustible gases. The lighter combustible gases burn first, the soot remaining in suspension as very tiny particles of incandescent carbon which give the flame its luminosity.

The authors call attention to the fact that the information presented in the paper is only relative, that is, it indicates how one factor affects another. The paper deals only with the means of averages of these effects. These means or averages form a basis upon which the actual temperatures can be estimated so that they can be given the right consideration in the design of a furnace.

After all, in the design of water-cooled furnaces the object of water-cooled surfaces is not to reduce the mean temperature below a certain point so that no fusion of ash would occur, but to place the water-cooled surfaces over the walls where destructive abrasion of refractory would take place. The calculated mean temperatures are of greater value in determining the location of the superheater and the extent of the superheating surface, and also in the calculation of the final temperature of the gases leaving the boiler so that the size of economizer or the air heater may be rationally determined.

G. H. Kaemmerling.[17] The authors have done a valuable service in compiling quantitative results of the application of radiation theory to a series of coal types which cover practically the entire heat-value range of usable solid fuels. The graphs presented make it possible to interpolate the radiation characteristics of nearly all commercial coals and lignites for various types of furnaces, and with the assistance of previous papers by Professor Wohlenberg, the complete range of furnace conditions for various fuel-air ratios may be determined.

It has been generally accepted in the design of furnaces for pulverized fuel that the intensity of combustion should be directly proportioned to the fusion temperatures of the ash. The present analysis demonstrates that the required variation in furnace performance between high-grade and low-grade fuels, as prescribed by ash-fusion points, has been overestimated.

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In a recent paper before the International Bituminous Coal Conference, E. G. Bailey pointed out the fact that coal ash has not one, but a number of fusion points, which are all brought into action in the furnace and which may vary widely from the composite ash-fusion temperature. When this is considered in conjunction with the characteristics of fuels brought out in the paper under discussion, it becomes at least permissible to speculate whether or not the question of suiting furnace design to ash fusion becomes so complicated in proportion to its importance as to make it impracticable.

It may be supposed that space limitations and the large amount of work involved in its preparation compelled the authors of this paper to restrict their consideration to uncooled refractory walls and bare water-cooled walls, the characteristics of each being approximately constant as heat-absorptive surfaces. The entire range between these extremes may be represented by the refractory-covered water wall, in which the heat absorptive capacity is directly proportional to the thickness, and also to the conductivity, of the refractory covering the tubes.

It has been the writer's privilege to see something of the operation of a recent water-cooled furnace installation in which the water walls are covered with silicon-carbide refractories. The range of operation of the furnace varied from a heat release of 10,000 to over 50,000 B.t.u. per cu. ft., assuming that the flame and furnace volumes were identical, or somewhat higher if Wohlenberg and Anthony's assumption is used. The upper figure corresponded to a boiler rating of over 500 per cent, the rated horsepower of the boiler being 500. No difficulty was experienced with the ash throughout the range of operation, although a uniformly high CO_2 of 16 per cent or better was maintained at the highest combustion rates attempted. The walls were at all times covered by a thin layer of porous slag, which adhered to but did not penetrate the face of the refractory, and could be peeled off in sheets when the furnace was cold without leaving the slightest sign of deterioration on the refractory faces. No hard slag was deposited on the boiler tubes. This fact may be ascribed to the use of a novel vertical turbulent burner, which fired downward and away from the boiler tubes. There was no indication that the limit of practicable heat release had been reached at the maximum rates attempted, with respect to ash handling, wall life, or burner performance.

The heat-transfer curves presented by Wohlenberg and Anthony show clearly that the total heat absorption by the furnace walls is increased to a minor degree only by increase of the fraction cold, due to the rapid falling off of rate of heat transfer per square foot as the proportion of cold area increases. This results in a decrease in heat-absorption efficiency of all the cold surfaces, including the front bank of the boiler.

In view of the limitations imposed upon furnace performance and adaptability by the use of either uncooled refractory or bare tube walls, the performance of refractory-covered water walls such as described becomes a matter of very considerable importance, and we may look forward with interest to the analysis of furnace conditions with water-cooled refractory walls which the authors of the paper have promised us.

Authors' Closure

The authors wish to express thanks to Messrs. Kaemmerling and Kreisinger for including in their discussions of the paper some valuable suggestions.

Mr. Kaemmerling points out the discrepancies between ash-fusion temperatures made in the laboratory and the possible behavior of the ash in the furnace. He then expresses some doubt concerning the practicability of designing furnaces with reference to ash fusion. It should be emphasized in this respect that even though our knowledge of the subject is rather fragmentary, each

new fact, even though it may contain information which does no more than relate one mean condition to another, nevertheless furnishes another bench mark, as it were, on which good judgment may be based. Finally when the whole picture has been circumscribed by such bench marks, judgment passes out of the picture, a condition which is probably never realized in engineering. Judgment, however, to be of any value at all must begin by being based on certain such bench marks and that is the light in which the data included in this paper may be viewed.

Both Messrs. Kaemmerling and Kreisinger point to the importance of water cooling in furnace walls for the purpose of protecting the necessary refractory parts. Mr. Kreisinger mentions the furnace envelope composed of bare water-cooled walls and other walls of refractory. Mr. Kaemmerling presents data from tests of a furnace in which certain refractory walls are water cooled. Both types of furnace envelope probably have their place, and, as Mr. Kreisinger says, an object of furnace design is to provide water-cooled walls in such places as to protect the refractory walls.

The question might be asked, Why put in any refractory walls at all? This is a matter of providing hot surfaces to improve combustion conditions. Whether, then, a furnace provided with hot and cold spots or one provided with a whole hot surface but at a somewhat reduced temperature as compared to the hot-spot part of the other type of furnace is the better is a question which practice alone can determine. Load curve, coal type, size of furnace, etc. are conditions that would have to be considered in each case before coming to a decision.

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Analysis of Heat Absorption in Boilers and Superheaters

By N. ARTSAY,[1] NEW YORK, N. Y.

MANY questions in boiler design, remodeling, and operation control require the knowledge of heat absorption in every part of the heating surface. Proper proportioning and placing of the superheaters, and estimates of performance of new types of boilers will also be helped by a method permitting an easy calculation of heat absorption in, say, each row of boiler tubes, including also the radiant surfaces.

Theory of heat exchange in boilers is scattered over a multitude of papers and articles; the basic ideas in them are conflicting; actual data of heat transfer pertain to tests on single tubes elaborated by different methods, while the data for total boiler surface cannot be used due to the inclusion of the radiant heat absorption. So far the author has not seen a uniform method of calculating heat absorption going from the radiant surface through the consecutive rows of boiler tubes, superheater tubes, and the last-pass rows, which would be simple enough for quick checking of tests, or inversely, which would be convenient for estimation of probable test results. Therefore the author feels justified in offering a method which permits calculation of heat absorption, row by row, or by tube banks from the radiant surface to the last row of tubes, and which contains only one basic empirical constant. This constant is valid theoretically for radiant surface as well as for the convection surfaces because it is arranged to represent the constant of molecular energy transformations in the outermost atomic layer of the water-cooled metal.

The principle of hydrodynamic analogy was extremely fruitful in the theory of elasticity in electrodynamics, and in the questions of heat transfer. The late Prof. Groum Grjymailo created a theory of furnaces on principles of hydrodynamics. All these applications are only different aspects of the more general theory of energy transformations between the potential and kinetic form. The cases where the law of dynamic similitude offer practical solutions of extremely complicated problems are innumerable, it is sufficient to mention the hydraulic testing laboratories, the experimental tanks for ship models, the aerodynamical laboratories, the experiments on underwater explosions, etc. Hence an attempt is made here to combine the hydrodynamic analogy and the law of similitude for the problems of heat absorption in boilers.

Radiation Absorption

Taken up seriously since the advent of powdered-coal firing, the question of radiant-heat absorption was dealt with by two different schools of engineering thought.

The more mathematically inclined school considers the radiation absorption with the aid of physical laws of radiation; the solutions are extremely complicated and transcend the power of the average engineer to handle the question with a quantitative discernment. The beauty of the Stefan-Boltzmann formula appeals to everybody, but in the hands of the multitude the formula is misused very much. In all these calculations the starting point taken is the "furnace temperature," which is often considered the same as that of the furnace gas. The author emphatically stresses the point that the furnace temperature is almost entirely irrelevant to the question of total radiation absorbed. It must be well understood that the furnace-gas temperature represents the residual heat after the fact that a certain amount of combustion radiation was absorbed.

This school, moreover, cannot avoid the serious source of errors in the highly questionable values of this temperature, and the latter enter in the fourth power.

The second school takes a more direct way and considers the radiation absorption from the standpoint of energy exchange. Orrok with his broad view on the boiler-furnace phenomena, and his flair for empirical relationships offered a modification of Hudson's formula for calculating the fraction of total available sensible heat cut off at the moment of combustion by the radiant-heating surfaces. This beautiful conception of the instantaneous transfer of radiant energy from the molecules of carbon and hydrogen combining with the molecules of oxygen to the water-cooled metal together with the simultaneous reradiation of the unabsorbed energy is in conformity with present quantum theories. The author had an opportunity to check Orrok's formula in the case of the Benson high-pressure boiler test where it was feasible to separate the total radiant-heat absorption, and the result was correct in a wide range of rating.

Convection

Next comes the convection with the very questionable furnace-gas temperature as the starting value. The heat-transfer formulas with their fractional powers for mass-flow velocities have no physical interpretation, the experimental values do not cover the whole furnace-gas range, and the extrapolations are of doubtful reliability. The placement and extent of heating surface of the superheaters is still more or less guesswork in spite of the immense amount of observed values.

The author offers to treat the heat-exchange problem from the standpoint of energetics. Instead of calculating temperatures and rates of heat transfer, the fractions of available heat content in one pound of gas absorbed by each stage is calculated, the first stage being the radiant surface, and the next stages the consecutive rows or banks of tubes. Thus the boiler in this sense may be regarded as a turbine, but without transformation of heat into mechanical energy, the equivalent of the latter being the heat transmitted to water or steam.

It is very fortunate that as explained in Appendixes 1 and 2, the interchange of heat energy in a boiler is analogous to the work of a fluid in a turbine. The validity of the law of similitude in hydrodynamics allows the writing of equivalent formulas for radiation and convection with the same intrinsic value of the single constant. For convection the value of the constant is changed in accordance with the changed number of active gas molecules. In radiation only the combining molecules of the fuel (C, CO, H_2, S) are effective, while in convection all the molecules of the gas are effective.

Formulas. The derivation of formulas for radiation and convection is given in the Appendixes 3 and 4. Let us denote:

H_o = total sensible heat in B.t.u. available in one pound of gas above the saturated-steam temperature, the heat

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the society.

being calculated after combustion but before radiation absorption. It is obtained from the calorific value of the fuel by deduction of H_2, H_2O, CO and unburned-carbon losses, and of the heat content between the inlet air and the saturation temperatures.

S_r = projected surface exposed to radiation, sq. ft.

q = weight of fuel burned in the furnace, lb. per sec.

Then the fraction μ_1 of H_o absorbed as radiant heat:

$$\mu_1 = \frac{1 + a}{1 + \frac{35\,(fq)^{2/3}}{\sqrt{S_r}}} \quad \ldots\ldots [1]$$

In this formula

$a = \frac{0.22\ T_s}{H_o}$; T_s = saturation temperature deg. fahr. absolute;

f = fuel factor.

For any fuel, $f = \frac{1}{8.67}\left(\frac{C}{12} + \frac{H_2}{2} + \frac{CO}{28} + \frac{S}{32} + \frac{H_n + C_m}{n + 12_m}\right)$ where the chemical symbols denote respective percentages of these constituents in the fuel analysis on the "as fired" basis. The constant in Formula [1] is selected in such a way that for "good bituminous coal" $f = 1.0$. For other fuels the factor f varies widely. For instance:

Anthracite		Good Bituminous Coal		Lignite	
H_2	2.23%	H_2	4.3%	H_2	2.5%
C	79.22%	C	78.0%	C	36.6%
S	0.54%	S	0.5%	S	0.6%
$N_2 + O_2$	18.01%	$N_2 + O_2$	4.8%	$N_2 + O_2$	10.0%
Ash		Ash	8.0%	Ash	10.3%
Moisture		Moisture	4.4%	Moisture	40.0%
$f = 0.892$		$f = 1.00$		$f = 0.498$	

Fuel Oil		Natural Gas		Blast-Furnace Gas	
C	85.00%	CO	1.97%	H_2	0.07%
H_2	12.30%	C_2H_4	1.37%	CO	26.00%
S	1.75%	C_2H_6	7.89%	CO_2	17.10%
$N_2 + O_2$ =	0.95%	CH_4	85.30%	N_2	56.80%
		H_2	1.63%		
		$CO_2 + O_2$	1.90%		
$f = 1.532$		$f = 0.759$		$f = 0.11$	

Coke (90 per cent C) $f = 0.865$

Acetylene (100 per cent C_2H_2) = 0.444

Hydrogen (100 per cent H_2) = 5.76

The analysis given is by weight for all fuels. Thus the fuel factor corrects the fuel weight according to the relative number of combustible molecules.

If the ultimate analysis of coal on "as fired" basis is not available, $f = C/0.83$ where C is the combustible fraction of the coal from the proximate "as fired" analysis.

A closer investigation of the Formula [1] will show that the rate of heat absorption is independent of the saturation temperature, indicating that the combustion radiation is superimposed on the saturation temperature level. In case of refractory-lined blocks covering the water-cooled surfaces, the reduction in the rate of radiant-heat absorption will be due to the surface atomic layer of the refractory which is restoring its radiation-absorbing ability much more slowly than the metals, and also due to the somewhat smaller number of atoms on a unit of surface. Thus for the refractory-lined blocks the constant in Formula [1] is different, also the furnace temperature will be higher. The corrections are given in Appendix 5. From the quantum theory it follows also that at the surface exposed to combustion radiation, the convection heat transfer is zero.

To calculate the heat absorption by convection in a boiler, the boiler must first be divided into consecutive stages or zones. The way of dividing the total heating surface is indicated in Appendixes 6, 7, and 8 where the tests of a straight-tube B. & W., a Stirling, and a Scotch marine boiler are checked. One of the stages of convection is the superheater. The available heat content in the gas H is reduced for this stage due to the higher temperature of metal. The correction for one pound of gas is simple: H corrected $= H - \frac{0.25\,(t - t_s)}{2}$ where

t = superheated steam temperature, outlet

t_s = saturated steam temperature, inlet.

The heat transfer by convection is in general so low that the temperature gradient through the metal can be neglected. The initial heat content above saturation temperature in gas entering the first convection stage H_c is easily found after deduction of the radiant heat absorption from H_o.

$$H_c = (1 - \mu_1)\, H_o$$

Let us denote further:

H_n = B.t.u. per pound in gas entering the nth convection stage (nth row of tubes for instance). The value of H_n corresponds to heat content above the saturation temperature

μ_{cn} = fraction of H_n possible to absorb in the nth convection stage

H_{an} = heat B.t.u. per lb. of gas absorbed by the nth stage

S_{cn} = total heating surface in sq. ft. of the nth convection stage, parts of the tubes between the baffles

G = gas flow by weight, pounds per second

e = enclosure factor.

For plain tubes and cross flow of gas

$$e = \frac{1}{1 + \frac{l - d}{\pi d}}$$

l = pitch of tubes

d = outer diameter of tubes.

For tubes with extended surface and crossflow

$$e = \frac{1}{1 + \frac{S_g}{S_c}}$$

S_g = gas passage in sq. ft.

S_c = total heating surface, sq. ft.

For banks of tubes with parallel flow of gas, $e = 1.0$.

As derived in Appendix 4,

$$\mu_{cn} = \frac{1}{1 + \frac{C}{\sqrt[4]{\mu_{cn}}}} \quad \ldots\ldots [2]$$

for which:

$$C = \frac{18.5\,(eG)^{2/3}\,\sqrt[4]{1 + aH_o/H_n}}{\sqrt{S_{cn}}} \quad \ldots\ldots [3]$$

$$H_{an} = e\mu_{cn}\, H_n \quad \ldots\ldots [4]$$

First C is calculated from [3], then from the graph of solutions given on the diagram Fig. 3, μ_{cn} is read, and finally H_{an} is computed. The direct solution of [2] is too complicated but the graph gives the direct reading.

The majority of boilers contain many consecutive rows of tubes, and to avoid excessive arithmetic several rows may be taken at once by averaging S_c, and assuming an intermediate

value for H_n, the error thus made will be insignificant. If for one averaged row,

$$H \text{ absorbed} = \eta H_n$$

where η is calculated from [2], [3], and [4], then for m rows:

$$H_a = H_n [1 - (1 - \eta)^m] \quad [5]$$

where H_a is the heat absorbed per pound of gas by m rows, and H_n is heat content in one pound of gas entering the first of the m rows of tubes.

For the coal-fired boilers it is advisable to reduce the heating surface by about 20 per cent to account for the accumulation of flying ash or soot. For all fuels and types of heating surface it is recommended to take only the surface actually swept by the gases, neglecting the dead pockets.

The checking of boiler tests given in the Appendixes 6, 7, and 8 shows a fair agreement between the calculated and the observed values. It is easy to show that steam boilers in general transmit to water a very large fraction of the sensible heat in gas available above the saturation temperature; it runs 90 to 99 per cent depending upon the rating and degree of tube cleanliness. This figure, varying very little, may be linked by some simple function with the rating or any other valuation of the rate of steaming, and consequently the need for the aforesaid method may be questioned from the standpoint of overall performance. But most interesting things are learned from the distribution of heat absorption between different parts of the heating surface including superheaters, and it is here that the usefulness of the method comes in.

Appendix 1—Energy Transfer by Radiation and Convection

The method elaborated here is planned on the following lines in conformity with the quantum theories of radiation which regard a quantum of energy as dynamically definite entity, a "molecule of energy."

When the combustible molecules of a fuel combine with the corresponding number of oxygen molecules, certain quanta of energy are simultaneously radiated in all directions. These quanta are large compared with the quanta which are being absorbed, and reradiated by the furnace gas and refractories, and being far less numerous they are "crowded out" on the excited molecules of gas and refractories. Hence, these large quanta suffer multiple reflections, and only when they strike, during the reverberations, the water-cooled metal surface which is continuously restoring its absorbing ability, a fraction of every one of them is absorbed, while the remaining radiant energy, in reduced quanta produces the thermal level of the furnace through the medium of CO, CO_2, and H_2O. The important point is that the time during which the large combustion quanta are many times reflected by the molecules (excited to the furnace quantum level) is many millions of times shorter than the time during which the part of generated gas is present in the furnace. Hence, *all the combustion quanta radiated are reflected from the gaseous molecules; they will reach successively every square unit of the furnace envelope in a time very short compared with the presence of the generated gas in the furnace, and also every quantum of combustion radiation will eventually strike the surface of water-cooled metal.* The proof of the statement in italics by the quantum theory would be too complicated, but a simple calculation will indicate its plausibility.

A furnace 20 × 20 × 20 ft. will have a volume of 8000 cu. ft. and a projected surface of 2400 sq. ft. At usual furnace temperatures there are roughly 1.5×10^{20} molecules of gas per cubic foot while each square foot of metal exposes roughly 1.82×10^{16} atoms of iron. Hence every quantum wave train of radiation reverberating in the furnace has to pass each time 1.2×10^{24} molecules of gas, and to reflect from 4.3×10^{19} atoms of iron and refractory. The absorbing ability of gaseous molecules here is apparently many million times less than the absorbing ability of the metal walls. However, it is known that their absorbing abilities are of the same order, and hence this result can be explained only by the assumption that the gaseous molecules are busy with the smaller, but much more numerous, quanta of gaseous and refractory radiation and therefore unable simultaneously to absorb and reradiate the large combustion quanta. In other words, it may be stated that *the furnace gas is in a degree opaque to the gaseous and refractory radiation of the furnace-temperature level, and transparent to the larger quanta of combustion radiation, at the same time the refractories act toward the latter as perfect mirrors, and the cooled-metal surfaces receive the full impact of radiation from the burning fuel.*

These ideas are equivalent to the conception that the first atomic layer on the water-cooled metal surfaces has the furnace-gas temperature, while the next layer of the same thickness of one atom has the temperature defined by the rate of heat conductance. Thus one understands easier the selective partial absorption of the larger combustion quanta by the water-cooled metallic surface. The sharp drop of temperature from the exposed atomic layer to the next layer of the same thickness, of one atom, is explainable by the theories of electronic excitation. It is, however, necessary to note that the first atomic layer is maintained at furnace temperature not because of absorption of the gaseous radiation but because of reradiation of the unabsorbed fractions of the combustion quanta, and the furnace-gas heat content is the consequence of this reradiation. Thus from an engineer's standpoint the path of heat energy is as follows:

The combustion quanta leave the burning molecules, reverberate in the furnace being reflected by the molecules of gas excited to the furnace temperature level, find the cooled metallic surfaces, enter the first atomic layer and are partially absorbed, the unabsorbed fractions being reradiated back into the furnace gas whose excitation level permits their absorption by still lesser fractions and after many reverberations. The lesser furnace-gas quanta cannot be absorbed by the water-cooled walls because the first atomic layer is excited to the same level, or in other words, it has the furnace-gas temperature, and the process of radiation absorption is a purely selective one during the many million reverberations.

It can now be considered that the radiant-heat absorption by water-cooled surfaces is an energy transfer directly between the combining molecules of the fuel and the water-cooled surfaces, the solid angles, the extent and shape of refractories, and the composition of furnace gas having nothing to do with it. Only the degree of decomposition of $2CO_2$ into 2CO and O_2 has an influence on the values of combustion quanta. It is obvious also that the heat transfer by convection on surfaces exposed to radiation of combustion is zero. Thus a sharp distinction is drawn between the radiation from the combining molecules, and the general residual radiation. It is quite unfortunate that the majority of radiation estimates based on the Stefan-Boltzmann law take this residual radiant energy, better to say "radiant-exhaust" energy, as the initial state or the "radiant-inlet energy."

Many experiments and models on gas flow in banks of tubes show a clear streamline-velocity field somewhat distorted by local turbulence, and superimposed on the unseen molecular-velocity field. In the presence of a heat-absorbing surface the flow is distorted by friction, and a violent turbulence is generated in the "gas film," gradually subsiding toward the undisturbed flow. A simple rough calculation would show that if the gas in the film

moves relatively to the absorbing surface with one-half of the velocity, still the amount of heat passing by in the film is sufficient to account for the known values of heat transfer by convection at the thicknesses of the film less than $^1/_8$ in., if the rate of heat exchange between the film and the metal is comparable to that by radiation. *Hence the major difference between the heat transfer by radiation and by convection from the standpoint of energetics is in the carriers. In radiation the carriers are the combustion quanta whose number is equal to the number of combining molecules while in convection the carriers are the molecules of gas whose number is in a definite ratio to the combining molecules.* The minor difference will be considered in the derivation of formulas.

Thus in making a comparison between the two phenomena of heat transfer it is seen that the process of combustion in radiation is equivalent in the convection to the diffusion of heat into the gas film, and the turbulent mixing with the fresh gas from the main flow. In order that the same type of relationship holds in both cases it must be assumed that the possible rate of heat exchange between the film and the main flow is greater than the heat exchange between the film and the metal. This assumption can be substantiated by estimating the ratio between the number of gas molecules in contact with the surface of metal and the number of molecules of gas in the gas layer having the thickness of the mean free path of the molecule.

From the viewpoint of energetics, all the forms of energy are subject to the same laws of transformation or exchange, and hence, the heat transfer can be compared with the most convenient of the well-known processes. The peculiarity of radiation

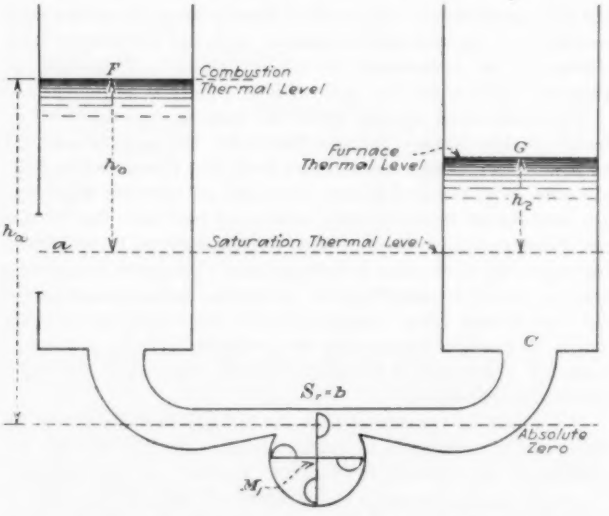


FIG. 1 HYDRODYNAMIC ANALOGY MODEL FOR RADIATION

is in the independency of the rate upon the "back pressure," the rate of radiation is a function of the absolute temperature. In hydrodynamics there is the same condition for the gas flow when the velocity is above the critical value, hence the most convenient form of analogy will be the hydrodynamic one. Thus to this apparent break in the continuity of heat-energy exchange process may be fitted the break in the hydrodynamic energy transformation occurring at critical velocities.

Appendix 2—Hydrodynamic Analogy

Fig. 1 shows the model for radiation, Fig. 2 is for convection. The turbine wheels between the two capacities represent the first atomic layer of the water-cooled metallic surface which absorbs the heat.

The combustion of q lb. of fuel per second, which evolves the flow of G lb. of gas per second, corresponds to the inlet of Q lb. per sec. of a fluid in the models. $Q = gG$ where g is acceleration of gravity. Further the fluid pressures or heads h will be set proportional to heat contents in one pound of gas H in such a way that $Qh = JGH$ where J is the mechanical equivalent of heat. In these conditions the work produced by the turbine wheels M_1 and M_2 of the two models will correspond to heat transmitted to water. The radiation model (Fig. 1) has the combustion

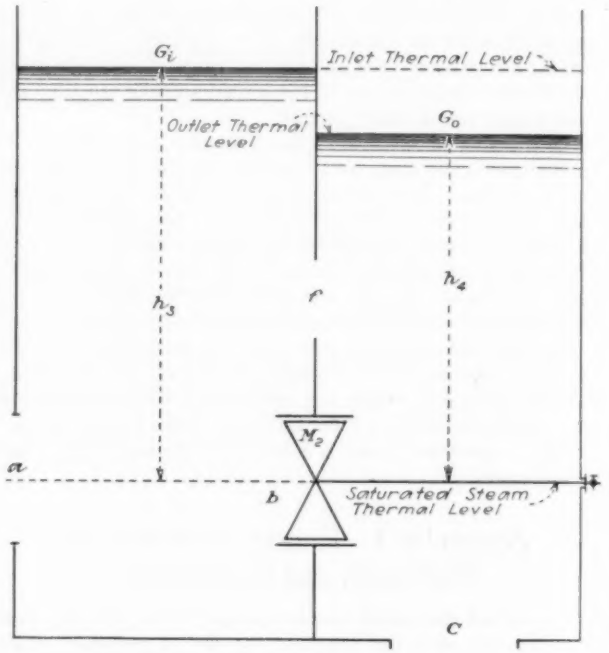


FIG. 2 MODEL FOR CONVECTION

orifice a, the area of which is proportional to the number of combining molecules or to the rate of firing. The turbine nozzle b is of constant area = S_r = heating surface (projected) exposed to radiation, C is the outlet orifice into the convection zones. The combustion of q lb. of fuel per sec. is equivalent to the inlet of Q lb. per sec. of the fluid in the reservoir F against the head h_o, which is the flame, and the total head in this reservoir is h_{oo}, corresponding to the total heat content in one pound of the gas above the absolute-zero temperature. In order to represent the radiation absorption it must be assumed that the turbine M_1 is of the impulse type utilizing the full velocity produced by the absolute head h_{oo} and that the velocity is higher than the critical value for the fluid, and hence independent of the back pressure. This puts the level of the turbine at absolute zero. The reduced velocity of the fluid after leaving the turbine builds up the back pressure by means of the diffuser. The vessel G corresponds to the heat capacity of the gas in the furnace, and the back pressure (h_2 above the saturation level) corresponds to the thermal level in the furnace gas. The head h_o corresponds to the total available sensible heat per pound of gas above the saturation temperature.

In the convection model (Fig. 2) the vessels G_1 and G_0 represent the heat capacities of the gas respectively entering and leaving the convection zone. The orifice a represents the inlet-gas orifice and its area is proportional to the mass flow of gas; the orifice C is for the outlet. The orifice b is constant, and its area is equal to the convection heating surface S_c sq. ft. M_2 is a reaction turbine the work of which is equivalent to the heat

transmitted to water. The phenomenon of convection depends upon the difference of temperatures (heads), and hence a reaction turbine must be assumed in which the velocity of the fluid is below the critical value. The furnace is an almost complete enclosure for the radiation, while in the convection zone the clearances between the tubes across the flow constitute orifices of leakage shown on Fig. 2 by f. For a bank of tubes with parallel flow of gas the enclosure is practically complete. The heads h_3 and h_4 represent the mean heat contents in one pound of gas at the inlet and the outlet, respectively. Thus the whole convection surface of a crossflow boiler of the B. & W. type will be thermally a turbine with as many reaction stages as there are rows of tubes between the baffles of the gas passage, and the radiant surface will be thermally a single-impulse wheel.

Appendix 3—Radiant-Heat-Absorption Formula

Take the bucket efficiency of the impulse wheel = 100 per cent. The power developed by the wheel = $Q\ (h_o - h_2)$.

Denote by μ_1 the fraction of total available heat (above saturation) in one pound of gas H_o absorbed by radiation. Then:

$$J\mu_1 H_o = Q(h_o - h_2)$$

also

$$\mu_1 = \frac{h_o - h_2}{h_o} = \frac{1}{1 + \frac{h_2}{h_o - h_2}} \quad \ldots\ldots\ldots\ldots [6]$$

Consider first the nozzle which is the combustion orifice whose area is proportional to the rate of firing. If the combustion efficiency (degree of dissociation of CO_2) is kept constant the head h_o and the velocity of the jet will be constant. Let us denote for one pound of gas:

$$h_{oo} = h_o + \frac{JC_pT_s}{g} = h_o\ (1 + a)$$

which thermally is equivalent to

$$H_{oo} = H_o + C_pT_s = H_o\ (1 + a)$$

where T_s is the absolute saturation temperature and C_p the mean specific heat of gas between the absolute zero and the saturation temperature.

Let us take now two wheels, identical in design but built on

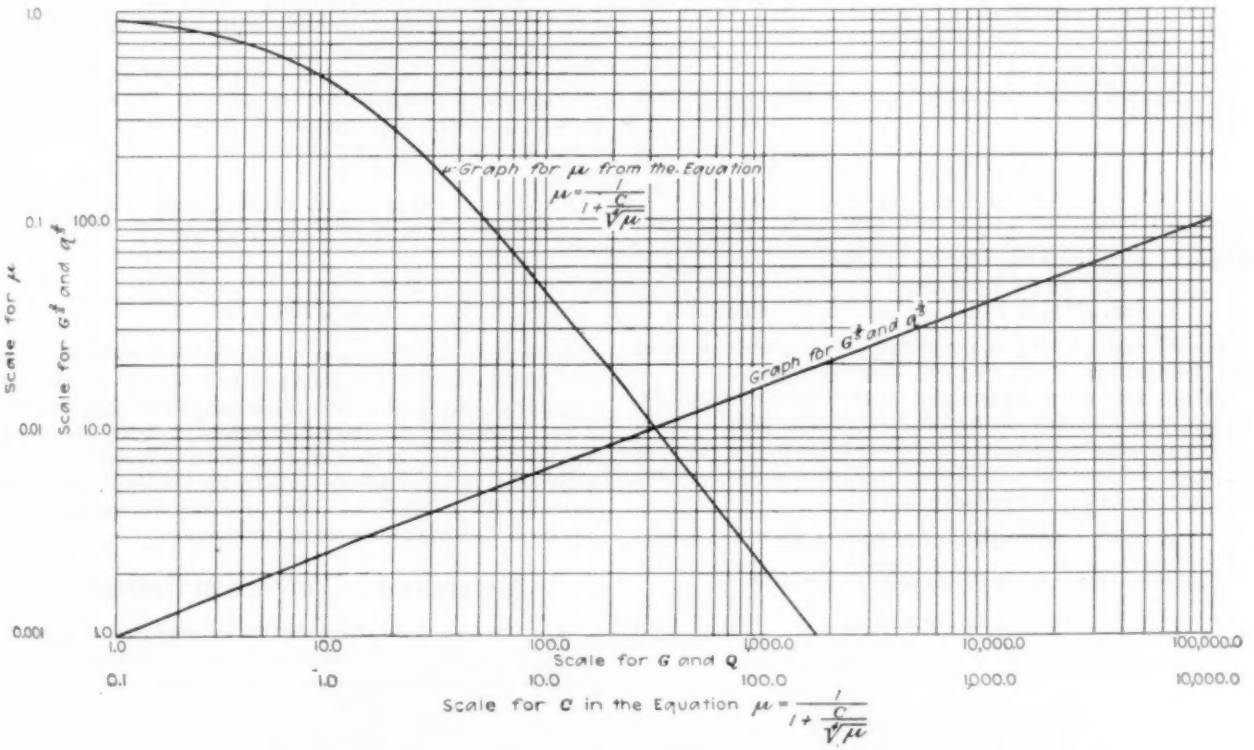


FIG. 3 GRAPHICAL SOLUTIONS OF BOILER AND SUPERHEATER PERFORMANCE

different scales, the relative linear scale being s. Under corresponding hydrodynamic conditions the flows will vary as $s^{5/2}$ or $s = \left(\frac{Q_2}{Q_1}\right)^{2/5}$.

The heads will be of course in the linear ratio s.

Let the first wheel be allowed to run at different speeds when the amount of flow is variable, then at Q discharge, taking the wheel alone:

$$\frac{h_2 + ah_o}{h_o - h_2} = K_1 s = K_2 Q^{2/5} \quad \ldots\ldots\ldots\ldots [7]$$

This will be valid if the speed and the bucket position adjust themselves in such a way that the effective outlet area is increasing in the ratio $s^2 = \left(\frac{Q_2}{Q_1}\right)^{4/5}$. The last requirement corresponds to the requirement of maximum action. If another wheel similar in design but with a larger area of the nozzle S_r be put in the same hydrodynamic conditions, the scale will be reduced by $\sqrt{\frac{S_r}{S_{ro}}}$.

Hence in general

$$\frac{h_2 + ah_o}{h_o - h_2} = K_3 \frac{Q^{2/3}}{\sqrt{S_r}} \quad [8]$$

Combining (8) and (6) we get:

$$\mu_1 = \frac{1 + a}{1 + \frac{K_3 Q^{2/3}}{\sqrt{S_r}}} \quad [9]$$

Substituting the thermal values in [9]

$$\mu_1 = \frac{1 + a}{1 + \frac{K_4 Q^{2/3}}{\sqrt{S_r}}} \quad [10]$$

This formula differs very little from the well-known Orrok formula:

$$\mu_1 = \frac{1}{1 + \frac{A}{27}\sqrt{\frac{C}{S_r}}}$$

where S_r = radiant surface (projected) in sq. ft.
A = lb. of air per lb. of coal
C = lb. of "good bituminous coal" fired per hour.

The difference is only in the exponent for the rate of firing and the results given by both formulas are very close.

Taking Orrok's formula as representing in the closest way the experimental data and choosing the adjustment point at 6 lb. of coal per hour per square foot of projected radiant surface, 15 lb. of air per pound of coal, 200 sq. ft. radiant surface and 390 deg. fahr. saturation temperature, we get

$$K_4 = 35.0 \text{ at } H_o = 710 \text{ B.t.u. per lb.}$$

If the rate of firing is given in pounds fuel per hour:

$$K_4 = 1.32$$

The coal considered here is of the following analysis:

H_2	4.3%
C	78.0%
(S)	0.5%
$N_2 + O_2$	4.8%
Ash	8.0%
Moisture	4.4%
Total	100%

The number of combining molecules per pound is proportional to a sum b:

$$b = \frac{C}{12} + \frac{H_2}{2} + \frac{CO}{28} + \frac{S}{32} + \frac{H_nC_m}{n + 12_m}$$

where the chemical symbols denote respective percentages of these constituents in the fuel analyzed as fired.

For the "good bituminous coal" $b = 8.67$, hence for firing rate q_1 of any other fuel, the q in the radiation formula: $q = fq_1$ where the fuel factor

$$f = \frac{1}{8.67}\left(\frac{C}{12} + \frac{H_2}{2} + \frac{CO}{28} + \frac{S}{32} + \frac{H_nC_m}{n + 12_m}\right)$$

Hence:

$$\mu_1 = \frac{1 + a}{1 + \frac{35\,(fq)^{2/3}}{\sqrt{S_r}}} \quad [11]$$

It is impossible to derive the constant K_4 theoretically, but the upper and lower limits for the constant can be estimated.

The number of iron atoms per square foot of projected cylindrical surface is 2.84×10^{16}. The number of combustible molecules per pound of the "good bituminous coal" is 2.31×10^{22}. The frequency of atomic vibrations for iron is 1×10^{13}. The maximum frequency of radiant energy during combustion of C to CO, V_1, is computed at 7.05×10^{17}. The frequency V_2 corresponding to 2800 deg. K (5040 deg. fahr. abs.) can be computed from the expression:

$$hv_2 = \frac{3}{2}RT$$

where R = gas constant = 1.372×10^{-16} ergs, h = Planck's constant, $V_2 = 8.8 \times 10^{13}$.

The constant K_4 is the relative weight of the units in terms of atomic oscillations and waves of radiation. The upper limit for K_4 is defined by the frequency V_1, while the lower limit will be fixed by the frequency V_2.
Hence:

$$(K_4)_{max} = \frac{(2.31 \times 7.05 \times 10^{22} \times 10^{17})^{2/3}}{(2.84 \times 10^{16} \times 10^{13})^{1/3}} = 72.0$$

$$(K_4)_{min} = \frac{(2.31 \times 8.8 \times 10^{22} \times 10^{13})^{2/3}}{(2.84 \times 10^{16} \times 10^{13})^{1/3}} = 1.98$$

The mean is 37, quite close to the empirical value 35. The value of the constant depends upon the molecular processes of liberating the quanta of radiation about which very little is known so far. The higher frequency V_1 is computed on the assumption that the whole heat of molecular oxidation is radiated without the recoil energy, and this accounts for the too high value which runs up to X-rays, while the lowest frequency corresponds to the position of the maximum in the infra-red region.

Appendix 4—Convection Formula

As in the case of radiation the bucket efficiency is taken as 100 per cent. Let G pounds of gas, each containing H_c B.t.u. available above saturation temperature, enter the convection zone per second.

$$H_o = H_o\,(1 - \mu_1)$$

The thermal equilibrium is attained not instantaneously but in a time somewhat shorter than the time of gas passage through the row of tubes. To account for this we have to change the impulse wheel of the radiation to a reaction wheel shown on Fig. 2. The difference between the reaction and the impulse wheels is in the velocity of the hypothetical fluid passing the wheels. In the case of the impulse wheel the velocity is $V_i = \sqrt{2gh_{oo}}$ (see Fig. 1) which is independent of the rate of firing. For the reaction wheel the velocity will be $V_r = \sqrt{2g(h_o - h_2)}$.

The ratio of velocities $\frac{V_r}{V_i} = \sqrt{\frac{h_0 - h_2}{h_{oo}}} = \sqrt{\frac{\mu_1}{1 + a}}$

Fig. 2 represents the model for convection.
Denoting:

G_i = gas volume inlet
G_o = gas volume outlet
a = gas inlet orifice
c = gas outlet orifice
b = heating surface orifice = S_c sq. ft.
f = "leakage" orifice
M_2 = reaction wheel representing the action of the first atomic layer on S_c

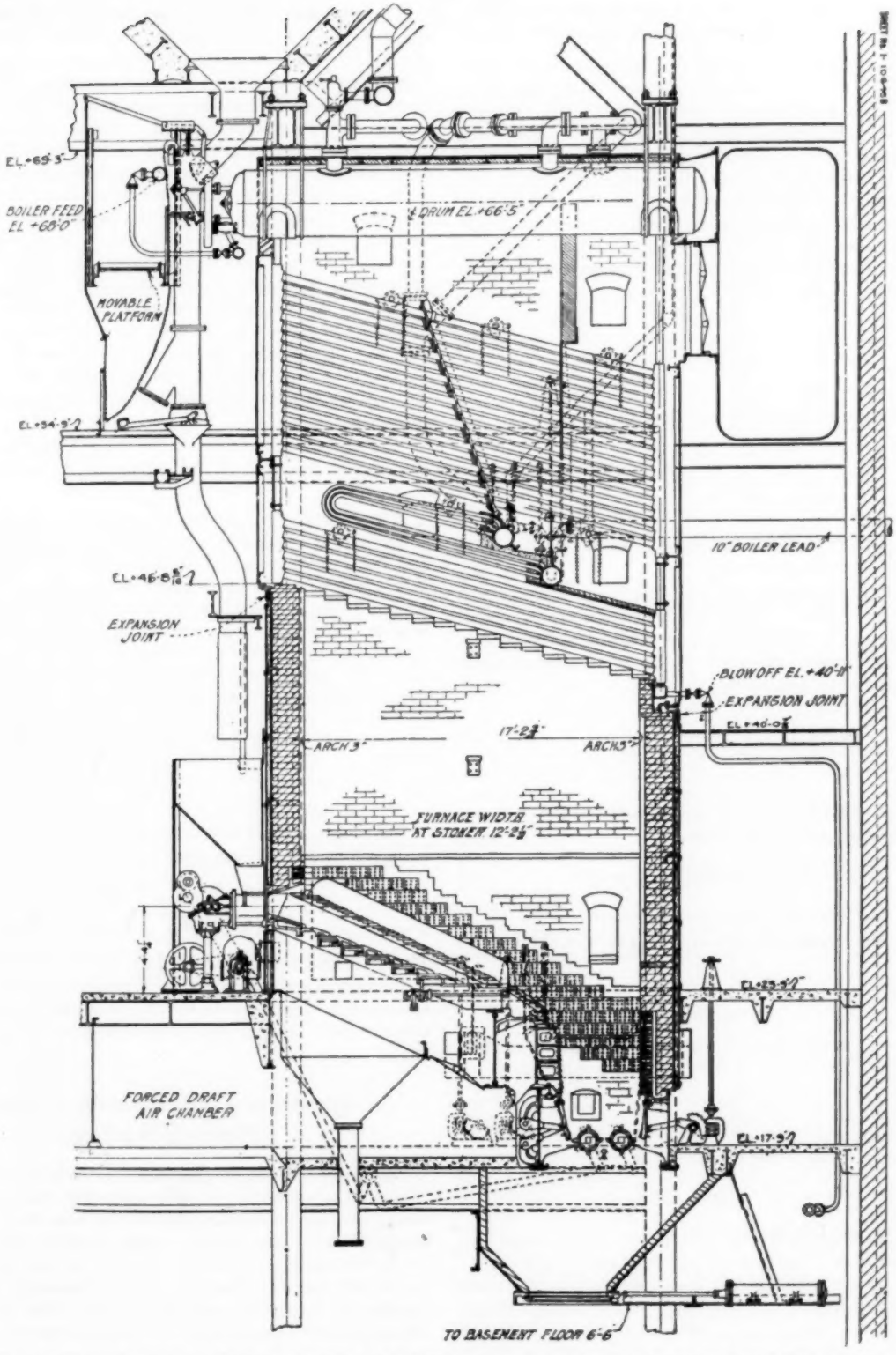


FIG. 4 CROSS-SECTION OF BOILER, STOKER, AND FURNACE AT FIFTY-NINTH STREET POWER HOUSE OF THE INTERBOROUGH RAPID TRANSIT COMPANY

$h_3 = \frac{JH_e}{g}$ = head representing inlet heat content in one pound of gas above saturation temperature, B.t.u. per lb.

h_4 = head representing outlet heat content. The "leakage" orifice is introduced here to account for the incompleteness of the enclosure. In a parallel flow boiler (fire-tube boiler, for instance) the enclosure is practically complete, but in a crossflow boiler the clearance between the tubes into the next stage constitutes the orifice of leakage. If l is the distance between centers of the tubes and d the tube diameter, the enclosure factor

$$e = \frac{l}{l + \frac{l - d}{\pi d}} \quad \ldots\ldots [12]$$

For a case of tubes with extended surface:

$$e = \frac{l}{l + \frac{S_g}{S_c}} \quad \ldots\ldots [13]$$

where S_g is the gas passage area, sq. ft. and S_c is the heating surface. Evidently:

$$S_c\,(1 - e) = f = \text{orifice of leakage}$$

Thus only the fraction e of gas is effective in convection.

Let μ_c be the wheel's efficiency or the fraction of available heat absorbed by the surface S_c.

$$\mu_c = 1 - \frac{h_4}{h_3} = \frac{1}{1 + \frac{h_4}{h_3 - h_4}} \quad \ldots\ldots [14]$$

The amount of fluid passing the wheel is eQ and the work produced by the wheel per pound of flow, h_e:

$$h_e = eh_3\mu_c \text{ or thermally, } H_e = eH\mu_c \quad \ldots\ldots [15]$$

which is the heat available above saturation at the inlet.

Using the same considerations as in the case of the impulse wheel

$$\frac{h_4}{h_3 - h_4} = \frac{K_6\,(eQ)^{2/3}\sqrt[4]{1 + a\frac{h_e}{h_3}}}{\sqrt[4]{\mu_c}\,\sqrt{S_c}} \quad \ldots\ldots [16]$$

or thermally

$$\frac{H - H_e}{H_e} = \frac{K_6\,(eG)^{2/3}\sqrt[4]{1 + a\frac{H_e}{H}}}{\sqrt[4]{\mu_c}\,\sqrt{S_c}} \quad \ldots\ldots [17]$$

connecting [14] and [17] we get:

$$\mu_c = \frac{1}{1 + \frac{C}{\sqrt[4]{\mu_c}}} \quad \ldots\ldots [18]$$

$$C = \frac{K_6\,(eG)^{2/3}\sqrt[4]{1 + a\frac{H_e}{H}}}{\sqrt{S_c}} \quad \ldots\ldots [19]$$

$$H_e = eH\mu_c \quad \ldots\ldots [20]$$

C is calculated directly, μ_c is read from the graph of solutions for [18] given on the diagram of Fig. 3, and then H_e is easily computed. The constant K_6 is intrinsically the same as that for radiation except that the number of molecules per pound of gas is different from the number of combustible molecules per pound of coal, and also the frequency of collisions of gaseous molecules with the wall differs from the radiation frequency. Here the hydrodynamic model helps. At the same amounts and potentials of hydrodynamic energy, the efficiencies of the wheels differ only due to their type. Hence the heating effect by radiation and by convection must be the same per pound of gas with H_e available heat content. For radiation the initial heat content H_e is being produced by combustion of C and H_2 molecules in the coal. For convection the same heat content may be regarded as produced, for instance, by a mechanical agitation of N_2 and O_2 molecules. Hence, on the average, 13 lb. of gas are equivalent to one pound of fuel for which the fuel factor is $100/(30.5 \times 8.67) = 0.378$ where 30.5 is the molecular weight of furnace gas.

$$\text{Thus } K_6 = 35 \times \left(\frac{1}{0.378 \times 13}\right)^{2/3} = 18.5$$

It is gratifying to note that the derivation of the constant for convection is impossible without the hydrodynamic analogy, and the correctness of its value as shown by the checking of actual boiler tests, which support the validity of the considerations. There are some cases (Brown-Boveri boiler) where combustion takes place at a pressure higher than the atmospheric. The frequency of collisions between the molecules of gas and the atoms of metal is proportional to pressure, hence, if p is pressure in pound per square inch, the formula [19]

$$C = \frac{18.5\left(eG\frac{p}{14.7}\right)^{2/3}\sqrt[4]{1 + a\frac{H_e}{H}}}{\sqrt{S_c}} \quad \ldots\ldots [21]$$

The constant $K_6 = 18.5$ depends upon the metal of which the surface is made. If some other metal than iron is used

$$K_6 = 18.5\sqrt{\frac{25}{K}} \times \sqrt[3]{\frac{7.85\omega}{56\delta}} \times \sqrt{\frac{v_i}{v_m}} \quad \ldots\ldots [22]$$

Here K is heat conductance in B.t.u. per hr. per ft. per deg. fahr., δ = density in respect to water, ω = molecular weight, v_i = atomic frequency for iron $= 1 \times 10^{12}$, v_m = atomic frequency of the other metal. Usually this correction of the constant is very slight.

Appendix 5—Refractory-Protected Heating Surfaces

Sometimes the furnace tubes are covered with refractory-lined blocks. Carborundum is used often for this purpose. The main constituent of carborundum is CSi (carbide of silicon). Its density is 3.12, and atomic weight is 40.4. The number of atoms is 2.44×10^{16} or 3.82×10^{16} per sq. ft. of projected cylindrical surface. The frequency of molecular oscillations is close to that of the diamond, 3.0×10^{12}. Thus if the heat conductance of carborundum was the same as that of iron, the constant $K_4 = 35\sqrt{\frac{2.84 \times 10^{28}}{3.82 \times 3.0 \times 10^{28}}} = 17.4$. But the lower degree of conductance reduces the number of effective atoms in the ratio of conductances. For iron the conductance may be taken at 26.0 B.t.u. per hr. per ft. per deg. fahr. For carborundum at high temperature it runs up to 2.5. Hence the constant

$$K_4 = 17.4\sqrt{\frac{26}{2.5}} = 56.$$

Usually the radiant surface is protected by such blocks only partially. Let S_1 be the projected heating surface covered with blocks, and S_2 the projected bare-tube surface. The order of applying the formula is as follows:

(1) The fuel rate for the block-covered surface is reduced to:

$$q_1 = \frac{qS_1}{S_1 + S_2}$$

(2) Fraction of radiant heat absorbed on the block-covered tubes is

$$\mu_1{}^b = \frac{1 + a}{1 + \frac{56\ (fq)^{2/3}}{\sqrt{S_1}}}$$

The residual heat H_c above the saturation is computed.

$$H_c{}^b = (1 - \mu_1{}^b)\ H_1$$

(3) Absorption of the primary radiation on bare tubes is calculated for the full rate of firing q

$$\mu_1 = \frac{1 + a}{1 + \frac{35\ (fq)^{2/3}}{\sqrt{S_2}}}$$

(4) The reduced quanta reradiated from the block-covered surfaces are absorbed on the bare surface, the latter dealing with the total number of quanta, proportional to q. Hence the total heat absorptions A in B.t.u. per hour are:

For the block-covered surface, $A_1 = \frac{GS_1}{S_1 + S_2} \times \mu_1{}^b H_e$

For the bare surface, $A_2 = \left(H_e + \frac{S_1}{S_1 + S_2} H_c{}^b\right) \mu_1 G$

Thus covering by blocks of a part of heating surface results in a somewhat increased thermal load on the unprotected bare tubes

So far the rate of heat absorption on the block-covered tubes is independent of the thickness of the refractory lining. The greater the thickness the higher the temperature of the second atomic layer in the refractory, and when it reaches the furnace-gas temperature or that of the first atomic layer, the rate of heat absorption remains constant (independent of the rate of firing). This limit can be found by a trial and error method.

The furnace temperature t_f in degree fahrenheit is

$$t_f = \frac{GH_e - (A_1 + A_2)}{C_p G} + (T_s - 460)$$

where C_p is the specific heat of furnace gas, it may run up to 0.28, and T_s = saturation temperature, deg. fahr. abs.

Appendix 6—Test Results at Interborough Rapid Transit Company

In a paper presented before the A.S.M.E. in 1926, H. B. Reynolds, J. M. Taggart, and R. S. Lane give results of tests on a Babcock & Wilcox boiler at the Fifty-ninth Street Station of the Interborough Rapid Transit Company, New York City. Fig. 4 shows the boiler. It is 24 tubes high and 21 tubes wide with a heating surface of 11,400 sq. ft. (10,560 sq. ft. in tubes), and superheating surface of 1485 sq. ft. Of the latter about 90 per cent is effective.

Distribution of Heating Surface

1 Projected surface exposed to radiation = 259 sq. ft.

2 First convection zone, 6 rows of 440 sq. ft. $5\frac{1}{2}$ rows are effective.

3 Superheater, 6 rows of 222 sq. ft. Total gas passage into second bank, across the superheater, is 58 sq. ft., of which 35.6 sq. ft. is between superheater tubes; hence the superheater utilizes only 61.5 per cent of gas.

4 First pass second bank, 9 rows of 220 sq. ft. and 9 rows of 200 sq. ft.

5 Approximately 40 per cent of gas sweeps the bottoms of boiler drums. Heating surface = 242 sq. ft. Enclosure factor = 0.5.

6 Second pass, 14 rows of 128.5 sq. ft.

7 Parallel flow part between the second and third passes, 792 sq. ft.

8 Third pass, 14 rows of 106 sq. ft. and one row of header nipples.

In calculations the soot correction was taken as 20 per cent reduction of tube surface which is equivalent to a rise in constant's value to 23.1. For the superheater tubes the reduction was taken 15 per cent, due to smaller diameter of tubes. Throughout the boiler surface the corrected enclosure factor was 0.770 and for superheater 0.842. Tests 7, 9, and 15 were selected as giving the lowest unaccounted-for losses.

The result of the calculation is as shown in Table 1.

The graph of the test gives a constant superheat of 200 deg. fahr. The gas readings for tests 7 and 15 are somewhat off,

TABLE 1

Test number	7		9		15	
Results	Calc.	Obs.	Calc.	Obs.	Calc.	Obs.
Rating developed, per cent	.....	174	.....	228	.....	340
Radiation and unacc. for, per cent	.....	2.79	.....	0.59	.....	6.18
Overall boiler and superheater efficiency, per cent	82.1	82.14	82.0	82.05	73.0	74.36
Flue-gas temperature, deg. fahr.	498	495	543	539.5	623	574
Superheat, deg. fahr.	185	212.7	202	177.6	215	203.2
Steam temperature, deg. fahr.	586	613.7	600	575.6	616	604.2
Fuel factor	1.04	....	1.10	....	1.11	....
Radiation absorption, μ	0.332	....	0.297	....	0.255	....

and hence the calculated values which depend upon the gas flow.

The flue-gas temperature readings are always lower than the calculated values; this may be attributed to air leakage. Of course the temperature of steam depends upon the moisture in saturated steam; test results do not contain moisture data and it was assumed 0.5, 1.0, and 1.0 per cent, respectively. In the case of the low rating in test 7 it is quite possible that the resistance to gas flow through the first bank was insignificant, and hence more than 61.5 per cent of gas passed the superheater, according to the natural direction of the flow. (See Fig. 4.) This will make up the shortage of calculated superheat at test 7.

Appendix 7—Test Results at Detroit Edison Company

In a paper presented before the A.S.M.E. in December, 1922, Paul W. Thompson gives results of tests on a 2365-hp. Stirling "W" boiler at Connors Creek Station of the Detroit Edison Company. Beside having the flow of gas mostly parallel to the tubes, this boiler is interesting because it was tested with two sets of baffling. (Figs. 5 and 6.)

The present method must be able to show the effect of change in baffling and consequently three tests were checked for each system of baffling.

The calculations were made in the following way for tests 43, 40, 42, 64, 63, and 66.

Distribution of Heat Absorbing Surface. Baffling "A."

(*a*) Projected surface exposed to radiation = 1165 sq. ft. For one side:

(*b*) First bank in the furnace minus surface exposed to radiation = 3800 sq. ft. It was assumed that 5, 10, and 15 per cent

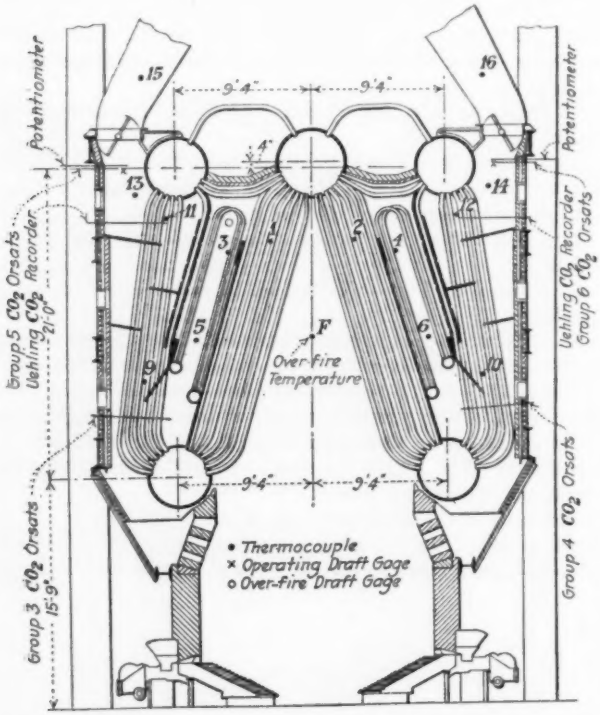


FIG. 5 BAFFLE ARRANGEMENT "A" OF DETROIT EDISON COMPANY BOILER

of gas penetrates in this bank on each side of the boiler on tests 43, 40, and 42, respectively.

(*c*) First pass: Crossflow consists of seven rows of 286 sq. ft. each. Six rows of 250 sq. ft. are effective, one row being fully loaded with radiation.

(*d*) Second pass: The circulating tubes (430 sq. ft.) are neglected due to the dead pocket produced by draft. The superheater (1495 sq. ft.) is divided into two stages: 350 sq. ft. first stage, crossflow, and 1145 sq. ft. second stage, parallel flow.

(*e*) Third pass. Inlet: 5 effective rows of 147 sq. ft. each, crossflow, then 3157 sq. ft. with parallel flow, and finally 5 rows of 123 sq. ft. each as crossflow outlet.

Baffling "B."

(*a*) Projected surface exposed to radiation same as "A" 1165 sq. ft.

(*b*) The convection on two rows exposed to furnace gas was neglected because the surface is loaded with radiation.

(*c*) First pass: 7 rows of 136 sq. ft. each; 6 rows of 85 sq. ft. were taken as effective.

(*d*) Second pass: The return tubes were neglected and the superheater was considered in two crossflow stages of 250 sq. ft. and 1245 sq. ft. The first bank of tubes in the second pass was taken in three stages. Two rows of 520 sq. ft. crossflow, 2340 sq. ft. parallel flow; two rows of 260 sq. ft. crossflow outlet.

(*e*) The third pass was not changed. Same as for baffling "A."

The heat absorption per pound of gas was calculated in every one of the mentioned stages. The steam temperature was calculated by using the ratio of superheater to boiler absorption, neglecting the effect of moisture in saturated steam, which is very small in such boilers, at moderate ratings. (See Table 2.)

Actual flue gas was cooled by air leakage, and moreover the

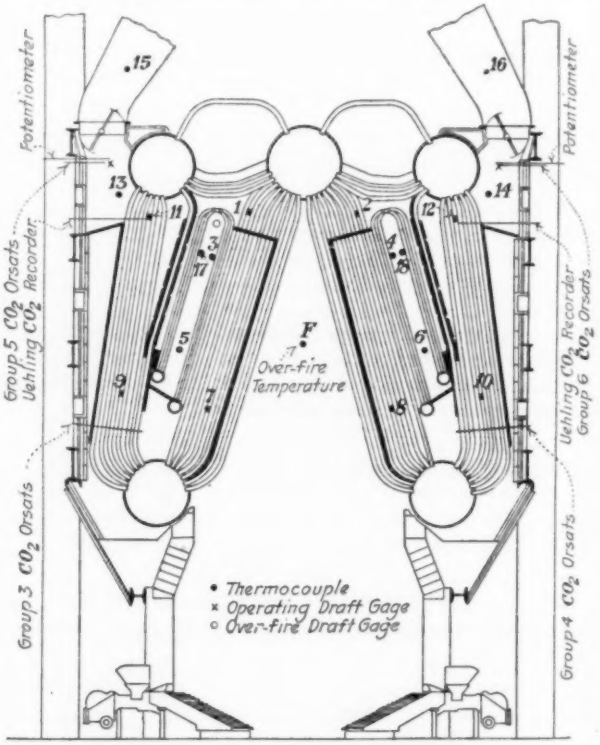


FIG. 6 BAFFLE ARRANGEMENT "B" OF DETROIT EDISON COMPANY BOILER

downcoming water in the third pass was cooler than the saturated steam due to relatively cold feed. The over-fire temperatures are doubtful figures and are given here to show the order of differences. The high calculated superheat at both high ratings is due to the neglect of moisture effect. The data of the test do not contain moisture determinations. The gas readings during these tests were better for the 64, 63, and 66 tests, and hence there is better agreement in general between the calculated and observed values.

Appendix 8—Tests on a Scotch Marine Boiler

The Bureau of Mines Bulletin No. 214 contains results of tests made on a Scotch marine boiler by John Blizard, A. R. Mumford, and R. A. Sherman. The boiler was tested on coal and oil,

TABLE 2 SUMMARY OF TEST AND CALCULATED VALUES

Test number	43		40		42		64		63		66	
	Calc.	Test	Calc.	Test	Calc.	Test	Calc.	Test	Calc.	Test	Calc.	Test
Rating developed, per cent	.....	108	.....	153.5	.....	226	.....	59	.....	106	.....	212.5
Radiant and unacc., per cent	.....	2.3	.....	0.6	.....	2.6	.....	2.1	.....	0.1	.....	1.9
Overall boiler and sph. eff., per cent	77.0	77.3	77.0	77.9	70.8	72.9	79.9	80.9	83.0	83.5	75.1	75.8
Flue gas temperature, deg. fahr.	573	566	645	614	765	726	446	407	490	466	646	619
Superheat, deg. fahr.	160	173	194	195	248	216	188	185	201	200	265	244
Steam temperature, deg. fahr.	560	573	592	592	646	614	586	583	599	598	663.5	642
Overfire temperature, deg. fahr.	.....	...	1860	1612	2080	1835	.....	...	2100	1561	2270	1769
Fuel factor	0.922	...	0.928	....	0.915	....	0.957	...	0.917	....	0.920	....
Radiation absorption, μ	0.525	...	0.469	....	0.420	....	0.585	...	0.504	....	0.416	....

and being also of entirely different type from those discussed, it presents unusual interest for checking the hydrodynamic similitude method.

The heating surface is distributed as follows:

(1) Furnaces and combustion chambers 702 sq. ft. when fired with coal and 947 sq. ft. when fired with oil.

(2) Inner surface of the fire tubes = 2075 sq. ft. to which must be added 1300 sq. ft. of retarders, except test 13 which was run without retarders.

From the multitude of tests analysis will be made of 13, and 12 with coal firing and 26, 50, and 33 with oil firing.

The calculations were made in the following way: Radiant surface was taken as that of furnaces and combustion chambers. The inner surface of the fire tube was taken as convection zone. Due to difference in power in which G and S enter into convection formula, the tubes were taken individually, which makes a considerable difference. Furthermore, the flow of gas referred to convection surface is very low in this case, and the streamline velocity field cannot be maintained, hence a general turbulent flow must be assumed.

Any tube may be divided into any number of sections in series, which will, of course, give different rates of absorption, increasing with the number of stages. For the condition of maximum action the number of stages must be also a maximum and the shortest section must be determined where the conditions of flow approach the forced streamline field. Considering the rolling of gas in the tube, the least length ought to be $\pi d/2$, where d is inner diameter. Actual dimensions of tubes in the boiler require accordingly 28 sections or stages.

The results of calculations and the observed values are given in Table 3.

TABLE 3

Test number	13		12		26		50		33	
Results	Calc.	Obs.	Calc.	Obs.	Calc.	Obs.	Calc.	Obs.	Calc.	Obs.
Fuel fired per hr. lb.	.....	1250	.....	1650	.....	835	.....	1105	.....	1595
Fuel	Coal	Coal	Coal	Coal	Oil	Oil	Oil	Oil	Oil	Oil
Fuel factor	0.958	....	0.910	....	1.46	...	1.477	....	1.445	....
Efficiency, per cent	75.0	75.0	79.0	80.0	82.5	82.5	80.5	81.5	78.3	78.5
Flue gas temp. deg. fahr.	625	635	609	582	543	533	610	574	706	695
Radiation absorption	0.693	....	0.650	....	0.706	...	0.650	....	0.605	....

Radiation from the hot retarders in the fire tubes was considered as equivalent to the full metallic contact.

Appendix 9—Further Analysis of the Heat-Absorption Formulas

The fraction of heat content in one pound of gas absorbed as radiation is

$$\mu_1 = \frac{1 + a}{1 + \frac{35\,(fq)^{2/3}}{\sqrt{S_r}}}$$

Let us reduce q to a very small amount $q \approx 0$ (a candle for instance) leaving $G \approx 0$. This would mean practically a circulation of hot gas through the furnace. In these conditions the formula indicates $\mu_1 = 1 + a$. This is impossible because it means cooling of the gas to the absolute zero temperature. Hydrodynamically (Fig. 1), these conditions are equivalent to a flow against the head. From this it is seen that there exists a certain limit to the application of the radiation formula. In order that the heat is carried from the burning fuel to the water by radiation the outlet velocity head from the impulse wheel on the hydrodynamic model must be sufficient to counterbalance the head of saturation level. This critical velocity head or a critical minimum rate of combustion will define also the maximum value of μ. From Fig. 1 it is seen that $\mu_{max} = \frac{h_o}{h_{oo}} = \frac{1}{1 + a}$, hence the critical rate of combustion may be calculated from the expression:

$$\frac{1}{1 + a} = \frac{1 + a}{1 + \frac{35\,(fq)^{2/3}}{\sqrt{S_r}}}$$

When the rate of combustion is below the critical value then for calculating heat absorption in the furnace instead of the radiation formula it is necessary to use the convection formula:

$$\mu_1 = \frac{1}{1 + \frac{18.5\,G^{2/3}\,4}{\sqrt{S_r}}\sqrt{\frac{1 + a}{\mu_1}}}$$

which is suitably re-written for the case. This critical rate of combustion is low. For instance, in a 1500-hp. boiler S_r = 900 sq. ft., H_o = 1000 B.t.u. per lb., T_s = 460 + 440 = 900 deg. fahr., f = 1.0, $a = \frac{0.22 \times 900}{1000} = 0.198$ and q critical = 0.088 pounds of coal per second or 315 pounds per hour.

The wide range of the radiation formula can be substantiated in the conditions of the other extreme. With a cylinder of a Diesel engine, at 24 in. diameter the mean surface exposed to radiation during the combustion will be about 7.5 sq. ft. The power developed in one chamber is 650 hp. at 0.45 lb. of oil per hr. The nominal combustion rate is $\frac{0.45 \times 650}{3600} = 0.0813$ lb. per sec. Fuel factor f = 1.4. Actual combustion takes place at 10 per cent of the stroke, or on 37 deg. turn, hence the equivalent rate of coal combustion is $\frac{0.0813 \times 1.4 \times 360}{37} = 1.11$ lb. per sec. Taking the jacket water at 200 deg. fahr. = 660 deg. abs. and H_o = 1200 B.t.u. per lb.: $a = \frac{0.22 \times 660}{1200} = 0.121$. The fraction of generated heat absorbed as radiation

$$\mu_1 = \frac{1.121}{1 + \frac{35 \times 1.11^{7/8}}{\sqrt{7.5}}} = 0.0782 \text{ or } 7.8 \text{ per cent.}$$

The figure is well within the reasonable limits of jacket loss, leaving a large margin for convection during the remainder of the power stroke. In the convection zones of a boiler, the radiation from hot CO_2, CO, H_2O gas plays a considerable part in the total heat transfer. In the convection formula the decrease in the rate of heat transfer when the gas cools down is reflected by the factor $\sqrt[4]{1 + \frac{aH_o}{H}}$ in the denominator term. The value of it increases as H, the current heat content, is reduced. In actual boilers the value of heat transfer coefficient in the first pass is about 50–70 per cent higher than in the last pass. Take, for example, a row of tubes of 225 sq. ft. total heating surface in the first pass and in the last pass, 50 lb. of gas per second, enclosure factor $e = 0.8$, $a = 0.2$, $H_o = 1200$, $H = 700$ in the first pass, $H = 50$ in the last pass above saturation level. Using convection formulas [2], [3], [4], and the graph, we get in the first pass $\mu_c = 0.086$ and in the last pass $\mu_c = 0.056$; the difference is 54 per cent.

The velocity of gas in boilers runs from 10 to 50 ft. per sec. between the tubes. The overall heat transfer is usually considered as proportional to velocity in power between 0.5–0.8. However, these experimental values are for relatively low velocities.

By equating the heating effect estimates of the two kinds, $G\mu_c = KG^x$, it is seen how the exponent x changes if the coefficient K is a constant. The same row of tubes is taken, 225 sq. ft. with 75 sq. ft. area of gas passage. At a value 40 cu. ft. per lb. for specific volume of the gas, the order of velocity at 15 lb. per sec. is 8.6 ft. per sec. and at 150 lb. per sec. it is 86 ft. per sec.

At $a = 0.2$, $e = 0.8$, the convection formulas and the graph give for $G = 15$, $\mu_c = 0.135$; for $G = 150$, $\mu_c = 0.044$.

If, at $G = 15$, $x = 0.6$, then $K = 0.399$, and for $G = 150$, the exponent $x = 0.561$. If we set, on the other hand, the exponent as constant with the value $= 0.5$, the coefficient K will vary from 0.523 to 0.540, respectively, or an exponential formula would differ from the hydrodynamical model's about 3 per cent in the range

FSP-51-38

A Machine for Making Creep Tests at High Temperatures

By GLEN D. BAGLEY,[1] LONG ISLAND CITY, N. Y.

The demand for materials suitable for operation under stress at high temperatures has made it necessary to investigate the properties of various alloys with regard to their resistance to creep. Various types of machines have been developed for this purpose. This paper describes the apparatus developed by the Union Carbide and Carbon Research Laboratories. Its principal features are a compound lever system of applying the load, similar to that used in regular tensile machines, and a simplified method of measuring the extension of the specimens. The compound lever system makes it possible to determine the load accurately and to use standard 0.505-in. diameter specimens without employing excessive weights, even at relatively low temperatures. The system used for measuring extension is such that the construction of the machine is simplified and the convenience of operation improved. The accuracy is ample for all creep tests except those in which extremely small creeps over long periods of time must be measured.

THE industrial development of many processes has led to the use of higher and higher temperatures, so that the limitation of the process has been found in the properties of

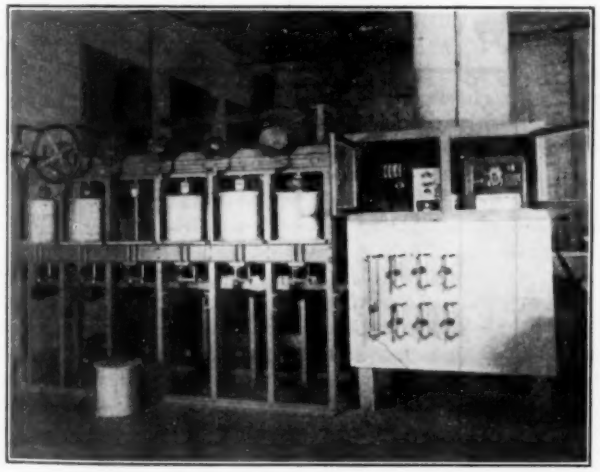


FIG. 1 FRONT VIEW OF THE COMPLETE MACHINE

available materials. Notable improvements in efficiency and capacity might be effected if better materials could be obtained. The governing property in most cases is the slow extension or creep which sometimes occurs at remarkably low stresses when the period of application is long. The only method at present available for determining the value of a given material in this respect is to make long-time tensile tests at elevated temperatures.

The machine described in this paper was developed at the Union Carbide and Carbon Research Laboratories for the purpose of conveniently carrying out a program of such high-temperature creep tests, in connection with the Joint Research Committee on Effect of Temperature on Properties of Metals, sponsored by the American Society for Testing Materials and The American Society of Mechanical Engineers. Since these tests require so long a time, it is essential that the machine give accurately reproducible testing conditions and that it be convenient to operate.

The primary conditions which the machine was to meet were decided upon as follows:

1 To apply a load of 75,000 lb. per sq. in. to a specimen 0.505 in. in diameter
2 To maintain a uniform temperature over the length of the reduced section of the test piece within plus or minus 2 deg. cent. for any desired length of time
3 To measure the creep of the specimen without applying measuring devices to the part of the specimen inside of the furnace.

The appearance of the machine which was developed is shown in the photographs Figs. 1, 2, and 3 and the cross-sectional drawing Fig. 4.

A system of weights which apply tension to the specimen

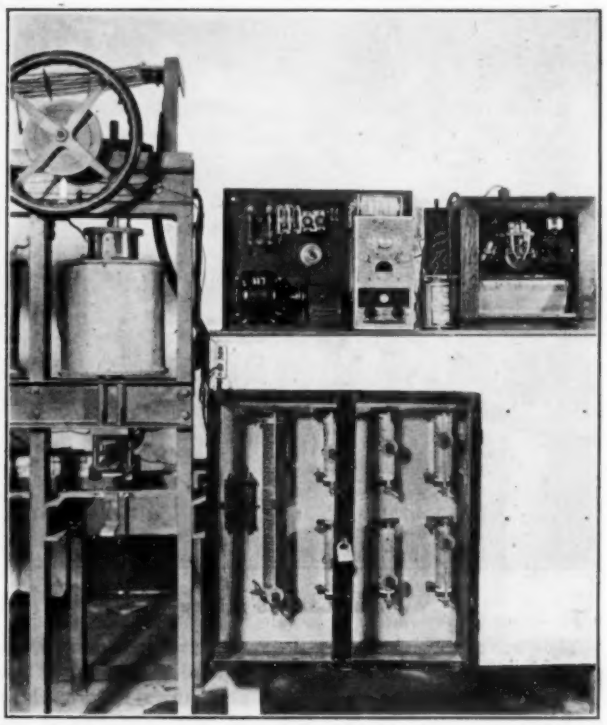


FIG. 2 TEMPERATURE-CONTROLLING MECHANISM AND TEMPERATURE RECORDER

through a compound lever operating on knife edges was chosen for loading. This arrangement has the advantage of a minimum amount of friction combined with compact size and convenient

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

operation. In order to produce a stress of 75,000 lb. per sq. in. in the 0.505-in. diameter specimen, a total load of 15,000 lb. is required at the specimen. This is obtained with the compound lever as shown, which has a total multiplying effect of one hundred, by applying weights amounting to 150 lb. This weight is low enough to be conveniently adjustable, and yet the ratios of the individual levers are only ten to one, which makes it quite easy to adjust them accurately. For convenience the levers were constructed with two of the knife edges attached to one side and one to the other, and it is therefore necessary to maintain them quite close to the horizontal position. The travel of the weights was limited so that the levers could only turn through a small angle, and in addition a small level was attached to the top of the primary lever to indicate when it was accurately horizontal. An electrical contact was also attached to this primary lever so that by moving the upper head of the machine through the hand wheel, worm and worm gear, and screw, the lever could be adjusted from the front of the machine by observing a small indicating light connected in series with the electrical contact. A swivel joint moving in the plane of the framework on the machine was provided at the bottom of the specimen, and a universal joint allowing motion in all directions was attached between the top of the specimen and the adjusting screw. This universal joint was found necessary to insure accurate axial loading of the specimen. This system provides a satisfactory means of applying loads up to 75,000 lb. per sq. in. to the specimens, although this maximum load is required only at relatively low temperatures. For higher-temperature tests the primary lever is used alone, with the weights suspended directly from it. In this case a correction is made for the loading effect of the secondary lever.

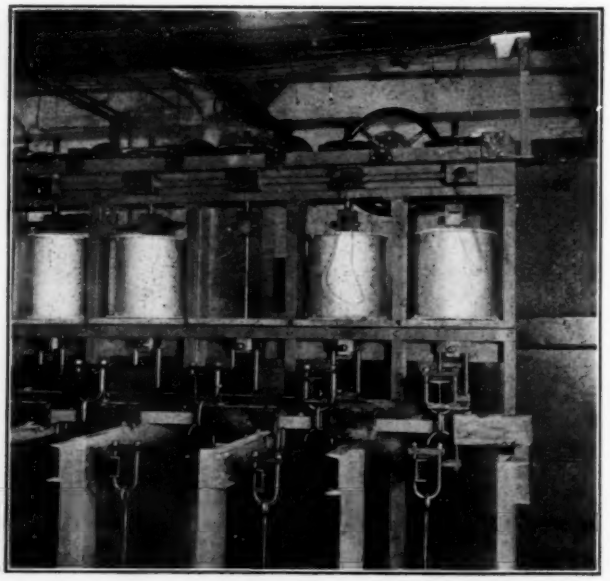


FIG. 3 LOADING SYSTEM

The furnace for maintaining the specimen at the desired temperature is constructed of an alundum tube slightly larger than the specimen; this is fastened inside of a cylindrical sheet-steel case with asbestos-board heads. Sil-O-Cel heat insulation is used between the alundum tube and the furnace shell. The furnace is attached to the upper universal joint so that it moves with the upper end of the specimen, maintaining a tight contact so that there will be no circulation of air through the furnace and past the specimen. This is quite important, as a construction which would allow relative motion between the furnace and specimen would be likely to produce variations in the temperature of the specimen. A Wilson-Maeulen temperature controller was chosen to maintain the temperature constant. A single controlling couple of chromel-alumel wire is used to control the temperature on one furnace. The controller turns the power off and on for all six of the furnaces, according to the temperature of this couple. The temperatures of the other five furnaces are adjusted to be equal to that of the master furnace by series resistances. A Leeds

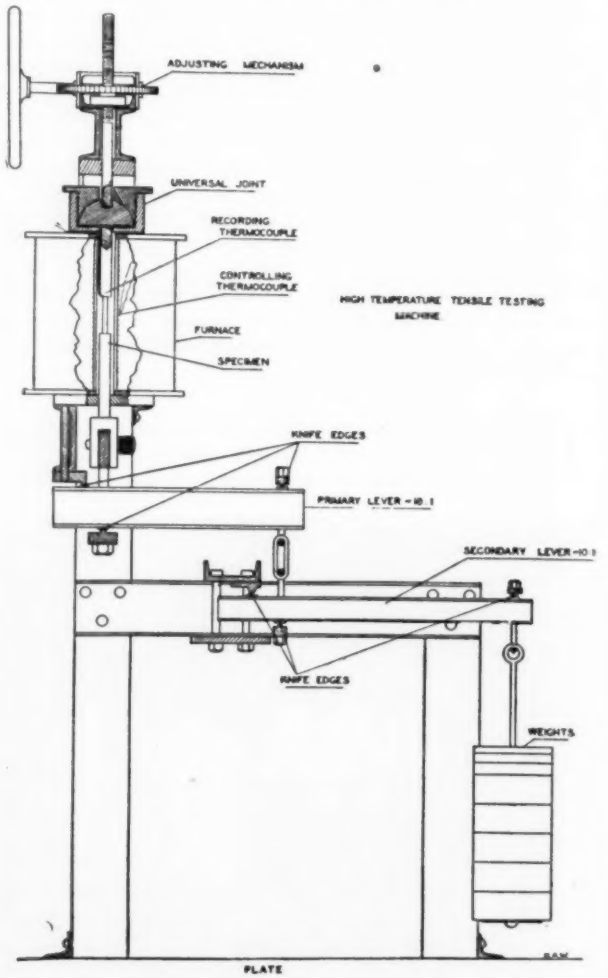


FIG. 4 SECTION THROUGH ONE OF THE ELEMENTS OF THE MACHINE WITH THE FURNACE BROKEN AWAY TO SHOW THE SPECIMEN

& Northrup six-point temperature recorder makes a record of the temperature in each furnace from a recording couple which is inserted in a small hole in the upper shoulder of each specimen. The series resistances are varied until the recorder shows the temperatures of all six of the furnaces to be equal. The controlling couple is not attached to the specimen, but is placed on the outside of the furnace tube in close contact with the winding, from which it is separated by a thin layer of alundum cement. The temperature calibrations on the dial of the controller are entirely disregarded and it is merely set to give the desired temperature at the specimen, the temperature of the winding being from 100 to 200 deg. cent. higher than that of the specimen. If the controlling couple is attached directly to the specimen, a wavy temperature line is produced on the recorder due to the time lag in the transmission of the heat from the winding through the alundum tube to the specimen. With the controlling couple attached

directly to the winding this variation is entirely eliminated, since the temperature of the winding itself is maintained accurately constant. The winding on the furnace is spaced with the turns farther apart at the center of the furnace than at the ends to compensate for the heat conduction through the ends of the specimen. By experimenting with various spacings, an arrangement was found which gave a sufficiently uniform temperature over the length of the reduced section of the specimen.

Methods of measuring the small amounts of creep which occur in tests of this kind offer the most difficulty in operation. The attachment of any form of extensometer directly to the specimen necessitates a less satisfactory form of furnace construction and carries possibilities of changes in the measuring instrument itself due to long-continued operation at high temperature. Optical methods also necessitate special furnace construction which is likely to produce non-uniformity of temperature and involve difficulties in distinguishing the marks on the specimen at high temperatures, especially in cases where considerable oxidation occurs. The method adopted in this machine was to make the large part of the specimen of sufficient section so that its elastic limit could not be exceeded with the load which the reduced section could support. This made it certain that all of the creep would take place in the reduced section of the specimen, and it could then be measured by the motion of the upper head of the machine, since the loading lever is maintained horizontal. The vertical shaft which is connected to the upper half of the universal joint is supported from the top of the machine by a threaded nut which rests on a thrust bearing. This nut is turned by a worm wheel attached to it, which in turn is rotated by a worm attached to a shaft carrying a hand wheel. A calibrated dial is also attached to this shaft and arranged so that the movement of the upper head of the machine is indicated in ten-thousandths of an inch. The furnace is adjusted to a constant temperature, a load applied, and the lever leveled. A zero reading is then taken on the dial and any creep occurring in the specimen is indicated by the movement required on this dial to bring the lever back to the level position. This method has been checked several times and has been found to be sufficiently accurate for any test where creeps of less than one ten-thousandth of an inch in 2 in. are not important.

This machine has given satisfactory results for the purposes for which it was designed, and this description is submitted in the hope that it may prove useful to others who are facing similar problems and promote suggestions and criticisms which will assist in the development of this type of testing with consequent improvement of materials for high-temperature purposes.

Discussion

J. J. KANTER.[2] The construction of creep-testing machines, as well as conducting the tests over extended periods of time, presents many difficulties which any plan of attack may not completely solve. While the equipment in question embodies a number of excellent and original features, it would appear that possibly more emphasis should be placed on certain of the vital refinements to creep testing.

The heart and soul, so to speak, of accurate and dependable creep-testing apparatus is the temperature regulation and control. In order to conduct tests with the highest degree of accuracy and efficiency each unit should be entirely independent of the others for its temperature control. Although a number of investigators consider the using of one temperature controller for several tests a feasible plan, the failure of the single controller to function naturally jeopardizes all of the tests operating in the units. It is extremely difficult to keep a temperature controller in continuous operation without the danger that at some time when no one is present to attend it something will go wrong—motor stopping, breaking of galvanometer suspension, contactor failure, control-thermocouple failure, or even the master furnace burning out. When using an individual controller for each unit, if any of the failures occur but one test is affected.

Another difficulty of maintaining constant temperature in a battery of units using but one controller would seem to arise when discontinuing a test in the master furnace. Control would necessarily have to be established through a second furnace, necessitating a certain amount of rheostat adjustment to maintain within ± 2 deg. cent. of the desired temperatures. It would be of interest to hear more about the adjusting of the series rheostats required in conducting tests with this arrangement.

In regard to creep measurement, the author suggests that extensometer and optical methods give rise to objectionable furnace-design features. While this is true to a certain extent, careful survey has shown that, with the furnace construction necessitated, simultaneous temperature variation over the reduced section of the test piece need not be more than ± 5 deg. cent. total variation.

Presuming that the author has carefully surveyed his furnaces for temperature uniformity over a reduced section of the test specimen by means of calibration test bars, it would be of interest to know what the variations, if any, amount to at the various testing temperatures.

The optical method of creep measurement is objected to on the ground that difficulty is experienced in distinguishing specimen marks, particularly at high temperature. While this is quite true when features upon the specimen itself are used as gage marks, platinum targets suitably attached serve the purpose quite satisfactorily even at temperatures as high as 1400 deg. fahr. and in the presence of heavy scaling.

The leveling of levers as a precision means of determining creep, it seems, would be open to some of the same objections as an extensometer for high-temperature testing—chiefly that external variations in temperature cause change in relative positions of the parts. Discussion of the effects of such variation upon reproducibility of readings and also the torque transmitted to the test piece through the thrust bearing should not be overlooked.

E. R. FISH.[3] With the increase in boiler pressures and size of units, coupled with great improvements in the fuel-burning apparatus, boiler manufacturers are faced with new problems.

Improvements in burning fuel have resulted in much higher gas temperatures. Higher steam pressure means higher steam temperatures and much greater metal thicknesses of the pressure parts. The increased dimensions of boilers involve stresses of considerable magnitude other than those due strictly to the pressure. As there are important changes in the physical characteristics in the materials used in boiler construction beginning at 650 or 700 deg. fahr., and as all the factors mentioned accumulate so as to make somewhat uncertain the actual stresses to be provided for, it is highly important that the behavior of the materials be thoroughly understood.

Among the several phenomena is the so-called creep effect which takes place, the magnitude of which varies with the loading and temperature. It is important that boiler designers have definite and reliable information on this. Not only are comparative figures for varying temperatures and loads useful, but the absolute quantities which can be used in making calculations also are needed. The development of testing machines which give accurate results is one of the important phases of the accumulation of the needed data.

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[3] Combustion Engineering Corporation, New York, N. Y. Mem. A.S.M.E.

FSP-51-39

Evolution of the Slag-Tap Furnace at the Charles R. Huntley Station of the Buffalo General Electric Co.

By H. M. CUSHING,[1] BUFFALO, N. Y.

The handling of low-fusion-ash coals at high ratings and with high efficiencies is one of the stiffest problems ever presented to a furnace designer. This paper tells how in their attempt to design a furnace that would burn any coal offered on the Buffalo market the engineers developed the slag-tap furnace for burning powdered coal, after ten years' experience in driving stoker-fired boilers at 300 to 400 per cent of rating. Because of the greater freedom in selecting coals for a powdered-coal-fired furnace and on account of the greater capacities that could be obtained from the boilers by that method of firing, also because boilers so fired could be made to respond more readily to sudden changes in load incident to peak-load stand-by plant operation than by any other method of coal firing, powdered-coal firing was chosen for the 60,000-kw. 1926 extension to Charles R. Huntley Station.

To obtain data for designing the furnaces under four cross-drum boilers which were purchased to deliver 200,000 lb. of steam per hour, an experimental furnace was installed under one of them. Maximum ratings were in mind and 275,000 lb. per hr. evaporation was obtained.

The paper shows how these experiments, on a furnace designed to remove the ash in the dry form, led to the development of the slag-tap furnaces for the 1926 extension and again for the 1928 extension, where the ratings were increased still further.

The performance of the slag-tap furnace has exceeded the expectation of its designers. Some of its advantages from both the design and operating standpoint are here enumerated:

It requires a minimum of building volume
It will burn efficiently and at high ratings an unusually wide variety of coals
It produces quick ignition of the fuel
It prevents infiltration of air into the bottom of the furnace
It requires a minimum of ash-handling labor, and
The ash handling is remarkably free from dust and dirt.

THE evolution of the slag-tap furnace, which was first used at the Charles R. Huntley Station in Buffalo about three years ago, has played a very important part in the development of the powdered-coal-fired furnace. Fourteen furnaces so equipped are in daily operation, and twenty more are in the process of manufacture and erection. These furnaces will supply boilers which have an aggregated steaming capacity in excess of 10,000,000 lb. per hour.

The conditions which surrounded the development of the slag-tap furnace were as follows: The Buffalo General Electric Co., in the fall of 1925, decided to purchase four 12,500-sq. ft. cross-drum boilers, 11 tubes high, 44 tubes wide, with 20-ft. tubes 4 in. in diameter, and overdeck superheaters. The existing boilers, 12 in number, were all stoker-fired with an unusually large ratio of grate area to boiler area which permitted 300 to 400 per cent of rating to be obtained for peak-load periods, which might be continuous for weeks at a time. The decision had been reached to fire the new boilers added in 1926 with powdered coal by the direct-fired system. All four boilers were to be installed at once, and one furnace was to be rushed to completion for test in order to get data for the design of the furnaces which must be installed in time for the next high-load season. It was the aim of the designers to build a furnace that would efficiently burn, at high ratings, any coal normally offered in the Buffalo market.

Much of the coal which comes into the Buffalo market has ash of a relatively low fusion temperature, 1800 to 2300 deg. fahr., and a heating value of about 12,500 B.t.u. Some coals which come into this market at approximately the same price range have a higher heating value of about 13,700 B.t.u., and ash of a 2600-deg. fusion temperature, but these cannot be successfully burned on the stokers at the Huntley Station on account of their coking qualities, which prevent free flow of air through the fuel beds.

Ten years of experience driving stoker-fired boilers at 300 to 400 per cent of rating demonstrated that the performance of the furnaces depended largely on the quality of the coal being burned. The qualities of a coal which affect its burning characteristics could not be determined by laboratory tests, which give proximate or ultimate analysis and ash-fusion temperatures only. Often in comparing two coals, the one which showed the highest heating value by laboratory tests has shown the poorest performance on the stokers and has evaporated less water per pound of coal.

None of the engineers of the coal companies who visit this territory have been able to predict the performance of a coal from its chemical ingredients as shown by its analysis. The analysis and ash-fusion temperatures serve only as guides in predicting how a coal will burn. Certain coals burned freely and efficiently when being consumed at the rate of from 20 to 25 lb. per sq. ft. of grate area per hour, but when the rating was pushed up by burning twice this amount per hour, the increased furnace temperatures matted the coal together into a solid mass. In some cases this matting was a direct result of the low-fusion temperature of the ash, and in other cases it was due entirely to the coking qualities of the coal. In any event the poor fireman was helpless when the coal fed to his furnaces changed to a coal of either of these characteristics. In spite of all precautions a carload or so of this character would come through now and then, and it always seemed to appear at a time when extra large amounts of steam were required. The very narrow and definite limits which the stoker-fired furnaces had placed on our coal purchases caused the engineers of the Buffalo General Electric Co. to give much study to this angle of the problem.

The narrow limits referred to above for stoker-fired furnaces apply to a lesser extent to the conventional powdered-coal furnace. With the refractory-lined furnace the engineer was forced to burn a very high-fusion-ash coal if he operated his furnaces at their maximum efficiency, i.e., with a minimum of excess air. He might choose the other expedient of sacrificing efficiency by lowering the furnace temperature by the use of large amounts

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[1] Chief Engineer, Buffalo General Electric Co. Mem. A.S.M.E.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

of excess air. In any event he was forced to operate his furnace at a lower temperature than that of the ash fusion of the particular coal which he was burning, as no refractory so far dis-

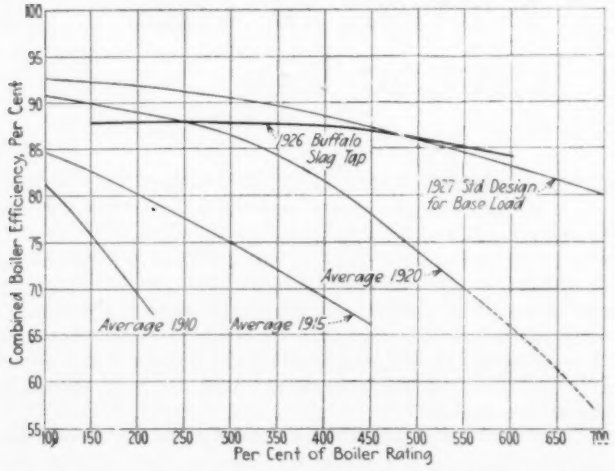


FIG. 1 AVERAGE EFFICIENCIES FOR STEAM-GENERATING UNITS, SHOWING IMPROVEMENTS MADE IN THE FUEL-BURNING AND HEAT-ABSORBING EQUIPMENT

covered will withstand the erosive action of molten ash running on its surface.

This is very clearly shown in Fig. 1 of a paper[2] presented by E. G. Bailey before the Annual Meeting of the Society in December, 1927. Mr. Bailey's paper also shows how sensitive the various powdered-coal furnaces are to this question of the ash-fusion temperature. It also shows how the ratings of the existing furnaces of the conventional refractory-lined type would have to be reduced if they were obliged to burn coals with lower ash-fusion temperatures.

All furnaces before the advent of the slag-tap furnace were designed for, and their successful operation depended upon, the removal of the ash in the dry form. Operators soon learned by bitter experience that if they did not so control the furnace temperatures that the ash was thrown down in a dry form, they would very shortly be forced to take the unit off the line until the solidified slag could be removed by pick and shovel.

Powdered-coal firing was chosen for the 60,000-kw. 1926 extension to the Charles R. Huntley Station on account of the greater capacities that could be obtained from the boilers by that method of firing, and because boilers so fired could be made to respond more readily to sudden changes in load incident to peak-load stand-by plant operation than by any other method of coal firing. The high ratings were shown to be economical for this plant, based on the assumption that the boiler-efficiency curves would drop off as rapidly as the 1920 projected stoker-fired curve, Fig. 1, shown by dotted line, would indicate. This figure shows in curve form plotted against rating, the average boiler efficiency obtainable in the different periods in the development of fuel-burning equipment.[3] Note the sharp decline in efficiency shown by the curves for the earlier periods. The narrow range of reasonably good combustion was caused by the low-set boilers of that day. The high point on the highest curve was obtained by providing a large amount of heat-absorbing surface beyond the furnace. The low-load-factor operation of the Huntley Station would justify the costs of only a moderate amount of heating surface. The actual efficiencies obtained on the 1926 boiler units appear in heavy solid line. The remarkably flat curve is due in part to the slag-tap furnace, as the high temperatures obtained permit the completion of combustion before the gases are chilled by contact with the boiler tubes. This minimizes losses from carbon carried over, or unburned gases. These tests[4] were reported before the Society in October, 1927.

The Buffalo General Electric Co.'s engineers, in laying out the experimental well-type furnace with its 8-ft.-square well,

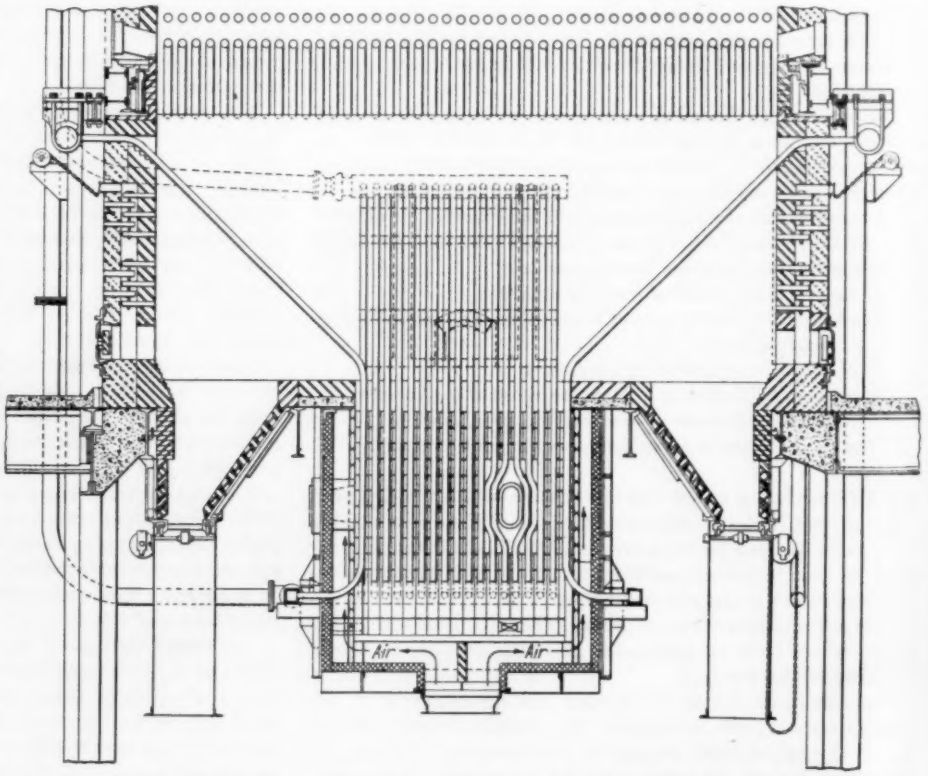


FIG. 2 CROSS-SECTION OF EXPERIMENTAL FURNACE NO. 13, SHOWING HOPPERS INTENDED FOR THE DRY REMOVAL OF THE ASH

hoped to be able to remove the ash in a dry state. They thought that the terrific velocity of the gases resulting from firing 20,000

[2] "Some Factors in Furnace Design for High Capacity," by E. G. Bailey. *Mechanical Engineering*, vol. 49, no. 12, December, 1927, p. 1300.

[3] Reproduced in part from Fig. 4 of the 1927 A.I.E.E. Power Generation Report.

[4] See Fig. 13, "Direct-Fired Powdered-Fuel Boilers With Well-Type Furnaces at Charles R. Huntley Station," by H. M. Cushing and R. P. Moore. Presented at the First National Fuels Meeting at St. Louis, Oct. 10–13, 1927. Trans. A.S.M.E., vol. 50 (1928), no. 8, p. 63.

to 35,000 lb. of coal an hour in this small well might prevent any accumulation of ash therein, and that the ash would all be carried up into the larger furnace above, where, as the gas expanded, it would settle down around the edges and later be hoed or shoveled into ash hoppers provided in the ends. (See Fig. 2.) A dry furnace did actually exist at first; but this was due to too cold a furnace. Only 3 in. of ashes had accumulated in the bottom of the well by the time that 3,000,000 lb. of water had been evaporated. Most of this was slag that had dropped from the boiler tubes above. A partial covering of tile was at this time put over the wall tubes above the well. After 4,300,000 lb. of additional water was evaporated, the furnace was tapped for the first time.

TABLE 1 COMPARISON OF BUFFALO SLAG-TAP FURNACES

Boiler No.	Heating surface, sq. ft.	Burner type	Method of firing	Furnace vol., cu. ft.	Excess air, per cent	Maximum lb. steam per hr., actual[1]	Heat liberation, B.t.u. per cu. ft. per hr. Total furnace	Heat liberation, B.t.u. per cu. ft. per hr. Below top of burners	Time coal enters until its gases leave Total elapsed sec.	Vertical speeds in ft. per sec. Furnace	Vertical speeds in ft. per sec. Well
Old 13	12,515	Calumet	Tangential	5,076	20	275,000	77,800	1,000,000	0.85	17	122
14–16	12,515	Calumet	Tangential	10,200	20	275,000	37,100	234,000	1.35	31	20
New 13	12,515	Cross-tube	Opposed	11,000	20	275,000	34,400	245,000	1.19	24	..
17–20	13,806	Calumet	Tangential	11,626	20	330,000	39,200	250,000	1.28	27	19

[1] These maximum evaporations are 10 per cent more than the continuous rating except for the experimental furnace where the continuous rating was cut down to 150,000 lb. evaporation on account of the severe punishment of the well walls at higher ratings.

While large quantities of ash came down around the edges of the larger furnace, both in powder and granular form, molten ash soon appeared in the bottom of the well and was tapped off as shown in Fig. 3. Note how the white-hot stream of slag, as it dropped some 7 ft. from the furnace bottom, disappeared into the pile of slag on the floor, the surface of which had been chilled by spraying water upon it. The cloud of steam in the background came from this cooling water. The molten slag having been quenched at one point to stop its flow from underneath this pile would soon break out at another point and proceed across the floor like a fiery monster. It was very fascinating to watch the action of this slag as it was tapped on to the boiler basement floor. One could spend hours watching the weird but fascinating spectacle.

The 1926 boilers and furnaces were bought to deliver 200,000 lb. of steam per hour. This expected output was immediately exceeded and evaporations up to 275,000 lb. of steam per hour were actually obtained from this boiler equipped with an 8-ft.-square well and a total furnace volume of 5076 cu. ft.

The volume of the well was 680 cu. ft. At the high ratings, 77,800 B.t.u. per cu. ft. were being liberated in the furnace and more than 1,000,000 B.t.u. per cu. ft. below the top of the burners. This latter value serves as a measure of the local punishment that was being inflicted on the furnace by assuming for the sake of comparison that all of the heat in the coal was liberated below the top of the burners. The heat-liberation values for the powdered-coal furnaces, other than for Buffalo and the experimental furnace at Bayonne, shown in Table 1 of Mr. Bailey's paper, above referred to, range from 12,200 B.t.u. minimum, (19,650 B.t.u. average) to 30,000 B.t.u. maximum, with maximum heat liberation below the top of the burners of 56,000 B.t.u. for Calumet and 57,000 B.t.u. for Narragansett, to compare with the corresponding values for furnace No. 13 of 77,800 B.t.u. and 1,000,000 B.t.u., respectively.

The extremely high values from No. 13 furnace were obtained as a result of the desire on the part of the designers to find the limit of punishment that the materials would withstand. For the other three 1926 furnaces, Nos. 14, 15, and 16, a furnace volume was chosen such that the heat liberation below the top of the burners was 234,000 B.t.u., and for the total furnace volume, 37,100 B.t.u.

In making these comparisons it might be of interest at this time to add the values for the four new 1928 furnaces and the rebuilt furnace No. 13. The 1928 furnaces were installed under boilers with 10 per cent more surface and were expected to produce 20 per cent more output. The furnace volumes were increased only 14 per cent; hence the corresponding heat-liberation values below the top of the burners became 250,000 B.t.u. and for the entire furnace volume 39,200 B.t.u. See Table 1. The rebuilt No. 13 furnace has a volume of 11,000 cu. ft. and comparative values for the heat release are: 245,000 B.t.u. below top of burners, and 34,400 B.t.u. for the entire furnace.



FIG. 3 ONE OF THE EARLY SLAG-TAPPING SCENES WHERE THE MOLTEN ASH FROM NO. 13 FURNACE IS DROPPED ON TO THE BOILER BASEMENT FLOOR

It became evident within two weeks after boiler No. 13 was first started that if the low-fusion-ash coals, which the plant was receiving at the time from mine-stripping operations, were to be burned, designs for the new furnaces must provide for the handling of the ash in the molten form.

It was demonstrated also, in the tests on No. 13 furnace, that the ash could still be removed in the molten form while operating with only one of the two pulverizing mills which normally supply this boiler. The maximum rating, as stated before, was 275,000 lb. of steam per hour, and the minimum with one mill operating, 60,009 lb.

The engineers realized that in deciding on a design to handle the ashes in a molten form they might be falling somewhat short of their original goal of designing a furnace capable of burning any coal offered on the Buffalo market. They reasoned, however, that the furnace conditions would automatically adjust themselves to give a higher temperature and the ash could still be handled in a molten form if they were required to handle the higher-fusion-ash coals, which so far had not been burned in furnace No. 13. Time would not permit of such a test before a decision had to be made for the design of the furnaces for the three other boilers.

Each engineer interested in this development had a different idea as to how this molten slag could be handled. They all agreed, however, that one or more of the schemes suggested offered a sufficiently practical solution to the problem to warrant the adoption at this time of the slag-tap design of furnace.



FIG. 4 LEFT: GRANULAR SLAG FROM SLUICE DISCHARGE; LEFT CENTER: SLAG TUBE FORMATION; RIGHT CENTER: MINERAL OR SLAG WOOL; RIGHT: SOLID BLOCK OF SLAG

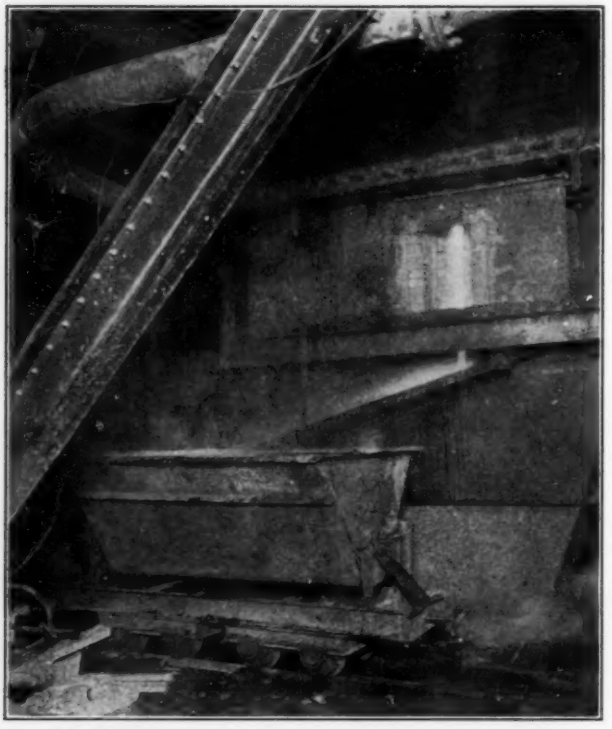


FIG. 5 TRIAL SLUICING OF THE SLAG FROM NO. 13 FURNACE INTO AN ASH CAR

At this stage of the development the engineers began to see the possibilities of obtaining much greater efficiencies at the extremely high loads than the performance of other furnaces had led them to expect. The adoption of the slag-tap furnace promised to be a help in that direction. It was argued that the hot bottom would increase the speed of ignition of the entering coal and air. The three T's of combustion again appear: Time, Temperature, and Turbulence. The Buffalo designs had slashed the time for combustion to a minimum, allowing the coal and its products to remain in the furnace only one and one-quarter seconds. It was necessary, therefore, to play up turbulence and temperature to a maximum. The hot bottom would unquestionably improve the temperature factor. The only point against the adoption of a slag-tap design was that it might possibly be necessary to choose only the low-fusion-ash coals for the new furnaces. For reasons previously stated this was not considered much of a handicap to the proposed new design.

To give a better picture of the problem, the following analyses of coal and slag are given, together with a few of the characteristics of the slag:

Coal (Dry Basis)	Per Cent
Fixed carbon	53.29
Volatile matter	34.66
Ash	12.05
Sulphur	2.85

Slag Analysis	Per Cent
Silica (SiO_2)	44.52
Aluminum oxide (Al_2O_3)	18.64
Iron oxide (Fe_2O_3)	31.35
Titanium oxide (TiO_2)	0.76
Phosphorus pentoxide (P_2O_5)	0.25
Calcium oxide (CaO)	1.33
Magnesium oxide (MgO)	1.10
Sulphur trioxide (SO_3)	1.80
Undetermined	0.25

Fusing point of ash, about 1950 deg. fahr.

When chilled into a solid state the slag had a very low coefficient of heat transmission, a fact that made it serve as a good heat insulation for the floor of the well. The lower part of the slag on the floor solidified due to the passing of secondary air beneath the bottom of the well. This low thermal conductivity also added to the problem of handling.

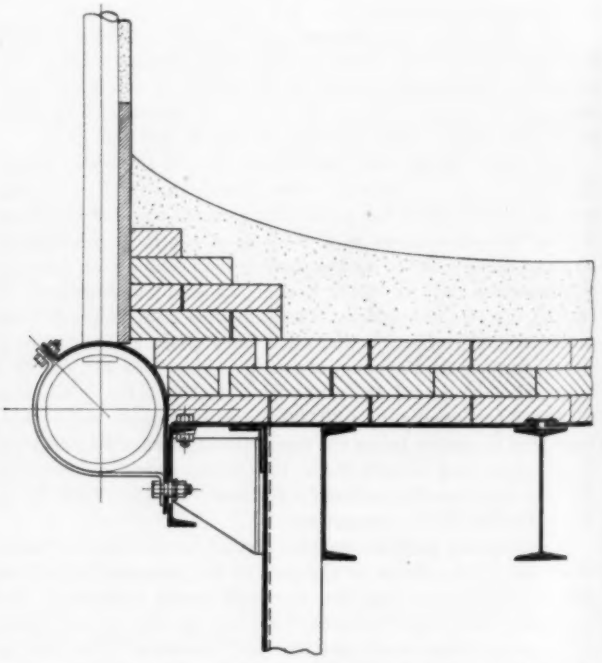


FIG. 6 DESIGN SLAG-TAP FLOOR SHOWING SLAG SEALS AT HEADERS

At a high temperature and below its melting point the slag was tough and viscous, like molten glass. When cool it was a dense black solid, often with a coppery opalescence at a fracture; it was hard enough to cut glass and weighed approximately 200 lb. per cu. ft. When running from the slag spouts into the open air the outer part of the stream congealed and the flow

continued through the center, forming a tube or pipe, one of which is shown in Fig. 4.

While the new furnaces were in the process of construction, experiments were made of dropping the slag directly from the furnace into a concrete buggy filled with water. To our great surprise no explosion occurred and there was little sputtering. The slag stream proceeded to form its own insulating tube to the bottom of the buggy and built up as if there were no water on top of it. The sheet-metal casing on the bottom of the buggy became red hot long before the surface of the water showed any signs of boiling. When the buggy was half-full of slag it was rolled to one side, and some fifteen minutes later it was emptied; the interior of the slag still being red hot.

Sluicing was then tried out by placing an 8-ft. length of 12-in. channel iron, sloped at an angle of about 30 deg., under the spout of the original boiler No. 13, and placing two $^3/_4$-in. water nozzles at the top so as to sweep the channel clean. (See Fig. 5.) The lower end discharged into an ash car.

These tests on sluicing led to observations that were of great value in arriving at the final solution. The slag stream was broken into small pieces by the mechanical force of the jets, and these pieces in turn tended to shatter into smaller pieces, owing to the rapid chilling effect of the water made possible by the exposing of greater surface areas per unit volume of slag. These facts indicated the necessity of using comparatively large volumes of water and were responsible for hydro-jet handling.

The design of the slag-tap bottom which has been installed substantially as shown on all eight slag-tap furnaces now in operation at the Huntley Station is shown in Fig. 6. A sheet-metal plenum chamber with angle and channel stiffeners is built on and rigidly supported from the boiler basement floor. This plenum chamber supplies the secondary air to the powdered-

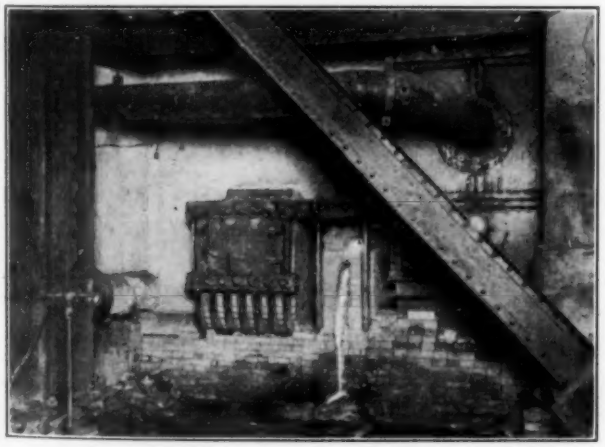


Fig. 7 Original Design of Slag Spout and Access Door

coal burners. The sides of the plenum chamber were made of concrete for furnaces Nos. 14, 15, and 16, but the later ones are all steel. The roof of the chamber is built very strong to support the furnace bottom with the load of from 24 to 30 in. of slag which might accumulate on it, also to take some thrust from the furnace side walls as they move up and down past this bottom. The maximum movement at this point amounts to $^3/_4$ in. Upon the 12-in. I-beams spaced 24 in. apart, $^1/_4$-in. steel plates are laid. Then three layers of a good grade of firebrick are laid—not too tightly—with their smallest dimension vertical. Around the edges four additional layers of firebrick are corbeled up to form a basin. About 6 in. of burned dolomite is then spread over the top. Slag from one of the other furnaces was used in place of the dolomite on the last furnace built, remodeled furnace No. 13, with equally good results. Special attention is called to the design of the slag and air seals, where the relative movement occurs between the furnace walls and the floor. It is very important that these are carefully assembled so that the parts cannot become unbolted and allow the molten slag to have a free path to the outside, which it is sure to find if cracks are allowed to open up. The cases where molten slag has gotten out on the floor have been caused by nuts not having

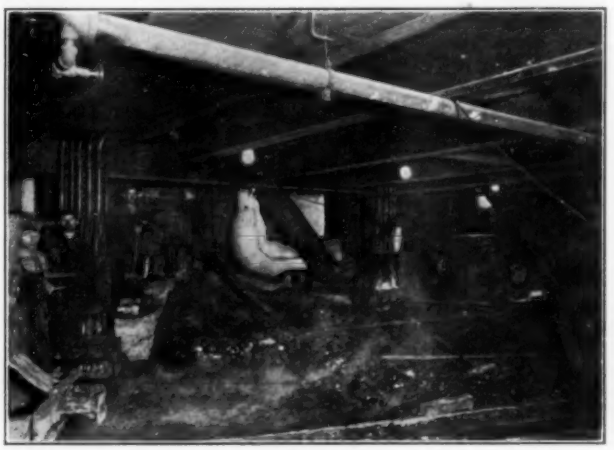


Fig. 8 Mass of Slag on Floor to Be Removed by Pick and Shovel

been put on in the first place, or having become loose and fallen off in operation. In the original job the workman could see no need of installing the nuts on the bolts which hold the bottom plates to the outside channels of the end walls, as it was impossible for these bolts to get out of place. He did not realize that as the boiler got hot it would push the lower header down about $^3/_4$ in. and carry this channel with it. The furnaces operated for months and nothing happened until after several consecutive 24-hour runs of one of the boilers at high rating, the slag broke through one evening, a couple of hours after the slag had been tapped very low, and several tons ran out on the floor to the consternation of the operators. The boiler was cut off the load as soon as possible and the hole soon froze shut. Being desirous of completing a test then in progress the boiler was put on the load the next morning, but at the end of another 48-hour run, broke out in another spot. The slag is a very good heat insulator and never has it broken out twice in the same place, nor has it ever broken out since the seals were properly put together.

In the original design the access door for the furnace was put at the bottom and adjacent to the slag-tap opening, so as to make use of a common header construction. Fig. 7 shows this construction. Back of the access door and in line with the furnace wall a 9-in. course of carborundum brick was laid up after each internal inspection to seal the opening. These bricks wasted away rather rapidly and the Bailey blocks adjacent to this opening could not withstand the punishment. These blocks had a great deal larger ratio of exposed surface in the furnace to tube-clamping surface than the other blocks in the well.

Note in Fig. 7 the slag as it starts to run on the floor and the loosely laid-up bricks protecting the lower parts of the plenum chamber. Fig. 8 shows the job which confronted the pick and shovel gang the next morning, with some 15 tons of ashes to be removed from the boiler basement floor.

The original tapping spout for the slag was made of firebrick supported on the circulating tubes for the water-wall headers,

as shown in the same Fig. 7. Fig. 9 shows the next step in this development. A hinged cast-iron spout permits the operator to raise the spout to stop the flow of slag, or to reduce the volume of an unusually large flow. The slag never sticks to smooth cast iron. As it cools its high coefficient of contraction makes its removal easy. The later designs of spouts are also of cast iron. They are attached to the water walls and equipped with

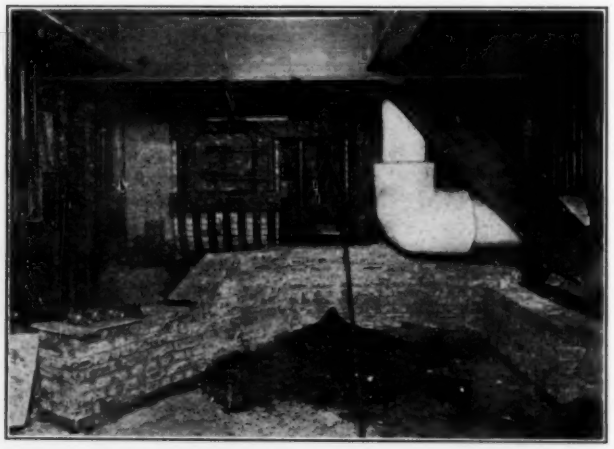


FIG. 9 HINGED CAST-IRON SLAG SPOUT

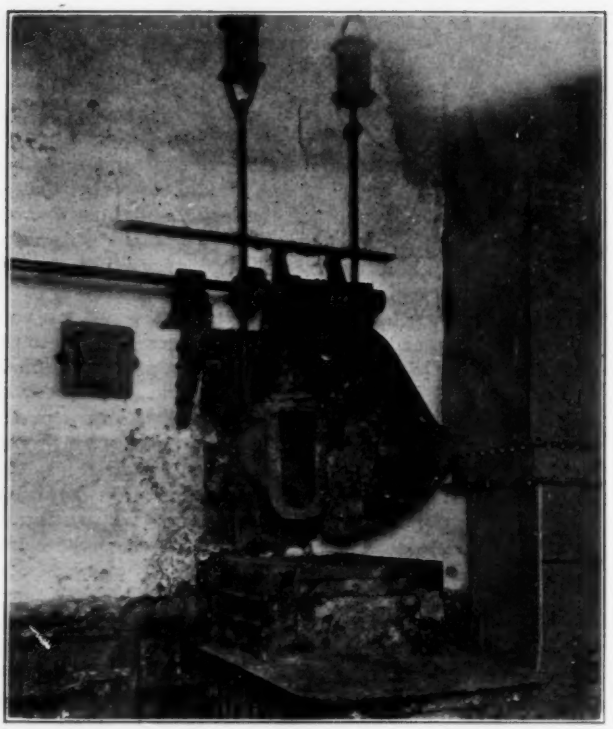


FIG. 10 WORM DRIVEN SLAG SPOUT GATE

chain-operated worm-wheel-driven cut-off gates operated from a point remote from the tapping spout as shown in Fig. 10. The gate as designed can be easily backed off by loosening a few bolts, which permit opening it should there be any binding from contact with the slag. It was expected that the gate would bind on account of the accumulation of molten slag behind it. This has not been the case. Although streams of slag 6 in. in diameter have been easily closed off with this gate, the gate has always been opened easily as soon as the slag behind it cooled. The cooling of the slag behind the gate leaves it in a very brittle condition, filled with shrinkage cracks which materially assist in clearing the tap hole for the next tap.

At the same time that the new design of tapping spout and shut-off gate were installed, the access door was removed from the bottom of the furnace and placed up in the V-shaped section of brickwork just below the main boiler tubes and above the water walls in the side walls of the boiler setting. An aluminum ladder especially designed to permit its use while the boiler walls are still hot has been designed. (See Fig. 11.) It is 30 ft. long and weighs 100 lb., and is flexible and can be easily inserted into and removed from the furnace.

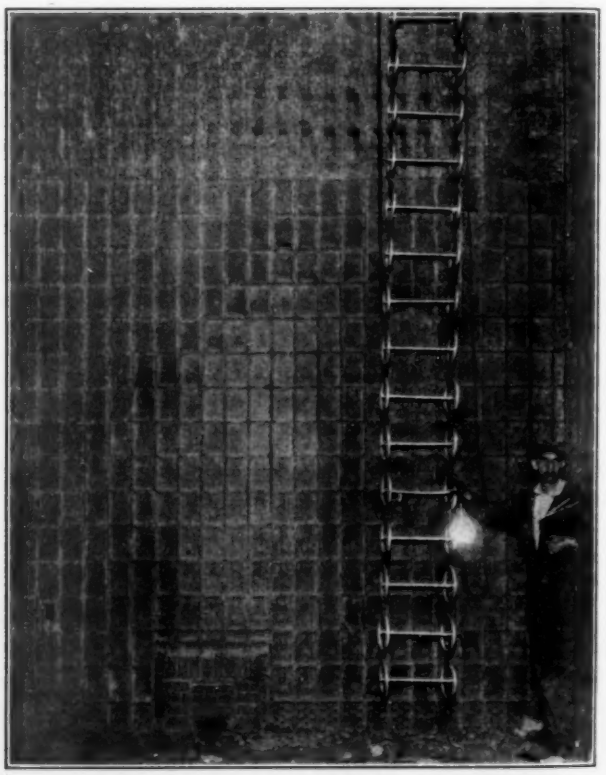


FIG. 11 FLEXIBLE LADDER FOR FURNACE ACCESS

As late as May, 1927, no definite plans had been agreed upon for the practical handling of this molten ash. Numerous suggestions had been made as to the best method of disposal of this slag, such as refractory-lined larry cars to handle the ash in a molten state; pouring the slag into water-jacketed cars; pouring the slag into water-filled dump cars direct from the furnace, somewhat as shown in Fig. 8; also various methods of sluicing the slag into a drag-chain conveyor which was run in water at the first part of its travel and then carry the chilled slag up an incline to the standard ash car used at the plant.

Not until all three of the new slag-tap furnaces Nos. 14, 15, and 16 had been put in service and the series of boiler tests had been completed, as reported at the St. Louis A.S.M.E. meeting[4] above referred to, was a satisfactory scheme of handling the slag worked out. Sluicing the slag had been seriously considered for some time, but the existing building beams would not permit of the installation of such a scheme if the existing practice of building sluiceways must be adhered to. Practice said

that right-angle turns introduced into such a system would lead to trouble. The manufacturer of this equipment, after satisfying himself that none other than right-angle turns would make a practical job at the Huntley Station, offered to furnish the material for a trial installation to be built above the boiler-room basement floor. The trial installation proved such a success from the start that the manufacturer was given orders to proceed with the manufacture of four equipments based on this first day's trial. The temporary equipment sluicing the slag into a hole in the yard continued in service until the permanent equipment was received. Fig. 12 shows the design of the sluicing system. The slag spout is not the latest design. The molten slag falls from the slag spout on to a water table formed by four primary jets of water, where it is cut into small pieces and partly chilled. Falling from here into a U-shaped cast-iron trough, it is then carried crosswise the building with the aid of two carrier jets for a distance of approximately 10 ft., and still further chilled in the process, owing to the additional cold water. This flow hits a cast-iron smashing plate, where it drops about 6 ft. and is turned at right angles into another U-shaped cast-iron trough running lengthwise the building and discharging

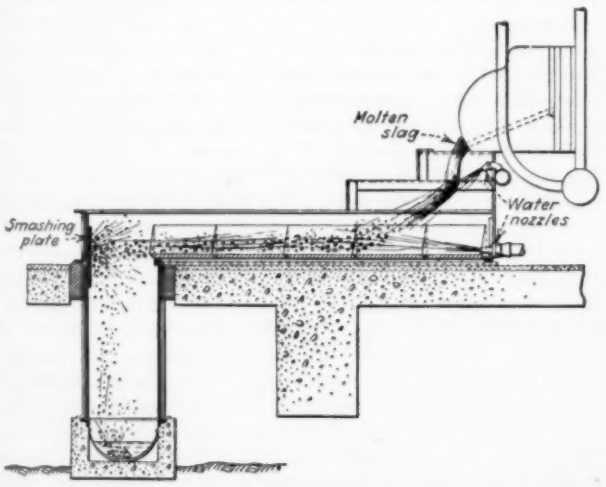


FIG. 12 SECTIONAL LAYOUT OF SLUICING SYSTEM

into a sump pit. Two additional carrier nozzles working in tendem are placed just behind the point where the slag strikes the lower sluiceway. The slag is deposited in a fine granular state at the discharge of the chute. Although pieces of slag 18 to 20 in. long, of a formation similar to the tubular specimen shown in Fig. 4, are often dropped into the sluicing system, they are hot enough though black in color to be shattered to bits in passing through the sluice. A grating is placed over the pit at the discharge from each sluice to catch pieces too large to be handled by the 4-in. diameter 760 g.p.m. vertical manganese-steel pumps, which are installed under the boiler basement floor. Two pumps are installed and are used alternately or together if a number of the boilers are being tapped at the same time to pump this slag a distance of 1000 ft. to the yard. Fig. 13 shows this discharge in service. All elbows in the suction and discharge piping are made of manganese steel. The discharge pipes are 6 in. in diameter and made of steel for the straight runs. The pipes are pitched to make them self-draining and are not insulated. Note that the material discharged spreads over a wide area instead of forming a pile beneath the pipes as had been expected. A pressure of 140 lb. is used at the sluice nozzles.

It is remarkable that this installation, which is the first to be designed for handling molten slag from boiler furnaces by the hydro-jet system, should have proved so successful from the start. Had not the low furnace-floor levels of boilers 14, 15 and 16 made the handling of this slag in the molten state by means of slag cars extremely difficult, this happy solution for handling it might not have been found.

Fig. 14 shows the molten slag discharging into the present sluicing system. This photograph was taken before the location of the access door was changed.

The question is often asked as to how the slag-tap design affects the slagging of the main boiler tubes. In the author's opinion, it lessens the slagging of the boiler tubes in a properly



FIG. 13 DISCHARGE FROM SLAG PUMPS

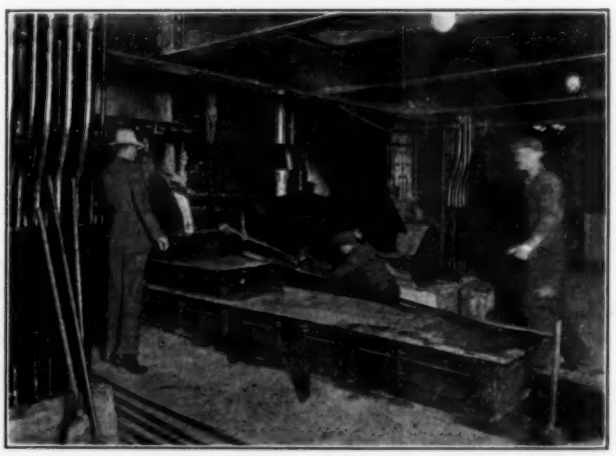


FIG. 14 PRESENT SLUICEWAY IN OPERATION

designed furnace in permitting the combustion to be completed lower down in the furnace, on account of the quicker ignition of the fuel.

The handling of low-fusion-ash coals at high ratings and with high efficiencies is one of the stiffest problems ever presented to a furnace designer. The paper has so far only dealt with the development of the slag-tap bottom for a slag-tap furnace. The walls themselves present an even greater problem. Obviously a slag-tap bottom cannot be successfully operated with any furnace whose walls will not withstand the action of molten slag running down its surfaces. This rules out the air-cooled refractory-faced wall for such service. It is the author's belief that only such walls are suitable for this service as will permit the molten ash to run down their surfaces, thus maintaining

the exposed part of the adjacent wall surface at a temperature at or above the ash fusion. The water-cooled refractory-lined walls that live under this service do so by virtue of the fact that the surface of the wall is maintained below the freezing point of the ash by its heat-conducting qualities, which carry the heat away from the surface exposed to the fire so fast into the water inside of the tubes that the wall-surface temperature is always kept below the ash-fusion temperature. This causes a layer of frozen ash to form over the wall, protecting it against further action of the slag and providing a self-renewing surface for the slag to flow over, for when the slag removes frozen slag by the erosive action of mechanical friction, a new layer is immediately formed. To make this self-renewing action possible the wall surface must no doubt be quite a lot cooler than the actual freezing point of the slag.

The formation of ash on wall blocks is very interesting. Examine a smooth water-cooled iron surface which may have an accumulation of frozen slag on it, and it will be found that it is very insecurely attached and will drop off at a touch if it is not otherwise attached to enable it to span this smooth surface. There seems to be a thin, fluffy slag between the bare iron and the solid slag, as if the first particles of ash to adhere to the iron were sufficiently cooled in approaching the relatively cold surface to deposit as fluffy ash.

The performance of the slag-tap furnace has exceeded the expectation of its designers. It has many advantages from both the design and operating standpoint; for instance:

It requires a minimum of building volume
It will burn efficiently and at high ratings an unusually wide variety of coals
It produces quick ignition of the fuel
It prevents infiltration of air into the bottom of the furnace
It requires a minimum of ash handling labor, and
The ash handling is remarkably free from dust and dirt.

Discussion

HENRY W. BROOKS.[5] Far-sighted engineering good judgment plus the courage of his conviction characterizes a contribution to fuels-engineering history by Mr. Cushing and his staff, which ten or twenty years from now even more than today we shall look back on as one of the outstanding developments of the fuels-engineering arts. For at least fifty years past, ever since man began to appreciate the efficiency possibilities of high-temperature furnaces, every effort has been concentrated, in boiler-furnace practice at least, either on the prevention of slag or the minimizing of its destructive effects.

Suddenly Mr. Cushing and his associates, based on their study of experiments, which though carefully done, were *only* experiments, decided to cast away all previous experience in this regard and to solve a difficult problem in an entirely new way. As it was the writer's privilege to work with Mr. Cushing and his associates during these days, perhaps no one else can so well appreciate to just what extent Mr. Cushing hazarded his engineering reputation on the success of this project. Once, however, Mr. Cushing had seen the character and extent of the experiments on the new form of combustion at Fullerton and had had an opportunity to study the results of this type of furnace at the Sherman Creek Station in New York, he became convinced that to generate peak-load power as an auxiliary to the cheap Niagara Falls hydroelectric power and to maintain anywhere near reasonably consistent costs required heroic treatment. He became convinced that the only possible means of accomplishing this, which the then state of the art permitted, was the type of furnace he adopted. To have talked of stations of initial cost of the usual $100 to $125 per installed kilowatt in this case would have been ridiculous, for the fixed charges alone would have necessitated selling at a loss on the basis of the low rates in the Niagara district. The problem put up to the engineers of the Buffalo General Electric Company was to design an efficient and workable plant at nearly half the usual accepted power-plant cost. They did it. They succeeded beyond their own expectations. In doing so, I am of the firm belief they have initiated a new era in the combustion of low-grade coals. Consider for the moment another view on this subject. Unquestionably the principal reason for the being of pulverized coal has been its claimed ability to burn with greater efficiency the lower grades of coals. Can this claim really have said to have been true prior to the Huntley installation? As one whose job for many years has been to cure "sick" powdered-coal jobs all over the United States, I am in position to answer this question unequivocally in the negative. Whenever low-ash-fusion coals had been encountered previously, trouble in large quantities always confronted the operator. If he obtained any decent rating out of his boilers, he did so at the expense of his dry-gas loss, for there was but one known method of combating wall punishment in those days and that was to decrease the furnace temperature by increasing the CO_2. Literally dozens of furnace walls, both solid and air-cooled, in my own experience, had buckled and in many cases fallen in, while in countless other cases wall erosion, due to slagging, had increased furnace-maintenance cost vastly beyond the savings which the pulverized-coal method made possible. The success of Mr. Cushing's furnaces once and for all definitely demonstrated the large-scale feasibility of abolishing the problem which for a time threatened the very existence of the newly developing powdered-coal art. As will be observed from the paper, this translation of scientific possibility into practical operating fact was not accomplished by a mere wave of the hand. No really successful technical accomplishment ever is, in spite of the natural enthusiasm of inventors. Fortunately, in this case the new-construction program had preceded the actual demand for steam by a sufficient period to permit ample experimentation before the equipment would be called on for steady service. As was expected, many minor difficulties had to be overcome. The fundamental basis, however, on which the design was made had been thoroughly proved, so that the application of calm, intelligent engineering thinking, with patience and perseverance, finally solved the problem. That the problem is solved and to the satisfaction of every one connected with it is demonstrated by the second installation of the same type as the first.

Financiers and company operating executives are notably hard-boiled when it comes to sinking hundreds of thousands of dollars into relatively new and untried projects. The fact that they did so the first time might have been attributable to incorrect judgment to which we are all prone. The fact that they did it the second time, however, coupled with world-renowned success of the property they direct, means that they must have had thorough proof that their initial judgment had been correct. Any one who has seen and studied the Huntley Station will not question this.

In conclusion, I think I am violating no confidence in telling you that the initial cost on this peak-load station, even including all of the experimental work which has gone on there, has been approximately one-half the initial cost of the average of all similar generating plants which have been built in the United States in the last four years. The initial cost of this plant is so far lower than its nearest competitor along these lines that Mr. Cushing has only preserved his engineering veracity at the expense of reticence on a subject vitally important to the entire engineering field. Furthermore, based on monthly operating logs which I

[5] Consulting Engineer, New York, N. Y. Mem. A.S.M.E.

had the privilege of seeing when last in Buffalo, taken at random since the initiation of this new development, I find, much to my surprise, that the overall station economy in B.t.u. per kilowatt-hour is exceeded by but few stations in the United States. I should be violating a confidence if I told you just how low these figures are, but I believe that I can say that they are within less than 2000 B.t.u. of the most efficient station now existing in the world. As I happen to know that this latter station cost initially more than two and one-quarter times as much as the Huntley addition and as all of us are familiar with what a large bearing the fixed charges have on the total average cost of generated central-station power, I think it can be safely said that, on an equivalent coal price, Huntley Station is today perhaps the lowest-cost generating station in the world. I may also add my observation that the accomplishment of this is at least 90 per cent attributable to the ability and foresight of Mr. Cushing and his associates.

E. J. Billings.[6] The author is to be congratulated upon this very frank report of his experiences and the fact that he has always given freely to the engineering profession the benefit of his experiences in handling ash from a boiler furnace in liquid form.

Liquid-ash removal or the slag-tap furnace is recognized by the leading engineers as a development which has permitted the art of pulverized-coal firing to overcome one of the most difficult problems connected with this method of burning coal. The limitations of pulverized-coal firing as pointed out by the author have not in the past primarily been in the pulverizing and burning equipment but in the furnace design. The slag-tap furnace with proper water-cooled construction now makes it possible to design suitable furnaces that are capable of handling coals having wide ranges of moisture, ash, and ash-fusing temperature.

The merits of this method of ash removal are evidenced by the fact that there are 17 slag-tap furnaces of one particular manufacturer's design that are now in daily operation and 26 others in the process of design and erection.

The results of an investigation of a large block of slag taken from the center portion of a slag-tap furnace floor after the boiler had been operated over a period of $2^1/_2$ years are of considerable interest. The block was approximately 18 in. thick from the top surface of the slag. Various analyses were made from samples taken at different elevations, as shown in Table 2.

TABLE 2 ANALYSIS OF BLOCK OF SLAG

Distance from top, in.	Per cent					Fusing temp., deg. fahr.
	SiO_2	Al_2O_3	FeO	CaO	MgO	
1/2	39.03	23.86	34.30	1.40	1.41	2045
5	39.63	22.90	33.86	1.54	1.34	2045
9	38.50	23.78	35.06	2.40	1.64	2065
13	28.06	25.58	32.14	9.70	3.73	2140
16	29.80	27.05	18.30	10.20	13.44	3000

It will be noted that down to the 9-in. elevation the fusing temperature remained substantially constant, and at the 16-in. elevation slag penetration has not been sufficient to lower the high-fusion filler, which in this case was burned dolomite. The rapid increase in CaO and MgO below the 9-in. elevation also attests this same condition.

The furnaces at Buffalo have a plenum chamber beneath them which permits circulation of the air used for combustion on the under side of the steel plate supporting the furnace floor. Although this is an expedient for this particular burner and furnace arrangement, it is not essential that the floor be cooled in this manner. If the supporting steel is arranged so that there are no air pockets formed on the under side of the furnace floor, natural circulation of boiler-room air is sufficient to keep the floor-supporting members from being overheated.

The flexible ladder which the author mentions for furnace access is a very ingenious piece of equipment. One man can handle this ladder very conveniently.

F. B. Allen.[7] Mr. Cushing has quite thoroughly outlined the method of molten-ash handling employed at Buffalo. It so happens that the writer, together with other engineers of the company with which he is associated, spent considerable time at Buffalo in connection with this problem; in consequence, there are certain details that he feels might be emphasized.

The fundamental principle consists in tremendously increasing the area of molten ash, at the same time reducing it to smallest practical size while it is in the molten state. After this has been done it is of course relatively easy to chill each particle below its fusion point, because

(*a*) As there are literally millions of particles per ton, the total area per ton is extremely large, and as all or most of this area comes in contact with water, the heat release is tremendous.

(*b*) As all or most of the particles are very small, their maximum thickness is insufficient to allow the surface of the particle to be chilled while the center of the particle remains molten.

Although as Mr. Cushing says the ash-handling system has been successful from the first, yet minor difficulties have developed which had to be met as they appeared, such as:

1 It is important that the water table discharge parallel with the flow of the slag down the spout. It should not be at right-angles to this flow, because the quantity of slag discharge varies tremendously; in consequence, the path of the ash will vary nearly a foot from maximum to minimum discharge, and if the primary jets forming the water are not parallel with the flow, the slag may not reach or may partially override the table.

2 Care should be taken that the water quantities are sufficient for the maximum slag discharge, and as this is perhaps twice the average it is probable that considerable power could be saved by the installation of valves that would allow of the gradual reduction of water quantities after the first floods of slag had subsided.

3 Horizontal manganese-steel pumps were originally installed for pumping the disintegrated ash particles together with the water out of the plant. These operated very well, but had the disadvantage that (as they necessarily were installed) they had to be primed before starting, and occupied valuable floor space on the mill floor. Later, therefore, vertical pumps were installed. Detail troubles developed in these pumps. It was found that the stuffing box along the shaft gave trouble, as it was impossible to adequately inspect it. The pump was redesigned so as to omit this. Other troubles followed, most of which resulted from difficulty with the bearing below water. These are being met as they develop.

It was also found very difficult to operate two pumps through a common discharge line, and a second discharge line, one for each pump, was later installed.

From the standpoint of a manufacturer of ash-handling machinery the writer can say that the most outstanding feature of the installation made by Mr. Cushing and his engineers is its absolute cleanliness. There simply is no dust from the ashes, as none of the ash is in the form of dust.

Another thing that clearly stands out is the thoroughness with which Mr. Cushing and his engineers attacked the ash problem. Every furnace of the slag-tap design put in operation since Buf-

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[7] President Allen-Sherman-Hoff Company, Philadelphia, Pa.

falo, so far as is known by the writer, uses substantially the same ash-handling principles.

Author's Closure

The data presented by Mr. Billings, as to the composition of slag from various depths in the floor of the slag-tap furnace, are very interesting and show how little effect the composition of the floor has on the fusing point of the main body of ash. The tests made on one of the Baffalo furnace bottoms confirms the information given by Mr. Billings, that forced circulation of air underneath the steel-plate floor is unnecessary provided free unpocketed natural circulation is available. In this experiment the supporting steel I-beams were insulated with $^{7}/_{8}$-in. matched lumber, sealed with tar paper. Thermocouples placed on the top flanges of the I-beams adjacent to the steel-plate bottom showed a maximum temperature of 200 deg. fahr., which maximum temperature was more than 50 deg. hotter than the operating temperature and occurred as a gradual rise when the furnace was cut out of service. The furnace was operated during this test from ten to fourteen hours each day for a week at about 450 to 500 per cent of rating.

The principles underlying the successful handling of molten slag by hydro-jet system are clearly set forth in Mr. Allen's comments. The manganese-steel horizontal pumps of which Mr. Allen speaks were in service for more than 18 months, handling all the slag from four furnaces, and have been in intermittent service during the last nine months, handling slag from eight furnaces. It is interesting to note that in the handling of this highly abrasive material, no pump parts, with the exception of bearings, have had to be replaced.

FSP-51-40

Proprietary Air-Cooled Refractory Walls

Evolution of the Boiler Furnace—Forms of Patented or Proprietary Air-Cooled Refractory Walls Now Available—Data on Performance of Air-Cooled Walls in Service

By C. S. GLADDEN,[1] DETROIT, MICH.

This paper discusses briefly (1) the evolution of the boiler furnace and the increasing tendencies toward higher ratings of boiler operation and the attendant increasing maintenance troubles with refractories; (2) describes some of the proprietary air-cooled walls now available for the consideration of designing engineers and boiler-plant operators; (3) gives the operating experiences of certain engineers with air-cooled refractory walls of different types, and (4) presents the author's opinion concerning the conditions suitable for air-cooled refractory-wall installations.

THE author of this paper does not pretend to present all the facts and figures that may or may not be available to definitely establish rules for determining when air-cooled furnace walls should or should not be used for boiler furnaces. Engineers interested in combustion problems differ widely in their opinions concerning this subject, and this difference of opinion is often justified by the particular angle or point of view from which they approach the subject.

This paper, then, will attempt to trace, in a sketchy fashion, the evolution of the boiler furnace, describe the forms of patented or proprietary walls now available, and present certain data concerning the performance of walls which have been in service.

EVOLUTION OF THE BOILER FURNACE

About 160 years ago Watt was engaged in improving and in developing new features of the first steam engine. The boilers furnishing the steam for these engines are not described in engineering literature, but they are supposed to have been of the cylindrical shell type, fired externally with coal in furnaces constructed with some sort of refractory materials.

About 20 years after the historic date, 1769, that Watt began his improvements of the Newcomen pumping engine, we read of the first successful attempts, by James Rumsey, to construct water-tube boilers in the United States, although patents for several forms of boilers of this type had been previously brought out in England. Numerous forms of water-tube boilers were developed during the next 60 or 70 years, but the art of boiler construction at that time had not advanced sufficiently to place their manufacture among commercially successful enterprises. On the other hand, the shell type of boiler was comparatively easy to build.

The boiler used by Fulton in the *Clermont* and by Stephenson in the first locomotive was designed with a single flue lengthwise through the lower part of the main boiler shell. Soon after the development of the first locomotive and the first steamboat, horizontal return tubular boilers and Scotch marine-type boilers were constructed. The return tubular boiler required a furnace and setting constructed of refractory walls, while furnaces of the internally fired boilers were water-cooled.

For a considerable period of time the horizontal return tubular boilers were extensively used in connection with steam generation for stationary practice, and not until the first Babcock & Wilcox boiler was brought out in 1867 did the manufacture of water-tube boilers become commercially successful. Both the horizontal return tubular boilers and the early water-tube boilers were set in brick settings with refractory furnace walls. The boilers were set low, close to the firebed; smokeless combustion was not then given serious consideration. During the same period of time the boilers used in connection with transportation, for locomotives and steamboats, were entirely of the internally fired water-cooled firebox type. Thus, we have the two types of construction of furnace walls being developed simultaneously in the two fields of steam-power development.

The next forward step in steam generation was that of mechanical stoking. The primary purpose of this phase of steam-generation development was the elimination of manual labor in stoking, but another very important result was the ability to obtain increased evaporation per square foot of heating surface, since the combustion of more coal per square foot of grate area became possible. However, maintenance troubles were experienced with the refractories in the furnaces.

In 1875 Thomas Murphy began the experimental development work of the Murphy stoker on his dredge operating on the Great Lakes. In 1880 the first successful installation of the Murphy stoker was made in one of the plants of the Cambria Steel Company. In 1888 Wm. R. Roney introduced the first Roney natural-draft stoker, which was extensively used for 30 years. In the same year, 1888, Evans Jones invented the underfeed retort type of stoker which has since been used in many of the smaller boiler settings. The introduction of the inclined underfeed type of stokers, pioneered by the American Ship Windlass Company in the plant of the Narragansett Brewing Company at Providence, R. I., in 1905, materially changed boiler-furnace conditions and requirements. Again the rate of combustion per square foot was increased, boilers were raised to higher elevations above the fuel bed, furnace volumes were increased, and smokeless combustion was achieved. The furnace walls required still greater care and engineering skill in design and construction to avoid expensive maintenance.

The introduction of pulverized fuel has again altered the service required of furnace refractories. The competition of pulverized-fuel combustion with the burning of fuel on modern underfeed stokers has served to bring about a constantly increasing rate of evaporation per square foot of heating surface in the boiler, larger furnace volume, and an ever-increasing number of problems in the design of the furnace walls. The use of fuel oil and refuse wood from furniture and automobile-body manufacturing plants has also created severe conditions for furnace refractories to meet.

Until recently, wherever refractory-arch construction was required, the arches were supported only by the skewbacks from which they were sprung. As the development of the stoker became more and more perfected and higher boiler ratings were being obtained, increasing difficulties with sprung arches in the combustion chambers were encountered. The Laclede Christy Company introduced a suspended arch with a chain-grate

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

stoker in 1907; M. H. Detrick installed the first Detrick arch in 1911; the first Bigelow arch was installed in 1916; the Liptak double-suspended arch in 1918, the DeWolf arch in 1920, the American arch in 1921, and the McLeod & Henry arch in 1922. Considerable savings were effected in the maintenance cost of arch construction. Since the World War, nearly all well-designed boiler furnaces requiring arch construction have been built with suspended arches.

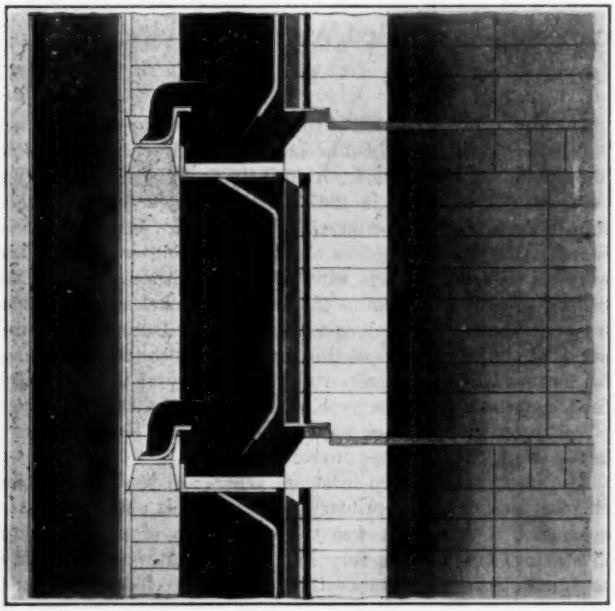


FIG. 1 CROSS-SECTION OF DETRICK WALL

The next important development in furnace-wall construction was that of the supported side wall, which may be air cooled. These walls proved quite satisfactory for the operation of boilers at ratings less than 250 per cent.

With the introduction of pulverized coal and boiler ratings in excess of 250 per cent the air-cooled refractory side-wall construction has not in all cases proved entirely satisfactory. So-called water-cooled side walls have been introduced, in which the refractory walls are protected with tubes through which water passes in connection with the circulation in the boilers. These tubes may be placed adjacent to the wall on the inside of the furnace, the entire furnace wall may be constructed of tubes with a small amount of refractory and insulating material on the outside, or the tubes may be arranged with strips of steel fin-like in form welded to the tubes to present a solid iron surface to the fire. We also find a combination of water-cooled and air-cooled walls in which the water-cooled surfaces are subjected to the more severe conditions and the air-cooled surfaces to the less severe conditions within the furnace. Thus, in stoker-fired installations with bent-water-tube boilers we find designs in which water-cooled walls are used for the bridge wall and for the side walls in the zone in contact with burning fuels. Above these zones of most severe conditions, and in the front wall many modern installations are using air-cooled suspended side-wall construction.

An interesting illustration of the development of boiler furnaces is furnished at the River Rouge plant of the Ford Motor Company, the boilers for which were originally designed to generate 250,000 lb. of steam per hour and were constructed with solid refractory walls, the secondary air for combustion being ad-

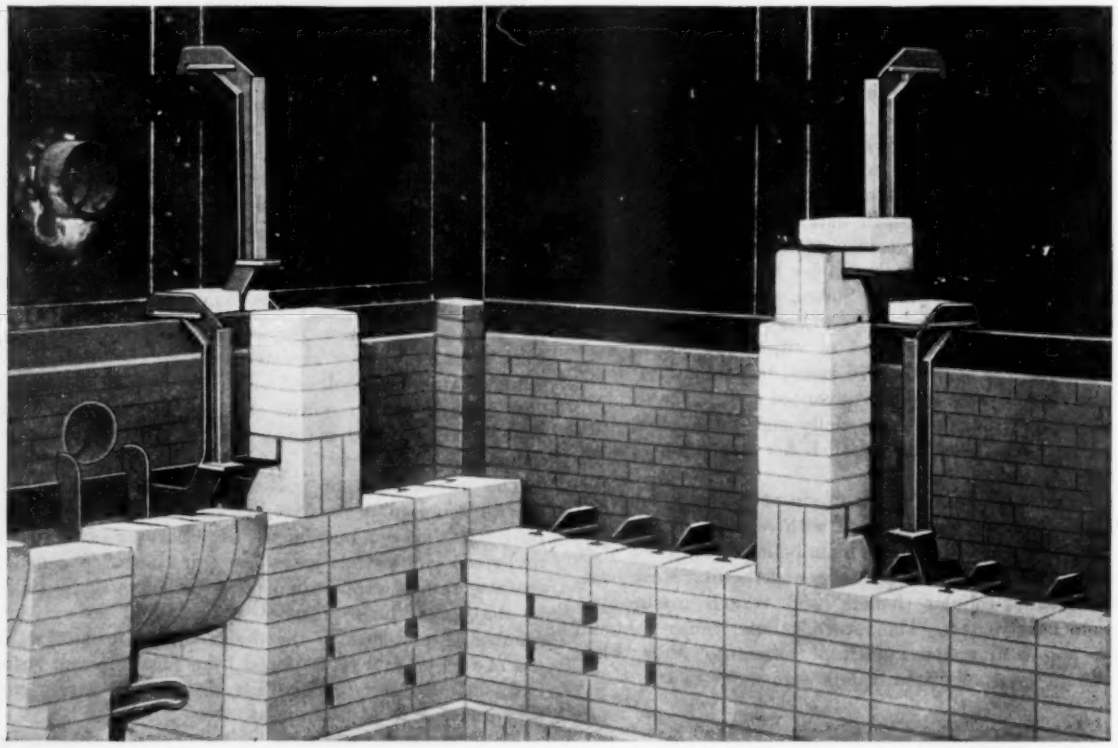


FIG. 2 GENERAL ASSEMBLY OF DETRICK WALLS

mitted through ports in both front walls of the double Ladd boilers, entering the furnace in the immediate vicinity of the pulverized-fuel flames. When it became necessary to increase the steam production of this plant a change in the furnace settings was made in which the side-wall construction was replaced by water-cooled construction. Here the air for combustion is preheated in air preheaters and is admitted with the pulverized fuel through specially designed nozzles around the burner and through ports in the front walls. With this construction the same boilers have been increased in capacity to produce 500,000 lb. of steam per hour.

AIR-COOLED REFRACTORY FURNACE WALLS

Air-cooled refractory walls may be segregated under two general classifications; namely,

1 Walls in which special refractory shapes are definitely held in position by structural steel and cast-iron members which also support the weight of the refractories; and

2 Walls in which special refractory shapes for furnace lining are bonded to the main wall but are not supported thereby.

The wall construction of the first group will now be described.

The Detrick Wall

The M. H. Detrick Company's first wall was installed in the

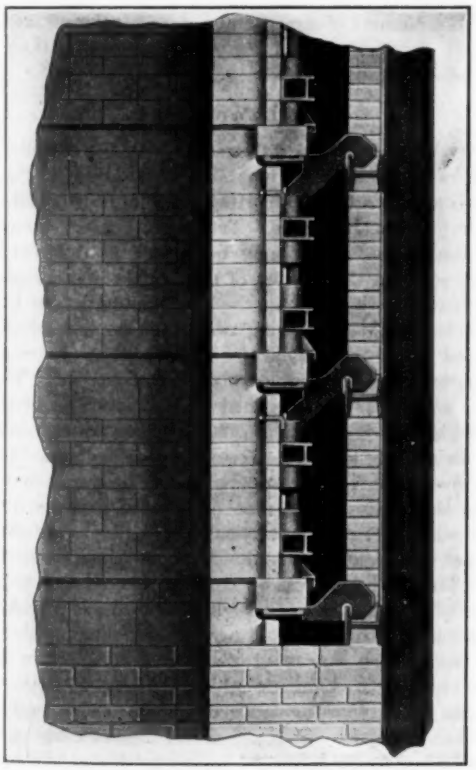


Fig. 3 Cross-Section of American Arch Company's Wall

plant of the Binghamton Light, Heat & Power Company at Binghamton, N. Y., in 1924.

The design and methods of construction of the Detrick wall are indicated by Figs. 1 and 2. This wall requires three forms of blocks for its construction. As will be seen by referring to the illustrations, it consists essentially of independent sections, each $10^1/_2$ in. wide by 30 in. high. The wall sections are placed side by side in horizontal bands. The wall tiles are placed adjacent to each other with plastic clay applied in the vertical joint. Each joint is sufficient for any expansion due to heat that may occur; however, the precaution of laying up corners in such a manner that expansion may take place freely is usually followed. The face tile are $10^1/_2$ in. wide by 3 in. high and are

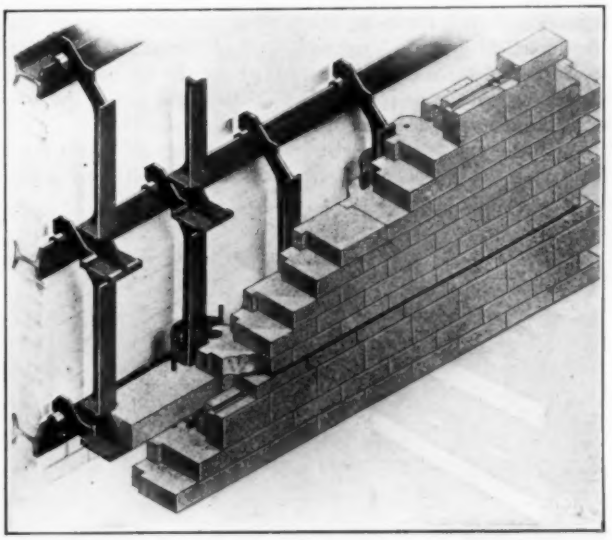


Fig. 4 General Assembly of American Arch Company's Wall

10 in. in depth, and the distance from the face of the tile to the slot which receives the T-head of the cast-iron bracket support is $7^1/_4$ in.

The cast-iron hangers are arranged with a horizontal shelf near the base to support each integral section independently of all other sections. They are carried on horizontal girders attached to vertical columns which support and maintain the alignment of the entire wall. They also serve as heat conductors from the grooves in the tiles to the cooling air surrounding them. The upper end of the hanger hooks over the edge of the horizontal girder, and the lower end thrusts against the girder below. No bolts or fastenings are used, so that the hangers are free to move should the expansion of the wall be sufficient to cause such displacement.

The hangers are provided with suitable flanges for supporting baffle tiles or plates which may be placed horizontally between the hangers to form horizontal lanes or passages for the air. If these baffles are omitted, vertical circulation of air may be obtained. The outside of the air passage or the outside face of the furnace wall is usually a single course of red brick, although steel plate or other material may be used.

The American Wall

The American air-cooled side wall has been applied to a considerable number of important boiler installations. Its general construction is shown in Fig. 3, and the method of bonding and of attaching alternate courses to the cast-iron supporting bars is indicated in Fig. 4. The wall is made up of independent horizontal sections 30 in. high, each section consisting of nine courses of special blocks which are attached to the cast-iron hangers. Five different shapes of refractory blocks are required for this construction. For each section there is first a course of supporting tile resting on shelves at the lower end of the hangers; then seven courses of 3-in. × 9-in. × 9-in. tile with

offset vertical joints in two shapes, one with a hole in a projection in the back of the tile to receive the pin of the retaining yoke located at each alternate hanger, and the other with a straight back surface; then a thicker course of one special shape grooved in the top surface and held in position by tee anchors; and finally, at the top, a course of sealing tile with a tongue or ridge to fit into the groove of the tile below. All courses bond over the course below and the corner construction is bonded without provision for a slip joint, the horizontal expansion of each tile being taken up in the vertical joint. The offset vertical joint is designed to reduce the infiltration of air. It will be noted that the effective depth of the tile extends from the inside face to the pin in the cast-iron yoke and is the same as the full depth of the tile, or 9 in.

The lower end of each cast-iron hanger rests against the top of the hanger below. The hangers hook over 5-in. H-beam girders

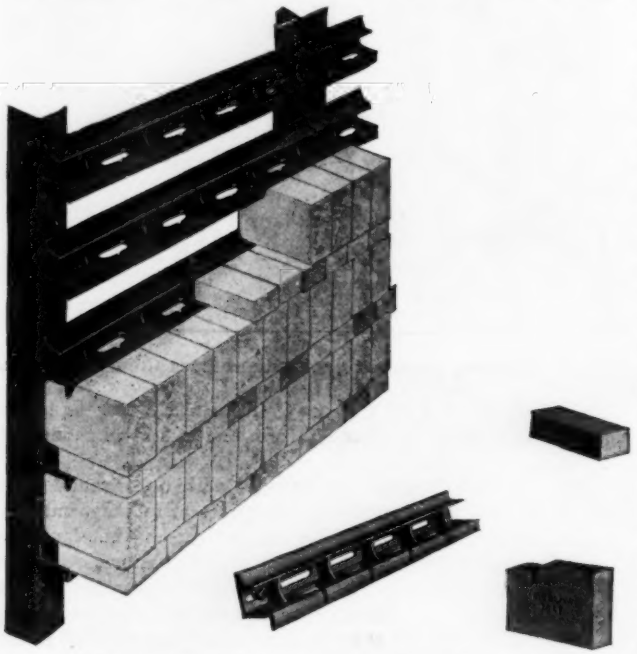


FIG. 5 BIGELOW SUSPENDED FURNACE WALL

which are attached to vertical steel members. The yokes which surround the back of the hangers and hold the tile in place are not attached to the hanger but are free to move with any vertical expansion. Air lanes may be arranged as desired with suitable baffling between the furnace tile and the exterior curtain wall or covering.

THE BIGELOW WALL

The first installations of Bigelow air-cooled side walls were placed in service in the oil-still furnaces of the Freedom Oil Company in April, 1925, and were first installed in boiler-furnace construction in the same plant in November of the same year. They have since been successfully installed in the furnaces of many boilers, the largest to date being 1510 hp. The construction of this wall is illustrated by Fig. 5. It consists of a single special type or design of block and standard firebrick. Each block is independently supported and is therefore free to expand in all directions within the limits of the vertical and horizontal joints. The blocks are supported in horizontal bands as shown in the illustration by means of horizontal bracket castings bolted to vertical steel members, which may be either I-beams or built-up columns with angles as flanges. They are laid in courses spaced $10^3/_4$ in. center to center vertically, with one course of standard firebrick laid as headers between each course. The special blocks are 11 in. in depth, and the distance from the face of the cast-iron bracket to the inside of the wall is approximately $7^1/_2$ in. The block is 4 in. in width and 8 in. high. The $^1/_8$-in. joints on top and underneath the course of blocks are filled with a special plastic fireclay. The vertical joints are formed by dipping the blocks in this special fireclay batter and placing the tile in the wall firmly together. To remove or repair any section of this wall it is only necessary to remove the firebrick above and below the block, when the block may be taken out by tilting it upward. This construction involves vertical joints between the blocks 8 in. high. The blocks are either hand molded or dry pressed. The latter process insures accuracy in size, and the vertical joints are reduced to a minimum and yet are sufficient to allow for the expansion of each block.

The air circulation, when I-beams are used as supporting members for the wall, is vertical. The outside of the column may be covered with steel plate, which may be insulated if desired, or with a thin brick wall. The natural chimney effect of the air between the vertical columns heated by the furnace lining is sufficient to circulate the air required for cooling the walls, and is controlled by adjustable dampers or predetermined orifices. In many cases the air after circulating past the heated walls is recovered and used for combustion. If horizontal circulation is desired a built-up supporting member may be used, constructed with two pairs of angles riveted to steel crosslacing or spacing plates, spaced as desired to form horizontal circulation. The amount of preheat absorbed by the air varies from 50 to 150 deg. fahr., depending on the rating and the extent of air-cooled side wall used.

THE DEWOLF WALL

The DeWolf air-cooled wall was first used in a boiler-furnace setting in 1924. Its design is shown in detail in Fig. 6. In this construction the supporting tiles are attached directly to the vertical steel I-beams. Turning to Fig. 6, reference No. 1 shows the beams, which may be spaced at different centers. The difference in the spacing of the centers is accomplished by using three lengths each in the two standard face blocks, so that for the minimum spacing two of the smallest blocks would be placed end to end, while for the maximum spacing two of the largest blocks would be placed end to end. The intermediate spacing is accomplished by combinations of the three sizes of blocks. Reference No. 2 illustrates the cast-iron clamp which secures the hanger brick to the steel column. Reference No. 3 shows the regular A-type hanger brick, each of which engages the ends of four face brick with grooves fitting the tongue which forms the vertical edges of the anchor brick. Reference No. 4 illustrates the A2 type of supporting hanger brick. The A2 bricks rest on continuous angles or clips which are riveted or bolted to the vertical steel beams. The A2 hanger brick are longer than the A hanger bricks, thus forming a ledge in the front which carries all of the face brick above the ledge to the top of the 32-in. or 40-in. section. Reference No. 5 illustrates the supporting angle or clip which supports the A2 hanger brick. Reference No. 6 shows the assembly of the face brick attached to the lower A2 hanger brick and which are supported on the lower courses which in turn are carried by A2 hanger brick further down. Reference No. 7 shows the face brick which are attached to the A hanger brick and are supported by A2 hanger brick further down. The facing brick for both A and A2 hanger brick are 4 in. in height and are $7^1/_8$ in., $9^1/_8$ in., and $11^1/_8$ in. in width. This wall construction, therefore, is developed with eight forms of special blocks to accommodate

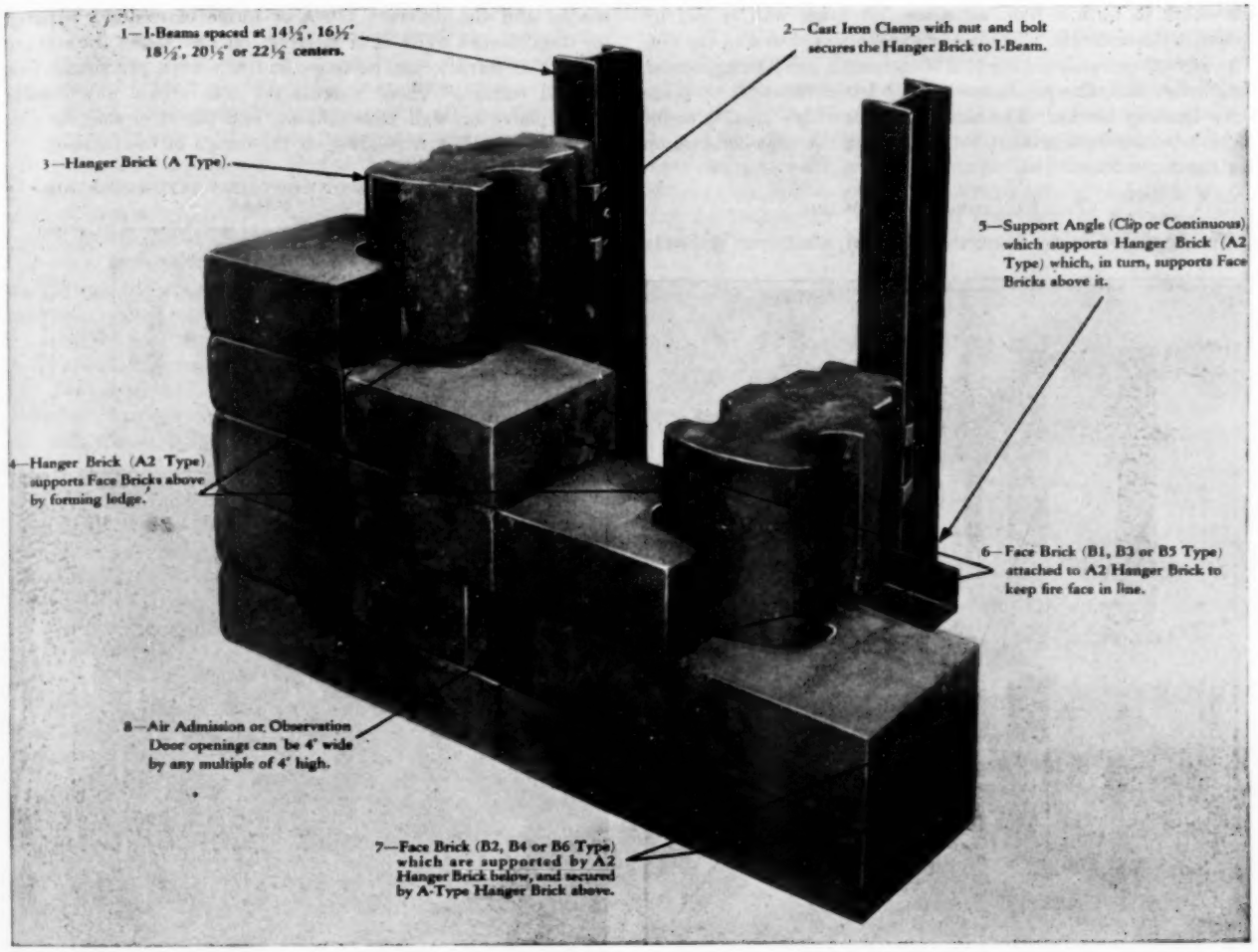


FIG. 6 CONSTRUCTION OF DEWOLF WALL

various spacings of vertical I-beams. If the vertical I-beams can be definitely spaced on the same centers, not over four blocks of different shapes will be required. No horizontal expansion joint is provided between the horizontal sections as it is considered that this is not necessary. In laying the wall plastic fireclay is placed between the horizontal and vertical joints of the face brick, but care is taken that no clay is placed in the joints between the face brick and the hanger brick.

The outside of the vertical beams is covered with steel plate to form vertical air ducts between the wall and the cover plates. The chimney effect or draft produced by the heated air is sufficient to cool the wall if proper openings are left at the top and bottom of the ducts. The air may be drawn through the ducts by a fan and, when heated, may be used for combustion.

THE MCLEOD & HENRY WALL

This wall design is the most recent of the supported air-cooled walls, the first installation having been completed last year. The construction of this wall is shown in Fig. 7. The wall is supported by cast-iron brackets in horizontal bands or sections 24 in. high, with a packed expansion joint between each horizontal section. Three shapes of wall tiles are required, the regular tile being 4 in. high, 12 in. wide, and 10 in. deep. The tiles are held in position by cast-iron clips which slip over the head of the vertical T-bars and fit into suitable recesses in the upper face of each tile. The cast-iron brackets are bolted to the upper flange of standard 9-in. channel beams, and the two braces of each bracket rest against the lower flange of the channel. The brackets are of two designs, one to form a 9-in. air space between the back of the refractory blocks and the face of the vertical steel columns in which the channels are attached, and

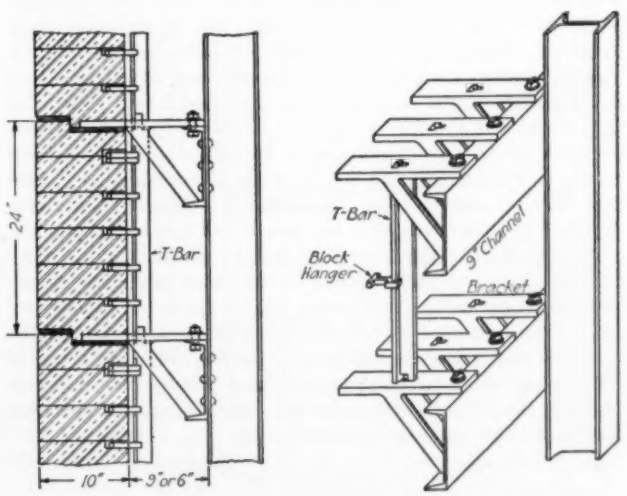


FIG. 7 CONSTRUCTION OF MCLEOD & HENRY COMPANY'S WALL

the other to form a 6-in. air space. A brick wall is laid up between the columns. The T-bar assembly is shown in the cut. The advantages claimed for this construction are its ruggedness, simplicity, and the small amount of labor required to place the refractory blocks. The blocks are laid with a small amount of high-temperature cement buttered along the edge of block on the furnace side.

The Laclede-Christy Wall

The Laclede-Christy Company installed what was probably

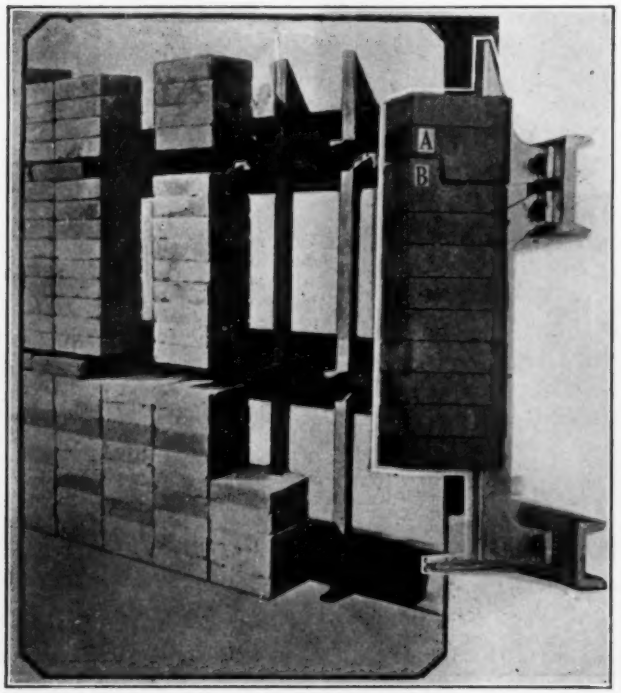


Fig. 8 Laclede-Christy Wall

the first steel-supported side-wall construction in this country in a boiler setting for the St. Joseph Lead Co., in Missouri, in 1920. This was a short curtain wall in a furnace for a 768-hp. Stirling boiler fired with pulverized coal. In this case the refractory blocks were held in alignment by steel I-beams and were supported on structural steel beams. Very little development work was then done on this side wall construction until recently. The design now offered is illustrated by Fig. 8. The wall blocks are 3 in. by $10^1/_2$ in. face and 10 in. deep, and are identical with those used in the Laclede-Christy suspended arch construction. The horizontal bands or courses between supports are of any height desired to suit the structural-steel design of the furnace up to 6 ft. high. The cast-iron hanger bars therefore vary in length from about 2 ft. 6 in. to a maximum of 6 ft. The bars are bolted at the top and bottom ends to the horizontal steel members. Two courses of sealing tile "A" and "B" as shown in the insert in Fig. 8 are laid between the sections or groups of tiles carried by the hangers. Refractory tile "A" is supported by a foot or shelf at the lower end of the cast-iron hanger and is separated from tile "B" at the top of the section below by an expansion joint filled with plastic refractory material. The blocks are laid with plastic bonding clay in the horizontal and vertical joints.

Comparison of Different Walls

Table 1, giving the comparison of the number of refractory shapes and the different kinds or forms of castings required for the different walls, is of interest. It also shows the relative weights of refractories, castings, and steelwork per square foot of wall surface. These weights do not include any outside cover plates or wall construction, and the steel weights may vary considerably, depending on the design of the furnace.

TABLE 1 COMPARISON OF CERTAIN FEATURES OF AIR-COOLED WALLS

Wall type	Number of special refractory shapes	Number of kinds of castings	Avg. weight per sq. ft. of wall: Refractory blocks, lb.	Cast iron, lb.	Supporting steel, lb.
Laclede-Christy	3	1	92	15.0	15
Detrick	3	1	100	18.0	20
American	5	3	107 (*c*)	24 (*b*)	19
De Wolf	4 (*a*)	1	120	3.0	13
Bigelow	1	1 (*a*)	103	13.4	10
McLeod & Henry	3	3	108 (*c*)	17	18

(*a*) For uniform spacing of vertical columns.
(*b*) Estimated weights.
(*c*) Based on 130 lb. per cu. ft.

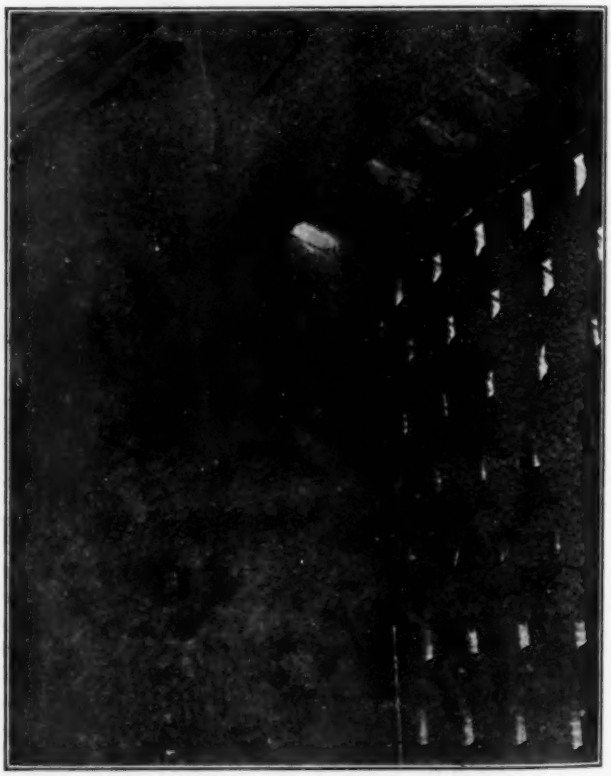


Fig. 9 Interior of Furnace for 1112-Hp. Stirling Boiler, Sioux City (Iowa) Gas and Electric Company

Of interest also is the remarkable increase in the air-cooled supported refractory wall surface installed by four of the principal manufacturers during each of the last five years:

Year	Total sq. ft. of air-cooled walls installed each year
1924	35,358
1925	44,505
1926	106,686
1927	117,289
1928	197,379

Bonded Air-Cooled Walls

Air-cooled walls are sometimes built for the smaller boiler furnaces by constructing hollow spaces or flues in the interior

of the wall and arranging for a circulation of air through the flues or ducts. This may be accomplished by using regular 9-in. standard firebrick or special refractory blocks bonded into the wall. Although this type of wall is sold by the manufacturers of the special shapes for furnaces having comparatively high walls, it is the author's belief that walls higher than 10 or 12 ft. should not be constructed in this manner. The inner lining of such a wall on the furnace side would have a greater vertical expansion than the main outer wall to which it is bonded. This would be likely to result in breaking the bond and allowing the interior lining to fall away from the outer wall.

Among the constructions of the bonded air-cooled walls the following may be mentioned:

1 *The American Arch Company* offer special air-cooled blocks which are bonded to the main wall with standard 9-in. firebrick. These blocks form horizontal and vertical air passages.

2 *The Bigelow-Liptak Company* offer a design of air-cooled refractory bonded furnace wall having a lining constructed of one course of standard firebrick which is bonded to the main wall with special bonding tile. In this construction the bonding tile will support the lining for repair work, but this supporting feature is not relied upon to carry any of the weight of the lining wall when in regular service. The bonding tile are so designed that they may readily be replaced.

3 *The Combustion Engineering Corporation* offer a design in which the furnace-lining wall is built of 9-in. standard firebrick in sections of any desired height between horizontal courses of bonding tile. The bonding tile are 5 in. thick, 9 in. wide, and 15 in. deep, laid in courses between the sections of lining wall, thus forming horizontal air passages between the lining and outer wall construction. The bonding tile are held to the outer wall by hooks which engage in holes in the bonding tile and pass through the main wall with anchor plates and nuts on the outside.

4 *The Drake Non-Clinkering Furnace Block Company*, in addition to their perforated carborundum blocks for admitting air to the furnace, also produce a carborundum air-cooled lining for furnace walls. In this construction the special wall lining blocks are of two shapes, all tongued and grooved together and held to the main outer wall by cast-iron anchors laid in the wall. These anchors support the lining laterally, but do not support the weight of the lining wall.

5 *The Furnace Economy Company* have produced special refractory blocks which, when laid in the wall in pairs, resemble two square tile attached to each other by a cylindrical section. The pairs are bonded in double courses so that the cooling air in passing through the spaces in the wall impinges upon the cylindrical surfaces and thus has greater cooling effect.

6 *The McLeod & Henry Company* supply special refractory furnace lining blocks of two designs. One design is 9 in. wide by 10 in. high by 6 in. thick, with 9 pyramidal projections 3 in. high on the back. These are laid in double courses with bonding tile hooked to the main wall to hold them in line laterally. Another design is similar to the one described in (3), having bonding tile and firebrick lining wall. In this case solid refractory blocks 10 in. in depth are substituted for the 9-in. double-course wall.

OPERATING SERVICE CONDITIONS OF AIR-COOLED WALLS

There exists a vast fund of interesting and pertinent information concerning the performance of air-cooled walls in service. But the combination of variable factors entering into the conditions of service render it a most difficult and laborious task to gather data to definitely establish facts or to set up principles which may be used in the design and operation of furnaces. The following results of operation of furnaces with air-cooled walls obtained at certain plants are presented with the hope that the information will be of interest:

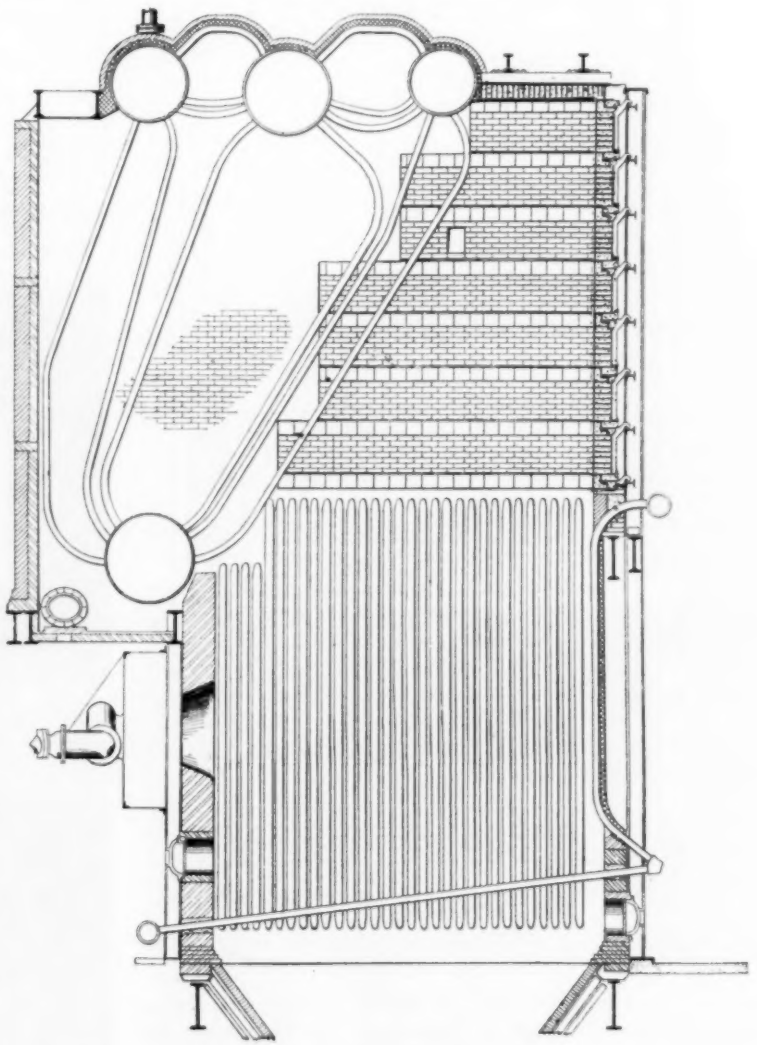


FIG. 10 SETTING FOR 2500-HP. HEINE BOILERS, GULF STATE SERVICE CORPORATION, HOLT, ALA.

THE DETRICK WALL

Detrick walls have been installed in the furnaces for many types of boilers and for average ratings of from 175 per cent to nearly 300 per cent, and for peak ratings as high as 425 per cent where no water-cooled walls were also installed. The first installation at Binghamton operates on 8-hr. peaks at 325 per cent of rating. At this plant the average rate of heat liberation is 10,750 B.t.u. per cu. ft. of furnace volume per hr.; at the higher ratings it is as high as 16,190 B.t.u. per cu. ft. per hr.

The fusion temperature of the ash is 2260 deg. fahr., and slagging of the side walls occurs if the CO_2 is over 14 per cent. No serious trouble or maintenance expense has been encountered from spalling and erosion. The air circulates horizontally around the side walls and enters through ports in the front walls in the immediate vicinity of the long pulverized-fuel flames; the temperature of the air is about 200 deg. fahr. The photo-

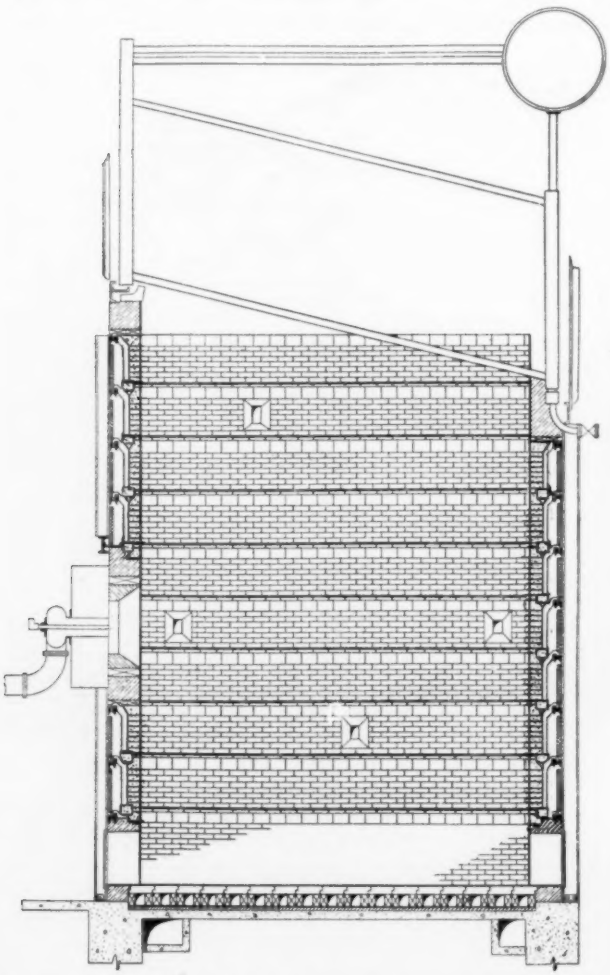


FIG. 11 SETTING FOR 614-HP. WALSH & WEIDNER BOILER, E. I. DU PONT DE NEMOURS & COMPANY, NEWBURGH, N. Y.

graph of another furnace which is reproduced in Fig. 9 was taken after a considerable period of operation, with ratings varying from an average of 200 per cent to a maximum of 350 per cent for one hour. The CO_2 content of flue gases is maintained at $13^1/_2$ per cent when operating at 200 per cent of rating and 11 per cent at 300 per cent of rating. The fuels burned in pulverized form in three boilers at the Big Sioux Station have widely varying characteristics. Table 2 gives in condensed form the proximate analysis of some of the fuels used during the past year. Practically the same amount of slagging results with the different coals, and this largely depends on the watchfulness of the operator in adjusting the amount of excess air at different ratings to prevent slagging and maintain the highest practicable CO_2 content in the furnace gases. No spalling has been experienced, but erosion of the front walls and of the side walls in the vicinity of the path of the flame has occurred. The front wall of each furnace has been repaired once since June, 1925, with replacements of refractories and castings, but the side walls have not been renewed. The outside burners have been tilted away from the side walls, additional air has been admitted through the arch between outside burners and the side walls, and repairs where necessary consisting of ganister and fireclay have been applied with a cement gun after each period of operation. Boilers have thus been kept in service for 2550 steaming hours before being taken out of service for inspection. This furnace is designed with an air-cooled ash hopper, water-cooled bridge wall, and is proportioned for a heat liberation of 18,000 B.t.u. per cu. ft. per hr. at 350 per cent of rating.

TABLE 2 TYPES OF FUELS BURNED AT BIG SIOUX STATION

Location of Mines	Moisture	Volatile	Carbon	Ash	Sulphur	B.t.u.
Mahaska Co., Iowa	25.8	25.4[a]	28.3[a]	20.5	3.05	7,349
LeFlore Co., Okla.	3.5	21.5	61.0	14.0	1.34	12,492
Johnson Co., Ark.	2.1	13.0	74.9	10.0	2.03	13,397
Musselshell Co., Mont.	12.2	41.4	38.5	7.9	1.08	11,198
Petroleum coke breeze	5.8	...	...	0.5	1.08	15,177
Saline Co., Ill.	8.8	35.0	43.0	13.2	3.23	11,333
Adair Co., Mo.	15.0	23.3	41.7	20.0	...	8,700
Union Co., Ky.	3.5	34.2	48.1	14.2	2.9	12,190

[a] Estimated values based on data in Keystone Coal Buyers Catalog, 1928.

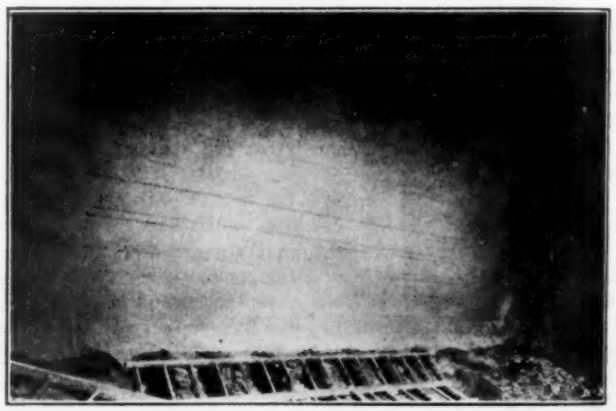


FIG. 12 INTERIOR VIEW OF BIGELOW FRONT WALL FOR 1510-HP. BABCOCK & WILCOX BOILER, EDISON ELECTRIC ILLUMINATING COMPANY, EDGAR STATION, BOSTON, MASS.

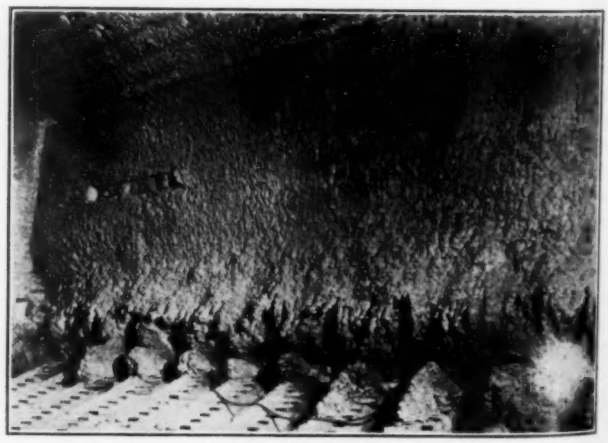


FIG. 13 INTERIOR VIEW OF SOLID-REFRACTORY FRONT WALL FOR 1510-HP. BABCOCK & WILCOX BOILER, EDISON ELECTRIC ILLUMINATING COMPANY, EDGAR STATION, BOSTON, MASS.

A study of the designs of seven of the larger installations of the Detrick wall in which less than 10 per cent of the heating surface is installed as water walls or screens indicates that the furnaces were proportioned, as to volume, for an average maxi-

mum heat liberation of 18,600 B.t.u. per hr. per cu. ft. of furnace volume.

The American Wall

Fig. 10 shows an interesting application of the American side-wall construction in the furnaces for three 2500-hp. bent-tube boilers fired with short-flame pulverized-coal burners set in the bridge wall under the mud drum. The lower portion of the furnace is designed with water-cooled walls and water screen, and the upper portions are built with air-cooled refractory walls.

Another noteworthy installation is shown in Fig. 11. This furnace-wall construction is for a 614-hp. cross-drum boiler fired with a low-velocity or short-flame pulverized-coal burner. In this case the four walls are entirely constructed with air-cooled refractory walls, and the furnace bottom is also air cooled. This furnace is designed for operation of boiler at 200 per cent average and 250 per cent maximum ratings. High furnace temperatures and slagging occur when the CO_2 in maintained at about 14.5 per cent, but slagging ceases when the excess air is increased to reduce the CO_2 to not over 13.5 per cent. When the boiler is operating at about 175 per cent of rating the air for cooling the side walls is heated to about 240 deg. fahr., and is used partly as primary air for drying

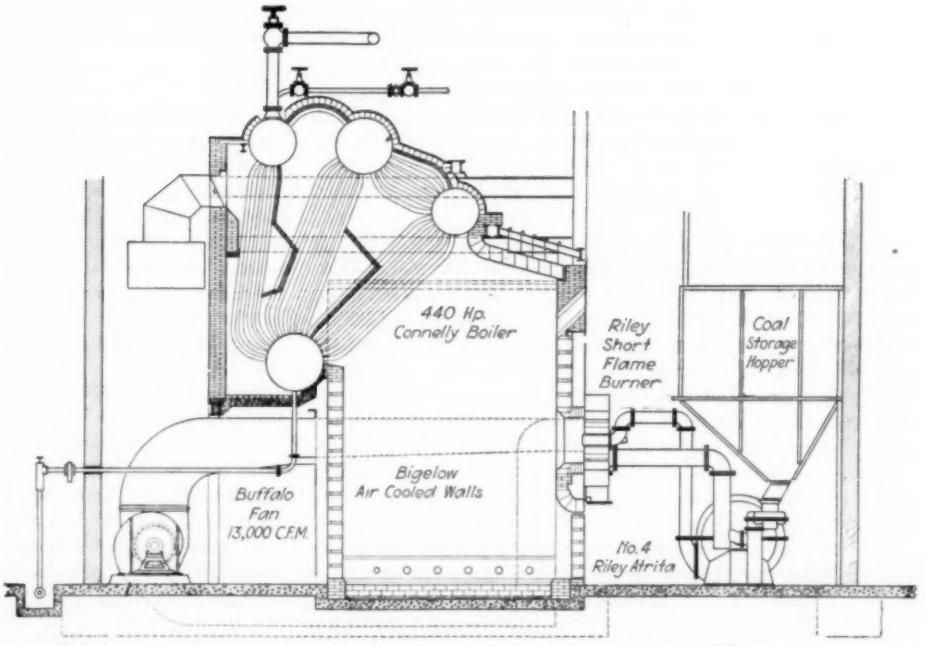


Fig. 14 Setting for 440-Hp. Connelly Boiler, General Motors of Canada, Ltd., Oshawa, Ont.

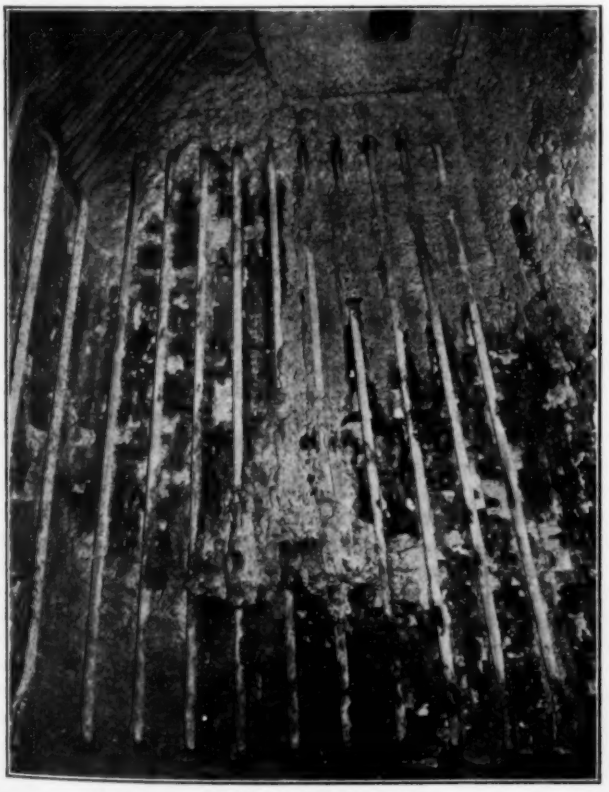


Fig. 15 DeWolf Side Wall for 1122-Hp. Bigelow-Hornsby Boiler, Rochester Gas & Electric Corporation, Rochester, N. Y.



Fig. 16 Interior of Furnace for 600-Hp. Connelly Boiler, Syracuse University, Syracuse, N. Y.

and conveying the pulverized coal from the unit pulverizer and partly as secondary air admitted to the furnace through the short-flame burner. At certain times this boiler has been operated at normal rating with coal containing 10 per cent moisture as received by the unit pulverizer. The heat liberation in this furnace at 250 per cent of rating is at the rate of 13,300 B.t.u. per cu. ft. per hr.

The Bigelow Wall

A typical illustration of an installation of the Bigelow wall is shown in Fig. 12, a photograph taken after 3000 hours of service. This furnace is for a Babcock & Wilcox cross-drum boiler of 1510 boiler hp., steam pressure 1250 lb., and has been operated at 350 per cent of rating with CO_2 at about 13 per cent and air preheated to approximately 350 deg. fahr. The furnace is fired by multiple-retort stokers and is constructed with Bailey water walls on the two sides and bridge wall and a Bigelow front wall. The coal burned is New River nut and slack, taken from the Tug River and Sewall seams in West Virginia. The refractory walls coat over with a glaze of slag, and there is no trouble with erosion or spalling.

Fig. 13 shows a solid refractory wall in a furnace operating with the same fuel and rating conditions after 2600 hours of service. This furnace is identical with that shown in Fig. 12 as to dimensions and stoker equipment, but the side walls and front wall are solid refractory, not air cooled or water cooled.

This comparison shows in a marked degree the effect of air cooling and lower furnace temperature on the prevention of slag adhesion to the interior surface of furnace walls.

The author's experience with the Bigelow wall in a furnace burning both pulverized coal and kiln-dried wood chips, shavings, and sawdust may be of interest. Fig. 14 shows one of two 440-hp. Connelly boilers, each one set with furnaces fired with a low-velocity short-flame burner in the front wall and with four ports through the upper part of the front wall for admitting the wood fuel. The four side walls are of Bigelow wall construction. The air for side-wall cooling is admitted through ports at the top and bottom of the vertical air ducts in the side and rear walls, and is drawn off horizontally along the center of the walls to an air preheater. The air, preheated to a temperature of about 400 deg. fahr., is then forced either to the pulverized-coal burner or, when burning wood, through the small ports at the bottom of the furnace, where it enters at high velocity and rapidly burns any of the larger pieces of wood that do not burn in suspension.

Wood is burned during the working hours of the wood mill, the boiler operating at about 200 to 250 per cent of rating; CO_2 is maintained at between 12 and 14 per cent. At night and over week ends pulverized coal is burned, maintaining approximately the same ratings. Once in three weeks the furnace is closed down and the slag and accumulated ashes are removed from the furnace bottom which is neither air cooled nor water cooled. The wood fire is 200 to 300 deg. hotter than the coal fire, so that practically all the ash deposited as such when burning coal is soon melted into a slag after the wood fire has been maintained for some time. The combination of wood and coal ash and high furnace temperature creates a most severe condition for erosion and fluxing action on the side walls. After about two years of this service the side-wall blocks opposite the burner had eroded away about 5 in. from the original surface, and it was decided to repair the walls—for the first time. This erosion was at the rate of about 0.2 in. per month. At another plant in which similar kiln-dried wood refuse was burned at somewhat lower boiler ratings, the furnaces suffered an erosive action which carried away from 1 in. to $1^1/_4$ in. of refractory surface from the furnace walls per month. These walls were constructed of solid firebrick and were arranged so that a 9-in. lining could be frequently replaced.

The DeWolf Wall

A noteworthy installation of the DeWolf wall is that of a furnace for a Bigelow-Hornsby boiler of 1122 boiler hp. rated capacity. (Rating includes the economizer section of the heating surface.) This furnace (Fig. 15), however, is designed with tubes spaced on 18-in. centers adjacent to the interior surface of the air-cooled walls to produce sufficient cooling effect to permit operation at high ratings. The furnace is fired with pulverized coal, using high-velocity long-flame burners and has been in operation for three years, with ratings varying between 50 and 400 per cent. The photograph reproduced in Fig. 15 was taken after the furnace has been in operation for 2 years and 10 months, during which time 21,930 tons of coal had been burned.

Fig. 16 illustrates another furnace constructed with the DeWolf wall which had been operating 9 months when the photograph was taken, at ratings between 50 and 325 per cent. The walls are said to be free from cracks and in excellent condition. This furnace is for a 600-hp. Connelly boiler, fired with pulverized coal, and is not provided with water-tube cooling along the face of the walls.

GENERAL CONCLUSIONS

In selecting a type of wall for boiler furnaces the engineer should consider the service requirements of the boiler, the type of equipment selected for burning the fuel, and the economics of the plant operation, including fixed charges on the investment.

If the furnace is to be stoker fired and is to be designed for a moderate maximum heat liberation of not more than 25,000 B.t.u. per cu. ft. of furnace volume per hour, air-cooled walls should prove quite satisfactory. If the maximum heat liberation is greater than 25,000 B.t.u. per cu. ft. per hour, then water-cooled bridge walls should be considered and a certain amount of water cooling of the side walls, depending on the maximum rate of combustion. The air after passing through the side-wall cooling ducts should be returned to furnace for combustion, thus reducing the radiation losses.

When pulverized coal, natural gas, fuel oil, or kiln-dried wood refuse is to be burned, the problem is somewhat different. Lower rates of heat liberation per cubic foot of furnace, the length and form of the flame, the load factor of the boiler, the fusion temperature of the ash, and the design of the boiler with respect to the extent and position of radiant heating surface exposed to the furnace should be carefully studied. The furnace walls should have little, if any, water-cooling surface if the load factor is low and the boiler must be operated at times at low ratings. On the other hand, if a high load factor is possible and the boiler will at all times be operated at high ratings, more water cooling of the side walls is justified. In general the furnace for rates of heat liberation of less than 15,000 B.t.u. per cu. ft. per hr. may be constructed with air-cooled side walls and ash hoppers, provided the furnace bottom is sufficiently depressed below the flame travel to prevent slagging of the ash. If the rate of heat liberation is between 15,000 and 20,000 B.t.u. per cu. ft. per hr. a water screen in the furnace bottom and a certain amount of water cooling of one or more of the side walls is advisable. If the rate of combustion liberates more than 20,000 B.t.u. per cu. ft. per hr., a still greater amount of radiant heating surface in the side walls is necessary.

Furnace constructed with air-cooled walls, especially if without water-cooling protection, should be so designed that there will be no impingement of the flame upon the wall, thus preventing erosion.

It is the author's belief that when a careful study of all the economic factors of construction and operating costs of both air-cooled and water-cooled furnace walls has been made the saving in fixed charges and in loss of carbon in the ash refuse in the case of furnaces with air-cooled refractory walls is greater than the saving in wall maintenance and in the loss of heat through excess air in the case of furnaces with water-cooled walls. Present engineering practice indicates that there is a distinct field for air-cooled supported side-wall construction and another field for water-cooled side-wall construction, and that in many cases combinations of the two types of construction may be used to advantage.

Discussion

W. A. Shoudy.[2] Mr. Gladden has presented a very complete survey of the air-cooled walls that are available in this country. The impartial way in which he has presented the various types of walls will make the paper of considerable value to the designer of boiler houses. He has also pointed out the important fact that the decision to use either water-cooled or air-cooled furnace walls must be made upon an economic rather than a technical basis.

It is unfortunate that Mr. Gladden has used the term "per cent rating" in the discussion of boiler furnaces. Unless the type of boiler is specified, the use of this term may lead to wrong conclusions in furnace design. The writer believes that any just comparison of furnace operation should be in B.t.u. per cubic foot per hour or a similar unit.

It is very interesting to note the rather wide use of air-cooled walls, and particularly the very rapid development, as shown in the data on the sixth page. It is evidence of the fact that the water-cooled wall has not replaced the refractory, but rather has stimulated its development, and we now find that there is a field for each type of furnace. There is no sharp line of demarcation between the two fields, but in general the field of the water-cooled furnace is that of the high-capacity, high-load-factor installation, and that of the refractory furnace, lower capacities. It might be better to substitute for the term "capacity" high-B.t.u. liberation. We have several installations of high-steam-capacity units using air-cooled walls, some of them delivering as high as 300,000 lb. of steam per hour, but these installations operate with B.t.u. releases of 20,000 B.t.u. per cu. ft. per hour and under. The same steaming capacities can be obtained with refractory walls as with water walls; consequently the decision must be based on a comparison of the total cost of the installation and a determination as to whether the increased fixed charges due to the more expensive water walls are not greater than the slightly higher maintenance cost of the refractory wall.

There have not yet been sufficient data collected to determine definitely the limitations of the air-cooled wall. During the past two years the writer has been investigating substantially all of the furnaces installed by one manufacturer. Operating records are not always fully reliable; consequently, only 15 installations totaling 48 boilers could be selected as truly representing air-cooled furnace wall practice. The principal data concerning these installations is given in Table 3 and covers boilers which have been operating as many as 17,000 hours and others which have consumed over 500,000 tons of coal.

The most satisfactory basis of comparison seems to be that of the cost of repairs expressed in cents per ton of coal burned. These costs vary from nothing to nearly five cents. The majority, however, show less than two cents per ton. The B.t.u. release per cubic foot is given as maximum and average. The term "maximum" is not fully satisfactory, because in some cases it applies only to extremely short peaks, whereas in others it represents regularly recurring high peaks.

The results, however, indicate that in general the air-cooled wall can be used for B.t.u. releases as high as 20,000 and that with a small area of water-cooled wall 25,000 B.t.u. per cu. ft. per hour is not out of reason. Before the B.t.u. releases can be determined, however, full knowledge of the character of the fuel must be obtained.

The fusing temperature of the ash determines the limitation of the correctly designed air-cooled sectionally supported walls; that is, the fluxing action of the ash on the refractory determines its life. An ash that fuses below 2100 deg. fahr. will cause trouble with any refractory wall unless burned with about 40 per cent excess air. Air cooling alone will not suffice. When size of unit or load factor will not warrant the high cost of water-cooled walls and low excess air is necessary, partial water cooling such as a bridge wall or side wall will give a very satisfactory compromise.

The evils of high excess air or low CO_2 have been overemphasized. There is an optimum per cent excess air that varies with each installation. Smoke may be one limitation. It is obviously illogical to design a furnace for low excess air if it cannot be obtained without smoke. In such cases the air-cooled wall is fully as satisfactory and much cheaper than the water wall, and in some cases to be preferred, because combustion is improved by the hot walls.

When a boiler is used without economizer or air heater, an increase in excess air results in a lower efficiency because of the higher exit-gas temperature, but with economizers or air heaters the reduction in efficiency is negligible, for the following reasons:

An increase in excess air lowers furnace temperature, thus reducing absorption by radiation, and the transfer from the gas by convection is reduced because of the lower temperature difference between water and gas. This results in a high exit-gas temperature, but on entering the economizer the heat transfer by the economizer is increased because of the greater temperature difference and the greater velocity of gas due to the increase in gas volume.

Consequently, an increase of the per cent in excess air of from 30 to 40 per cent results in so small a difference in final exit-gas temperature that it is negligible. The gas weight has increased about seven per cent and the stack loss about seven per cent of 10 or 11 per cent, or the efficiency has been reduced about nine-tenths of one per cent. However, this loss sometimes is an actual gain because the increase in air for combustion may eliminate all loss from CO or reduce carbon losses to the stack.

The air-cooled wall to be satisfactory must meet certain definite conditions. Every refractory wall is air-cooled, but the built-up wall is so thick that the cooling action is negligible. To obtain any cooling effect at all, the wall must be relatively thin. Nine or ten inches is probably the maximum thickness with which air cooling is effective, and seven or eight inches is as thin as it can be made without weakening the construction. Too thin a wall will not be sufficiently gas-tight.

A refractory wall fails for one of five reasons: by melting, softening, spalling, fluxing, or expansion. Melting and spalling can be corrected by the selection of the refractory. Fluxing is a function of ash-fusing temperature and furnace temperature and can be controlled. Softening removes the support from the upper rows of brick, causing the wall to fall into the furnace. Expansion develops bad cracks. If the lower course of brick can be relieved from the weight of the wall, slight softening will not be serious. This has been one reason for the success of the sectionally supported wall. There is never more than 30 in. of brick above any tile.

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If the refractory is so supported that there may be slight movements due to expansion, serious cracks will not develop. The sectionally supported wall being in such short sections makes it possible for a tile to move under the change in temperature. This relief of expansion strains prevents the formation of cracks.

If these few fundamentals are borne in mind in the selection or design of an air-cooled wall, the cost of maintenance will be almost negligible. One of the reasons for the trend toward the limitation of the use of the refractory has been the fact that these principles have been lost sight of. Improperly designed walls have failed, and the entire class of wall has been condemned with the improperly designed ones.

TABLE 3 MAINTENANCE RECORD OF 48 BOILERS USING PULVERIZED COAL, DETRICK FURNACES, AS OF JULY 1, 1926

Plant	Operating hours, average per boiler	Total tons of coal burned all boilers	Fusing temperature of ash	Per cent of excess air	B.t.u. release per cu. ft. Maximum	B.t.u. release per cu. ft. Average	Water cooling in per cent of boiler surface	Total cost of repairs in cents per ton of coal fired
A....	16,912	68,043	2,240	40	16,100	10,750	0.0	1.1
B....	9,097	151,000	2,150–2,350	35	18,500	13,000	3.6	1.27
C....	9,923	295,000	2,100–2,300	30	15,200	10,000	2.6	1.41
D....	2,800	517,800	2,700	40	20,500	15,600	0.0	4.78
E....	8,400	36,750	2,700	30	13,300	10,700	3.6	0.041
F....	2,040	3,008	2,825	35	18,300	8,700	0.0	0.0
G....	4,990	5,775	2,495	45	17,100	11,000	0.0	0.0
H....	9,355	68,455	2,250	30	25,000	16,750	0.0	2.49
I....	5,200	21,024	2,240	30	20,800	13,200	7.0	0.95
J....	4,750	3,200	2,000–2,200	20	31,000	21,000	14.9	1.87
K....	22[a]	244,791	2,300	30	18,000	12,000	13.8	0.0
L....	3,723	63,612	2,400	20	17,500	11,700	14.9	0.0
M....	3,123	45,600	2,400	20	25,300	22,000	11.5	1.81
N....	7,608	25,759	2,700	30	11,500	....	5.0[b]	0.0
O....	9,257	271,020	2,000	40	27,000	16,400	3.35	0.0

[a] Months. [b] Superheater.

R. J. GAUDY.[3] Mr. Gladden has produced on this subject exactly what is needed in this and many other engineering items. In this paper the development, history and physical comparisons, and operating experience are brought together in a complete manner.

No engineer can be an exponent of water walls or air walls, as the elements of analysis, as well stated by Mr. Gladden, will positively control the selection based upon true economy.

In Mr. Gladden's conclusion he has set forth several dividing lines of release on satisfactory service. These are very valuable figures, as in checking several installations, installed by us and by others, the satisfaction in each case falls upon the side of the demarcation established by Mr. Gladden's findings and experience.

A series of articles edited in exactly the way this paper was written and covering various equipments and materials would produce a reference work of inestimable value.

G. G. HOLLINS.[4] It is essential to design the supporting steelwork in these walls so that it can expand freely and uniformly and not cause distortion. The designs of the walls should be scrutinized very carefully to see that the steelwork will not be warped by unequal heating.

It takes a relatively small temperature difference between the two flanges of an I-beam to cause considerable bowing of the beam if of any length. For instance, in the case of a beam 20 ft. long a temperature difference of 20 deg. per inch depth will cause the beam to bow in the center approximately one inch, and if the beam is 30 ft. long the bowing will be approximately two inches. The bowing of the beam of course will be toward the hot side. It is suggested that steel members of any length be of ample cross-section so as to permit free flow of the heat and equalizing of the temperature, and that the air circulation be directed so as to obtain as nearly as possible uniform cooling of the steel.

OLLISON CRAIG.[5] The heat release per cubic foot should depend not only on the type of furnace—that is, whether solid refractory wall, air-cooled structurally supported wall, or wholly or partially water-cooled wall—but also should depend on the fusion temperature of the ash of the coal to be burned.

In the March, 1928, issue of *Mechanical Engineering* there was published a tabulation by H. W. Brooks citing figures for heat releases under various conditions of character of wall and fusion temperature of ash. It is impossible to fix on a definite numerical value for a particular case. This tabulation, however, sets up a very good set of averages and makes an excellent guide to use in practical design. It is quite possible to depart considerably from these figures without radical changes in results, and the figures in the tabulation should be considered as indications only.

Less heat release per cubic foot simply means a larger furnace volume for a given rate of fuel burning. We can imagine that at a given rate of fuel burning with given fuel-burning equipment, a certain flame having a certain outline will be formed. It is probable that a larger furnace for a given rate of fuel burning is for the purpose of having the furnace walls at a greater distance from the surface of the flame. It would seem apparent that the lower the fusion temperature of the ash, the greater this distance should be, and that the refractory surface should be at a greater distance than the water-cooled surface.

In choosing between air-cooled structurally supported walls and partially or wholly water-cooled walls the decision should be based on a number of factors, but one factor which should receive considerable weight is that of load factor. The less the load factor the less money that can be spent for protecting the wall by water cooling. If the load factor is low it will be found more economical to spend money in the form of coal during the short periods of peak loads rather than in the form of fixed charges for water-wall construction. In the first case the charge is only during the period of peak loads, whereas in the second case the charge is continuous. It would be advisable in any given case to make a set-up of costs in order to determine which would be the most economical.

H. E. JOHNSON.[6] Referring to Mr. Shelby's suggestion that the depth of tile in an air-cooled wall could be made somewhat less, it may be of interest to know that in a construction having heavy iron brackets or radiating ribs, the most efficient depth of the tile shape is probably about four inches. This refers to heat conduction and absence of slag. There have been a number of Laclede-Christy arches on which the tile burned down to within two or three inches of the iron work, and the arch performance was very satisfactory in that condition. However, as an added factor of safety for such emergencies as tile breakage and other mechanical contingencies, it is found practical to make the tile deeper.

One of the points which is often overlooked in determining the practicability of installing air-cooled walls is the fact that they can be repaired more rapidly and easily than the straight refractory walls. In these days when production is worth so much, and plants are designed to run at the greatest capacity for the capital invested, shutdowns are costly. When every hour that the boiler is off the line costs money, a few hours or a day saved in repairs may effect an economy which is more than might be surmised at first thought.

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[5] Consulting Engr., Riley Stoker Corp., Worcester, Mass. Mem. A.S.M.E.

[6] The Rex Clay Products Co., Detroit, Mich.

Author's Closure

Referring to Professor Shoudy's discussion, the author appreciates the necessity of adhering to consistent and proper terms in connection with boiler-plant design and operation. The various terms relating to boiler capacities do not necessarily relate to furnace performance, except that perhaps the per cent of rating is commonly understood by average boiler operators to give a relative idea of the capacity performance of the boiler. If this factor is high, then the furnace must be liberating more heat than if this factor is low. The boiler and the furnace are necessarily two parts of a common unit, and the furnace may be large or small relative to the capacity of the boiler for which it is furnishing the heat. Professor Shoudy is correct that the terms relating to capacity performance in the discussion of boiler furnaces should be confined to the rate of heat liberation per cubic foot of furnace volume per hour. This factor again is only relative as affecting the service required of the refractories used in the furnace walls as the heat absorbed by the walls will vary due to the proximity of the inner surfaces of the walls to the flame; or to the reflected heat from opposite walls, if the furnace be narrow; or to the cooling effect of radiant-heat-absorbing surfaces in the boiler or furnace. However, this term is undoubtedly as good a unit of furnace operating capacity as can be used for general consideration of the design and operation of boiler furnaces.

Table 3, given in Professor Shoudy's discussion, contains valuable information concerning the maintenance cost of the furnaces in which there is no water-cooling surface as compared with furnaces that also have water-cooling surfaces. The author has compared the five furnaces given in Professor Shoudy's tabulation in which no water cooling is installed in connection with the air-cooled walls, with the ten furnaces in which the per cent of water cooling in the furnace walls ranges from 2.6 per cent to 14.9 per cent of that in the boiler-heating surface. It is found that the maintenance cost per ton of coal fired in furnaces without water-cooling protection averages 1.67 cents per ton of coal fired, while the average maintenance cost for the group of air-cooled furnaces in which water-cooling protection is also provided is 74 cents per ton of coal fired. Whether this saving in maintenance cost by providing water-cooling protection is offset by the additional fixed charges of the cost of providing this surface in the furnace, is a matter of economic study. It is appreciated that the fusing temperature of the ash and the labor cost of removing the ashes from the bottom of the furnace also enter into the justification for the installation of water screens.

Professor Shoudy's data, given in Table 3, furnish the following data with regard to average operating practice in the rate of heat liberation in furnaces constructed with air-cooled refractory walls:

	B.t.u. liberation per cu. ft. Average of maximum	Average of normals
Five plants in which air-cooled furnace walls are not provided with water-cooling surface	19,400	12,570
Ten plants in which water-cooling surface is installed in the furnaces of air-cooled refractory wall construction	19,810	14,440

Table 4 lists 15 plants in which a similar segregation of furnaces with air-cooled walls has been made, nine plants having no water cooling whatever, and six being provided with radiant-heat-absorbent surfaces in the side walls, or with water screens, or both. It should be noted that each furnace in the latter group is installed in a public-utility plant and is evidence of the conservative values of heat release per cubic foot of volume that are used by public-utility engineers.

Professor Shoudy points out that the use of economizers or air preheaters eliminates to a certain extent the desirability of maintaining as small a percentage of excess air as possible. With this equipment in connection with the boiler setting having air-cooled furnace-wall construction, the operator may control the slagging and erosion of the furnace walls with greater ease than if this equipment is not present. As Professor Shoudy has indicated, the following advantages of supported air-cooled refractory-wall construction may be cited; the low maintenance cost compared with fixed charges of a smaller furnace constructed with water-cooled walls, the ease of making rapairs, and the elimination of difficulties from sagging walls due to softening or spalling of the firebrick.

Referring to Mr. Craig's discussion, the author realizes the influence that the fusing temperature of the ash should have on the furnace design. The choice between structurally supported air-cooled walls and water-cooled walls, or the extent of water-cooling protection for refractory walls, depends not only on the economic features of the installation, but also on the minimum rating at which the furnace may have to operate. It is the author's belief that a lower minimum heat release per cubic foot of furnace volume may be obtained with refractory walls than with water-cooled walls in any given setting.

The factors of heat release published by H. W. Brooks (*Mechanical Engineering*, March, 1928), as cited by Mr. Craig in his discussion of this paper, are of real practical value to the designing engineer. These values agree quite closely with the operating practice of many of the plants cited in Professor Shoudy's discussion (Table 3) and with the data given in Table 4.

As has been pointed out in the discussion, wherever possible the natural circulation of air through the channels of air-cooled walls should be utilized. That is, for vertical baffling of the air channels the heated air should be taken off at the top of the air-cooled section rather than at the bottom to insure complete circulation past all portions of the air-cooled wall surfaces. The heated air, if not recovered, and used for combustion purposes, will result in an appreciable loss in furnace efficiency, and should therefore be utilized wherever possible.

TABLE 4 HEAT LIBERATION OF CERTAIN BOILER FURNACES

Plant	Furnace volume	Heat liberation B.t.u. per cu. ft. per hr. Maximum	Average
Air-Cooled Wall Construction Without Water-Cooling Surface			
1	14,660	12,400 (8 hr.)	....
2	14,200	10,600	8,300
3	12,916	17,300	14,000
4	9,300	19,900 (8 hr.)	....
5	7,970	18,100	11,100
6	6,630	13,900	12,100
7	4,950	14,300	10,000
8	4,480	13,400	12,500
9	2,760	18,500	11,100
Averages		15,380	11,300
Air-Cooled Wall Construction With Water-Cooling Surface in Side Walls or Water Screen			
10	31,250	11,800	9,600
11	25,140	18,200	9,100
12	13,800	14,500	7,250
13	12,916	22,200	18,800
14	9,100	22,000	20,000
15	6,000	29,800	22,300
Averages		19,750	14,500

FSP-51-41

Surface Heat Transmission

By R. H. HEILMAN,[1] PITTSBURGH, PA.

In view of the dearth of reliable data on the emissivity coefficients of various surfaces at the lower temperatures met with in refrigeration and heat-insulation work, the author undertook an investigation of the subject at Mellon Institute of Industrial Research, Pittsburgh. The paper describes that investigation, and among other things gives particulars regarding the methods employed in determining surface coefficients; the radiometer used; the determination of emissivity coefficients; derivation of an equation for convection; convection losses from various geometrical shapes; total heat loss from various shapes and surfaces, etc. Several charts are included in the paper which make it possible for the engineer readily to determine the total heat loss from various surfaces, and which eliminate all need for calculations for surface temperatures up to 700 deg. fahr.

A CAREFUL perusal of the existing literature indicates that there are few reliable data on the emissivity coefficients of various surfaces at the lower temperatures met with in refrigeration and heat-insulation work. However, a careful study has been made at Mellon Institute on the total transfer of heat from bare pipes. In this investigation the total surface transfer obtained from the experiments on the bare pipes has been separated into its two components, radiation and convection.

Tests have also been conducted on circular disks and on a long, thin ribbon in a vertical position. Knowing the relative amounts of radiation and convection from these various shapes, it is possible to derive formulas which will give the heat loss by convection as a function of geometrical shape, position, temperature excess, etc. Having these formulas for convection from the various shapes, we can then determine accurately the total heat loss from any shape and for any type or nature of surface by adding to the convection loss the loss due to radiation from the well-known Stefan-Boltzmann law, and the emissivity of the various surfaces experimentally determined.

Method Used in Determining Surface Coefficients

The method used in this investigation to determine the emissivity factors for various surfaces is somewhat similar to the method used by Shakespear[2] in determining the black-body radiation constant.

The total rate of heat loss was measured under given temperature conditions for a lampblack-covered brass disk, heavily silver plated, containing an electric heating element, and later under the same conditions, except that the disk surface was highly polished. The difference between these two measurements gives the difference in radiation for lampblack and polished silver surfaces, since the convection is independent of the nature of the surface. At the same time the power measurements were taken, the ratio of radiation from the lampblack surface to that of the highly polished silvered surface was obtained from the deflections of a sensitive radiometer. Having the difference in radiation from the power measurements and the ratio of the two radiations from the radiometer, we can solve for the absolute value of each.

At the same time we can calibrate the radiometer so that we can determine directly from the radiometer readings the actual losses by radiation from any surface to which the radiometer is exposed. This method of procedure allows us not only to determine the radiation loss or radiation coefficient from various surfaces, but enables us also to measure the amount of heat lost from the same surface by convection, so that we can accurately determine the convection losses for various surfaces as functions of temperature, geometrical shape, and position.

Description of Radiometer

The radiometers usually employed by physicists generally require considerable extra equipment when they are to be used for determining radiation over a considerable temperature range.

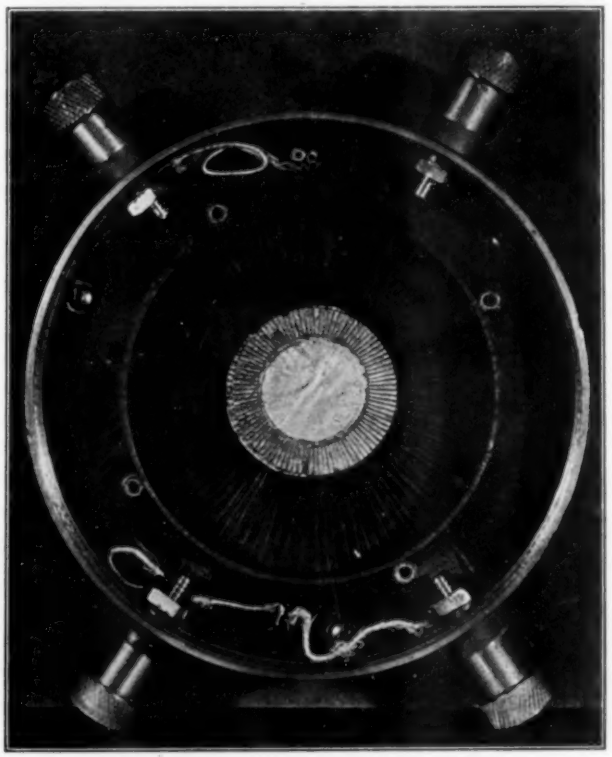


Fig. 1 Rear of Receiver

In order to keep the deflections of the galvanometer usually employed on the scale, it is necessary to employ various devices, such as varying the distance of the radiometer from the radiation source, the use of shunts in the galvanometer circuit, or the employment of sectored disks to cut down the radiation, etc.

To eliminate these devices which have a tendency to cut down the accuracy of the measurements, the author has constructed a sensitive thermopile which can be used directly on a Leeds & Northrup Type K potentiometer, which latter is sensitive enough to give direct readings at temperatures below 100 deg. fahr. and to as high a temperature as is desired.

A full-size photograph of the rear side of the receiver is shown in Fig. 1, while the complete thermopile in its mounting with the radiation disk is shown in Fig. 2.

The receiver contains 96 thermojunctions of copper and con-

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[2] Proc. Roy. Soc., London, vol. 86A (1912), p. 180.

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stantan wire, No. 44 B. & S. gage copper and No. 40 B. & S. gage constantan. The receiving disk consists of a sheet of tin approximately $^7/_8$ in. in diameter and 0.0005 in. thick, to the back of which is cemented with very thin shellac, 96 copper segments approximately 0.024 sq. in. in area and 0.0003 in. thick. The hot junctions were soldered to the copper segments with a very small, specially constructed soldering iron, the amount of solder deposited on each segment being extremely minute. The cold junctions were soldered to small brass pins contained in a circular micarta holder. In constructing this receiver the lightest materials obtainable were used in order to keep down the heat capacity. The copper to which the hot junctions were soldered was obtained by electrolytically depositing a very thin film of

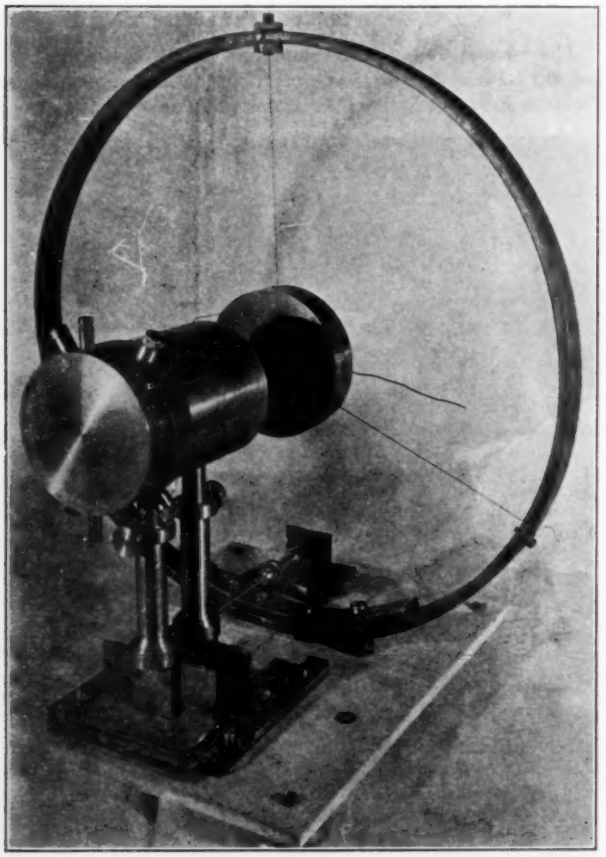


FIG. 2 THERMOPILE USED IN THE INVESTIGATION

copper on an aluminum sheet, after which the copper was stripped from the aluminum sheet.

The front side of the receiver was coated with a thin film of electrolytically precipitated platinum black, which produces a receiving surface having a very low reflection coefficient and a very high absorption coefficient.

When this receiver is placed at a distance of 3.75 in. from a radiation source of lampblack at a temperature of 300 deg. fahr., the e.m.f. generated by the receiver amounts to 5400 microvolts.

A copper-constantan thermocouple is soldered to one of the cold-junction pins, which enables one to determine at any time the temperature of the receiving disk.

DETERMINATION OF EMISSIVITY COEFFICIENTS

As mentioned previously, the method used in this investigation was to measure the total heat dissipated by two surfaces at the same temperature and also to determine the ratio of the radiations emitted from the surfaces. The theory of the determinations is somewhat as follows:

Let the radiator be lampblacked and maintained at temperature T_1 deg. abs.

If we expose this surface to the radiometer, the receiving surface of which consists of a platinum-blacked disk at temperature T_2 deg. abs., we shall get a certain deflection D_1.

Let R_{B1} be the energy radiated per hour from a unit area of the blackened radiator at temperature T_1 when exposed to the blackened cold surface of the receiver at temperature T_2, and let R_{B2} be the energy radiated per hour from a unit area of the blackened receiver at temperature T_2 to the hot blackened radiator at temperature T_1. Then $D_1 = M(R_{B1} - R_{B2})$, where M is a constant.

Now let the lampblack be removed from the radiator and let the surface of the radiator be polished, raised to temperature T_1, and exposed, as before, to the radiometer. Let the deflection be D_2. Then if R_{P1} be the energy radiated per hour from a unit area of the polished surface at T_1 to the blackened receiver at T_2,

$$D_2 = M(R_{P1} - R_{P2})$$

hence

$$\frac{D_1}{D_2} = \frac{R_{B1} - R_{B2}}{R_{P1} - R_{P2}}$$

Let

$$\frac{D_1}{D_2} = K$$

then

$$\frac{K-1}{K} = \frac{(R_{B1} - R_{B2}) - (R_{P1} - R_{P2})}{R_{B1} - R_{B2}}$$

Now, let C represent the combined conduction and convection losses per hour from the blackened radiator at temperature T_1, and H_{B1} the total loss of energy per hour from the blackened radiator at temperature T_1; then

$$H_{B1} = C + A(R_{B1} - R_{B2})(1 - b)$$

where A is the area of the radiating surface of the radiator and b is the reflection coefficient of the cold receiver.

In this case the factor $(1 - b)$ is almost negligible, as the reflection factor for platinum black is approximately 0.025 and the area of the receiver is very small in comparison with the area of the radiator. Test readings also confirm this assumption, as practically no change in temperature of the radiator could be detected when the radiometer was placed in front of the radiator.

Similarly, when the lampblack is removed and the radiator polished,

$$H_{P1} = C + A(R_{P1} - R_{P2})(1 - b).$$

Let

$$H_{B1} - H_{P1} = E$$

then

$$E = (1 - b)A\{(R_{B1} - R_{B2}) - (R_{P1} - R_{P2})\}$$

$$= (1 - b)A\frac{K-1}{K}(R_{B1} - R_{B2})$$

Let

$$(R_{B1} - R_{B2}) = a(T_1^4 - T_2^4)$$

then

$$E = (1 - b)A\frac{K-1}{K}a(T_1^4 - T_2^4)$$

so that

$$a = \frac{K}{K-1} \times \frac{E}{(1 - b)A(T_1^4 - T_2^4)} \quad \text{[1]}$$

The use of lampblack on the radiator presents various difficulties in determining the constants of the radiometer. It is usually assumed by various investigators that the emissivity coefficient of a lampblack surface is approximately 98 per cent of that of a black body. However, it has been found during the course of this investigation that the emissivity of a lampblack surface varies considerably from 98 per cent of black-body radiation, especially when the lampblack is deposited over a polished silver surface. Values as low as 70 per cent and upward to 96 per cent have been obtained, depending upon the thickness of the lampblack deposited. In the first experiment it was found that the emissivity coefficient of the lampblack surface was approximately 70 per cent of black-body radiation, although the appearance of the surface indicated that the radiator was absolutely black as far as the reflection of light could be observed. It was reasoned that so low an emissivity coefficient must indicate that the thickness of the lampblack was not sufficient. The actual thickness of the lampblack on the radiator was then measured with a microscope and found to be approximately 15 microns. The radiator was then resooted with a deposit of soot from natural gas bubbled through benzene. The thickness of the lampblack deposited was measured with the microscope and found to be approximately 72 microns or approximately 0.0028 in. It was to be expected that a thickness of 0.0028 in. of lampblack, which has a very low thermal conductivity, would cause a drop in temperature between the surface of the plate and the surface of the lampblack. In order to be able to calculate the temperature drop through the lampblack, it was necessary to know the conductivity of the lampblack used. The density of the lampblack deposited was carefully determined in order that conductivity tests could be run on lampblack having approximately the same density as that on the radiator. However, it was found that the density of the lampblack deposited was considerably lower than that of any lampblack available.

Two conductivity tests were run on lampblack. In the first test the lampblack had an apparent density of 0.343 gram per cu. cm. The lampblack was then fluffed as much as possible and a second test conducted with an apparent density of 0.263 gram per cu. cm. The apparent density of the lampblack removed from the radiator was 0.0311 gram per cu. cm. While the density of the two samples tested was considerably higher than that on the receiver, an inspection of Fig. 3 indicates that there is little difference in the two samples tested, and that the true conductivity of the lampblack removed from the receiver is probably slightly lower than that indicated by the lower curve.

The calculated temperature drop through the 0.0028-in. deposit of lampblack on the radiator, when the metal is at a temperature of 500 deg. fahr., is approximately 19 deg. fahr.

This value is somewhat surprising, as it has usually been considered by various investigators that the drop through the lampblack is negligible. However, there is no question that such a drop does exist, and it is very necessary to take it into account in order to obtain accurate results. It has been found that 40–50 microns thickness of lampblack is required on a polished metal surface to give an emissivity coefficient approaching that of a black body.

It is very desirable when calibrating the instrument to calibrate it from surfaces having as high emissivity values as possible for the one surface and as low as possible for the other surface, and also to be able to determine the constants of the radiometer over the entire temperature range through which it is to be used. It has been found during the course of this investigation that the radiation from the surface of asbestos paper approaches very nearly that of a black body. The main objection to the use of asbestos paper is that the temperature range of the instrument cannot be carried as far as with lampblack. However, it is believed by the author that for lower-temperature work a thin sheet of asbestos paper can be used directly to calibrate a radiometer. For the benefit of future investigators who may wish to conduct radiation experiments, the emissivity factors as obtained during this investigation are given for asbestos paper for temperature ranges of 100 to 700 deg. fahr.

It has been found that there are a great many materials used in engineering practice which have emissivity values very close to 98 per cent of black-body radiation, and it is believed that a great many of these surfaces could be used as standard radiators in preference to lampblack, which has been used almost entirely by former investigators.

The emissivity values for asbestos paper were determined in the same manner as those for lampblack, with the exception that the actual surface temperature of the asbestos paper, which was approximately 0.015 in. thick, was measured with a No. 40 B. & S. gage copper-constantan thermocouple fastened to the asbestos surface with a thin strip of asbestos-paper tape. In this case, as also in the case of the lampblack, the increased area due to the thickness of the paper was corrected for in making the final calculations.

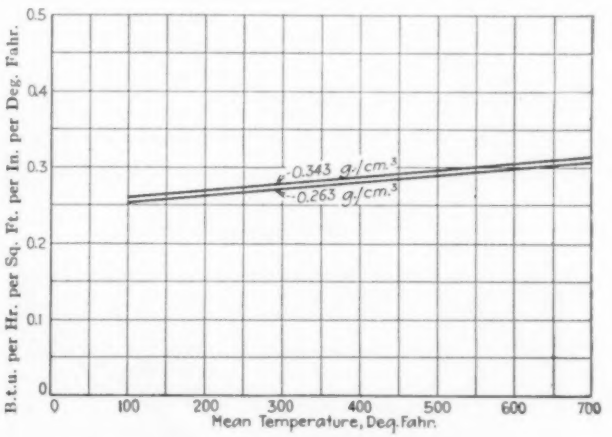


FIG. 3 THERMAL CONDUCTIVITY OF LAMPBLACK

The emissivity values obtained for various surfaces, and the approximate values for 0.0028-in. thickness of lampblack on a polished surface are shown in Table 1.

TABLE 1 EMISSIVITY VALUES FOR VARIOUS SURFACES

Surface	Temperature, deg. fahr. 100	200	300	400	500	600	700
Polished silver	0.0221	0.0252	0.0292	0.0315	0.0295	0.0308	0.0312
Lampblack	0.945	0.945	0.945	0.945	0.945	0.945	0.945
Asbestos paper	0.930	0.934	0.943	0.955	0.929	0.938	0.943
Rough steel plate	0.945	0.950	0.955	0.961	0.969	0.975	0.975
Aluminum-surfaced roofing	0.216	....	....	....	....	....	....
Polished brass	0.096	0.096	0.098	0.098	0.096	0.096	....
Flat black lacquer	0.96	0.98	....	....	....	....	....
Black lacquer	0.80	0.95	....	....	....	....	....
White lacquer	0.80	0.95	....	....	....	....	....

The following calculations show how the emissivity values of Table 1 were obtained. The data used in these calculations are taken from those obtained with the radiator surface at 300 deg. fahr. Power corrections have been made for slight changes in room temperature and for losses due to exposed power leads and supports.

CALCULATIONS FOR ASBESTOS PAPER AND POLISHED SILVER

Temperature of receiver, 80.75 deg. fahr.
Area of radiator, 0.1885 sq. ft.
Deflection by asbestos surface, 5200 microvolts

Deflection by silver surface, 139 microvolts
$K = 37.4$
Energy required to maintain asbestos surface at 300 deg., 39.7 watts, corrected for area
Energy required to maintain silver surface at 300 deg., 18.16 watts
Ratio of receiver area to exposed area of radiator, 0.117.

Then

$$a = \frac{1.027 \times 21.54 \times 3.415}{0.1885(760^4 - 540.75^4)0.997}$$

$$= 16.22 \times 10^{-10} \text{ B.t.u. per sq. ft. per hr.}$$

The black-body radiation according to the latest determinations is $17.23 \times 10^{-10} (T_1^4 - T_2^4)$ B.t.u. per sq. ft. per hr.

The emissivity e of the asbestos surface, therefore, has the value $16.22/17.23 = 0.943$. Radiation from the asbestos surface is $16.22 \times 10^{-10} \times 24.81 \times 10^{10} = 402.5$ B.t.u. per sq. ft. per hr.

Radiation and convection per sq. ft. of asbestos surface are

$$\frac{29.7 \times 3.415}{0.1885} = 719 \text{ B.t.u. per sq. ft. per hr.}$$

Convection from asbestos surface equals

$$719 - 402.5 = 316.5 \text{ B.t.u. per sq. ft. per hr.}$$

Radiation and convection from the polished silver surface equal

$$\frac{18.16 \times 3.415}{0.1885} = 329 \text{ B.t.u. per sq. ft. per hr.}$$

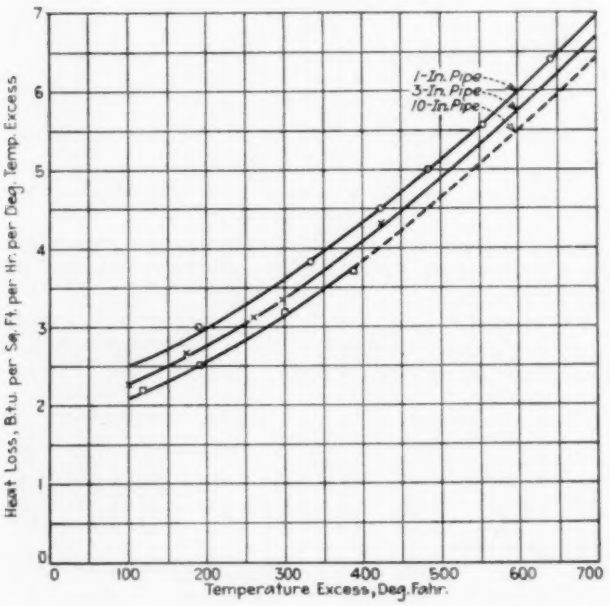


FIG. 4 TOTAL HEAT LOSS FROM BARE PIPES

Radiation from the polished silver surface is

$$329 - 316.5 = 12.5 \text{ B.t.u. per sq. ft. per hr.}$$

Then e for the polished silver surface $= \frac{12.5}{17.23 \times 24.81} = 0.0292.$

HEAT LOSS BY RADIATION AND CONVECTION FROM HORIZONTAL CYLINDERS

The heat loss from nominal 1-in., 3-in., and 10-in. steel pipes has been determined by the author.[3] The experimental curves corrected to a standard room temperature of 80 deg. fahr. are shown in Fig. 4. The total heat loss as obtained from these corrected curves has been separated into its two components, radiation and convection, by the following method.

The emissivity of a rough steel plate having the same characteristic surface as the steel pipes was carefully determined with the radiometer for temperatures of 100 deg. fahr. to 700 deg. fahr.

The heat loss by radiation was then calculated by use of the Stefan-Boltzmann formula

$$R = 17.23 \times 10^{-10} \times e(T_1^4 - T_2^4) \quad \ldots\ldots [2]$$

The radiation loss was then subtracted from the total experimentally determined loss, which gave the loss by convection.

The results of these calculations are given in Table 2.

The heat loss by convection in B.t.u. per hr. per sq. ft. as a function of temperature excess, deg. fahr., pipe to room, is plotted in Fig. 5.

The heat loss by radiation per unit area for the steel surface or

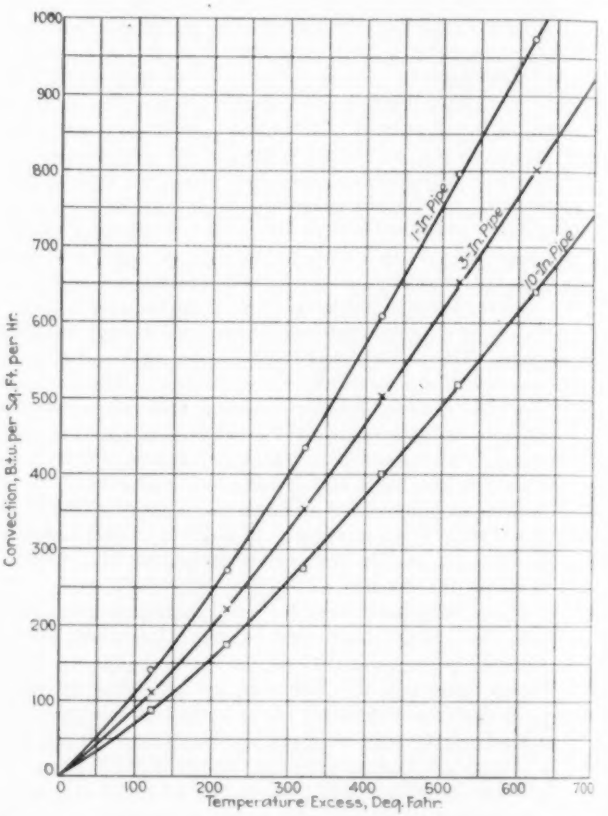


FIG. 5 HEAT LOSS BY CONVECTION FROM THREE SIZES OF PIPE

TABLE 2 HEAT LOSS FROM BARE STEEL PIPES—DATA CORRECTED TO 80 DEG. FAHR. ROOM TEMPERATURE

Pipe temperature, deg. fahr.		200	300	400	500	600	700
Temperature difference, deg. fahr.		120	220	320	420	520	620
Pipe diameter, actual							
1.315 in.	Total	312	682	1200	1885	2772	3871
	Radiation	171.5	409	765	1276	1977	2897
	Convection	140.5	273	435	609	795	974
3.5 in.	Total	283	631	1120	1780	2631	3698
	Radiation	171.5	409	765	1276	1977	2897
	Convection	111.5	222	355	504	654	801
10.75 in.	Total	259	583	1040	1676	2496	3537
	Radiation	171.5	409	765	1276	1977	2897
	Convection	87.5	174	275	400	519	640

[3] R. H. Heilman, *Ind. & Eng. Chem.*, vol. 16 (1924), p. 451.

for any surface whose emissivity is known can be readily calculated for the surface in any shape or position or for any temperature excess. However, the calculation of the heat loss by convection for the same conditions is an entirely different matter and presents many difficulties, some of which will have to be solved experimentally. The experimental work can be reduced considerably by the use of dimensional analysis.

One of the objects of this paper is to modify the formulas for horizontal cylinders which have been derived by dimensional reasoning, so as to be able to calculate more accurately the heat loss from horizontal cylinders of any diameter desired.

Free Convection From Horizontal Cylinders

Perhaps the most noteworthy contribution to engineering literature on free convection is the paper by C. W. Rice[4] presented before the A.I.E.E. in June, 1923. In this paper Rice adopts the film theory developed by Langmuir,[5] who states that the reason for the existence of a film around a hot body may be seen as follows: Consider a horizontal wire maintained at a given temperature in a fluid; the fluid adjacent to the wire will become heated and rise, while the cooler fluid of greater density will flow into its place. Thus a convection current is set up by the difference in density between the hot and cold fluids.

At the surface of the wire the fluid is stationary due to viscosity. As we proceed from the surface of the wire the velocity of the convection currents increases until a distance is reached at which the critical-velocity conditions in the fluid are exceeded and the streamline flow bursts into turbulent motion. The discontinuity between the streamline and the turbulent motion constitutes the outer boundary of the film. At the inner boundary the fluid has the temperature of the hot surface and at the outer boundary the temperature of the ambient fluid.

Langmuir's assumption thus reduces the complex problem of free convection to one of conduction in the steady state.

The question of how the film thickness varies with the size and shape of the body and the properties of the ambient fluid is determined by the method of dimensions plus experiments.

The development of the general expression for effective film thickness as given by Rice is as follows: Assume that the film thickness will depend on the temperature difference, the coefficient of density change per degree of temperature change, the acceleration of gravity, and the density and viscosity. Thus the driving force of the convection currents will be proportional to the difference in density produced by the expansion times the acceleration of gravity, and it is the viscosity which opposes this driving force. The motion will obviously depend upon the shape of the hot body. The film thickness may also depend upon the rate at which heat is transmitted through the film, and the specific heat. The addition of these variables also allows us to include the effects of heat carried by slow mass motion within the film. We shall therefore assume that the ratio of film thickness to the size of the body is given by the following expression:

$$B/D = K(\alpha g)^x \ \Delta t^y D^z \mu^p \rho^n k^m c_p^r \quad \ldots\ldots [3]$$

where B = film thickness (L)
D = a linear dimension (L)
K = a numerical constant
α = coefficient of density change per deg. cent. (θ^{-1})
g = acceleration of gravity (LT^{-2})
Δt = temperature difference, deg. cent. (θ)
μ = viscosity of fluid ($L^{-1}MT^{-1}$)
ρ = density of fluid (ML^{-3})
k = heat conductivity of fluid ($HL^{-1}T^{-1}\theta^{-1}$)
c_p = specific heat per gram at constant pressure ($HM^{-1}\theta^{-1}$)

Substituting the dimensions of the various factors in Equation [3] we obtain

$$0 = (LT^{-2}\theta^{-1})^x(\theta)^y(L)^z(L^{-1}MT^{-1})^p(ML^{-3})^n (HL^{-1}T^{-1}\theta^{-1})^m(HM^{-1}\theta^{-1})^r \quad \ldots\ldots [4]$$

from which the following system of simultaneous equations is obtained:

$$\left.\begin{array}{ll} \text{By Length:} & 0 = x + z - p - 3n - m \\ \text{By Mass:} & 0 = p + n - r \\ \text{By Time:} & 0 = -2x - p - m \\ \text{By Heat:} & 0 = m + r \\ \text{By Temperature:} & 0 = -x + y - m - r \end{array}\right\} \quad \ldots\ldots [5]$$

The solution of the simultaneous equations in [5] gives the following relations:

$$\left.\begin{array}{ll} x = n/2 & p = -n - m \\ y = n/2 & r = -m \\ z = 3n/2 & \end{array}\right\} \quad \ldots\ldots [6]$$

From physical considerations we know that the film thickness should increase with the viscosity of the fluid. We therefore change the sign of n and substitute the relations [6] in Equation [3] and obtain

$$\frac{B}{D} = K\left[\frac{v}{(\alpha g\ \Delta t)^{1/3}D^{1/3}}\right]^n\left(\frac{k}{\mu c_p}\right)^m \quad \ldots\ldots [7]$$

where $v = \mu/\rho$ = kinematic viscosity of the fluid.

In order to use this equation we must determine experimentally the values of K, m, and n.

Experimental data on convection in liquids are needed to accurately determine the value of m. For ideal gases this factor does not vary greatly from one gas to another, and for air it may therefore be neglected. However, Rice uses a value of $^1/_4$ for m.

The value for n has been determined by various investigators from tests on heat loss as a function of density of gas. The best average values seem to lie between 0.5 and 0.55. Tests conducted by the author on a silver disk in a very large vacuum oven with pressures of 0.3 to 1 atmosphere indicated a value of approximately 0.53.

The constant K will vary with every system of similar bodies such as cylinders, spheres, flat surfaces, etc., and must be determined experimentally for each system. This constant, according to the results of various investigators, varies between very wide limits.

Derivation of Equation for Convection From Horizontal Cylinders

In accordance with Langmuir's film theory, Rice assumes that the heat lost from a cylinder can be calculated as a problem of conduction in the steady state.

The heat loss will therefore be directly proportional to the conductivity of the film and the temperature drop through it, and inversely proportional to its thickness. For large surfaces such as commercial pipes the film thickness will be small in comparison with the diameter of the pipe, and it is unnecessary to use the logarithmic mean thickness.

The loss by convection is then given by

$$W_c = (A/B)k_{avg.}\ \Delta t \quad \ldots\ldots [8]$$

where
A = area of surface in cm.^{2}
B = effective film thickness in cm.

[4] Chester W. Rice, Trans. A.I.E.E., vol. 43 (1924), p. 131.
[5] Irving Langmuir, Trans. Am. Electrochem. Soc., vol. 23 (1913), p. 318.

k = heat conductivity for the average of surface and ambient temperature in watts cm.$^{-1}$ deg. cent.$^{-1}$
Δt = temperature difference in deg. cent.

The effective film thickness from [7] is

$$B = KD(k/\mu c_p)^m [v/(\alpha g \Delta t)^{1/2} D^{1/2})]^n \text{ cm} \ldots\ldots\ldots [9]$$

where
K = experimental constant depending on the system of similar bodies under consideration
D = diameter of cylinder in cm.
μ = viscosity of fluid at average temperature, cm.$^{-1}$ gram sec.$^{-1}$
c_p = specific heat at constant pressure for average temperature, joule gram^{-1} deg. cent.$^{-1}$
m, n = experimental exponents
$v = \mu/\rho$ = kinematic viscosity of fluid for average temperature, cm.^{2} sec.$^{-1}$
g = acceleration of gravity = 980 cm. sec.$^{-2}$
$\rho = \rho_0 (1 + \alpha t)$ gram cm.$^{-3}$ at temperature t deg. cent.
ρ_0 = density of fluid at 0 deg. cent.
$\alpha = 1/273$ for the case of ideal gases.

Substituting Equation [9] in [8], the general expression for thin films and moderate temperature differences is obtained.

$$W_e = (Ak\Delta t/KD)\ [(\alpha g \Delta t)^{1/2} D^{3/2}/v]^n\ (\mu c_p/k)^m \ldots\ [10]$$

For air at atmospheric pressure Jeans[6] gives

$$\mu = \mu_0 \left(\frac{T}{273}\right)^{0.754}$$

$\mu_0 = 0.0001705$ in c.g.s. units (independent of pressure)
$\rho_0 = 0.00129$ gram per cu. cm. at 0 deg. cent. and 76 cm. Hg
$v = \mu/\rho = 7.05 \times 10^{-6}\ T^{1.754}$ cm.^{2} sec.$^{-1}$
$k = 3.46 \times 10^{-6}\ T^{0.754}$ watts per cm. per deg. cent. at 400 deg. K.

$$\frac{\rho}{\mu} = 7.56 \left(\frac{273}{T}\right)^{1.754}$$

$\mu c_p/k = 0.74$

For large horizontal cylinders in gases and liquids Rice uses the following constants for the three factors K, n, and m in the general equation: $K = 2.12$, $n = 1/2$, and $m = 1/4$. Substituting these values in [10] gives

$$W_e = 0.472A[(\mu c_p/k)(\alpha g/D)]^{1/4}\ (1/v)^{1/2} k\, \Delta t^{5}/_{4} \text{ watts} \ldots [11]$$

Using the constants given above for air and substituting in [11] gives

$$W_e = 0.000785A(1/D)^{1/4} p^{1/2} (1/T_{\text{avg.}})^{0.123}\, \Delta t^{5}/_{4} \text{ watts} \ldots [12]$$

where $A = \pi DL$ = area in sq. cm.
p = absolute air pressure in atmospheres
$T_{\text{avg.}}$ = average of cylinder and ambient absolute temperature, deg. K.

Equation [12] expressed in engineering units is as follows:

$$H_e = 1.015\ (1/D)^{0.25} p^{1/2} (1/T_{\text{avg.}})^{0.123} (\Delta t)^{1.25} \text{ B.t.u. per sq. ft. per hr.} \ldots [13]$$

where D = cylinder diameter in inches
$T_{\text{avg.}}$ = average of cylinder and ambient temperature, deg. fahr. abs.
Δt = temperature difference (deg. fahr.) between surface and air.

If we calculate the heat loss from a 3.5-in. horizontal pipe at a temperature of 580 deg. fahr. with room temperature at 80 deg. fahr. from Rice's modified equation, we have

$$H_e = 1.015 \times (1/3.5)^{0.25} \times 1^{1/2} \times (1/750)^{0.123} \times (420)^{1.25} = 626 \text{ B.t.u. per sq. ft. per hr.}$$

Similarly for the 1.315-in. and the 10.75-in. diameter pipes, we have 799 and 473 B.t.u., respectively.

The experimentally determined values for the three pipes as taken from Fig. 5 are 608, 500, and 398 B.t.u., respectively. This indicates that the values as calculated from Rice's modified equation are 24, 20, and 16 per cent higher than the experimental values.

Similar calculations for lower temperatures indicate slightly lower percentages, and for higher temperatures, slightly higher percentages.

Changing the value of K in Rice's equation would bring the calculated and experimental values in line for only one pipe size and at only one pipe temperature. It is readily apparent that the exponent for the factor $1/D$ is the one which is the most out of line.

By calculation it is found that the exponent for the shape factor which best fits the experimental results for the horizontal pipes is 0.2 instead of 0.25. However, if we wish to change the exponent for the shape factor to 0.2 and yet maintain the general form of Equation [10], it will be necessary to change the value of n from 0.5 to 0.533.

Changing n to 0.533 in [10] results in a change of the exponents of Δt from 1.25 to 1.266, v from 0.5 to 0.533, and g from 0.25 to 0.266. Letting $C = K$ times the numerical constant obtained in [12], the resulting final equation is

$$W_e = CA \left(\frac{1}{D}\right)^{0.2} p^{0.533} \left(\frac{1}{T_{\text{avg.}}}\right)^{0.181} \Delta t^{1.266} \ldots\ldots [14]$$

The average value for C as obtained from the experiments on the horizontal pipes in engineering units is 1.016, or

$$H_e = 1.016 \left(\frac{1}{D}\right)^{0.2} p^{0.533} \left(\frac{1}{T_{\text{avg.}}}\right)^{0.181} \Delta t^{1.266} \text{ B.t.u. per sq. ft. per hour} \ldots\ldots [15]$$

It is generally accepted that the value for Δt is very close to 1.25. However, it has been found during the course of this investigation that the exponent is not always constant but generally varies slightly with the temperature excess. For example, if we take the convection curve for the 3-in. pipe shown in Fig. 5 and analyze it carefully by means of the equation $y = ax^b$ or $\log y = \log a + b \log x$, taking off values from the experimental curve and solving the resulting simultaneous equations we find that the average value of b from 100 to 350 deg. temperature excess is 1.19 and from 350 to 700 deg. is 1.23, with an average for the entire curve of approximately 1.21.

Similar analysis from the curve obtained by Langmuir on a silver disk (part of which is shown in Fig. 6) gives from 100 to 700 deg. a value of 1.23, and from 700 to 1100 deg. a value of 1.19.

Also analysis of a curve obtained by the author on a silver disk 6.55 in. diameter and 0.8 in. thick gives values of b from 100 to 350 deg. of 1.23, and from 350 to 700 deg. of 1.31.

Analyses of other curves obtained by various investigators show the same characteristics. It is therefore readily apparent that while the variation with temperature excess can be forced to follow an exponential straight line having a given slope, with little error, in order to determine accurately the true heat loss it is necessary to introduce another factor into the equation which will take care of this variation. The factor $(1/T_{\text{avg.}})$ serves to fill in this gap.

[6] J. H. Jeans, The Dynamical Theory of Gases, Cambridge Univ. Press, 2nd edition, 1916, p. 317.

If we take Equation [15] and apply it to the experimental curves given in Fig. 5, we find that it fits almost exactly over the entire range of the curves.

Convection Losses From Various Geometrical Shapes

Vertical Cylinders. Apparently no experiments have been conducted on vertical cylinders to determine accurately the effect of diameter on the heat loss. Griffith and Davis[7] conducted experiments on a 6-in.-diameter pipe to determine the effect of height on the convection loss. They obtained a minimum loss for a height of approximately 3 ft. and a maximum loss for the shortest height tested, which was 1.8 in.

The longest cylinder tested was 8 ft. 8 in. in height, and the exponent for the variation of convection with temperature for this particular height was 1.34.

Rice,[4] using the value of 1.333, calculates the value of n in the general equation and obtains a value of 0.666. Using this value, the shape factor in Equation [10] cancels out entirely, which indicates that for long vertical cylinders the loss per unit area is independent of size.

It is very probable that this assumption is not correct, as Griffith and Davis obtained various values for the exponent of Δt ranging from 1.34 to 1.17 for the various heights tested, with an average value of 1.24 for the eight different lengths. Using any one of the exponents with the exception of 1.333 in Equation [10] will cause the shape factor to appear in the equation. Also, as was noted for the case of the horizontal cylinders, changing the exponent of Δt from 1.25 to 1.266 caused a change in the exponent for the shape factor from 0.25 to 0.20. Thus a slight change in the exponent of Δt causes a very important change in the exponent of $(1/D)$ or $(1/H)$.

To obtain some experimental data on the effect of size on the loss from vertical cylinders, a 1-in. and a 3-in. pipe each 3 ft. long were placed in a vertical position and tested. These tests resulted in two separate total-heat-loss curves being obtained, which were parallel to each other and also parallel to the curves for the horizontal pipes but somewhat above them. These tests prove conclusively that for heights of 3 ft. the diameter of the pipe has a decided effect on the heat loss per unit area. Since Griffith and Davis obtained practically the same heat loss per unit area for the 3-ft. and 8-ft. heights, it is reasonable to expect that for heights above 3 ft. the shape factor would enter into the equation as for the 3-ft. tests. Therefore, for vertical cylinders 3 ft. in height or over, it is suggested that Equation [14] be used in calculation of the convection losses. The results of the experiments conducted on the two vertical pipes indicated the same exponent for $(1/D)$ and also a value of 1.235 for C. Therefore, for long vertical cylinders in air at atmospheric pressure, Equation [14] becomes

$$H_c = 1.235 \left(\frac{1}{D}\right)^{0.2} \left(\frac{1}{T_{\text{avg.}}}\right)^{0.181} \Delta t^{1.266} \quad \ldots\ldots [16]$$

Plane Surfaces. To determine the effect of height on plane vertical surfaces, experiments were conducted on three electrically heated brass disks heavily silverplated and highly polished, in order to reduce the radiation effect as much as possible. The dimensions of the disks were as follows: 3.47 in. diam., 0.758 in. thick; 6.55 in. diam., 0.80 in. thick; and 9.97 in. diam., 0.90 in. thick.

The test results showing the convection loss in B.t.u. per sq. ft. of surface is given in Fig. 6. Taking the vertical height as equal to the square root of the area of one of the sides, calculations show that here again the exponent for the shape factor is 0.20. The experimental results for the three disks indicate a value of 1.394 for C. If we substitute this value for C in Equation [14] and calculate the convection loss for various temperature excesses we find that the equation fits the experimental curves with very little error for the entire temperature range.

The effect of shape factor for vertical walls is a very important item in engineering industry. For this reason further investigational work should be carried out on larger plane surfaces than were used during this investigation. While the experiments conducted prove conclusively that the effect of size is very evident for diameters up to 10 in., they do not indicate how far the exponent can be used for larger diameters or vertical heights.

Griffith and Davis[7] state that for heights above 8 or 12 in., the heat loss per unit area is, roughly, independent of size. However, they obtain a greater heat loss per unit area for a vertical plate approximately 50 in. square than was obtained during this investigation for a vertical disk 10 in. in diameter or than Langmuir[5] obtained for a vertical disk 7.5 in. in diameter. It is very probable that Griffith and Davis did not have the correct emissivity coefficient when calculating the radiation correction. In the case of the 10-in. disk and Langmuir's 7.5-in. disk, the radiation correction was always lower than 3.5 per cent for the entire temperature range of the experiments.

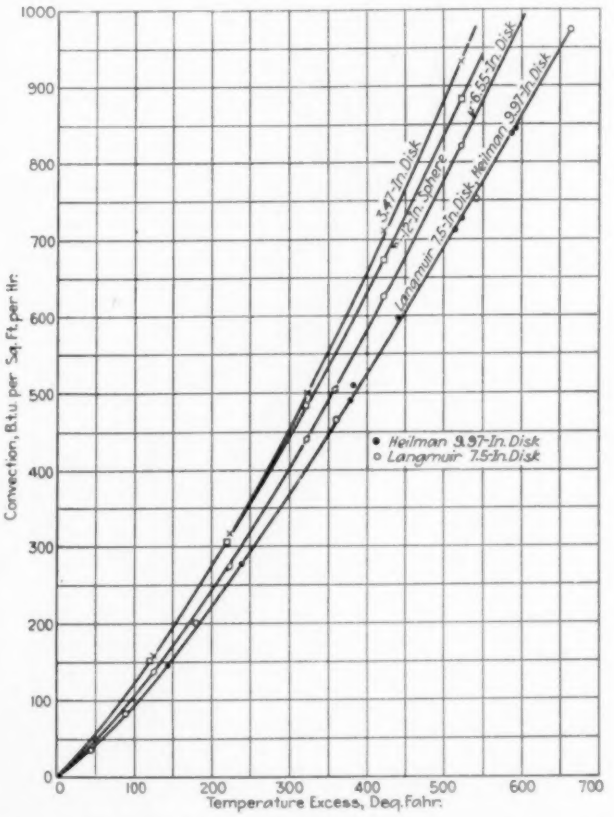


Fig. 6 Heat Loss by Convection From Spheres and Disks

To obtain further data on the effect of height on convection losses a test was conducted on a strip of Nichrome IV ribbon 0.50 in. wide, 0.05 in. thick, and varying in length from 2 in. to 9 ft. This ribbon was suspended in a vertical position and heated by passing a current of electricity through it. To take care of lead-wire losses, small voltage taps were silver-soldered to the ribbon at a distance of 6 in. from each end in one case, and at a distance of 2 in. from each end in the other tests.

[7] Special Report 9, Department of Scientific and Industrial Research, London, England.

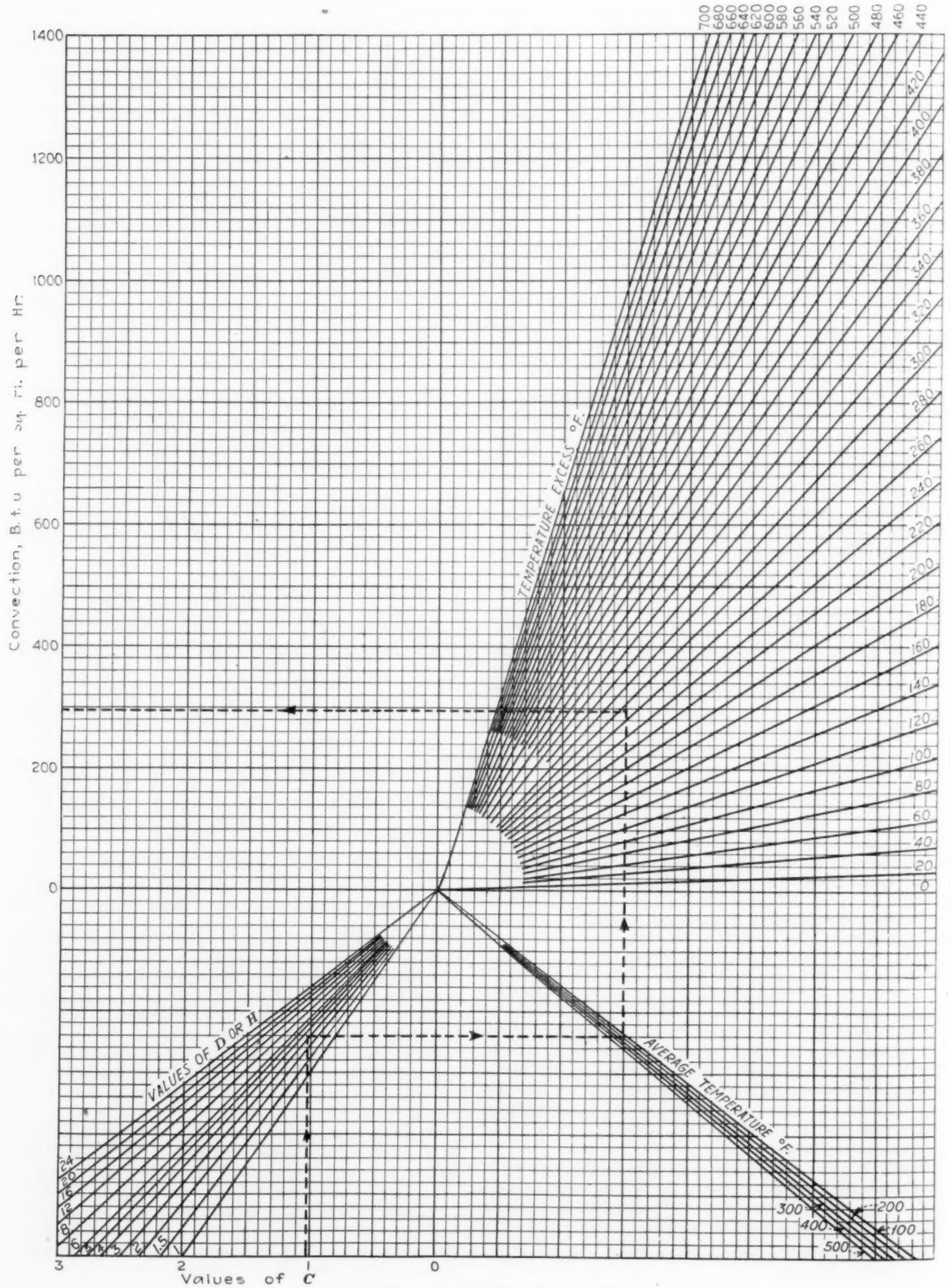


FIG. 7 HEAT TRANSMITTED BY CONVECTION

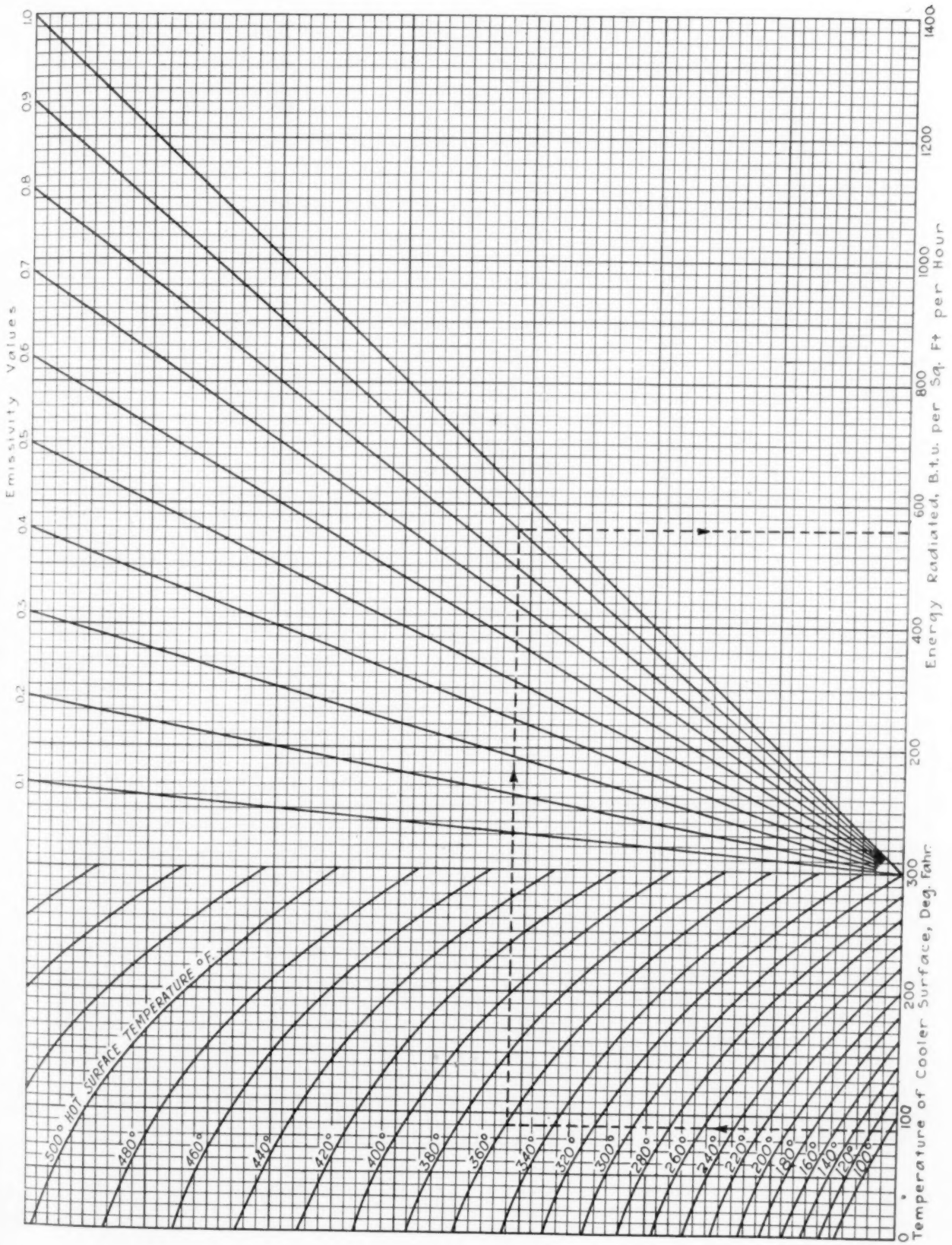


FIG. 8 HEAT TRANSMITTED BY RADIATION

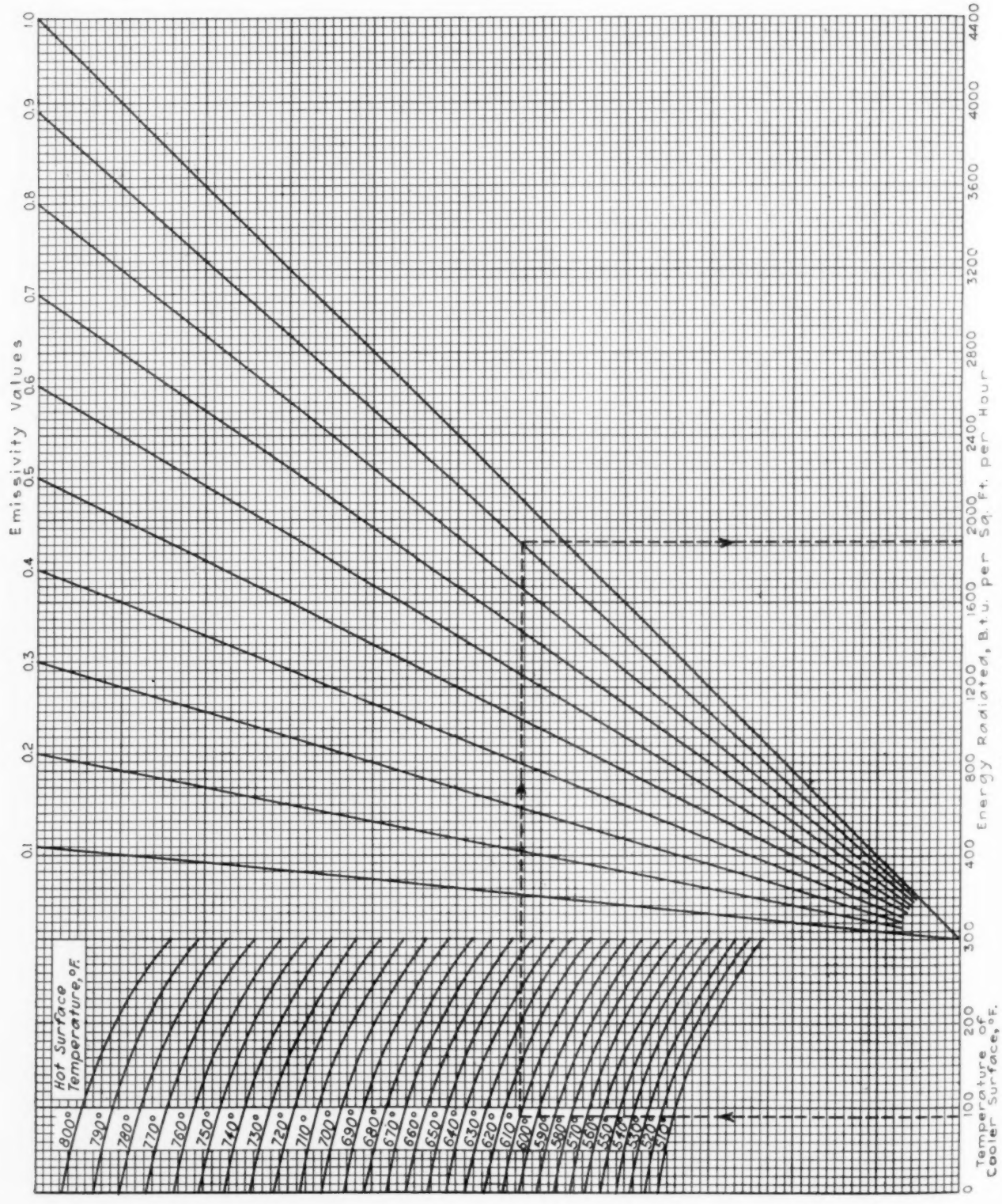


FIG. 9 HEAT TRANSMITTED BY RADIATION

For temperature excesses up to 350 deg. fahr., the heat loss per unit area was roughly independent of length for lengths above 2 ft.

While these tests throw some additional light on the vertical height at which the heat loss becomes constant for a vertical wall, they do not indicate conclusively the exact height, as the modification of the convection currents about a thin vertical ribbon of very small width in comparison to height is probably somewhat different from what it is for a vertical wall having a width comparable to its height.

Additional tests on larger plane surfaces are needed to definitely settle this question.

However, until further experimental data have been obtained, it is suggested that Equation [14] with a value of 1.394 for C be used for vertical surfaces, assuming the heat loss to become constant for heights greater than 24 in., or $(1/24)^{0.2} = 0.549$.

The heat loss from a plane surface in a horizontal position facing upward is greater than for the same surface in a vertical position, also the heat loss in a horizontal position facing downward is less than for the same surface in a vertical position. Langmuir found the convection upward from a 7.5-in. disk to be about 12 per cent greater than for the disk in a vertical position. He also found the convection downward to be about one-half as much as the convection upward.

Griffith and Davis found the convection upward from a 50-in-square plate to be about 28 per cent greater than for the vertical position, and the convection downward to be about 34 per cent less than the convection in the vertical position.

Here again the size of the plate would have a marked effect on the modification of the convection currents and the consequent convection losses in the various positions.

More experimental work is needed on various-size plates in the three positions to determine accurately the effect of size and position.

If we take the percentages obtained by Griffith and Davis for the present, we shall have for the value of C in our equation, 1.79 for the horizontal position facing upward and 0.89 for the horizontal position facing downward.

Spheres. One test was run on a 12-in-diameter cast-iron sphere during this investigation. The convection loss from this sphere is shown in Fig. 6. The shape of the curve is slightly different from that of the vertical disks, but the curve can be very closely approximated by Equation [10], using 1.82 for the value of C. The sphere shows a very great rate of heat loss in comparison with the other shapes tested, the factor C being slightly higher than obtained for the horizontal plate facing upward.

Total Heat Loss From Various Shapes and Surfaces

It is seen from the discussion preceding that the heat loss by free convection from the surfaces of the principal geometrical shapes met with in engineering practice can be approximated with great accuracy in most cases by the equation

$$H_c = C \times \left(\frac{1}{D}\right)^{0.2} \times \left(\frac{1}{T_{avg.}}\right)^{0.181} \times \Delta t^{1.266} \text{ B.t.u. per sq. ft. per hr.}$$

The heat loss by radiation can be calculated by means of the equation

$$H_r = 17.23 \times 10^{-10} \times e(T_1^4 - T_2^4) \text{ B.t.u. per sq. ft. per hr.}$$

Table 1 in the first part of the paper gives values of e for some of the surfaces met with in practice. It was the author's intention to include a great many more surface values in this paper. However, it is his opinion that practically all the surfaces encountered in engineering practice (other than lustrous or bright metallic surfaces) have practically the same emissivity value as was found for asbestos paper or the rough steel plate. Some of the surfaces included in this list are brick, stone, wood, plaster, paper, cloth, various colored paints, etc.

It should be mentioned here that these values are intended only for what might be termed "black" heat radiation, as no attempt has been made to determine emissivity values for solar radiation during this investigation.

The results of the emissivity determinations show a somewhat greater heat loss by radiation than was formerly believed to exist for the various surfaces commonly met with in practice. For instance, a commercial rough steel pipe in a room at 70 deg. fahr. with the pipe surface at 100 deg. fahr., loses over 60 per cent of the total heat loss from the surface by radiation. The same pipe at a temperature of 1000 deg. fahr. loses about 86 per cent of its heat by radiation.

To enable the engineer to readily determine the total heat loss from various surfaces the author has included several charts which eliminate all calculations for surface temperatures up to 700 deg. fahr. These charts have been carefully drawn and should give an accuracy of 1 per cent or less in most cases, except for the very low temperature excesses.

The dotted lines show the method of procedure for a 3-in. O.D. horizontal cylinder at a temperature of 360 deg. fahr. with a room temperature of 90 deg. fahr. or a temperature excess of 270 deg. and average temperature of 225 deg. Emissivity value, 0.9.

Referring to Fig. 7, the dotted line shows 298 B.t.u. loss by convection and Fig. 8 shows 560 B.t.u. loss by radiation, or a total loss of 858 B.t.u. by radiation and convection.

The following table gives the values of C to use in Fig. 7 for the various shapes.

Horizontal cylinders	1.016
Long vertical cylinders	1.235
Vertical plates	1.394
Horizontal plates facing upward	1.79
Horizontal plates facing downward	0.89
Spheres	1.82

As mentioned previously, the actual convection loss per unit area from small horizontal plates and short vertical cylinders decreases as the size of the plate and the height of the cylinder increase. The exact size or height at which the loss becomes constant is not known. Until further experimental work is done it is suggested that 24 in. be taken as the limit.

In the case of short vertical cylinders, assuming that the effect of height varies as the 0.2 power for lengths up to 24 in. and is independent of height for lengths above 24 in., the approximate convection loss can be obtained from Fig. 7 as follows:

Take, for example, the heat loss by convection from a 2-in-diameter cylinder 12 in. in height, at a temperature of 360 deg. fahr. with room temperature of 90 deg. fahr. According to Equation [14], the loss from a horizontal cylinder varies directly as the shape factor or $(1/D)^{0.2}$, therefore, for a short vertical cylinder the loss should vary directly as the shape factor and also directly as the height factor or $(1/D)^{0.2} \times (1/H)^{0.2}$, or for the 2-in-diameter cylinder 12 in. in height, we have $2 \times 12 = 24$. Using the value of 24 for D or H in Fig. 7 and a value of 2.33 for C, which is the value to use for short vertical cylinders, we proceed from 2.33 for C vertically to 24 for D or H, thence horizontally to 225 deg. fahr. average temperature, thence vertically to 270 deg. fahr. temperature excess, and thence horizontally to 450 B.t.u. loss by convection from the short vertical cylinder. When the product of $D \times H$ for short vertical cylinders is greater than 24, it will be necessary to assume the probable position of this product above the values of D or H

Fig. 9 Heat Transmitted by Radiation

shown in Fig. 7, as the values are given in the diagram up to only 24 in. so as not to conflict with the statement given above that the loss per unit area probably becomes constant for values of D or H above 24 in.

ACKNOWLEDGMENT

In conclusion, the author wishes to express his thanks to Mr. R. W. Ortmiller, who has assisted in securing the data for this paper.

Discussion

W. J. KING.[8] This paper constitutes a valuable contribution to the literature of heat transmission. In addition to adding to our knowledge of the subject, it calls attention to the thin spots where more information is badly needed.

The determinations of the radiation emissivities of various surfaces were particularly interesting. The methods and care used in these measurements inspire confidence in their accuracy. The results are confirmed by some similar tests which have been made recently at Schenectady, except that we found considerably lower values for our particular kind of steel surface. It is very difficult to specify exact values for the emissivity of metallic surfaces because it varies so much with the state of oxidation. We found values as low as 0.68 for a steel plate as received, and 0.34 for the same plate brightened up with fine sandpaper. Likewise, the emissivity of aluminum paint may vary with the amount and kind of vehicle from 0.28 for spar varnish to 0.55 for amyl acetate. We also confirm the statement that most nonmetallic surfaces encountered in engineering practice have an emissivity of about 0.95, at temperatures below incandescence. Apparently the two values of 0.80 for black and white lacquer at 100 deg. are misprints.

Are the conductivities for lampblack in Fig. 3 per inch or per foot of thickness?

It is unfortunate that the determinations of the convection components of the heat loss from horizontal pipes were made with steel pipes having a high emissivity, instead of with polished silver surfaces. At the higher temperatures the radiation component is several times the convection. If the emissivity of the original pipe happened to be 0.90 instead of 0.95, as determined for an ostensibly similar steel plate several years later, the convection in the case of the 10-in. pipe at 700 deg. would have been apparently 156 B.t.u. more, representing an error of 24.4 per cent. This same uncertainty detracts from the value of Griffith and Davis' work on the convection from lamp-blacked cylinders.

In re-deriving Mr. Rice's formula for convection from horizontal cylinders, the author seems to have fixed his attention upon Rice's *approximate* equation using the value of $n = 0.5$ in Equation [7] as convenient for rough calculations, and appears to have completely overlooked the fact that Rice gave $n = 0.54$ and $K = 2.60$ as the *exact* values. Using these latter values in Equations [7] and [10] gives exactly the same results as Equation [15]. Although the author might have spared himself this repetition, he has nevertheless independently confirmed Rice's work, so that the formula for free convection for long horizontal cylinders seems to be rather well established.

In the section on convection from vertical cylinders, the author mentions Rice's conclusion that for long, vertical cylinders the loss per unit area is independent of size, but says that, "It is very probable that this assumption is not correct, as Griffith and Davis obtain values of the exponent of Δt ranging from 1.34 to 1.17 for the various heights tested, with an average value of 1.24 for the eight different lengths." It should be observed that Rice specifies *long*, vertical cylinders, whereas the last four of Griffith and Davis ranged from 1.83 in. to 11.3 in. in height, or 0.266 to 1.65 diameters.

Apparently both Rice's and the author's formulas for vertical cylinders are not quite correct, and in some cases may result in very considerable errors. Rice's formula does not contain the diameter as a factor in the case of long cylinders, and the author's method is dimensionally unsound in the case of short cylinders. It is suggested that since the loss from a horizontal cylinder varies as the shape factor or $(1/D)^{0.2}$ (dimensionally $L^{-0.2}$), for a short, vertical cylinder, the loss should vary directly as the shape factor and also directly as the height factor or $(1/D)^{0.2} \times (1/H)^{0.2}$ (dimensionally $L^{-0.2} \times L^{-0.2} = L^{-0.4}$). Since all the other exponents remain the same, this latter method cannot be used without destroying the dimensional equality. For long cylinders, since the coefficient in Equation [16] is greater than that in [15], it would appear that the convection would be greater in the vertical than in the horizontal position, at atmospheric pressure, which does not seem reasonable and is contradicted by Rice's results.

Although experimental data to confirm it are lacking, it seems probable that for short horizontal or vertical cylinders both the length and diameter enter into the equation in such a way that the sum of their exponents is equal to something close to —0.2. When horizontal the effect of diameter predominates, and when vertical the effect of height predominates. For the horizontal position, as a rough guess, the length may enter the equation as $(1/L)^{0.05}$ and the diameter as $(1/D)^{0.15}$; and conversely for the vertical position. Upon plotting the graphs of these two functions the shapes of the curves suggest that they would account for the known experimental facts for both long and short cylinders, as they become nearly constant for large dimensions and change rapidly for small values.

Likewise, for vertical planes it is probable that both height and width enter into the formula, with the effect of height predominating. In the horizontal position the exponents of the two dimensions must be the same. This would account for the discrepancy between the ratios for convection in the vertical and horizontal positions as found by Langmuir and by Griffith and Davis.

It is to be hoped that the author will continue his experiments with vertical pipes and with plane surfaces. If he does so with the same thoroughness which characterizes his measurements of radiation emissivities, the aforementioned difficulties will no doubt be straightened out in due course.

CHARLES H. HERTER.[9] The author's contribution on surface heat transmission is most interesting and valuable. He deserves much credit for endeavoring to determine separately the effects of radiation and convection. This important work should be continued. After perusing the first paragraph one expects to find emissivity coefficients applicable to heat insulation and refrigeration in which fields the temperature differences diminish to 50 and as low as 5 deg. fahr.

In Figs. 5, 6, and 7 the convection curves terminate at 0 for 0 deg. difference, but in Fig. 4 the curves for total heat loss apply only to temperature differences from 100 to 700 deg. Engineers must therefore continue to make their own assumptions as to the heat-absorbing capacity of dry refrigerating surfaces at the low differences usually met with. It is conceded that at 30 deg. temperature difference between pipe and cold air the value of K per degree difference is greater than at 10 deg. difference because the convection is weak at small differences; but the exact rate of change in the curve is not well established. In refrigerating practice the problem is further complicated by the presence of condensed moisture and snow of varying degrees of density on

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the cooling coils, also by the direction of air flow along or across these pipes, by their grouping, and finally by their surface temperature, which may not be as constant or uniform as one would wish for best effect.

If therefore these heat and temperature measurements could actually be extended into the field of lower temperatures, to produce these missing data, all engineers concerned with air conditioning and the preservation of perishable products would greatly appreciate any efforts along these lines.

In 80 per cent of these applications horizontal cylindrical pipes down to $^1/_2$ in. outside diameter are used; the remainder may be flat surfaces, mostly vertical, but in no cases in practice are spheres used for heat transmission, even though a sphere is a geometrical shape of particular fascination to a physicist.

F. G. HECHLER.[10] It seems unnecessary to say that this paper is a welcome contribution to the literature on surface transmission of heat and that it should help materially in correcting some popular misconceptions. One of these deals with the heat transmission through air spaces as commonly used in walls of dwellings. Although the paper does not refer to air spaces, the data should with slight modification be applicable.

It has long been the practice in this country to calculate the heat transmission through such an air space by assuming that the resistance of the space is equivalent to that of two additional surfaces under still-air conditions. The value of the surface coefficient for still-air conditions is assumed to be 1.34, which is the mean value obtained from a few tests. This gives for the thermal resistance of an air space $2 \div 1.34 = 1.49$, a value which is quite generally used for calculating the heat losses through walls, though it does not appear to be based either on tests or a rational analysis of the problem. During the past few years data from various sources have become available which show that the value 1.49 is entirely too high and that if for simplicity and convenience a single value is desirable, the value 1.00 would be much better. In a well-insulated wall where the total thermal resistance is high the effect of the air space is masked and it makes little difference whether we use 1.00 or 1.49; but for uninsulated walls with low thermal resistances the error may be large; for example, in a wall plastered inside and stuccoed outside, both on metal lath, the heat transmission calculated with the factor 1.49 will be about 20 per cent too low.

In vertical air spaces convection and conduction are closely connected, their relative values depending largely on the width of the space. For very narrow spaces convection is greatly restricted, but conduction increases so rapidly that the total heat transfer gets larger. However, for spaces wider than about 0.5 in. there appears to be but little change with width. In practice air spaces are seldom less than 0.75 in. nor more than 4 in. wide, and it is interesting to calculate the heat transfer across such a space with the data now available.

Some years ago the Bureau of Standards published such a table in which the radiation values were based on an emissivity coefficient of 70 per cent of that for a black body. Shortly afterwards the results obtained by Griffith and Davis in England indicated that this coefficient should have been much higher. This is confirmed by Mr. Heilman, who finds that for most building materials the coefficient should be 93 per cent. This of course magnifies the effect of radiation.

The average temperature in building walls as well as the temperature range in the air space is somewhat below the useful range of the curves given in the paper, so that it is necessary to calculate the desired data from the equations for convection and radiation. It is also of interest to compare the equations given by Heilman with those proposed by Griffith and Davis. If we choose a mean temperature of 40 deg. fahr. (500 deg. abs.) in order to reduce Heilman's Equation [14] for convection to its simplest form we have the following comparison:

	Hc, convection	Hr, radiation
Heilman	$0.248\ \Delta T^{1.266}$	$16.024 \times 10^{-10}(T_1^4 - T_2^4)$
Griffith and Davis	$0.30\ \Delta T^{1.25}$	$16 \times 10^{-10}(T_1^4 - T_2^4)$

where ΔT represents the temperature excess and T_1 and T_2 are the absolute temperatures in degrees fahrenheit. The two equations for radiation loss are almost identical, but those for convection differ considerably, though it should be noted that the difference decreases with increasing values of ΔT. The coefficient as given by Heilman depends on the average absolute temperature, but for the small temperature range found in walls this has little effect on the coefficient which is 0.252 at 0 deg fahr. and 0.246 at 70 deg. fahr.

The interesting fact, however, is the rapid increase of the convection loss with increasing temperature differentials. For example, with a mean temperature of 40 deg. fahr. the value of Hc for $\Delta T = 1$ deg. is 0.248 B.t.u., while for $\Delta T = 10$ deg. and the same average temperature $Hc = 4.58$ B.t.u., which is equivalent to 0.458 B.t.u. per deg.; this is nearly twice as great as for $\Delta T = 1$. Changing the temperature differential, however, has no effect on the radiation loss if the mean temperature remains constant.

The radiation loss is independent of the distance between the surfaces and applies therefore to air spaces in walls. The convection and conduction losses, however, depend on the width of the space; the foregoing equations for Hc apply only for free-air convection and not to the condition found in air spaces where convection currents are greatly restricted. Fortunately Griffith and Davis did some experimental work to determine the convection and conduction losses in air spaces of different widths, and from their data it appears that the sum of conduction and convection in air spaces is about 60 per cent of the value for free convection. Applying this factor to Heilman's equations the results given in Table 3 have been calculated for average temperatures varying from 20 deg. to 70 deg. and for values of ΔT of 1 and 10 deg.

TABLE 3

Average temperature deg. fahr.	T_1	T_2	Hc free air, total	Hc^1 air space, per degree	Hr per degree	Ht total, per degree	Air space resistance, $= \frac{1}{Ht}$
20	480.5	479.5	0.250	0.16	0.71	0.87	1.15
20	485.0	475.0	4.613	0.28	0.71	0.99	1.01
30	490.5	489.5	0.249	0.15	0.75	0.90	1.11
30	495.0	485.0	4.594	0.28	0.75	1.03	0.97
40	500.5	499.5	0.248	0.15	0.80	0.95	1.05
40	505.0	495.0	4.576	0.27	0.80	1.07	0.94
50	510.5	509.5	0.248	0.15	0.85	1.00	1.00
50	515.0	505.0	4.576	0.27	0.85	1.12	0.89
60	520.5	519.5	0.247	0.15	0.90	1.05	0.95
60	525.0	515.0	4.557	0.27	0.90	1.17	0.86
70	530.5	529.5	0.246	0.15	0.96	1.11	0.90
70	535.0	525.0	4.539	0.27	0.96	1.23	0.81

Hc = convection loss in free air, Equation [14].
Hc^1 = convection and conduction loss in air space = $0.6Hc$.
Hr = radiation loss.
$Ht = Hc^1 + Hr$.

The surprising feature to many who have not given the matter much attention is the important part that radiation plays in the heat transfer across air spaces at these low temperatures. Radiation accounts for about 80 per cent of the total and convection and conduction for only 20 per cent when the value of ΔT is 10 deg. However, if ΔT increases, without changing the value of the average temperature, the latter losses increase rapidly, as may be seen by comparing the values for temperature differences of 1 deg. and 10 deg.

The last column gives the resistance of the air space. These values should be compared with the conventional value of 1.49.

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It is significant that only at the lowest temperatures and for 1 deg. temperature difference is the resistance greater than 1.00. For this part of the country the average temperature of an air space is probably about 40 deg., and the value of ΔT will be in the neighborhood of 10 deg., though this may vary considerably with the kind of wall, so that for ordinary calculations the thermal resistance of an air space may be taken as 1.00. If the actual temperatures are known or if they can be accurately predicted, the appropriate value for the resistance may be calculated.

P. Nicholls.[11] The author and the company which is supplying the funds are to be congratulated that a serious attempt is being made to put the general case of the transfer of heat from surfaces in contact with air on a more assured basis, and it is to be hoped that the work will be continued by the author and his results confirmed by others.

The correctness of the deductions for the loss by convection depends entirely on the accuracy of the measurement of the radiation. In view of this it would have been better if the experimental work on the measurement of the radiation had been published in greater detail either in this or a previous paper. It can be presumed that every care was taken, but one is justified in asking for a fuller treatment with a discussion of the possible sources of error and, as far as possible, attempts to evaluate their magnitude. Confidence in the radiation values is necessary for the acceptance of the deductions on the convection law.

The following are some points in the radiation measurement not dealt with:

(1) What was the exact design of the radiator; presumedly it lost heat from all its surfaces? What assurance was there that the loss of heat from the part of its surface exposed to the receiver was the same as that from the rest of the surface?

(2) Apparently the assumption was made that the radiation loss of the radiator to other objects than the disk of the receiver was the same as that to the disk; how far was this shown to be true?

(3) Are the radiometer deflections dependent on the temperature of the cold side of the junctions, or what assurance was there that the temperature of the cold junctions remained constant during an observation?

(4) It would seem necessary that the rate of loss of heat from the disk should be proportional to the rate it receives heat; would it not be preferable to water-cool the enclosing case of the receiver so that its temperature could be the same for all tests? Also to use a water-cooled enclosure to surround the radiator, and to make its interior surface of the same finish as that of the exterior of the radiometer? Would it not be still better to give all surfaces a platinum black finish so that the radiator would have full exposure to that finish?

(5) Assurance is desirable that the radiation constant of the steel surface used in the radiation tests corresponded exactly with that of the steel pipes used in the total heat-loss tests, because the derivation of their convection loss depends on the values being the same.

Some of these questions would not have much influence on the results obtained, but information on them is desirable; the suggestions are not made critically as it is recognized that the work is still in its early stages. The incompleteness of the details of test procedure and of the data obtained as given in the report of the similar work done by the English Department of Scientific and Industrial Research militated against its availability and usefulness.

The author's main experimental results are on horizontal cylinders; since he will have much data on loss from canvas coverings, would it not be well to treat those surfaces as he has done the bare pipes?

One would not imagine that the diameter would have as much influence on heat transfer from vertical as from horizontal cylinders, although apparently it is suggested that the same index for D be used; however, to remove doubt, the summation of results in the table giving values for C could well be expanded to include the value to be used for D for each shape. It is also suggested that the experimental data do not warrant the number of significant figures used for any of the values of C except for the horizontal cylinders.

The form of the diagrams used for the computations is most convenient, and their use will probably give results more nearly corresponding to actual heat transfer than are usually arrived at by other methods of computation.

A general treatment of heat transfer by radiation for the use of engineers is needed, particularly to explain the effect of the situation and the size of the surface relative to its surroundings. The most complete treatment in English is given in Bulletin No. 1 of the Department of Heating and Ventilating of the University of London, "The effect of the shape and surroundings of a hot surface on the radiation from it," by A. H. Barker and M. Kinoshita. The theoretical treatment is complete although the supporting experimental data are not conclusive.

Author's Closure

In reply to Mr. Nicholls' discussion, the author agrees with him that a fuller discussion of the design and the experimental work with the radiometer would have been desirable. However, it was felt that the results of the experiments would be of the most value to the engineering profession, and to include both in the same paper would have made the paper too long.

If time will permit a complete paper on the radiometer and experimental determinations will be prepared in the near future. For the present, however, the author will try to answer a few of Mr. Nicholls' questions.

(1) The radiator was constructed of $^1/_4$ in. thick brass, so as to assure a uniform temperature throughout its surface. It consisted of a circular disk approximately 3.5 in. in diameter and 0.75 in. thick, with a cavity approximately 3.25 in. in diameter by 0.25 in. thick. This cavity was completely filled with an alundum plate containing a uniformly wound heating element. The lead wires were brought out through small holes and also the three supporting wires so that there were no obstructions or protruding screws.

The whole outer surface of the receiver was heavily silver-plated and highly polished so that the radiation from any part of the external surface would be exactly the same. When the radiator was lamp-blacked care was taken to see that a uniform thickness of lampblack was deposited over the entire surface. The thickness was checked by the use of a microscope, thus leaving little chance for an error in the rate of heat lost from the surface exposed to the receiver, in comparison to the heat lost to the walls of the hood in which the entire apparatus was enclosed.

(2) The disk of the receiver was painted with platinum black, while the walls of the hood were lined with fibrous insulating material whose emissivity value was approximately the same as that of the platinum blacked receiver.

(3) The radiometer deflections are dependent on the temperature of the cold side of the junctions. The temperature of the cold junctions increases as the temperature of the radiator and the heat loss from the radiator increases. However, the exact temperature of the cold junctions was measured at the time the deflections were taken, and the true temperature of the receiver could then be determined from the number of junctions in series.

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The emissivity values were thus calculated from the true temperature of the radiator and the true temperature of the receiver.

(4) In the preliminary experiments, the receiver was provided with a water-cooled enclosure. However, the water temperature could not be maintained constant to closer than 0.1 deg. fahr., and continually rose and fell within that range owing to the operation of the thermostat. Owing to the large number of junctions in the receiver, a variation of 0.1 deg. fahr. causes considerable variation in the deflections. The final experiments were run without the water-cooling as it was found that the temperature of the hood could be held much closer than the water-cooled enclosure could be held. As mentioned previously, the radiator had full exposure to approximately black-body conditions. In any event the area of the radiator was insignificant in comparison to the area of the walls of the hood, and the amount of energy reflected back from the walls to the receiver would be proportional to the ratio of the receiver area to the hood area; and when we consider that the walls have a total reflection of only about 2 per cent and dividing this 2 per cent by the ratio of the receiver area to the wall area the energy received back would be far below 0.01 per cent. In fact, the hood walls could have been lined with polished silver without affecting the accuracy of the experiment to any great extent.

(5) The steel surface used in the emissivity tests was identical in emissivity value to that of the steel pipes, as both were later checked and both were considerably oxidized.

The author appreciates the fact that Mr. Nicholls' discussion is not in the nature of a criticism of the results obtained, but is merely to bring out some of the points of the constructional details and experimental procedure. The author feels that the accuracy of the experiments can be summed up by the fact that his tests check almost exactly with the tests by former investigators who have experimentally determined the black-body radiation constant. This constant has been determined by a great many investigators and has also been calculated mathematically, so that there is little doubt that it will ever be changed 1 per cent, no matter how many investigators re-determine it.

In reply to Mr. King, the author is gratified to know that his results have been mainly confirmed by the experiments of the General Electric Company.

The values of 0.80 for black and white lacquer at 100 deg. are not misprints. These values are for lacquers having a glossy surface. At 100 deg. a value of 0.80 was obtained, while at 200 deg. the value had increased to 0.95. This increase is accounted for by the fact that at a temperature of 200 deg. fahr. the lacquers had become oxidized to a considerable extent and had consequently lost most of their glossy finish, and were consequently radiating at almost as high a rate as the flat black lacquer. The results of these tests were included to show that there was a difference in radiating power between a glossy lacquer or paint and a flat lacquer or paint and also that color had no effect on "black-heat" radiation.

The conductivities for lampblack in Fig. 3 are per inch of thickness.

In regard to the emissivity of the steel pipes, the 3-in. and 1-in. pipes were rechecked at the time the radiation tests were conducted and were found to give exactly the same losses as were obtained several years before.

The author cannot agree with Mr. King's statement that Rice gave the exact value of $n = 0.54$ and $K = 2.60$ in his paper, as he nowhere makes this statement in his paper. In fact, Rice gives the impression that $n = 0.5$ and $K = 2.12$ are the more exact values to use, as he states, "The line of slope $n = 1/2$ has been drawn in such a manner as to give most weight to the last 17 points, since the accuracy of determining (B/D) $(\mu C_{p/k})^{1/4}$ is greater for thin films, assuming equal accuracy in determining the watts convection. Thus for $n = 1/2$ we take $K = 2.12$ in.

It seems strange that if Rice knew that the exact values of n and K were 0.54 and 2.60, respectively, he would go to all the trouble to calculate his equations using the values of 0.5 and 2.12. Throughout his entire paper, including the Summary, he uses the value of n as $1/2$. Any one reading his paper would certainly gain the impression that $1/2$ was the value to use for n.

Using 0.54 and 2.60 for n and K results in the following equation: $W_c = 0.000852\ (1/D)^{0.19}\ p^{0.54}\left(\frac{1}{T_{avg.}}\right)^{0.193}\ \Delta t^{1.27}$ which is not exactly the same as Equation (15) but only approximately so.

The author still maintains that Rice's equation for *long* vertical cylinders is not correct as his equation absolutely ignores the effect of diameter.

While Mr. King is right in stating that the last four of Griffith and Davis' cylinders ranged from 1.83 in. to 11.3 in. in height, he entirely neglects the fact that they obtained practically the same heat loss per unit area of cylinders for 3 ft. and 8 ft. heights. From the results of their tests, a cylinder 3 ft. in height can be considered a *long* cylinder as their graph shows that the loss becomes constant at a height of approximately 60 centimeters, or approximately 2 ft.

In regard to the effect of both diameter and height for short cylinders, the author realizes that an equation containing both diameter and height is dimensionally unsound, and he was therefore very careful not to include such an equation in the paper. However, he cannot agree with Mr. King's statement that the combined effect of diameter and height for short cylinders results in an exponent of approximately 0.2, but believes that the exponent for each dimension or height and diameter should be about 0.2 as mentioned in the paper. If this is true, as experimental tests seem to indicate is the case, then the method outlined in the latter part of the paper is mathematically exact.

In regard to long vertical and horizontal cylinders, the author can see no reason why the vertical cylinder should not lose more heat than the horizontal. As mentioned in the paper, actual tests on a 1-in. and a 3-in. pipe proved conclusively that such was the case. The author expects to continue the work on large vertical and horizontal planes and hopes that some reliable data will be obtained on these shapes.

In regard to Mr. Herter's discussion, it will be very hard to apply the results of these tests to refrigeration work, where the pipes and surfaces are covered with frost or moisture of varying extent.

The discussion by Professor Hechler is very timely and interesting and should help to clear up some of the misconceptions on the transfer of heat through air spaces. The author always felt that it was not logical to use two surface resistances in calculating the resistance of an air space. Since the loss of heat through an air space is usually mainly by radiation, it is not logical to use a surface coefficient of 1.34 for both surfaces. This is theoretically unsound, owing to the fact that radiation takes place from the hotter surface to the cooler surface and the amount transmitted is tied up in both surfaces at the same time and cannot be separated.

The calculations of Professor Hechler showing that the resistance of an air space is usually less than 1.00 have been confirmed by tests conducted at Mellon Institute on air spaces of 0.5 in. to 4 in. These tests were conducted for heat flow horizontally across the space and also vertically upward and downward. By the method of tests, it was possible to separate radiation, convection, and pure conduction. These tests indicated that in vertical air spaces less than 1 in. in width there was practically no heat transmitted by convection.

Professor Hechler's table has also been confirmed by Rowley

and Algren,[12] who found that the resistance of vertical air spaces increased with the width of the space up to about $^3/_4$ in. and became practically constant for widths greater than $^3/_4$ in. They also found that for mean temperatures from 20 to 150 deg. fahr. the resistance was always less than 1.00. The resistance of a 1.5-in. air space at 20 and 150 deg. fahr. mean temperature was 0.975 and 0.64, respectively. For a $^1/_2$-in. air space at the same temperature, the resistances were 0.92 and 0.58, respectively. The temperature differences in these tests were greater than 10 deg. fahr. in all cases, and for the highest mean temperatures the temperature differences were as much as 150 deg. fahr. This accounts for the fact that the resistances obtained by Rowley and Algren are somewhat lower than the values calculated by Professor Hechler for 1 and 10 deg. fahr. temperature difference.

In comparing the author's equations with those of Griffith and Davis, a much closer agreement will be found if the fact is considered that Griffith and Davis used the old black-body radiation constant of 16×10^{-10} instead of the latest value of 17.23×10^{-11}, as used by the author. The old value is about 7 per cent lower than the later values.

[12] Journal A.S.H.&V.E., Jan., 1929, p. 17.

FSP-51-42

The Laws of Similarity for Orifice and Nozzle Flows

A Summary of the More Important Theoretical Results of the Author's Twenty Years of Work on the Flow Through Orifices, Nozzles, and Venturi Tubes

By JOHN L. HODGSON,[1] EGGINGTON, BEDFORDSHIRE, ENGLAND

IN 1909 the author commenced a research which had for its objects the establishment of the square-edged orifice[2] placed between two flanges in a pipe line as an accurate secondary standard of flow measurement, and the development of the necessary theoretical relations which would enable the discharge coefficient for an orifice placed between two flanges in a pipe line, a nozzle, or a venturi tube, when calibrated with any one fluid such as water, to be used as a basis for determining the coefficient of discharge Ω when a geometrically similar[3] orifice, nozzle, or venturi tube was used to measure the discharge of any other fluid such as steam, air, or oil. An account of this work as far as it was then developed was published in 1917 in a paper entitled "The Commercial Metering of Air, Gas, and Steam"[4] which was presented to the Institution of Civil Engineers.

The 1917 paper dealt largely[5] with the comparison of air and water calibrations of the same orifice,[6] nozzle, or venturi tube, and Figs. 17, 32, 33, and 35 of that paper gave the results of such tests plotted against an "elastic-flow criterion" (P_2/P_1) and a "viscous-flow criterion" $(V_1d_2W_1/)\mu$.[7] Figs. 32 and 33 of the paper showed the failure of the viscous-flow criterion when there was appreciable expansion of the fluid at the orifice or nozzle, and Fig. 32 predicted the way in which the elastic-flow criterion would break down when the flow conditions were such that viscosity became of major importance.

In the tests carried out prior to the writing of the 1917 paper the author had not been able to work down to very low values of $V_1d_2W_1/\mu$ with either water or air owing to the small "heads" produced when the velocity was small. Fig. 35 of the 1917 paper therefore showed the $\Omega:V_1d_2W_1/\mu$ curve for air and water[8] rising steeply as it approached the origin, though it was obvious from theoretical considerations that when $V_1d_2W_1/\mu$ was zero Ω must also be zero.[9]

An attempt to complete the curve by using extremely sensitive differential pressure-measuring devices having failed on account of the difficulty, with apparatus of the size then used, of holding the flow steady and eliminating small temperature changes for a sufficient time before each point was taken to enable the minute pressure differences available to produce their proper effect, the author finally succeeded in obtaining a complete curve for one orifice by using various mixtures of glycerine and water. This enabled the viscosity to be varied over a large range, and an easily measurable head to be obtained at each discharge. The complete curve down to the zero value of $V_1d_2W_1/\mu$ was first published by the author in April, 1921.[10] Later in the same year a full account[11] of the research by which the curve was obtained was given by W. N. Bond, who had assisted in the work.

In April, 1922, the author published the results of some further tests on orifices and venturi tubes covering a large flow range,[12] and in 1925 he presented a second paper to the Institution of Civil Engineers entitled "The Orifice as a Basis of Flow Measurement."[13] This paper gave the results of many further tests, and also dealt at some length with the theory of orifice discharge, and with the use of flow criteria which enabled the values of the coefficient of discharge obtained at various flows of one given fluid to be used for any other fluid whose viscosity, density, and elastic properties were known. The following paper and its appendixes summarize and bring up to date the theory of orifice and nozzle flow developed in the 1917 and 1925 papers above referred to.

The Types of Flow

Three properties of a fluid, namely, its viscosity, density, and compressibility, affect its flow through an orifice, nozzle, or venturi tube. In the case of fluids such as compressed air or steam at the pressures usually employed, these three properties normally give rise to six main types of flow, the first five of which

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[1] Messrs. George Kent, Limited, Luton, England.

[2] Since square-edged orifices placed between two flanges in a pipe line are now almost universally used in the measurement of steam flows, it may be of interest to put on record that the first of such orifices was furnished by the author to Mr. Holbrook Gaskell of the United Alkali Company in 1910; this orifice being calibrated by means of water flows.

[3] It should be noted that the "geometrical similarity" should extend to the positions of the pressure holes, to the relative roughness of the walls of the pipe, to the upstream face of the orifice, and to the upstream and downstream conditions if these are irregular.

[4] Minutes of Proc. Inst. C. E., vol. 204, pp. 108–193.

[5] This paper also gives details of the first large compressed-air meters that were evolved. These meters, which were designed in connection with the Rand Air Power Scheme, were required to measure the kilowatt-hours U in the air delivered according to the relation

$$U = (\Omega\phi)K_2N(P_1 - P_2)^{1/3}(P_1^{1/3}\log P_1/K_3)$$

where K_2 and K_3 are numerical constants.

The first of these meters was installed by the author in 1910, and they are still functioning with an overall accuracy of ± 1 per cent. They measure about 400,000,000 horsepower-hours per year.

[6] Gaskell's tests on the relation between water and steam flows (see vol. 197 of the Proc. Inst. C. E.) were based on the author's then unpublished work, as is explained in Appendix 7 of "The Commercial Metering of Air, Gas, and Steam."

[7] The tests made by the author in 1916–17 were the first tests which proved experimentally for water and air that, provided there was no appreciable expansion of the fluid in passing through the nozzle or orifice, the value of the coefficient of discharge depended only upon the value of the viscous-flow criterion $V_1d_2W_1/\mu$. The viscous-flow criterion is very often wrongly called the Reynolds criterion. It was Lord Rayleigh, and not Prof. Osborne Reynolds, who generalized the theorem. (See *Phil. Mag.*, 1892, p. 59; 1899, vol. 48, p. 231; and the Report of the Advisory Committee for Aeronautics, 1909–10, p. 38.) Reynolds simply showed by his experiments that $VcdW/\mu$ (where Vc is the "critical velocity" and d the diameter of the pipe) was constant for all fluids whatever might be their density W or viscosity μ. (See Phil. Trans., 1883, vol. 174, p. 935, and Reynolds' Collected Papers, vol. 2, p. 51.)

Contributed by the Special Research Committee on Fluid Meters and presented at the Annual Meeting, New York, N. Y., Dec. 3 to 7, 1928, of The American Society of Mechanical Engineers.

[8] The values of the abscissas of this curve are incorrect. The multiplier 10^6 should be 10^2, if V_1 and W_1 are measured in f.p.s. units, and μ is measured in c.g.s. units.

[9] Compare Fig. 4 and see Footnote 21.

[10] Minutes of Proc. Inst. C. E., vol. 212, p. 272, Fig. 15.

[11] "Viscosity in Orifice Flows," Proc. Physical Soc., vol. 33 (1921), p. 225.

[12] Trans. Inst. Naval Architects, vol. 64 (1922).

[13] Selected Engineering Papers, No. 31, Proc. Inst. C. E.

follow one another in succession as the difference of pressure across the orifice is gradually increased from zero upward, or as the pressure ratio is gradually decreased from unity toward zero. These six types of flow may be considered to depend on:

1 *Viscosity only* (streamline flow), where the flow is directly proportional to the difference of pressure across the orifice.

2 *Both viscosity and density.*

3 *Density only*, where the effects of changes of viscosity and compressibility are so small as to be inappreciable, and where the flow through the orifice is proportional to the square root of the difference of pressure across it.[14]

4 *Both the density and the law of compression for the gas* under the particular conditions which obtain at the orifice. For this type the equation of the flow is more conveniently expressed in terms of the pressure ratio, than of the pressure difference, across the orifice.

5 *Both the density and the compressibility of the gas.* For this type the flow is directly proportional to the pressure on the upstream side of the orifice and is independent of the downstream

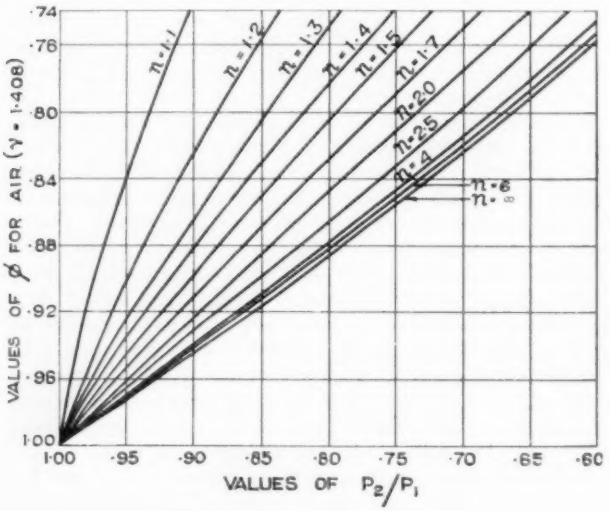


FIG. 1 P_2/P_1 vs. ϕ FOR AIR

pressure, the velocity at the vena contracta being equal to the velocity of sound in the gas at the pressure and density which obtain there.[15]

6 *Density, compressibility, and viscosity.* This type of flow occurs, for example, when the ratio of the respective diameters of the main and orifice is large, and air at low pressure is being measured. In this case the effects of both compressibility and viscosity have to be considered together.

It is only the first of these six types of flow which has a clearly defined and commonly accepted name, viz., streamline or viscous flow. The second type must necessarily involve viscosity with or without turbulence, the third type turbulence, while the last three come within what may be termed the "elastic" region of the flow.

THE DISCHARGE-FORMULA AND COEFFICIENT CURVES

At first glance it would appear necessary to use six different types of formulas to represent these types of flow. The present paper shows that it is possible by means of one formula and two curves of coefficients to represent all six types of flow for any

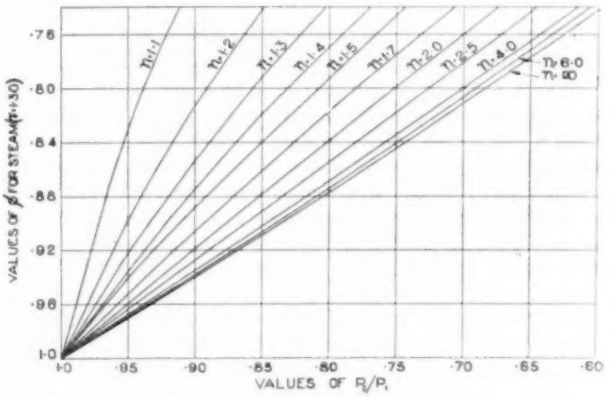


FIG. 1*a* P_2/P_1 vs. ϕ FOR STEAM

orifice of given proportions and situation for any one easily compressible fluid such as air, and for all relatively incompressible fluids such as water. It is further possible to make the results approximately true for all easily compressible fluids, as well as for all relatively incompressible fluids.

Since pressure differences are more easily measured than pressure ratios, the most convenient formula[16] to use from the point of view of practical metering is

$$Q = (\Omega\phi)N[2g(P_1 - P_2)W_1]^{1/2} \quad \ldots\ldots [1]$$

where

$$N = a_1/\sqrt{n^2 - 1} = a_1/\sqrt{(a_1/a_2)^2 - 1} \quad \ldots\ldots [2]$$

and

$$\phi = \frac{\left\{\left(\frac{1}{1-R}\right)\left(\frac{\gamma}{\gamma-1}\right)(1 - R^{(\gamma-1)/\gamma})R^{2/\gamma}\right\}^{1/2}}{\left(\frac{n^2 - R^{2/\gamma}}{n^2 - 1}\right)^{1/2}} \quad \ldots\ldots [3][17]$$

which is the ratio of the adiabatic discharge equation[18] to the liquid discharge equation.

$$Q = \Omega N[2g\,(P_1 - P_2)\,W_1]^{1/2} \quad \ldots\ldots [4]$$

which equation applies without elaboration to flows of the third type.

Values of ϕ for various values of P_2/P_1 and n, calculated for $\gamma = 1.408$ (the value for air) and for $\gamma = 1.3$ (the approximate value for superheated steam), are plotted in Figs. 1 and 1*a*.[19] These curves show that, in the limit when P_2/P_1 is equal to unity, ϕ is also equal to unity. They further show that when n is large

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[14] For this type of flow, which is the type that most frequently occurs in actual metering, it is only necessary to take account of the density of the fluid. The effects of its viscosity and compressibility may be neglected.

[15] It will be seen later that Flow Types 1 to 5 may be broadly defined as follows:

Type 1, $\sqrt{Q/d_2\mu}$ lies between 0 and 1.5.

Type 2, $\sqrt{Q/d_2\mu}$ lies between 1.5 and 60.0.

Type 3, $\sqrt{Q/d_2\mu}$ is above 60.0, for incompressible fluids.

Type 4, P_2/P_1 lies between 0.96 and the value corresponding to the critical velocity.

Type 5, P_2/P_1 has a value above that corresponding to the critical velocity.

[16] Derived in Appendix No. 2.

[17] Equation [24], Appendix No. 2.

[18] Equation [16], Appendix No. 2.

[19] The following table gives the value of d_2/d_1 corresponding to the values of ϕ, β, and δ plotted in Figs. 1, 1*a*, 17, 18, and 19.

n	1.1	1.2	1.3	1.4	1.5	1.7
d_2/d_1	0.953	0.9125	0.877	0.845	0.8165	0.767
n	2.0	2.25	2.5	4.0	6.0	9.0
d_2/d_1	0.707	0.667	0.6322	0.500	0.408	0.333

(say, greater than 6), the value of ϕ is practically independent of the value of n, but that when n is small, the value of ϕ is very sensitive to the value of n chosen.

As no means exist which enable the exact size of the vena contracta for an orifice in a pipe line to be ascertained (even if any definite vena contracta occurs), the individual values of Ω and ϕ cannot be separated in the case of an orifice for which n is small.[20] As such separation of the values of Ω and ϕ is of theoretical interest only, there is no objection from the point of view of practical metering to plotting values of the combined

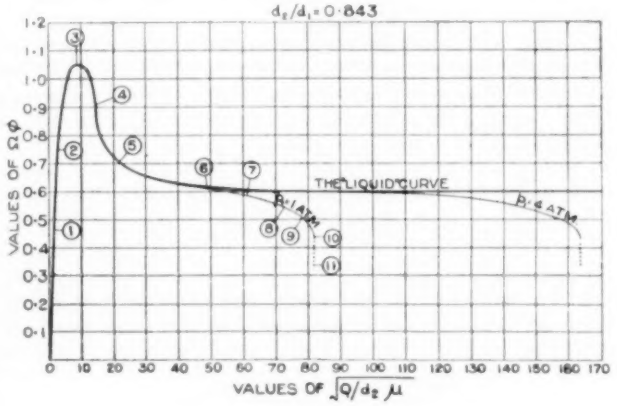


FIG. 2 $\Omega\phi$ VS. $\sqrt{Q/d_2\mu}$ FOR $d_2/d_1 = 0.843$

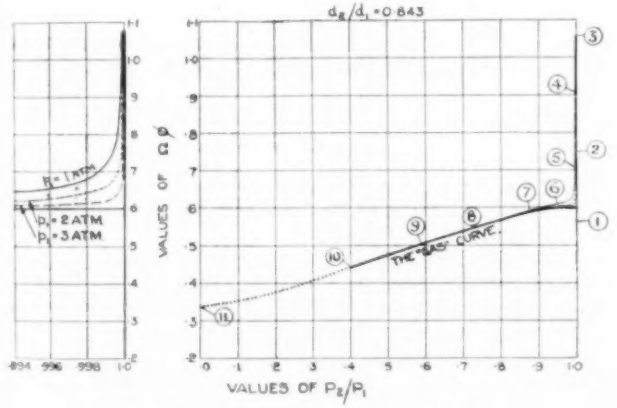


FIG. 3 $\Omega\phi$ VS. P_2/P_1 FOR $d_2/d_1 = 0.843$

coefficients $\Omega\phi$ instead of endeavoring to ascertain the value of Ω itself. For high values of P_2/P_1, where ϕ is practically unity, this procedure is actually equivalent to plotting Ω alone. But, as will be seen on reference to Fig. 1, for the lower values of P_2/P_1, where ϕ is considerably less than unity, the value of $\Omega\phi$ is considerably less than the value that Ω would have, could that value be ascertained. It is for this reason that the value of $\Omega\phi$ falls off as P_2/P_1 is diminished in the curves plotted in Figs. 3 and 6.

If, keeping p_1 equal to 1 atmosphere, an orifice is tested with air (say, a 0.8032-cm. diameter orifice in a 0.952-cm. main for which $d_2/d_1 = 0.843$) for all values of P_2/P_1 between unity and zero, two curves can be plotted. One, the curve numbered 1 to 11 in Fig. 2, connecting $\Omega\phi$ with the viscous-flow criterion[21] $\sqrt{Q/d_2\mu}$; the other, the line numbered 1 to 11 in Fig. 3, connecting $\Omega\phi$ with P_2/P_1. Points in the two curves which represent the same flows are numbered with the same figures.

If now tests are carried out at some higher values of P_1, say, 2, 3, or 4 atmospheres, it will be found that, owing to the diminishing effects of compressibility, portions of these curves will be shifted to the right in both Figs. 2 and 3. (See the curves marked 1 or 4 atmospheres in Fig. 2, and 1, 2, and 3 atmospheres in Fig. 3.) Thus, for every value of p_1 tested, a different set of higher values of $\sqrt{Q/d_2\mu}$ is obtained for the curves shown in Fig. 2, and a different set of higher values of P_2/P_1 for the curves shown in Fig. 3. In a similar way the size of the orifice and the temperature and viscosity of the air will also be found to affect the value of $\Omega\phi$ in both curves.[22]

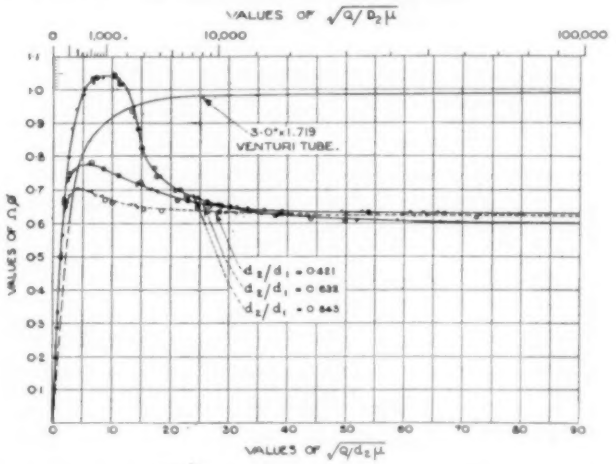


FIG. 4 $\Omega\phi$ VS. $\sqrt{Q/d_2\mu}$ FOR THREE ORIFICES AND A VENTURI TUBE

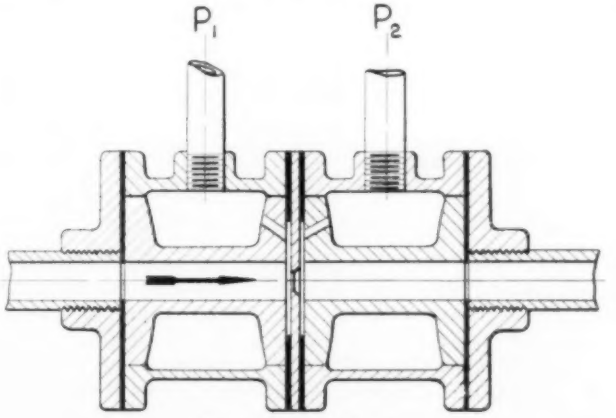


FIG. 5 A TEST ORIFICE

It is possible, however, to express the whole information about the flows through any particular orifice (or group of similar orifices) by means of two curves which, respectively, eliminate compressibility and viscosity effects. This can be done by testing the orifice with a relatively incompressible fluid such as water, or a mixture of glycerine and water, in order to obtain a basis for

[20] Figs. 1 and 1*a* show that, when n is large, the value of ϕ can be obtained with fair accuracy, even though the exact size of the vena contracta, and hence of n, is not known. In such cases a value of Ω can be calculated.

[21] The reason why the viscous-flow criterion, $V_1 d_2 W_1/\mu$, has been converted into $\sqrt{Q/d_2\mu}$ is given in Appendix No. 3 where this criterion is fully discussed.

[22] It is shown in Appendix No. 5 that the general case is to plot the curves connecting values of $\Omega\phi$ and $f(P_2/P_1, \gamma)$ for different values of $P_1 W_1/\mu^2$. Compare also Minutes of Proc. Inst. C. E., vol. 104, p. 190, Fig. 34.

estimating the compressibility effects. The results of such a test are shown by the thick line in Fig. 2, which may be termed the "liquid" curve. For the orifice tested this curve appears to approach to a limiting value of 0.600. If the line $\Omega\phi = 0.600$ is drawn in both diagrams, the distance of the liquid curve above or below this line in Fig. 2 represents the viscosity effect, since the line $\Omega\phi = 0.600$ represents the discharge over that portion of the flow range where the viscosity or compressibility effects are both negligibly small (flow type 3), and the discharge follows the square-root law.

The deducting of this amount at each corresponding point from the curve shown in Fig. 3 gives the thick line, which shows values of $\Omega\phi$ for which the compressibility effects have been separated from the viscosity effects, and which may be called the "gas" curve.

The value of $\Omega\phi$ can be ascertained from the liquid and the gas curves by adding (for the particular values of P_2/P_1 and $\sqrt{Q/d_2\mu}$ chosen) the positive or negative differences of $\Omega\phi$ from the limiting liquid value (0.600) and the thick curves in Figs.

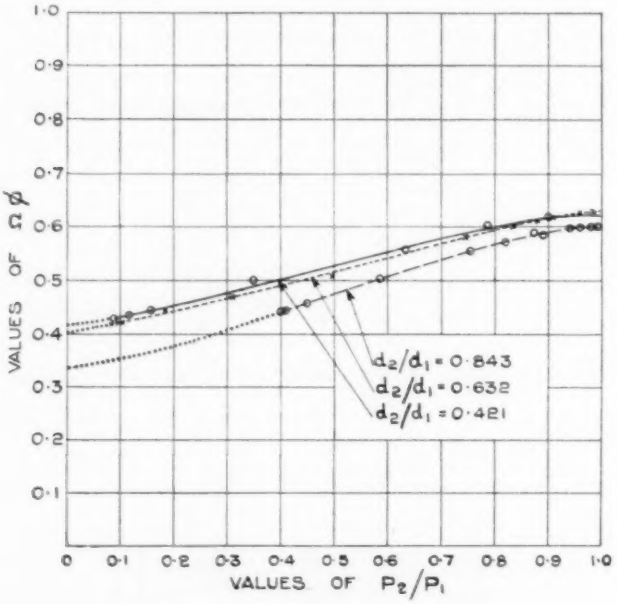


FIG. 6 $\Omega\phi$ VS. P_2/P_1 FOR THREE ORIFICES

2 and 3. Thus, for the points on the thick curves corresponding to the point 6, $\Omega\phi = 0.600 + 0.015$ (from Fig. 2) $+ 0$ (from Fig. 3) $= 0.615$, whereas for the points corresponding to 8, $\Omega\phi = 0.600 + 0$ (from Fig. 2) $- 0.045$ (from Fig. 3) $= 0.555$.

The curves plotted in Fig. 4 show the values of $\Omega\phi$ (i.e., Ω) for incompressible fluids for square-edged orifices of the proportions and sizes described in Appendix No. 8 and illustrated in Fig. 5; while the curves shown in Fig. 6 give the gas curves for these orifices for various values of P_2/P_1.

The extensions of the curves down to $P_2/P_1 = 0$, which are shown in Figs. 2, 3, and 6, have been calculated by the method derived in Appendix No. 6.

Actually the values plotted in Fig. 6 only apply to air. If, however, instead of plotting values of $\Omega\phi$ against P_2/P_1, they had been plotted against[23]

$$\frac{(P_1/P_2)^{(\gamma-1)/\gamma} - 1}{(\gamma-1)\left[1 - \frac{1}{n^2}(P_2/P_1)^{2/\gamma}\right]}$$

[23] See Appendix No. 4, Equation [38].

the "gas" curves determined from this further curve would have been true within small limits for any elastic fluid for which the value of γ was known.[24]

The upper curve in Fig. 7 shows, for various values of d_2/d_1, the limiting values of $\Omega\phi$ when $\sqrt{Q/d_2\mu}$ is large for the liquid curves plotted in Fig. 4; while the lower curve shows the probable limiting value of $\Omega\phi$ when P_2/P_1 = zero[25] for the gas curves plotted in Fig. 6.

Equations are developed in Appendix No. 5, which show that:

1 When measuring gaseous flows at high pressure, the viscosity effects may be neglected unless extremely small pressure differences are being dealt with.

2 The "gas" curve can be determined for the lower values of P_2/P_1 without reference to the liquid curve by testing the orifice with gas at moderately high pressures (say, 4 to 6 atmospheres) or by deducing the results for a high-pressure test from a low-pressure test.

THE VALIDITY OF THE VISCOUS-FLOW CRITERION

The experimental points on the curves in Fig. 4 (covering flow types 1, 2, and 3) which have circles round them were obtained by passing air through the orifice out of or into a liquid-sealed bell of known capacity, and making due allowance for the temperature, pressure, and humidity of the air. The remaining points were taken by means of mixtures of glycerine and water

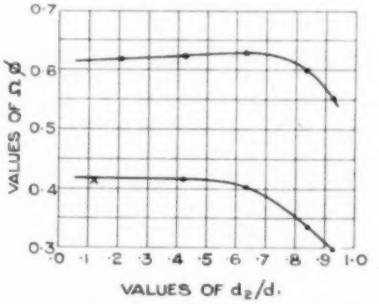


FIG. 7 $\Omega\phi$ VS. d_2/d_1 FOR FOUR ORIFICES

for which the viscosities at the particular temperature worked at were carefully ascertained for each point taken.[26] The great value of the viscous-flow criterion as a basis for coordinating experimental data and for predicting the discharge of geometrically similar orifices is shown by the close agreement between the liquid and gaseous points of the curves plotted in this figure.

It should be remembered, however, that these results were obtained under test conditions, using smooth pipes upstream and downstream of the orifice. If in practice the pipes vary in roughness, the coefficients of discharge for otherwise geometrically similar orifices will not be the same, especially for the smaller pipes where a given change in the roughness produces the greatest effect, which effect results in slightly increasing the value of Ω for flows of type 3, but not appreciably for flow of type 1.

Also coefficient curves such as those shown in Fig. 4 cannot be used to predict the discharge at low rates of flow of such highly viscous oils as tend to jellify when at rest. The actual viscosity of these oils at the instant of discharge depends upon

[24] In any particular case, the variables which may have to be dealt with are n (or d_2/d_1), d_2, Q, W_1 and μ, P_2/P_1 (or $P_1 - P_2$), P_1, and $\Omega\phi$. Curves to suit any particular set of unknowns may be derived by the method given by the author in "Tests on Model Propellers," Proc. Inst. Automobile Engineers, vol. 11 (1917), pp. 270–275.

[25] Calculated by Equation [42], Appendix No. 6.

[26] See Appendix No. 8.

their previous physical treatment, which usually cannot be known with precision.

It is therefore desirable that when these oils have to be measured by means of orifices or nozzles, they should be heated to such a temperature that their viscosity is both sufficiently reduced, and brought to a definite value.

The Importance of Establishing Flow Criteria

The importance, from the point of view of producing accurate meters at a reasonable price, of establishing accurate relations between the discharge of water on the one hand, and such fluids as gas or steam on the other, through similar nozzles and orifices, will be understood when it is remembered that the power required for calibrating any given nozzle or orifice in a closed circuit varies inversely as the square root of the density of the fluid circulating.

Table 1[27] shows that the cost for power when using water as the calibrating fluid is only $^1/_{28.6}$ part of the cost when air at atmospheric pressure is used, and $^1/_{925}$ part of the cost when superheated steam is used. The economy effected in the cost of calibration is actually much more marked than is indicated by the figures in the table, as the capital outlay and the staff required are much less in the case of the more efficient plants.

The table gives, according to the type of calibration plant used, the approximate power in kilowatts required for testing a meter which has a capacity of 6 lb. of air per second at 80 lb. per sq. in. (abs.) and 60 deg. fahr. and which causes a friction drop of 1 lb. per sq. in. in a 6-in. main.

The table has been calculated on the following assumptions:

(*a*) That where the meter is calibrated in a closed circuit, there is an additional fall of pressure of 1 lb. per sq. in. due to friction in the circuit.

(*b*) That the efficiency of the compressing or circulating plant is 0.64.

(*c*) That 20 lb. of steam, if used for the purpose of calibrating meters, would produce 1 kw-hr.

TABLE 1 COMPARISON OF POWER REQUIRED FOR VARIOUS TYPES OF CALIBRATION PLANTS

		Kilowatts	Relative figures
I	Discharging steam at 80 lb. per sq. in. (abs.) and 100 deg. fahr. superheat through the meter and a valve into a condenser	666	925
II	Discharging air at 80 lb. per sq. in. (abs.) through the meter and a valve to atmosphere	598	830
III	Discharging steam at 20 lb. per sq. in. (abs.) and 100 deg. fahr. superheat through the meter and a valve into a condenser	348	483
IV	Circulating air at 14.7 lb. per sq. in. (abs.) in a closed circuit	20.63	28.6
V	Circulating air at 80 lb. per sq. in. (abs.) in a closed circuit	8.82	12.2
VI	Circulating water in a closed circuit	0.721	1.0

Practical Considerations

When using orifices for flow-measurement purposes, an orifice for which d_2/d_1 is less than 0.75 should preferably be chosen, as such an orifice has for liquid flows a practically constant coefficient of discharge over a wide range of flow. Fig. 4 shows that even for the curve $d_2/d_1 = 0.843$, the value of $\Omega\phi$ becomes constant for all values of $\sqrt{Q/d_2\mu}$ above 40. The values of $\sqrt{Q/d_2\mu}$ usually employed in air, steam, and water metering lie between 60 and 900, while the values of P_2/P_1 usually lie between 1.0 and 0.96.

Further, when d_2/d_1 is greater than 0.75, the discharge for any given head becomes much more sensitive to variations in the roughness or in the diameter of the pipe, and to bad upstream or downstream conditions, so that it becomes much more difficult to insure that the test conditions are reproduced in practice with sufficient accuracy.

It is preferable to use a segmental orifice which concentrates the obstructing area symmetrically around the pressure holes instead of increasing d_2/d_1 beyond 0.75, but this arrangement is still sensitive to the sources of error enumerated above. These sources of error are considerably reduced by using the orifice carrier shown in Fig. 8.[28] Here a conical upstream portion, whose widest part is greater and whose narrowest part is less than the

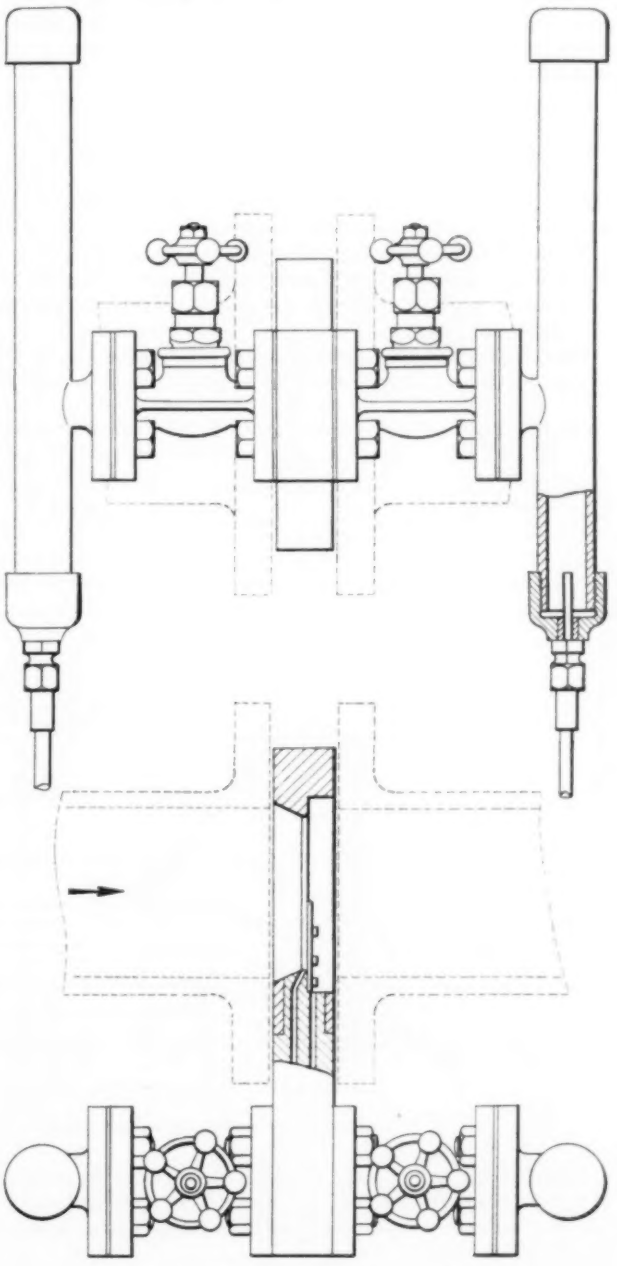


Fig. 8 The Hodgson-Type Orifice Carrier

[27] Reproduced from the author's 1917 paper.

[28] This type of orifice carrier was first put on the market in 1916. It took its present form in 1920. It was first described in the Transactions of the Institute of Naval Architects, p. 185 and Fig. 1, plate 10, vol. lxiv, 1922. Modifications of it have been developed for use in vertical and inclined mains.

nominal diameter of the pipe, eliminates errors due to small variations in that diameter, to slight errors in centering, and to the presence of obstructions such as rivet heads immediately upstream or downstream of the orifice. The drilling of the pressure holes in the orifice carrier also eliminates errors caused by careless drilling and failure to remove the burrs. Fig. 8 shows a segmental orifice plate in place, and illustrates the author's standard-type orifice carrier, cooling pots, and valves for steam measurement.

If the upstream velocity is high, and the value of d_2/d_1 approaches 0.75, the variations in the value of $\Omega\phi$ at the lower velocities should be allowed for as in Appendix No. 7, case 3.

If the value of $\Omega\phi$ used is obtained from tests made on a similar orifice, the pipe upstream and downstream of the orifice should be free from bends, tees, control cocks, etc., unless these were reproduced to scale when the similar orifice was being calibrated.[29] For standard work, a cellular grid should be placed upstream of the orifice to take out the swirl, the presence of which increases the coefficient of discharge.

When measuring gaseous flows at low pressures and at small pressure differences, due allowance should be made for the effects of viscosity as shown in Figs. 3 and 4. If, however, the pressure of the gas is moderately high, say, above 3 or 4 atmospheres, the effects of viscosity may be neglected, and curves similar to those shown in Fig. 6 should be used.

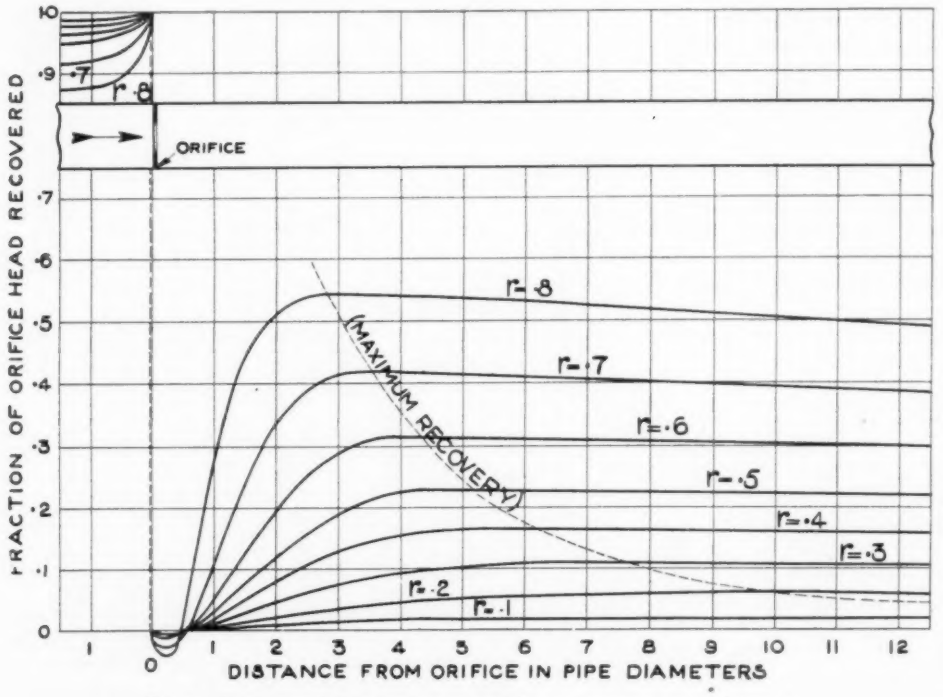


FIG. 9 PRESSURE DISTRIBUTION IN THE NEIGHBORHOOD OF AN ORIFICE

It is not always possible to put the pressure holes in the plane of the orifice,[30] or to make the orifice with square edges or as a circular hole in a thin plate. The relations developed in this paper, however, apply generally to all types of orifices, nozzles, and venturi tubes. They have been found to hold within the limits of commercial accuracy for such diverse fluids as water, thick oil, chlorine, steam, coal gas, etc.; and, whenever it has been possible to make tests with scientific accuracy, they have been found to hold exactly. Such scientific tests have been made by the author on orifices installed in mains from $^3/_8$ in. up to 44 in. in diameter.

A reference to Fig. 9 will show that as a much reduced head is obtained when the downstream pressure hole is drilled in the plane of "maximum recovery," a smaller orifice can be used to produce a given head if the downstream pressure hole is placed in this plane. It is often very tempting to take advantage of this fact when the velocity is high in order to use instruments which will only work up to a standard maximum head. It is, however, undesirable to place the downstream pressure hole in this position unless the orifice can be calibrated in situ, as when this is done the head produced is partly due to the roughness of the pipe immediately downstream of the orifice, and this is not under the control of the designer.

The author prefers the square-edged orifice to any form of nozzle, as the former can be accurately reproduced without difficulty, so that when once the discharge coefficients have been determined for a given orifice, any geometrically similar orifice which has the pressure holes in the same relative position as the original orifice can be constructed and used with confidence. Provided the surface of the upstream face of the orifice is reasonably smooth, the relative roughness of the downstream face does not appreciably alter the value of the coefficient.[31] Monel metal is generally the best material to use, as the squareness of the edge of the orifice does not deteriorate even in superheated steam. The orifice plate must be of sufficient thickness to resist distortion under the difference of pressure (which may include the action of water hammer) to which it will be subjected. Plates thinner than $^1/_{16}$ to $^1/_8$ in. should not be used for accurate work for mains exceeding, say, 3 in. in diameter; and such plates should be beveled off at an angle of 45 deg. on their downstream side so as to leave a thin square edge. A *sharp* beveled edge is both undesirable and unnecessary as it is easily damaged by erosion or impact, and is difficult to measure up accurately.

In the case of orifices used for the measurement of water, a small hole should be drilled through the orifice plate at the top to allow air to pass (see Fig. 13*a*), while in the case of orifices used for the measurement of air or steam the hole should be drilled at the bottom to allow drainage. The area of these holes should be taken as part of a_2.

THE VENTURI TUBE

This device originated in the expanding adjutages used by the

[29] It is, of course, not possible to use an orifice for flow-measurement purposes when the flow is regulated by a valve immediately adjacent to it.

[30] Fig. 9, which is further referred to in Appendix No. 9, shows the way in which the pressure varies along a pipe line in the neighborhood of an orifice. This figure was given in the 1917 paper.

[31] Undue roughness of the upstream face of the orifice slightly increases the coefficient of discharge for flows of type 3, but not appreciably for type 1.

ancient Italians to increase their allowance of irrigation water. It was first developed as one of the most valuable of all devices for the bulk measurement of water by Mr. Clemens Herschel.[32] It consists of a converging upstream cone which produces the fall of pressure required to operate the meter, and a diverging downstream cone which recovers about $^8/_9$ of the fall of pressure produced. The plane of the throat pressure holes is placed between these upstream and downstream cones, and the position of this plane, the angles of the cones, and the changes in the curvatures of the contour of the wall of the tube bridging the cones vary with the experience or fancy of the particular designer.

The curves plotted in Fig. 10 show tests obtained on a number of venturi tubes produced by different makers or experimenters. Some of these tubes have no walls bridging the gap between the upstream and the downstream cones, others have parallel walls;

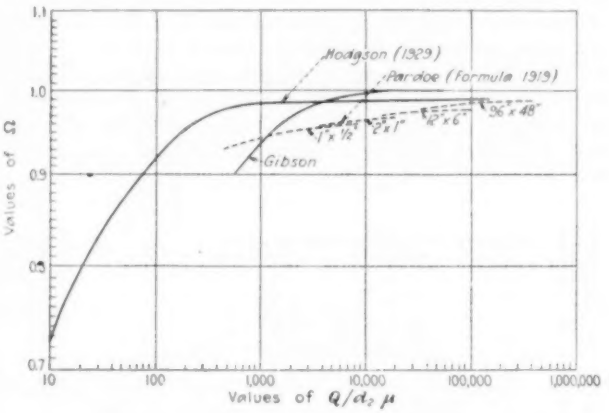


FIG. 10 VARIOUS VENTURI COEFFICIENTS

while others have smooth contours in which there are no abrupt changes of curvature, and in which the throat pressure is taken immediately the maximum convergence of the walls has been effected.[33]

Where the object is to achieve the minimum variation of the coefficient of the venturi tube over the largest possible range of flow, it is obvious that the internal contour of the tube and the plane at which the throat pressure holes are placed must be carefully considered. Even when this is done, it is not possible to obtain with a venturi tube so long a range of fairly level coefficients as it is with an orifice for which d_2/d_1 is small. This can be seen on reference to Fig. 4 on which the test of the Kent 3-in. × 1.719-in. venturi tube which is plotted in Figs. 10 and 11, has been replotted for comparison.[34]

A point of interest in connection with the venturi coefficient is that it is well known that large tubes have higher coefficients than small tubes of the same general proportions. This is apparently contrary to the laws of similarity developed in this paper. It is partly due to the fact that the larger tubes work at higher values of $Q/d_2\mu$. This reason for the apparently higher coefficients of the larger venturi tubes will be seen on reference to Fig. 11.

THE EFFECT OF PULSATIONS IN THE FLOW

If a pulsating *liquid* flow is being measured, the only pulsation error to be allowed for is the *velocity* pulsation error, which is due to the fact that a damped manometer reads the mean of the pressure differences across the orifice; whereas the flow depends upon the square root of these pressure differences. This error should either be reduced to a negligible amount by the provision of air chambers, break-pressure tanks, etc., or of an undamped differential-pressure measuring device which will give equal movements for equal increments or decrements in the flow, and be of sufficiently small inertia to follow the pulsation; the mean of its readings being taken.

It is not usually possible to measure pulsating *gaseous* flows accurately by means of an orifice; as, in addition to the error described above, pressure waves travel up and down the pressure pipes and cause spurious readings. The author has succeeded in eliminating the effect of these pressure waves by splitting up each pressure pipe into several pipes of capillary bore (or the equivalent of this, such as filling the pipes with coarse sand or fine shot) so as to insure that only viscous flows exist in the pressure pipes. This method is not practicable unless the pressure pipes can be kept free from dirt or condensation of moisture (as by heating and filtering).[35]

The best way to measure a compressor discharge with an orifice is therefore to expand the air down to atmospheric pressure (when the pulsations will be damped out by throttling) before inserting the orifice. Alternatively, sufficient receiver capacity and throttling must be introduced between the source of pulsation and the measuring orifice virtually to damp out all the pulsations. In the case of steam flows, the pressure waves in the pressure pipes cause no trouble, as the latter are in this case

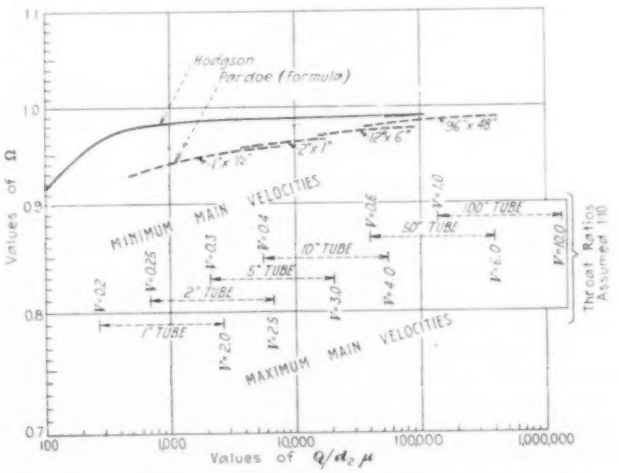


FIG. 11 INCREASE IN THE VENTURI COEFFICIENT WITH d_1

filled with water, which is not appreciably compressed and for which μ is about 60 times that of air; so that viscous damping is obtained without unduly constricting the size of the pipes, and the *velocity* pulsation only has to be dealt with.

It is shown in Appendix No. 10 that, if certain assumptions are made in order to simplify the investigations, the pulsation error due to changes of *velocity* (as distinct from the error caused by pressure waves in the pressure pipes) is the same for the same values of the criterion CFL/ZQ.

Fig. 12 shows a set of pulsation-error curves calculated for a single-cylinder double-acting steam engine at various cut-offs. The units used in that figure are C, cu. ft.; F, cycles per sec.;

[32] See Proc. A.S.C.E., December, 1887.

[33] Compare p. 130 of the author's 1917 paper.

[34] It is of interest to note for purposes of comparison that according to the Stanton and Pannell researches (see Phil. Trans. Royal Soc., vol. 214, pp. 199–224) the flow in smooth pipes of diameter d_2 is definitely viscous for values of $\sqrt{Q/d_2\mu}$ up to 11.2, and is definitely turbulent for values of $\sqrt{Q/d_2\mu}$ down to 15.2.

[35] In meters of the gate and turbine type (described in the author's 1917 paper) the error due to the pressure pulsations does not arise, as there are no pressure pipes.

L, lb. per sq. in.; Z, lb. per sq. in. divided by lb. per cu. ft.; and Q, lb. per sec. These calculated curves indicate that, when the velocity-pulsation error is small, large percentage increases in the value of CFL/ZQ do not affect the pulsation error by more than a fraction of 1 per cent; whereas, when the velocity pulsation error is large, a comparatively small percentage de-

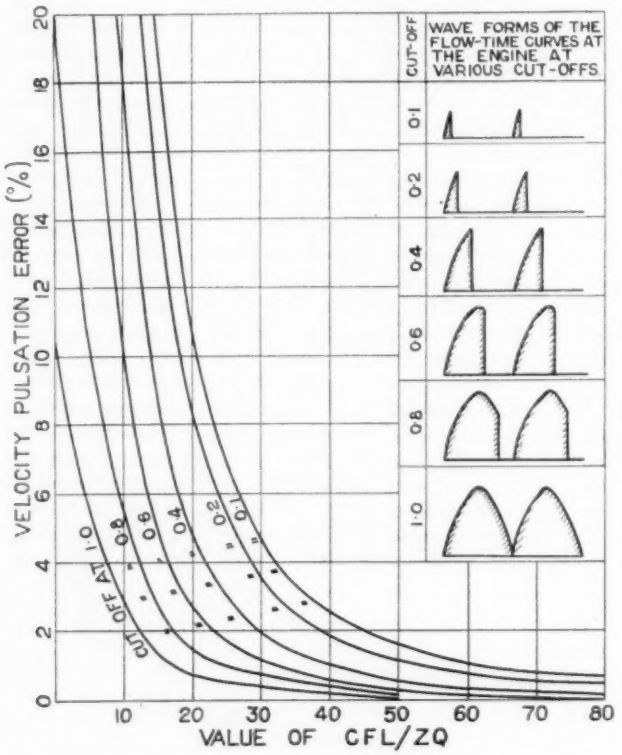


FIG. 12 PULSATION-ERROR CURVES FOR SINGLE-CYLINDER ENGINE

crease in the frequency of the pulsation or the quantity passing may easily affect the pulsation error by 10 or 20 per cent.

Fig. 13 shows how a "velocity-time" wave of square form may be smoothed out, and the meter error reduced to a negligible amount, by increasing the value of CFL/ZQ.

It will be seen from the above investigations that (1) if accurate measurement is required the pulsation error should be kept small, even if allowed for; (2) it is possible to calculate the capacity and loss of pressure required to cause the velocity pulsation error to be small in any particular case; and (3) if the pulsation error is large, and an attempt is made to correct the meter reading by "rating," considerable inaccuracies may be introduced.

Other deductions which follow from a consideration of the CFL/ZQ criterion are:

1 A large meter error may often be reduced to a negligible amount by trebling or quadrupling the value of CL. This may be done by providing for additional throttling, and by putting the meter further from the source of pulsation so as to secure additional capacity. It will be seen that in order to reduce the pulsation error, it is equally efficacious to increase C or L. The energy loss is least when the pulsation is reduced by increasing C, while, if a "capacity" has to be specially provided, the capital cost is greater.

2 If the rate of flow is reduced because the engine slows down (the wave form remaining the same) the meter error will be increased as F and L are reduced, since L falls off more rapidly than Q.

3 Similarly when a steam engine governs, the meter error increases as the flow is reduced. The increase in the error is greatest when the engine governs on the "cut-off" (instead of "on the throttle"), as the wave form is then altered for the worse. If there is an appreciable steam-chest capacity on the engine side of the throttle, the pulsation error may actually be reduced when the engine governs on the throttle.

4 If a throttling orifice which consists of a valve (instead of a fixed orifice) is provided between the metering orifice and the "capacity" and this valve is shut down (either automatically or by hand) as the flow is reduced so that the original L is maintained, the pulsation error will remain constant at all flows, provided that F and Q fall off in the same ratio and the wave form remains the same.

5 It will be seen on reference to Figs. 12 and 13 that if the pulsation error for any flow wave form is large, any small change in the value of CFL/ZQ will cause a large change in the pulsation error, whereas if the pulsation error is reduced to 1 or 2 per cent there may be large variations in the value of CFL/ZQ without causing any appreciable change in the overall error of the meter. It is therefore far better to reduce the pulsation error to a small amount by increasing CFL/ZQ than it is to rate the meter by actual calibration for a large pulsation error.

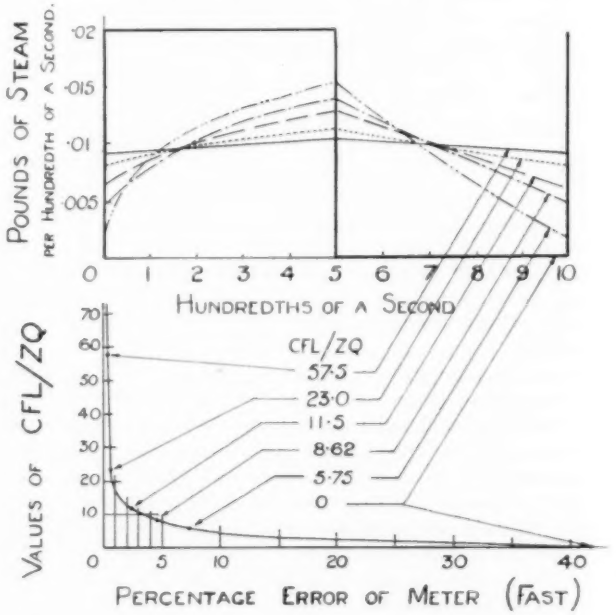


FIG. 13 PULSATION-ERROR CURVES FOR SQUARE WAVE FORM

6 It will be seen that for given values of F, L, Z, and Q the amount that the meter reads fast is (roughly) inversely proportional to C^2, or, for given values of C, L, Z, and Q, to F^2, and so on.

It should be understood that the CFL/ZQ criterion, being deduced from premises which simplify the actual conditions, cannot be used to predict the *exact* pulsation error in any given case, and also that it only holds over a limited range of conditions. For instance, it does not hold when the rate of flow is so great that the loss of pressure L no longer follows the square-root law.

THE CALCULATION OF AN ORIFICE

If a_1 and a_2 are measured in square inches; and h, the observed head in inches of water (0.03609 lb. per cu. in.), is substituted for $(P_1 - P_2)$, Equation [1] becomes

$$Q = 0.127\ (\Omega\phi)\ N\sqrt{hW_1}\ \text{lb. per sec} \quad \ldots\ldots [5]^{35a}$$

N is defined in Equation [2]. It is a term which includes the pipe diameter d_1 and the orifice ratio d_2/d_1 or the ratio s/d_1 as defined in Figs. 14 and 15*a*, i.e., all the actual numerical particulars about the orifice.

The value of W_1, the density of the fluid in pounds per cubic foot at the pressure and temperature which exist at the upstream side of the orifice, will be found for steam or other fluids in the various tables available. It may be calculated for dry air or other gases from the formula

$$W_1 = 2.7\ \Delta p_1/T_1 \quad \ldots\ldots [6]$$

where Δ = specific gravity of the gas relative to dry air at the same temperature and pressure
p_1 = pressure in lb. per sq. in. (abs.)
T_1 = temperature in deg. fahr. (abs.)

In order to ascertain the correct value of $\Omega\phi$ to use, the values of $\sqrt{Q/d_2\mu}$ and P_2/P_1 must be worked out, and the appropriate curves, such as those given in Figs. 4 or 6, consulted.

If the value of P_2/P_1 lies between 1.0 and 0.96,[36] and the value of $\sqrt{Q/d_2\mu}$ is above 60 (which will generally be the case when the orifice is used for metering), and the flow is definitely

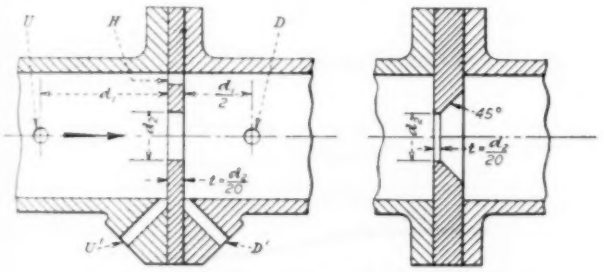


FIG. 13*a* PROPORTIONS OF SQUARE-EDGED ORIFICES

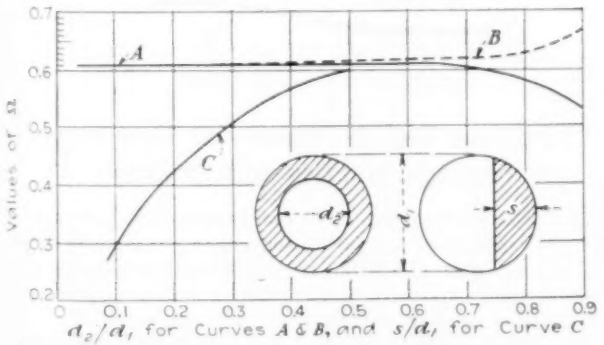


FIG. 14 Ω vs. d_2/d_1 FOR CIRCULAR AND SEGMENTAL ORIFICES

known to be of type 3; a curve similar to the upper curve in Fig. 7 for which ϕ is unity and Ω depends only upon the orifice ratio d_2/d_1 may be used.

Such curves connecting Ω and d_2/d_1 for square-edged circular and segmental orifices in thin plates[36a] placed in 6-in. to 12-in. diameter mains (or with their downstream edges beveled off at 45 deg. as described in the section "Practical Considerations" and illustrated in Fig. 13*a*) with their pressure holes in the plane of the orifice, are shown at *A* and *C* in Fig. 14. The curve when the pressure holes are respectively d_1 upstream and $d_1/2$ downstream of the orifice is shown at *B*.

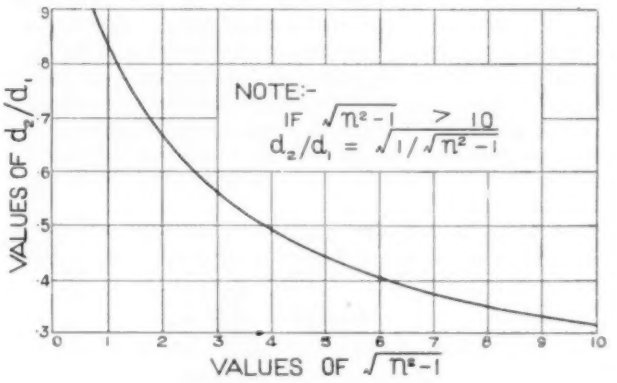


FIG. 15 d_2/d_1 vs. $\sqrt{n^2-1}$

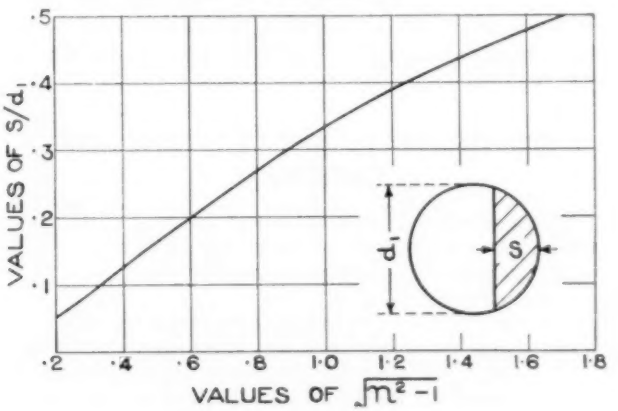


FIG. 15*a* S/d_1 vs. $\sqrt{n^2-1}$

If it is required to find d_2, given the other data, Equation [5] should be rewritten in the form

$$\sqrt{n^2-1} = \frac{0.127\ \Omega\phi\ a_1\sqrt{hW_1}}{Q} \quad \ldots\ldots [7]$$

If ϕ is put equal to unity and an approximate value of Ω (say, 0.61) is assumed, the numerical value of $\sqrt{n^2-1}$ can be calculated from the known data, and the orifice ratio d_2/d_1 (or s/d_1) obtained from curves such as are given in Figs. 15 and 15*a*. If the numerical value of $\sqrt{n^2-1}$ works out to be greater than 10, $\sqrt{n^2-1}$ is equivalent to $\sqrt{n^2}$; i.e., to $(d_1/d_2)^2$. Hence d_2/d_1 can be taken as $\sqrt{1/\sqrt{n^2-1}}$, and obtained by means of a slide rule. The value of d_2 (or s) so obtained will be sufficiently near for all practical purposes. If, however, the curve of coefficients shows that the value of Ω corresponding to the value of d_2 (or s) chosen is appreciably different from 0.61, a more correct value of Ω should be assumed, and a new value of d_2 (or s) worked out.

The value of Ω corresponding to the value of d_2 (or s) actually

[35a] Another form of this equation, derived from Equation [20], is:
$Q = 0.669\ \Omega N(p_1W_1)^{1/2}d$ ..... [5*a*]
$= 0.3250\ \Omega N(p_1W_1)^{1/2}$ ..... [5*b*]
$= 0.3005\ \Omega N(p_1W_1)^{1/2}$ ..... [5*c*]
Equation [5*a*] is generally true; Equation [5*b*] applies to air for which $\gamma = 1.408$, while Equation [5*c*] applies to steam for which $\gamma = 1.130$.

[36] Reference to Fig. 6 will show that if slightly lower values of Ω had been plotted in Fig. 14 so that the deviations of Ω from a mean value had been plus or minus instead of being only minus, it would have been possible to work down to values of P_2/P_1 of 0.9 without appreciable error. It is not generally realized that in those cases where the upstream pressure is more than a few atmospheres, an orifice can be used with reasonable accuracy down to $P_2/P_1 = 0.9$ without correcting for ϕ or its equivalent.

[36a] When the thickness of the parallel part of the orifice does not exceed $d^2/20$.

chosen must, of course, be read off the coefficient curves and substituted in Equation [5] before this is used.

The above procedure, which in special cases can advantageously be shortened by the use of purely graphical methods, is quite general, and enables any set of coefficient curves to be used.

The Units Used

The English system of f.p.s. units has been used throughout the paper, except where otherwise stated. All the values of the viscosity μ have been calculated in these units. In Appendix No. 9 tables of absolute viscosities in f.p.s. units, and the conversion factors between absolute f.p.s. and c.g.s. units are given.[37]

It will be seen from these tables that 10^6 times the absolute f.p.s. unit of viscosity forms a convenient practical unit, and the author has proposed[38] to call this the *megapoisel* (10^6 poisels) after the physician Poiseuille, who first determined the absolute viscosities of fluids.

There seems no reason why engineers who have to work in English units should not also use viscosities tabulated in these units.

Conclusion

In concluding this paper, which is based on the work of twenty years, the author wishes to mention the names of some of those who have been or are his assistants; notably Mr. F. Gray, Mr. W. N. Bond (now Dr. Bond), Mr. E. W. Hovenden, B.Sc., Mr. N. H. Hunt, Mr. H. E. Dall, and Mr. J. R. Ratcliff, B.Sc. Most of these are still with his firm, Messrs. George Kent, Limited, of Luton, England.

Appendix No. 1—List of Symbols and Constants

Q = discharge in pounds per second
P_1, P_2 = pressure in pounds per square foot (abs.[39])
p_1, p_2 = pressures in pounds per square inch (abs.)
R = P_2/P_1
R_1 = the value of P_2/P_1 which satisfies Equation [22]
h, H = heads in inches of water or mercury, respectively (weight of 1 cu. in. of water at 60 deg. fahr. = 0.03609 lb.; weight of 1 cu. in. of pure mercury at 60 deg. fahr. = 0.4893 lb.)
W, W_1 = densities in pounds per cubic foot
A_1, A_2 = areas in square feet
D_1, D_2 = diameters in feet
d_1, d_2 = diameters in inches
a_1, a_2 = areas in square inches
n = $\dfrac{\text{area of upstream}}{\text{area of orifice, or of nozzle throat}}$[40] = a_1/a_2
V_1, V_2 = velocities in feet per second
s = chord width in inches
T, T_1 = temperature in degrees fahrenheit (abs.) = 459.5 + thermometer reading
Ω = discharge coefficient = $\dfrac{\text{actual discharge}}{\text{theoretical discharge}}$[41]
$(\Omega\phi)$ = $\dfrac{\text{actual discharge}}{\text{theoretical discharge not allowing for compressibility effects}}$
Δ = specific gravity of the gas relative to air at the same pressure and temperature
γ = ratio of the specific heats of gases
= 1.408 for air
= 1.3 for superheated steam
g = acceleration due to gravity
= 32.2 ft. per sec. per sec. at Greenwich
β, ϕ, δ, ψ, and α represent terms which are defined in Equations [20], [24], [25], [44], and [47], respectively
μ = viscosity in f.p.s. units
ν = kinematic viscosity = μ/W
Y, K, and K_1 are numerical constants
χ = coefficient of contraction for an orifice
M, L, and T represent the dimensions of mass, length, and time in Appendix No. 11.

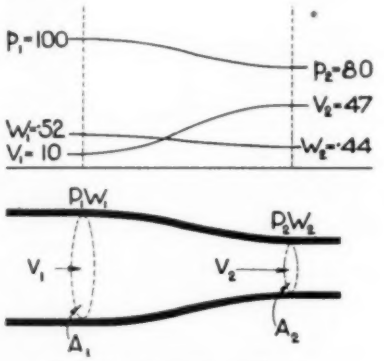


Fig. 16 Diagram of Nozzle

Appendix No. 2—Theory of Discharge Through Nozzles and Orifices[42]

Let A_1 and A_2 in Fig. 16 be two cross-sectional areas in a converging nozzle or in the converging stream lines at an orifice at which the stream lines are parallel and therefore normal[43] to the sections considered; and at which the velocities, pressures, and densities are, respectively, V_1, P_1, W_1, and V_2, P_2, W_2.

If a steady flow be maintained, the same weight of fluid will pass each cross-section in unit time.

If we assume that no energy is supplied or abstracted from the fluid between the two sections, there will be the same energy in the fluid at each section.

[37] To convert $\sqrt{Q/d_2\mu}$ to f.p.s. or c.g.s. units, multiply by $\sqrt{12}$, (=3.464), i.e., by the square root of the number of inches in one foot. The viscous-flow criterion is non-dimensional, and hence has the same value in any *consistent* system of units. For practical purposes it is more convenient to work with d_2 in inches rather than in feet; hence the units in which $\sqrt{Q/d_2\mu}$ is plotted in Figs. 2, 4, 10, and 11.

[38] Proc. Inst. Mining Engineers, 1927, vol. 74.

[39] The suffix "1" is used to denote a point upstream of the orifice, nozzle, or venturi tube; while the suffix "2" is used to denote the plane of the orifice or of the throat of the nozzle or venturi tube, except when otherwise indicated in the text.

[40] In the case of a venturi tube this is usually termed the throat ratio. Note that for orifices, a_2 is to be taken as the area of the actual hole in the plate.

[41] By "theoretical discharge" is meant the discharge calculated on the assumption that there is no loss of energy due to friction, that the stream lines are parallel at the cross-sections between which the differential pressure is measured, that the velocity of flow is uniform across both of these sections, and that compressibility effects have been allowed for by introducing ϕ or its equivalent.

[42] Equations [16], [20], [24], [26], and [22], which involve n, β, ϕ, δ, and relate the maximum values of β to the value of n, respectively, were derived in 1908–9, and first published in the 1917 paper. The methods of evaluating the full St. Venant equation developed in Appendix No. 2 are now almost universally adopted.

[43] The volume passing any cross-section in unit time is only equal to the velocity of the fluid multiplied by the area of the section when the stream lines are perpendicular to the section.

In the case of an incompressible fluid, the velocity at A_2 will be greater than the velocity at A_1 in the ratio of A_1/A_2. This involves an increase in the kinetic energy and a fall in the potential energy of the fluid between A_1 and A_2, since the total energy which passes each of the cross-sections is the same.

If the fluid considered be a gas, the ratio of the increase of velocity will be greater than A_1/A_2, owing to the increase of volume which takes place when the pressure energy, and consequently the pressure falls.[44]

When $(P_1 - P_2)$ is large in relation to P_1,[45] the change in volume which is consequent upon the fall in pressure between A_1 and A_2 causes the discharge formula for gases to differ materially from that which holds for liquids.

In the body of the paper it is shown how to deduce the gaseous discharge from the known liquid discharge through a nozzle or orifice, provided that the velocity of the gas is not sufficiently high to cause the gas to expand appreciably as it passes through the nozzle or orifice.

The general discharge formulas for nozzles[46] and orifices may be derived as follows:

If the energy per pound of fluid is assumed to be constant, the potential energy liberated between A_1 and A_2 will be equal to the kinetic energy gained.

Also, since there is no net gain or loss of energy, the expansion will be adiabatic.

Hence, the potential energy liberated per pound of fluid between A_1 and A_2 is

$$\frac{P_1}{W_1}\frac{\gamma}{\gamma-1}\left\{1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right\} \quad \ldots\ldots [8]$$

The kinetic energy gained per pound of fluid between A_1 and A_2 is

$$\frac{V_2^2 - V_1^2}{2g} \quad \ldots\ldots [9]$$

Equating,

$$V_2^2 - V_1^2 = 2g\frac{P_1}{W_1}\frac{\gamma}{\gamma-1}\left\{1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right\} \quad \ldots\ldots [10]$$

For adiabatic expansion

$$\frac{W_2}{W_1} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}} \quad \ldots\ldots [11]$$

Now

$$Q = A_1V_1W_1 \quad \ldots\ldots [12]$$

$$= A_2V_2W_2 \quad \ldots\ldots [13]$$

Therefore

$$V_1 = V_2\frac{A_2}{A_1}\frac{W_2}{W_1} = V_2\frac{1}{n}\left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}} \quad \ldots\ldots [14]$$

Equation [14] in [10] gives

$$V_2^2\left\{1-\frac{1}{n^2}\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}}\right\} = 2g\frac{P_1}{W_1}\frac{\gamma}{\gamma-1}\left\{1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right\} \quad [15]$$

Equations [11] and [15] in [13] give

$$Q = A_2V_2W_2 = A_2V_2W_1\left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}$$

$$= \Omega A_2\left[\frac{2gP_1W_1\frac{\gamma}{\gamma-1}\left\{1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right\}\left\{\frac{P_2}{P_1}\right\}^{\frac{2}{\gamma}}}{1-\frac{1}{n^2}\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}}}\right]^{1/2} \quad \ldots [16]$$

This is the general discharge formula required; where Ω is an experimental discharge coefficient introduced to allow for slight errors in size and faults in manufacture, the formation of a vena contracta in the case of orifices, and the effects of viscosity and of deviations from Boyle's law.

The actual volume passing (measured at P and T, and W) may be obtained from the relation

$$C = Q/W \quad \ldots\ldots [17]$$

Equation [16] being cumbersome to use, its evaluation may be simplified by combining it with the simpler formula obtained on the assumption that the fluid is incompressible, and that W is constant.

To derive this formula, the loss of potential energy is equated to the gain of kinetic energy per pound of fluid between A_1 and A_2, thus:

$$\frac{P_1 - P_2}{W_1} = \frac{V_2^2 - V_1^2}{2g} = \frac{V_2^2 - \left(\frac{V_2}{n}\right)^2}{2g} = \frac{V_2^2}{2g}\left(1-\frac{1}{n^2}\right)$$

Therefore

$$V_2 = \left\{(P_1 - P_2)\frac{2g}{W_1}\frac{n^2}{n^2-1}\right\}^{1/2} \quad \ldots\ldots [18]$$

Inserting Equation [18] in [13] and multiplying Equation [13] by Ω, we have

$$Q = \Omega A_2\left[(P_1 - P_2)\,2gW_1\frac{n^2}{n^2-1}\right]^{1/2} \quad \ldots\ldots [19]$$

This is the approximate equation corresponding to Equation [16]. Combining Equations [16] and [19], we have, after reduction,

$$Q = \Omega A_2\left[\frac{2gn^2}{n^2-1}P_1W_1\right]^{1/2}[\beta] \quad \ldots\ldots [20]$$

$$\text{where } \beta = \left[\frac{\frac{\gamma}{\gamma-1}\left\{1-R^{\frac{\gamma-1}{\gamma}}\right\}\{R\}^{\frac{2}{\gamma}}}{\frac{n^2-R^{2/\gamma}}{n^2-1}}\right]^{1/2} \quad \ldots\ldots [21]$$

and

$$R = P_2/P_1 \quad \ldots\ldots [21a]$$

Equation [20] enables Q to be calculated in terms of P_1 and P_2/P_1. It is the most suitable equation to use for high-velocity discharges.

The curves connecting P_2/P_1 and β for air discharge ($\gamma = 1.408$) and for superheated steam discharge ($\gamma = 1.3$) are plotted for various values of n in Figs. 17 and 18.

Upon reference to Equation [21], it will be seen that β is a maximum when

$$\frac{\left(1-R^{\frac{\gamma-1}{\gamma}}\right)(R)^{2/\gamma}}{n^2-R^{2/\gamma}}$$

is a maximum, i.e., when

[44] Corresponding changes in the velocity, pressure, and density are shown in Fig. 16. They have been calculated on the basis that $P_1 = 100$, $W_1 = 0.52$, $V_1 = 10$, and $A_1/A_2 = 4$.

[45] Say, when $\frac{P_1 - P_2}{P_1} = \frac{1}{50}$

[46] For the purposes of the discharge formula, which does not deal with recovery of pressure, a venturi tube may be considered as a special form of nozzle.

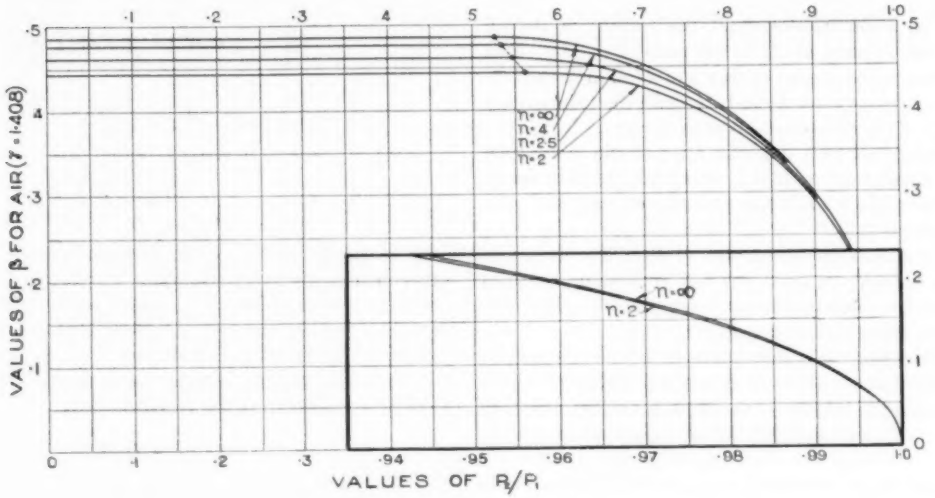


FIG. 17 β VS. P_2/P_1 FOR AIR

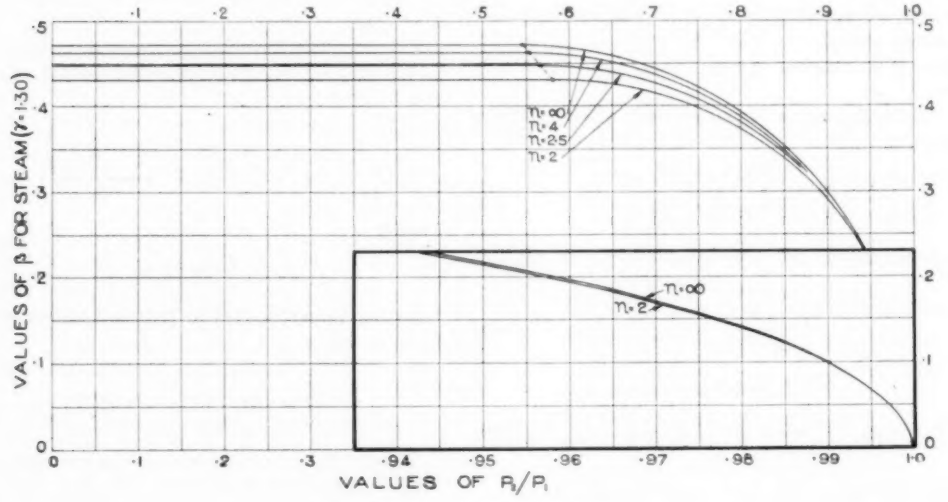


FIG. 18 β VS. P_2/P_1 FOR STEAM

$$\frac{\gamma+1}{R^{2/\gamma}} - \frac{2}{R^{\frac{\gamma+1}{\gamma}}} - \frac{\gamma-1}{n^2} = 0 \quad \ldots\ldots\ldots [22]^{47}$$

and that the discharge, which is proportional to β, apparently diminishes for values of P_2/P_1 which are less than the value of R_1 which satisfies Equation [22]. Actually, however, the pressure at the narrowest section of the issuing jet cannot fall below P_1R_1; as when this velocity is reached the velocity of discharge becomes equal to the velocity of wave propagation at the temperature and pressure which exist at this cross-section, and any external pressure less than P_1R_1 is unable to transmit itself backward along the issuing jet and affect the discharge.

For values of the external pressure less than P_1R_1 the weight discharge therefore becomes directly proportional to P_1, β remaining constant at the value which obtains when $P_2/P_1 = R_1$.

The curves connecting β and P_2/P_1 are therefore drawn parallel

[47] When $n = \infty$, this reduces to $R = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \ldots\ldots [22a]$

and the value of β given in Equation [21] reduces to

$$\beta = \left(\frac{\gamma}{\gamma+1}\right)^{1/2}\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \ldots\ldots [22b]$$

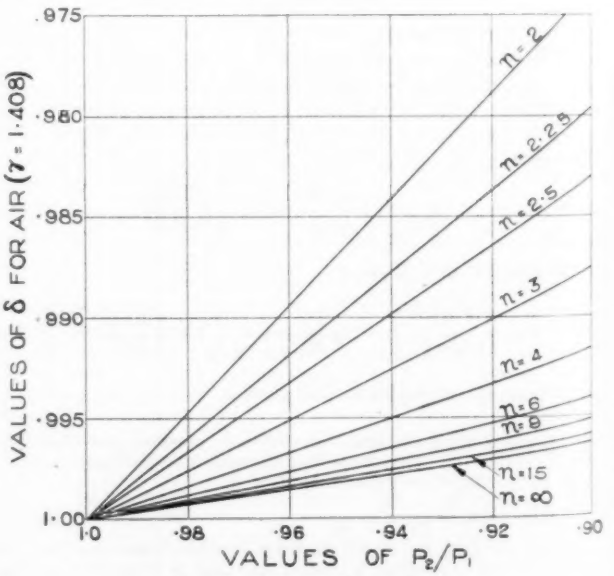


FIG. 19 δ VS. P_2/P_1 FOR AIR

to the P_2/P_1 axis for values of P_2/P_1 which are less than those which satisfy Equation [22].

In most cases that occur in actual metering even the highest velocities which have to be dealt with are comparatively low, so that the value of P_2/P_1 will be found to lie between 0.96 and 1.

Under these circumstances the pressure difference $(P_1 - P_2)$ can be measured much more accurately than the pressure ratio P_2/P_1.

It is therefore necessary to express the discharge Equation [20] in terms of the pressure difference.

This can be done in two ways, the first of which involves what is practically P_1 and $(P_1 - P_2)$ and the second what is practically P_2 and $(P_1 - P_2)$.

These equations, which are easily derived from Equation [20], are:

$$Q = \Omega A_2 \left[\frac{2gn^2}{n^2 - 1} (P_1 - P_2) W_1 \right]^{1/2} [\phi] \quad \ldots\ldots [23]$$

where

$$\phi = \left[\frac{\left(\frac{1}{1-R}\right)\left(\frac{\gamma}{\gamma-1}\right)\left(1 - R^{\frac{\gamma-1}{\gamma}}\right)(R)^{2/\gamma}}{\frac{n^2 - R^{2/\gamma}}{n^2 - 1}} \right]^{1/2} \quad \ldots\ldots [24]$$

and

$$Q = \Omega A_2 \left[\frac{2gn^2}{n^2 - 1} (P_1 - P_2) P_2 \frac{W_1}{P_1} \right]^{1/2} [\delta] \quad \ldots\ldots [25]$$

where

$$\delta = \frac{\phi}{R^{1/2}} = \left[\frac{\frac{1}{R} \cdot \frac{1}{1-R} \cdot \frac{\gamma}{\gamma-1}\left(1 - R^{\frac{\gamma-1}{\gamma}}\right) R^{2/\gamma}}{\frac{n^2 - R^{2/\gamma}}{n^2 - 1}} \right]^{1/2} \quad \ldots\ldots [26]$$

By Equations [21] and [24]

$$\beta = \phi(1 - R) \quad \ldots\ldots [27]$$

Hence, by Equation [26]

$$\beta = \delta[R(1 - R)]^{1/2} \quad \ldots\ldots [28]$$

and

$$\phi = \beta \left(\frac{P_1}{P_1 - P_2} \right)^{1/2} \quad \ldots\ldots [29]$$

$$= \delta \left(\frac{P_2}{P_1} \right)^{1/2} \quad \ldots\ldots [30]$$

Curves connecting P_2/P_1 and ϕ for $\gamma = 1.408$ and 1.3, i.e., corresponding to air and steam discharges, respectively, and connecting P_2/P_1 and δ for $\gamma = 1.408$ are plotted for various values of n in Figs. 1, 1*a*, and 19, respectively.

It will be noticed, on reference to these figures, that when $n > 3$, δ can be taken as unity for values of P_2/P_1 which lie between 0.96 and 1.0; while the corresponding values of ϕ differ considerably from unity.

Equation [25], which involves δ, is therefore in certain cases a more convenient equation to use than Equation [23], which involves ϕ. Equation [23] is however the most generally useful equation to employ as P_1 is usually more constant than P_2.

Appendix No. 3—The Flow Criterion for Viscous Flows

IF IT is assumed that the gaseous or liquid flow through an orifice depends only upon the viscosity μ, the density W, the average velocity V at the orifice, and the diameter d_2 of the orifice, it can be shown by the method of dimensions that the flow conditions are similar for identical values of

$$Vd_2W/\mu \quad \ldots\ldots [31]$$

If $4Q/\pi d_2$ is substituted for Vd_2W, the flow criterion becomes

$$Q/d_2\mu \quad \ldots\ldots [32]$$

The relation between $\Omega\phi$ and $Q/d_2\mu$ can be plotted in various ways, such as directly, or logarithmically. A very convenient way, which has been adopted in most of the figures of the present paper, is to plot $\Omega\phi$ against $\sqrt{Q/d_2\mu}$.[48] This method of plotting causes the liquid curve in the turbulent and the viscous regions (flow types 1 and 3) to be straight lines. It also enables the viscous portion of the curve to be followed right down to zero flow; which logarithmic plotting does not do.

It will be obvious that when the above criterion is applied to gases it can only hold at flows which are so small that compressibility effects are negligible.[49]

It is, however, of interest to compare the values of $\Omega\phi$ and $\sqrt{Q/d_2\mu}$ for liquid and gaseous flows through an orifice over the whole range of values of P_2/P_1 for the gaseous flow.

This has been done at two pressures in Fig. 2.

If

$$2gW_1 = KP_1 \quad \ldots\ldots [33]$$

is substituted in Equation [1] this becomes

$$Q = (\Omega\phi)N\sqrt{(KP_1 - P_2)P_1} \quad \ldots\ldots [34]$$

where

$$\phi = f(P_2/P_1, \gamma)$$

= unity in the limit when $P_2/P_1 = 1$.

It will be seen from Equation [34], and from the curves plotted in Fig. 1 that if P_2/P_1 (and consequently ϕ) differ from unity, the rate of discharge Q will vary with $\sqrt{P_1}$ for given values of P_2/P_1 and $(P_1 - P_2)$. Hence the plotting of corresponding values of Ω and $\sqrt{Q/d_2\mu}$ when the compressibility effects are appreciable is meaningless unless all the results are reduced to some standard pressure P_0 (or density W_0). This can be done by multiplying the value of $\sqrt{Q/d_2\mu}$ obtained by test for any value of $\Omega\phi$ by $\sqrt{P_0/P_1}$ (or, more generally, by $\sqrt{W_0/W_1}$).

The value of $\sqrt{Q/d_2\mu}$ at which the gas curve deviates appreciably from the liquid curve in Fig. 2 shows where compressibility effects begin to be of importance, and the viscous-flow criterion no longer holds.

The straight vertical portions of the constant-pressure curves shown in that figure correspond to flows for which no further diminution in the downstream pressure increases the flow, so that $\sqrt{Q/d_2\mu}$ is constant.

The terminal point of the vertical portions corresponds to the value of $P_2/P_1 = 0$, when $\Omega\phi$ has its minimum value. The method of determining this minimum value is described in Appendix No. 6.

Appendix No. 4—The Approximate Flow Criterion for Elastic Flows

IF IT is assumed that the effects of viscosity are negligible, and that the flow of gas through a constriction of diameter d_2 depends only upon the velocity V_2 (actually it depends upon some value of the velocity between the upstream velocity V_1 and the velocity at the constriction, V_2), the density W_2, and

[48] See footnote 37.

[49] This point was brought out in Figs. 32 and 33 of the 1917 paper.

the adiabatic elasticity P_2 at the constriction, it can be shown by the method of dimensions that the flow conditions are similar for identical values of

$$\frac{W_2 V_2^2}{\gamma P_2} \quad \text{.....[35]}^{50}$$

If $n = \infty$, we have by Equation [15]

$$V_1^2 = 2g \frac{P_1}{W_1} \frac{\gamma}{\gamma - 1} \left\{ 1 - \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} \right\}$$

$$= 2g \frac{P_1}{W_1} \left(\frac{P_2}{P_1} \right)^{1/\gamma} \frac{\gamma}{\gamma - 1} \left\{ 1 - \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} \right\} \text{.....[36]}$$

Since

$$W_2/W_1 = (P_2/P_1)^{1/\gamma}$$

Therefore

$$\frac{W_2 V_2^2}{\gamma P_2} = \frac{W_2}{\gamma P_2} \cdot 2g \frac{P_1}{W_1} \left(\frac{P_2}{P_1} \right)^{1/\gamma} \frac{\gamma}{\gamma - 1} \left\{ 1 - \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} \right\}$$

$$= 2g \frac{(P_1/P_2)^{(\gamma-1)/\gamma}}{\gamma - 1}$$

Hence

$$\frac{\left(\frac{P_1}{P_2} \right)^{(\gamma-1)/\gamma} - 1}{\gamma - 1} \quad \text{.....[37]}$$

may be taken as the flow criterion for elastic flows when $n = \infty$.

Various steam and air tests compared over the elastic flow region on the basis of this criterion have been found to agree within the limits of experimental error.

If n is finite, Equation [37] becomes

$$\frac{\left(\frac{P_1}{P_2} \right)^{\frac{\gamma-1}{\gamma}} - 1}{(\gamma - 1) \left\{ 1 - \frac{1}{n^2} \left(\frac{P_2}{P_1} \right)^{2/\gamma} \right\}} \quad \text{.....[38]}$$

Appendix No. 5—The Relation Between the Viscosity Effect and the Other Factors Affecting Flow

WHEN calibrating an orifice with a gaseous fluid at high values of P_2/P_1, the value of $\Omega\phi$ for any given high value of P_2/P_1 will be found to depend upon the value of P_1 at the time of the test, as shown in Fig. 3.

The value of P_2/P_1 for any given value of $\Omega\phi$ for any value of P_2 can be found from the test results as follows:

It has been shown in Appendix No. 3 that if the compressibility effects are negligible, $\Omega\phi$ and $\sqrt{Q/d_2\mu}$ will have corresponding values.

Hence, $d_2\mu$ may be introduced into Equation [1], which may be rewritten as follows:

$$\frac{P_2}{P_1} = 1 - \frac{\frac{1}{2g} \left(\frac{Q}{d_2\mu} \cdot \frac{1}{\Omega\phi} \right)^2}{\left(\frac{N}{d_2\mu} \right)^2 P_1 W_1}$$

$$= 1 - \frac{\frac{1}{2g} \left(\frac{Q}{d_2\mu} \cdot \frac{1}{\Omega\phi} \right)^2}{\left(\frac{\pi}{4} \right)^2 \frac{n^2}{n^2 - 1} \left(\frac{1}{\mu} \right)^2 \frac{AP_1^2}{T_1}} \quad \text{.....[39]}$$

if a substitution is made for n, and AP_1/T_1 is put equal to W_1. If d_2n, μ, and T_1 are constant, Equation [39] reduces to

$$\frac{P_2}{P_1} = 1 - \frac{\frac{1}{B} \left(\frac{Q}{\Omega\phi} \right)^2}{P_1^2} \quad \text{.....[40]}$$

where B is a constant.

Equation [40] which has been used in calculating the curves marked 1, 2, and 3 atmospheres in Fig. 3 enables the value of P_2/P_1 to be calculated for any particular value of P_1 for each pair of values of Q and $\Omega\phi$ obtained from the test of an orifice. It should be noted that the results plotted apply only to the orifice used. If general results are required, Equation [39] should be used, and the values of $\Omega\phi$ for particular values of n plotted against P_2/P_1 or $f(P_2/P_1, \gamma)$ for various values of $P_1^2/\mu^2 T_1$ or $P_1 W_1/\mu^2$.

In a similar way it can be shown that for liquid flows general results for any particular value of n can be expressed by plotting the relation[51] between Ω and $(P_1 - P_2)$ for various values of

$$(W/\mu)^2 \quad \text{.....[41]}$$

Equation [40] shows that for given values of Q and $\Omega\phi$, P_2/P_1 approaches unity as P_1 is increased. This indicates:

1 That when measuring gaseous flows at high pressure, the viscosity effects may be neglected unless extremely small pressure-differences are being worked with.

2 That what the author has termed the gas curve can be determined for the lower values of P_2/P_1 without reference to the liquid curve by testing the orifice with gas at moderately high pressures (4 to 6 atmospheres), or by deducing the results for a high-pressure test from a low-pressure test by means of Equation [39].

Appendix No. 6—Extrapolation of the $\Omega\phi$ vs. P_2/P_1 Curves Given in Figs. 3 and 6

THE author has found experimentally that, when P_2/P_1 for an orifice is small (say, below 0.3 or 0.2), the ratio P_1/Q_1 becomes constant; that is, the weight discharge becomes proportional to the upstream pressure as in the case of a shaped nozzle. If it is assumed that P_1/Q_1 remains constant right down to the limiting value $P_2/P_1 = 0$, it is easy to derive an equation which enables the $\Omega\phi$ vs. P_2/P_1 curve to be completed.

Thus, if Q_1 is put equal to P_1/Y, where Y is the experimentally determined constant value which P_1/Q_1 reaches at low values of P_2/P_1, this value of Q_1 may be substituted in Equation [1], giving

$$\Omega\phi = \frac{1}{YN\sqrt{2g}} \left(\frac{1}{1 - P_2/P_1} \right)^{1/2} \quad \text{.....[42]}$$

$$= \frac{1}{YN\sqrt{2g}} \text{ in the limit when } P_2 = 0 \text{.....[43]}$$

[50] $\frac{W_2 V_2^2}{\gamma P_2} = \left(\frac{V_2}{V_s} \right)^2$, where V_s is the velocity of sound in the medium of density W_2 which exists at the constriction.

[51] Compare case 3, Appendix No. 7.

Appendix No. 7—Use of the Curves Given in Fig. 4 for Measuring the Flow of Viscous Fluids

THE curves given in Fig. 4 can be used for determining the flow of oil or other viscous fluids.

There are three cases:

Case 1. When the value of $\sqrt{Q/d_2\mu}$ is greater than, say, 60, and the flow is of type 3, Formula [1] should be used. The values of $\Omega\phi$ are given in the upper curve of Fig. 7.

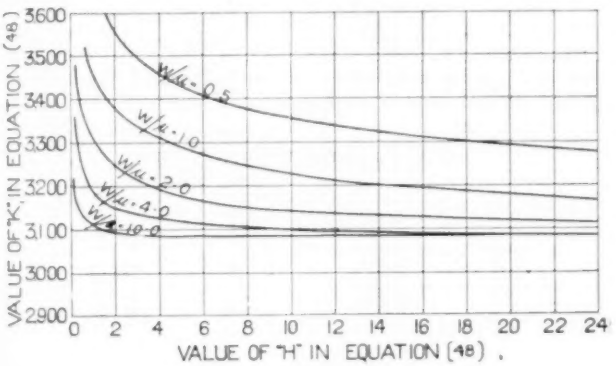


FIG. 20 K vs. W/μ

Case 2. When the value of $\sqrt{Q/d_2\mu}$ is less than, say, 1.5, and the flow is viscous.

It will be seen on reference to Fig. 4, that since the $\Omega\phi$ vs. $\sqrt{Q/d_2\mu}$ curves for each value of d_2/d_1 are straight lines[52] for values of $\sqrt{Q/d_2\mu}$ less than 1.5, it is permissible to write

$$\frac{\sqrt{Q/d_2\mu}}{\Omega\phi} = \psi$$

or

$$Q/\psi^2 d_2\mu = (\Omega\phi)^2 \quad \ldots\ldots [44]$$

where ψ is a constant whose value depends upon the slope of the particular line considered.

But by Equation [1]

$$(\Omega\phi)^2 = Q^2/2gN^2(P_1 - P_2)W_1 \quad \ldots\ldots [45]$$

Equating Equations [44] and [45], we have

$$Q = 2g\left(\frac{N}{\psi}\right)^2 \frac{(P_1 - P_2)W_1}{d_2\mu} \quad \ldots\ldots [46][53]$$

The values of ψ for various values of d_2/d_1 for orifices similar to the orifice used in these tests (which were obtained by plotting to an enlarged scale the data on which the right-hand end of the curves shown in Fig. 4 are based), are given in Table 2.

TABLE 2

d_2/d_1	ψ
0.0515	5.7
0.4210	6.5
0.6320	7.5
0.8340	8.7

Case 3. When the value of $\sqrt{Q/d_2\mu}$ lies between, say, 1.5 and 60, and the flow is of type 2.

In any actual test in which an orifice is used to determine the flow, the quantities which are observed are: the orifice-head, the viscosity, and the density. Hence the simplest procedure is to plot values of α in the equation

$$Q = \alpha\sqrt{(P_1 - P_2)} \quad \ldots\ldots [47]$$

(where α is the value of $(\Omega\phi)N\sqrt{2gW_1}$) for the particular orifice used and for the particular rate of flow passing) against $(P_1 - P_2)$ for various values of W/μ.

An alternative method is to plot the curve connecting values of α and $(P_1 - P_2)\ (W/\mu)^2$. In this case only one curve has to be drawn.

Fig. 20 shows, for various values of W/μ, the curves connecting the value of K_1 in the formula

$$D = K_1\sqrt{H/W} \quad \ldots\ldots [48]$$

where D denotes the discharge in pounds per minute, H the head across the orifice in inches of mercury (at 60 deg. fahr.) immersed in oil of specific gravity 0.8, for a 5.23-in. diameter orifice of the type shown in Fig. 5 in a 9-in. main (i.e., an orifice for which $d_2/d_1 = 0.581$).

It should be noted that, since for any particular fluid W/μ varies with the temperature, the values of W/μ plotted in Fig. 20 can be replaced by temperature values. Further, if it is assumed that W is constant over the range of the experiments, K_1 can be replaced by $K_1\sqrt{H}$, i.e., by D. This enables D to be read directly off a group of curves such as Fig. 20, plotted for different temperatures, when H and the temperature are known.

Alternatively, when W is constant, a single curve connecting K_1 and H/μ can be plotted.

Appendix No. 8—Test Methods Used

THE tests on which the results given in Figs. 2, 3, 4, 6, and 7 are based were carried out with a square-edged orifice 0.8032 cm. in diameter and 0.0914 cm. thick, mounted concentrically in mains 0.952, 1.27, and 1.904 cm. in diameter, corresponding to the following values of d_2/d_1 and N:

TABLE 3

Diameter of main, cm.	d_2/d_1	N
0.952	0.843	0.7203
1.270	0.632	0.5526
1.904	0.421	0.5146

The mean diameter of the orifice was measured by means of a microscope fitted with a micrometer attachment, a plug gage being unsatisfactory. The orifice was mounted as shown in Fig. 5 so that an annular gap 0.04 cm. in width was left at each side between it and the ends of the main. The pressure pipes communicated with these gaps. The orifice was beveled off on the downstream side at an angle of 45 deg., and the length of the parallel part was 0.04 cm.

For the water-glycerine tests, the flow circuit consisted of a tank placed 10 ft. above the orifice; a vertical pipe down to the level of the orifice containing a control cock; a horizontal length of main containing the orifice and of the same diameter as the main at the orifice and a short upward vertical length discharging through a curved spout into an open tank which was connected to a pump delivering the discharged liquid back into the upper tank.

The flow was held steady by means of the indications of a manometer tube which was connected to an opening of large bore which was made in the top of the main upstream of the orifice. The temperature of the fluid was taken on the downstream side of the orifice. The rate of flow was measured by being intercepted at the spout for a known time and weighed. The densities of the mixtures of glycerine and water were

[52] Owing to the small scale to which the figure is drawn these lines all seem to coalesce. Actually they are separate.

[53] This equation can be used to determine the viscosity μ or the kinematic viscosity μ/W.

measured by means of calibrated hydrometers, while their viscosity was determined by means of a capillary-tube viscometer. Tubes of several bores were used in the viscometer and each tube was long enough to ensure that the end effects would be negligible.

Both the water-glycerine and the air tests were carried out in buildings so built or so chosen that they had windows only on the north side, so as to keep temperature changes small. Spiral motion of the fluid in the pipe upstream of the orifice was prevented by the use of a cellular grid. One size of orifice, with different pipe diameters, was used throughout the tests, in order that the flow range to be measured should be as small as possible.

Appendix No. 9—Calculation of Head Recovered at an Orifice

THE curves plotted in Fig. 9[54] show, for flows of type 3, how the pressure varies in the vicinity of circular orifices of various orifice ratios. It will be seen that the recovery of pressure is greatest for the large values of d_2/d_1 and that the point of maximum recovery of pressure travels further downstream as the ratio d_2/d_1 is reduced. It is worthy of note that in cases where d_2/d_1 is large, more than half the orifice head is recovered without any special means being taken, such as the fitting of a diverging cone, to effect such recovery.

The reason for this is that the ratio

$$\frac{(V_2 - V_1)^2/2g}{(V_2^2 - V_1^2)/2g} = \frac{\text{head lost in shock}}{\text{head across orifice}} \quad \ldots\ldots [49]$$

diminishes, for a constant value of $(V_2^2 - V_1^2)/2g$, as V_1 is made more nearly equal to V_2, i.e., as the value of d_2/d_1 is increased. V_2 and V_1 are the velocities at the vena contracta and in the main, respectively.

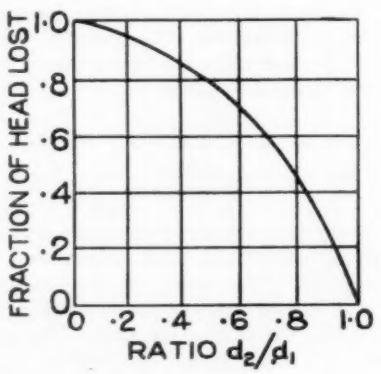


FIG. 21 HEAD LOST VS. d_2/d_1

The fraction of the head recovered by an orifice can be calculated with fair accuracy as follows, if the coefficients of discharge Ω_1 for heads taken between a point one diameter upstream of the orifice and the vena contracta are known:

Fraction of head recovered is

$$\frac{(V_2^2 - V_1^2)/2g - (V_2 - V_1)^2/2g}{(V_2^2 - V_1^2)/2g} = 2V_1/(V_2 + V_1) \quad \ldots [50]$$

But

$$V_2 = \frac{n}{\chi} V_1 \quad \ldots\ldots [51]$$

where χ is the coefficient of contraction for the orifice, and

$$\chi = \frac{\Omega_1 n}{\sqrt{\Omega_1^2 + n^2 - 1}} \quad \ldots\ldots [52]$$

since

$$\frac{\Omega_1}{\sqrt{n^2 - 1}} = \frac{1}{\sqrt{(n/\chi)^2 - 1}}$$

Substituting [51] and [52] in [50], we have, fraction of head recovered is

$$\frac{2\Omega_1}{\Omega_1 + \sqrt{\Omega_1^2 + n^2 - 1}} \quad \ldots\ldots [53]$$

The values of $d_2/d_1, \Omega_1$ as obtained from Fig. 9 and the calculated and experimental values of the fraction of the head recovered are tabulated in Table 4.

TABLE 4 COMPARISON BETWEEN ACTUAL AND CALCULATED HEAD RECOVERED AT ORIFICES

d_2/d_1	Ω_1	Fraction of head recovered	
		Calculated	Experimental
0.8	0.622	0.620	0.619
0.7	0.628	0.500	0.456
0.6	0.622	0.379	0.332
0.5	0.616	0.272	0.234
0.4	0.612	0.180	0.169
0.3	0.609	0.1048	0.105
0.2	0.608	0.0476	0.056
0.1	0.608	0.0122	0.018

Fig. 21 shows the relation between the value of d_2/d_1 (or the equivalent value, $\sqrt{a_2/a_1}$, based on the area ratio) and the fraction of the head lost by an orifice. It is interesting to note that for a simple segmental orifice for which the area ratio is equivalent to $d_2/d_1 = 0.96$, the recovery of pressure is the same as for a venturi tube; and that if the equivalent area ratio of such orifices exceeds 0.96 the recovery of pressure is greater than for a venturi tube.

Appendix No. 10—The *CFL/ZQ* Relation

THE following elementary proof shows that if a meter measures the mean[55] of the difference of pressure across an orifice through which a pulsating flow is passing, the velocity-pulsation error for the cycle will depend upon the numerical value of the pulsating flow criterion CFL/ZQ

where C = the capacity between the points BB and O, in Fig. 22
F = the frequency with which the wave form repeats itself
L = the loss of pressure across the orifice O at the mean flow Q
Z = the product of the specific volume of the steam at the pressure and temperature which exist in the receiver C, and its absolute pressure p_2 (or more exactly $\delta p/\delta W$, where W is the density of the steam at pressure p)
p = the mean absolute pressure in the receiver during the cycle.

A diagram of the arrangement of boiler, receiver, engine, and measuring orifice O is given in Fig. 22, while Fig. 23 shows the wave forms of the flow at BB and O, and the pressure changes in the receiver at various points in the cycle. In this figure l is the loss of pressure across the orifice at any instant.

[54] First published in the Proc. Inst. C. E., vol. 204, p. 132.

[55] The lowest frequency at which the CFL/ZQ relation holds in any particular case depends upon the damping of the meter being such that it is capable of giving a mean value of the differential pressure produced at this lowest frequency. It will be obvious that what is a mere "pulsation" to a heavily damped meter may be a "surge" to one that is lightly damped.

The following assumptions have been made in order to simplify the investigation:

1 That the boiler pressure p_1 is maintained constant
2 That the meter shows the mean value of the differential pressure for the cycle
3 That all the loss of pressure between the engine and the boiler occurs at the orifice O
4 That this loss of pressure varies as the square of the flow, i.e., as Q^2.

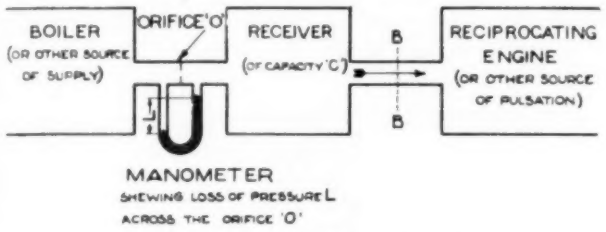


FIG. 22 PULSATING-FLOW DIAGRAM

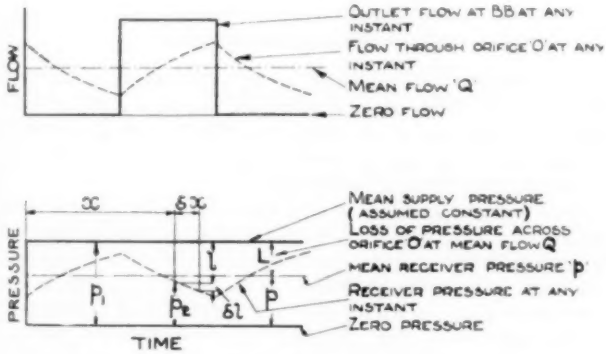


FIG. 23 PULSATING-FLOW FIGURE

The pulsation error will remain the same as long as the ratio of the square root of the mean pressure differences to the mean of the square root of the pressure differences for the cycle remain the same, i.e., as long as

$$\frac{(\Sigma l)^{1/2}}{\Sigma (l)^{1/2}} = \text{constant} \quad \ldots\ldots [54]$$

i.e., so long as δl, the change in head across the orifice O, remains the same for any given fraction of the cycle x, or so long as the ratio of l to L at each point of the cycle remains unaltered.

If S be the weight of steam in the receiver of capacity C at any pressure p_2, then

$$p_2 = \frac{Z}{C} S \quad \ldots\ldots [55]$$

so that if S be the weight of steam lost by the receiver in the time corresponding to the interval δx, then by Equation [55], the corresponding fall of pressure, δl in the receiver will be given by the relation:

$$\delta l = \frac{Z}{C} \delta S \quad \ldots\ldots [56]$$

Now, if Z, F, L, and p are kept constant, and the mean flow Q is doubled by increasing the size of the orifice O, Equation [56] shows that δl will remain constant if C is doubled. Hence if the pulsation error is to remain the same:

$$C/Q = \text{constant} \quad \ldots\ldots [57]$$

If F, L, Q, and p are kept constant, S will be constant, since Q is constant. Hence, by Equation [56], δl and the pulsation error will remain constant if

$$Z/Q = \text{constant} \quad \ldots\ldots [58]$$

If the frequency F be doubled, and Z, L, Q, and p remain the same, only half the flow S will pass through the orifice in the same fraction of the cycle x. Hence if δl is to remain the same, C must be halved, i.e.

$$FC = \text{constant} \quad \ldots\ldots [59]$$

Similarly, if C, L, Q, and p remain the same, Z must be doubled when F is doubled if δl is to remain the same, i.e.,

$$F/Z = \text{constant} \quad \ldots\ldots [60]$$

If Z, F, Q, and p are kept constant, and the loss of pressure across the orifice O at the mean flow Q is doubled by reducing the size of the orifice, δl must be doubled if the $l_1 x$ curve is to retain the same form. That is, ZS/C must be doubled. But, since Z and Q (and hence δS) are constant, this means that C must be halved, i.e., that

$$LC = \text{constant} \quad \ldots\ldots [61]$$

Lastly, if Z, F, L, and Q remain the same, and p is doubled, δS will be the same, since Q is unaltered. Hence, by Equation [56], δl will be unaltered if C remains the same. Hence, the pulsation error is independent of p, although dependent upon Z

$$\ldots\ldots [62]$$

Combining results of Equations [57], [58], [59], [60], [61], and [62], we have:

$$\frac{CFL}{ZQ} = \text{constant} \quad \ldots\ldots [63]$$

Appendix No. 11—Some Notes on Viscosity

THE absolute viscosity of a fluid is defined as *the tangential force per unit area between two parallel planes, unit distance apart in the fluid, which move past one another with unit velocity.*

The dimensions of viscosity are M/LT.

Values of absolute viscosity are usually given in c.g.s. units.

To convert from viscosity in absolute c.g.s. units to viscosity in absolute[56] f.p.s. units, multiply by 0.0672, or by 1/14.88. To convert from viscosity in absolute c.g.s. units to viscosity in gravitational f.p.s. units, multiply by $0.0672/g$ or by $1/14.88g$.

The ratio μ/W or ν is defined as the kinematic viscosity.

The dimensions of kinematic viscosity are L^2/T, and the conversion factor from c.g.s. to f.p.s. units is 0.0010755, or 1/929.03.

TABLE 5 VALUES OF W AND μ FOR AIR AND WATER AT VARIOUS TEMPERATURES, IN ABSOLUTE F.P.S. UNITS[57]

Temperature, deg. fahr.	W Dry air at 30 in. barometer	W Pure water	$10^6 \times \mu$ Air	$10^6 \times \mu$ Water
40	0.07925	62.43	11.72	1037.0
50	0.07770	62.42	11.91	882.0
60	0.07620	62.37	12.09	760.4
70	0.07480	62.30	12.27	659.6
80	0.07345	62.21	12.46	578.8
100	0.07080	62.00	12.83	460.3
120	0.06835	61.71	13.19	378.3
140	0.06610	61.38	13.56	317.7

[56] It is *not* desirable to tabulate viscosities in gravitational or engineers' units, as such values involve the value of g, which varies from place to place.

[57] As stated in the body of the paper, the author proposes to call the unit of viscosity in f.p.s. units the *poisel*, after Poiseuille, who first determined the absolute viscosities of various fluids. The *megapoisel* (10^6 poisels) forms a convenient practical unit.

TABLE 6 VALUES OF μ FOR SATURATED OR SUPERHEATED STEAM[58] AT VARIOUS TEMPERATURES IN F.P.S. UNITS

Temperature, deg. fahr.	$10^6 \times \mu$	Temperature, deg. fahr.	$10^6 \times \mu$
32	6.05	400	11.77
50	6.33	500	13.34
100	7.11	600	14.90
200	8.67	700	16.45
300	10.22		

TABLE 7 MISCELLANEOUS VISCOSITIES

Substance	Temperature, deg. fahr.	$10^6 \times \mu$
Town-gas at 30-in. barometer and 70 per cent wet ($W = 0.4$)	60	9.15
Mercury	68	1,069
Paraffin oil	66	1,340
Olive oil	59	66,500
Machine oil	66	67,200
Rape oil	68	107,000

Discussion

A. A. BARNES.[59] The author's researches over a long period of years in relation to the metering of water, air, gas, and steam are summarized in the present paper and therefore a tremendous amount of detail has been compressed into a small space; this fact makes the paper somewhat difficult to follow at times. Speaking, however, as a water-works engineer, the writer feels that a great future lies before the "orifice gage" for measuring the flow of water in pipes of all sizes in a simple, economical, and straightforward manner and to a degree of accuracy quite exceptional. The criticism, then, is that the formulas are not presented in a form suitable for use from a "water" point of view. No doubt it is necessary and desirable that the questions of viscosity, compressibility, and density of the various elements should be embodied in the general formulas, in order to illustrate the remarkable manner in which air, gas, steam, and water all obey one fundamental law, but, this point having been proved, it seems a pity that the simplified formula for water only has not then been presented.

Existing water meters must work throughout the year over fairly large ranges of temperature, and recording charts take no account of changes in viscosity but are based upon an average density of water of about 62.4 lb. per cu. ft. It is preferable to see the discharge of water represented by the usual unit, cubic feet per second, and, further, to separate in the formula $Q = AV$ the constituents for area and velocity. It is true that those elements are embodied in the final formula, Mr. Hodgson's Equation [5], but they are not evident at first sight. Rearranging that formula for the use of water engineers, the steps are as follows:

$$Q = 0.127\ (\Omega\phi)N\ \sqrt{hW_1}\ \text{lb. per sec.} \quad [5]$$

First divide by W to convert to cubic feet per second, give W the value of 62.4 lb. per cu. ft., and allocate to ϕ its value = 1 for water. Then

$$\text{Discharge} = \frac{0.127\ \Omega N\ \sqrt{h}}{\sqrt{62.4}}\ \text{cu. ft. per sec.}$$

Second, to represent the area of the pipe in the formula, take this from the numerator of the author's coefficient N:

$$N = \frac{a_1}{\sqrt{\left(\frac{a_1}{a_2}\right)^2 - 1}} \quad [2]$$

Hence

$$\text{Discharge} = \frac{a_1 \times 0.127 \times \Omega \times \sqrt{h}}{\sqrt{62.4} \times \sqrt{\left(\frac{a_1}{a_2}\right)^2 - 1}}\ \text{cu. ft. per sec.}$$

Third, from Fig. 14 take the value of Ω to be 0.602, which covers in practice all cases in which the ratio of diameter of orifice to diameter of pipe is not more than 0.7. This gives practically complete contraction and can always be arranged. Hence

$$\text{Discharge} = \frac{a_1 \times 0.127 \times 0.602 \times \sqrt{h}}{\sqrt{62.4} \times \sqrt{\left(\frac{a_1}{a_2}\right)^2 - 1}}\ \text{cu. ft. per sec.} \quad [64a]$$

The formula then becomes very simple:

$$\text{Discharge} = \frac{a_1 \times 0.009678\ \sqrt{h}}{\sqrt{\left(\frac{a_1}{a_2}\right)^2 - 1}}\ \text{cu. ft. per sec.} \quad [64b]$$

in which h = differential head in inches
a_1 = area of pipe in square inches
a_2 = area of orifice in square inches.

The writer made a full-scale orifice experiment on the Elan Aqueduct for Birmingham under ideal conditions, and a short description may not be out of place since it illustrates a practical application of the author's work to sizes and volumes which he himself has not been able to attempt.

On a straight length of a pipe siphon 17 miles long, a brass orifice machined to an exact diameter of 42 in. was pinched between two flanges of a concrete-lined steel pipe, 60 in. in internal diameter. The orifice plate was $^5/_{16}$ in. thick; the opening was machined square for $^1/_{16}$ in. and then chamfered off downstream at an angle of 45 deg. Pressure holes were drilled radially into the 1-in. thick flanges immediately at the orifice, and a brass pipe with an internal diameter of $^5/_{32}$ in. was screwed into each of these, whence copper pipes were led to the gages. The float pipes were taken up to the hydraulic gradient; the water columns therefore gave the differential head without any reduction. The quantity of water passing was controlled by an overflow weir, 19 ft. long, at the inlet to the siphon; this quantity could not vary because the total friction head in the 17 miles of pipe was about 61 ft. The temperature of the water was 52 deg. fahr., and the quantity was steady for 36 hours. The differential head at the orifice was 9.125 in. and the quantity passing was 24,820,000 imp. gal. per day or 46.037 cu. ft. per sec. The discharge was read at submerged rectangular orifices, which had been standardized with similar quantities of water by a series of accurate experiments into a brick-lined reservoir of 100 million gallons capacity.

Computing from the above data with the author's Equation [5] as rearranged in Equation [64b]

$$\text{Discharge} = \frac{2827.44 \times 0.009678 \times \sqrt{9.125}}{\sqrt{\left(\frac{2827.44}{1385.44}\right)^2 - 1}}$$

$$= 46.464\ \text{cu. ft. per sec.}$$

This agrees within 0.9 per cent with the observed reading of 46.037 cu. ft. per sec. and forms a most satisfactory check of the formula far outside the range of the experiments upon which it was based.

The author states that he has made scientific tests on mains up to 44 in. in diameter. It would be important to know if

[58] The viscosity of steam depends only upon its temperature, and is independent of the degree of superheat.

[59] Bartley Reservoir Works, Northfield, Birmingham, England.

any details of such large-scale tests could be given, together with a short description of the methods of calibration which were adopted.

F. N. Connet.[60] The basic invention of the venturi meter by Clemens Herschel in 1887 was followed by the invention and development of the earliest type of registering device by the Builders Iron Foundry of Providence, R. I. Shortly afterward patent licenses for both the meter tube and register were granted to Messrs. George Kent, Ltd., London, England, who revised the mechanical details of the register to conform to European practice. From the first there has been the most helpful cooperation between the two companies, and the experience acquired by each has been at the disposal of the other.

In the early days both companies believed that the most useful field for the venturi meter was that of municipal water works, and this is evidenced by the fact that the average inlet diameter of the first hundred meters made by Builders Iron Foundry was 24 in. This does not include ten 2-in. and 3-in. meters supplied to college laboratories.

It was early found that the capacity range needed for water-works service was about in the ratio 1:6. In other words, the minimum measuring capacity did not have to be less than about one-sixth, or 17 per cent of the maximum. However, the registers actually functioned satisfactorily down to one-thirteenth, or 8 per cent of the maximum.

The exact value of the flow coefficient at extremely low and unused rates was therefore of little practical importance except for small experimental meters where the interest was more academic than useful.

From the very beginning Mr. Herschel encouraged additional experimentation, especially with large meters, and he wisely insisted on following strictly his original venturi-tube proportions so that all subsequent tests would supplement existing data.

One feature of Mr. Herschel's throat design was the short cylindrical portion at the center of which the vent holes were drilled. This cylinder was joined to the upstream cone by a smooth transition curve having a long radius.

This design has been strictly adhered to in the many thousands of meter tubes now in use, and a study of experimental data from many sources does not tempt the Builders Iron Foundry to adopt any modification of the original design. The practice of locating the throat vent holes as nearly as possible at the juncture of the inlet curve and the throat cylinder has some undesirable features because a slight error in location affects the effective throat diameter and appreciably alters the differential, thus making advisable a flow-measurement calibration of each individual meter tube. On the other hand, meter tubes made according to Mr. Herschel's original design prove to be hydraulically identical when machined within measurable tolerances of the intended dimensions. Tests on these tubes having throats even as small as 2 in. in diameter show practically flat coefficients down to about one-tenth of the maximum rate.

Experiments made in America did not fully confirm the author's conclusion that the coefficient curve is less flat when the throat vent holes are at the middle of the cylinder instead of at the end of the transition curve.

It is not easy to obtain extremely accurate readings when the differentials are very low, and even a slight temperature difference in the two small pressure pipes may introduce an error that appears large when plotted upon logarithmic paper where the readings are magnified unduly only at the left-hand edge of the paper. Sometimes a more accurate perspective of values is obtained by plotting tests on the natural scale.

[60] Chief Engineer, Venturi Meter Dept., Builders Iron Foundry, Providence, R. I. Mem. A.S.M.E.

It is a pleasure to study through any of Mr. Hodgson's writings, technical or otherwise, and his ability is shown again in the present paper. He has achieved the art of condensing long and complicated truths into a few lucid sentences.

Ed S. Smith, Jr.[61] The author has long been an able pioneer and authority in fluid-meter theory. The present summary of his work using the method of "similarity" is welcome and most timely as assisting the general acceptance in this country of this method—which is the only rational one—as applied to venturi tubes, nozzles, and orifices. This method has been extensively used in working from models, in airplane and ship design,[62] and has more recently been very valuable in oil heat-exchanger design and pipe-line design in this country. Although widely used abroad, in this country there has been inertia in accepting the method as the basis for fluid-meter coefficients.

The present paper deals primarily with this method and brings out the necessity of using it in connection with fluid meters when measuring the various fluids used in modern industry. Whereas the majority of fluid meters have practically constant coefficients over their operating range, this method shows the actual departures from constant coefficients under certain conditions. The method is invaluable for use with all fluid meters since it definitely establishes the *limits* for constancy of coefficients, thereby serving as the fundamental basis for all fluid metering.

The coefficients presented graphically by the author were, the writer believes, only intended to illustrate the application of this method, and probably would not be recommended by him for use in commercial metering. A paper[63] on metering viscous oils with American venturi tubes presented by the writer introduced this important method of calibrating fluid meters into actual use in this country, although it contained but meager data. The present American practice is better illustrated by the summary[64] prepared by Dr. E. Buckingham, which pre-

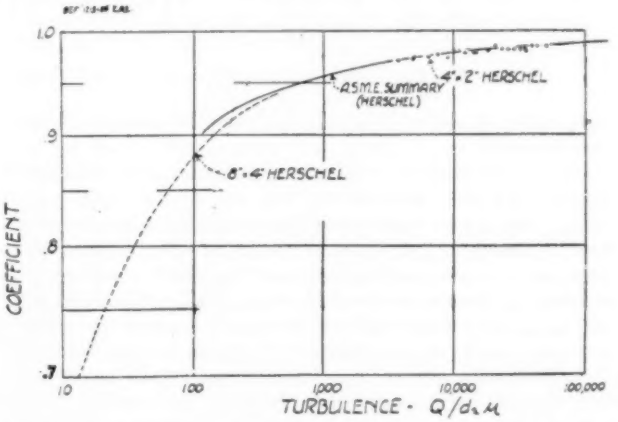


Fig. 24 Coefficient for Venturi Tubes as Designed by Mr. Herschel

sents a large number of calibrations of venturi tubes following the design of Clemens Herschel, which has been standard in America since his invention of this meter. Further data obtained at various hydraulic laboratories for the Builders Iron Foundry on such standard tubes reinforce Dr. Buckingham's

[61] Hydraulic Engineer, Builders Iron Foundry, Providence, R. I. Jun. A.S.M.E.

[62] "The Practical Application of Modern Dynamics to Marine Propulsion," by Stephen E. Slocum, Ph.D., *J. Am. Soc. Naval Engrs.*, vol. 39, pt. 1, Feb., 1927, pp. 1–38.

[63] Trans. A.S.M.E., vol. 45 (1923), p. 67. Ed S. Smith, Jr., "The Oil Venturi Meter."

[64] A.S.M.E. Fluid Meter Report, Part 1, 1924.

summary which, in the opinion of the writer, presents the most accurate available data for these tubes, except for very small sizes (2-in. throat or smaller).

Fig. 24 herewith presents the A.S.M.E. summary of coefficients for venturi tubes as designed by Mr. Herschel. In addition, there are plotted data points obtained by Prof. W. S. Pardoe at the University of Pennsylvania hydraulic laboratory, and witnessed by the writer, upon a standard 4-in. × 2-in. venturi tube, of Mr. Herschel's design, manufactured by Builders Iron Foundry. To give approximately the performance of venturi tubes at much lower Reynolds numbers, there is presented a previously unpublished calibration upon an 8-in. × 4-in. venturi tube, of Mr. Herschel's design, also manufactured by Builders Iron Foundry.

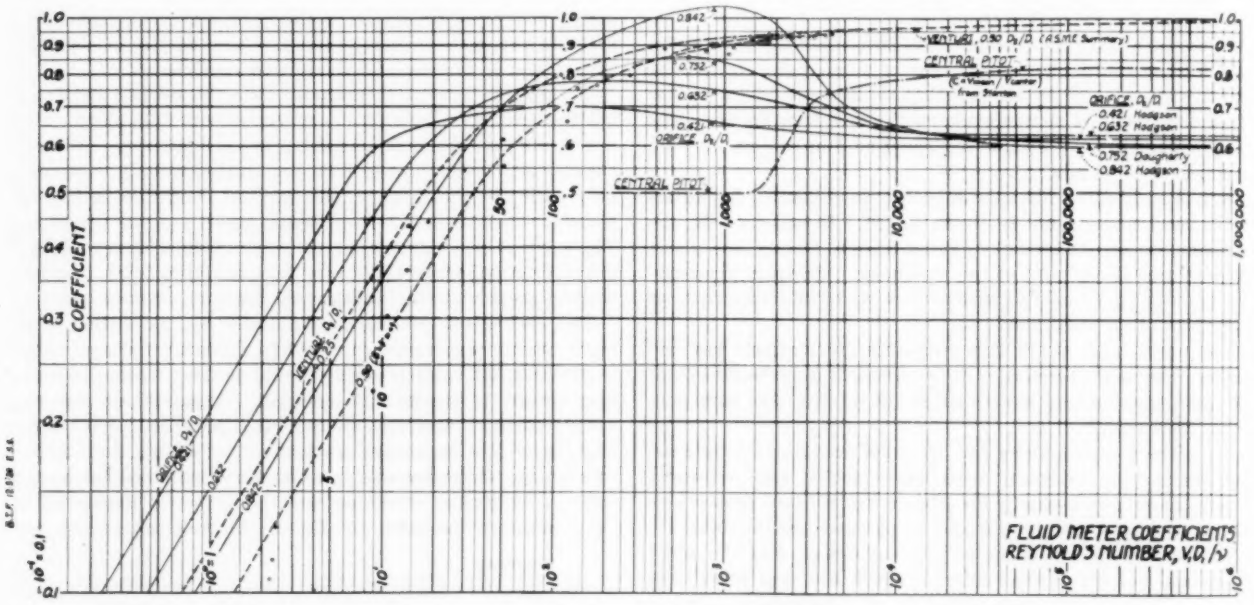


FIG. 25 COEFFICIENT FOR VENTURI TUBES, THIN-PLATE ORIFICES, AND CENTRALLY LOCATED PITOT TUBE

Fig. 25 illustrates the coefficients for venturi tubes, thin-plate orifices with close connections, and a centrally located pitot tube. This shows that the orifice coefficients, for the ratios usually employed, rise at the low rates considerably more than those for the corresponding venturi tubes fall at low rates, considering the change on the percentage basis. (Having smaller loss generally, commercial venturi meters use more differential and have smaller throats than orifice meters for the same rates. Also the orifice size is considerably larger than its effective "throat" or vena contracta.) Further, the coefficients for orifices for various ratios of d_2/d_1 spread out over a wide range when measuring moderately viscous oils; in other words, the venturi coefficient is far better established in general and changes through a much smaller range than orifice coefficients, especially when measuring moderately viscous oils.

As a matter of interest, it will be noted that the centrally located pitot tube has a "flat" coefficient of 0.50 for all values below the lower critical velocity. Although it would seem to form an ideal meter in this range, still it has been found too sensitive to operating difficulties to be reliable in the stream-line region.

In Appendix No. 9, it is noted that some large-ratio orifices may have a smaller loss than some venturi tubes. However, venturi tubes, having substantially the same loss, form reliable meters unaffected by upstream irregularities, while the large-ratio orifices may be highly subject to errors from upstream irregularities.

Logarithmic plotting has the great advantage over square-root plotting of Reynolds numbers that units can be changed as required by merely tracing the coefficients upon shifting the base, whereas the square-root plotting requires recalculation of each test point. Further, on logarithmic paper one can extrapolate the curves as close to zero as necessary since the coefficients show as a straight line on this paper at low Reynolds numbers. This fact is covered in Fig. 25.

In American practice it seems preferable to use the absolute units for both absolute and kinematic viscosities instead of the names "poise" and "poisel," since engineers are not yet generally familiar with those quantities. The kinematic viscosity is much more convenient for oil work than the absolute viscosity. If c.g.s. units are used, it will be found that the kinematic viscosity of water at 68 deg. fahr. is about 0.01 sq. cm. per sec. This is an extremely convenient unit, as the kinematic viscosity of an oil can thus be readily compared with that of water; in other words, the specific kinematic viscosity of the oil is instantly seen when the kinematic viscosity is given as above. Further, the conversion to the f.p.s. unit of square feet per second for kinematic viscosity is obvious, whereas the absolute viscosity involves force and mass units which are less generally agreed upon. The American oil refiner has long been familiar with the conversion from Saybolt seconds to square centimeters per second and this practice is well established in this country. Further, the density in c.g.s. units practically agrees with "specific gravity" for oils and is consequently most convenient for the refiner. For general American practice, the form of Reynolds number which seems most easily used and understood is "turbulence" equals Q/dy, where Q is the rate of flow in U. S. gallons per minute, d is the diameter of the pipe in inches, and y is the kinematic viscosity in square centimeters per second.

J. M. SPITZGLASS.[65] For the last few years the writer has been an ardent reader of Mr. Hodgson's classical papers on the

[65] Mechanical Engineer and Vice-President, Republic Flow Meters Co., Chicago, Ill. Mem. A.S.M.E.

measurement of fluid flow in pipes. When he visited his home last year he told Mr. Hodgson that he would have enjoyed his work much more if the author had given the details pertaining to that work. The present paper is very complete in that respect, and the statements throughout are clear, concise, and to the point. And whether one agrees with the author or not, he knows exactly where he stands and on what his conclusions are based.

The curves in Figs. 2, 3, and 6 are a true picture of the five types of flow involving the three properties of the fluid. The graphical presentation of the quantities is made up partly from experimental work and partly from mathematical computations based on thermodynamic expressions, but the whole picture is so genuine that it is hard to distinguish where experiment ends and theory begins, or vice versa. The matter for discussion is what effect will this paper have upon further developments in the subject of fluid flow.

The author's analysis puts the burden of variation on the existing properties of the fluid. Leaving out the compressibility, which is a matter of mathematical computations, he has varied the viscosity, the density, and the total flow, all of which are properties of the fluid being measured. He has elected to use one size of orifice for all his experiments so that the insertion of the quantity d_2 in his criterion presents a variation in the performance only through the geometrical similarity of the orifice ratio. This is equivalent to advising us that we take care of the properties of the fluid and let the construction of the device be taken care of by geometrical similarity. This geometrical similarity of construction is, in the writer's opinion, not entirely sufficient to ascertain the same results. The roughness, for instance, is a factor which may not be similar for various sizes. In the writer's experiments on the flow of fluids he found that a given percentage in orifice ratio would act differently in different sizes of pipes; therefore he maintains that results obtained from small sizes of pipes will not be duplicated in larger sizes even though the same geometrical proportions are apparently maintained. He does not believe the author is justified in using the results from Fig. 4, which were obtained from tests on pipes less than 2 cm. in diameter, for predicting the flow in a 9-in. main as he does in Fig. 20.

It is to be noticed that in picturing the properties of the fluid and the types of flow the author entirely avoided reference to inertia or the velocity of the fluid, which are really quantities that are included in the criterion. (This omission perhaps is the reason for certain statements which appear contrary to fact.) The writer maintains that the first and most important property in the measurement of fluid is the inertia of the flow or the conversion from pressure to velocity, and vice versa. In most of the devices used the inertia is present in any type of flow. The author refers to the first type as "viscosity only where the flow is directly proportional to the difference of pressure across the orifice." However, even in stream-line flow we cannot neglect the law of conservation of energy, and the increase in velocity through the orifice will require a pressure drop proportional to the square of that velocity in addition to the pressure drop that will overcome the resistance of friction or the effect of viscosity. Therefore even stream-line flow through an orifice is not directly proportional to the difference of pressure across the orifice, because part of that difference is used up to increase the velocity of fluid passing through the orifice. The statement of direct proportionality in stream-line flow has reference only to drop of pressure over a length of pipe where the initial and the final velocity of the flow are the same at both ends.

There are a few other salient points that appeal to the writer in Mr. Hodgson's paper.

1 In his footnote 3, the author warns us that geometrical similarity should be extended to the tap spacing, the roughness of the pipe, the upstream face of the orifice, and the upstream and downstream conditions. To these may be added the thickness of the orifice, and this will give five items for additional experimental investigation.

2 In Figs. 1 and 1*a*, the author shows the correction factor for compressibility of gases and superheated steam. These curves should correspond to those given in Table 1 on page 53 of the Fluid Meter Report as the values of the factor Y for venturi tubes. The writer has compared Equation [3] of the paper with the corresponding Equation [177] of the Fluid Meter Report and found that they read exactly alike, still the figures in Table 1 are somewhat higher than in the author's curves. The shortage of time did not give the writer a chance to investigate the difference. This should be taken up later.

3 In figuring the factors for compressibility the author did not take into consideration the effect of contraction. That is, he treats the orifice the same as a nozzle, which is not strictly correct. In the first paragraph on page 3, he refers to the combined coefficient as a quantity that was obtained directly by experiment. This relates to points 1 to 11 shown on Fig. 3. It would be interesting to have the details of the quantities involved in those points if they were actually carried out by experiment.

4 In Fig. 5 it is seen that the construction of the orifice plate and connections are such as to utilize part of the initial velocity, thereby increasing the amount of the differential pressure corresponding to a given flow. This should be taken into consideration when comparing the results with an orifice plate where the pressures are obtained at points perpendicular to the stream of the fluid.

5 On page 6, at the top of the second column, Mr. Hodgson says, "Such scientific tests have been made by the author on orifices installed in mains from $^{3}/_{8}$ in. up to 44 in. in diameter." The writer would like to ask Mr. Hodgson to give some additional data on some of the scientific tests. He would appreciate having a sketch showing an outline of the method of testing and the actual quantities involved so that it could be compared with other methods and the results of other experimenters.

6 In the third paragraph on page 7, the author calls attention to the fact that the venturi coefficient increases when the size of the pipe is increased for the same general proportions. He says, "This is apparently contrary to the laws of similarity developed in this paper, but it is due to the fact that the larger tubes pass flows at higher velocities." This is equivalent to saying that the blame is not to be put on the increased diameter but on the increased velocity, and, inasmuch as both the diameter and the velocity are equally represented in the Reynolds criterion, the deviation on account of one is just as contrary to the law as the deviation on account of the other.

It is a well-known fact the venturi coefficients increase with the increase in diameter of the pipe, whereas orifice coefficients decrease with increased diameter of the pipe. In the case of the venturi the reduced effect of the friction in larger sizes reduces the total differential for a given flow, and therefore increases the coefficient in the same proportion. In the case of the orifice the frictional resistance has no direct effect on the coefficient because the orifice is practically frictionless. However, when laminar flow is approached the distribution of velocities through the pipe changes considerably, approaching the parabolic form which exists in strictly laminar flow. This condition changes the effect of contraction because the rim of the orifice cuts off comparatively lower velocities. The effect of contraction is reduced, the area of the vena contracta is increased, and therefore the coefficient is increased with the decrease of velocity and diameter.

In the case of the author's tests, embodied in Fig. 6, the effect

appears to be reversed, as the lowest coefficient is shown for the smallest size of pipe (0.952 cm. diameter). This was probably due to the high ratio and the very close connections of the pressure taps. These connections utilize part of the velocity of approach, which is more significant at high ratios. It would be interesting, therefore, to have also the details of the piping, whether it was regular commercial tubing or specially prepared smooth specimen. In our experience small commercial sizes do not give satisfactory results.

The author did not investigate the relation between the theoretical value of ϕ, the compressibility factor, and the corresponding value of this factor as shown by his experiments. As the matter stands now with curves 1 and 1*a* given for reference, one is liable to think that Fig. 6 is made up of the product of experimental hydraulic Ω and compressibility factor ϕ obtained from these curves. For this purpose the writer prepared the

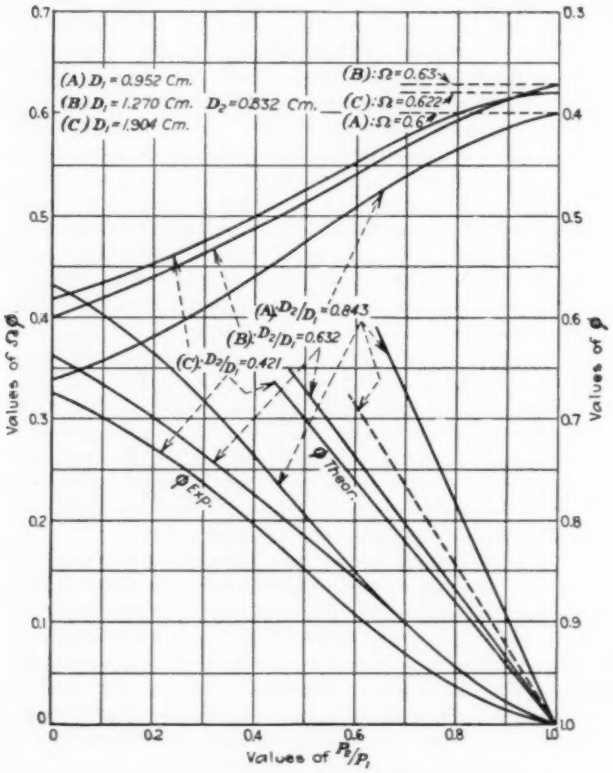


FIG. 26 MODIFICATION OF THE AUTHOR'S FIG. 6

enclosed modification of the author's Fig. 6 (see Fig. 26). This figure shows the comparative values of ϕ theoretical, obtained from Fig. 1, and the experimental obtained when dividing $\Omega\phi$ by the limiting value of Ω. For Case *A*, where the ratio is very high, the value of ϕ is greatly reduced when the contraction is allowed for in the theoretical equation. This is shown by the dotted line which was obtained by correcting the denominator in the author's Equation [3] to read

$$\left(\frac{\frac{n^2}{x^2} - R^{2/\gamma}}{\frac{n^2}{x^2} - 1}\right)^{1/2}$$

instead of

$$\left(\frac{n^2 - R^{2/\gamma}}{n^2 - 1}\right)^{1/2}$$

as given by the author. In Cases *B* and *C* the correction is insignificant, and by correcting Case *A*, as shown, there appears to be an approximate relation of one to two in the difference between ϕ and unity in all cases.

R. E. SPRENKLE.[66] I should like to ask the author if the values of $\Omega\phi$ against d_2/d_1 as shown in Fig. 7 and as obtained with the test orifices installed as in Fig. 5, were duplicated when orifices of the character shown in Fig. 8 were used. An idea as to the magnitude of these differences, if such were found, would be highly useful. Also, is a concentric orifice used with the orifice carrier, and if so how does its coefficient vary from that of the test orifice set up under like conditions of flow?

It is assumed that the pressure connections were made in the plane of the orifice in obtaining the data shown in Fig. 7. If so, how does the author reconcile the perfectly flat curve *a* in Fig. 14 with the upper curve in Fig. 7 which shows an increase of approximately 4 per cent on a flow basis between zero and 0.65 for d_2/d_1?

It is pointed out that various American experimenters have found it inadvisable to damp or throttle the meter pressure pipes, as suggested in appendix No. 10. They also believe from their experiments—notably those of Professor Judd—that the pressure pulsation is greater than the velocity pulsation. It should also be borne in mind that the effect of capacity *C* is at its greatest when it affords an abrupt volume enlargement over the pipe size adjacent to it, instead of a gradual volume enlargement.

F. C. JOHANSEN.[67] In 1928 the writer made a series of experiments, similar to those described in Mr. Hodgson's paper, on the flow of water through diaphragm orifices mounted in pipe lines. The results of the work were expected to be of value mainly in connection with laboratory research, and the scope

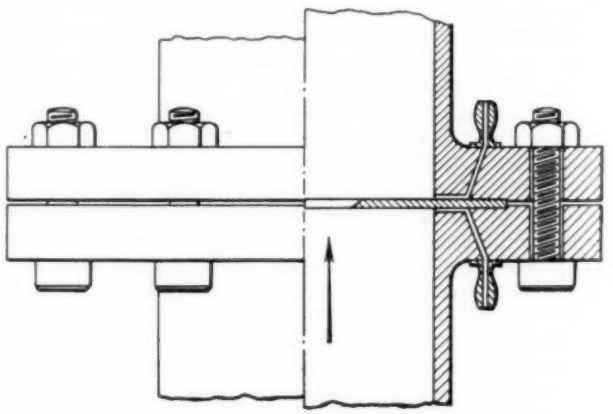


FIG. 27 MR. JOHANSEN'S TEST ORIFICE

of the experiments was therefore restricted to values of Reynolds number below about 50,000. Altogether four sizes of orifice were studied, the diameter ratios being, respectively, 0.2, 0.4, 0.6, and 0.8. The orifice plates were mounted in a smooth brass pipe of 3.2 cm. bore in the manner shown in Fig. 27. The apertures were sharp-edged and beveled at 45 deg. on the downstream side, while the thickness of the plate was arranged to be 0.08 of the orifice diameter in each case. With regard to the situation of the pressure holes for the measurement of discharge coefficient, Mr. Hodgson's practice of arranging these as close as possible to the diaphragm was adjudged to provide the most

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[67] National Physical Laboratory, England.

definite solution of a somewhat controversial feature, and accordingly was followed. It may be remarked that not the least advantage of this location arises from the facility with which the work of different investigators may be compared.

In addition to the determination of discharge coefficients over a range of Reynolds numbers extending downward to flows of a purely laminar type, the writer paid some attention to observations of the pressure gradients upstream and downstream of the orifice, and to variations of these gradients accompanying variations of orifice diameter ratio and rate of flow. For the measurement of the smallest pressures involved, a two-liquid modification of the well-known Chattock tilting manometer was devised, in which carbon tetrachloride (of specific gravity about 1.605) and water were the fluids used. When the meniscus of separation was viewed through a microscope magnifying about 25 diameters and provided with a graticule eyepiece, differential heads could be measured to within $^1/_{1000}$ of a centimeter of water. The maximum head measurable with this gage being about 1 cm. of water, a similar gage, containing mercury in contact with water, was required for the larger flows.

As regards discharge coefficients, four curves similar to those of Mr. Hodgson's Fig. 4 were derived and show good agreement, the interesting phenomenon of maximum coefficients appreciably greater than those which hold over the major portion of the $\sqrt{Q/d_2\mu}$ range being very clearly exhibited. The writer found that his maximum values, plotted against diameter ratio, fell on a smooth curve to contain Mr. Hodgson's three values and one further value due to Dr. Bond,[68] who worked with a

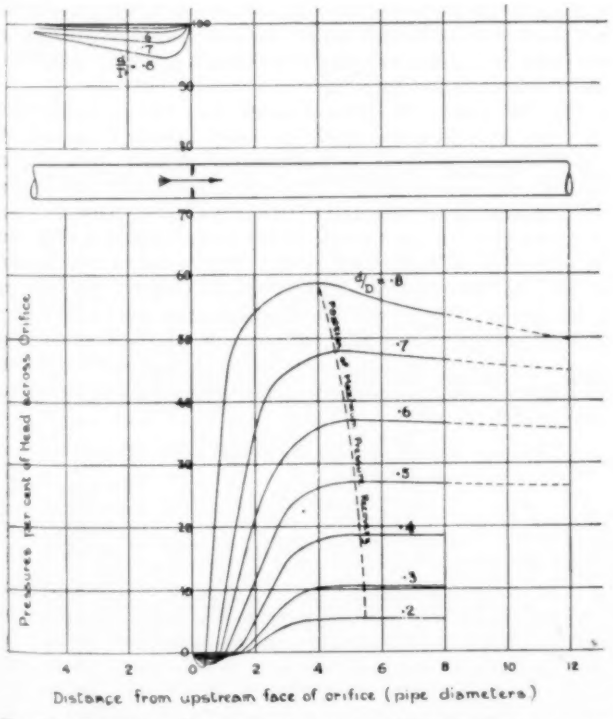


FIG. 28 PRESSURE DISTRIBUTION IN THE NEIGHBORHOOD OF AN ORIFICE IN A SMOOTH BRASS PIPE 3.194 CM. BORE

diameter ratio of 0.0515. Similarly the values of $\sqrt{Q/d_2\mu}$ at which these maxima occur, when plotted against diameter ratio, fall on one curve for all three series of experiments. These large discharge coefficients may possibly be explained to some extent by the consideration that, at the rates of flow for which they occur, the distribution of velocity upstream of the orifice is far from uniform, with the result that the true kinetic head upstream is in excess of that based on the mean flow to an extent greater than that which relates to the jet at the orifice.

In the region of purely turbulent flow, the writer's sharp-edged orifices were found to have discharge coefficients in close agreement with those of Mr. Hodgson, as shown in his Fig. 7, the value of the coefficient increasing from 0.608 at $d_2/d_1 = 0.2$ to 0.624 at $d_2/d_1 = 0.7$.

Mr. Hodgson's pressure-distribution curves in Fig. 9 are mainly confirmed by the writer's experiments, a selection from which relating to Reynolds number 10,000 is reproduced for comparison in Fig. 28. Almost the only feature at variance is the position of the section of maximum pressure recovery, which, for the writer's small-orifice ratios, tended to a limiting distance of about five pipe diameters downstream of the orifice. It should be realized that the broken line indicating these positions in Fig. 28 has been derived from the actual observations of pressure distribution and is much more clearly defined than the shape of the curves of Fig. 28 would appear to warrant. It is thought that the positions of maximum pressure recovery shown in Fig. 28 are correct to about half a pipe diameter. One further feature of the writer's pressure-recovery curves which calls for comment is the pressure gradient just downstream of the maximum, which is appreciably steeper than that which occurs beyond about eight pipe diameters downstream of the orifice. This effect appears most clearly in the curve for diameter ratio 0.8 of Fig. 28. Since completing his experiments the writer has noticed a similar effect in the report of Jakob and Kretzschmer's[69] work on orifices.

The scope of the writer's experiments did not extend to the investigation of the variation of the pressure phenomena near an orifice with the roughness of the pipe line, although it was evident from Spitzglass's work that this variant was of importance—a conclusion which Jakob and Kretzschmer's research has amply confirmed. Mr. Hodgson's present paper is not, of course, concerned with that aspect of the orifice-flow problem, but it would be of interest to learn whether his experience over a wide range of size of orifice meter has enabled him to arrive at any definite conclusions regarding the effect of pipe roughness on discharge coefficient.

G. M. CLARK.[70] The writer has been particularly interested in reading the very valuable contribution of the author to later knowledge on the behavior of the flow of fluids through orifices and nozzles, as they had first met about twenty years ago, when the air meters designed by the author were first put into service for the Victoria Falls & Transvaal Power Company in South Africa for the metering of the supply of compressed air to the gold mines of the Witwatersrand. The sale of this compressed air gives a revenue of about one million dollars a year at a rate that is equivalent to a supply of electric energy at 1 cent a kilowatt-hour. It was therefore important that the meters should secure a high order of accuracy. This was attained at that time with an accuracy of about 1 per cent, and the meters have been in constant service for almost 20 years. These meters are of the venturi type, and their indications are checked at the mines by the purchasers' meters of the weighted-gate type,[71] also designed by the author, and these comparisons show that both groups of meters have continued to exercise their functions with the greatest satisfaction.

At the same period the author first brought out the plate orifice almost in its present-day design, and it was first used

[68] See footnote 11 of Mr. Hodgson's paper.

[69] Forschungsarbeiten, heft. 311, fig. 96.

[70] Messrs. G. M. Clark & Partner, Cairo, Egypt.

[71] See Fig. 8, p. 128, Proc. Inst. C. E., vol. 204.

in South Africa for measuring the output of the compressors supplying this system. The air is thus measured both into and out of the pipe-transmission system, and there is a constant check of the different types of meter. The accuracy of the measurement of compressed air by this means is quite as high as that obtained by the use of any but the very best instruments for the measurement of electric power, and in testing motor-driven compressors it was not unusual for a discrepancy in the electric measurements to be found through the consistency of the air measurements, rather than the reverse.

Besides being used for compressed air, orifices were also at that time in use for metering steam and water flows, such as turbine-condensate discharges, and were thus instrumental in forming the basis of the economical operation of one of the largest power-supply companies. Not only are the meters relatively consistent, but as there is on the mines a large displacement meter for measuring the flow of compressed air, the orifices form a system of absolute measurement also in terms of the fundamental units of volume and weight. The coefficients thus obtained in South Africa were in practical agreement, say to about one-half of 1 per cent, with those given from work done entirely independently in England. The system of measurement has thus the added advantage of being a reproducible one.

To those who have not followed the work step by step during the last twenty years, it may appear from the form of the equations that the method of determining the required coefficients for any particular set of circumstances is of considerable complication. This is due only to the completeness of the paper itself, which deals with the six types of flow that may occur (see beginning of the second page), whereas in practice only one type of flow will be used at the time, and the equations, aided by the appropriate diagrams, are exceedingly convenient.

Although the author has written very fully and given complete references to other papers, these may not always be available, and it is to be hoped that the author will expand this paper at an early date into book form. He will then be able to deal not only with the theoretical part, which is the principal theme of the paper under review, but will also be able to give some of the interesting data upon which his conclusions have been founded. Thus, for example (see the sixth page, top of second column) the author refers to the many experiments that have been made on the flow of very diverse fluids on orifices varied in diameter by 120 to 1. Some of the test methods have been described in Appendix 8, but it might be possible for the author to include some more details of this nature and a short table giving the results for the different fluids employed, more especially perhaps as regards viscous flow, for there has been ample experience during the last twenty years of the measurement of compressed air and of water on a very large scale with great accuracy.

In conclusion it may be of interest that the same plate-orifice method that was started so many years ago is now to be used for the testing of the largest air compressor yet constructed that is now in process of erection in South Africa and that its coefficient has been calculated by the methods of this paper.

Author's Closure

Mr. A. A. Barnes regrets that special formulas to suit the requirements of water engineers and others are not given in the paper. The author considers that in a general paper such as the present one it is undesirable to give special formulas, provided that the formulas which are given are so set out that they can readily be adapted to suit special requirements. It is obvious from Mr. Barnes' contribution to the discussion that he has found no difficulty in deriving from the data given in the paper a special formula that will suit his needs. There is, however, considerable objection to any formula such as Mr. Barnes' Equation [64*b*] in which the numerical constant masks physical quantities which are subject to variation. Thus the constant in his formula masks the value of $(\Omega\phi)$, which will be seen from Fig. 14 to have a value of 0.608 over the larger part of its range, as against the value of 0.602 correctly chosen by Mr. Barnes for his special case. It also masks the value of $W = 62.4$, which only applies for water at the particular temperature of 52 deg. fahr. (See Table 5.)

It is quite as easy and much more satisfactory to work from a general formula such as Equation [5].

Thus, from the data given by Mr. Barnes:

$(\Omega\phi) = 0.602$ (by Fig. 14)

$$N = 2827.44 / \sqrt{\left(\frac{2827.44}{1385.14}\right)^2 - 1}$$

$$= 1590$$

$h = 9.125$

$W = 62.4$ (by Table 5)

Putting these values in Equation [5], we have

$$Q = 0.127 \times 0.602 \times 1590 \times 9.125 \times 62.4$$

(= 2900 lb. per sec.)[72]

Dividing this result by 62.4, the weight of 1 cu. ft. of water at 52 deg. fahr., we have

Discharge = 46.464 cu. ft. per sec.

which is Mr. Barnes' result.

By working from Equation [5], as in the foregoing, all the assumptions as to coefficients and physical constants are clearly set down, and there is no chance of using a formula under conditions where it does not apply.

The test which Mr. Barnes quotes is a very valuable one. It would be almost impossible to obtain greater steadiness of the flow conditions than he had during that test. The author has been in correspondence with him since his contribution to the discussion was sent in and is satisfied that there was no meniscus error in the reading of his manometer and that the volume of the brick-lined reservoir used (which was semicircular in plan and set out with a theodolite) was accurately known to a fraction of 1 per cent. He therefore proposes to take Mr. Barnes' coefficient of 0.5966 for a square-edged orifice of ratio $d_2/d_1 = 0.7$ placed in a smooth 60-in. main as one of the values on which a table of variation of orifice coefficients may be drawn up. (See Table 8.)

TABLE 8 VARIATION OF $(\Omega\phi)$ WITH PIPE DIAMETER

Diameter of pipe in inches	$(\Omega\phi)$
1	0.620
2	0.614
9	0.608
24	0.601
60	0.597

It will be noted that Mr. Barnes' value of $(\Omega\phi)$ is about 0.9 per cent lower than the value given in Fig. 14, which applies to orifices placed in 6-in. to 12-in. internal-diameter cast-iron or steel pipes. This is because the ratio of the central to the mean velocity is greater in the case of the smaller and relatively rougher pipes, a point which is dealt with in the second column of the fourth page of the paper. Other values than those given in Table 8 will, of course, apply to pipes of greater or less specific roughness than that of ordinary commercial cast-iron or steel pipes. The values given in Table 8 are, however, typical of the variation of $(\Omega\phi)$ with pipe diameter. The changes in the value of $(\Omega\phi)$ due to variations in the specific roughness of the pipe in which the orifice is placed are, however, more marked in the

[72] It is not necessary that this step should be worked out.

case of orifices whose diameter ratio is greater than 0.65 and for the segmental orifices whose curve of coefficients is plotted at B in Fig. 14.

In reply to Mr. Ed S. Smith, Jr.: (1) It is interesting to note and to put on record that the method of similarity is now largely used in connection with many other engineering problems than those concerned with airplane, ship, heat-exchanger, pipe-line, and orifice design. It is used in the study of the vibrations of structures,[73] aeroplane wings,[74] rotating shafts,[75] turbine disks,[76] air screws, etc.,[77] in the study of boiler circulations,[78] in examining the effects of pulsations of pressure[79] in exhaust and intake manifolds, in the planning of harbors[80] and the intakes[81] to hydraulic power plants, in charting the distribution of fluid velocities,[82] in investigating the laws of flow through half-drowned sluices[83] and open channels,[84] in the proportioning of centrifugal pumps[85] and surge tanks,[86] in the design of propellers,[87] dynamometers[88] and high-speed vehicles,[89] in the selection of suitable materials for road surfaces[90] and canal beds,[91] in arranging the acoustics[92] and illumination[93] of buildings, and so on.[94]

The first to use the method were probably architects and sculptors when planning temples and large-scale figures. There are many temple and figure models made by the ancient Egyptians in the Museum Boulaq at Cairo.

Froude was the first to apply the method to ship design. A reference to Mr. Clemens Herschel's classical paper of 1887,[95] which introduced the venturi tube for the measurement of water, will clearly show that he expected to obtain similar coefficients for similar venturi tubes of widely differing linear dimensions when these were calibrated with the same fluid, water. The author was the first to apply the method of similarity to geometrically similar venturi tubes, nozzles, and orifices, using different fluids such as water, glycerine, oil, gas, air, and steam; to deduce the viscosity criterion $Q/d_2\mu$ by the same mathematical method which Lord Rayleigh was at the time applying to aeronautical problems, and to examine the upper limits of its application when it was used in connection with compressible fluids, such as air. He was also the first to find the necessity for and to deduce a similarity criterion, when the fluid passing through the orifice or nozzle was appreciably compressed and viscosity effects were negligible. This portion of the work was mainly done in 1909 and 1910 in connection with the large air meters referred to in footnote 5 and in Mr. G. M. Clark's contribution to the discussion.

The values of the coefficients for the orifices given in Fig. 4 only apply to the particular orifice shown in Fig. 5. General coefficients for square orifices with a short length of parallel part are given in Fig. 14.

Mr. Smith's Fig. 25 is useful in indicating the change in coefficient to be expected for a venturi tube or orifice for a given volume discharge through the pipe. It is not, however, so convenient as Fig. 4 when it is necessary to design orifices or venturi tubes for particular flows. In this case d_1 is known and d_2 has to be found.

Both logarithmic and square-root plottings have their special advantages, and both have been used in the paper. The logarithmic plotting has the great disadvantage that it gives a magnified effect at the low values of (ϕ) which are not often used, and cramps the plotting at the high values, where high accuracy is required. This defect is seen in Fig. 25.

The author does not agree that it will be found more convenient to use kinematic viscosity rather than absolute viscosity even in those cases where liquid such as oil is being measured. It is certainly not so convenient to use in those cases where gaseous flows have to be dealt with. The only generally satisfactory way of dealing with gaseous flows is on a weight basis, i.e., to state the flow as so many pounds per second. If the flow is stated on a volume basis, complications with regard to the possible pressure and temperature are immediately introduced. It is quite customary to specify oil flows in gallons per second at a given temperature. This is equivalent to specifying the flow on a weight and not on a volume basis. If, then, the flow is stated on a weight basis, as in most cases it is or should be, the only other factor to be taken into account when calculating the value of the viscosity criterion, besides the linear dimension of the measuring orifice, is the absolute viscosity of the fluid. It therefore does not seem necessary or desirable to introduce the idea of kinematic viscosity in this connection.

Mr. Smith's suggestion that it is more difficult to convert absolute viscosity from one set of units to another than it is to so convert kinematic viscosity does not seem to be true. Kinematic viscosity is absolute viscosity divided by density so that the conversion is actually simpler in the case of absolute viscosity. In any case, the dimensions of each type of viscosity and the necessary conversion factors are given in Appendix 11 of the paper.

In concluding the reply to Mr. Ed S. Smith, Jr., the author desires to add a note of high appreciation of the work which Mr. Smith has done in bringing the method of dynamical similarity, so far as it affects venturi and orifice flows, to the notice of American engineers.

The author feels highly honored that Mr. J. M. Spitzglass should have gone through the paper with such extreme care.

The questions of the roughness of the pipe upstream of the orifice and of the upstream face of the orifice itself, and the importance, when making use of the method of dynamical similarity, of obtaining pipes and orifices of the same relative roughness, has been referred to in various parts of the paper and in footnote 31. The effect of roughness in the pipe upstream of the orifice on the value of the coefficient of discharge has already been dealt with in the reply to Mr. Barnes.

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[73] Proc. Inst. Mech. Eng., 1928, vol. II, p. 747.

[74] Technical Report of the Aeronautical Research Committee, 1928–29, No. 1, 155.

[75] Technical Report of the Aeronautical Research Committee, 1918–19, vol. 2, No. 551, p. 691.

[76] Journal Inst. E. E., 1921, vol. 59, p. 565. Proc. Roy. Soc., Series A, 1921, vol. 94, p. 272; and 1922, vol. 101, p. 133. Trans. Amer. Soc. Mech. Eng., 1924, vol. 46, p. 31.

[77] Proc. Roy. Soc., series A, 1925, vol. 107, p. 451.

[78] *Engineering*, 1896, vol. 61, p. 39. Trans. Amer. Soc. Mech. Eng., 1915, vol. 37, p. 1011.

[79] Technical Report of the Aeronautical Research Committee, 1924–25, vol. 2, no. 957, p. 661; and *Phil. Mag.*, 1925, vol. 50, p. 553.

[80] Proc. Inst. Mech. Eng., March, 1929.

[81] Proc. Inst. Mech. Eng., March, 1929.

[82] Proc. Inst. Mech. Eng., March, 1929.

[83] Proc. Inst. Civil Eng., 1924, vol. 218, p. 72.

[84] Proc. Inst. Mech. Eng., March, 1929.

[85] *Engineering*, 1926, vol. 122, p. 374.

[86] Proc. Inst. Civil Eng., 1925, vol. 219, p. 161.

[87] Proc. Inst. Automobile Eng., April, 1916; Journal Royal Aero. Soc., May, 1929.

[88] Proc. Inst. Automobile Eng., April, 1926.

[89] See *Engineer*, 1928, vol. 145, p. 146; and *Railway Engineer*, 1928, vol. 49, p. 257. *Engineer*, 1898, vol. 86, p. 164. "The Resistance of Express Trains," by C. F. Dendy Marshall. Technical Report of the Aeronautical Research Committee, 1924–25, vol. 2, 962, p. 674. Trans. Amer. Soc. C. E., 1912, vol. 75, p. 982.

[90] See "Modern Roads," by H. P. Boulnois.

[91] Charlottenburg Model Tank. Investigations, 1910–11.

[92] Journal of Scientific Instruments, 1926, vol. 3, p. 393; Architects' Journal, 1926, vol. 63, p. 565; and Proc. Phys. Soc. Lond., 1926, vol. 38, p. 234. Phil. Mag., 1927, vol. 3, p. 177.

[93] Dept. of Scientific and Industrial Research, Illumination Research Technical Paper No. 6.

[94] One of the best general discussions of the principles of dynamical similarity and their practical application is given in Glazebrook's Dictionary of Applied Physics, Vol. 1, pp. 81–96.

[95] See Proc. of the Amer. Soc. C. E., Dec. 21, 1887.

Another factor than roughness entered into the tests made by Mr. Spitzglass; namely, that for all sizes of pipe which he tested he used the same length (instead of the same proportional length) of upstream after a right-angled bend. This alone would cause the flow distribution to be different in the case of every orifice tested. When using the $Q/d_2\mu$ criterion it was necessary to ensure that the flow distributions as well as all the geometrical proportions of the orifices compared were similar. It is because variation in the relative roughness is one of the factors which alter the similarity of the flow distribution for the various sizes of pipe worked with that such variation is of importance.

The reason why the author has referred to the first type of flow as dependent upon viscosity only is that his division of the flows through orifices and nozzles into six principal types is based on practical considerations rather than upon hair-splitting theory. Thus, when measuring viscous flows by means of an orifice, the pressure difference required to overcome the viscosity of the fluid as it passes through the orifice is so much greater than the pressure difference required to accelerate it that the latter can be practically neglected. Similarly, when forcing a compressible fluid such as air through an orifice, the compressibility effects can be neglected when the change in the static pressure is small.

The values plotted in Figs. 1 and 1*a* are correct. Their method of calculation is illustrated by the following example:

Thus when $P_2/P_1 = R = 0.70$ and $n = 2$, we read off the curve for $n = 2$ in Fig. 1 that $\phi = 0.775$. This value is calculated from Equation [3] in the following steps:

Putting $R = 0.7$, and $\gamma = 1.408$, we have

$$\begin{aligned} 1/(1 - R) &= 3.33 \\ \gamma/(\gamma - 1) &= 3.45 \\ (\gamma - 1)/\gamma &= 0.29 \\ 2/\gamma &= 1.421 \end{aligned}$$

whence

$$\begin{aligned} R^{(\gamma-1)/\gamma} &= 0.70^{0.29} \\ \text{or} \qquad \log 0.70 &= 1.8451 - 1 \\ &= -(0.1549) \\ 0.29 \log 0.70 &= -(0.1549) \times 0.29 \\ &= -0.0449 \\ &= 0.9551 - 1 \\ &= \log 0.9018 \\ (1 - R^{(\gamma-1)/\gamma}) &= 0.0982 \\ R^{2/\gamma} &= 0.70^{1.421} \\ 1.42 \log 0.70 &= 0.7800 - 1 \\ &= \log 0.6026 \\ (n^2 - R^{2/\gamma}) &= 4 - 0.6026 \\ &= 3.3974 \\ (n^2 - 1) &= 3.0 \end{aligned}$$

whence

$$\begin{aligned} \phi &= \left\{ \frac{3.33 \times 3.45 \times 0.0982 \times 0.6026 \times 3}{3.3974} \right\}^{1/2} \\ &= 0.775 \end{aligned}$$

With reference to points 1–11 in Fig. 3, the points 1–10 were obtained by experiment; point 11 was calculated by Equation [43], using the method described in Appendix 6. Points 1–5 and along the liquid curve were obtained by tests with water and glycerine, as described in Appendix 8, while points 6–10 were obtained by calibration against a standard orifice with high-pressure air with atmospheric pressure at the outlet of the pipe in which the orifice was placed. The test results were then reduced to a common upstream pressure of 1 atmosphere by the method given in Appendix 5.

Some further details of the test methods used are given in the general reply to Mr. Barnes, Mr. Clark, and Mr. Spitzglass.

The author is in disagreement with Mr. Spitzglass that the principal cause of the higher coefficients of large venturi tubes is due to their specific roughness being less. Years ago he thought that this was so, and in order to eliminate the effects of variation in roughness he had perfectly smooth tubes of various sizes made up and tested. The larger tubes still had the larger coefficients at the flows worked with. It was only when he had determined a sufficient length of the Ω vs. $Q/d_2\mu$ curve for one tube to appreciate that the shape of the venturi discharge curve for any one tube tested over a sufficiently large range was generally as shown by the curves in Figs. 4, 10, and 11, and had noticed that the larger tubes worked at higher values of $Q/d_2\mu$ that the true explanation became apparent.

Mr. Spitzglass points out that the orifice, unlike the venturi tube, is practically frictionless. Though, as stated in footnote 31, the coefficient and discharge can be slightly increased by making the upstream face of the orifice rough, there is no difficulty in making this face of sufficient smoothness to avoid any appreciable increase in the coefficient due to roughness of the plate. This frictionlessness of the orifice, with its streamlines which are constrained only at the square edge, which is so easily made to specification and inspected, makes it more satisfactory than any form of nozzle can be as a secondary standard of flow measurement. Fig. 14 shows that if the value of d_2/d_1 for a square-edged orifice with a very short length of parallel part is kept below 0.65, the value of the coefficient of discharge may be taken as 0.608. Theory shows that if the convergence of the streamlines of a "perfect fluid" is taken into account, the term $\pi/(\pi + 2)$ should be introduced into Equation [1] for such orifices and for sharp-edged orifices. As the numerical value of $\pi/(\pi + 2)$ is 0.611, this means that if the discharge equation had been based on the full theory of the orifice instead of upon a modified nozzle theory, the coefficient of discharge of orifices for which d_2/d_1 is less than 0.65 would have been $0.608/0.611 = 0.996$, or only 0.4 per cent different from theory.

Mr. Spitzglass has given only part of the reason for the rise in the coefficient of orifices as the $Q/d_2\mu$ value is decreased. The increasing effect of viscosity suppresses the effects of momentum, and so reduces the convergence of the streamlines passing through the orifice. The vena contracta, and in consequence the coefficients of discharge, are thus both increased. The author has tests which show that this effect occurs even when the orifice ratio is ten times less than the lowest value, 0.421, given in Fig. 4. He first called attention to the variation of the coefficient of contraction with viscosity during the discussion of a paper at the Institution of Civil Engineers in 1921.[96]

Mr. Spitzglass' Fig. 26 is interesting. If an attempt is made to separate out Ω and ϕ, the value of ϕ should, of course, be calculated from the assumed value of the diameter of the vena contracta and not from the diameter of the orifice. But as it is impossible to calculate the diameter of the vena contracta for an orifice without making assumptions as to the effects of compressibility and turbulence, it has seemed to the author best not to endeavor to effect any such separation for orifices, but to treat $(\Omega\phi)$ as one variable, though in the past he has made many attempts on the lines of Mr. Spitzglass's Fig. 26.

The P_2/P_1 vs. ϕ curves for air and steam which he and his assistants prepared in 1909–10 have been included in the paper only because of their value in calculating out the discharge through nozzles, for which Ω usually lies between 0.94 and 0.99.

With regard to the tests given in Fig. 6, the piping used was smooth-drawn brass tubing for 20 diameters upstream and 10

[96] See the author's contribution to the discussion of MacDonald and Hurst on "The Measurement of the Discharge of the Nile Through the Sluices of the Assuan Dam," Proc. Inst. of Civil Eng., vol. 212, 1921, part 2.

diameters downstream of the orifice, selected commercial wrought-iron tubing being used upstream and downstream of this. All except the round-dotted portions of Fig. 6 were obtained by extrapolation, using the method given in Appendix 6. The actual experimental points in Fig. 6 are shown marked by circles or crosses.

In reply to Mr. Sprenkel, the coefficients for self-contained orifices with the steadying cone are not the same as for an orifice in a straight pipe except when the hole is very small, when the coefficient is about 0.61. The coefficients for orifices used with the steadying cone shown in Fig. 8, as well as being different from the coefficients given in Figs. 4 and 14, also vary with each size of main, as the cones are of equal length, but of different diameters, so that geometrical similarity is not preserved.

The reason why the curve *A* in Fig. 14 differs from the other curve in Fig. 7 is that the orifice shown in Fig. 5 had a relatively long parallel portion, was made of fairly thick material, and was of constant width for all ratios of d_2/d_1. The orifices whose coefficients are given in Fig. 14 had, on the other hand, a very thin parallel portion so that they might almost be considered sharp-edged, and the thickness of the plate increased as the diameter of the pipe increased, instead of remaining constant. Further, as the curve given in Fig. 14 was intended for general use, and aimed at being accurate only to within plus or minus 1 per cent, any small deviations from exact straightness were faired out before drawing it up.

The author can find no reference in Appendix 10 of any suggestion to throttle the meter pressure pipes. A reference is made to this possibility on the seventh page, second column, but it is stated there that the "method is not practicable unless the pressure pipes are kept free from dirt or condensation of moisture."

In the discussion of Messrs. Judd and Pheley's paper in 1922,[97] the author stated that the pressure change should be considered as a symptom rather than as a cause. He has now (1929) nothing to add to that statement.

[97] Proc. Amer. Soc. Mech. Eng., Dec., 1922.

It is stated on the seventh page of the paper, and again in Appendix 10, that a number of assumptions have had to be made in order to derive the CFL/ZQ criterion. If the criterion is used bearing these clearly in mind, it will be found to be an extremely valuable guide to a mass of otherwise almost wholly uncoordinated experimental facts. During the past eight years the author has applied this criterion to many hundreds of cases of pulsating flows, and always with complete success. Mr. Sprenkel is, of course, correct in suggesting that a capacity which causes eddies or otherwise retards the flow produces a greater damping effect than one which does not do so. The CFL/ZQ criterion takes this effect into account to a considerable extent because eddies or vortices produced in the chamber providing the capacity C cause an increase in the value of the loss of pressure L.

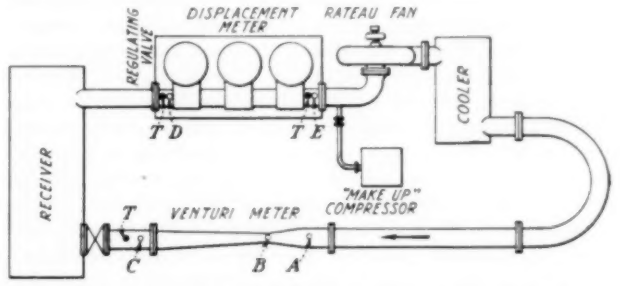


FIG. 30 ARRANGEMENT OF RAND AIR-METER CALIBRATION PLANT
(The pressure of the air was taken by a sensitive pressure gage at *A*; differential pressures were taken between *A* and *B*, *A* and *C*, *A* and *D*; *D* and *E*; temperatures were taken at the points marked *T*.)

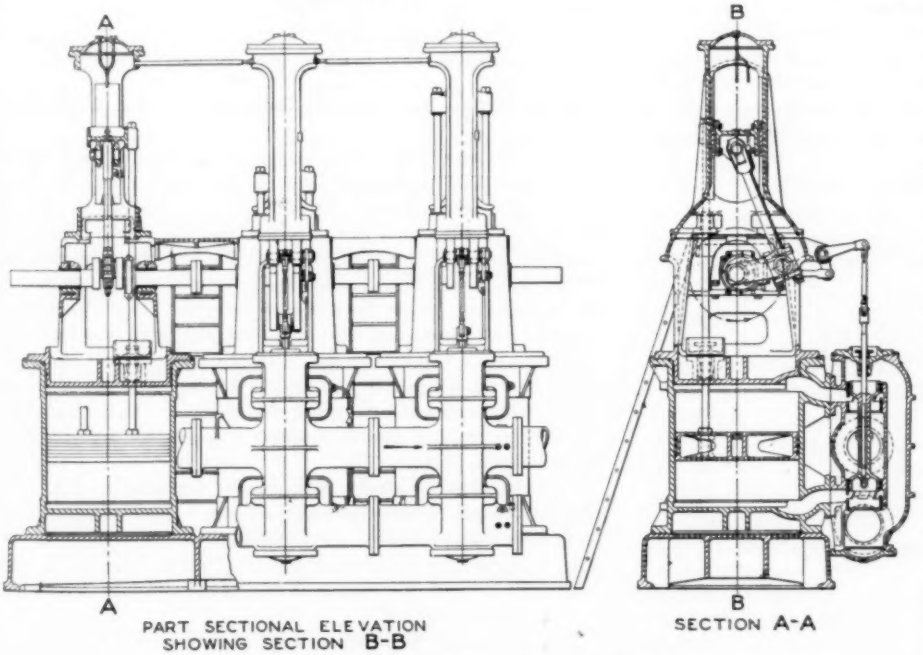


FIG. 29 THE RAND DISPLACEMENT METER

Mr. Barnes, Mr. Clark, and Mr. Spitzglass ask that further details shall be given as to test methods used. In this connection the following notes may be of interest:

The meters referred to in footnote 5 and also the first large orifices made were calibrated on a plant which was designed and erected specially for the work. The following illustrations and description are taken from the author's paper to the Institution of Civil Engineers in 1927:[98]

"The meters required in connection with the Victoria Falls and Transvaal power scheme, together with various sizes of venturi tubes, nozzles, and orifices, were calibrated in a special plant, which was designed and built at Erith by Messrs. Fraser and Chalmers, in consultation with Mr. A. M. Robeson, Mr. A. Smart, and the author. The plant was erected in the first instance to the author's specification in 1909, and, after being placed at the disposal of Messrs. Kent for about six months by Messrs. H. Eckstein and Company at Erith, was taken down and re-erected at Ferreira Deep on the Witwatersrand. It is now the standard air-calibration plant for South Africa.

"The air to be measured is passed through a large displacement meter [Fig. 29] which is built somewhat on the lines of a

[98] Proc. Inst. C. E., vol. 204, pp. 108–193.

steam engine. It has three cylinders 36 in. in diameter, and the stroke is 27 in. The admission and discharge of the air to the cylinders are controlled by means of piston valves which are set to cut off exactly at the top and bottom of the stroke. The stroke volume is 95.156 cu. ft., and the meter therefore passes 50 lb. weight of air per revolution at 100 lb. per sq. in. (abs.) and 513 deg. fahr. (abs.); or roughly, 1 ton per minute at the maximum speed at which it is designed to run. This displacement meter forms part of a closed air circuit [Fig. 30] wherein the air is circulated by a single-stage Rateau fan, which is driven by a 50-hp. motor at 2500 r.p.m., and produces a difference of pressure of about 2.5 lb. per sq. in. when the air is at a pressure of 100 lb. per sq. in. The pressure in the circuit is maintained by a small 'make-up' compressor, requiring about 30 hp., which delivers into the circuit between the displacement meter and the Rateau fan. After leaving the fan the air passes through a cooler, so that the temperature in the remainder of the circuit may be kept constant. It is then passed successively through the meter to be tested and the displacement meter."

The pressure of the air was taken by a sensitive pressure gage at *A*. Differential pressures were taken between *A* and *B*, *A* and *C*, *A* and *D*, and *D* and *E*. Temperatures were taken at the points marked *T*.

With this plant it was possible to obtain very accurate measurement of the air, as there was no leakage at all from the displacement meter, and there was practically no leakage across it owing to the small difference of pressure required to operate it. The slight pulsations which occurred when the displacement meter was running at its maximum speed were damped out by means of the receiver and regulating valve shown.

The cost of this plant at pre-war prices was about £4000.

Another plant on which very valuable tests were taken in 1913 was the test plant at Ragged Chutes in Northern Ontario, which was placed at the author's disposal for several weeks by the courtesy of the Northern Ontario Light and Power Company.

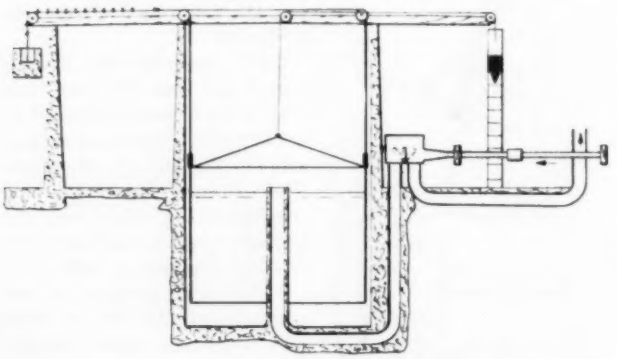


FIG. 31 TEST HOLDER AT RAGGED CHUTES

This plant is illustrated in Fig. 31. It consisted of a gas holder of 250 sq. ft. cross-section and 3000 cu. ft. capacity. The air passing through the orifice to be tested was held at the desired pressure by means of the two valves shown, and was discharged into the gas holder at practically atmospheric pressure. A throw-over valve with a deep water seal enabled the air passing through the orifice to be discharged into the gas holder or to atmosphere as desired. In its middle position it enabled the gas holder to discharge to atmosphere, the high-pressure air supply being of course, cut off. Compensation for the change in buoyancy of the gas holder due to immersion was made by means of weights attached to the cord carrying the compensating weight.

The movement of the gas holder was indicated on a steel tape marked in tenths and hundredths of a foot, a weighted pointer being attached by a metal cord to the center of the gas holder. It was easily possible to read the position of this pointer to an accuracy of one cubic foot.

The gas holder was housed in a special heat-proof building having windows only on its northern side. This arrangement eliminated changes of temperature due to the varying intensity of the sun's rays, which is a very frequent source of inaccuracy in the measurement of gaseous quantities by means of gas holders.

The source of air was a Taylor hydraulic compressor which delivered air that was perfectly dry and free from the slightest pulsation. Several thousands of horsepower were available from the compressor, which utilized the whole discharge of the

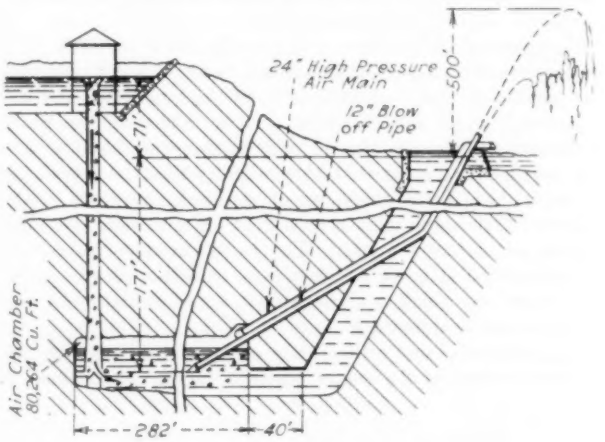


FIG. 32 TAYLOR HYDRAULIC AIR COMPRESSOR

Montreal River. Fig. 32 shows the main features of this compressor.

It will be seen that a dam was built across the river above the original fall at Ragged Chutes and that a vertical shaft some 242 ft. deep was sunk into the rock. A horizontal passage 322 ft. in length which had a raised roof for 282 ft. led from this vertical shaft to a rising shaft at the lower side of the fall. These shafts and their connecting passages were of sufficient size to take the whole discharge of the river. Air was sucked down by the river in its fall, and then separated out from the water at about 110 lb. per sq. in. in the underground chamber, which had an air capacity of 80,264 cu. ft. A relief pipe was provided through which excess air could escape. Various governing and controlling devices ensured a supply of high-pressure air at uniform pressure.[99]

On this plant extremely careful tests were taken with thin-plate orifices of various diameter ratios in a piece of bored 6-in. main which was specially taken to Ragged Chutes for the purpose. The orifices both before and after the air calibration were calibrated by means of water at the Messrs. Kents' hydraulic-test plant at Luton.

This hydraulic-test plant consists of a large tank in the floor of the test shop, a centrifugal pump delivering water to a large tank placed in a water tower, and a down-pipe passing from this tank to the meter to be tested, which meter discharges through a control valve into the tank in the floor of the test shop. The plant is so arranged that the quantity of water passed can be measured either in the large tank in the water tower or in smaller tanks before it is passed into the tank in the floor of the test

[99] It would seem highly desirable that such a plant as that which existed at Ragged Chutes in 1913 should be acquired for test purposes by those interested in the accurate measurement of gaseous fluids.

shop. The plant is capable of measuring flows up to 100,000 gal. an hour. The various tanks used were originally calibrated and are periodically checked by means of weight measurements.

The author has also made tests on orifices and venturi tubes against drum-type gas meters of sizes varying from one-tenth of 1 cu. ft. per revolution to 750,000 cu. ft. per hour. Such meters are calibrated against small, temperature-shielded gas holders, and will measure accurately to within plus or minus 1 per cent under test conditions.

A number of check tests on orifices have been taken from time to time for the satisfaction of clients. The first square-edged orifice which the author installed in South Africa in 1911 was checked for the satisfaction of his clients against a venturi tube which had been previously calibrated against the displacement meter shown in Fig. 29. In this test the orifice was put in series with the venturi tube with a sufficient length of piping in between them. Air at 100 lb. pressure was then passed through both, and simultaneous readings were taken over a period of time. Later on, for the satisfaction of another client, twelve orifices identical with the one referred to were placed in parallel with sufficient lengths of piping upstream and downstream of them, and the combined discharge at 100 lb. pressure was then passed through an orifice placed in a 44-in. main. Simultaneous readings were taken on twelve manometers connected across the twelve 12-in. orifices in parallel and upon a manometer connected across the orifice in the 44-in. main.

The cost of this set of tests worked out at about £50 an hour.

In the test results plotted in Fig. 6, which show the values of $\alpha\phi$ for discharges through orifices at high-pressure ratios, the air was measured by passing it through calibrated orifices working at high static pressure, but at low-pressure differences, and placed in mains much larger in diameter than the main in which the orifice itself was placed. These calibrated orifices were fed with air from a large receiver, which in turn was supplied by an air compressor. The ends of the pipes which contained the test orifice were open to atmosphere.

In addition to the tests enumerated, the author has made a large number of tests on the flow of steam at various degrees of wetness and superheat through orifices by condensing the steam and weighing it. Such condensate tests have been taken on mains up to 12 in. in diameter. He has also tested very small orifices for gaseous discharge by discharging gases such as oxygen and chlorine through them from steel bottles containing the compressed gas, the bottles being placed on a weigh bridge and flexibly connected to a pipe containing the orifice.

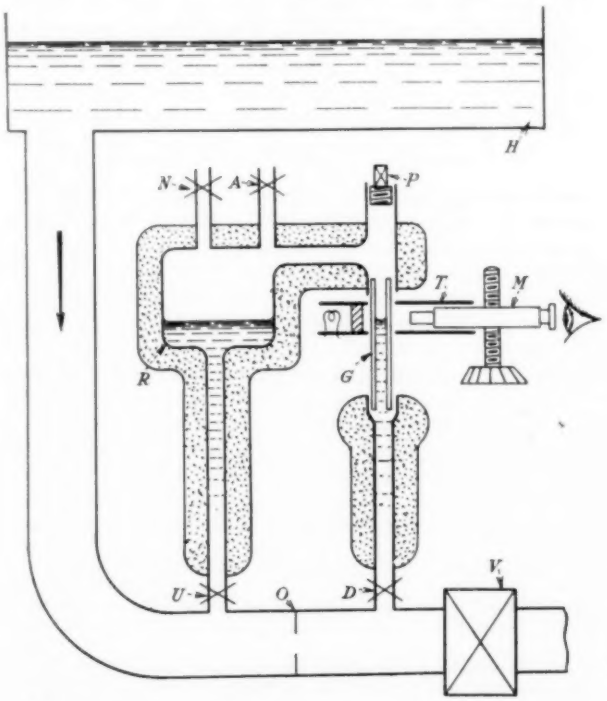


FIG. 33 DIAGRAM OF MICROMANOMETER FOR LIQUID FLOWS

Mr. Johansen's contribution is interesting in showing by its agreement with the author's results obtained by the use of viscous fluids that extremely small differences of water pressure can be measured with accuracy. Since the paper was written, the author and one of his assistants (Mr. H. E. Dall) have again attacked the problem of measuring very small heads of water with accuracy, and have found it possible, with the apparatus shown diagrammatically in Fig. 33, to read accurately down to $^1/_{5000}$th of a centimeter.

The apparatus consists of an inverted single-tube manometer containing a reservoir R and a gage glass G. This manometer is connected across the orifice to be tested, O. The pressure pipes are brought off at the side of the main and not at the top, as shown in the diagram, in order to avoid air bubbles rising into the manometer. Liquid is supplied from an overhead tank H, and the flow is controlled by a valve V placed on the downstream side of the orifice. The valves U and D control the flow of liquid into the manometer, and a plug P makes the interior of the gage glass accessible. Before a series of tests are taken the gage glass is carefully cleaned with hydrochloric acid or other suitable chemical. Air is pumped in with a bicycle pump through the valve A, and the valves U and D are then opened. Any excess air is allowed to escape through the needle valve N until the meniscus in the gage tube G comes to the desired reading on the micromanometer M.

When the flow is turned on, the level of the meniscus in the gage tube G falls, while that in the reservoir R rises slightly, the amount of rise being easily calculated when the ratio of the area of the reservoir to the area of the gage glass is known, and it is remembered that the volume of air trapped at the top of the reservoir remains constant so long as the static pressure on the orifice and the temperature of this air remain constant. The static pressure is maintained constant by having the supply tank H of large area so that the fall of level during a test is an imperceptible fraction of the total static head and by ensuring that the surface of the water in the supply tank and the tank itself are free from disturbance and vibration. Thus, if the tank is in a tower in a windy situation it will be found impossible to take accurate tests on a windy day owing to the variation in static pressure caused by slight waves set up in the tank due to the rocking of the tower.

The internal diameter of the gage-glass tube is approximately 0.3 in., the magnification of the microscope 40 diameters, and the micrometer screw $^1/_2$ millimeter pitch. The meniscus is shielded from extraneous light by a tube T through which the gage glass passes and which slides on the object glass of the microscope. The meniscus is illuminated by a lamp which is attached to this shield. This ensures that the particular portion of the meniscus on which the microscope reading is taken does not change its position through variations in the relative position of the source of illumination. (Actually the lamp is placed about 6 in. away from the meniscus and not close up, as shown, in order to prevent error due to heating the meniscus. The light is further passed through a glass plate before reaching the gage tube in order that the heat rays may be absorbed.)

The air in the reservoir is shielded from temperature changes, as any variation in the temperature of this air will cause its expansion or contraction, with consequent movement of the

meniscus. In order to ensure that no such contraction or expansion has taken place a zero reading is taken before and after each test. The pressure pipes leading from the orifice are also shielded from temperature changes in order that the density of the liquid in these pipes shall not alter and so affect the reading. In order to avoid error due to any difference of density of the liquid in the pressure pipes, these are made short and laid with a very slight slope so that the meniscus is practically at the same level as the center line of the orifice.

With this apparatus it is possible to read accurately down to $^1/_{5000}$th of a centimeter of water and to estimate the reading to $^1/_{10,000}$th of a centimeter.

The foregoing notes also cover a point raised by Mr. Connet.

The author is indebted to Mr. Johansen for his careful repetition of the tests on which Fig. 9 of the paper was based. The results given in Fig. 9 were taken with a 6-in. orifice in a rough cast-iron pipe about 1909 in the early days of orifice work, while those by Mr. Johansen were taken in a smooth brass pipe in 1929 after 20 years' improvement in technique. These facts may explain the slight difference between the results. At the same time it should be noted that the pressure distribution curves vary with the value of $Q/d_2\mu$, so that a complete statement of what happens cannot be made on a single set of curves which apply only over a limited range of $Q/d_2\mu$ values.

Mr. Johansen's question as to the effect of roughness of the pipe upstream of the orifice has already been answered in replying to Mr. Barnes.

Mr. G. M. Clark's contribution to the discussion is of special interest as he was responsible for the drafting of that part of the Victoria Falls & Transvaal Power Company's contract which dealt with the basis on which the compressed air was to be measured and sold. This contract provided for the measurement of the air on an energy basis so that the readings of the air meters could be directly comparable with the readings of the electricity meters and the whole of the heat balances of the plant could be drawn up on an energy basis. The coal, for instance, was rated at so many kilowatts, instead of at so many B.t.u. per pound. The "air unit," in terms of which the meter had to register, was defined as the "quantity of air which would be compressed from the mean atmosphere and temperature on the Rand (12.086 lb. per sq. in. abs. and 60 deg. fahr.) to the pressure of delivery by the expenditure of one kilowatt-hour of energy in an isothermal compression process of an agreed overall efficiency." This "air unit" is also defined by the equation given in footnote 5.[100] Mr. Clark was also responsible for the acceptance of the first large compressed air meters which the author designed some twenty years ago, and which are referred to in footnote 5. In these days when research workers all over the world are intensively studying the orifice as a method of measurement it is gratifying to have testimony as to the high accuracy of the work done twenty years ago when every step made was a step in the dark and involved risk of failure. The large displacement meter referred to by Mr. Clark is shown in Fig. 29.

In concluding his reply to the discussion, the author desires to make reference to Mr. Clemens Herschel, whose classical work on the venturi tube first made the accurate measurement of fluids in closed conduits possible.

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[100] See also Proc. Inst. C. E., vol. 204, p. 109.

FSP-51-43

Low-Temperature Carbonization of Southern Appalachian Coal

By LEO HOLDREDGE,[1] KNOXVILLE, TENN.

After discussing the principal reasons for the low-temperature carbonization of bituminous coal, the author describes briefly the four types of methods used or proposed for low-temperature carbonization, and then gives details regarding a laboratory experiment carried out at the University of Tennessee Engineering Experiment Station, the primary purpose of which was to produce a smokeless fuel in a form suitable for domestic use. A summary of the results of ten tests of a high-volatile northern Tennessee steam coal is included which gives quantities of semi-coke, gas, tar, and tar products produced, together with their gravities, heating values, etc. The coke obtained is easily ignited, burns smokelessly even in an open grate, and, the author believes, should find a market as a domestic fuel.

A CENTURY ago Sir Robert Peel said that the future belonged to the nation which could produce the most coal. In 1910 a German writer remarked, "The future belongs to the nation which makes the best uses of its coal resources." Both of these expressions are overstatements if they are taken at their face value. Many other elements besides its supply and use of coal enter into the civilization of any nation. If other things are equal, the future will belong to the nation which possesses and efficiently uses an abundant coal supply.

Low-temperature carbonization of bituminous coal is one of the proposed methods for making efficient use of our coal resources. The fuel resources, the practice most commonly followed in the use of fuel, the economic conditions, and the demand for a raw material for making synthetic chemicals, will determine when low-temperature carbonization is an efficient method of using our coal supplies.

There are three principal reasons for the low-temperature carbonization of bituminous coal:

1 Production of a free-burning smokeless fuel
2 Recovery of the by-products
3 Increase of the value of coal as mined.

Production of Free-Burning Smokeless Fuel

The value of a smokeless fuel can best be appreciated by observing the damage done by smoke. Definite data on the damage from smoke cannot be compiled, but it certainly is enormous. Estimates of from $500,000,000 to $550,000,000 annual loss in the United States have been made by investigators who used surveys of the United States Government as the bases for their calculation. This damage may be considered under four divisions: (*a*) Damage to vegetation, (*b*) damage to structures, (*c*) damage to health, and (*d*) economic damage.

Carbon dioxide from the air is assimilated by the plant by means of the stomatal openings in the leaves. Oxygen is eliminated. As the animal inhales the oxygen and exhales carbon dioxide, so the plant takes in carbon dioxide and gives off oxygen. Sunlight is necessary for this life process of the plant.

Soot, the solid matter of smoke, consists principally of free carbon, tarry matter, and ash. The soot clings so tenaciously to anything with which it comes in contact that it is not even removed by hard rains. It is deposited on the leaves of the plant and interferes with the vital interchange of carbon dioxide and oxygen. If the leaves were covered with the tarry matter until the openings were completely closed, the plant would die. The partial closing of these openings interferes with the vital processes of the plant so that growth is retarded.

It is also known that sunlight is necessary for plant growth. Smoke greatly reduces the intensity of the sunlight and consequently retards the plant growth.

In normal country air there are four parts of carbon dioxide to ten thousand parts of air. This carbon dioxide ratio is increased in the areas where smoke is produced. This factor itself would increase plant growth, but it is more than counterbalanced by the tarry deposits on the leaves and the decreased intensity of sunlight due to the smoke. But if the coal were burned without smoke, the carbon dioxide ratio would be increased still more and the tarry deposits would be absent. This would stimulate plant growth.

The damage that smoke causes to buildings is twofold. The acids in the smoke react with the materials out of which the building is constructed. In course of time this damages the building in usefulness and appearance. The smoke also mars the beauty of the building by reducing all parts of it to a dull drab color. Light shades and pleasing contrasts in color with which the architect has beautified the building are covered with grime and soot.

But the greatest damage done by smoke is the damage to health. The following extract from the report of the Anti-Smoke Committee of the St. Louis Medical Society indicates the extent of this damage.

> The permanent deposits of carbon in the lungs and the thickening of the air passages must reduce the normal lung capacity. This in turn definitely lowers the general vitality and resistance to disease. Chronic diseases of the nasal sinuses, throat, and deeper air passages of the lungs are aggravated, prolonged, and often rendered incurable. Ordinary colds occur more frequently and tend to progress into more serious conditions. The failure of proper interchange of gases in the lungs also causes mental sluggishness and depression. These definite effects upon our bodies and minds, in addition to the economic losses to the community through the necessity of constant cleansing of every article in our households, offices, and business places seem to us to warrant a united and determined effort to find a solution of the smoke problem. We must rid ourselves of our worst personal and civic nuisance. Many persons leave our city or fail to come here on account of smoke.

Not only is there damage to vegetation, buildings, and health by smoke, but there are also large economic losses. The amount of carbon in the smoke which gives it the black color may not be large. The amount of invisible hydrocarbons which are present in the smoke often represent a loss which is several times as large as the loss in the visible smoke. Smoke is evidence of incomplete combustion. Lack of smoke is a strong indication of complete combustion.

The decreased heat conductivity due to deposits of soot represent a large loss. Carbon in any form is a good non-conductor of heat. In the loose, fluffy form of soot it is one of the best-known non-conductors. Sooty heating surfaces are found with

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

smoke. Clean heating surfaces are the rule where no smoke is produced.

The constant cleaning of draperies, clothes, and indeed every article about the home, office, and business house, causes a large loss. Not only is there the cost of the cleaning, which is large, but there is also the larger loss due to decreased length of usefulness of the articles themselves. This loss caused by smoke is certainly of no small size.

In most cities the total amount of smoke made by the domestic users of coal is larger than that made by either the railroads or the industries. Smokeless cities will be a dream and a vision until the household user is furnished a fuel which he can and will burn smokelessly. That is where low-temperature carbonization of coal should help. It provides a smokeless fuel which is free burning and can be used in the ordinary household appliances.

Recovery of By-Products

The second reason for low-temperature carbonization of bituminous coal is for the recovery of the by-products. In opening the International Conference on Bituminous Coal held in Pittsburgh, November 15–18, 1926, President Baker of Carnegie Institute of Technology said:

> In less than a generation the present method of shipping coal to be burned in its raw state under boilers hundreds of miles from the mines will appear to have been primitive and rudely unscientific. Just what form the developments of fuel technology are going to take one may not forecast. Of one thing we are certain, and that is that the changes will be radical, and it may not be too much to hope that this present International Conference on Bituminous Coal may mark a turning point, at least in this country, in the science of the utilization of coal.

The development of the internal-combustion engine in the last few decades has caused a very large demand for low-boiling motor spirits. The principal source of this fuel at present is petroleum. In 1924 the petroleum refineries in the United States produced 8,960,000,000 gallons of gasoline. Gasoline from natural gas amounted to 934,000,000 gallons. The quantity of motor benzol produced from coal in the same year is not known exactly, but 100,000,000 gallons is perhaps about right. The petroleum refineries produced 2.26 times as much gasoline in 1924 as they did in 1919.

The world's known supply of petroleum will not last many decades at the present rate of production. It is reasonably certain that other deposits of petroleum will be discovered and that better methods of recovery will increase the production from the present fields. But in spite of this, there is a shortage of oil in some parts of the world at present. Just when this shortage might become acute enough to interfere seriously with the industry and commerce of any nation would be difficult to forecast.

Table 1, from "The Conversion of Coal into Oil," by Fischer (English translation by Lessing), shows the coal resources of the world.

TABLE 1 COAL RESOURCES OF THE WORLD

Continent	Total deposits, including proved, probable and possible, million tons	Proved deposits, million tons
Europe	784,190	274,189
North America	5,073,431	414,804
South America	32,097	2,087
Asia	1,279,586	20,502
Australia	170,410	4,073
Africa	57,839	500

Coal is being produced at a rate of 1,250,000,000 tons per year. From Table 1 it may be seen that the world's supply of coal will last for thousands of years at the present rate of production. On a heat basis the annual production of oil amounts to about 14 per cent of the coal production. Or in other words, an increase of 14 per cent in the amount of coal mined would equal all the oil produced as far as heat is concerned.

Low-temperature carbonization of bituminous coal can supply some of the world's need for a low-boiling motor fuel, and thus supplement a limited petroleum resource with a fuel from an almost unlimited coal supply.

The by-products of low-temperature carbonization may become of greater use in the fields of chemistry than in the fuel field. Coal tar is a raw material for a very large number of organic chemicals. With the increase in the number and importance of the synthetic organic compounds, the amount of coal tar used in the chemical industries is sure to become greater.

Increase of the Value of Coal as Mined

The free-burning smokeless fuel is of greater value and commands a higher price than a coal which produces smoke when burned under the conditions which obtain in domestic heating. This is best seen by a comparison of the prices on anthracite, low-volatile bituminous, and high-volatile bituminous coals. On September 1, 1927, the prices at Minneapolis, Minnesota, were as follows:

Anthracite		Pocahontas		Kentucky	
Egg	$13.20	Block	$8.00	Block	$6.00 to $7.00
Stove	13.60	Mine run	5.75	Mine run	5.75
Nut	13.20	Screenings	4.75	Screenings	4.75
Pea	11.05				

All prices f.o.b. Duluth, Minnesota.

These prices are quoted from *The Black Diamond*, September 3, 1927, issue.

In the same issue of that journal the spot prices at eastern Kentucky mines were given as follows:

Block	$2.50 to $3.25
Lump	$2.00 to $2.50
Egg	$2.00 to $2.50
Nut	$2.00 to $2.25
Mine run	$1.50 to $1.75
Screenings	$1.10 to $1.35

Low-temperature carbonization of bituminous coal should become of practical economic value if it can take low-priced mine run, or nut and screenings from bituminous coal mines and convert that raw material into a clean, free-burning, smokeless semi-coke which will replace the anthracite coal which is becoming quite scarce.

Methods of Low-Temperature Carbonization

The methods used or proposed for low-temperature carbonization may be divided into four general types, depending upon the type of retort used and the method of applying heat.

1 Externally heated retorts in which the coal is in thin layers and is not stirred during carbonization
2 Externally heated retorts in which the coal is stirred during carbonization
3 Internally heated retorts with the coal filling the retort and the heat supplied by means of hot gases or superheated steam
4 Internally heated retorts with the coal pulverized and heated in suspension by means of hot gases.

The method which is best to use in any given case will depend upon the coal which is to be used and the products which are to be made.

If a coking coal is to be used and a lump semi-coke is desired, the externally heated retort, in which the coal is quiescent during carbonization, may be used. The great difficulty in this

type of retort is in transferring the heat to the coal at the low temperature gradient which is necessary. Coal is a very poor conductor of heat. This necessitates the use of thin layers of coal. As a result the output is low and the investment is large for the yearly output.

With coal of from 32 to 35 per cent volatile matter, such as is common in the Southern Appalachian coal field, a gas yield of from 2500 to 4000 cu. ft. per ton may be expected. This gas should have a heat value of about 1000 B.t.u. per cu. ft. The tar yield would be above 30 gal. per ton. (Laboratory yields much higher than this have been secured.)

The externally heated retort in which the coal is stirred during carbonization seems suited for carbonizing non-coking coals. This method should result in faster heating than where the coal is not stirred. The semi-coke would be in a pulverized form, and briquetting would be necessary if a domestic fuel were to be produced.

A coking coal is too sticky and gummy when heated to use in a retort of this type unless special devices are used to prevent the coal from sticking to the sides of the retort. The K.S.G. system has apparently overcome this difficulty by use of two concentric cylinders. The cylinders are slightly inclined. The raw coal enters the lower end of the inner cylinder and by means of vanes on the inside of the cylinder is carried to the upper end of the cylinder as it slowly rotates. In the inner cylinder the coal is warmed, but the real carbonization occurs as the coal descends the outer cylinder. A suitable lump semi-coke is said to be made by this process.

The internally heated retort with the coal filling the retort and the heat supplied by hot gases or superheated steam would seem to have some advantages. As was previously mentioned, coal is a very poor conductor of heat. The hot gas or steam could carry the heat to every part of the mass. Certain difficulties have arisen with the use of these retorts. If the heat is carried to the charge by means of flue gas there is a very large yield of a gas low in heat value. If superheated steam is used the tar will be very intimately mixed with several times its own weight of water. The tar is not easily separated from this water.

The Turner process, which is an outgrowth of the process for distilling Scottish oil shale, uses superheated steam. The process has one unique feature. A series of valves are operated so that the steam builds up a pressure of a few pounds above atmospheric pressure. Then the admission valve is suddenly closed and the exhaust valve at the same time opens wide, so that the pressure at once falls. This process is then repeated every few seconds. By this process the inventor claims that no non-condensible gases are produced, and that a satisfactory semi-coke is made.

The McEwen-Runge system carbonizes the pulverized coal with hot gas as it falls through a vertical retort. The coking properties of the coal are destroyed in a preheating process so that the semi-coke is left in a pulverized form. It is used as a powdered fuel for boilers, and is said to be better than raw coal for this purpose.

This outline of the methods of low-temperature carbonization is intentionally very brief. For more complete information reference is made to numerous articles in the technical press and to the recent books on the subject.

LABORATORY EXPERIMENT AT THE UNIVERSITY OF TENNESSEE ENGINEERING EXPERIMENT STATION

The primary purpose of the experiment was to produce a smokeless fuel which would be in a form suitable for domestic use. In addition to this there was a desire to determine the coking characteristics of representative Tennessee coals, when coking at low temperatures; it was also desired to learn enough about the tar produced so that its most profitable use could be predicted.

Preliminary Tests

Some preliminary tests were made in order to determine, if possible, the amount of volatile matter which could be left in the coal, and at the same time have a smokeless fuel. Five 5-gram samples of coal in crucibles were heated in a muffle furnace. The coal used for these tests had 34.57 per cent volatile matter. Four tests of this kind were made. The amount of volatile matter left in the samples was as follows: 12.49 per cent, 15.09 per cent, 16.74 per cent, and 20.34 per cent. Each sample was then pulverized and burned in a crucible. The samples with 12.49 per cent and 15.09 per cent of volatile matter showed no smoke whatever. Smoke was barely perceptible on burning the sample with 16.74 per cent volatile matter. The sample with 20.34 per cent volatile matter was quite decidedly smoky on burning. It should be noted that the amount of volatile matter as given above is the percentage of the original sample. The amount of volatile matter in the semi-coke itself of these same samples is 14.28 per cent, 17.77 per cent, 20.11 per cent, and 25.53 per cent. From these tests it is concluded that the volatile content of the coal used for these tests must be reduced to 18 or 20 per cent before the coal will burn in the crucible without smoke.

Another series of tests was made with a mixture of hard foundry coke and raw coal. Each was pulverized and then the mixtures were burned in the crucible. This mixture would not burn in the crucible without smoke whenever the amount of volatile matter in the mixture was over 3 per cent. From these two series of tests it is concluded that it is not so much the amount of volatile matter in the coal as its form which causes the smoke.

Coal Used

Eight of the coals tested were from the northern Tennessee Coal field. One gas coal from southern Kentucky was tested. A few tests were run on each of these coals, and then a series of tests were made on one coal which had the following analysis:

Ash	6.56 per cent
Moisture	1.68 per cent
Volatile matter	35.71 per cent
Fixed carbon	56.65 per cent
Heating value	13,916 B.t.u. per lb.

This coal was taken from the thickest seam in the state. The coal is at present sold entirely as a steam coal. It is quite strongly coking, but it is too high in sulphur to be used for making metallurgical coke.

All the coals used were high-volatile coals, since this kind of coal predominates in the Southern Appalachian field.

Apparatus Used

The first apparatus used was designed so that the coal was heated by superheated steam. The first part of this apparatus was a superheating coil in which low-pressure steam was highly superheated. From the coil the steam entered a steam jacket. The retort with the coal was within this steam jacket. The retort was perforated so that the steam had free access to the coal. From the retort a pipe led to a condenser. The tar, gas, and condensed steam then flowed to a wash bottle, from which the gas was led to a gas holder.

Two difficulties arose in the use of this apparatus. In the first place, it was not possible to get sufficient heat to the coal, even though auxiliary burners were used to heat the retort and the superheating coils were kept at a dull red heat. This difficulty could have been easily overcome, however, by the use of more insulation so as to better conserve the heat. The other

difficulty was that the water and tar would not separate by gravity, even though left undisturbed for weeks at a time. Since at best the volume of water would be several times the volume of the tar, it seemed best to change to a type of apparatus where this difficulty would not be met.

While using the first apparatus one interesting characteristic of the coal was noticed. In one of the tests sufficient heat was secured apparently to fuse completely about two inches of coal on the bottom of the retort. The coke had a volatile matter content of 34.14 per cent. It was quite friable and burned with considerable smoke. This, together with other observations of a like nature, indicates two things: First, that the coal fuses before a large amount of the volatile matter is driven off; and

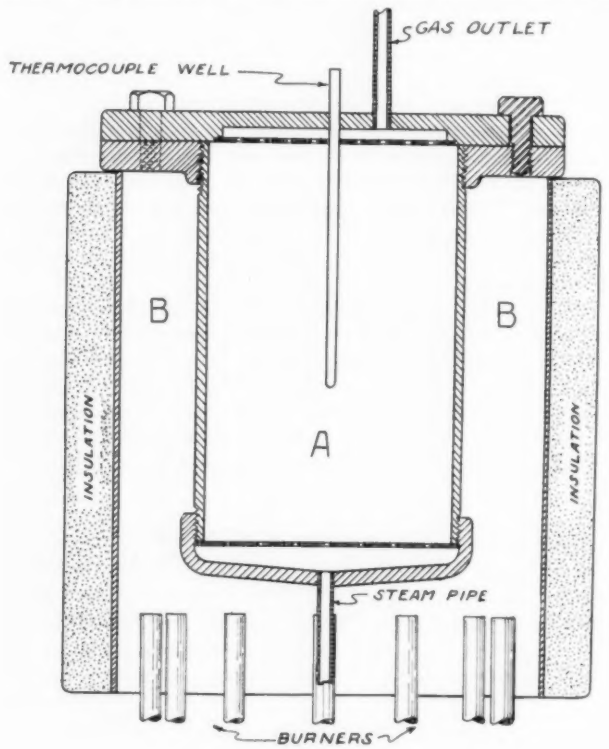


FIG. 1 THE RETORT

second, that exceedingly low temperatures will not produce a semi-coke hard enough to stand handling.

The apparatus finally used is shown in Fig. 1 and was built out of pipe fittings. The retort *A* is a piece of 4-in. pipe. On the lower end is a pipe cap, in the center of which a hole is drilled and tapped for the steam pipe. A plate with a large number of small holes is fitted in the lower end of the retort. This arrangement permits steam to enter the retort and be evenly distributed. At the same time the cap keeps the lower end of the retort from being overheated.

A flange is fitted on the upper end of the retort. A blind flange fastened to it by eight cap screws has a recess cut in its lower face in which a perforated plate is inserted, leaving a space above the plate as shown. The gas comes through the holes in the perforated plate and goes out through the gas line which leads from this space. A tube also protrudes through the plate into the retort. The thermocouple is placed in this tube. Other holes through both of the flanges permit the escape of the flue gases from the heating burners. The retort *A* is placed in the heating chamber *B*, which is well insulated. The heat is applied by the burners as shown in the figure.

In the first tests with this apparatus superheated steam was supplied through the steam pipe until the steam would blow through the gas outlet. In later tests the apparatus was used without steam.

The gas line extended to a bottle placed in cold water (Fig. 2). This gas line and cooled bottle acted as a condenser. The tar was deposited in the bottle and the gas passed on to a gas holder.

At the end of each run the gas was carefully measured and the reading of the meter corrected to reduce it to an equivalent volume at 60 deg. fahr. and 30 in. of mercury pressure. The tar was distilled and separated into various fractions as indicated later.

The carbonizing temperature was a maximum of 1000 deg. fahr. It took approximately three hours to reach this temperature. While the apparatus was designed for the use of steam, the tests here recorded were made without steam. Experiments with steam were not successful, but the condensing apparatus is being rearranged and other tests are to be made with the use of steam.

SUMMARY OF TESTS (10 TESTS)

Semi-coke per ton of coal			1393 lb.
Gas per ton of coal			3613 cu. ft.
Dehydrated tar per ton of coal			37 gal.
1	Dehydrated Tar:		
	a	Viscosity at 100 deg. fahr.	525
		at 210 deg. fahr.	40
	b	Specific gravity	1.04
	c	Flash point	140 deg. fahr.
	d	Burning point	175 deg. fahr.
	e	Heating value per lb.	16,600 B.t.u.
2	Dehydrated Tar, Less Oils to 100 Deg. Cent.:		
	a	Viscosity at 210 deg. fahr	49
	b	Specific gravity	1.06
	c	Flash point	255 deg. fahr.
	d	Burning point	280 deg. fahr.
	e	Heating value per lb.	16,500 B.t.u.
3	Dehydrated Tar, Less Oils to 225 Deg. Cent.:		
	a	Viscosity at 210 deg. fahr.	1570
	b	Specific gravity	1.089
	c	Heating value per lb.	16,400 B.t.u
	d	Flash point	312 deg. fahr.
	e	Burning point	350 deg. fahr.
4	Pitch at 290 Deg. Cent.:		
	a	Melting point	111 deg. fahr.
	b	Specific gravity	1.120
	c	Twist point	75 deg. fahr.
5	Pitch above 325 Deg. Cent.:		
	a	Melting point	450 deg. fahr.
6	Tar Products:		
	Motor fuel (0 to 210 deg. cent.) per ton of coal		4.4 gal.
	Burning oil (210–270 deg. cent.) per ton of coal		4.6 gal.
	Light lubricating oil (270–325 deg. cent.) per ton of coal,		4.4 gal.
	Heavy lubricating oil (325–350 deg. cent) per ton of coal,		2.8 gal.
	Grease per ton of coal		12 lb.
	Pitch per ton of coal		90 lb.
	Tar acids (phenol and cresols) per ton of coal		22 lb.
	Heavier tar acids per ton of coal		44 lb.
	Heating value of motor fuel per lb.		18,480 B.t.u.
	Specific gravity of motor fuel		0.833
	Heating value of burning oil per lb.		18,172 B.t.u.
	Specific gravity of burning oil		0.905

USE OF PRODUCTS

The coke produced is easily ignited and burns with a short blue flame. It burns without smoke even in an open grate. The grate fire has a very pleasing appearance. The semi-coke should find a market as a domestic fuel, since it takes no special appliances or procedure to use it. It is not an artificial anthracite; its density is much less. If used in stoves or furnaces designed

for anthracite, the semi-coke would require more frequent firing.

The gas was not scrubbed to remove light oils. If such scrubbing were done, about three gallons of light oil per ton of coal would be expected. This would reduce the value of the gas but would very greatly increase the amount of motor fuel recovered. The gas has a heating value of approximately 1000 B.t.u. per cubic foot. It is very high in luminants. The most obvious use of such a gas would be as an enriching agent for mixing with a lean gas such as water gas.

The tar might be used in several different ways. The large amount of tar acids suggests that they should be removed and used as preservatives for timber, as disinfectants, or as one of the raw materials for making such materials as bakelite. Other uses for the tar acids would be as raw materials for making explosives, dyes, and medical compounds.

The light oils both from the tar and that which could be recovered from the gas would make an excellent motor fuel. The lubricating oils and greases would be used as similar products from petroleum are used. The pitch could be used as the pitch from high-temperature tar is used.

Under present conditions the removal of the motor fuel and then the use of the remainder of the tar as liquid fuel for burning under boilers or for Diesel-engine fuel seems the most probable way of employing it. It is easy to paint pictures of the wonderful possibilities of coal tar as a raw material for the chemical industries, and its use in such a way is likely to increase rapidly in the future. Further tests will likely show that low-temperature tar is more suitable for that purpose than high-temperature tar. But the fact remains that at present a large part of the tar must find a use as fuel and the producer of low-temperature tar must expect that the large part of his tar will be used as fuel and will command the price it is worth in open competition with other liquid fuels.

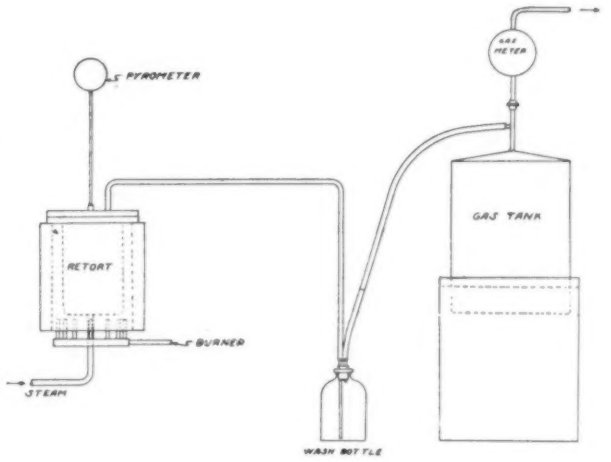


FIG. 2 ARRANGEMENT OF APPARATUS

FSP-51-44

A Laboratory Slagging Test for Boiler-Furnace Refractories

By R. K. HURSH,[1] URBANA, ILL.

This paper gives particulars of a simulative service test of refractories with coal-ash slag made in a laboratory furnace in which the test brick form the lining of a cylindrical chamber rotated about a vertical axis. Powdered slag or a mixture of slag with powdered coal is fed at a uniform rate through the gas burner which heats the furnace, impinging on the vertical faces of the brick and flowing out through the bottom of the furnace. Temperature and atmospheric conditions are controlled and the test brick receive equal slag treatment. Comparison of slag effect on the refractories is made by volume loss per unit area of exposed surface.

Tests have been made with slags representing coals used at five power stations where field tests were conducted, and the results are presented by the author. He finds that the relative effects of these different slags at a furnace temperature of 2900 deg. fahr. varies with the composition and fusibility of the coal ash. Further, the erosion of the refractory is dependent on the viscosity of the slag and increases in direct proportion to the furnace temperature. Tests made at temperatures corresponding to those in the boiler furnaces produce similar action on the refractories, as indicated by petrographic examinations. The relative resistance of refractories of different composition and physical structure is not the same for different slags.

IN AN INVESTIGATION of the slagging effect of coal ash on refractories, a laboratory testing method was sought which would determine the relative resistance of different bricks to such slags in terms which might be correlated with the results of service in boiler furnaces. The simulated service test was found most applicable for this purpose. Difficulty was found in obtaining equal heat treatment and uniform slag application on test bricks under conditions representative of boiler-furnace operation. A furnace[2] and a testing method have been developed which give a very satisfactory simulated service test.

THE LABORATORY SLAGGING-TEST FURNACE

General Description of Furnace

A general view of the furnace and its auxiliary equipment is shown in Fig. 1. A cylindrical furnace chamber is supported vertically on rollers and is rotated about its axis during the test. It is heated to any desired temperature with controlled atmospheric conditions by a blast burner using city gas and air, both at high pressure. Powdered slag is fed at a uniform rate through the burner and impinges with the blast on the vertical faces of the standard-sized test bricks which form the lining of the chamber. The melted slag flows down over the faces of the bricks and out through a port in the middle of the furnace bottom.

By rotating the furnace, each test brick is brought successively and repeatedly into position before the burner. Identical temperature conditions and uniform slag treatment for the several bricks are thus insured. A close control of both temperature and furnace atmosphere can be maintained to simulate conditions in the boiler furnace.

Furnace Details

Details of the furnace and equipment are shown in Fig. 2. Sixteen test bricks T are set on end with the $2^1/_2$-in. × 9-in. faces lining the chamber. These alternate with special arch bricks U to form a circle of $14^1/_2$ in. inside diameter. The lining is carefully laid up with thin mortar joints of fireclay and fine grog. The lining is backed by $2^1/_2$ in. of silo-cel brick placed on end inside the steel jacket E.

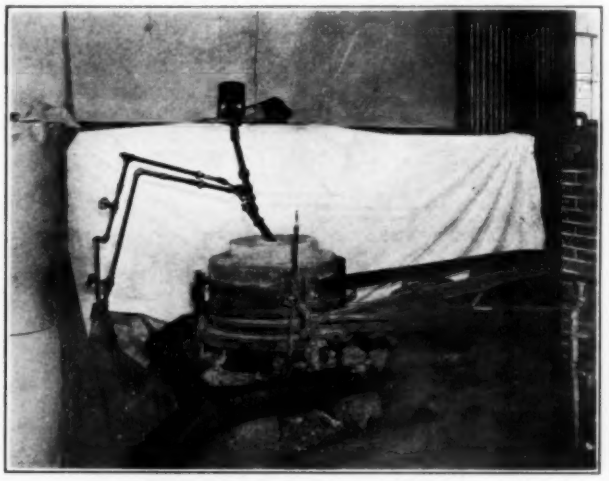


Fig. 1 Exterior of Slagging-Test Furnace

The bottom blocks of the chamber are supported by a cast-iron tire rim C and eight radial arms A. A hole 5 in. in diameter is left in the center for outflow of the melted slag.

The furnace is supported on rollers and is rotated at 13 r.p.m. by means of a belt passing around the steel belt band B. A series of guide rollers hold the furnace in position on the supporting rollers.

In order to protect the rollers from the heat of the gases and the melted slag at the bottom of the furnace, an octagonal screen of insulating blocks I is fastened to the supporting piers and channels. The rollers are also cooled by an air blast from a 12-in. sirocco fan through four branch pipes on a main air pipe surrounding the furnace. Further cooling of the furnace jacket, belt band, and tire rim is effected by a spray of water from a hose nozzle on the furnace jacket. This also protects the belt from excessive heating.

Slag from the bottom of the furnace is quenched in a drip pan placed between the supporting piers. A constant water level is maintained in the pan by a stream from valve Z. The regulation

[1] Department of Ceramic Engineering, University of Illinois.

[2] R. K. Hursh and C. E. Grigsby, "A Laboratory Furnace for Testing Resistance of Fire Brick to Slag Erosion," Circular 17, University of Illinois Engineering Experiment Station.

Contributed by the A.S.M.E. Special Research Committee on Boiler-Furnace Refractories and presented at the Rochester Meeting, Rochester, N. Y., May 13 to 16, 1929, of The American Society of Mechanical Engineers. (The results herein presented are a part of an investigation conducted by the Engineering Experiment Station of the University of Illinois, of which M. S. Ketchum, Dean of the College of Engineering, is the director, and in the Department of Ceramic Engineering, of which Prof. C. W. Parmelee is the head. The work is being done in cooperation with the Special Research Committee on Boiler-Furnace Refractories of The American Society of Mechanical Engineers, of which Mr. C. F. Hirshfeld is Chairman.)

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

of water level is necessary because of the cooling effect on the bottom opening of the furnace. Excessive cooling causes the slag to congeal and choke the slag exit.

The height of the furnace chamber is increased to $11\frac{1}{2}$ in. by a course of circle bricks V to permit placing the burner in such a position that the flame and slag will impinge at the desired height on the test bricks. A crown block W is provided with a 5-in. hole

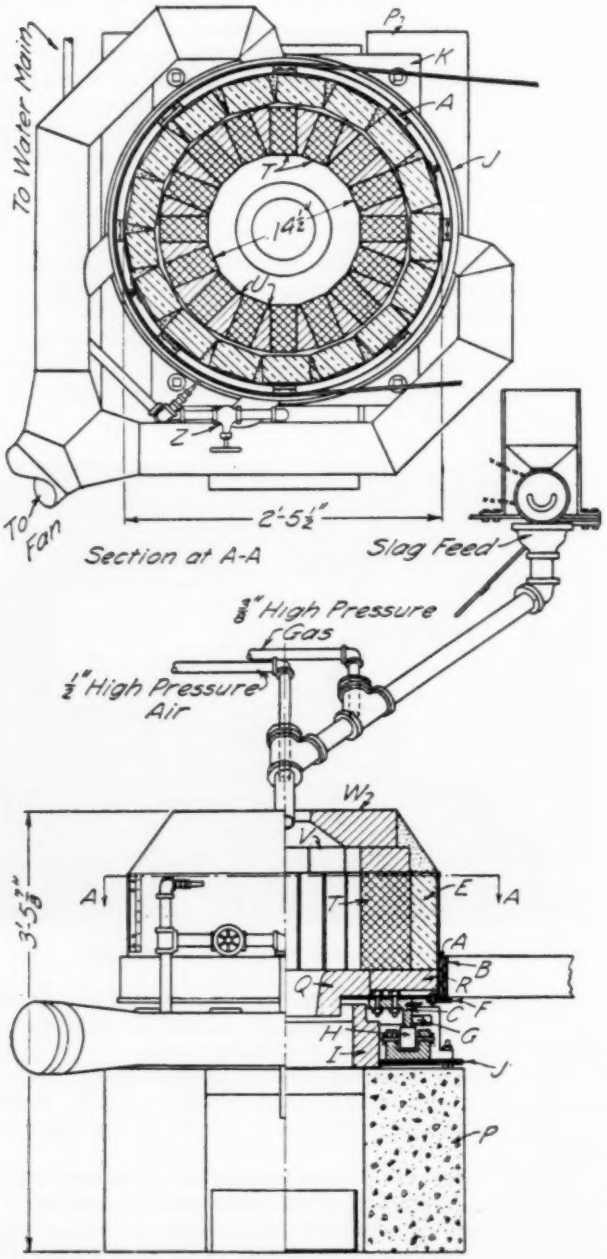


FIG. 2 DETAILS OF SLAGGING-TEST FURNACE

in the center which flares out to 12 in. on the underside to permit proper direction of the flame. The crown block is carefully centered in setting up the furnace for a test to insure uniform flame and slag impingement on all the bricks.

FEEDING OF SLAG

It is required that the burner shall deliver an adequate and properly proportioned mixture of gas and air and at the same time serve as a nozzle to direct a continuous stream of powdered slag against the faces of the brick. The burner is made up of standard pipe fittings. High-pressure air is supplied through a $\frac{3}{8}$-in. nozzle inside the upper $1\frac{1}{2}$-in. Y and, by its inspirating effect, draws in additional air through the slag feed opening above. City gas, at 10 lb. pressure, is supplied through a $\frac{1}{2}$-in. pipe which projects through the lower $1\frac{1}{2}$-in. Y into the mixing nozzle of the burner. The powdered slag, fed into the upper pipe, is carried in suspension by the air and gas mixture and is projected with the flame on the faces of the test bricks.

The slag, ground to pass a 40-mesh screen, is fed to the burner by a small screw feeder from which it discharges through a slot at one end. The screw, a $\frac{3}{4}$-in. shaft with a round-bottom thread of $\frac{1}{2}$ in. pitch, is driven at a fixed speed, ranging from 20 to 40 r.p.m., by a roller chain from a worm-gear speed reducer which is belted to a $\frac{1}{8}$-hp. motor. The speed of the screw is regulated to give a desired rate of slag feed.

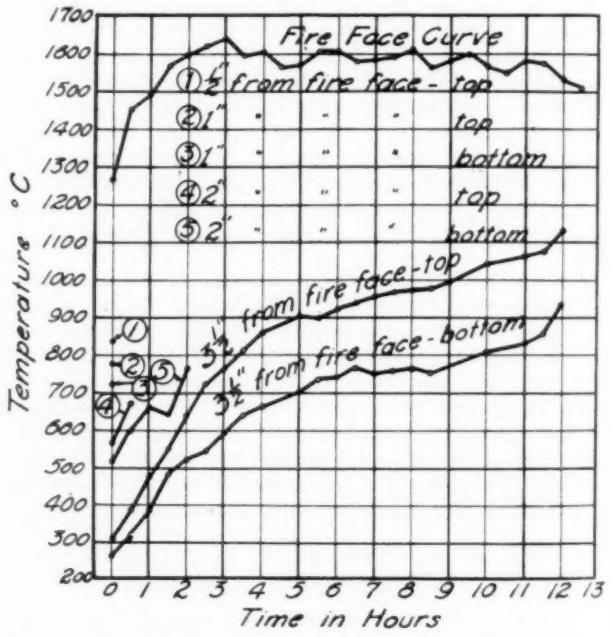


FIG. 3 TEMPERATURES IN TEST BRICK

TESTING PROCEDURE

The furnace is heated, while rotating, to a determined temperature before slag is introduced. This temperature is then maintained for a period of 24 hours during which slag is fed at a constant rate through the burner. Temperature readings are taken by means of an optical pyrometer sighted on the middle of the face of the test brick.

Gas samples are taken by means of a water-cooled sampling tube inserted through the top opening of the furnace. Atmospheric conditions can be varied over a considerable range, but can be maintained fairly constant with either oxidizing or reducing conditions. It is customary, in these tests, to maintain slightly reducing conditions to correspond with ordinary gaseous atmospheres in contact with the walls of boiler furnaces.

The temperature gradient through the test brick can be determined by inserting thermocouples at different depths when building up the lining, passing the leads out through the joint in the jacket. The rotation of the furnace is stopped while readings are being made. The curves in Fig. 3 show such readings taken during one of the tests. Variation of the temperature gradient

TABLE 1 COMPARISON OF ANALYSES OF SLAGS PREPARED FOR LABORATORY TESTS WITH THOSE OF COAL ASH OBTAINED AT TIME OF FIELD TESTS

Sample from	Laboratory	Dry slag[1] Combustible, per cent	Dry slag[1] Ash, per cent	Composition of ash[1] SiO_2	Al_2O_3	Fe_2O_3	CaO	MgO	SO_3	Alkalies	Softening-point determinations, deg. fahr. S.P.	S.I.	F.I.	P.C.E.[2] value	
Commonwealth Edison Co., Chicago, Ill.	Lab.	15.5	84.5	57.5	23.6	8.6	5.1	1.0	1.9	2.3	2310	190	210	8–9	4
	Field	..	..	45.0	19.3	18.9	6.6	0.5	6.3	3.4	...	...	...	...	...
Ill. Elec. Power Co., East Peoria, Ill.	Lab.	29.5	70.5	40.4	13.5	12.4	22.4	1.0	8.8	1.5	2090	40	70	1–2	01
	Field	..	..	36.4	11.2	16.5	17.6	0.8	15.3	2.2	...	...	...	...	...
Penna.-Ohio P. & L. Co., Lowellville, Ohio.	Lab.	37.0	63.0	57.7	29.8	7.3	1.9	0.7	0.6	2.0	2690	420	170	15–16	8–9
	Field	..	..	55.9	29.7	8.3	2.0	0.6	1.2	2.3	...	...	...	...	...
Cleveland Elec. Illum. Co., Cleveland, Ohio.	Lab.	38.4	61.6	45.3	22.2	25.0	3.7	0.9	1.6	1.3	2260	210	80	9–10	...
	Field	..	..	42.3	20.7	29.2	2.8	0.7	1.5	2.8	...	...	...	...	...
Union Elec. Light & Power Co., Cahokia, Ill.	Lab.	22.4	77.6	57.0	26.2	10.3	2.2	1.0	0.6	2.7	2350	210	320	10–11	6
	Field	..	..	54.1	25.5	9.7	3.6	1.1	3.0	3.0	..	...	...	...	...

[1] Determinations by Bureau of Mines.
[2] Determinations by Ceramic Engineering Department, University of Illinois.

can be effected by changing the character or amount of insulation backing the test brick.

The interior of the furnace at the end of a test is shown in Fig. 4. Three bricks each of five brands were used in this test.

Measurement of Test Brick

The volume of each test brick is measured before and after testing by means of a mercury balance.[3] Depressions or irregularities along the edges of the slagged bricks, due to chipping while removing them from the furnace, are filled with plastic clay before making the final volume measurement. The difference between the initial volume and the residual volume after testing is taken as the loss due to slag erosion. Some error is involved in this due to shrinkage of the brick during the test. This increases the apparent erosion to a slight extent, but it is offset by the volume of the slag layer on the face of the brick, for which no correction is made. The slag layer, however, is in no case of very great thickness in these tests. The furnace temperature is usually maintained for 15 min. after the end of the slagging period to allow excess slag to run off and so minimize the error in measurement of erosion loss.

To eliminate errors in comparison of erosion loss due to variations in size of bricks the results are given in terms of cubic centimeters loss per square centimeter of exposed face. Comparative ratings of different brands of brick may be made by the ratio of their unit losses to that of some brand adopted as a standard for such comparison.

TEST WITH COAL–ASH SLAGS

Slag has been prepared from the ash of coals used at several plants at which studies of the furnace operation were made by the U. S. Bureau of Mines. Samples of the coals were burned on a hand-fired grate at the laboratory, and the cinder collected and ground to pass 20-mesh. The slags so prepared were used for tests in which refractories were included which were in use at the respective plants during the field tests.

It has been found desirable in some of these tests to use a mixture of equal parts of the coal ash and of the pulverized coal from which it was prepared. By use of this mixture, a higher concentration of reducing gases could be maintained in the furnace during some of the tests. Comparison of the effect of different atmospheric conditions on the slagging action could thus be made over a greater range than was possible with coal ash alone. To make these tests comparable with those in which only the slag was fed through the burner, the amount of coal and slag mixture was made sufficient to supply the same weight of ash as in the cases of slag alone.

A comparison of the analyses of the slag prepared for the laboratory tests with those of the coal ash obtained at the time of the field tests is shown in Table 1. All the analyses were made at the U. S. Bureau of Mines Laboratories. Softening-point determinations of the laboratory slags are also shown under both oxidizing and reducing conditions. In the latter determination the atmosphere was maintained at practically the same conditions as in most of the slagging tests.

There is a close similarity in the analyses of laboratory and field samples of ash from the Lowellville, Cleveland, and Cahokia coals. In the case of the East Peoria material the laboratory ash shows higher contents of SiO_2 and CaO with less Fe_2O_3 than the field sample. The variation is probably not greater than occurs regularly in sampling of this coal, and is not considered to be of any special significance in test results.

The two samples from the Chicago station show very decided



Fig. 4 Interior of Furnace After Test

differences. At the time the coal was sent for laboratory tests there was some question as to whether or not it corresponded to the coal used during the field tests. It was evident that the laboratory sample did not represent the usual slag in these furnaces since it was much less fusible and its action on refractories was much less severe than was to be expected. A later sample of coal from the same station was of an entirely different character and conformed to expectations in its action in the slagging tests.

In a majority of the laboratory tests a temperature of 1600 deg. cent (2912 deg. fahr.) has been arbitrarily chosen for the slagging period. This relatively high temperature was adopted to produce a considerable amount of erosion of refractories in a period of 24 hours. It is not desirable to extend the time for a laboratory test much beyond this period. Since the temperature selected exceeds that in most of the boiler furnaces, the question may be raised as to whether or not laboratory results would fairly represent those of service. The slagging action will be definitely more severe at the higher temperature, but results indicate that, for comparing the slag resistance of different refractories, the higher

[3] A. E. R. Westman, *J. Am. Cer. Soc.*, vol. 9 (1926), p. 311.

temperatures give more distinct differences in erosion loss but in much the same relative proportions as the lower temperatures.

Relative Effects of Different Slags

The results of tests with slags from the different coals, at approximately the same temperature and atmospheric conditions, are shown in Table 2. Test No. 23 was made with ash from Chicago coal which did not correspond with that used in the field tests. The ash used in test No. 30 was from a second lot of coal which came from the same mine as that used during the plant tests. The difference is quite evident in the relative effect on the refractories.

TABLE 2 RESULTS OF TESTS WITH SLAGS FROM THE DIFFERENT COALS, AT APPROXIMATELY THE SAME TEMPERATURE AND ATMOSPHERIC CONDITIONS

	Chicago No. 1	Chicago No. 2	East Peoria	Lowellville	Cleveland	Cahokia
Test No.	23	30	22	24	19	25
Avg. temp., deg. fahr.	2906	2913	2918	2930	2925	2893
Avg. per cent CO	2.5	2.7	2.4	1.8	2.8	2.3
Time, hr	24	24	24	24	24	24
Lb. slag	77	75	69	50	83½	64
Brick brand	Erosion loss in cu. cm. per sq. cm.					
15	0	1.83	1.81	0	0.84	0
17	0.04	2.10	2.31	0	0.99	0.14
19	..	2.22	2.05	0	1.41	0.23
16	0.18	2.33	2.26	0.13	1.38	0.17
13	..	2.31	1.62	0	1.92	0.07
18	0.10	2.37	1.79	..	1.42	0.02
21	0.26	2.29	1.70	0	1.59	0.17

The erosion losses of brick of the different brands is low for the Chicago No. 1, Pennsylvania-Ohio, and Cahokia slags at 1600 deg. cent., intermediate in value for the Cleveland, and high with East Peoria and Chicago No. 2 slags. Both of the last named show high CaO content as compared with the others. Both of these slags were much more fluid than the other samples at 1600 deg. cent. They attacked the brick much more vigorously and tended to cause channeling in the eroded faces to a much greater extent than the others.

The Cleveland slag was somewhat less fluid than the East Peoria and Chicago No. 2 slags, but much more so than that from the other coals. While the Fe_2O_3 content is much the highest of the group, the CaO content is intermediate between the high-lime and low-lime slags. The erosion losses due to the Cleveland slag are somewhat lower than those of the East Peoria and Chicago No. 2 tests, but much in excess of the other three.

The three slags showing the low erosion effects vary considerably in the contents of Fe_2O_3 and CaO, but are similar in the content of SiO_2, which is well above that of the slags which showed greater erosive effects. The Al_2O_3 content of these three slags is also somewhat above that of the more fluid slags. The high SiO_2 content would greatly increase the viscosity of the slags, and even small increases in Al_2O_3 should aid in this effect. In general, the erosive effects of the slags may be said to correlate with the chemical composition and with the fluidity at the furnace temperatures.

It was noted that the results of the laboratory test, as regards the relative erosive effects of slags from East Peoria and Cahokia coals, did not correspond to the observed conditions during the field tests. At the East Peoria station little trouble was encountered due to slagging of the furnace walls, while at Cahokia the slagging effect of the ash was severe. The ash analyses show that the fused ash from Cahokia coal should be much more viscous than that from East Peoria, and there is also a difference of 260 deg. fahr. in the softening temperatures. Hence, the foregoing results probably reflect very well the relative activity of the two slags under like operating conditions.

In normal furnace operation at the two plants, it is shown by Sherman and Taylor[4] that the gas temperature near the wall face at the point of maximum erosion was 2750–2780 deg. fahr. in the Cahokia furnace and 2050–2420 deg. fahr. in the East Peoria furnace, an average difference of around 500 deg. fahr. The atmospheric conditions in the East Peoria furnace were only very slightly reducing, with less than 1 per cent CO, while the gases in the Cahokia furnace showed approximately 4 per cent each of H_2 and CO. These differences in operating conditions are sufficient to account for the apparent discrepancy in the behavior of the two slags.

The appearance of bricks from the tests listed in Table 2 with the six different slags is shown in Figs. 6 and 7. The marked difference in effect of the two slags from Chicago coal is quite evident. The high viscosity of the ash from the Lowellville coal has produced an appreciably thicker slag coating than that of the more fluid slags.

Effect of Temperature on Slag Action

The results of tests made with three coal-ash slags at different slagging temperatures are shown in Table 3. In each case the lowest temperature used approximates that reported as the average in normal operation at the respective plants. A test with the East Peoria slag at 2400 deg. fahr. was not completed due to a furnace breakdown after 7½ hours' operation.

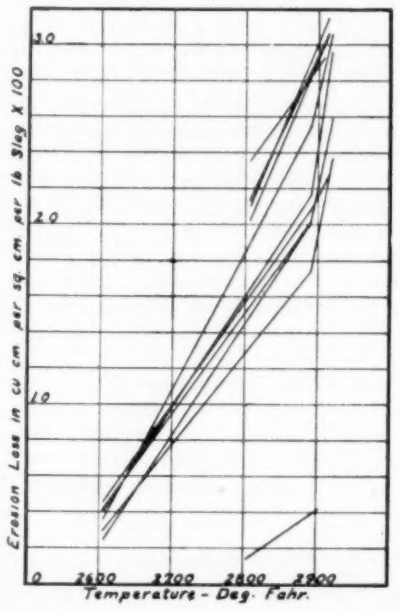


Fig. 5 Effect of Temperature on Erosion by Slags

To better simulate service conditions, the tests with Cahokia slag were made with more strongly reducing conditions. This necessitated the use of a mixture of equal parts of powdered coal and coal ash. The rate of feeding was increased so as to introduce approximately the same amount of actual slag as in previous tests with powdered slag alone.

The amount of erosion loss in the case of each of the slags increases rapidly as the furnace operating temperature is raised. A study of the relationship between the viscosity of coal-ash slags and temperature should be of interest in this connection. It is apparent that the viscosity is a primary factor in the relative erosion of the refractories at different temperatures. A rough

TABLE 3 EROSION LOSSES OF VARIOUS BRANDS OF FIREBRICK

Slag used	East Peoria			Chicago No. 2		Cahokia 50 coal	Cahokia 50 slag
Test No.	27	21	22	28	30	34	36
Avg. temp. deg. fahr.	2610	2890	2920	2800	2910	2800	2900
Avg. per cent CO	2.1	3.5	2.4	2.2	2.7	7.4	6.2
Brick brand	Erosion loss, cu. cm. per sq. cm.						
15	0.28	1.49	1.81	1.65	1.83	0	0.38
17	0.41	1.58	2.31	1.69	2.10	0.05	0.55
19	0.46	1.56	2.05	2.01	2.22	0.19	0.71
9	0.40	1.86	2.26	1.82	2.33	0.15	0.72
13	0.43	1.28	1.62	1.65	2.31	0.11	0.72
18	0.49	1.49	1.79	1.79	2.37	0.16	0.78
21	0.50	1.22	1.70	1.65	2.29	0.18	0.75
30	0.52	1.96	2.35	..	..	..	..
22	..	..	..	1.92	2.44	..	..
27	..	..	..	..	..	0.14	0.63

[4] R. A. Sherman and E. Taylor, "Refractories Service Conditions in Furnaces Burning Powdered Illinois Coal With Long-Flame Burners." Trans. A.S.M.E., vol. 50 (1928), pp. 125–140.

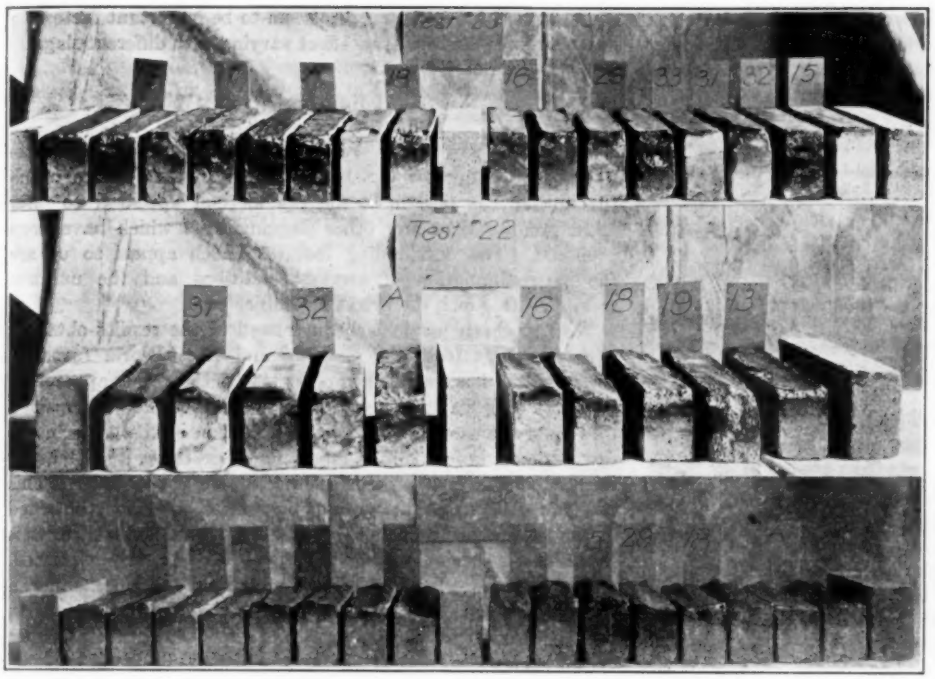


FIG. 6 COMPARATIVE EFFECT OF SLAGS

comparison of the effects on several brands of firebrick is shown by the curves in Fig. 5 for the three slags. The erosion losses of the bricks given in Table 3 have been calculated as cubic centimeters per square centimeter of exposed face per pound of slag fed to the furnace. Tests at one or two additional temperatures will show whether the relation is linear or parabolic in character.

It is apparent from these data that for each slag there is a definite temperature below which the slag will build up on the refractory without producing an appreciable amount of erosion. This temperature will depend directly on the composition of the slag and its viscosity-temperature relationship. A study of the viscosities of coal-ash slags at different temperatures should be of considerable interest in this connection.

EFFECT OF FURNACE-GAS COMPOSITION

The comparative effects of slag action with different degrees of reducing conditions in the furnace gases have not yet been determined. It has been shown[5] that in softening-point determinations on coal ash the furnace atmospheric conditions have a very appreciable effect due to reduction of Fe_2O_3 to FeO. At the higher temperatures encountered in boiler furnaces and in these tests the effect may not be so marked, since a part of the iron of the slag is in the ferrous form due to the equilibrium conditions between FeO, Fe_2O_3, and O_2 at these temperatures. Further tests are to be made with differing atmospheric conditions.

[5] A. C. Fieldner and A. E. Hall, "The Fusibility of Coal Ash and the Determination of the Softening Temperature." Bull. 129, U. S. Bureau of Mines (1918).

COMPARISON OF TEST RESULTS WITH SERVICE RESULTS

Microscopic examinations have been made of thin sections from bricks subjected to the laboratory tests along with samples of the same brands taken from furnaces during the field tests. These have been compared to determine the similarity or dissimilarity in the crystalline phases appearing in the slag coatings and at the slag contact on the bricks. Similarity in this respect would indicate that the conditions in the laboratory test were sufficiently like those in the boiler furnace to make the results of the laboratory tests comparable with service conditions.

In a progress report Rexford[6] states that "The field samples all contain much mullite and small quantities of minor minerals, particularly magnetite. The test samples also contain mullite, but rarely show any minor minerals except a little magnetite at

[6] E. P. Rexford, "Petrographic Comparison of Slagged Samples From the Test Furnace With Field Samples." Progress report, Oct. 27, 1928, U. S. Bureau of Standards, Columbus, Ohio.

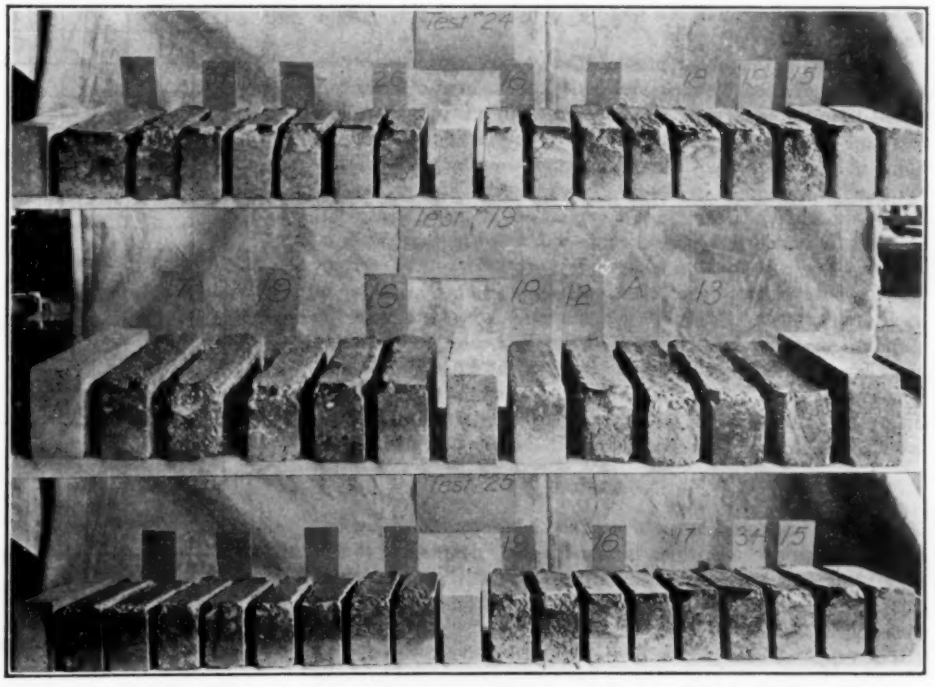


FIG. 7 COMPARATIVE EFFECT OF SLAGS

times. So, in general we may say that, for all practical purposes, the test furnace is duplicating the field service so far as the slag formation is concerned unless the minor minerals have considerably more importance than is apparent at present."

TABLE 4 RELATIVE RATING OF REFRACTORIES BY COMPARISON WITH BRAND 18

Slag used	—Chicago No. 2—			—East Peoria—			—Cleveland—		
Test No.	28	30	Avg.	18	19	Avg.	21	22	Avg.
Brand				Dry Pressed Brick					
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
19	1.12	0.93	1.02	1.05	1.14	1.10	0.90	0.99	0.94
9	1.02	0.98	1.00	1.25	1.26	1.25	0.96	0.96	0.96
				Stiff Mud Brick					
21	0.93	0.97	0.95	1.01	0.95	0.98	1.19	1.12	1.15
13	0.93	0.98	0.95	0.85	0.90	0.87	1.02	1.26	1.13
				High-Alumina Brick					
15	0.93	0.77	0.85	1.00	1.01	1.00	0.51	0.59	0.55
17	0.95	0.89	0.92	1.06	1.29	1.17	0.60	0.70	0.65

Comparison of the East Peoria field samples with those of the laboratory tests showed a discrepancy which was undoubtedly due to higher temperature in these tests than existed in the boiler furnace. Examination of samples from the incomplete test at 2400 deg. fahr., with this slag showed crystalline development very similar to that in the field samples. This temperature approximates the average gas temperature near the side wall in the field tests. Hence it appears that the same crystal phases appear in the cooled laboratory-test slags as in those from the boiler furnace when the operating temperatures are the same.

Comparison of Refractories

Comparison of the resistance of different refractories may be made by reference to some one brand as a standard. For present purposes, bricks of brand 18 were selected as a basis of comparison since these brick appeared to be quite uniform in quality and in their resistance to slagging action. The data in Table 4 show the average losses of two bricks each of several brands in tests with three different coal-ash slags. These tests were selected because the amount of erosion produced during the period of the test was sufficient to give comparative losses. In tests with more refractory slags the erosion was so slight that a fair comparison of the resistance of different bricks was not obtainable in the 24-hour slagging period.

Each figure in the table represents the average loss of two bricks of a brand divided by the average loss in volume of bricks of brand 18 in the same test. Ratings less than unity indicate less loss and better resistance than that of the comparison brand. Some of the refractories show considerable variation in the resistances of individual bricks to a given slag. This may be due to non-uniformity in structure, composition, or firing treatment.

There is evidently a considerable difference in the effects of different coal-ash slags on a given type or brand of refractory, which would indicate that chemical composition is a factor of some importance. Bricks of higher alumina content than the standard flint-clay refractories show much greater resistance to the Chicago and Cleveland slags than to the East Peoria slag. The structural character of stiff-mud bricks seems to be a factor of greater importance in the case of East Peoria coal ash, which is notably high in lime content, than in that of the Cleveland slag.

CONCLUSION

The data presented indicate that the laboratory slagging test reproduces service conditions to a satisfactory degree and that it may be used as a means of comparing the relative resistances of different types of refractory to the slagging action of coal ash in boiler furnaces.

The range of composition and structural character of the refractories included in these tests is not sufficient to give a basis for general conclusions as to the relative merits of the different types. Both composition and structure seem to be important factors in slag resistance, their relative effect varying with different slags.

Discussion

Ralph A. Sherman.[7] We of the Bureau of Mines have followed with much interest the tests with the furnace which the author has described, for the method appeared to offer more promise than any other slagging tests which have been proposed. The outstanding features which appeal to us are the close simulation of service conditions and the uniform treatment to which the bricks are subjected.

We have been particularly interested in the results obtained with the slags from the Cahokia and East Peoria furnaces. As the author has stated, the first runs with these slags apparently showed a discrepancy between the operating and the test results, but when the temperature and composition of the gases in the test furnace were made comparable with those in the boiler furnace, the degree of erosion was in the right order. This illustrates the value of the data on service conditions obtained in the field phase of this investigation, for without them it is doubtful whether this could have been satisfactorily explained.

Another point which is forcibly brought out is the manner in which a few changes in furnace design at the East Peoria station brought about satisfactory costs of brickwork maintenance even with an extremely erosive ash. The results of the author's

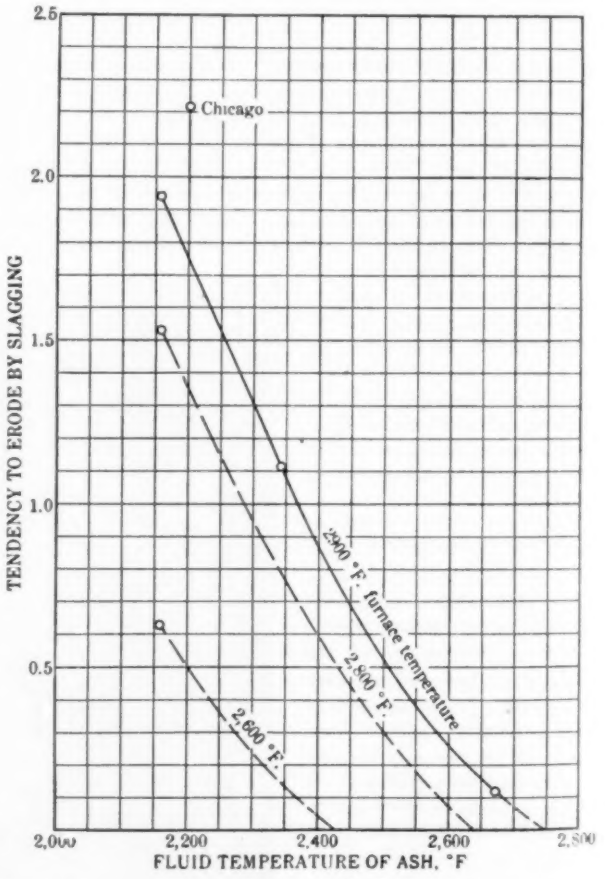


Fig. 8 Tendency of the Ash of Coal to Erode Plotted Against the Fluid Temperature of the Ash

[7] Fuel Engineer, Pittsburgh Experiment Station, U. S. Bureau of Mines, Pittsburgh, Pa. Assoc-Mem. A.S.M.E.

tests indicate that at similar temperatures the erosion by the East Peoria slag would be two to three times as great as by the Cahokia slag, but the actual erosion at East Peoria was made less instead of greater than that at Cahokia by slight changes in the design of the burner, by changes in the design of the ports for the admission of secondary air, and by the introduction of air at the side wall through ports in the burner arch, all of which did not lower the combustion or boiler efficiency.

Another point of interest, particularly with the Cahokia slag, is the great difference in the erosion caused by a change in temperature of but 100 deg., and illustrates the care which an operator must take to remain within safe operating limits.

P. NICHOLLS.[5] With our present limited knowledge of the relation between viscosity, temperature, and the chemical analysis of coal ash it is doubtful whether one could safely predict the probable slagging effect of a given coal from its ash analysis alone. Although doubt is sometimes cast also on the reliability of the values given by the standard gas-furnace fusion test, yet it is the most used method, and its values for most coals are available; it is also to some extent a direct measure of viscosity. It would therefore seem logical, for the time being at least, to attempt to relate the tendency of a coal to slag to its ash-fusion temperatures.

This was done from the data given in the paper. It was assumed that the data given in Table 1 for the laboratory sample of the Chicago ash were for what is called Chicago No. 1 and that those for Chicago No. 2 are not included, but if the ash were similar to that collected in the field the values would be: Softening temperature, 2000 deg. fahr.; softening interval, 60 deg. fahr.; fluid interval, 200 deg. fahr. Plots showed that the fluid temperature gives a more consistent curve for the quantity of slagging than does the softening temperature; this would be expected because the softening temperature represents a comparatively high viscosity. Each type of brick would have its own slagging-fluid temperature curve, but the paper shows that the variations are not extreme and the curves of slagging against face temperature tend to be parallel. One can therefore speak of the tendency of a coal ash to slag independent of a particular refractory.

In Fig. 8 the tendency of an ash to erode by slagging is plotted against the fluid temperature of the ash. Only values from the author's tests having the same reducing atmosphere of about 2.5 per cent CO were used. There are several points to fix the 2900 deg. fahr. temperature of the surface; the point for the Chicago coal on the assumed fluid temperature of 2200 deg. fahr. is shown, but it is not included in the curve. There is only one point each for the 2800 and 2600 deg. fahr. wall-face temperatures, and the dotted curves indicate the form one might expect these curves to take.

It is not safe to generalize much from the limited data now available, but the establishing of such a relationship would be useful in judging the relative availability of coals; variations from such curves would be expected, but the knowledge of prevailing furnace conditions in a particular furnace will rarely be exact.

The new tool developed at the University of Illinois should prove very useful not only in the study of refractories but also to coordinate data from laboratory studies of coal ashes and slags with this larger scale application.

AUTHOR'S CLOSURE

Dr. Nicholls has pointed out an apparent relationship between the fluid temperature of a coal ash and its comparative slagging effect on refractories that merits further study. A general relationship would be expected since the fluidity of the slag is of primary importance in its action on firebrick. When data for individual brands of firebrick are plotted, as in Fig. 8, the curves will be found to flatten out at the higher temperatures. There is also some question whether the points for the East Peoria slag fall in line satisfactorily with those for the other coal ashes. This ash was exceptionally high in CaO, and its comparative action on different brands of brick was not in the same order as that of other slags. Some classification of ashes with respect to their chemical compositions might be necessary in applying the correlation suggested.

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FSP-51-45

A Study of Crystalline Compounds Formed in Slags on Boiler-Furnace Refractories[1]

By T. A. KLINEFELTER[2] and E. P. REXFORD,[2] COLUMBUS, OHIO

For several years the action of boiler-furnace refractories under various service conditions has been investigated. The present method of investigation used by the Bureau of Standards at Columbus is similar to that used by the Geophysical Laboratory in studying the lime-alumina-silica system, with the addition of iron. It is known as the quenching method. The apparatus consists of two furnaces, a preheater and a quenching furnace automatically controlled. The length of time required for each melt depends on its composition. Each melt determines an equilibrium point, a number of which are necessary for determining the point sought, i.e., the temperature at which the primary phase for that composition disappears and the charge becomes a clear glass. A number of the latter points determine a surface known as an "isopleth." In working out the system one component is held constant, in this case iron, and the others are varied. In conjunction with the above, field slags, the data of which are known, are being quenched and their results compared with those of the component system.

The paper is a progress report of the work done to date, of the methods used and the difficulties encountered in a laboratory study of the equilibrium relation of oxides composing the slags on boiler-furnace refractories at high temperatures. No data or conclusions are presented.

FOR several years the action of slags on boiler-furnace refractories under different service conditions has been investigated cooperatively by the A.S.M.E. Special Research Committee on Boiler-Furnace Refractories, the University of Illinois, the Bureau of Mines, and the Bureau of Standards. A previous report[3] on this problem pointed out the necessity of a fundamental study of the system involving the principal components found in the slags formed on boiler settings.

When the general investigation started, chemical and petrographic analyses were made of slags sent in from various boiler settings, but these failed to give the desired information. Some further work was done by making mixtures similar in composition to the minerals found in the slags, and determining the fusion points of small cones prepared from them. Since the methods tried failed to give information on the course and rate of reactions between coal ash and clay refractories at high temperatures, a method believed to be satisfactory for the purpose was adopted. This method is similar to that used by Rankin and Wright[4] at the Geophysical Laboratory for the study of the lime-alumina-silica system. It is known as the quenching method, because samples having once reached equilibrium under a given set of conditions are chilled or frozen before a change of conditions due to cooling can affect the phases present in the sample. Phases which are crystalline at the temperature of equilibrium are crystalline at room temperatures, while those which are liquid at equilibrium temperature are glass at room temperatures. It was felt that this procedure would be all the more appropriate since slags in general contain lime, alumina, and silica as major constituents, and in addition significant amounts of iron oxide. Minerals containing other oxides such as magnesia and soda have been found in some of the slags, but usually in small quantities. Because the complexity of a system involving six or more oxides would increase the amount of work greatly, it was decided to limit the study to the lime, alumina, silica, and iron oxide. The system becomes five-component if both the ferrous and ferric forms of iron are present in these slags, as they probably are.

The apparatus is essentially that described in the article mentioned. It consists of two furnaces, one for preheating and the other for quenching the melt after equilibrium is attained, the latter being automatically controlled by a Roberts[5] temperature regulator.

The charge is sintered in the preheater, ground and resintered several times to insure complete uniformity, and finally introduced into the quenching furnace. The temperature in this furnace is carried well above the point at which equilibrium is to be established so as to insure complete fusion within a reasonable time. The furnace is then cooled slowly until the temperature is reached at which the quenching is to be made, and it is held at this temperature sufficiently long to make certain of complete equilibrium in the charge.

The length of time to attain equilibrium depends on the composition of the charge. Some of the high-silica charges require 6 to 8 hours, while others lower in SiO_2 require only 15 to 20 minutes.

At the proper time the charge is dropped from the furnace into a mercury bath, freezing or quenching it suddenly from the temperature to which it has been subjected. A microscopic examination of the chilled materials reveals the phases present before the charge was quenched.

Each of these melts determines an equilibrium point, since the phases are in equilibrium for that temperature and composition. We are particularly interested in the temperature at which the first crystalline phase (called the "primary phase") appears on cooling a charge of a given composition. This point cannot be predicted precisely, but may be obtained by using the method of trial and error. Three to fifteen equilibrium determinations are often necessary. Charges of the same composition are first heated to such high temperatures that they are completely liquid, then to successively lower temperatures, until a point is reached at which the primary crystalline phase first appears. This procedure is checked by heating charges, first at a rather low temperature, then at successively higher temperatures, until the last trace of the primary crystalline phase disappears. The final result gives a desired point on the solubility surface. By determining a sufficient number of these points and passing a warped plane through them, we obtain the solubility surface for this particular part of the system. Points above this surface represent solutions that are unsaturated with respect to the primary phase, while points below this surface represent saturated solutions.

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[1] Publication approved by the Director of the Bureau of Standards, U. S. Department of Commerce.

[2] Bureau of Standards, Columbus Branch.

[3] S. J. McDowell, "A Laboratory Study of Slag Erosion," *Mechanical Engineering*, vol. 49, no. 5, 1927.

[4] *American Journal of Science*, 39, pp. 1–79 (1915).

[5] H. S. Roberts, *Jl. Opt. Soc. Am.*, vol. 11, no. 2, August, 1925.

Contributed by the A.S.M.E. Special Research Committee on Boiler-Furnace Refractories and presented at the Rochester Meeting, Rochester, N. Y., May 13 to 16, 1929, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

In working out a four-component system one component is held constant and the other three are varied. This gives us what might be termed a level ("isopleth," in physical-chemical terminology) with respect to the component held constant. After covering the triaxial field at intervals in this way, the component held constant is increased or decreased and again made a constant, and the other three are again varied so as to cover the field once more. We now have two levels with rather different contours.

In the system involving the slags in question, the first study was made with the iron oxide constant at 10 per cent by weight, and the anorthite, silica, and mullite varied to cover the field included by these three components. The work with 10 per cent iron oxide as the constant is nearing completion. Since the slags frequently contain higher percentages of iron oxide than 10 per cent, it will be necessary to work out several more levels before correct conclusions can be drawn.

Since adopting the quenching method, some of the earlier determinations made on the field slags have been checked. It has been found unsatisfactory to base conclusions on petrographic examinations of slowly cooled samples since the phases present in such samples are not necessarily those present at the operating temperatures. It is therefore more desirable to reheat the sample to the operating temperature at which it was formed, quench it, and then make the determination.

Another consideration of the greatest importance in this study is whether the iron oxide under operating conditions is present in the ferrous or ferric state. It is known in general that a highly reducing atmosphere greatly decreases the life of clay refractories. Reducing conditions cause the ferric iron present in the refractories to change to the ferrous state. We have some evidence that a slag high in ferrous iron develops very little mullite. Other evidence indicates that a slag containing a large quantity of mullite is more viscous. Because of its high viscosity the rate of flow and of corrosive action of this slag is relatively low.

To investigate this matter thoroughly it would be necessary to make all the equilibrium determinations first under highly oxidizing conditions and then to duplicate all of them under highly reducing conditions. The melts at present are made under oxidizing conditions, but it is almost impossible at times to prevent a certain amount of reduction due to the high temperatures. A determination of the ferric-ferrous iron ratio of the charges will undoubtedly give a number of cases with a high ferrous ratio and therefore some indication of the phases present in parts of the field under reducing conditions.

All the slags collected for test, except one, are normal boiler-furnace slags and may be considered as lying within the limits of the system under investigation. This one exception contains significant amounts of constituents other than lime, alumina, silica, and iron oxide. The operating temperature of the furnace in which this slag was produced was relatively low, a condition that may account for the fact that, although the slag contains no mullite, it does not corrode the refractory appreciably.

While the field under investigation does not cover this exceptional case, the work done so far does show definitely that the case is clearly outside the field, and also points out fairly clearly the direction in which the field of that slag lies.

The study of the system is fairly well started and is reaching a stage where, with the data from the field samples, explanations of some points are apparently indicated.

A brief description of the system being investigated and of the procedure involved has been given. It can readily be seen that the work is time-consuming, but the few results obtained justify the belief that further work will provide data of value in predicting the behavior of coal-ash slags of known composition when in contact with clay refractories at temperatures encountered during boiler-furnace operation. When this study of equilibrium conditions at high temperatures in this system is completed, it is hoped that it will be necessary only to know the approximate composition of the slags and the operating conditions, especially the temperatures and the character of the furnace atmosphere, to determine whether clay refractories will have a satisfactory life. On the other hand, wasteful service tests will not be necessary to ascertain if coal from a specified district may be used under the conditions obtaining in a furnace.

FSP-51-46

Heat-Insulation Practice in the Modern Steam-Generating Plant

By L. B. McMILLAN,[1] NEW YORK, N. Y.

This paper outlines briefly the engineering principles of heat-insulation design. Especial emphasis is given to the fact that the heat dissipated by so-called "radiation" losses is from the most valuable portion of the heat, and that these losses frequently result in the lowering of heat potential or temperature head, with a resulting loss in the value of all of the heat as measured by its remaining effectiveness.

A distinction is drawn between the cost of average heat and the value of a particular portion of the heat in terms of what it will accomplish in the way of useful results. The higher value of high-potential heat is demonstrated, and a specific evaluation in the case of heat in superheated steam used for power generation shows that the high-potential increments of such heat may be 1-1/2 or even 2 times as valuable as the average heat content of the steam.

Results accomplished by the insulation of various units of power-plant equipment are discussed and typical installations are shown.

The factors involved in determining the most economical thickness of insulation are discussed and a chart is given on which each of the factors may be taken separately into account. The resulting economical thickness may quickly and conveniently be determined directly from the chart.

THE insistent demand for higher economies has resulted in the devoting of the greatest care to the design and operation of steam power plants in order that, for the conditions of the particular installation, the maximum overall economy will be secured. A marked feature of this development has been the advent of steam temperatures and pressures, which only ten years ago would have been considered highly speculative. But these and many other revolutionary improvements in power-plant design have proved highly successful, and along with them heat insulation, always an important item, has become a still more vital factor in power-plant economy. And insulation is no longer treated as merely an incidental, but is made the subject of careful engineering design, just as are the other items in the more complex power station of today. The object of this paper, therefore, is to point out some of the more important features of heat-insulation design, and to present them in a manner conveniently useful to power engineers.

An outstanding feature of recent progress in the heat-insulation field has been the increasing realization of the fact that, large as they are, direct fuel economies are by no means the sum total of the beneficial effects secured by the minimizing of heat losses from heat-using equipment. In industry in general, greater attention is being given to the effect of insulation in increasing production through the maintenance of heat potential, in improving quality of product through the maintenance of greater uniformity of temperature, and in increasing the productivity of labor through the maintenance of improved working conditions in the vicinity of heated equipment.

Nor are these indirect effects lacking in the steam power plant. Results there may not be classifiable under the aforementioned three headings, but the insulation of pulverized-coal bunkers and piping to prevent caking of the coal due to condensation, and the insulation of turbine-room roofs, cold-air ducts and condenser-water piping to prevent the troublesome drip due to condensation of moisture from the air are typical examples of cases where there are other considerations which completely overshadow that of heat saving.

The Higher Value of High-Potential Heat

The power station furnishes also one of the most convincing examples of the value of the maintenance of heat potential. The heat conserved by insulation is the most valuable portion of the heat. It is the cream, so to speak; the high-potential heat that costs most to produce and that does the most effective work.

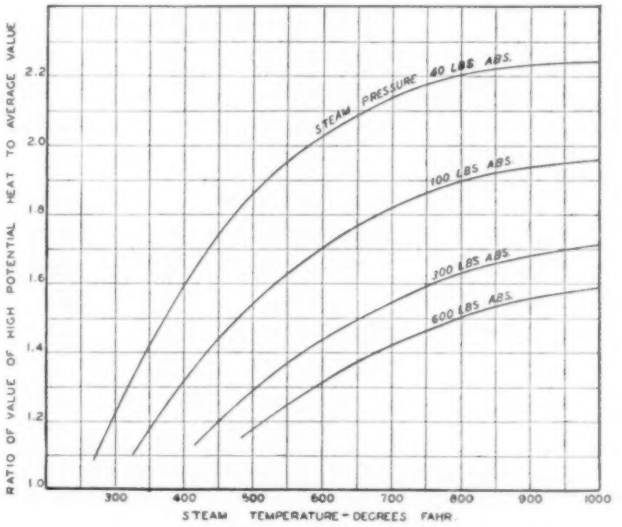


FIG. 1 RATIO OF VALUE OF HEAT AT STATED TEMPERATURE TO AVERAGE VALUE

(Based on expansion to condenser pressure of 1 lb. abs.)

Power engineers do not need to be told that a given number of B.t.u. at high potential will produce more power than the same number at lower potential; or that maintaining the required superheat will make all of the heat units more effective in the turbine than if dissipation of temperature head is permitted to degrade all of the heat; or that there is plenty of heat in the condensing water but useless because it lacks the required potential to make it capable of accomplishing useful work. But the attempt to evaluate the heat in terms of its potential is worth while, and seems not to have received the attention it deserves.

Fig. 1 represents the results of such an evaluation. It is at once apparent that in some cases the value of the last increment of heat at the top of a particular temperature range is between 50 per cent and 100 per cent greater than the average value for that range (the temperature range is that of the complete cycle from condenser temperature up to the given temperature). This is reasonable,

[1] Deceased. Formerly Consulting Engineer, Johns-Manville Corporation. Mem. A.S.M.E.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

because no extensive analysis is needed to show that the effectiveness of the last increment of heat in doing useful work must be much greater than the average when that average is penalized by the low effectiveness of the first increments of heat added in the cycle to water at condenser temperature. The availability of these first increments of heat in the cycle to do useful work is but

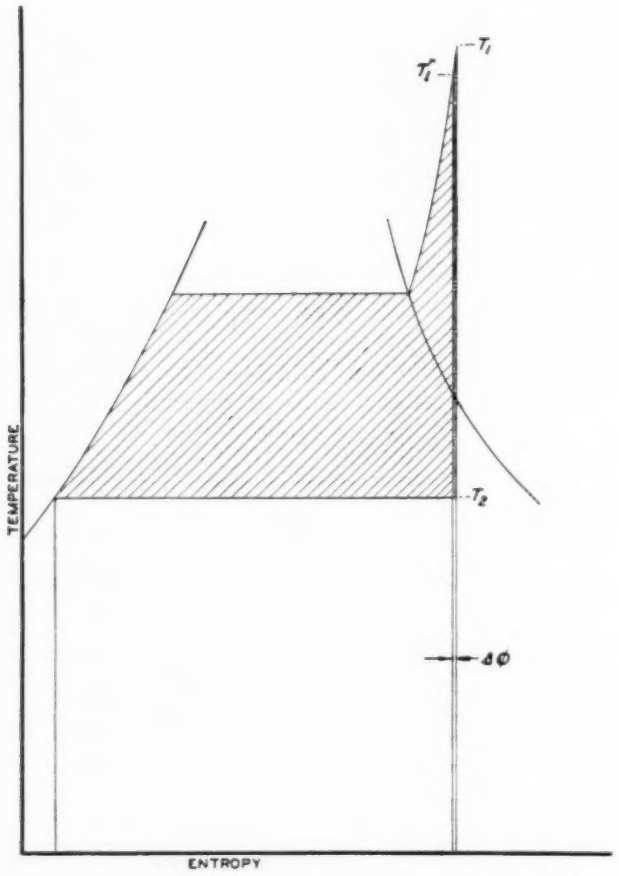


FIG. 2 TEMPERATURE-ENTROPY DIAGRAM OF STEAM-POWER-PLANT CYCLE

little above zero. Hence the average effectiveness must be lower than that of the last increment added at high potential. All of this is clearly obvious in the temperature-entropy diagram, Fig. 2, in which the area $[(T_1 + T_1')/2]\,\Delta\phi$ is the increment of heat which may or may not be lost by radiation and convection losses according to the effectiveness of the insulation. It will be noted also that the convertability of this increment of heat into useful work approaches that of the ideal cycle where efficiency is $(T_1 - T_2)/T_1$.

Furthermore, it may not be said that the value ratios in Fig. 1 are only theoretical. They are very conservative and intensely practical. No claim is made that the high-potential increment of heat is converted into work at ideal efficiency, but surely the inclusion of this increment *does not increase* nozzle losses, bucket losses, and windage. These items take a proportionately greater toll from the average than from the high-potential heat; therefore, for actual operating conditions the *ratios* in Fig. 1 are most conservative.

From Fig. 1 it will be noted that the relative value of the last increment as compared with the average value of heat decreases as the pressure is increased. This does not mean that the actual value of that increment is any less. It is the same, but since the average value of all of the heat is increased at the higher pressures, the ratio of value of the high-potential increment to the average is, of course, decreased.

It might be said that there is nothing new about this, and that the value of adding superheat to steam and of preventing excessive temperature drop has long been recognized. This is true, but temperature drop as such does not tell the whole story. It is commonly felt, if there are only a very few degrees drop from the superheater to the turbine, that the situation is satisfactory, and little thought is given to the dollar value of these few degrees when applied to literally billions of pounds of steam handled per year. When the dollar value is considered it usually is in terms of the fuel cost of the average and not of the high-potential heat. Yet it is this more valuable high-potential heat which figures in "radiation" losses—heat which would be available without further fuel cost, equipment cost, or operating cost.

The suggestion is sometimes offered that heat losses from equipment in power stations are not actually lost because they go to heat the building, to supply preheated air to the boiler furnaces, etc. But heat is not required to heat the building all of the time, and, still more important than that, such heat as is needed for that purpose can be obtained from bled steam having

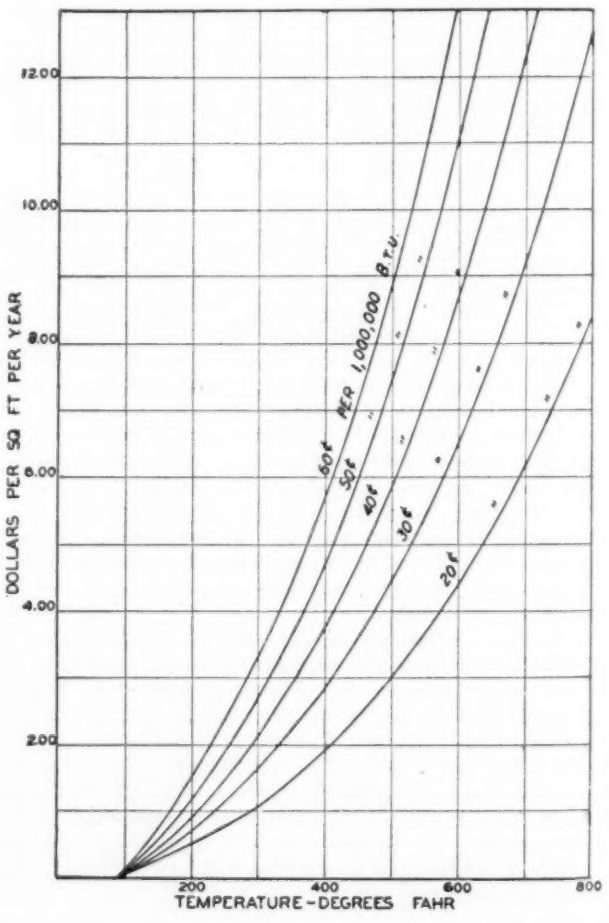


FIG. 3 ANNUAL COST OF HEAT LOSSES AT DIFFERENT VALUES OF HEAT PER 1,000,000 B.T.U.

approximately only one-fourth the value of the high-potential heat yet just as good for heating purposes. Likewise the cream of the heat from the superheated-steam lines is a somewhat expensive source of heat for preheating combustion air

Cost of Heat Losses

In spite of the fact that it does not tell the whole story, a striking picture of the cost of heat losses from bare surfaces is obtainable by reference to Fig. 3. In this chart the equivalent losses in dollars per square foot per year of 8760 hours are shown for various values of heat per 1,000,000 B.t.u. This chart serves to show more nearly the true situation than would simply figuring the fuel equivalent of the heat loss, because the curve for value of heat may be chosen so as to represent the true value and not simply the fuel cost. In selecting the curve to be used in connection with superheated steam for power generation, account should be taken of the higher value of high-potential heat as demonstrated in Fig. 1.

The dollar value of heat losses from bare surfaces may be still more forcibly illustrated by a specific example. From Fig. 3 it is readily apparent that the loss from 1 sq. ft. of bare surface, at a temperature 300 deg. fahr. above room temperature and at a value of heat of 30 cents per 1,000,000 B.t.u., is $2.60 per year. Therefore 100 sq. ft. of such surface would lose $260 worth of heat a year. If the equipment is in use 7200 hours per year instead of 8760, the loss in spite of this reduced period of operation is still 7200/8760 = $214 per year for the 100-sq. ft. area. If the surface in question is on boiler drums, headers, pipe fittings, or other apparatus where the surface temperature of the metal is nearly the same as the inside temperature, insulation which will save more than 90 per cent of this loss may be applied at a cost considerably less than one year's savings.

Even if there are only a few square feet of surface at any one point, such as on bare boiler drum heads, pipe fittings, and flanges, the total loss from a number of such small areas may be

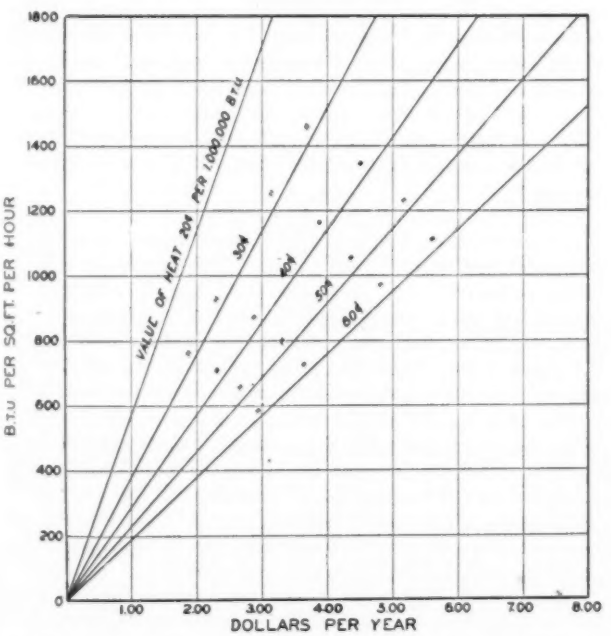


Fig. 4 Conversion Chart—B.t.u. per Hour to Dollars per Year

startlingly large. This emphasizes the desirability of minimizing the heat losses from such surfaces, which are frequently left bare even when adjacent piping is effectively insulated.

In the case of superheated-steam piping the extent of surfaces left bare is usually quite small, but, because of the higher temperatures and the greater value of the heat, the losses from such small areas may be much greater than would at first glance appear to be likely or possible. For example, the loss from one square foot of bare surface at 700 deg. fahr., and at a value of heat of 30 cents per 1,000,000 B.t.u., is $9.30 per year. If the steam pressure is 300 lb. abs., reference to Fig. 1 shows that the value of the high-potential increment of heat for power generation purposes is 1.55 times the average value. Therefore, the actual loss is $1.30 × 1.55 = $14.40 per square foot per year.

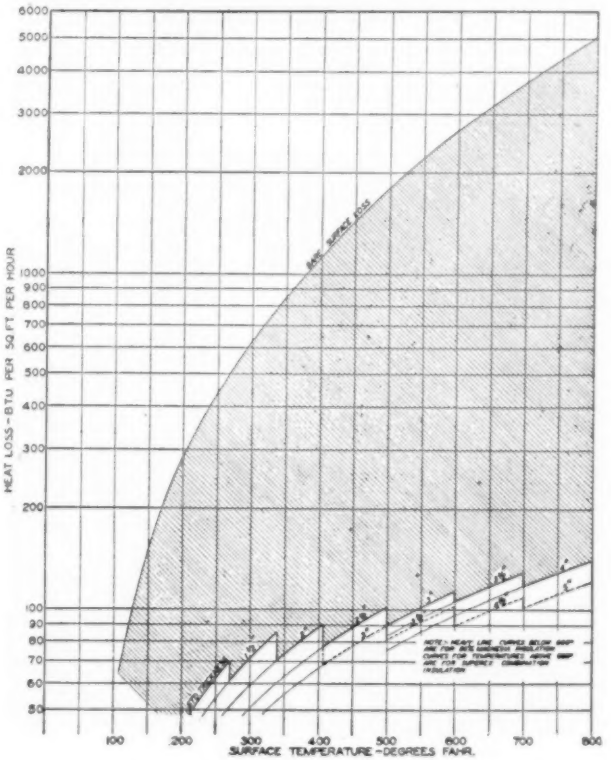


Fig. 5 Heat Losses Through Typical Insulations on 8-In. Pipe as Compared With Bare-Pipe Losses

The foregoing examples of losses and savings are based on still-air conditions. The effect of air circulation over a bare surface is to increase enormously the rate of heat loss. An air velocity of 10 miles per hour will at least double the rate of loss from such surfaces. On the other hand, the losses from insulated surfaces, where the resistance to heat flow is mostly inherent in the insulation itself, are increased but slightly in proportion by air circulation. Where the increase in heat loss from a bare surface is of the order of 100 per cent the increase in loss from a well-insulated surface, with insulation tightly applied and thoroughly sealed so that air can circulate only over its surface, is of the order of less than 10 per cent. Therefore, where surfaces are exposed to air circulation the results accomplished by insulation are proportionately much greater than those illustrated.

Where heat-loss data are available in terms of B.t.u. per hour, as in the case of Figs. 5, 7, 8, and 13 shown further along in this paper, Fig. 4 furnishes a convenient means of converting them to their equivalents in dollars per year of 8760 hours. Where the period of operation is less than the full year, values from Fig. 4 should be multiplied by the proportionate part of the year represented by the operating period.

Typical Insulation of Steam-Plant Equipment

Piping. Steam is the finished product of the boiler plant. When it leaves the superheater outlet, or the boiler nozzle if only

saturated steam is produced, it is a commodity against which all costs of production have been charged. Hence the desirability of delivering that product to the consumer with a minimum of loss en route has made the subject of pipe insulation a matter of primary interest since the early days of steam-power development.

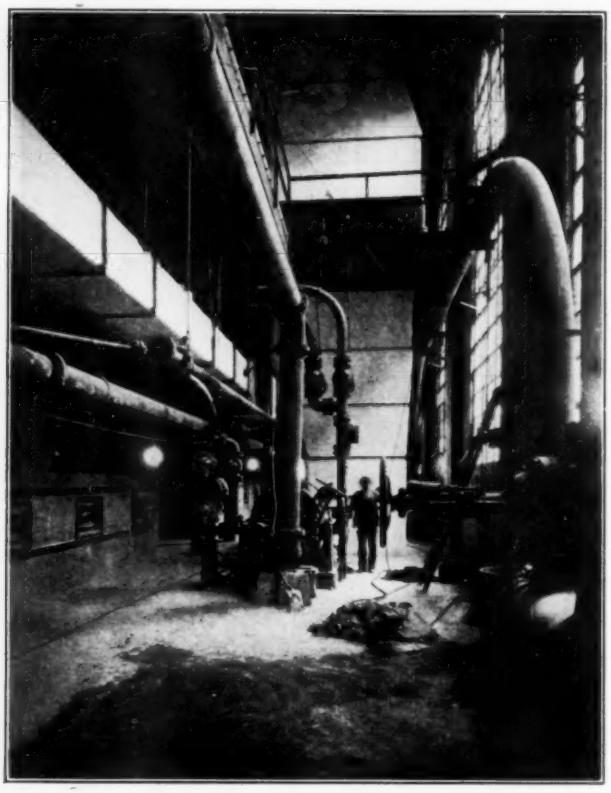


FIG. 6 INSULATED PREHEATED-AIR DUCTS, AMERICAN GAS & ELECTRIC CO., APPALACHIAN POWER CO., CABIN CREEK, W. VA.

The progressively increasing steam temperatures and the increasing costs of fuels have added still further to the relative importance of this item of power-plant equipment.

Likewise these same factors have led to the substitution of rational engineering design for the empirical methods of selection formerly used. The matter of economical thickness of insulation has been made the subject of extensive investigation, and thicknesses are now chosen with assurance that they are really economical—that they are sufficient to provide adequate reduction in heat losses and yet not so thick as to necessitate unjustified first cost. And materials have been developed which may be used with the assurance that they will provide effective results at temperatures far beyond those of present-day steam-plant practice.

But the high-pressure and high-temperature-steam mains represent only a part of the entire piping system of a modern plant. Feedwater piping, auxiliary headers, water-wall circulation piping, connections from boiler nozzles to superheaters, soot-blower piping, drip piping, etc., constitute items less important per unit of area, but when total area is considered, become of major importance in the pipe-insulation problem. Even cold-water lines must be included among the classes of piping requiring insulation if troublesome drip from condensation of moisture from the air is to be avoided.

Fig. 5 shows graphically the relative heat losses from bare and insulated pipes at various temperatures up to 800 deg. fahr. In comparing the relative positions of the curves it must be borne in mind that the vertical scale is logarithmic and that for a true comparison of ordinates the values read from the heat-loss scale must be compared. Over the greater portion of the range of the chart, the actual differences between the heat losses from bare and insulated pipes are much greater than the apparent differences between lengths of the ordinates of the respective curves.

The thicknesses of insulations represented by the heavy solid lines are typical of present-day practice in the ranges of temperature indicated and for pipe sizes $4^1/_2$ in. and larger. The heavy dotted-line curves are representative of the heat losses through the thicker insulations, which would be required for best economy at the higher temperatures by the considerations demonstrated in Fig. 1 as to the higher value of high-potential heat in superheated steam used for power generation. Both sets of thicknesses represented by the heavy-line curves are based on the same average or base value of heat, and all other conditions are the same except that the additional value of high-potential heat is not taken into account in determining the thicknesses represented by the solid-line curves. Undoubtedly, the future tendency in insulation design will be toward taking this feature more thoroughly into account.

For pipe sizes smaller than $4^1/_2$ in. the thicknesses of insulation are graduated downward as illustrated in Fig. 17. The actual procedure in the determination of economical thicknesses of insulation will be discussed more fully in connection with Fig. 17.

Preheated-Air Ducts. The preheating of the air used for combustion before admitting it to the furnace is a comparatively recent development, and, because of the fact that the heat in the air is usually recovered from what might otherwise be considered as waste heat, as, for example, the heat in the flue gases, it is often looked upon as having been obtained at little or no cost. But it does actually cost something. At least the fixed charges on the equipment required to recover the heat must be charged against it, and something more besides. But after all, *it is the value of the heat and not its cost that is important.* The major part of the cost has been expended when the equipment is put in, and that cost is incurred whether or not the heat is utilized after it is recovered.

On the other hand, the *value* depends upon the extent to which the preheated air improves the entire process of steam generation. The installation is put in not only for the purpose of imparting to the air heat which it would otherwise have to receive in the furnace, but also to improve furnace and boiler efficiency because of the higher temperatures of combustion resulting from the use of preheated air. Therefore, at the very least, the value of the preheat in the air is not less than its equivalent in fuel. Actually it must be somewhat more than that. The true measure of the value of such heat is the cost of its replacement at the point of use. Or, more specifically, its value is equal to the cost of fuel required to replace its effect—the cost of the additional fuel required to produce a given quantity of steam using air at lower temperature, over and above what would be required for the same production with air supplied at the higher temperature.

The function of insulation is, of course, to keep the heat in the air once it has been put there, and to maintain its effectiveness by minimizing temperature drop in the air from the time it leaves the preheater until it is admitted to the furnace. The heat conserved is the most valuable portion of the heat content of the air. Again the importance of maintaining temperature head and the higher value of high potential heat are very apparent.

Fig. 6, reproduced from a photograph, shows an insulated primary-air duct in the foreground and an insulated vertical connecting preheated-air duct in the background. Figs. 7 and 8 show the heat losses which may be expected from various thicknesses of typical insulating materials. Fig. 7 applies to the tem-

perature range up to 600 deg. fahr., and Fig. 8 to that from 600 to 1000 deg. fahr.

Boilers and Auxiliary Equipment. The practice in connection with the insulation of boiler drums, drum heads, other exposed portions of the boiler shell, feedwater heaters and other auxiliary equipment is so thoroughly standardized that little attention need be given it here. A notable exception to this general statement is that adequate attention has not been given to the matter of minimizing the losses of heat from the tube headers of water-tube boilers. Perhaps the reason is that they are not considered as exposed surfaces because of the doors which shut them off from view. But these steel doors offer comparatively little resistance to heat flow, and, while the losses from the headers are not as great as if the doors were not there, they are still much greater than they need be. Lining the doors with suitable insulation provides resistance to heat flow many times as great as that of the air space between the headers and the doors; because, at the temperatures involved, large air spaces are very ineffective in limiting heat flow. The heat dissipated from the tube headers is heat which has already undergone the process of being made available for steam generation. It is at the same potential, and if not lost would be of the same degree of usefulness as the heat in the saturated steam produced by the boiler. Its value is therefore just as great, and conservation at this point is a desirable step toward more economical operation.

The data on heat transmission through insulation, given in Fig. 7, for flat surfaces apply to boiler (not to boiler settings, of course) and auxiliary-equipment surfaces with a degree of accuracy sufficient for most engineering purposes, because the radii of curvature are usually quite large. If somewhat higher accuracy is required, this may be obtained by considering that the values from Fig. 7 apply to the mean area of the inner and outer surfaces of the insulation instead of to the actual area of the metal surface. If theoretical accuracy is desired, the rates of heat transfer from cylindrical and spherical surfaces of drums may be calculated by the use of the customary formulas for heat transfer through materials on such surfaces. However, such accuracy is rarely required.

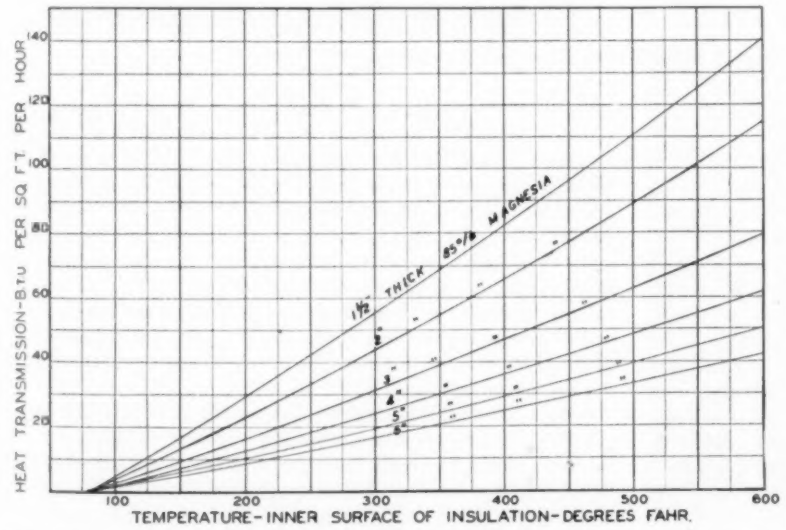


FIG. 7 HEAT TRANSMISSION THROUGH INSULATION ON FLAT SURFACES—TEMPERATURE RANGE UP TO 600 DEG. FAHR. (AIR TEMPERATURE 80 DEG. FAHR.)

Boiler Furnaces and Boiler Settings. The insulation of boiler settings and of the newer types of boiler furnaces are more recently developed fields which present many interesting problems and possibilities. In Fig. 9, typical sections of four modern types of boiler-furnace walls are illustrated diagrammatically. The sections are for the most part self-explanatory except that in Fig. 9-D the detailed arrangement of the refractories around, between, or back of the tubes is not shown. There are a number of well-known types of water-cooled walls differing considerably in design, but so far as insulation is concerned the general principles governing its application are closely similar in practically all cases. First, the walls provide, in addition to the water tubes,

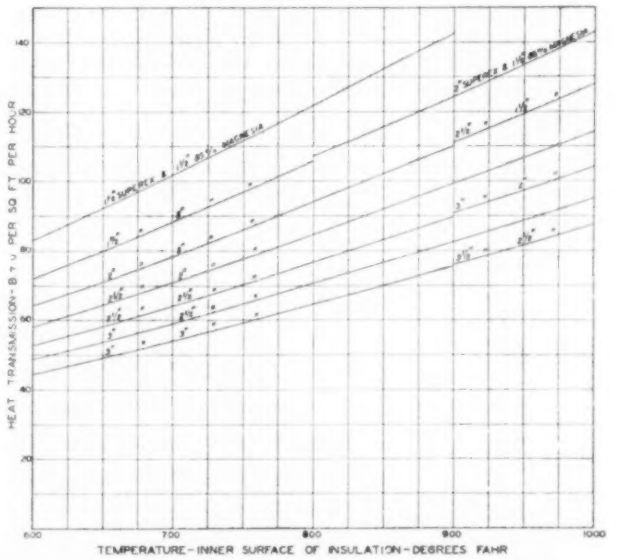


FIG. 8 HEAT TRANSMISSION THROUGH INSULATION ON FLAT SURFACES—TEMPERATURE RANGE 600 TO 1000 DEG. FAHR. (AIR TEMPERATURE 80 DEG. FAHR.)

some arrangement of refractories (cast iron would be considered a refractory in this sense) which protects the insulation from being subjected to the excessive temperatures that would penetrate between the tubes if such provision were not made. Then there is the insulation itself, and finally an outside protective and air-sealing finish for the insulation, which consists of asbestos wood, steel plate, or hard-finish insulating cement. Fig. 10 illustrates a typical insulated water-cooled wall on which the protective finish for the insulation consists of asbestos-wood panels with steel battens gasketed to provide air-tightness.

An installation of the type of wall represented by Fig. 9-A is illustrated in Fig. 11. The insulation is applied against the outside of a solid firebrick wall, and the protective and air-sealing finish consists of asbestos-wood panels over the greater part of the area. On the extreme upper portions, however, hard-finish insulating cement provides the protective finish and the air-sealing medium.

The veneer wall construction illustrated in Fig. 9-B is often used in connection with the insulation of existing boiler settings as well as on new work. In this case, however, the bonding courses shown in the diagram are not used and the veneer course is secured to the existing wall by means of metal fastenings. In such cases a hard-finish insulating cement is usually applied over mesh-wire reinforcement to cover the metal fastenings and

provide a suitable finish. For new walls, where the bonding courses may be used, types of insulating brick are available which provide a durable and attractive finish without the use of other means of finishing.

There are a number of designs of air-cooled walls; therefore Fig. 9-C is simply diagrammatic of the principles involved; and no attempt is made to show details of wall construction. However, in the main these walls consist of an inner refractory wall, a space or spaces for the circulating air, and an outer insulating casing. In general, the principles already discussed under preheated-air ducts, in connection with the value of retaining the heat in preheated air, apply also to air-cooled boiler-furnace walls. In Fig. 12 is illustrated an air-cooled wall in the course of construction, and the application of the insulation to the inside of the casing is clearly shown.

There is still another type of insulated construction, not illustrated in Fig. 9, which is used with good results in the settings of smaller boilers of capacities up to 500 hp. This is the so-called "core-wall" type, where a course of insulating brick is used between a firebrick lining on the furnace side and a course of common brick on the outer side of the wall.

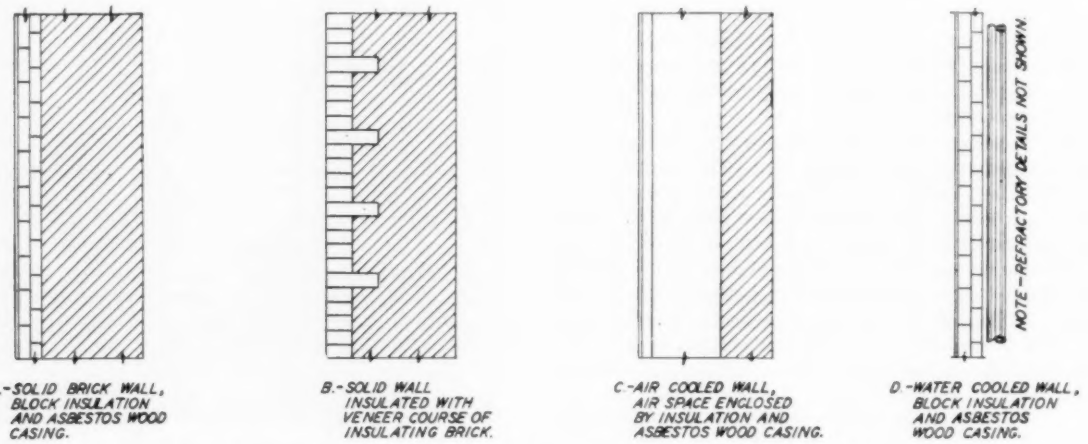


FIG. 9 TYPICAL SECTIONS OF FOUR MODERN TYPES OF BOILER-FURNACE WALLS

Results Accomplished by Insulation on Boiler Furnaces. The relative heat losses from solid refractory walls with and without insulation are shown in Fig. 13. The savings represented by the differences between the curves for insulated and uninsulated walls are very conservative since the conductivities of firebrick used in the computations were purposely taken in the low rather than the high portion of the range of firebrick conductivities. The values used range from 6.0 B.t.u. per square foot per degree temperature difference per inch of thickness per hour at a mean temperature of 1000 deg. fahr., to 8.77 B.t.u. at 2600 deg. fahr. mean temperature. Many investigators have shown conductivities at least 25 per cent higher than these, and higher conductivities of the firebrick would result in higher losses from uninsulated wall than those shown in Fig. 13. The losses through insulated walls would be increased somewhat by such higher firebrick conductivities, but to a much smaller extent; therefore, under such conditions the savings would be greater than those shown. Where insulating brick is used the thickness is usually $4^1/_2$ in., and the losses through walls insulated in this manner are closely approximated by the curve for 4-in. thick block insulation.

Insulation may be applied in such manner as to minimize air infiltration as well as heat-transmission losses. Because of wide variations in their magnitudes on settings without insulation, it is not possible to evaluate as definitely the extent of air infiltration losses, but frequently the savings effected by insulation in reducing these losses is at least as great as those due to its effect in minimizing heat-transmission losses.

On both air-cooled walls and water walls the savings by insulation are proportionately much greater than on solid refractory walls, and a given thickness of insulation applied to either of these types of walls will bring the total loss down to a point far below what it would be if the same amount of insulation were applied on a solid refractory wall. This is due to the fact that the portion of the heat flow stopped by the insulation is immediately picked up and carried away by the cooling medium, instead of tending to increase the temperature head back of the insulation as it does on refractory walls. The fact that the losses from cooled walls can be reduced by insulation to a very low point is most fortunate, because, with the greatly increased wall areas that accompany the larger furnace volumes now used, if the heat transmission per unit area could not be made small, the resulting total losses would become a serious factor in limiting economy.

Breechings, Flues, and Stacks. Where there are no air preheaters or other means for utilizing the heat in the gases, the principal reason for insulating breechings and flues is to prevent excessive heating of the boiler room. There is also a corollary advantage, however, in the improvement of draft due to the minimizing of temperature drop in the gases before they reach the stack. Also, insulation is often placed inside of the breeching in order to provide a protective lining to guard the steel from corrosion. In many cases, such as that illustrated in Fig. 14, the insulation is used to serve the double purpose of protecting the steel and of conserving the heat content of the gases for utilization at some other point, as, for example, in an industrial process requiring the use of such heat.

Where there are economizers or air preheaters or both, there are still greater advantages resulting from the insulation of uptakes and flues. In such cases the heat saved has a real dollar value, and the maintenance of temperature head increases the effectiveness of heat recovery in the economizer or air preheater. While the ducts or breechings carrying the spent gases from the recovery apparatus to the stack are at lower temperatures, and the value of the heat saved is a much smaller item, the very fact that they are at lower temperatures increases the tendency toward corrosion due to condensation of moisture on the steel and increases the importance of insulating for the purpose of minimizing corrosion troubles.

Steel stacks are often provided with insulating linings for the purpose of protecting the steel from such corrosive action. Of

course, in the case of stacks located in buildings there is a further very important requirement for insulation in order that overheating of spaces adjacent to the stacks may be avoided. The stack shown in Fig. 16 happens to be an industrial installation, but it was chosen as an illustration because, being a close-up, it shows clearly the method of insulating, and the same method is widely used for the insulation of boiler-plant stacks.

Economical Thickness of Insulation

When the decision has been reached that a given equipment should be insulated, the solution is not yet complete. The question remains as to how effectively it is to be insulated. Among the factors to be considered are, of course, the adaptability of the material to the conditions of temperature and usage to which it will be subjected, its durability, and the relative permanence with which it retains its initial insulating value. However, with these questions decided, there still remains the question as to what thickness would be most effective. Obviously, the greater the thickness the lower will be the heat loss, but as thickness is increased, fixed charges are also increased; therefore, the

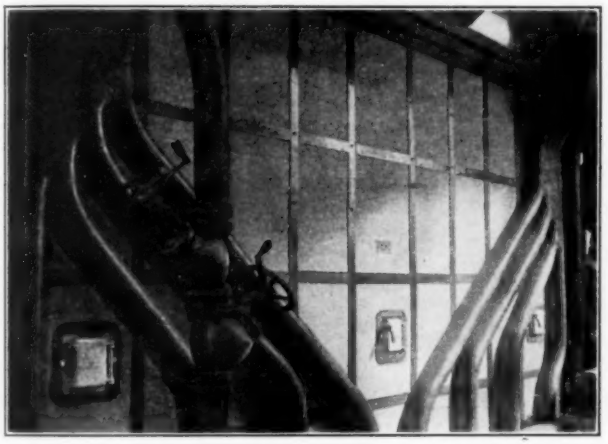


Fig. 10 Insulated Water-Cooled Wall, Virginia Electric & Power Co., Norfolk, Va.

thickness at which the sum of these two costs is a minimum is that which is most economical. At this point all of the insulation pays a satisfactory return on the investment represented by its cost. This is true even of the last increment of thickness, whereas the insulation as a whole may pay a return many times the required minimum.

Return on the investment *properly interpreted* is of course a most important consideration, but, as is true in connection with any percentage item, correct interpretation is most essential. The frequent misinterpretation of the significance of such terms as "return on investment" and "years required to repay first cost" is by no means confined to insulation. In the case of any equipment, the one which pays the highest *per cent* return on the investment, or which repays its first cost in the shortest length of time, is not by any means necessarily the most economical for the purpose. It is more likely to be the least economical. Nor is this the paradox which at first sight it might appear to be. The very fact that an equipment pays an exceptionally high rate of return on its cost, several hundred per cent, for example, is a very good indication that there may be, and most likely is, an opportunity to make still further investments at rates of return which, while not as high as in the first instance, are still highly attractive.

All of this is so very elementary that further discussion should hardly be necessary, but because of the frequent misinterpretation of the principles involved, not only in the case of insulation but also in connection with other equipment affecting operating economy, a typical example is given which will serve to illustrate these principles still more clearly. It will be noted from Table 1 that the percentage returns on the investments are all very high, but it will be noted further that as the thickness is increased the percentages are decreased. Likewise, as thickness is increased

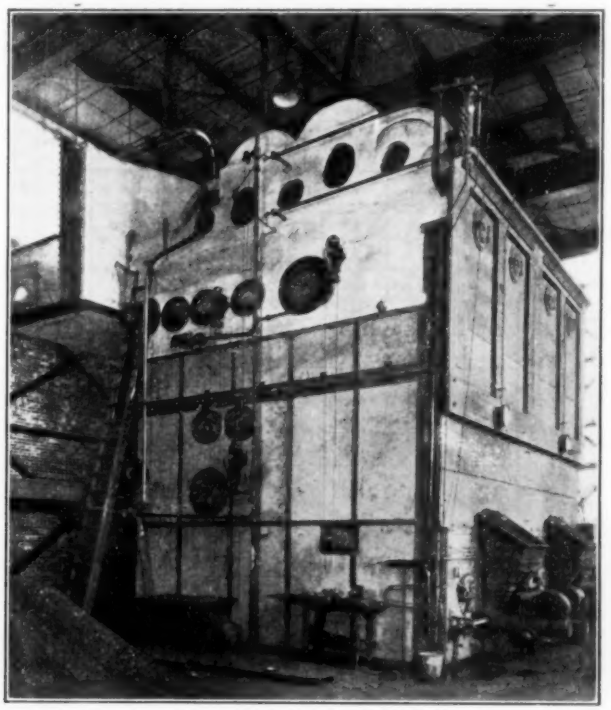


Fig. 11 Insulated Boiler Setting With Solid Refractory Walls, Worthington Pump & Machinery Co., Buffalo, N. Y.

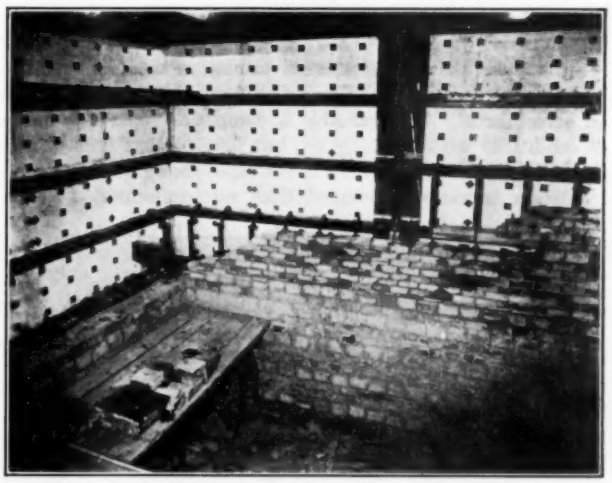


Fig. 12 Application of Insulation in Air-Cooled Wall, Worcester Salt Co., Silver Springs, N. Y.

the length of time required to repay first cost is increased. Therefore, if the trends in either of these items could be considered as a deciding factor, even 1-in. thickness would be too great since the trends indicate increasing percentage returns as the thickness approaches zero.

Yet a return of 1000 or 1200 per cent is not bad, so something

TABLE 1 PER CENT RETURN ON INVESTMENT AND TIME REQUIRED TO REPAY FIRST COST

[Based on a loss from bare, flat surface of 1600 B.t.u. per sq. ft. per hour (corresponding approximately to the loss at a temperature difference of 400 deg. fahr.) and value of heat at $0.50 per 1,000,000 B.t.u. Insulation is provided with 1/2-in.-thick asbestos-cement finish in addition to thickness shown. Also the block insulation is applied in two layers where thickness is 3 in. or more. These items account for the non-proportionality of cost to thickness.]

Thickness of insulation, in.	Annual loss per 100 sq. ft.	Annual saving	First cost of insulation	Per cent annual return (gross)	Years to repay first cost
0	$701.0	...	...	...	...
1	65.0	$636	$ 50	1272	0.08
2	38.0	663	65	1020	0.10
3	27.0	674	85	793	0.13
4	21.0	680	100	680	0.15
5	17.0	684	115	595	0.17
6	14.5	686.5	130	528	0.19

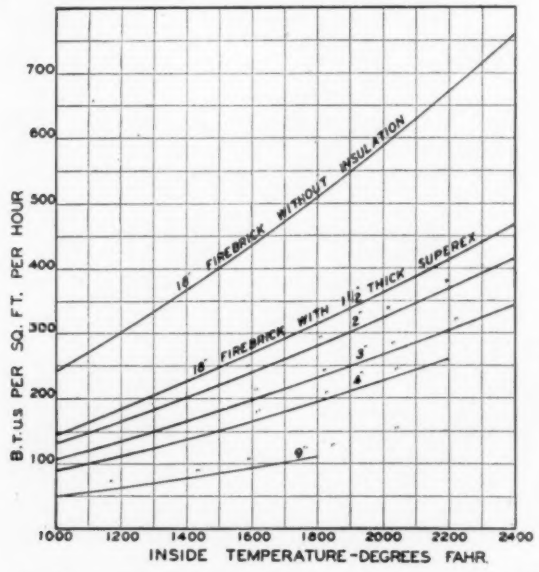


FIG. 13 HEAT TRANSMISSION THROUGH INSULATED AND UNINSULATED FURNACE WALLS—18-IN. FIREBRICK (AIR TEMPERATURE, 80 DEG. FAHR.)

is evidently wrong with that method of comparison. Such returns show that at least some insulation must be justified. This illustrates clearly the limit of usefulness of the term "return on investment." It is valuable in comparing separately *one set* of conditions with *one other set*, but beyond that it is of little further use. Thus, this item in the table shows that 1-in.-thick insulation is more economical than no insulation. Likewise,

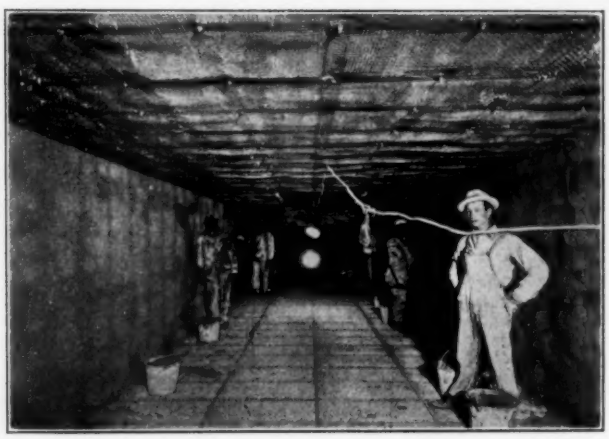


FIG. 14 TYPICAL FLUE-LINING INSULATION, FORD MOTOR CO., IRON MOUNTAIN, MICH.

it shows that 6-in. insulation and each of the other thicknesses shown is better than none at all, but it does not show that 6-in. is more economical than 1-in. or vice versa. To determine which of the insulations is the better suited to the conditions they must be compared with each other. This is done in Table 2,

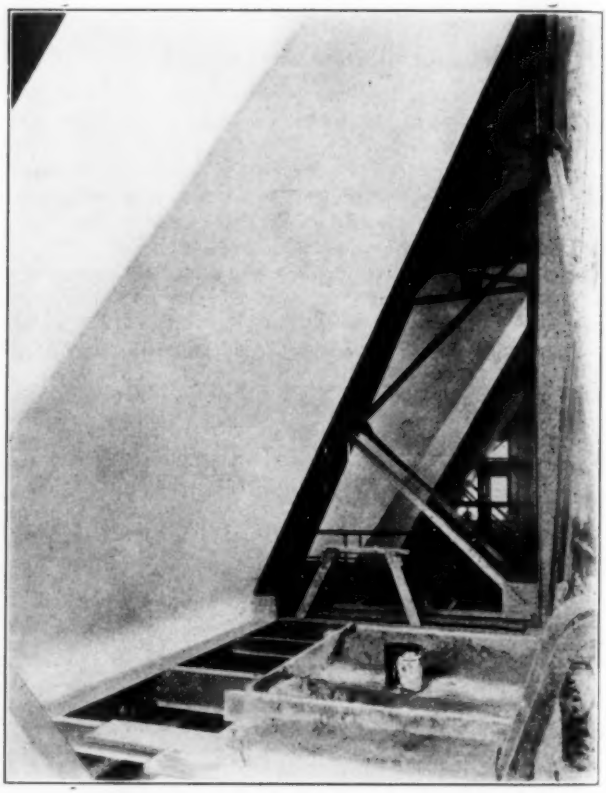


FIG. 15 TYPICAL EXTERNALLY INSULATED FLUE, CONNECTICUT LIGHT & POWER CO., DEVON STATION



FIG. 16 TYPICAL STACK-LINING INSTALLATION, ILLINOIS PACIFIC GLASS CO., LOS ANGELES, CALIF.

and it is at once apparent that 2 in. thick is more economical than 1 in. thick since the *additional* saving pays a return of 180 per cent on the *additional* cost. Likewise, 3 in. thick is preferable to 2 in., but by a decreased margin, and so on down to the point

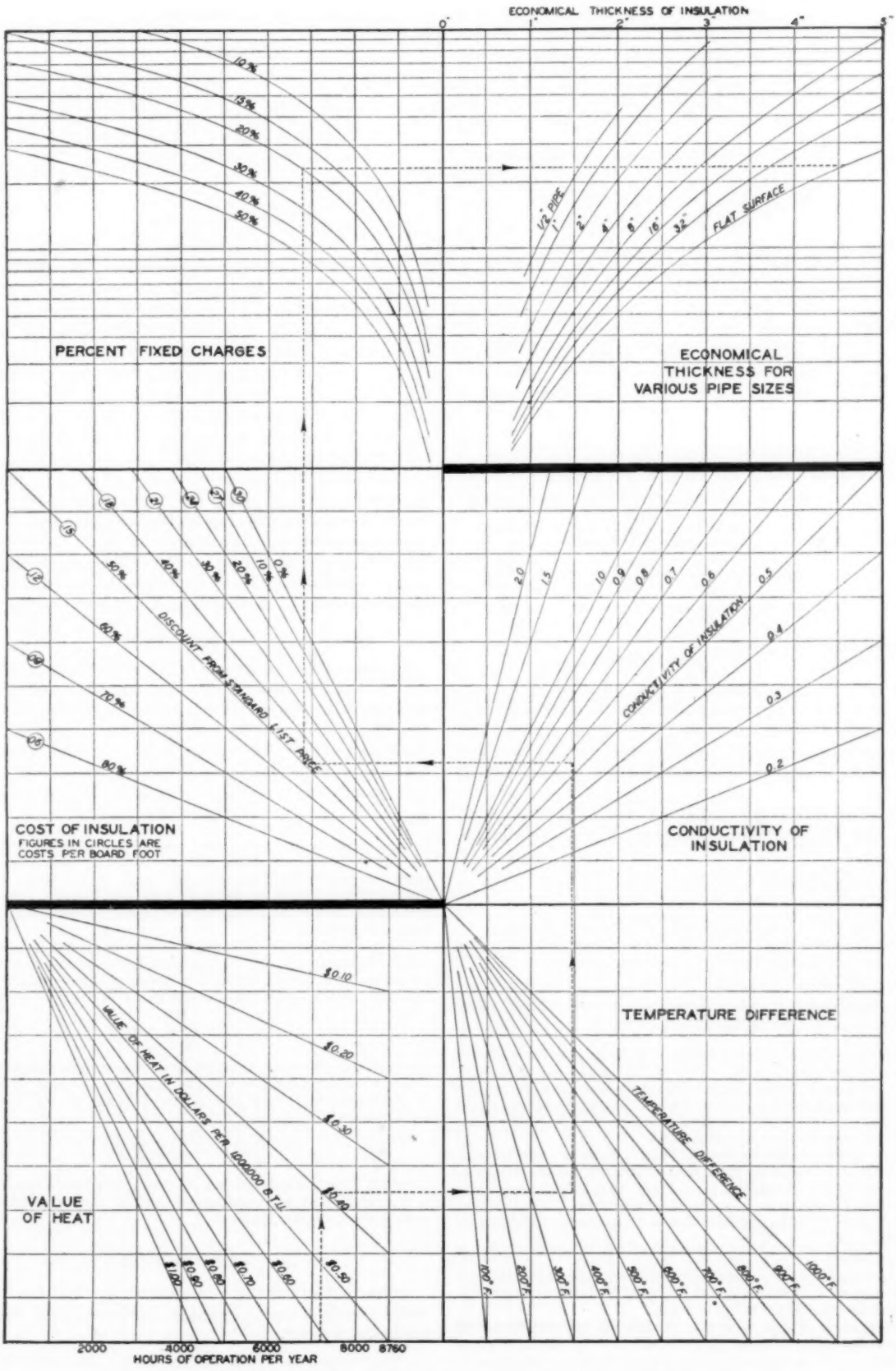


FIG. 17 CHART FOR DETERMINING ECONOMICAL THICKNESS OF INSULATION

where the additional saving by added thickness no longer pays the required minimum return on the additional cost. In this example, if such minimum acceptable return is 20 per cent, the most economical thickness is 5 in., since the additional saving by 6-in.-thick insulation fails to pay the required return on the additional cost, whereas the *last inch* of the 5-in. thickness pays 27 per cent on its cost.

But there is a far more direct and convenient method of determining the economical thickness. This is based simply upon determining the total cost, heat loss per year plus cost of insulation per year (first cost times per cent annual fixed charges), and choosing the thickness at which the sum of these two items is the lowest. This method is illustrated in Table 3, which is based on the same example as that employed in Tables 1 and 2.

TABLE 2 PER CENT RETURN ON ADDITIONAL INVESTMENT REPRESENTED BY EACH INCH OF ADDITIONAL THICKNESS

Thickness of insulation, in.	Annual loss per 100 sq. ft.	Annual saving by additional 1 in. thickness	First cost of insulation	Cost of 1 in. additional thickness	Per cent return on additional investment
1	$65.0	..	$50	..	..
2	38.0	$27.0	65	$15	180
3	27.0	11.0	85	20	55
4	21.0	6.0	100	15	40
5	17.0	4.0	115	15	27[1]
6	14.5	2.5	130	15	17

[1] The most economical thickness for th stated conditions is 5 in.

TABLE 3 TOTAL ANNUAL COST: HEAT LOSS PLUS INSULATION COST

Thickness of insulation, in.	Annual loss per 100 sq. ft.	First cost of insulation	Annual cost on insulation[1]	Total annual cost
1	$65.0	$ 50	$10	$75.0
2	38.0	65	13	51.0
3	27.0	85	17	44.0
4	21.0	100	20	41.0
5	17.0	115	23	40.0[2]
6	14.5	130	26	40.5

[1] Annual fixed charges = 20 per cent.
[2] The most economical thickness for the stated conditions is 5 in.

In an earlier paper [Trans. A.S.M.E., vol. 48 (1926), pp. 1269–1318] the author presented a rational method of determining economical thicknesses of insulation. The chart Fig. 17 is based upon the equations developed in that paper, but it has been simplified in such a manner that the entire solution may be obtained graphically.

The principal factors involved in determining the economical thickness and the order in which they are taken into account are as follows:

1 Hours of Operation per Year
(8760 hours for continuous operation)
2 Value of Heat in Dollars per 1,000,000 B.t.u.
(Not necessarily the average cost of heat, but its value at the given potential. See Fig. 1)
3 Temperature Difference
(Temperature inside of equipment to be insulated minus temperature of surrounding air[2])
4 Conductivity of Insulation
(B.t.u. per square foot per degree temperature difference per 1 in. thick per hour)
5 Cost of Insulation, per cent discount from standard list
(Curves are also marked with respective costs per square foot 1 in. thick to provide for materials whose costs are not figured in terms of discount from standard list)
6 Per Cent Annual Fixed Charges
(Usually about 15 per cent in the classes of work under discussion)
7 Shape of Surface or Pipe Size
(Curves are given for cylindrical surfaces of various diameters and a curve is also shown for flat surfaces.)

In order to use the chart, start at the lower left-hand corner and proceed to the right to a point representing the given number of *hours of operation per year;* then proceed vertically to the line representing the given *value of heat;* thence horizontally, to the right, to the line representing the given *temperature difference;* thence vertically to the line representing the *conductivity* of the given material; thence horizontally, to the left, to the line representing the given *discount* on that material (or given cost per square foot, board measure); thence vertically to the curve representing the required *per cent annual fixed charges;* thence horizontally, to the right, to the curve representing the given *shape;* thence vertically to the scale at the top of the sheet, where the *economical thickness* may be read off directly. The dotted line on the chart illustrates its use in solving a typical example. The actual thickness used will, of course, be the available commercial thickness nearest to the thickness as read from the chart.

Conclusions

No attempt has been made in this paper to present a lengthy discussion of insulation details. The central thought has been to set forth as clearly as possible the principles involved, and to present the subject in its true perspective. Especial emphasis has been given to the importance of maintaining temperature head and to the higher value of high-potential heat. To whatever extent these principles have been made more clear, to that same extent the effort has been worth while.

[2] Where there is but little resistance to heat flow between the surface to be insulated and the source of heat, as in the case where the source of heat is steam, water, or rapidly moving gases on one side of a thin metal plate or pipe wall and insulation is to be applied directly to such surface, the resulting thickness read from the chart is at once the correct economical thickness.

However, it must be borne in mind that, if there is a considerable resistance to heat flow other than that offered by the insulation itself, as in the case of insulation on a brick furnace wall, correction must be made for the effect of such other resistances. For example, if the insulation is to be applied to a wall having a resistance of $2.0 \left(\frac{\text{thickness}}{\text{conductivity}} = 2.0\right)$ and the conductivity of the insulation is 0.6 B.t.u. per sq. ft. per deg. temperature difference per 1 in. thick per hour, the correction which must be subtracted from the thickness obtained from the chart is $2.0 \times 0.6 = 1.2$ in.

The explanation of this correction is that the rational equations upon which the chart is based give the thickness of insulating material which would be required if there were no insulating value in the construction other than that offered by such material itself. If other resistances to heat flow are already present, naturally less additional insulating value will be required, and the correction simply involves deducting the thickness of insulation equivalent in insulating value to the resistances already present. (In the chart as plotted, deduction of 0.3 in. has already been made for outside surface resistance.)

For flat surfaces, the rational equation for economical thickness is quite simple; therefore, for furnace walls and other cases where temperatures involved are beyond the range of the chart, the following equation for flat surfaces may conveniently be used:

$$x = \sqrt{\frac{ak}{b}} - Rk$$

in which x is the most economical thickness, k is the conductivity, b is the cost of insulation *per inch thickness per year*, R is the sum of the resistances of all *other* elements in the construction, including surface resistance, and

$$a = \frac{Y(t_0 - t_a)M}{1{,}000{,}000}$$

in which Y is hours operation per year, t_0 is inside temperature, t_a is temperature of surrounding air, and M is the value of heat in dollars per 1,000,000 B.t.u.

Discussion

Irvin G. McChesney.[3] A glimpse of actual conditions met with by users of pipe covering is of interest in the discussion of the author's paper.

[3] Superintendent, Power Testing Section, Rochester Gas & Electric Corp., Rochester, N. Y. Jun. A.S.M.E.

TABLE 4

Hours operation per year:
High pressure—8760 hr.
Low pressure—7000 hr.

Value of heat in dollars per million B.t.u. $0.24

Temperature difference:

High pressure overhead	430 deg. fahr.
underground	350 deg. fahr.
Low pressure overhead	220 deg. fahr.
underground	140 deg. fahr.

Conductivity of insulation:

Brand	A	B	C	D	E	F	G	H
High pressure overhead.....	0.47	0.51	0.46	0.48	0.51	0.64	0.48	0.58
High pressure underground..	0.49	0.52	0.48	0.50	0.53	0.67	0.49	0.61
Low pressure overhead.....	0.43	0.49	0.40	0.46	0.47	0.56	0.46	0.54
Low pressure underground...	0.45	0.50	0.43	0.47	0.49	0.59	0.47	0.55

Discount off list:

Standard, per cent.....	25	60	45	40	63	55	56	62½
Thick pipe covering, per cent	25	60	47	40	63	50	56	62½

To guide purchasing policy it is necessary to compute economic thickness and minimum cost of coverings having suitable physical characteristics.

In a problem concerning a large heat-distribution system, testing and computations were carried out as described by the author in the 1926 Transactions.

A brief summary of data used in computations is given in Table 4.

For brevity only the results of extreme service conditions are considered; high pressure overhead and low pressure underground.

Bare pipe not considered, there remains the problem of balancing one brand of covering against another on minimum cost.

The author's chart will serve for approximation of economic

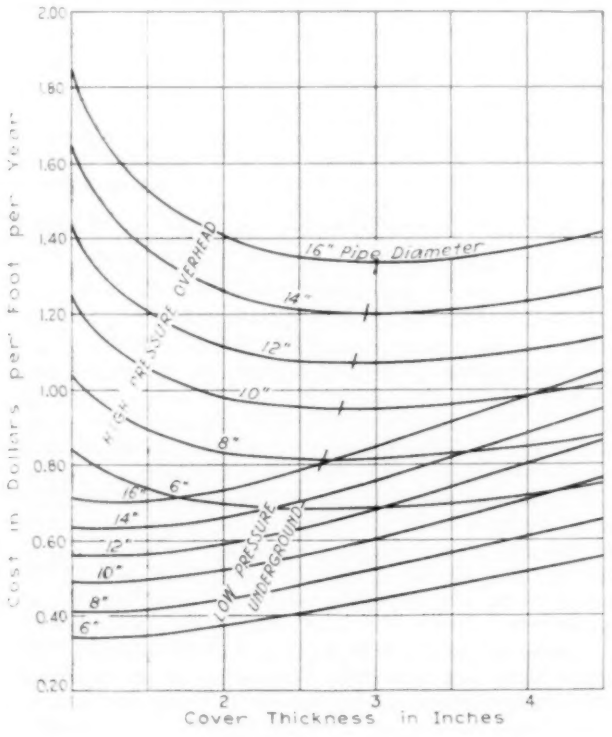


FIG. 18

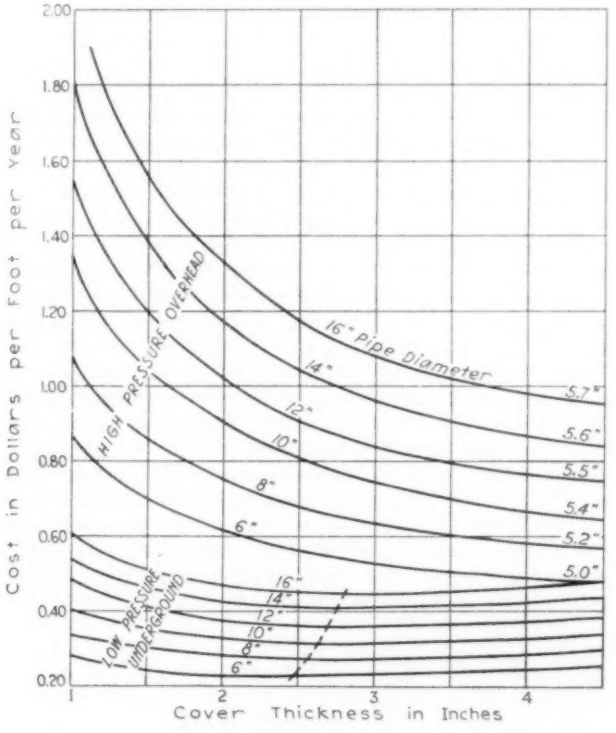


FIG. 19

TABLE 5 ECONOMIC THICKNESS AND YEARLY COST PER FOOT OF PIPE FOR VARIOUS BRANDS OF STEAM PIPE INSULATING MATERIAL

(*a*) High pressure—overhead

Nominal pipe diameter	4		6		8		10		12		14		16	
Economic thickness and cost	In.	$	In.	$	In.	$	In.	$	In.	$	In.	$	In.	$
Brand of cover:														
A	2.2	0.61	2.4	0.76	2.5	0.91	2.6	1.06	2.7	1.21	2.8	1.36	2.8	1.53
B	2.5	0.51	2.9	0.68	3.2	0.74	3.5	0.85	3.7	0.96	3.8	1.09	3.9	1.28
C	2.8	0.48	3.3	0.59	3.7	0.70	4.1	0.80	4.3	0.90	4.6	1.01	4.7	1.12
D	2.4	0.54	2.6	0.68	2.7	0.81	2.8	0.95	2.9	1.07	3.0	1.20	3.0	1.33
E	3.0	0.49	3.4	0.60	3.7	0.71	3.8	0.82	4.0	0.92	4.2	1.05	4.2	1.16
F	3.0	0.60	3.4	0.75	3.7	0.88	3.9	1.01	4.0	1.04	4.2	1.27	4.3	1.44
G	2.8	0.49	3.1	0.61	3.2	0.73	3.4	0.84	3.5	0.94	3.6	1.07	3.6	1.20
H	4.5	0.39	5.0	0.48	5.2	0.56	5.4	0.65	5.5	0.73	5.6	0.80	5.7	0.92

(*b*) Low pressure—underground

Nominal pipe diameter	4		6		8		10		12		14		16	
Economic thickness and cost	In.	$	In.	$	In.	$	In.	$	In.	$	In.	$	In.	$
Brand of cover:														
A	1.0	0.31	1.1	0.39	1.2	0.47	1.2	0.55	1.3	0.62	1.3	0.70	1.3	0.77
B	1.4	0.26	1.6	0.32	1.7	0.39	1.7	0.46	1.8	0.52	1.8	0.58	1.8	0.65
C	1.1	0.23	1.3	0.30	1.5	0.36	1.6	0.43	1.6	0.49	1.7	0.56	1.7	0.62
D	1.0	0.26	1.1	0.34	1.2	0.41	1.2	0.49	1.3	0.56	1.3	0.63	1.4	0.70
E	1.5	0.24	1.6	0.31	1.6	0.37	1.7	0.44	1.8	0.50	1.9	0.56	1.9	0.62
F	1.2	0.28	1.4	0.37	1.5	0.44	1.6	0.52	1.7	0.60	1.7	0.67	1.7	0.74
G	1.2	0.25	1.4	0.32	1.5	0.38	1.6	0.45	1.6	0.51	1.7	0.58	1.7	0.64
H	2.3	0.19	2.5	0.24	2.6	0.27	2.7	0.32	2.7	0.36	2.8	0.41	2.8	0.45

thickness in many cases, but fails to give minimum cost, which is the criterion of brand selection, where physical properties are satisfactory.

As the author brought out in oral discussion the nature of the minimum point is important.

Figs. 18 and 19 demonstrate the nature of minimum points.

Fig. 18 represents a covering of low conductivity and high fixed charges, and Fig. 19 relatively high conductivity and low fixed charges.

The author's consideration of high potential heat adds an important variable to power plant pipe covering design.

EDGE TAYLOR COPE.[4] It is interesting to note the development which has occurred in the design of steam-pipe covering in the past five years. Mr. McMillan is to be congratulated on bringing together and expressing in such a clear manner the essential facts in the matter of determining economical thickness of steam-pipe covering.

Fig. 17 is especially good in that it takes into consideration all the factors entering into the determination of economical thickness of steam-pipe covering. The writer has checked the values of covering thickness as obtained by the use of this chart against the values of thickness in use at Trenton Channel Plant and finds the two sets of values in substantial agreement over the whole range of steam temperatures and pipe sizes. The covering thicknesses for Trenton Channel Plant were calculated according to the procedure outlined in a paper presented by Mr. W. A. Carter and the writer in 1925, at the Milwaukee meeting of the Society.[5]

Since Trenton Channel Plant was one of the early ones in which compound covering on steam pipes was used, it was thought worth while to check up the temperature of covering as it operated against the temperatures as calculated. A section of 16-in. main steam piping near one of the 50,000-kw. units was selected for a temperature test. Calibrated iron-constantan thermocouples were used to read temperatures. The arrangement is shown in Fig. 20. At a later date a similar test was made on a 12-in. superheated steam line in the same plant. The readings are as follows:

Nominal pipe size, in.	12	16
Total thickness of covering, in.	4	3⅝
Thickness of inner layer (high-temperature-resisting), in.	1⅛	2
Thickness of outer layer (85% magnesia), in.	2	2
Temperature of outside of pipe, deg. fahr.	698	700
Temperature at outer surface of inner layer of covering, deg. fahr.	422	450
Temperature at outside of covering, deg. fahr.	133	124
Calculated temperature at outside of covering, deg. fahr.	126	121

It is to be noted that there is some difference between the actual and the calculated temperatures of the outside surface of the covering in each case. This can be accounted for in part by the fact that it was not possible to buy covering of standard thickness that exactly agreed with the thicknesses shown by the design. A thickness of inner layer (high-temperature-resisting covering) greater than that shown by the design was selected in each case. This covering is not quite so good an insulator (a better heat conductor) as 85 per cent magnesia. The result was that the total covering thickness was made up of a greater portion of the poorer insulator than was shown by the design. The heat conduction of the covering is a little greater than was hoped for, and the outside temperature is a little higher than the design called for.

It is worth noting, however, that the values of mean absolute thermal conductivity were used as supplied by the manufacturers of the various covering materials. The writer is of the opinion that the results obtained at Trenton Channel are excellent evidence of the accuracy of the values given and of the uniformity of the material supplied at different times. The 16-in. pipe was installed and tested in 1924, and the 12-in. pipe was installed in 1927 and tested in 1928.

There is another item which the writer would like to note in regard to this matter of pipe covering. Many engineers have an exaggerated idea of the loss resulting from poor insulation under modern conditions in large plants. Let it be understood that this is no effort to belittle the importance or value of adequate covering for steam piping, because the writer believes in it thoroughly. We ought, however, to keep a proper perspective. Let us take a concrete case and make a few calculations. Let us assume a 16-in. pipe carrying superheated steam at, say, 700 deg. fahr. and passing 400,000 lb. of steam per hour. Let us suppose that this pipe is bare and in air maintained at 80 deg. fahr. This last assumption probably would be true at the start of our imaginary conditions, but would not last for very long because of the loss of heat from the pipe. This would be the worst condition, so let us assume that it could happen.

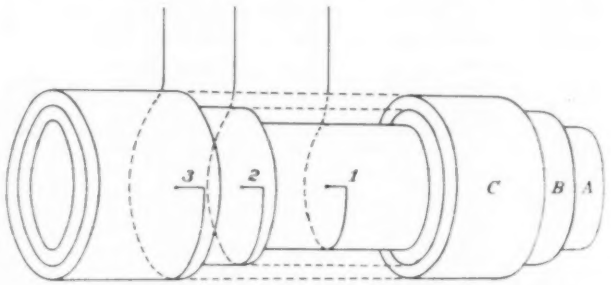


FIG. 20 ARRANGEMENT FOR MEASURING THE TEMPERATURE OF PIPE AND OF COVERING

(*A*, pipe; *B*, high-temperature-resisting layer; *C*, 85 per cent magnesia layer. Position of thermocouples: 1, hot junction peened into shallow drill hole in pipe; 2, hot junction laid between 85 per cent magnesia and high temperature-resisting layers; 3, hot junction between 85 per cent magnesia layer and canvas covering.)

Taking the value of heat loss from this bare pipe from Heilman's tests[6] and taking the temperature of the pipe equal to the steam temperature (the worst condition, not quite, but nearly true), the loss per 100 ft. of bare 16-in. pipe would be about 1,400,000 B.t.u. per hr. Now, 400,000 lb. of steam are passing this 100 ft. of pipe each hour. The average loss per pound of steam, therefore, would be about 3.7 B.t.u. This would account for a loss of temperature of the steam of approximately 7½ deg. fahr. The manufacturer tells us that a drop of 11 deg. fahr. in superheat temperature will produce an increase of 1 per cent in the turbine-water rate. We must conclude that the worst condition, while undesirable, is not really a tragedy.

Now let us consider the insulated 16-in. pipe which the writer tested at Trenton Channel. The outside temperature was 133 deg. fahr., and the outside diameter was 24 in. (4 in. of covering). At the best loading of the 50,000-kw. turbine served by this 16-in. pipe there were about 400,000 lb. of steam passing per hour. Let us calculate the loss of heat from this pipe by using Heilman's formula for the loss of heat from canvas-covered surfaces.

$$h = \frac{147.6\, D^{0.81}(tc - ta)}{272.5 - (tc - ta)}$$

[4] Detroit Edison Co., Detroit, Mich. Assoc-Mem. A.S.M.E.

[5] "The Rational Design of Pipe Covering for Pipes Carrying Steam up to 800° F." W. A. Carter and E. T. Cope. Trans. A.S.M.E., vol. 47, 1925, p. 255.

[6] "Heat Loss From Bare and Covered Wrought-Iron Pipe at Temperaturesup to 800 Deg. Fahr." R. H. Heilman, Trans. A.S.M.E., vol. 44, 1922, p. 301.

where h = B.t.u. lost per linear foot of pipe per hour.
D = outside temperature of covering in deg. fahr.
tc and ta = temperature of the surface of the covering and of the air, respectively, in deg. fahr.

The loss from 100 ft. of pipe would be about 46,800 B.t.u. per hr.

The loss per pound of steam passing would be 46,800 ÷ 400,000, which equals 0.117 B.t.u. The loss of temperature would be about 0.24 deg. Obviously the increase in turbine-water rate because of such a small temperature loss would be negligible.

The economical thickness of covering must be that thickness which saves enough coal annually to balance the allowed annual fixed charge on the cost of the covering installed.

E. F. MUELLER.[7] The presentation in the paper of the question of economic thickness of insulation is in such a simple and usable form that it seems desirable to carry the argument somewhat further and to state the conclusion in a different form. The paper suggests the conclusion that the thickness of insulation corresponding to the minimum total annual cost is the most desirable.

The characteristics of a maximum or minimum may be, however, as important as or more important than the fact that such a maximum or minimum exists. Not all maxima and minima are alike. A maximum may be as sharp as the point of a needle or as broad and flat as the top of a table, and the use to be made of our knowledge as to the existence of the maximum may be decidedly different in the two cases.

The arithmetical, graphical, or analytical processes used in determining a minimum can do no more than give the logical result of the assumptions originally put into the analysis. Yet it seems to be a well-developed trait of human nature, once the existence of a maximum or minimum has been established by analysis, to consider such a maximum or minimum as an optimum to be sought at any cost for its own sake, without regard to the nature of the maximum or to other factors not included in the analysis. This tendency appears to be the more pronounced when the demonstration is not completely understood. It is unfortunately more common in research than in engineering work and occasionally leads to much loss of time and effort.

When a maximum or minimum having desirable characteristics has been found, it is important, therefore, not to stop with this discovery, but to examine the values on either side. If the maximum is sharply defined, it may be well to attempt to attain it, but if it is found to be very broad, so that essentially the same value as the maximum is found over a wide range, a value considerably removed from the maximum indicated by the analysis may be dictated by other considerations.

The proposition just stated is of very general applicability, and the data of Table 3 of the paper are merely an excellent illustration. The conclusion from Table 3 and the discussion leading up to it might be stated as follows:

"If only one kind of insulation is to be used, and if the costs of heat loss and of insulation are those given in the table, then, for fixed charges of 20 per cent, the 5-in. thickness is the most economical."

However, an examination of the very small differences between the estimated total costs of 4-in., 5-in., and 6-in. thicknesses shows that even such a highly restricted conclusion is hardly warranted by the data, but that we may conclude:

"As far as total annual costs can be estimated in this case, the 4-in., 5-in., and 6-in. thicknesses are equally economical, and the choice between them may be based on other considerations."

[7] Physicist, Bureau of Standards, Washington, D. C.

Furthermore, considering the inherent uncertainties in estimated costs and that the maximum may be very appreciably displaced by comparatively small changes in the initial assumptions, a feature which it may prove profitable to examine more in detail, it seems that we may conclude that considerations of total annual costs allow a choice between thickness from 3 to perhaps 8 in., and within this range the choice may be dictated by considerations other than total annual costs. Such other considerations will doubtless present themselves in embarrassing abundance to any one charged with designing a modern steam-generating plant. By way of illustration, such factors as the probable period of usefulness of the plant, the use of two or three thicknesses only instead of a large number in the whole plant, the trend in fuel costs, the stability of the industries served by the plant, the availability of capital, could well be taken into account in choosing between the several equally economical thicknesses indicated by the analysis of total annual costs.

The gist of this discussion is given in the statement that an analysis based on total annual costs gives not the most economical thickness but a range of thicknesses, all about equally economical, from which a choice may be made.

HERBERT B. REYNOLDS.[8] The author has called attention to a very important point which should be considered in providing for heat insulation, and that is the fact that high-potential heat of steam is more valuable than the average heat value. In fact, I think it actually has a higher value than the curves in Fig. 1 would indicate.

In order to demonstrate the relative value of high-potential heat and the average value in a practical case, I have studied the heat losses which would occur from 200 ft. of bare 18-in. steam line or its equivalent between boilers and a 35,000-kw. turbine. A pressure of 300 lb. abs. and a superheat of 200 deg. fahr. at the boilers were assumed, while the average steam flow was taken at 250,000 lb. per hr. Using the data for bare-pipe losses as given in Fig. 5, it was found that the actual loss of heat due to radiation would amount to 0.83 per cent of the total heat passing through the line. However, this heat loss would cause a reduction in the superheat at the turbine throttle of 21.4 deg. fahr., which in turn would cause an increase in the steam consumption of the turbine of 2.14 per cent, allowing 1 per cent increase for each 10 deg. decrease of superheat. It will thus be seen that the ratio of the value of high-potential heat to average value is 2.14 ÷ 0.83, or 2.58, while Fig. 1 gives this ratio as 1.45 for the pressure and temperature assumed. This higher value of the high-potential heat of course does not exist during the time when steam lines are under pressure but with no flow of steam through them.

P. NICHOLLS.[9] The outstanding urge advanced by the author is that insulation should be bought on the basis of the computed economical thickness, and that figures cannot lie. There is no question of the principle of the economic thickness being correct and that one can consider its predictions as final. Instead of questioning its principles it is the function of the engineer to examine the numerical values used for the various factors included in the general equation by which the economic thickness is determined. Some are definite and others are based on the future and involve probability, and this is where the engineer can use his good judgment.

The initial cost is fixed, and so in all probability is the temperature difference; past experience indicates that in some

[8] Mechanical Engineer, Interborough Rapid Transit Co., New York, N. Y. Mem. A.S.M.E.

[9] Supervising Engineer, Fuels Section, Pittsburgh Experiment Station. U. S. Bureau of Mines.

types of plants steam pressures or the superheat may be increased which would warrant the use of a thicker insulation. One probably tends, however, to assign too high a temperature difference to furnace and other walls, and takes maximum instead of average of time and the area conditions. The initial conductivity of the insulation is known, and, in general, insulation materials used on hot surfaces do not increase their inherent conductivity; on the other hand, all other probabilities will tend to give loss of insulation value, such possible causes being poor workmanship, mechanical damage or failure, cracks, and air infiltration.

The value of the heat used should be the average during the life of the insulation, which is looking into the future; although we generally think of the cost of fuel as increasing, yet past experience indicates that improvements in operation or other means have decreased the unit cost; each type of plant will require special consideration to predict the future probability.

Fixing the value to be used for the per cent fixed charges involves the most guessing; the insulation salesman will be optimistic and make it low, whereas most of the probabilities will tend to increase it. Repairs or changes in the construction insulated, shutdowns, and obsolescence cause increase, and different values should be used for furnaces than for steam pipes, for example.

Finally, there is the general decision as to whether one would prefer to have the money now or to invest it for the future.

The chart developed by the author is a time saver, but it could well have included the factor of percentage of time the insulation is in service. There should also be an accompanying sheet with concise tabulated guidance for selecting values for the various factors in relation to types of plant and apparatus insulated, for the translation of pipe insulation to board feet, and other data which sales experience has shown to be useful.

The paragraph on air-cooled and water walls probably tends to give the credit for savings to the insulation instead of to the wall construction. Naturally a given thickness of insulation will give a lower heat loss than it would if used on a refractory wall, but that is why the money is put into the air or water construction, and the one that would be expected to save on the cost of insulation.

F. L. Wagner.[10] No mention has been made in this paper of the cost of applying covering in various thicknesses. This cost of application will vary tremendously with the nature of the lines and equipment to be covered, their location, accessibility, and several other factors.

The writer feels that this application cost should be duly considered in selecting the economical thickness of insulation for any given set of lines or equipment.

This labor cost can readily be included with the material cost, as shown on Fig. 17, by taking the discount from standard list price of the insulation applied, instead of the discount from list price of the insulation itself.

[10] Armstrong Cork & Insulation Co., Inc., Rochester, N. Y.

FSP-51-47

Studies of Moisture at High Rates of Evaporation

By A. R. MUMFORD,[1] NEW YORK, N. Y.

In this paper the author gives particulars of studies made in an attempt to reduce or eliminate the moisture in steam from boilers operated at the high evaporative rate of 25 lb. per sq. ft. of boiler and furnace surface, using 90 per cent make-up and full chemical treatment in the boiler for scale elimination. From these studies he concludes that:

1 The concentration of the hydroxyl radical is a factor in the production of wet steam which acts independently of the rate of evaporation, or of the concentration of other solid or dissolved chemical substances, and which has a critical value varying with the boiler design.

2 The production of dry steam at high rates of evaporation is partly a problem of reentrainment of the separated water due to inadequate drainage.

3 If it is assumed that the physical condition of the water-steam mixture on any boiler heating surface varies only with the rate at which heat is being absorbed by that surface, then the problem of moisture elimination is reduced to one of steam-path design applicable to boilers in general.

THE purpose of this paper is to describe certain studies of a specific moisture problem and to draw conclusions as to the causes of moisture entrainment from some features which were common to these studies. The problem was the reduction or elimination of moisture in steam from boilers operated at rates of 25 lb. evaporation per square foot of boiler and furnace surface, with 90 per cent make-up and with full chemical treatment in the boiler for the elimination of scale. The conclusions drawn from these studies are:

1 The concentration of the hydroxyl radical is a factor in the production of wet steam which acts independently of the rate of evaporation, or of the concentration, within the range of these studies, of other solid or dissolved chemical substances, and which has a critical value varying with the boiler design.

2 The production of dry steam at high rates of evaporation is partly a problem of reentrainment of the separated water due to inadequate drainage.

3 If the assumption that the physical condition of the mixture of steam and water on any boiler heating surface varies only with the rate at which heat is being absorbed by that surface is acceptable, then the problem of moisture elimination is reduced to a problem of steam-path design applicable to boilers in general.

This paper will of course offer data to substantiate the foregoing conclusions, but the author frankly offers the conclusions only as an incentive to discussion which, it is hoped, will awaken interest in the basic causes of wet steam and perhaps indicate the lines which future investigations should follow.

The author has had no intimate connection with a problem as baffling as that of moisture in steam at high boiler capacities recently completely solved at the Kips Bay Station of the New York Steam Corporation. The studies started in the early winter of 1927–1928 and were continued, under increasing stress, until January, 1929, when the essential solution was reached, the refinement of which is still in progress.

[1] Engineer, New York Steam Corporation. Mem. A.S.M.E.

[2] Presented at a meeting of the Metropolitan Section of the A.S.M.E., New York, N. Y., April 1, 1929.

The first series of studies constituted a search for variables, other than capacity, on which the moisture quantity depended. Water level was found to be a very minor factor in the problem within the range of normal operation. One or two studies showed reasonable and consistent agreement between variations in water level and variations in moisture, but for the greater number of the studies it was found that the amount of water-level variation normally occurring was not sufficient to affect the calorimeters, and the minor variations in this level were neglected during the remaining studies.

Rate of evaporation as a factor in the moisture problem was

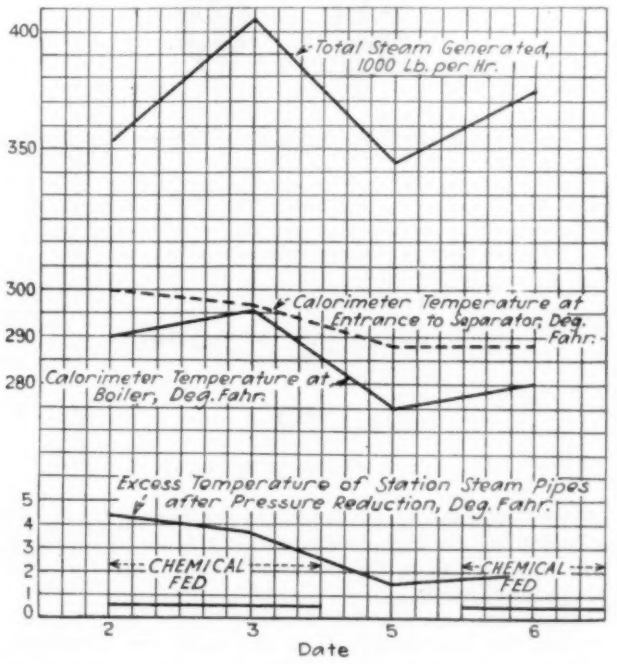


FIG. 1 RESULTS OF FOUR-DAY TEST TO STUDY THE EFFECT OF CHEMICAL ADDITION ON MOISTURE

neglected somewhat in the early studies because the calorimeter indications were not consistent. Some studies indicate less moisture at high rates than had previously been found at lower rates. These indications directed the studies toward the search for some other variable which was independent of rate of evaporation and on which the quantity of moisture depended.

The first variable studied was the design of calorimeters and sampling nozzles. It was found that the number of measurements of moisture in a given steam main varied directly with the number of different calorimeters installed, and also directly with the number of different locations of the same design of calorimeter and sampling pipe. There was erosion of the blading on the feed-pump turbines, yet there were calorimeters which indicated no moisture and others which indicated more moisture than the expansion of the steam could evaporate. Although it was felt that absolute accuracy in the measurement of the quantity of moisture in steam was practically beyond the reach of operating tests, we were not prepared for the unexpected

ability to indicate almost any desired quantity of moisture by slight alterations in calorimeter location or design. That this variation in moisture indication was a serious blow to our established understanding of the sensible conduct of such an investigation was evidenced by the fact that we actually spent considerable time searching for any factors other than moisture which might have caused the apparent erosion of the turbine blading. However, the calorimeter problem was eventually solved by standardizing both design and location on the basis

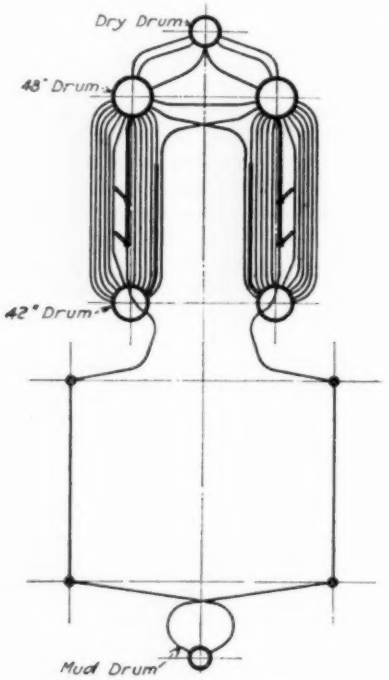


FIG. 2 ELEVATION OF BOILER

of the highest moisture indication of the various types. This standardizing was done in the hope that when such a calorimeter was made to indicate dry steam the turbine buckets would no longer show erosion.

About this time a second variable, that of the addition of chemical for scale prevention in the boiler, was investigated, and then temporarily abandoned as one of the factors in the moisture problem. The operators of the station found that there was apparently some connection between the rate of addition of chemical and the condition of the steam at gland leaks and trycocks in various locations. A test of four days' duration was then made, during which the operators varied the rate of chemical feed from zero to maximum without the knowledge of the test men so that personal opinions could be completely eliminated. The results are shown in Fig. 1, and indicate that the moisture was slightly greater when the chemical was fed at zero rate than when fed at maximum rate.

The curve at the top of Fig. 1 indicates the average rating on each of the four days of the test. The next two curves indicate the average calorimeter temperature at two points on each of the four days. The dotted curve shows the temperature in a calorimeter on the steam main just before the turbine separators, and the full curve shows the temperature in calorimeters in the risers from the boilers. The steam should be drier at the turbine separator than at the boiler riser by the amount of water drained from the mains through the drip lines, and therefore the relative position of the two temperature curves is reasonable. The two curves show no unreasonable difference in trend and therefore are corroborative. Our suspicion of calorimeters was based on the sampling error, so in order to eliminate it, in so far as possible, we took advantage of the drop in pressure from the station header to the street mains. This drop in pressure was about 130 lb. per sq. in., and was sufficient to evaporate approximately 1.25 per cent moisture. If the moisture was less than this amount then the pipe ought to be at a higher temperature than that corresponding to the saturation temperature. The surface temperature of the pipe was generally a few degrees above the saturation temperature equivalent to the operating pressure, and therefore the moisture in the steam before expansion was less than 1.25 per cent unless some unstable condition of supersaturation existed at the point of temperature measurement. Whatever the possible errors in the method, the curve showing the excess temperature has the same shape as the calorimeter curves above, and therefore we have further confirmation of the fact that there was more moisture in the steam on the third day than on any of the other three days. Chemical was fed to the system beginning at noon on the first day and continued for 36 hours. There was then a gap of 24 hours in which no chemical was fed to the boilers, and the test was ended with a 24-hour period in which chemical was fed to the system. The difference in the quantity of moisture indicated by calorimeter temperatures was so slight as to preclude any satisfactory conclusion, but at least it could not be said that the results indicated that the on or off position of the chemical feed valve was a factor in the moisture problem. We have said that the investigation of the effect on moisture of the addition of chemical to the feedwater was temporarily abandoned after this test, and this was the case, but in the light of the later studies it is evident that the abandonment was a mistake, because the chemical factor which influenced the moisture happened to have been kept below the critical point found later and was therefore not evident as a factor. Later when the studies, interrupted by construction work, were continued, the critical value of the hydroxyl radical was found, but in the interim the use of me-

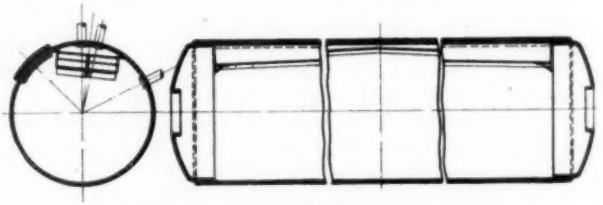


FIG. 3 BOX BAFFLE UNDER RISER TUBES

chanical means such as baffles, etc. to reduce the moisture was studied.

The only feature common to all of the baffle studies was the failure of each study to indicate an improvement in the moisture conditions. For this reason these studies are described only in so far as the type of baffle is concerned, and no attempt is made at this point in the paper to indicate why each type of baffle not only failed to solve the problem, but failed to improve the indications of the moisture condition sufficiently to warrant further study. Later an explanation of the failures will be offered to promote discussion.

Fig. 2 is an elevation sketch of the boiler and furnace presented to fix the locations of each of the six drums of the system. The top drum is 30 in. in diameter and has finally come to live up to its name of "dry drum." The next lower drums are each 48 in. in diameter and contain both steam and water. These drums are referred to as the 48-in. drums. The next lower drums are the 42-in. water drums, in which no work for these studies was done. The bottom drum is the source of water

supply for some of the furnace-wall screens and is referred to as the mud drum.

Fig. 3 shows the nature of the first baffle tried. This was a sheet-metal trough designed to force all steam rising from the boiler tubes to move to the ends of the 48-in. drums before entering the riser tubes and moving to the dry drum. Access to this trough was therefore through the ends, and egress through the top. The thought involved in this design was to change the path of the steam so that gravity would have time to remove the moisture while it was traveling horizontally. Apparently, however, it was ineffective.

Fig. 4 shows a baffle which was placed over the discharge opening of each upper water-wall header in the 48-in. drums. These baffles, of which there were four per boiler, were designed to prevent the steam and water discharged by the water walls from rising directly to the open ends of the box baffle shown in Fig. 3. The steam was deflected toward the center of the drum where it reversed its direction of flow, and mingling with the steam from the rest of the boiler, traveled back to the ends of the drum and entered the open ends of the box baffle. The thought was that the reversal of direction of flow would remove some of the moisture, but apparently the centrifugal force involved was not sufficient for the purpose.

Fig. 5 shows a louver-type baffle which was installed at each end of each 48-in. drum just inboard of the open end of the upper box baffle. This baffle was designed to induce the moisture to

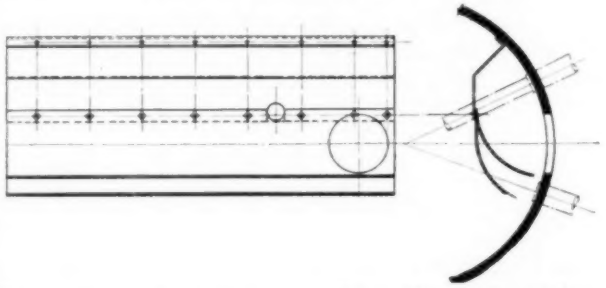


Fig. 4 Baffle Over Discharge From Upper Water-Wall Header

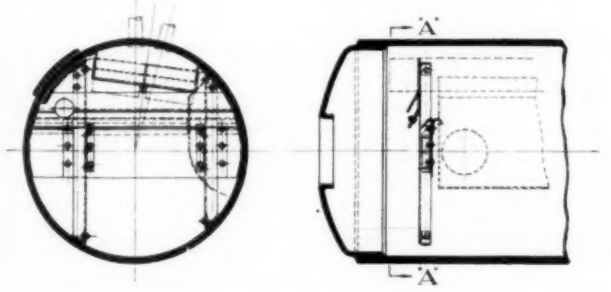


Fig. 5 Louver Baffle

give up its momentum in one direction and accept a new momentum in a new direction differing from that to be taken by the steam. Its action, however, was not of sufficient intensity to cause the indication of the calorimeters to change for the better from a moisture standpoint.

Fig. 6 shows some extension nipples which were connected to those tubes in the baffle row through which the steam and water from the bottom screen, front wall, and arch on each side reached the steam space of the 48-in. drums. In the belief that the long length of travel, due to the series connection of these portions of the furnace heating surface, might result in extremely high velocities at the exit of those tubes in the baffle row, which were connected at their lower end by internal drum connections to the discharge from the arch screen, extension nipples were added to avoid disturbance of the water in the 48-in. drums. One or both of two things must have been true, either the fluid issuing from the tubes was not as high in proportion of steam as was

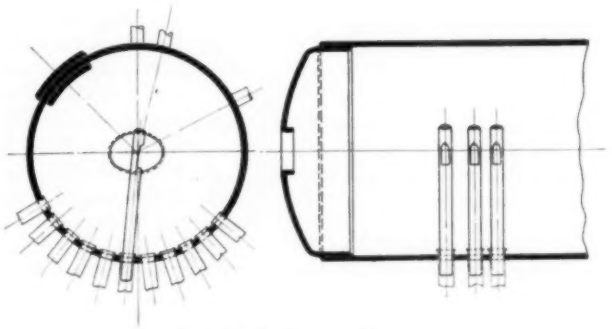


Fig. 6 Extension Nipples

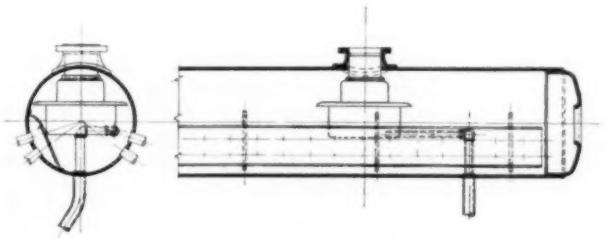


Fig. 7 Dry-Drum Deflector Baffle

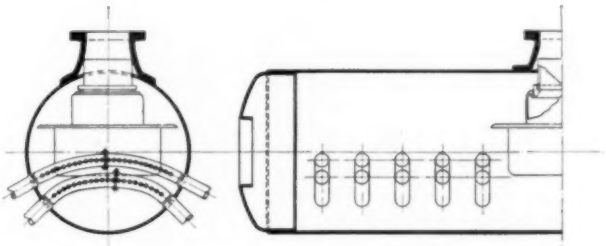


Fig. 8 Perforated Internal Connectors in Dry Drum

thought, or the density of the fluid in the 48-in. drums was so low as to be practically the same as the discharge from the wall tubes, because the calorimeters did not indicate any improvement. These nipples were installed after the baffles shown in Figs. 3, 4, and 5 had been removed.

Fig. 7 shows a simple deflector-type baffle which was installed over the ends of the riser tubes on each side of the dry drum. The designed purpose of this baffle was to deflect the moisture in the steam to the bottom of the dry drum where the drum drains would permit it to return to the 48-in. drums. This installation did not improve the reading of the calorimeters.

Fig. 8 shows a series of perforated internal connectors installed in the dry drum to join the ends of the delivery tubes on each side of the drum. The thought was that the difference in density between the moisture and the steam would cause the moisture to be trapped in these tubes and returned against the low velocity of steam in the riser tubes, to the 48-in. drums. A possible reason for the failure of these perforated connections as well as the failure of the deflecting baffle shown in Fig. 7 is that the amount of water separated from the steam in this drum exceeded

in amount the capacity of the existing drainage tubes, and therefore had no choice but to be reentrained and thus leave the boiler through the steam nozzle.

The author feels that the foregoing studies are logical applications of previous engineering experiences brought forward by several engineers who at one time or another were consulted on the problem. Most of the baffles represented types which had accomplished really satisfactory results elsewhere, and which could be reasonably expected to improve the conditions encountered at this plant, but they did not do so. All tests were made with care, and only one of the known variables affecting moisture was studied at a time.

Therefore, after having mentioned those studies the results of which were purely negative, it is more pleasant to describe others which produced positive results, and which eventually explained the success of the changes which solved the moisture problem.

Effect of the Concentration of Hydroxyl on Moisture

The first conclusion drawn from these moisture studies was that there was a critical value of the concentration of hydroxyl above which the production of dry steam was, to say the least, extremely difficult, and below which the hydroxyl seemed to have little or no effect.

Substantiation of this conclusion can be found in the data of practically every study made during the investigation, but rather than offer data which are only repetitive, but three studies are presented.

In each figure the average calorimeter temperature is plotted as a full line near the top of the chart with its ordinates at the left. Immediately below the temperature curve is a dash-line

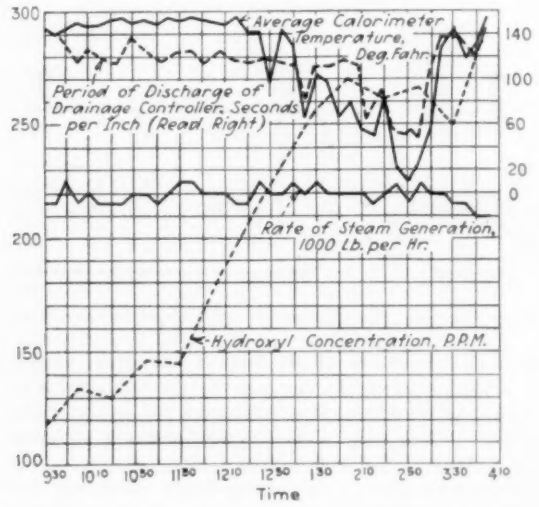


Fig. 9 Effect of Hydroxyl Concentration on Moisture

curve showing the number of seconds during which the drainage tank, for the station steam lines, filled one inch. This curve is merely a substantiation of the calorimeter temperature curve, inasmuch as the amount of water drained from the steam mains ought to vary with the amount of moisture in the steam. The ordinates for this curve are at the right of the chart. The lower full line indicates the rate of steam generation by the boiler under test. The dotted curve shows the concentration of hydroxyl in the boiler water.

In Fig. 9 are plotted the data of the first test revealing the relation of hydroxyl concentration to moisture. The calorimeter temperature was steady until about noon, when the conditions became unstable, and definitely indicated more moisture by 1:00 p.m. The steam increased in moisture until about 3:30 p.m. when it began to get dry. The calorimeter indications were substantiated by the record of the drainage tank, and the boiler rating is shown to have been held constant so that changes in the hydroxyl concentration must have been responsible for the variations. When the concentration of hydroxyl reached a value somewhere between 200 and 250 parts per million, the calorimeters gave indications of an unstable condition. As the concentration increased to about 260 p.p.m. the calorimeter temperature continued to drop, but as soon as the concentration decreased the temperatures rose again. The author left the

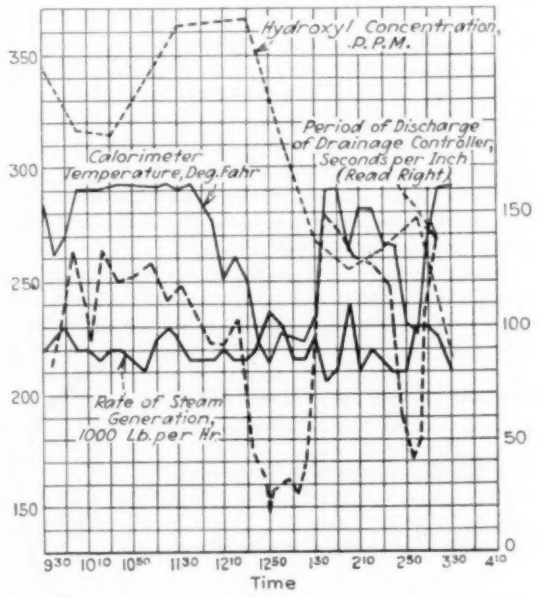


Fig. 10 Effect of Hydroxyl Concentration on Moisture

calorimeters at about 3:45 p.m. on this particular test to observe the nature of the leakage at the turbine valve glands. The conditions were apparently good but just after four o'clock moisture was indicated, and a hurried trip to the calorimeters showed a low temperature which was in agreement with the existing high hydroxyl concentration. The slope of the hydroxyl-concentration curve was changed by blowing down either through the steam nozzle or the blowdown valves. Evidently a lag exists in the relation between the concentration of hydroxyl at the steam-releasing surface and at the water-sampling point which, especially when the concentration is changing rapidly, causes a lag between the concentration and the calorimeter temperature.

In Fig. 10 the study was started with a high initial hydroxyl concentration which was reduced near the middle of the study and further reduced at about 3:30 p.m. The effect on the calorimeter temperature is evident and is confirmed by the period of the drainage tank. The concentration of hydroxyl was higher during the early stages of the study than the author believes it can be successfully carried without trouble. This condition has occurred on other tests but very infrequently, so that he feels that the discrepancy arises from non-uniform concentration due to circulation rather than from any other factor in the problem.

In Fig. 11 the calorimeter temperature, confirmed by the drainage tank, falls in irregular steps as the hydroxyl concentration rises, until the moisture increases sufficiently to stop further increase in hydroxyl concentration. The blowdown at about 12:30 p.m. reduced the concentration sufficiently to permit of

fairly constant calorimeter temperatures for the remainder of the study.

In the opinion of the author there is a critical concentration of hydroxyl the value of which, for these particular boilers, is somewhere between the limits of 220 and 250 p.p.m. None of the apparatus designed for moisture removal which was tested during these studies is sufficiently powerful to dry the steam when the hydroxyl concentration exceeds the critical value mentioned. The question which naturally occurs is, What is the value for other boilers? The author is able to offer Fig. 12 showing some of the data taken on a moisture test of a horizontal water-tube boiler of the box-header type. This boiler was fed with water containing a high concentration of sodium hydroxide, and measurements of steam quality as well as purifier-trap discharge were made to determine the effect of the hydroxide on steam quality.

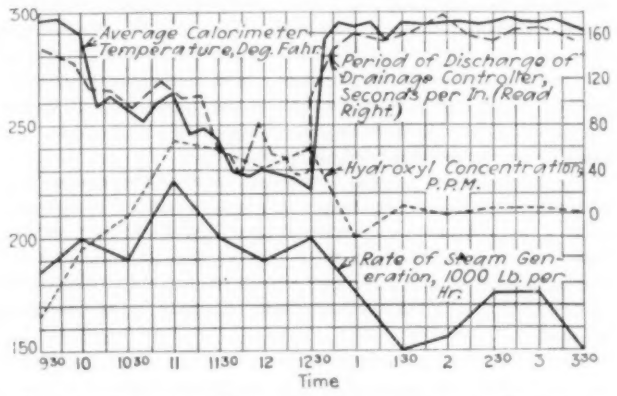


FIG. 11 EFFECT OF HYDROXYL CONCENTRATION ON MOISTURE

The trap discharge was mixed with cold weighed water and the temperature rise of the latter was measured. In this manner, assuming that the trap discharge contained only the heat of the liquid at boiler pressure, the quantity of moisture removed by the purifiers was ascertained. The mixture of trap discharge and weighed water constituted the boiler feed, so that the only loss of chemical was through the steam nozzle. Weighed quantities of sodium hydroxide were added to the feedwater and fed to the boiler. Assuming that the water capacity of the boiler was constant for the period of the test because the rate of steam generation was approximately constant, the concentration of hydroxyl in the boiler was computed from the known amount added. In Fig. 12 the full curve at the bottom shows the computed moisture in steam as indicated by a calorimeter. The dotted curve shows the purifier-trap discharge computed as a percentage of the steam generated. The dash-line curve shows the approximate hydroxyl concentration in the boiler, and the full curve at the top shows the rate of steam generation. The readings of water level are not shown but were of no significance because the water level was held practically constant during the entire test.

For the first two hours of the test the calorimeter read practically constant and indicated 0.5 per cent moisture. A constant calorimeter temperature is sometimes made the basis for the assumption of dry steam inasmuch as it is reasonable to assume that moisture is not present as a constant percentage, but this chart makes no allowances for this assumption nor for any base error in the calorimeter. Instead of the plotted value of 0.5 per cent, the moisture was more probably from 0.0 to 0.25 per cent.

The curve showing the trap discharge has an upward trend as the concentration of hydroxide increases, with intervening variations in almost exact duplication of the rating curve until the hydroxyl concentration reaches about 700 p.p.m., when there is a very sudden increase in the trap discharge to a peak of about 27 per cent of the steam generated.

Although no more moisture was leaving the boiler through the steam nozzle at this point of high trap discharge than had left it previously, the operating conditions became hazardous due to the hammering of the trap lines. The boiler rating was decreased rapidly and the trap discharge reduced by closing a by-pass so that part of the moisture was discharged through the steam nozzle.

The sharp change in the slope of the trap-discharge curve at about 700 p.p.m. hydroxyl concentration indicates that this concentration is critical for this particular boiler.

Neglecting any discrepancy in the accuracy of the measurements concerned with the determination of the concentration of hydroxyl, it is evident that in the bent-tube boiler the concentration of hydroxyl ion above which moisture in the steam increases in amount to a very excessive extent, is much lower than the similar critical concentration for the straight-tube header-type boiler. This indication of a critical hydroxyl concentration on the straight-tube boiler has not been confirmed by repeated observations, as has the value for the bent-tube boiler, but observations of moisture have shown excellent conditions with hydroxyl concentrations of 300 to 400 p.p.m., and previous attempts to build up the hydroxyl concentrations to the value of 1000 p.p.m. for corrosion-study purposes were entirely unsuccessful and led to the study shown in Fig. 12.

The author has found that the concentration of hydroxyl has varied independently of the concentration of other dissolved solids in the boiler water of the bent-tube boiler, and yet when the hydroxyl concentration has reached the range of the critical

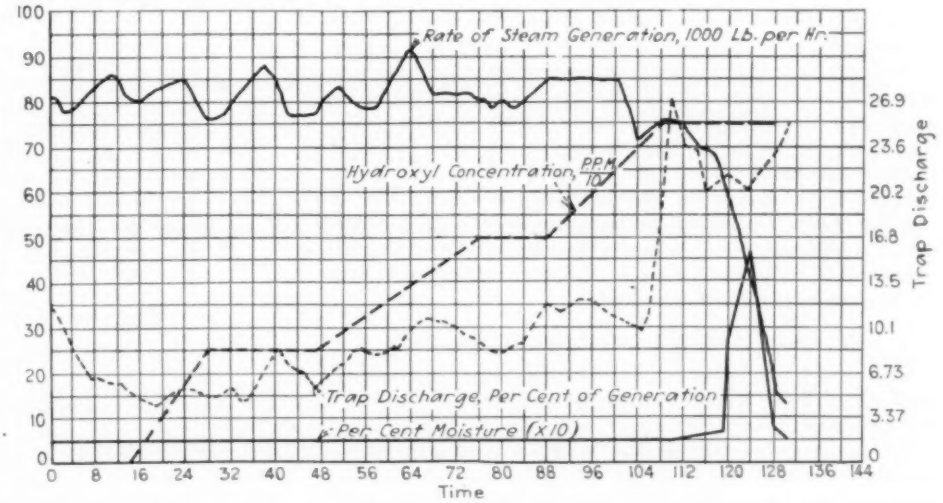


FIG. 12 EFFECT OF HYDROXYL CONCENTRATION ON MOISTURE—STRAIGHT-TUBE BOILER

value the amount of moisture in the steam has almost invariably increased. The effect of the critical concentration of hydroxyl has been the same in boilers just filled with raw water as it was on boilers which had been in the line for weeks, so that the effect of hydroxyl must have been independent of the concentration of suspended matter, which must have varied under the above conditions.

In the foregoing paragraphs it has been shown that the concentration of hydroxyl is a factor in the production of wet steam which acts independently of the capacity and the concentration of other solid or dissolved chemical substances, and which does depend on the boiler design. It may be said at this point, to forestall possible criticism, that several hundred concentrations of the raw water and the boiler water showed practically no organic matter when made acid and subjected to

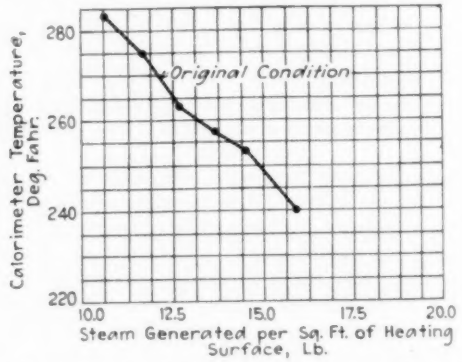


FIG. 13 CALORIMETER TEMPERATURE VARIATION WITH RATE OF GENERATION

ether extraction, so that saponification is not the obvious reason for the action of hydroxyl in this case. The physical or chemical reasons for such findings as have been indicated by the foregoing studies is, in the author's opinion, a good question to lay at the door of the laboratory specialists in these subjects because the complexity of the problem is beyond the range of power-plant work in the field.

The chemical side of the moisture problem, namely, the economic maintenance of low hydroxyl concentration, is at present under investigation, and the indications are that eventually the result will be to maintain a proper hydroxyl concentration without the existing excessive blowdown.

EFFECT OF RATE OF EVAPORATION ON MOISTURE

The second conclusion drawn from this series of studies is that the moisture is partly a problem of reentrainment due to inadequate drainage.

The problem of purifier drainage in plants whose percentage of make-up is of the order of 90 per cent is far more of a problem than in plants whose make-up is 5 or 10 per cent of their generation. Usually the high-make-up plant is a high-moisture plant, and one in which the exhaust from non-condensing auxiliaries is used to heat the raw water. The purifier discharge would generally be too great a quantity for an efficient continuous blowdown, and could not be returned to the feedwater heater without upsetting the heat balance. For the foregoing reasons internal drainage of purifiers was held to be a requirement of these studies and no thought was given to external drainage.

Fig. 13 shows the relation between moisture and rate of evaporation at the beginning of the capacity studies. The function can be said to be linear and shows a calorimeter temperature of 309 deg. fahr., at a 7.5-lb. rate of evaporation per square foot of boiler and furnace heating surface, dropping to the lowest calorimeter indication at a 17.5-lb. rate.

The condition of the unit at this time was exactly as originally installed. There were no baffles or other apparatus in the 48-in. drums, and only the original purifiers in the dry drum. These purifiers had proved their ability to dry steam containing as much as 27 per cent water during the study shown in Fig. 12 when the drains were connected to outside traps. As installed in the bent-tube boiler, however, the difference in elevation of the dry drum and the 48-in. drum was depended upon to provide sufficient head for purifier drainage without resort to traps and their attendant nuisances. We may call this natural drainage, and Fig. 13 indicates how effective this drainage was in removing water from the purifiers. The confidence warranted by past experiences, however, led to the application of means for draining the purifiers to a more complete degree than was possible by the natural head alone.

Fig. 14 shows the relation between the rate of evaporation and moisture after the drains from the purifiers had been equipped with a hydrokineter or aspirator supplied with part of the feedwater as a motivating fluid. There was an indicated decrease of approximately $1^1/_2$ per cent in the quantity of moisture at practically all rates of evaporation, due only to the change in drainage.

The hydrokineter or aspirator is working under somewhat unusual conditions when placed in the steam drum of a boiler, inasmuch as with any practical excess pressure on the feedwater passing through it, it is unable materially to reduce the pressure in its suction line. Any pressure reduction is accompanied by a temperature drop and a resultant transfer of heat from the steam space to the fluid being drained, so that evaporation

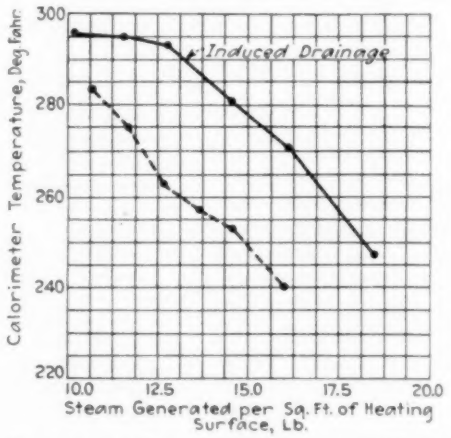


FIG. 14 CALORIMETER TEMPERATURE VARIATION WITH RATE OF GENERATION

takes place, replacing a comparatively inelastic fluid with an extremely elastic one. With this thought in mind the resistance between the purifiers and the aspirators was reduced by duplicating the drainage area of the purifiers and drain piping up to a short distance from the aspirators.

Figure 15 shows the results of decreased drainage resistance. The steam was commercially dry up to rates of evaporation of 17.5 lb. per sq. ft. The improvement was greater at the higher rates than at the lower, although there was theoretically as much room for improvement at the lower rates as there was at the high rates. The only possible conclusion was that there was a base error in the calorimeters which caused them to indicate about 1 per cent moisture when the steam was actually dry. Based on this assumption steam was thereafter called less than $^1/_2$ of 1 per cent when the calorimeters read 305 deg. fahr., or more, although the steam tables indicated that this temperature corresponded to about 1 per cent moisture.

About this time the dryness of the dry drum was investigated by installing some test-gage glasses. The first indications were that water was flooding the drums, and that the drainage system was entirely inadequate for the load placed upon it at these rates of evaporation. Water levels of sufficient height to cover the steam-riser tubes have been measured in the dry drum so that there was little or no possibility of steam leaving the boiler without reentraining moisture. The drainage area was increased to four times the original area, and this increase was sufficient to necessitate only an inch or two of water in the dry drum to provide the entrance loss into the drainage tubes.

Thus far in the studies of the relation of the rate of evaporation to moisture we have not actually inserted into the path of the steam any obstacles to the passage of water. However, because we have made those existing obstacles effective by removing the obstructed water from them, we have in effect installed obstacles which have changed the relation between the rate of evaporation and moisture.

Work was outlined to progress along these lines in the belief that the solution of the moisture problem rested in the separation of the moisture from the steam in the dry drum. The experimental work, however, had to adapt itself to operating requirements to an increasing extent as the winter load came on, and therefore progress was slow. Just after a particularly promising new arrangement of purifier drainage had made conditions worse, because of poorly welded joints at inaccessible locations, the makers of the boiler brought forward a new type of baffle for installation in the 48-in. drums. This baffle fitted into some ideas which had grown out of previous failures and consisted of a perforated plate placed at an angle in each 48-in. drum as indicated in Fig. 16.

The results with this baffle were so pleasing, as shown in Fig. 17, that all further work on the mechanical side of the moisture problem was dropped. The calorimeter temperature remained practically constant at ratings up to 22.5 lb. per sq. ft. of heating surface, and its value was such that the moisture can be said to be less than $^1/_2$ of 1 per cent.

Possible Explanation of Mechanical Results

We have seen that reasonable appearing obstructions to the passage of water mixed with steam failed to reduce the moisture; we have seen that a critical value exists for the concentration of hydroxyl which does not vary appreciably with the rate of working the boiler but which does vary with the boiler design and which has a pronounced effect on the amount of moisture in the steam; we have seen progressive improvement result from the effective use of existing obstructions to the passage of moisture; and finally we have seen the solution of the difficulty result from the installation of an obstruction which in most respects, other than results, does not promise the effectiveness of some of the baffles tried in the early studies. The application of a similar

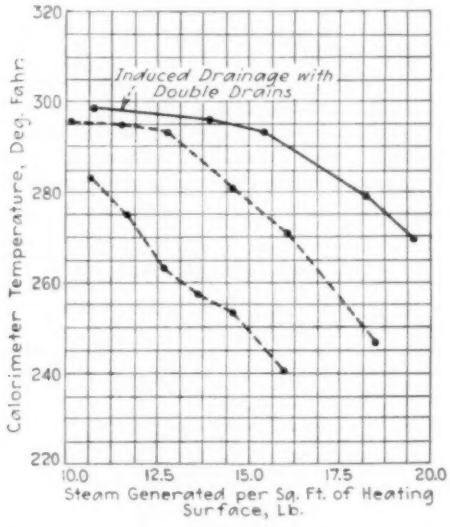


Fig. 15 Calorimeter Temperature Variation With Rate of Generation

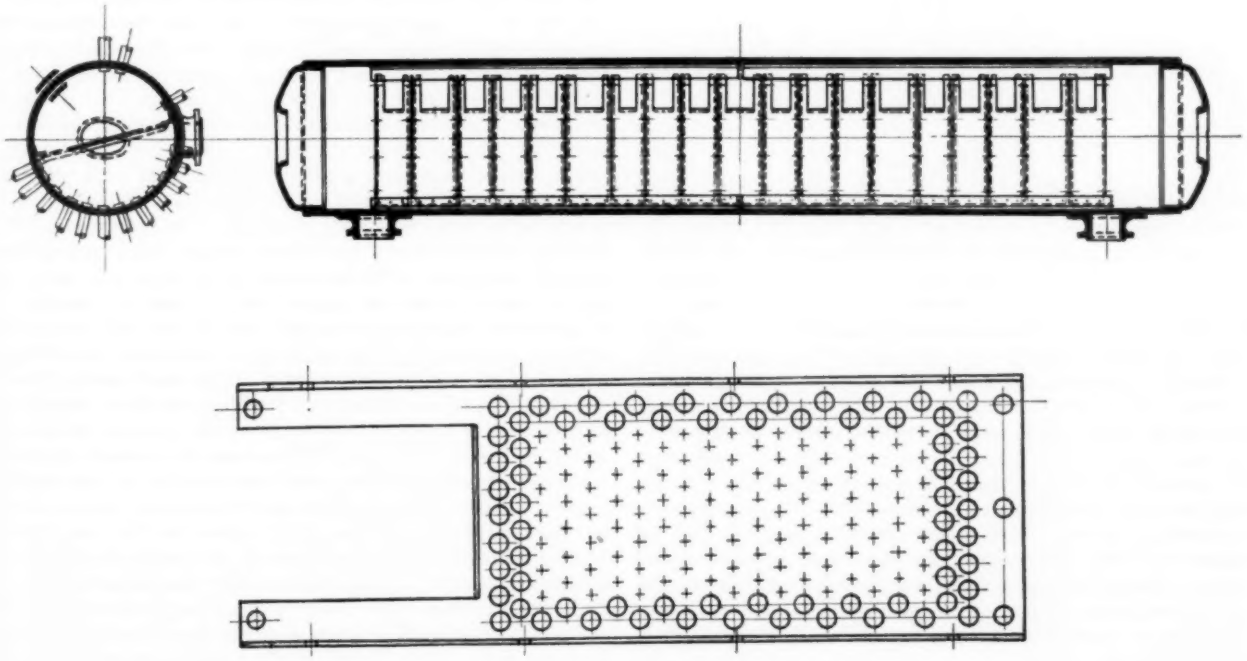


Fig. 16 Perforated Baffle

baffle to another type of boiler might or might not solve the moisture problem, and if we are to learn anything about the problem we must guess as reasonably as possible why one change improved conditions while another made no improvement.

In any consideration of separation of steam from large drops of water, space—or its equivalent, a decrease in velocity—is of paramount importance and probably of greatest physical effect. If separation is to be effective, the difference in density between the fluids is of importance, but of greater importance is the size of the droplets of water mixed with the steam. It is far more difficult to separate an emulsion than a mixture of larger particles of the same specific density from a given fluid, because as the particle size decreases the weight per unit surface area decreases very much faster, and because the surface area influences the resistance to the motion of the particle through the fluid. To expedite such a separation it is necessary to impinge the fluid on a target and coalesce the small particles into larger ones

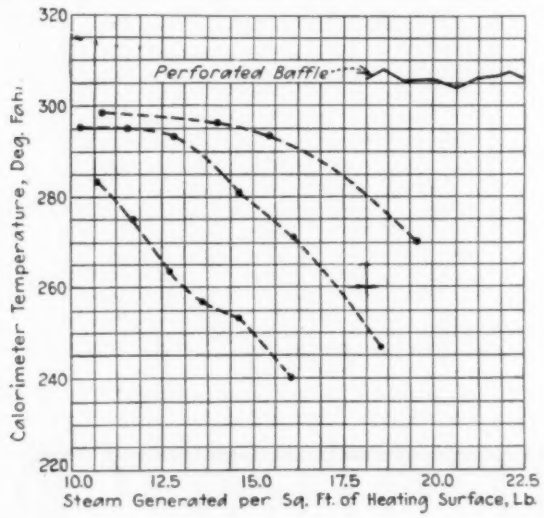


FIG. 17 CALORIMETER TEMPERATURE VARIATION WITH RATE OF GENERATION

of less surface area per unit weight, or to apply some fairly powerful centrifugal force to the fluid to help the coalescing. At Kips Bay or at any station with high rates of steam generation, it is extremely likely that the fluid delivered to the steam space is of the nature of an emulsion. The time necessary to separate the liquid and gaseous phases is probably greater than can be accomplished by simple enlargement of passages resulting in velocity decreases. Therefore the increase in drum sizes is probably not the answer to moisture. The impingement of the fluid on the walls of the tubes connecting the steam drums to the dry drum is effective in coalescing enough of the water in the emulsion to permit the larger droplets to separate in the dry drum, and, if the drainage is sufficient to return this moisture to circulation in the boiler, ratings of 15 lb. per sq. ft. can be carried with only this impingement as provision for drying the steam. With these thoughts in mind, the separation of the emulsion into its components phases calls for increased drum drainage and increased impingement action. The introduction of steam into the dry drum must take place above the level of water accumulation in the dry drum, otherwise reentrainment of of moisture will take place to an extent sufficient to overload any known purifier equipment.

If the foregoing theories or explanations of observed facts are acceptable, then the basic cause of moisture in steam is velocity in upcomers rather than the point at which upcomers are introduced into the steam drum. The velocity of fluid in upcomers as understood in this paper covers not only the physical velocity of the fluid but the nature of the fluid, which latter is dependent to a large extent on its velocity. In this case, therefore, velocity is intended to carry the significance of speed and emulsion.

To substantiate these explanations it is only necessary to refer to other boiler designs whose characteristics are well known compared to the design at Kips Bay. In any horizontal tubular boiler the velocities and the nature of the fluid is almost surely the same as at Kips Bay at points on the heating surface where the heat transmission is as great. However, if the path of the fluid from the lower row of tubes is followed, it can be readily seen that the fluid is subject to successive impingements on the walls of sinuous headers, on the staybolts of box headers, on the tubes leading to the steam spaces, and by meeting the fluid injected into the steam from other tubes. In addition the upper third of the tube bank invariably acts as downcomer area and returns the coalesced water to the boiler circulation without its passing into the steam space. Some such possible and probable explanation of the action of horizontal boilers is necessary to explain the ease with which the final separation is secured in such boilers. In vertical tubular boilers few are operating at ratings comparable with those at Kips Bay, but in all of them where moisture has been a problem the use of impingement has reduced the problem to one within the range of purifiers.

In one type of bent-tube boiler the impingement has been obtained by passing the steam from the front to the middle and thence to the rear drums through connecting tubes, and permitting that part of the emulsion which had coalesced to return to the circulation as it dropped out in the middle or rear drum. In cases in which baffles have been unsuccessful, it can be pointed out that the impingement was not serious enough or that insufficient provision for prevention of reentrainment was made. The next most important consideration to impingement is the removal of the coalesced water by drainage. The successful separation of the water from the steam after the emulsion has been coalesced necessitates a decrease in velocity in order that gravity may act, or if it is impossible to decrease the velocity sufficiently, some centrifugal action must be used to cause the separation in a smaller space.

The water-cooled walls of a furnace usually operate with an average rate of evaporation of 25 to 30 lb. per sq. ft. of exposed area. In the author's opinion, this rate of evaporation is sufficient to emulsify the fluid flowing through these tubes. In order that coalescing of such an emulsion takes place, it is necessary to provide a tortuous path so that the fluid will be subject to successive impingement actions after it has been removed from the heated zone. Increase in space or decrease in velocity, it seems, if the results observed are of any significance, will be insufficient to cause separation. However, because of the increased resistance of a tortuous path and the resulting retarding action on the circulation, a balance must be reached between impingement and resistance, and it will probably be found safer to provide most of the impingement between the steam drums and the so-called dry drums.

The foregoing thoughts are based on the assumption that the condition of the fluid in any boiler is the same at all points of the heating surface where the rate of evaporation is the same, and is of the nature of an emulsion of fine fog increasing in emulsification or fineness as the rate of evaporation increases. If this assumption is acceptable, then the moisture problem can be solved by designing the steam path so that the mist is coalesced into droplets separable by gravity. This coalescence can be accomplished by impinging the fluid at high velocity on targets or by combing the moisture through small apertures. The only other requirement is that sufficient drainage be provided to remove the water as soon as it is separated.

Discussion

N. Artsay.[2] The problem of moisture in saturated steam may be considered from the theoretical side. Many physical facts, laboratory experiments, and theoretical studies, especially those of Wilson and Stodola, indicate that the behavior of ideally pure water at a continuous rise of heat content will follow the Van der Waals' isotherms, i.e., the metastable conditions during the evaporation will be actually present.

In Fig. 18, that of the standard temperature-entropy diagram, the heavy full line represents the behavior of ideally pure water, the region A will correspond to the unstable state of superheated water, and the region B to the unstable state of undercooled vapor. On the pv diagram (Fig. 19) the metastable states are shown more clearly and are denoted by the same letters.

In a boiler the steam and entrained water present a mixture of the stable and unstable phases. The percentages of the two phases will depend upon the degree of water purity, the absence of nuclei of condensation, and the speed of evaporation or rate of heat absorption per unit of circulating fluid below the water level in the boiler drum. The entrained portion will depend upon the degree of impingement and turbulence produced above the water level in the boiler drum. The character of the moisture in commercial steam produced by the two causes will be entirely different. The entrained moisture consists of relatively large droplets having the temperature of saturated steam, while the moisture produced by the metastable phase will be in the form of minute globules of water at a temperature higher than that of saturated steam. Moreover the entrained moisture can be completely evaporated by a sufficient degree of nozzle throttling, while the metastable moisture can be eliminated only by throttling through a porous plug.

It must be borne in mind that the minute globules of water are in thermodynamic equilibrium with the surrounding superheated steam. Wilson's experiments on undercooled water vapor indicate that the maximum degree of undercooling in a mixture of dustless air and water vapor is 8. This means that in order to obtain condensation the absolute partial pressure of the vapor must be lowered adiabatically eight times from that corresponding to saturation pressure in order to break down the unstable state in the absence of extraneous nuclei. In Fig. 19 this represents $p_1 = 8p_2$ numerically. In the presence of extraneous nuclei the condensation occurs sooner in the sense of pressure; for commercial steam ejected from nozzles Stodola found the degree of undercooling to be from 3.2 to 3.5. The conditions of Wilson's experiments with a dustless air-vapor mixture means a persistent condition within the range of stable undercooling. Stodola's jet tests indicate that the degree of undercooling is lessened by the presence of nuclei but that it can persist also for an appreciable time. Wilson found also that ionized molecules can be centers of condensation, or are equivalent to extraneous nuclei like dust particles, for instance.

The reverse process takes place during evaporation. A perfectly pure water in contact with a polished surface will be superheated (points C) to the limit of the unstable state and then a certain part of it will flash into undercooled steam (points D). When the bubble is separated from the wall and begins to rise in the stream of circulating mixture of water and steam bubbles, gradually the undercooled steam will be heated up to the true saturation temperature if the time of travel in the water is sufficient, taking into account the size of the bubble. There are certain considerations which indicate that the heat transfer between the undercooled steam of the bubble and water is low, and hence in the case of very pure water we may expect a certain amount of undercooling in the drum if the boiler is perfectly clean and the circulation is rapid. This undercooling when it ceases after a time will produce a moisture content in the form of minute globules inseparable by jet throttling but effective in the adiabatic expansion.

At 300 lb. per sq. in. pressure the 3.0 degree of undercooling will give 2 per cent moisture, and at the 8.0 degree the moisture will rise to about 4.2 per cent. These are the limiting figures if no heating of the bubble takes place, and they may be read directly on the Mollier chart.[3] It is worth noting that at the critical pressure of 3226 lb. per sq. in. there is no separation of moisture because the metastable phase is impossible. Hence Wilson's limit for undercooling of 8 must locate the maximum value of the saturation curve on the Mollier chart, the value being 3226/8 = 403 lb. per sq. in. abs., while the Keenan chart puts this point around 420 lb. per sq. in., or the points coincide within less than 0.1 B.t.u. per lb. of heat content in the steam. This condition means zero content of both the metastable

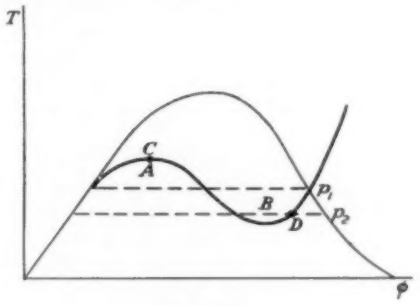


Fig. 18

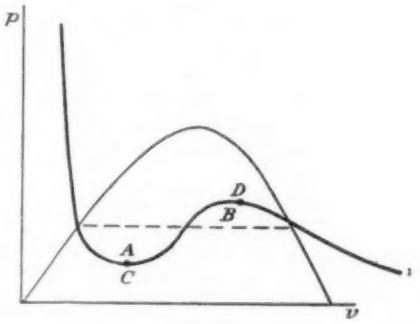


Fig. 19

phases. Wilson's value for the limit of undercooling is a corrected experimental value, but there are also theoretical considerations which indicate that it is uniform for all liquids with more or less symmetrical molecules.

Thus we come to the conclusion that a clean boiler with ideally pure water will deliver saturated steam with moisture content between 0 and 4.2 per cent if running at 300 lb. per sq. in. absolute pressure. This moisture will be in the form of minute globules of water, and impossible to detect by nozzle throttling but detectable in an adiabatic expansion. Now let us take an actual boiler evaporating water containing solid particles in suspension. The rough walls of the boiler tubes will prevent the superheating of water but will not prevent the undercooling of steam in an expanding bubble. Simultaneously the size of the bubbles will be very much reduced. The presence of solid particles and of ions in the bubbles will reduce the degree of undercooling, and we can reasonably expect a drier steam from an actual boiler if the steam is tested for moisture in an adiabatic

[2] Consulting Engineer, New York, N. Y.

[3] Up to 1000 lb. per sq. in. in abs. For higher pressures the heat of the metastable phase is in the form of superheated moisture; this was observed on certain high-pressure boiler tests.

expansion. But in an actual boiler there is another feature which counterbalances the effect of rough walls and of solid particles: the bottom tubes or ends of tubes are usually 10 to 40 ft. below the level of the water in the drum, which means a rise of the boiling point of from 1 to 4 deg. fahr. for boilers carrying 300 lb. pressure. Hence the water which passed these low points in circulation will rise again in a slightly superheated state.

The excess heat will produce additional evaporation which will increase the size of the bubbles already formed, and also produce new minute bubbles around nuclei furnished by the suspended solid particles. The latter fact is most important. The greater the number of nuclei per unit volume, the smaller will be the size of these minute bubbles and the more difficult their separation from the water. It is easy to see that at a certain specific concentration of nuclei in the water volume and a corresponding rate of heat absorption per unit mass of circulating fluid, the latter may become emulsified, causing so-called priming. We see also that the chance of priming is the greater the less the height of the boiler, because the bubbles will be smaller due to less superheating of water. These considerations if applied to the drum itself will furnish the explanation of foaming.

In all these considerations we must remember that the smaller the bubble of steam the more difficult is its separation from water and its ascension to the releasing surface in the drum, and hence the greater the chance of its being drawn down by the circulation, which recirculation will increase the emulsification. As mentioned before, according to Wilson, the ions act as nuclei in condensation, and we can assume that they act as nuclei in the evaporation of superheated water.

The moisture observations of the author made by throttling calorimeters must be accounted for by the entrained moisture, while the effect on the turbine blades is the combined effect of entrained moisture plus the moisture of adiabatic expansion separated on the much more numerous nuclei of the moisture generated by the described process in the superheated water and undercooled steam.

The existence of a critical value for the hydroxyl content may be explained in the following manner. First the hydroxyl content or degree of ionization must be regarded as additional to the nuclei furnished by the solid particles. With rising concentration of hydroxyl the effect of the latter will produce, first, the hidden moisture from the metastable phases and the minute bubbles of steam, which will tend to increase the degree of emulsification of water. At this period the entrainment of large droplets is approximately constant if the rating is maintained, the size of bubbles is appreciable, and their coalescence is possible. Beginning with the "critical" value of concentration, the bubbles formed around the nuclei are too small to coalesce, and broken films of emulsified water will be thrown into the drum, these films being easily picked up by the moving steam, and the entrained moisture content will rise sharply. The coalescing of the bubbles is greatly affected by *turbulence* in the boiler tubes, and hence the greater the velocity of steam-water mixture in the tubes, the larger the bubbles of steam will be. In general, apart from the size and design of the boiler, the velocities in the tubes will be an inverse ratio to the mean density of water-steam mixture in the boiler. The horizontal-tube boilers usually have a reversed flow in part of the top tubes, and this means a recirculation of steam in the circuit which lessens greatly the average density of the mixture. Hence boilers of the B. & W. type will give a larger-sized bubble, and the degree of emulsification will therefore be less and the steam drier. This consideration is amply substantiated by the observations of the author.

GEO. A. ORROK.[4] The author has given us a most interesting account of the solution of a troublesome problem which all operating men meet from time to time. We may lay the trouble to bad feed, improper dry pipes, or circulation overloads, but in the end we come back to the concentration of salts in solution in the boiler water. We have long known that at low pressures and ratings concentrations as high as 3000 grains per gallon are perfectly feasible, while at 250 to 300 lb. and evaporations of 10 lb. per sq. ft. of surface, 300 grains per gallon is perhaps the maximum allowable. The salts in boiler water are nearly always alkaline, a condition conducive to the formation of colloids, which may cause a major portion of the trouble. The author reports that no signs of saponification were found, but colloidal solutions exist without either soaps, starch, or other organic compounds, and they also occupy the ground between solutions and saturated gases. The steam might thus carry large percentages of liquid and solid matter in colloid form not susceptible to any ordinary separating action. Since colloids are solutions of particles coarser than molecular size, we avoid trouble by keeping boiler-water concentrations as low as possible, using mud-drum and surface blow-downs, deconcentrators, and other devices. If by any means these very fine particles can be agglutinated, the colloids will be broken up and the solids can then be removed by filtration. It would appear that the reducing of the hydroxyl concentration was one of these methods.

W. A. SHOUDY.[4] There is little that can be added to this paper, except by way of substantiation. It was the writer's good furtune to be associated with the author during the last period of the investigation described, and he consequently wants to take this opportunity of expressing his appreciation of the manner in which it was carried on, and the belief that the author has done toward solving this important problem.

Like every one else associated in the work, the writer immediately offered a solution based upon sound engineering information as it existed last year, but that solution was tried, with a resulting higher moisture content than existed previously.

The fact that there are now on the market so many steam purifiers is evidence enough that moisture in steam from boilers operating at high rates of evaporation is a serious problem. There are very few published data upon which the engineer may base any conclusions for solving this problem. In the author's opinion, the moisture problem is far more serious than we are willing to admit. The majority of boilers are equipped with superheaters, and these, except in extreme cases of moisture, tend to hide the trouble. To be sure, eventually this moisture content shows up in the superheater as scaling or other troubles, but quite frequently the operator is not aware of this fact for a considerable period of time.

The problem which the author discusses is probably one of the worst that any boiler operator has had to meet. These boilers were operating under extremely high rates of evaporation, which alone has a tendency toward production of high moisture content, but when added to this high rate of evaporation we have water which is 95 per cent make-up, and with full chemical treatment, we have a combination which is very difficult to analyze. Without the careful, painstaking elimination of variables in this analysis as carried on by the author, no solution would have been reached.

We have always attempted to eliminate moisture from steam by a sudden change in direction, when the steam was moving at a relatively low velocity. The elimination of the water was obtained by impinging on a baffle, so arranged that the path of the steam could not again pick up the water.

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[4] Consulting Engineer, New York, N. Y. Mem. A.S.M.E.

In the writer's opinion the author has brought out the fact that the size or condition of the drop of water varies with different types of boilers. If these drops are extremely small, they cannot be separated by any standard methods except by a reduction in velocity so low as to make the separator impractical. If these drops can be made large enough to fall out by gravity, baffles and dry pipes will function.

The second important point which is brought out is that the hydroxyl radical has a greater influence upon priming than has concentration. We have all known that the degree of concentration was not necessarily an indication of priming, and although the author has not proved that the hydroxyl radical is the only reason for priming, he has indicated that at least under one set of conditions it is a direct indication of priming. It is to be hoped that studies of different waters will be made by others interested in the subject, and it is possible that such studies may result in a different control of priming.

G. K. Saurwein.[5] The writer had had experience at a number of boiler houses in by-product coking plants where very little condensate is returned to the boilers. Most of these plants are located in the South and have bad water supply. The water is treated by the hot softening process and filtered before it is fed to the boilers. The concentration in the boilers is controlled by continuous blowdown through orifices in bypasses to the blow-off valves. Blowdown rates as high as 8 per cent of the boiler feed are used, but this clears up all trouble from salts carried over into the superheaters, piping, and steam-using equipment.

The fuel here is surplus gas and coke breeze which in most cases does not warrant the installation of heat-reclaiming equipment. Simple heat exchangers can be applied to recover the heat in the blowdown, thus giving good boiler operation without this waste of heat.

Without having seen the curves and sketches of the paper, the writer would say that he agrees with its findings as expressed in the conclusions at the opening of the paper.

As the concentration increases, the ebullition tears away and the steam carries with it particles of the water, and the only way to remove these particles from the steam current is to provide sufficient baffles upon which the drops may impinge and coalesce into larger drops which will gravitate back into circulation. The steam velocity, moreover, must be sufficiently low so that these drops will not be reentrained. A velocity of 1000 or 1200 ft. per minute will entrain good-sized drops.

Gravity and centrifugal force never "go to sleep on the job" despite the author's discouragements. But they are working against tremendous odds in the turbulence that exists in the steam-relieving spaces in the boiler at these high steaming rates.

At evaporative rates of 4 to 5 lb. per sq. ft. customary a few years ago, the boiler constructions offered then served well enough. The lamentable fact is that the leading boiler manufacturers still offer the same old construction for steaming rates five times as high. The user is left to work out of his trouble by installing special drum baffles, etc., or is compelled to purchase auxiliary devices for drying his steam. If he wants superheat, his difficulties are further increased.

There is great need for development work on the part of the boiler manufacturer to make his equipment suitable for present methods of operation.

Author's Closure

The nature of the chemical conditioning of the boiler water at Kips Bay Station is such that the dissolved material is thrown out of solution under conditions which are not conducive to crystalline growth. The result is that finely divided suspended material is always present, and this would undoubtedly provide innumerable nuclei for the development of minute bubbles of steam. This condition caused the author to consider the fluid circulating in the boiler as having the nature of an emulsion with the accompanying difficulties of separation.

The action of ionized hydroxide may be considered as an addition to the number of nuclei, but the effect of an ionized particle may be somewhat different from that of a solid particle in suspension. The continuation of studies since the presentation of the paper has indicated that with chemical reduction of hydroxyl concentration and consequent relative increase in the concentration of suspended matter, the critical value of hydroxyl concentration has not changed. If ionized particles had nearly the same effect as solid particles, the author believes that a relative increase in the concentration of suspended matter ought to reduce the critical value of the hydroxyl concentration. Such a change has not yet become apparent at Kips Bay Station.

The existence of slightly superheated particles of water in contact with steam as indicated by Mr. Artsay may explain the results shown in Fig. 1. Using the reducing valves as throttling calorimeters indications were that the steam from the boilers was commercially dry although conditions at other points in the system indicated that the term "commercially dry" would not include the amounts of moisture found in the steam.

The possibility of colloidal material in the boiler water becoming a factor in the moisture problem was one of the phases which the author shied away from in this work. It was known that some material, possibly colloidal, was present in the concentrated boiler water, but the intricate studies involved in determining what part the colloids played in the problem were beyond the range of practical work in the field. Under conditions where operation must continue and only such technical work as is essential is practically permissible some attractive phases of such an investigation must be omitted and handed on to the laboratory specialists who have the equipment and time to determine properly how each phase fits into the general problem.

The moisture problem discussed in this paper was productive of many discouraging failures. Mr. Shoudy came in on the problem at the low ebb of experimental results when the non-productive nature of the gradual elimination of one variable at a time seemed likely to extend the studies over an uneconomically long period. The author wishes to express his thanks and that of his co-workers to Mr. Shoudy for his support of the program, and his helpful suggestions in the realization of the solution.

The value of results from an individual plant with particular governing conditions is small unless the results are interpreted and broadened by similar studies under other conditions. The author hopes that such studies will be made and the results interpreted so that the boiler designers may apply them to the improvement of their product with especial reference to capacity.

The thought that low velocity is essential to proper separation of entrained water is the correct basis for attack on the problem of moisture in steam. However, an examination of the cost of boilers indicates that approximately 80 per cent of the cost is represented by drums in a multidrum boiler. To reduce velocities the drums must be increased in diameter and shell thickness if the tube spacing is maintained. The problem of changing design so that lower velocities may be realized is extremely involved, and may offer serious obstacles under existing competitive conditions.

The author believes that the problem can be solved without increasing drum sizes by proper design of steam path so that moisture particles will be coalesced into separable drops without the expense of velocity reduction.

[5] Engineer, Lehman Hall, Harvard University, Cambridge, Mass. Mem. A.S.M.E.

FSP-51-48

Salt Lake City's Fuel and Smoke Problem

A Review of the Current Situation

By GEO. A. ORROK,[1] NEW YORK, N. Y., and W. H. TRASK, JR.,[2] SALT LAKE CITY, UTAH

This paper is an attempt to set forth the smoke and dust conditions in Salt Lake City, discussing the various plans by which an improvement might be secured. In this connection, district heat, gas firing with natural gas and coke-oven gas, and the use of the products of low-temperature carbonization and the by-product coke oven are discussed and evaluated. The connection between the prosperity of Salt Lake City, the State of Utah, and its coal-mining industry is indicated, and certain conclusions looking toward the solution of the problem are stated. The whole discussion in the paper is preparatory to a discussion on the floor of the meeting which it is hoped will clarify the state of opinion regarding this problem in Salt Lake City.

SALT LAKE CITY is unique in many ways. Its population, 155,000, is the largest in the Interior Basin. It is the capital of the state of Utah and has a large industrial population. Within its borders are 25,000 homes, about 14,000 of which are occupied by their owners. The city is laid out on the valley floor at the mouth of the City Creek Canyon, down which flows City Creek, with the outlying ranges of the Wasatch to the north and east, while to the west the valley slopes gently across the Jordan River to Salt Lake, 16 miles away. The site is attractive; the soil, when given the proper amount of water, is most fertile, and it is no wonder that the early pioneers, after passing the defiles of the Wasatch, thought they had reached the promised land. The original city plan, ten-acre-square blocks, with wide streets and the creek diverted to throw a stream of water down each side of each street, has been extended far beyond the original boundaries to meet the expanding growth of the city and its change from an agricultural patriarchate to an industrial city. The early household fires of sagebrush, with mountain mahogany and other woods laboriously carted from the mountains, have been replaced by the boiler and industrial furnace fed with coal from Carbon County, just as the toiling prairie schooners which slowly traversed the dusty trails have been replaced by the Iron Horse fed with the same Carbon County coal.

The climate of the Salt Lake Valley is the ordinary 40th parallel climate of the north temperate zone modified by the altitude, about 4350 ft. above sea level, and the desiccation of the Great Interior Basin: cold winters, warm summers, little rainfall, and that mostly in the three winter months. In earlier times the cool mountain winds rushing in to take the place of the hot air rising from the heated plains swept the dust and smoke over the lake to the desert beyond. But when December came and the plain was frozen there was no return breeze, and the sagebrush smoke hanging low over the city is remembered by many who were there as late as 1890. The smoke from Utah coal was noticeable at that time, when less than 2 per cent of the Utah coal mined up to date had been produced. But thinking of the winter months of 1890 brings the sagebrush smoke of that time to mind.

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Presented at the Semi-Annual Meeting, Salt Lake City, Utah, July 1 to 4, 1929, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

Annual Fuel Consumption of Salt Lake City

Salt Lake City now burns over 700,000 tons of Utah coal per annum, and the smoke nuisance in the winter months is of course immeasurably worse than in 1890. Winter storms may disperse the cloud for a time, but the city is not free from trouble until the mountain winds start again in the spring with the warming up of the plain. While the smoke problem is the worst in winter, the dust problem from coal is also of major importance. Experiments have shown that from 1 to 3 per cent of the coal fired in furnaces of all kinds goes up the chimney into the air, and household fires are even worse offenders in this respect than some industrial plants. The economic loss from smoke is not large, of the order at maximum of 1 per cent, but the dust thrown out is largely unburned coal and coke with some ashes, and in some experiments amounted to over 4 per cent of the combustible. In large plants most of the dust can be caught by dust catchers of various forms, and well-run industrial plants with good dust-catching devices reduce this loss to less than $^1/_2$ of 1 per cent. Smoke, on the other hand, is most difficult to catch; its formation must be prevented. Modern industrial furnaces are designed to do this in a measure, but any high-volatile coal introduced into a hot furnace will smoke even when a most intimate mixture of air and the products of coal distillation is secured. The black, tarry vapors thus produced are persistent and most troublesome. Cleaning fires, which must be done at intervals during the day, always increases the amount of smoke and dust, although proper supervision can reduce this source to a minimum.

The City Council of Salt Lake has an ordinance prohibiting the production of black smoke, defined as No. 3 on the Ringelmann scale. Inspection of plants, observations from the smoke tower with telephone connections to the visible plants, and active cooperation with owners and operators of plants and railways together with the installation of stokers have reduced the smoke from industrial sources since 1919 by 93 per cent. It is considered that 70 per cent of the smoke at present comes from household fires, with comparatively slight chances of much reduction except through a change of fuel. Leaving aside the industrial use which is roughly 400,000 tons per year, there are 300,000 tons of coal burned by 25,000 families and served by at least 50,000 chimney flues from kitchen stoves, furnaces, and open grates. It is obviously impossible to exercise even a general supervision over such a number of prospective offenders, particularly as it is dark when they are most likely to offend. This is the problem as it stands today. And it is the purpose of this paper to place before this meeting and discuss the possible solutions to this problem.

Problem of the 300,000 Tons of Domestic Coal Used in a Year

The 300,000 tons of domestic coal heats the 25,000 buildings, runs the culinary departments for 25,000 families with some assistance from the gas and electric companies, and furnishes many "comfort" fires. The 400,000 tons of industrial coal supplies the locomotives and fires the boilers of a hundred or more industrial plants, mills, brickyards, smelters, and other metallurgical industries, laundries, printing establishments, etc., the district heating stations, and the gas works. These industrial establishments are few in number as compared with the 25,000

buildings, and their chimneys are well marked and can be kept under rigorous inspection. Moreover, stokers, dust catchers, and smoke-abatement devices are in use, and their numbers and efficiency can be augmented. It would appear that it is with the household use of 300,000 tons that this discussion is most concerned. And here a new element has entered into the problem. The Public Utilities Commission last January granted permission to the Western Public Service Corporation to lay pipe lines and import natural gas into Utah, and these lines will be brought into Salt Lake, the price to the user to be 50 cents per 1000 cu. ft. for all gas used above 5000 cu. ft. per month. If all householders would at once change their furnaces and stoves from coal-burning to gas-burning apparatus, all of the dust and

Coke Fuel Available

At Provo, 45 miles south of Salt Lake City, the Columbia Steel Company has a bank of coke ovens where they manufacture by-product coke for use in metallurgical furnaces. Small-sized by-product coke, nut and stove sizes, is a most excellent household fuel for furnaces, stoves, and open grates. Over 8,000,000 tons of this coke was sold in the United States for domestic fuel in 1927, and the use is increasing at a rapid rate wherever it is possible. An additional bank of 40 ovens installed at Provo or nearer Salt Lake City would easily supply the entire domestic consumption. Such a bank of ovens would cost with all auxiliaries roughly $2,500,000, or if built next to the Columbia ovens, using their by-product plant, might cost $1,750,-



Fig. 1 Airplane View of Salt Lake City Taken From the Northwest

much of the smoke from the 300,000 tons of domestic coal would be eliminated. The use of natural gas is not absolutely non-smoke-producing, since an inadequate supply of air results in the production of lampblack. As a matter of interest, 10 per cent of the entire natural-gas production is used in the manufacture of lampblack, and the gas flame is made to smoke for that purpose. Under this plan the Salt Lake domestic-fuel bill instead of being roughly $2,500,000 would be $4,000,000 at least, not considering the cost of changing over to gas burning, which might be set off against the present ash-removal and handling cost. There is no doubt, however, but that the production of smoke would be practically eliminated.

000. Incidentally the coal-mining industry would be called upon to furnish 150,000 tons more coal per annum, and about 3,000,-000,000 cu. ft. of 560-B.t.u. coke-oven gas would be available for metallurgical and domestic use. The cost of the coke at Salt Lake City should not exceed the $8.50 now paid for domestic lump, and no change would be needed in the furnaces, stoves, and grates. Smoke would be very much reduced, but the dust would still be present though in somewhat diminished quantity. If the time of coking could be shortened so that the 2 per cent of volatile matter would be increased to 4 to 6 per cent, a free-burning fuel would be secured for domestic use. This is quite possible in practice, increasing the output and reducing the

cost of operation. Coke is also a good fuel for industrial furnaces, but the price differential would probably prevent its use.

The Utah Oil Refining Company, as a by-product of its business of refining petroleum, markets about 10,000 tons of oil coke per year. This petroleum coke with its 5 to 6 per cent of volatile matter and little ash content is a most excellent smokeless fuel for household purposes. It should be crushed to the small stove size ($1^1/_2$-in. lumps) for the best results. More than 1,000,000 tons of this fuel is sold in the United States per annum. This is a luxury fuel for stoves and open grates and is readily absorbed. Coke-oven gas (560 B.t.u. per cu. ft.) is now largely used in city gas distribution, either by itself or mixed with coal gas, water gas, or natural gas. Being obtained as a by-product from the manufacture of metallurgical coke, the price at which it is sold to the city gas companies is quite low, averaging not far from 20 cents per 1000 cu. ft. Even when distribution costs are added, the public has usually benefited by a lower gas rate where this gas has been introduced. Oil gas has a much higher B.t.u. value, does not mix as well with other forms of gas fuel, and has not been so available for city gas use.



FIG. 2 HUNTS POINT COKE-OVEN PLANT, CONSOLIDATED GAS CO. OF NEW YORK

POSSIBILITIES OF LOW-TEMPERATURE-CARBONIZATION PRODUCTS

Many low-temperature-carbonization processes are in use in Europe and at least half a dozen are commercial, although few dividends have been paid. In this country no process has as yet been commercially successful, although a number have made good fuel and scientifically have been workable processes. The McEwen-Runge process now under trial at Milwaukee is not suited to the production of a household fuel, but the K.S.G. process under construction at New Brunswick, N. J., has this particular end in view though the main product is gas. However, none of these processes have been tried on Utah coals, and he would be a bold prophet to claim success in an untried field in a shorter time than eight or ten years. In England, Sir Geo. Beilby, starting in 1918 on the Fuel Research process, carried it along till his death in 1923. His successor, Dr. C. H. Lander, stated last November that he has 40 ovens in service, but does not expect to be able to call the process commercially successful short of three years. Meanwhile, five or six plants of constantly increasing size have been built, tried out, and replaced until he hopes the present plant of 40 ovens at Kingston-on-Thames will show a commercial profit and pay dividends. The financing in this case, started by the English Government as a war measure, has been carried on partly by Government funds and partly by the London Gas Light Company. The coke produced is an excellent fuel, suitable for the English grate fires, and they have had no difficulty in disposing of their available product. Whether the process would work on Utah coals

is not known. Dr. Lander also says that in general the by-products have shown no advantage over the by-products from the high-temperature coke oven.

Among the American processes the carbo-coal process also originated as a war measure and was financed by the Government. After the improvement of the retort by McIntire with the aid of private financing, a considerable amount of good household fuel was turned out and sold at or about the anthracite price as an anthracite substitute. This process is scientifically workable but has never been able to make its own way and is now shut down. We may conclude that low-temperature carbonization, especially with Utah coals, would have too much experimental work attached to it to bring relief within a reasonable time.

Trent-Process Briquets

During the war another very interesting type of process became known through the experimental work at the Bureau of Mines. This is the Trent process, which takes advantage of the fact that finely powdered carbon is readily wet by certain grades of oil while earthy particles remain suspended in the aqueous solution. The coal is finely ground with about 50 per cent of water, the resulting pulp is thickened, from 5 to 10 per cent of oil is added, and the whole mass is violently agitated. The coal particles are agglutinated into a pasty mass called the amalgam, while the water and earthy particles drain away. The amalgam is a fair fuel in this state, but is smoky owing to the oil which is used as a binder. The amalgam is then passed through a brick machine under considerable pressure, extruded through a nozzle, and cut off into bricks or small briquets. These briquets pass through a retort, which drives off a large portion of the oil, to be condensed and used over again, leaving a hard, semi-porous briquet which burns like the best anthracite coal. This briquet will stand transportation, is low in ash, and makes an ideal household fuel for stove or grate. The experiments at the Bureau of Mines have shown that some Utah coals are suited to this process.

Experimental plants of some size have been installed at Alexandria, Va., Providence, R. I., Toledo, Ohio, and Newark, N. J. The amalgam has been sold in bulk, amalgam bricks weighing 20 lb. have been wrapped in paper and sold for household fuel, and partially coked and fully coked briquets have been sold as an anthracite substitute. However, only the fully coked briquet is smokeless, and with this fuel the dust is much reduced. The complete process has not been tried out on Utah coals, and it is still a question whether in coking the briquet the smoke-forming constituents of the Utah coal would be broken up and driven off to such an extent as to leave a smokeless fuel. Such a plant capable of delivering 300,000 tons of briquets per annum might cost $1,250,000, would use at least 350,000 tons of slack coal which might cost $3.50 per ton, and would require a supply of oil equivalent to about $1 per ton of briquets produced. The margin between this cost and the price of domestic fuel would appear to be sufficient to insure a profit for the undertaking.



Fig. 3 K.S.G. Process Plant, Combustion Engineering Corp., at New Brunswick, N. J.

District Steam Heating

Many communities noted for their freedom from the dust and smoke nuisance are served by the district steam-heating systems which have been increasing in numbers during the last few years. Freedom from smoke and dust means that every building in the community must take the steam service as well as the gas and electric service, consequently many of these steam-heating companies are run in conjunction with the gas and electric company. With the district steam system, the heating of the houses is automatically controlled, the householder being relieved of the firing and ash problem. The cost to the house-

holder is usually from one-third to one-half higher than if he ran his own furnace, the extra cost in lieu of the firing and ash-removal costs. The steam is generated at high efficiency in a large station where the smoke and dust nuisance can be nearly eliminated. Distribution costs are necessarily high, particularly if the city is spread out, and the lengthening mains become more and more costly as the distance increases. Distances of three or four miles are not prohibitive, and many of the smaller industrial plants, hotels, laundries, etc., beside the householders find such a system very attractive. The four district heating plants now operating in Salt Lake City are all in the business district, have a capacity of about 6000 hp., and serve only a small portion of the city. This service could be much further developed, to the mutual advantage of the inhabitants of the city and the steam supply company.

The Householder's Attitude Toward Smoke Abatement

Looking on this problem from the viewpoint of the ordinary citizen, the man in the street so called, the average householder, who owns his six- or eight-room detached house and may be said to be the representative average taxpayer, we find his attitude somewhat disturbing. He sees the locomotive smoke and the industrial chimneys and he does not like it. He sees his neighbors' chimneys also, and when the wind is right the smoke comes in his own windows, and he does not like that. His wife protests that the washing problem becomes worse every day, yet he runs his furnace as usual, has his grate fire for comfort in the evening, pays a man to take his ashes to the sidewalk, and boasts that his total cost for coal and ashes does not exceed $130 per year. He would not buy a better grade of coal at $1 extra per ton. The chances are against his paying $100 for a larger gas connection and the changes to his furnace which might be necessary so that he could use gas and thus avoid smoke. His coal costs him roughly 35 cents per 1,000,000 B.t.u. Substituting gas at present rates he would pay $3.24 for the first 1,000,000 B.t.u. and he sees no economy in that, although he will use the gas stove for some things. With natural gas at 50 cents per 1000 cu. ft. his heat is likely to cost 75 cents per 1,000,000 B.t.u., and he looks askance at the 200 per cent increase. The average citizen is public spirited, but doubling his fuel cost is to him a large price to pay in addition to the 60 per cent increases in his other budget expenses since 1914 which he has had to absorb. District heat at 75 to 90 cents per 1,000,000 B.t.u. would probably interest him, since he might save the cost of his ash removal. If the price was $1.00 to $1.25, a more likely figure, he probably would not consider it. But if he could get smokeless coke or briquets at his present coal price, good salesmanship plus the smoke ordinance would certainly secure his interest, and his public spirit would do the rest.

The industrial use of 400,000 tons may be divided into two portions: the really large plant which can afford to install modern methods of combustion and dust catchers and which are readily amenable to inspection and supervision, and the larger number of smaller installations for which there are three remedies, namely, central-station service, using the smokeless fuel at some additional cost, or combination with other neighboring plants to make an aggregation large enough to be able to afford modern furnaces, good combustion, and dust catchers. Either one of these three remedies would reduce smoke and dust and aid the smoke-prevention service in its work. Modern furnaces and good combustion are greatly aided by the efficient stokers which can now be obtained. These stokers are applicable not alone to the larger sizes of steam boilers, but are now manufactured for small boilers as well. More than 160 stokers are at present installed under the 30,000 hp. of boilers in industrial and heating plants, and about 300 small stokers are in use in domestic heating plants. Installations of this kind are of great use in minimizing the smoke nuisance.

The coal reserves of Utah are estimated by the U. S. Geological Survey to be more than 150 times the yearly coal production of the United States, the work of the last seventy years having depleted these reserves to the extent of less than one-tenth of one per cent. The annual production is not far from 5,000,000 tons, with a value of over $12,500,000. The producing companies, eighteen in number, employ more than 5000 men. The railroad investment in tracks, locomotives, and other equipment serving these mines for coal transportation only is estimated at around $35,000,000. With the allied trades and supply houses it is altogether probable that one-eighth of Utah's population is interested in the well-being of the coal producers, and this fact may explain the protest made against granting the natural-gas pipe-line franchise before the Public Utility Commission.

Conclusions

From the foregoing discussion and its implications we may conclude:

1 That no drastic ordinance or strict enforcement of legal prohibitions of smoke can be of much help in the present circumstances.

2 That no great spending of city tax funds is indicated.

3 That, given the present smoke ordinance and a fund sufficient for vigorous enforcement in a reasonable way, the natural competition between fuels and the economic law of supply and demand will insure a reasonable solution of the problem.

4 One indicated solution under present knowledge is the by-product coke oven supplying domestic coke to householders, coke-oven gas to the gas company, and increasing the use of a most valuable home product, Utah coal. The increase in the number of small household stokers is indicated, and the use of gas in the household will also increase. Competition between supplies and the possibility of mixing coke-oven gas and natural gas will give the gas company a chance to approach nearer the coal price with their services, to the mutual advantage of the company and the citizens of Salt Lake City.

With an assured market for domestic coke and coke-oven gas and the constantly increasing metallurgical industry which is indicated in the reports, the remaining by-products, tar, benzol, and ammonia, should not lack a market; and even if the price obtained should be low, the margin between raw coal and finished products would appear to be above the average. The financing of such a proposition should obviously be handled by the coal industry since it so closely concerns their output and markets, and they should participate in the profits. The city of Salt Lake and its citizens will profit by a smokeless atmosphere, a cheaper gas service, and the extension of the comforts of civilization which follow every major improvement in living conditions.

Discussion

Lewis C. Karrick.[2] This paper, offering an analysis of Salt Lake City's fuel and smoke problem and discussing various fuels and coal-treating processes and other smoke-abatement methods, presents an indefinite and discouraging picture for the Utah coal trade and for those citizens seeking a clear atmosphere. This is not a logical deduction from a thorough analysis of the technical and economic factors peculiar to Utah coals and markets, which favor the complete elimination of all smoke and simultaneous reduction of heating costs. It should have been noted that Utah coals are unique and possess remarkable proper-

[2] Washington, D. C.

ties for producing large yields of rich gas and oils and popular forms of semi-coke not requiring expensive briquetting. The natural-gas enterprise requires for its success that no gas be manufactured for local distribution, and therefore the natural-gas enterprise will operate as a barrier to the success of coal-treating plants by absorbing the revenue due from the sale of by-product gas. It will cost the poor man three times as much to heat with natural gas as by low-priced coal. Furthermore, the very nature of Utah coals permits immediate successful commercial operation of well-known processes only slightly modified in order to supply all the popular semi-coke and gas as rich as natural gas, needed in eliminating all the smoke from Salt Lake City, Ogden, and adjacent towns. These conclusions result largely from studies by myself and associates at the Bureau of Mines Inter-mountain Station, beginning in 1919, assisted by state appropriations made available by the late Governor Simon Bamberger.

J. Billeter.[4] This paper is a real contribution to the solution of Salt Lake City's fuel and smoke problem. Some exception must be taken to the suggestion made that the problem can be largely solved by the use of domestic coke. My experience in the City Smoke Department has shown me that by far the most difficult problem of smoke abatement is that which has to do with residences. The use of fuel for this purpose comes under four distinct headings; namely, cooking ranges, hot-water heaters, grates, and stoves and furnaces.

Coal ranges for cooking have been largely replaced by gas and electricity. At present, coal ranges are found only in the poorer district, where, however, they are used quite extensively. The only coal substitute in the solid state at present is coke. This is unsuitable, for the reason that it is hard to ignite, slow burning, and in short is not at all the kind of fuel the housewife wants to use. Consequently, gas and electricity will have to be depended upon to replace the burning of coal, unless at some future date a more suitable, solid, smokeless fuel (made from Utah coal) is obtainable.

During the winter it is customary to heat the hot water needed by installing water coils in the combustion chamber of the furnace and connecting them with the hot-water tank. This arrangement is quite satisfactory in the average home. During the summer, when furnaces are not used, other provisions must be made. Heaters fired by coal (called monkey stoves) and by gas and by electricity are available. All three types of heaters are used extensively. Here again a solid fuel in the form of coke is not available, for the same reasons given as for cooking ranges. Coal must be replaced by gas and electricity or a better-adapted solid smokeless fuel.

The utility of grate fires consists not so much in the heating of a house as it does in the lending of "atmosphere" to a room and making it "cozy like." The wealthier classes use grate fires considerably more than the middle classes, while the poorer classes do not use them at all, as they cannot afford them. Coke will burn satisfactorily, although in many cases the fire has to be started with coal, and due to the indifference of the public, is replenished by coal. The use of logs in grate fires is so small as to be neglected.

Stoves are used to heat one or two rooms, while furnaces are designed to heat an entire house. Burning coke in stoves has not proved practical, because the combustion chamber is too small for satisfactory coke firing. Wherever possible, furnaces are installed and stoves discarded. A great variety of different furnaces, from anthracite to soft-coal burners, may be found. Over 90 per cent are warm-air furnaces. All sizes of coal are used, from lump to slack coal, with larger coal, however, greatly predominating. Over 250 oil burners, over 100 gas furnaces, and over 250 stokers have been installed in residences. Coke has been used by some for many years with very good results.

The sale of coke has trebled within the last few years. Notwithstanding this fact, the gain in permanent customers has been slow. There are several reasons for this.

Many customers have failed to use coke a sufficient length of time to acquaint themselves with the proper method of firing it. Coke manufacturers have failed to provide a uniform high-grade fuel. At one time they deliver to a customer a satisfactory fuel and perhaps at another time one difficult to burn.

In the morning, substituting a slow-burning coke fire for the convenience of a quick, hot coal fire has not been considered very attractive by the public. Then again, there are those residences that cannot burn coke on account of having furnaces that are too small or chimneys that are inadequate. This applies particularly to a number of old houses.

Petroleum carbon (coke) has been available for a number of years. But it was not until last year that, with the help of an active sales campaign, petroleum carbon was sold in any quantity. For best results it must be used in small sizes. At times it has been coated with oil to make it easier to ignite. This practice is objectionable, because the oil drains off, leaving stains on the basement floor. Petroleum carbon is equally as difficult to ignite as coke, and consequently has the same objections.

Coke has a wide field of useful application, but it also has its limitations. These limitations are believed to be of sufficient magnitude to rule it out as a complete solid-fuel substitute for domestic coal. An easier ignitible, freer, and quicker-burning fuel will have to be provided where coke is not suitable.

The piping in of natural gas from Wyoming will have an important bearing upon the solution of the problem. Through a policy of rendering unusual service and carrying out construction work in a highly efficient manner, the natural-gas people have succeeded in winning the public's good will and confidence. Natural gas will be new in Salt Lake City. Only a few people have lived in localities where natural gas has been used. The extent of the popularity and success of the natural gas remains to be seen.

The availability of natural gas will have a strong tendency to unite the coal producers, but at first it may retard any great fuel development, as the success or failure of the natural gas must first be established. Its popularity will no doubt have a deciding influence in determining the feasibility of building a smokeless fuel plant as also the kind of process that would have to be adopted.

In the meantime the installation of stokers and similar devices will be greatly encouraged.

Lloyd Garrison.[5] The problem of smoke production in Salt Lake City is not essentially different from that found in scores of similar American cities. Smoke production is a factor which has heretofore been considered a necessary evil accompanying industrial development and the universal demand for comfort in the home.

Salt Lake City, while it produces no more smoke than other cities of like size, is so located that the smoke produced is not readily dissipated. Situated within a cove of the mountains, and having very slight air movements during the hours of greatest smoke production, the city finds itself with a rather aggravated condition. In 1919 this condition became so bad that concerted action was taken by the citizenry to abate the nuisance. The results so far obtained from this effort, while still leaving much room for improvement, have been very gratifying.

[4] Engineer of Smoke Abatement, Salt Lake City Corporation, Salt Lake City, Utah. Jun. A.S.M.E.

[5] Consulting Engineer, Utah Coal Producers Association, Salt Lake City, Utah.

A survey of conditions was made by Mr. Osborn Monnett, and a campaign of smoke abatement was based on his findings. Mr. H. W. Clark was placed in charge of the Smoke Department of the city. Mr. Clark directed his efforts chiefly toward reduction of the smoke produced by the larger and medium-sized heating plants and the railroads. Corrections in plant design, instruction of firemen, supervision of new plants, installation of mechanical stokers, and rigid supervision all contributed to the success of Mr. Clark's work. Weather Bureau observations of the number of total smoky hours and total hours of dense smoke show conclusively that a material reduction in the total amount of smoke produced in the city was effected. As a matter of fact, this reduction was 42 per cent during the period from 1920 to 1928, inclusive, although during the same period the number of residences increased 35 per cent.

For several years the Smoke Department was unable to devote much time to residence work because of lack of funds. However, during the past two seasons the Chamber of Commerce has been quite active and has been able to secure funds that have enabled the Smoke Department to give considerable instruction to residence owners in proper firing methods. The public of Salt Lake City is much in sympathy with the efforts being made to abate the smoke nuisance, and the records show a fine spirit of co-operation. Also there have been developed within the last few years automatic stokers adapted to residence plants, and many of these have been installed.

Stokers, oil burners, and gas furnaces all contribute their share toward smoke elimination. However, the writer, while chairman of the Smokeless City Committee of the Chamber of Commerce two years ago, made a careful investigation of the whole situation as shown locally and in other cities, and came to the conclusion that the greater portion of the population will continue to use solid fuel. The reason for this is that coal is materially cheaper than other fuels, no change in equipment is necessary, equipment for its use is cheaper than for other fuels, and it is considerably more reliable than competitive fuels. It therefore became apparent that no solution of the smoke problem locally or elsewhere can be expected until a prepared smokeless solid fuel becomes generally available at a reasonable cost. There are a number of possibilities along this line. Several processes have developed to a point which indicates that a low-temperature coke can be produced economically from Utah coals which are particularly well adapted to low-temperature treatment. Also it may be possible to produce briquets of raw, untreated coal which are smokeless.

The coal industry of Utah is keenly interested in any process or method that will successfully produce a solid, smokeless fuel. Such a development will not only make it possible for the coal industry to render a service to the public for which there is an increasing demand, but will also effect desired economies in operation.

J. D. Roberts.[6] As to the most logical remedy for the deplorable smoke nuisance and fuel problem which exists in the great Salt Lake Valley, it seems that a great deal of time and thought has been given to a possible means of reducing this evil without either curtailing the production of coal or offending the sensibilities of the coal producers. In reality the remedy is at hand in the form of natural gas, which will be turned into the mains of the Utah Gas and Coke Company early in September. This fuel, which has cleared the sky lines of many cities throughout the Southwest and has made them more pleasant places in which to live, will be available to practically all of the inhabitants of Salt Lake City, Ogden, and to those towns lying between Salt Lake City and Ogden and west and south of Salt Lake City, at a price equal to the present market price of coal.

The authors state that the use of natural gas is not absolutely non-smoke producing. This statement is true of any fuel containing carbon, but it is a well-known fact that it is practically impossible to produce any smoke by its use in domestic appliances, and the amount of smoke emitted by industrial users is less than negligible, clear stacks being the rule rather than the exception. A visit to Tulsa, Oklahoma City, Dallas, Fort Worth, Houston, or any other city in the Southwest will demonstrate this fact beyond contradiction.

The writer would like to have a detailed explanation of the authors' comparison of the cost of coal and the equivalent costs of natural gas. Under the proposed schedule of rates, the cost to domestic consumers for 1,000,000 B.t.u. is nothing like 75 cents; rather it is from 55 to 58 cents; and it is neither reasonable nor sound engineering practice to compare natural gas and coal on a B.t.u. basis. The fuel bill of the average citizen will not be doubled in cost as stated by the authors, and it is not seen how the reputed 60 per cent increase in other budget expenses since 1914 has anything to do with the present question.

The authors evidently followed the same line of reasoning when they stated that the domestic fuel bill would be raised from $2,500,000 to $4,000,000 per year. This might be possible if the two fuels were compared strictly on their heating values.

The argument that the Columbia Steel Company, of Provo, by installing an additional battery of 40 coke ovens, could supply Salt Lake City with coke-oven gas is untenable. In the first place, the Columbia Steel Company is not a public utility, and it might be averse to becoming responsible over a long period of time for the domestic fuel supply of Salt Lake City. The amount of coke produced at Provo under any conditions would depend upon the market price for coke and its by-products and also the price of other competitive fuels. In addition, it is most improbable that the Columbia Steel Company could afford to sell coke-oven gas to the Utah Gas and Coke Company at such a price as would amortize the present capital investment in pipe lines, wells, and gas properties, which are now the property of the Western Public Service Corporation, owners of the Utah Gas and Coke Company. The authors mention a price for coke-oven gas of 20 cents per 1000 cu. ft. as a possible price to be obtained in this territory. This price would not nearly satisfy the foregoing requirements, and it is the writer's impression that this price is considerably less than that for which coke-oven gas is sold to the Utah Valley Gas and Coke Company, distributors of coke-oven gas in Provo, Springville, and Spanish Fork, the three towns in the vicinity of the Columbia Steel Company's plant.

Numerous attempts to finance low-temperature carbonization plants in Utah have failed, the reason for which is that, as the authors state, in this country no process has as yet ever been commercially successful.

District steam heating would, as stated by the authors, offer a relief from the smoke nuisance, but evidently distribution costs in the residential districts are prohibitive; otherwise, the present plants, which are owned by very wealthy corporations, would have been expanded long ago.

Natural gas for industrial purposes will be sold at rates competitive with other fuels on the market. The displacement of more than 75 per cent of the coal used for industrial purposes, other than in railroad locomotives, together with the use of natural gas in more than 85 per cent of the homes in Salt Lake City, will be the greatest factor in removing the smoke pall from the Salt Lake Valley that we will ever see.

The authors state that the average householder will look askance at the large increase in his heating bill. I might say

[6] Operating Vice-President, Utah Gas and Coke Co. Mem. A.S.M.E.

that we are now contracting for house heating at the rate of twenty-five per day. This does not appear as though the average householder is greatly worried over an increase in his fuel bill.

It is possibly true that one-eighth of Utah's population is affected to some extent by the production of coal, but their interest in the well-being of the coal producers does not seem to extend to their pocketbooks.

With the coming of natural gas, a smoke-prevention ordinance with "teeth" in it can be drafted and vigorously enforced, since there will then be on the market two smokeless fuels, natural gas and refinery coke, at a price within reach of the average citizen, and there should then be no acceptable excuse for polluting the air with coal smoke. The present smoke ordinance is ineffective, as it is a hard matter to inflict a fine for a smoky chimney when there is no smokeless fuel available in any quantity.

Natural gas is not a new and untried fuel. It will do whatever any other fuel will do and better. Its use as a household fuel and to fire the industrial boilers and factories is general in a number of our larger and most progressive cities. Salt Lake City is not watching an experiment. It is merely awaiting a certainty.

FSP-51-49

The Formation and Thermal Effects of Calcium Sulphate Boiler Scale

By EVERETT P. PARTRIDGE[1] and ALFRED H. WHITE,[2] ANN ARBOR, MICH.

A fundamental study of the manner in which calcium sulphate scale forms upon an evaporative surface, a redetermination of the solubility values for calcium sulphate in the boiler temperature range, and measurements of the rate of formation and heat conductivity of calcium sulphate scales produced in an experimental boiler, are described in abstract in this paper.

Evidence is presented to show that the initial deposition of calcium sulphate scale takes place as the result of the evolution of bubbles, either of dissolved gas or of steam, from an evaporative surface, and that the continued growth of scale is dependent upon the fact that the solubility of calcium sulphate decreases with temperature increase. The work of the authors thus presents a new explanation of the mechanism of initial scale formation, supports Hall's theory of the growth of scale, negates the colloidal theory of scale formation, and offers a fundamental basis for the study of scale-prevention methods. Photomicrographs of the early stages of deposition and growth of calcium sulphate scale are shown, as well as photomicrographs of scales produced under varying conditions.

From the results of their own work and from that of other investigators, the authors present a discussion of the present knowledge concerning calcium sulphate scale. While the effect of scale upon heat utilization in modern boilers is believed to be very slight, calculations based on the heat conductivity of anhydrite scale indicate the importance of preventing overheating of tubes due to the insulating effect of scale deposits within them.

1—THE CRYSTAL FORMS OF CALCIUM SULPHATE AND THEIR SOLUBILITIES

The Forms of Calcium Sulphate

RECENT developments in crystallography[3] recognize the existence of three different forms of calcium sulphate:

Gypsum	$CaSO_4.2H_2O$
Hemihydrate	$CaSO_4.\frac{1}{2}H_2O$
Anhydrite	$CaSO_4$

The form which has frequently been referred to as "soluble anhydrite" has been found to possess the same crystal structure as hemihydrate.[4,5] Substances are known which lose water of hydration without change of crystal form, and hemihydrate apparently belongs to this class. "Soluble anhydrite" may therefore be considered as a partially or completely dehydrated hemihydrate, which, in spite of its loss of water of hydration, possesses the distinctive properties, such as crystal form and solubility,[6] of its parent substance.

It is necessary to consider all three of the forms of calcium sulphate in connection with scale formation, although anhydrite is the only form which is stable in the boiler temperature range. Both gypsum and hemihydrate are found in scale produced during short-time tests (up to 48 hours) with solutions boiling under atmospheric pressure. Hemihydrate alone is present in scale produced from solutions boiling under a gage pressure of 25 lb. per sq. in. during tests of similar duration. The occurrence of these forms of calcium sulphate in scale is undoubted, but experiment has shown that they must exist as metastable phases, since gypsum crystals in contact with water at the atmospheric boiling point are gradually replaced by crystals of anhydrite.[5]

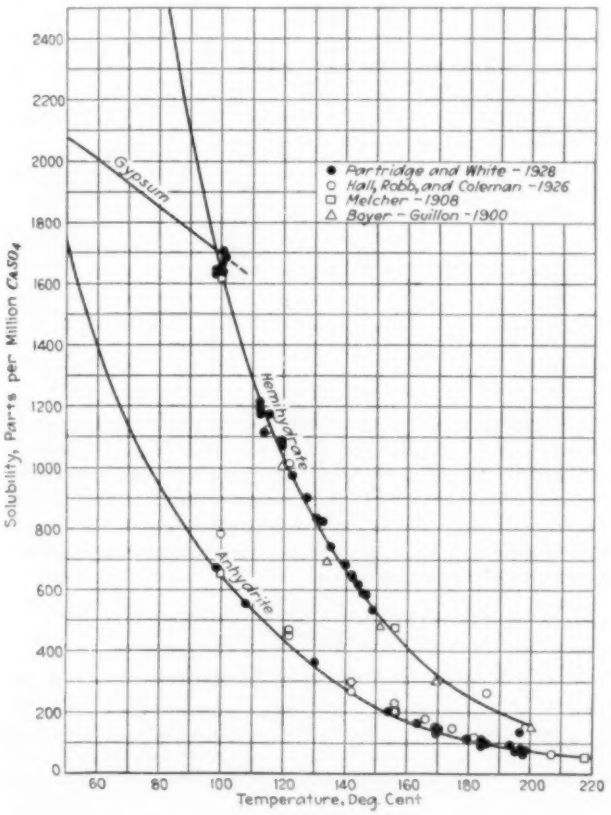


FIG. 1 SOLUBILITIES OF CALCIUM SULPHATE

All three forms of calcium sulphate give scale deposits which are essentially similar in being crystalline, hard, compact, and continuous. The differentiation between the three forms can be made only by crystallographic methods or X-ray analysis.

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[1] This paper is an abstract of parts of the dissertation presented by Dr. Partridge in partial fulfilment of the requirements for the Ph.D. degree at the University of Michigan. The work was supported through a fellowship established by The Detroit Edison Company.

[2] Professor of Chemical Engineering, University of Michigan.

[3] Linck, G., and Jung, H., "Concerning the Dehydration and Rehydration of Gypsum." *Zeit. anorg. Chem.*, vol. 137, pp. 407–417 (1924).

[4] Jung, H., "X-Ray Experiments on the Dehydration Products of Gypsum." *Zeit. anorg. Chem.*, vol. 142, pp. 73–79 (1925).

[5] Ramsdell, L. S., and Partridge, E. P., "The Crystal Forms of Calcium Sulphate." *Am. Mineralogist*, vol. 14, pp. 58–73 (1929).

Contributed by the Joint Research Committee on Boiler Feedwater Studies and presented at the Rochester Meeting, Rochester, N. Y., May 13 to 16, 1929, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[6] Chassevent, L., "Research on Calcium Sulphate." *Ann. Chim.*, vol. (10) 6, pp. 313–351 (1926).

The Solubilities of the Forms of Calcium Sulphate

The solubility values for calcium sulphate will be considered in this paper only in the range above the atmospheric boiling point. Gypsum goes over readily to hemihydrate above 98 deg. cent. or 208 deg. fahr. Although hemihydrate is metastable with respect to anhydrite, it apparently persists over considerable periods of time in contact with water at temperatures from 100 to 135 deg. cent. or 212 to 275 deg. fahr. The solubility curve of hemihydrate must therefore be considered as well as that of anhydrite.

The solubilities of hemihydrate or of anhydrite have been determined by Boyer-Guillon;[7] Melcher;[8] Hall, Robb, and Coleman;[9] and the authors.[10] An early determination by Tilden and Shenstone may be disregarded, although credit for pioneer work belongs to these investigators. Fig. 1 shows the solubility curves determined by the points of the four more recent series of data. In plotting the anhydrite curve, only the minimum individual values obtained by Hall, Robb, and Coleman have been used, since at the lower temperatures their other data fall considerably above the values obtained by Melcher and by the authors. Table 1 gives values at intervals of 10 deg. cent. taken from a large-scale plot of Fig. 1.

TABLE 1 SOLUBILITY OF HEMIHYDRATE AND ANHYDRITE BETWEEN 100 AND 220 DEG. CENT.

(The values in this table are taken from curves determined by the data of Boyer-Guillon;[7] Melcher;[8] Hall, Robb, and Coleman;[9] and the authors.[10])

Temperature		Solubility expressed as parts $CaSO_4$ per million parts solution	
Deg. cent.	Deg. fahr.	Hemihydrate	Anhydrite
100	212	1645	650
110	230	1290	535
120	248	1030	435
130	266	830	350
140	284	665	280
150	302	530	222
160	320	415	176
170	338	325	140
180	356	255	112
190	374	205	92
200	392	165	76
210	410	..	64
220	428	..	55

The significance of the negative slope of the solubility curves of calcium sulphate with respect to the formation of anhydrite scale upon boiler evaporative surfaces has been pointed out by Hall and his co-workers.[11,12,13] In a subsequent section of this paper an attempt will be made to correlate quantitatively the rates of scale formation at various temperatures with the corresponding solubility-curve slopes.

2—THE MECHANISM OF FORMATION OF CALCIUM SULPHATE SCALE

Theories of Scale Formation

Two theories of scale formation have been described in the last five years. The first of these may be called the theory of colloidal precipitation; the second, the theory of deposition at the hot surface.

The theory of colloidal precipitation,[14,15] as described by French and by E. M. Partridge, holds that scale is formed initially as a fine dispersion of solid particles throughout a boiler solution as the latter attains saturation. These particles, suspended in the moving currents of boiler water, strike at random upon the evaporative surfaces, become accidentally attached to them, and gradually form a hard, crystalline mass. Up to the present time

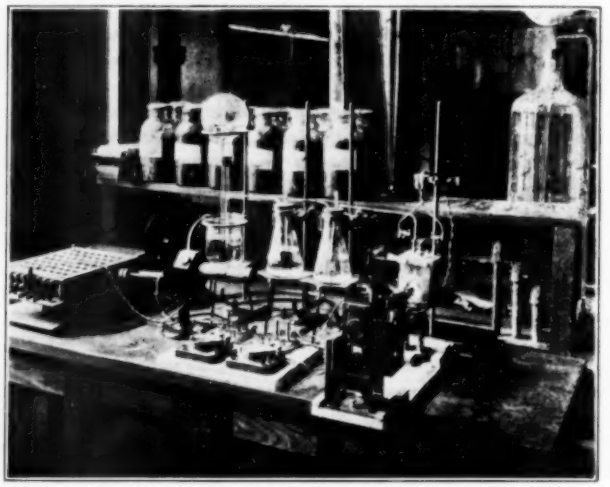


Fig. 2 Apparatus for Photomicrographic Study of Mechanism of Scale Formation

no experimental evidence has been adduced in favor of this theory.

The theory of deposition on the hot surface, advanced by Hall and his co-workers,[11,12,13] is almost exactly opposed to the previously described theory. According to Hall, scale forms only at a surface at which heat transfer is taking place, and its formation is dependent upon the fact that the film of fluid in contact with a heated metal surface must actually be at a somewhat higher temperature than the body of the boiler solution. If the body of solution is, however, saturated with calcium sulphate, such an increase in temperature at the hot surface must cause the deposition of the excess solid material directly at this surface, rather than in the body of solution. Hall has provided experimental verification of the soundness of his theory, both on a laboratory and on a plant scale.

Experimental Work

Reference has previously been made to the fact that the authors have produced scale containing either gypsum and hemihydrate, hemihydrate, or anhydrite, and that in each case the properties of the different scale samples were markedly similar. This similarity makes it possible to study the mechanism of scale formation at the atmospheric boiling point, where the rate of formation is rapid, and a metastable deposit of gypsum is produced, with a good degree of assurance that the scaling process is fundamentally the same as that which takes place in a boiler which is depositing anhydrite scale under modern operating conditions of temperature and pressure.

A photomicrographic study of scale formation on a hot surface

[7] Boyer-Guillon, A., "Study of the Solubility of Calcium Sulphate." *Ann. Conserv. Arts et Métiérs*, vol. (3) 2, pp. 187–211 (1900).

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[10] Partridge, E. P., and White, A. H., "The Solubility of Calcium Sulphate from 0 to 200°C." *Jl. Am. Chem. Soc.*, vol. 51, pp. 360–370 (1929).

[11] Hall, R. E., "A System of Boiler-Water Treatment Based on Chemical Equilibrium." *Ind. & Eng. Chem.*, vol. 17, pp. 283–290 (1925).

[12] Hall, R. E., *et al.*, "A Physico-Chemical Study of Scale Formation and Boiler-Water Conditioning." Bulletin 24, Mining and Metallurgical Investigations, Carnegie Institute of Technology, 239 pp. (1926).

[13] Hall, R. E., "Water Treatment for Continuous Steam Production." *Mechanical Engineering*, vol. 46, pp. 810–817 (1924).

[14] French, D. K., "Internal Treatment of Boiler Water—Proper and Improper." *Ind. & Eng. Chem.*, vol. 15, p. 1239 (1923).

[15] Partridge, E. M., "The Sphere of Boiler Compound–Colloid Chemistry and Scale Prevention." *Jl. Am. Water Works Assn.*, vol. 12, pp. 288–294 (1924).

was made by the authors, using solutions of calcium sulphate boiling under atmospheric pressure. The apparatus is shown in Fig. 2. The principle of the metallographic microscope was utilized by making the evaporative surface upon which the scale was to be formed a highly polished deposit of chromium plate upon a brass base. This metallic mirror formed the front face of a brass box containing an electrical-resistance heating unit. The brass box was immersed in calcium sulphate solution in a brass cell having a glass window on one side, through which a microscope was focused on the mirror surface on which scale formation was expected. Light was supplied by an arc lamp, the beam from which was reflected upon the mirror surface and in turn reflected back through the microscope. A small moving-picture camera adjusted to take single instantaneous exposures and an observation ocular were attached to the microscope so that the process of scale formation could be photographed without interruption of visual observation.

Scale formation was first studied with calcium sulphate solutions saturated at room temperature and containing dissolved air. When these were used it was found that the heat from the evaporative surface caused small bubbles of air to come out of solution on the metal surface before the boiling point was reached. As each of these bubbles grew, it was finally detached from the surface by its own buoyancy, disclosing a ring of crystals which marked the line at which the solid surface, the solution, and the gas bubble had been in contact. The appearance of such a ring, immediately after the departure of the bubble under which it was formed, is shown in Fig. 3(*b*). The small crystals deposited in each ring as the result of bubble evolution grew rapidly, as shown in (*c*) and (*d*), Fig. 3, forming an interlocking network of needle crystals which rapidly obscured the original rings.

In order that the normal course of scale formation might be more closely reproduced in the laboratory, the procedure was then changed to the following. The apparatus was filled initially with distilled water, heat was supplied as before, and the water in the cell was boiled continuously with a make-up of calcium sulphate solution which was deaerated and heated to the boiling point in a series of flasks, vented to the atmosphere, through which it passed slowly on its way to the cell. This deaerating system appears in Fig. 2, together with the automatic-feed solution reservoir.

Starting with distilled water in the cell, between two and three hours were required for the cell solution to arrive at saturation. The first traces of crystal deposition from the initially unsaturated and well-deaerated solutions were, however, strikingly similar to those previously obtained from the evolution of bubbles of dissolved gas. Very fine rings first appeared on the surface, as in (*a*), Fig. 4, and from these rings small needle crystals grew into the solution as before. This growth is shown in the successive pictures in Fig. 4.

Since the extremely low concentration of dissolved gases in the cell solution could not account for the ring deposits produced from the initially unsaturated solutions, it seemed probable that these rings were produced by the evolution of steam bubbles rather than gas bubbles, and that the fundamental mechanism by which calcium sulphate formed scale had been observed.

Some experiments were also made with the evolution of dissolved-gas bubbles from calcium sulphate solutions which were made up to a concentration slightly below the saturation value. It was found that crystal rings were formed by bubble evolution, but that immediately upon the departure of each bubble the crystal ring deposited by it commenced to redissolve, disappearing completely within a few seconds.

A New Theory of Calcium Sulphate Scale Formation

The work of the authors gives the first experimental evidence of the mechanism of initial scale formation, which is shown to depend upon the evolution of bubbles from an evaporative surface. This surface is necessarily hotter than the body of solution, and tends to superheat the film of solution next to it, with the result that small areas of the film suddenly vaporize, forming minute steam bubbles which are actually in contact with the heated surface. The line of contact between each bubble and the surface is ring-shaped. Outside of each ring the heated surface is in contact with the normal fluid film, but within each ring the surface is in contact with dry vapor. The resistance to heat flow of this vapor is much greater than that of the normal film of solution, hence the surface tends to overheat slightly beneath each bubble. This local overheating causes evaporation to take place from the very thin layer of solution surrounding the dry surface. As evaporation into the bubble continues, the latter grows and is finally detached from the metal surface by its own buoyancy, the solution closes over the dry spot on the surface, and the normal fluid film is again established.

When such a process of bubble evolution takes place at a surface in contact with a solution saturated with calcium sulphate, the fluid film at the heated surface must be supersaturated by the increase in temperature at that point. As the bubble forms, the thin layer of this supersaturated solution at the line of contact between the bubble and the surface is subjected to increased evaporation, and is forced to deposit minute crystals in close contact with the metal surface. From the time of their first deposition, these crystals are always in contact with a solution supersaturated with calcium sulphate, at first by evaporation of water into the bubble, and, after the departure of the bubble, by the return of the normal fluid film. They thus tend to grow continuously as long as heat is being transferred across the evaporative surface.

The continued growth of the crystals of calcium sulphate after their initial deposition confirms the theory of scale formation stated by Hall, while all of the experimental evidence contradicts the colloidal theory of scale formation. In addition, the new theory advanced by the authors allows a logical explanation of the formation of the thin calcium carbonate or calcium phosphate scales which are customarily formed upon the evaporative surfaces of boilers using soda ash or sodium phosphate for internal conditioning. Calcium carbonate is known to increase in solubility with increase in temperature, and calcium phosphate is believed to have a similar characteristic; hence, according to Hall's theory, these substances should not form scale on hot surfaces, since they would be more soluble in the fluid film at the hot surface than in the body of the boiler solution. A characteristic property of these scales is the fact that they do not appear to increase in thickness over long periods of time. From consideration of the results obtained with the evolution of gas bubbles from slightly undersaturated solutions, this anomalous scale formation can be explained as the equilibrium between two opposed actions. On the one hand, the evolution of steam bubbles at the evaporative surface deposits small crystals at the minute solid-liquid-vapor interfaces formed under the bubbles, while on the other hand, the condition of undersaturation produced in the fluid film by its increase in temperature over the body of the boiler solution is continually tending to cause these minute deposited crystals to redissolve.

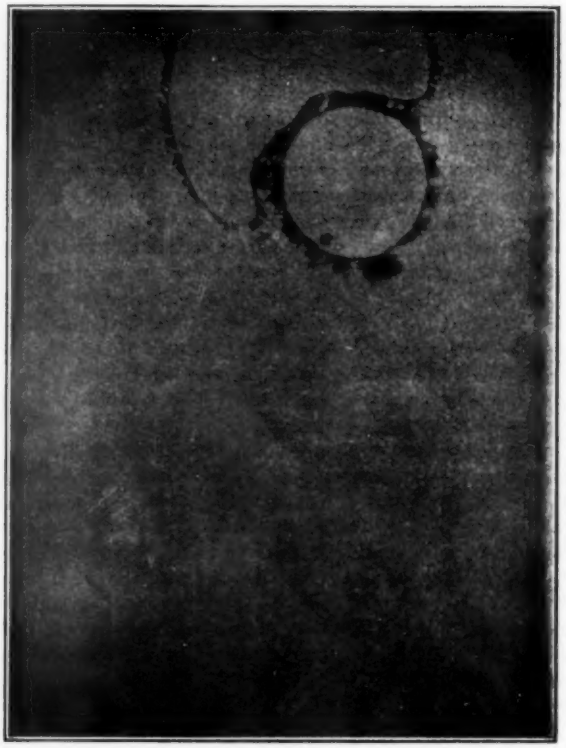
3—THE THERMAL EFFECTS AND RATE OF FORMATION OF CALCIUM SULPHATE SCALE

Previous Investigations

Very little quantitative information is available concerning the resistance to heat flow across boiler heating surfaces caused by the formation of scale on these surfaces. Several series of experi-

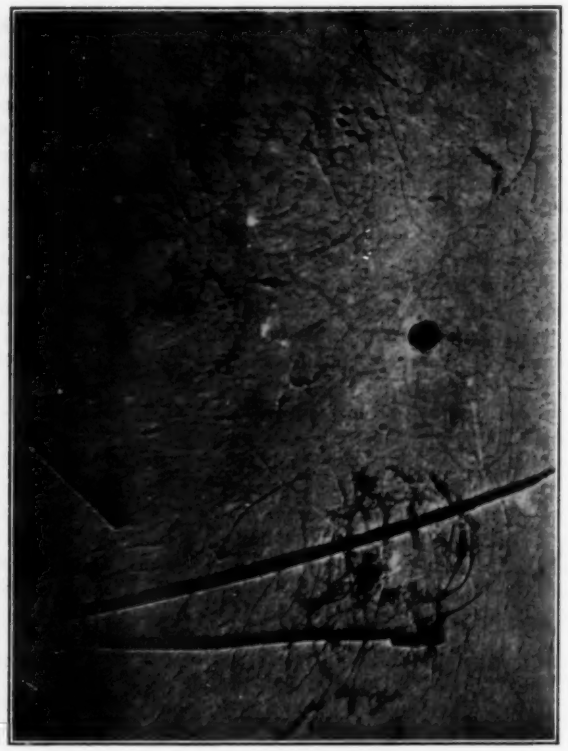


(*a*) Bubble attached to surface. ×75



(*b*) Ring of crystals left by bubble. ×75
[2 min. later than (*a*).]

FIG. 3 DEPOSITION OF CALCIUM SULPHATE SCALE CRYSTALS BY BUBBLE EVOLUTION AT



(*a*) First visible traces. ×75
(Note rings faintly outlined on surface.)



(*b*) Growth of crystals. ×75
[1.5 min. later than (*a*).]

FIG. 4 DEPOSITION OF CALCIUM SULPHATE SCALE CRYSTALS BY BUBBLE EVOLUTION AT

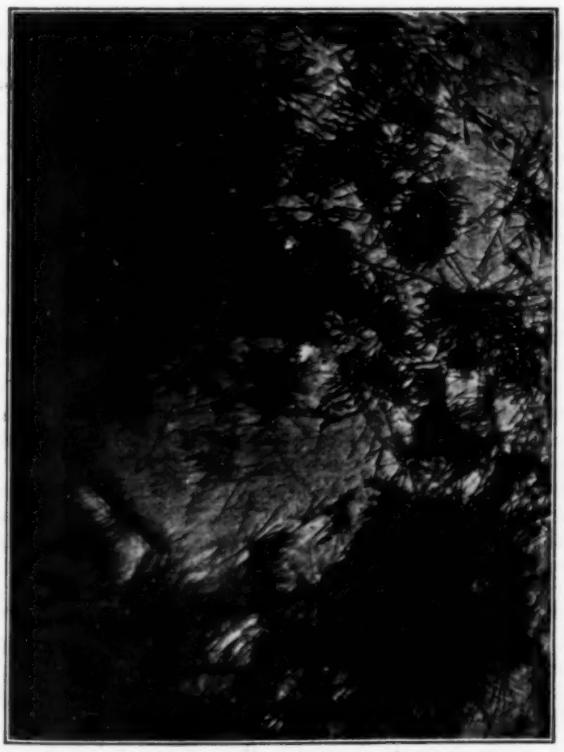


(*c*) Other rings left by bubbles. ×75
[6 min. later than (*a*).]

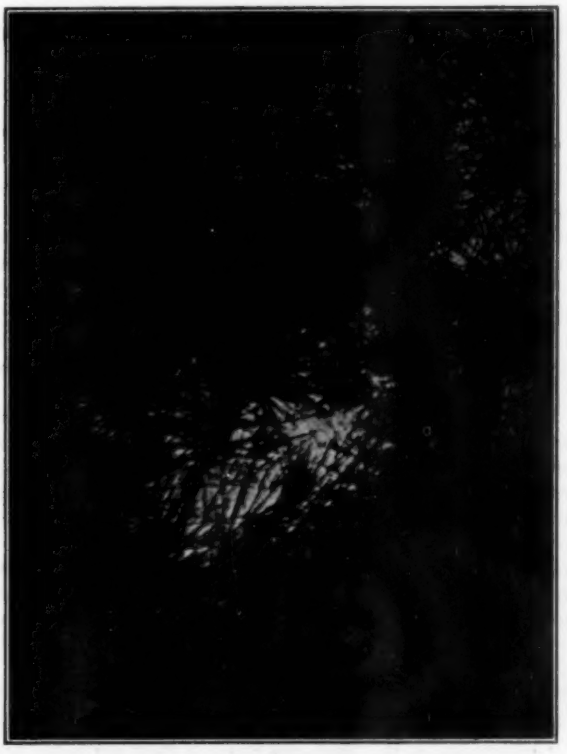


(*d*) Growth of crystals. ×75
[34 min. later than (*a*).]

EVAPORATIVE SURFACE—I: INITIALLY SATURATED SOLUTION CONTAINING DISSOLVED AIR



(*c*) Continued growth. ×75
[7 min. later than (*a*).]



(*d*) Continued growth. ×75
[13 min. later than (*a*).]

EVAPORATIVE SURFACE—II: INITIALLY UNSATURATED AND COMPLETELY DEAERATED SOLUTION

ments were made at the University of Illinois between 1898 and 1905,[16] which indicated decreases, varying up to 15 per cent, in the rate of heat transfer across locomotive boiler tubes which had formed scale in service, and had then been removed and tested under laboratory conditions. The scale formation on these tubes had taken place under widely varying conditions, and the data obtained are dependent upon too many unevaluated factors to be used as conclusive evidence at the present time. One hand-picked excerpt[17] from these data indicates the results much less accurately than it does the bias of the person who compiled it. Individual determinations in the original data vary widely, due both to experimental error and to great variation in the physical structure and composition of the scales tested.

In the original calculations from the test data, arithmetical mean average temperature drops were used. The authors have recalculated the data, using logarithmic mean temperature drops, and have averaged the results obtained for all scales of a given thickness, without any attempt at consideration of other factors or weighting of individual determinations. The results are given in Table 2, which expresses the decrease in overall conductivity of scaled 2-in. boiler tubes expressed as a percentage of the overall conductivity of clean tubes, against the thickness of the scale deposit on the tubes.

TABLE 2 DECREASE IN OVERALL CONDUCTIVITY OF SCALED BOILER TUBES AS A FUNCTION OF SCALE THICKNESS

(Recalculated, using logarithmic mean temperature drops, from the data of Schmidt and Snodgrass, University of Illinois Engineering Experiment Station Bulletin No. 11. Determinations for different scales of the same thickness have been averaged together without attempt at an evaluation of the factors of scale composition and structure and without weighting of individual values.)

Thickness of scale, in.	Average decrease in conductivity, per cent	Number of scales tested	Number of determinations
0.02	2.9	2	5
0.03	4.1	5	9
0.04	6.8	7	11
0.05	4.6	3	5
0.06	6.2	3	8
0.07	4.7	5	8
0.08	8.6	2	2
0.09	8.1	3	6
0.10	...	...	...
0.11	15.8[a]	1	2
0.12	...	...	...
0.13	6.6[a]	1	2

[a] Probably poor values, due to small number of scales tested.

Comparative tests on a locomotive, also reported by Schmidt and Snodgrass, showed a decrease in heat utilization of 9.55 per cent when the principal heating surfaces were covered with a scale averaging 0.05 in. in thickness. These investigators refer to earlier railroad operating tests which indicated decreases in fuel utilization of from 11 per cent to 22 per cent for scaled locomotives as compared with similar ones run with clean boilers. These road tests are, however, to be given little weight as quantitative determinations.

The high values for heat loss due to scale reported in some of the individual tests at the University of Illinois have naturally been emphasized by the proponents of feedwater-treatment systems.[18] Croft, also of the University of Illinois, has recently protested against misinterpretation of the older data, and has published the results of a more accurate test.[19] He found, by heat-transfer tests on a 4-in. No. 9 B.w.g. boiler tube operated at 118 lb. per sq. in. gage pressure, that 0.1-in. and 0.2-in. thicknesses of scale caused respective losses in heat utilization of 1.67 and 2.99 per cent at an operating rating of 90 per cent, and losses of 3.83 and 5.98 per cent at a rating of 225 per cent.

The most complete experimental data on the effect of scale on heat utilization are those published by Reutlinger.[20] This work was done under the supervision of Eberle, who was probably the first investigator to show that figures for heat losses due to scale had been greatly exaggerated. In 1909 he was saying:[21]

> From the results of the experiments it is to be concluded that a scale deposit averaging 5.5 mm. (0.22 in.) in thickness and of average heat conductivity influences the heat utilization in a boiler but very little, so that the determination of this influence by comparative boiler tests in a reliable manner is not possible. As long as the average scale thickness of 5 mm. is not exceeded, which can probably be regarded as the rule for all regulated operation, the decrease in heat utilization due to the scale deposit is not likely to exceed 5 per cent, even with a scale of very low conductivity; with scale deposits of this thickness and medium conductivity this influence is likely to amount to only 2 to 3 per cent.

The work of Eberle and Reutlinger on heat utilization losses due to scale has been quoted by Hempel,[22] but has not been widely disseminated.

Hellemans,[23] more recently, states that both theoretical calculations and actual comparative boiler tests show that the heat losses claimed to be due to scale are exaggerated. In an actual boiler test the removal of scale varying in thickness from 0.75 to 12.0 mm. (0.03 to 0.47 in.) had no appreciable effect on the efficiency of heat utilization.

Thus far, attention has been directed to the effect of scale upon heat utilization. While it is conceivable that this effect may have been important in early types of boilers, it appears that it is very small in boilers of modern design. In considering the heat effects of scale in the latter, emphasis must be shifted from the viewpoint of heat utilization to that of overheating of the boiler metal. It is precisely the heat-transfer surfaces exposed to the most severe conditions on which calcium sulphate or silicate scales will form most rapidly. While a scale deposit in a front-row boiler tube can have no effect upon the absorption of radiant energy by the metal of the tube, it does offer, as it grows in thickness, a steadily increasing resistance to the flow of heat from the tube metal to the liquid within the tube. As a consequence, the temperature of the tube metal must increase in order to drive through the increasing resistance the constant flow of heat it is receiving by radiation. Not a very great amount of scale is necessary to raise the tube temperature to a point at which the steel will fail.

It is interesting to note that, while the conclusions of Eberle and of Reutlinger concerning the slight effect of scale upon heat utilization have not been quoted in this country, the values for increase in heating surface temperature due to scale, taken from Reutlinger's same extensive article, are used quite generally.[24,25] These latter values and a separate determination by Croft will be discussed in connection with the results of experiments by the present authors, which will now be reported. This latter work was intended to determine the actual thermal conductivities of calcium sulphate scales produced upon an evaporative surface, with accurate measurement of the temperature of the surface

[16] Schmidt, E. C., and Snodgrass, J. M., "Effect of Scale on the Transmission of Heat Through Locomotive Boiler Tubes." Bulletin 11, Univ. of Ill. Eng. Expt. Sta., 24 pp. (1907).

[17] "Boiler Scale Prevention." *Elec. World*, vol. 73, p. 1218 (1919).

[18] Applebaum, S. B., "Reducing Costs of Fuel and Boiler Maintenance." *Elec. World*, vol. 71, pp. 349–352 (1918).

[19] Croft, H. O., "What Boiler Scale Does to Heat Transmission." *Power Plant Eng.*, vol. 31, pp. 1001–1002 (1927).

[20] Reutlinger, E., "The Influence of Boiler Scale on the Economy and Safety of Operation of Heating Equipment." *Zeit. Ver. Deut. Ing.*, vol. 54, pp. 545–553 (1910).

[21] Eberle, C., "The Influence of Boiler Scale on Heat Transfer." *Zeit. Bayerischen Revisions-Verein*, vol. 13, p. 61 (1909).

[22] Hempel, H., "Tests Concerning the Influence of Boiler Scale." *Oest. Zeit. Berg.-Hüttenw.*, vol. 58, pp. 267–268 (1910).

[23] Hellemans, *Chim. et. Ind.* (Spec. No. Sept., 1926), pp. 300–308.

[24] "Steam Boiler Engineering." Heine Safety Boiler Co., 27th ed., p. 511 (1920).

[25] "Finding and Stopping Waste in Modern Boiler Rooms." Cochrane Handbook, Cochrane Corporation, 3rd ed., p. 716 (1928).

itself and of the temperature drop from the tube surface to the solution. This work was done in an experimental boiler at several different pressures between atmospheric pressure and 150 lb. per sq. in. gage pressure, and afforded data not only for determinations of conductivity and tube-surface-to-solution temperature drops, but also for the testing of a theoretical expression for the rate of scale formation as a function of the slope of the solubility curve of the scaling substance. This relation will be described in a subsequent section of this paper.

Experimental Work on Heat Conductivity of Calcium Sulphate Scale

Scale-formation tests were conducted in an experimental boiler unit, shown in Fig. 5, in which it was possible to maintain a constant rate of heat transfer across a small tube surface which was in contact with calcium sulphate solution. The temperatures of the tube surface and of the solution were measured by separate thermocouples, and the difference in temperature was also obtained directly by a differential connection. Accuracy in the measurement of the tube surface temperature was obtained by the use of a thermo element of the plated-surface type.[26]

Scale was formed during runs varying in length from 12 to 200 hours. The pressures used were atmospheric, and 10, 25, 50, 100, and 150 lb. gage. The rate of heat transfer was maintained constant during all runs at a value of 18,720 B.t.u. per sq. ft. per hour, by means of constant electrical power input to the tube used for scale formation, and by the use of an air thermostat in which the boiler was suspended. At the end of a run, whenever scale had been formed in an appreciable continuous deposit, the thickness of the scale and the weight deposited per unit of area were measured.

At the beginning of a run the average temperature drop from the tube surface to the solution was found to be 5 deg. cent. or 9 deg. fahr. This temperature difference was due entirely to the resistance to heat flow of the fluid film at the tube surface. The gradual formation of scale upon the tube surface interposed an additional increasing resistance to heat flow which resulted in an increased value for the tube-surface-to-solution temperature drop. As scale formed, the surface in contact with the solution necessarily became rougher than the original smooth metal surface, and consequently the thickness of the fluid film and its resistance to heat flow probably increased somewhat. This would result in a corresponding increase in the temperature drop across the fluid film, but since it was not possible to determine experimentally the amount of this change, and since its magnitude in most cases must have been small, it has been assumed that the temperature drop across the fluid film remained constant at its original value. By subtracting this value found at the beginning of a run with a clean tube from the temperature-drop value found at the end of a run, the temperature drop due to the scale deposit was obtained. From this last value, and from the measured thickness of scale and the known rate of heat transfer, the heat conductivity of the scale was calculated. The data are shown in Table 3 for the six samples of scale which had a measurable uniform thickness.

The last column of Table 3, which gives the heat-conductivity coefficient for the various samples of scale in terms of the customary units as B.t.u. per hour per square foot per degree fahrenheit per foot thickness of material, shows values ranging from 1 to 2 for five of the six samples. The apparent high value of the scale formed at 10 lb. pressure is due to the fact that overall thickness was measured. The unusual structure of this scale, together with the structure of other typical samples, is shown in the photomicrographs of Fig. 6. While the total thickness of the 10-lb. scale was 0.173 in., the thickness of the continuous layer was only 0.028 in., the remainder consisting of individual, unconnected needle crystals growing out radially from the surface. If the layer of solid material is assumed to be the only resistance to heat flow, the value of the heat-conductivity coefficient for this layer comes out as 0.95. From this consideration, which places the value for the 10-lb. scale in line with the others, it seems probable that the actual coefficient for calcium sulphate scales lies between 1 and 2. Within the experimental error of the determinations, the variation between the different forms of calcium sulphate is not distinguishable.

[26] Othmer, D. F., and Coats, H. B., "Measurement of Surface Temperature." *Ind. & Eng. Chem.*, vol. 20, pp. 124–128 (1928).

TABLE 3 HEAT CONDUCTIVITY OF CALCIUM SULPHATE SCALES PRODUCED IN EXPERIMENTAL BOILER

Gage pressure, lb. per sq. in.	Duration of run, hr.	Thickness of scale, in.	Temperature drop through deg. scale, fahr.	Heat-conductivity coefficient, B.t.u. per hr. per sq. ft. per deg. fahr. per ft.
0	12	0.021	31.0	1.06
10	48	0.173[a]	46.0	5.87[a]
25	48	0.056	50.0	1.75
25	48	0.044	47.0	1.46
50	48	0.019	14.4	2.06
150	200	0.0032	5.4	0.93

[a] Note unusual structure of scale, shown in Fig. 6. Although the total thickness of scale is 0.173 in., less than a quarter of this is solid material, the remainder consisting of needle crystals projecting into the solution. If the thickness of the solid deposit is considered alone, the heat-conductivity coefficient becomes 0.95.

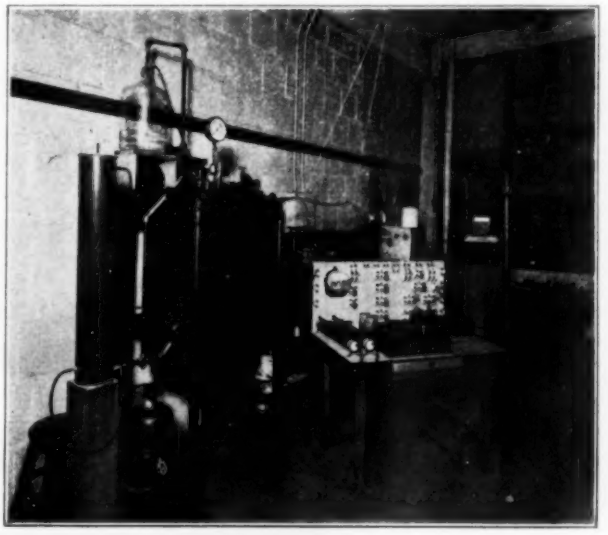


Fig. 5 Experimental Boiler Unit

Discussion of Heat Conductivity of Scale

Reutlinger's data on the conductivity of scale[20] were obtained in an apparatus with a horizontal cast-iron heating surface, and show very high temperature drops from this heating surface, when perfectly clean, to the body of the boiling solution. Thus at a heat-transfer rate of 10,850 B.t.u. per sq. ft. per hour, the temperature drop from heating surface to solution is 30 deg. cent. or 54 deg. fahr. The present authors, using a vertical tube with a polished nickel surface as their heating unit, found a tube-surface-to-solution temperature drop of only 5 deg. cent. or 9 deg. fahr. for a heat-transfer rate of 18,720 B.t.u. per sq. ft. per hour. In the latter case, convection past the heated surface was undoubtedly much more rapid, and the fluid film at the surface was much thinner than in Reutlinger's apparatus, conditions which account for some of the discrepancy between the figures.

The actual temperature drops through Reutlinger's two calcium sulphate scales are quite comparable with the values obtained by

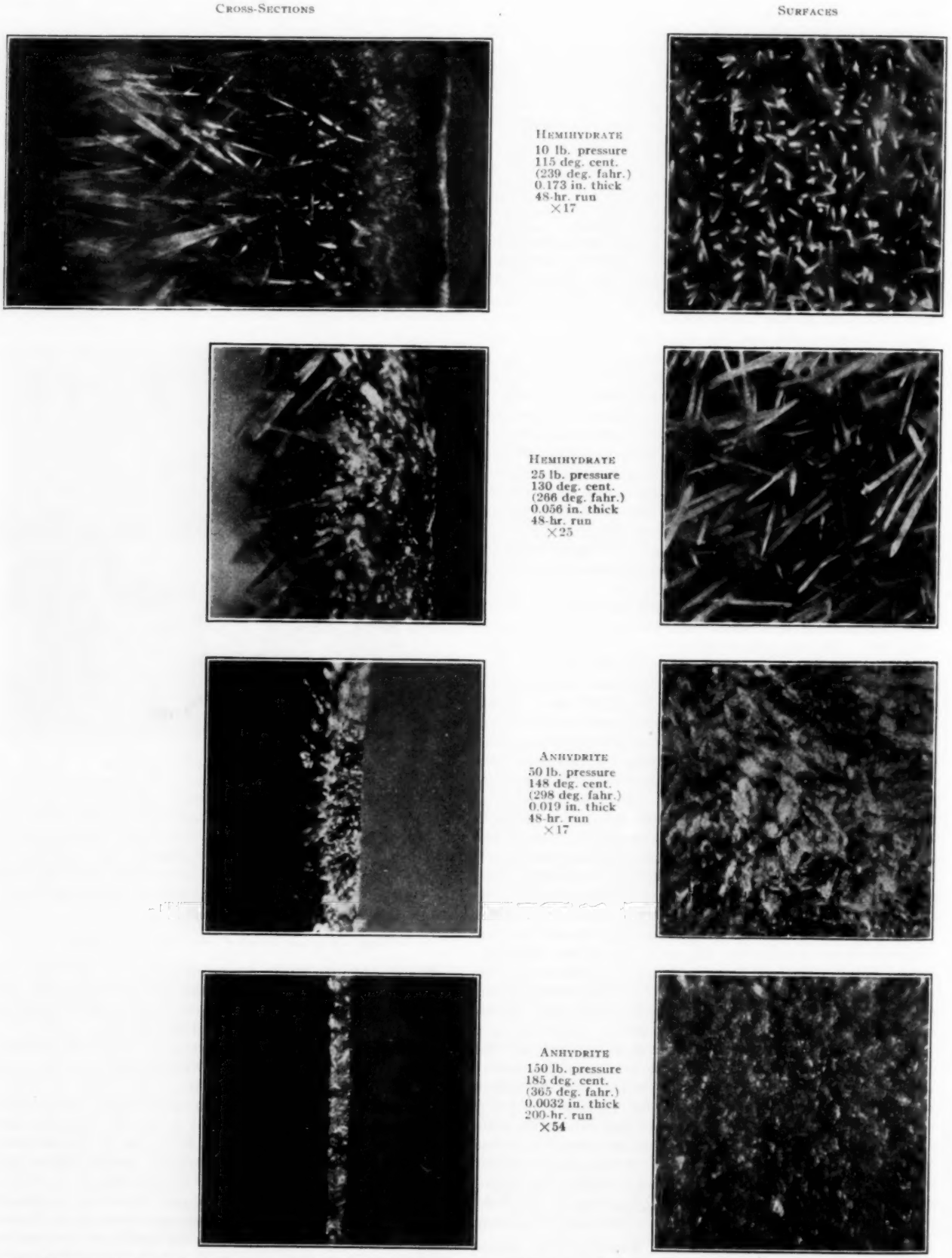


Fig. 6 Comparison of Cross-Sectional and Surface Appearance of Scales Produced at Various Pressures in Experimental Boiler

the present authors, the heat-conductivity coefficients coming out respectively as 1.27 and 1.94 B.t.u. per sq. ft. per hour per deg. fahr. per ft. thickness of material.

The single determination by Croft,[19] previously referred to, showed a temperature drop of 20.6 deg. cent. or 37 deg. fahr. from the clean tube surface to the solution when operated at 90 per cent of rating, or a heat-transfer rate of 3013 B.t.u. per sq. ft. per hour. This temperature drop across the fluid film is even higher, relative to the rate of heat transfer, than those reported by Reutlinger, and seems somewhat excessive to the present authors. A scale deposit of 0.1 in. thickness on the tube used by Croft increased the tube temperature 8.3 deg. cent. or 15 deg. fahr. The conductivity, as calculated by Croft, is equal to 1.75 B.t.u. per sq. ft. per hour per deg. fahr. per ft. thickness of material, a value well in line with those of Reutlinger and of the present authors. All of the values reported fall between 1 and 2, while the average is 1.46.

The front-row tubes and water walls of boilers at the present time are subjected to very severe radiation. It is probable that a large percentage of the heat transferred across these surfaces is received by direct radiation, a condition which would tend to maintain a constant rate of heat input to the metal of the tubes, regardless of the increasing resistance to the conduction of heat away from the metal imposed by a layer of scale depositing within the tubes. If this assumption of constant rate of heat input is made, and the value of the heat-conductivity coefficient for anhydrite scale is taken as 1.5, it follows that a scale deposit of 0.01 in. will cause an increase in the metal temperature beneath the scale of approximately 3.1 deg. cent. or 5.6 deg. fahr. for each 10,000 B.t.u. per sq. ft. per hour passing across the metal surface. For a boiler operating at a heat-transfer rate of 70,000 B.t.u. per sq. ft. per hour across the front-row tubes, each 0.01 in. of calcium sulphate scale would cause an increase in the tube-surface temperature of approximately 22 deg. cent. or 39 deg. fahr.

At 150 lb. per sq. in. gage pressure, the boiler-water temperature is 185 deg. cent. or 365 deg. fahr. Anhydrite scale 0.10 in. thick would, from these figures, cause a tube-metal temperature of 401 deg. cent. or 753 deg. fahr. for the heat-transfer rate cited. From the results of experiments with the formation of scale at 150 lb. pressure in the laboratory boiler, it seems probable that a scale 0.10 in. thick could be formed in 70 days at the heat-transfer rate of 70,000 B.t.u. per sq. ft. per hour, provided the boiler water were continuously saturated with respect to calcium sulphate.

At present there is a double tendency toward the use of higher pressures and of higher ratings for steam boilers. The first tendency narrows the limits for the safe increase of tube temperature over boiler-water temperature, and at the same time increases the probable penalty for tube failure. Its effect in decreasing the rate of scale formation is not known above 200 lb. gage pressure, while the second tendency toward higher ratings tends to increase the rate of scale formation. The net result is that it becomes increasingly important to prevent scale formation as boiler pressures and temperatures are increased. For example, 0.07 in. of scale at 600 lb. pressure, and 0.04 in. of scale at 1500 lb. pressure, will each cause an actual tube temperature equivalent to that caused by 0.10 in. of scale at 150 lb. pressure, the rate of heat transfer being the same as given, 70,000 B.t.u. per sq. ft. per hour, in all cases.

In the preceding calculations the temperature drop across the fluid film has been neglected. While the work of Reutlinger and of Croft indicates that its value may be appreciable, the experiments of the present authors suggest the great influence which convection and surface conditions may have upon its magnitude. By neglecting it in the calculations, the tube-temperature-increase figures are made very conservative. In the calculations the additional assumption has been made that the heat conductivity of anhydrite scale is constant over the temperature range involved. The rate of heat transfer quoted, 70,000 B.t.u. per sq. ft. per hour, is a conservative estimate for the condition in tubes exposed to severe radiation in modern boiler settings.[37]

These estimates as made indicate the part that scale may play in causing the failure of boiler tubes exposed to radiant heat. Another factor which may enter into the failure of tubes from overheating is the possible complete evaporation of water within these tubes, a condition which would result in a tremendous increase in the resistance to heat flow from the tube metal. That such a condition ever exists is not definitely known. As long as the tubes are wetted by a foam, overheating will not occur; on the other hand, only a brief period of complete vaporization would be necessary to cause failure.

Rate of Formation of Calcium Sulphate Scales

During the course of the scale-formation tests it was discovered that the rate of formation was much lower at the higher pressures. It did not seem probable that this effect was actually the result of variation in pressure, therefore an explanation was sought in the corresponding variation in temperature, and in the effect of this variation upon the solubility of calcium sulphate.

Since it was not known at the time which of the forms of calcum sulphate might be present in the scale samples from the various runs, refractive index measurements and X-ray determinations of crystal structure were made on portions of these samples for purposes of identification. These tests showed the results given in Table 4.

TABLE 4 CRYSTAL FORMS OF CALCIUM SULPHATE PRESENT IN SCALE FORMED AT VARIOUS PRESSURES IN EXPERIMENTAL BOILER

Gage pressure, lb. per sq. in.	Temperature		Duration of run, hr.	Crystal form of calcium sulphate
	Deg. cent.	Deg. fahr.		
0	100	212	12	Gypsum and hemihydrate
10	115	239	48	Hemihydrate
25	130	266	48	Hemihydrate
50	148	298	48	Anhydrite
100	170	338	48	Anhydrite
150	185	365	200	Anhydrite

The process of scale formation was then considered from a theoretical standpoint. Given a solid surface across which heat is being continuously transferred to a solution saturated with respect to calcium sulphate, and given an initial deposit of crystals upon this surface due to bubble evolution or to incomplete removal of old scale, further growth of these crystals depends upon the degree of supersaturation produced in the fluid film at the surface by the transfer of heat across it. This degree of supersaturation depends upon the change of solubility of calcium sulphate with the change in temperature from that of the body of solution to that of the somewhat hotter fluid film. For a given rate of heat transfer and constant surface conditions of steam evolution and convection, this temperature gradient would be the same for all temperatures at which the boiler was operated, but the change in solubility over this temperature gradient would vary greatly with the actual solution temperature. For example, by referring to the solubility curve of Fig. 1, it is seen that at 110 deg. cent. or 239 deg. fahr., at which temperature hemihydrate scale is formed, the change in solubility with a 5 deg. cent. or 9 deg. fahr. increase in solution temperature is —140 parts per million, while at 185 deg. cent. or 365 deg. fahr., corresponding to a pressure of 150 lb. gage, under which conditions anhydrite is the form of calcium sulphate produced in scale, the change in solubility for the same increase in temperature is only —12 parts per million.

[37] The Detroit Edison Company. Private communication to the authors.

It is difficult to conceive of experimental refinements which would allow the measurement of the actual temperature gradient from the very thin fluid film at the hot surface to the body of solution, but without knowledge of this actual value it is impossible to state exactly the degree of supersaturation produced in this film at any given temperature of operation. A close approximation may be obtained, however, by considering the degrees of supersaturation produced at different temperatures, but at the same rate of heat transfer, to be proportional to the negative values of the slopes of the solubility curve at these temperatures, using in each case the solubility curve for the form of calcium sulphate produced as scale. The degree of supersaturation produced in the fluid film may, however, be considered as the driving force in the continued growth of scale from the initial nuclei. On this basis the theoretical statement may be made that the rate of scale formation is a direct negative function of the slope of the solubility curve of the scaling substance. If the following notation is adopted: namely,

D = quantity of scale formed
S = solubility of scaling substance
T = temperature
t = time, and
c = a constant,

then the foregoing relation may be expressed mathematically as

$$\frac{dD}{dt} = -c\frac{dS}{dT}$$

This equation was tested with the data obtained from the work with the experimental boiler. Table 5 presents the data. The slopes of the solubility curves were obtained by graphical differentiation of the solubility data of Table 1. The rate of scale formation is expressed in three ways: as the slope of the line representing the increase in temperature difference between the tube surface and solution, or in other words, as the rate at which the resistance to heat flow increased; as the weight of scale, corrected to $CaSO_4$, deposited per square foot of surface per day; and as the thickness of scale deposited per day.

TABLE 5 CORRELATION OF RATE OF SCALE FORMATION WITH SLOPE OF SOLUBILITY CURVE AT VARIOUS TEMPERATURES

(Rate of heat transfer, 18,720 B.t.u. per sq. ft. per hr.)

Gage pressure, lb. per sq. in.	Duration of run, hr.	Solubility slope, p.p.m. per deg. cent.	Rate of scale formation: Slope of temp. drop, deg. cent. per hr.	Rate of scale formation: Weight of scale, lb. per sq. ft. per day	Rate of scale formation: Thickness of scale, in. per day
0	12	−34.0 (H)[1]	1.17	....	0.042
10	48	−27.0 (H)	0.71	0.320	0.086
25	48	−18.3 (H)	0.55	0.279	0.028
25	48	−18.3 (H)	0.55	....	0.022
50	48	− 5.3 (A)[2]	0.063	0.032	0.0096
100	48	− 3.2 (A)	0.013	0.0032	...
150	200	− 2.0 (A)	0.011	0.0102	0.00038

[1] (H) Hemihydrate; [2] (A) Anhydrite.

Due to the difference in structure of scales produced at different temperatures, referred to previously, and shown in photomicrographs in Fig. 6, the thickness of the scales from the experimental boiler is not a good criterion of the rate of scale formation for comparison of values over a wide range of temperatures. When the figures for the slope of the temperature-drop-increase lines from the scale-forming runs at different temperatures are plotted against the values for the slope of the solubility curve at these temperatures, a straight-line relationship is indicated, as shown in Fig. 7. Similarly, when the rate of scale formation as pounds per square foot per day is plotted against the slope of the solubility curve, a straight line probably expresses the best relation, although the fact that the value at atmospheric pressure was not determined makes this relation less conclusive than the previous one. Like points in each set of data fall in similar positions, however, indicating that the slope of the temperature-drop increase is a fair measure of the actual rate of scale formation.

The probable error in the location of points on the rate-of-scale-formation axis in Fig. 7 is rather large, since the scale forming on the tubes was found to break off occasionally during a run. On this account the values for rate of scale formation cannot be taken as exactly quantitative. The data do, however, indicate the probability of a direct proportionality between the rate of scale formation and the negative of the solubility slope.

4—SUMMARY AND DISCUSSION OF SCALE FORMATION

On the basis of the work of other investigators and of that reported in this paper, the authors present the following discussion

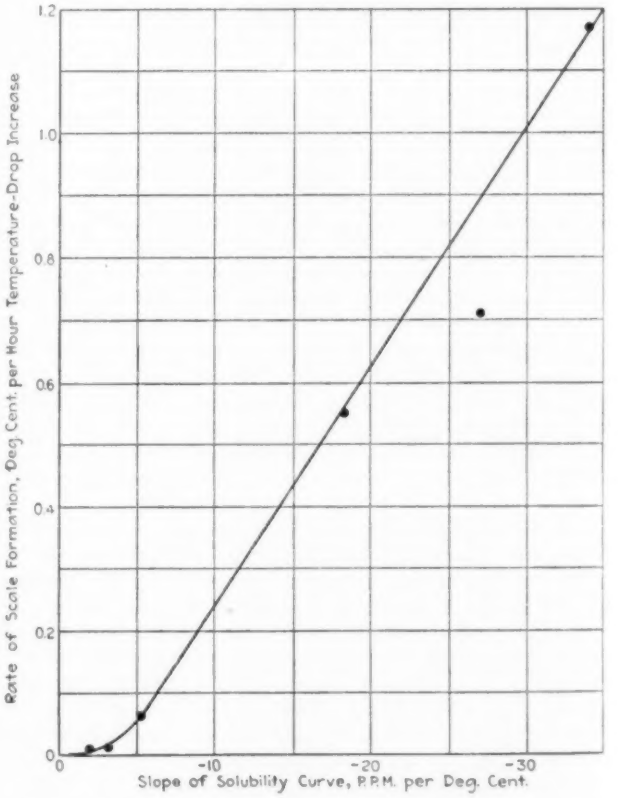


FIG. 7 RATE OF SCALE FORMATION AS A FUNCTION OF THE SLOPE OF THE SOLUBILITY CURVE

which is intended to summarize the present state of knowledge concerning the formation of calcium sulphate scale.

Calcium sulphate is deposited as scale only at heating surfaces, and does not result from the precipitation of minute crystals throughout a boiler solution with the subsequent attachment of these particles to the heating surface. The only way in which calcium sulphate may actually be precipitated in suspension is in the case of a feedwater which contains a concentration of this substance greater than the saturation value corresponding to the temperature at which the boiler is operating. For example, referring to the solubility curves of Fig. 1, if a boiler operating at 150 lb. pressure—that is, at 185 deg. cent. or 365 deg. fahr.—were supplied with a feedwater containing more than 100 parts per million of calcium sulphate, it is entirely accurate to say that minute crystals of anhydrite might be thrown out of solution throughout the body of the boiler solution. These crystals would, however, play no part in the formation of scale, other than by some accidental inclusion in the scale which would be simul-

taneously forming at the heating surfaces as the result of a quite different mechanism.

The actual scale growth depends upon two simple fundamental actions. The first of these is the formation of crystal nuclei on the hot surface by the evolution of bubbles. The actual amount of material deposited in this way by each bubble is infinitesimal, but these minute crystals are of immense importance because they serve as the initial foundation upon which the scale is built.

Since the boiler solution is saturated with calcium sulphate, these crystals on the hot surface are in contact with a fluid film slightly supersaturated, due to the increase in temperature in the film. The crystals thus cannot redissolve, but, on the contrary, tend to grow from contact with this supersaturated film. This process continues uninterrupted as long as the main body of the solution remains saturated with calcium sulphate and heat is transmitted from the hot surface to the solution. While it is probable that small amounts of material continue to be deposited by bubble evolution, nearly all of the growth of the scale subsequent to the initial deposition of nuclei may probably be attributed to the process of crystal growth from the supersaturated fluid film.

Exactly the same dual mechanism of scale formation may result in the growth of calcium hydroxide scales from feedwaters high in bicarbonate and low in sulphate or from waters overtreated with lime, since calcium hydroxide belongs to the class of substances whose solubility decreases with increase in temperature. It is probable that calcium silicate and magnesium silicate scales are also formed in this manner,[12] although solubility data for these substances at boiler temperatures are lacking.

In boilers treated internally with soda ash or with sodium phosphate the formation of thin scales which do not increase in thickness over long periods of time is reported. Since calcium carbonate is known to increase in solubility with temperature increase, the fluid film of solution at a hot surface in this case would be undersaturated rather than supersaturated, and hence any calcium carbonate crystals deposited on the surface would tend to redissolve rather than to grow. It is probable that calcium phosphate is similar to calcium carbonate in this respect. The thin scales reported may quite possibly be the result of the deposition of material by bubble evolution at a very slow rate which is almost balanced by the re-solution of the deposited crystals in the unsaturated fluid film.

Ideas advanced from time to time concerning the use of colloids and of electricity in the prevention of scale must be revised in the light of the present knowledge of calcium sulphate scale formation. Both colloidal and electrical systems for the prevention of scale have been based on the theory that scale-forming materials were precipitated throughout the boiler solution as finely divided, probably colloidal particles bearing electrical charges. The addition of protective colloids was said to cause these particles to settle out as sludge instead of forming hard scale, while electrical treatment was reputed to prevent these charged particles from approaching or adhering to the oppositely charged heating surfaces of the boiler. But it is evident from the work described in this paper that the growth of scale depends upon an entirely different mechanism than the one postulated in support of these scale-prevention systems, and any actual results from the use of colloids or electrical treatment must be ascribed to a different type of action.

The authors have made some laboratory tests which are incomplete at the present time, but which indicate definitely that many of the electrical systems proposed for the prevention of scale have no effect in stopping or decreasing the deposition of calcium sulphate scale from solutions boiling under atmospheric pressure. The use of several types of colloidal materials has also been investigated, with results that indicate that certain of these materials are adsorbed on the calcium sulphate crystals *after initial deposition* and strongly modify the physical characteristics of the scale, in some cases to such an extent that the scale flakes off from the hot surface under the influence of ordinary convection currents. It is therefore probable that some colloids may have beneficial effects in minimizing scale, although the theory advanced in support of their use has been shown to be inaccurate.

The authors have modified the solubility curves for hemihydrate and anhydrite slightly as the result of their determinations, and have advanced the theory that the rate of scale formation is proportional to the negative of the slope of the solubility curve of the scaling substance at the temperature of operation. It is probable that the rate of scale formation is also directly proportional to the rate of heat transfer across the scaling surface, although this relation has not been tested. A third factor which probably exerts an influence upon the rate of scale formation is the velocity of the boiler solution through the tubes.

Current suggestions to the effect that scale will not form upon evaporative surfaces made from specific materials are negated by the fact that the authors have produced exactly similar scales under similar conditions upon surfaces of polished steel, oxidized steel, chromium and nickel and silver plate, and glass. It was also incidentally discovered that in no case did calcium sulphate by itself produce a scale which appeared to be bonded to any degree to the surface upon which it had formed. The scales invariably cracked off cleanly from the surface. What has been referred to as the "adherence" of calcium sulphate scale is probably more accurately a bonding effect resulting from corrosion of the metal surface, combined with the considerable mechanical strength of the scale, which makes its removal from the insides of tubes far from perfectly smooth, a rather difficult piece of work.

Data available in the literature indicate that the effect of scale upon heat utilization in modern boilers is very slight. The prevention of scale in modern boiler practice is, however, becoming increasingly important from the standpoint of the elimination of tube failures due to overheating. Rather small amounts of scale formed on areas exposed to severe radiation will cause overheating of the metal and subsequent failure.

Discussion

HENRY D. FISHER.[23] A recent experience in our plant is very satisfactorily explained by the facts developed in this paper. The New Haven Pulp and Board Company recently installed a 760-hp. Bigelow-Hornsby boiler to operate at 400 lb. pressure, equipped with integral economizer, plate-type air heater, water-cooled bridgewall, and front wall and arch. The total tube area in boiler and economizer is 9234 sq. ft. and heating surface in the air heater 600 sq. ft. It is fired with a Westinghouse underfeed stoker with clinker grinder, total area 207 sq. ft., and has a furnace volume of 4800 cu. ft. single boiler up to 84,000 lb. of steam per hour.

The boiler was put under load March 18, 1929, and operated at ratings of 60,000 to 65,000 lb. of steam per hour, the load being very steady 120 hours per week, with a negligible load over week ends, until April 19, when the average was raised to 70,000 lb. per hour, with a two-hour period somewhat in excess of this. The week-end inspection showed that the fifth and ninth tubes from the left side of the bridgewall were blistered, though no rupture or leakage occurred, and the boiler was taken out of service and these tubes were replaced, as shown in Fig. 8. All the tubes were turbined, but comparatively little scale was found except in the bridgewall tubes. In these in the bare portion the scale was 0.034 to 0.049 in. thick on the side of the

[23] Plant Engineer, New Haven Pulp and Board Co., New Haven, Conn. Mem. A.S.M.E.

tube toward the fire, the thicker scale occurring toward the upper end of the tube, but becoming thinner at the sides, and on the back, against the brick wall, there was a space of $1^1/_2$ in.

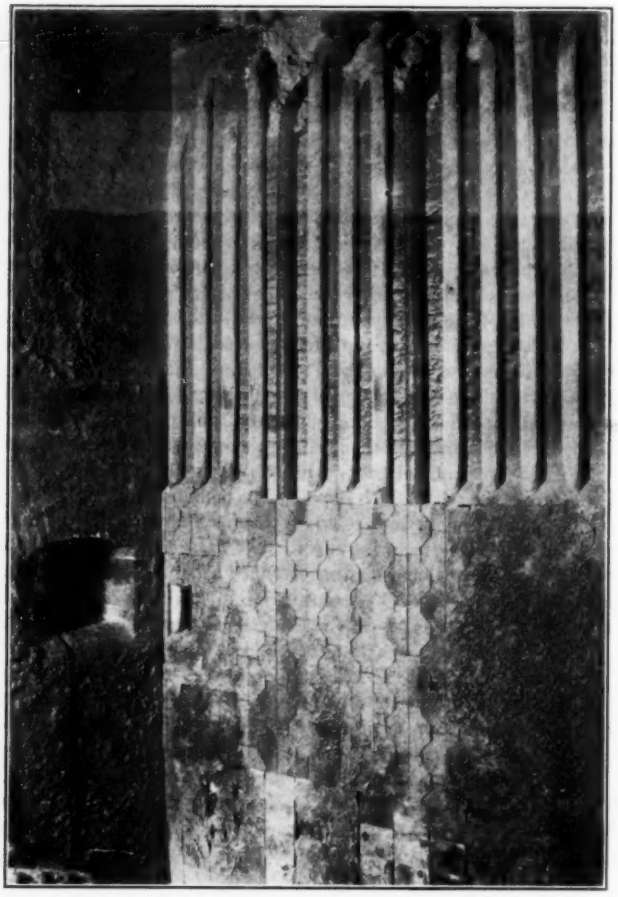


FIG. 8 SHOWING BRIDGEWALL TUBES THAT WERE BLISTERED AND REPLACED

on the circumference which showed none at all. In the covered section of the tube in the Nygaard blocks (carborundum) just below the bare section, the scale was too soft to separate in coherent chips for measurement, but did not appear to amount to more than 0.010 in. on the fire side and 0.002 or 0.003 in. against the wall, there being, however, apparently more deposit on the back of the tube than in the bare portion. Below the Nygaard blocks the tubes were covered in the square cast-iron plates machined to fit the curve of the tube and bolted up with clips. In this section the scale was a little thicker and more coherent, chips from the fireside measuring 0.012 to 0.015 in. and tapering down to a mere painting of probably 0.002 or 0.003 in. at the back.

The normal ash level in the clinker-grinder pit is about the bottom of the third cast-iron block, and it is rather surprising that the faces of some of them fused as badly as they did considering the small amount of scale in that part of the tube. Some of the blocks on the blistered tubes were also in bad shape and were replaced along with the tubes.

The source of contamination was traced to leaky process-water heaters, allowing well water to mix with the condensate. This contains considerable sulphate of lime and some magnesium salts as well as lime carbonate, and it seems quite likely that the disproportionate deposition on the front of the bridgewall tubes arose from the rapid concentration of the water in these, particularly that in contact with the front part of the tube at the high rate of evaporation occurring in them. That this was very high is evidenced by both superheat and flue-gas temperatures being almost 100 deg. fahr. below the guarantees at this rating, resulting, too, in the flue-gas temperature leaving the air heater being only about 300 deg. fahr. and the preheated air temperature in the windbox 200 to 210 deg. fahr. All this would seem to indicate a very large proportionate heat absorption in the furnace.

It is proposed to cover the remainder of the bridgewall tubes, as a careful internal inspection each week end while the boiler was in operation had shown no indication in any visible portion of the boiler tubes of the existence of such a dangerously scaled condition, and manufacturing conditions are too little under the control of the operating force to ensure against a repetition of such contamination, although evaporators were provided for supplying distilled make-up and the chances of contamination were believed much more remote than the event showed.

AUTHORS' CLOSURE

E. P. PARTRIDGE. The experience cited by Mr. Fisher offers a practical demonstration of the effect which a small amount of scale may have upon a boiler surface exposed to extreme heat radiation. The distribution of scale is particularly interesting, since the thickness of the deposit in different regions seems to have been proportional to the probable rate of steam evolution in these regions. I would be very glad to obtain information, samples of scale, or small sections of scaled tube from any one having a similar experience.

FSP-51-50

Present Status of Tests for Refractories

By STUART M. PHELPS[1] and J. SPOTTS McDOWELL,[2] PITTSBURGH, PA.

Tests for refractories may be divided into three general types: durability, suitability, and control. Durability tests can be made only in industrial furnaces, under actual operating conditions. Suitability tests may be carried out in the laboratory, under conditions which simulate those of service as closely as possible. Control tests, for maintenance of uniformity of product, should be relatively simple and require only a short length of time to perform. The pyrometric cone equivalent gives an index of refractoriness, but is not a measure of the actual temperature which a material will withstand under long-continued heating. The standard load test is not an index of refractoriness, but is a good control test, especially for brick to be used under load at high temperatures. Difficulties of interpretation affect the value of reheat and spalling tests. The most useful specifications are those of the control type.

MODERN developments in furnace design and operation have been such as to impose increasingly severe service upon the refractory linings. Operating and maintenance costs are in no small measure dependent upon the selection and use of lining materials which are adapted to the service conditions. Continuity of operation, together with the frequency and extent of repairs, is determined by the degree to which the refractory material is adapted to its environment. Among the most important environmental conditions are intensity of the heat, temperature fluctuations, action of slags and reducing gases, static loads, and furnace design. These conditions impose upon the refractories the requirements of resistance to fusion, spalling, slag erosion, and distortion. As there are wide variations in furnace practice, there must be corresponding variations in the properties of the lining materials if best results are to be secured.

General Types of Tests

The study of the properties of refractory materials and of suitable methods of testing them has long engaged the attention of investigators. Chemical analysis and resistance to fusion were the first subjects investigated; later came studies of distortion under load, slag erosion, spalling, and constancy of volume at high temperatures. In much of this early work it was regarded as of fundamental importance that the tests be relatively simple, inexpensive, easily duplicated, and require only a short length of time to perform. The general adoption of tests of this character has greatly facilitated the study of refractories. Specifications based upon them have made possible a study of the possibility of correlating laboratory results with those of service. The data accumulated during a period of several years have demonstrated that while in some cases the correlation is fairly satisfactory, laboratory results frequently offer no criterion as to the probable life of a refractory material. This is not surprising, since the conditions under which the tests are made are usually not comparable with those of service. It is now becoming generally realized that more than one type of testing is required, depending upon the purpose for which the data are required. The three general types may be classified as follows:

1 Durability tests
2 Suitability tests
3 Control tests.

Durability tests can be made only under actual service conditions. To be of greatest value, ample data must be obtained both as to the properties of the refractory material under investigation and the conditions in the furnace during the test period. Results obtained upon merely a few brick are usually of little if any value. In order to obtain conclusive results, it is essential that an entire section of the furnace be constructed of the material under test.

Suitability tests should simulate service conditions as closely as possible within the limitations of laboratory procedure. The usual short-time tests can rarely be regarded as measures of the suitability of a refractory material in service, as they do not give sufficient consideration to the time factor and to other important variables. The development of suitable simulative service tests is now receiving the attention of various groups of investigators, and much progress along this line during the next few years may be anticipated. The A.S.M.E. Special Research Committee on Boiler-Furnace Refractories has made contributions of much value in this connection. The work on slag erosion being conducted for the committee by the University of Illinois is of special importance. The Refractories Fellowship of the Mellon Institute is conducting extensive research upon service spalling tests.

Control tests are indispensable to the manufacturer of refractories and extremely useful to the consumer. Their primary function is maintenance of uniformity of product based upon certain standards arbitrarily established as a result of experience. Control work logically begins with the raw material and follows through the various stages of manufacture to the finished product. For plant control or control of shipments it is desirable that the tests be such as can be quickly conducted with relatively simple apparatus. On unburned material, the tests commonly applied are chemical analysis, refractoriness (pyrometric cone equivalent), moisture content, screen analysis, and porosity and shrinkage under definite heat treatment; on finished brick, chemical analysis, refractoriness, porosity, density, mechanical strength, rigidity, and constancy of volume at elevated temperatures as well as resistance to rapid thermal changes.

A brief discussion will here be given of some of the more important tests.

Pyrometric Cone Equivalent

The P.C.E. test is one of the oldest and most widely used of all the present tests. It is conducted by heating in a suitable furnace a small portion of the sample, ground and molded into the form of a small tetrahedron or "cone," together with a series of standardized cones of definite compositions. These fuse at definite temperatures, the intervals between cones being about 20 deg. cent. They must, however, be heated at a definite rate to cause fusion at known temperatures; if heated more slowly, they fuse at much lower temperatures. It is essential that all P.C.E. tests be conducted with the same rate of heating over a total period of only a few hours. It is obvious that the index of refractoriness supplied by the P.C.E. is not necessarily applicable to conditions where the time of heating may be weeks or months.

The results should always be given in standard cone numbers

[1] Senior Fellow, Refractories Fellowship, Mellon Institute of Industrial Research.

[2] Director of Research, Harbison-Walker Refractories Company.

Contributed by the A.S.M.E. Special Research Committee on Boiler-Furnace Refractories and presented at the Rochester Meeting, Rochester, N. Y., May 13 to 16, 1929, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

and not in temperatures, as the temperatures are accurate only for the standard rate of heating and the results cannot be used to predict the actual temperature which a material will withstand under long-continued heating.

P.C.E. determinations are conducted in a short time and give fairly consistent results. For a given laboratory the results should check within plus or minus one cone, and for most materials within less than half a cone. The test is of great value in connection with the control of raw materials. Maintenance of uniformity of quality is facilitated by the preparation of charts of the type shown in Fig. 1. Although valuable in the control of finished products, the pyrometric cone equivalent cannot be safely used as an index to their refractoriness without supplementary data. This may be shown by considering a brick made of fused alumina bonded with a low-grade clay. The P.C.E. of this mixture would be high, but its heat-resisting qualities in service would be low. This example is extreme, but the principle often applies. Examples are given in Table 1, which gives refractoriness values for a number of "super" refractories.

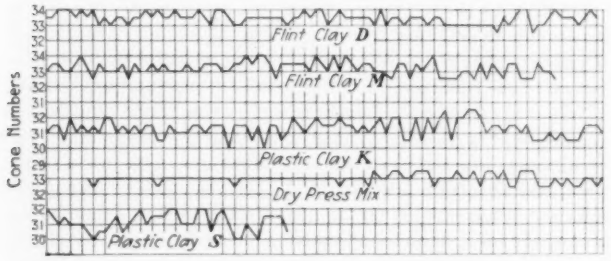


FIG. 1 P.C.E. DATA ON WEEKLY SAMPLES

TABLE 1

Sample	P.C.E.	Subsidence in load test at 1500° C. under 50 lb. per sq. in., per cent	Bending of brick under load with a span of 7 in., temperature 1600° C. mm.
1	37	0.2	1.0
2	37	2.2	1.6
3	36–37	0.4	2.8
4	Above 38	5.8	Broke

Sample 4 shows the highest P.C.E. value, but is poorest in both the load and bending tests.

STANDARD LOAD TEST

The test is conducted by heating the brick at a definite rate, with a load of 25 lb. per sq. in. applied on end, to 1350 deg. cent., which is maintained 1½ hours. The data are of considerable value when properly used, although the interpretation now given the results differs considerably from that of some years ago. The test was originally regarded as a determination of "quality;" it is now known that the results are affected not merely by the character of the raw material from which the brick were made, but also by the fineness of texture, the process of manufacture, and the temperature of firing. For a given brand of fireclay brick the deformation may vary from 2 to 10 per cent, depending upon the degree of burn.

Recent tests conducted in a special load-test furnace[3] in which ten brick may be tested at one time under identical conditions show that the variation between samples is relatively large. The data presented in Table 2 are of interest, since the ten brick tested from each lot are as uniform as may be expected from commercial products. It is obvious from this that a figure determined by testing only one specimen is of little value as an indication of the true characteristics of a brand.

[3] Technologic Paper No. 2, American Refractories Institute.

TABLE 2 PERCENTAGE OF SUBSIDENCE IN LOAD TESTS CONDUCTED AT 1350 DEG. CENT. UNDER A PRESSURE OF 25 LB. PER SQ. IN.

Sample	Brand A, Cone 32–33 Brick Result	Brand A, Cone 32–33 Brick Deviation	Brand B, Cone 30 Brick Result	Brand B, Cone 30 Brick Deviation
1	3.8	+0.7	8.2	−0.4
2	3.7	+0.6	8.8	+0.2
3	3.4	+0.3	9.7	+1.1
4	3.2	+0.1	10.8	+2.2
5	3.1	0.0	5.6	−3.0
6	2.9	−0.2	11.3	+2.7
7	2.9	−0.2	8.5	−0.1
8	2.8	−0.3	8.5	−0.1
9	2.3	−0.8	7.8	−0.8
10			6.8	−1.8
Mean	3.1	...	8.6	...
Standard deviation	...	0.5	...	1.7

Fig. 2 presents a view after testing of the brick shown as "Brand B" in Table 2. The bricks at each end of the row were not included in the test; by comparison the variation in the subsidence of the brick which were tested can be noted. The cone plaques in front of each brick were upon the respective brick during the test and show the uniformity of the furnace temperature.

Load-test conditions, in which the brick are heated throughout, are not comparable with conditions in the wall of a boiler setting, in which only one end of the brick is exposed to the heat, resulting in a temperature gradient between the inner and outer faces. The lack of concordance to be expected between test data and behavior in service is illustrated by an experiment conducted at the Mellon Institute. Brick of a brand which showed unusually high deformation in the standard load test were laid up in a small furnace in the form of a panel 9 in. thick, with the outside surface thoroughly insulated to simulate the most severe conditions



FIG. 2 BRAND "B" AFTER LOAD TEST

of service. A pressure of 25 lb. per sq. in. was applied to the wall, the furnace temperature being 1350 deg. cent. The brick showed no measurable deformation under these conditions, although under standard load-test conditions, at the same temperature and pressure, other brick of the same lot deformed 22 per cent.

The load test, like many others, has its greatest usefulness in control work. It is frequently useful as a means of classifying various brands of brick into groups, for service in which rigidity at high temperatures is an essential requirement.

THE REHEAT TEST

The object of the reheat test is to determine the constancy of volume of brick at a temperature higher than that at which they were burned. In the standard method they are heated in 4½ hours to 1400 deg. cent., and maintained at that temperature for 5 hours. From measurements taken before and after testing, the percentage of linear change is calculated.

Recent studies have shown that many brick of high refractory value exhibit in the reheat test expansion phenomena similar to those of overfiring. The cause of this so-called "secondary expansion" is obscure. It renders difficult the interpretation of reheat test data, for one cannot always be sure whether an abnormally high expansion results from overfiring, and is therefore an indication of inferior quality, or whether it is merely a

"secondary expansion" which has no significance in service. However, a brick which shrinks in the test will probably do so in service.

The data presented in Table 3 show that there is no appreciable difference in the linear change of high-grade brick heated to 1400 deg. cent. in 4½ hours or in 60 hours. Comparison of these changes with those produced by 98 days' continuous heating in a commercial tunnel kiln shows that there is no relation between the figures in the standard reheat test and change of volume in service.

TABLE 3 PERCENTAGE OF LINEAR CHANGE OF FIRECLAY BRICK IN REHEAT TESTS

Brand	Regular reheat test, 4½ hr. to reach 1400° C., 5 hr. at 1400° C.	Special reheat test, 60 hr. to reach 1400° C., 5 hr. at 1400° C.	98 days of continuous heating in tunnel kilns Cone 13 kiln (1300° C.)	Cone 17 kiln	Periodic kiln Cone 12	Periodic kiln Cone 15
1	+0.3	+0.4	+3.1	+ 5.3	...	...
2	+0.4	+0.6	−4.7	− 6.9	...	...
3	+0.8	+0.9	−1.4	+ 1.8	...	...
4	+1.0	+1.1	−0.6	− 2.1	...	...
5	−0.4	−0.2	−0.2	−11.8	...	...
6	−1.2	−1.2	−3.0	− 5.9	...	...
7, green brick	+1.3	+1.1	...	...	−1.0	...
7A, soft burned	+1.3	+1.1	...	...	...	0.0
8, green brick	+1.6	+1.3	...	...	−1.0	...
8A, soft burned	+1.1	+0.6	...	...	...	−0.8

Additional studies are planned to learn the effect of furnace atmosphere upon the results. Furnace atmosphere has not been given the consideration which its importance merits in test procedures.

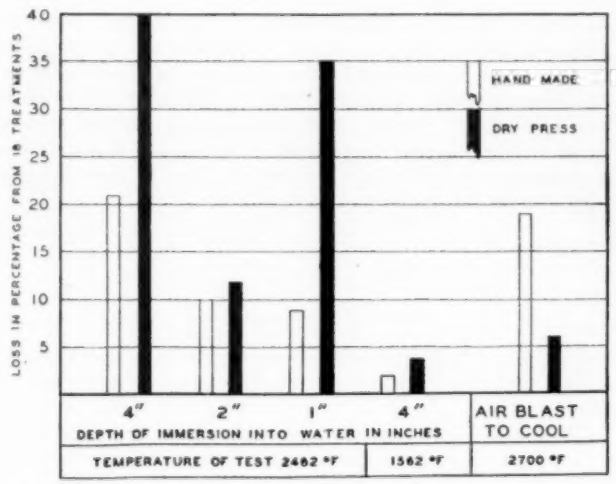


FIG. 3 BEHAVIOR OF TYPICAL HAND-MADE AND DRY-PRESSED BRICK IN FIVE TYPES OF SPALLING TESTS

WATER-IMMERSION SPALLING TEST

The water-spalling test has been the subject of much controversy. It originated at a time when the only type of spalling considered was that which results from rapid temperature changes or thermal shock. It is now realized that this cause of spalling is often of minor importance, and therefore that any test which gives only an index of thermal spalling is inadequate.

The water-dip test in itself is not satisfactory for the thermal-spalling phase. It is too severe in its method of cooling the heated ends of the brick and is subject to large errors. The character of vitrification produced in service at the hot ends of the brick is not simulated. The most severe thermal changes encountered in service will rarely spall clay brick unless they become vitrified to a measure through the action of heat, slags, or fluxes in the gases.

Fig. 3 gives a comparison of two brands of fireclay brick, one hand-made and the other dry-pressed, tested in five different ways. It will be observed that, according to two methods of testing, the hand-made brick appear to be superior to the dry-pressed; in two, about equal; and in one, inferior. Obviously such contradictory results cast doubt upon all the methods indicated. The service spalling test, now in process of development at the Mellon Institute, offers great promise of a close correlation of laboratory data and life in service. A section of a furnace wall is simulated by applying the necessary insulation and pressure upon ten brick laid up in a panel in the manner illustrated in Fig. 4. The surface of the panel is first heated so as to produce vitrification comparable with that of furnace walls; subsequently, it is cooled by means of an air blast.

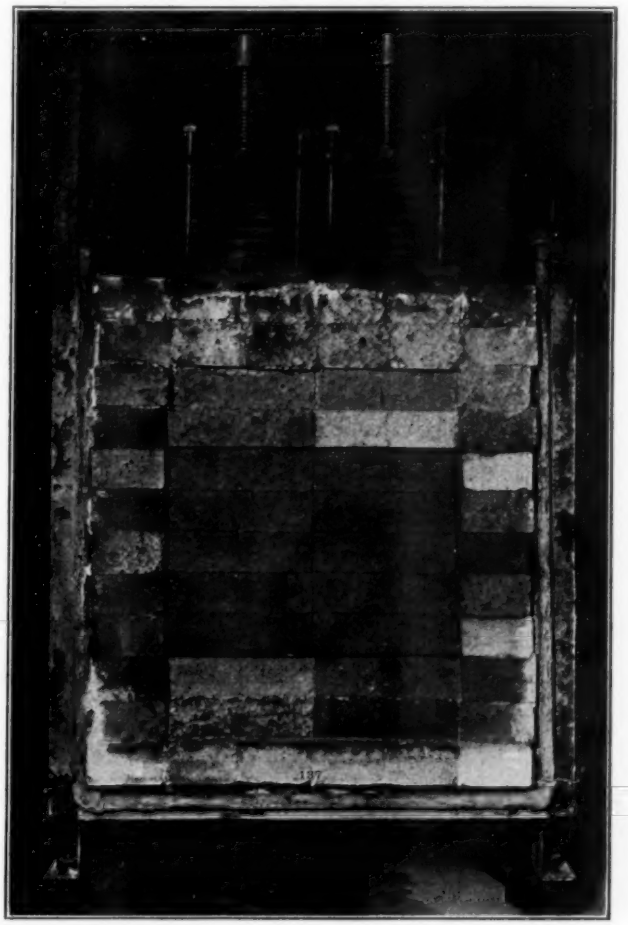


FIG. 4 TEST BRICK ARRANGED IN PANEL FOR SERVICE SPALLING

USE OF TESTS IN SELECTION OF REFRACTORIES

Analysis of the service conditions in a given furnace will reveal the requirements which the brick should meet. Requirements differ in different furnaces, and often in different parts of the same furnace. In many cases all of the properties desired cannot be incorporated in one refractory material. For example, there is no refractory material that is resistant to oxidation and that has also the exceptional rigidity of silica brick at high temperatures and the high resistance to spalling of many fireclay brick.

In the case of fireclay brick, due regard must be given to the

fact that certain properties have a relationship which cannot be changed except within narrow limits. An example of this is afforded by the effect of the degree of firing upon fireclay brick, as shown graphically in Fig. 5. The relations shown are not fixed and invariable, but are merely general tendencies. They are further illustrated by brands 1 and 2 of Table 4. The pronounced increase in cold strength, rigidity at high temperatures, and constancy of volume in the reheat test, resulting from hard burning, is apparent. The elasticity and resistance to thermal shock, however, have been decreased. This detrimental effect of hard burning upon thermal spalling often prevents the use of hard-burned brick in service where they would otherwise be desirable.

TABLE 4 TESTS OF FIRECLAY BRICK

Brands	Pyrometric cone equivalent (P.C.E.)	Load test, 1350° C., 25 lb per. sq. in.; percentage of subsidence	Reheating, 1400° C. for 5 hr.; percentage of linear change	Spalling, 1350° C. into 2 in. of water; number of treatments for 10% loss	15% loss	20% loss	Transverse strength, lb. per sq. in.	Porosity, per cent	Percentage chemical analysis Silica	Alumina
1 Soft burn	32–33	13.3	+1.7	17	45	47	900	16.2	51.4	42.3
1 Hard burn	32–33	5.4	+0.5	7	13	15	622	17.0	51.4	42.3
2 Soft burn	32–33	10.9	+1.5	24	37	46	790	24.1	58.8	36.9
2 Hard burn	32–33	5.6	+0.9	20	29	40	465	22.0	58.8	36.9
3	32–33	4.2	+0.8	11	14	16	1224	17.7	55.8	37.6
4	32–33	8.4	−0.5	11	12	14	732	18.8	60.5	35.0

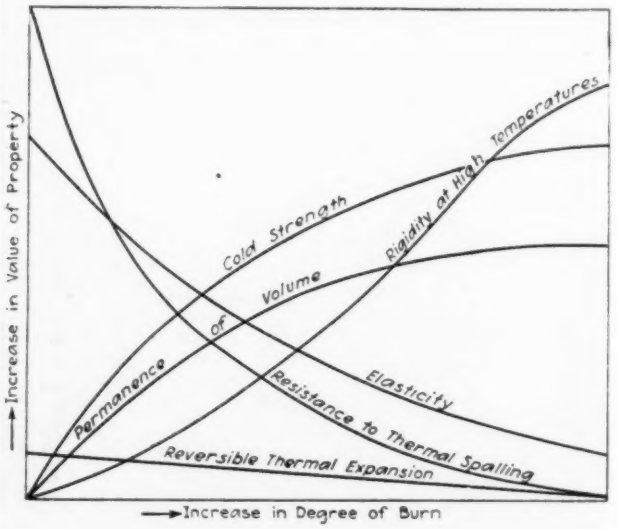


FIG. 5 CHART SHOWING EFFECT PRODUCED BY VARYING THE DEGREE OF BURN OF CLAY BRICK

The brands of brick listed in Table 4 are known to give good service in boiler furnaces. The data given show that, while all have the same P.C.E. value, their behavior in other tests is very different. This condition illustrates the difficulty of so drafting specifications as to include the various brands which will give good service and at the same time to have the requirements specific.

Various specifications used for the selection of refractories have served the highly useful function of demonstrating wherein laboratory methods are imperfect or the results improperly interpreted. They have stimulated investigations which are leading to the development of more suitable test procedures.

The most useful specifications are those of the control type. Service tests are required for conclusive evidence as to the best type of refractory for a given practice. Once this is definitely known, a specification with reasonable limits and tolerances may be drafted based upon the properties of the material known to give satisfactory service. Proper consideration must be given to the degree of precision of the laboratory methods. Limits and tolerances which are too extreme in either direction will not lead to adequate control. Such specifications, written with a broad understanding of the methods of testing and of the service requirements, will stimulate research and lead to more definite correlation of laboratory and service data.

FSP-51-51

Study of Some Factors in Removal of Ash as Molten Slag From Powdered-Coal Furnaces[1]

By RALPH A. SHERMAN,[2] P. NICHOLLS,[3] and EDMUND TAYLOR,[4] PITTSBURGH, PA.

The purpose of this investigation, which was conducted at the Toronto, Ohio, station of the Pennsylvania-Ohio Power and Light Co., in cooperation with Stevens & Wood, Inc., New York, was to study those factors governing the application of the slagging method of ash removal which are related to the characteristics of the ash of the coal burned. The history of the development of and the merits of slagging furnaces are briefly discussed, and the results of observations made when burning three coals, whose ash had fluid temperatures of 2400, 2600, 2800 deg. fahr., in a furnace fired with horizontal burners and having bare water tubes set on three walls, are presented. The following studies were made: (1) The condition of the slag in the hearth and on tapping; (2) the temperature of the slag; (3) the temperature and composition of the gases near the slag bed; (4) the relation of the composition and fusion temperatures of the slag and the dust carried out of the furnace to those of the original ash; (5) the percentage of the ash fired which was carried out of the furnace and stack; (6) the relative temperatures of the gas entering the boiler tubes in two furnaces, one with a lower water-tube slag screen and the other with a slagging bottom; and (7) a brief study of the amount of flux necessary to add to a given ash to obtain a fluid slag.

THIS paper presents the results of an investigation of the removal of ash as molten slag from furnaces burning powdered coal at the Toronto, Ohio, station of the Pennsylvania-Ohio Power and Light Company, in cooperation with Stevens & Wood, Inc., New York City. The purpose of the investigation was to study those factors governing the application of the slagging method of ash removal which are related to the characteristics of the ash of the coal burned.

SLAGGING FURNACES

Development

The removal of ash always presents difficulties in the burning of coal, whether on stokers or in powdered form. In the early trials of powdered coal the tendency of the ash to melt in the flame and then solidify in the lower part of the furnace was one of the most serious problems encountered and led to the development of water-tube slag screens, water-cooled ashpits, and water-cooled furnaces which are intended to absorb heat from the ash and thus prevent its deposition in the molten state. Even with these devices, however, it is rare with some coal and with some conditions of operation that difficulty is not encountered with accumulation of solidified slag in the ashpit and consequent difficulty in its removal. Fig. 1 shows an extreme case of this accumulation which necessitated the removal of the boiler from service once a month in order to mine out the slag.

Because of this common tendency of the ash to become fluid it was natural that attempts should be made to remove it in the molten state. Anderson[5] has stated that the removal of ash as molten slag was of necessity the original method used at the Oneida Street station in Milwaukee, Wis., before screen tubes were added. Blizard[6] reported that in 1919 an installation in Seattle was designed for this process, but inquiry has shown that it has never been followed as a practical measure. Experience with this method of ash removal in France was reported at the Second Congress on Industrial Heating in 1928. Weiss and Munier[7] reported their experiences which date back to 1920 and state:

> We were forced to conclude that the utilization of powdered coal with molten ash was not of general application because it was successful only with certain classes of coals having easily fusible ash and was not suited to the class of coals which we were using.



FIG. 1 EXTREME EXAMPLE OF SLAG ACCUMULATION BELOW WATER-TUBE SLAG SCREEN OF POWDERED-COAL BOILER

Schereschewsky[8] reports that the discussion which arose in earlier years in France between the partizans of molten ash and pulverulent ash seems to have disappeared, and now only 22 boilers totaling about 52,000 sq. ft. of heating surface, or about 5 per cent of the total powdered-coal installations, are operating with removal of molten ash.

The early experience with slagging furnaces was therefore not particularly favorable, so that the trials at the Charles R. Huntley[9] Station of the Buffalo General Electric Company and at the Toronto, Ohio, station, while not an innovation, were at least new in their successful application in the United States.

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Presented at the Rochester Meeting, Rochester, N. Y., May 13 to 16, 1929, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[5] Anderson, John, discussion of paper, "Direct-Fired Powdered-Coal Boilers With Well-Type Furnaces at Charles R. Huntley Station," Trans. A.S.M.E. Fuels and Steam Power Section, vol. 50, no. 8, p. 63, 1928.

[6] Blizard, John, "Preparation, Transportation, and Combustion of Powdered Coal," Bull. 217, Bureau of Mines, p. 91, 1923.

[7] Weiss and Munier, "Notes on the Firing of Boilers With Powdered Coal," *Chaleur et Industrie*, vol. 9, no. 102, p. 105, 1928.

[8] Schereschewsky, "Firing of Boilers by Powdered Coal in France, *Chaleur et Industrie*, vol. 9, no. 102, p. 85, 1928.

[9] Cushing, H. M., and Moore, R. P., "Direct-Fired Powdered-Fuel Boilers With Well-Type Furnaces at Charles R. Huntley Station," Trans. A.S.M.E. Fuels and Steam Power Section, vol. 50, no. 8, p. 63, 1928.

Merits

The Fuller-Lehigh Company recently stated[10] that they have equipped 11 boiler units with slagging furnaces and have 15 units under construction; and further, that

....experiences with some furnaces during the past two years and with other furnaces that are now going into operation indicate

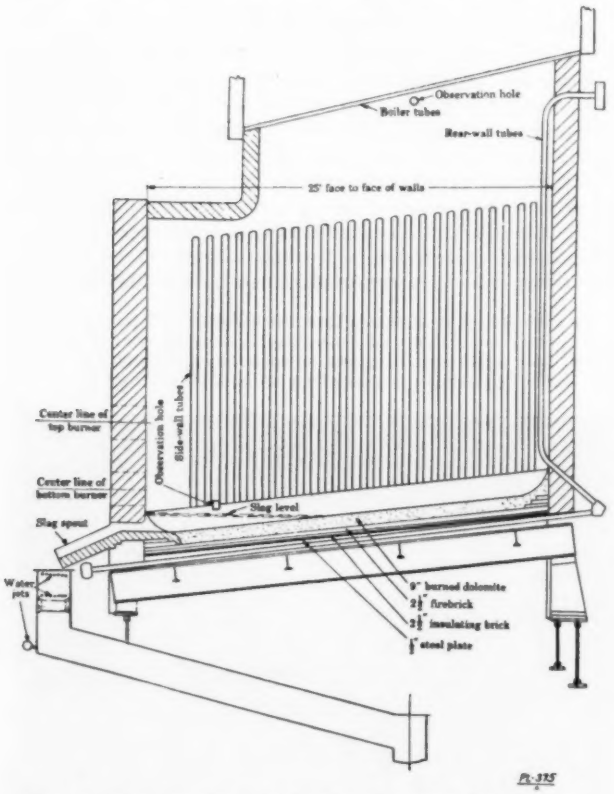


Fig. 2 Side-Sectional Elevation of Boiler No. 4 With Slagging Bottom, Toronto Station

that this is an ideal method of handling ash from pulverized-coal furnaces....With horizontal turbulent burners in the lower part of the furnace, there is no difficulty in handling ash having a fusing temperature as high as 2500 or 2600 deg. in a satisfactory molten condition. With very high temperatures in the floor and walls immediately surrounding the ash, turbulence can be had in this space with the flames impinging directly upon the floor without any fear of damage. Other advantages of this furnace are reduction in height of building and the use of a simple ash-removal system. The ash is removed in a very satisfactory manner by means of a water jet with a minimum of labor, and the boiler room is kept clean and free from dust.

INVESTIGATION OF SLAGGING METHOD

Origin of Investigation

The trials of the slagging method at the Toronto station were occasioned by the usual difficulty of the accumulation, under certain conditions of operation, of masses of solid slag in the ash-pits which could not be removed by the regular sluicing system. A slag hearth was installed first in the furnace of boiler No. 5, which was equipped with Bailey refractory-faced water walls and was fired with Bailey Calumet vertical inter-tube burners. A second installation was made in the furnace of boiler No. 4, which had refractory walls protected on the side and rear by bare water tubes and was fired with horizontal Peabody Toronto burners. The ash has been removed from these two furnaces by tapping off the molten slag approximately once every 24 hours under all normal conditions of boiler operation since April, 1928.

As the designing and constructing engineers of the Toronto Station, Stevens & Wood, Inc., desired information on the relation of the characteristics of the coal ash for various coals to those of the slag formed, they requested the Bureau of Mines to cooperate with them in an investigation.

Analysis of Problem and Scope of Investigation

The two available measures of the properties of coal ash are the "cone" test and chemical analysis, between which no definite relation has been established. The fluid temperature determined by the cone test is probably some indication of the temperature which must be attained in the hearth of a boiler furnace in order that the ash may be removed as molten slag. However, the temperature of the slag must be higher than that at which it will just flow, so that the loss of heat at the walls, at the tap hole, and in the spout will not cause freezing.

The average maximum temperature attained in boiler furnaces does not often exceed 2800 deg. fahr.,[11] and the temperature of the hearth will naturally be lower, probably by at least 100 deg. It would be expected, therefore, that the upper limit of fluid



Fig. 3 Interior of Furnace of Boiler No. 4, Toronto Station
(Showing powdered-coal burners and top of slag bed.)

temperature of an ash which would flow satisfactorily would be 2500 to 2600 deg. if that deposited on the hearth had the same composition and fluid temperature as the ash fired to the furnace.

Data have been presented, however, which indicate that the

[10] *Power Plant Engineering*, vol. 33, p. 104, 1929.

[11] Fahrenheit scale used throughout this paper.

ash which is deposited at the bottom of the furnace may differ in composition and fluid temperature from that in the original coal. Sherman and Taylor[12] have reported that the softening temperature was lower and the content of iron and lime was higher in the material which collected on the water screens and on the walls than in the original coal ash in two furnaces burning powdered Illinois coal with long-flame burners. Jacobus and Bailey[13] reported that they had found the softening temperature of the material remaining in the furnace lower, and that of the dust leaving the furnace higher, than that of the original ash. They reported a large number of samples from various installations but did not give their composition.

The principal requirement in this investigation was therefore to determine the relation in the Toronto slagging furnaces between the composition and characteristics of the coal ash, the slag from the hearth, and the dust carried out of the furnace. In addition data were obtained on the amount of dust carried out of the furnace, the temperatures in the slag bed, and on the temperature and composition of the gases above the hearth.

Data were obtained for all of these factors in the furnace of boiler No. 4 when burning three coals, the two normally burned at the station, one from a strip mine in the Ohio-Pittsburgh No. 8 bed, hereafter called strip-mine coal, the other from underground mines in the Pennsylvania Pittsburgh No. 8 bed, hereafter called Pittsburgh coal, and a special coal brought in for the tests from the Georges Creek district of the Pittsburgh Big Vein, hereafter called Georges Creek Coal. Observations were made in the furnace of boiler No. 5 only when burning Pittsburgh coal.

In boiler No. 4 the action of the slag bed was observed and data were taken over two periods of 6 days each when burning the strip-mine coal, over two periods of 5 days each when burning the Pittsburgh coal, and over one period of 4 days when burning the Georges Creek coal; in boiler No. 5 data were taken over a period of 8 days.

DESCRIPTION OF FURNACES

Boiler No. 4. Fig. 2 is a side-sectional elevation of the furnace of boiler No. 4, and Fig. 3 is a photograph of the front wall which shows the powdered-coal burners and the top of the slag bed. The boiler operated at 375 lb. gage pressure; it had 20,470 sq. ft. of heating surface, and the area of heating surface of the bare tubes set in front of the refractory of the side and rear walls and over the ashpit was 2100 sq. ft. The volume of the furnace above the slag bed was 12,750 cu. ft.

The square in the lower front corner of the furnace, Fig. 2, marks the hole through which samples were taken from the slag bed and various observations were made; the circle just below the tubes marks the hole through which dust samples were taken and temperature measurements were made.

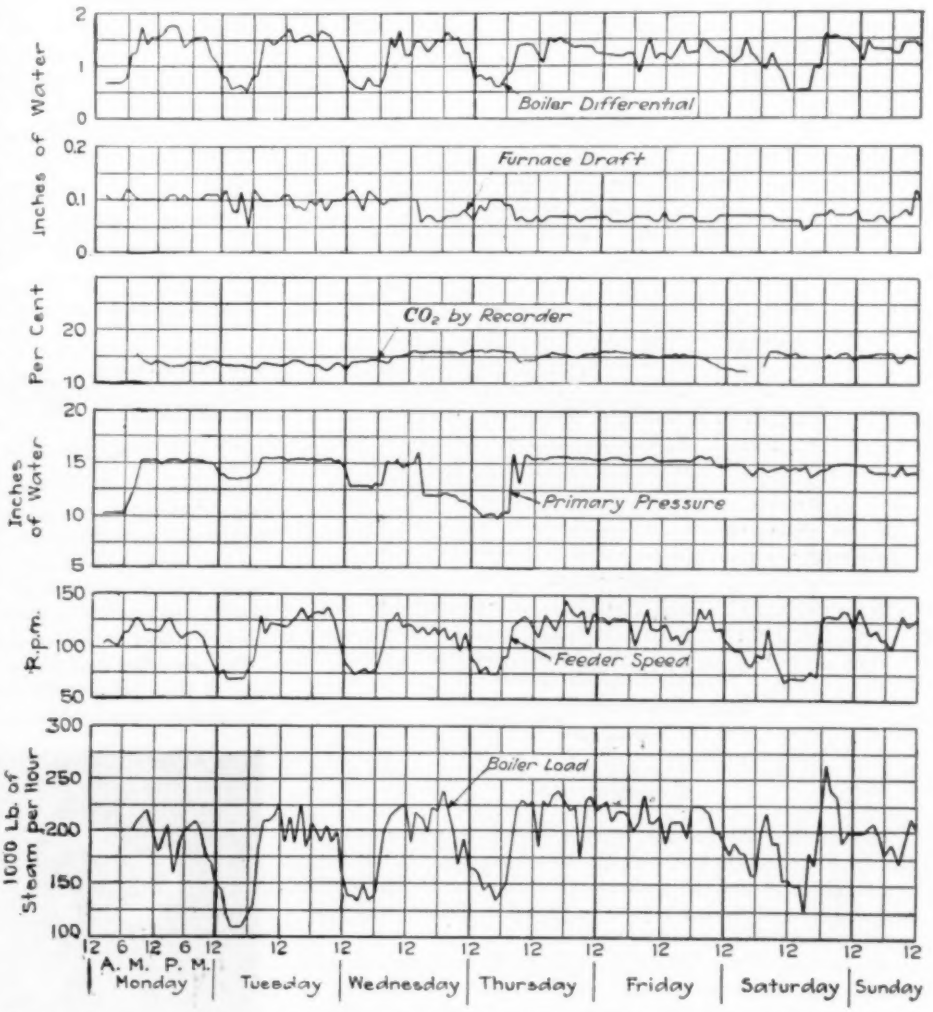


FIG. 4 TYPICAL LOG BOILER OPERATION, BOILER NO. 4, TORONTO STATION, NOV. 12 TO 18, INCL., 1928

The normal, maximum, and minimum loads on the boiler were respectively 210,000, 250,000, and 140,000 lb. of steam per hour; at these loads the rates of heat liberation were approximately 21,000, 26,000, and 13,000 B.t.u. per cu. ft. of combustion space per hour. Fig. 4 is a typical week's log of the boiler.

The hearth was made up, as indicated in Fig. 2, of a steel plate, $2^1/_2$ in. of insulation, $2^1/_2$ in. of firebrick, and 9 in. of burned dolomite placed on the sloping water-tube slag screen. The dolomite was only loosely packed on the bottom. Because of the slope of the hearth the area for slag accumulation was limited to about half the area as the height of the slag was then at the bottom of the burner openings. In order that the slag could be completely drained from the hearth it was necessary to place the

[12] Sherman, R. A., and Taylor, Edmund, "Refractories Service Conditions in Furnaces Burning Powdered Illinois Coal With Long-Flame Burners," Trans. A.S.M.E. Fuels and Steam Power Section, vol. 50, no. 15, pp. 125–140, 1928.

[13] Jacobus, D. A., and Bailey, E. G., "Removal of Ash and Loss of Carbon From Boiler Furnaces," Second International Conference on Bituminous Coal, Pittsburgh, Nov., 1928.

tap hole at the lowest point, in a front corner, which was in an unfavorable position relative to the high-temperature zone of the furnace.

Boiler No. 5. Fig. 5 is a side-sectional elevation of the furnace of boiler No. 5. This boiler operated at 400 lb. gage pressure; it had 21,620 sq. ft. of boiler heating surface, and the area of the refractory-faced blocks on the front, side, and rear walls was 2578 sq. ft.

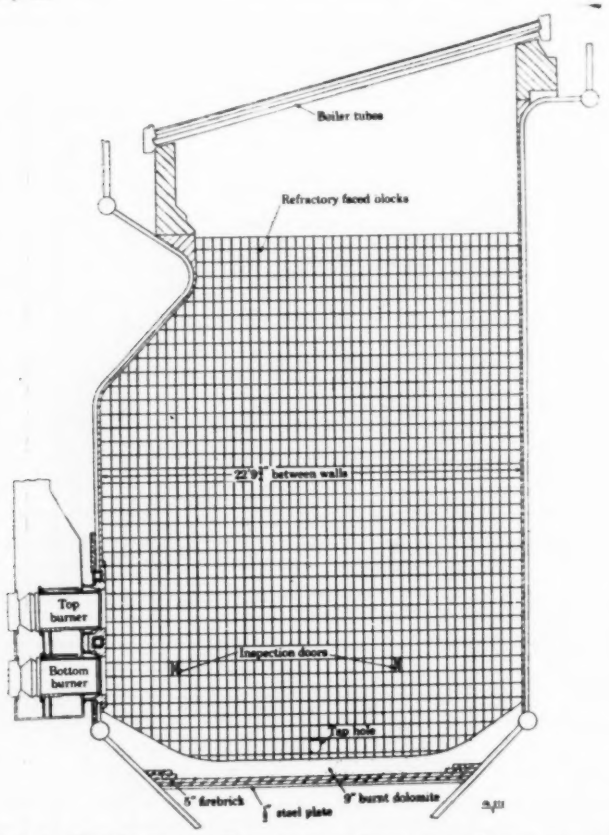


FIG. 5 SIDE-SECTIONAL ELEVATION OF FURNACE OF BOILER NO. 5, TORONTO STATION

The hearth of this furnace, which was of similar construction to that of No. 4, was installed above the ash hopper. It was horizontal, which allowed the accumulation of slag over the entire area and also allowed the tap hole to be placed in the side wall halfway between the front and rear walls.

At the time of the tests Peabody Toronto burners had been installed in this furnace. These were similar to those used in No. 4, except that provision was made for the admission of tertiary air around the burner on the furnace side.

The coal for both furnaces was pulverized in roller mills and fed to the burners from an overhead bunker by screw feeders. The coal and primary air entered tangentially to the barrel of the burner, and the secondary air entered in the center. The air and coal were mixed by their rotary motion in the barrel, which was rifled; the coal ignited near the face of the burner. The air was heated in a plate-type heater to about 300 deg. at normal load.

The tap holes were plugged with fireclay between tappings; on tapping, this was removed with a bar and sledge, and the slag flowed in the refractory-lined spout to fall into jets of water where it was granulated by the sudden cooling and was carried out by the water through the ash sluice. Fig. 6 shows the operation of tapping furnace No. 4.

CHARACTERISTICS OF COAL AND ASH

Table 1 gives the proximate and ultimate analyses, and calorific value on the dry basis, the size of the powdered coal, and the initial deformation, softening, and fluid temperatures of the ash of the three coals burned. The analyses are of composite samples made up of the indicated number of daily samples.

TABLE 1 PERCENTAGE ANALYSES OF COAL AND ASH

	Coal		
	Strip-mine	Pittsburgh	Georges Creek
No. of daily samples included.	4	4	2
Proximate Analysis:			
Volatile matter	37.9	35.2	20.3
Fixed carbon	50.2	55.0	69.3
Ash	11.9	9.8	10.4
	100.0	100.0	100.0
Ultimate Analysis:			
Hydrogen	5.0	5.1	4.6
Carbon	71.5	75.6	78.6
Nitrogen	1.4	1.5	1.8
Oxygen	6.9	6.7	3.2
Sulphur	3.3	1.3	1.4
Ash	11.9	9.8	10.4
	100.0	100.0	100.0
Calorific value, B.t.u. per lb.	13,050	13,680	13,880
Size of powdered coal, per cent:			
On 100 mesh	17.2	18.9	16.4
Through 100 mesh	82.8	81.1	83.6
Through 200 mesh	64.7	62.4	66.4

	Min.	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.
Initial deform. temp., deg.	2000	2050	2110	2300	2320	2350	2550	2600	2640
Softening temp., deg.	2120	2180	2240	2420	2470	2510	2680	2700	2730
Fluid temp., deg.	2360	2400	2520	2570	2640	2730	2790	2790	2800

METHOD OF INVESTIGATION

Handling of Coal. In burning the three coals care was taken to insure that only one coal was reaching the boiler. A raw-coal bunker was carefully cleaned, filled with the coal to be burned, and the level in the powdered-coal bunker was brought to the lowest possible point before the pulverizing mill was started. Thus it was possible to know within an hour when the new coal was being fired to the furnaces.

The powdered coal was sampled at half-hour intervals at the base of the cyclone collector and accumulated over 24 hours. Analyses were made of the daily samples.



FIG. 6 TAPPING MOLTEN SLAG FROM BOILER NO. 4, TORONTO STATION

Measurement of Temperature. The temperature of the furnace gases was measured by means of a platinum, platinum-rhodium thermocouple which was protected and supported in a water-cooled tube. The hot junction and $1^1/_2$ in. of the wires were directly exposed to the gases.

The temperature of the slag was measured by means of a plati-

num, platinum-rhodium thermocouple protected from contact with the slag by a closed-end fused-silica tube; it was supported in a water-cooled tube for measurements in the slag bed of the furnace and in an iron pipe for measurements in the flowing slag in the spout. All thermocouples were frequently checked by comparison with a standard thermocouple.

Sampling Gases. Samples of gases were taken from the furnace through water-cooled sampling tubes, collected over an acidified, saturated solution of salt, and analyzed in an orsat apparatus for CO_2, O_2, CO, H_2, and CH_4.

Sampling Slag. Because a large amount of slag from the coal previously burned always remained in the furnace, in the slag bed, and on the walls and tubes, the samples for analysis and comparison with the composition of the coal ash and flue dust were not taken until at least two days after a change in the coal.

Samples of the slag flowing in the spout were taken with an iron ladle, and samples from the furnace by dipping a water-cooled copper tube into the molten slag.

Sampling Dust From Gases. Fig. 7 shows the water-cooled tube used for sampling dust from the gases within the furnace below the boiler tubes and the collector and filter in which the dust was removed from the gas. The construction of the sampling tube is clearly shown; nickel is used for the inner tube because it is corroded less than copper by the sulphur acids in the gas.

The gas carrying the dust enters the tube at the left which curves toward the side of the bottle so that a large part of the dust is thrown out on the bottom of the bottle. The remainder of the dust is removed as the gases pass through the parchment thimble and the volume of gas is measured by an orifice meter. The dust which settles in the sampling tube is removed and added to that collected in the bottle.

Fig. 8 shows the apparatus used for sampling dust from the gases at the discharge of the induced-draft fan. The gas and dust are drawn through the 1-in. knife-edged pipe and enter the col-

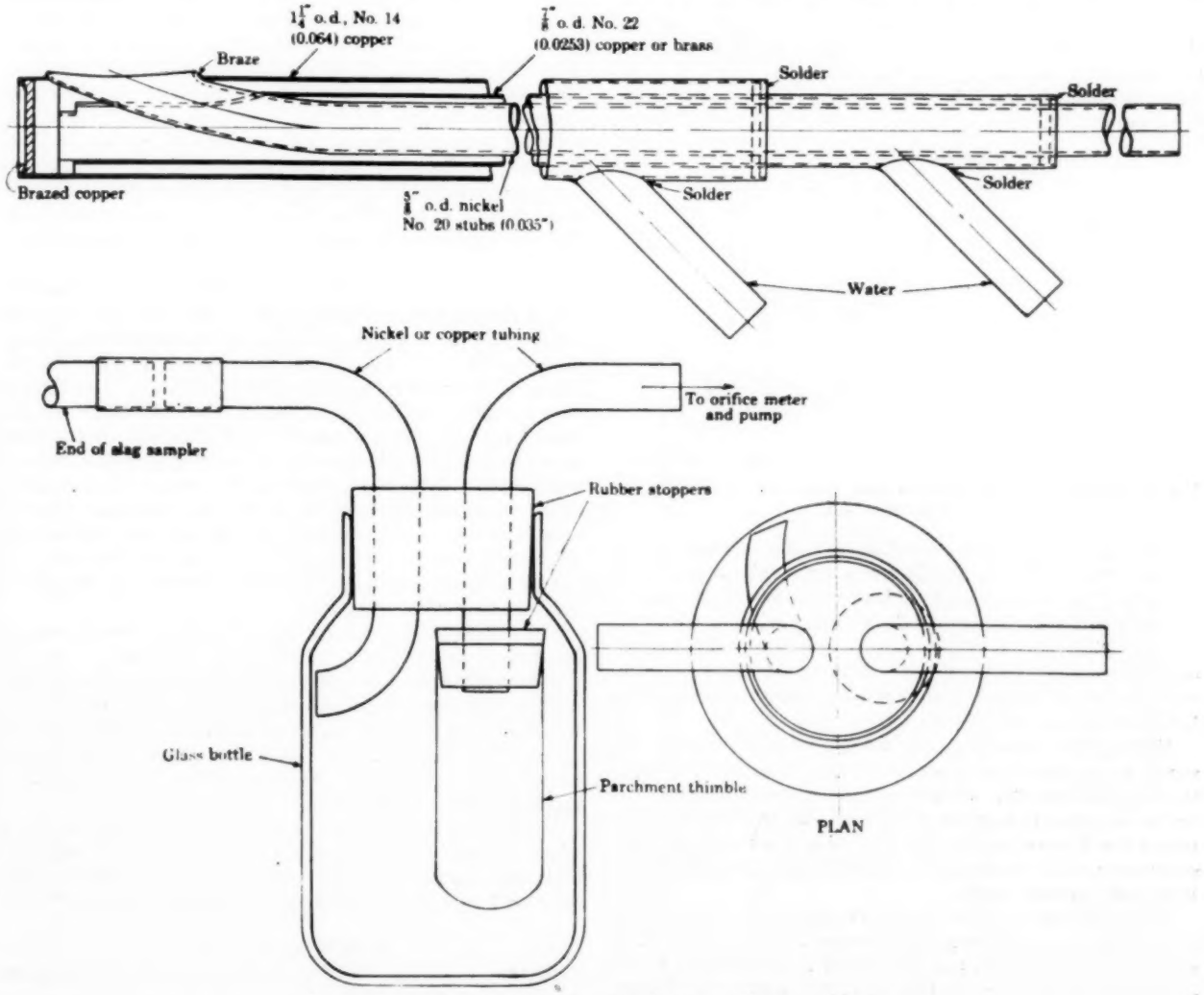


FIG. 7 SAMPLER AND COLLECTOR FOR SOLIDS IN FURNACE GASES

lecting bag made of feather-proof ticking which is supported in the welded metal container. The volume of gas is measured by an orifice meter. This method of dust sampling was developed by the test engineers of the Toronto station and the results presented in this paper were obtained by them.

With both types of samplers the open end of the sampling tube faced the direction of gas flow and the rate of flow was set so that the linear velocity of entrance to the tube was the same as that of the gas at the point of sampling. The velocity of the gases in the furnace was determined by means of a water-cooled pitot tube; the velocity in the stack was calculated from the rate of firing,

the flue-gas analysis, and the area of the flue; it was checked by a pitot tube at one load and the calculated value was found to be correct.

The efficiency of the ticking bag as a filter for dust was determined by drawing a part of the gas through the parchment thimble collector after it had passed through the bag. The weight of dust caught by the thimble was 0.6 per cent of the total collected; the efficiency of the bag was therefore 99.4 per cent.

DISCUSSION OF OBSERVATIONS

Condition of Slag in Hearth

Strip-Mine Coal in Boiler No. 4. The ash which collected on the hearth of the furnace when burning this coal was in the form of a thinly fluid slag and was tapped without difficulty under all observed conditions of operation from a rate of steaming of 140,000 to 250,000 lb. per hour. The top of the bed had a smooth, fluid appearance, and slight ripples could often be seen as the flame swept across it.

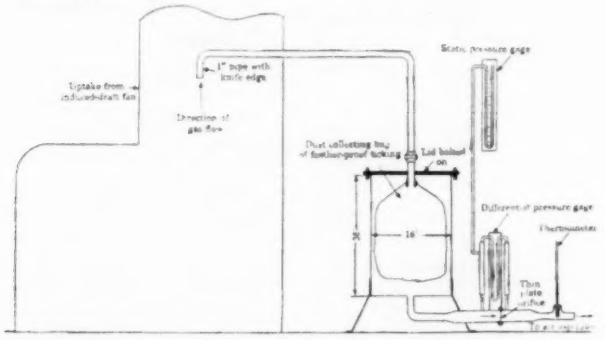


Fig. 8 Assembly of Apparatus for Sampling Flue Dust in Stack, Toronto Station

When the slag bed was cold after the boiler had been out of service for several days, the slag was not usually tapped until the second day of operation; difficulty was sometimes encountered at the first tappings until the solid slag in the corner at the tap hole had melted. Under normal operating conditions the slag became thinly fluid in that corner to the side and front walls, and only 5 to 15 minutes of work was necessary to open the tap hole and start the flow of slag.

Although the stream of slag tended to freeze over in the spout, an occasional pull with a rod removed the crust and kept the slag flowing freely. On some occasions, when too large a hole had been opened in tapping or when the slag level was high, the rate of flow became greater than the jets of water could cool and carry away, and it was necessary to decrease the size of the stream by spraying it with water.

Pittsburgh Coal in Boiler No. 4. Both periods of the burning of Pittsburgh coal followed periods when the boiler was out of service for several days, and were therefore started with a cold slag bed remaining from the burning of strip-mine coal. During each run, slag was tapped from the furnace on the first trial, the second day after the start, but thereafter the slag became hard in the bed near the tap hole and a hole was broken back to where it was fluid enough to run, only with a great deal of labor. The boiler load was increased, the excess air was decreased, air hammers were used, wood fires were built in the tap hole to prevent chilling of the slag by the cold air drawn in, and the combined efforts of several men were required almost continuously during the daytime to start and maintain the flow of slag.

The bed did not present the smooth appearance which it had when burning the strip-mine coal; it was rough and filled with small pieces of unfused ash. Feeling with a rod showed that there was molten slag in the center of the furnace, but toward the walls it was not thin enough to flow. The first run with this coal was closed because of the failure of a portion of the furnace arch. Inspection of the furnace showed that there was an accumulation of unfused and partly fused ash in the back of the furnace to a depth of 5 to 6 ft. This did not occur when burning the coal the second time, although the slag was not more fluid. The second run was closed because the operators decided that they could not tap the slag without excessive labor. They felt, however, that they could have removed the slag if the tap hole had been in the side of the furnace in a hotter zone.

Pittsburgh Coal in Boiler No. 5. As this boiler had been out of service for repairs before the test, a mixture of Pittsburgh and strip-mine coal was fired for two days to insure the favorable condition of a fluid bed of slag before firing Pittsburgh coal only. During the 8 days' run the slag was fluid and was tapped without difficulty, except on the fourth day, when no attempt had been made to tap the slag for two days over a week-end. The slag appeared to be fluid enough to run in the center of the bed, but was hard at the walls and around the tap hole. The furnace temperature was raised by decreasing the excess air and in a few hours the slag had melted at the tap hole and was tapped easily. The test was closed because of the necessity of repairs to the superheater.

Georges Creek Coal in Boiler No. 4. Observations on the condition of the slag bed were made continuously from the start of the burning of this coal, which was fired after a period when strip-mine coal had been burned and the slag bed was thinly fluid. Within an hour and a half after the coal was started into the furnace a crust of sintered and unfused ash had formed on the surface of the bed. This increased in thickness, and several hours after the start it could scarcely be penetrated with a rod. A small amount of slag was tapped from the furnace on the first day of the run which was, presumably, the slag remaining from the strip-mine coal. After the first day the bed was solid at any point that could be reached with a rod and no more slag was tapped during the remaining three days in which the 900 tons of this coal available were burned.

The bed was very rough on top and much of the ash was unfused; it was slightly sticky at times but never fluid.

Mixtures of Coals. Although no extended observations were made with definite mixtures of coals, the change-over periods from the Pittsburgh and Georges Creek coals to the strip-mine coal in boiler No. 4 were observed. Each time, the top of the slag bed became fluid within a few hours after the change, and a successful tapping could be made within 8 to 12 hours.

The furnace was also seen under normal operating conditions when indeterminate mixtures of the Pittsburgh and strip-mine coals were burned. The operators reported no particular difficulty with tapping of slag with any normal mixtures of coals.

Temperature of Slag

Table 2 is a summary of the temperatures measured in the flowing slag in the spout and in the slag in the bed of boilers Nos. 4 and 5.

TABLE 2 SUMMARY OF TEMPERATURES IN SLAG

	Slag in spout			Slag in bed		
	Min.	Avg.	Max.	Min.	Avg.	Max.
Coal		Boiler No. 4				
Strip-mine.....	2540	2590_5	2630	2415	2560_{11}	2645
Pittsburgh.....	2415	2580_3	2670	2410	2540_3	2670
Georges Creek[a]	..	..	..	2165	2390_3	2645
		Boiler No. 5				
Pittsburgh.....	2400	2510_3	2725	2460	2570_3	2680

Note: Subscript following average temperature denotes number of determinations averaged.

[a] Slag not fluid. Temperature of top of bed.

In boiler No. 4 the minimum temperature measured in the flowing slag was 125 deg. lower with the Pittsburgh coal than with the strip-mine coal, but the average and maximum temperatures were practically the same. As the slag did not flow when burning the Georges Creek coal, temperature measurements could not be made in the spout.

The maximum temperatures measured in the slag bed of boiler No. 4 were very nearly the same when burning the three coals. The average temperature in the bed of the strip-mine coal slag was about the same as with the Pittsburgh, and 170 deg. higher than for the Georges Creek coal. As the slag was not actually fluid when burning the Georges Creek coal, the thermocouple tube could only be placed on the slag and not in it, consequently the temperatures recorded are not as accurate as those for the other coals.

In boiler No. 5 the average temperature of the slag flowing in the spout was somewhat lower and the maximum temperature was somewhat higher than in boiler No. 4. The average temperature measured in the slag near the tap hole in the bed of boiler No. 5 was higher than when burning Pittsburgh coal in boiler No. 4; both the average and maximum temperatures were nearly the same as those in the bed of strip-mine coal slag in boiler No. 4.

ging bottom was installed. Similar data were not taken on boiler No. 5.

There was considerable variation among the determinations made at similar ratings of the percentage of ash carried by the gases below the boiler tubes. This would be expected because the area of the sampling tube was very small compared to the area of the furnace, and because the velocity and direction of gas flow varied from moment to moment. The agreement of the results of separate determinations at similar ratings of the percentage of ash carried in the gases at the stack was excellent.

Both the softening and fluid temperatures were similar for the several samples with each of the three coals, both below the tubes and in the stack. The softening temperatures of the samples from below the tubes were generally somewhat lower than of the samples from the stack, but the fluid temperatures of the samples were more nearly the same.

TABLE 3 AMOUNT, COMPOSITION, AND SOFTENING TEMPERATURES OF DUST CARRIED IN GASES BELOW BOILER TUBES AND IN STACK, BOILER NO. 4, TORONTO STATION

Dust below tubes											Dust in stack									
Boiler load, 1000 lb. steam per hr.	Sample No.	CO_2 in gas, per cent	CO in gas, per cent	Comb. in dust, per cent	Ash per 100 cu. ft. of[a] gas. gm.	Ash, percentage of total ash	Solid carbon, percentage of total C in gas	Initial deformation temp., deg.	Softening temp., deg.	Fluid temp., deg.	Sample No.	CO_2 in gas, per cent	CO in gas, per cent	Comb. in dust, per cent	Ash per 100. cu. ft. of[a] gas, gm.	Ash, percentage of total ash	Solid carbon, percentage of total C in gas	Heat loss in solid C, per cent	Initial deformation temp., deg.	Softening temp., deg.	Fluid temp., deg.
Strip-Mine Coal—Slagging Bottom																					
148	97	13.1	0.0	9.6	13.8	45	0.8	2060	2230	2410	100	12.4	0.0	25.4	7.73	26	1.5	1.2	2090	2170	2460
155	98	14.0	0.0	11.4	14.5	44	1.0	2090	2230	2460	101	12.6	0.0	23.9	7.87	26	1.4	1.1	2060	2230	2490
171	96	12.6	0.0	18.2	20.4	67	2.5	2090	2190	2490	99	12.2	0.0	12.6	11.93	41	1.0	0.8	2070	2240	2460
205	84	16.0	0.0	24.7	11.1	29	1.6	2060	2170	2390	87	13.4	0.0	21.7	9.53	30	1.4	1.1	2110	2300	2500
209	83	16.0	0.0	16.0	16.0	42	1.3	2070	2270	2540	86	13.5	0.0	20.2	9.87	31	1.3	1.1	2120	2330	2500
216	82	15.1	0.0	15.5	17.6	49	1.5	2070	2250	2560	85	13.3	0.0	26.5	9.53	30	1.8	1.5	2110	2310	2500
224	89	16.7	0.7	20.2	29.6	70	3.0	1990	2180	2410	92	13.4	0.0	18.9	8.98	28	1.1	0.9	2120	2310	2520
233	91	16.7	0.5	24.2	17.8	43	2.3	2050	2180	2420	94	13.4	0.0	17.3	7.15	22	0.8	0.6	2090	2260	2490
238	90	13.8	0.7	27.1	19.7	56	3.5	2070	2240	2470	93	14.0	0.0	16.7	10.32	31	1.1	0.9	2090	2230	2460
Average						49		2060	2220	2460						29			2100	2260	2490
Pittsburgh Coal—Slagging Bottom																					
145	58	...	...	38.6	18.3	...	...	2180	2270	2540	64	11.9	0.0	34.3	6.77	31	2.0	1.7	2320	2420	2690
149	59	17.1	0.0	39.4	19.6	60	5.0	2180	2270	2520	65	11.1	0.0	37.4	6.28	30	2.4	1.9	2220	2360	2630
171	57	13.0	0.0	40.3	18.8	74	6.5	2220	2390	2640	63	12.5	0.0	41.2	9.13	39	3.5	2.9	2350	2440	2650
212	54	13.5	0.0	45.2	21.3	79	8.4	2150	2210	2530	60	14.2	0.0	32.8	9.00	34	2.2	1.8	2320	2420	2650
215	55	15.2	0.0	19.5	20.2	71	2.2	2190	2300	2550	61	14.5	0.0	24.4	9.37	35	1.5	1.2	2350	2420	2650
221	56	10.7	0.0	50.1	21.5	96	12.6	2180	2270	2540	62	14.5	0.0	29.4	9.55	35	1.9	1.6	2320	2400	2690
235	48	...	...	33.6	9.4	...	...	2220	2390	2640	51	14.2	0.0	24.0	11.56	44	1.8	1.5	2230	2410	2620
235	49	15.1	0.0	38.4	18.7	64	5.2	2180	2380	2540	52	14.8	0.1	26.6	11.11	40	1.9	1.5	2290	2380	2610
235	50	14.9	0.0	35.2	22.7	79	5.5	2250	2380	2610	53	14.7	0.0	27.1	11.03	40	1.9	1.6	2350	2480	2610
Average						75		2190	2320	2570						35			2310	2410	2640
Georges Creek Coal—Slagging Bottom																					
157	115	16.0	0.0	27.3	14.8	48	2.4	2520	2670	2780	118	12.4	0.0	13.2	8.64	37	0.7	0.6	2770	2800	2900
159	116	17.1	0.0	30.1	16.6	50	2.9	2650	2770	2870	119	12.9	0.0	12.6	8.35	34	0.7	0.6	2770	2800	2880
200	122	16.3	1.1	38.1	24.6	71	5.8	2530	2650	2830	125	13.7	0.0	20.1	9.85	38	1.3	1.1	2650	2740	2830
207	121	15.4	1.7	52.4	31.3	86	12.5	2530	2650	2860	124	14.2	0.0	28.2	12.06	45	2.3	1.9	2590	2740	2830
237	109	11.6	0.0	34.3	17.4	76	5.3	2630	2740	2880	113	12.7	0.0	21.6	10.75	45	1.6	1.4	2670	2800	2880
237	...	...	...	...	...	...	...	...	...	...	112	13.2	0.0	30.9	12.35	49	2.9	2.4	2570	2710	2870
238	110	15.2	0.0	33.1	19.1	64	4.2	2570	2710	2880	114	13.3	0.0	24.8	10.41	41	1.8	1.5	2710	2810	2880
Average						66		2570	2700	2850						41			2680	2770	2870
Strip-Mine and Pittsburgh Coal—Dry Bottom																					
147	...	..	...	..	..	..	..	..	..	..	...	12.4	0.0	7.3	10.96	44	0.5	0.4	..	..	..
185	...	..	...	..	..	..	..	..	..	..	...	13.2	0.0	7.0	13.26	50	0.5	0.4	..	..	..
193	...	..	...	..	..	..	..	..	..	..	...	14.1	0.0	12.9	11.92	42	0.9	0.7	..	..	..
228	...	..	...	..	..	..	..	..	..	..	...	14.4	0.0	8.8	13.85	47	0.7	0.5	..	..	..

[a] At 70 deg. and 29.92 in. Hg.

Amount of Dust in Gases

Table 3 gives the amount of ash, percentage of combustible, and fusion temperatures of samples of dust taken from the gases below the boiler tubes and in the stack at the discharge of the induced-draft fan for each of the three coals at three boiler loads on boiler No. 4 with the slagging bottom, and with a mixture of strip-mine and Pittsburgh coal on the same boiler before the slag-

Fig. 9 shows graphically the values for the percentage of ash fired, carried in the stack gases, and for the percentage heat loss in carbon in the dust as taken from Table 3. The relative position of the curves for the boiler with the slagging bottom conforms with the relative fluid temperatures of the ash; the highest percentage of ash was carried out of the setting with the Georges Creek coal whose slag had the highest fluid temperature, and the lowest percentage of ash was carried out with the strip-mine coal whose slag had the lowest fluid temperature. The curve of the percentage of ash carried in the gases when the furnace had a dry bottom falls above all those with the slagging bottom. The coal burned when the samples were taken with the dry bottom was a mixture of Pittsburgh and strip-mine coals. The curve for a similar mixture with the slagging bottom would probably fall be-

tween the curves shown for the two coals. Hence the percentage of ash carried out with the slagging bottom would be approximately one-third, or 15 per cent of the total ash in the coal, less than with the dry bottom.

It is recognized that these values, even with the dry bottom, are lower than the usual values reported, but in view of the efficiency of filtering which was found and the excellent agreement of the results, they are reported with the belief that they are representative of the amount of ash carried at the point of sampling in the stack during normal operation. As the samples were taken from only one point in the stack, in the center, the results may possibly be low due to stratification; for the same reason the percentages carried by the gases below the boiler tubes may be high. A difference between the values at the two points would of course be expected because of deposition in the passes of the boiler. However, as the samples were taken at the same points for all three coals and for the dry and slagging bottoms, the relative results should be correct.

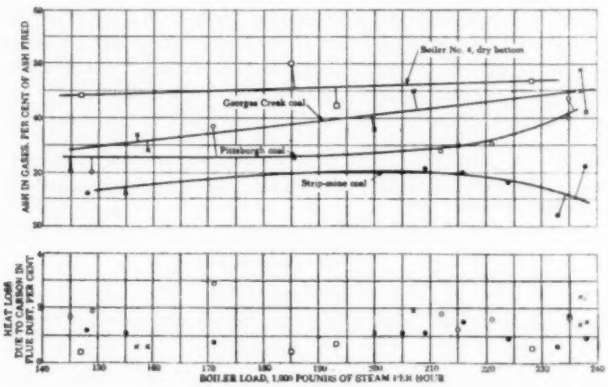


FIG. 9 PERCENTAGE OF ASH FIRED CARRIED OUT OF STACK WITH GASES AND PERCENTAGE HEAT LOSS IN CARBON IN DUST, BOILER NO. 4, TORONTO STATION

The heat loss in carbon in the dust which went out of the stack, for all samples but two, was less than 2 per cent. There appears to be no relation between this value and the coal or the rate of steaming. However, the average loss for the furnace with the dry bottom, shown by the squares in Fig. 9, was less than with the slagging bottom.

COMPOSITION OF ASH, SLAG, AND DUST

Table 4 gives the chemical analyses of the original ash, slag from the bed, and dust from below the boiler tubes and from the stack of the three coals tested in boiler No. 4 and of the coal ash and slags from boiler No. 5.

COMPOSITION OF SAMPLES FROM BOILER NO. 4

The analyses of the dusts were made on composites of two or three samples taken simultaneously at each point during the low-rate tests; the slag samples were taken from the top of the bed following the period of dust sampling, and the ash analyses were made on composites of two to four daily samples.

The outstanding difference in the composition of the three coal ashes was in their ferric oxide content. This was 23.8 per cent for the strip-mine ash with a fluid temperature of 2400 deg., 10.6 per cent for the Pittsburgh ash with a fluid temperature of 2640 deg., and 5.4 per cent for the Georges Creek ash with a fluid temperature of 2790 deg. The lime content of the strip-mine and Georges Creek ashes was approximately similar, but that of the Pittsburgh ash was much greater. The content of both the silica and alumina increased as the fluid temperature increased.

The ferric oxide content of the slag of all three coals was greater than that of the ash, by about one-sixth for the strip-mine coal, by almost one-half for the Pittsburgh coal, and by more than twice for the Georges Creek coal. The lime content of the slag did not change greatly from that of the ash for any of the coals. The softening temperature of the slag was lower than that of the ash for all three coals: it was lower by more than 300 deg. for the Georges Creek coal, but with this coal the fluid temperature of the slag was only 120 deg. lower than that of the ash; for the other two coals it was practically the same for the ash and slag.

The composition of the dust samples from the two positions was very similar for each coal, except that the silica content was 1.3 to 3.6 per cent higher in the dust from the stack than in that from below the tubes. With this exception the composition of all dust samples did not greatly differ from the composition of the coal ash, and the softening and fluid temperatures were also nearly the same as those of the coal ash.

COMPOSITION OF SAMPLES FROM BOILER NO. 5

The composition of the two samples of coal ash from this boiler was similar, but differed from that of the Pittsburgh coal ash from boiler No. 4 in that the alumina content was somewhat higher, the ferric oxide content lower, and the lime content higher. The softening temperatures of both samples and the average fluid temperature was lower than of those from boiler No. 4.

The increase in the ferric oxide content of the slag over that of the ash was greater than in boiler No. 4, but the percentage was very nearly the same. The softening temperatures of the slags were not much lower; but the fluid temperatures were lower than those of the coal ash, and also lower by about 100 deg. than that of

TABLE 4 COMPOSITION AND SOFTENING TEMPERATURES OF COAL ASH, SLAG, AND DUST SAMPLES

Sample No.	Description	Per cent SiO_2	Al_2O_3	Fe_2O_3	CaO	MgO	Initial deform. temp., deg.	Softening temp., deg.	Fluid temp., deg.
	Boiler No. 4								
	Strip-mine coal								
a	Coal ash	46.3	24.4	23.8	1.7	0.8	2050	2180	2400
102	Slag from bed	47.4	22.2	28.0	1.5	0.4	2000	2080	2430
96–97–98[a]	Dust below tubes	46.8	24.5	21.1	1.5	0.4	2080	2220	2450
99–100–101[a]	Dust from stack	49.9	24.0	21.3	1.3	0.5	2070	2210	2470
	Pittsburgh coal								
a	Coal ash	51.7	26.9	10.6	3.9	0.8	2320	2470	2640
66	Slag from bed	52.5	26.3	14.7	4.8	0.4	2130	2360	2630
57–58–59[a]	Dust below tubes	50.5	28.2	9.8	4.3	0.4	2190	2320	2570
63–64–65[a]	Dust from stack	54.1	28.0	9.8	4.4	0.6	2300	2410	2660
	Georges Creek coal								
a	Coal ash	56.2	33.5	5.4	1.1	0.6	2600	2700	2790
117	Slag from bed	55.6	29.1	11.7	1.2	0.3	2180	2370	2670
115–116[a]	Dust below tubes	54.7	30.3	8.2	1.0	0.5	2640	2720	2830
118–119[a]	Dust from stack	56.0	31.9	6.8	0.8	0.6	2770	2800	2890
	Boiler No. 5								
	Pittsburgh coal								
145	Coal ash	53.6	30.5	7.0	5.2	1.0	2270	2370	2610
147	Coal ash	51.7	30.4	8.4	4.6	1.1	2200	2340	2520
136	Slag from bed	51.5	28.2	13.1	4.7	0.9	2250	2360	2480
140	Slag from bed	51.2	28.0	14.1	4.6	0.9	2090	2220	2490

[a] Analysis of composite sample.

the slag from boiler No. 4, although the composition of the slags from the two boilers was nearly the same. The fluid temperatures of three other samples of slag from boiler No. 5 were 2420, 2490, and 2530 deg., which shows good agreement and confirms that the slag deposited from Pittsburgh coal on the bed of No. 5 boiler had a lower fluid temperature than that taken from the bed of No. 4.

Table 5 gives the composition of three additional samples of slag from the bed, two from the side-wall tubes, and one from the boiler tubes from boiler No. 4 when burning Pittsburgh coal. These analyses are very similar to those of the corresponding samples given in Table 4 for the same coal. In the samples from the bed the content of ferric oxide was higher than in the coal ash, whereas in the slag from the tubes it was nearly the same or somewhat less.

Combustible Content of Slag Samples

When ignited preparatory to chemical analysis, some of the samples of slag showed a gain in weight and others showed a loss. A gain on ignition results from the oxidation of ferrous or metallic iron in the slag, and loss may result from the burning out of combustible matter or from the decomposition of carbonates or sulphides. Samples which showed an appreciable loss on ignition were tested for carbonates or sulphides with negative results.

Because of the possibility of gain in weight due to the oxidation of iron or ferrous oxide the loss on ignition cannot be used as a measure of the combustible content of the slag; therefore it was determined by analysis for hydrogen and carbon, as in the ultimate analysis of coal, by combustion in oxygen and collection of the water and carbon dioxide formed.

TABLE 5 COMPOSITION OF SLAG FROM BURNING PITTSBURGH COAL, BOILER NO. 4

Sample No.	Description of sample	Per cent: Loss on ignition	SiO_2	Al_2O_3	Fe_2O_3	CaO	MgO	Initial deform. temp., deg.	Softening temp., deg.	Fluid temp., deg.
15	Slag from bed	0.7	53.2	28.8	11.7	4.7	0.6	2350	2380	2590
16	Slag from bed	1.6	53.5	28.5	11.8	4.7	0.7	2350	2380	2590
17	Slag from bed	0.3[a]	53.4	26.9	15.0	3.8	0.8	2340	2390	2540
20	Slag from left side wall tubes	6.5	53.4	29.3	10.8	3.7	0.6	2200	2330	2630
21	Slag from right side wall tubes	0.4	54.4	29.4	10.5	4.0	0.7	2100	2130	2620
22	Slag from boiler tubes, first pass	1.3	55.2	29.8	9.3	3.7	0.7	2110	2350	2530

[a] Gain on ignition.

TABLE 6 COMBUSTIBLE CONTENT OF SLAG SAMPLES

Sample no.	Type sample	From	Boiler no.	Sample	Per cent: Moisture	Loss on ignition	Hydrogen	Carbon
44	Grab	Bed	4	Strip-mine	0.1	3.8	0.5	4.2
102	Grab	Bed	4	Strip-mine	0.1	1.0	0.4	2.3
88	Grab	Bed	4	Strip-mine	...	...	0.2	2.0
95	Grab	Bed	4	Strip-mine	...	...	0.4	4.7
66	Grab	Bed	4	Pittsburgh	0.1	8.0	0.7	7.8
117	Grab	Bed	4	Georges Creek	0.2	8.9	0.8	8.6
153	Avg [a]	Spout	4	Strip-mine and Pittsburgh	...	...	0.3	2.7
136	Grab	Spout	5	Pittsburgh	...	0.3[b]	0.1	0.8
138	Grab	Spout	5	Pittsburgh	...	0.9[b]	0.2	0.8
140	Grab	Spout	5	Pittsburgh	...	0.8[b]	0.2	0.6
141	Grab	Bed	5	Pittsburgh	...	0.2[b]	0.1	0.3
154	Avg [c]	Spout	5	Strip-mine and Pittsburgh	...	...	0.2	0.7
155	Avg [d]	Spout	5	Strip-mine and Pittsburgh	...	...	0.4	4.6
137	Grab	Spout	7	Pittsburgh	...	...	0.1	0.7

[a] Sampled periodically over entire period of tapping.
[b] Gain on ignition.
[c] First half of tapping period.
[d] Last half of tapping period.

Table 6 gives the results of this determination on 14 samples. All contained carbon; the content varied from 0.3 to 8.6 per cent. The samples which contained the most carbon were slags from the Pittsburgh and Georges Creek coals burned in boiler No. 4. As previously noted, these slags were very viscous and probably enclosed particles of carbon. However, the strip-mine slag, which was very fluid, contained up to 4.7 per cent carbon, and sample No. 153, which was an average of an entire period of tapping, contained 2.7 per cent carbon.

No investigation has been made of the form in which the carbon is present in the slag. There is a possibility that it may be in solution in metallic iron which has been reduced from the oxide or silicon carbide may have been formed. In the larger percentages, however, it is considered probable that it is deposited on the surface of the slag bed as particles of coke and is possibly prevented from burning by being covered by slag. This conception is partly confirmed by samples 154 and 155. The sample representing the first half of the tapping period, which was, probably, principally the lower part of the molten slag, contained but 0.7 per cent carbon; the sample representing the last half of the period, which was probably the top of the slag bed, contained 4.6 per cent carbon.

The heat loss resulting from these percentages of carbon is not large. Assuming 10 per cent ash in the coal and 50 per cent of the ash fired collected as slag in the hearth, 3 per cent carbon in the slag would mean a loss of less than 0.2 per cent of the heat in the coal.

TABLE 7 COMPOSITION OF SLAG AT VARIOUS DEPTHS IN BED BOILER NO. 4

Sample No.	Depth of sample, in.	Per cent: Loss on ignition	SiO_2	Al_2O_3	Fe_2O_3	CaO	MgO	Initial deform. temp., deg.	Soft temp., deg.	Fluid temp., deg.
8	0–4	0.6[a]	47.2	25.7	24.7	1.8	0.6	2020	2080	2150
9	4–10	0.9	43.7	24.4	29.5	1.7	0.6	1980	2110	2380
10	10–12	1.2[a]	40.6	22.3	36.1	1.5	0.7	2000	2150	2350
11	12–15	0.3[a]	40.2	22.9	33.3	2.6	1.5	1990	2040	2340

[a] Gain on ignition.

Not enough samples have been taken and analyzed to show the average percentage of carbon which may be in the slag, but the results do show that the molten slag may carry unburned carbon and that the possibility of loss cannot be ignored.

Composition of Slag at Various Depths in Bed

As previously stated, the hearths were made up of burned dolomite which was only loosely thrown in. During the early weeks of operation of the furnace the company's engineers observed that the slag contained material which appeared to be dolomite. It was also thought that a great deal of dolomite was probably chemically dissolved by the slag. Before the start of the present tests samples of slag were taken in boiler No. 4 at four depths in the bed remaining from the burning of strip-mine coal.

Table 7 gives the analyses of the four samples. The principal differences in the first three samples was the increase in the ferric oxide content and the corresponding decrease in silica with increase in distance from the top of the bed. The content of lime and magnesia by which the presence of dolomite may be detected was practically constant and normal for slag from that coal.

The fourth sample at 12 to 15 in. from the top contained more lime and magnesia than would come from the coal ash, and therefore there were traces of dolomite left at that depth.

Because of the uneven surface of the slag bed it was impossible to measure accurately the height of the top of the slag above the original level of the dolomite, but it was estimated to be 10 to 12

TABLE 8 TEMPERATURE OF GASES AT LOWER OBSERVATION HOLE NEAR FACE OF BURNERS, AT VARYING DISTANCES FROM SIDE WALL, BOILER NO. 4 WITH SLAGGING BOTTOM, TORONTO STATION

Boiler load, 1000 lb. steam per hr.	Boiler diff., in. of water	Furnace draft, in. of water	Feeder speed, r.p.m.	CO_2 by recorder, per cent	Primary air pressure, in. of water	Distance from side-wall tubes, in. 6	12	18	24	30	36	42	48	54	Average temp.[a]
Strip-mine coal															
220	1.67	0.085	145	15.4	15.6	1860	2200	2570	2445	2535	2580	2680	2765	2765	2570
230	1.58	0.075	132	15.5	14.5	2170	2245	2280	2310	2385	2520	2590	2680	..	2430
Pittsburgh coal															
145	1.05	0.100	78	..	11.7	2060	2655	2670	2745	2815	..	..	..	..	720
150	0.80	0.055	85	..	12.0	2510	2670	2800	2865	2865	..	..	..	..	2800
190	1.50	0.100	100	..	..	2290	2675	2740	2795	2800	..	..	..	..	2750
200	1.35	0.100	105	16.0	..	2465	2450	2525	2655	2725	..	..	..	..	2590
205	1.34	0.080	102	15.8	..	2110	2610	2655	2630	2580	..	..	..	..	2620
220	1.56	0.120	115	15.0	12.5	2825	2840	2865	2865	2855	..	..	..	..	2860
230	1.54	0.120	135	15.2	13.0	2785	2895	2980	3010	2965	..	..	..	..	2960
Georges Creek coal															
159	0.85	0.070	99	15.2	15.9	2385	2740	2795	2835	2865	2880	2755	2740	..	2800
200	1.21	0.070	142	15.8	15.5	2170	2510	2695	2710	2655	..	..	..	..	2640
238	1.73	0.080	143	15.8	15.7	2385	2370	2355	2460	2475	2505	2355	2265	..	2400

[a] Not including reading at 6 in.

in. During operation the slag level was not drained down to the depth at which the samples were taken, therefore the actual effective bottom was principally slag.

Temperature of Furnace Gases

Table 8 presents a summary of the temperatures measured in the gases in boiler No. 4 at the lower observation hole, which was approximately 4 ft. from the face of the burners. As no general temperature survey of the furnace was made these temperatures represent neither the maximum, minimum, nor average temperatures of the furnace gases.

TABLE 9 COMPARISON OF TEMPERATURES BELOW BOILER TUBES IN BOILER NO. 3 WITH DRY BOTTOM AND BOILER NO. 4 WITH SLAGGING BOTTOM, TORONTO STATION

	Boiler No. 3	4	3	4
Boiler load, 1000 lb. steam per hour	160	165	195	200
Furnace draft, in. of water	0.08	0.13	0.09	0.11
Boiler differential, in. of water	1.14	1.22	1.31	1.38
Primary air pressure, in. of water	16.1	15.1	17.0	15.6
Feeder speed, r.p.m.	125	125	138	140
Temp. of gas entering air heater[a], deg. fahr.	556	566	562	589
Temp. of gas leaving air heater[a] deg. fahr.	423	425	422	425
CO_2 in last pass, by recorder, per cent	14.0	15.0	16.1	15.2
CO_2 at induced-draft fan discharge, by orsat analysis, per cent	11.9	12.0	11.9	12.1
Temperature of gas below tubes[b], deg. fahr.:				
2 ft. from side wall	1850	2275	1990	2310
3 ft. from side wall	1870	2260	2025	2315
4 ft. from side wall	1880	2240	2030	2300
Average	1865	2260	2015	2310

[a] By recording thermometer.
[b] Average of 4 determinations at each distance.

Visual observation of the flames with the three coals indicated that the flame of the Georges Creek coal, whose volatile content was 20 per cent, was longer than with the other two coals, whose volatile content was 35 to 37 per cent, when other furnace conditions were similar. The difference between the temperatures measured with the Pittsburgh and Georges Creek coals confirm this indication of difference in the length of the flame for with similar rating, excess air, and primary air pressure the temperatures were lower with the Georges Creek coal.

The principal, independently variable factor which governs the length of the flame is the pressure of the primary air. Fig. 10 shows the average temperature of the gases at 18 to 48 in. from the side wall at the lower observation hole and just below the tubes with different pressures of primary air when burning Georges Creek coal with a boiler load of approximately 210,000 lb. of steam per hour. All other conditions as furnace draft, boiler differential, and excess air were maintained as constant as possible. The increase in temperature near the burners shows that the length of flame decreased with the increase in air pressure. The corresponding decrease in temperature below the boiler tubes was due to the longer time that the gases had for transfer

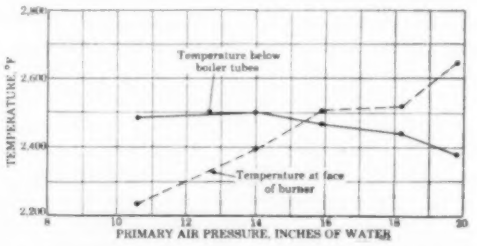


Fig. 10 Variation of Temperature at Face of Burner and Below Boiler Tubes With Pressure of Primary Air, Georges Creek Coal, Boiler Load Approximately 210,000 Lb. of Steam per Hr. Boiler No. 4, Toronto Station

of heat to the water-cooled surface after combustion was complete.

Comparison of Temperatures Below Tubes in Furnaces With Dry and Slagging Bottoms

Table 9 gives the temperature of the gases measured about 6 in. below the boiler tubes at similar loads and other conditions in boilers No. 3 and 4. The boilers and furnaces were exactly alike except that the slag hearth in No. 4 covered the lower water-tube slag screen. The area of radiant-heat absorbing surface was therefore approximately 166 sq. ft. less in No. 4 than in No. 3 boiler.

The average temperature below the tubes in No. 3 at a load of 160,000 lb. of steam per hour and 11.9 per cent CO_2 at the stack was 395 deg. lower than in No. 4 at a load of 165,000 lb. per hour, and 12.0 per cent CO_2. The temperature was 295 deg. lower in No. 3 than in No. 4 at loads of 195,000 and 200,000 lb. per hour, when the percentages of CO_2 were 11.9 and 12.1, respectively. These temperature differences of 395 and 295 deg. has decreased

TABLE 10 PERCENTAGE COMPOSITION OF GASES NEAR SLAG BED, WHEN BURNING PITTSBURGH COAL, BOILER NO. 4

Boiler load	Boiler diff., in. of water	Furnace draft, in. of water	Feeder speed, r.p.m.	CO_2 by recorder, per cent	Primary press., in. of water	Per cent CO_2	O_2	CO	CH_4	H_2	N_2
150	1.0	0.075	90	..	12.0	9.7	1.1	9.5	0.0	5.4	73.3
170	1.10	0.09	75	15.0	11.0	9.4	9.2	0.0	0.0	0.0	81.4
180	1.5	0.1	..	15	..	10.7	4.6	2.0	0.4	0.7	81.6
190	1.26	0.08	105	16.5	..	9.6	8.7	0.0	0.0	0.0	81.7
210	1.54	0.12	108	15.0	12.5	11.4	6.2	0.5	0.0	0.2	81.7

to 10 and 27 deg. at the boiler outlet, and to 2 and 3 deg., respectively, at the air-heater outlet. Therefore, although the difference in gas temperature at the entrance to the boiler was relatively great, the heat-absorbing surface in the boiler and air heater was great enough so that the final efficiency of heat absorption in the two boilers was approximately the same.

Composition of Gases Near Slag Bed of Boiler No. 4

Only a few samples of gas were taken from the furnace as a complete survey of the gas composition was not considered within the scope of the investigation. Table 10 gives the composition of five samples taken in boiler No. 4 through the lower observation hole, 4 ft. from the front wall. To obtain an average sample the sampling tube was moved back and forth 12 to 40 in. from the side-wall tubes.

The composition of the samples taken varies considerably and

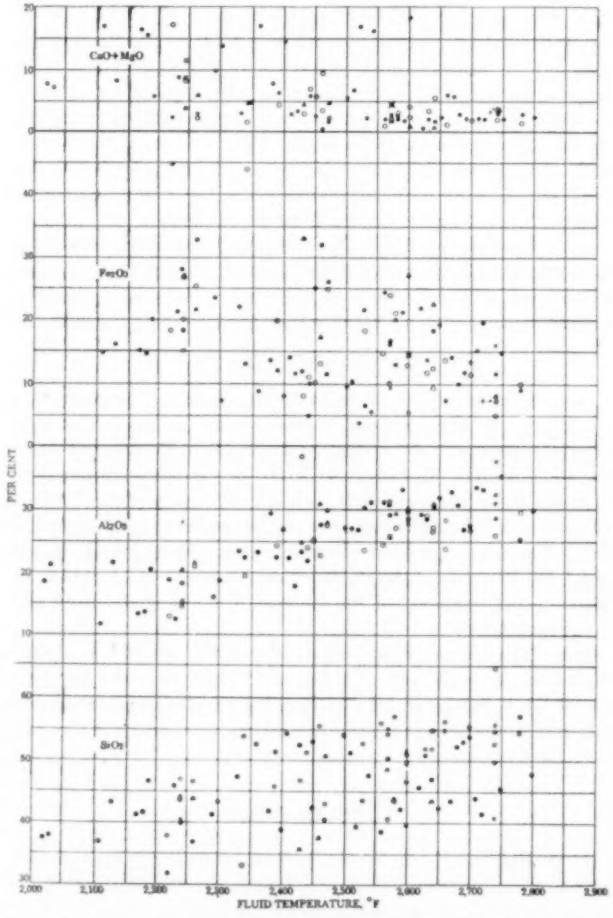


Fig. 11 Relation of Fluid Temperature of 120 Coal Ashes and Slags to Chemical Composition

not enough were taken to determine the average value. The presence of as much as 9.5 per cent of CO and 5.4 per cent H_2 in one sample and lower amounts in two other samples shows that the atmosphere over the hearth may be strongly reducing at times, which is advantageous in the formation of fluid slags from ash containing iron.

Addition of Fluxes to Coal Ash

The addition of fluxes has been suggested as a means to extend the advantages of the slagging method of removal of ash to include coals whose ash has a high fluid temperature.

Iron, lime, and the alkalis are the three fluxes that occur in the ash of coal in appreciable amounts. The quantity of alkalis in ash is not often determined; the data that are available show that the quantity of alkalis in the ash is less than in the coal, and that when coal is burned the greater part of the alkalis is carried away in the gases; the same action would be likely to occur if alkalis were added as fluxes to the coal, or bed of slag.

Iron and lime are the fluxes that are usually considered, but data relating them to the fusion temperatures of the mixtures are incomplete: a study of the silica-aluminum-iron-lime system is being made by the Bureau of Standards jointly with the Special Research Committee on Boiler-Furnace Refractories of the A.S.M.E.

TABLE 11 SOFTENING AND FLUID TEMPERATURES BY STANDARD CONE TEST OF MIXTURES OF PITTSBURGH COAL ASH AND SLAG WITH FERRIC OXIDE

Description of sample	Softening temperature, deg.	Fluid temperature, deg.
Original coal ash	2460	2560
Coal ash, fused at 2700 deg. for 30 min.	2280	2650
Fe_2O_3 mixed with coal ash, per cent:		
2.5	2390	2550
5.0	2350	2520
10.0	2240	2390
15.0	2260	2380
20.0	2230	2340
Coal ash mixed with Fe_2O_3 and fused at 2700 deg. for 30 min., per cent:		
2.5	2220	2650
5.0	2200	2570
Slag from bed	2340	2470
Slag mixed with 20 per cent Fe_2O_3	2020	2350
Slag mixed with 20 per cent Fe_2O_3 and fused at 2700 deg. for 30 min.	2100	2350

Attempts to derive relations from the analysis and fusion temperature of coal ash have not been successful because of undetermined factors or because the number of determinations is not enough. Fig. 11 shows such a plot of chemical composition against fluid temperature. The plot shows a general tendency for the Fe_2O_3 and the CaO + MgO to decrease with increasing temperature; plotting either the sum of these three fluxes or the ratio of the total acids to the total bases gives a somewhat better agreement, but neither method establishes a definite enough guide to determine the additional quantity of fluxes to add to assure the necessary reduction of the fluid temperature. As a rule, iron occurs in greater amount in the ashes with a low fluid temperature than does lime; high content of iron was the characteristic of the strip-mine coal used in this investigation.

A thorough investigation of the possibility and economical feasibility of adding flux to coal ash to render it fluid would necessitate a method of estimating the fluidity of the mixtures and many tests in laboratory and plant. However, the results of a brief study which has been made of the softening and fluid temperatures of mixtures of ferric oxide with Pittsburgh coal ash and slag by the standard gas-furnace cone test are of interest. The ash used was from a sample of the Pittsburgh coal obtained during these tests. One portion of it was fused at 2700 deg. for 30 minutes in a platinum crucible heated in a gas furnace, and the resultant slag was ground fine and a cone test made to determine whether the fusion temperature had been changed by the melting. Other portions of the ash were then mixed with increasing percentages of Fe_2O_3 and the fusion temperatures determined without previously fusing the mixture. In addition, two of these mixtures were fused at 2700 deg. for 30 minutes and the fusion temperatures of the slag determined.

Table 11 presents the results of this study. They show that fusion of the ash lowered the softening temperature but not the fluid temperature. Likewise the fluid temperatures of the fused mixtures of the ash or slag with the ferric oxide were not lower than those of the mixtures which were only ground together. Additions of 2.5 or 5 per cent of ferric oxide did not appreciably lower the fluid temperature of the ash; 10 per cent lowered it 170 deg. to 2390 deg. and further additions lowered it little more. The addition of 20 per cent ferric oxide to the slag lowered the fluid temperature from 2470 to 2350 deg.

These tests indicate that the addition of 10 per cent of ferric oxide to the ash might decrease the fluid temperature of the Pittsburgh coal ash enough to give a slag fluid enough to flow from the

furnace. As the coal contained approximately 10 per cent ash, and assuming 50 per cent of the ash was deposited on the hearth, 0.5 per cent or 10 lb. of ferric oxide per ton of coal or approximately 1600 lb. per furnace per day would be required. Ferric oxide is available in the form of iron ore which contains about 70 per cent ferric oxide or as mill scale.

More or less than this amount might be required, depending on the amount of the flux which was carried out with the gases, and the rate of combination and mixing with the ash on the hearth. It would be necessary to add the flux either continuously, or at regular intervals in proportion to the rate of firing coal. The mechanical means of adding the flux might offer some difficulty, but this feature will not be discussed in this paper.

SUMMARY OF OBSERVATIONS

As the data given in this paper are based on the burning of the three coals in furnaces which were not originally designed for use with a slagging bottom, the results of the observations cannot be generalized too freely, and it may be expected that the actual numerical values would be affected by the types of furnace and burners; on the other hand, the relative order of the actions should be the same. The unfavorable conditions for the removal of the slag from boiler No. 4 have been pointed out: the proportional area of the bed exposed to the cooling action of the walls was large, the position of the main part of the bed under the face of the burners was not favorable relative to the zone of maximum combustion, and the tap hole was in a corner where the cooling action was great and where a long tunnel was necessary.

1 A selective separation of the constituents of the ash occurred with all coals which increased the percentage of ferric oxide in the slag compared with that in the ash of the coal. The percentage increase was greater as the percentage of ferric oxide in the original ash was lower.

2 The softening temperatures of the slags were lower than those of the original ash, but the fluid temperature did not decrease as much; the decrease of these fusion temperatures increased as those of the original ash increased and was about 120 deg. for the fluid temperature of the Georges Creek ash.

3 The percentage of the total ash fired which passed to the stack with the slagging-bottom furnace increased with increase in fluid temperature of the original ash. The amount carried out of the furnace even at the highest rating and with the least fluid ash was less than 50 per cent of that fired.

4 For similar coals the percentage of ash which passed to the stack with the slagging-bottom furnace was approximately one-third less than with the dry-bottom furnace.

5 All samples of slag analyzed contained carbon, although not enough were tested to determine the average content. The percentage heat loss was small, but the possibility of loss by unburned carbon in the slag cannot be neglected.

6 Comparisons of the temperature of the gas below the boiler tubes in two furnaces exactly alike, except that one was equipped with a water-tube slag screen and ash-collecting hopper and the other with a slagging bottom installed above the slag-screen tubes, showed that the temperature, at similar boiler loads and excess air, was 300 to 400 deg. higher in the furnace with the slagging bottom. The temperatures at the outlets of the air heater were, however, nearly the same.

7 During these tests the slag from the ash of the Pittsburgh coal could not be successfully removed as molten slag from the furnace of boiler No. 4, although as high a temperature was attained in the bed of slag as with the coal from the strip-mine. It was, however, on the border line of successful removal and the operators felt that it could have been tapped if the tap hole had been in a more favorable position.

8 The removal of the molten slag was successful when burning Pittsburgh coal in boiler No. 5 and the reasons for this, as compared with the non-success in No. 4, apart from the more favorable position of the tap hole, were probably that there was a somewhat higher average temperature of the slag and also that the average fluid temperature of the slag was about 100 deg. lower.

9 The chemical composition of the slags of the Pittsburgh coal from the beds of No. 4 and No. 5 boilers were nearly the same, although those of the coals differed appreciably: data are not available to explain why the fusion temperatures of the two slags differed.

10 The slag of the Georges Creek coal could not be removed from the furnace of boiler No. 4. The top of the bed was hard and the temperatures measured were those near the surface; the maximum temperatures observed were as high as those in the molten slag of strip-mine and Pittsburgh coals, but the average was 150 deg. lower; the position of the bed near the front wall of the furnace was less favorable with the low volatile Georges Creek coal because its longer flame placed the zone of maximum intensity of combustion farther from the burners.

11 A brief study of fluxes for Pittsburgh coal ash showed that the addition of ferric oxide in an amount of 10 per cent of the slag deposited should lower the fluid temperature so that it could be successfully tapped as molten slag from the furnace investigated.

ACKNOWLEDGMENTS

The cooperation with Stevens & Wood, Inc., was carried on through P. B. Rice, general engineer, who, with A. Carnegie, superintendent of power, Pennsylvania-Ohio Power and Light Co., F. C. Beattie, chief engineer, C. H. Davis, efficiency engineer, P. Kelly, boiler-room engineer, and all employees of the Toronto station rendered every assistance in the conduct of the investigation. The dust samples in the stack were collected by L. A. Orr and J. W. Freeman of the efficiency department. C. L. Aldeman, assistant fuel engineer, of the Bureau, was engaged during part of the investigation.

The chemical analyses were made in the general analytical laboratory of the Bureau of Mines at Pittsburgh under the direction of W. A. Selvig, and the coal analyses and fusion temperature determinations in the coal-analysis laboratory under the direction of H. M. Cooper.

Discussion

JOHN BLIZARD.[14] In Table 9 the authors compare the temperatures of thermocouples just below the boiler tubes when operating with a water screen and with a slagging bottom. They show that with a slagging bottom these temperatures are from 300 to 400 deg. higher than they are with a water screen.

This difference in temperature is greater than might have been expected, and if these temperatures really represent the actual temperatures of the gases entering the boiler tubes, then the furnace with the water screen must be credited with absorbing the additional heat equivalent to the lower temperature. The lower temperature with the dry bottom may be of great practical importance, since it will render the ash less likely to slag the boiler tubes.

I should like to know whether there was any greater tendency to slag the tubes of the boiler with the slagging bottom, and if so, whether this tendency may not to some extent militate against the removal of the slag in liquid form.

Since the temperatures in the furnace are reduced by the extent, position, and shape of the heating surface in the furnace, it is important to know how the heating surface of the water walls and water screen were computed.

[14] Head, Research Department, Foster Wheeler Corporation, New York, N. Y. Mem. A.S.M.E.

There are several ways of computing this surface. Some regard the projected area as the best measure; others use the total area; and the computation of the equivalent surface of indirect surface such as cast-iron blocks or fins may be carried out in many ways.

In the chart in Fig. 9 the carbon loss from a furnace with bare tubes is seen to be small and does not change with the rate of combustion. Since this rate of combustion varied between 13,000 and 25,000 B.t.u. per cu. ft. per hour, these results should go far to allay the fears of a few who hesitate to install bare tubes in a furnace for moderate rates of heat liberation.

The paper is a notable contribution to the literature of the subject of boiler furnaces, and the authors may rest assured that users and designers of powdered-coal furnaces will find their work of great value.

W. S. Johnston.[15] From the time the Toronto station went into initial operation until early in 1928, the ash from the pulverized-coal-fired furnaces was removed in the dry state by means of the conventional ash hoppers and hydraulic jets commonly used for such purposes. During that period there were four furnaces of the Lopulco design, and in October, 1927, two Bailey furnaces were added to the station. When the hopper-bottom type Bailey furnaces were fired with strip-mine coal some difficulty was experienced with accumulations of slag along the sides and tops of the hopper walls. The accumulation of slag greatly increased when Pittsburgh coal was fired, and could not be completely removed by the suggested method of increasing the furnace temperature by decreasing the excess air, increasing the CO_2, and thus fuse and melt the slag. With six boiler units in the plant there was a large amount of expensive manual handling of slag and ashes.

In the spring of 1928 it was decided to try a molten-slag bottom in the furnace of boiler No. 5, then fired with Calumet inter-tube burners, because of the reputed greater ease in tapping molten slag than in handling dry ash. This facility was very quickly established after one month's operation. A similar bottom was projected for boiler No. 4, being placed directly over the original floor-screen tubes, to determine (1) if slag could be melted and tapped from a furnace fired horizontally with bare furnace-wall tubes in front of refractory walls on three sides and (2) the effect of a molten-slag bed in contact with a refractory wall on the fourth side without any cooling protection.

The foregoing briefly outlines the reasons for changing to molten-slag furnace bottoms. Some troubles were encountered before the operators learned by experience exactly the correct method of handling molten slag. But the amount of labor required for disposing of refuse material was greatly decreased. What follows is offered, not as a criticism of the paper, but as supplementary comments which may convey some additional information to those interested in the subject.

When the flat bottoms were first installed at Toronto, there was not much information available as to how they should be constructed. It was assumed that a fluid-tight bottom carried on a heavy steel structure, because of the weight of the slag, would be necessary, and that it must be suitably lined with refractory material. Some diligence was used to secure a seal around the edges of the bottom adjacent to the furnace walls. The first bottom installed was lined with two courses of firebrick on top of a $^1/_2$-in. steel plate. When the second bottom was built, the lower course of firebrick was replaced by one course of sil-o-cel. In furnace No. 5 the steelwork was located above the original hopper bottom which closed it in on the front and back. The original brick walls of the ash hopper closed in the sides. This placed the supporting structure in a hot location that could not be ventilated. An attempt was made to take its temperature and it was found to run about 600 deg. None of the steel, though, was found to be seriously affected. In the light of experience it is believed that the insulating material, while desirable, is not essential.

Dolomite was placed on top of the brick. However, there is evidence that the molten slag has probably entirely melted the dolomite and may have melted some of the firebrick. What probably has occurred is that at some point in the furnace bottom a plane of equilibrium has been established. Below this plane the room air chills the slag below its melting temperature, and above the plane the slag begins to take the fluid state. Of course, initially it is necessary to provide refractory material, but after a period of operation the slag makes its own bottom which is practically impervious.

The location of the slag tap hole is of importance. On furnace No. 5 there was opportunity to place the hole wherever it was desired, and, more or less arbitrarily, it was located on the center line of a side wall. However, in the case of furnace No. 4, with its solid brick walls and water-cooled tubes on three sides, it was thought easiest, from the construction standpoint, to locate the tap hole in the solid brick front wall. Experience over a number of months indicated that the slag cooled somewhat below its fluid temperature by flowing through an 18-in. brick wall and over a brick-lined spout and impeded, at least, the initial tapping.

Considerable of the difficulty mentioned under the head of "Pittsburgh Coal in Boiler No. 4," in the paper by Ralph A. Sherman, P. Nicholls, and Edmund Taylor, was largely due to this fact. Within the last few months the location of the slag tap hole on this boiler has been changed to the south side of the furnace in place of the third side-wall water tube from the front which has been removed. The hole is at a much hotter point of the furnace, and it is now very easy to draw off the slag. In fact slag from Pittsburgh coal is tapped with almost the same facility as when strip-mine coal is fired. Probably the best material for a slag spout is cast iron; the slag does not stick to it, and it minimizes the cooling effect which results if the flow of slag down the spout is choked. The use of some kind of a gate at the outer end of the spout is highly desirable as it gives the operators a ready means of controlling the flow. A gate will not be frozen in place even though it is used to stop a flow of molten slag. Apparently the slag does not adhere to the iron or steel. In Mr. Cushing's paper it is mentioned that it is very easy to remove any solidified slag in the spout. This is true. It would seem to the inexperienced that this presents some difficulties, but actually it does not.

As soon as the slag leaves its bed, it must be moved to the water sprays as quickly as possible and kept at as high a temperature as is feasible. When the slag meets the water, two changes take place. The slag is cooled, and in the cooling process it is disintegrated. Thereafter, additional water continues the cooling process, carries disintegration further, and keeps the slag moving. Nozzles for the bottoms of chutes should be located so that the spray of water sweeps the surface of the bottom. This means a distance of but $^1/_4$ or $^1/_2$ in. and not 1 in. or more, as otherwise the slag will deposit and accumulate on the bottom below the sheet of water. Without a doubt, disposal of the refuse from a pulverized-coal-fired furnace may be easily accomplished in the molten state, but the degree of success depends upon, and is a direct function of, the fluid temperature of the slag.

If the ash is removed in the dry form through the conventional ash hoppers and hydraulic jets, only a portion of the total accumulation is removed each day, and finally the boiler units must be taken off the line for a thorough cleaning. On the other

[15] Engineer, Stevens & Wood, New York, N. Y. Mem. A.S.M.E.

hand, it is possible and practical with molten-slag removal to withdraw all of the slag accumulating over any period, provided it will flow, and it is not necessary to take a unit out of service for a general cleaning because of the slag in the furnace bottom. It has been found that molten slag can be handled at the rate of 7000 lb. or 35 cu. ft. in an interval of 10 minutes. As the accumulation at Toronto never exceeds about 20 tons, all of the slag may be removed within an hour. The average daily accumulation is between 4 and 5 in., and hence it requires only about three-quarters of an hour each day to remove the molten slag from one furnace.

Mr. Sherman's paper brings out the fact that the average temperatures of the slag beds from strip-mine and Pittsburgh coals were only about 170 deg. higher than for the Georges Greek coal. While the fluid temperature of the slag from strip-mine coal and Pittsburgh coal may be nearly the same, yet the viscosity of the Pittsburgh coal slag is higher than that from strip-mine coal, so that the latter does not flow quite as easily as the former. However, it does bring out the fact that possibly Pittsburgh coal slag has a fluid temperature very near the critical, i.e., its temperature is about the limit at which flow will occur.

One of the most interesting facts developed by the Bureau of Mines tests is the amount of ash discharged to the stacks. Prior to the tests, no accurate determination was made at Toronto of the amount disposed of in this way, but it was presumed to be at least two-thirds of the total ash. The tests indicate that it amounts to only about one-half of this quantity, or one-third of the total ash fired. It is interesting to note that the quantity of ash increased as better coal was burned, and it would be well if this subject were investigated further to confirm this conclusion.

When slag bottoms were first installed at Toronto, it was presumed the carbon loss in the ash would be greatly decreased. When the first slag was analyzed, only a trace of unburnt carbon was found. It is a considerable surprise that the Bureau tests show that there is some carbon in the slag, as a large number of slag analyses have shown that only a trace is present.

The addition of fluxes to the slag bed in order to make it flow and to decrease its fluid temperature partially solves a problem. From the figures given in the Sherman paper, it would seem as though a large quantity of flux must be used. With an all-water-cooled furnace without doors immediately above the bottom, it is likely to prove difficult to introduce the flux. One of the most desirable features to incorporate in the design of an all-water-cooled furnace with a molten slag bottom is a suitable access door in the lower part of the furnace, and thus avoid the necessity of entering the furnace above the wall tubes. Light ladders are available, but these are not as desirable as a door located near the bottom of the furnace or just above the slag floor. However, for an all-water-cooled furnace, the manufacturers of such equipment do not yet seem to be in a position to offer a suitable door.

R. M. HARDGROVE.[16] The authors have presented some very valuable data on the operation of slag-tap furnaces at Toronto, and are to be congratulated on the very thorough manner in which they have conducted these tests.

That traces of carbon were found in the slag is not to be wondered at, for it is always the last trace of any element that is difficult to eliminate in any chemical reaction.

A study of the ash from underfeed stokers shows carbon in the hard, vitrified pieces of slag as indicated by the first three items of Table 12, each group containing an average of over one per cent carbon as determined in the combustion train, and we might expect traces of carbon in the slag from slag-tap furnaces. However, it is usually only a trace, and if the authors are to be criticized at all, it will be because they have left the impression that the average carbon content in daily operation is as high as the few samples tabulated in the paper.

In Table 12 are given the average determinations of 49 samples from Toronto, 27 from Buffalo, and 3 from Oklahoma Gas and Electric, all taken during regular daily operation. The average carbon content for Toronto was 0.161 per cent and for Buffalo was 0.080 per cent. The samples marked "Ladle" were caught in a ladle as the slag came from the spout, while those marked "Sluice" were disintegrated slag caught from the sluice during the entire tapping period and then quartered down. The "sluice" samples are therefore more representative and in both plants average lower than the "ladle" samples.

The highest carbon content in any of the 79 samples tested was 1.25 per cent, so that a carbon content under 0.2 per cent can be expected under regular daily operating conditions.

Most samples gain in weight due to a change from Fe and FeO to Fe_2O_3 as mentioned in the paper, and this gain is roughly proportional to the iron content. The samples quoted in Table 12 have the following iron content:

	Toronto	Okla. Gas	Buffalo
Iron as Fe_2O_3	13.46	16.63	34.73
Avg. gain in weight plus carbon, per cent	0.875	0.697	1.44

The gains in each case are less than the theoretical change from FeO to Fe_2O_3, but more than the theoretical change from Fe_3O_4 to Fe_2O_3, which indicates that the iron exists partly as FeO and partly as Fe_3O_4, with some metallic iron Fe in the slag as tapped.

This method of ash removal can therefore be depended upon to give a loss of carbon in the ash at least as low as the best forms of dry ash removal from pulverized-fuel furnaces, and usually much lower.

The small percentage of ash passing up the stack as given in Table 3 of the paper is very remarkable, which bears out the writer's experience on other installations where the slag removed was measured and the quantity passing up the stack was found

[16] Research Engineer, Fuller Lehigh Co., Fullerton, Pa.

TABLE 12

Plant	Boiler no.	Method of obtaining samples	Number of samples averaged	Change in weight on ignition, per cent			Carbon by combustion train, per cent		
				Highest	Lowest	Average	Highest	Lowest	Average
Stoker A	.....	Picking, only hard, vitrified pieces from ashpit sample	5	−4.05	−0.10	−1.16	4.05	0.05	1.31
Stoker B	.....		2	−0.05	+0.10	+0.02	2.10	0.12	1.11
Stoker C	.....		11	−11.75	+0.95	−1.16	10.87	0.16	1.48
Pulverized Coal, Slag-Tap Furnaces:									
Toronto	4	Sluice	2	+0.75	+0.75	+0.75	0.095	0.027	0.061
Toronto	5	Ladle	21	−0.75	+1.50	+0.85	1.25	0.00	0.21
Toronto	7 and 8	Ladle	16	−0.15	+1.10	+0.63	0.71	0.00	0.145
Toronto	7 and 8	Sluice	10	−0.25	+1.40	+0.56	0.42	0.02	0.105
			49			+0.714		Average...	0.161
Buffalo	.....	Ladle	10	+0.55	+1.80	+1.06	0.13	0.02	0.092
Buffalo	17–20	Sluice	17	+1.10	+1.96	+1.54	0.15	0.02	0.073
			27			+1.36		Average...	0.080
Okla. G. & E.	.....	Ladle	3	+0.25	+0.65	+0.48	0.29	0.140	0.217

to be materially less than on installations using a form of dry ash removal. This result is also influenced by the character of the walls, as a refractory-covered surface giving a wet coat of slag will entrain more of the ash than a cold, dry surface.

Authors' Closure

In answer to Mr. Blizard's question, the area of the heating surface in the furnace was computed as the projected exposed area of the tubes.

The tendency to slag the tubes of the boiler was greater in both of the furnaces with slagging bottoms than in the furnaces with dry bottoms. It must be remembered, however, that these two furnaces were not originally designed for slagging operation; in furnaces which are designed with slagging bottoms the amount of radiant heat-absorbing surface in the furnace can and must be increased in order that the temperature of the gases entering the tubes will be lowered enough to prevent slagging of the tubes of the boiler.

Mr. Johnston has added some interesting and valuable comments on the history of and operating experience with the slagging furnaces at Toronto. The designing and operating engineers at this station deserve much credit for their work in applying this method of ash removal to other types of furnaces and coals than those which had previously been attempted.

Mr. Hardgrove has contributed some valuable additional data relative to the carbon content of slag from this type of furnace. The authors did not intend to convey the impression that the carbon loss might be as high in daily operation as the highest samples reported; for, as stated, not enough samples had been taken to show the average percentage of carbon which might be in the slag. The conclusion which was drawn was that, as all the 14 samples analyzed contained carbon, the possibility of loss could not be ignored although it would be expected to be low. Mr. Hardgrove's contribution of the analyses of these 79 samples is therefore very helpful, and his data confirm the expectation that the average carbon loss should be almost negligible.

FSP-51-52

Design of Steam Piping to Care for Expansion

By W. H. SHIPMAN,[1] SYRACUSE, N. Y.

This paper endeavors (1) to arrive at the fundamental principles of design, (2) to present a complete mathematical theory of piping subjected to expansion, and (3) to reduce the very complicated results to a simple, usable basis.

The mathematical analysis of expansion stresses and reactions is presented for discussion as the basis of a proposed code of practice for the design of steam piping. The mathematical analysis is verified by experimental work, part of which has been previously published, and part of which is original. Further check by experiment is desirable, and the cooperation of other engineers is requested in building up an absolutely dependable body of data upon which a code of practice may be based.

A STUDY of the flexibility of piping is important:

1 To avoid excessive moments at the joints which would damage the joints and cause leaks.

2 To avoid excessive reactions which would damage anchors and apparatus.

3 To avoid excessive stresses in the pipe.

Present methods of design consist largely of empirical rules and judgment of the designer as to the looks of the layout. Long experience with moderate pressures and temperatures has developed designers who can tell by the eye alone whether or not a given pipe line is sufficiently flexible. The higher temperatures now in use and the heavier pipe walls used with higher pressures have rendered this method of design highly untrustworthy. The following study was undertaken to reduce the subject, so far as practical, to a rational basis.

PART I—FUNDAMENTAL PRINCIPLES TO BE CONSIDERED IN DESIGN

Eight fundamental principles of design are here proposed, and elaborated throughout the paper. Although many of these principles may appear axiomatic to the practical designer, the literature reveals no previous mention of them as such.

Principle No. 1—Entire Pipe Line the Expansion Bend. As far as possible avoid expansion bends and design the entire pipe line to take care of its own expansion. Where the shape required to clear obstructions does not give sufficient flexibility it is true that more pipe must be added, but it does not follow that conventional bends are always the best solution. The rule should be to put all parts of the entire pipe line to work taking the expansion. The use of expansion bends should be the exception.

Principle No. 2—Supports and Anchors. Stresses and reactions are always increased by restraint from free movement. The secret of obtaining maximum flexibility lies in computing or estimating the shape of the pipe line after expansion, and then placing the supports (and anchors when possible) so that they will not interfere with this natural movement. Formulas are given for computing the movement of all points in the line.

Principle No. 3—Shape Efficiency. That shape is most efficient in which the maximum length is working at the maximum safe stress. See Fig. 22 for typical study. The application of this principle is limited somewhat by Principle No. 4 in that a less efficient shape will sometimes keep the joints away from points of high moment.

Principle No. 4—Moment at Joints. Excessive bending moment at joints is more likely to cause trouble than excessive stresses in pipe walls. This is witnessed by the fact that failures of pipe walls are rare while joints leak by the thousand, every new power plant having some leaky joints until the piping has set. This statement holds only for A.E.S.C. Standard piping and joints. It would be possible to put in more bolt area and stiffer flanges until the critical point would be pipe wall rather than joints.

Principle No. 5—Cold Springing. Piping should be designed with a reasonable amount of cold springing to facilitate erection, to reduce expansion stresses, and to reduce the distortion of the system. However, caution must be used where cold springing is relied upon to take care of expansion since there is always danger that the piping may grow or slide through anchorages, or that the joints may rotate under the effect of torsion, or that the amount of cold springing actually obtained may be far less than was expected.

Principle No. 6—Flattening of Curved Portions. Reactions and stresses are greatly influenced by flattening of the cross-section of curved portions of the pipe line.

Principle No. 7—End Conditions. Reactions and stresses are very greatly influenced by end conditions. A method of taking account of this factor is presented.

Principle No. 8—Variation of Physical Properties With Temperature. Working stresses should decrease with increasing temperature. Modulus of elasticity E decreases with temperature; mean coefficient of expansion α increases with temperature; and, within the range from 0 to 750 deg. fahr., the product αE is approximately constant for low carbon steel at the value 190 lb. per sq. in. per deg. fahr.

PART II—SURVEY OF PUBLISHED WORK

The following abstracts of the best-known works on piping published during the last twenty years show a rapid development of the subject. The abstracts are arranged in order of date of publication, and the most significant contributions are indicated in each case. Marbec, Lorenz, and Carlier do not seem to have been used by the other American writers on the subject.

(*A*) Bantlin, Prof. A., "Formänderung und Beanspruchung federnder Ausgleichröhren," *Zeit. V.D.I.*, vol. 54, no. 2, 1910, pp. 43–49. Prof. Bantlin found on tests of several thin-walled, short-radius, lyre-type bends made of seamless steel tubing that the flexibility of the bends was as much as five times that indicated by the common theory for solid curved bars. Bantlin found no such discrepancy for a cast-iron bend, and suggested that the discrepancy for steel tubes might be due to wrinkles and irregularities produced in the manufacturing process. References (*B*), (*C*), (*D*), (*G*), (*J*), (*K*) and the present work account for this discrepancy on another basis.

(*B*) von Karman, Dr. Th., "Über die Formänderung dünnwandiger Rohre," *Zeit. V.D.I.*, vol. 55, 1911, p. 1889. Dr. von Karman developed formulas which showed that the dis-

[1] The Atmospheric Nitrogen Corporation.

Contributed by the Power Division and presented at the Annual Meeting, New York, N. Y., Dec. 3 to 7, 1928, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

crepancies observed by Prof. Bantlin were explainable by a theory which considered the effect of flattening of the cross-section on the stress distribution and flexibility. The flattening referred to is that produced by bending due to expansion and not that produced by the manufacture of the bend. Karman's analysis employed the theory of least work under the assumption that the mean circumference of the pipe remains unchanged during the deformation of the cross-section.

(*C*) Marbec, M., *Bulletin Association Technique Maritime*, vol. 22, 1911. Marbec developed by the method of least work a formula for the relative flexibility of straight and curved pipes under the assumption that the deformed cross-section takes the shape of an ellipse.

(*D*) Lorenz, Prof. H., "Die Biegung Krummer Rohre," *Dinglers Polytechnisches Journal*, Sept. 14, 1912, p. 577. Prof. Lorenz developed, by an application of Castigliano's theorem, a formula for the relative flexibility of straight and curved pipes, under the assumption that the deformed cross-section is doubly symmetrical, making no assumption as to the form of the curve.

(*E*) Crane Company, "Pipe Bends, Their Growing Use and Efficiency," *The Valve World*, Oct., 1915. This report gave the results of extensive tests on full-size pipe bends. The design data, proposed as a result of these tests, differs materially from more recent data (see Fig. 15), but is often quoted in catalogs and handbooks and is widely used by engineers.

(*F*) Crocker, S., and Sanford, S. S., "The Elasticity of Pipe Bends," MECHANICAL ENGINEERING, 1922, p. 547. This work was a distinct step in the solution of the problem, and gave many mathematical solutions with check tests. Though it is stated that one end is assumed free, an analysis shows that both ends of all symmetrical bends were treated as if free. The stresses and reactions are considerably greater than commonly believed, but are still too small, as the author himself points out in a later article (reference *H*), because in practice pipe ends are more nearly fixed than free.

(*G*) Carlier, Henri, "L'Aptitude elastique des tuyauteries a vapeur au point de vue dilatation," 1920, 1923, 1925, 1927. (Copies obtainable from author, Ingenieur, Ressaix (Hainaut) Belgium.) The first three publications contained detailed solutions, by the free-body method, of many complicated single-plane shapes, all with both ends fixed, and with a flexibility factor *K* used in connection with all curved members. The fourth publication covered a typical two-plane shape. These works represent the most complete mathematical treatise published to date on pipe expansion.

(*H*) Crocker, S., "How to Lay Out Power Plant Piping—V," *Power*, Aug. 3, 1926, p. 172. In this article the author stated that even the high values given in his previous paper (reference *F*) are too low, so that data widely published in handbooks and catalogs are even farther from the true conditions. For example, the maximum bending stress in a U-bend is 1.5 times that previously published (Par. *F*), and the thrust is 3 times as great.

(*J*) Hovgaard, Wm., "The Elastic Deformation of Pipe Bends," *Jour. Math. and Phys.*, vol. 6, no. 2, Nov., 1926, pp. 69–118. In addition to illustrating a graphical method of computing thrust and moment for certain hinged-end bends, Hovgaard, independently derived Karman's equation for the relative flexibility of curved and straight pipes, and showed that four of the five full-size pipe bend tests considered verified Karman's equation almost exactly, while the fifth bend was about 20 per cent more flexible.

(*K*) Wahl, A. M., "Stresses and Reactions in Expansion Pipe Bends," Trans. A.S.M.E., 1928, FSP-50-49. Wahl derived formulas by Castigliano and free-body methods for reactions and moments for several single-plane pipe bends under various combinations of end conditions, and showed that these formulas, using Karman's flexibility factor *K*, are checked by several experiments on small model U-bends. One of the most useful parts of the paper is a chart for determining the maximum resultant stress (using Karman's analysis and the Guest or shear theory of failure) in terms of the conventionally computed maximum bending stress.

PART III—THEORETICAL ANALYSIS

A—Introduction

The determination of deflections, moments, reactions, and stresses in pipe lines by ordinary mathematical methods is tedious and requires a working knowledge of integral calculus and the use of much algebra. The methods herein developed facilitate the computation of the required unknown quantities for pipe lines in one plane, and by eliminating calculus operations, make the solution depend on substitution in formulas only. With the assistance of curve sheets as illustrated under Part V, the work may be greatly simplified and shortened.

It has been impossible to include complete derivations of formulas but the author will be glad to supply these upon request.

B—Notation

A, B, C, F, G, H = values of integrals given in Table 1
a, b, c, d, e, f, g = distances on pipe sketches
0, 1, 2, 3, 4, 5, 6, 7, 8, etc. = points on pipe line
A_o = area, sq. in.
D = outside diameter of pipe, in.
E = modulus of elasticity, lb. per sq. in.
F_x = horizontal force, lb.
F_y = vertical force, lb.
h = vertical projection of pipe line between anchors, in.
I = moment of inertia, quartic in.
K = relative flexibility of curved and straight pipes (see Part III, J).
L = horizontal projection of pipe line between anchors, in.
M = bending moment at any point on the pipe, in-lb.
M_0 = moment reaction at left-hand end of any one-plane pipe line
M_1 = moment reaction at right-hand end of any one-plane pipe line
N = ratio of dimensions of geometrically similar pipe lines
P = vertical force at any point on a pipe line, lb.
p = any point on a pipe line
Q = horizontal force at any point on a pipe line, lb.
R = radius of center line of pipe bend, in.
r = mean radius of cross-section of pipe, in.
s = distance along center line of pipe, in.
S_t = tensile stress, lb. per sq. in.
S_s = shearing stress, lb. per sq. in.
T = temperature rise, deg. fahr.
t = thickness of pipe wall, in.
U = elastic energy, in-lb.
V = $\sqrt{(F_x^2 + F_y^2)}$
x, y, z = coordinates of any point on pipe line, in. Left end of pipe line is origin unless otherwise specified
α = mean coefficient of linear thermal expansion, in. per in. per deg. fahr.
β = ratio of the maximum resultant stress in pipe to the commonly computed bending stress (see Fig. 19 and Part III, J and K).
Δ = deflection, in.
Δx = x-component of deflection, in.
Δy = y-component of deflection, in.
η = factor depending on shape of pipe line
θ, ϕ = angles shown in pipe sketches

λ = factor for curved pipes = $(tR)/r^2$
μ = factor depending upon radius of bend
π = 3.1416

C—Development of Fundamental Equations for Any One-Plane Pipe Line

The expression for the elastic energy is

$$U = \frac{K}{2EI}\int M^2 ds \quad \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

Use of this expression involves the following assumptions:

1 Energy due to direct compression or tension is negligible.
2 Energy due to direct shear is negligible.
3 Neutral axis of cross-section is at center of gravity of cross-section.
4 Pipes originally curved are more flexible than pipes originally straight, and the relative flexibility K is a function only of the ratio tR/r^2.

An additional assumption will be needed later:

5 Moment at any point on a pipe line is not affected by deflections of the pipe line.

Let point p of Fig. 1 be any point, with coordinates x and y, of any one-plane pipe line between points 0 and 1. The reaction at each end may be represented by a force consisting of two components F_x and F_y, and a moment M_0 or M_1.

Equilibrium of the forces on the part $0p$ of the line gives for the moment M at any point.

$$M = M_0 + F_y x - F_x y \quad \ldots\ldots\ldots\ldots\ldots\ldots [2]$$

Moments will be considered positive when they act in a clockwise direction on the left-hand end of any portion or in a counterclockwise direction on the right-hand end as shown in Fig. 1

According to Castigliano's theorem, the components x and y

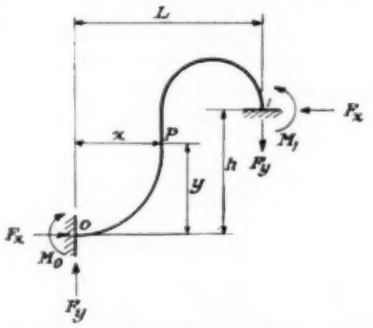


Fig. 1 Single-Plane Pipe Line Between Any Two Points 0 and 1

of the deflection of 0 relative to 1, and the angular deflection $\Delta\phi$ of the tangent to the elastic line at 0 relative to the tangent to the elastic line at 1 are given by the equations:

$$\Delta x = \frac{\partial U}{\partial F_x}; \quad \Delta y = \frac{\partial U}{\partial F_y}; \quad \Delta\phi = \frac{\partial U}{\partial M_0} \quad \ldots\ldots [3]$$

Substituting Equation [1] in Equations [3]

$$\Delta x = \frac{K}{2EI}\int 2M\frac{\partial M}{\partial F_x}ds$$

$$\Delta y = \frac{K}{2EI}\int 2M\frac{\partial M}{\partial F_y}ds$$

$$\Delta\phi = \frac{K}{2EI}\int 2M\frac{\partial M}{\partial M_0}ds \quad \ldots [4]$$

Differentiating M in Equation [2] with respect to F_x, F_y, and M_0,

$$\frac{\partial M}{\partial F_x} = -y; \quad \frac{\partial M}{\partial F_y} = x; \quad \frac{\partial M}{\partial M_0} = 1 \quad \ldots\ldots [5]$$

Substituting [5] in [4]

$$\Delta x = -\frac{K}{EI}\int My ds \qquad \Delta y = \frac{K}{EI}\int Mx ds$$

$$\Delta\phi = \frac{K}{EI}\int M ds \quad \ldots [6]$$

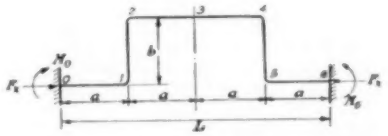


Fig. 2 Example of Use of Table 1

The values of F, G, and H for the symmetrical expansion bend shown in Fig. 2 may be obtained from Table 1 as follows:

Part of pipe		Value of F	Value of G	Value of H
0 to 1		0	0	a
1 to 2		$b^2/2$	$b^3/3$	b
2 to 3		ab	ab^2	a
3 to 4		ab	ab^2	a
4 to 5		$b^2/2$	$b^3/3$	b
5 to 6		0	0	a
Total	0 to 6	$2ab + b^2$	$2ab^2 + 2b^3/3$	$4a + 2b$

Substituting in general Equations [16] and [17] for symmetrical lines, and simplifying:

$$M_0 = \frac{EI\alpha TL}{2ab + b^2/3} \text{ and } F_x = \frac{EI\alpha TL}{ab^2 + b^3/6}$$

Equations [6] are fundamental for any one-plane pipe line with any condition of end fixedness. For most cases they are identical with those developed by a slightly different method by Carlier (reference G) and used throughout his work.

D—Development of General Equations for Any One-Plane Pipe Line With Fixed Ends

Since the tangents at the ends do not move, $\Delta\phi = 0$. Substituting Equation [2] in Equations [6]

$$\Delta x = -\frac{K}{EI}\left[M_0\int y ds - F_x\int y^2 ds + F_y\int xy ds\right] \quad \ldots\ldots [7]$$

$$\Delta y = +\frac{K}{EI}\left[M_0\int x ds - F_x\int xy ds + F_y\int x^2 ds\right] \quad \ldots\ldots [8]$$

$$\Delta\phi = 0 = \frac{K}{EI}\left[M_0\int ds - F_x\int y ds + F_y\int x ds\right] \quad \ldots\ldots [9]$$

Equations [7], [8], and [9] contain six different integrals. Each of these integrals multiplied by K may be represented by a single symbol, such as A, B, C, F, G, and H. For example, let $K\int x ds = A$. For any given pipe line consisting of several curved and straight parts joined together, $A = K\int x ds$ may be determined by integrating for the separate parts of the pipe line and adding the results algebraically.

Equations [7], [8], and [9] may be written:

$$\Delta x = -\frac{1}{EI}\left[M_0 F - F_x G + F_y B\right] \quad \ldots\ldots [10]$$

$$\Delta y = \frac{1}{EI}\left[M_0 A - F_x B + F_y C\right] \quad \ldots\ldots [11]$$

$$\Delta\phi = 0 = M_0 H - F_x F + F_y A \quad \ldots\ldots\ldots\ldots [12]$$

For simple circular arcs and straight pipes in one plane the values of A, B, C, F, G, and H are given in Table 1.

Solving Equations [10], [11], and [12] for M_0, F_x, and F_y and putting $\Delta x = \alpha T L$ and $\Delta y = \alpha T h$, where α = coefficient of expansion, T = temperature difference between hot and cold pipe line, and h and L are the vertical and horizontal distances shown in Fig. 1:

TABLE 1 FOR A, B, C, F, G, AND H

Section	$A = K\int x\,ds$	$B = K\int xy\,ds$	$C = K\int x^2\,ds$	$F = K\int y\,ds$	$G = K\int y^2\,ds$	$H = K\int ds$
s (x_1,y_1) (x_2,y_2)	$\frac{s}{2}(x_1 + x_2)$	Ay	$s\left(\frac{s^2}{3} + x_1 x_2\right)$	sy	Fy	s
s (x_1,y_2) (x,y_1)	sx	$\frac{A}{2}(y_1 + y_2)$	Ax	$\frac{s}{2}(y_1 + y_2)$	$s\left(\frac{s^2}{3} + y_1 y_2\right)$	s
s (x_2,y_2) (x_1,y_1)	$\frac{s}{2}(x_1 + x_2)$	$\frac{A}{3}(y_1 + y_2) + \frac{s}{6}(x_1 y_1 + x_2 y_2)$	$s\left(\frac{s^2}{3} + x_1 x_2\right)$	$\frac{s}{2}(y_1 + y_2)$	$s\left(\frac{s^2}{3} + y_1 y_2\right)$	s
R (x,y)	πKRx	$A\left(y + \frac{2R}{\pi}\right)$	$A\left(x + \frac{R^2}{2x}\right)$	$\left(\pi y + 2R\right)KR$	$Fy + \left(2y + \frac{\pi}{2}R\right)KR^2$	πKR
(x,y) R	πKRx	$A\left(y - \frac{2R}{\pi}\right)$	$A\left(x + \frac{R^2}{2x}\right)$	$\left(\pi y - 2R\right)KR$	$Fy - \left(2y - \frac{\pi}{2}R\right)KR^2$	πKR
R (x,y)	$\left(\frac{\pi x}{2} - R\right)KR$	$Ay + \left(x - \frac{R}{2}\right)KR^2$	$Ax + \left(\frac{\pi R}{4} - x\right)KR^2$	$\left(\frac{\pi y}{2} + R\right)KR$	$Fy + \left(\frac{\pi R}{4} + y\right)KR^2$	$\frac{\pi KR}{2}$
(x,y) R	$\left(\frac{\pi x}{2} + R\right)KR$	$Ay + \left(x + \frac{R}{2}\right)KR^2$	$Ax + \left(\frac{\pi R}{4} + x\right)KR^2$	$\left(\frac{\pi y}{2} + R\right)KR$	$Fy + \left(\frac{\pi R}{4} + y\right)KR^2$	$\frac{\pi KR}{2}$
(x,y) R	$\left(\frac{\pi x}{2} + R\right)KR$	$Ay - \left(x + \frac{R}{2}\right)KR^2$	$Ax + \left(\frac{\pi R}{4} + x\right)KR^2$	$\left(\frac{\pi y}{2} - R\right)KR$	$Fy + \left(\frac{\pi R}{4} - y\right)KR^2$	$\frac{\pi KR}{2}$
R (x,y)	$\left(\frac{\pi x}{2} - R\right)KR$	$Ay - \left(x - \frac{R}{2}\right)KR^2$	$Ax + \left(\frac{\pi R}{4} - x\right)KR^2$	$\left(\frac{\pi y}{2} - R\right)KR$	$Fy + \left(\frac{\pi R}{4} - y\right)KR^2$	$\frac{\pi KR}{2}$
R $45°$ (x,y)	$\left(\frac{\pi x}{4} - \frac{R}{\sqrt{2}}\right)KR$	$Ay + \left[0.2929x - \frac{R}{4}\right]KR^2$	$Ax + \left(.06427R - \frac{\sqrt{2}x}{2}\right)KR^2$	$\left(\frac{\pi y}{4} + .2929R\right)KR$	$Fy + \left[.2929y + .1427R\right]KR^2$	$\frac{\pi KR}{4}$
R $45°$ (x,y)	$\left(\frac{\pi x}{4} - \frac{R}{\sqrt{2}}\right)KR$	$Ay - \left[0.2929x - \frac{R}{4}\right]KR^2$	$Ax + \left(.06427R - \frac{\sqrt{2}x}{2}\right)KR^2$	$\left(\frac{\pi y}{4} - .2929R\right)KR$	$Fy - \left[.2929y - .1427R\right]KR^2$	$\frac{\pi KR}{4}$
(x,y) R $45°$	$\left(\frac{\pi x}{4} + \frac{R}{\sqrt{2}}\right)KR$	$Ay + \left[0.2929x + \frac{R}{4}\right]KR^2$	$Ax + \left(.06427R + \frac{\sqrt{2}x}{2}\right)KR^2$	$\left(\frac{\pi y}{4} + .2929R\right)KR$	$Fy + \left[.2929y + .1427R\right]KR^2$	$\frac{\pi KR}{4}$
(x,y) R $45°$	$\left(\frac{\pi x}{4} + \frac{R}{\sqrt{2}}\right)KR$	$Ay - \left[0.2929x + \frac{R}{4}\right]KR^2$	$Ax + \left(.06427R + \frac{\sqrt{2}x}{2}\right)KR^2$	$\left(\frac{\pi y}{4} - .2929R\right)KR$	$Fy - \left[.2929y - .1427R\right]KR^2$	$\frac{\pi KR}{4}$
θ_1 θ_2 R (x,y)	$\left[x(\theta_2 - \theta_1) - R(\sin\theta_2 - \sin\theta_1)\right]KR$	$Ay\left[x(\cos\theta_2 - \cos\theta_1) + \frac{R}{2}(\sin^2\theta_2 - \sin^2\theta_1)\right]KR^2$	$Ax - \left[x\left(\sin\theta_2 - \sin\theta_1\right) - \frac{R}{4}(\sin 2\theta_2 - \sin 2\theta_1) - \frac{R^2}{2}(\theta_2 - \theta_1)\right]KR^2$	$\left[y(\theta_2 - \theta_1) - R(\cos\theta_2 - \cos\theta_1)\right]KR$	$Fy - \left[y(\cos\theta_2 - \cos\theta_1) + \frac{R}{4}(\sin 2\theta_2 - \sin 2\theta_1) - \frac{R}{2}(\theta_2 - \theta_1)\right]KR^2$	$(\theta_2 - \theta_1)KR$

NOTE: Equations for shapes not given may be obtained by algebraic addition to those given. All measurements are from left-hand end of pipe line.

Table 1 has two fields of use:

1. For the original derivation of formulas similar to those given under Part V.
2. For solution of complicated special cases. See Part III, R for typical examples.

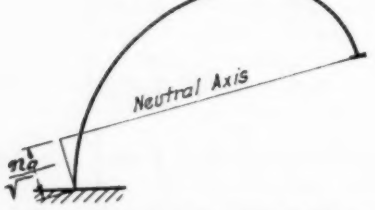


FIG. 3 POSITION OF NEUTRAL AXIS FOR PIPE LINE WITH RIGHT END HINGED AND LEFT END PARTIALLY FIXED

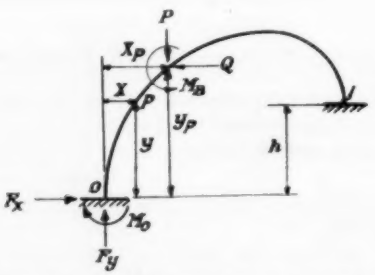


FIG. 4 PIPE LINE WITH FORCE AND MOMENT APPLIED BETWEEN THE ENDS

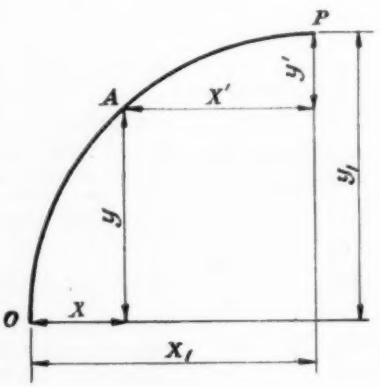


FIG. 5

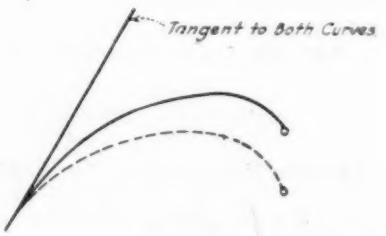


FIG. 6

$$M_0 = \frac{EI\alpha T[F(Bh + CL) - A(BL + Gh)]}{2ABF + HGC - A^2G - B^2H - CF^2} \quad [13]$$

$$F_x = \frac{EI\alpha T[H(Bh + CL) - A(AL + Fh)]}{2ABF + HGC - A^2G - B^2H - CF^2} \quad [14]$$

$$F_y = \frac{EI\alpha T[H(BL + Gh) - F(AL + Fh)]}{2ABF + HGC - A^2G - B^2H - CF^2} \quad [15]$$

For symmetrical lines with fixed ends $h = 0$, $F_y = 0$ and Equation [11] ceases to exist. Solving Equations [10] and [12] together, simpler expressions for M_0 and F_x involving only the integrals F, G, and H are obtained:

$$M_0 = \frac{EI\alpha TLF}{GH - F^2} \quad [16]$$

$$F_x = \frac{EI\alpha TLH}{GH - F^2} \quad [17]$$

For the special case of a symmetrical fixed-end line with center of gravity located a distance above the origin equal to half the maximum height of the line

$$M_0 = \frac{EI\alpha TLh'}{4G_c} \quad [18]$$

$$F_x = \frac{EI\alpha TL}{2G_c} \quad [19]$$

where h' = maximum height of bend
G_c = the integral G for the upper half of the bend, using the neutral axis as origin.

E—General Equations for Any One-Plane Pipe Line With Hinged Ends

In this case, the resultant thrust always passes through the hinges, and $M_0 = 0$

$$F_x = \frac{EI\alpha TL(L^2 + h^2)}{Ch^2 - 2BhL + GL^2} \quad [20]$$

$$F_y = \frac{EI\alpha Th(L^2 + h^2)}{Ch^2 - 2BhL + GL^2} \quad [21]$$

For symmetrical lines with hinged ends $h = 0$, $F_y = 0$, and

$$F_x = \frac{EI\alpha TL}{G} \quad [22]$$

For symmetrical lines with hinged ends the angular rotation of either end is

$$\phi_1 = -\frac{\alpha TLF}{2G} \quad [23]$$

F—Development of General Equations for Any One-Plane Pipe Line With Partially Fixed Ends

The fixity of either end of any pipe line may be defined as a function of the amount of rotation of that end by assuming that the fixity of a fully fixed end is unity and that the fixity of a hinged end is zero. As an aid in visualizing this effect and as an aid in the mathematical derivation, we shall also assume that the distance of the neutral axis from an end is proportional to the fixity of that end if the other end is hinged. In estimating the fixity of an end, then, we mentally compare the rotation of that end with the rotation of the same end if it were hinged and assign a corresponding value of the fixity. Thus, if we estimate that the end can rotate one-third as much as if it were free, we assign the end a fixity of two-thirds.

The neutral axis, line of thrust, or line of zero moment, for a symmetrical pipe line fully fixed at both ends, is a horizontal line through the center of gravity of the line. For a hinged-

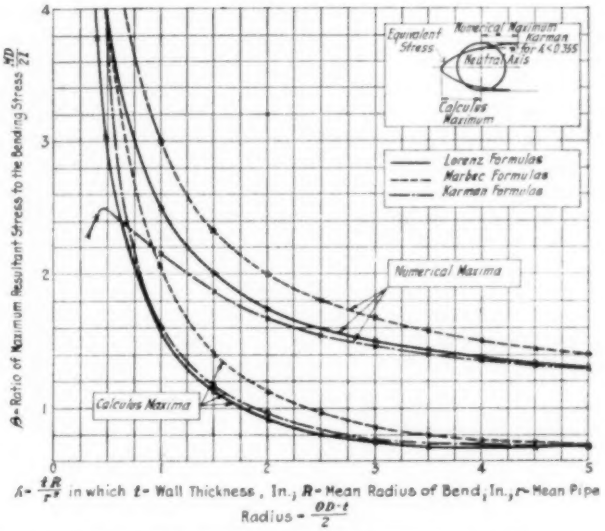


Fig. 7 Values of Equivalent Stress Constant β as Given by Various Investigators

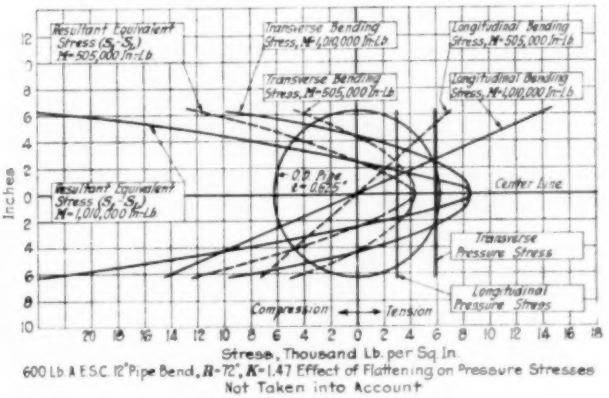


Fig. 8a Stress Distribution in Typical Pipe Bend

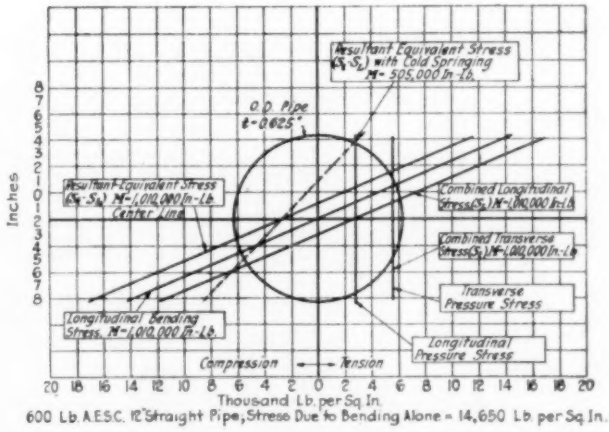


Fig. 8b Stress Distribution in Typical Straight Pipe

end pipe line it passes through the hinges. For intermediate cases with equal fixity at the ends the neutral axis will be between these two extremes. If one end is hinged and the other is in some degree fixed, the neutral axis will appear as in Fig. 1. The following theory is based on the assumption that the distance M_0/V of the neutral axis from the partly or wholly fixed end is proportional to the fixity of that end, the other end remaining hinged. If the partly or wholly fixed end remains unchanged as to fixity and the hinged end has its fixity increased from zero to some other value, the value of M_0/V for the originally fixed end will change, hence M_0/V is then a function of the fixities of both ends, whether they are equal or unequal.

Under the above assumption we have the equation pertaining to the left end of the bend $M_0/V = m_0(K_0 - K_1m_1)$ and that pertaining to the right end of the bend, $M_1/V = m_1(K_2 - K_3m_0)$

where M_0 = moment at left end
M_1 = moment at right end
m_0 = fixity at left end
m_1 = fixity at right end
V = resultant force on the bend $= \sqrt{(F_x^2 + F_y^2)}$

K_0, K_1, K_2, K_3 are constants depending on the form of the bend. The M/V term in each is the distance of the neutral axis from the end in question. Note that each distance is a function of the fixity at both ends.

By a treatment similar to that used in the development of the fundamental equations, we derive the formulas that follow.

Case I—General. Supports at different levels; non-symmetrical with respect to an axis perpendicular to line joining ends of pipe line, fixity at ends equal or unequal; or symmetrical with respect to an axis perpendicular to line joining ends of pipe line, fixity at ends unequal.

$$F_x = \frac{EI\alpha T(L^2 + h^2)(L\sqrt{L^2 + h^2 - \psi^2} - \psi h)}{W(hA - LF)(L^2 + h^2) + \sqrt{L^2 + h^2 - \psi^2}(h^2C - 2hLB + L^2G) + \psi(hLC + h^2B - L^2B - hLG)} \quad [24]$$

$$F_y = \frac{EI\alpha T(L^2 + h^2)(\psi L + h\sqrt{L^2 + h^2 - \psi^2})}{\text{Same Denominator}} \quad [25]$$

$$M_0 = \frac{EI\alpha T(L^2 + h^2)^2 W}{\text{Same Denominator}} \quad [26]$$

where

$$\psi = m_1(K_2 - K_3m_0) - m_0(K_0 - K_1m_1) \quad [27]$$

$$W = m_0(K_0 - K_1m_1) \quad [28]$$

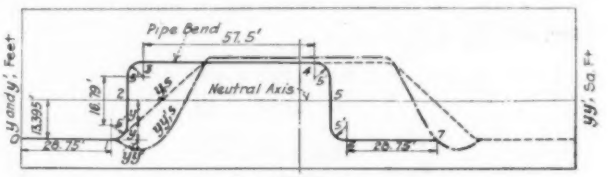


FIG. 9 GRAPHICAL SOLUTION OF A PIPE BEND

$$K_0 - K_1 = \frac{F(Bh + CL) - A(BL + Gh)}{\sqrt{[H(Bh + CL) - A(AL + Fh)]^2 + [H(BL + Gh) - F(AL + Fh)]^2}} \quad [29]$$

$$K_0 = \frac{h^2(B - LF - hA + hLH) + hL(C - 2AL + L^2H - G + 2hF - h^2H) - L^2(B - LF - hA + hLH)}{\sqrt{[h(B - LF - hA + hLH) + L(C - 2AL + L^2H)]^2 + [h(G - 2hF + h^2H) + L(B - LF - hA + hLH)]^2}} \quad [30]$$

$$K_2 - K_3 = \frac{(Bh + CL)(F - hH) + (BL + Gh)(HL - A) + (AL + Fh)(Ah - FL)}{\sqrt{[H(Bh + CL) - A(AL + Fh)]^2 + H[(BL + Gh) - F(AL + Fh)]^2}} \quad [31]$$

$$K_2 = \frac{L(Gh + BL) - h(Bh + CL)}{\sqrt{(Bh + CL)^2 + (Gh + BL)^2}} \quad [32]$$

If the bend is symmetrical with respect to an axis perpendicular to the line joining the ends of the bend, then

$K_0 = K_2$, $K_1 = K_3$, $\psi = K_0(m_1 - m_0)$, and $W = m_0(K_0 - K_1m_1)$

hence find K_0 from [32] and K_1 from [29].

Having found the constants needed, solve for ψ and W and obtain M_0, F_x, and F_y from Equations [24], [25], and [26].

Case II. Supports at different levels; symmetrical with respect to an axis, perpendicular to line joining ends of pipe line, fixity at ends equal. In this case $m_1 = m_0 = m$, hence $\psi = 0$. Then

$$F_x = \frac{EI\ \alpha\ TL(L^2 + h^2)}{W(hA - LF)\sqrt{L^2 + h^2} + h^2C - 2hLB + L^2G} \quad [33]$$

$$F_y = \frac{EI\alpha Th(L^2 + h^2)}{\text{Same Denominator}} \quad [34]$$

$$M_0 = \frac{EI\alpha TW(L^2 + h^2)^{3/2}}{\text{Same Denominator}} \quad [35]$$

$$K_0 = \frac{L(Gh + BL) - h(Bh + CL)}{\sqrt{(Bh + CL)^2 + (Gh + BL)^2}} \quad [36]$$

$$K_0 - K_1 = \frac{F(Bh + CL) - A(Gh + BL)}{\sqrt{[H(Bh + CL) - A(AL + Fh)]^2 + [H(Gh + BL) - F(AL + Fh)]^2}} \quad [37]$$

Case III. Supports at same level; non-symmetrical with respect to a vertical axis, fixity at ends equal or unequal; or symmetrical with respect to a vertical axis, fixity at ends unequal. This is the same as Case I with $h = 0$. Hence

$$F_x = \frac{EI\alpha TL\sqrt{L^2 - \psi^2}}{G\sqrt{L^2 - \psi^2} - WLF - B\psi} \quad [38]$$

$$F_y = \frac{EI\alpha TL\psi}{G\sqrt{L^2 - \psi^2} - WLF - B\psi} \quad [39]$$

$$M_0 = \frac{EI\alpha TL^2 W}{G\sqrt{L^2 - \psi^2} - WLF - B\psi} \quad [40]$$

$$K_0 - K_1 = \frac{CF - AB}{\sqrt{(CH - A^2)^2 + (BH - AF)^2}} \quad [41]$$

$$K_0 = \frac{L(LF - B)}{\sqrt{(C - 2AL + HL^2)^2 + (LF - B)^2}} \quad [42]$$

$$K_2 - K_3 = \frac{(CF - AB) + L(BH - AF)}{\sqrt{(CH - A^2)^2 + (BH - AF)^2}} \quad [43]$$

$$K_2 = \frac{BL}{\sqrt{B^2 + C^2}} \quad [44]$$

If the bend is symmetrical with respect to a vertical axis, then $K_0 = K_2$, $K_1 = K_3$, $\psi = K_0(m_1 - m_0)$, $W = m_0(K_0 - K_1 m_1)$, K_0 results from [44], and $K_0 - K_1 = F/H$.

Case IV. Supports at same level; symmetrical with respect to a vertical axis, fixity at ends equal. This is the same as Case II with $h = 0$.

Hence

$$F_x = \frac{EI\alpha TL}{G - WF} \quad [45]$$

$$F_y = 0$$

$$M_0 = \frac{EI\alpha TLW}{G - WF} \quad [46]$$

$$K_0 = \frac{BL}{\sqrt{B^2 + C^2}} \quad [47]$$

$$K_0 - K_1 = F/H \quad [48]$$

$$W = m(K_0 - K_1 m) \quad [49]$$

For symmetrical lines with equal fixities at the ends it may be assumed that the moment at the ends varies directly as the fixity (i.e., $M_0 \propto m$). This approximation is believed to be sufficiently close for practical purposes in the simpler shapes.

G—General Equations for Effect of Force or Moment Applied at Any Point of Any One-Plane Pipe Line With Fixed Ends

Equations [50] to [58] make possible the solution of pipe lines with branches, since the branch may be replaced by P, Q, and M_B. Simultaneous solution of the equations for the branch and the main line can be performed by noting that P, Q, and M_B are the same for the branch and the main line. The integrals A, B, C, F, G, and H are found from Table 1 by adding up the values of the integrals for each part of the entire pipe line. The integrals A_1, B_1, C_1, F_1, G_1, and H_1 are similarly found from Table 1 by adding up the integrals for the part of the pipe line between points p and 1, i.e., for the part of the pipe line to the right of the load.

H—Development of General Equations for Deflection of Any Point in Any One-Plane Pipe Line

Let P be a point with coordinates x_1, y_1 referred to O on a pipe line whose left end is at O, and A any point between P and O. The y projection of the deflection due to bending, of P relative to O is given by $+(1/EI)\int_P^O Mx'ds$ and the x projec-

(*a*) Due to Vertical Force, P

$$M_0 = \frac{P[(C_1 - x_pA_1)(BF - AG) + (B_1 - x_pF_1)(AB - CF) + (A_1 - x_pH_1)(CG - B^2)]}{2ABF + HGC - A^2G - B^2H - CF^2} \quad [50]$$

$$F_x = \frac{P[(C_1 - x_pA_1)(BH - AF) + (B_1 - x_pF_1)(A^2 - CH) + (A_1 - x_pH_1)(CF - AB)]}{\text{Same Denominator}} \quad [51]$$

$$F_y = \frac{P[(C_1 - x_pA_1)(GH - F^2) + (B_1 - x_pF_1)(AF - BH) + (A_1 - x_pH_1)(BF - AC)]}{\text{Same Denominator}} \quad [52]$$

(*b*) Due to Horizontal Force, Q

$$M_0 = \frac{Q[(B_1 - A_1y_p)(AG - BF) + (G_1 - F_1y_p)(CF - AB) + (F_1 - H_1y_p)(B^2 - CG)]}{\text{Same Denominator}} \quad [53]$$

$$F_x = \frac{Q[(B_1 - A_1y_p)(AF - BH) + (G_1 - F_1y_p)(CH - A^2) + (F_1 - H_1y_p)(AB - CF)]}{\text{Same Denominator}} \quad [54]$$

$$F_y = \frac{Q[(B_1 - A_1y_p)(F^2 - GH) + (G_1 - F_1y_p)(BH - AF) + (F_1 - H_1y_p)(AG - BF)]}{\text{Same Denominator}} \quad [55]$$

(*c*) Due to Moment, M_B

$$M_0 = \frac{M_p[A_1(BF - AG) + F_1(AB - CF) + H_1(CG - B^2)]}{\text{Same Denominator}} \quad [56]$$

$$F_x = \frac{M_p[A_1(BH - AF) + F_1(A^2 - CH) + H_1(CF - AB)]}{\text{Same Denominator}} \quad [57]$$

$$F_y = \frac{M_p[A_1(GH - F^2) + F_1(AF - BH) + H_1(BF - AG)]}{\text{Same Denominator}} \quad [58]$$

tion by $-(1/EI)\int_P^O My'ds$ where x' and y' are measured from P as origin. From Fig. 5, $x' = x_1 - x$ and $y' = y_1 - y$, where x and y are the coordinates of A referred to O as origin. We have by substitution in the above formulas,

$$dy_1 = (1/EI)\int_O^P M(x_1 - x)ds$$

and

$$dx_1 = -(1/EI)\int_O^P M(y_1 - y)ds$$ where dy_1 and dx_1 are the y and x displacements of P relative to O, and x and y are measured from O. Due to a rise in temperature T, a y deflection of amount $\alpha T y_1$ and an x deflection of amount $\alpha T x_1$ are also produced, so that our final expression for the y and x deflections of any point P relative to the left end become

$$dy_1 = \frac{1}{EI}\int_O^P M(x_1 - x)ds + \alpha T y_1 \quad \ldots\ldots [59]$$

$$dx_1 = -\frac{1}{EI}\int_O^P M(y_1 - y)ds + \alpha T x_1 \quad \ldots\ldots [60]$$

Case I—Unsymmetrical Line With Fixed Ends. In the notation of Part III *D*, Equations [59] and [60] become

$$dy_1 = \frac{1}{EI}[M_0(x_1H' - A') - F_x(x_1F' - B') + F_y(x_1A' - C)] + \alpha T_{y1} \quad \ldots\ldots [61]$$

$$dx_1 = -\frac{1}{EI}[M_0(y_1H' - F') - F_x(y_1F' - G') + F_y(y_1A' - B')] + \alpha T x_1 \quad \ldots\ldots [62]$$

Where A', B', C', etc. are the integrals from the left-hand end of the line up to the point P where the deflections are desired.

Case II—Symmetrical Line With Fixed Ends. In this case $F_y = 0$ and

$$dy_1 = \frac{1}{EI}[M_0(x_1H' - A') - F_x(x_1F' - B')] + \alpha T_{y1} \quad \ldots [63]$$

$$dx_1 = \frac{1}{EI}[M_0(y_1H' - F') - F_x(y_1F' - G')] + \alpha T_{x1} \quad \ldots [64]$$

In particular, the vertical deflection of the middle point of a symmetrical fixed-end pipe line is given by

$$dy_1 = \frac{\alpha TL}{GH - F^2}(B'H - A'F) + \alpha T_{y1} \quad \ldots\ldots [65]$$

Case III—Unsymmetrical Line With Hinged Ends. In this case $M_0 = 0$ and

$$dy_1 = \frac{1}{EI}[-F_x(x_1F' - B') + F_y(x_1A' - C')] + \alpha T_{y1} \quad \ldots [66]$$

$$dx_1 = -\frac{1}{EI}[-F_x(y_1F' - G') + F_y(y_1A' - B')] + \alpha T_{x1} \quad \ldots [67]$$

Note that for hinged ends these deflections are relative to the left end, that is, when plotted as offsets to the original curve they will give a figure similar to the dotted curve of Fig. 6. This dotted curve is the deformed line rotated so that the tangents at the left end coincide. In less technical language, the deflections given by Formulas [66] and [67] will give the correct shape of the deformed line but not in its correct position.

Case IV—Symmetrical Line With Hinged Ends. Here $F_y = 0$ and the formulas become

$$dy_1 = \frac{1}{EI}[-F_x(x_1F' - B')] + \alpha T_{y1} \quad \ldots\ldots [68]$$

$$dx_1 = -\frac{1}{EI}[-F_x(y_1F' - G')] + \alpha T_{x1} \quad \ldots\ldots [69]$$

As in Case III, these deflections correspond to the dotted curve of Fig. 6.

In particular, the actual vertical deflection of the middle point of a symmetrical hinged-end pipe line (above its original position) is given by

$$dy_1 = \frac{F_xB'}{EI} + \alpha T_{y1} = \alpha T\left(\frac{LB'}{G} + y_1\right) \quad \ldots\ldots [70]$$

Case V—Symmetrical or Unsymmetrical Line With Partially Fixed Ends. For a pipe line that is unsymmetrical or for which the fixity at the ends is different, use Equations [61] and [62] and for a symmetrical pipe line with equal fixity at the ends, use Equations [63] and [64]. In either case, the deflections will correspond to a curve of the form shown in Fig. 6.

J—Flexibility Constant, *K*

In the common bending theory no account is taken of the deformation of the cross-section of the beam. In the case of pipes with comparatively thin walls, consideration of this deformation becomes necessary, as such pipes prove much more flexible during bending than can be accounted for by the ordinary theory.

It has been shown by several investigators that a factor K which may be called the flexibility constant, introduced into the ordinary bending formulas, will allow for this added flexibility. See Part II for brief description of mathematical work and Fig. 13 for plot of equations. Test data by Hovgaard seems to indicate that the distribution of longitudinal stress is more nearly in accordance with the Karman than with the other theories. However, since the test data are so meager and since the Lorenz formula appears to be more reasonable from a mathematical standpoint, as shown by Table 2, we shall use the Lorenz formula in the remainder of this paper.

TABLE 2 COMPARISON OF THEORIES OF LORENZ, KARMAN, AND MARBEC

Basis of comparison	Lorenz	Karman	Marbec
When $\beta = 0$	K = infinity	$K = 10$	K = infinity
β curve	One curve over entire range	2 curves	One curve
Simplicity of formulas	1st choice	3rd choice	2nd choice
Apparent agreement with test data (see Fig. 13)	1st choice	2nd choice	3rd choice
Assumption forming basis of theory	Deformed cross-section doubly symmetrical, no assumption as to shape of curve	Mean circumference remains unchanged	Cross-section takes shape of ellipse

K—Stresses at Any Point in Any One-Plane Pipe Line

At any cross-section of a pipe line subject to a rise in temperature T and an internal pressure p, there are six more or less important stresses to be considered. These are:

1 A longitudinal stress S_l parallel to the axis of the pipe, due to bending. This stress varies from zero at the neutral axis of the cross-section to a maximum at, or near, the fibers farthest from the neutral axis.

2 A transverse stress S_t perpendicular to the axis of the pipe, due to bending, and produced by the tendency of the cross-section to flatten. This stress is computed on the assumption of linear distribution of the bending stress across the pipe wall.

(*Text continued on p. 425*)

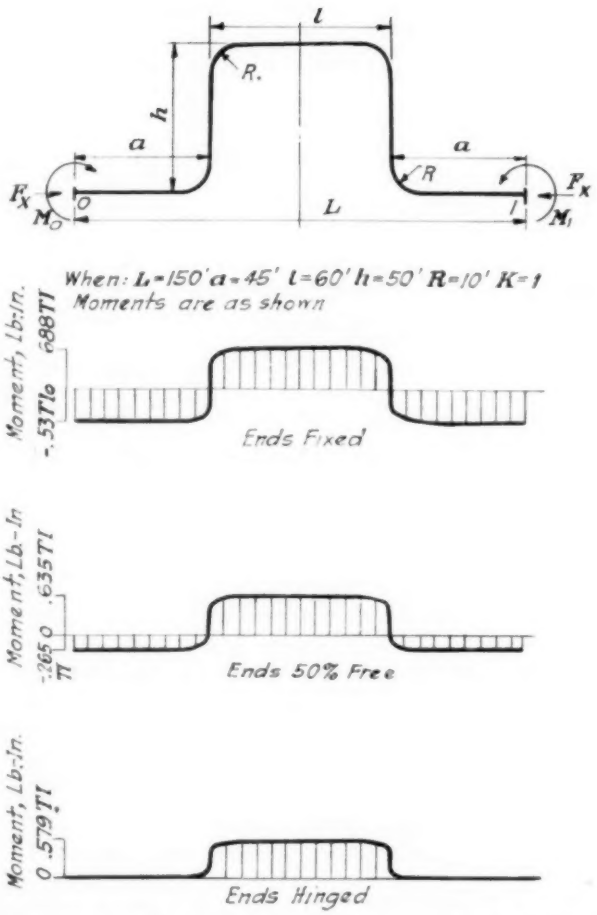


FIG. 10a MOMENT DIAGRAMS FOR VARIOUS END CONDITIONS, ONE-PLANE PIPE LINES

Formulas for Fig. 10a

$$F = h(h - 4R + l + \pi KR)$$

$$G = KR(1.42R^2 - 2.28hh + \pi h^2) + h\left(hl - 4hR + \frac{2h^2}{3} + 2R^2\right) - \frac{4R^3}{3}$$

$$H = 2\left(a + h + \frac{l}{2} - 4R + \pi KR\right)$$

Ends fixed:

$$M_o = M_1 = \frac{EI\alpha TLF}{GH - F^2}$$

$$F_x = \frac{EI\alpha TLH}{GH - F^2} \text{ and } F_y = 0$$

Ends partly free:

$$M_o = M_1 = \frac{EI\alpha TLF\,(1 - n)}{GH - F^2}$$

$$F_x = \frac{EI\,\alpha\,TLH\left(1 - \frac{nF^2}{GH}\right)}{GH - F^2} \text{ and } F_y = 0$$

where n = degree of freedom in the approximate formula.

Ends hinged:

$$M_o = M_1 = 0$$

$$F_x = \frac{EI\alpha TL}{G} \text{ and } F_y = 0$$

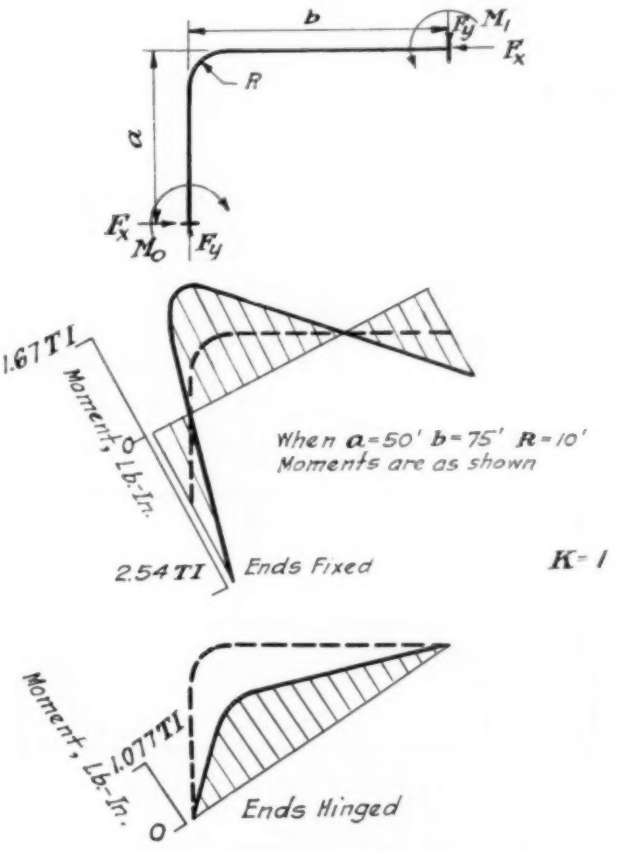


FIG. 10b MOMENT DIAGRAMS FOR VARIOUS END CONDITIONS, ONE-PLANE PIPE LINES

Formulas for Fig. 10b

$$A = 0.57KR^2 + \frac{b^2 - R^2}{2}$$

$$B = (0.57a - 0.07R)KR^2 + \frac{a(b^2 - R^2)}{2}$$

$$C = 0.355KR^3 + \frac{b^3 - R^3}{3}$$

$$F = (1.57a - 0.57R)KR + \frac{a^2 + 2a(b - 2R) + R^2}{2}$$

$$G = (1.57a^2 - 1.14aR + 0.355R^2)KR + \frac{a^3 - 6a^2R + 3a^2b + 3aR^2 - R^2}{3}$$

$$H = 1.57KR + a + b - 2R$$

Ends fixed:

$$M_0 = \frac{EI\alpha T[F(Ba + Cb) - A(Bb + Ga)]}{2ABF + CGH - A^2G - B^2H - CF^2}$$

$$F_x = \frac{EI\alpha T[H(Ba + Cb) - A(Ab + Fa)]}{2ABF + CGH - A^2G - B^2H - CF^2}$$

$$F_y = \frac{EI\alpha T[H(Bb + Ga) - F(Ab + Fa)]}{2ABF + CGH - A^2G - B^2H - CF^2}$$

$$M_1 = M_0 - F_x a + F_y b$$

Ends hinged:

$$M_0 = M_1 = 0$$

$$F_x = \frac{EI\alpha T(b^2 + a^2)b}{a^2c - 2Bba + Gb^2} \text{ and } F_y = \frac{EI\alpha T(b^2 + a^2)a}{a^2C - 2Bba + Gb^2}$$

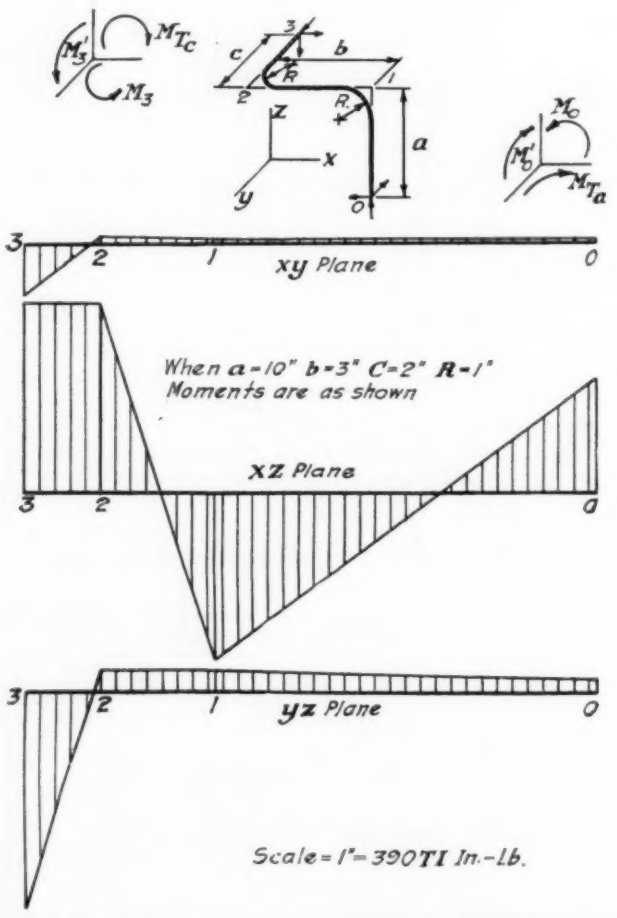


FIG. 10c MOMENT DIAGRAMS FOR TYPICAL PIPE LINE IN MORE THAN ONE PLANE

Formulas for Fig. 10c

$$F_x = \alpha TEIG_2$$

$$F_y = \alpha \frac{TEI}{A_1}(c - G_2H_1)$$

$$F_z = \frac{\alpha TEI}{F_2}(a - B_2G_2)$$

$$M_0 = \frac{\alpha TEI}{C_1F_2}\{G_2[aF_2(c_1 - a) - bB_2(2a + b - C_1)] + ab(2a + b - c_1)\}$$

$$M_3 = \frac{\alpha TEI}{A_1(2c + H)}\{G_2[A_1c(c + H) + bH_1(H - b)] - bc(H - b)\}$$

$$M_{1a} = \frac{\alpha TEI}{A_1(2c + H)}\{G_2[bH_1(b + 2c) + A_1c^2] - bc(b + 2c)\}$$

$$M_{1b} = \frac{\alpha TEI}{2(a + c) + \frac{5}{2}(b - 2R)}\left\{ac\left(\frac{a}{A_1} + \frac{c}{F_2}\right) - G_2\left(\frac{a^2H_1}{A_1} + \frac{C^2B_2}{F_2}\right)\right\}$$

$$M_{1c} = \frac{\alpha TEI}{C_1F_2}\{G_2[-a^2F_2 - bB_2(2a + b)] + ab(2a + b)\}$$

$$M'_0 = F_y a - M_{1b}$$

$$M'_3 = F_x C - M_{1b}$$

$$A = \frac{b^2 + R^2(1.14K - 1) + 2.5b(c - R)}{2}$$

$$B = \frac{b^3 + bR^2(3.42K - 3) - R^3(2.13K - 2) - 2c^3}{6}$$

$$N = \frac{c^2 + R^2(1.14K - 1)}{2}$$

$$F = B - \frac{a^3 - c^3}{3}$$

$$G = A + 1.25b(a - c)$$

$$H = 2b + \frac{5}{4}(a - R)$$

$$A_1 = \frac{bG(b + 2c)}{H + 2C} - F$$

$$B_1 = \frac{a^2 + R^2(1.14K - 1)}{2} = c + \frac{a^2 - c^2}{2}$$

$$C_1 = 2b + 2a + \frac{5}{2}(C - R)$$

$$F_1 = \frac{cN(c + H)}{H + 2c}$$

$$G_1 = \frac{a^3 + c^3 + R^3(3.42K - 3)(a + c) - R^3(4.26K - 4)}{6}$$

$$H_1 = 0.071KR^3 - \frac{c^2G}{H + 2c}$$

$$A_2 = \frac{1}{A_1}\left[0.07KR^3 - \frac{bN(H - b)}{H + 2c}\right]$$

$$B_2 = 0.07KR^3 - \frac{a^2A}{C_1}$$

$$C_2 = \frac{B_1(b^2 + 2ab - bC_1) + 0.07KR^3C_1}{A(b^2 + 2ab) - BC_1}$$

$$F_2 = \frac{A}{C_1}(b^2 + 2ab) - B$$

$$G_2 = \frac{(b - cA_2 - aC_2)}{aB_1\left(1 - \frac{a}{c_1}\right) + F_1 - G_1 - A_2H_1 - B_2C_2}$$

(*Continued from p. 422*)

It is a maximum at either $y = o$ or $y = r$. At $y = o$, it is compressive on the inside of the wall and tensile on the outside of the wall. The reverse is true at $y = r$.

3 A transverse compression S_{tc} due to the resultant forces on the top and bottom halves of the bend. This compressive stress is a maximum at the neutral axis of the cross-section and decreases to zero at the top and bottom of the pipe.

4 A transverse tension S_{tp} due to the internal pressure. This tensile stress is constant over the entire cross-section under the assumption of a thin cylinder.

5 A longitudinal tension S_{lp} due to the internal pressure. This tensile stress is also constant over the entire cross-section.

6 A longitudinal compression S_{ln} due to the direct thrust N in the pipe. This compressive stress is constant over the entire cross-section and is ordinarily small.

Of these stresses, 1, 2, and 3 are variable over the cross-section while 4, 5, and 6 are considered constant over the cross-section. Formulas for 1, 2, and 3 will depend upon the assumptions used by the investigator and consequently there are three sets of formulas corresponding to the work of Karman, Marbec, and Lorenz.

Formulas for Brittle Materials, Such as Cast Iron. The maximum principal stress may be either tensile or compressive. Ordinarily, stresses 4, 5, and 6 are not taken into account, and it is then necessary to compute only max. S_l (tensile or compressive), and max. S_t + max. S_{tc} (compressive) and compare

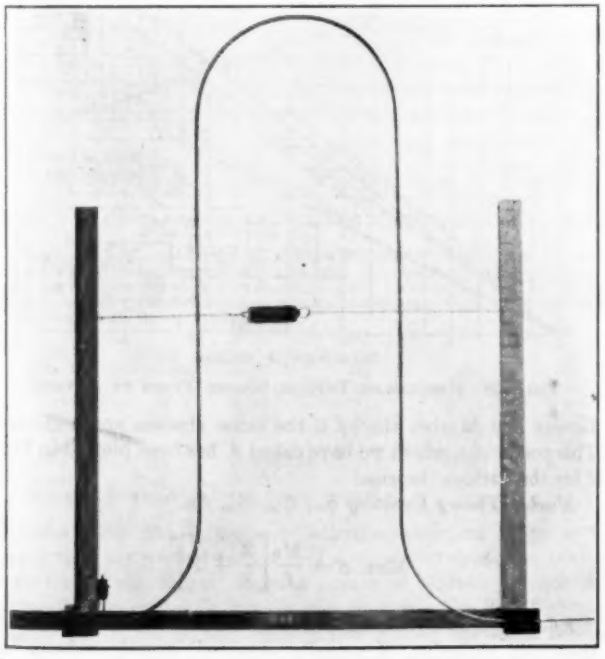


Fig. 11*b*

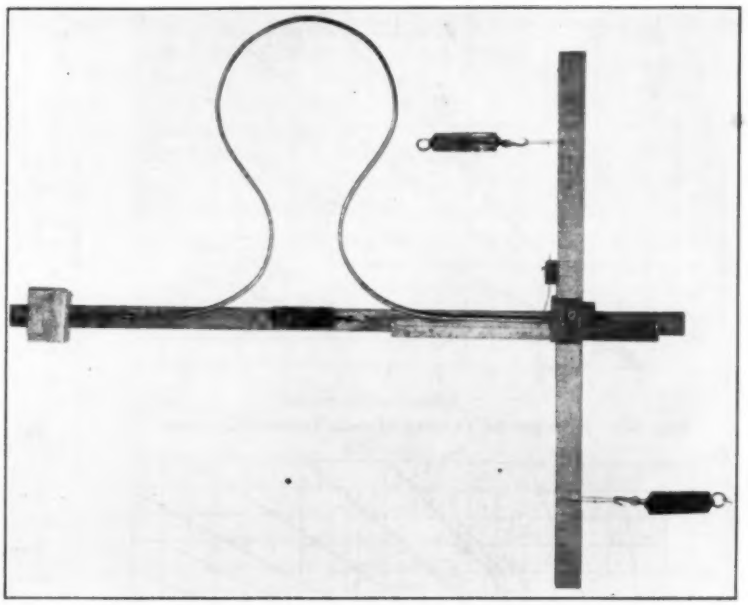


Fig. 11*a* Typical Model Test Set-Ups Employed

these values, using the greater. In this case, using the formulas of Karman with values of S_{tc} by Hovgaard;

$$\text{max. } S_l = \frac{Mr}{I}\frac{4}{9\sqrt{2}}\left[\frac{(5 + 6\lambda^2)^{3/2}}{1 + 12\lambda^2}\right] \quad [71]$$

$$\text{max. } S_t + \text{max. } S_{tc} = \frac{Mr}{I}\frac{2}{R}\left[\frac{9\lambda R + r + 6\pi\lambda^2}{1 + 12\lambda^2}\right] \quad [72]$$

For the same case using the formulas of Lorenz:

$$\text{max. } S_l = \frac{Mr}{I} \quad [73]$$

$$\text{max. } S_t + \text{max. } S_{tc} = \frac{Mr}{I}\left[\frac{3}{2\lambda} + \frac{r}{R}\right] \quad [74]$$

Formulas for Ductile Materials, Such as Steel, Lorenz Theory. If we omit the relatively small stress S_{tc} and the stresses S_{tp}, S_{lp}, S_{ln}, and restrict values of λ to those <6, we have the maximum equivalent stress in the pipe always equal to

$$\frac{Mr}{I}\left[\frac{3}{2\lambda} + 1\right] \quad [75]$$

Karman Theory, Omitting S_{tc}, S_{tp}, S_{lp}, S_{ln}. This case has been developed by Wahl and the formulas are:

$$S_1 = \frac{Mr}{I}\left[\frac{8\lambda - 36\lambda^3 - \left(32\lambda^2 + \frac{20}{3}\right)\sqrt{\frac{4}{3}\lambda^2 + \frac{5}{18}}}{1 + 12\lambda^2}\right] \quad [76]$$

$$S_2 = \frac{Mr}{I}\left[\frac{8\lambda - 36\lambda^3 + \left(32\lambda^2 + \frac{20}{3}\right)\sqrt{\frac{4}{3}\lambda^2 + \frac{5}{18}}}{1 + 12\lambda^2}\right] \quad [77]$$

$$S_3 = \frac{Mr}{I}\left[\frac{-12\lambda^2 - 18\lambda + 2}{1 + 12\lambda^2}\right] \quad [78]$$

with the rule

For $0.3 < \lambda < 0.335$, use S_1 or S_2

For $\lambda > 0.335$, use S_2 or S_3

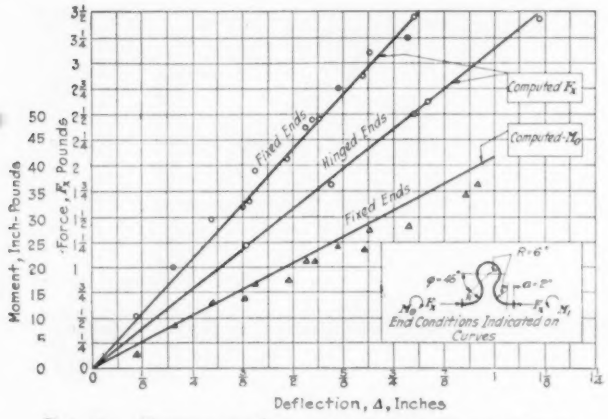


FIG. 12a RESULTS OF TYPICAL MODEL TESTS BY AUTHOR

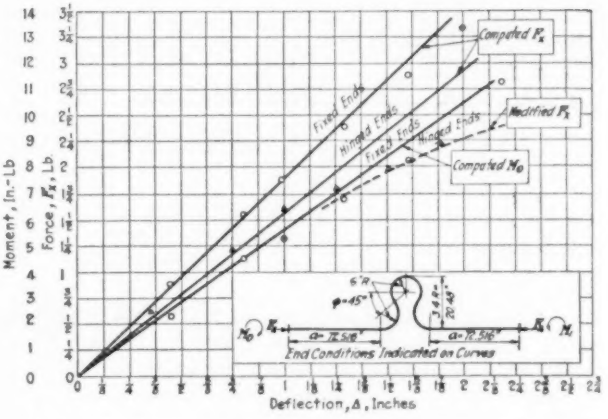


FIG. 12b RESULTS OF TYPICAL MODEL TESTS BY AUTHOR

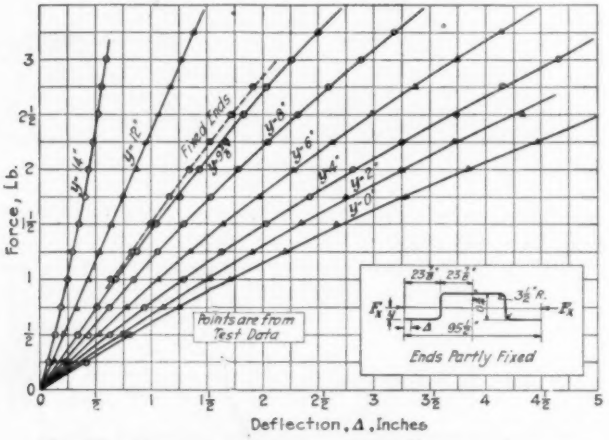


FIG. 12c RESULTS OF TYPICAL MODEL TESTS BY AUTHOR

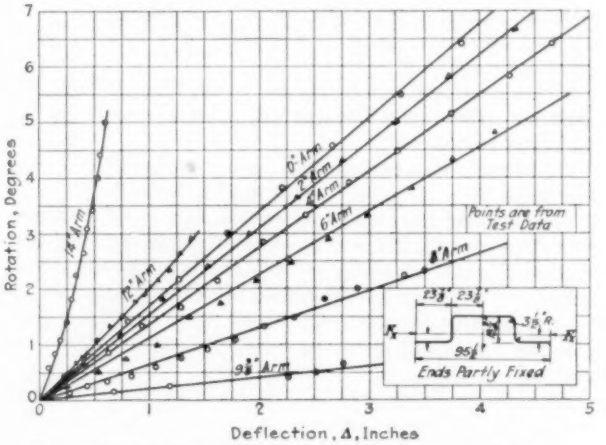


FIG. 12d RESULTS OF TYPICAL MODEL TESTS BY AUTHOR

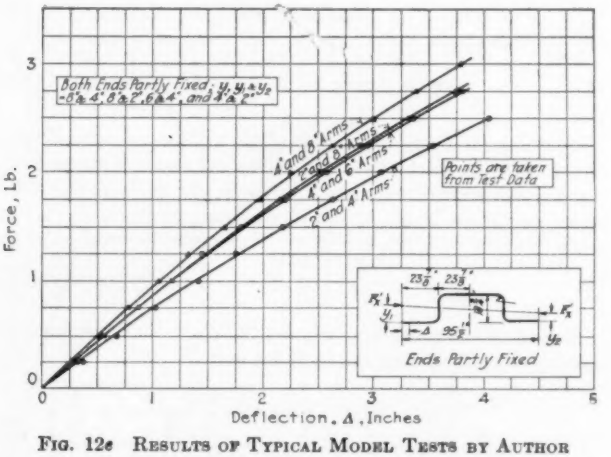


FIG. 12e RESULTS OF TYPICAL MODEL TESTS BY AUTHOR

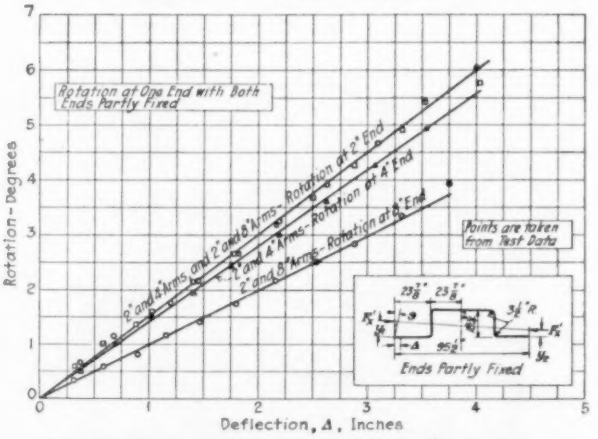


FIG. 12f RESULTS OF TYPICAL MODEL TESTS BY AUTHOR

TABLE 3 COMPARISON OF THEORY WITH TESTS OF FIGS. 12c AND 12e

Δx, in.	F_x (observed), lb.	F_x (computed), lb.
Equal fixities, 8-in. arm at each end		
7/16	0.5	0.55
27/32	1.00	1.07
1 1/16	1.25	1.35
Unequal fixities, arm 8-in. at left, 4-in. at right		
17/32	0.5	0.55
25/32	0.75	0.81
1 1/16	1.00	1.09

It is obvious that the coefficient of Mr/I in the foregoing equations can be plotted as a function of λ and values of this coefficient taken from the curve so obtained. This is equally true for the Lorenz and Marbec theory if the same stresses are neglected. This coefficient, which we have called β, has been plotted in Fig. 7 for the various theories.

Marbec Theory Omitting S_{1c}, S_{1p}, S_{1p}, S_{1s}.

$$\text{Max. } S = \frac{Mr}{I}\left[\frac{2}{\lambda} + 1\right] \qquad [79]$$

Effect of Cold Springing of Pipe. The curves of Fig. 8 show that cold springing (if actually obtained) greatly reduces the stresses in pipes.

Effect of Steam Pressure on Amount of Flattening. It has been shown by Wahl that this effect is negligible.

Summary. Adopting the Lorenz theory (see Part III, J) we have the equation for maximum stress in a pipe bend of a ductile material in the case where $\lambda < 6$, and where S_{tc} is omitted:

$$S = \frac{Mr}{I}\left[\frac{3}{2\lambda} + 1\right] + S_{tp} - S_{ip} + S_{in} \ldots\ldots [80]$$

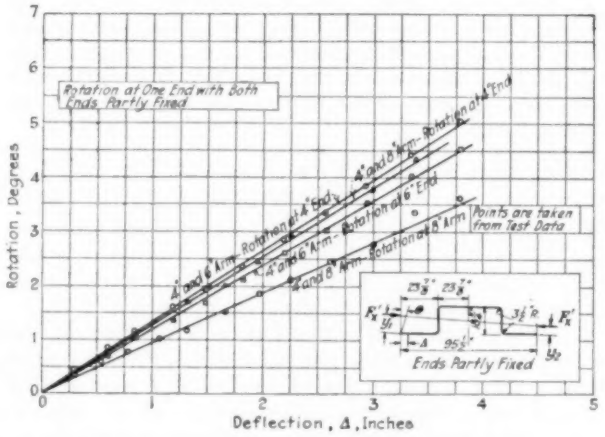


FIG. 12*g* RESULTS OF TYPICAL MODEL TESTS BY AUTHOR

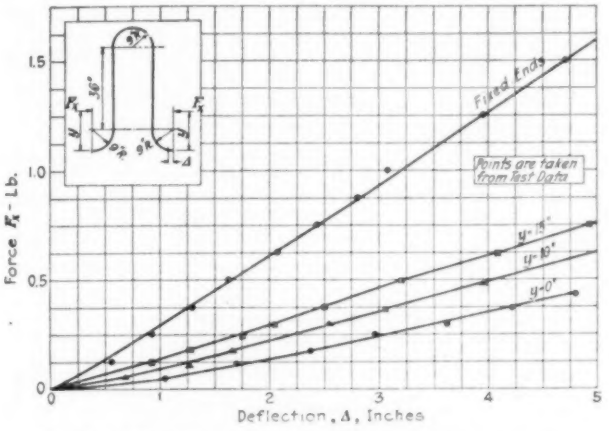


FIG. 12*h* RESULTS OF TYPICAL MODEL TESTS BY AUTHOR

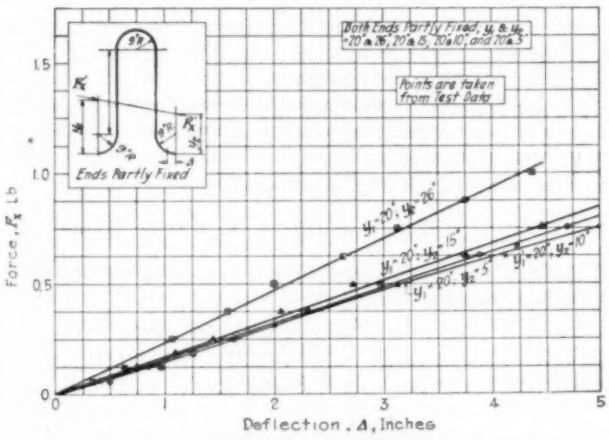


FIG. 12*j* RESULTS OF TYPICAL MODEL TESTS BY AUTHOR

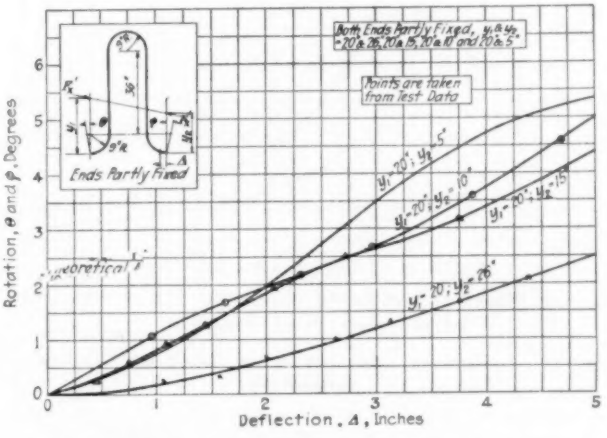


FIG. 12*k* RESULTS OF TYPICAL MODEL TESTS BY AUTHOR

TABLE 4 COMPARISON OF THEORY WITH TESTS OF FIGS. 12*h* AND 12*j*

Test data are consistently lower than the computed data, due probably to the distortion of the model. However, this difference is usually small, and the test results in the main show good agreement with the theory. The quantities m_0 and m_1 are the computed fixities at the left- and right-hand ends, respectively.

Left arm, in.	Right arm, in.	F_x computed, lb.	F_x observed, lb.	m_0	m_1
20	26	0.252	0.229	0.74	0.96
20	15	0.185	0.172	0.73	0.55
20	10	0.168	0.160	0.73	0.37
20	5	0.155	0.150	0.73	0.18
15	15	0.163	0.150	0.55	0.55
10	10	0.135	0.124	0.37	0.37
Free	Free	0.099	0.086	0	0
Fixed	Fixed	0.333	0.322	1	1

Referring to Fig. 7, an actual calculus maximum (slope = 0) exists near the neutral surface and a numerical maximum occurs near the outside fibers. For the Lorenz or Marbec theories the numerical maximum is always the greater. For the Karman theory the curve of equivalent stress follows the form shown dotted in the figure, and it becomes necessary to compare the calculus and numerical maxima in order to determine the greater. Both calculus and numerical maxima equal unity for all theories when λ equals infinity (i.e., for a straight pipe).

Study of Stress Distribution. Fig. 8*a* illustrates the stress distribution in a curved pipe under the more exact theory and indicates that it is much more complicated than has been generally realized. The longitudinal stress due to bending is added to the longitudinal stress due to pressure giving the curve of combined longitudinal stress. In a similar manner the transverse stress due to bending is added to the transverse stress due to pressure, giving the curve of combined transverse stress. The resultant combined (equivalent) stress ($S_t - S_l$) is then obtained according to the maximum shear theory of failure, for ductile material.

Fig. 8*a* also shows the stresses due to bending when reduced one-half by cold springing. The pressure stresses are, of course, not changed by this means.

The same stresses are shown for a straight pipe in Fig. 8*b*. In this case the distribution is the same as in a beam, there being no flattening effect. This flattening effect would be much greater for piping with thinner walls than for the 600-lb. piping used in the illustration.

It is common practice to stress piping up to or beyond the elastic (or creep) limit as is witnessed by the common experience that some joints in a new installation will leak until the piping has set. This practice has proved satisfactory since that particular portion of the metal which is overstressed, yields (or creeps), thus relieving the moment on the line.

L—Study of Errors Due to Assumptions in Derivation

For the model pipe lines of Fig. 12, and others not published the magnitude of the errors involved in the assumptions of the derivation in Part III have been computed to illustrate concretely their importance.

a Direct compression. Error less than 0.1 per cent.

b Direct shear. Error less than 0.1 per cent.

c Effect of deflection on moment. In most cases the error was less than 5.0 per cent. In some long tangent free-end tests the error was cumulative with increasing force and reached as great a value as 20 per cent. However, the latter cases are not approached in practice and the error in practice is believed to be less than 5.0 per cent. This error is in the opposite direction to the other errors and will, in general, balance them so that the resultant error in practical installations can be considered small and on the safe side.

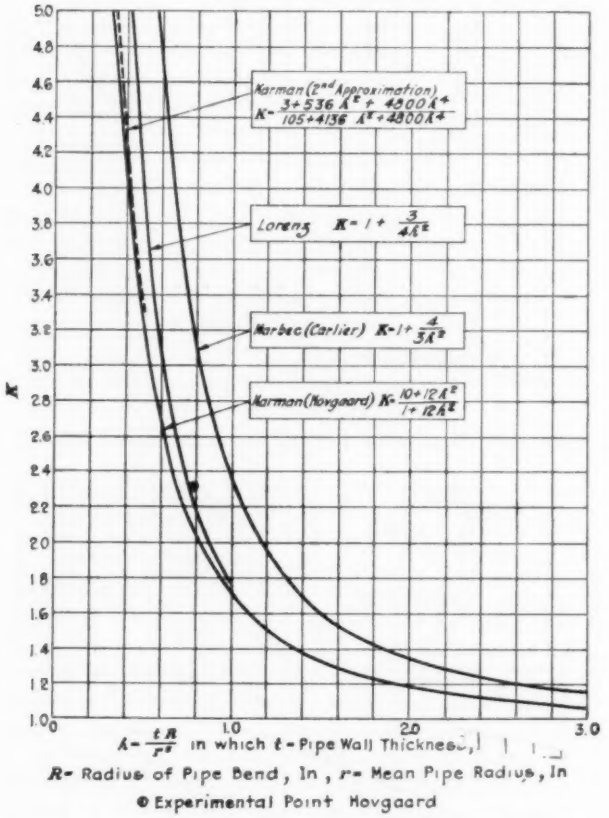


Fig. 13 Flexibility Constant K as Given by Various Investigators

d Location of neutral axis of cross-section of pipe. The neutral axis of a straight bar subjected to bending passes through the center of area of the cross-section. For a curved bar, however, this is no longer strictly true, but it may be shown that the error due to this cause is usually only a fraction of one per cent for pipe bends.

M—Graphical Method for Any One-Plane Pipe Line With Any End Conditions

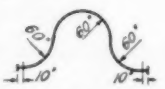
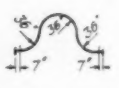
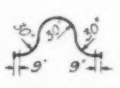
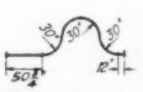
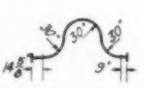
Hovgaard (Reference *J*) gives illustrations of graphical solutions for several hinged-end bends. An illustration is here given, for a symmetrical fixed-end pipe line, of a method which will suggest solutions for any one-plane pipe line.

Referring to Fig. 9, the neutral axis of the pipe line will first be determined. In general, the distance n of the neutral axis from the line 0–7 through the ends is $n = (K\int yds)/(K\int ds)$. In other words, the neutral axis passes through the center of gravity of the pipe line if the ends are fixed and if K for all parts can be assumed to be the same. Assuming $K = 1$, from symmetry of the pipe line the neutral axis bisects each of the lengths b; therefore $n = b/2$. Replotting the pipe line against s instead of x, the (y, s) curve is obtained. As shown on the figure, at any distance s from the left-hand end, the distances y and y' can be read directly. Multiplying these two distances together at each point a curve is plotted whose ordinates are proportional to yy' and hence to $F_x yy'$ or M_y. The area under this curve for the entire pipe line is $\int Myds$, and from Equation [6]

$$\int Myds = EI\alpha TL = F_x \int yy'ds \quad \ldots\ldots\ldots [81]$$

Thus the desired relationship between F_x and αTL is obtainable from the area under the yy' curve, and

$$F_x = -\frac{EI\alpha TL}{\int yy'ds} \quad \ldots\ldots\ldots [82]$$

Test Specimen						
Author and Author's Fig. No.	Crocker Fig. 17	Crocker Fig. 18	Crocker Fig. 19	Crocker Fig. 20	Crocker Fig. 21	Crocker Fig. 22
Pipe size and thickness	10″ pipe; t = 0.336″	6″ pipe; t = 0.418″	6″ pipe; t = 0.27″	6″ pipe; t = 0.274″	6″ pipe; t = 0.285″	6″ pipe; t = 0.41″
F_x (test)	964 Δ	2600 Δ	2080 Δ	1760 Δ	2550 Δ	603 Δ
F_x (computed) free ends	902 Δ	1990 Δ	1432 Δ	1476 Δ	1596 Δ	460 Δ
F_x (computed) fixed ends	2590 Δ	5480 Δ	3960 Δ	3490 Δ	4290 Δ	1375 Δ
K (test) ends assumed free	2.21	1.00	1.49	1.79	1.27	1.00
K (computed) Lorenz	2.36	1.31	2.17	2.13	2.04	1.32
Fixedness (computed) Lorenz K assumed, per cent	7	28	37	18	46	31

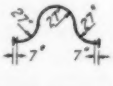
Test Specimen						
Author and Author's Fig. No.	Crocker Fig. 23	Crocker Fig. 24	Bantlin	Bantlin	Bantlin	Hovgaard Fig. 13
Pipe size and thickness	4″ pipe; t = 0.237″	3¼″ boiler tube; t = 0.12″	8″ pipe; t = 0.262″	5″ pipe; t = 0.165″	8″ C. I. pipe; t = 0.78″	6″ pipe; t = 0.265″
F_x (test)	235 Δ	226 Δ	487 Δ	156 Δ	2150 Δ	408 Δ
F_x (computed) free ends	177 Δ	158 Δ	550 Δ	183 Δ	2165 Δ	411 Δ
F_x (computed) fixed ends	536 Δ	428 Δ	1420 Δ	453 Δ	6630 Δ	1276 Δ
K (test) ends assumed free	1.02	1.00	....	....	....	....
K (computed) Lorenz	1.35	1.43	7.41 (R = 32.7″) 3.91 (R = 22.0″)	4.86 (R = 31.6″) 2.13 (R = 17.1″)	1.56 (R = 32.7″) 1.25 (R = 22.0″)	2.28 (R = 30.6″) 2.21 (R = 29.6″)
Fixedness (computed) K assumed, per cent K Lorenz	32	36	0	0	0	

Fig. 14 Comparison of the Author's Theory With Previously Published Test Results

$$M_0 = F_x n \quad \ldots\ldots\ldots\ldots\ldots\ldots [83]$$

In Fig. 9 with $s = 40$ ft., $y = 7.6$ ft., $y' = 5.8$ ft., $yy' = -44.1$ For the entire line $\int yy'ds = 25{,}700$ cu. ft. Hence $F_x = -(EI\alpha TL)/25700$ and $M_0 = (EI\alpha TL)/1920$. These numerical values would, however, be appreciably different if $K = 2$, as it does in the curved portion of certain standard pipe bends. The correct value of K can, however, be included in plotting the yy' curve by multiplying the values by K wherever they occur. If the resulting discontinuities in the yy' curve are smoothed out by eye, the result probably corresponds closely to the experimentally observed values, since a discontinuity of K where a straight part joins a curved part is a practical impossibility.

The graphical method for an unsymmetrical line consists either of trial to get a minimum area in the moment diagram or of a combination of the graphical method with the equations of Table 1.

The graphical method for a pipe line with partially fixed ends consists of the application of the theory presented in Part III, F.

N—Analysis of Pipe Lines in More Than One Plane

The exact analysis of pipe lines in more than one plane requires a very large amount of labor. There are in general twelve unknown quantities; three moments and three reactions at each end. Six equations are afforded by statics, leaving six more to be determined from consideration of the elastic deformation of the structure. On each branch of a multiplane pipe line there will be two bending moments in planes at right angles, and a torsional moment, hence the stress is produced by combined bending and torsion. The resulting bending moment on each section is equal to the square root of the sum of the squares of the two bending moments. This combined with the torsional moment by the ordinary formula for combined bending and torsion gives the stress in that section.

A general method for pipe lines in space has been developed, but has not been reduced to practical simplicity at this time.

In Fig. 10c the bending and torsional moments in a typical multiplane pipe line have been obtained and plotted. For the proportions chosen in the numerical example, the maximum resultant bending moment occurs at the right-hand end.

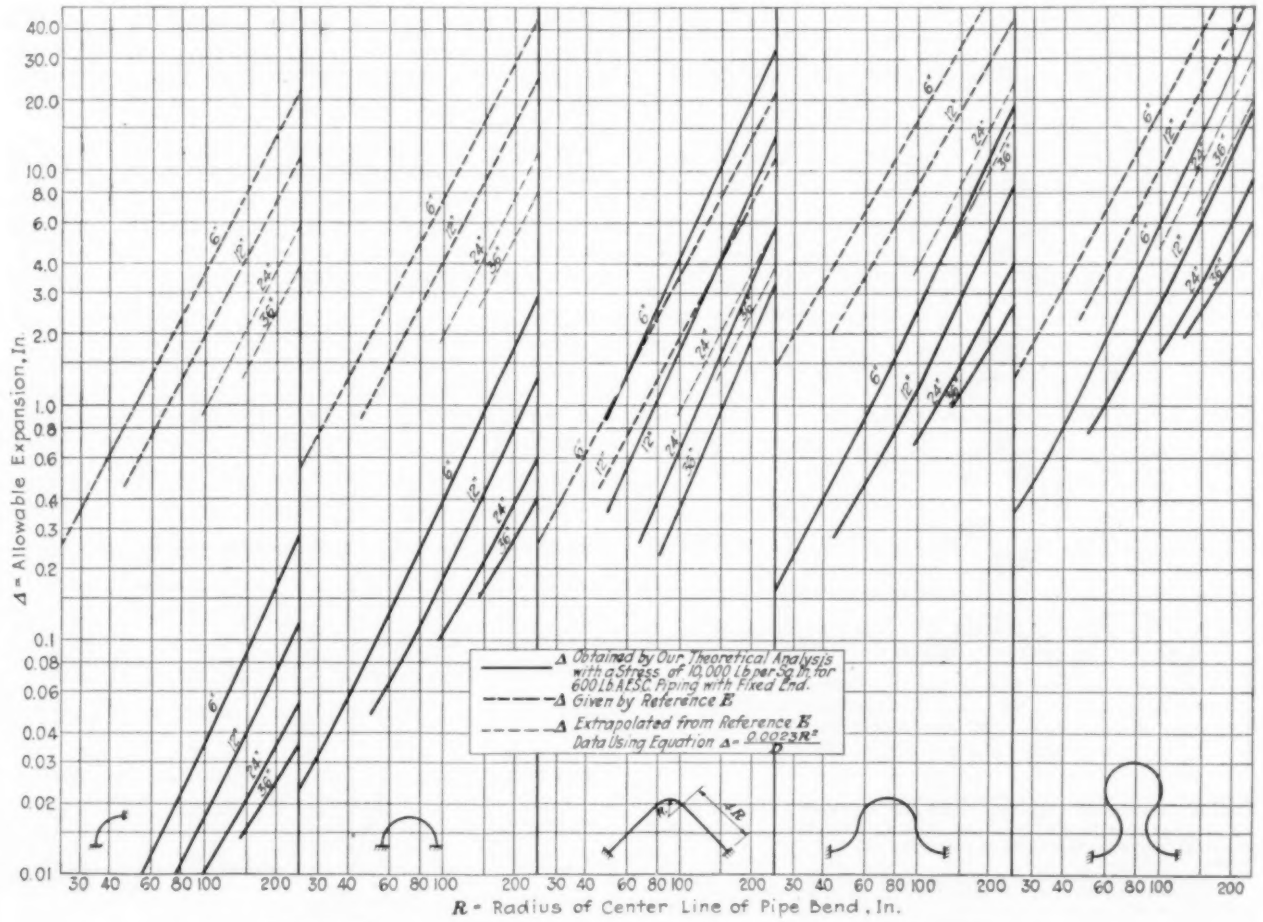


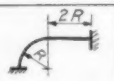
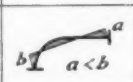
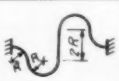
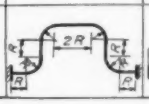
Fig. 15 Comparison Between Current Practice and Proposed Formulas

P—Moments and Reactions in Geometrically Similar Pipe Lines

Designate the ratio of each dimension (excepting those of the cross-section) on a given pipe line [1] to the corresponding dimension on a smaller but geometrically similar pipe line [2] by N. In the integrals in Equations [7], [8], and [9], every value of x, y, and s, and the ratio M_0/F_x for pipe line [1] is N times the corresponding value in pipe line [2]. With a given deflection,

$$\frac{F_{y1}}{F_{y2}} = \frac{F_{x1}}{F_{x2}} = \frac{1}{N^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots [84]$$

and

No	Shape	Moment Diagram	M_o = Moment at Left End	F_x
I			$\frac{\mu_1 \Delta I}{K}$	$\frac{\mu_2 \Delta I}{K}$
II			$\mu_1 \Delta I \eta$	$\mu_2 \Delta I \eta$
III			$\mu_1 \Delta I \eta$	$\mu_2 \Delta I \eta$
IV			$\frac{0.095 \mu_1 \Delta I}{K}$	$\frac{0.04 \mu_2 \Delta I}{K}$
V			$\frac{0.006 \mu_1 \Delta I}{K}$	$\frac{0.00098 \mu_2 \Delta I}{K}$
VI			$\frac{0.0142 \mu_1 \Delta I}{K}$	$\frac{0.0039 \mu_2 \Delta I}{K}$
VII			$\mu_1 \Delta I \eta$	$\mu_2 \Delta I \eta$
VIII			$\mu_1 \Delta I \eta$	$\mu_2 \Delta I \eta$
IX			$\mu_1 \Delta I \eta$	$\mu_2 \Delta I \eta$
X			$\mu_1 \Delta I \eta$	$\mu_2 \Delta I \eta$

Note:- To Determine Maximum Moment Draw Moment Diagram. Moment Arm = $\frac{M_o}{F_x}$ for Symmetrical Bends
Then S Max. = M Max. $\frac{r\beta}{I}$

FIG. 16 FORMULAS FOR REACTIONS FOR COMMON TYPES OF BENDS

I = moment of inertia, Δ = deflection (or expansion) to be absorbed, and μ_1, μ_2, and η are constants given by Fig. 17.

NOTES:
1 Moment diagrams are not to a comparative scale.
2 Check both curved and straight portions for maximum stress.
3 Values of η for partial fixedness, while given for Case VIII only, may be worked out for other cases, if desired.
4 Curves of I or of $(r\beta)/I$ could be prepared for A.E.S.C. pipe.
5 E assumed to be 29×10^6.

$$\frac{M_{01}}{M_{02}} = \frac{1}{N^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots [85]$$

For comparing two full-size pipe lines, however, it is more convenient to consider a given temperature change rather than a given deflection, and

$$\frac{F_{y1}}{F_{y2}} = \frac{F_{x1}}{F_{x2}} = \frac{1}{N^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots [86]$$

and

$$\frac{M_{01}}{M_{02}} = \frac{1}{N} \quad \ldots\ldots\ldots\ldots\ldots\ldots [87]$$

The behavior of full-size pipe lines can be readily pre-determined from the behavior of models by means of these equations. They also indicate the factor by which to multiply the dimensions of an unsatisfactory layout to obtain without trial a satisfactory layout.

R—STUDY IN APPLICATION OF RESULTS OF THEORETICAL ANALYSIS TO TYPICAL PIPE LINES

Figs. 10*a* and 10*b* give equations resulting from Table 1 and moment diagrams for two typical one-plane pipe lines. Fig. 10*c* gives equations and moment diagrams for a typical multiplane line. The moment diagrams may be obtained graphically as is shown under Part III M or they may be computed algebraically from the equations accompanying Figs. 10*a*, 10*b*, and 10*c*.

On the moment diagrams, the line bounding the moments is not the pipe line although in some cases it is of similar proportions and shape. In each moment diagram the perpendicular distances from the neutral axis to the limiting line are the moments on the pipe.

Familiarity with these diagrams will suggest the drawing of others by inspection for other cases and will be found very helpful in design. It should be noted that the outer fiber toward the neutral axis is always in compression and that the outer fiber away from the neutral axis is always in tension. For the multiplane line it should be noted that if rotation of flanges on gaskets

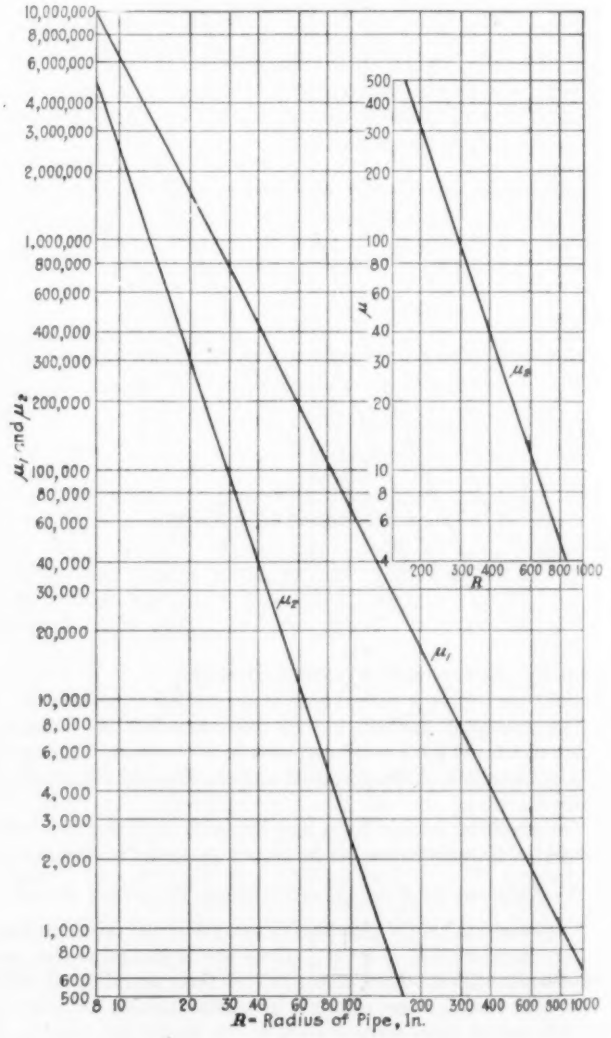


FIG. 17*a* VALUES OF CONSTANT μ

is allowable and is obtained in practice, the expansion stresses are greatly reduced since a rotation of 10 minutes makes a deflection of $^3/_8$ in. at a 10-foot radius and this may equal the total expansion to be absorbed.

PART IV—COMPARISON OF THEORY WITH EXPERIMENT AND WITH CURRENT PRACTICE

A—Load-Deflection Tests by Author on Solid Models

These tests were made on $^5/_{16}$-in. solid, cold-drawn bars and thus eliminate the variable flexibility constant K. See Fig. 11 for photograph of typical set ups, and Fig. 12 for plot of typical tests.

The actual diameter of the rod averaged 0.311 in. and this value was used in the computed curves. Tests of the material showed an average value of E of 29,000,000 lb. per sq. in.

Figs. 12*a* and 12*b* compare computed and observed values of reactions for a lyre-shaped model with and without tangents. It is seen that results are in close agreement for small values of

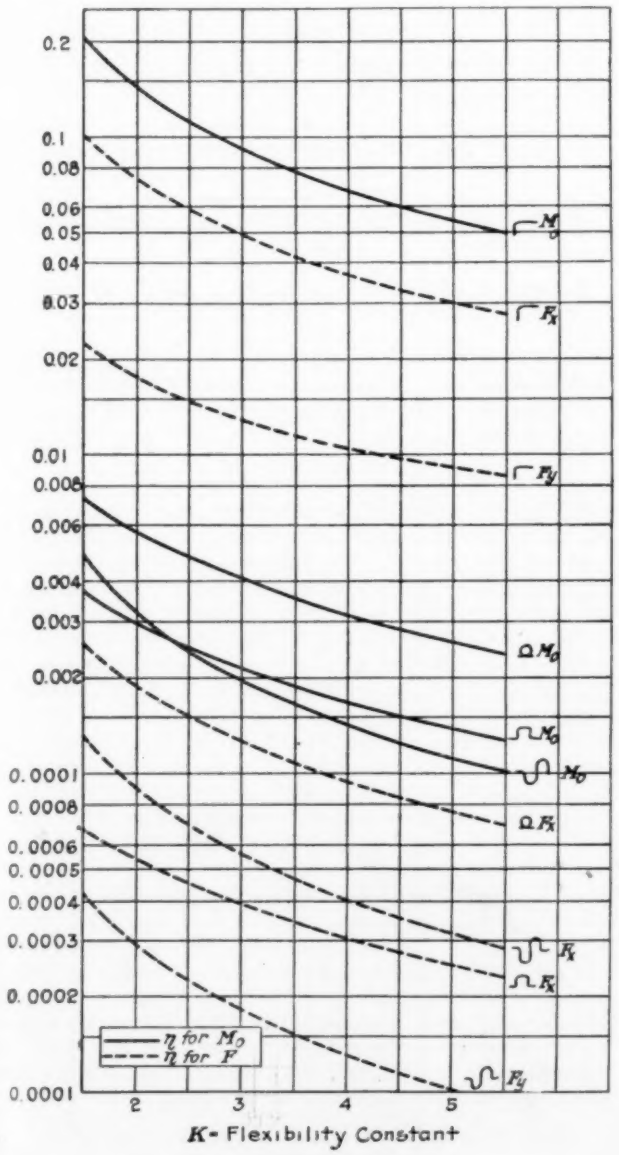


Fig. 17*b* Values of Constant η

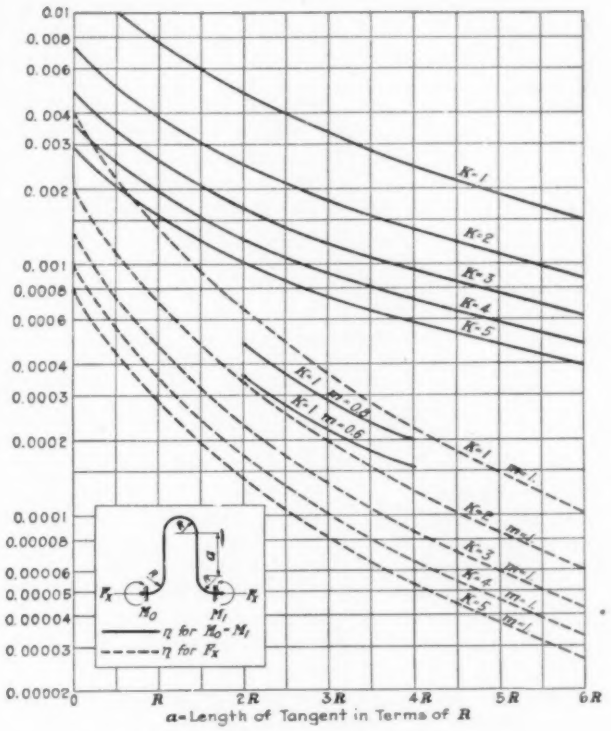


Fig. 17*c* Values of η for Hairpin Bends

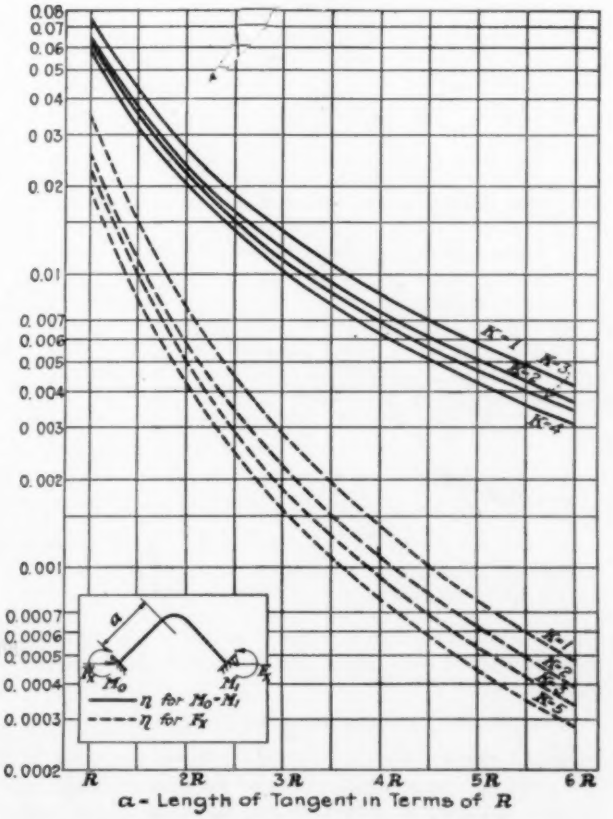


Fig. 17*d* Values of η for Quarter Bends With Tangents

deflection. For larger values of deflection theory and experiment are still in close accord with fixed ends, but with hinged ends the flexibility is greater than indicated by theory, especially for long tangents. However, the larger values of deflection are greater relative to the length of the pipe line than would be obtained in practice, so that within practical limits the results check closely. These larger values of deflection produce a distorted shape so different from the original that close agreement between theory and test cannot be expected. In practice, ends are more nearly fixed than hinged, which also is on the side of accuracy.

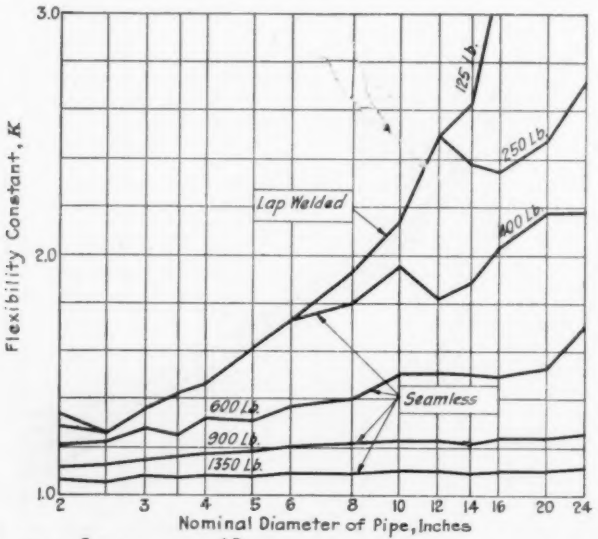
Figs. 12, *c* to *j*, show the results of experiments to determine the effect of partially fixed ends. Values of force computed by the theory of fixity previously presented lie on lines drawn tangent to the experimental curves at the origin, again showing that for practical values of Δ the theory closely agrees with experiment.

Figs. 11*a* and *b* show photographs of two of the model pipe lines.

Sixty-four-ounce spring balances were used in measuring force and moment and a steel scale in measuring deflection. Specimens were suspended by long strings, thus eliminating friction. Spring scales were re-calibrated constantly and readings taken with both increasing and decreasing loads.

One end of the model shown in Fig. 11*a* was fixed by keeping the moment arm parallel to a series of pencil lines drawn on the sheet of paper under the apparatus, and the other was held in a wooden block with a steel bushing or in a vise.

Forces applied, as in the hairpin bend-test set-up, at any distance from the ends of the pipe determine the neutral axis. The moment at any point in the pipe equals the force applied



$K = 1 + \frac{3}{4\lambda^2}$; in which $\lambda = \frac{tR}{r^2}$; t = Pipe Wall Thickness; In.; r = Mean Pipe Radius = $\frac{OD - t}{2}$; $R = 6D$

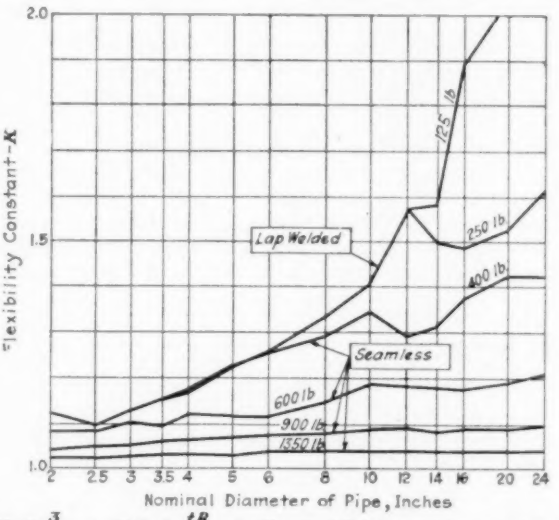
FIG. 18*a* VALUES OF FLEXIBILITY CONSTANT K FOR A.S.A. STANDARD PIPE WITH $R = 6D$

times the distance from the line of action of the force to the point (measured perpendicular to the line of action, or neutral axis).

B—PREVIOUSLY REPORTED TESTS

Fig. 13 shows the scarcity of test data on the value of K. The point by Hovgaard is considered the best published. The Crocker and Sanford data were not plotted since the ends of specimens were apparently partially fixed (see Fig. 14). The Wahl data were not plotted because of uncertainty as to end conditions of curved portions and the Bantlin data were not plotted because each specimen had two different radii.

Fig. 14 shows that the theory presented under Part III accounts for the apparent inconsistencies between previously reported tests. The free-end tests of Bantlin and Hovgaard show good agreement and the Crocker and Sanford tests are shown



$K = 1 + \frac{3}{4\lambda^2}$ in which $\lambda = \frac{tR}{r^2}$; t = Pipe Wall Thickness, In., r = Mean Pipe Radius = $\frac{O.D. - t}{2}$; $R = 10D$

FIG. 18*b* VALUES OF FLEXIBILITY CONSTANT K FOR A.S.A. STANDARD PIPE WITH $R = 10D$

to have partially fixed ends as would be expected from the method of making the tests. The assumption was made in computing Fig. 14 that both ends of all specimens were of equal degree of fixity.

C—CURRENT PRACTICE

Fig. 15 is a comparative study of the data used in current practice with that proposed as a result of the rational theory presented in Part III. The allowable deflections heretofore used are empirical and incomplete in that they give the same values for 125-lb. to 600-lb. piping and do not consider the influence of straight parts of the pipe lines. According to current practice a semicircle should have twice the allowable deflection of a quadrant of the same radius while the addition of straight portions of pipe would have no effect. Rational theory shows that this may give misleading results, as shown graphically by the first three bends here compared. A change from a quadrant to a semicircular bend actually multiplies the allowable deflection by about ten and the addition of straight portions with length equal to four times the radius of the bend, multiplies the quarter-bend values by about 100. That is, when a quadrant or semicircle is designed so that both ends are approximately fixed current practice would give unsatisfactory results, but when accompanied by sufficient additional flexibility in adjacent straight portions the data is satisfactory. It would be possible to make the results coincide by adding the proper length of straight pipe in each case, and this sometimes occurs in practice. Partial fixedness of the ends or thinner walls than used in the comparison or a combination of these two conditions would also in general bring them in closer agreement.

PART V—PROPOSED CODE OF DESIGN PRACTICE

To be of practical value rational formulas which take account

of all conditions must be derived and must be converted into workable formulas, tables, or graphs that the average engineer and draftsman can use. The following data are proposed as a preliminary code of practice. The importance of such a code is witnessed by the fact that the designer of most structures and machines can add metal and keep on the safe side when in doubt, whereas added metal in steam piping means added stiffness, and at least partial defeat of the object of safety in mind.

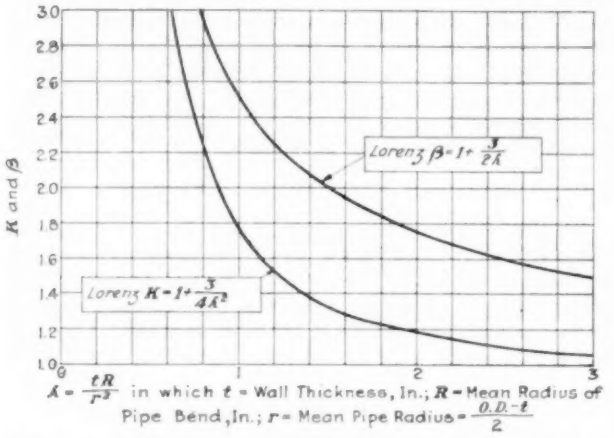


FIG. 19 VALUES OF FLEXIBILITY CONSTANT K AND EQUIVALENT STRESS CONSTANT β

A—PROCEDURE IN DESIGN

1 Make preliminary layout, from practical considerations, of the entire pipe line; and with principles of Part I in mind, locate anchors.

2 Decide on shape for best practical efficiency, by inspection, or study similar to Fig. 22, keeping joints away from points of high moment. Familiarity with the moment diagrams will be found of great help in performing much of the design by inspection.

3 Check the layout for reactions, stresses, and moments at joints, by means of the data given in Fig. 16 and Table 1. Consider, as far as practical, the entire section of the line between the anchors. Decide upon the amount of cold springing, the maximum being one-half of the total expansion. As noted under Part I, cold springing must be relied upon with caution to reduce stresses and reactions.

4 Design and place supports to allow the pipe line to move freely. This can be done most intelligently after computing or estimating the shape after deflection.

B—SUPPORTS AND ANCHORS

a Supports. Supports should be designed and placed to interfere as little as possible with the free movement of the entire pipe line. A study of the shape of the entire line before and after expansion is the starting point. A designer with this information before him is in position to choose more intelligently the type of supports and to decide upon their position. The following types are suggested as showing the trend of recent design:

1 Long rods with turn-buckle adjustments
2 Spring supports on long rods
3 Counterweight supports on long rods
4 Spring supports on rollers
5 Cylindrical rollers (rollers with curvature to shape of pipe restrict movement)
6 Ball- and roller-bearing supports; see Carlier (Ref. *G*) for details.

b Anchors. Anchors serve two main purposes:

1 They avoid excessive thrust on apparatus such as turbines, boilers, and tanks
2 They avoid excessive displacements of the pipe line on expansion.

If an anchor does not directly serve one of these purposes, it should not be used. In cases of doubt it is suggested that the use of a limited-travel-type anchor will allow some movement and also insure that the amount of movement is within the desired limits.

The distance between an anchor and the apparatus which it is desired to protect should be as small as possible, since the apparatus must absorb the expansion of the length of pipe between itself and the anchor. Otherwise sufficient flexibility must be provided between the anchor and apparatus to keep the reactions within the allowable values.

C—PROPOSED DESIGN DATA

Fig. 16 is proposed to replace the values commonly quoted in handbooks and catalogs (Reference *E*), which now appear to be incorrect in several points and to have no rational basis. The proposed values make possible the computation of stresses and reactions, for given deflections, by means of a simple slide-rule operation, taking certain constants from the curves of Fig. 17. Solutions for other shapes than those given may be obtained, as were these, from the data given in Part III.

Illustrative Problem. Taking a 12-in., 000-lb standard pipe with fixed ends, and, for simplicity Case V of Fig. 16,

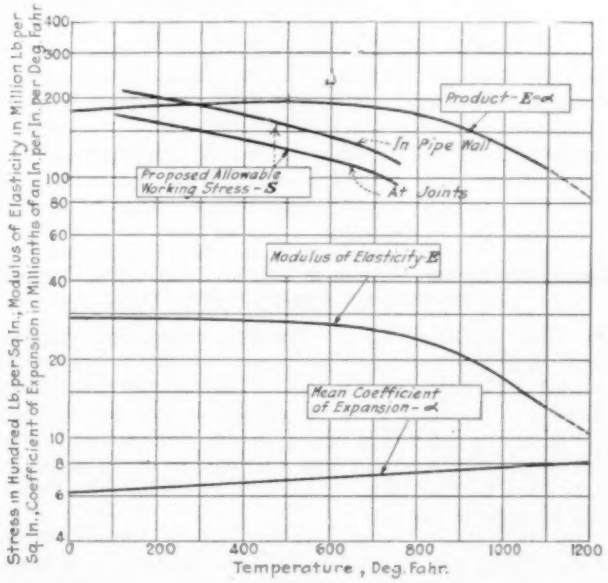


FIG. 20 ALLOWABLE STRESS, MODULUS OF ELASTICITY, AND COEFFICIENT OF EXPANSION FOR MILD STEEL PIPING

(Proposed allowable-working-stress curves are parallel to the proportional limit by French (Bur. Std. No. 296). Coefficient of expansion from Marks' Mechanical Engineers' Handbook. Modulus of elasticity from Trans. A.S.M.E., vol. 46, p. 405.)

O.D. = 12.75, t = 3/8, r = 6.06, temp. rise = 750 deg. fahr.
Assume $R = 6D$ = 72 in. I = 437 $E = 29\ (10)^6$
$\lambda = 1.225$ $K = 1.50$ $\alpha = 0.0000065$
From Fig. 17*a* $\mu_1 = 127{,}600$ $\Delta = 0.995$ in.
$\mu_2 = 6{,}370$

$$M_0 = \frac{0.006\ \mu_1 \Delta I}{K} = 222{,}000 \text{ lb-in.}$$

$$F_s = \frac{0.00098\,\mu_2\,\Delta I}{M} = 1810 \text{ lb.}$$

$\beta = 2.22$

Max. stress $= Mr\beta/I = 6840$ lb. per sq. in.

If the pipe be cold sprung to the extent of one-half the expansion, the reactions and stresses are all reduced one-half. This computation, for the sake of simplicity, ignores the flexibility of tangents at the ends of the lyre. It is recommended that the effect of the tangents be considered in design.

Suppose, that, instead of being fixed, the ends are only 50 per cent fixed. An accurate solution can be obtained by using the theory of fixity given under Part III. An approximate solution will be obtained by assuming the moment proportional to the fixity which, of course, will reduce the stress to one-half

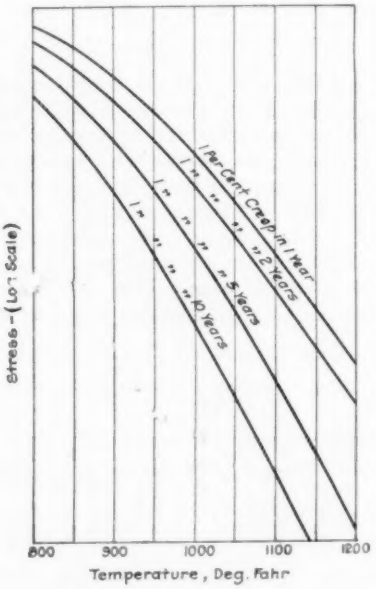


FIG. 21 SUGGESTED METHOD OF PLOTTING ALLOWABLE WORKING STRESSES ABOVE ABOUT 800 DEG. FAHR.

the value previously found. For more complicated cases of fixity, or in any case where an accurate solution is desired, the method of Part III should be used. The condition of fixed ends assumed in the figure gives the greatest stresses and reactions and should be assumed in case of doubt. Curves for the solution of partially fixed-end pipe lines may be drawn similar to those given for fixed ends. A few such curves for partial fixedness are given in Fig. 17*c*.

Fig. 18 makes it unnecessary to compute values of K for A.E.S.C. Standard Pipe when the radius of the bend R is between 6 and 10 times the mean diameter of the pipe cross-section. Intermediate values may be interpolated. Knowing the value of K from this figure, β can be easily obtained from Fig. 19.

It will be unnecessary to compute values of λ, since after obtaining the value of K from Fig. 18, the corresponding value of β will lie on the same vertical line in Fig. 19. Thus, from Fig. 18, the value of K for a 12-in., 600-lb. pipe with $R = 72$ in. is 2.21. Locating this value of K on Fig. 19, the corresponding value of $\beta = 2.26$.

Note that the value of β depends upon the curvature of the pipe at the point where M is taken. If the pipe is straight at this point, then $\beta = 1$; if not, it must be taken from the curve. For this reason the maximum stress in the pipe may occur in the curved section even though M does not reach its greatest value there. Hence, in case of doubt, the stress should be calculated using the greatest moment in the straight section with $\beta = 1$, and should be calculated again using the greatest moment in the curved section with the proper value of β.

The curves of Fig. 21 are intended to illustrate the suggested method of plotting allowable working stresses vs. temperature, and do not represent proposed values. Above some point (apparently around 800 deg. fahr. for carbon steel) creep phenomena take place and allowable stresses depend entirely upon the life desired, that is, upon economic considerations. Having set the maximum amount of creep at which the piping is to be retired, say, 3.0 per cent, the allowable stresses can be quickly obtained from such data as proposed above. A stress chosen from the "1 per cent creep in 5 years" curve would then mean that the piping should be retired in 15 years.

D—Typical Study of Shape Efficiency

For this brief study, three types of pipe lines are considered: Cases V, VIII, and X of Fig. 16. In all three cases we shall assume a constant total length of pipe of 884 in. and a constant horizontal length $L = 576$ in. Case V will then have a radius of 46.7 in. and straight portions of 222 in. on each end. Case VIII will have a radius of 72 in. and straight portions of 144 in. on each end. Case X will have a radius of 72 in. with no added length at the ends. An 8-in., 600-lb. standard pipe with fixed ends is used.

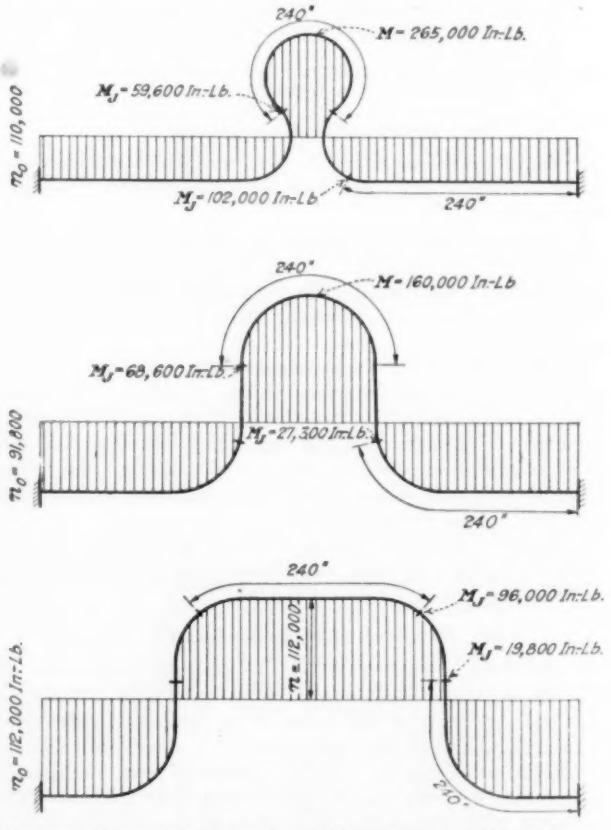


FIG. 22 TYPICAL STUDY IN SHAPE EFFICIENCIES OF PIPE LINES

Fig. 22 shows each case with neutral axis and location of maximum moments. In order to determine conditions at the joints of the pipe, a 20-ft. length of pipe has been assumed and the moments at the joints have been calculated. These are also shown in Fig. 22.

The results show that of the types considered, Case X is the most efficient as regards maximum stress. With the assumed pipe length Case VIII gives the least moment at the joints.

Conclusions

An attempt has been made:

1 To arrive at the fundamental principles of design.

2 To present a complete mathematical theory of piping subjected to expansion.

3 To reduce the very complicated results to a simple usable basis in the form of a proposed code of practice.

It is hoped that this very complicated subject has been reduced to such simple fundamentals as will enable designers to understand and visualize the design of steam piping to care for expansion much as they now do such simple elements as beams, so that much of the design can be accomplished by intelligent inspection of layouts on the drafting board.

It is the desire of the author to follow up the presentation of the data by checking it with interested persons against actual installations, and to cooperate in tests on full-size specimens so that a check may be made on the completeness of the theoretical analysis.

Acknowledgments

The great amount of tedious work involved in producing and checking this paper was performed chiefly by Messrs. H. E. Jenks, H. S. Keating, A. L. Porter, W. D. Washburn, and R. N. Williams, without whose cooperation the large amount of data presented would have been impossible. Acknowledgment is made to Mr. W. F. Ryan for valuable suggestions. The debt to authors of previous publications on the subject is obvious.

Discussion

A. Bantlin.[2] It would appear from the paper that along with many other important questions which have to be taken into consideration in the design of steam piping, the author considered the question of change of shape and stresses in the elastic pipe bends. I have already dealt with this question in the *Zeitschrift des Vereines deutscher Ingenieure*, 1910, p. 43 (compare also Forschungsheft No. 96). The tests on which that investigation was based were carried out with funds supplied by the V.D.I. In evaluating these tests, begun nearly 20 years ago, with the calculating methods then available, it is important not to forget that the investigation for the first time of a question as involved as that of highly curved pipe bends was very difficult. Not even then were equations for the change of shape of curved pipes established which would give values conforming to those obtained by actual measurements. The present paper corroborates my experience that the solution of a difficult scientific problem does not result from the first effort but from repeated endeavors. I came to this conclusion two years ago with respect to Professor Hovgaard's paper on "The Elastic Deformation of Pipe Bends," which came to my attention at that time.

Since my first investigation and report a number of German, English, French, and American investigators [compare (*B*) to (*K*), given in this paper] have successively dealt with the important problems which it involved, which is only natural if it is remembered that in the meantime the steam pressures and temperatures employed have greatly increased. Both of these factors have had a great influence on the design of complete piping installations as well as on the material and design of individual elements of the piping system. The renewed interest in elastic expansion members, an important element in piping systems, is therefore entirely justified. The mathematical treatment of the subject, as the author states, is quite a complicated matter, and, as any one who dealt with it knows, calls for a thorough knowledge of the matters dealt with; yet, in spite of the skill which has been applied, mathematical treatment has not given a specific solution of the problem. Notwithstanding the fact that the mathematical analysis has been to some extent simplified by various workers, among them the author himself, through the employment of curves and tables, these investigations, though they give a certain insight into the forces acting and the deformations produced thereby and indicate the measures which the engineer must employ in designing expansion equalization members and as a matter of fact the entire piping, still the fundamental problem has not been solved, namely, that of the actual as opposed to the calculated stressing of the pipe bends. As regards these actual stresses even today we are not yet in a position to draw reliable conclusions.

I might express the conviction that further extension of our knowledge regarding the determination of the actual magnitudes of the stresses must be expected mainly from further experimental work in which direct measurements of the actua! stresses in the bodies under test will be made. Instruments now available permit such direct measurement of stresses, and obviously a great step forward will be taken when it becomes possible to determine with certainty by test the true values of the maximum stress.

Such tests should be carried out only on pipe bends and piping of such dimensions and shapes as are actually used in commercial engineering, which means that none of these tests should be made on models.

I do not expect much from tests on models. In fact, the actual picture is likely to be falsified by such tests; that is, the results will never be exactly such as to make it possible to transfer reliably the data obtained from model tests to the design and construction of equipment for actual use. Tests of full-size commercial bends and piping require the expenditure of considerable sums, but are justified because of the valuable information which they afford designers and engineers.

Andrew A. Bato.[3] There has been written on the problem of temperature stresses in pipe lines material enough to fill a book; nevertheless the author has succeeded in summarizing what has been done in the past, and has also presented the results of his own work at the same time, the latter being a most valuable contribution to the theoretical as well as the practical handling of the design of steam pipe lines. This contribution is the theorem that the distance of the "line of zero moments" from an end is proportional to the fixity of that end, if the other end is hinged; also the application of the theorem to cases in which both ends are only partly fixed. To prove this theorem the author performed 44 series of experiments on 10 models, with 34 series out of the 44 on partly fixed models, all of them proving very closely the correctness and the usability of the theorem. By "fixity" the author designated what might be called "angular fixity." A further step will evidently be the introduction of "linear fixity," and the combination and influence of the two on each other. A good example of the connection between the two fixities is that of a pipe connected at one end to a main header. If the header is supported in such a manner that it will not bend nor yield to the linear expansion of the pipe line, the end of the latter will have a greater tendency to twist the header, as in the case in which the header can be twisted as well as bent.

Table 1 giving integrals for the various sections of pipe lines evidently also satisfies a demand felt in other quarters, as a proof of which the writer may mention that he too prepared a similar

[2] Professor, School of Technology, Stuttgart, Germany.

[3] Consulting Engineer, East Orange, N. J. Mem. A.S.M.E.

table while working for a public-utility corporation. The experience obtained at that time led him to point out that it should be impressed upon the minds of practical piping designers that this table has been derived by using the same methods, and is therefore just as reliable and is to be used in the same way as, for instance, the table in handbooks for the various cases of supported beams carrying loads.

In this connection it may be pointed out that the problem of temperature stresses has been worked out very elaborately for steel and concrete bridges, for simple arches as well as combinations resting on high columns, so that the bending of the latter has to be considered too, a case analogous to pipe lines with branches. The finest and most complete presentation of these problems in the American and English literature can be found in the books of Hool and Kinney on building structures. The circumstance that the temperatures to be considered in building structures are much lower than those occurring in pipe lines makes no difference in the mathematical methods. The loads in the case of bridges correspond to the weights of the pipe lines and their contents as well as the reactions of hangers, anchors, etc. The equations given in Part III, Section G, of the paper evidently cover these questions too, although the fact is not mentioned in the text.

In connection with the problem of the three-dimensional pipe line, that is, a pipe line in more than one plane, the writer made an experiment that in many cases may lead to a great reduction in the "very large amount of labor," mentioned by the author when dealing with this problem. As an example he took a pipe line consisting of three straight sections of different lengths, at right angles to each other, similar to that shown in Fig. 10 *c*. The case was assumed to be that of a pipe line with both ends fixed in every respect, angular and linear, and expanding under the influence of temperature. Six equations were developed, three corresponding to Equations [7] and [8] of the paper, that is, giving the linear deflections at one end as functions of the forces and moments acting at that end and of the elastic qualities of the pipe line; and three corresponding to Equation [9] of the paper, giving the three angular deflections at the same end of the pipe line in the three planes of the coordinate system, all set equal to zero, but otherwise on the same principle as the first three equations giving the linear deflections. The calculations naturally included the torsional deformations. The fact that pipes of a circular cross-section were assumed simplified the calculation somewhat, as for this case the polar moment of inertia to be considered for torsion is equal to twice the moment of inertia figuring in the bending. Furthermore the modulus of elasticity for shear used in calculating torsion is a fraction of the modulus for tension, consequently the members for torsion in the equations could be handled together with those for bending. The check that the calculations were correct was found in the fact that the resultant of the three forces was in the same plane as the resultant of the three moments. A little reflection shows that this could not be otherwise. The interesting fact about the result was that when the resulting force was combined with the resulting moment, which was made possible by the fact that they were in the same plane, the force thus shifted passed through the center of gravity of the pipe line. If it can be proved mathematically for every case that the resulting force, even when causing torsion, passes through the center of gravity or elastic center if both ends are fixed in every way, the work of calculating would be reduced to the solution of three equations.

A special merit of the paper lies in the thorough study of what has been done in the past, as a consequence of which the bibliography as known to the American public heretofore has been enriched by three works of fundamental importance.

The writer has not checked the formulas contained in the paper, partly because of insufficient time and partly because they had been checked as stated by the author.

The examples shown by the author seem to indicate that in most cases provisions for expansion in the general layout of the pipe line are preferable to the application of expansion bends. Nevertheless, even if expansion bends will be done away with, ordinary 90-deg. and 45-deg. elbows and bends will always be used in great numbers. A knowledge of the factor K for these bends is of a great importance in the design of pipe lines. As the value of this factor depends on the various methods of manufacturing it would be natural that the manufacturers would determine these values by experiments and publish them in their catalogs. Instead of doing this they have displayed lately, great indifference as to what has been going on in the development in the more correct methods of design, and all that has been produced and published has been due to the efforts and enthusiasm of those using pipe lines or designing them for their own use. These are the men who do the pioneering work, and the author has secured for himself the place of a leader with his work which he described in the paper.

HENRI CARLIER.[4] I am very much interested in the very fine work which the author has done, and it gives me real pleasure to make this humble contribution to its discussion.

GENERAL CONSIDERATIONS

In steam-piping installations it is advisable to take local conditions into consideration in determining a layout that will give sufficient flexibility. I agree with the author that so-called expansion bends should be used only under exceptional conditions. The flexibility of these devices is insignificant in comparison with that which can be obtained by considering the layout as a whole.

The supports and anchors can be determined only after a study of the layout. They should be designed so as to allow all movements resulting from the elastic deformation of the piping.

As to the problem of joints I cannot quite agree with the explanation given in the paper. The joints must be constructed so as to give perfect rigidity. The number of bolts must be sufficiently great. Joint leaks cannot occur then.

In other words, the joints should be tightened to such an extent that the stresses due to expansion will not further compress the packing material (metallic or metallo-plastic joint).

As far as possible pipe joints should be made near the inflection points (where the bending moment = zero). It will always be advantageous to utilize the elastic properties of the piping in the cold state by shortening it during erection, an amount which should be less than one-half of the expansion.

The author's solution of the problem of the flattening of curved parts does not appear to me quite correct.

In making a bend a so-called "ovalization" results, and during bending tests it will therefore pay to verify from point to point the various sections and to measure later the changes in shape of successive sections at the time when the bending stresses come into effect. In the expansion formula (elastic deformation) $\Delta = A/EI$, the decrease in I due to flattening corresponds to an increase in elastic flexibility.

The value of I depends on the care exercised in manufacturing. The bend may be more or less perfectly made (more or less irregular, with flat parts present).

On the other hand, if we consider a well-made and regular bend, we have the right to say that flattening under the effect of bending will occur at a point A, Fig. 23, near the point of tangency, and once this flattening has occurred there is no

[4] Engineer, Ressaix (Hainaut), Belgium.

reason why it should not be accentuated, if the expansion effects demand it. Therefore, as the writer sees it, the use of a coefficient $K = \varphi(\lambda)$ in which $\lambda = tR/r^2$ will not be sufficient to rectify the theoretical formulas.

To sum up the foregoing, the writer would recommend:

1 The use of thicker tubes.

2 The use of large radii of curvature (from 6 to 10 diameters). In this manner the tendency to ovalization at the bends will be reduced, and as a consequence uncertainties in computed results will be eliminated.

Fixing the ends of the piping diminishes the flexibility. Computations, however, should be made on the basis of thorough fixedness. If this is not entirely realized in practice, the flexibility will be greater than that computed and the safety correspondingly greater.

Method of Computation

The method of computation presented by the author is very elegant, but it seems to me difficult to derive therefrom formulas and construct diagrams for the use of designing draftsmen. In practice it is seldom possible to apply directly and simply formulas of elastic deformation derived from various well-determined cases. The conditions of the problem generally require a complete special analysis of the plans. In certain horizontal stretches following after vertical ones it is necessary to introduce bending due to the weight of the former. It is necessary, then, that those in charge of the design should have a perfect knowledge of the analytic theory.

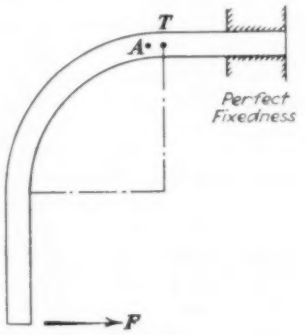


Fig. 23

In making the computations one must not make use of too optimistic hypotheses in order to protect oneself against surprises. On the other hand, it is less important to arrive, say, at the result that the intensity of stress S in the metal of the pipe is 3.755 kg. per sq. mm. (5340 lb. per sq. in.) than to the fact that $S<4$ kg. (5688 lb. per sq. in.).

The exact value to be assigned to E for high temperatures should be determined. Without having the exact data at hand, nevertheless I am under the impression that tests were made at a temperature of 450 deg. cent. (842 deg. fahr.) and that E was found to equal 16,000 (22,750,000) instead of 20,000 (28,500,000). The reduction in the value of E corresponds to an increase in flexibility. It is to be noted here, however, as the author says, that the elastic limit decreases with temperature and that, in consequence of this, the allowable stress S should be reduced in case of high pressures and high superheat temperatures.

Some Considerations as to the Installation of Steam Piping Systems

In spite of superheating, when the turbines stop running the steam condenses in the piping. To avoid the nuisance of trapped water during starting up, it is necessary to have the piping constructed with sufficient slope from the boilers to the turbines, so as to reduce the effect of low points forming pockets where the water due to condensed steam may accumulate.

In connection with this, a calculation of the sag produced by bending due to the weight of the piping (between two supports) will not be devoid of interest.

Let us consider a horizontal pipe resting on uniformly spaced supports of span A (Fig. 24a).

Let p be the weight of pipe per running meter.

At the middle of each span there will be produced a sag $f = \dfrac{pa^4}{384EI}$ with $\mu = \dfrac{pa^2}{12}$ and $\mu_M = \dfrac{pa^2}{24}$.

The spacing of the supports plays an important part. Indeed, let us suppose that for certain local reasons it has been decided to decrease the distance between certain supports (Fig. 24b) by shortening certain spans (i.e., $b>a$).

The conditions of resistance to bending can be improved while the sag will be found materially increased. Let us take the specific case defined by $b = {}^1/_2a$.

Let us designate the moments at A, B, and C, by μ_A, μ_B, and μ_C.

The theorem of three moments gives

$$\mu_A a + 2\mu_B(a + b) + \mu_c b = {}^1/_4 p\,(a^3 + b^3)$$

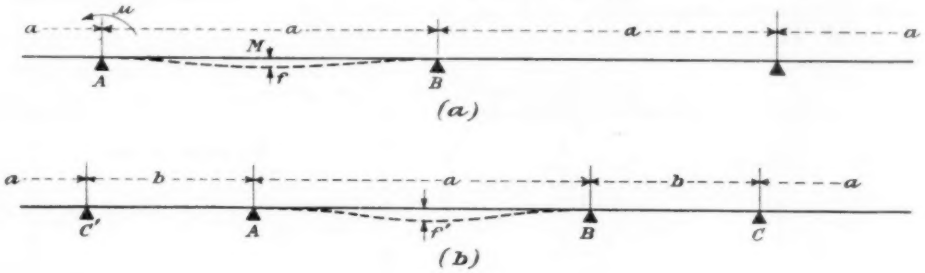


Fig. 24

but since $\mu_A = \mu_B = \mu_C = \mu$ and $b = {}^1/_2a$, therefore

$$\mu' = \frac{pa^2}{16}$$

Designating by $R = pa/2$ the part of the reaction $p(a + b)/2$ at A producing the deformation of AB, we have for AB (see Fig. 25),

$$\frac{d^2y}{dx^2} = \frac{\mu - p\,(a/2)x + px^2/2}{EI}$$

Integrating,

$$\frac{dy}{dx} = \frac{\mu x - (pax^2)/4 + px^3/6}{EI} + C$$

For $x = a/2$, $dy/dx = 0$ and $C = -\dfrac{\mu a/2 - pa^3/16 + pa^3/48}{EI}$

$$= \frac{pa^2/16 \times a/2 - pa^3/16 + pa^3/48}{EI}$$

$$= \frac{pa^3/96}{EI}$$

whence

$$\frac{dy}{dx} = \frac{\mu x - pax^2/4 + px^3/6 + pa^3/6}{EI}$$

and

$$y = \frac{\mu x^2/2 - pax^2/12 + px^4/24 + pa^3x/96}{EI} \text{ with } C' = 0$$

y is a maximum for $x = a/2$, and

$$y_{max} \text{ or } f' = \frac{(pa^2/16) \times (a^2/8) - pa^4/(12 \times 8) + pa^4/384 + pa^4/192}{EI}$$

$$= \frac{pa^4}{192EI}$$

we see that $\mu = pa^2/16 < pa^2/12$, and that $f^1 = pa^4/192\,EI = 2f$.

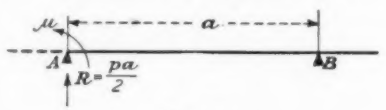


FIG. 25

These considerations suffice to demonstrate the importance of investigating the spacing of the supports.

Unsupported bends require a specially careful analysis also for the purpose of avoiding low points.

If one confines himself to the problem of resistance, it will be sufficient to impose the condition $\mu_C = \mu_A$, the tangents at B and B' (Fig. 26) being horizontal, and to seek for relations connecting a, b, and c.

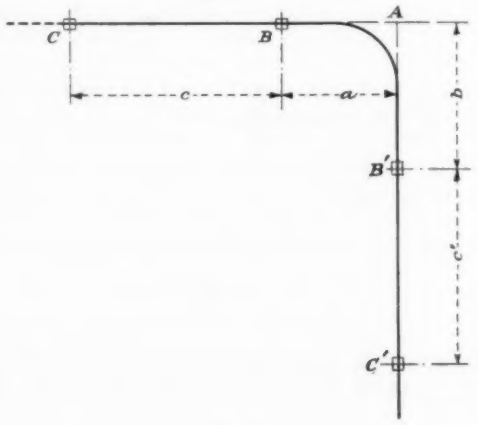


FIG. 26

The condition $a > b$ leads us to consider a stress P at the end of a and a reaction equal to P at the end of b.

The connection between a and b brings about fixedness at A (μ_A). Under the influence of μ_A, b is subjected to torsion.

The differential equation of the deformed length a (Fig. 27) is:

$$\frac{d^2y}{dx^2} = \frac{\mu_B - Rx + px^2/2}{EI}$$

Whence

$$\frac{dy}{dx} = \frac{\mu_B x - Rx^2/2 + Px^3/6}{EI} + C$$

($C = 0$ if we assume that at B, $dy/dx = 0$.)
For $x = a$,

$$\frac{dy}{dx} = \frac{\mu_B a - Ra^2/2 + pa^3/6}{EI}$$

Now at A the section suffers rotation caused by the torsion of the branch $b = \frac{5}{4}\frac{\mu_A(b-r)}{EI}$ in limiting b to the right-hand portion $b - r$; whence the equation

$$\mu_B a = -R\frac{a^2}{2} + \frac{pa^3}{6} = -\frac{5}{4}\mu_A (b - r)$$

μ_A takes its negative sign with respect to μ_A with the positive sign.

Analogous reasoning with reference to the branch b gives (P changes in direction—see Fig. 28):

$$Q = pb - P$$

$$\mu'_B b - Q\frac{b^2}{2} + \frac{pb^3}{6} = -\frac{5}{4}\mu'_A(a - r)$$

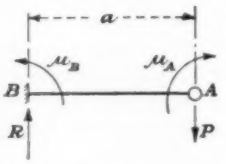


FIG. 27

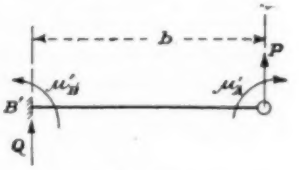


FIG. 28

The moment equations are as follows:

$$\mu_B - Ra + pa^2/2 - \mu_A = 0$$

$$\mu'_B - Qb + pb^2/2 - \mu'_A = 0$$

$$\mu_B = 1/12pc^2 \text{ and } \mu'_B = pc'^2/12;\ C > < C'$$

We finally obtain:

$$\frac{1}{12}pc^2a - (pa + P)\frac{a^3}{2} + \frac{pa^3}{6} = -\frac{5}{4}\mu'_A(b - r)$$

$$\frac{1}{12}pc'^2b - (pb - P)\frac{b^2}{2} + \frac{pb^3}{6} = -\frac{5}{4}\mu'_A(a - r)$$

$$\frac{1}{12}pc^2 - (pa + P)a + \frac{pa^2}{6} - \mu_A = 0$$

$$\frac{1}{12}pc'^2 - (pb - P)b + \frac{pb^2}{2} - \mu'_A = 0$$

The unknowns are μ_A, μ'_A, P, a, and b.

Eliminating μ_A, μ'_A, and P, there will remain an equation of a relation connecting a and b.

Under the above conditions ($dy/dx = 0$ for B and B'), what will be the value of vertical displacement of fictitious point A?

To solve this problem, a and b are imposed. The above equations permit determining μ_A and μ'_A.

The elastic equation of deformed part a is

$$\frac{d^2y}{dx^2} = \frac{\mu_B - Rx + px^2/2}{EI}$$

$$\frac{dy}{dx} = \frac{\mu_B x - Rx^2/2 + px^3/6}{EI}$$

$$dy = \frac{\mu_B x^2/2 - Rx^3/6 + px^4/24}{EI}$$

$$f_A = \frac{\mu_B a^2/2 - Ra^3/6 + pa^4/24}{EI}$$

NOTE. If either C or C' or both of these spacings, other conditions remaining unchanged, are decreased, the conditions of resistance will not be found materially modified, but the equili-

brium of moments will rapidly cause the descent of point A, manifestly creating a low point.

The section B and B', or both, are turned through an angle α, and, other conditions remained unchanged, point A consequently descends a supplementary distance αa. It will be advisable to give the pipes a slope as shown in Fig. 29. Here J and K are joint flanges. For the drilling of bolt holes, it should be noted that flange K must be turned with reference to J through an angle corresponding to the inclination of branch AB'.

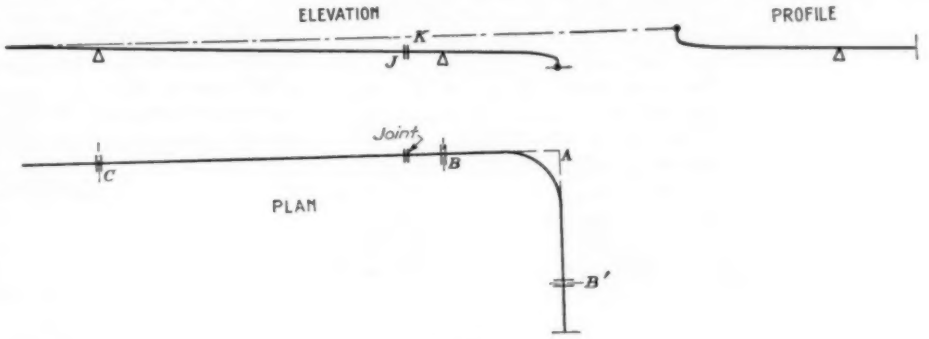


FIG. 29

SPECIAL SUPPORTS

Figs. 30–32 show three types of support. Fig. 30 shows a support (suspension) for a horizontal pipe at a point relatively close to the vertical branch $a > h\alpha T$ so that the pipe will still have a slope after the expansion of the vertical branch; α is the coefficient of expansion. The spring (shown enlarged at the right of Fig. 30) must be designed to have great flexibility in order to absorb the expansion and balance it against the dead weight suspended.

Fig. 31 shows a swivel-joint support for the vertical branch of a piping system having five branches, a, b, c d, and h. The swivel joint permits:

1 Torsion of branch h under thrust of branches a and b.

2 Rotation with bending in plane MN under thrust of $b(b>a)$ and at the same time in a plane resulting from the effect of the expansion of c and d. a and b may not be in the same plane; in this case a and b do not afford the same flexibility for absorbing the expansion of branches c and d.

Fig. 32 shows a support for a horizontal pipe which permits rotation in every direction.

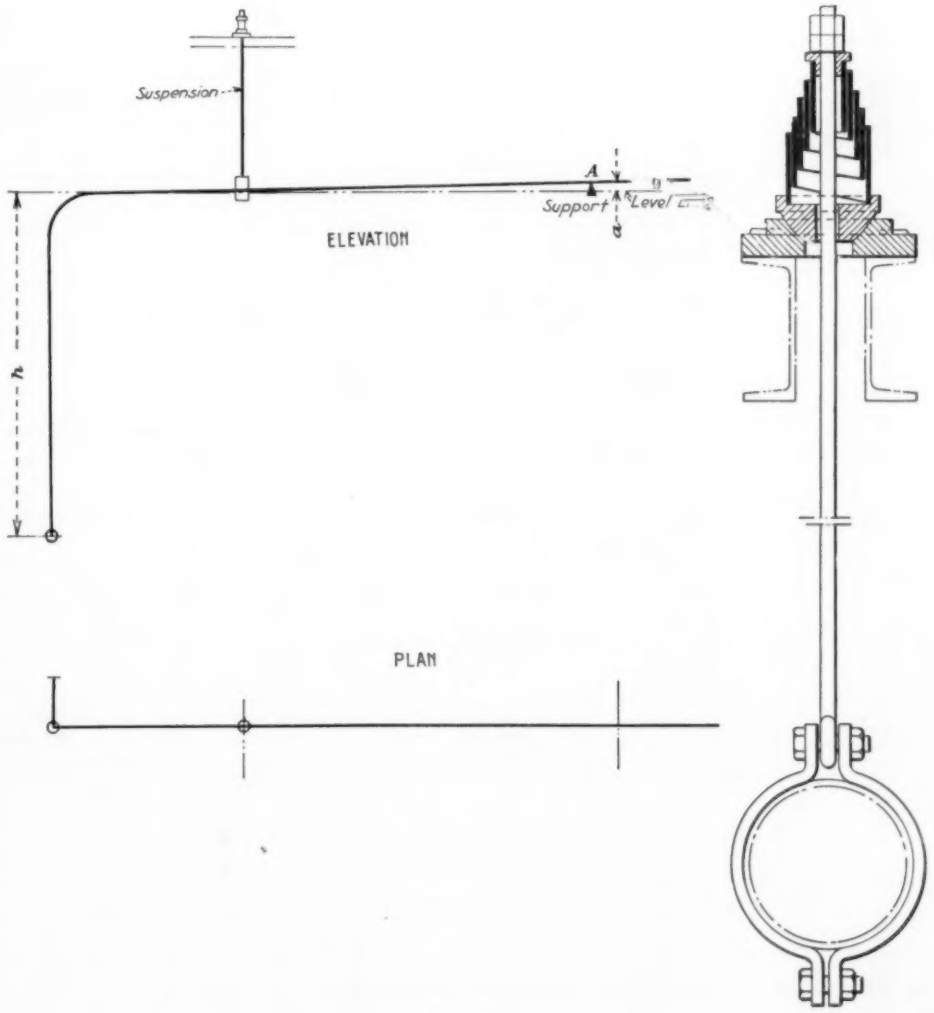


FIG. 30

LETTER FROM HENRI CARLIER

I have received and studied the very interesting paper which you presented at the Annual Meeting in New York last December.[5] Difficulties which I encountered in translating it and also the short time between the Annual Meeting and the time I received your letter prevented me from entering into the discussion to the extent that I desired, nevertheless I sent to the meeting a brief communication of which you are undoubtedly aware. In that discussion I stated that it seemed to me to be difficult to work out a method that would be useful to designers. I would now recall that statement and inform you that I have succeeded in finding a simple method which permits solving all the curves considered in my 1925 book. You are of course aware of the enormous amount of work which I have done in the last few years in connection with the study of elastic deformation of steam piping. Within the last few months I have studied Mr. Marbec's method and have come to utilize the single point

[5] Letter from Mr. Carlier to the author dated March 7, 1929.

and the central force (resultant of internal elastic force) but without finding it necessary to look for the axis and the *principal* moments of inertia.

The different formulas of my 1925 book, with the exception of formulas [55], [58], [66], and [72], have been checked by the new method. This applies to such cases as that shown in

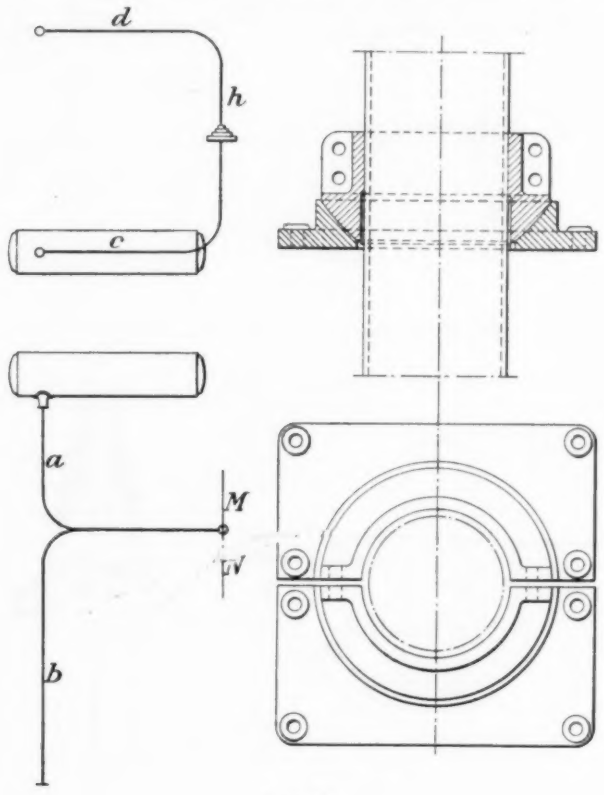


FIG. 31

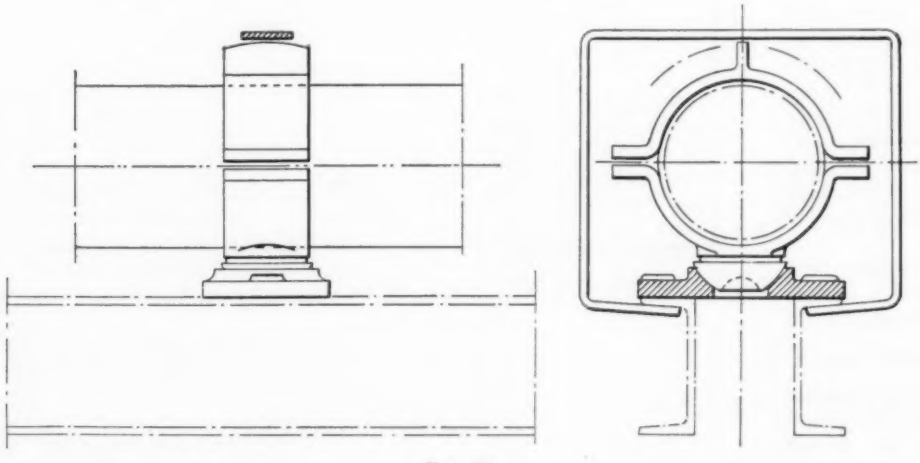


FIG. 32

Fig. 33 in which h is smaller than the sum of the transverse elements.

Integration leads to the formulas referred to above, but the general formula

$$\Delta y = \int (x_1 - x)\frac{M}{EI}\,ds$$

cannot be applied here because the branch C reenters into the curve.

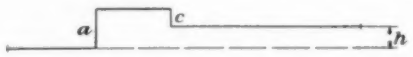


FIG. 33

As to what I said in reference to formula [56], p. 93, Fig. 52, of my book of 1925, let us take the following example, formula [64] (see Fig. 34):

Formula [66] gives $106.09\,P' - 28.75\,P = h\alpha T$. This expression is erroneous. My new method gives directly

$$\frac{1}{EI}(109.08\,P' - 7.75\,P) = h\alpha T$$

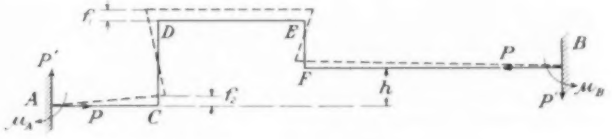


FIG. 34

and I have verified it by the Marbec method. Moreover it can be verified by the following analytic solution:

Referring to Fig. 35, let us consider separately

1 The elastic system AC, in which the transverse displacement of $C = f_2$, and

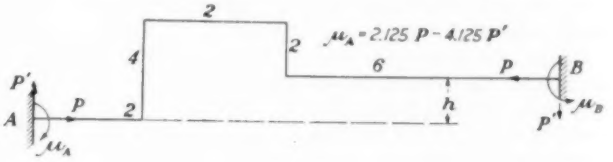


FIG. 35

2 The system $BFED$ in which the transverse displacement of $D = f_1$.

$$f_1 - f_2 = h\alpha T$$

$$f_2 = \frac{1}{EI}(4.25P + 6.91P') \quad f_1 = \frac{1}{EI}(102.17P' - 3.5P)$$

$$f_1 - f_2 = \frac{1}{EI}(102.17P' - 3.5P - 4.25P - 6.91P'), \text{ and}$$

$$\frac{1}{EI}(109.08P' - 7.75P) = h\alpha T$$

I have thought that considering the value of the results of my investigation, it should be possible for me to obtain some return therefor. I am now engaged in revising my new notes. These latter will not be published, but a limited number of copies will be reproduced for the use of Belgian and French designers to whom I shall offer my method and for a certain subscription price. Possibly there may be engineers in the United States who would like to avail themselves of the same opportunity. In this new work I show that the central point is the center of gravity of the curves in the case of piping fixed at both ends. I also show how the central point is displaced when one of the extremities turns through a certain angle (case studied in my pamphlet published in 1927).

ELBERT C. FISHER. With the high pressures and temperatures now being used in modern practice, it is very essential that the piping system should be designed so that the anchors may be reduced to the minimum, and that the system be made as flexible as possible without imposing excessive strains at any point in the system.

With the amount of theoretical data, as well as the results of tests that have been made in the last ten years and presented before the Society at various times, the proper design of a piping system, based on the above data, must be worked out by an engineer well acquainted with this subject, as well as equipped with a good knowledge of mathematics.

The author's paper makes his theoretical analysis available for the practical problems encountered in design of piping systems.

[6] Engineer, Design, Public Service Production Co., Newark, N. J. Mem. A.S.M.E.

TABLE 5 SUMMARY OF GENERAL AND SPECIAL FORMULAS FOR THE SIMPLE AND THE KARMAN BENDING THEORY FOR THE MAXIMUM VALUE. (PIPE FIXED AT ONE END AND FREE AT THE OTHER END)

Case	Value: Required	Value: Given	General formulas Karman bending theory	Karman correction factor based on Karman bending theory	Special formulas for bends consisting of an arc or arcs Karman bending theory
A	Deflection	Thrust	$\Delta_K = K_H \frac{H}{EI}\int y^2 ds$	$K_H = \frac{10 + 12\varphi^2}{1 + 12\varphi^2}$ (a) φ = 0.3 to 1.5	$\Delta_K = K_H C_H \left(\frac{H}{EI}\right) R^3$
B	Deflection	Maximum longitudinal bending stress	$\Delta_K = K_S \left(\frac{f_b}{E}\right)\frac{2}{d_1}\frac{\int y^2 ds}{y_{max.}}$	$K_s = \frac{9}{\sqrt{10 + 12\varphi^2}}$ (b) $\varphi > 1.5$ $K_S = \frac{5 + 6\varphi^2}{6\varphi^2 - 1}$	$\Delta_K = 2K_S C_S \left(\frac{f_b}{E}\right)\left(\frac{R^2}{d_1}\right)$
C	Thrust	Maximum longitudinal bending stress	$H_K = 2K_T \left(\frac{I}{d_1}\right)\frac{f_b}{y_{max.}}$	(a) φ = 0.3 to 1.5 $K_T = \left(\frac{1 + 12\varphi^2}{10 + 12\varphi^2}\right)\left(\frac{9}{\sqrt{10 + 12\varphi^2}}\right)$ (b) $\varphi > 1.5$ $K_T = \left(\frac{1 + 12\varphi^2}{10 + 12\varphi^2}\right)\left(\frac{5 + 6\varphi^2}{6\varphi^2 - 1}\right)$	$H_K = 2K_T C_T \left(\frac{I}{Rd_1}\right) f_b$
D	Maximum longitudinal bending stress	Thrust	$f_{bK} = \frac{H}{2K_T}\left(\frac{d_1}{I}\right) y_{max.}$	(a) φ = 0.3 to 1.5 $\frac{1}{K_T} = \left(\frac{10 + 12\varphi^2}{1 + 12\varphi^2}\right)\left(\frac{\sqrt{10 + 12\varphi^2}}{9}\right)$ (b) $\varphi > 1.5$ $\frac{1}{K_T} = \left(\frac{10 + 12\varphi^2}{1 + 12\varphi^2}\right)\left(\frac{6\varphi^2 - 1}{5 + 6\varphi^2}\right)$	$f_{bK} = \frac{H}{2K_T C_T}\left(\frac{Rd_1}{I}\right)$
E	Maximum transverse bending stress	Thrust	At top of O. S. Dia. $[f_{bK}]_{t_1} = \frac{[\psi_{r_1}]_t}{[K_T]_{to}}\frac{H}{2}\left(\frac{d_1}{I}\right) y_{max.}$	$\frac{1}{[K_T]_{to}} = \frac{3}{4\varphi}\frac{24\varphi^2 + 1}{12\varphi^2 + 1}$ $[\psi_{r_1}]_t = \left[1 - 2\left(\frac{1}{Kr_1}\right)^2\right]$	$[f_{bK}]_{t_1} = \frac{[\psi_{r_1}]_t}{[K_T]_{to}}\frac{H}{2C_T}\left(\frac{Rd_1}{I}\right)$
F	Maximum combined bending stress	Thrust	At top of O. S. Dia. $[f_{bK}]_{G_1} = \frac{1}{[K_T]_G}\frac{H}{2}\left(\frac{d_1}{I}\right) y_{max.}$	$\varphi > 0.35_{+1}$ $\frac{1}{[K_T]_G} = 1 + \frac{18\varphi - 3\left(1 - \frac{1}{4\varphi}\right)}{1 + 12\varphi^2}$	$[f_{bK}]_{G_1} = \frac{1}{[K_T]_G}\frac{H}{2C_T}\left(\frac{Rd_1}{I}\right)$

TABLE 6 KARMAN CORRECTION FACTORS FOR 4-INCH TO 20-INCH PIPE FACTORS BASED ON 6-INCH PIPE DIAMETERS FOR BENDING RADIUS (R)

Pipe size	d_1	d	r	r_1	δ	I	R	φ	K_H	0.3 to 1.5φ K_S	>1.5φ K_S	0.3 to 1.5φ K_T	>1.5φ K_T	0.3 to 1.5φ $\frac{1}{K_T}$	>1.5φ $\frac{1}{K_T}$	$\left[\frac{1}{K_T}\right]_{t_0}$	$[\psi_{r_1}]_t$	$\left[\frac{1}{K_T}\right]_G$
ST4	4.5	4.026	2.131	2.25	0.237	7.233	24	1.252	1.454	1.675		1.151		0.867		1.168	—1.226	1.017
XH4	4.5	3.826	2.08	2.25	0.337	9.61	24	1.869	1.209		1.30		1.074		0.928	0.792	—1.336	0.723
XXH4	4.5	3.152	1.913	2.25	0.674	15.28	24	4.419	1.038		1.050		1.011		0.986	0.337	—1.766	0.326
ST5	5.563	5.047	2.652	2.781	0.258	15.16	30	1.10	1.579	1.817		1.148		0.868		1.317	—1.200	1.126
XH5	5.563	4.813	2.594	2.781	0.375	20.67	30	1.671	1.260		1.380		1.094		0.912	0.885	—1.298	0.798
XXH5	5.563	4.063	2.406	2.781	0.75	23.63	30	3.886	1.050		1.066		1.015		0.984	0.385	—1.668	0.378
ST6	6.625	6.065	3.172	3.312	0.28	28.14	36	1.001	1.691	1.918		1.133		0.881		1.442	—1.180	1.212
XH6	6.625	5.761	3.096	3.312	0.432	40.19	36	1.622	1.276		1.40		1.096		0.907	0.910	—1.286	0.818
XXH6	6.625	4.891	2.88	3.312	0.864	66.33	36	3.749	1.053		1.072		1.017		0.981	0.399	—1.644	0.381
ST7	7.625	7.023	3.662	3.812	0.301	46.52	42	0.942	1.773	1.981		1.115		0.894		1.523	—1.162	1.267
XH7	7.625	6.625	3.562	3.812	0.50	71.37	42	1.655	1.265		1.388		1.096		0.911	0.892	—1.290	0.804
XXH7	7.625	5.875	3.375	3.812	0.875	107.5	42	3.226	1.071		1.097		1.023		0.976	0.462	—1.548	0.439
ST8	8.625	8.071	4.174	4.312	0.277	63.35	48	0.762	2.128	2.184		1.024		0.972		1.844	—1.134	1.468
ST8	8.625	7.981	4.151	4.312	0.322	72.49	48	0.896	1.846	2.029		1.098		0.908		1.595	—1.154	1.312
XH8	8.625	7.625	4.062	4.312	0.50	105.7	48	1.454	1.341	1.512		1.126		0.886		1.0104	—1.252	0.898
XXH8	8.625	6.875	3.875	4.312	0.875	162.0	48	2.797	1.094		1.130		1.032		0.967	0.533	—1.472	0.502
ST9	9.625	8.941	4.641	4.812	0.342	107.6	56	0.859	1.913	2.072		1.081		0.922		1.657	—1.146	1.352
XH9	9.625	8.625	4.562	4.812	0.50	149.6	56	1.297	1.424	1.638		1.148		0.869		1.128	—1.222	0.988
ST10	10.75	10.192	5.235	5.375	0.279	125.9	60	0.610	2.647	2.367		0.892		1.117		2.231	—1.106	1.685
ST10	10.75	10.136	5.221	5.375	0.307	137.4	60	0.675	2.391	2.287		0.956		1.045		2.050	—1.118	1.585
ST10	10.75	10.02	5.192	5.375	0.365	160.7	60	0.812	2.010	2.126		1.057		0.945		1.762	—1.142	1.408
XH10	10.75	9.75	5.125	5.375	0.50	212.0	60	1.142	1.540	1.778		1.153		0.865		1.272	—1.196	1.095
ST11	11.75	11.0	5.687	5.875	0.375	217.0	66	0.765	2.122	2.183		1.028		0.514		1.837	—1.134	1.465
XH11	11.75	10.75	5.625	5.875	0.50	280.1	66	1.044	1.477	1.704		1.159		0.865		1.399	—1.180	0.877
ST12	12.75	12.09	6.21	6.375	0.330	248.5	72	0.616	2.611	2.356		0.898		1.107		2.215	—1.106	1.677
ST12	12.75	12.0	6.187	6.375	0.375	279.3	72	0.705	2.292	2.522		0.980		1.017		1.973	—1.126	1.544
XH12	12.75	11.75	6.125	6.375	0.50	361.5	72	0.959	1.747	1.962		1.122		0.889		1.498	—1.162	1.249
14	13	14	6.75	7	0.50	483.8	84	0.922	1.804	2.000		1.108		0.902		1.554	—1.15	1.286
16	15	16	7.75	8	0.50	731.9	96	0.799	2.039	2.143		1.0495		0.952		1.769	—1.131	1.429
18	16.750	18	8.6875	9	0.625	1289.0	108	0.895	1.845	2.031		1.098		0.910		1.599	—1.146	1.314
20	18.750	20	9.6875	10	0.625	1787.0	120	0.799	2.039	2.143		1.0495		0.952		1.769	—1.064	1.429

Fig. 16, for finding bending moments for various bends with fixed ends, is not of much value in power-house piping design, but would be of greater value in finding the stress in flanges for short-length bends connecting various parts on the same piece of apparatus, or sometimes where the connections between pieces of apparatus are placed closely together.

With the type of flanges used in the present design for high-pressure service, the stresses in the flanges are usually very small; that is, if the piping layout is properly designed.

The writer has taken a considerable interest in the articles regarding the flexibility of pipe that have appeared from time to time. However, the four articles by J. R. Finniecome published in *The Engineer* (London), August 17, 24, and 31 and September 7, 1928, seem to have covered this problem in a most thorough and practical manner.

Mr. Finniecome, after a thorough investigation, has adopted Karman's theory and in the last article he gave a table of formulas which also includes the Karman correction formulas. These formulas, given herewith in Table 5, are such that a power-house designer can easily determine the thrust and strain which will occur in a power-house piping system.

If, in some cases, Mr. Finniecome's analysis should slightly disagree with the exact theory, the writer nevertheless believes it could be considered as close enough for all practical purposes for checking any piping system.

In order to simplify Mr. Finniecome's analysis, the writer has prepared Table 6, which gives Karman's correction factors to be used for the different standard sizes of pipe ranging from 4 in. to 20 in., and based upon a radius of six pipe diameters for bends.

Δ_K = Karman deflection.
K_H = Karman deflection, thrust correction factor.
H = force producing deflection due to temperature change.
$\varphi = \delta R/r^2$ = pipe factor.
K_s = Karman deflection-stress factor.
f_b = theoretical longitudinal bending stress due to thrust.
$1/K_T$ = Karman stress-thrust correction factor.
K_T = Karman thrust-stress factor.
H_K = Karman thrust.
f_{bK} = Karman longitudinal stress.
$[f_{bK}]_{l_1}$ = transverse stress at top of outside diameter of cross-section for y_{max}.
$[\psi r_1]_l$ = Z—transverse stress factor for $Z = r_1$.
Z = distance from neutral axis.
$1/[K_T]_{l_o}$ = ratio of transverse bending stress at neutral axis to theoretical longitudinal stress.
$[f_{bK}]_{G_1}$ = combined Karman stress at top of outside diameter of cross-section.
y_{max} = maximum normal distance of arc element from direction of thrust.
R = radius of pipe bend.
d_1 = outside diameter of pipe.
d = bore of pipe.
r = mean radius of cross-section.
δ = wall thickness of pipe.
I = moment of inertia.

$$\left.\begin{array}{l}\text{Theoretical deflection thrust factor} = C_H = \dfrac{\int y^2 ds}{R^3} \\ \text{Theoretical deflection stress factor} = C_S = \dfrac{\int y^2 ds}{y_{max} R^2}\end{array}\right\} \text{for bends consisting of an arc or a number of arcs of the same mean radius}$$

$C_T = C_S/C_H$ = theoretical thrust stress factor.

$1/[K_T]_G$ = ratio of Karman combined stress to theoretical longitudinal stress for $z = r$.

W. Hovgaard.[7] The paper covers practically the whole field of the design of pipe bends, and should prove of great value to practical engineers.

[7] Professor of Naval Design and Construction, Massachusetts Institute of Technology, Cambridge, Mass.

Fundamental equations for the calculation of deflections, moments, and reactions in plane pipe lines are given and are applied to a great number of specific cases. The value of the integrals which enter into the calculations are tabulated, but the formulas given for the more general cases on the sixth and seventh pages seem rather cumbrous, and it is believed preferable there to use graphical methods. It may not be out of place to remark that the same problem is discussed in a general way in a paper just published by the writer.[8]

In the discussion of stresses in ductile materials, the author gives preference to the formulas of Professor Lorenz on account of their simplicity. It seems, however, that since these formulas disregard the flattening of the pipe section and assume the stresses to be proportional to the distance from the neutral axis, they cannot be correct. The author makes use of the maximum shear theory in calculating the "equivalent" stress, as does also Mr. Wahl in his investigation. According to this theory the criterion of strength is the difference between the algebraically greatest and smallest principal stresses, while the middle stress is assumed to be without influence on strength. This theory has been shown in recent years to be not quite correct, and a new formula has been proposed by Dr. H. Hencky[9] and others. This formula is based on the principle that plastic flow occurs when the elastic work of deformation (not including the volume strain) reaches a certain constant value, whatever be the stresses by which it is produced. In two-dimensional stress as in the wall of pipe bends, flow will occur according to this theory when:

$$S_1^2 - S_1 S_2 + S_2^2 = S_0^2$$

where S_0 is the yield point of the material. Thus, according to this theory, the equivalent stress is equal to the square root of the expression on the left-hand side of the equation. This theory has found excellent confirmation by various tests, notably by those made in Zürich by Professor M. Ros and A. Eichinger.[10]

Quite recently the writer has made tests on pipe bends of 6-in., 9½-in., and 14-in. diameter, with the special object of determining the stresses in the pipe at the point where a permanent set occurs in the bends. It was hoped in this way to obtain a criterion for the strength of pipe bends which can be used in practice with a factor of safety as determined by experience. Measurements were made of the deflection of the bends, that is, the approach of the bends to each other; the stresses were determined on a section at the top of the bend where the bending moment was a maximum. The longitudinal strains were measured directly with Berry strain gages; the transverse strains were computed from measurements with a micrometer caliper of the diameters all around the pipe section. The stresses were calculated by taking into account both longitudinal and transverse strains from the formulas:

$$S_1 = \frac{m^2 E}{m^2 - 1}\left[\epsilon_1 + \frac{\epsilon_2}{n}\right]$$

$$S_2 = \frac{m^2 E}{m^2 - 1}\left[\epsilon_2 + \frac{\epsilon_1}{n}\right]$$

where ϵ_1 and ϵ_2 are the observed strains and m is Poisson's constant. The "observed" stresses so determined were compared with the "calculated" stresses obtained by the theoretical formulas. Also the equivalent stress, using the Hencky formula as well as that of the maximum-shear theory, was calculated.

[8] W. Hovgaard, "Deformation of Plane Pipes," *Jl. Math. and Phys.*, Mass. Inst. Tech., vol. 7, no. 3, October, 1928.

[9] *Zeitschrift für angewandte Mathematik und Mechanik*, Bd. 4, Heft 4, 1924.

[10] Second International Congress of Tech. Mech., Zürich, 1926.

It was found, contrary to expectation, that at the load where the pipe began to show a measurable permanent set, the transverse as well as the equivalent stresses were far in excess of the limit of plasticity. Evidently none of these stresses could be used as a criterion of strength. It must be noted that the transverse and equivalent stresses exist only on the surface and extend at first over limited zones, so that they cannot be expected to influence the behavior of the bend as a whole until the zones of plasticity have spread over greater areas and to greater depth below the surface.

The longitudinal stresses, on the other hand, were just within the limit of elasticity when the bends began to show a permanent set, whence it would appear that these stresses, which exist with moderate variations throughout the whole thickness of the pipe wall, might serve as a measure of strength. The tests, however, are not considered sufficiently comprehensive to justify this generalization. They covered a relatively limited range of λ values, comprising only pipes for moderate steam pressures. Further tests are required, notably tests on pipes for high pressures, say, 500 lb. per sq. in.

A full description of the tests here referred to and their analysis will shortly be published in the *Journal of Mathematics and Physics* of the Massachusetts Institute of Technology under the title, "Further Research on Pipe Bends."

HAROLD E. JENKS.[11] The paper presents a very complete collection of exact formulas and is unique in that the solution of a one-plane pipe line of any shape is reduced to a practical problem.

The first fundamental principle given in the paper—that the entire pipe line (between anchors) is carrying the expansion and that the use of expansion bends should be the exception and not the rule—is finding increasing application in modern design. Hence in the future we may expect to analyze fewer conventional bends and more odd shapes, partly curved and partly straight. Table 1 and Formulas [13] to [22], inclusive, provide a solution for the reactions in any one-plane pipe line with fixed or hinged ends. These formulas, together with formula [2], which enables the maximum moment to be found, and formulas for the flexibility constant K and for stress are all that are needed.

The point that in practice the ends of pipe bends are usually more nearly fixed than hinged is of extreme importance. With the exception of Carlier's notable work and one or two examples in Wahl's article, all mathematical analyses which the writer has seen up to the present time have ignored this important factor. Under fixed-end conditions the maximum stresses in a pipe line may be as much as 50 per cent greater than under hinged-end conditions and at an entirely different point in the line. The reactions may vary even more than this. Hence, where any doubt exists as to the amount of fixity of the ends, they should be assumed fully fixed so that a variation from these conditions will result in an error on the safe side. Hinged ends make excellent test specimens as their analysis may be relatively simple, but they are of infrequent occurrence in design. A series of articles by Mr. Finniecome recently published in *The Engineer* (London) contains much valuable information, but here again all calculations are based on hinged-end conditions. The same disregard of end fixity leads him to what are probably erroneous conclusions regarding the value of the flexibility constant K, as will be explained below.

The values of the flexibility constant K and corresponding stress formulas as obtained to date by various methods of analysis are presented by the author, Mr. Shipman, who adopts the Lorenz formulas in his work. According to the Lorenz theory, the longitudinal stress in the curved portion of a pipe bend is proportional to the distance from the neutral axis—a straight-line distribution. This does not, however, mean that flattening of the pipe has been disregarded, since the moment from which the stress is found is a function of the flexibility constant. The Karman theory, on the other hand, results in a longitudinal stress which has not a straight-line distribution. Because of the apparent verification by Professor Hovgaard of the Karman distribution of longitudinal stress, the writer believes that the Karman theory represents the closest approximation to the conditions actually obtaining in practice. The Karman flexibility constant $\frac{10 + 12\lambda^2}{1 + 12\lambda^2}$, as at present used, is admittedly erroneous for small values of λ. Karman's second approximation,

$$\frac{3 + 536\lambda^2 + 4800\lambda^4}{105 + 4136\lambda^2 + 4800\lambda^4}$$

is better, but for small values of λ still gives results that are considerably too low.

The necessity for analyzing several pipe lines where λ was approximately 0.05 recently led the writer to extend the Karman theory to cover such cases; in other words, to find the nth approximation. The theory is much too involved for presentation here, but the result may be expressed as follows: For all values of λ the true value of the Karman flexibility constant is given by

$$K = \frac{10 + 12\lambda^2 - j}{1 + 12\lambda^2 - j}$$

where j is a complex function of λ varying from unity when $\lambda = 0$ to zero when $\lambda =$ infinity. The numerical value of j may be found to any desired degree of accuracy by obtaining the ratio of two determinants of the nth order and is given in the following table:

λ...	0	0.05	0.1	0.2	0.3	0.4	0.5	0.75	1.00
j...	1	0.7625	0.5684	0.3074	0.1764	0.1107	0.07488	0.03526	0.02026

The error due to using the formula $(10 + 12\lambda^2)/(1 + 12\lambda^2)$ instead of the true value amounts to 72 per cent when $\lambda = 0.05$, 7 per cent when $\lambda = 0.3$, and 1.2 per cent when $\lambda = 0.5$. For values of λ greater than 0.5 the error is less than 1 per cent, hence the formula $(10 + 12\lambda^2)/(1 + 12\lambda^2)$ is sufficiently accurate for such values.

A curve plotted from the above table will enable intermediate values of j to be found with accuracy; hence the true curve of K for all values of λ may also be plotted.

The true value of K as given above is in approximate agreement with Mr. Finniecome's suggested value for values of λ less than unity. The Crocker and Sanford data used by Mr. Finniecome in arriving at a suggested curve for values of λ greater than unity have been shown by the author to represent a probable fixity of from 7 to 46 per cent, which would account for Mr. Finniecome's test curve falling below the true curve. The quantity, and with few exceptions the quality, of available test data do not in the writer's opinion warrant a definite statement that the value of K by any theory has or has not been checked.

The more complete theory also requires the derivation of new stress formulas, taking into account the effect of longitudinal and transverse strains according to the theory of elasticity. For values of λ above 0.5, the following formulas give the true stresses:

Longitudinal stress at outer surface of pipe

$$S_1 = \frac{KMr}{I}\left[\sin\phi + \frac{4K - 4}{K}\left(\frac{\lambda}{2m}\cos 2\phi - \frac{1}{3}\sin^3\phi\right)\right]$$

[11] Atmospheric Nitrogen Corporation, Syracuse, N. Y. Formerly Assistant Professor of Mathematics, U. S. Naval Academy.

Transverse stress at outer surface of pipe

$$S_2 = \frac{KMr}{I}\left[\frac{\sin\phi}{m} + \frac{4K-4}{K}\left(\frac{\lambda}{2}\cos 2\phi - \frac{\sin^3\phi}{3m}\right)\right]$$

Longitudinal stress at middle surface of pipe

$$S_3 = \frac{KMr}{I}\sin\phi\left(1 - \frac{4K-4}{3K}\sin^2\phi\right)$$

In these formulas ϕ is the variable angle of the pipe cross-section, being zero at a distance R from the center of the pipe bend and 90 deg. at a distance $R + r$ from the center, and m = the reciprocal of Poisson's ratio, usually taken as 10/3 for steel. The value of ϕ for a maximum value of stress can be found by differentiation with respect to ϕ. Construction of curve sheets will facilitate the use of these formulas.

For values of λ of 0.5 or lower the above formulas or any heretofore published are not sufficiently accurate. In such cases the exact formulas are very complicated and will not be presented here, but values computed from them are given in the following table from which curves may be plotted:

λ	0.05	0.1	0.2	0.3	0.4	0.5
K	34.6	17.31	8.68	5.73	4.20	3.29
C_1	7.94	4.97	3.10	2.33	1.864	1.557
C_2	13.61	8.47	5.24	3.94	3.19	2.675
C_3	6.34	3.99	2.51	1.877	1.500	1.260

The maximum values of S_1, S_2, and S_3 are given by $C_1\frac{Mr}{I}$, $C_2\frac{Mr}{I}$, and $C_3\frac{Mr}{I}$, respectively.

Professor Hovgaard's recent valuable experiments indicate that S_3 may be the true criterion of pipe strength rather than "equivalent stress" as heretofore used, and he tentatively recommends that this stress be kept below 20,000 lb. per sq. in. for a cold pipe. The maximum value of S_3 for $\lambda > 0.5$ is given by

$$S_3 \max = \frac{KMr}{3I}\sqrt{\frac{K}{K-1}} \text{ when } K > \frac{4}{3}$$

$$= \frac{Mr}{I}\left(\frac{4-K}{3}\right) \text{ when } K < \frac{4}{3}$$

which with the values of C_3 given above would, under this criterion, make limiting stress calculation a relatively simple matter.

C. T. Mitchell.[12] Since the author expresses a desire to aid in promoting the practical application of the theories presented in his paper, he will be interested to know that a graphical method of analysis, identical in theory and similar in application to the one he briefly outlines, has been for some time in active use in one of the offices in New York City. The method was developed from formulas which the writer published about eighteen years ago[13] for the determination of neutral axes in rigid framed structures. As applied to pipe structures, it proves surprisingly simple. The pipe between anchors is laid out to scale, with square corners substituted for the bends. The ends are assumed to be fixed, which requires, as the author points out, that the axis of zero bending moment shall pass through the center of gravity of the structure. This axis is laid out and the bending-moment diagram plotted for each straight member, the member itself forming the central axis of the diagram. The sum of the areas of all the moment diagrams times the distance of the center of gravity of each from the neutral axis supplies the only unknown quantity necessary to determine the thrust at the anchors or the bending stress at any point in the structure. A correction is finally made to allow for the bends by multiplying the computed thrust by the ratio of the length of assumed square-cornered structure to the length of the actual structure with round corners.

A simpler method than this for attaining anything like accuracy is hardly possible. It requires no tables or formulas other than those necessary to compute the moment of inertia of the pipe section. In the stress of production which prevails in most work, it is a veritable time saver. Most pipe structures are changed a dozen times in the course of design. Sometimes a small change in dimensions has a serious effect on the expansion stresses. Elaborate recalculation with each shift in design is not practical. The graphical method, being rapid in execution, aids the designer in acquiring a valuable sense of proportion.

So much for the practical aspect of this graphical method. Its limitations as to accuracy appear to be the following:

In always assuming 100 per cent rigidity at the anchors, it sometimes indicates greater rigidity in the structure than actually exists. But the results are on the side of safety.

It takes no account of the increased flexibility of bent over straight pipe. In this respect the results are again on the side of safety. It is not accurate for certain structures composed entirely of bends. However, structures of this nature are extremely rare in practice. By far the major part of power-plant piping is made up of straight lengths connected by bends or fittings.

In other words, the graphical method appears to sacrifice a small amount of accuracy in favor of uniformity and simplicity. High degree of accuracy in problems of this nature is, of course, unattainable, due to the uncertainty regarding the rigidity of the anchors and also as to the amount of flattening of cross-section that occurs in the bending of the pipe. To be on the safe side in these matters requires a little more pipe, the cost of which is probably negligible.

The writer would very much like to learn the author's opinion regarding the allowable stress on a pipe joint. In the diagram, Fig. 20, of the paper, the author submits a curve giving proposed allowable working stresses at pipe joints ranging from 10,000 to 20,000 lb., according to temperature. It is assumed that this refers to the stress in the pipe at the joint. These figures seem very high. The writer would suggest that in high-pressure work (600 to 1200 lb.) the pipe stresses at the joints due to bending be maintained at not over 5000 lb. per sq. in. for temperatures above 700 deg. fahr.

A. M. Wahl.[14] The writer believes the present paper to be a valuable contribution to the literature on the design of steam piping. The various principles enumerated on the first page deserve special recognition because of their importance. Principle No. 2, which states that supports and anchors should be placed so as not to restrain free movement of the pipe line, is especially important, while the limited-travel anchors mentioned may be of very great help in eliminating undesirable reactions in many practical cases.

It is the writer's belief that the Karman theory as developed in his previous paper[15] will give results more nearly in accordance with actual pipe-bend test results than either the Lorenz or Marbec theories. He bases this opinion on the following facts: Karman derived his formulas in a rational manner, using Ritz's method of specifying the deformation of the cross-section in the

[12] With Stevens & Wood, Inc., New York, N. Y.

[13] *Engineering News*, May 26, 1910.

[14] Mechanics Section, Research Department, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa. Jun. A.S.M.E.

[15] A. M. Wahl, "Stresses and Reactions in Expansion Pipe Bends," Trans. A.S.M.E., paper FSP-50-49, 1928.

form of a trigonometrical series with undetermined coefficients and later determining these coefficients to make the potential energy a minimum. Lorenz, on the other hand, used an approximate method and obtained a straight-line law of longitudinal stress distribution over the cross-section. This type of stress distribution, however, is not correct, as is shown experimentally by the strain measurements on actual pipe bends both by Hovgaard and the writer. Strain measurements made by the writer on 10-in. diameter turbine expansion pipe bends in collaboration with the South Philadelphia Works of the Westinghouse Com-

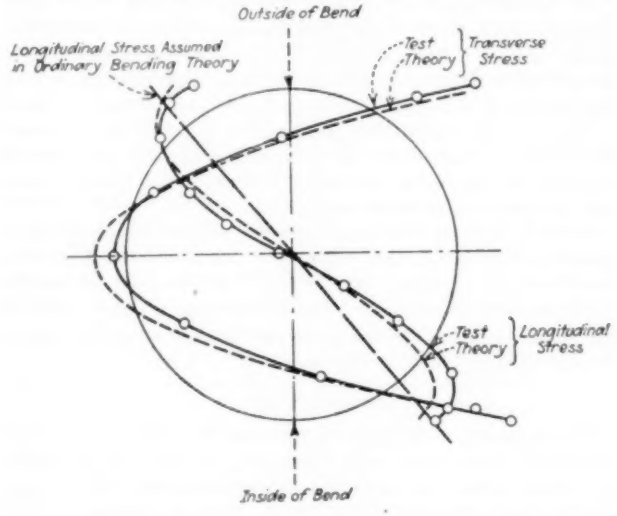


FIG. 36

pany give a stress distribution over the pipe cross-section as shown in Fig. 36. The full lines show the stress distribution found by test, while the dotted lines show that calculated theoretically using the Karman theory and its development by the writer. It may be seen that the distribution of stress closely follows the Karman theory, while the linear law given by Lorenz is entirely incorrect.

A further reason for believing the Karman theory to be more nearly correct than the Lorenz theory is given by the values of k found by the writer in the paper mentioned above. In Fig. 37 the curves of Fig. 13 of the paper have been replotted, and the experimental points obtained by the writer indicated. It may be noted that these points follow the Karman curve very closely. The author states that the writer's data "were not plotted because of uncertainty as to end conditions of curved portions." However, the writer cannot see where any uncertainty exists.

In his tests the U-bends were loaded as shown in Fig. 38. There was practically a hinged-end condition at the ends of the U-bend at c and c' because of looseness in the clamps attached to these points. Hence, at the ends b and b' of the curved portions, we have practically a moment Pa acting plus a direct load P. Under these conditions the loading at the ends b and b' is completely defined. The deflection between these points may then be computed, using the formulas for Cases 4 and 6 in the writer's paper.[14]

It should also be noted that the chart given by the writer in Fig. 25 of his paper for equivalent stress in curved pipes was also given later by Finniecome.[16] However, it must be realized that there is not a great deal of difference between the values of stress and deflection given by the Karman and Lorenz formulas. This is apparent by reference to Figs. 7 and 13 of the author's paper.

In the manufacture of pipe bends there is also produced an initial ellipticity of the pipe cross-section. This ellipticity, though small, is nevertheless sufficient to greatly change the stress distribution produced by internal pressure. Tests made by the writer in collaboration with the South Philadelphia Works of the Westinghouse Co. show that where the ellipticity of the cross-section of a 10-in. pipe bend is as small as $^1/_8$ in., the bending caused by internal pressure may be sufficient to induce compressive stresses at certain points on the outside of the pipe wall. At other points, under these conditions, stresses equal to twice the direct stress produced by internal pressure may arise. The stress set up by this initial ellipticity of the pipe bend may be approximately computed by using formula [49] of the writer's paper, remembering that M_0 is the bending moment per inch length. Hence, the stress is

$$S = \frac{6\,M_0}{t^2}$$

where t is the thickness of the pipe wall. This stress due to bending must be superimposed on that due to direct circumferential tension.

When this stress, which may be quite high, under the increasingly high steam pressures now coming into use, is superimposed on the stresses due to bending moments, flattening of the cross-section, etc., the resultant equivalent stress may be beyond the yield-point of the material. For this reason, it is not surprising to the writer that pipe bends so often undergo permanent set when placed in service. This permanent set may annihilate the effect of cold-springing, allowing excessively high reactions to be set up on turbines, anchors, etc. For these reasons, in the future, it might be desirable to specify limits

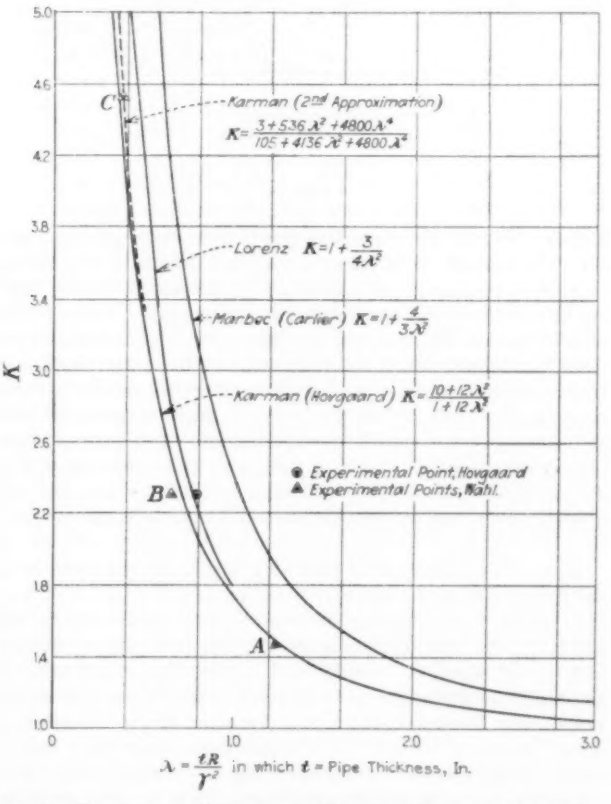


FIG. 37 FLEXIBILITY CONSTANT AS GIVEN BY VARIOUS INVESTIGATORS

(R = radius of pipe bend, in.; r = mean pipe radius, in.)

[14] *The Engineer* (London), Sept. 7, 1928.

to the permissible ellipticity produced in pipe bends during fabrication. At least this would tend to eliminate another uncertain factor from the problem.

On the whole, the writer believes that some such code as proposed by the author should replace the tables and formulas given in our present handbooks and catalogs. These ordinary formulas are obviously not rational, in the light of our recent knowledge of this subject. Whether the code proposed by the author is the best remains to be seen, but at least it is an improvement on those ordinarily used.

In the writer's opinion, the big question in designing pipe bends is whether the ends of the pipe can be considered fixed or

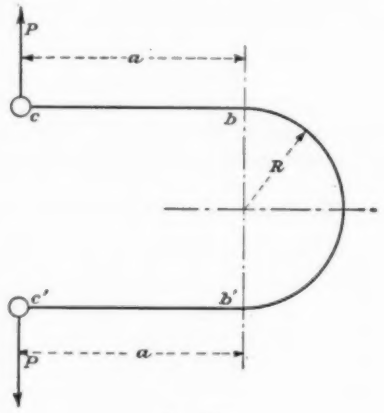


FIG. 38

hinged. On the answer to this question depends, in large measure, the amount of stress and reactions set up in the piping. In view of the indeterminate nature of the end conditions existing in most practical installations of pipe bends, it seems to the writer that the safest course to pursue would be to assume the worst conditions, which would usually be fixed-end conditions. In certain cases, however, such as those where unrestrained lateral movement of the pipe line is allowed, it might be permissible to assume hinged-end conditions in calculation. It is thus apparent that a certain amount of good judgment as to the proper conditions to assume must be exercised by the pipe-line designer when applying the various theoretical formulas to his problem.

KARL D. WOOD.[17] The following points do not seem to be clearly emphasized by the text of the paper.

1 Table 2, which compares several theoretical formulas for the factor K which expresses the relative flexibility of straight and curved pipes, is a unique and outstanding contribution to the American literature on this subject, since the work of neither Lorenz nor Marbec appears to have come to the attention of other American writers. While the table shows the Lorenz theory to be the most reasonable, Fig. 13 shows it to be supported by only one experimental point, while no data are given supporting the other theories. This indicates a very profitable direction for future research, since significant results should be obtainable at small expense.

2 The quantitative treatment of fixedness of ends of pipe lines, as shown in the last line of Fig. 14, appears to be also a novel and valuable idea. While the table attempts to verify values of both fixedness and flexibility factor K, and therefore verifies neither, it indicates that simple experiments may be conducted in the future which will check both factors by holding one of them constant at a time.

3 While Fig. 15 shows that current practice for quarter-bends with fixed ends allows 100 times as much expansion as the proposed well-verified theory indicates would produce a safe stress, this need not imply that such pipe bends have failed, but rather that such bends have been subjected to excessive working and therefore short life.

4 In the proposed Code of Design Practice, the typical study of shape efficiency is based on an assumed constant total length of pipe. An assumption of constant yearly cost of pipe would, however, probably be a more practical basis of comparison, and not much more difficult than the one used. Principle No. 3 in Part I which summarizes this study is nevertheless valid and important; namely, that the maximum length of pipe should work at the maximum safe stress; and with this principle as a guide, a meaningful result can be obtained.

In spite of these criticisms, however, the writer feels that engineers will recognize this paper to be one of the most valuable studies of pipe expansion ever published, and he believes that the Society would do well to consider the issuance of a code of pipe-expansion practice similar to its other codes in the light of the material presented in this paper.

AUTHOR'S CLOSURE

The following comments are made in the belief that a brief summary of the entire case, attempting to voice the consensus of opinion of all who took part in the discussion, will be more helpful than a detailed study of each of the discussions.

Part I—Fundamental Principles to Be Considered in Design. This stands approved.

Part II—Survey of Published Work. To those given should now be added the splendid discussions of this paper and the work mentioned by Professor Hovgaard and Mr. Carlier.

Part III—Theoretical Analysis. The fixed-end and partial-fixity-of-ends theory stands approved. The work of Hovgaard and of Wahl points to the Karman as the correct theory for the flexibility constant, and the work of Jenks to the nth approximation of this theory.

Part IV—Comparison of Theory With Experiment and With Current Practice. The tests of small solid models verify the mathematical theory without disturbance from such factors as the flattening effect and lack of complete end fixedness which always exists in practice. The next step, which has been partially carried out as noted in the discussion, is a series of tests to check the flexibility constant. The final step, which it is hoped interested parties everywhere will undertake, is the check on actual layouts as used in practice. It is felt that a thorough theoretical groundwork has been laid for these tests and that the main factors to be considered have been pointed out.

Part V—Proposed Code of Design Practice. For the simpler shapes given, the solution is so simple that there is no longer excuse for the empirical and guess methods in quite common use. For the more complicated shapes mathematical and graphical solutions are available.

The author feels that the design of piping to care for expansion is much nearer the standardized basis on which most other branches of design rest than ever before, and the author expresses his thanks to his collaborators and to those who have contributed so much in the discussion for their large part in the achievement.

[17] Assistant Professor of Mechanics, Cornell University, Ithaca, N. Y.

FSP-51-53

Design and Proportions of Economizers and Air Preheaters

By HOSEA WEBSTER,[1] NEW YORK, N. Y.

Thirty years ago 200 lb. working pressure and 40 lb. of coal burned per square foot of grate surface were the maximum. Superheat had not come into use. Since that time there has been a rapid development or evolution in the design of steam-generating and fuel-burning equipment. Today the rate of evaporation per square foot of boiler heating surface and the rate of fuel burning per foot width of furnace have increased 1000 per cent, and 1400 lb. steam pressure and 800 deg. steam temperature have passed the experimental stage, with indications that both pressures and temperatures may go higher where conditions of service and of fuel cost warrant. Modern high pressures and temperatures and high rates of fuel combustion have not only required much greater care in the selection, fabrication, design, and construction of pressure parts and furnaces and fuel-burning equipment, but also an entire rearrangement and distribution of heat-absorbing equipment. The temperature of the gases leaving the stack is an approximate measure of the overall efficiency. Recovery of the heat contained or carried in the gases leaving the steam generator proper may be accomplished by an economizer or an air preheater or both. Economizers of vertical cast-iron tubes decidedly antedate the increase in working pressures which required the use of steel tubes, now mostly horizontal and of the return-bend type, and because of the greater susceptibility to interior corrosion, may be fed only by water practically oxygen free. The introduction of pulverized coal requiring preheated air for proper combustion has greatly extended the use of air preheaters, of which three types are in use: tubular, plate, and regenerative. While the additional efficiency of a generating equipment runs from 3 to in some cases 7 per cent with economizers, it runs from 5 to 10 and sometimes more where air preheaters are used. Service and operating conditions may warrant the use of either or both, each plant being an individual problem of station economics.

CONCURRENT with and contributing in no small degree to the marvelous industrial and power development in this country since the end of the World War have been the improvements in the design and efficiency of steam-generating equipment which have made possible the coincident advances in design, capacity, and efficiency of prime movers. These improvements have been in the furnace and fuel-burning equipment and in the design, fabrication, materials, and disposition of steam-generating and heat-absorption surfaces.

Thirty years ago most boilers were hand-fired, with a few natural-draft chain grates and overfeed stokers. With the introduction of the underfeed stoker with under-the-grate forced draft came an increase in the rate of fuel burning per square foot of grate surface. Then, as now, the limit to the steam-generating capacity of a boiler fed with good water was the limit to the amount of fuel which could be burned under it. Furnace performance was stated in pounds of fuel burned per hour per square foot of grate surface.

Furnace performance as to input and capacity when the fuel is liquid, gas, or pulverized coal is generally stated in B.t.u. per hour per cubic foot of the furnace volume required for complete combustion within the furnace.

Reduced to its lowest terms, the problem of steam-generator design is, how far does it pay to go by reducing the temperature of stack gases in utilizing the heat which can be developed in burning the fuel available for the work to be done or power to be developed.

Manifestly, then, the first step is to determine upon the design of the furnace and fuel-burning equipment which will get the best combustion results from the fuel available, having in mind of course the variations in the maximum, minimum, and average rates of demand for the output of the installation.

An input of $2^1/_2$ to 3 million B.t.u. per hour per foot width of furnace is about the limit for a hand-fired stationary boiler. Underfeed and chain-grate stokers with forced blast can maintain an input of 20 million B.t.u. per hour per foot width of furnace, while recent designs of double-ended pulverized-coal furnaces are being offered for as high as 35 million B.t.u. input per hour per foot width of furnace. At these higher rates there are serious or difficult problems of a physical, chemical, and mechanical nature, requiring particular care in design and construction.

The increase in the rate of fuel input, at first gradual, but of late more rapid, was at first accompanied up to a certain point by a corresponding increase in the area of boiler heating surface per foot width of furnace.

This advance in methods of fuel-burning equipment has decidedly complicated the design of the generating equipment, the problem being how far and in what direction it pays to go in investing in heat-absorbing equipment to reduce stack losses and gain higher consequent overall efficiency.

In the case of central-heating and power stations where fuel cost is a considerable part of the selling price of the product, the problem is naturally more complicated than in an isolated plant where fuel cost is too small a part of the selling price of the product to warrant the complications of equipment and the operation costs required for high overall plant efficiency.

A high ratio of power or equivalent B.t.u. output to B.t.u. equivalent of fuel input is predicated upon a reduction of the temperature of the gases leaving the steam generator by using them to heat the feedwater by an economizer or to heat the air required for combustion by means of an air heater, or by both working in series.

The necessity for reheating the exhaust from high-pressure turbines has brought out the convection reheater which, utilizing the furnace gases as they leave the boiler proper, really is a form of economizer in that it takes heat from the furnace gases which is utilized in the low-pressure turbine.

While in isolated plants, notably in New England textile mills where the power requirements were constant, economizers with cast-iron elements receiving water at or around 150 deg. from auxiliary-feed heaters have for years been in general use, the higher pressures and higher rates of steam generation have brought into use economizers made up of steel elements. Although prior to the adoption of pulverized coal they were coming into general use in central-power-station practice, the use of extracted steam in heating the feedwater up to near the temperature due to the pressure carried in the boiler in order to maintain

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[1] Sales Manager, Babcock & Wilcox Company. Mem. A.S.M.E.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

station heat balance has somewhat reduced their use, they still appear in many station designs.

Today the design and construction of an economizer is not as simple as it looks. Subject to extremes of temperature and to various destructive chemical reactions inside and outside, the surfaces and flow area for both gases and water must be so designed and disposed as to give a maximum of heat transfer with a minimum of first cost and of space requirement for the necessary heating surface. Resistance to the flow of water to the boiler and draft loss must be kept within reasonable limits, and proper provisions must be made against exterior and interior accumulations which tend to reduce heat transfer efficiencies.

There should be no joints in the surface exposed to the furnace gases.

Two distinct types of economizer are on the market: vertical tube and horizontal tube. The former is sometimes an integral part of the boiler, but usually independent, and is usually cross-baffled to give several passes of the gases over the heating surface, usually entering the top and leaving at the bottom, the tubes being expanded into a lower drum which receives the feedwater and into an upper drum from which the heated feedwater passes to the boiler.

Before high pressures required the use of steel tubes, all economizers were made up of vertical cast-iron tubes pressed at each end into cast-iron headers. Mechanical scrapers keep the exterior surfaces free from soot.

Because it takes up less space there developed a demand for a horizontal steel-tube economizer, of which the return-bend type is coming into more general use. Disposed in horizontal rows one row above the other, the flanged ends of the tubes in each row are joined at one end to a tube above and at the other end to a tube below by a flanged return bend, making a male and female gasketed joint.

The topmost row of tubes is connected at one end to a forged-steel outlet header as is the lowermost row to a forged-steel inlet header. The tubes are carried at each end by a supporting plate which serves as an end casing for the economizer. The tubes pass the supporting plate by a gas-tight joint so that the return bends have their joints outside the gas passage and can be readily examined with the economizer in service.

Because of the consequent higher rate of heat transfer, economizers should operate on the counterflow principle, with gases entering at the top and feedwater entering the bottom and flowing upward.

An upward flow of gases with a downflow of water is apt to cause an annoying and possibly destructive water hammer and should be avoided.

To avoid external corrosion the area of heating surfaces and temperature of feed must be so proportioned that at any rate of operation the temperature of the tubes will be above the dewpoint of the gases, as any accumulation of moisture on the exterior of the tubes quickly absorbs any sulphurous gas coming from the fuel.

To avoid interior corrosion the feedwater must be kept free or practically free from air or carbonic-acid gas. The oxygen content of the water entering the economizer or boiler should be kept below the equivalent of 0.05 cc. per liter. Several deaerators and feedwater heaters are on the market which keep the oxygen content down to or below this point.

One advantage of the horizontal-tube return-bend type of economizer is that if corrosion does occur it is localized and requires replacement of only a small part of the surface.

Steaming economizers are in successful and satisfactory use. Steam is formed at high rates of evaporation in the boiler, with consequent high temperatures of outgoing gases.

Ordinarily an economizer should be so proportioned with respect to the surface of the boiler that the outgoing feedwater will be at least 50 deg. below the temperature due to the boiler pressure, but if conditions of station heat balance require multistage bleeding with feed temperature approaching that of the boiler water, the steaming economizer is fully justified and several are in use.

While not absolutely so, the temperature of the stack gases is indicative of the overall efficiency of the unit.

For the proper combustion of pulverized coal the requirement of moderately heated primary air and higher temperature secondary air has led to the use of air preheaters utilizing the heat in the flue gases and cooling them prior to their entrance to the stack. Experience has shown that their use with liquid and gaseous fuel and also with stokers in most cases pays a profitable return upon their first cost.

While much more are used than ever before, air heaters are by no means a novelty, as the records show their use abroad over a hundred years ago in metallurgical furnaces.

All the early English patents covering air preheaters, of which over fifty had been issued by 1839, based their claims on smoke reduction, and the use of preheated air to get better plant-combustion efficiency does not appear to have been considered until about 1870. The first installation in this country was in a New England plant in 1878. In marine service preheated air has been in use for years, with forced or induced draft installations, but it required the modern increases in fuel costs, demand for high efficiencies, and high rates of steam output to make their installation seem worth while in stationary boilers.

The ideal air heater gives a maximum heat-transfer rate with minimum draft loss, minimum leakage, minimum space requirements, and ease of access for cleaning and repair.

The first two factors are intimately related, and the forms of gas and air channels have a bearing on the transfer rate, which is primarily a function of gas and air velocity, which in turn governs the draft loss and air-flow resistance or friction.

As in the case of economizers, consideration must be given in their design to cost of power required to overcome the frictional resistance.

There are so many variable factors involved that no universal rule or formula can be devised for the proportions of economizers and air heaters.

Each case is a complicated problem requiring individual solution to determine how far and in what direction it will best pay to go in reducing the temperature of stack gases with an adopted method of burning the fuel available.

There are three distinctive types of air heaters: tubular, plate, and regenerative. All have their advocates, and often space requirements or limitations govern the type selected.

The tubular and plate heaters are usually vertical, utilizing the countercurrent principle in gas- and air-flow direction.

Vertical tubular air heaters are made up of vertical steel tubes expanded at their upper and lower ends to generally horizontal plates. The fit at one end of the tubes is usually just loose enough to permit any expansion and contraction which may occur as a result of uneven distribution of gases or air over the surface.

The gases entering the bottom flow inside the tubes to the top. The tubes are enclosed in a casing which serves as the air flue, down which and over the outside of the tubes the air passes.

There are several forms of plate air heaters which fundamentally consist of vertical parallel plates spaced from $1/2$ to 1 in. apart and so joined at their vertical edges that the air passages alternate with the gas passages. The air and gas inlets and outlets are so disposed as to give countercurrent flow. They usually require less height than the tubular type of equal capacity. In some designs the plate ends are joined by welding; in others, by gasketed joints.

As with tubular and plate heaters there are several designs of the regenerative type on the market. Usually they are made up of metal elements so disposed that they are first heated by the flue gases and then give up the heat to a succeeding flow of air. In some designs the elements pass alternately into the gas and air passages; in others, the elements are stationary and are alternately bathed in flue gas and in air.

In design and material, provision must be made to withstand the temperature extremes of entering hot gases and relatively cold air and consequent expansion and contraction, as well as for cleaning the surfaces over which furnace gases pass, with oftentimes resulting soot and dust deposits which rapidly reduce the heat-transfer rate. As cooling the furnace gases below the dewpoint of some of their chemical constituents results in rapid corrosion of the tubes or plates, care must be used to avoid too much surface.

While the additive efficiency characteristic of an assumed economizer can be exactly calculated, such is not the case with an air heater, as there are two factors involved—first, the saving due to the heat brought back to the furnace; second, that due to the resulting increased furnace temperature, with consequent higher combustion efficiency.

Opinions differ as to the amount of increase in furnace temperature resulting from increasing incoming-air temperature, but it is probably from one-fifth to one-third of a degree for each degree increase in entering air. It is probable that some fuel characteristics enter into the problem.

With most bituminous or semi-bituminous coals there is a temperature at which they will begin to soften. This characteristic limits the temperature of the primary air for pulverized-coal equipment and of the air passing through the fresh coal bed of both chain-grate and underfeed stokers where the temperature is high enough to cause the coal to mat and to cut off or greatly reduce the air flow. Another limitation to air temperature with stokers is the necessity of avoiding overheating the grates and other structural details.

In the case of stokers, preheating the air unquestionably results in better and quicker ignition, shorter flame, and a marked decrease in the loss from unconsumed carbon in the ash and higher possible rates of fuel input.

Inasmuch as they utilize furnace heat which has not been used in generating steam, convection reheaters which resuperheat the exhaust steam from high-pressure turbines on the way to the low-pressure turbine throttle are really a form of economizer. As modern practice calls for reheating the steam to 700 to 800 deg., the gases coming from the boiler must have the high temperature characteristic of high rates of evaporation. The temperature of the gases leaving the reheater depends upon the temperature and B.t.u. content of the incoming steam. Usually the outgoing-gas temperature warrants the use of both a relatively small economizer and an air heater of the highest efficiency.

A number of designs are in successful use, all tubular in construction and varying greatly in the distribution of surfaces and gas- and steam-flow areas which depend primarily upon the temperature and pressure of the incoming steam and upon the temperature of the outgoing steam.

The flow area of the tubes and headers must always be large enough to avoid any material pressure drop of the steam passing from the inlet to the outlet.

Rates of heat transfer with economizers and air heaters vary, of course, with the variation in the output of the equipment. In the earlier days of cast-iron economizers, 2 B.t.u. per square foot per degree of temperature difference was the average. The rate of transfer varies with the velocity flow of the fluids over the heating surface, which in turn is accompanied with varying frictional resistance, to overcome which consumes power.

TABLE 1 SERVICE CONDITIONS AND PERFORMANCE CHARACTERISTICS OF PRESENT-DAY TYPICAL COMBINATIONS OF BOILER, ECONOMIZER, AND AIR HEATER PER FOOT WIDTH OF FURNACE

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Working pressure, lb.	200	200	200	200	325	350	350	400	400	475	500	1350*	285	350
Fuel and type of furnace†	Pulverized	Pulverized	Underfeed stoker	Underfeed stoker	Pulverized	Underfeed stoker	Chain-grate stoker	Underfeed stoker	Pulverized	Underfeed stoker	Pulverized	Pulverized	Pulverized	Oil
Per Foot Width of Furnace:														
Boiler surface, sq. ft.	550	550	550	550	450	1000	810	880	1130	925	1350	275	1450	664
Economizer surface, sq. ft.	...	257.5	...	257.5	190	270	...	630	...	...	...	525	380	...
Air heater surface, sq. ft.	800	...	800	...	420	1020	1610	...	1200	1130	2300	1220	1250	700
Output and Input:														
Thousand lb. steam per hr.	2.55	2.55	2.55	2.55	2.0	8.3	3.0	2.0	3.5	5.75	11.5	3.35	4.75	2.7
Output million B.t.u. per hr.	2.77	2.77	2.77	2.77	2.2	9.6	2.35	3.3	4.1	6.7	12.0	3.35	7.60	2.95
Fuel input million B.t.u. per hr.	3.15	3.17	3.24	3.29	2.8	11.0	2.70	3.7	4.65	7.57	14.0	3.80	8.50	3.45
Lb. steam per hr. per sq. ft. of boiler	4.5	4.5	4.5	4.5	4.4	5.0	3.0	3.1	3.0	6.1	8.5	12.55	5.0	4.0
Draft Loss, In. H_2O:														
Boiler	0.16	0.2	0.33	0.42	0.17	1.42	0.27	0.23	0.28	0.77	3.0	0.4	0.27	0.17
Economizer	...	0.31	...	0.52	0.10	0.52	...	0.18	...	0.60	...	...	0.35	...
Air heater	0.19	...	0.31	...	0.15	0.56	0.14	...	0.29	...	1.55	0.21	0.28	0.12
Total	0.35	0.5	0.64	0.94	0.42	2.50	0.41	0.41	0.57	1.37	4.55	0.61	0.90	0.29
Resistance through air heater, in. H_2O	0.14	0.22	0.29	...	0.13	1.71	0.15	...	0.32	0.67	1.05	0.21	0.16	0.13
Efficiencies:														
Boiler	81.5	74.9	77.6	77.6	79.7	79.8	79.4	82.2	82.5	79.9	79.6	82.7	80.0	80.3
Boiler and economizer	...	81.5	...	71.5	75.9	74.5	73.5	78.5	79.1	67.6	75.7	78.7	70.9	69.9
Boiler and air heater	87.6	83.3	85.4	81.0	83.6	82.8	...	88.9	85.7	...	...	...	85.2	79.7
Boiler and economizer and air heater	...	87.2	...	...	80.4	...	87.3	...	88.2	87.5	85.7	88.6	...	85.6
Stack temperature, deg. fahr.	292	308	286	324	241	276	209	232	242	249	329	225	270	277
	...	449	449	456	313	335	344	413	333	347	435	325	374	589
	466	...	...	...	84.8	85.1	...	...	...	...	...	...	84.8	...
	...	...	...	...	87.5	...	...	...	...	...	...	...	84.7	...

* Steaming economizer.
† Bituminous coal, cols. 1 to 13; oil, col. 14.

Proper proportions of velocities and flow areas and their relations to fuel cost, load factors, and first cost are factors which enter into the design of economizers and air heaters and steam reheaters.

Table 1 gives a comparative analysis of a number of typical combinations based upon their performance per foot width of furnace. It will be noted that the additive efficiencies due to the air heater considerably exceed those characteristic of economizers.

Some of the lower efficiencies of stoker-fired units are due to the assumption of a progressively increasing loss due to unconsumed carbon in the ash, which with more recent designs using preheated air is not warranted. Measurement of unconsumed carbon in the ash is not so easy to determine with pulverized coal and has been assumed to be relatively small. In making up the table, no account has been taken of the cost from car to burning equipment.

In conclusion it may be said that the improvements in design of fuel-burning equipment have resulted in an increase of rates of combustion which is bringing about an evolution in the design of the heat-absorbing equipment, which is progressing toward a standardization of details and proportions and away from more expensive and, to an extent, experimental installations.

While higher pressures, higher temperatures, and greatly increased rates of fuel input have not required any material changes in the fundamental design of the steam-generating and heat-absorbing equipment, they have required much advance in refinement of details of materials and construction to durably withstand the sudden and extreme changes in temperature and pressures characteristic of modern operating and service conditions.

For the isolated or industrial plant an equipment consisting of a properly proportioned combination of steam-generating or boiler-heating surface with an air heater, because of its simplicity and low cost of operation and upkeep, will pay a good return upon the first cost.

For central stations where the factors involved are more numerous and complicated, economizers or air heaters or both have become necessary, primarily because of the relation of fuel cost to selling price of the product.

Attention is almost daily called to the fact that while the last ten or fifteen years have witnessed an increase in the cost of most everything else, the cost of electric power has remained almost without change, and in many cases has been reduced. To this condition the improvements in design and construction of the steam-generating equipment and its accessories have contributed in no small degree.

FSP-51-54

Progress in Steam-Power Engineering

Contributed by the Power Division

Executive Committee: Vern E. Alden, *Chairman*, W. F. Ryan, *Secretary*, John A. Hunter, F. M. Gibson, and V. M. Frost

DURING the past year we have seen a great deal of intensive work in the consolidation of gains incident to past progress and the raising of the average standard of steam-station design and operation throughout the country, although there have been no strikingly new developments.

The developments, which during the last 15 years have made it possible to reduce by 40 per cent the fuel consumption per kilowatt-hour in steam-electric generating stations and to effect a reduction of more than 50 per cent in fuel consumption for power generated by the mercury-vapor process, are ample evidence of the intensive work done by hundreds of designing and operating engineers. One of the most important factors which has contributed to rapid progress is the free exchange of ideas and the pooling of operating experience through the medium of the A.S.M.E. and the Prime Movers Committee of the National Electric Light Association. Steps have been taken during the last two years by the A.S.M.E. Power Division to provide for the freer exchange of ideas between those of our members who are associated with the industries. It should be a source of satisfaction to the many designing and operating engineers, who by their work have contributed to the progress in steam-power engineering, to know that they have had a substantial part in bringing about the present era of prosperity in this country, resulting in no small measure from the abundant supply of cheap power with which the efforts of each man in industry have been supplemented. It should be no less a source of satisfaction that the rapid progress has been so largely the result of full cooperation and the pooling of ideas and operating experience, for where else than in the United States could this have happened?

Some Major Objectives Attained

It is interesting to take stock of some of the major objectives actually attained:

1 The types of fuel preparation and burning equipment which are available permit the efficient burning of coal varying in grade from lignite to anthracite.

2 Fuel-preparation and burning equipment is available which is sufficiently flexible so that coal of widely varying grades may be burned in connection with a single installation.

3 Fuel-burning equipment, boilers, and boiler accessories are available which permit the generation in a single boiler unit of the steam required for a 75,000-kw. turbine.

4 Single-cylinder turbines of 75,000-kw. capacity, tandem-compound turbines of 160,000-kw. capacity, and cross-compound turbines of from 200,000-kw. to 300,000-kw. capacity are available.

5 Equipment is available for operation at steam pressures up to 1400 lb. per sq. in. gage and steam temperatures up to 750 deg. fahr. (There is considerable evidence to indicate satisfactory operation at even higher temperatures.)

6 Equipment is available for the reheating of steam withdrawn from the lower stages of the turbine, using either the heat from flue gases or the latent heat from high-pressure steam.

7 There is available equipment for heating feedwater by means of the regenerative cycle, using from three to five stages of heating.

8 Furnace water-wall construction is available which eliminates to a great extent the use of refractories in the portions of the furnace walls, which are subjected to the highest temperatures.

9 Condensing equipment is available which will maintain high vacuum with low investment and operating costs.

10 Economizers and air heaters are available by means of which the losses in the flue gases are reduced to very low values; and the investment and maintenance incident to the use of this equipment are reasonable.

11 There is available commercial equipment for operation on the mercury-vapor–steam cycle, each unit in the boiler house being capable of developing up to 20,000 kw. capacity.

12 There is available combustion-control equipment which will maintain day in and day out in the boiler house, operating efficiencies closely approximating test efficiencies.

13 For the average power station it has been possible to reduce the make-up water to approximately 1 per cent, which may be easily evaporated with an inappreciable loss due to the degrading of heat. For boiler plants requiring a high percentage of make-up, equipment is available which makes it possible to maintain conditions of operation and a degree of reliability and availability of boiler units closely approximating that for boilers operating on distilled water.

14 The coal- and ash-handling equipment now available make very low labor costs obtainable.

It is possible, assuming favorable conditions, to produce power by using the straight steam cycle with 12,500 B.t.u. per kw-hr. of net send-out, and, assuming the same favorable conditions, to produce power with from 11,000 to 10,000 B.t.u. per kw-hr. of net send-out by using the mercury-vapor–steam cycle of generation.

We have available today all of the equipment and the knowledge required for the assembly of this equipment and for its operation to permit us to build highly efficient generating stations, varying in size from the superpower station of 1,000,000-kw. capacity to the industrial plant of 1000-kw. capacity.

Problems Suggested for Study

It is interesting to speculate as to the possibility of future developments in research which will lead to the reduction in fuel consumption per unit of output. It will, however, be more profitable and certainly more effective in the reduction of the costs for power available to our generation, if we concentrate on a study of the best use of the knowledge and the equipment which are now available to us. That we may make further appreciable reductions in the total cost of power and steam, an intensive study of certain problems is urged. The responsibility for this study lies with the engineers who design the generating stations and boiler plants, the engineers who design the equipment, and the engineers who operate these plants. The responsibility for making major decisions rests with the executives of the operating companies, but the responsibility for proper guidance of the executives lies with the engineers. The problems suggested for study are:

Reliability. The need for steam-station equipment which will be available for operation during the largest possible percentage of the hours in the year transcends all other needs. Definite progress has been made in supplying this need. The almost universal use of the closed system of ventilation for generators has re-

duced the outage hours by at least one-third. The use of water-cooled walls for boiler furnaces has, by the elimination to a large extent of refractories and the reduction in the maximum rate of heat absorption by surface which "sees" the fire, reduced the outage time for boiler units by at least an equal amount. Some progress has been made in reducing the outage time for turbines, but there is still room for further improvement. It is mainly to the turbine designer and the engineer carrying on research in blade structures and metallurgy that we must look for further help in improving the reliability of the turbine.

The improvement of design details in connection with major equipment; the careful selection of auxiliary equipment; the division of the auxiliaries for each large major piece of equipment into two or more units; the careful design of piping, giving full weight to the operating man's viewpoint; and the carrying through of continuous and adequate inspections which will permit the scheduling of much of the maintenance and repair work for convenient periods when equipment is not needed for operation; these and other related work will go far toward reducing the time during which equipment is not available for service.

Investment. The fact that the fixed charges allocated to each kilowatt-hour on an average exceed the fuel costs by at least 50 per cent emphasizes the possibilities of a reduction in the total cost of power through a reduction in the investment. During the last three years the emphasis has been more largely placed on the reduction of investment, and worthwhile results have been obtained.

Reserve Capacity and the Date of Starting New Power-Station Extensions. On an average, the sum of the fixed charges and maintenance on reserve capacity constitute at least 10 per cent of the total cost of power on the station bus bars, and their combined value is equal to 25 per cent of the fuel cost. The importance of reducing the investment in reserve capacity to the lowest figure consistent with the maintenance of reliable service is self-evident.

The accurate prediction of the growth of power loads so as to defer extensions as long as possible and the advance planning of work which will make high-speed construction schedules possible will permit the reduction in fixed charges on unproductive investments.

Size of Units and Stations. No more important problem confronts the power-station engineer than the choice of size of units to be used in the turbine room and boiler house. Choice of units which are either too large or too small will increase the total cost of power on the station bus bars. The decision as to the ultimate size of a given station is one which can seldom be made with exactness at the time that the initial installation is made. This question is directly related to the geographical distribution of the load and the design of the transmission system.

Interchangeability of Equipment and Production in Development Costs. There is probably no single factor which will contribute more to the simplification of operation and maintenance, and the reduction of the fixed charges allocated to each kilowatt-hour generated, than the use, in a station, of main generating units having interchangeable parts. The argument for interchangeability of parts holds with almost equal force for other equipment in the station, and particularly for the important auxiliaries, such as boiler feed pumps, condensate pumps, and circulating pumps. In order to obtain interchangeability, it is justifiable in most cases to make the decision as to size of units on the basis of operating conditions and the system load as they will be when the station is half completed, rather than on the basis of conditions during the first year of operation.

A high degree of interchangeability of equipment in a given station and also interchangeability of equipment in different stations operated by the same company will tend toward reduced investments through reduced development and reduced manufacturing costs.

Quality of Fuel Burned. In many cases two or more kinds of fuel are available. Careful studies must be made to determine the wisdom of designing the fuel-preparation and burning equipment for burning one of these kinds of fuel or of making provision for burning several different kinds of fuel. The continuity of supply and the probable trend of costs of fuel must be considered.

Location. The choice of location for a new power station bears definitely on the cost of power delivered to the user, as it affects the cost of construction, the cost of fuel, the cost of operating labor, and the investment in the transmission system.

Simplification and Ease of Operation. Much can be done in the way of simplifying operation by uniformity of design throughout a given station and by the elimination of details which can readily be dispensed with. The designing engineer should, in making each decision, give due weight to the viewpoint of the man who must operate the station. The designing engineer should in every case transmit to the operating organization a clear-cut statement as to the provisions which he has made for various operating contingencies and not depend upon the art of mind-reading, which is not now well developed among operating engineers.

There is a crying need in connection with our newer stations for better operation. The concentration of responsibility for operation in the hands of fewer men, the vastly increased value of the avoidable losses under the control of each operating man, and the unavoidable complexity of the equipment which we must use to obtain a fuel economy of 15,000 B.t.u. per kw-hr., or lower, all justify and make imperative a high degree of intelligence in the men who operate our stations. Intelligence is not enough, however. These men should be given every opportunity to acquaint themselves with the details of construction and the operating principles which underlie each piece of equipment. One of the most important opportunities of which we are now in large measure failing to avail ourselves is that of making provision so that to a greater extent the results of the work which is being done by the N.E.L.A. Prime Movers Committee and the A.S.M.E. Power Division shall filter down to these men who are actually operating the stations. Their help is needed if power costs are to be further reduced.

These problems are intimately interrelated. They form the basis for an intelligent attack on any power problem whether it involves the construction and operation of a 1,000,000-kw. superpower station, or a boiler house for an industrial plant. An intimate understanding of the fundamental principles of steam-station engineering and a knowledge of the work done by other engineers, for which there is no substitute, is presupposed. Clear thinking, which will enable the engineer to pick the solution called for by a particular set of conditions, and a full realization that the same type of solution and the same kind of results are not to be expected in connection with a 1000-kw. station and a 500,000-kw. station, are the necessary equipment of the engineers who are to effect further reductions in power costs.

It is questionable if any branch of engineering presents a more interesting problem, or one which is more important to our whole country, than the production of an abundant supply of reliable power at the lowest cost. The splendid results already obtained make each additional reduction in cost of power increasingly difficult. Difficulties have, however, always served as the incentive and not the obstacle to engineering work.

VERN E. ALDEN, *Chairman.*

Progress in Hydraulics

Contributed by the Hydraulic Division

Executive Committee: Ely C. Hutchinson, *Chairman*, H. L. Doolittle, *Secretary*, R. L. Thomas, W. M. White, and L. F. Moody

THE progress report of the Hydraulic Division for 1927, submitted by Mr. Ely C. Hutchinson, was a comprehensive survey of the hydraulic field. Since the preparation of that report there have been no outstanding new developments or trends in machinery or methods, and few changes in practice. The intervening period has, however, witnessed intensive and successful effort along several of the lines of development noted last year, and particularly an increased striving to approach the goal of perfection in economy of design and operation. Hence this year's report must be in large part a follow-up or bringing up to date of last year's report.

This condition is to be expected, as in the nature of things progress in hydraulics is slow as compared with that in some other branches of engineering, as, for example, steam-power generation. Hydraulic machinery of many types has reached an advanced stage of development. Theoretical maximum efficiencies have already been closely approached in the turbine field, recent reaction-turbine tests having shown efficiencies up to 94 per cent, although there is a larger margin for improvement in part-gate turbine efficiencies and in pumping equipment. Also changes must come rather slowly, as several or often many years are spent in the preliminary study, design, and construction of hydraulic works.

It seems to be a fact that there has been a slowing down of water-power development in the United States, although the same cannot be said of Canada, which in relation to demand is more richly endowed with low-cost water-power sites which do not have to compete with cheap local fuel supplies. The past year is notable for the important hydraulic-power plants which were put into operation, but these have been in the making for several years and are more or less familiar to hydraulic engineers. In spite of unusually favorable financial conditions, comparatively few important projects have been undertaken recently in the United States. This slowing down has been ascribed to two causes or groups of causes: first, political agitation and governmental interference or indecision, extraordinary taxation, and the withdrawal of some of our most important rivers and sites from the provisions of the Federal Water Power Act; and second, the economic condition brought about by the decreased cost of steam-generated power and the previous development of the cheapest and most accessible water-power sites. Also it is believed by some that the falling off is temporary, as the industry has always been subject to periodic fluctuations.

Economics of Hydraulic Power

The outstanding recent trend in the hydraulic-power field is economic rather than mechanical; that is, the best efforts of power engineers, manufacturers of equipment, and users thereof are being devoted to lowering the cost of water-power development, improving the economy thereof, and working out the best economic use of water power in coordination with steam power. With the growth of interconnection, the all-hydroelectric generating system is becoming a thing of the past in the United States. The combination of hydro and steam power makes possible the economic justification of many otherwise impracticable water-power projects. Formerly it was possible and customary to justify a hydro development, particularly a run-of-river project, in comparison with steam power by evaluating the operating replacement cost of the equivalent steam-generated energy, or often the saving in fuel cost alone. This is no longer possible on account of the great advances in the economy of generation at steam-electric plants. As the efficiency of hydraulic-power generation has already approached the theoretical maximum, it is not possible to meet steam competition by improvement in efficiency alone, and recourse must be had to lowering the investment cost through simplification of layout, elimination of nonessentials, and improvement in design; to the extensive overdevelopment of streams with respect to minimum dependable flow and continuous capacity; to the use of additional steam units to carry load increments until such time as the load has grown sufficiently to absorb the output of the next hydroelectric step; and to realizing, utilizing, and capitalizing the advantages of hydro other than its energy-replacement value. Among these may be mentioned the capacity value or peak service of even a small amount of hydro energy, as from a variable-flow stream plant during minimum flow, when properly applied; the mechanical ruggedness and quick-starting properties of hydraulic units; diversity of maintenance work and station trouble; improved reliability and distribution economy obtained by a strategic arrangement of points of supply from the independent hydro and steam sources; stability of annual cost; and sometimes the location value of hydro plants as, for example, a small local plant at the market end of a long transmission line the economy of which depends on the maintenance of a high line load factor.

Another former practice which is being and should be discarded is the requirement that the development of a water-power project must effect a handsome saving or profit as compared with an all-steam supply. It is encouraging to note that in the case of a recent important project the backers of the development stated at once that in dollars and cents the project might not effect any immediate saving, but pointed out that if the costs of hydro power and steam power are approximately the same, the desirability of conserving our fuel resources should throw the balance in favor of hydro power.

A noteworthy current activity is the revamping and in some cases the complete reconstruction of old hydraulic plants.

A phase of the hydraulic-power business which is receiving considerable thought is practice in drawing up contracts and purchasing equipment. As noted last year, the manufacturers have felt that, owing to competition and perhaps some slowing down of business, prices have been very low and have left little margin for experimental work. On the other hand, the users of equipment have pointed out the keen competition with steam power and the need for rock-bottom prices. A solution that has been offered is an improved form of bonus contract with less stress on high guaranteed efficiencies and a reward for efficiency in excess of the guarantee. An interesting example of such an arrangement which worked out satisfactorily to both parties is the recent replacement of the buckets on the original Big Creek impulse wheels of the Southern California Edison Company. The power company agreed to pay the manufacturer a certain amount for the new buckets and a bonus of so much per kilowatt for any increase in output obtained with the new buckets over

that obtained under like conditions with the old buckets. A considerable increase in efficiency and capacity was obtained. The manufacturer immediately devoted his bonus to further experimental work, as a result of which still more efficient buckets were developed.

There is an encouraging tendency in the West to effect cooperation between irrigation districts and power companies. In several cases the utility company owns and operates the power plant built in conjunction with the irrigation dam, paying the district a stipulated charge for the released water.

There is no cause for pessimism as to the future of water-power development. Necessity is a good stimulant. The working out of the various problems mentioned, together with the undoubted popularity of water power, should insure the development of large additional amounts.

Dams

The failure of the St. Francis dam in California has received much publicity. The causes have been thoroughly investigated and are well understood, but cannot be discussed here. The failure has undoubtedly shaken the confidence of the public in the stability of existing dams, although the fault was in the foundation rather than in the dam itself. The lesson seems to be that the work of municipalities should be subjected to supervision just as much as privately owned dams. It is to be hoped that in the effort to remedy the situation and restore public confidence in the safety of dams, extreme legislation will not be passed unnecessarily increasing the size and cost of the structures.

The completion of the principal tests on the Stevenson Creek Arch Dam and the publication of data and findings in the Proceedings of the A.S.C.E., May, 1928, Part 3, under the auspices of the Engineering Foundation, are an important achievement and have done much to put the design of thin-section, and particularly arched, concrete dams on a scientific basis. As described in the test report, tests made by Professor Beggs with a celluloid model one-fortieth of actual size indicated the probability that deflections in an arch dam can be predicted accurately from tests on models. Since the printing of that report, tests have been conducted with a concrete model at Boulder, Colo., which confirm the previous important findings regarding prediction of deflections in full-sized arch dams.

As a result of research work and experience there is increasing confidence in less-than-gravity-section types of dams. Several multiple-arch dams have recently been constructed. The Coolidge dam of the U. S. Indian Service on the Gila River, Ariz., recently completed, is a unique multiple-dome type.

There is an increasing use of rock-fill dams with thin reinforced-concrete faces.

Much study has been given to the design of dam aprons to prevent erosion of the foundation at the toe. The Conowingo and Norwood dams are notable examples of apron design based on experimental research.

There is an increased tendency to install permanent and substantial crest gates on dams rather than renewable flashboards. Clear-crest dams have become the exception rather than the rule. There still remains considerable room for improvement in the design of deep-crest gates and their operating mechanism to insure simple and reliable operation and low maintenance as well as investment cost. Recent floods in several sections of the country, particularly in California, with runoffs in many cases twice as great as any previously recorded, have shown the necessity for more liberal design and substantial construction of crest gates.

In 1928 two especially large roller crest gates or rolling dams, each 115 ft. long and 18 ft. high, were put into operation at the Bellows Falls plant of the New England Power Company on the Connecticut River.

As a means of rapidly draining reservoirs in times of emergency, an alternative for additional crest gates is the installation of discharge valves. There is an increasing demand for an inexpensive valve of large capacity. Such a valve need not be efficient. It will be called upon to function perhaps only once in several years, but at such a time must be capable of discharging a maximum quantity of water in a minimum time.

A feature of masonry dams with respect to which there is considerable difference in practice is the provision of inspection galleries, particularly for lower-head dams. Standardization in this respect may not be desirable, but there appears to be room for the dissemination of information and the establishment of principles of good practice.

An important subject of current investigation is the proper proportioning of concrete and methods of mixing and pouring, especially in northern climates. Attention has been called to a number of dams recently repaired which are less than ten years old.

Important Hydraulic-Power Developments

An unusual number of hydroelectric plants which are noteworthy either for size or special features or both have been put into successful commercial operation recently. Among these may be mentioned the Conowingo development on the Susquehanna River, containing the largest turbines in physical size yet built on this continent; the Louisville development on the Ohio River, the largest semi-automatic plant in operation and containing the largest propeller-type wheels in physical size that have been installed in the United States; the Norwood development on the Yadkin River, with several unique features; the Bucks Creek plant on the Feather River, with the highest head yet developed in America (2562 ft.); the Big Creek 2A plant, containing impulse units of record capacity (56,000 hp.); the Lake Chelan development in the State of Washington; the Upper Tallassee development in Alabama, the Bellows Falls development on the Connecticut River; the Rocky River development in Connecticut, the first important plant in America to use in large part pumped water; the small automatic plant built on "stilts" over the Kentucky River at Dam No. 7; the Glines Canyon plant in the State of Washington, the Spaulding No. 3 plant in California, and the Alouette plant in British Columbia, all notable automatic plants; and the extension of the Great Falls plant, Manitoba, containing propeller-type wheels notable for size and high head (56 ft.). Data regarding these installations may be found in the reports of the Hydraulic Power Committee of the National Electric Light Association on "Bibliography of Hydraulic Subjects and Manufacturers Statements" for 1927 and 1928. Special features will be mentioned hereinafter.

Important projects now under construction are the Paugan development on the Gatineau River, the Alcoa development (Chute-à-Caron) on the Saguenay River, and the Back River development in Canada; Jordan Dam (Lock 18) on the Coosa River; the Saluda, Waterville, and Oxford developments in the Carolinas; the Lower Tallassee development in Alabama; and the Fifteen Mile Falls development on the Connecticut River.

Turbines and Accessories

The propeller-type turbine continues to hold a prominent place in hydro-plant equipment. The completion of the Great Falls, Manitoba, plant, with a total installation of six high-speed wheels each rated at 28,000 hp. at 56 ft., after a lengthy experience with the first units, is worthy of note.

Probably the outstanding current development in the turbine field is the adjustable-blade runner, which with manual operation,

requiring the shutting down of the unit for adjustment, makes possible increased output under reduced head during flood flows, when efficiency is not a consideration. The largest units of this type in America will be the wheels rated at 8800 hp. at 26 ft. and 85.7 r.p.m. in the Back River plant in Canada, now under construction.

When the turbine-blade angle is changed automatically, high efficiency is obtained at part load, thus overcoming the principal disadvantage of the propeller-type turbine. The first installation of this type in America will be at the Devils River plant of the Central Power & Light Company in Texas, with a wheel rating of 1900 hp. at 33 ft. and 277 r.p.m. Based on laboratory model tests, the manufacturers expect the wheels to exceed 80 per cent efficiency from 575 to 2300 hp., and 90 per cent from 950 to 1575 hp.

In reviewing the year's progress, special mention must be made of advance in the design of impulse wheels, which is of particular importance in the West. Official tests of the Bucks Creek wheels showed efficiencies of 85 per cent or better over a considerable range.

The increase in cooperation between power companies and irrigation projects has emphasized the importance in such cases of obtaining high efficiencies over a wide range in head. The Melones plant of the Pacific Gas & Electric Company is a notable accomplishment in this respect.

Governors are being equipped with new control devices to adapt them for automatic or semi-automatic plant operation. The electric governor drive is being used in an increasing proportion of new installations.

Many turbines have recently been equipped with automatic air admission valves for increasing part-gate efficiencies and particularly for facilitating operation of the direct-connected generators as synchronous condensers for power-factor correction and voltage regulation.

The effort to reduce the cost of low-head turbines has led to the construction of turbine parts such as distributer rings, stay vanes, and head covers of welded plate and structural steel.

Repair of pitted turbine buckets by electric welding has become standard practice. Efforts to find a cheap substitute such as type metal, which will serve as a cheaply applied but durable filler, have as yet been unsuccessful.

Several additional rubber guide bearings have been installed on large turbine shafts, and those previously installed continue to give excellent service with negligible wear.

There is an increasing use of large butterfly valves instead of head gates in turbine intakes. The principal advantage is the tight seal which can be obtained, with consequent reduction in leakage during shutdown. The largest valves of this type, with a diameter of 27 ft., are in operation at Conowingo. These valves are sealed with rubber rings or tubes $\frac{1}{2}$ in. thick and $3\frac{1}{2}$ in. outside diameter imbedded in the casing around the valve disk and held in place by steel keepers. These seal rings have exceeded all expectations. A test with normal head pressure on the gate and a hydraulic pressure of 80 lb. per sq. in. applied to the sealing tube showed a leakage of less than $\frac{1}{2}$ cu. ft. per sec. A test on a $12\frac{1}{2}$-ft. butterfly valve at Rochester, N. Y., on a unit operating at a head of 130 ft., showed a leakage of less than $2\frac{1}{2}$ gal. per min. Another test on a 102-in. valve under a pressure of 100 lb. per sq. in. showed a leakage of 4.1 gal. per min.

Cylinder gates outside of the wicket gates have been used in at least two recent turbine installations to reduce leakage to a minimum.

Other Features of Hydroelectric-Plant Design, Construction, and Operation

There has been a notable development and use of equipment for the semi-automatic or remote control of individual generating units within a station, as at Louisville, and for the automatic operation of a plant as a whole, particularly when it is suitably situated with reference to a larger master station. Automatic operation facilitates the development of isolated water-power sites and serves in some measure to meet the decreasing cost of steam power. Particular interest centers on plants in which there is no governing equipment. Important automatic plants which have been put into operation recently have already been mentioned.

The overlapping of the heads of plants in series on the same stream is becoming more common. Under this arrangement headwater at the lower plant is normally high enough to cause backwater in the tailrace of the upper plant.

As a means of reducing the first cost, the so-called outdoor or semi-outdoor type of station has recently been adopted in several instances. The most notable example is the Norwood plant, with a close-fitting individual removable plate-steel housing for each generator. Another interesting feature of the Norwood plant is the improvement of overall plant efficiency by the installation of two different types of Francis unit. There are two relatively high-speed units and one with lower speed and capacity but higher part-gate efficiencies for load and frequency regulation.

In recent years much study has been given by many companies to the working out and following up of schedules or rules for the most economical distribution of load among units and stations. In the last analysis all of these schemes depended on the human element in that the operator was relied upon to carry out the proper loading. Recently a method has been developed for the automatic control of the loading of generating units and plants. The device, which in effect is a mechanical operator, is at the time of writing being installed at the Norwood plant. The equipment consists of instruments for the control, through the operation of the synchronizing motor on the turbine governor, of the load on a unit, while the governor continues to care for sudden load changes in the usual manner. In effect the device does what a skilled operator would do after a governor action in "touching up" the synchronizing motors on the units affected in order to restore or maintain the proper load. The system can be applied to distribution of load between stations, hydro or steam, as well as among individual units in the same station.

Another automatic-control device which functions through the governor-synchronizing motor is a frequency control. This seems to have been worked out independently in somewhat different form in two or three places.

Last year's report pointed out that the troublesome problem of penstocks for high-head plants has apparently been solved. It is interesting to note that the three plants in America with the highest heads—Balch, Bucks Creek, and Big Creek 2A, with heads of 2381, 2562, and 2419 ft., respectively—have used band-reinforced steel pipe, consisting of a welded steel core around which are shrunk seamless steel bands. With these bands the thickness of the lap-welded core can be kept within limits which insure good welds.

A need has been felt for an improved design of intake structure and gate mechanism for the control of a flow tunnel leading from the bottom of a deep reservoir. Vibration trouble has been experienced with two different types of regulating gate at part-gate openings.

Much attention is being given to power-house-crane design, long-distance water-level gages, continuous or intermittent water measurement by the use of venturi sections, etc., the removal of trash on racks by mechanical rakes, the prevention of ice accumulation on racks and turbine parts by electric heating, electric means of keeping fish from entering intakes, and mechanical fish elevators.

Pumping Equipment

The Rocky River generating and pumping plant mentioned in last year's report has been in successful operation since the spring of 1928. As the electrical capacity of the plant will not be required by the system until next year, full use has not been made of the plant this year. The reservoir was filled to an elevation 10 ft. below the maximum for the initial project. The pumps have handled several billion cubic feet of water. This project, the first of its kind in America, is probably the forerunner of numerous generating plants using in part pumped water. The project is fully described in a paper by E. J. Amberg, published in the *Journal of the A.I.E.E.*, August, 1928.

Progress in centrifugal-pump design has been similar to that in turbine design in the application of high-specific-speed runners of propeller or semi-propeller type in the range of low or moderate heads.

Testing and Research

There has been a great increase in recent years in laboratory experimental work and field testing. Several power companies as well as all the principal manufacturers now have well-equipped laboratories. There is an increasing use of models, not only for testing turbine performance but also for determining the proper design and layout of intakes, casings, draft tubes, valves, and even of an entire project, with power house, dam, tailrace, river channel, etc., all reproduced to scale.

The practice of making a field acceptance test of large turbines or of at least one of two or more similar units is becoming general, and the operating value of actual performance data is widely recognized. From June, 1920, to September, 1928, more than 175 individual turbine tests were made in the United States and Canada by the Gibson pressure-time method. About 120 wheels have been tested in the United States and Canada by Prof. C. M. Allen, using the salt velocity method, in addition to which many tests have been made by this method by power companies on the Pacific Coast.

The importance of research work, both in the laboratory and in the field, is more generally recognized.

A monumental work which should be of particular interest to those who wish to follow hydraulic research, is the report on "Hydraulic Laboratory Practice" which is being published by The American Society of Mechanical Engineers, sponsored and edited by John R. Freeman. This work consists of a translation of "Die Wasserbaulaboratorien Europas" (The Hydraulic Laboratories of Europe), with extensions of several of the original chapters to include recent researches and experiments, several appendixes describing European and American laboratories, and three appendixes treating of the principle of similitude as applied to experiments with models of hydraulic structures. It is hoped that the book will be available about the time this report is presented. The Society and hydraulic engineers everywhere are greatly indebted to Mr. Freeman for this work.

Bibliography

A complete bibliography of hydraulic literature may be found in a recent serial report of the Hydraulic Power Committee of the National Electric Light Association on "Bibliography and Manufacturers' Statements." It would be unnecessary repetition to present a bibliography with this progress report.

Other recent reports of the N.E.L.A. Committee which are of special interest to the members of this division are "Waterwheel Testing and Operating Records of Plant Discharge," March, 1928, and "Mechanical Reliability of Hydro-Electric Units," July, 1928.

Acknowledgment

The undersigned wishes to acknowledge his indebtedness and express his thanks to the many members of the Society and other engineers and manufacturers whose contributions and suggestions have been freely used in the compilation of this report.

Ralph L. Thomas.

Discussion

Byron E. White.[1] Referring to the subject of dams, the report states: "The lesson seems to be that the work of municipalities should be subjected to supervision just as much as privately owned dams. It is to be hoped that, in the effort to remedy the situation and restore public confidence in the safety of dams, extreme legislation will not be passed unnecessarily increasing the size and cost of the structures."

The writer is convinced that the following should be included so that an additional lesson may be learned from this failure:

Not only should the work of municipalities, as well as other public or semi-public enterprises, be subject to supervision by state or national commissions or competent authorities, but they should avail themselves of the services of one or preferably more consulting engineers, engineering firms, or geologists specializing in the particular type of construction or type of investigation involved and thoroughly competent to pass intelligent and final judgment upon the merits of the particular case. Privately owned works are usually constructed only after having obtained a sufficiency of competent technical advice respecting the particular engineering and geological problems presented. This is especially true of the larger and more prosperous companies, though often disregarded by smaller organizations.

It is to be hoped that this disaster, together with others which are still fresh in mind, will suffice to impress upon all engineers and executives, whether of privately or publicly owned works, the necessity, as well as the desirability, of assuring themselves so far as is humanly possible of the safety of their proposed structures, by adequate preliminary investigation, followed by competent supervision and inspection and, if necessary, further investigation during construction and even after completion.

The moral as well as legal advantage which will result from having taken all possible precautions against improper or dangerous design and construction should be obvious to any one having a sense of responsibility to his principals as well as to the public.

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Tests on Small Rotary Pumps

By DIPL.-ING. F. ASCHNER and DIPL.-ING. L. MATTHEUS, BERLIN, GERMANY

GEAR pumps and other small rotary pumps are used for pumping lubricating oil, cooling water, and other liquids to all kinds of machines and especially to machine tools and steam and internal-combustion engines. Published information on the behavior of these pumps under a variety of pressures, speeds, and oil viscosities is very scarce, even though forced lubrication by means of such pumps is the basis of modern engine construction. The Mechanical Laboratory of the Technical University of Breslau has attempted to remedy this lack of data by performance and efficiency tests on these pumps under various working conditions. These tests were made on the usual types of lubricating pumps: rotary piston pumps as used on the Hispano-Suiza machine (Fig. 3) and gear pumps for machine tools and automobile engines as constructed by Ludwig Loewe, Berlin and Breuer, Hoechst a/M (Figs. 5 and 11).

The Loewe gear pump used had gears with pitch diameters of 1.575 in. and a theoretical discharge Q_{tA} of 0.87 cu. in. per revolution and was operated at various discharge pressures P_d and at speeds between 250 and 1000 r.p.m. with an oil viscosity of 250 sec. Saybolt (corresponding to a temperature of 120 deg. fahr. for the oil used in all the tests as may be seen from Fig. 4). Test results are shown graphically in Figs. 6 to 10.

The Breuer gear pump used had gears with pitch diameters of 0.95 in. and a theoretical discharge Q_{tA} of 0.19 cu. in. per revolution. Tests were run at various discharge pressures P_d between 0 and 42 lb. per sq. in., various speeds between 250 and 2500 r.p.m., and various oil viscosities between 1200 and 32 sec. Saybolt (corresponding to temperatures between 85 and 190 deg. fahr., as may be seen from Fig. 4). Some test results for a speed of 2000 r.p.m. are plotted in Fig. 12.

A Hispano-Suiza rotary piston pump with a cylinder diameter of 1.1 in., a piston diameter of 0.95 in., a cylinder length of 1.55 in., and a theoretical oil discharge Q_{tA} of 0.6 cu. in. per revolution was also tested. The range of discharge pressures P_d was 0 to 210 lb. per sq. in., the range of velocities was 250 to 2500 r.p.m., and the range of viscosities was 1200 to 70 sec. Saybolt (corresponding to 85 to 190 deg. fahr. from Fig. 4). Graphs of test results are given in Figs. 13 to 19.

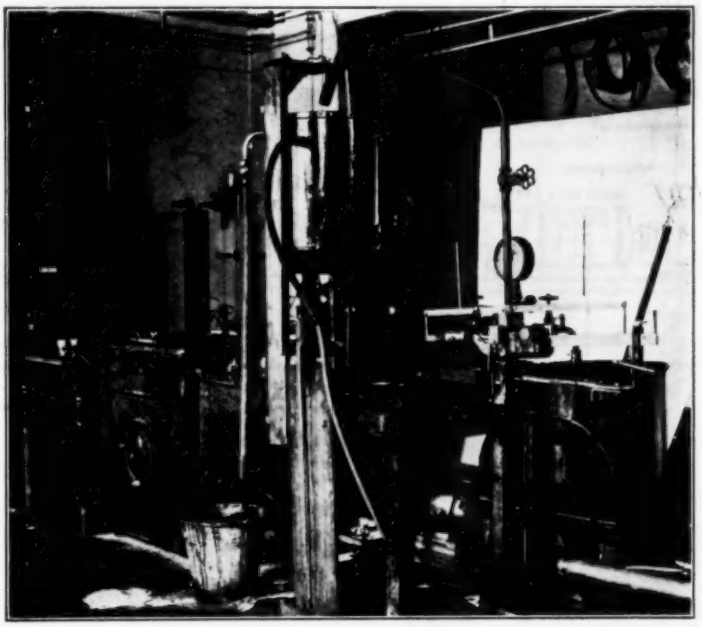


Fig. 1 Photograph of Test Apparatus

Contributed by the Hydraulic Division and presented at the Annual Meeting, New York, N. Y., December 3 to 7, 1928, of The American Society of Mechanical Engineers.

Comments on Test Results

Values of volumetric efficiency of the pump (actual oil discharged per minute ÷ theoretical pump displacement per minute) are important design data since each moving part of the main engine requires a definite quantity of oil per minute for proper lubrication and certain engines (e.g., internal-combustion types) will be seriously affected by too much lubrication. The net efficiency of the pump (power output ÷ power input) is of less interest because the power input to the pump is very small in proportion to the power output of the main engine.

The volumetric efficiency is influenced by two principal factors, the viscosity of the pumped oil and the clearances within the pump that are due to design, manufacture, and wear. An important factor in connection with gear pumps is the effect of eddies formed at the discharge side by the operation of the gears. These, however, reduce the net efficiency only. At the higher oil temperatures and consequent decreasing viscosities, the sluggishness of the oil decreases and the influence of clearances increases. Increased pressures P_d produce a decrease in volumetric efficiency in all cases. At higher speeds the influence of oil tenacity increases and the influence of clearance decreases. It is interesting to note that the influence of the oil temperature on the volume pumped Q_r is more constant and distinct than that of the viscosity. In nearly all cases the change of Q_r caused by the 35-deg. temperature change between 85 and 120 deg. fahr. (1200 and 250 sec. Saybolt from Fig. 4) equaled the change of Q_r caused by the 35-deg. temperature change between 120 and 155 deg. fahr. (250 and 110 sec. Saybolt), other conditions being equal.

The curves give the best and clearest representation of the test results, both for individual data and general characteristics. Therefore only a short discussion of the results will be given.

The Breuer gear pump for automobile engines was tested at low pressures only. At each oil temperature and speed, the pressures at which the actual discharge and the volumetric efficiency become zero were measured. At 2000 r.p.m. this limit changed from P_d equals 70 lb. per sq. in. at a viscosity of 1200 sec. Saybolt to P_d equals 14 lb. per sq. in. at a viscosity of 32 sec. Saybolt. These higher values at greater viscosities indicate that the obtainable pressure P_d depends on the clearance losses caused by the back pressure of the oil. At $P_d = 0$, the volumetric efficiency is nearly independent of the oil vis-

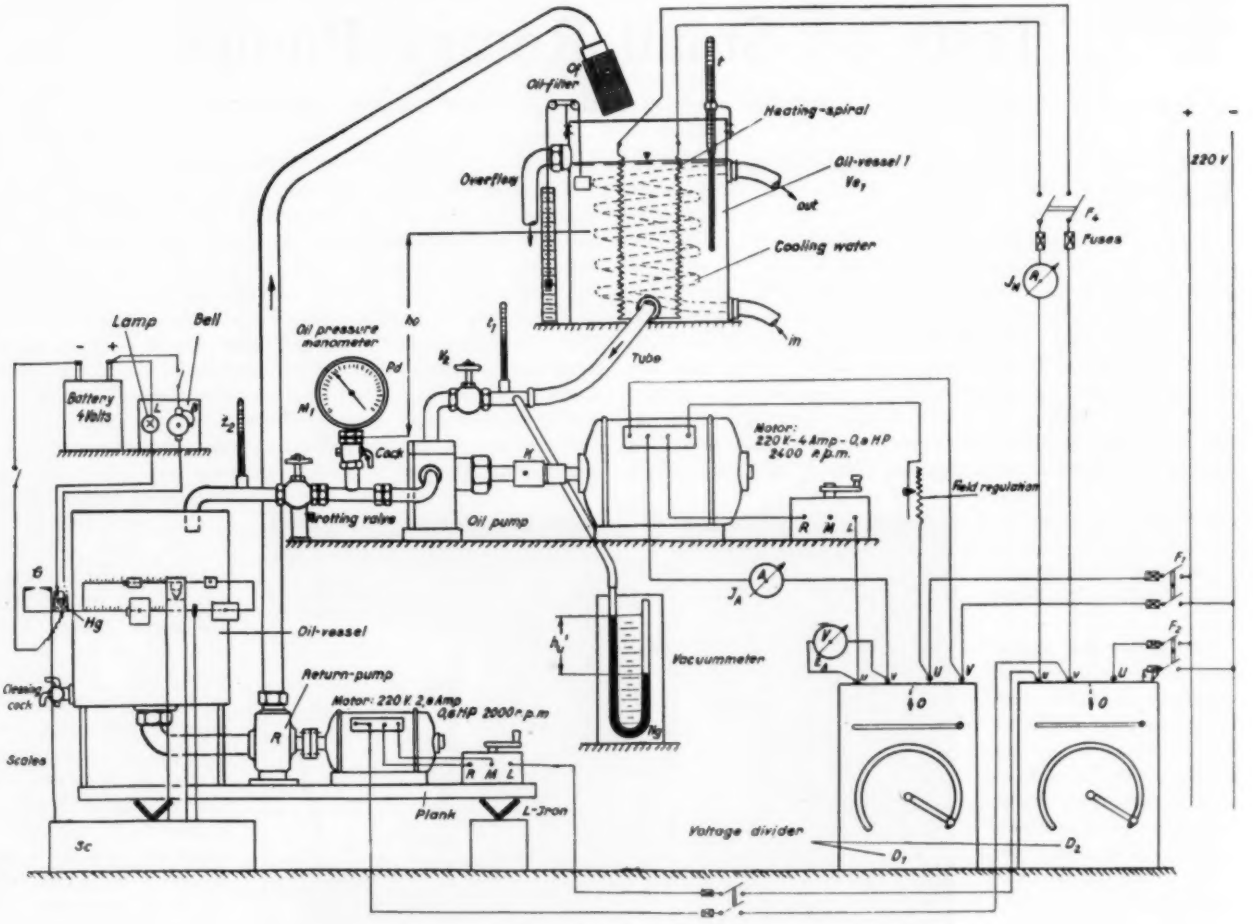


FIG. 2 DIAGRAM OF TEST APPARATUS

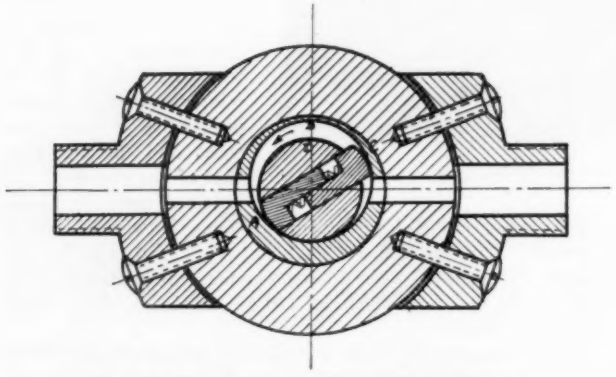


FIG. 3 HISPANO-SUIZA ROTARY PISTON PUMP

cosity and is always about 80 per cent except at lower speeds where there are maxima in excess of 90 per cent at a viscosity of 110 sec. Saybolt. At higher speeds (above 1000 r.p.m.) and at all pressures the volumetric efficiency increases with the viscosity as is shown in Fig. 12. As the discharge pressure P_d increases, the volumetric efficiency decreases, especially at low speed and low viscosity. At the testing speeds the maximum volumetric efficiency was not reached. The test results of the other two pumps need only be discussed in so far as they differ essentially from the Bteuer pump results. The volumetric efficiency of the Loewe gear pump is constant at speeds above 900 r.p.m. for pressures between 0 and 56 lb. per sq. in. At higher pressures maximum volumetric efficiencies were not reached. The highest pressure at high speed was in excess of

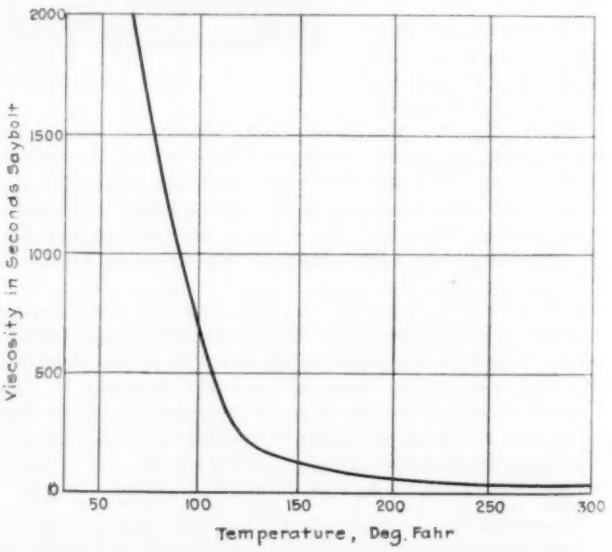


FIG. 4 VARIATION OF VISCOSITY WITH TEMPERATURE FOR OIL USED IN TESTS

170 lb. per sq. in. Speeds above 1000 r.p.m. caused excessively noisy operation.

During the tests of the Hispano-Suiza rotating piston pump, speeds higher than 1000 r.p.m. caused the pressure to mount to about 275 lb. per sq. in. at each oil temperature. This increased oil pressure produced by this kind of pump is caused by the tightening effect of the springs pressing the blades to the cylinder wall. The highest values of discharge pressure

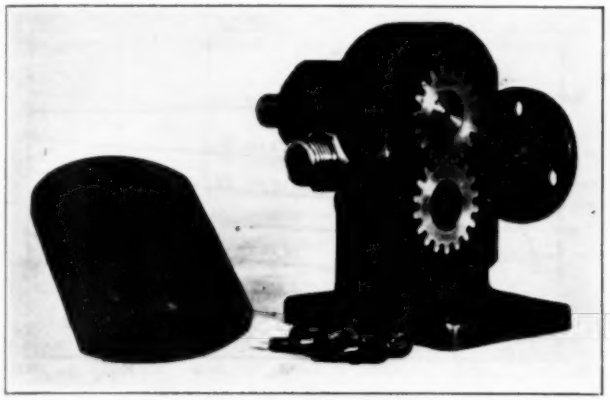


FIG. 5 LOEWE GEAR PUMP

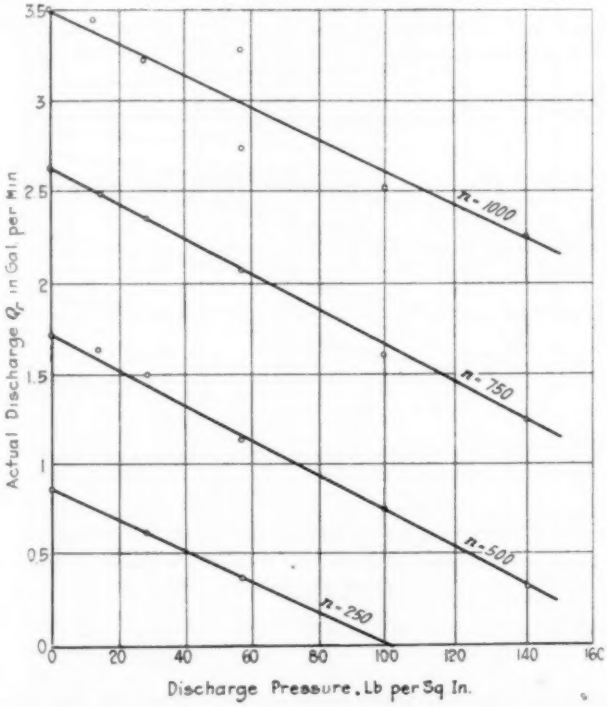


FIG. 6 VARIATION OF ACTUAL DISCHARGE Q_r AGAINST DISCHARGE PRESSURE P_d IN THE LOEWE GEAR PUMP FOR SEVERAL CONSTANT SPEEDS n

P_d at low speed were obtained (Fig. 16). At 500 r.p.m. these pressures vary from 250 lb. per sq. in. at a viscosity of 1200 sec. Saybolt to 200 lb. per sq. in. at a viscosity of 70 sec. Saybolt, i.e., the influence of the oil viscosity on the pressure of this pump is much smaller than it is on the pressure of gear pumps. However, excessive wear of the cylinder caused by the high blade pressure may occur. The volumetric efficiency shows a distinct maximum at all pressures and speeds between 1000 r.p.m. at a viscosity of 1200 sec. Saybolt and 2000 r.p.m. at 70 sec. Saybolt (Figs. 17 to 19), a phenomenon which may be attributed to the tenacity of the oil which cannot follow the rapid motion

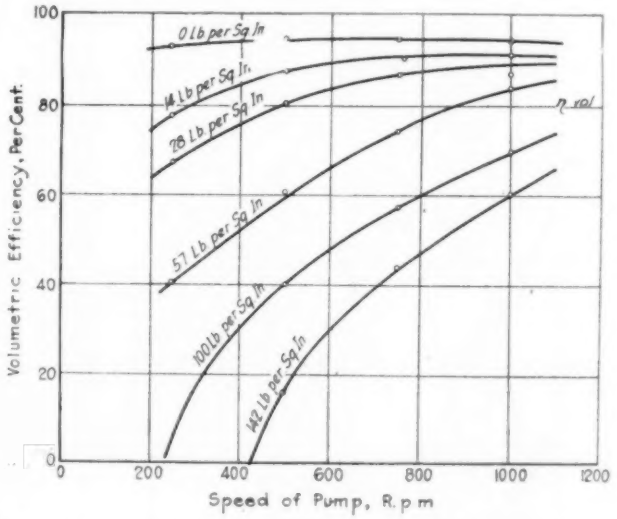


FIG. 7 VARIATION OF VOLUMETRIC EFFICIENCY OF THE LOEWE GEAR PUMP AGAINST SPEED AT SEVERAL CONSTANT DISCHARGE PRESSURES

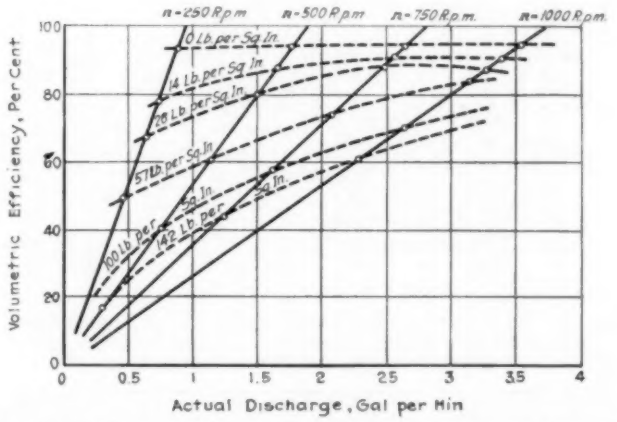


FIG. 8 VARIATION OF VOLUMETRIC EFFICIENCY AGAINST ACTUAL DISCHARGE IN THE LOEWE GEAR PUMP AT SEVERAL CONSTANT SPEEDS n AND DISCHARGE PRESSURES

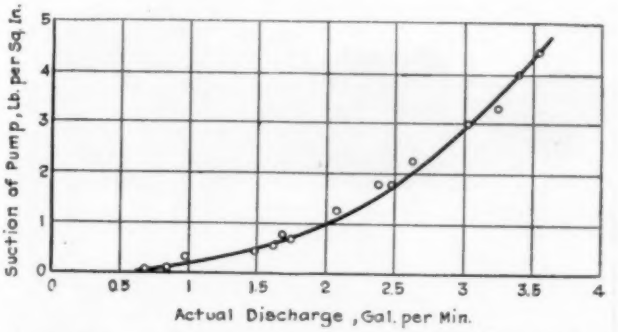


FIG. 9 VARIATION OF SUCTION OF LOEWE GEAR PUMP AGAINST ACTUAL DISCHARGE

of the blades. As this maximum is not reached in gear pumps, these latter are better adapted for the higher working speeds. Figs. 17 to 19 show curves of volumetric efficiency plotted

against actual discharge for various constant discharge pressures and speeds. These curves are useful for design purposes because they give volumetric efficiency and actual discharge for a variety of working conditions.

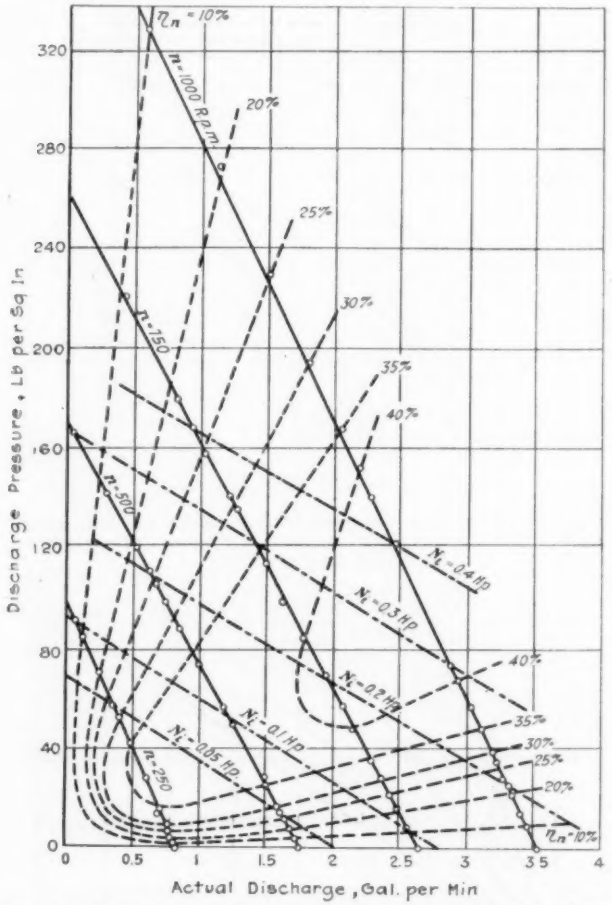


FIG. 10 VARIATION OF DISCHARGE PRESSURE OF LOEWE GEAR PUMP WITH ACTUAL DISCHARGE FOR VARIOUS SPEEDS n, NET EFFICIENCIES η_n, AND POWER INPUTS N_i

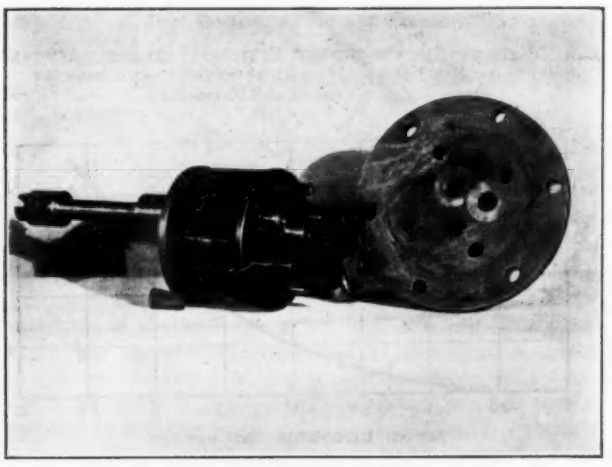


FIG. 11 BREUER GEAR PUMP

In spite of the high values of vacuum that often occur at the suction side of the pump these pumps have little suction when starting until they are primed. If convenient the oil should run to the suction side of the pump with at least a small pressure. The vacuum measured during the operation of the pump is proportional to the square of the oil velocity (Fig. 9). The power characteristic was obtained by a method used in testing centrifugal pumps (Fig. 10). Table 1 gives data for the most probable working conditions of these pumps.

The influence of viscosity has already been discussed. The volumetric efficiencies of the Loewe and Hispano-Suiza pumps are high at usual pumping conditions. The lower volumetric

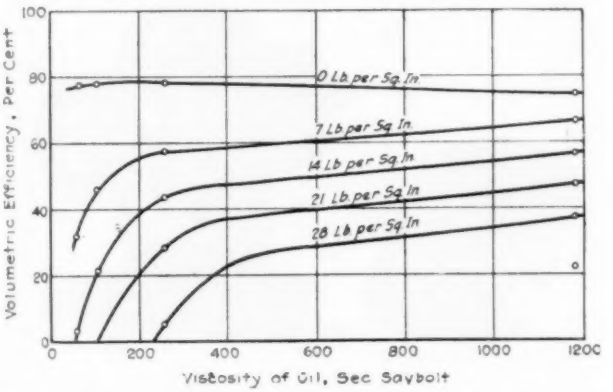


FIG. 12 VARIATION OF VOLUMETRIC EFFICIENCY WITH VISCOSITY FOR THE BREUER GEAR PUMP AT SEVERAL DISCHARGE PRESSURES. SPEED = 2000 R.P.M.

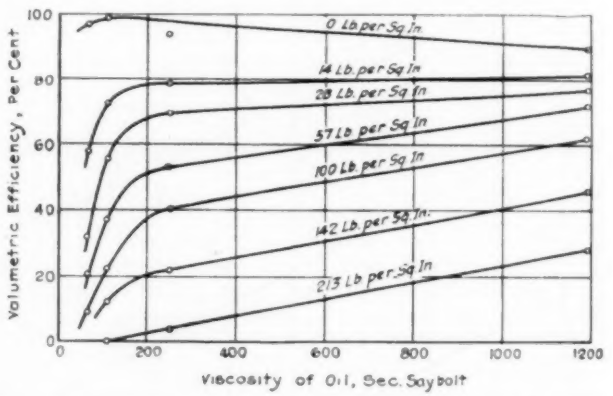


FIG. 13 VOLUMETRIC EFFICIENCY OF HISPANO-SUIZA PUMP PLOTTED AGAINST VISCOSITY OF OIL USED. VARIOUS DISCHARGE PRESSURES. SPEED = 500 R.P.M.

TABLE 1 USEFUL PUMP DATA

Loewe gear pump operated at 500 r.p.m. Viscosity = 250 sec. Saybolt

Discharge pressure, lb. per sq. in.	Volumetric efficiency, per cent	Net efficiency, per cent
0	88.0	...
14	87.4	29.4
28	80.0	36.0
56	60.6	37.6

Breuer gear pump for automobile engines operated at 2000 r.p.m. Viscosity = 250 sec. Saybolt

Discharge pressure, lb. per sq. in.	Volumetric efficiency, per cent	Net efficiency, per cent
0	78.0	...
7	56.9	2.8
14	43.0	3.9
21	27.2	3.8

Hispano-Suiza rotary piston pump operated at 1000 r.p.m. Viscosity = 250 sec. Saybolt

Discharge pressure, lb. per sq. in.	Volumetric efficiency, per cent	Net efficiency, per cent
0	90.0	...
14	79.2	18.95
28	74.1	29.5
56	65.6	36.0
98	56.4	29.0

efficiency of the Breuer pump can be accounted for by the small dimensions of the pump and the clearance losses which are relatively great. This kind of pump therefore should not be employed if high oil pressures are needed. A particularly good tooth formation is desirable. The influences which determine these efficiencies can hardly be calculated in a theoretical way, since the clearances depend in large measure on the design and manufacture of the pump; but, since nearly all pumps of this size resemble one of the tested types in design and construction, conclusions can be made from the test results as to the general characteristics of gear and rotary piston pumps.

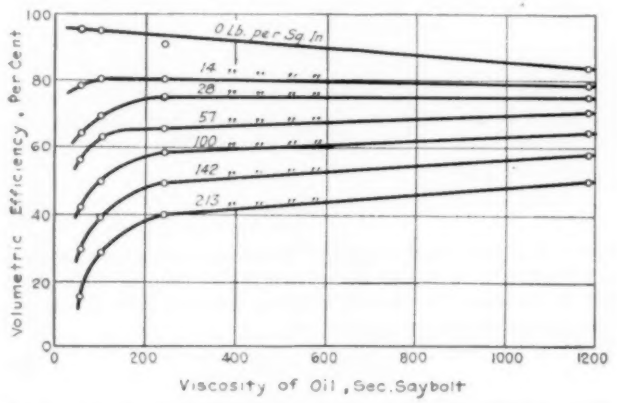


FIG. 14 VOLUMETRIC EFFICIENCY OF HISPANO-SUIZA PUMP PLOTTED AGAINST VISCOSITY OF OIL USED. VARIOUS DISCHARGE PRESSURES. SPEED = 1000 R.P.M.

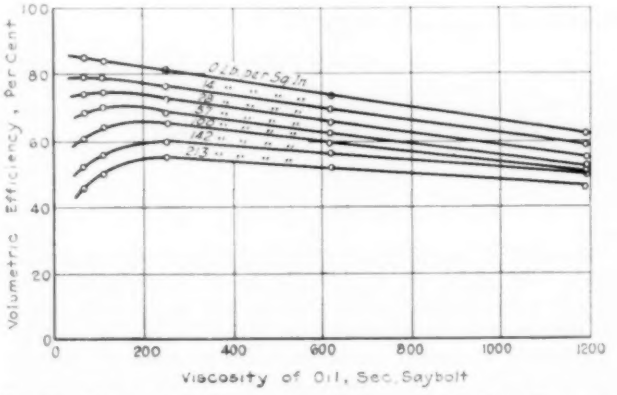


FIG. 15 VOLUMETRIC EFFICIENCY OF HISPANO-SUIZA PUMP PLOTTED AGAINST VISCOSITY OF OIL USED. VARIOUS DISCHARGE PRESSURES. SPEED = 2000 R.P.M.

ARRANGEMENT OF APPARATUS

Figs. 1 and 2 show the general arrangement of the apparatus used in conducting the test. The pump under test is directly coupled to an electric motor, the speed of which is controlled by the rheostat D_1. In the vessel Ve_1 the oil can be heated by an electric heater connected to the rheostat D_2, or it can be cooled by a water coil so that the required oil viscosity can be obtained at all times. The oil pressures on the pump can be changed by throttling of the valve V_1. Manometers M_1 and M_2 show the pressure and the vacuum produced by the pump, respectively; and thermometers indicate the temperature of the oil before entering and after passing through the pump and in the vessel Ve_1. After having passed through the pump the oil is weighed in the vessel Ve_2 by the scales Sc. The time for filling the vessel is measured by a stop watch and, since the specific weight of the oil is known, the output of the pump per

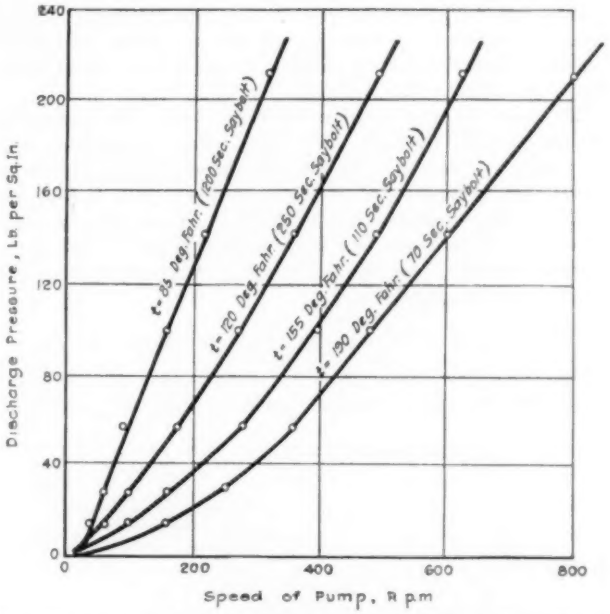


FIG. 16 HIGHEST DISCHARGE PRESSURES OF HISPANO-SUIZA PUMP PLOTTED AGAINST SPEED. VARIOUS OIL TEMPERATURES t AND VISCOSITIES. ACTUAL DISCHARGE AND VOLUMETRIC EFFICIENCY EQUAL ZERO

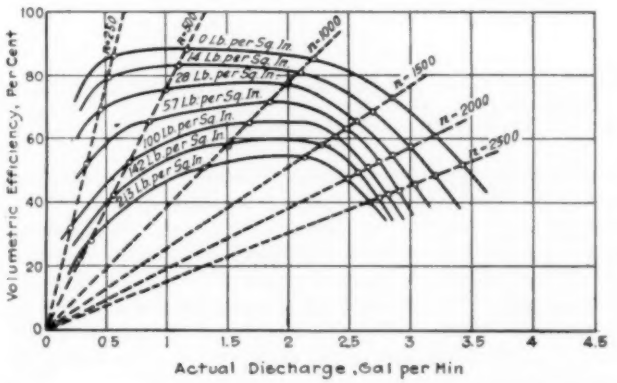


FIG. 17 VOLUMETRIC EFFICIENCY OF HISPANO-SUIZA PUMP PLOTTED AGAINST ACTUAL DISCHARGE FOR VARIOUS DISCHARGE PRESSURES. VARIOUS SPEEDS n. TEMPERATURE = 85 DEG. FAHR.

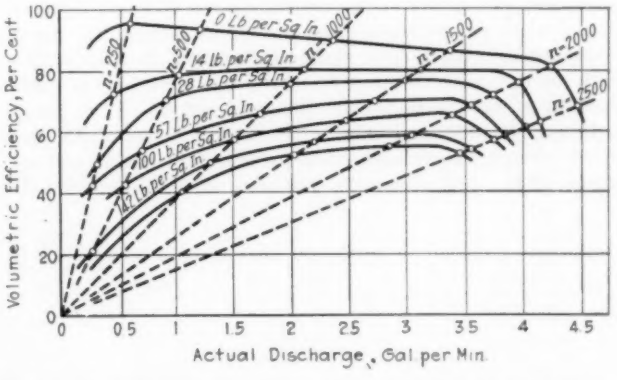


FIG. 18 VOLUMETRIC EFFICIENCY OF HISPANO-SUIZA PUMP PLOTTED AGAINST ACTUAL DISCHARGE FOR VARIOUS DISCHARGE PRESSURES. VARIOUS SPEEDS n. TEMPERATURE = 120 DEG. FAHR.

minute can be accurately calculated. The balancing of the scales is indicated by lamp L and bell B. From the vessel Ve_2 the oil is transferred by pump R to vessel Ve_1. All of these details are shown in Figs. 1 and 2.

The same oil was employed for all tests. Its properties (specific weight, viscosity, flash point) were determined, and the viscosity was varied by change of temperature according to the test requirements. Preliminary tests showed that the oil output and consequently the efficiency did not depend upon the distance of the vessel Ve_1 above the suction inlet of the pump. A small descent of the oil to the pump is necessary to avoid frequent need of priming.

Methods of Calculation and Preliminary Tests

The volumetric efficiency is the actual discharge per min. Q_r divided by the theoretical pump displacement per min.

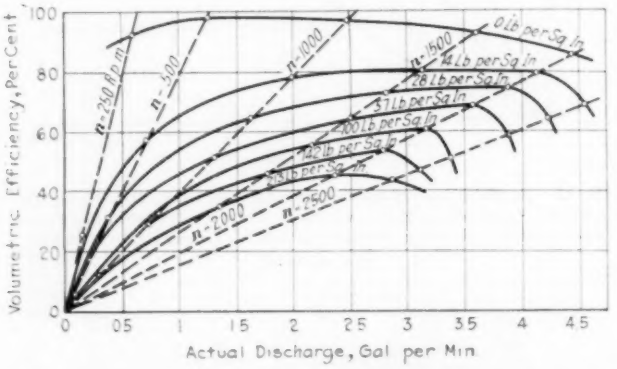


Fig. 19 Volumetric Efficiency of Hispano-Suiza Pump Plotted Against Actual Discharge for Various Discharge Pressures. Various Speeds n. Tempbrature = 190 Deg. Fahr.

Q_{th}, the result being expressed as a percentage. The theoretical output Q_{th} of rotary piston pumps is sometimes calculated incorrectly. The usual rotary piston pumps have a displacement piston eccentrically placed in a cylinder which is always divided into two chambers by blades forced out by springs (Fig. 3). The contents of the displacement chamber are discharged twice for each revolution. Calling the contents of the section $ABCD$ K sq. in. and the length of the chamber l in., the theoretical output in cu. in. per min. at n r.p.m. is given by

$$Q_{th} = 2Kln \quad [1]$$

The theoretical volumetric discharge of a gear pump is obtained by correct calculation of the double volume of liquid expelled by the teeth of one wheel from the spaces between the teeth of the other wheel. The clearance between the top of the teeth on the one wheel and bottom of the teeth on the other wheel must not be neglected. The real output of the pump Q_r in gal. per min. can only be determined by test. As previously stated the pumping time for a certain quantity of oil (20 or 40 lb.) was accurately measured by a stop watch. If this time is T seconds for 40 lb. and if γ is the specific weight of the oil in lb. per gallon, then

$$Q_r = \frac{40}{(T/60)\gamma} = \frac{2400}{T\gamma} \quad [2]$$

In these tests the effort was made to secure complete characteristics of the pumps, also data as to their power demand and net efficiency. The net efficiency equals the quotient, expressed as a percentage, of the power output of the pump divided by its power input. Since the driving motor and pump are directly coupled, the power input of the pump N_i equals the power output of the electric motor, or

$$N_i = \eta_e EI/746 \quad [3]$$

where N_i is the power input of the pump in hp., E is the motor voltage, I is the motor current in amperes, and η_e is the percentage efficiency of the electric motor. The power output of the pump is given by

$$N_o = Gh/550 \quad [4]$$

where N_o is the power output of the pump in hp., G is the rate of pumping oil in lb. per sec., and h is the head of oil in feet, that is, equivalent to the oil pressure p, which is in lb. per sq. in. The relation between p and h is

$$p = 12h\gamma/231 \quad [5]$$

where γ is the specific weight of the oil in lb. per gallon and 231 is the number of cubic inches in one gallon. Solving for h

$$h = 19.25p/\gamma \quad [6]$$

G can also be expressed in terms of γ

$$G = Q_r\gamma/60 \quad [7]$$

where Q_r is the actual discharge of the pump in gal. per min. Substituting Equations [6] and [7] in Equation [4]

$$N_e = \frac{Q_r(19.25p)}{60(550)} = pQ_r/1715 \quad [8]$$

The oil used in all the tests was tested for change of specific weight and of viscosity with change of temperature. The specific weight changed from 7.75 to 6.75 lb. per gal. when the temperature was increased from 65 to 230 deg. fahr. The viscosity as a function of temperature is shown in seconds Saybolt in Fig. 4. The flash point of the oil was 250 deg. fahr. Since the oil had a Saybolt viscosity of 1200 sec. at 85 deg. fahr., the operation of the pumps at lower oil temperatures was of no interest. The viscosity most frequently used in practice is about 250 sec. Saybolt and, consequently, this viscosity is of the most interest from the research standpoint. The test of the Loewe pump conducted at only one oil temperature was therefore carried out on oil at 120 deg. fahr. (viscosity of 250 sec. Saybolt from Fig. 4). Above 200 deg. the change of viscosity is small. The highest temperature used in the test was 190 deg. fahr.

HYD-51-3

New Aspects of Maximum Pressure Rise in Closed Conduits

By S. LOGAN KERR,[1] PHILADELPHIA, PA.

This paper discusses the various elements entering into the determination of the true maximum rise in pressure which may occur in closed conduits. It shows that, contrary to the usual belief, the maximum rise in pressure will not necessarily be produced when the full flow in a given conduit is cut off, but is a function of the initial flow from which the deceleration starts. The necessity of establishing the relation between the average conduit velocity and the rate of gate travel for various proportional strokes is cited.

Comparison is made of the computations by several of the accepted methods based on the elastic wave theory, and a translation of important studies made by M. Maurice Gariel in France is included. The application to practical cases is outlined and specific examples are worked out in detail.

A general summary of water-hammer computations is given with several simple formulas for the rapid calculation of the critical conditions.

IN CONDUITS carrying fluid under pressure where the flow may be reduced with any degree of rapidity, the resultant change of momentum will cause an appreciable increase of pressure with respect to time. This is called water hammer, and its occurrence in water-works distribution systems, aqueducts, and hydraulic-turbine penstocks is of particular interest. The safety of these conduits is of prime importance and their design should be adequate to withstand both the normal static pressure and the maximum instantaneous rise in pressure resulting from water hammer.

The investigation of the safety of penstocks in several recent hydroelectric plants has resulted in the finding of some very interesting relations affecting the determination of the true maximum rise in pressure. Contrary to the usual belief, the maximum rise in pressure will not necessarily be produced when the full flow in the given conduit is cut off. Research in this direction brings to light the fact that water hammer is a function not only of the rate at which the flow is decelerated, but is also a function of the initial opening of the control apparatus from which the closure starts. The purpose of this paper is to discuss the various elements entering into the determination of the true maximum pressure rise under such conditions.

Historical Note

The rise in pressure in closed conduits, caused by the deceleration of flow, has occupied the attention of engineers and scientists for a number of years. Numerous theories have been advanced and many formulas have been developed to solve for the maximum value of this rise in pressure. The first research, which later led to the development of the elastic-wave theory, was made by Joukovski in Moscow, Russia, and was published in English in the Proceedings of the American Water Works Association in 1904.[2] An amplification of this work, made by Lorenzo Allievi in Italy, has been published in Italian, French, and German, and recently has been translated into English.[3] Studies have been made in this country by Johnson,[4] Gibson,[5] Durand,[6] and others.

Recently Quick[7] presented a comparison of the elastic-wave theory and the various water-hammer formulas published in English up to that time. His studies also include a simple chart for the rapid solution of water-hammer problems.

Certain important researches in this field have been made in France by de Sparre[8] based on the fundamental principles as laid down by Joukovski and Allievi. De Sparre developed very simple and accurate formulas for the solution of water-hammer problems where the maximum rise in pressure does not exceed one-half of the normal head on the conduit.

Later, the mass of problems connected with the question of water hammer was taken up in great detail by Camichel,[9] Eydoux, and Gariel[10] who, by theoretical research and by more than three thousand tests made both in the laboratory and in important power plants, reached certain conclusions of highly practical importance. The work of Gariel, particularly, is of interest and will be discussed in greater detail in this paper.

General Theory

The experiments conducted by Joukovski showed that pressure waves travel up and down the length of the conduit as a result of the partial or complete stoppage of the flow of water in a closed conduit, and that their amplitude and speed of propagation are a function of the compressibility of the fluid and the elasticity of the walls of the conduit. The velocity of the pressure wave is commonly called a and is determined from the following formula:

$$a = \frac{12}{\sqrt{\frac{w}{g}\left(\frac{1}{K} + \frac{d}{Ee}\right)}} \qquad [1]$$

or for steel pipes

$$a = \frac{4660}{\sqrt{1 + \frac{d}{100b}}} \qquad [1a]$$

where

a = velocity of the pressure wave in feet per second
g = acceleration due to gravity in feet per second per second
d = diameter of the pipe in inches

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[2] "Water Hammer," Joukovski, translated by Miss O. Simin. Proceedings American Water Works Association, 1904, p. 341.

Contributed by the Hydraulic Division and presented at the Annual Meeting, New York, N. Y., December 3 to 7, 1928, of The American Society of Mechanical Engineers.

[3] "Theory of Water Hammer," Lorenzo Allievi, translated by Eugene E. Halmos, under auspices of A.S.M.E. and A.S.C.E.

[4] Trans. A.S.C.E., vol. 79 (1915), p. 277, and vol. 83 (1920), p. 754.

[5] Ibid., vol. 83 (1920), p. 707.

[6] Hydraulics of Pipelines, 1921.

[7] "Comparison and Limitations of Various Water-Hammer Theories," R. S. Quick. *Mechanical Engineering*, vol. 49, no. 5a, Mid-May, 1927, p. 524.

[8] Houille Blanche, 1904, and Bulletin Speciale No. 1 du Comité Technique de la Société Hydrotechnique de France.

[9] "Etude Theorique et Experimentale des Coups de Belier," Camichel, Eydoux and Gariel. A publication of the University of Toulouse, France, and under the auspices of La Société Hydrotechnique de France.

[10] Gariel, *Revue Générale de L'Electricité*, vol. 4, September 21, 1918, pp. 403–411, and October 5, 1918, pp. 483–491.

e = thickness of the walls of pipe in inches
k = the voluminal modulus of elasticity of water, 294,000
E = modulus of elasticity of the material of the pipe walls = approximately 29,400,000 for steel
w = weight of one cubic foot of water in pounds.

Joukovski determined that the maximum rise in pressure in a given conduit would occur when the total flow was stopped in a period of time equal to that required for the pressure wave to travel from the point of control to the point of relief and return.

This time is commonly called μ, and is determined from the formula

$$\mu = \frac{2L}{a} \quad \ldots\ldots [2]$$

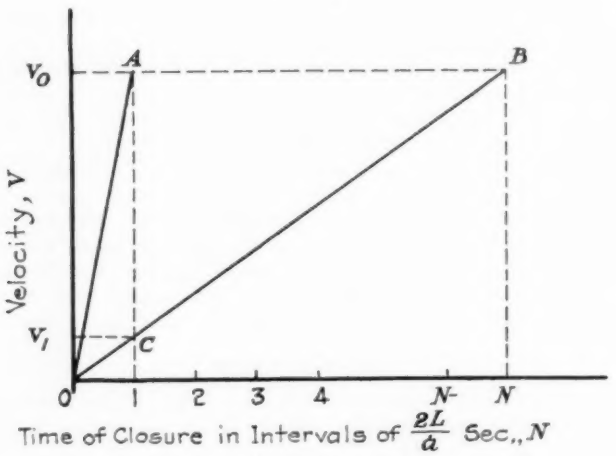


FIG. 1

where

L = length of the conduit in feet
a = velocity of the pressure wave from Formula [1].

Under these conditions the maximum rise in pressure is given by the relation

$$h = \frac{aV_0}{g} \quad \ldots\ldots [3]$$

where

h = the rise in pressure above normal in feet
V_0 = the initial velocity in feet per second
a = the velocity of the pressure wave
g = the acceleration due to gravity.

If the closure is made in a time less than $2L/a$ sec., assuming that the conduit is uniform in cross-section and in thickness, the pressure rise will not be greater, but will occur proportionately sooner and will be transmitted undiminished for a given distance along the conduit.

In commercial design, it is almost invariably the practice to prevent the deceleration of full flow from occurring at any such rate and a time is selected for the full closure of the control apparatus that permits a rise in pressure usually not in excess of one-half the normal head. In certain installations, higher values of pressure rise are permitted, but in a larger number the tendency has been toward more conservative design.

Critical Conditions

Referring to Fig. 1, if V_0 is the initial velocity in a given conduit, the line OA represents the relation of the time of closure with respect to velocity for the full flow to be cut off in $2L/a$ sec. or $N = 1$. This is the condition outlined under the heading of "General Theory" and the rise in pressure is given by Formula [3].

If this same velocity V_0 is to be decelerated in a longer time than $N = 1$, then the relation between initial velocity and time of closure is given by the line OB, where N may be any number of intervals greater than 1.

In the second case where $N > 1$, if the closure starts from some intermediate velocity V_1, less than the full velocity V_0 such that the time of closure will occur in exactly $2L/a$ sec., this condition may be shown as the line OC, Fig. 1. The rise in pressure will then be

$$h_1 = \frac{aV_1}{g} \quad \ldots\ldots [3a]$$

Thus in any closed conduit where the full flow is decelerated in a time greater than $2L/a$ sec., the deceleration of some intermediate initial flow will occur in exactly $2L/a$ sec., and the rise in pressure in this case will be given by the Joukovski formula, aV/g.

This interesting condition has not, in general, been previously appreciated. Investigation shows that the pressure rise occurring due to such a closure from a partial gate opening will be more severe than that which occurs due to closure from full gate opening in a proportionately longer time.

Practical Significance

Since the principal application of water-hammer theory is required in connection with the design of conduits feeding hydraulic turbines, this phase of the problem will be used as the basis of the examples given herein.

The use of hydraulic turbines in commercial service necessitates the rapid movement of the control apparatus, and almost any change in loading may occur during the operation of the unit. It is possible for a turbine to be operating at no load and pick up large amounts of power very rapidly; or the turbine can be operated at full gate and have any or all of its load dropped off; or it can be operating at partial gate openings and the same changes in load may occur. Thus, there is a very wide range of conditions which may take place during the normal functioning of the turbine.

The matter is still further complicated by the fact that the discharge, and hence the velocity in the conduit, may not vary uniformly with the amount of gate opening; the rate of gate travel may not be a uniform or linear function with respect to time; and the time required for partial strokes of the gate mechanism may not represent direct proportions of the full gate stroke.

From the point of view of safety of the plant and of the equipment therein, it becomes imperative to analyze just what conditions can be critical and what operations of the turbine governor and control mechanism will give the maximum pressure rise for which the water conduits must be designed.

General Assumptions

The complex nature of the problem renders it essential to make certain assumptions in order to facilitate the analysis and to avoid a great deal of computation. The special features of the more involved cases requiring detailed analyses will be taken up in another section of this paper.

Initially the studies will be based on the following assumptions for the sake of simplicity:

1 That the conduit considered is of constant thickness and of constant diameter.

2 That there are no surge tanks, air chambers, or relief valves in the conduit being considered.

3 That the loss in head due to friction is negligible.

4 That the closure of the turbine gates is a straight-line function with respect to time and no cushioning effect is present at the end of the stroke nor "dead time" at the beginning.

5 That the time required to effect a partial stroke of the gates is a direct proportion of the time required for the full stroke. That is, the closure from half gate to zero is effected in one-half the time required for the closure from full gate to zero, and that a closure from one-quarter of full gate to zero is made in one-quarter of the full gate time and so on.

6 That the discharge through the orifice and hence the average velocity in the conduit is considered to be a direct linear function of the opening of the orifice, there being no variation in the coefficient of discharge.

7 With the formulas of de Sparre, there is one further assumption necessary; the rate of closure will be such that the rise in pressure will not exceed one-half of the normal head on the conduit.

Methods of Determination

Any of the accepted methods of water-hammer computation may be used in the determination of the critical condition. They include the series of formulas developed by Allievi, Gibson, and Quick; the arithmetic integration method of Gibson; the charts prepared by Allievi and Quick; or the formulas of de Sparre when used as outlined by Gariel. These methods are most conveniently applied to uniform relations of quantity, gate motion, and time with respect to each other as outlined under the heading of "General Assumptions," but by certain corrections can be applied to non-uniform relations as well.

The series of formulas of Allievi, Gibson, and Quick and the arithmetic integration method of Gibson provide the most accurate solution for all conditions, since the actual form of the pressure curve with respect to time is developed. They entail a great deal of computation and, for this reason, are made use of only in special cases where the shape of the pressure-time curve is required.

The charts of Allievi and Quick offer a more rapid and convenient method of solution and give the maximum pressure rise for any given condition. They have some limitations, however, due to the fact that they are not available universally and, when reproduced in publications, cannot usually be read with great accuracy. This is particularly true of the Allievi chart for conditions involving fairly rapid closures from small gate openings, a serious inconvenience in determining the critical conditions for the problem now under consideration. The chart prepared by Quick is likewise subject to inaccuracies in interpretation due to the necessity of estimating between coordinate lines of all three functions involved.

The other solution, by using the formulas of de Sparre, is limited to closures that will produce pressure rises not in excess of one-half the normal head. Since the general tendency is to consider this amount of excess pressure as the limit for good design, these formulas may be used for most practical problems. They provide a rapid and accurate means of solving these problems and are recommended for use generally within their correct range of application.

Of the foregoing methods, all are familiar to American engineers, with the exception of the various formulas of de Sparre and the studies made by Gariel. The writer has been able to translate and present in this article, through the kindness of M. Maurice Gariel in granting the necessary permission, the essential parts of his solution of this problem using the formulas of de Sparre.

For convenience, the general assumptions as already outlined will be used as a basis of this analysis, and in addition the symbols in the formulas will conform to those used by Quick. The original symbols employed by Gariel will also be given, except in the conventional use of h for the rise in pressure instead of η_M and H_0 for initial steady head on the conduit instead of y_0.

In addition to the definitions given under "General Theory," the following are included for the sake of clearness:

h_{max} = pressure rise for instantaneous closure = aV_0/g ft.

P = pressure rise as a proportion of $h_{max} = \dfrac{h}{\dfrac{aV_0}{g}} = \dfrac{gh}{aV_0}$

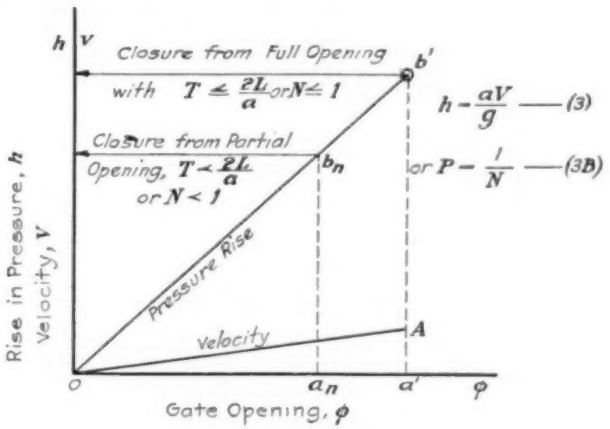


Fig. 2 Case 1

K = pipe-line constant = $\dfrac{h_{max}}{2H_0} = \dfrac{aV_0}{2gH_0}$ = ρ of Allievi

N = time constant or number of $2L/a$ intervals in time of closure = $aT/2L = \theta$ of Allievi

T = time of gate travel in seconds

ϕ = gate opening factor where $V = \phi\sqrt{H}$.

Subscripts 0 and n denote respectively initial conditions and condition at any time during closure or designating number of intervals of $2L/a$ sec.

Gariel Discussion

The following is a translation from Gariel's discussion:

Three cases should be considered—

1 Complete closure of the gates starting from full opening in a time T less than $2L/a$ ($N = 1$)

2 With a time T greater than $2L/a$ ($N \geqq 1$) and the expression $\dfrac{aV_0}{2gH_0} \leqq 1$ ($K \leqq 1$)

3 With a time T greater than $2L/a$ ($N > 1$) and the expression $\dfrac{aV_0}{2gH_0} \geqq 1$ ($K \geqq 1$)

Case 1 $T \leqq 2L/a$ ($N \leqq 1$)

Let V_0 be the velocity in the conduit corresponding to the full opening of the gates, the maximum water hammer h_{max} will be given by

$$h_{max} = \frac{aV_0}{g} \quad \ldots\ldots\ldots\ldots\ldots\ldots [3]$$

Consider next the turbine operating at partial load and let V_n be the corresponding velocity in the conduit, evidently $V_n < V_1$ and the water hammer h_{max} corresponding to a complete closure starting from this opening will be given by

$$h_n = \frac{aV_n}{g} \quad \ldots\ldots\ldots\ldots\ldots\ldots [3a]$$

If, as assumed in all this study, the discharge is directly proportional to the gate opening, the velocity V_n will be proportional to these same openings.

In Fig. 2 the openings ϕ of the distributor are plotted as abscissas and the velocities V of the flow in the conduit as ordinates.

The curve representing V as a function of ϕ is a straight line OA passing through the origin.

Let Oa' be the segment representing the full opening; on the ordinate of a', plot the value of the maximum rise in pressure corresponding to the complete closure from full-open position.

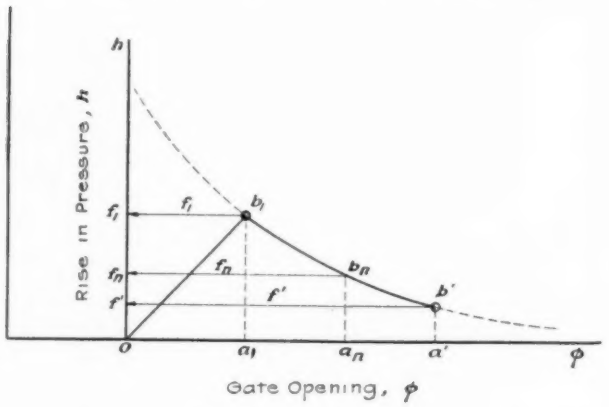


FIG. 3 CASES 1 AND 2

f_1 = closure from partial opening in time $T = 2L/a$ or $N = 1$
f_n = closure from partial opening $N > 1$ and $K < 1$
f' = closure from full opening $N > 1$ and $K < 1$
$P = \frac{1}{N + K(N-1)}$ Eq. [4b]

This maximum is equal to $\frac{aV_0}{g} = a'b'$

Let Oa_n be the segment representing any partial opening whatever; on the ordinate of a_n plot the value of the maximum water hammer corresponding to the full closure from some partial opening Oa_n. This maximum will be equal to $\frac{aV_n}{g} = a_nb_n$. Since the velocities V_n are proportional to the gate opening, the water hammer $\frac{aV_n}{g}$ will also be proportional and the line Ob_nb' is a straight line.

It may be seen, in the case where the time of full closure of the gates from full opening is less than, or at most equal to, $2L/a$, the maximum water hammer is produced on closing from full-open position, and for partial openings it decreases linearly with the value of opening from which the movement starts.

Case 2 $T > 2L/a$ and $\frac{aV_0}{2gH_0} < 1$ $(N > 1, K < 1)$

The phenomenon changes its behavior completely if the time of full closure of the gates is greater than $2L/a$ sec.

Consider first the case where $\frac{aV_0}{2gH_0}$ is less than 1 $(K < 1)$. This is the condition found with high-head plants. The maximum water hammer for full closure from full opening is given by

$$h = \frac{2LV_0}{gT} \cdot \frac{1}{1 + \frac{aV_0}{2gH_0}\left(1 - \frac{2L}{aT}\right)} \quad \ldots\ldots\ldots [4]$$

Employing the same graphic representation as previously, plot on Fig. 3 $a'b' = h$.

Consider some partial opening Oa_n corresponding to a velocity V_n in the conduit and let T_n be the time of complete closure starting from this opening. The maximum water hammer which will be produced during this closure will be given by

$$h = \frac{2LV_n}{gT_n} \cdot \frac{1}{1 + \frac{aV_n}{2gH_0}\left(1 - \frac{2L}{aT_n}\right)} \quad \ldots\ldots\ldots [4a]$$

By rearranging this expression it will be found that the equation of an equilateral hyperbola results, having for one asymptote the axis of T_n and the other parallel to the axis of h.

Having been given, by hypothesis, that the law of closure is assumed to be linear, this same hyperbola plotted with another scale of abscissas represents the variations of the maximum rise in pressure as a function of the opening from which the closure starts. This hyperbola is shown on Fig. 3. It is essential to note that this hyperbola does not represent the phenomenon of maximum water hammer except in one portion determined from one of its limits. It has been shown above that Formula [4] does not apply unless time T_n is greater than $2L/a$ and the expression $\frac{aV_0}{2gH_0}$, $(k \geqq 1)$.

Starting from the opening Oa_1 such that the total closure is effected in the time $2L/a$, all openings smaller will fall under Case 1 and the curve representing the maximum is reduced to the straight line Ob_1. For the closure b_1f_1 effected in the time $T_1 = 2L/a$ or $N = 1$ the maximum rise in pressure is equal to aV_1/g. For closures starting from openings greater than the critical opening Oa_1, the maximum pressure rise will be quite appreciably less than a_1b_1. The greater the opening from which the closure starts the less the pressure rise will be. It will reach a minimum if the velocity V_0 is such that the expression K is equal to 1. Starting from the point where K is greater than 1, the preceding considerations no longer apply and the solution falls into the third case.

Case 3 When $T > 2L/a$, $\frac{aV_0}{2gH_0} > 1$ or $N > 1, K > 1$

This is, in general, the case for low-head plants. The rise in pressure h is given for complete closure from full opening by the formula

$$h = \frac{2LV_0}{gT} \cdot \frac{1}{2\left(1 - \frac{LV_0}{2gTH_0}\right)} \quad \ldots\ldots\ldots [5]$$

and for full closures starting from partial openings

$$h = \frac{2LV_n}{gT_n} \cdot \frac{1}{2\left(1 - \frac{LV_n}{2gT_nH_0}\right)} \quad \ldots\ldots\ldots [5a]$$

It can be seen that the quantities V_n and T_n come into the expression only by the ratio V_n/T_n. As the closure has been assumed linear, this ratio is constant and hence the rise in pressure is constant from any opening from which the closure starts.

Taking an installation in which for full opening $T > 2L/a$ and $\frac{aV_0}{2gH_0} > 1$ or $N > 1$ and $K > 1$, the diagram of the maximum pressure rise may be shown as a function of the gate opening from which the closure starts.

Let Oa' be the full opening, then $a'b'$ is the maximum rise in pressure corresponding to the full closure.

If Oa'_n is a partial opening such that $K > 1$, the maximum water hammer corresponding to the partial closure f'_n will be the same as for the full opening, hence $a'_nb'_n = a'b'$ and the

curve of the maximums will be a straight line parallel to axis of ϕ.

It may readily be shown that for all conditions where the rise in pressure is not in excess of one-half the normal head, all closures in a time of $2L/a$ sec. will fall in the case where $K < 1$, hence there will always be a condition for Case 2 if the time of closure is greater than this critical time.

In the case of the diagram shown in Fig. 4 where $K > 1$ for full gate opening, there must be a partial opening a_c corresponding to a time greater than $2L/a$ for which $K = 1$. For all openings less than a_c, $K < 1$ and the solution will fall under Case 2.

Where the gate opening Oa_1 represents a closure in exactly $2L/a$ sec. ($N = 1$) the rise in pressure will be a maximum, and for closures from gate openings less than this value, the solution will fall under Case 1.

Fig. 4 represents a complete picture of three possible cases for water hammer in any conduit.

Quoting directly from Gariel:

> All the formulas and conclusions given herein apply to conduits having constant thickness of walls and constant diameter. The tests which we made at Soulon with MM. Camichel and Eydoux have shown that the general theories of MM. Allievi and de Sparre can be applied with very close approximation to conduits with constant diameter but variable thickness (a condition most frequently found in hydraulic installations) in all cases where the duration of closure is greater than $2L/a$, or slightly less than this value (a being taken as the average velocity of the pressure wave for the various sections of the conduit).

Conversion to Nomenclature of Quick

Formulas [3], [4], and [5] can be expressed in the terms as given by Quick.

For Case 1 where $K < 1$ and $N < 1$, Formula [3] reduces to

$$P = 1 \quad \ldots\ldots [3b]$$

indicating for any time of closure equal to or less than $2L/a$ sec. the maximum possible pressure rise for the given velocity will be attained.

It is interesting to note that this is identical with the formulas of Vensano and Warren as simplified by Quick for the case where $N = 1$.

In Case 2 where $K < 1$ and $N > 1$, Formula [4] reduces to

$$P = \frac{1}{N + K(N-1)} \quad \ldots\ldots [4b]$$

In this case if $N = 1$, the formula gives the same result as Formula [3b] and for $K = 1$ the same result as Formula [5b].

In Case 3 where $K > 1$ and $N > 1$, Formula [5] reduces to

$$P = \frac{1}{2N - K} \quad \ldots\ldots [5b]$$

At $K = 1$ this gives the same result as Formula [4b]. And at the respective limits of application, if $N = 1$, then $P = 1$, and if $K = 1$, then $P = \dfrac{1}{2N-1}$.

In each case care should be taken to determine P and hence h for the correct initial velocity from which the closure starts.

Typical Application of Theory to Specific Problems

It will be seen from the foregoing discussion that three basic formulas may be used for the solution of virtually all water-hammer problems which occur in normal practice. It would be of interest to apply these three formulas to a specific case and compare their solution with the results obtained from the chart prepared by Quick. A comparison will also be made between the results secured by both these methods and those determined by the arithmetic-integration method of Gibson giving the development of the actual shape of the curve of pressure with respect to time, from which the maximum values can be obtained (Table 1).

The specific example selected shows very clearly the points brought out in the previous discussion and at the same time represents a typical condition which may be found in actual practice.

The problem is based on a single unit operating under a head of 233 ft. with a penstock 600 ft. in length, of uniform cross-section, with the thickness of the pipe walls also uniform and a velocity of the pressure wave equal to 3000 ft. per sec. By assuming that the discharge is a direct linear function of the amount of gate opening, and that the rate of gate movement is also linear with respect to time, the various conditions will

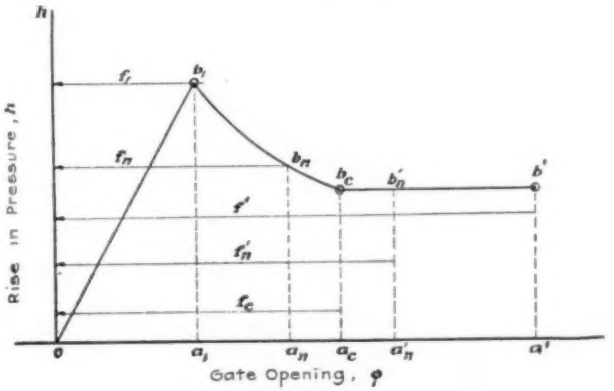


Fig. 4 Cases 1, 2, and 3

Case	Closure	K	N	Rise in pressures
1	f_1	<1	=1	$P = 1$ Eq. [3b]
2	f_n	<1	>1	$P = \dfrac{1}{N + K(N-1)}$ Eq. [4b]
2,3	f_c	=1	>1	$P = \dfrac{1}{2N-1}$
3	f'_n	>1	>1	
3	f'	>1	>1	$P = \dfrac{1}{2N-K}$ Eq. [3b]

coincide with the general assumptions as outlined at the beginning of this discussion. This same problem will later be applied to cases where the discharge is not a linear function with respect to time, and the effect of a non-uniform rate of gate movement will also be discussed.

The full-load velocity is taken as 10 ft. per sec., and since the critical time $2L/a = 0.4$ sec., a closure in 4 sec. will make $N = 10$ intervals of $2L/a$ sec. The relation of quantity, velocity, and gate opening with respect to time for this example is shown in Fig. 5.

The value of K at full load is equal to 2.0, and hence the point of inflection of the envelope curve will occur at one-half gate opening at the point where K is equal to 1. N is equal to 1 at 0.1 gate, which represents an appreciable amount of gate opening on the turbine.

TABLE 1 COMPARISON OF RESULTS OF COMPUTATIONS BY VARIOUS METHODS

Initial velocity, ft. per sec.	Time of closure intervals	Rise in pressure in feet — Formulas of de Sparre-Gariel	Charts of R. S. Quick	Arithmetical integration, Gibson
10	10	51.8	51.8	51.5
9	9	51.8	51.8	..
8	8	51.8	51.0	..
7	7	51.8	51.0	..
6	6	51.8	51.2	..
5	5	51.8	53.8	54.1
4	4	58.2	59.6	..
3	3	66.5	67.4	68.0
2	2	77.7	78.5	..
1	1	93.3	93.2	93.26
0	0	0.0	0.0	0.0

Referring to Fig. 6, the values from Formula [3*b*] are plotted as well as those obtained from Formulas [4*b*] and [5*b*]. It will be noted that the respective formulas give the same result at the points corresponding to the end of the limits in which they are applicable. Thus we have a curve *AB* for Formula [5*b*], *BC* for Formula [4*b*], and *CD* for Formula [3*b*].

Taking the solution of these same closures from the chart prepared by Quick and in each case plotting the maximum value as an ordinate opposite the abscissa corresponding to the initial point from which the closure starts, it will be found in every case from *A* to *B* that the solutions from the charts of either Allievi or of Quick agree exactly with those secured from the Formula [5*b*]. Between the points *B* and *C* there are light discrepancies, but not serious. It is interesting to note, however, that at point *C* all of the solutions agree exactly. Points *C* correspond to a velocity of 1 ft. per sec. and the closure occurs

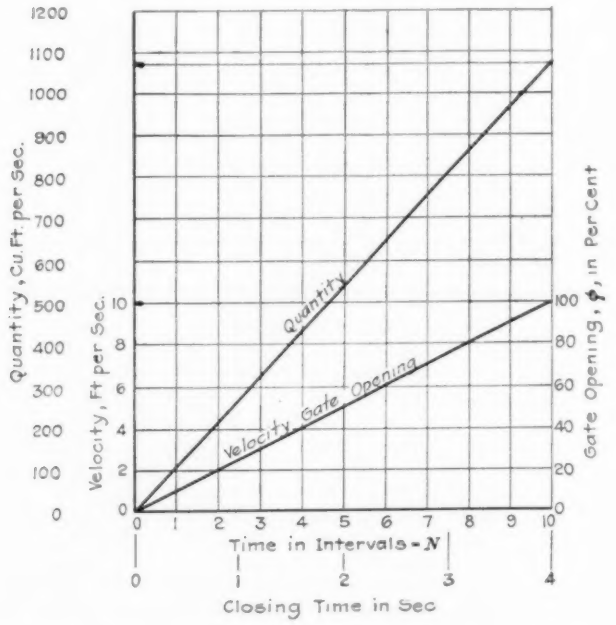


Fig. 5 Uniform Relation of Quantity, Gate, and Time

in 0.4 sec., resulting in a pressure rise of 93.26 ft. From *C* to *D*, the pressure rise is naturally reduced uniformly due to the fact that the initial velocity is directly proportional to the time of closure, since the closing time is less than $2L/a$ sec. and the pressure rise is directly proportional to the velocity destroyed.

Further to confirm these values, the various curves of pressure with respect to time for different closures at a uniform rate are shown on this same figure. Designating each of these curves in terms of the corresponding value of *N*, it will be noted that the curve $N = 10$ reaches a maximum of 51.5 ft. during the fourth interval after the closure begins, and from the fourth to the tenth interval the value remains constant. This compares with the rise of 51.8 ft. secured from the formulas or from the charts and is in substantial agreement therewith.

With the closure starting from $N = 5$ with a velocity of 5 ft. per sec. and a closure effected in 2.0 sec., the shape of the curve of pressure with respect to time changes materially and reaches the maximum of 54.1 ft. during the second interval, then decreases, and finally remains constant for the last two intervals. For the curve $N = 3$, the closure is effected in 1.2 sec. and the maximum is reached at the end of the first interval, followed by an abrupt surge downward during the second interval and upward during the third.

For the closures from $N = 10$ to include $N = 5$ the initial velocity was such that *K* has always been greater than unity and in each case it may be seen that the maximum rise in pressure is approximately the same although it is reached earlier in the stroke as the value of *K* approaches unity.

Starting at point $N = 5$ the shape of the curve of pressure with respect to time changes abruptly. The curve shown as $N = 3$ is a striking example of this fact. The maximum is reached at the end of the first interval of time and is equal to 68.0 ft. instead of 51.5 ft. as secured before, an increase of 32 per cent.

Reaching the critical point of $N = 1.0$ and a value of $K = 0.2$, the curve of pressure with respect to time increases rapidly and reaches a peak at the end of the closure, which of course is the end of the first interval of time. A maximum pressure of 93.26 ft. results, or 81 per cent in excess of that secured for the closure from full gate. At $N = 0.5$ the maximum rise is just one-half of that value, namely 46.53 ft.

This indicates in a very striking manner the possibility of securing pressure rises far in excess of those corresponding to the full gate closure.

Comparison With Energy Formula

It is interesting to note at this time a comparison between the conventional formulas which have been used in the past for determining the maximum rise in pressure from an energy basis.

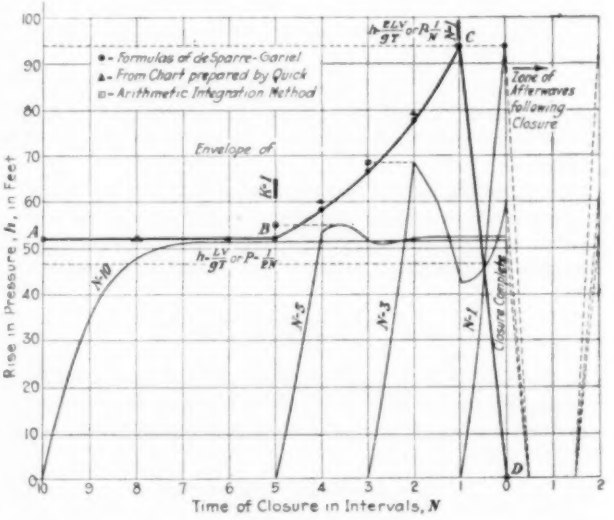


Fig. 6 Typical Problem With Uniform Relations of Quantity, Gate, and Time

The early formula of $h = LV/gT$ gives a value of only 46.63 ft. which is less than that resulting from full gate closure. The other formula, which is sometimes called Michaud's formula, $h = 2LV/gT$, gives exactly the maximum pressure rise occurring at the critical point $N = 1$ where the value of 93.26 ft. was reached.

This brings out a rather unusual relation that may be verified by any number of similar examples, namely, that this simple formula $2LV/gT$ does give the actual maximum rise in pressure which may be obtained in any given conduit at some point due to a closure from some partial gate opening.

Gariel draws particular attention to this relation and mentions that this latter formula was first developed by Michaud

in France[11] about fifty years ago, and that it is possibly the oldest of formulas for water hammer and one which has occupied the attention of engineers for some time. Experience has shown that this formula should be corrected for certain specific closures since it gives results which are too high in every case, except for closures in the critical time $N = 1$. The studies which have been made from time to time confirm the conclusion that this formula is incorrect and show that a variable correction factor must be applied depending upon the given conditions. For this reason the formula has been practically abandoned by engineers.

It is interesting to take this formula, which has so often been discredited, and substitute for the value of T the time required for a pressure wave to travel from the point of control to the point of relief and return, namely, $2L/a$ sec., and let the velocity V be equal to the amount cut off in this time.

$$h = \frac{2LV}{gT} = \frac{2LV}{g} \cdot \frac{a}{2L} = \frac{aV}{g}$$

The formula is reduced to the basic theory as developed by Joukovski and Allievi.

The various researches conducted by Camichel, Eydoux, and Gariel show that this formula should not be abandoned and that while empirical in its origin and inaccurate in its application, it is actually in close agreement with the most complete modern theories, and furthermore has been carefully verified experimentally.

It will be seen that the relation of V/T in this formula is constant, and so long as the relation of velocity to time is a linear function, this ratio will have the same value regardless of the initial opening.

It may not necessarily, and usually will not, give the correct value of rise in pressure corresponding to the deceleration of the full quantity in any given case, but it will give the maximum that must occur in any given conduit where the gate motion is uniform with respect to time and where the discharge is also a uniform function in respect to gate opening.

This offers a simple but effective means of computing the maximum possible rise in pressure in any given conduit for any particular value of velocity and time, provided that the time of closure is equal to or greater than $2L/a$ sec.

It should be emphasized, however, that this condition is true only with the uniform gate motion with respect to time and with uniform variation of discharge with respect to gate motion.

It is interesting to convert both of these two simple relations to the nomenclature used by Quick.

Thus

$$h = \frac{LV}{gT} \text{ reduces to } P = 1/2N \ldots\ldots\ldots [6]$$

This is the Formula [5*b*] if the time N is large with respect to the pipe constant K, indicating that for slow closures the simple "acceleration" relation is approximately correct.

Also

$$h = \frac{2LV}{gT} \text{ reduces to } P = 1/N \ldots\ldots\ldots\ldots [7]$$

which is identical with the formulas of Vensano and Warren, and represents the limit of Formula [4*b*] as K decreases with respect to N. For the case where $N = 1$, then $P = 1$ and it becomes identical with Formula [3*b*].

If a column of water is considered to be a rigid body, then the relation in Formula [7] will result from a mathematical analysis of the complete deceleration of the flow, showing the agreement between the elastic-wave theory and the rigid-column theory at this one point where the closure is made in $2L/a$ sec. One might be led to conclude that at some particular rate of closure the water in the conduit would behave as a rigid column. Since it has been shown that this relation is true only at $N = 1$, then the further conclusion might be reached, namely, that the water in the conduit for such a closure actually would be a rigid column.

[11] Michaud, Bulletin de la Société Vaudoise des Ingenieurs et des Architectes, 1878, nos. 3 and 4. Also Bulletin Technique de la Suisse Romande, February 10 and 15, 1903.

Partial Closures

The question naturally arises in studying this phenomenon of critical closures, if the closure from 10 per cent to zero produces such an unusual rise in pressure, does not a closure from 100 to 90 per cent in the same time produce the same rise? It

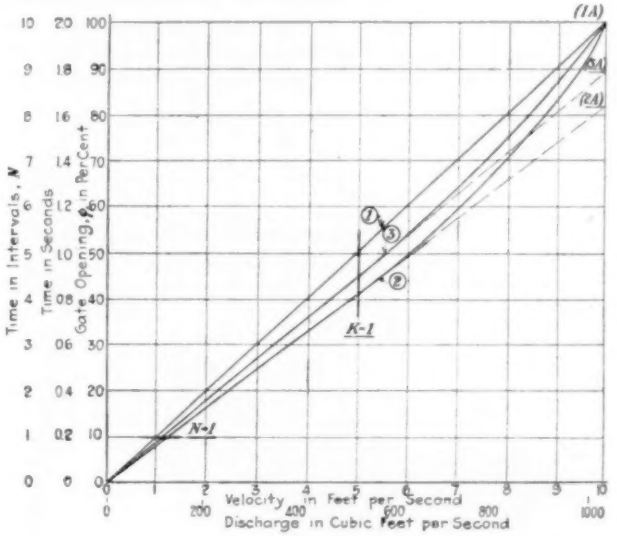


Fig. 7 Typical Relations of Gate Opening and Discharge

can be stated confidently that such cannot be the case. By referring to Fig. 6, if the closure stopped at 90 per cent on the curve $N = 10$, the rise in pressure would equal only 35 ft., the after waves would have this amplitude, and the pressure would not increase above this value.

In the case of $N = 5$, the pressure rise for a 10 per cent stroke starting from 50 per cent gate would be about 53 ft.; for a stroke starting from 30 per cent 68 ft.; and from 10 per cent gate about 93 ft.

Thus the rise in pressure for partial closures from partial gate openings is also a function of the initial gate from which the movement starts; the same as for complete closures.

After-Waves

Although the gate movement stops, the pressure waves set up due to closure will continue to oscillate until they are damped out by internal friction. They form "after-waves" with a fixed period of $4L/a$ sec. and with an amplitude usually equal to the final pressure rise existing at the instant the gates have stopped moving. For non-uniform motion, since the amplitude and wave form will be affected by the sequence and amplitude of the individual waves during closure, the after-waves may take peculiar forms. This can only be determined by the detailed computation of the individual waves by the arithmetic integration method, or by the use of the series of formulas of Allievi, Gibson, and Quick.

Non-Uniform Discharge—Gate Relation

In actual practice the ideal condition of uniform relation of velocity gate and time is rarely encountered, since the discharge generally tends to increase directly with the gate opening up to a certain point and then continues to increase, but at a slower rate with additional gate openings until a point is reached where a further opening of the turbine gates results in no increase in discharge.

Assuming for the time being that the movement of the gates with respect to time is still uniform, it becomes apparent for the same discharge at full gate opening that the discharge at fractional gate openings will be greater than would be the case if the discharge varied uniformly with respect to time, or as a linear function rather than a variable function as is actually found in practice.

In Fig. 7 are shown two non-uniform gate-discharge relations which would be typical for a turbine operating under the conditions given in Fig. 5. The uniform relation of discharge and gate is shown as Curve 1 for comparison. Curve 2 is the usual

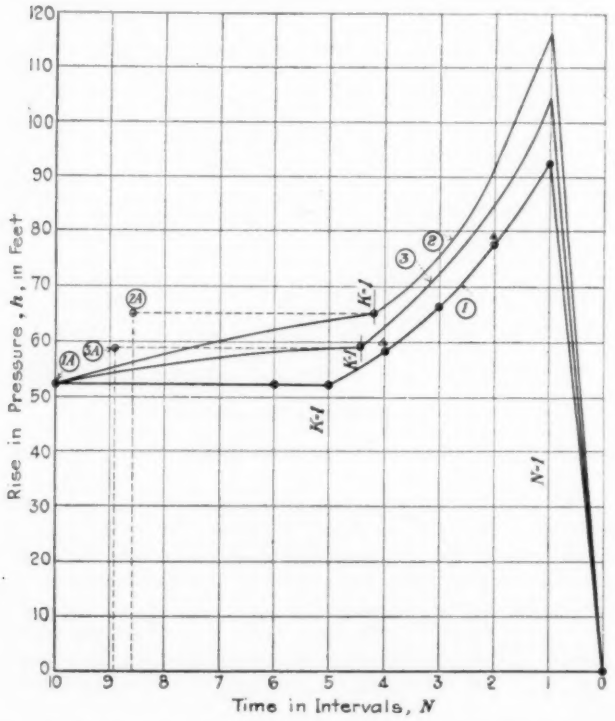


Fig. 8 Variation in Pressure Rise for Different Gate Opening-Discharge Curves

form of discharge gate relation with full gate fixed by the extreme maximum power that could be obtained from the unit. Curve 3 is the same type of unit, but with full gate fixed as the upper limit of the range of economic operation.

It will be noted on the three curves that the point where $K = 1$ occurs at the same discharge, but at different gate openings, while the point where $N = 1$ occurs at the same gate opening, but the discharge is much greater for the non-uniform relations than for the uniform relation.

Taking these three curves, and applying the Formulas [3*B*], [4*B*], and [5*B*], the envelopes of the maximum values of pressure rise may be computed. The results of these computations are given in Fig. 8 as Curves 1, 2, and 3, corresponding to the respective curves in Fig. 7.

It is interesting to compare the values of maximum pressure rise resulting from Curves 2 and 3 with the previous values obtained in Fig. 6, and shown as Curve 1 in Fig. 7. The maximum rise in pressure due to the closure from full gate is the same in all three cases, but for closures from smaller gate openings the discharge is sufficiently greater for the non-uniform relations to increase the pressure rise appreciably.

At the point where $K = 1$, the discharge is the same in all three cases, but the rise in pressure is greater with non-uniform motion since this discharge occurs at smaller gate openings, and the time of closure is correspondingly smaller.

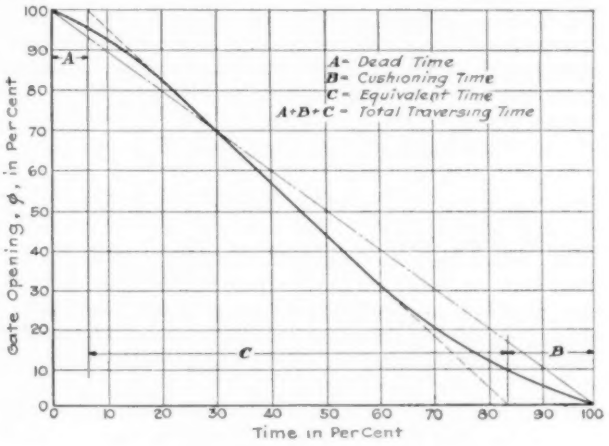


Fig. 9 Typical Gate Motion
(A = dead time; B = cushioning time; C = equivalent time; $A + B + C$ = total traversing time.)

At the point where $N = 1$, the gate opening is the same for all three cases, and the time of closure is also the same, but the discharge being successively greater, the pressure rise will also be greater at this point.

The development of this envelope may be simplified greatly by noting an interesting relation.

The curve of discharge with respect to gate opening is really a straight line for some distance starting from zero, and as the point of maximum efficiency of the turbine is approached the discharge tends to decrease for a given increment of gate opening and causes the relation to depart from the straight line and assume a curved form. If, then, the straight portion of the curve is extended until the value of full gate discharge is reached, the curve of discharge with respect to gate becomes a uniform relation, and it is necessary only to compute, as with the example in Fig. 6, the critical points by the Formulas [3*b*], [4*b*], and [5*b*]. The corresponding values of time and gate openings are used, and the envelope curve determined from the rise in pressure at the equivalent full gate $2A$ and $2B$ and the points $K = 1$ and $N = 1$.

Non-Uniform Gate Motion With Respect to Time

If the third variable is considered, namely the rate of gate movement with respect to time, still further corrections must be made. In the first place, it will be of interest, as well as for the sake of clearness, to indicate and define the various factors entering into gate movement. There is first the total time elapsed between the initial movement of the gates and their coming to rest. This may properly be called the "total traversing time" of the governor as shown in Fig. 9. It will be noticed that there is a lag of the gates in their initial movement which is present until the inertia of the mechanism has been overcome, after which the gates are usually moving at a uniform rate for the

major portion of the stroke, and then a cushioning effect begins which slows down the movement toward the end of the stroke. This characteristic is found in most governors and gate-operating mechanisms, although it has been the practice of some manufacturers to omit the cushioning effect at the end of the stroke.

If the straight portion of the curve, corresponding to the maximum rate of movement, is extended as a straight line until it intersects the initial and final values of gate opening, a new time results which represents that required for the same amount of gate travel at a uniform rate corresponding to the maximum rate attained during the stroke. This may be designated as the "equivalent time." It is this time which actually determines the maximum rise in pressure, since the maximum rise in pressure is a function of the rate of gate movement and not of the total time of travel. Where the gate motion is of this general form, then the equivalent time is approximately 75 to 85 per cent of the total traversing time.

The difference between the total traversing and the equivalent time is made up of two parts, the first being the additional time required to accelerate the mechanism, usually called the "dead time," and the "cushioning time" occurring at the end of the stroke.

The maximum rate of gate movement is attained in the major portion of the gate stroke for practically every closure, except those starting from very small gate openings. Having once determined the time and the corresponding gate travel required for acceleration and for the deceleration of the parts, it is possible to fix the minimum time and hence the minimum amount of gate stroke during which this maximum rate will be reached, which may affect the rate of closure for $N = 1$ and reduce the pressure rise at this point.

Ideal Gate Motion

Instead of an envelope curve such as that shown in Fig. 6 or Fig. 8, a uniform relation of maximum pressure rise can be obtained by adjusting the rate of gate motion for different portions of the stroke, so that the time of closure will limit the actual maximum rise in pressure to any given value.

Referring again to Fig. 6, the maximum pressure rise is constant from A to B. The extreme maximum is reached at C and then decreases uniformly to zero at D. If the time of closure for the discharge corresponding to 10 per cent gate in this particular problem is increased, so that a closure takes place in approximately twice this time, the point C will occur at $N = 2$ and the maximum value will be about the same as that secured in the range from $N = 10$ to $N = 5$. The actual pressure rise for $N = 1$ will be somewhat less than 51.5 ft. due to the slowing up of the gate motion in this range.

Between B and C the closures should take place at progressively slower rates until the minimum rate of motion between C and D is reached. The ideal relation of gate opening with respect to time of closure is shown in Fig. 10, and the letters designating the respective points corresponding to those given in the envelope curve in Fig. 6. The slope of the line AB is determined from the first derivative of the relation of velocity with respect to gate opening for normal closures, such that the rise in pressure in this range of operation will equal the desired amount. The slope of the line CD is determined from another relation of velocity with respect to time to reduce the value of pressure rise to the amount permitted by the rate from A to B. The curve between B and C will take somewhat the form as shown, and sufficient points can be determined at intermediate values of gate opening to establish the shape of this curve.

There are a number of ways in actual practice of securing this form of gate motion, among them being a throttling valve which is operated by the gate mechanism to reduce the flow of governor fluid to the operating cylinders within the range from B to D. This has also been accomplished by providing two governor pumps without accumulator tanks with the capacity of one pump sufficient only to give a rate of movement such as that shown at CD. The two pumps combined will increase the rate of travel to that shown from A to B, while the intermediate section from B to C is taken care of by the throttling of the discharge from the second pump. Still another method combines an improved design of levers and links with a specially designed cushioning effect built into the operating cylinders.

In any event, it is highly important to reduce the rate of gate travel in the critical portion of the stroke, or else to provide for additional strength in the water conduit sufficient to withstand the excess rise in pressure caused in this zone for uniform motion.

Speed Regulation

The relation to the problem of regulation is of interest, since these studies further emphasize the fact that the governor time or the rate of gate travel with respect to time in each installation should be determined from a consideration of the physical constants, such as the strength and allowable factor of safety of the conduit, the head on the plant, and the velocity in the flow line.

With these factors determined, the problem of speed regulation is a function only of the flywheel effect of the rotating element and the permissible speed variations for different load changes on the unit.

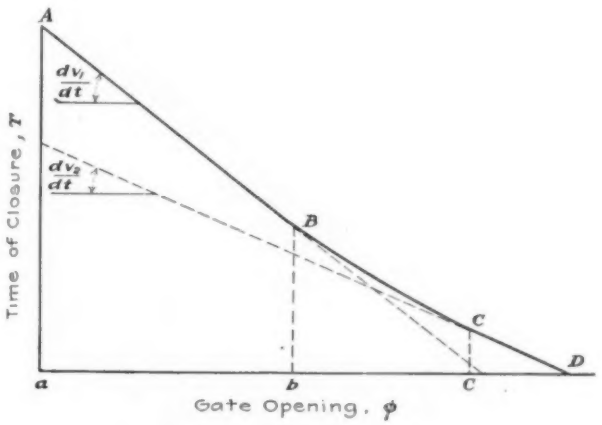


Fig. 10 Ideal Gate Motion

Improvements in regulation may come from devices which will reduce or limit the "dead time" by bringing the gate mechanism up to its maximum rate of travel without introducing any appreciable lag. This would tend to reduce the speed variations for small load changes, but the actual hydraulic conditions subsequent to this rapid travel for small load changes might actually cause a certain amount of instability due to the presence of the after waves.

Typical Application to a Given Installation

One of the most interesting applications of this theory was made in connection with the preliminary design of a recent hydroelectric plant, the Saluda development of the Lexington Water Power Co. This particular plant will operate under a maximum head of 180 ft. and the dam is of the earth-fill type with individual penstocks for each of the units carried through the base. Some of the units will be provided with surge tanks, while on others the surge tanks will be omitted. It is imme-

diately apparent that the safety of the plant depends primarily upon the safety of the conduits underneath the dam.

With this in mind, it was essential to investigate every possibility for excess water hammer. The studies were based on the application of a normal type of turbine to each conduit with a discharge-gate-opening curve somewhat on the order of that given in Fig. 7, Curve 2. The studies also assumed at first that the travel of turbine gates would be uniform with respect to time to simplify the computations. In Fig. 11*a* is shown, for a unit without a surge tank, the analysis of the pressure rise conditions for various closures from full gate to zero plotted with maximum pressure rise as ordinates and the per cent of maximum gate opening as abscissas.

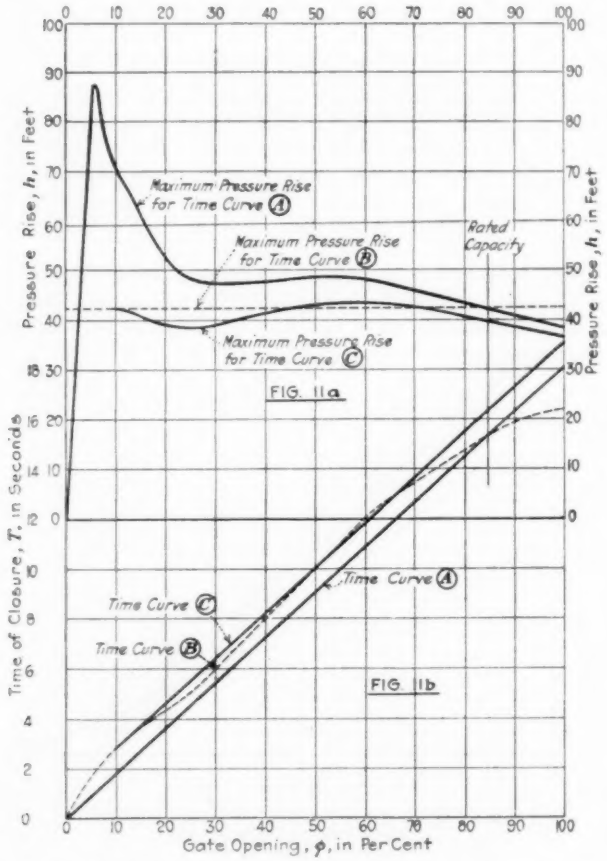


FIG. 11 WATER-HAMMER ANALYSIS OF UNIT WITHOUT SURGE TANK, SALUDA DEVELOPMENT

(Head = 180 ft.; Q = 3100 c.f.s.; avg. vel. = 15.85 ft. per sec.; L = 1179 ft.; a = 2545 ft. per sec.; T = 15.3 sec., uniform motion.)

Curve A results from the use of uniform motion with respect to time. A maximum pressure rise of 42.5 ft. above normal was fixed as the limit for safety, shown as Curve B in Fig. 11*a*. Computations were made to determine the correct time of closure starting from each gate opening to prevent the pressure rise from exceeding this value. The resultant curve of closing time with respect to gate opening is shown as B, Fig. 11*b*. It will be noticed that the allowable time of gate movement from 30 per cent to zero is very much longer than that resulting from uniform motion. It would not be practical, however, to have such an extremely variable rate of closure for different portions of the stroke, and a third curve shown as C in Fig. 11*b* was determined as being a commercial possibility. This curve is uniform with respect to time from 100 to 10 per cent, and from 10 per cent to zero a still slower rate is used. The corresponding maximum pressure rises for this form of gate motion are shown as Curve C, Fig. 11*a*.

The time as shown in Curve C represents the equivalent time as outlined in Fig. 9, and in order to convert these studies to the actual form of non-uniform gate motion, a certain increment of time was added to Curve C to take care of the "dead time" and "cushioning" time, thus giving the traversing time.

OTHER APPLICATIONS OF THE THEORY

The study of the water-hammer conditions which may occur in hydraulic turbine penstocks can be applied equally well to the problems arising from the variation of flow in closed conduits used in water-works service. In controlling the flow in such a conduit by means of a valve, the discharge is not appreciably affected by the closure of the valve until the area is reduced considerably and the actual loss through the valve assumes fair proportions. As this point is reached, the flow conditions in the control valve are similar to those occurring with free-discharge valves, since the velocity through the valve itself is very great. It is therefore important to analyze the rate of closures which occur, starting from relatively small openings of the valve, since these in turn cause the maximum variation in velocity in the conduit. The last portion of the stroke may change the flow in the conduit far more than the same amount of stroke at the fully open position.

The same methods of procedure can be used in analyzing this problem as are employed with hydraulic-turbine penstocks, but the loss in head through the valve for different openings becomes the determining factor and from this relation the variation in velocity and discharge through the valve can be established.

RELATION TO PREVIOUS PRACTICE

In view of the critical conditions of pressure rise for closures from partial openings, the question naturally arises as to why this particular characteristic has not caused an unusual amount of difficulty in existing plants.

There are several good reasons, the first of which is probably that some cushioning effect has been placed in most operating cylinders, which for low or medium heads will effectively prevent the difficulty due to the extreme maximum pressure rise occurring in $2L/a$ sec. On high-head plants there probably have been several instances where this phenomenon has caused difficulty, but it may have been attributed to a more rapid rate of gate motion than the design valve, rather than to an excess pressure rise caused by a partial closure at the normal rate of travel.

A second reason may be found in the general tendency of the rate of governor movement with respect to time to take a characteristic form such that, while the rate of gate motion during the middle section of the stroke, or in fact throughout most of the stroke, is uniform with respect to time, yet the fractional closures do not occur in direct proportion of the time such as a closure from one-half gate to zero in one-half the time, but may require 60 or 70 per cent of the full-load time for this travel. This becomes more pronounced at the smaller gate openings, and typical characteristics, such as those found in Fig. 12, Curves A or B, are found.

The partial opening of the control valves also has an effect upon the determination of the rate of movement, since for small-load changes the pilot valve of the governor is not necessarily raised its full travel, and hence there is only a partial port opening in the distributing valve which reduces the rate of gate movement proportionately by throttling the flow of oil through the governor. Some governor valves are built with variable-ported construction so that only a small part of the area is open when

the valve is lifted a short distance, and this opening increases both in height and in width with the stroke of the valve. With this sort of arrangement the rate of gate motion is reduced very materially for small movements of the gates and actually works for the safety of the plant as long as the gates were controlled by the governor alone. If for any reason a closure from a partial opening is made by raising the pilot valve manually or by other means, the gates would reach their maximum rate of movement even for the small strokes, and the water conduits would be subject to these pressure-rise conditions in such cases, and hence it is unsafe to rely on this means for guaranteeing the safety of the plant.

The general tendency recently has been to increase the governor time for full-load rejections, so that the pressure rise for this condition is decreased to possibly 25 or 30 per cent above normal,

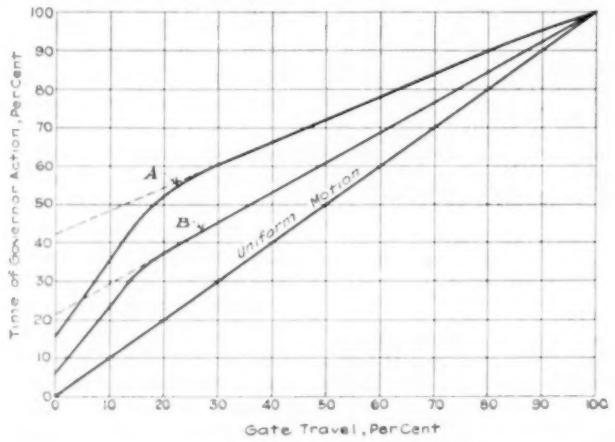


FIG. 12 TYPICAL GOVERNOR TIMING RELATIONS

instead of 50 per cent above normal. In such a case, even the critical pressure rise at some fractional gate opening would not reach greatly in excess of 50 per cent of normal head, and with the factors of safety held within reasonable limits the stress in the pipe line is not excessive.

CONCLUSIONS

Since the pressure rise due to the deceleration of flow in closed conduits is a function of the initial flow from which the deceleration starts, and since the retardation of fractional discharges may prove critical, it is imperative to investigate these conditions to insure the safety of the conduits under consideration.

The true relation of average conduit velocity with respect to gate opening or the opening of the control apparatus is essential for the accurate determination of such critical conditions.

Having once determined this relation, the simple formula of $2LV/gT$ will give the maximum rise in pressure which may occur in the conduit due to some particular closure, providing the motion of the control apparatus with respect to time is uniform and that relation of discharge with respect to gate opening and of gate opening with respect to time is also uniform.

For non-uniform conditions, if the maximum relation of velocity with respect to time can be established, then these two values will enter into this simple formula as a ratio and the true critical pressure rise can be determined in this manner.

For general practice Formulas [3*b*], [4*b*], and [5*b*] may be used but the critical condition will occur when $N = 1$. When $K = 1$ the pressure rise will be given by either Formula [4*b*] or [5*b*] and will correspond to the maximum rise at any point where K is greater than 1 with uniform motion.

The general knowledge in regard to water hammer and the means of correctly computing the maximum rise in pressure by the elastic-wave theory, coupled with the experimental confirmation of the elastic-wave theory, gives assurance to the designer that studies of the water-hammer conditions in accordance with these theories will be the correct basis for the determination of the strength of water conduits. The only concern should be to insure the correct application of these theories and to investigate each phase of the problem.

In the absence, however, of definite determination of the true maximum rise in pressure in accordance with the principles described in this paper, any plant may be in a dangerous condition unknown to the operating personnel.

Discussion

E. B. STROWGER.[12] The author has presented an interesting and useful analysis of the conditions of pressure rise in a hydraulic-turbine penstock for closures from full gate and from partial gate openings. The fact that the pressure rise due to a closure from a partial gate may considerably exceed that for the full gate condition emphasizes the necessity on the part of designers to investigate thoroughly the rise in pressure for the possible critical condition pointed out in the paper.

Recently the writer investigated four problems of water ham-

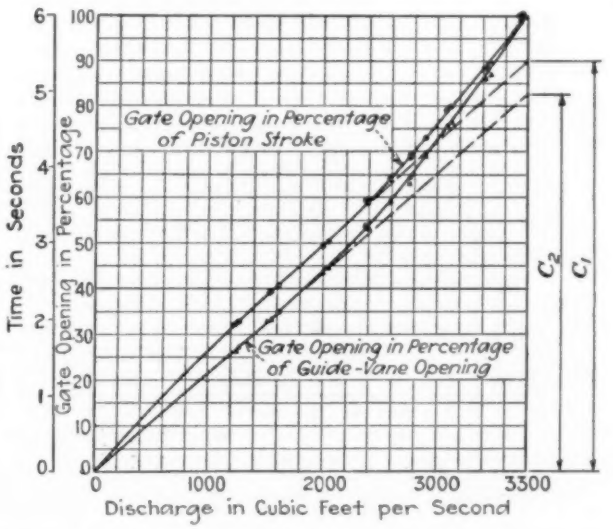


FIG. 13 COMPARISON OF DISCHARGE VERSUS PERCENTAGE OF PISTON STROKE AND DISCHARGE VERSUS PERCENTAGE OF MAXIMUM GUIDE-VANE OPENING FOR A 70,000-HP. HYDRAULIC TURBINE
(C_1 = equivalent time using piston stroke and C_2 = equivalent time using guide-vane opening.)

mer, each case of which showed a critical initial velocity at a partial gate opening which would produce a greater pressure rise than that due to the rejection of full load, assuming of course that both the discharge and the gate opening vary uniformly with respect to time, and also that the duration of closure is proportional to the gate opening. This is shown in Table 2, the last two columns of which give the critical velocity and the pressure rise produced by the destruction of this velocity. In these cases the pressure rise due to the destruction of the critical velocity varies from 109 per cent to 166 per cent of the pressure rise due to the destruction of the full-load velocity. As pointed out in the paper, fractional closures to zero usually occur in a time which is greater

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TABLE 2

$\frac{H_0}{2}$ ft.	V_0 f.p.s.	$\frac{2L}{a}$ sec.	T sec.	N	K	$\frac{aV_0}{g}$ ft.	h Gibson or Quick ft.	de Sparre-Gariel P	de Sparre-Gariel h ft.	de Sparre-Gariel approx. formula, % of error	V_1 f.p.s.	$\frac{aV_1}{g}$ ft.
$N > 1$, $K < 1$											$N = 1$, $K = 1$	
285	8.38	1.065	3.195	3.0	0.742	846.0	193.3	0.223	188.3	−2.6	2.79	281.0
285	8.38	1.065	4.260	4.0	0.742	846.0	138.0	0.1606	135.8	−1.6	2.095	211.0
285	8.38	1.065	7.455	7.0	0.742	846.0	74.6	0.0874	74.0	−0.8	1.197	121.0
825	5.0	21.05	40.0	1.9	0.1788	590.06	287.16	0.485	286.0	−0.4	2.63	310.5

TABLE 3

$\frac{H_0}{2}$ = 825 ft.
V_0 = 5 f.p.s.
$\frac{2L}{a}$ = 21.05 sec.
T = 40 sec.
N = 1.90 intervals
$\frac{aV}{g}$ = 590.06
K = 0.1788

n interval	0.0	0.50	0.85	0.90	1.0	1.50	1.85	1.90
t (sec.)	0.0	10.526	17.895	18.95	21.05	31.58	38.95	40.0
ϕ	1.0	0.7368	0.5526	0.5263	0.4737	0.2105	0.0263	0.0
2Σ	0.0	0.0	0.0	0.0	0.0	0.4701	0.8168	0.8686
$1 - 2\Sigma$	1.0	1.0	1.0	1.0	1.0	0.5299	0.1832	0.1314
ϕn^2K	0.17881	0.09708	0.05461	0.04953	0.04012	0.00793	0.00012	0.0000
$1 - 2\Sigma + \phi n^2K$	1.17881	1.09708	1.05461	1.04953	1.04012	0.53782	0.18334	0.13138
$2K(1 - 2\Sigma)$	0.35761	0.35761	0.35761	0.35761	0.35761	0.18950	0.06552	0.04698
$1 + \phi n^2K^2$	1.03197	1.01736	1.00976	1.00886	1.00717	1.00142	1.00002	1.0
$1 + 2K(1 - 2\Sigma) + \phi^2n^2K^2$	1.38958	1.37497	1.36738	1.36647	1.36479	1.19092	1.06554	1.04698
$\phi\sqrt{1 - 2K(1 - 2\Sigma) + \phi n^2K^2}$	1.1788	0.86203	0.64622	0.61522	0.55340	0.22975	0.02717	0.0
P_n	0.0	0.23505	0.40839	0.43431	0.48672	0.30807	0.15618	0.13138
h_n (ft.)	0.0	138.68	240.95	256.27	287.16	181.76	92.15	77.51

de Sparre-Gariel Solution
$N > 1$, $K < 1$

$$P = \frac{1}{N + K(N - 1)} = \frac{1}{1.90 + 0.1788(1.90 - 1)} = 0.485$$

$h_n = 0.485 \times 590.06 = 286.0$ ft.

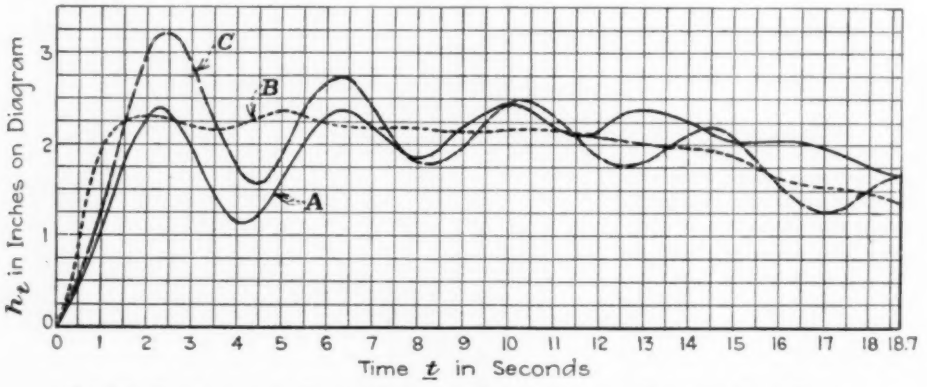


FIG. 14 COMPARISON OF GIBSON DIAGRAM WITH CALCULATED PRESSURE-TIME CURVE
(*A*, Gibson diagram; *B*, calculated pressure-time diagram with correct *a*; *C*, calculated pressure-time diagram with value of *a* taken at 10 per cent of correct *a*; area under three curves the same.)

than a direct proportion of the equivalent full gate traversing time, due to cushioning and due to the fact that the opening of the governor-control valve for small-load changes is a fractional value of the full opening. However, if the design is to take care of the contingency of a manual raising of the pilot valve by means of the load-limit device, a safety trip, or any other device capable of manually or automatically raising the pilot valve its full travel, then the pressure rise should be calculated in accordance with the foregoing assumption of a direct proportional duration of closure for the critical partial gate opening.

Table 2 gives a comparison of the time-pressure rise as by Gibson or Quick and the approximate values by the de Sparre-Gariel equations for four examples where $N > 1$ and $K < 1$ and shows a close agreement. Where the de Sparre-Gariel equations apply, their use simplifies the problem materially, as shown in Table 3, which gives both the approximate and the exact solution for a high-head, long penstock job with no surge tank, but with a slow-closing valve designed to close in 40 sec. The nomenclature used is that of Quick.

However, of five water-hammer problems recently investigated by the writer, for three of them the de Sparre-Gariel approximations could not be used, because the rise of pressure was greater than $H_0/2$. This condition is common with low-head plants, so that in these cases the arithmetic integration method, the method of Allievi, or the method of Quick must be used. As in the case of all approximate formulas, each problem must be investigated first to see if it is within the range of application of the approximate formula in question. Failing to do this may result in serious consequences so far as the penstock design is concerned, and for this reason the use of approximate formulas for water hammer is always hazardous in the hands of anybody but those of an expert.

The author does not specify how the gate opening of the turbine should be measured, but it is probably generally understood that the gate opening should be measured in percentage of maximum guide-vane opening and not in percentage of piston stroke of the regulating cylinder. It might be presumed that very little difference exists between the relation of discharge and guide-vane opening and that of discharge and percentage of piston stroke, but there may be considerable difference between the values of pressure rise determined by considering each of these relations. Fig. 13 gives the gate-opening-discharge curve of a 70,000-hp. unit as determined by the Gibson method of testing. Both the piston stroke of the regulating cylinder and the guide-vane opening measured at the vanes are plotted against the values of discharge. The tangents to the curves as shown will of course produce different values of the pressure rise. It will be noted that the guide-vane-opening relation produces a considerably greater rate of destruction of velocity near the end of the closure, and therefore in this case proves to be the more severe so far as pressure rise is concerned. In this particular case the type of operating gear used probably accounts for the wide difference between the curves. This illustration serves to show that, for a very accurate determination of pressure rise, accurate data concerning the turbine performance are necessary.

A Gibson test made on a 37,500-hp. unit produced the diagram of pressure and time as shown by the curve marked *A* in Fig. 14, in which the diagram has been replotted on a horizontal base line. Along with the diagram the chart of gate travel was taken by a recording device mounted on the regulating cylinder. Knowing

the quantity of water flowing in the penstock as determined by the Gibson diagram for the gate opening of the test run, and knowing the rate of gate travel, the pressure-time diagram marked *B* was calculated and compared with the Gibson diagram as to impulse area. The agreement was within $1^1/_2$ per cent, which was very close for the computation, which involved 93.5 intervals or 93 individual computations of pressure rise by the arithmetic-integration process and which neglected friction in the conduit. Another computation was made of the pressure-time diagram using a value for *a* equal to 10 per cent of the former value, e.g.,

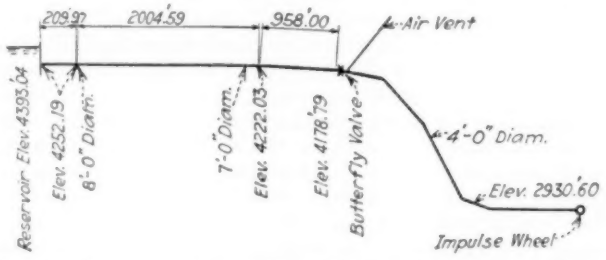


FIG. 15 PROFILE OF HYDRAULIC POWER PLANT WITH VALVE LOCATED ABOUT MIDWAY BETWEEN FOREBAY AND POWER HOUSE
(Control of flow assumed by valve after 67 per cent of its stroke is effected.)

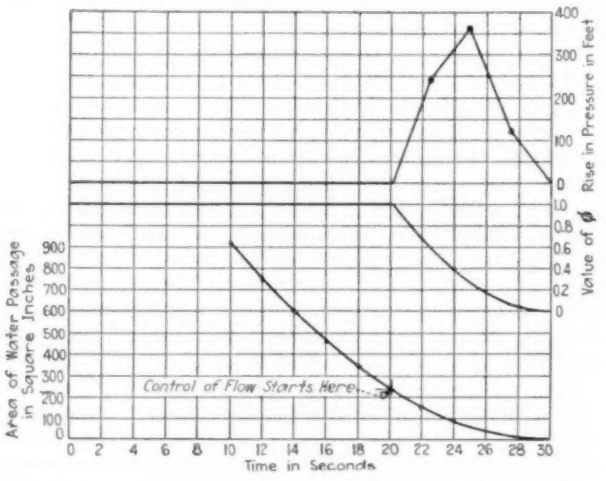


FIG. 16 RELATION OF GATE OPENING AND TIME AND OF PRESSURE RISE AND TIME FOR EXAMPLE OF FIG. 15

$a = 452$ instead of $a = 4520$, with the result that the area remained the same, but the maximum pressure rise increased by 23 per cent. This is shown by the curve *C* in Fig. 14. The Gibson diagram of course measures the impulse due to the closure, but does not necessarily give the correct pressure rise at any given instant. The agreement in area, however, is an indication of the accuracy of the calculation of the actual pressure-rise curve and is given for this purpose. The change in shape of the curve due to a large change in *a* is not great, so that if *a* is determined within reasonably close limits, an accurate determination is obtained in most cases.

Referring to waterworks service, the author states that "in controlling the flow in such a conduit by means of a valve, the discharge is not appreciably affected by the closure of the valve until the area is reduced considerably and the actual loss through the valve assumes fair proportions." The writer recently made an investigation of a water-power plant where this condition existed. Referring to Fig. 15, a butterfly valve was located about midway between the forebay and the impulse wheel. The valve operated under a head of about 215 ft. and the impulse wheel under 1400 ft. The discharge, which was to effect an automatic closure of the valve in 30 sec., was 236 c.f.s., this being 50 per cent greater than the discharge of the wheel at full gate. Under these conditions the valve will be closed to about one-third of its full opening before it assumes control of the flow, and if the full stroke is fixed at 30 sec., the flow will be shut off in about 10 sec., which results in an equivalent traversing time of about $7^1/_2$ sec. Fig. 16 shows the relation between time and the orifice or valve area which is denoted by ϕ. The maximum pressure rise of 361 ft., shown also in Fig. 16, results instead of a rise of 69.9 ft., which is based upon a complete destruction of velocity in 30 sec.

RAY S. QUICK.[13] The striking analysis by Mr. Kerr, showing that the maximum rise in pressure with uniform gate motion may be reached with closure starting from some partial opening rather than full opening, obviously is an important point to bear in mind in the consideration of practical hydroelectric water-hammer problems. While the general variations of the maximum pressure rise may be determined readily by use of the water-hammer charts now available, the limits are not apparent until considered in the light of Mr. Kerr's studies.

A direct confirmation of Mr. Kerr's conclusions for the case of a given head and uniform gate motion may be obtained by reference to Allievi's chart, Fig. 24,[14] by laying a straightedge along the lines of constant pressure rise and noting that they are practically radial through the origin for the major portion of the area of the chart. With uniform gate travel, it is obvious that so long as the line of constant pressure rise is truly radial the rise remains constant, since the initial gate opening and time are changed proportionally. As the origin of the chart is approached, the lines of constant pressure rise dip slightly below the radial, thus indicating that the percentage pressure rise with uniform motion is increasing until closure takes place in exactly one interval, after which the maximum rise reduces uniformly to zero.

In the upper left-hand part of the chart the lines of constant pressure rise reverse in inclination with respect to radial lines, indicating that maximum pressure rise is reached for closure from full gate opening, but this occurs under conditions where the rise in pressure is so many times the initial head that it would not likely be met with in actual practice.

In the region of the Allievi chart where the lines of constant pressure rise are radial, it is obvious that a slight error in the determination of the velocity of the pressure wave *a* is of no consequence, since this factor appears directly in both axes of the chart. Thus, fortunately, one may assume any reasonable value of *a* in a given problem and obtain the same answer for the rise in pressure in this region of the chart. Conversely, for a given allowable pressure rise, the time of gate closure may be established with the same degree of accuracy. However, when working in a region of the chart where the pipeline constant "rho" is less than unity, the lines of constant pressure rise diverge materially from the straight radial direction, thus introducing the possibility of some error for offhand values of *a*. As this embraces the region of high-head plants, care should be taken for such calculations and the true value of *a* closely estimated.

Consideration of the distribution of water-hammer stresses along the pipeline profile is important to bear in mind as it may be found that the factor of safety is a minimum at some point considerably upstream from the discharge orifice. In fact, a review of actual conduit failures in service would appear to bear this out. As the penstock usually dips steeply toward the power house in the lower portion of its length, special care should be taken to

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[14] "Theory of Water Hammer," L. Allievi, translated by E. E. Halmos, 1925.

safeguard this section, as a protection to the power house.

In conclusion, water-hammer stresses should be investigated and provided for not only at the discharge orifice or gate but along the conduit as well. The gate-operating mechanism and governor should be provided with adequate adjustments and safeguards of a "foolproof" nature so that the many modifying factors of practical operation may be allowed for. The addition of a governor-operated relief valve or bypass many times solves a perplexing practical operating problem in combined pressure and speed regulation of hydroelectric equipment. Experience shows that conservatism rather than economy should govern in the selection and arrangement of the plant equipment and penstocks.

EUGENE E. HALMOS.[15] Mr. Kerr has written on this subject before, and I consider this paper to be a sequel to his papers of 1924 and 1926. During the last ten or twelve years much has been written and published on water hammer. However, since Lorenzo Allievi's book one would have thought that there was nothing more to be said about this phenomenon. But Mr. Kerr has managed to find a new viewpoint, something that is worth while to bring before the profession.

Mr. Kerr brings out the fact that it is not sufficient to investigate and design a pipe line for the condition of closing the gate from full opening to zero in a specified minimum time, but it is extremely important that closures beginning at part-gate be studied.

One paragraph from Allievi's book has some reference to this matter. On page 17 he says: "It should be observed, moreover, that of the five elements which characterize a conduit under pressure, the first three, i.e., V_o, a, L, are constructive and in general invariable, whereas V_o and t are functional and are frequently modified, varying the flow of the pipe and the speed of the operation. By such modification of either of these two latter elements the pipe becomes a new conduit in which the phenomena of water hammer may present entirely different laws from those applying when the original assumptions are considered." This calls the designer's attention to the very danger pointed out by Mr. Kerr. We ought to be thankful that he has shown in a forceful manner the danger which may come by neglecting to investigate a pipe line for partial closure and has described the ingenious method by which such study may be carried out.

I do not quite agree with the author's recommendation of using de Sparre's formulas—first, because they are limited in application, as he has stated, and, second, because within that limit they are not accurate.

If in Table 1 the results of de Sparre's formula are compared with those of Gibson, which are derived by arithmetical integration, it will be found that even within these limits there are points where the results are almost 5 per cent off.

The author says this: "The charts of Allievi and Quick offer a more rapid and convenient method of solution and give the maximum pressure rise for any given condition. They have some limitations, however, due to the fact that they are not available universally and, when reproduced in publications, cannot usually be read with great accuracy. This is particularly true of the Allievi chart for conditions involving fairly rapid closures from small gate openings, a serious inconvenience in determining the critical conditions for the problem now under consideration."

I had the privilege of translating this work of Allievi's, and the book has been given gratis to the profession through the generosity of the owner. Anyone applying to the secretary of the several societies can obtain copies of his book.

It is true that in some of the handbooks which reprinted Allievi's chart, the chart which Allievi calls "General Synopsis" has been given. On this chart in the upper left-hand corner there is a blank space where the lines coming close together would make the diagram illegible.

With reference to this diagram I agree with Mr. Kerr that it is really hard to pick an accurate point; but Allievi, for the purpose of eliminating this defect, included another diagram, Fig. 23 in the book, in which he shows this region on such a large scale that there is really no difficulty at all to pick the necessary point with an accuracy which is almost as close as the arithmetical integration and certainly should not vary more than 1 per cent.

BYRON E. WHITE.[16] This paper is of outstanding importance and interest in the field of hydraulic engineering, and is particularly important to those engaged in the design and operation of hydroelectric power plants, especially where long closed conduits are involved. The principles brought out have a most important bearing on the speed or frequency regulation of hydraulic-turbine generating units.

The maintenance of very close speed regulation is most important, especially in textile industries, and is difficult to maintain with hydraulic-turbine equipment, no matter how sensitive the governors may be. It is usual to find considerable wavering of the line of recording frequency meters, when the governing is done exclusively by hydraulic-turbine governors. We have known in general the reasons why frequency fluctuations occurred, but the paper brings out and emphasizes some of these reasons still more strongly.

The importance of the modified gate-opening curve and the mechanism for bringing it about, as described partly by Mr. Kerr and partly by Mr. William Monroe White, ought to be forcibly brought to the attention of all who are interested in the governing of hydroelectric plants. This paper is an evidence that very important progress has been made in the art of so regulating and limiting pressure rises that we may have the pressures in our conduits limited to safe values and at the same time obtain reasonable speed regulation. Some of the overspeed that we sometimes experience is fully as much due to excessive pressure rise as it is to the volume of water which would normally pass through between the guide vanes of the turbine under normal pressure rise in the conduit.

ROBERT E. B. SHARP.[17] I should like to stress the importance of determining the value of a in wood-stave pipe lines. There is very little information on this subject, but what information is available shows that there is a wide variation in values of a that have been determined experimentally. This is only natural when the construction of wood-stave pipe lines is considered, and it is reasonable to expect that this value of a will depend upon a number of variable factors, such as the relation between thickness of staves and diameter of pipe, relation between spacing and diameter of bands and diameter of pipe, wood used in staves, and age of pipe line.

It would be reasonable to expect that this value of a would be lower for a new line than after it has aged and some compression has taken place in the staves and in the degree in which the staves have embedded themselves in the bands. Where the pipe line between the forebay and the hydraulic turbine is made up, as is sometimes the case, partly of a wood-stave section and partly of a steel section, it is essential that some data be available relative to the rate of travel of the pressure wave a of the wood-stave portion of the line in determining the fastest allowable rate of gate movement. Where a water-saving relief valve is installed, as is sometimes the case, the importance of taking into

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[17] Chief Engineer, I. P. Morris Corporation, Philadelphia, Pa. Mem. A.S.M.E.

account the true value of a is greater for gate openings than for gate closings.

Where a differential surge tank is employed between a wood-stave flow line and the turbine penstocks, the value of a may be so much lower than that assumed in the design and the length of the wood-stave pipe line so great that when loads are rejected from the turbines the ensuing rise of the water level in the riser of the tank may be sufficient to slow down the flow in the wood-stave pipe line such that $T<2L/a$. This results in a pressure rise in the flow line equal to AV/G, which may be destructive.

In view of the foregoing, a systematic effort to determine experimentally the value of a in large numbers of wood-stave lines and the collection of all existing data would be of real value.

Mason A. Stone.[18] The previous discusser has raised the question which I was about to ask as to the value of a, the velocity of pressure-wave propagation in wood-stave pipe. After a careful search through the literature of the subject I find only meager references.

In the latest handbook I found a statement that the velocity of the wave in wood-stave pipe is not predictable, which was most discouraging. A footnote referred to a paper presented before the A.S.C.E. in 1898 by Mark, Wing, and Hoskins. But that was before the correct theory of water hammer has been developed in the paper of Allievi; and while they made some experimental determinations of the pressure in wood-stave pipe lines, they begged the question of the velocity of propagation by saying that is was probably about the velocity of sound in water, which is of course quite wrong, as the elasticity of the staves greatly modifies this value.

There was also a paper presented before this Society in 1912 in which the author substituted the modulus of elasticity of wood for that of steel in a formula for a and laboriously tabulated the values obtained thereby for a wide range of pipe sizes.

Mr. Minton F. Warren criticized this formula in a paper before the A.S.C.E., suggesting its modification by the inclusion therein of an expression for the action of the steel bands. While it is rather late to be discussing a paper presented in 1912, it may be as well to point out that the elastic action of a circle of staves in which the material is not continuous is quite different from that of a continuous steel shell, especially for increased pressure in the pipe, and that therefore the values in this paper obtained by the use of E for wood are probably far from correct.

For instance, Yule's formula for a as modified by Warren gave me a value of about 3800 ft. per sec. for a 72-in. diameter wood-stave pipe. I am under the impression that this is very much too large for a wood-stave pipe and hope that some of the more experienced members may enlighten us on what values we should use for a.

It is very important that we should know them and that we should not be content with the statement that a is not predictable. The author has shown the importance of the water-hammer phenomena which depend directly on its value, and we should certainly be able to apply the results of his paper to wood-stave pipe, as this appears to be the most economical form of long, high-level conduit.

Elmer G. Hooper.[19] Attention has been called to an apparent discrepancy between actual water-hammer pressures and velocities in wood-stave pipes and the theoretical values obtained from accepted formulas. The explanation is that not all the factors influencing energy absorption in this type of conduit have been included in the formulas.

Water hammer, so-called, is the result of conversion of kinetic energy in a moving mass of water in potential energy. It may be expressed as the product of a force times a distance, $P \times D$, where P is one-half the product of the water-hammer pressure per unit area and the area of pipe cross-section and D is the *total movement* of the *mass center* of the water body while under the influence of this pressure. Since the amount of energy absorbed, or converted, in changing the velocity of a mass of water a given amount is constant, $P \times D$ is constant; hence anything that will change D will change P reciprocally. A conduit with comparatively "springy" or "stretchy" walls will develop a correspondingly large value of D with a proportionate reduction in P and in the unit pressure. The velocity of pressure wave is also a function of the "springiness" of the pipe, as is demonstrated in the development of Joukovsky's formula. The mathematical determination of "springiness" in a wood-stave pipe is somewhat more complex than for pipes with walls of simple homogeneous construction like cast iron and possibly steel pipe; Joukovsky's formula deals only with the latter. It is conceivable that this formula can be modified to include the other factors influencing the "stretch" of the pipe walls, though not without considerable study. The important factors are compression of water, compression of wood staves, stretch of bands, and deflection of staves between bands, the last two being influenced directly by dimensions of the units and the spacing of the bands. Such a formula would hold only up to the point where staves would separate and permit water to escape.

In the light of what has been said in the foregoing, the low excess pressures and low-pressure wave velocity in the wood-stave pipe as compared with the values computed from the formula are not difficult to understand.

Samuel Shulits.[20] The statement of the paradoxical fact that the maximum pressure rise due to a load rejection to zero load in a finite time greater than zero actually does not occur with maximum penstock velocity or maximum discharge, as so ably presented by Mr. Kerr, is to many engineers a "contradictio in adjecto;" and this latter truth has indicated the need of an intuitive perception of this interesting phenomenon. The fact of this pressure-rise phenomenon is mathematically so easily perceptible, but descriptively not always obvious. In line with this thought, the writer presents here a physical interpretation of the occurrence.

The purpose of a physical interpretation is in general to so illuminate a mathematical proof or an experimental observation that initial incredulous surprise is raised to an understanding, an intuition, or feeling that the described phenomenon can happen.

Physical Interpretation

With the first closing motion (or jerk) of the gate, the pressure would increase a certain amount due to the destruction of velocity according to the momentum law, under the ideal conditions that the rate of retardation of flow is proportional to the rate of diminution of gate opening and that water and penstock are rigid, inelastic bodies. Since these conditions are not nature-true, the ideal pressure rise does not occur. In its place a smaller pressure rise is realized because a diminution of the rate of retardation of the flow and a deformation of the elastic water and elastic penstock actually do take place.

This diminution of the rate of retardation of the flow suggests the use of a conception of a "relative acceleration." It is well known that the rise of pressure, which commences immediately

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[20] Freeman Traveling Fellow of the Boston Society of Civil Engineers at the Prussian Experimental Institution for Hydraulic Engineering and Shipbuilding, Berlin, Germany.

after the gates begin to move, increases the velocity of discharge through the gate opening. With no pressure rise, the velocity at the end of the first gate motion would be smaller than it is with the pressure rise. The difference between these two velocities divided by the time of this first-minute gate motion is a quotient that represents the "relative acceleration" due to increased pressure. This term will be used in this sense hereinafter. It should not be forgotten that the flow is being constantly decelerated during the closure, though at a smaller rate due to the relative acceleration.

Gate closure causes a change of kinetic energy into potential energy. If, during the closure, the total potential energy produced were in some manner during this closure reconverted into kinetic energy, then no pressure rise would be realized. With no such reconversion of energy, maximum pressure rise would result. (The elasticity of the water and penstock is here temporarily neglected for the immediate purpose of the analysis in this paragraph.) A partial reconversion would produce an intermediate value of the pressure rise. The latter is the case in question. The increased head due to the pressure rise causes a relative acceleration of the water to a velocity greater than that corresponding under normal conditions to the opening attained at the end of the small motion under consideration. Thus a part of the kinetic energy liberated and changed into potential energy by the gate closure is reconverted into kinetic energy during the closure.

For the foregoing process of relative acceleration, not all the kinetic energy initially available under normal conditions is needed. The remainder as converted into potential energy by the closure continues in this form and is stored by compression in the water and by expansion in the penstock wall.

It is essential at this point to consider the inertia of the water and the penstock. The column or body affected by the pressure rise is really the totality consisting of a mass of water and a surrounding container penstock. The moving mass of water acts to maintain its velocity against the stoppage compelled by the gate closure. This may be conveniently termed the inertia of the kinetic energy of the column. The water and penstock wall act to prevent the motion of deformation or to maintain their original shape as governed by the stresses in them. The shape and stresses depend on the amount of potential energy initially stored in the water and penstock under normal conditions. We may therefore conveniently speak of the inertia of the potential energy of the column. In the presence of both forms of energy, a predominance of either form of energy means a predominance of the interia of that form and a consequent predominant innate sluggishness toward a change to the other subordinate form of energy.

In the respective magnitudes of these two forms of energy, kinetic and potential, and especially in their mutual relationship, we have a measure of the actual pressure rise attainable. Since kinetic energy involves time and potential energy does not, it becomes immediately evident that the time, too, influences the pressure rise; for the kinetic and potential energy in the column together determine the pressure rise. In other words, the time influences the rate of change of kinetic energy into potential energy, or the *pressure rise*. This conception of the pressure rise as a rate is recognizable in the momentum law:

$$F\ (\Delta T) = M\ (\Delta V)$$

where F = the decelerating force
ΔV = the decrement of velocity
M = the mass
ΔT = an interval of time.

The pressure rise is the force F. The time influence is shown in the time of closure T or in N. This is self-evident.

What is a measure of the attainable rate of energy transformation? How can we measure the inertia influence? What can we use as a yardstick? Simply enough, the answer[21] is the constant K.

$$K = {}^1/_2 \sqrt{\frac{KE}{PE}} = \frac{aV_o}{2gH_o}$$

where KE and PE are respectively the kinetic energy and the potential energy contained by the conduit per unit length under the initial normal steady conditions of flow or with V_o and H_o, as defined by Mr. Kerr. In words, K is equal to one-half the square root of the ratio of the kinetic energy to the potential energy initially present in each unit length of the conduit. For the purpose of this discussion we will use the transposed form of this relation:

$$PE = \frac{KE}{4K^2}$$

Thus, the potential energy present at the start of gate closure varies inversely as the square of K, often called the "pipe-line constant." Between the kinetic and potential energy, then, there is always a natural balance, under the initial normal conditions, as formulated in the foregoing equation.

Greater values of K at larger gate openings are consequently intimately bound with a contained potential energy proportionately small in comparison with the contained kinetic energy, and vice versa. Another view is that with large K the weight is on the kinetic energy side of the balance, while with small K the weight is on the potential energy side. Consequently, with large K greater proportional relative accelerations occur, while with small K greater proportional storage of energy in the water and walls occur. The shorter the time of closure with small K, the greater the rate of energy transformation or storage in order to transform and store the same amount of energy, and therefore the greater the pressure rise.

The foregoing is the nucleus of the writer's interpretation, which will now be applied in illustrating the phenomenon presented by Mr. Kerr.

1 With 100 per cent gate opening and maximum K, we have present the minimum potential energy and the maximum kinetic energy in their proportional relation, together with the maximum time of closure. Of the available kinetic energy with its proportional preponderance and its obstinacy of inertia, a greater part insists on a relative acceleration of the water instead of decorously becoming potential energy immediately. Only a smaller part, then, of the kinetic energy is immediately transformed into potential energy. Now, because we have a larger time for this energy transformation during the time of closure, the result is a smaller rate of transformation of kinetic energy into potential energy. This means a smaller rate of storage of energy in the water and penstock wall or a *smaller pressure rise*, even though a greater proportional amount of kinetic energy is being converted into potential energy.

2 With a small gate opening—say, 10 per cent, to take Mr. Kerr's case—the conditions are proportionately small kinetic energy, proportionately large potential energy, and a short time of closure with small K. Only a smaller part of the available kinetic energy with its proportional subordination is free to accomplish a relative acceleration of the water. The proportional preponderance of the potential energy compellingly aids in these circumstances the net deceleration of the flow. A greater part of the kinetic energy is then transformed into potential energy.

[21] For derivation see "Theory of Water Hammer," Lorenzo Allievi, translated by Eugene E. Halmos, under auspices of A.S.M.E. and A.S.C.E., pp. 8 and 9.

Now, because we have a smaller time for the energy transformation during the time of closure, the result is a larger rate of transformation of kinetic energy into potential energy. This means a larger rate of storage of energy in the water and the penstock wall or a *larger pressure rise*, even though a smaller proportional amount of kinetic energy is being converted into potential energy.

Summary of 1 and 2. Therefore a maximum pressure rise at a smaller discharge than full discharge is possible in a finite time of closure smaller than the finite time of closure of full discharge. The mathematical and physical laws between the conditions of full discharge and those at a smaller discharge are shown by Gariel and de Sparre.

3 The basic relation $PE = \frac{KE}{4K^2}$ permits another conclusion. It states that with very small values of KE and K, PE approaches infinity, also that at zero gate the function is indeterminate. We know quite well the potential energy of our column of water at rest. It is certainly not infinite. It is also true that, with zero-gate opening closed in zero time, there is no pressure rise. Hence it follows from this consideration and from 1 and 2 that the rate of energy transformation or the *pressure rise* reaches a maximum at some gate opening greater than zero and smaller than 100 per cent and then decreases to zero at zero-gate opening. The possibility of a variation from a maximum value to zero in the region of $K = O$ is thus shown by this interpretive analysis.

4 The shape of the pressure-time curve with $N=3$ in Fig. 6 of Mr. Kerr's paper can also be explained. The initial rate of energy transformation or the initial pressure rise has been so large that the column of water receives a relative acceleration actually large enough to bring a visual relief-valve action into play, with a consequent decrease in pressure. With the decrease of this relief action, the pressure rise increases again. The values of K and N influence the shape of the curve.

The writer hopes to have shown that the phenomenon presented by Mr. Kerr is intuitively comprehensible by all. Many pressure-rise studies tend to bristle with constants, formulas, and mathematical abstrusities flung about the utter disregard of their physical significance. The writer uses the formula $PE = \frac{KE}{4K^2}$, but this is really a simple expression of the total physical state of affairs in any penstock; an expression usually concealed by practical necessity in $K = \frac{aV_o}{2gH_o}$.

General Remarks

It should be noted that the maximum pressure is a function not only of the initial gate opening or flow from which the deceleration starts (as Mr. Kerr summarizes), but also of the time in which this initial flow is decelerated. This is strikingly clear when one remembers that the true possible maximum-pressure rise is with maximum penstock and instantaneous closure; or, in Mr. Kerr's Fig. 4, the ordinate that would be obtained with $a'b'$ extended upward to meet ob' produced.

The Allievi chart for "fairly rapid closures from small gate openings" is actually available in Mr. Halmos's translation in Fig. 23. There values of K and N can be read with ease to 0.025.

The conditions described by Mr. Kerr are not unknown to American engineers who have had to deal with pressure-rise problems. Mr. R. D. Johnson of differential-surge-tank fame and one of the forerunners in American pressure-rise studies says:[22] "The highest possible surge above forebay level following a complete shutdown is not necessarily due to the shutting off of the maximum velocity obtainable, but may occur as a result of throwing off part load at a critical velocity," and also "it does not always follow that higher velocities call for higher surge tanks to catch all the water, for, as will now be demonstrated, there exists . . . a critical velocity which necessitates a higher surge tank than any greater or smaller velocity."

To Professor Dr. Adolph Ludin of the Technische Hochschule in Berlin-Charlottenburg and to Dr. Franz Eisner of the Prussian Experimental Institution for Hydraulic Engineering and Shipbuilding in Berlin, the writer is gratefully indebted for a critical reading of the foregoing and for many valuable suggestions.

Author's Closure

Several important points have been raised in the various discussions, and among them is the question of the effect of water hammer on speed regulation mentioned by Mr. Byron E. White. A careful study of the water-hammer conditions in any plant having an appreciable length of water conduit will show the danger of speeding up the action of the governor beyond the value fixed by conservative design. A further study combining speed regulation with water hammer will show that the speed regulation will be much better with a normal rate of governor action than with the governor speeded up, due to the abrupt form of pressure waves set up with a too rapid action of the turbine gates.

Mr. Quick emphasizes the need for the "foolproof" methods of fixing the gate movement. It is the author's opinion that all the adjusting devices for medium- and high-head plants should be set under the supervision of a competent engineer and then sealed in such a manner that no unauthorized person could tamper with them.

Too great emphasis cannot be laid on the point brought up by Mr. Strowger in regard to the relation of discharge and the rate of travel of the control mechanism. The question is not so much a matter of the stroke of the operating piston, the travel of an indicator, or the angular turn of a set of movable vanes, as it is a problem to find the rate of change of velocity in the water conduit. The rate of change of area of the control orifice and the rate of change, if any, of the coefficient of discharge of the orifice are the essential relations.

The question of the accurate determination of the velocity of the pressure wave for different types of conduits is vital to the accuracy of any water-hammer study in the critical zone where $K < 1$. As Mr. Quick points out, in the other ranges where K is greater than unity, the same accuracy is not required in the value of a, but with high-head plants or with long conduits under low heads, the true value of a is essential.

With steel penstocks not embedded in concrete, the value of a can be calculated with a great degree of accuracy, but when surrounded with concrete, the velocity is probably somewhat higher than for a simple steel conduit, and less than a nondeforming conduit such as a rock tunnel. A concrete-lined tunnel probably has the maximum rate of wave propagation, 4660 ft. per sec., but there apparently are no tests available to prove this assumption.

In the case of low-head conduits, particularly with wood-stave lines, as brought out by Mr. Sharp, Mr. Stone, and Mr. Hopper, the analytic determination of the value of a is not susceptible of an easy solution. The only means of securing reliable data is by test, and there have been only a very few such tests made. The two most reliable tests that have come to the writer's attention are those run at the Searsburg plant of the New England Power Company and at the South Falls plant of the Hydroelectric Power Commission of Ontario.

The conduit at Searsburg is over two miles long, and the head on the conduit is approximately 35 ft. at all points along the length. The diameter is uniform, and the measured value of a was about 600 ft. per sec.

[22] Trans. A.S.C.E., vol. 79 (1915).

At South Falls, however, the conduit is under a gradually increasing head, ranging from about 20 ft. at the top to slightly over 100 ft. at the bottom. The tests give a true weighted mean of the value of a, as the measurements taken were the time required for the wave to travel up the penstock and back again, that is $2L/a$ seconds. A number of runs were made and an average of 1400 ft. per sec. was secured from the most reliable of the measurements. The results indicate a value of about 300 or 400 ft. per sec. at the upper end, and perhaps over 2000 ft. per sec. at the lower end.

In the near future there will be several opportunities to test wood-stave pipe lines, and it is hoped that there will eventually be sufficient data collected to determine the value of a with reasonable accuracy.

In conclusion, the author wishes to include the remarks of Mr. Carroll F. Merriam[23] at the meeting: "It gives me great pleasure to read the following cablegram from M. Maurice Gariel, Directeur Général, Atèliers Neyret-Belier et Piccard-Pictet, Grenoble, France: 'Entirely in agreement with your paper. Letter follows. Best wishes. Gariel.' It gives me a great deal of personal satisfaction in having made the introduction between the author of this paper and M. Gariel which led to the presentation of this valuable material in clear and concise form. I have noticed in my contact with foreign engineers that they take a great interest in the proceedings of American engineering societies and I feel that this interest should be especially appreciated."

[23] Pennsylvania Water and Power Company, Baltimore, Md. Mem. A.S.M.E.

HYD-51-4

Dredge-Pump Pressures and Thrust Loads

By JAMES H. POLHEMUS,[1] and JAMES HEALY,[2] PORTLAND, ORE.

The Port of Portland engineering staff, desiring to determine the actual thrust loads and pressures acting on the impeller of a dredge pump, carried on some tests which should be of interest to members of this society.

Pressure gages were installed at controlling points in both the front and back heads of the 2700-b.hp. 30-in. pump of the dredge Clackamas and a series of tests run to determine the effect of varying lengths of pipe line, impeller clearances, and impellers with and without relief holes in the back shroud.

The results of the tests showed clearly:

1 That relief holes in the back of the impeller materially reduced the thrust load and did not reduce the efficiency of the pump to any noticeable degree.

2 That renewable clearance rings and small vanes on the outside of the impeller shrouds are necessary to maintain close clearances and control the thrust loads.

3 While large clearances produce low thrusts, close clearances do not create a prohibitive thrust and are necessary to maintain a high pump efficiency.

4 That the pressures on the front and back shrouds are not the same and are considerably less than the pump discharge pressure.

5 That the whirlpool pressures are not as high as the discharge pressure of the pump, and that the pressures in the lower half of the pump casing are somewhat greater than those in the upper half.

6 That it is not necessary to carry the same pressure on the water service to the pump-shaft seal as is the discharge pressure of the dredge pump.

7 That as might be expected the thrust load increased with the total head.

8 That the thrust load is greater for any given condition when pumping material rather than water.

9 That the thrust loads are less for any given heads with the smaller high-speed pump impeller as compared with larger slow-speed impellers.

DURING the early part of 1928, the Port of Portland ran a series of exhaustive dredge-pump-pressure tests on its modern high-powered Diesel-electric 30-inch dredge *Clackamas*.

This was an ideal dredge on which to carry out such a series of tests; the pump motor had both adjustable speed and power regulation so that it was possible to run these tests at various predetermined speeds and at known powers. The dredge pump also was of the latest and most efficient design for pumping large quantities of material through long pipe lines and against high net terminal lifts as necessitated by the Portland, Ore., work.

James Healy, master mechanic in charge of the upkeep of the Port of Portland's dredging plant, conducted the tests in the field and made the computations. C. V. Bjornvall of the Port of Portland engineering staff also assisted in this work.

Nature of Tests

The pump shell was bored both front and back at various locations and pressure or vacuum gages installed so that pressure and vacuum readings could be taken simultaneously of the pressures within the dredge pump, when operating against various lengths of pipe lines, as well as at various powers. Tests were also conducted with closed-back pump runners and those with relief holes in the back shroud.

The pumping medium was clear water, as it was found that if the tests were run when pumping material, the various gages soon became stopped up. At first some tests were made pumping through actual pipe lines, but to make the tests more complete and to be able to run them off with a minimum amount of dredge delay, two test nozzles representing two lengths of pipe line were used.

Object of Tests

Desiring to install ball-bearing or Kingsbury thrust bearings on the Port of Portland dredges, it was necessary to determine as closely as possible the actual thrust loads under various working conditions so as to be able to select the smallest bearing that would amply take care of the maximum thrust loads.

This was particularly important with this type of thrust bearing as its cost increases very materially as the thrust load increases. It is also desirable to keep down the weight and size of such items in modern dredge construction. It was desired to determine what effect various methods of pump design had upon the thrust load and to formulate a set of rules by which the accurate thrust load on any dredge pump could be closely determined.

It is most important that the thrust bearings of the dredge pump not only be large enough to take care of the maximum thrust loads, but also be of simple and rugged design, requiring small maintenance and little, if any, supervision when the dredge is in operation.

The modern pipe-line dredge of today uses 50 per cent more power to drive the dredge pump than was the case a few years ago, likewise the pipe-line pressures are 50 per cent greater, it not being an uncommon practice to operate with pipe-line pressures as great as 110 lb. per sq. in.

We have used both the Kingsbury and the marine horseshoe type of thrust bearings in connection with our dredge-pump developments and our objection to the horseshoe type of thrust bearing, though it is very much cheaper than a Kingsbury thrust bearing, is that it requires too much operating attention; the maintenance expense is high, and it is the cause of many expensive dredge delays. We have never used the SKF ball bearings for dredge-pump work but the U. S. Engineer Department has done so with very good results.

Another object of these tests was to determine accurately what water pressure should be maintained on the main pump-bearing seal. Naturally this problem is greatly simplified if a single-stage centrifugal pump with a large discharge volume at pressures less than 100 lb. per sq. in. can be used.

Description of the Dredge Pump

The 30-in. *Clackamas* dredge pump consists of one cast-steel ring and two cast-steel heads, which form the complete pump.

All the inside surfaces subject to wear are faced with renewable $^3/_4$-in. mild steel plate liners held in place with $^3/_4$-in. countersunk-head bolts.

The whirlpool is concentric with the impeller and is of rectangular section $27^1/_2$ in. between head liners and 8 ft. $10^1/_2$ in. in diameter. The distance between the periphery of the impeller

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to the ring liners is 18 in. The suction is 30 in. in diameter as is the discharge. In designing a new 30-in. pumping unit we probably would use a suction as large as 34 in.; however, in this case we adopted the 30-in. suction to keep all our dredge pumps standard.

The impeller is made of cast steel in one piece and is 70 in. in diameter. To use as much as 3000 hp. on a very long pipe line to build up 110 or 115 lb. pressure would require a runner somewhat larger in diameter, about 72 in. with the present maximum speed. The outer faces of the shrouds are finished to an overall width of 26 in. A ring $^5/_8$ in. thick and 30 in. inside diameter is riveted to the face of the front shroud for the purpose of maintaining a close clearance. This ring is renewed as the clearance is increased by wear.

Outside of this clearance ring and extending to the periphery of the impeller, renewable vane-shaped strips $^5/_8$ in. thick are riveted to the face of the front shroud.

The back shroud is also fitted with renewable vane-shaped strips and relief holes are provided to lessen the pressure on the back shroud and thus reduce the thrust load.

Direction of Thrust Load

The direction of the thrust in a dredge pump is always toward the suction side of the pump. The tests as run did not show any thrust in the other direction under any operating condition and the wear on all of the thrust bearings has shown this thrust to be in but one direction.

When the marine horseshoe type or "collar" type of thrust bearing is used, it is customary to provide equal bearing surfaces in each direction, as practically no extra expense is involved in doing this. On the other hand, with the Kingsbury type of thrust bearing, it would add undue complications and expense to provide for the maximum thrust in both directions.

We have heard it stated at various times by persons who might be expected to know, that a reversal of the thrust load did actually take place. We are quite sure from our practical operating experience, as well as the results of these tests, that if this is so it is of relatively small proportions. It is felt advisable, however, to provide for some thrust load in the opposite direction which assists in maintaining the fixed pump clearances and stops any floating of the pump shaft. With the Kingsbury thrust bearing we specify that the thrust elements have six shoes bearing against the side of the thrust collar toward the pump and two shoes on the opposite side.

Causes of Thrust Load

The thrust load on a dredge pump is caused by three general forces acting on the impeller:

a A force forward (toward the suction side of the pump) caused by the vacuum, built up by the pump, pulling upon that section of the impeller equal to the area within the path of the inner tips of the impeller vanes. A positive force.

b A force forward caused by the pressures inside the pump acting against the back shroud of the impeller. This is also a positive force.

c A force backward or in an opposite direction to forces *a* and *b* caused by the pressures inside the pump acting against the front shroud of the impeller. A negative force.

Methods of Computing the Thrust Load

The method usually used to figure the thrust load on a dredge pump, after assuming a maximum suction head and discharge head, has been as follows:

1 Compute the positive component of the thrust load *a* by multiplying the area of the impeller within the inner tips of the vanes by the suction head. (It is reasonable to assume about a 23-ft. suction head which amounts to approximately 10 lb. per sq. in. or a 20-in. vacuum.) Usually this area is the same as the area of the suction inlet, but in the more modern high-powered pumps this is not the case.

2 Compute the other positive component of the thrust load *b* by multiplying the area of the back shroud by the maximum pressure developed by the pump or the assumed maximum head.

3 Compute the remaining negative component of the thrust load *c* by multiplying the area of the front shroud by the maximum pressure developed by the pump or the assumed maximum head.

It was felt that this method produced a result which was considerably greater than the actual thrust load and that the pressures behind the front and back pump shrouds were not the same as the maximum pressure assumed for the dredge pump, and from previous dredge tests we knew that the pressure in the whirlpool chamber would be less than the discharge pressure of the pump.

With small and low-powered dredge pumps, where the ordinary marine horseshoe type of thrust bearing was used, this was not a serious matter as it only meant the adding of another collar or so. Now, with the more expensive type of thrust bearings, such as the SKF ball-bearing thrust bearing, or the Kingsbury thrust bearing, it is quite a different story when it is important to keep down sizes, weights, and costs.

As a matter of fact, these companies furnishing such thrust bearings for dredge pumps, in computing the total thrust load, assumed certain average pressures as acting on both the front and back head as well as the maximum discharge head and suction head which the dredge pump might be expected to develop.

We had not been able to obtain any real first-hand reliable information as to what these pressures were on the front and back shrouds of the pump impeller and just what factors caused them to vary. We noticed from time to time that various engineers in designing dredge-pump thrust bearings used widely different assumptions for relatively the same dredge pump and pumping conditions.

The thrust load for the *Clackamas* pump was figured by different persons all the way from 48,000 lb. to 160,000 lb. The results of our tests show that the maximum thrust load would be less than 60,000 lb. even with a discharge pressure of 110 lb.

In designing this thrust bearing we played safe and used the old method of not making any allowance for reduced pressures behind the pump impeller shrouds and adopted a maximum thrust load of approximately 100,000 lb. which we knew to be well on the safe side. If we had used a closed-back impeller, the maximum thrust load with 110 lb. pressure would have been considerably more.

Our experience in carrying on various dredge-pump tests, to determine other features of pump design, as well as to test the efficiency of various types of dredge-pump impellers, combined with our actual dredge operating experience, led us to believe that a close method of computing the thrust load of a dredge pump could be determined.

Unknown Variable Thrust-Load Factors

After assuming maximum suction and discharge pressures for the dredge pump, the only unknown factors required to determine the maximum thrust load are the average pressures being exerted on the front and back shrouds of the pump impeller.

We were satisfied that we could find a certain differential in pressures between the front and back shrouds for like diameters and areas. We also knew that we would not find the same pressures in back of the shrouds as in the whirlpool chamber. This shroud pressure was bound to be affected by the following features of the pump design:

1 Clearances between the pump impeller and pump casing, both front and back.

2 Whether the pump impeller was fitted with small vanes outside of the shrouds or not.

3 Whether there were relief holes in the back shroud of the impeller or not.

We have had experience with 20-in., 24-in., 27-in., and 30-in., dredge pumps operating with a maximum pipe-line pressure of 110 lb. and feel safe that this discharge pressure is the maximum to assume for a practical dredge pump. Likewise the adoption of a 23-ft. suction head, which is equivalent to approximately 10 lb. or 20-in. vacuum, is a liberal assumption to make when considering the subject of maximum thrust loads.

Method of Making Pressure Tests

Pressure gages were installed in both front and back heads as near as possible to the top center line and each gage on the front head was directly opposite the corresponding gage of the back head. (See Fig. 1 for the exact location of the gages, their designation, and distance from the center line of the pump shaft.)

To eliminate the pressure error that would be caused by leakage between the liner and the head, hollow countersunk bolts were installed through the liner and head, and gages were connected to the outer ends of these hollow bolts.

Tests were run pumping clear water through an 8-in. nozzle representing approximately 18,000 ft. of pipe line and through a 16-in. nozzle equivalent to approximately 1000 ft. of discharge pipe. The impeller used to make these tests was $^9/_{32}$ in. less in overall width than the distance between heads which allowed for three different clearance settings:

1 With $^1/_{32}$ in. in front and $^8/_{32}$ in. at back
2 With equal clearance front and back, and
3 With $^8/_{32}$ in. front and $^1/_{32}$ in. at back.

Tests at 250, 270, 290, 310, 330, and 350 r.p.m. were run with each clearance setting on the 8-in. nozzle which produced discharge pressures from 42 lb. at low speed to 92 lb. at high speed. We were not able to produce a higher discharge pressure as we could not fully utilize all of the power of the pump motor due to its limitation in speed. However, a large impeller or greater pump speed would utilize all of the power.

Tests at 250, 270, 290, and 300 r.p.m. were run with each clearance setting on the 16-in. nozzle with resulting discharge pressure of 41 lb. at low to 55 lb. at high speed. Simultaneous readings of all pressure gages were taken at each condition of speed, impeller clearance, and nozzle diameter and all properly tabulated. It was not possible to operate at a speed higher than 300 r.p.m. on the 16-in. nozzle because the motor reached its maximum power (2700). In our later 30-in. dredge pump designs, we now provide 3000 b.hp. and use a 72-in. diameter impeller.

In actual dredging practice the pump clearances are checked up once a week. The pump impeller is set with as close clearances both front and back as it is possible to obtain, which will vary from $^1/_{32}$ to $^1/_{16}$ in. Even with an adjustable pump shaft and an adjustable nose ring it may be expected to have clearances both in front and in back as great as $^1/_4$ in. if very sharp and abrasive sand is being pumped through long pipe lines where the pump pressures are high.

Recorded Test-Gage Readings

Tables 1 and 2 give the actual gage readings for each of the tests as conducted as well as the computed thrust loads in each instance. An open-back impeller was used for all runs from 1 to 28 inclusive.

From the actual pressure readings, as given in the tables, any one really interested in the subject of determining dredge-pump thrust loads can compute his own total thrust pressures and draw up a series of curves or formulate a set of laws by which the thrust load of any dredge pump can be accurately determined if it is known what the maximum pump pressure will be as well as the general dimensions and design of the dredge pump.

It is hoped that this paper will bring this subject to the attention of those interested in dredge-pump design and that it will create a free discussion of the subject and result in the greatest advantage being taken of these test records.

Computing Thrust Loads From Test Readings

The pressures as recorded for each condition, *A*, *B*, *C*, *D*, and *Y*, gages were plotted against their respective distances from shaft center and a curve drawn through them.

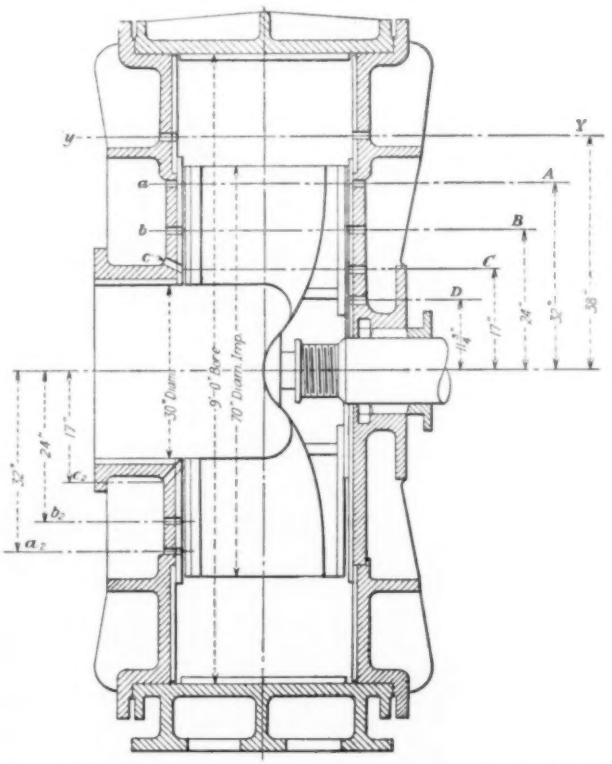


Fig. 1 Cross-Section of Pump Showing Location of Gages

The impeller shroud areas were divided into small area rings of equal width and the pressure as noted on the pressure curve at the radius of the center of width of each area ring was used as the average pressure per square inch for its respective area.

The product of the area and its respective average pressure is the total pressure acting on that area and the sum of the total pressures for all areas on their respective shrouds is the total pressure acting on that shroud. The same method was used for both the front and the back shrouds which make up the two thrust loads *b* and *c*.

The thrust load *a* is caused by the suction vacuum which acts over the area of the impeller whose outer diameter in this case is 30 in. or the path of the inner tip of vanes. The total thrust load is equal to the sum of the two positive thrust loads *a* and *b* minus the negative thrust load *c*.

Fig. 2 shows the method of computing the thrust load in detail for one test or running condition.

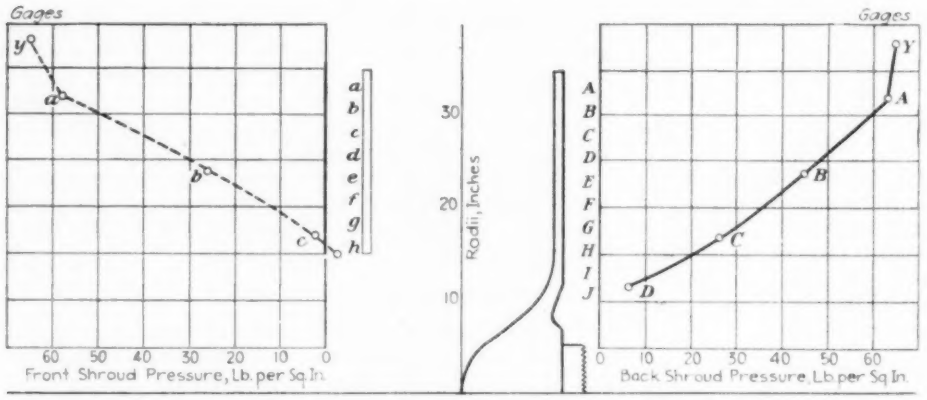


FIG. 2 DETAILED METHOD OF COMPUTING TOTAL THRUST LOAD ON DREDGE PUMP

(Approximate length of pipe line, 18,000 ft.; maximum discharge pressure, 92 lb. per sq. in., vacuum, 5 in.; pumping water)

Pressures	Lb. per sq. in.	Area	Sq. in.	Computations Lb.
A-a	3½	*A*	530	530 × 3½ = 1855
B-b	6½	*B*	490	490 × 6½ = 3185
C-c	11½	*C*	452	452 × 11½ = 5198
D-d	16	*D*	412	412 × 16 = 6592
E-e	19½	*E*	373	373 × 19½ = 7273
F-f	22½	*F*	334	334 × 22½ = 7515
G-g	24½	*G*	295	295 × 24½ = 7227
H-h	24½	*H*	255	255 × 24½ = 6247
I	15½	*I*	216	216 × 15½ = 3348
J	6	*J*	396	396 × 6 = 2376
Vacuum 5 in.	2½	Vacuum	707	707 × 2½ = 1767
				Total thrust load = 52583

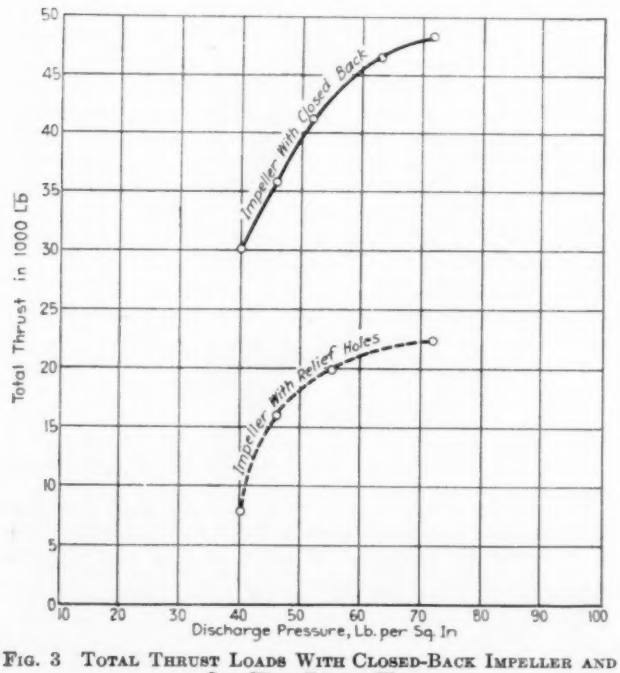


FIG. 3 TOTAL THRUST LOADS WITH CLOSED-BACK IMPELLER AND ONE WITH RELIEF HOLES

(Dredge *Clackamas* tests, Jan. 23, 1928; 70-in. diameter impeller, 3/32-in. clearance front and back; pumping water through 7640 ft. of pipe line with a net terminal lift of 27½ ft.; maximum pressure 72 lb. in both cases.)

EFFECT OF IMPELLER RELIEF HOLES

The *Clackamas* has a heavy marine horseshoe type thrust bearing and it was possible to run comparative dredging tests with both the closed-back type of impeller and the relief-hole type.

As far as dredging results and pump efficiencies were concerned there was no noticeable difference. The impeller with relief holes in the back shroud produced the same pipe-line pressures and vacuums with the same power as did the impeller with these relief holes plugged tight to make it a closed-back impeller. It was quite evident that one impeller would put just as much material through the pump into the fill as the other.

It was demonstrated that the thrust load was less with the relief holes in the impeller and we can see no good reason why relief holes should not be provided in the back shroud of all dredge-pump impellers.

TABLE 1 TESTS ON DREDGE *Clackamas* PUMP, FEBRUARY 21 1928, TO DETERMINE THRUST LOADS

(Pump-casing pressures with 8-in. nozzle and 100 ft. of line)

70-IN. IMPELLER, CLEARANCE, FRONT 1/32 IN., BACK 3/32 IN.						
No. of run	1	2	3	4	5	6
Pump r.p.m.	250	270	290	310	330	350
Pump amperes	1400	1750	2100	2650	3300	3550
Pump volts	515	510	505	500	490	500
Pump kilowatts	721	892	1060	1325	1617	1775
Pump brake horsepower	835	1078	1292	1643	2024	2230
Pump suction vacuum, in.	3.5	4	4.5	4.5	5	5
Pump discharge pressure, lb. per sq. in.	42	52	62.5	72	80	92
Front gages						
c, lb. per sq. in.	1	2	2	2	2	2
b, lb. per sq. in.	14	16	18	21	24	26
a, lb. per sq. in.	29	34	39	45	52	58
y, lb. per sq. in.	35	39	44	52	57	65
Back Gages						
D, lb. per sq. in.	3	3	3.5	4	4	6.5
C, lb. per sq. in.	14	16	18	20	23	26.5
B, lb. per sq. in.	22	26	29	34	39	45
A, lb. per sq. in.	32	37	43	50	56	63
Y, lb. per sq. in.	32.5	38	44	51	57	65
Total computed thrust load, lb.	26000	27400	31500	37300	39500	47600
70-IN. IMPELLER, CLEARANCE, FRONT 5/64 IN., BACK 5/64 IN.						
No. of run	7	8	9	10	11	12
Pump r.p.m.	250	270	290	310	330	350
Pump amperes	1350	1650	2000	2500	2850	3350
Pump volts	500	500	500	500	510	510
Pump kilowatts	675	825	1000	1250	1453	1708
Pump brake horsepower	774	984	1220	1562	1815	2141
Pump suction vacuum, in.	3	3.5	4	4	4	4
Pump discharge pressure, lb. per sq. in.	42	51	62	72	80	90
Front Gages						
c, lb. per sq. in.	5	6	7	8	9	10
b, lb. per sq. in.	17.5	20	24	27	30	33
a, lb. per sq. in.	29	34	39	45	51	57
y, lb. per sq. in.	33	38	45	51	55	62
Back Gages						
D, lb. per sq. in.	4	5	7	9	10	11
C, lb. per sq. in.	11	14	16	18.5	21	22
B, lb. per sq. in.	20	23	28	32	36	40
A, lb. per sq. in.	32	37	43	48.5	54	61
Y, lb. per sq. in.	32	38	44	50	56	62
Total computed thrust load, lb.	13,500	16,300	21,800	25,000	27,800	30,400
70 IN. IMPELLER, CLEARANCE, FRONT 3/32 IN., BACK 1/32 IN.						
No. of run	13	14	15	16	17	18
Pump r.p.m.	250	270	290	310	330	350
Pump amperes	1500	1850	2175	2550	3100	3575
Pump volts	510	500	500	500	500	500
Pump kilowatts	763	925	1087	1275	1550	1787
Pump brake horsepower	910	1088	1336	1578	1935	2248
Pump suction vacuum, in.	3	3.5	4	4	4.5	4.5
Pump discharge pressure, lb. per sq. in.	44	52	61	72	82	92
Front Gages						
c, lb. per sq. in.	6	8	10	12	14	15
b, lb. per sq. in.	19	24	28	31	35	40
a, lb. per sq. in.	28	35	41	47	54	60
y, lb. per sq. in.	32	39	45	50	56	63
Back Gages						
D, lb. per sq. in.	7	8	9	10	11	12
C, lb. per sq. in.	10	12	14	15	17	19
B, lb. per sq. in.	17	21	25	29	33	37
A, lb. per sq. in.	30	37	42	48	55	61
Y, lb. per sq. in.	31	38	44	50	57	64
Total thrust load, lb.	8400	8800	9000	9600	10440	10700

Table 3 gives the actual pressures and computed thrust loads for like tests with the closed-back impeller and one with relief holes in the back shroud. Fig. 3 is a graphic comparison of the resultant total thrust loads.

Relative Pressures Within Pump Casing

As will be noted all the gages were placed on the upper half of the pump as in this location they were easier to read and install. It would be impossible to install these gages on the lower half of the back head. However, to check the assumption that the pressures were relatively the same throughout the pump for any given distance out from the center of the shaft, similar gages were placed on the lower half of the front head and the pressures for these gages were from 2 to 3 lb. more throughout all the tests. (See Table 3.)

For all practical purposes it was felt that the gage readings on the upper half were accurate enough for computing the thrust loads in as close a manner as it would be practical to select or design a thrust bearing. It was felt that the differential in pressures between the front and back heads would remain constant for each relative position of the gages.

By referring to the tabulated pressures taken during these tests, it is clearly shown that the discharge pressure of a dredge pump is higher than the pressure within the whirlpool.

TABLE 2 TESTS ON DREDGE *Clackamas* PUMP, FEBRUARY 21, 1928, TO DETERMINE THRUST LOADS

(Pump-casing pressures with 16-in. nozzle and 100 ft. of line)

70-In. Impeller, Clearance, Front 5/32 In., Back 1/32 In.			
No. of run	19	20	21
Pump r.p.m.	250	260	290
Pump amperes	2775	3800	4150
Pump volts	500	500	500
Pump kilowatts	1387	1900	2075
Pump brake horsepower	1728	2390	2610
Pump suction vacuum, in.	14	15.5	17
Pump discharge pressure, lb. per sq. in.	41	45.5	52
Front Gages			
c, lb. per sq. in. vac.	0.5	1	1
b, lb. per sq. in.	14	16	18.5
a, lb. per sq. in.	23.5	27	31
y, lb. per sq. in.	28	34	37
Back Gages			
D, in. vac.	2	1	0.5
C, lb. per sq. in.	5	7	8
B, lb. per sq. in.	12.5	15.5	18.5
A, lb. per sq. in.	26	30	35
Y, lb. per sq. in.	27	31	36
Total thrust load, lb.	8700	11300	15200

70-In. Impeller, Clearance, Front 9/64 In., Back 9/64 In.				
No. of run	22	23	24	25
Pump r.p.m.	250	270	290	300
Pump amperes	2490	3200	4000	4280
Pump volts	500	500	500	500
Pump kilowatts	1245	1600	2000	2140
Pump brake horsepower	1535	2000	2520	2700
Pump suction vacuum, in.	14	16	18	18.5
Pump discharge pressure, lb. per sq. in.	41	46	52	55
Front Gages				
c, in.	5	6	7	7.5
b, lb. per sq. in.	10.5	12.5	13.5	14
a, lb. per sq. in.	22	27	31	32
y, lb. per sq. in.	29	35	39	40
Back Gages				
D, lb. per sq. in.	1	0.5	0.5	0.5
C, lb. per sq. in.	9	10.5	12	12.5
B, lb. per sq. in.	16	19	22.5	23.5
A, lb. per sq. in.	26	31	36	37
Y, lb. per sq. in.	27	32	37	38
Total thrust load, lb.	21900	24300	30100	30900

70-In. Impeller, Clearance, Front 1/32 In., Back 5/32 In.			
No. of run	26	27	28
Pump r.p.m.	250	270	290
Pump amperes	2775	3400	4125
Pump volts	500	500	500
Pump kilowatts	1387	1700	2062
Pump brake horsepower	1728	2130	2594
Pump suction vacuum, in.	14.5	16.5	18
Pump discharge pressure, lb. per sq. in.	42	47	52
Front Gages			
c, in. vac.	11	12	14
b, lb. per sq. in.	8	9	10
a, lb. per sq. in.	23	27	31
y, lb. per sq. in.	30	35	37.5
Back Gages			
D, lb. per sq. in.	0.5	1	2
C, lb. per sq. in.	10	11.5	13
B, lb. per sq. in.	17	20	23.5
A, lb. per sq. in.	27	31.5	36
Y, lb. per sq. in.	27	32	37
Total thrust load, lb.	26000	30300	36400

TABLE 3 DATA OF TEST MADE ON THE DREDGE *Clackamas* JANUARY 23, 1928, TO DETERMINE THE RELATIVE THRUST LOADS WITH CLOSED-BACK IMPELLER AND ONE WITH RELIEF HOLES

(Test run under regular dredging conditions, pumping both water and material through 7640 feet of pipe line with a net terminal lift of 27½ ft. Clearance of 5/32 in. both front and back.)

Tests With Closed-Back Impellers

(Pumping Water)

Run No.					1-*C*		2-*C*	3-*C*	4-*C*	5-*C*	6-*C*[1]	
R.p.m.					250		270	290	310	330	325	
Amperes					1950		2000	2500	3300	4200	4100	
Volts					520		520	510	500	500	500	
Vacuum, in.	7	7	8	8	9	9	10	10	12.5	12.5	27.5	
Pressure, lb. per sq. in.	40	40	46	46	52	52	63	62	72	72	75	
Front-head pressures, lb. per sq. in.												
Gage *y*					24		33	38	44	50	51	
Gage *a*					20		23	27	32	37	35	
Gage *b*					13		14	15	19	20	16	
Gage *c*					2		2	1	1	1	2	
Gage *a* 2					24		27	31	35	38		
Gage *b* 2					11		12	14	16	18		
Gage *c* 2					4.5		5	6.5	7.5	8.5		
Back-head pressures, lb. per sq. in.												
Gage *Y*					26		31	35	42	46	48	
Gage *A*					27		31	35	42	45	44	
Gage *B*					15.5		18	21	25	29	35	
Gage *C*					13.5		15.5	18	21	23	20	
Gage *D*					16.5		19.5	21	24	26.5	27.5	
Switchboard vacuum, in.					14		13	14	15.5	17.2	22	
Switchboard pressure, lb. per sq. in.					36		44	52	60	68	68	
Total thrust load, lb.					29921		35590	41092	46199	48057	60003	

Tests With Relief Holes in Back Shroud

(Same Impeller as Above With Plugs Knocked Out of Relief Holes)

(Pumping Water)

Run No.	1-*C*	2-*C*	3-*C*	4-*C*	5-*C*
R.p.m.	250	270	290	310	330
Amperes	1700	2100	2650	3350	4250
Volts	520	520	510	500	500
Suction-pipe vacuum, in.	7	8	9	10	12
Discharge pressure, lb. per sq. in.	40	46	55	63	72
Front-head pressures, lb. per sq. in.					
Gage *c*	2	2	1	1	1
Gage *b*	13	14	15	19	20
Gage *a*	20	23	27	32	37
Gage *y*	24	33	38	44	50
Back-head pressures, lb. per sq. in.					
Gage *D*	3	4	4.5	5	6
Gage *C*	1	3	4	4	4.5
Gage *B*	11	14	16	19	22
Gage *A*	27	32	36	41	46
Gage *Y*	31	37	43	49	54
Switchboard vacuum, in.	12	12.5	13.2	15	16
Switchboard pressure, lb. per sq. in.	38	44	52	60	68
Total thrust load, lb.	7615	15780	19700	18231	21985

[1] Pumping material.

Fig. 4 shows the thrust load plotted against discharge pressure with the impeller set for a clearance of 1/32 in. in front and 9/32 in. in back. Curve 1 is for the 8-in. nozzle, equivalent to approximately 18,000 ft. of pipe line, and Curve 2 is for the 16-in. nozzle, equivalent to approximately 1000 ft. of discharge pipe.

Fig. 5 shows the same thing except that the impeller clearance is equal both front and back, amounting to 9/64 in. Curve 3 is for the 8-in. nozzle and Curve 4 is for the 16-in. nozzle.

Likewise Fig. 6 shows the same thing but the impeller clearance is 5/32 in. in front and 1/32 in. in the back. Curve 5 is for the 8-in. nozzle and Curve 6 for the 16-in. nozzle. This close clearance in the back is just the reverse from the close clearance in the front as shown by Fig. 4.

A study of these curves shows that it is more important, as far as obtaining low thrust loads is concerned, to have a close clearance in the back rather than a close clearance in the front and a more open clearance in the back. Operating with close clearances both front and back, a high pump efficiency is maintained but not the lowest thrust load. Clearances as close as 1/32 in.

both front and back do not give as low a thrust load as the combination of $^{9}/_{32}$ in. front and $^{1}/_{32}$ in. back. It is obvious that the greater front clearance increases the value of the negative thrust load acting on the front shroud.

It will be noted that the total thrust load on the 8-in. nozzle varied from 10,600 lb. with all the clearance in the front, to 47,600 lb. with all the clearance in the back. With the clearances equal both front and back ($^{9}/_{64}$ in.), we got a total thrust load of 30,400 lb., which is approximately a mean between the two above extreme conditions, as might be expected.

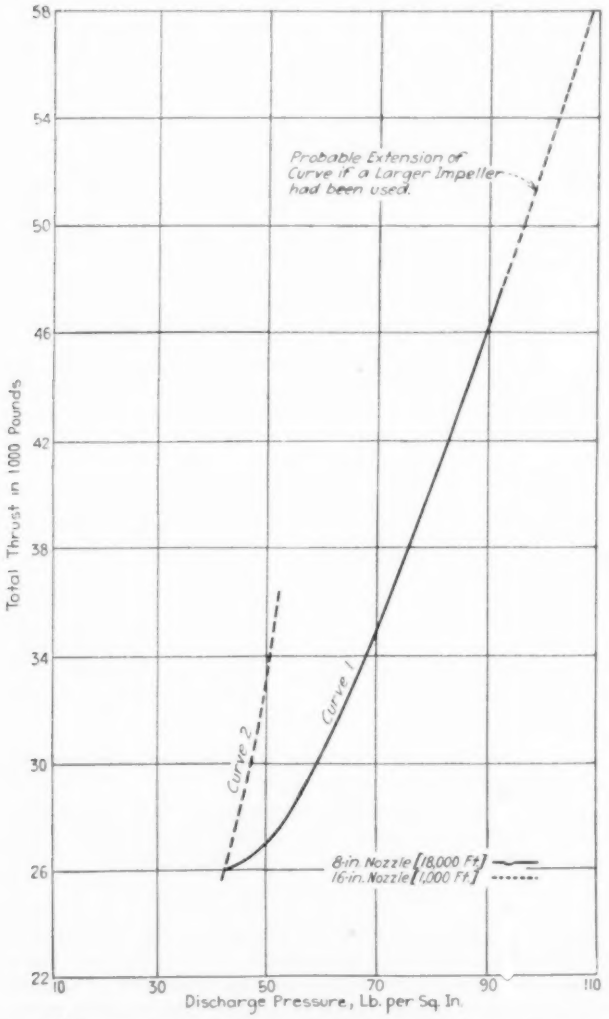


FIG. 4 THRUST-LOAD TESTS, 30-IN. DREDGE *Clackamas*, FEB. 21, 1928

(Front impeller clearance, $^{1}/_{32}$ in.; back impeller clearance $^{9}/_{32}$ in.)

A similar thrust-load relationship between the front and back clearances holds true for the 16-in. nozzle or an equivalent shorter length of discharge pipe line.

EFFECT OF PUMP HORSEPOWER

Fig. 7 shows the thrust load plotted against shaft horsepower for the 8-in. and 16-in. nozzles with both extremes of clearances. Curve 7 is for the 8-in. nozzle and Curve 8 is for the 16-in. nozzle with $^{1}/_{32}$ in. clearance in front and $^{9}/_{32}$ in. clearance in back. Curves 9 and 10 are the same with a front clearance of $^{9}/_{32}$ in. and a back clearance of $^{1}/_{32}$ in.

It is apparent from these curves that the thrust load increases with an increase of pump power for any given condition, due to the fact that the increased power increases both the discharge and suction heads.

EFFECT OF PUMP SPEED

Pump speed has no direct effect on the thrust load. The thrust load is caused by the pressures built up within the pumping casing, therefore, pump speed and horsepower are only incidental to bringing about the resultant pressures.

It is important to bear in mind, however, that the maximum thrust loads occur at top pump speeds which give the thrust bearing its greatest efficiency or capacity. Also high-speed dredge pumps have smaller pump impellers and the thrust load increases materially where it is necessary to increase the diameter of the impeller to raise the discharge or pump pressure.

Fig. 8 shows the relationship between impeller diameters and total thrust loads for the *Clackamas* dredge pump with a 70-in. diameter impeller, developing 92 lb. discharge pressure and a 5-in. vacuum, pumping water. Curve 11 represents the thrust load for

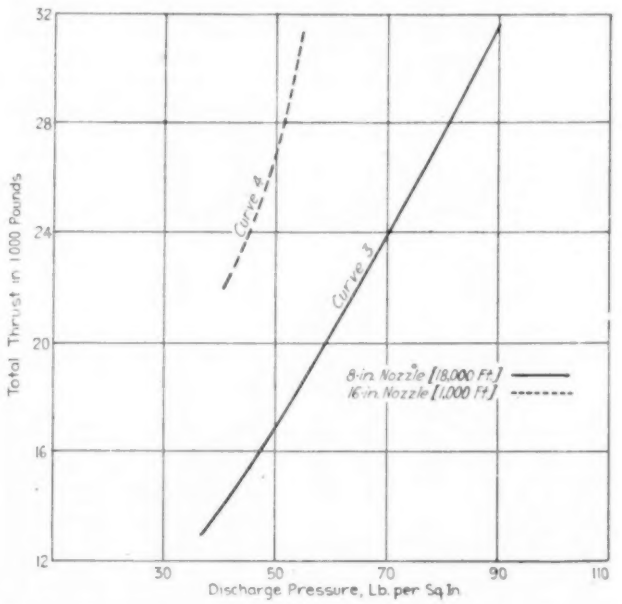


FIG. 5 THRUST-LOAD TESTS, 30-IN. DREDGE *Clackamas*, FEB. 21, 1928

(Front impeller clearance, $^{9}/_{64}$ in.; back impeller clearance, $^{9}/_{64}$ in.)

a $^{1}/_{32}$-in. clearance both front and back as might be expected when the pump was new, and Curve 12 is the thrust load for $^{1}/_{4}$ in. clearances after the clearance rings become badly worn. These curves not only show that the thrust load is considerably greater when large-diameter impellers are used to develop any given head, but also that the thrust loads are less with the $^{1}/_{4}$-in. clearances both front and back than with $^{1}/_{32}$-in. clearances.

EFFECT OF SIZE OF PUMP SUCTION

As the suction vacuum acts upon that part of the impeller within the suction area, any increase or decrease in the size of this area will naturally cause an increase or decrease in thrust load. This increase, however, is not great.

Assuming a maximum suction head of 23 ft., or approximately 10 lb. per sq. in., or 20 in. vacuum, the thrust load from this cause for dredge pumps having various size suctions is as shown in Table 4.

Though these tests pumping clear water did not show a combination of high suction vacuum and high discharge pressures,

TABLE 4 THRUST LOAD DUE TO SUCTION HEAD

Diameter of pump suction, in.	Area of pump suction, sq. in.	Thrust load at 10 lb. per sq. in., lb.
20	314	3,140
22	380	3,800
24	452	4,520
26	531	5,310
28	616	6,160
30	707	7,070
32	804	8,040
34	908	9,080

such a combination is possible in certain classes of dredging and, therefore, should be taken into consideration when determining maximum thrust loads.

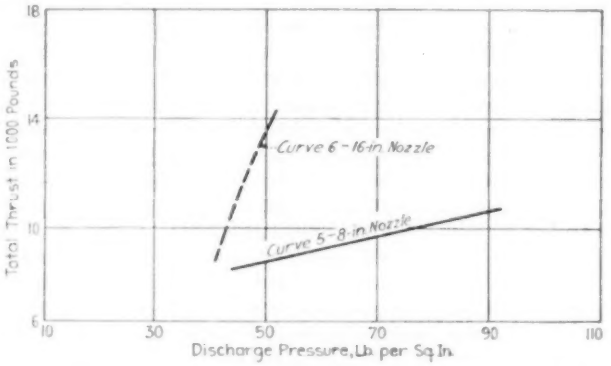


FIG. 6 THRUST-LOAD TESTS, 30-IN. DREDGE *Clackamas*, FEB. 21, 1928

(Front impeller clearance, 3/32 in.; back impeller clearance, 1/32 in.)

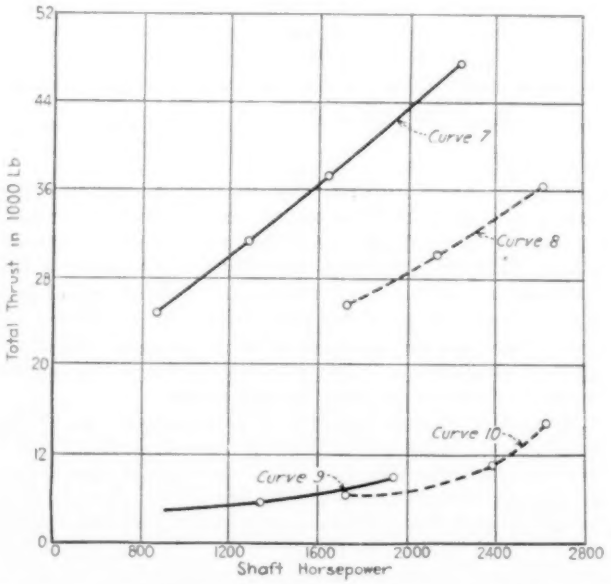


FIG. 7 RELATIONSHIP BETWEEN THRUST LOADS AND PUMP SHAFT HORSEPOWER, 30-IN. DREDGE *Clackamas*, FEB. 21, 1928

Curve 7, 8-in. nozzle, front clearance, 1/32 in.
Curve 8, 16-in. nozzle, back clearance, 3/32 in.
Curve 9, 8-in. nozzle, front clearance, 3/32 in.
Curve 10, 16-in. nozzle, back clearance, 1/32 in.

EFFECT OF LONG PIPE LINES

The effect of pumping through long pipe lines is to increase the thrust load as greater pipe-line pressures are required. This is clearly illustrated by studying the test data of the run on the 8-in. and 16-in. nozzles representing the two extreme conditions of pipe-line length.

It is not uncommon to find pipe-line pressures up to 110 lb. being used on all sizes of dredges from 20 in. up to 30 in. High terminal lifts also increase the pump pressures and as a practical guide in our work, we consider one foot of lift to be equivalent to approximately 100 ft. of 20-in. pipe, 75 ft. of 24-in. pipe, and 50 ft. of 30-in. pipe.

EFFECT OF PUMPING MATERIAL

It was shown that the thrust load was greater when pumping material than when pumping only clear water. This is due to the increased suction and discharge heads when pumping material. Refer to Table 3 giving data of test made on the *Clackamas* on January 23, 1928, to determine the effect of impeller relief holes.

RESULTS OF TESTS

Though these tests were not as scientifically conducted as they might have been, it was felt that they were run quite closely enough for all practical purposes. Likewise the computations were made in a simple manner understandable to the average dredge operator and close enough to determine the maximum thrust load on the dredge pump and show what the influencing factors were for increasing or decreasing the thrust load of a dredge pump.

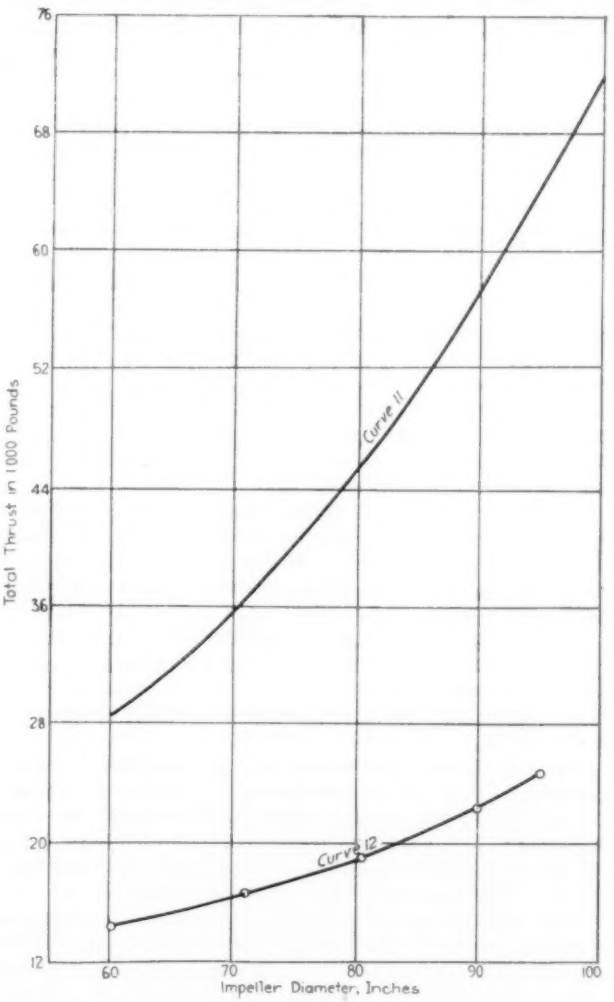


FIG. 8 RELATIONSHIP BETWEEN IMPELLER DIAMETER AND TOTAL THRUST LOAD, DREDGE *Clackamas*, FEB. 21, 1928

(Diameter of impeller for test, 70 in.; size of nozzle, 8 in.; discharge pressure, 92 lb., vacuum 5 in.; pumping clear water. Curve 11, impeller clearance 1/32 in. front and back; Curve 12, impeller clearance 1/4 in. front and back.)

It was clearly shown that a close pump clearance on the back and a more open clearance on the front produced the lowest thrust load for any given condition. However, such running clearances do not permit the obtaining of the best pump efficiencies and it is believed best to operate with close clearances both front and back of the impeller with a slightly increased thrust load.

The closed-back impeller creates a higher thrust load and has no practical advantages over the impeller with relief holes. Impellers with renewable vanes on their shrouds permit the maintenance of closer pump clearances and the carrying of lesser thrust loads. As naturally expected, the longer pipe lines requiring the higher heads produce the maximum thrust loads.

It was also clearly shown that the high-speed dredge pump with a relatively small pump impeller builds up less of a total thrust load than the slow-speed dredge pump with an impeller of larger diameter, all other things being equal. That a 110-lb. discharge pressure and a 23-ft. suction head (approximately 10 lb. or 20 in. vacuum) are reasonable maximum assumptions to assume for a modern high-speed high-powered dredge pump for sizes varying from 20 to 30 in. Although it is possible to set the pump impeller with a clearance of $^1/_{32}$ in. both front and back, it is reasonable to suppose that this clearance will increase as much as $^1/_4$ in. between repair periods but these tests show that this reduces the thrust load. (See Fig. 8.)

It was also demonstrated that relatively low water pressures on the main pump-shaft packing seal are sufficient even with closed-back impellers and high pump discharge pressures.

The practical use of these test data to the Port of Portland has been as follows:

1 It permitted the Port's engineering staff to compute closely the probable maximum thrust loads for all of its high-powered dredge pumps, varying in size from 20 in. to 30 in. and to select the most economical size of bearing.

2 It fully demonstrated the advantages of certain principles of dredge-pump design which the Port's engineers had always advocated, such as relief holes, outside shroud vanes, close clearances, high-speed small impellers, adjustable nose ring, adjustable pump shaft, etc.

3 It made clear to the operating force what can be done to reduce the thrust load, and what operating practices actually affect the thrust load.

4 It determined the proper water pressure to carry on the pump-shaft seal.

The data obtained from these tests permit a competent dredge-pump designer to compute the thrust loads for any given pump. No attempt has been made here to set forth the maximum thrust loads for the various sized dredge pumps in general use throughout the country as each pump is a problem in itself, depending upon its own characteristics and type of design.

These test data should be of value to the various manufacturers of thrust bearings in permitting them to develop a complete line of thrust bearings for dredge pumps to economically take care of all the standard sizes, such as 20-, 22-, 24-, 27-, and 30-in. pumps. Likewise this information should be of service to many of the designers and builders of dredge pumps.

Discussion

R. L. DAUGHERTY.[3] I should prefer to have the title of this paper use the term "centrifugal pump" rather than "dredge pump," because a dredge pump is merely one type or class of centrifugal pump. This changed title would make it clear that the principles and facts here set forth had a broader field of application.

[3] Professor of Mechanical and Hydraulic Engineering, California Institute of Technology, Pasadena, Calif. Mem. A.S.M.E.

What makes a dredge pump different from some other centrifugal pumps is that it is of necessity single suction in practically all cases, and also is usually operated only under relatively high pressures. Furthermore, because of the material it has to handle, certain features of its mechanical construction are naturally somewhat different from pumps handling clear water. But these factors affect the numerical result merely and not the principles involved.

The tests presented in this paper have the advantage of having been made on a pump of large physical dimensions. In measuring pressure distribution on the shrouds of an impeller as large as 70 in. in diameter, there is some gain in accuracy, because the dimensions involved in gage connections are so small in comparison with the impeller dimensions.

The total pressure on any area is given by $\int p dA$ where p is unit pressure and dA is an element of area. The authors have presented curves showing the variation of unit pressure as a function of radius of the impeller, and then used a simple arithmetical method of computing the total pressure. It would seem better to me to plot p as a function of the radius squared. The area bounded by such a curve, when expressed in proper units, is the value of the integral or the value of the total pressure. Furthermore, if the curves for pressure on each shroud are plotted on the same diagram as a function of the radius squared, the area between the two is the net thrust due to these two opposing factors.

There is one force causing thrust that the authors have overlooked, and that is the dynamic force due to the velocity of the water entering the impeller in an axial direction and then being deflected so as to leave in a radial direction. This force is W/gV, where W is pounds of water per second, V is velocity in feet per second in the suction pipe, and g is the acceleration of gravity in similar units. If A is the area of the pipe in square feet, this force is $\frac{62.4 \times A \times V^2}{g}$ and thus increases as the square of the discharge.

In this specific case with a 30-in. suction pipe and velocities ranging up to about 25 ft. per sec., as near as I can estimate from the data given, the thrust due to this dynamic effect reaches a value of 6120 lb. This is hardly negligible. The authors have considered a force due to the vacuum in the eye of the impeller, and in this specific case it had a maximum value of 7070 lb. It is seen that these two forces are of about the same magnitude. They are also opposite in direction, as the dynamic force on the impeller is directed away from the suction side.

In case a pump has a large rate of discharge and thus a large dynamic force acting and at the same time is working under a very low head, thus having a small thrust due to pressures on the shrouds, it might be possible for this dynamic force to be a predominating factor and thus cause the net thrust on the pump to be in the opposite direction from that which it was always found to be in the pump with which this paper is concerned.

By various combinations of clearance rings on both shrouds, and by different clearance distances, it is possible to vary the pressures on the two shrouds also in such a way as to cause a reversal of the net thrust. The authors simply failed to have the proper combination in this case to cause the thrust to be either zero or opposite in direction under some conditions.

In closing, I wish to emphasize the fact that this paper does not present an actual measurement of thrust. The experimental results consist simply of measurements of pressure distribution on the shrouds. The values of thrust given are merely computed values and are correct only if all factors have been allowed for.

I wish to thank the authors for having given to us, however, this valuable study of pressure distribution on the shrouds.

ELTON RISLEY.[4] Vanes on the outside of the runner shrouds add to the wear and tear and decrease the pump efficiency when the clearances get large. Balance holes may not noticeably decrease the efficiency of a 30-in. pump when used in connection with vanes on the outside of the runner shrouds, but would be more serious on a 16-in. pump without such vanes. We have reached the conclusion that the most economical method of taking care of thrust in a dredging pump, when both pump, efficiency, and maintenance are considered, is with an adequate thrust bearing. That is when pump efficiency is expressed in terms of the cost of power-plant installation and operation.

A. L. MCHUGH.[5] The authors have described a method of approximating the thrust on a dredge-type centrifugal pump by measuring the mean pressure at various diameters between the impeller shrouds and the casing and calculating the thrust directly from the values obtained.

The impeller shown is of the enclosed, side-suction type with shallow vanes fitted on outside of shrouds which rotate in the clearance space between impeller and casing and, by means of the forced vortex produced, reduce the pressure from the outer diameter of the impeller toward the shaft to a minimum. As the pressure reduction varies with the clearance between the vane and the side of the casing, it follows that as this clearance is increased the action of the vanes affecting the reduction of the pressure on the sides of the shrouds is materially reduced.

From Table 1, with maximum clearance on the back shroud, the thrust is maximum and is toward the suction. The clearance between the edge of the vanes and the casing is $^1/_4$ in., and the effect of using these vanes is considerably reduced.

The thrust resulting from the mean pressure on the rear shroud is toward the suction. The vanes on the front shroud have but $^1/_{32}$-in. clearance and operate much more efficiently,

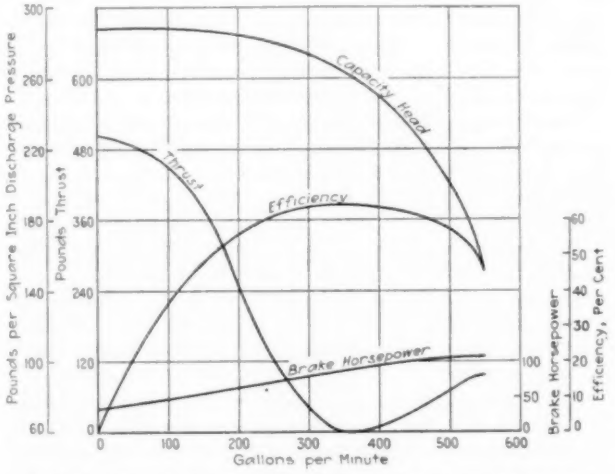


FIG. 9 THRUST CURVE FROM TEST ON A MULTISTAGE PUMP

and consequently the mean pressure back of the vanes is greatly reduced.

As the thrust due to the mean pressure on the front shroud is negative or away from the suction, the net thrust in the direction of the suction is increased in proportion as the amount of the mean pressure at the front shroud is reduced. Obviously, better balance would be effected under these conditions if the vanes were left off the front shroud. Partially balancing the pressure in the clearance space on each side of the impeller by means of holes through the rear shroud should not noticeably affect the efficiency on units of this size and will reduce the thrust materially.

Experimental results made in the laboratory of the company with which the writer is associated indicate that while the thrust may be approximated for points of best efficiency, there is a wide variation in the thrust when the pump is operated under a throttled condition or at a capacity beyond that at maximum efficiency.

The curve shown in Fig. 9 is a thrust curve from actual test on a multistage pump with single side-suction impellers which have balancing rings of the same diameter on each shroud and in which the chamber under the balancing ring from the shroud opposite the suction is connected through the hub by means of balancing ports. At the point of maximum efficiency it will be noted that the thrust is practically zero. As the pump is throttled the thrust increases very rapidly, and on this particular pump it was maximum at shutoff.

In order to measure the thrust a special attachment was built consisting of a pressure chamber covered with a rubber diaphragm on which rests a guided plunger. The thrust from the outer race of the ball bearing was transmitted directly to this plunger, and the pressure in the casing was read by means of a Bourdon gage. In order to build up the proper counterthrust an Alemite gun was used with a heavy oil to produce the proper pressure in the chamber behind the diaphragm. This device was calibrated before using on a platform scale, so that the thrust could be read directly from a curve by taking a reading from the gage.

On a single-purpose pump like a dredge pump, it might pay to mount the thrust bearing in a movable assembly so that the thrust could be read at any time by simply releasing a suitable locking device. If the thrust bearing is mounted on the shaft so it could be adjusted, the impeller could be shifted at times to minimize the thrust due to unequal wear.

The author mentions the fact that he has never used ball thrust bearings on his dredges. Most of the bearing failures in the past have been due to misapplication, and usually they were too small. The published ratings of ball bearings vary, and three of the larger ball-bearing manufacturers are now rating their bearings on a basis of 1000-hr. life. Halving the load, it is estimated the life is increased 10 times, and at one-quarter load the life will be approximately 100 times the full load. After having determined, therefore, the maximum thrust developed by the pump, the selection of the size of the ball bearing is simply a matter of the number of hours the bearing may be expected to operate before replacement.

In the absence of any definite method of measuring the thrust on a pump as large as the dredge pump described in this paper, the writer believes that it would be economy in the end to figure the thrust bearing on a basis of double the estimated thrust in order to insure continuous operation.

VICTOR J. MILKOWSKI.[6] The subject is of importance, not only to designers and operators of dredging pumps, but to designers of centrifugal pumps as well, especially pumps of the side-suction type, as similar reasoning and theory apply to them.

The writer has approached this subject in the course of his work both from a theoretical standpoint and from available test data. It is a pleasure to find in this case that the theory and practice are in accord, except for such discrepancies as are due to imperfections in equipment, etc. The assumptions in

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this paper as to the different external forces acting on the impeller and the assumption of the unknown factors affecting these forces are in accordance with the writer's studies and observations.

It will probably be best, first to compare the actual observations with the theoretical figures. The writer will select for this purpose test No. 12 (Fig. 10) with the same clearances on

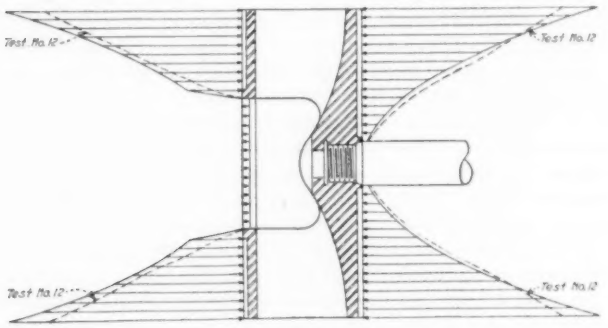


FIG. 10 DIAGRAM OF PRESSURES CAUSING THRUST ON DREDGE-PUMP IMPELLER

both sides of the impeller, no relief holes in the impeller, the pump operating at 350 r.p.m., and discharging through an 8-in. nozzle, the discharge pressure being 90 lb. and the suction vacuum 4 in.

With very close clearances between the impeller and the sides of the pump casing, and with the outside of the impeller shrouds being fitted with ribs or vanes so as to cause the water to rotate with the impeller, that portion of the pump may be considered as being a forced vertex, and the head or pressure at any point as being equal to $H = \frac{V^2}{2g}$, V being the velocity in feet per second and H the head in feet.

This formula can be expressed more conveniently as follows:

$$H = \left(\frac{\text{r.p.m.} \times 2\,\pi\,R}{480 \times 12}\right)^2$$

R being the radius in inches.

The pressures calculated in this manner compare with the actual observed pressures in test No. 12, as shown in Fig. 10 and as given thus:

	Theoretical pressure, lb.	Observed pressure, lb.
Back gages		
A	65	61
B	36	40
C	18	22
D	9	11
Front gages		
a	65	57
b	36	33
c	18	10

These theoretical and observed pressures agree quite closely. The lower pressures observed on the suction, or the front, shroud are undoubtedly due to leakage past the sealing ring at the suction. The higher pressures observed on the back shroud at points B, C, and D may be due to the sealing water supplied to the gland or to larger clearances at that point. A similar agreement is found to exist between the calculated pressures and the observed pressures for other tests as well.

The total head or pressure exerted at any surface is $H \times$ area.

In case of a revolving disk or ring with the variable pressures as mentioned, the total pressure or head can be expressed as an integral of

$$\int_{R_2}^{R_1} \pi\,RH$$

Since from a previous formula $H = \left(\frac{\text{r.p.m.} \times 2\,\pi\,R}{480 \times 12}\right)^2$, the total head equals the integral of

$$= \int_{R_2}^{R_1} 2\pi\,R \left(\frac{\text{r.p.m.} \times 2\pi\,R}{480 \times 12}\right)^2$$

$$= \int_{R_2}^{R_1} 2\pi\,R \left(\frac{\text{r.p.m.}}{916}\right)^2 R^2$$

$$= \int_{R_2}^{R_1} 0.0000075 \times \text{r.p.m.}^2 \times R^3$$

$$= {}^1/_4 \times 0.0000075 \times \text{r.p.m.}^2 \int_{R_2}^{R_1} 4R^3$$

$$= 0.000001875 \times \text{r.p.m.}^2 \times (R_1^4 - R_2^4)$$

In considering the different pressures acting on the outside of the impeller, this theoretical pressure shall be assumed to apply on the front shroud between R-35 and R-17. This, of course, would be in a negative direction, away from the suction. Between R-17 and R-15 it shall be assumed that the pressure changes to a vacuum and that the thrust produced on this ring is negligible. Between R-15 and R-0 the pressure would be uniform and would be equivalent to a vacuum of 4 in., or 5.2 ft. head per square inch. This pressure would be positive; that is, toward the suction.

On the back shroud it shall be assumed that the pressure would be in accordance with the same theory, but would extend from R-35 to R-5. This assumes that the ribs extend all the way down to the shaft and that close clearances are maintained all the way down. This probably is not in accordance with the facts, and any such error will reduce the calculated thrust. This thrust will also be in a positive direction.

The sum of these three thrusts will give the actual thrust to be provided for in the bearing. These assumptions are made more clear by referring to the diagram, Fig. 10.

The negative thrust on the suction shroud will then be $H = 0.000001875 \times 350^2 \times (35^4 - 17^4) = 325{,}500$ ft. $= 141{,}000$ lb.

The positive thrust on the suction due to the vacuum is then equal to $H = 15^2 \times \pi \times 4.52 = 3200$ ft. $= 1385$ lb.

The positive thrust on the back shroud is then equal to $H = 0.000001875 \times 350^2 \times (35^4 - 5^4) = 344{,}800$ ft. $= 149{,}000$ lb.

The total thrust is then the sum of $-141{,}000 + 1385 + 149{,}000$ lb. $= +9385$ lb.

The thrust as actually computed from observations in test No. 12 is 30,400 lb. The discrepancy between the theoretical figure of 9385 lb. and the actual figure of 30,400 lb. is due mainly to the leakage between the suction disk and the front shroud.

Comparing the pressures observed at A, B, and C with those observed at a, b, and c, and which should be the same at corresponding points if clearances were the same on both sides of the impeller and if no appreciable leakage could take place, it is found that there is a positive thrust on that part of the impeller between R-35 and R-17 of approximately 18,500 lb. This leaves only a discrepancy of about 2500 lb. to be accounted for, and this is probably due to larger clearances at the back shroud in the neighborhood of the shaft, lack of vanes at that point, etc. Possession of more details regarding the construction of the impeller and the pump would probably explain these discrepancies.

In figuring the thrust for pumps, the writer figures on a

20-in. vacuum over the suction, the difference between pressure on the front shroud and on the back shroud the same as figured in the foregoing, and on there being an unbalanced pressure of $1^1/_2$ lb. per sq. in. over the whole area of the front shroud, this unbalanced pressure acting in a positive direction. Applying these figures in this particular case gives the following results:

Thrust due to vacuum equals $15^2 \times \pi \times 22.6 = 16{,}000$ ft. $= +6930$ lb.

Pressure on the front shroud equals —141,000 lb.

Pressure on the back shroud equals +149,000 lb.

Unbalanced pressure over the front shroud equals $(35^2 - 17^2) \times \pi \times 1.5 = +4420$ lb.

Adding the foregoing figures, $-141{,}000 + 6930 + 4420 + 149{,}000 = +19{,}350$ lb.

This is the thrust load the writer ordinarily would figure on as the actual working load to be provided for in a pump of this design.

Regarding the possibility of reversal of thrust as mentioned in the paper, it can be seen readily by comparing tests Nos. 6, 12, and 18 with thrust loads of 47,600 lb., 30,400 lb., and 10,700 lb., respectively, that the thrust can easily be reversed by increasing the clearance between the front shroud and the suction disk. In fact, on test No. 18, with the proper sealing arrangement at the impeller intake, there would be a negative thrust of about 8000 lb., if it is assumed that the effect of the leakage is the same as in test No. 12; that is, about 18,500.

Regarding the question of putting relief holes in the back shroud of the impeller, that no doubt would relieve the thrust at that point; and as the vanes on the outside have similar pumping action as the vanes inside of the shrouds, the efficiency of the pump as a whole would not be reduced perceptibly, especially with as large a pump as in this case.

There are, however, practical considerations against this. It introduces wear between the back shroud and the hub disk and permits sand and grit to get into the stuffing box and packing. For these reasons the writer considers it best to eliminate the relief holes from the back shroud entirely.

The writer is tempted to make some observations on the efficiency of this pump. Taking test No. 12 with pump discharging through an 8-in. nozzle at 90 lb. pressure, or 208 ft. head, and assuming the nozzle to be 98 per cent efficient, it is found that $A = 50.27$ sq. in. V in ft. per sec. $= \sqrt{2g \times H} = \sqrt{2g \times 208} = 115.2$ ft. per sec.

The quantity, therefore, in gallons per minute is $115.2 \times 50.27/144 \times 60 \times 7.5 \times 0.98 = 17{,}650$ g.p.m.

The total head is then $H = 208 + 4.5$ ft. $= 212.5$ ft.

The water horsepower is $\dfrac{17650 \times 212.5}{3960} = 947$ hp.

Efficiency is then $947 \div 2141 = 44.2$ per cent.

Taking next test No. 25, with the pump discharging through a 16-in. nozzle with 55 lb. discharge pressure, or 127 ft. head, and $18^1/_2$ in. of vacuum on the suction, and assuming the nozzle to be 98 per cent efficient, gives $A = 201$ sq. in. V in ft. per sec. $= \sqrt{2g \times 127} = 90.7$ ft. per sec.

The capacity in gallons per minute equals $90.7 \times 201/144 \times 60 \times 7.5 \times 0.98 = 55{,}800$ g.p.m.

The total head equals 127 ft. + 20.9 ft. = 147.9 ft.

The water horsepower is then equal to $\dfrac{55{,}800 \times 147.9}{3960} = 2082$ hp.

Efficiency is then $2082 \div 2700 = 77.1$ per cent.

These figures do not impress the writer as being especially good for a pump of this size. A test on a 6-in. pump of recent design showed an efficiency of approximately 56 per cent and 70 per cent under conditions corresponding to the conditions as given.

The writer cannot agree with the deduction to be derived from curves 11 and 12, Fig. 8, and would be interested to know on what data these curves were based.

The thrust may be said to be a function of the vacuum and of the discharge pressure. These depend on the capacity and on the peripheral speed of the impeller, regardless of the diameter; therefore, the thrust is not dependent on the diameter in any way except for the greater possibility of the pressure not being equal on the two sides of the impeller.

Curve No. 12 would indicate that the larger the clearance the lower is the thrust load. Increasing the clearances between the impeller and the casing increases the pressure on the shrouds; and if carried far enough it would result in the full pressure developed by the pump being exerted on each shroud. This of course would give a far greater difference in pressures between the front shroud and the back shroud than the figure arrived at; that is, for forced vertex. There undoubtedly would also be greater leakage, which would increase the positive thrust still further.

No doubt a study of the data from which curves Nos. 11 and 12 were made up, together with a detailed study of the pump construction, condition of the pump, amount of wear, etc., would explain these discrepancies.

In conclusion, the writer would state his views on the matter as follows:

1 Vanes should be provided on the outside of the impeller shrouds and carried as far down as possible, and the clearances should be kept as small as possible in order to approximate the conditions of a forced vertex.

2 The clearance between the front shroud and the casing should be somewhat larger than between the back shroud and the casing to increase the negative thrust on the front shroud.

3 The pump should be fitted with an adjustable sealing ring at the suction intake to reduce the leakage at that point to a minimum, not only because of the effect on efficiency, but also on account of its effect on the thrust as well. This can readily be seen by a study of the diagram, Fig. 10.

4 No relief holes should be used in the back shroud for practical reasons, as enumerated in the preceding.

NELSON OGDEN.[7] The authors describe in detail the method they used for calculating the thrust loads from the observed pressures. Briefly, they plotted pressures against radii and summed up the loads calculated for a number of small annular areas. Their method is partly graphical and partly arithmetical. It may be interesting to examine a purely graphical method that is quicker and more accurate and permits closer analysis of the influences at work.

The method consists simply in plotting the observed pressures against the squares of their respective radii, thus permitting a direct determination of the mean net pressure by using a planimeter. This pressure, multiplied by the total area of the impeller, gives the thrust load.

The real problem, however, is to work out a rule for calculating the thrust in any dredge pump. It should be made possible for different persons to agree fairly well on the maximum load to be provided for. Certainly the range mentioned by the authors, 48,000 to 160,000 lb., sounds too much like guesswork.

Examination of a typical pressure diagram, of the sort now proposed (see Fig. 11) shows that the thrust determination can be no more accurate than the diagram, since the thrust is proportional to the mean width of the loop. It is not easy to see

[7] Frankford, Philadelphia, Pa. Assoc-Mem. A.S.M.E.

what really affects this "fatness," which is roughly a measure of the degree of unbalance. It appears that one could start out with given impeller dimensions and suction and discharge pressures and draw a variety of equally plausible diagrams, with mean widths varying at least in the ratio of 2 to 1. Thus, we seem still to be far from an exact rule for figuring thrusts.

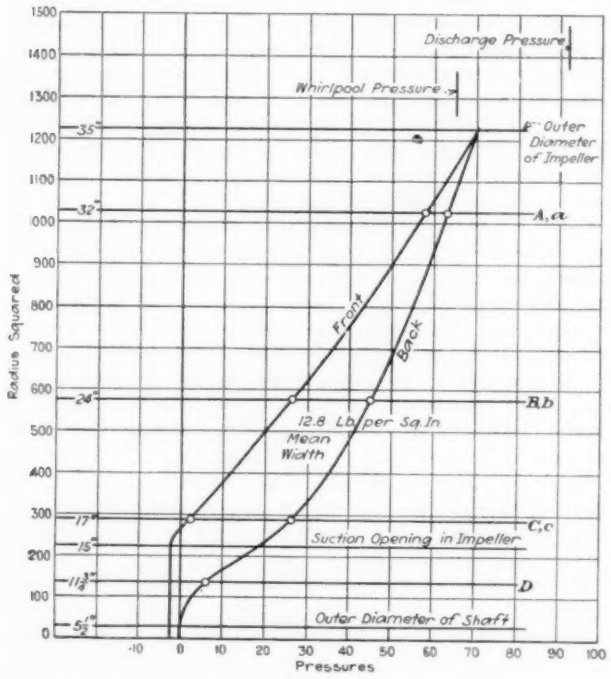


FIG. 11 GRAPHICAL METHOD OF CALCULATING THRUST LOADS

(Test run with 8-in. nozzle at 350 r.p.m.; 3848 sq. in. at 12.8 lb. per sq. in. = 49,200 lb. thrust.)

These remarks are certainly not intended as destructive criticism, but as an effort to show the need of further interpretation of the test results, and possibly new experimental investigation. It does seem that the authors have at least established general principles, such as the important effect of side clearances. The way has been paved for constructing characteristic curves as a guide in any specific problem.

ARTHUR B. LAKEY.[8] The authors have secured a wealth of authentic data concerning dredge-pump thrust loads, their causes and effects, and certain incidental or collateral conditions set up when it is sought to reduce the thrust loads.

One such collateral effect is reduction in the efficiency of the new pump when it is desired to lessen the net thrust load by using large side clearances. Another instance is accelerated wear favored by large initial clearances or by use of equalizing holes. The latter, even without notably affecting the new efficiency, seem to permit extensive short-circuiting, and hence rapid erosion of the runner. Pump efficiencies, through apparently omitted in working up the numerous tests reported, are so important that the authors have included them explicitly or by inference in three of their nine conclusions.

Attempts to supplement these test results by calculating the pump efficiencies show a need for more data on the resistances of the pipe lines and nozzles used. With additional information of this sort available, more reliable efficiency figures could be obtained, permitting study of the year-round mud-digging ability of a pump as affected by various thrust-producing conditions.

If extensive material-pumping tests appear desirable, the gage connections can be arranged for flushing with clear water under pressure prior to taking each reading.

The change in thrust load due to increasing the suction diameter of the impeller should be more generally appreciated. If this diameter is to be greater than the pipe-line size, the engineer should provide the extra thrust capacity that the excess involves.

If there is any one who can give an idea as to the relative

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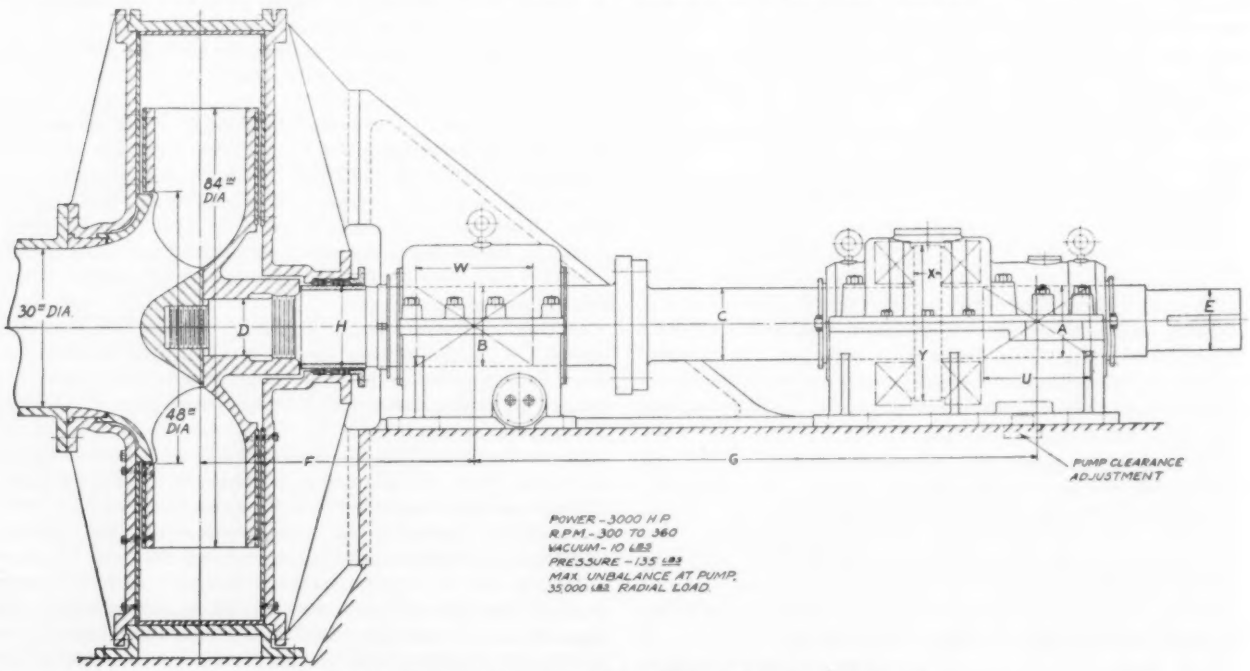


FIG. 12 TYPICAL ARRANGEMENT OF DREDGE PUMP AND BEARINGS

prevalence of rotors with external vanes as compared to those not so fitted, it is felt that the information will be most welcome. The authors' opinions are desired regarding the advisability of close radial fits at the outer diameter of the impeller. Such a construction, while a complication, would appear to help in combating short-circuit flow with its bad effects, and also to permit a closer approach to the whirlpool-chamber idea so largely used in moderate-power low-head pumps, with its reduced hydraulic losses.

Fig. 12 shows a pump arrangement followed largely on a number of recent dredges. The thrust bearing has about three times as much capacity on the working side as on the slack side. Inspection of such a dredge bearing recently, after extensive running, showed that the bearing surfaces had not been in metallic contact at any time during operation. Hence the 1 to 3 ratio advanced by the authors appears conservative.

The two journal bearings are designed to secure complete oil-film lubrication against large loads in any radial direction. They apply principles laid down in "A Graphical Analysis of Journal Bearing Lubrication," A.S.M.E. Proceedings 1923–4–5. The pedestal bearing near the impeller is self-aligning; that incorporated with the thrust bearing is rigid.

The maximum loads are taken as the algebraic sum of (*a*) the static loads and (*b*) a revolving load occasioned by a boulder filling an impeller passage and whirled at maximum revolutions per minute and at maximum radius. The greater unit journal pressures have been taken at about 100 lb. per sq. in.

The shaft diameter at certain points exceeds strength requirements to secure greater stiffness. The journals are purposely short to avoid needless friction and effects of shaft deflection.

All these bearings have suitable arrangement for automatic oiling and are thus independent of any oil-pumping system. With unusually high-powered dredge pumps it has worked out best to include an external cooler in the automatic oil system.

The "factor of ignorance" has been such that many dredges are over-supplied with thrust-bearing capacity. Taking the apparent extremes which have occurred within the writer's experience, we find on the basis of carefully estimated loads that the most daring case has relatively only one-fourth the thrust capacity of the most conservative.

The paper affords the best practical basis for attaining consistent yet safe thrust estimates that has reached the writer's attention. Engineers and operators who are faced with this problem should find the authors' results highly useful. These have been presented in a manner well adapted to stimulate discussion.

It is hoped that the authors may find it convenient to make further tests and to amplify the information already given. These tests may lead other experimenters to extend and supplement the investigations so well begun.

PIERCE J. MCAULIFFE.[9] The dredging fraternity is greatly indebted to Messrs. Polhemus and Healy for this paper, and all those who are interested in dredging problems will appreciate the work that has been done by the authors.

On the second page of the paper, under the heading, "Direction of Thrust Load," reference is made to occasional report of reversal of thrust load. It might be interesting, therefore, to add that I have seen a 10-in. dredge, working in a sand and gravel plant, where the thrust load was so nearly neutral that the impeller shaft was practically floating with only very slight movement toward the suction or away from the suction. The dredge pump was operating at 580 r.p.m. with discharge pressures ranging between 40 lb. and 45 lb. No measurements of thrust loads were made, but it was apparent from the action of the thrust bearing that the thrust load was very light. In the design of this pump there had been a conscious effort made to lighten the trust load.

The engineers of the Morris Machine Works, with which I am connected, have given much thought to the questions discussed in this paper. They agree with the views expressed by the authors in many respects, but they are at variance in other respects. Their conclusions are: It is possible to control or even to reverse the thrust on a dredging pump by (1) making the front shroud of larger diameter than the back shroud, (2) maintaining small clearance on the back side of the impeller, (3) allowing large clearance on the front side of the impeller. They do not consider relief holes necessary or desirable.

For the past several years none of the new designs of dredging pumps developed by Morris Machine Works has had relief holes in the impellers. We consider that relief holes produce circulation loss within the pump. Where circulation losses occur, wear on the pump parts becomes more active. Relief holes, through wear, increase in size, permitting increased circulation losses. Back liners have to be replaced more frequently. With the more rapid wear on the back liners, when relief holes are used, it is more difficult to maintain the small clearance on the back side of the impeller, which is one of the principal factors in reducing the thrust load.

To reduce the circulation losses which would otherwise occur with large clearance on the front side, an adjustable sealing ring is provided on the front side. This adjustable sealing ring is placed so as to control the clearance between the impeller and the suction disk at the suction opening of this impeller. As wear takes place the sealing ring is pushed in toward the impeller to keep the clearance a minimum.

We are convinced that circulation losses within the pump lower the pump efficiency. With the pressure and vacuum readings taken during the test it would be possible to make a sufficiently accurate computation of what the circulation losses amounted to if the size and number of the relief holes in the impellers had been stated.

During the discussion which followed the readings of the paper a criticism was made because the thrust load had been computed instead of being actually measured. I would like to add that I am thankful for the information presented. I realize how difficult it is to induce dredge owners to turn a plant such as this, with probable earning power of around $2500 per day, over to engineers for experimental purposes. I would also be very much interested in getting further information about a practical device for measuring the thrust load on a unit of the size referred to in the paper. What would the cost of such a device be? How could it be applied?

During the discussion two questions were asked concerning the design of the dredging pump presented to which I took the liberty of expressing my own views in answer. These questions were:

1 Is it customary practice to make dredging pumps of rectangular section as shown by Fig. 1?

2 Is it customary practice to have such an abrupt turn from suction piping into impeller as is shown by Fig. 1?

My answer to the first question is that by far the greater number of dredging pumps built have a whirlpool section beyond the perimeter of the impeller of circular shape or approaching the circular. From previous articles presented by Mr. Polhemus it is evident that he prefers the rectangular shape because it lends itself more readily to being fitted with casing liners. I believe that it is safe to say that more dredging engineers consider that it is cheaper in the end to do without casing liners and instead to make the casing very thick and to replace the entire casing when it is worn out rather than to have the more

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frequent shutdowns for renewal of casing liners. The casing on many dredging pumps is frequently made 5 in. thick to reduce the number of shutdowns necessary for replacement.

My answer to the second question is that recent designs of dredging pumps, as a rule, present a much easier entrance at the suction than is shown by Fig. 1. The cross-section of a 24-in. pump, shown by Mr. Ogden while reading the paper, had a much easier entrance at the suction, and such entrance is now followed more regularly than that in Fig. 1. There was another feature of the design presented by Mr. Ogden, which I think it well to comment on. The impeller vanes projected considerably into the suction opening of the pump. Lacking the other views of this same design, it would be my impression that the vanes of the impeller would converge to such an extent at the center as to restrict seriously the openings into the impeller. In the December 17, 1925, issue of *Engineering News-Record*, page 983, are shown two photographs of the accumulation of trash which was removed from the pump runner of one of Mr. Polhemus' dredges during a 24-hour run. These photographs will give an idea of why large passageways through dredging pumps are essential. A striking example of the improvement of output obtained by a dredging pump after it had been changed from one with rather restricted openings through

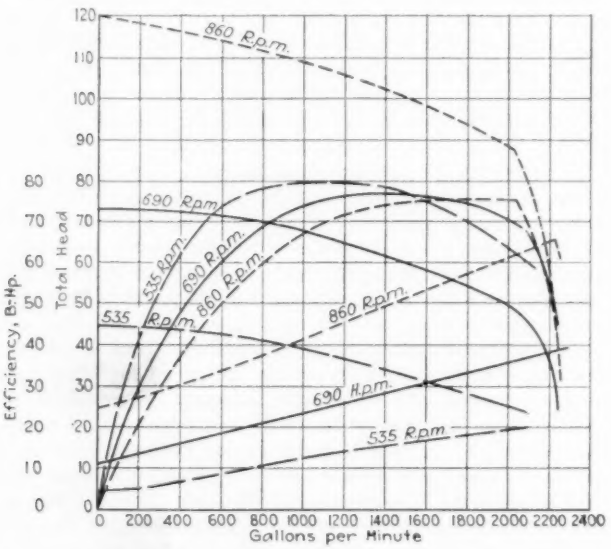


FIG. 13 TEST CHART OF 6-IN. DREDGING PUMP DESIGNED TO PASS OBJECTS UP TO $4^1/_8$ IN. IN DIAMETER

impeller to one with large openings will be found in the paper presented at the December, 1919, meeting of this Society by Mr. Walter J. White, of New Orleans.

In the discussion one inference I gained was that, while ruggedness had been provided for by dredge-pump designers, efficiency had been largely disregarded. This might have been true some years ago, but I am confident that it is not true today. I know that dredging pumps, particularly the smaller sizes, had rarely been credited with efficiencies higher than 50 per cent. To show that there has been an improvement in this respect I submit the test chart of a 6-in. dredging pump designed to pass objects up to $4^1/_8$ in. diameter (Fig. 13). This pump is one of a series, all of similar design, developed recently by Mr. V. J. Milkowski, chief engineer of the Dredging Department of the Morris Machine Works. The test was made on what has been adopted as the standard 6-in. medium-duty dredging pump. Other sizes of this same series are showing similar characteristics. As a matter of fact, some of the old designs, when tested at capacities above those usually credited to them, have shown surprisingly good efficiencies. According to my observations the low efficiencies obtained from dredging pumps have not been so much a question of design as one of application. Dredging pumps have been used too frequently at capacities too small to obtain good efficiencies.

LEWIS F. MOODY.[10] I fully agree with the points that discussors have accentuated covering the difference between the total thrust and that portion of the thrust which is calculated in the paper by measuring pressures in the clearance spaces. The title of the paper might mislead some readers to think that it gives rules for computing the total thrust on a pump.

As Professor Daugherty has pointed out, one factor is left out in the theoretical relation, that is, the impulse of the entering jet of water passing through the suction pipe. That water has to be turned at right angles and discharged radially. Therefore, right away we should include in the formula this expression WV/g, where W is the weight of water entering per second, V the velocity in feet per second, and g the acceleration of gravity. Even after we get that in we have not the whole story.

We know very well from many tests on hydraulic turbines and pumps of various types that when you measure the actual thrust you very often get quite a different result than when you attempt to compute by a certain method of analysis, such as this. Therefore, the final results that are given in this paper should not be mistaken for the total thrust of the pump, but only as an analysis of certain elements of that thrust.

Another point I might mention is that in looking at that first figure in the paper the design of this particular pump looks unnecessarily crude. We all realize that dredge-pump problems involve certain practical considerations which do not enter in ordinary centrifugal-pump designs—the handling of water containing large amounts of suspended material, the conditions of wear which have to be taken care of, and the necessity for a very simple design structure. Nevertheless, it rather offends the feelings of some of us who have been working in hydraulic design for a good while to see the attempt to make the water turn absolutely at a sharp right-angle at the transition from the suction pipe into the impeller. You will notice on the first design that there is an absolutely sharp angle at that point.

In the type of pump which Mr. Lakey showed in his discussion (Fig. 12) there has been a great improvement in the design at that particular point. A factor like that has an effect on the efficiency. Perhaps minor attention has been given to efficiency in many dredge-pump installations. As Mr. McAuliffe says, that problem is now being properly studied. But whenever you have an inefficient hydraulic design you always are in danger of other accompanying phenomena which are not any too good, such as localized erosion and pitting. I am inclined to agree with Mr. Knox that this dredge-pump problem should be attacked on properly theoretical hydraulic lines.

Another interesting thing that occurred to me is that I did not notice in the paper any comparison of the theoretical curve of pressure variations in these clearance spaces with the actual pressures attained. These two spaces are simply spaces bounded by two planes normal to the axis, and the variation of pressure should be in fairly close agreement with a parabolic relation. There was no attempt to compare the actual pressures with the theoretical curve of pressure in either of those spaces. I think it would be interesting to have that done.

I do not think that we should too strongly criticize an author for giving us what data he has been able to collect on tests just because we would like to have seen him obtain some more.

[10] Consulting Engineer, Cramp Morris Industrials, Philadelphia, Pa. Mem. A.S.M.E.

I think the authors should be complimented on submitting to us such results as they have been able to secure. At the same time, they do leave us an excellent field for further tests to find out what the actual results are on dredge pumps and on other types of centrifugal pumps.

Authors' Closure

While all suction dredge pumps are fundamentally of the centrifugal type, the term "dredge pump" was used in the title of the paper because the results as noted and discussed were obtained from actual tests on a dredge pump and we were concerned with the suction dredge pump only.

It remains for some one concerned with pumps designed for other than dredging service to determine whether or not the facts and principles as set forth in the paper are applicable to other fields. Likewise it remains for some one concerned with dredge pumps, who has the opportunity and available funds, to design, install, and operate a proper mechanism for measuring actual thrust in connection with dredge pumps of similar size.

It is agreed that more than one method of computing the data may be used, but as the method of obtaining the data was extremely simple and understandable to the average dredge operator, it was felt that the method of computation should also be simple for the same reason.

We appreciate Mr. R. L. Daugherty calling attention to our having left out the factor of the dynamic force of the water entering the impeller. However, as the thrust unit must be designed to accommodate a condition of maximum thrust and this condition occurs when discharge pressure is high and velocity of flow is low, it was felt this force would not be of sufficient magnitude to include in the computation and at least would be on the safe side. However, as the force due to suction vacuum is used and which is also low under a condition of maximum thrust, unquestionably the dynamic force of the flowing water should be included to complete the study.

Applying the correction factor of the dynamic force to Fig. 2, or the detailed method of computing total thrust load, we note that the suction vacuum is equal to 5 in.; this corresponds to a suction-pipe velocity of approximately 10 ft. per sec. in the dredge *Clackamas;* therefore

$$\frac{64.4 \times 4.9 \times 10^2}{32.2} = 980 \text{ lb.}$$

As this force is negative it must be subtracted from the original total positive, leaving a net correct total positive of 51,603 lb. instead of 52,583 lb.

Curves Nos. 11 and 12 were plotted from thrust loads computed from pressure data as obtained with the 70-in. diameter impeller operating with $^1/_{32}$ in. clearance front and back as Curve No. 11, and $^1/_4$ in. front and back as Curve No. 12, and applied to impellers assumed to be 60, 70, 80, 90, and 100 in. in diameter. That is, it was assumed that the pressures as read on the *y-Y* gages would be the same for all sizes if the gages were placed the same distance from the periphery of the impeller and when the pump is discharging against the same discharge pressure, as was the case with the 70-in. test impeller. Likewise, it was assumed that pressures on *A-B* gages would be the same if these gages were placed the same distance inside the periphery of the impeller; the *c-C* and *D* gages, if placed the same distance from shaft center and the *b-B* gages if placed midway between *a-A* and *c-C* gages.

These pressures were plotted against the proper radii for each of the various diameters and from the pressure curve drawn through, and the thrust load was computed in the manner described in the original paper.

As certain assumptions are used in this connection, the results are of course only approximate, but check with our findings by experience that impellers of large diameter do produce more thrust than those of like design and small diameter.

After a study of the discussions, it seems to be conceded by all that the most efficient dredge pump is one in which circulating losses are reduced to a minimum, also that thrust can be controlled by varying clearances as between impeller shrouds and heads. It seems in agreement and our tests have proved that close clearance at the back and more open clearance at the front produces less thrust, whereas the reverse condition produces high thrust.

In order to reduce circulation losses which occur between the the nose or seal ring and the suction disk of the impeller, it is necessary to maintain as small a pressure differential at this point as is possible. This can only be accomplished by the use of vanes running close to the front head and which eliminates the possibility of reducing thrust by an open front clearance and at the same time reduces circulation losses to a minimum at this point.

It is also in agreement that in order to reduce thrust it is necessary to maintain close clearance between the back head and back shroud vanes and that circulation losses at this point may be eliminated by the use of an impeller without relief holes. Therefore, it is reasonable to assume that a pump impeller without relief holes and with vanes on both front and back shroud running in close proximity to the casing heads will show the best efficiency although at the expense of a higher thrust load than would be the case if relief holes were used.

It was not intended in the paper to justify the use of relief holes except as they may affect thrust loads and which in the case of our test showed to be 118 per cent higher with relief holes plugged.

Mr. McAuliffe brings out in his discussion the reason why it was not practical to actually measure the thrust load. Not only would the testing apparatus be expensive, but the loss of dredging time would run into quite a considerable sum. It was for this reason that these tests were made to develop a means of computing the thrust loads on any type of dredge pump.

It is believed the data presented in this paper will give all interested parties sufficient information for them to determine the maximum size of thrust bearing for any given large high-powered dredge pump.

It is not often that one has the opportunity to make such a series of tests on a large dredge pump due to the expense involved in taking the dredge out of active service and the hesitancy of boring test holes in a new large pump casing. For these reasons it was felt this actual test data would be of interest to dredge-pump operators and builders.

The dredge-pump problem is quite different from that of a water pump or, in fact, any other type of pump, as the most important features are high output, low maintenance, and steady operation rather than high efficiency. The efficiency of this pump, however, was very good, the nozzle tests showing an efficiency of 61 per cent for a 2000-ft. pipe line and 55 per cent for a 10,000-ft. pipe line. Its performance under actual dredging operations has been most satisfactory even on pipe lines over two miles in length.

From a practical operating standpoint we feel that pump impellers with relief holes in the back shrouds are to be preferred to the closed back impellers. We find that the wear of either the relief holes or the back liner is not excessive and that relief holes do not reduce the output of the dredge.

The vanes on the outside of the impeller shrouds help the dredge operation in a practical way as they keep material from lodging between the impeller and the casing. They tend to reduce liner wear, and these impeller vanes themselves are

simple to renew. In fact, they and the various wearing rings must be renewed quite often if the pump clearances are to be kept close and uniform, which is most important if a large continuous output for the dredge is desired.

Adjustable nose rings are desirable in many instances especially where the unlined volute pump casing is used, though the practice of the Port of Portland has been to use renewable nose rings rather than adjustable ones, it being more practical to renew these rings from time to time as the pump is relined or repaired rather than to adjust the nose ring to insure a close fit while the pump is in operation. An effort is made to maintain reasonably close pump clearances at all times, and the dredge pumps are opened, inspected, and repaired or relined if necessary once a week, usually on Sundays.

The square casing with a full-lined volute as well as the abrupt suction entrance has been adopted to minimize the pump maintenance. The Port of Portland has found this type of dredge pump to be quite as efficient so far as practical dredging results are concerned as the rounded unlined volute or the pump with an eased suction entrance. The object which the Port of Portland has striven to obtain is to get the most number of cubic yards of sand into the fill for the least cost, and this type of pump has so far given the best results, though the Port has experimented with various shaped whirlpools and suction entrances. Where the material is clay, not abrasive, and has considerable trash in it, a less abrupt entrance is better. The object of this paper was not to discuss dredge-pump efficiencies and designs, but to give data relative to thrust loads and pressures inside the pump casing at various points, this being data which the usual pump tests do not show.

The latest dredge-pump design of the Port of Portland has an unlined volute and an adjustable nose ring on the suction side of the pump as well as an eased suction entrance.

The main object of the eased or flaring pump suction is to make it easier for the pump to pass sticks, rocks, and trash. The adjustable nose ring is used where the lighter renewable nose rings have to be replaced oftener than once a week due to the handling of very abrasive material. However, it might be of interest to know that the 30-in. pump with its abrupt suction entrance, as shown in the sketch, was a development based on the former use of the flared suction. This type of pump seemed to produce more yardage and was preferred to the flared suction type by the dredge operators when handling clean sand. When dredging clay or trashy materials, the flared type of suction is best, and the Port of Portland keeps both types of heads and runners on hand, and, in fact, on some of the pumps uses a combination front head permitting the use of either type.

For intermittent dredge operation or for the handling of light materials, the unlined volute is also preferable to the full-lined volute pump.

HYD-51-5

Thermodynamic Theory of the Air Lift

By ALEXEY J. STEPANOFF,[1] SAN FRANCISCO, CALIF.

The air lift as a method of pumping from deep wells is subject to no generally accepted theory that will serve as a basis for practical design. Treating the air-lift pump as hydraulic apparatus, the thermodynamic side of the phenomenon has been neglected, and this method will explain many points in the operation that now are obscure. In his conclusions the author decides that the efficiency of the air does not depend upon the form of the footpiece if the air-inlet devices do not obstruct the flow of water, if the openings for the air are sufficiently large to cause no impact, and if the air and water are so introduced as to secure uniform mixture. Attempts to introduce jet action of the air are due to misunderstanding the method of action. Slip losses are usually due to incomplete expansion of the air, the slip as such being not of much importance. No "single coefficient" experimental formula can express the variables involved in the air lift, as its operation depends upon factors which vary differently for various submergences, air-to-water ratios, sizes of pipes, etc.

1 Introduction

ALTHOUGH the air lift occupies a more and more prominent place among the methods of pumping liquids from deep wells, there is no generally accepted theory among the many proposed which will satisfactorily check with the experimental data and give a basis for practical design of the air-lift pump. All attempts to reconcile the discrepancies of the theories proposed with experimental data have failed, and until now many factors governing the operation of the air-lift pump are covered with the haze of mystery. The attempts to find some corrective factors for the existing formulas have sometimes gone so far as to lead to the introduction of terms that have no physical meaning. In the opinion of the author, the cause of such a situation lies in the fact that, considering the air-lift pump as hydraulic apparatus, the thermodynamic side of the phenomenon was neglected, while air under high pressure is best understood when treated thermodynamically.

Consideration of the air lift from a thermodynamical point of view has the advantage of not violating the scientific principles involved in the other theories. It throws light upon the operation of the air-lift pump which seemed obscure before and permits conclusions important from a practical point of view.

2 Operation of the Air-Lift Pump

As is well known, the air-lift pump consists of a bored well sufficiently deep to permit the liquid level to rise a certain distance above the bottom of the well. A pipe called the rising main or eduction pipe or lift tube is submerged in the liquid, and compressed air is introduced somewhere near the lower end.

Air bubbles rising through the water in the lift tube so reduce the specific gravity of the mixture, and therefore the weight of the column of the liquid, that the excess pressure at the base of the column due to external liquid pressure forces the mixture above the supply level and out of the top of the delivery pipe. This method of presentation of the operation of the air-lift pump was chosen here because it is easiest to understand; others will be given later. Different relative positions of the air pipe and water pipe are possible, but from the theoretical point of view they all are equal, though in practice certain combinations are more efficient than others.

3 Theories of the Air Lift

Among the most widely used theories of the air lift, those proposed by Prof. E. G. Harris, Dr. H. Lorenz, Prof. A. H. Gibson, and N. Swindin may be listed.

Professor Harris[2] considers the force of buoyancy of the air bubble as the motive force of the pump, analyzes the motion of the bubble, finds the relation between the size of the bubble, slip (relative velocity of the air bubble in respect to the water), and the head produced.

Dr. Lorenz[3] in developing his theory writes Bernoulli's equation for a differential head dh corresponding to a given flow in the discharge pipe, introducing variable pressure, density of mixture, and integrating between the limits $h = O$ and $h = h_s + h_d$ (Fig. 1), and thus he gets the relation between the variables involved.

Professor Gibson's theory, being simpler than these just given, will be quoted in detail as it is in his book:[4]

Let V_w = volume of water raised, in cubic feet per second
V_m = mean volume of air used, in cubic feet per second, during its passage through the rising main
V_a = volume of air used, in cubic feet per second, at atmospheric pressure P_a
P_1 = pressure at the base of rising main
h_d = net lift, in feet
h_s = submergence, in feet

Then assuming isothermal expansion of the air in the lift tube, we have:

$$V_m = \frac{P_1}{P_1 - P_a}\left(\log_e \frac{P_1}{P_a}\right)V_a$$

The mean specific gravity of the mixture in the tube is

$$\frac{V_w}{V_w + V_m}$$

Head producing flow $= h_s - (h_d + h_s)\dfrac{V_w}{V_w + V_m}$ feet of water, or

$$= h_s \frac{V_w + V_m}{V_w} - (h_d + h_s) \text{ feet of mixture in the lift tube}$$

$$= \frac{V_m}{V_w} h_s - h_d \text{ feet of mixture.}$$

Equating this to the sum of the friction head h_f and the kinetic head h_v we have, on reduction:

$$V_m = V_w \frac{h_d + h_f + h_v}{h_s}$$

hence

$$V_a = V_w \frac{P_1 - P_a}{P_a \log P_1/P_a} \cdot \frac{h_d + h_f + h_v}{h_s}$$

$$= \frac{V_w(h_d + h_f + h_v)}{34 \log_e P_1/P_a}$$

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] Harris, E. G., "Compressed Air," McGraw-Hill Book Co., New York, 1910.

[3] Lorenz, H., "Die Arbeitsweise and Berechnungen der Druckluft-Flusigkeitsheber." *V.D.I. Zeit.*, vol. 53, p. 545, April, 1909, or quotation by G. J. Davis and C. R. Weidner, "An Investigation of the Air Lift Pump," Bull. of Univ. of Wisconsin, no. 450, Eng. Series, vol. 6, no. 7.

[4] Gibson, A. H., "Hydraulics and Its Application," Van Nostrand Company, New York, 1926.

This relation enables the volume of free air per cubic foot of water to be determined in terms of h_f and h_s.

N. Swindin[5] contributes no new theory of the air lift, but throughout the proposed design procedure the losses are itemized; among the "so-called losses" such items as "acceleration" loss and slip loss are introduced.

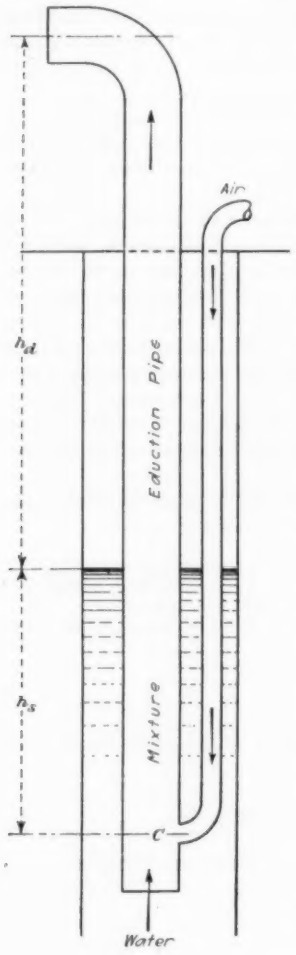


FIG. 1

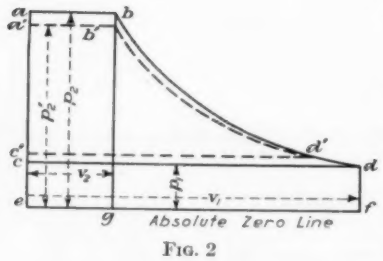


FIG. 2

THERMODYNAMIC THEORY OF THE AIR LIFT

Now the phenomenon will be approached from a different point of view. To make the matter clearer it is better to start from the air compressor and follow the air in all intermediate steps in its way from the compressor intake to the pump discharge. The work of compression of air consists of three parts: W_1, the work of charging cylinder; W_2, the work of raising the pressure of the air from some pressure P_1 and volume V_1 to some pressure P_2 and volume V_2 (Fig. 2); and W_3, the work of forcing the compressed air out of the cylinder.[6] W_1 is done on the side of the piston opposite to that on which W_2 and W_3 are performed in the same cycle; hence the total work of compression will be:

$$W = W_2 + W_3 - W_1 \ldots [1]$$

Assuming no clearance volume and isothermal compression, the work per pound of air

$$W_1 = P_1V_1 \text{ ft-lb} \ldots\ldots [2]$$

and

$$W_3 = P_2V_2 \ldots [3]$$

But

$P_1V_1 = P_2V_2$, hence

$$W_1 = W_3 \ldots [4]$$

And the total work of compression

$$W = W_2 = P_1V_1 \log_e P_2/P_1 = RT_1 \log r \ldots\ldots [5]$$

R being for air = 53.35; T_1 = temperature of air at intake, and r = ratio of compression.

On the PV diagram (Fig. 2) $W_1 = P_1V_1$ is presented by the area *cdfe*; $W_3 = P_2V_2$ is represented by the area *abge*, and $W = W_2 = P_1V_1 \log_e P_2/P_1$ by the area *bdfg* or *abdc*, as is evident from Equations [1] and [4].

If the compressor were equipped with an indicator, the indicator card obtained would be, in general, of the shape shown on Fig. 2 by *abdc*, and to a certain scale will represent the work of compression (from which the work of compression can be calculated) which if expressed in horsepower will be the indicated horsepower of the compressor, and which theoretically will be equal to $W = RT \log r$ per pound of air.

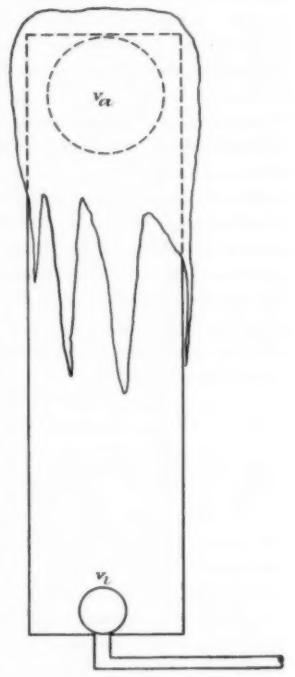


FIG. 3

The energy spent on compression is stored in the air and can be utilized in different ways. An air engine could be operated by the compressed air, and in an ideal case work done by this engine would be equal to that of compression. The indicator diagram of such an engine will be exactly the same as that of the compressor. Air would be admitted along the line *ab*, after which isothermal expansion would begin from *b* to *d* and exhaust from *d* to *c*.

But in the actual case, even assuming isothermal expansion, there are inevitable losses. For our purpose it is important to note that there is a loss of pressure from the air reservoir to the cylinder of the air engine. Instead of pressure P_2 there will be a pressure P'_2 lower than P_2, area *abd'b'a'* being energy lost due to the drop of pressure between the air compressor or receiver and the air engine. Also the pressure P_1 for an actual engine will be higher than that for the compressor by a certain amount, area *cdc'd'* being the loss due to incomplete expansion of the air in the air cylinder and back pressure during the exhaust.

The air-lift pump may be looked upon as an air engine and a displacement pump in one. If one pound of air is introduced at the pressure P_s, the volume being V_s, it is evident that during admission into the pipe the air will do the same work as if it were in the cylinder of the air engine; that is, P_sV_s. This work will be done by raising the water by displacement for the distance V_s/A, where A is the area of the pipe. The work done will be equal to the weight of water raised (Ah_sW) times the distance V_s/A through which the water is raised = $(Ah_sw)V_s/A = h_swV_s = P_sV_s$, where h_s is the submergence and w is the specific weight of the water. The same result may be obtained in the following way:

The height of the water column in the rising main above the water level outside the rising main will be V_s/A; thus the work done is equivalent to raising the volume of water V_s through the distance h_s, which will give again $V_swh_s = V_s(wh_s) = P_sV_s$ ft-lb., neglecting friction losses, of course.

It should be noted that when the air pressure is applied to the water during the starting period in the lift tube it will act upward and downward, but as the mass of water column in the eduction pipe is less than that outside of the pipe, the former will start moving, the outside column being brought in slight oscillation until the flow is established completely.

Air in the form of fine bubbles, or pistons occupying the whole cross-sectional area, begins to rise in the eduction pipe. While rising, the air expands isothermally, and finally escapes

[5] N. Swindin, "Modern Theory and Practice of Pumping," E. Benn, London, 1926.

[6] Carpenter, R. C., and Diederichs, H., "Experimental Engineering," p. 954.

with water or alone at the discharge. Expanding air does work exactly in the same manner as it would in the cylinder of the air engine, no matter whether it stays in one place or moves with water or slips through the water, because as soon as there is an increase in volume there is work done on the water. If the air during its rising completes its expansion or escapes at atmospheric pressure, the work done would be $W_2 = RT\log_e P_s/P_a$, P_a being atmospheric pressure and T temperature of water in the well. Thus the air gives out completely the work done upon it $W_3 + W_2$, as it would in an ideal air engine; from this work $W_1 = P_aV_a$ must be subtracted corresponding to the exhaust stroke of the air engine, which is the work done on the air, the net work done being:

$$W_3 + W_2 - W_1 = P_sV_s\log_e P_s/P_a = RT\log_e r.$$

Thus it would appear that moving the air does not change the availability of the work of expansion of the air, the only objectionable effect of the slip being the tendency of air bubbles to coalesce and rush out of the rising main at a pressure higher than atmospheric, without completing its expansion, the loss being due to incomplete expansion rather than to slip. Of course, a certain amount of work is consumed by internal friction of air and water during the slip, but later it will be shown that this loss is of small importance.

The work done by the air is utilized for raising V_w cubic feet of water per second through the distance h_d and in various losses such as friction losses, entrance loss, and velocity-head loss. These losses can be accounted for by introducing corresponding heads lost: h_f = head lost due to friction in the rising main; h_e = entrance loss head; h_v = velocity head. Head lost in the elbows may be included in friction losses by introducing the equivalent lengths of pipe in addition to the actual length of the rising main.

Thus, we may equate:

$$W(RT\log_e P_s/P_a) = V_w.w\,(h_d + h_f + h_e + h_v)$$

where W = weight of air used per second in pounds and w = specific weight of water = 62.3 lb. per cu. ft.

Air in the air-lift pump performs the discharge stroke of the ordinary displacement pump, the static head outside of the rising main completing the suction stroke by filling the space liberated by the air, the pump being "uniflow;" as reciprocating members are absent.

It is frequently thought that air bubbles rising through the water do no work, but just slip. This is not true in an actual case; air rising through the water column expands, and in expanding it does work. The work available from the rising air bubbles is a function of a particular installation.

Suppose a standpipe full of water and that volume V_1 of air under pressure P_1 sufficient to overcome the static pressure due to water column is introduced into the pipe near the bottom; immediately a volume of water V_1 will overflow from the standpipe, and water will continue to overflow until the air bubble rises to the top of the pipe, the total volume of water displaced being $V_1\ P_1/P_a$ where P_a is atmospheric pressure. If a series of bubbles is introduced into the pipe, water will overflow, but larger and larger volumes of air would be required to maintain the flow, which at least would stop, though the air bubbles would rise through the water. But this does not mean that air does no work. It does, but the conditions are similar to those of a centrifugal pump at impending delivery, or for any pump with the suction more than 34 ft. Energy is applied to the water, but it is spent on whirling water and finally in heat. Properly speaking, the static head outside the rising main does the same for the air lift as the static atmospheric pressure does for all pumps; it delivers water which is raised by the pump, or air, in the air lift. No pump can deliver water unless the water is delivered to it by atmospheric pressure.

5 Work Equation From Consideration of Force of Buoyancy and Difference of Densities of Water in the Rising Main and Outside of It

Now the problem may be looked upon from a different point of view. Suppose in the air-lift pump a volume V_s of air is introduced near the bottom, it must be at the pressure $= h_s.w$. This air possesses a certain amount of energy equal to the force of buoyancy $= V_sw$ multiplied by the distance h_s through which this force may act, in the same manner as a weight W lb. elevated a distance h ft. possesses the energy equal to Wh ft-lb. In addition the air has a certain amount of energy of compression, which increases the volume of air during its rise along the rising main; thus the force of buoyancy will increase also until at the top it will be V_aw, V_a being equal to V_sP_s/P_a. The mean volume of air during its rise will be

$$V_m = \frac{P_aV_a\log_e\dfrac{P_s}{P_a}}{P_s - P_a}$$

hence the mean force of buoyancy will be

$$\frac{(P_aV_a\log_e r)\,w}{P_s - P_a} = V_mw$$

and the total work done by the air will be equal to

$$\frac{P_aV_a}{P_s - P_a}(\log_e r).w.h_s$$

but $h_s.w = P_s - P_a$; hence the total work done by the air is $P_aV_a\log_e r$. This work is utilized in raising water and overcoming the various losses, or

$$P_aV_a\log_e r = V_w.w.h_d + \text{losses}$$

which is the same equation as the foregoing, deduced by thermodynamic reasoning.

Professor Gibson, considering the decrease in density of mixture in the rising main, deduced the equation (as already quoted from his book) as

$$V_a = \frac{V_w\,(h_d + h_f + h_v)}{34\log_e P_s/P_a}$$

$$= \frac{V_w\,(h_d + h_f + h_v)}{P_a/w\log_e P_s/P_a}$$

or

$$V_aP_a\log_e\frac{P_s}{P_a} = V_w.w(h_d + h_f + h_v)$$

which is, again, the same equation as the preceding two.

From this equation the relation between volumes of water and air may be obtained as soon as h_f, h_e, and h_v are determined.

6 Work Equation in Complete Form

The following equation of work was developed:

$$W\left(RT\log_e\frac{P_s}{P_a}\right) = V_w.w.(h_d + h_f + h_e + h_v)\ldots\ldots\ [1]$$

where h_d = the net lift, h_f = head lost due to friction in the rising main and fittings, and can be expressed as follows:

$$h_f = \frac{f(h_d + h_s + h')\, v^2}{2\, g.m} \quad \ldots\ldots\ldots\ldots\ldots\ldots [2]$$

where m = the hydraulic mean depth, h' = additional length of pipe to compensate the friction loss in elbows, v^2 = the mean square of the velocity in the tube, which is equal to

$$\frac{1}{(P_s - P_a)A^2} \int_{P_a}^{P_s} \left\{ P_a^2\, V_a^2 \frac{1}{p^2} + V_w^2 + 2\, P_a V_a V_w \left(\frac{1}{p}\right) \right\} dp$$

which upon integration and reduction takes the form

$$\frac{1}{A^2}\left(V_w^2 + \frac{2V_wV_a}{r-1} \log r + \frac{V_a^2}{r} \right) \quad \ldots\ldots\ldots\ldots [3]$$

where A = area of the pipe and r = ratio of expansion of air = P_s/P_a. As an approximation, the value of V^2 may be taken as

$$V^2 = \left(\frac{V_a/2 + V_w}{A} \right)^2 = \frac{1}{A^2}\left(V_w^2 + V_aV_w + \frac{V_a^2}{4} \right) = \left(\frac{V_a + 2V_w}{2A} \right)^2$$

$h_s = k\frac{V_1^2}{2g}$, k being an energy coefficient for entrance loss and V_1 velocity of water below the air inlet

$= V_w/A$

$$h_v = \frac{1}{2g}\left(\frac{V_a + V_w}{A} \right)^2$$

Substituting these terms in Equation [1], we get the equations from which the values of V_a per unit of water can be obtained in terms of h_d, h_s, f, k, and A.

The value of coefficient f has been determined by Professor Gibson for a mixture of air and water, the friction head being expressed in feet of column of the mixture in the eduction pipe, from the published data of a large number of trials on such pumps. In every case f had a value approximately six times that obtained for the flow of water alone at the same velocity. It is evident that the value of f determined in this manner would take account not only of friction loss but also losses due to incomplete expansion of air and slip. The attempts to separate these losses were not successful. The chief reasons are the absence of data concerning friction of mixtures of liquids and air conducted under conditions excluding expansion and slip (horizontal pipes provided with means for intimate mixing of air and liquid). As soon as these data shall be determined for different ratios of liquid and air and different velocities of mixture, there will be no difficulty in separating this loss from those due to incomplete expansion and slip.

APPLICATION OF THE THERMODYNAMIC THEORY

It was shown that by thermodynamic reasoning we can deduce the same equation connecting the lift, submergence, and volumes of air and water as by other methods, but the thermodynamic reasoning enables us to go further and make a few conclusions important for practical application.

7 Form of the Footpiece

The "footpiece" in the air lift is understood to be the lower part of the pump; i.e., the end of the rising main and the arrangement for introducing air. Considering the air lift as an air engine and displacement pump, we have seen that if all losses above the footpiece are neglected, the work given out by the air is equal to the total potential energy of compressed air stored in it, and hence any ingenuous design of the footpiece cannot save any energy lost above the footpiece. The requirements for the design of the footpiece are the same as those for any engine, i.e., that the area of the opening must be sufficient to introduce the necessary quantity of air with the same velocity as that of the water, thus avoiding shocks or throttling, and then the air-introducing devices should not obstruct the flow of water or mixture. From the point of view of these requirements the best forms of the footpiece would be those shown in Fig. 4, *A* and *B*. For the purpose of reducing losses due to incomplete expansion and uniform flow at the intake it is important to introduce air in the form of fine bubbles, intimately mixed with water; hence the total area for the air inlet should be divided into a great number of small holes. As to the form of the lower part of the rising main, the same rules apply as those for any pump; to decrease the entrance loss the trumpet form has definite advantages.

There are many footpieces in use attempting to involve the jet action of the air in the air lift. Their appearance is due to confusing the method of operation of the air lift with that of an injector or jet pump. It seems that there never were any serious attempts to base the theory of the air lift on the jet principle; it should be remembered that at usual proportions of air to water the weight of air is negligibly small as compared with that of water. Thus the velocity of air must be very great to make appreciable effect upon the water. But an increase in velocity can be obtained only by restricting the area of the inlet and by an increase of pressure of the air back of this restriction, which means an increase of the load on the compressor. This increase in pressure must be considerable, as the volume of air at the pressure existing at the footpiece is comparatively small. As the conversion of the pressure energy into kinetic energy is always accompanied by the losses due to friction in the nozzle or orifice, it is evident that all these appliances only consume power, which was very well shown by extensive experiments by the Westinghouse Air Brake Company (1800 different experiments), Professors Davis and Weidner, and others.

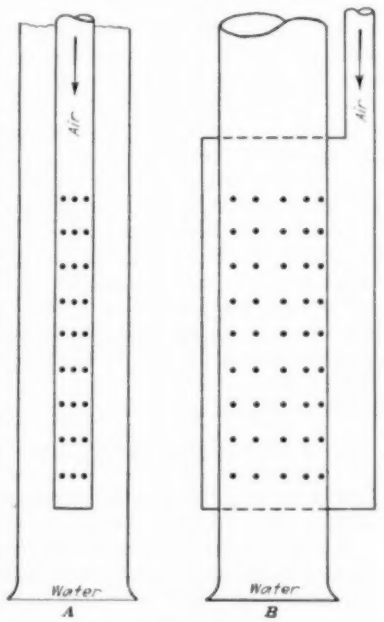


Fig. 4

From the point of view of thermodynamics it makes no difference if the air is introduced in small bubbles or large bubbles, but taking into account that large bubbles at the footpiece interfere with the flow of water and are liable to coalesce at the top and form streams discharging directly into the atmosphere without completing expansion, the smaller sizes of air bubbles uniformly distributed give better results.

The elongation of the rising pipe below the air inlet is important for two reasons: (1) If air is admitted in large quantities, the air bubbles may get outside of the pipe and be lost; and (2) when the pump is started, the air pressure acts upward and downward, and to start the pump it is essential to have a

certain excess of water column outside of the pipe, which insures steady and quick operation of the pump at the start.

8 Slip

It has been pointed out already that if the expansion of the air is completed or if air escapes at atmospheric pressure the slip has no effect upon the efficiency of the pump except for small losses due to internal friction of water and air. In other words, if the air escapes at atmospheric pressure it carries with it no energy put into it by the compressor. Very interesting and instructive experiments were conducted at the University of Wisconsin by Professors Davis and Weidner upon the behavior of the air bubbles in the air lift. They used $1^1/_4$-in. glass pipe for the rising main. The results of their observations are exactly the same as those of Professor Josse in Germany,[7] who used 3-in. glass pipe.

These tests showed that when air is admitted at a very slow rate the bubbles take the form of convex lenses with sharp edges, with the convex surface upward, the lower surface being quite flat; the bubbles occupied almost all the cross-section of the pipe. When the rate of discharge air was increased, the bubbles took the form of pistons separated by cylinders of water, and in both cases the flow was steady and uniform without any disturbances. Near the top, the bubbles rushed through the water pistons, and the water slipped down, forming larger water pistons. When these water pistons were long enough to prevent the air from rushing through them, they were discharged. The shape of the bubbles shows that they are moving under pressure from below; if there should be a slip, they would take the form of least resistance, i.e., elongated in the direction of motion.

From this we see that there is little tendency for the air bubbles to slip, except near the top of the discharge pipe, and that water slipped down does not slip through all the pipe, because the pipe along its full length is filled with alternate water and air pistons moving steadily and uniformly. The slight disturbances are only changes of distribution of water and air pistons, and only the discharge of air at a pressure higher than atmospheric produces a loss.

It should be noted that the foregoing refers to the air lift when the flow is established and the water overflows at the top. The behavior of air bubbles and pistons is quite different when there is no flow or when the supply of water at the bottom is not sufficient. Then, naturally, the air is forced out through the water layers, thus dissipating its energy on friction and discharging at a higher pressure. This condition is similar to that of any pump without a sufficient supply of water on the suction side. In this case the air lift has the advantage in comparison with the displacement pumps in that no damage occurs, the only result being loss of energy.

In case the air is mixed with water very intimately and remains in the form of bubbles at the discharge, its pressure is higher than atmospheric, due to surface tension.

It is important to separate these two losses—namely, loss due to slip and loss due to incomplete expansion—for the analysis, as they have different effects on the efficiency at different submergences. The first one is proportional to the length of the rising main or time required for air to pass the rising main. The second is dependent upon the ratio of water to air. Its absolute value is independent of the submergence, being controlled by the conditions at the discharge.

9 Effect of the Submergence Upon the Efficiency of the Air Lift

It was found by many experimenters that for constant input in foot-pounds the efficiency of the air lift increases with the increase of the submergence until a certain maximum is reached, after which the efficiency begins to drop.

Thus far no explanation of this fact has been given, this point being one of the mysteries of the air lift. And it seems really mysterious, for, with an increase of submergence, the lift being constant, the length of the pipe increases for both the air and water pipes; hence friction losses increase and loss due to slip should increase also, because the time for the air to pass the eduction pipe is greater. Then what makes the efficiency increase?

The answer is quite simple if we look upon the air lift as an air-engine-displacement pump and construct its indicator card, as explained, for two submergences. The increase in the submergence requires an increased air pressure. Thus if we construct indicator cards for one pound of air the height of the diagram for the greater submergence will be greater than for the smaller submergence, the lower parts of the cards being the same for both cases. It was shown that the area *cdlk* (Fig. 5) is loss due to incomplete expansion of the air, or discharge of air at a pressure higher than that of the atmosphere. This loss is constant for one pound of air, if the method of introducing the air is the same for all submergences, and it depends only upon the volume of air at the discharge. The work done per pound of air for the smaller submergence is represented by the area *efdc;* that for the greater submergence will be represented by the area *abcd*. The loss due to incomplete expansion is the same in both cases; thus it is evident that this loss expressed in per cent of the input will be less for the large submergence than for the small one.

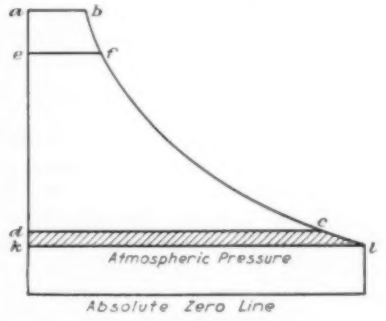


Fig. 5

If the increase in losses due to friction in the air pipe and the rising main could be estimated accurately it would be possible to predetermine the point of maximum efficiency from the study of indicator cards for different submergences.

10 Effect of the Increase of the Diameter of the Rising Main Upon the Efficiency

From the experiments of the Westinghouse Air Brake Company, Professors Davis and Weidner, and others, it was found that for the same submergence the efficiency of the air lift increased with the increase of the diameter of the eduction pipe. With the increase of the diameter of the rising main the velocity of the mixture decreases, thus decreasing the friction loss and velocity-head loss. Now, the decrease of velocity of mixture for a constant length of the eduction pipe means that the time for water and air to pass from the footpiece to the discharge will be greater, hence the slip loss should increase; the fact that efficiency increases with the increase in diameter of the eduction pipe clearly shows that the friction loss constitutes a more important part of all losses than the slip (the loss due to incomplete expansion being constant for both cases).

11 Effect of the Ratio of Air to Water Upon the Efficiency

All experimenters agree that the efficiency of the air lift is highest for the lowest rates of pumping, provided the continuous

[7] Josse, E., "Druckluft-Wasserheber," *V.D.I. Zeit.*, vol. 42, p. 981, Sept. 1898.

discharge is established; or, efficiency decreases as the input increases. If the slip is the main source of the losses, the opposite results would be obtained for the lower rates of pumping, because the low rate of pumping means low velocity, hence greater time for the air to slip. Then it would seem that the slip would be less for greater inputs of greater rates of pumping, because greater input at a constant submergence means a larger quantity of air; but with greater quantities of air, air bubbles tend to form pistons separated by layers of water which move uniformly without slipping until they reach the top of the pipe. Thus for greater inputs, the time for the bubbles to slip is less and the tendency to slip is less, hence it would be natural to expect that efficiency would increase. But experience shows the opposite, which is again the indication that losses ascribed to the slip are really due not to slip but to some other cause.

12 Effect of Introducing Compressed Air Above the Surface of Water in the Well

One of the mistakes resulting from a misunderstanding of the principle of operation of the air lift is that of introducing compressed air above the water level in the well. It has been shown that the role of the static pressure outside of the eduction pipe is exactly the same for the air lift as that of atmospheric pressure in the case of any other type of pump; that is, to supply water raised by the pump. The same line of reasoning as in the case of the air lift would lead to the conclusion that the application of the air pressure upon the surface of the water in the well or open pond would be advantageous for any kind of pump, especially for centrifugal pumps, which are so sensitive to a variation in the suction head. The advocates of this measure overlook the fact that the water level in the well is just an indication of the water level of the underground water which controls the flow of water into the well, and the attempts to increase static pressure in the well have no effect upon the static head of this underground reservoir. Hence it is natural that experiments conducted by Davis and Weidner could not reveal any effect of the additional static pressure in the well.

13 Effect of the Climate Upon the Efficiency of the Air Lift

There is no uniformity in calculating the efficiency of the input to the air-lift pump. The efficiency of the air lift as usually determined is the ratio of water horsepower, $Qwh_s/550$, to the input or air horsepower. The latter expression is differently interpreted by different writers; sometimes the air horsepower means the input in horsepower calculated from the theoretical air pressure at the footpiece, or $h_s.62.3/144$ lb. per sq. inch; sometimes as the same based upon the air pressure at the well, and sometimes the air horsepower is taken as actual indicated horsepower of the air compressor. For comparison of different air-lift pumps between themselves and with other types of pumps the last method of calculating the input should be used as it is more definite, while in the case where an air compressor supplies air to many air-consuming machines or is located too far from the well, the second method—i.e., the air horsepower based on the air pressure at the well—is preferable. The first method is impracticable for actual tests because the air pressure at the footpiece cannot be directly obtained. That calculated from the submergence may differ considerably from the actual, as it depends upon the design of the footpiece and the rate of discharge of the air. For theoretical consideration or design the input may be taken as based on the theoretical air pressure, but for calculating the probable efficiency of the pump the air pressure at the surface of the ground should be used, which would lead to the more accurate consideration of the air pressure losses in the air pipe, neglected in many instances. Besides the losses due to friction in the air pipe in calculating the efficiency, or comparing efficiencies of different air-lift pumps, or analysis of losses occurring in the air lift, the temperature of the surrounding air, the temperature of the air at compressor discharge, and the temperature of the water in the well should be taken into account. It should be remembered that in cooling air each five degrees on the Fahrenheit scale results in a loss of energy of about one per cent. No doubt the discrepancies in results of many tests are due to the neglect of this fact. In all data published on the air lift the data concerning the temperatures of water, air, etc., are absent.

While it is always assumed that the compression of air in the air compressor is isothermal, it is never isothermal in practice even with efficient cooling. Thus air leaves the compressor cylinder at a temperature higher than that of the surrounding air, which requires more power for compression. Air during the passage through the receiver and pipes begins to cool, and at the moment of its entrance into the footpiece it is cooled to the temperature of the water in the well, which might be considerably lower than that of the surrounding air at the surface of the ground. The average temperature of the ground water in the deep wells is about 50 deg. fahr., and varies little during the year. The temperature of the surrounding air might reach 90 or 100 deg. fahr., or even higher during the summer time and the temperature of the air at the compressor discharge will be still higher. Thus in this case the loss due to the cooling of the air will not be less than 10 per cent, assuming isothermal compression. For adiabatic compression, this loss will be still higher. It is therefore evident that, other conditions being equal, the efficiency of the air lift depends upon the climate and the efficiency of cooling the compressor.

In the most favorable conditions, assuming the surrounding air at 70 deg. fahr., water at 50 fahr., and isothermal compression, the loss due to cooling exceeds those due to entrance resistance and velocity head.

The foregoing discussion suggests the utilization of the air discharged from the well at the air-compressor inlet. This may be done in practice by installing a special air separator working at atmospheric pressure on the discharge pipe. Thus the same air is used over and over, and works on a closed cycle.

An interesting modification of this system from the theoretical point of view is the air-lift system with a booster; the compressor inlet is connected to the booster. This system consists of the air-lift pump discharging a mixture into a closed vessel on the surface of the ground. Here air is separated from the water and occupies the upper part of the vessel, while the water is forced through the outlet at the bottom by the air pressure maintained in this chamber. The compressor intake is connected to this chamber, using cooled air at a pressure higher than that of the atmosphere. This system forms a completely closed circuit similar to the condensing-steam-engine plant. The advantages of this system are (1) economy due to using cooled air, (2) the pressure of the air escaping without completing its expansion, or slip, is utilized by the compressor, and (3) the part of the eduction pipe above the surface of the ground carries only normal water at a lower velocity, therefore with smaller losses. Small additional losses are present due to velocity head and entrance loss at the booster.

There are no data available upon the actual operation of pumps of this kind, although several patents were given out in 1924. The complications in this system might arise from the fact that air might carry with it too much water into the compressor and, in turn, water will carry with it a portion of the air from the system, thus requiring a make-up of the quantity of air circulating.

14 Conclusions

1 Treating the air-lift problem thermodynamically has a definite advantage over the other methods, explaining many points of the operation of the air lift which are obscure when treated from other points of view.

2 The efficiency of the air lift does not depend upon the form of the footpiece if the following conditions are fulfilled: (*a*) That the air-inlet devices do not obstruct in any way the flow of water; (*b*) that the air-inlet openings are sufficient to give a velocity of air approximately equal to that of the water (no impact); (*c*) that the method of introducing the air insures uniform and intimate mixture of air and liquid.

3 All attempts to involve kinetic (jet) action of air in the air lift are due to misunderstanding of the physical side of the phenomenon, and are influenced by the method of action of an injector and a jet pump working on quite different principles.

4 Losses usually ascribed to the slip are chiefly due to incomplete expansion of air; the slip as such is of little importance.

5 Introducing compressed air above the surface of water in the well has no effect on the output of the pump, except possible loss of air through the leaks.

6 The motive force in the air lift is the potential energy of the compressed air, the static head outside of the rising main performing the same function as the atmospheric pressure in any other types of pump.

7 The efficiency of the air lift based on the indicated horsepower of the compressor or the air horsepower calculated from the pressure at the well depends upon (*a*) the method of compressing the air, (*b*) the temperature of the surrounding air, (*c*) the size of the air duct, and (*d*) the losses at the air inlet due to local obstruction in the form of nozzles, etc.

8 No "single coefficient" experimental formula can express the relation between the variables involved in the air lift, as the operation of the air lift depends upon several factors which vary differently for various submergences, air to water ratios, sizes of pipes, etc.

Some Interesting European Hydraulic Turbine Research

By BLAKE R. VAN LEER,[1] WASHINGTON, D. C.

IT IS the purpose of this paper to bring to attention certain turbine researches and their results which, although publicly known in Europe, are not generally known to the engineering profession in America.

The attitude of the hydraulic-turbine industry in Europe seems slightly different from that found in America. American engineers have a long list of brilliant accomplishments to their credit, but of recent years these have been brought about by dire necessity. It appears that most hydraulic engineers with American firms must convince their management that hydraulic experimentation and research are absolutely necessary before any money is forthcoming with which to conduct these researches.

In a great many European factories a definite research staff is maintained always at work on research problems, and if there are not any immediate problems, the solutions to which are required by the factory or by the practice, the staff is set to work on a problem which may yield information of benefit. Since labor, and especially technically trained and skilled labor, is much cheaper in Europe than in America, it is possible to maintain this highly technical work at a much less cost than in this country. Hence, from a standpoint of quantity of research and data collected on the subject of hydraulic turbines, Europe seems to have assembled the most. Furthermore, the majority of our American contributions have been the result of a few months' thought and experimentation, sometimes perhaps a few years, but abroad the tempo is vastly different. Europeans work with patience and thoroughness for years upon a problem.

Extensive Use of Models

As is well known to all hydraulic engineers and designers, our turbine theory is very imperfect. Consequently, after the theory has given a lead, the only sure way of ascertaining the truth is by investigation. Hence it is common practice here as in Europe to employ models for experimentation. This is done not only on account of its great cheapness, ease, and convenience, but also because our knowledge of the laws of similarity has increased so much in recent years that these tests are quite accurate within engineering limits. However, this use of models is much more common and extensive in Europe.

Dr. Thoma on Hydraulic Experimentation

The best article thus far published giving the European viewpoint on experimental research is "Experimental Research in the Field of Water Power," by Prof. Dr. Ing. D. Thoma, of the Technical University of Munich. Dr. Thoma very forcibly points out the value of experimental research when he says: "Theory, even if logically deduced, is unable to explain the complex facts of reality on account of the simplifying and not always exact assumptions underlying all theories. . . . These theories must be supplemented by experimental research. . . . The whole German turbine design has been based for a long time upon experimental research."

The weight of this statement is all the greater because Dr. Thoma is thoroughly grounded in all the turbine theories and is one of Germany's greatest teachers of theories. Although the paper quoted is over two years old, it is well to point out some research contributions that it mentions.

The first is the method of using parameters to designate vane surfaces. This makes it possible to classify, designate, and compare research data concerning very complex surfaces. Dr. Lawaczeck is given credit for this contribution.

The second is the value of having a standard method of designating similar things. Such a scheme makes it possible to compare results of research in different countries. Dr. Thoma suggests the use of absolute coefficients which would be the same regardless of the systems of measurements used.

The third contribution is that the conditions which exist for cavitation, as first shown by H. B. Taylor and L. F. Moody at the Hydroelectric Conference at Philadelphia in 1922, namely $\sigma = \frac{Ha - Hs}{H}$, can be used in model cavitation tests provided the proportion $\frac{H_1}{D_1} = \frac{D}{H}$ be maintained. This offers an extensive field for cavitation experiments with models.

The fourth is the visual study of hydraulic research problems, which is much more extensively practised in Europe than in America. By means of glass walls, stroboscopes, etc., one can frequently see when trouble is starting and what produces it when it is impossible to predict it by any known mathematical means.

Dr. Thoma has also listed the hydraulic-machinery laboratories of Germany. These are given in Table 1.

TABLE 1 EUROPEAN HYDRAULIC-MACHINERY LABORATORIES

Name of laboratory	Location	Person in charge	Year built	Experimental head, meters	Maximum quantity water, liters per second	Number of test stations
Briegleb Hansen & Co	Gotha	...	1913	30.0	120	1
			1914	4.5	1200	2
F. Schichau	Danzig	...	1920	11.1	2400	2
			1921			
J. M. Voith	Hermaringen	Dr. Hahn	1908	2.0–5.8	10000	2
	Heidenheim	Dr. Hahn	1917	6.0–100	450	5
			1919	2.0–3.0	280	1
Technische Hochschule, Charlottenburg	Berlin	Prof. Foettinger	1904–6	1–400	50–2500	5
Technische Hochschule, Danzig	Danzig	Prof. Schulze	1910	4.3–28	130	2
Technische Hochschule, Darmstadt	Darmstadt	Prof. Wagenbach	1903–4	5.5–300	7.5–1000	5
Technische Hochschule, Dresden	Dresden	Prof. Lewicki	1900–2	4–100	26–1200	6
Technische Hochschule, Hanover	Hanover	Prof. Oesterlen	1921	5–30	100–900	6
Technische Hochschule, Karlsruhe	Karlsruhe	Prof. Spannhoke	1923–4	4.5	125	1
Technische Hochschule, Munich	Munich	Prof. Thoma	1911–4	5.5–18	1100–2200	5

[1] Assistant Secretary, American Engineering Foundation; formerly Assistant Professor of Mechanical Engineering, University of California. John R. Freeman Scholar, A.S.M.E. 1927–1928. Mem. A.S.M.E.

Contributed by the Hydraulic Division and presented at the Annual Meeting, New York, December 3 to 7, 1928, of The American Society of Mechanical Engineers.

In addition to these laboratories, there are others at Budapest; Vienna; Gratz; Prague; Stockholm; Gothenburg; Copenhagen; Padua; Milan University; Riva Construction Company, Milan; Winterthur; Zurich University, Escher Wyss, Zurich; Geneva, Ateliers des Charmilles; Grenoble University, Electro-Technique Grenoble; Toulouse; Paris; Teddington; Edinburgh; Manchester; etc. These constitute the chief laboratories of Europe which are continually making contributions to the advancement of hydraulic-turbine research.

There are numerous minor mechanical details in European practice that are of interest to American engineers. An excellent treatment of these will be found in "Observations on European Water Power Practice," by A. J. R. Houston, *Journal* of the Boston Society of Civil Engineers, October, 1926, vol. xiii, no. 8.

Efficiency Formulas

In the use of model turbines European laboratories apply the

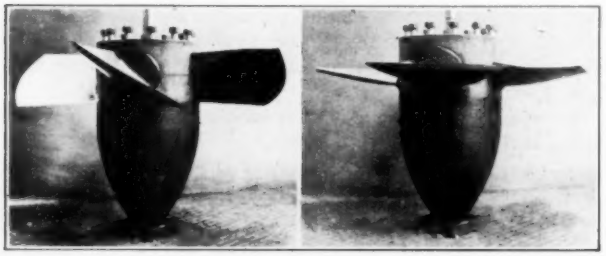


Fig. 1 Kaplan Turbine Runner
A—Blades in position for maximum load or quantity. B—Blades in position for minimum load or quantity.)

Moody, the Cammerer, and the Staufer formulas for predicting the maximum efficiency which may be expected of a full-scale turbine from a model test. These formulas are:

1 *Moody formula:*

$$E = 1 - (1 - e)\left(\frac{d}{D}\right)^{1/4}\left(\frac{h}{H}\right)^{1/10}$$

2 *Cammerer formula:*

$$E = 1 - (1 - e)\frac{1.4 + \sqrt{\frac{1}{D}}}{1.4 + \sqrt{\frac{1}{d}}}$$

Where E = maximum efficiency of the full-scale turbine
e = maximum efficiency of the model
D = diameter of the full-scale turbine
d = diameter of the model
H = head on the full-scale turbine
h = head on the model turbine.

In the Aufkirchen plant (Germany) the Cammerer formula gave a predicted efficiency of 91.1 per cent and the Moody formula of 93.7 per cent. The actual acceptance test showed an efficiency of 92.6 per cent, or between the two, indicating that the Moody formula gives results a little too high and the Cammerer formula a little too low. There also has been published recently another formula known after the man who proposed it as the Staufer formula. It is exactly the same as the Moody formula except the exponent for the ratio of heads is $^1/_8$ instead of $^1/_{10}$. In the case of the Aufkirchen plant this formula would give still higher results than the Moody formula. The amount of data at hand does not seem to warrant making a differentiation between an exponent of $^1/_{10}$ and $^1/_8$.

Draft-Tube Research

In many of the European hydraulic laboratories one finds that extensive research either has been made or is in process on draft tubes. All sorts, sizes, and shapes seem to have been tried, and the consensus of opinion seems to be that no one type of draft tube can claim superiority over others in all cases. The selection is dependent upon many factors, and if a maximum overall efficiency is to be obtained, the draft tube must be designed with the turbine runner.

Cavitation

Researches on the subject of cavitation are now in progress in several hydraulic laboratories in Europe. The foremost contributors to this work seem to be Professors Foettinger, of Berlin; Dahl, of Sweden; Oesterlen, of Hanover; and Thoma, of Munich.

Cavitation, including its prediction, prevention, or elimination, is today the foremost problem in the field of hydraulic-turbine design, and that laboratory which is not equipped to carry on research in this field is soon to find itself outclassed by its competitors. As pointed out, the chief means of making these studies is at present the visual one. The cheapest and perhaps most effective method of studying cavitation is the use of the stroboscope upon models, and the stroborama is a finer, better instrument for this work.

There are several theories as to the cause of cavitation which will be found discussed in "Turbines and the Cavitation Problem," by Elov Englessan, *Canadian Engineer*, April 13, 1916. The Swedes seem to have made the most rapid strides in finding metals that will resist cavitation, but the real problem is to find a method of eliminating it where high-speed Kaplan and propeller runners are used. This is necessary because cavitation very materially reduces efficiency.

Probably the best-equipped cavitation laboratories in the

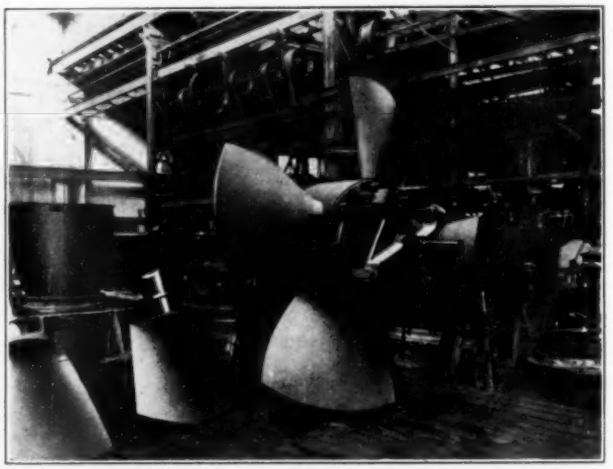


Fig. 2 Kaplan Turbine Runner, by J. M. Voith, Heidenheim, Germany
(Mechanism is shown for moving the blades of the runner while it is in motion.)

world today are those at Kristinehamn and the University of Stockholm, Sweden; J. M. Voith's in Heidenheim, Germany, and the Escher Wyss laboratory now under construction at Zurich, Switzerland.

Two European Turbines

There are two types of turbines that are distinctly European. Not only have they been developed in European laboratories,

but as yet they have not been used in America. They are the Kaplan and the Banki turbines.

The Kaplan turbine, invented by Professor Kaplan, of Brunn, about 1910, is a high-specific-speed propeller type of turbine, the propeller blades of which may be adjusted externally by means of a servomotor actuated by the governor while the turbine is in motion (Fig. 2). The Kaplan turbine has been used for heads up to 56 ft., as at the Wiesenberg plant in Czechoslovakia, and Mr. Elov Englessan, chief designer for Verkstaden, Kristinehamn, Sweden, is the authority for the statement that "50 feet can be regarded as a safe upper limit for the present, but for a head of this magnitude both the specific speed and the turbine has been written by Prof. Viktor Kaplan in "Die Entwicklung des Kaplan-Laufrades," 1927–1928, Wasserkraft Jahrbuch, pp. 414–424. Here it is shown that the Kaplan turbine was conceived as a propeller turbine, but gradually evolved into a propeller turbine with movable blades.

The Kaplan turbine has the characteristics of propeller-type turbines, and in addition it has the advantage that its blades can be moved so that it at all times operates with its blades in a position of maximum efficiency, and therefore it gives a much higher efficiency at partial loads and overloads than does either the Francis or the simple propeller-type turbine (Figs. 3 and 4). This has been clearly shown by a number of European experi-

TABLE 2 KAPLAN TURBINE INSTALLATIONS IN EUROPE

Plant and location	Number of units	Date in operation	Maximum head, ft.	Q sec-ft.	n r.p.m.	N hp.	Total hp.
Siebenbrum,[1] Steyrermuhl, Austria	3	1923	20.5	583	250	1100	3300
Auerhammer, Saxony, Germany	2	1925	15.7	71	500	102	204
Gratwein, Vienna, Austria	2	1925	27.9	1380	167	3540	7080
Unterturkheim, Stuttgart, Germany	1	1925	13.1	869	125	1065	1065
(1) Klosterbuch, Saxony, Germany	1	1926	13.0	1000	125	1185	1185
(2) Klosterbuch, Saxony, Germany	1	1925	14.1	435	187	565	565
Rotteln, Haagen, Baden, Germany	1	1925	13.1	265	250	320	320
City Electric Plant, Nurtingen, Wurtemberg, Germany	1	1926	10.3	282	187	270	270
Donaustetten, Ulm, Austria	2	1926	26.2	780	214	1885	3770
Liebenstein, Kennelbach, Austria	1	1926	7.8	407	187	295	295
Sofienhutte, Vienna, Austria	1	1926	15.1	560	167	780	780
Lorrach, Baden, Germany	1	1926	11.8	277	215	303	303
Schopfheim, Baden, Germany	1	1926	10.7	212	250	210	210
Ladenburg Nechar Corp., Stuttgart, Germany	1	1927	14.8	374	214	509	509
Gotzinger Achen Power Plant, Wagingersee, Munich, Germany	1	1927	22.6	229	375	475	475
Lilla Edet, Sweden	1	1925	21.3	5300	62	11200	11200
Boberullersdorf, Hirschberg, Germany	1	1927	...	433	300	2000	2000
Wasserkraft bauamt, Waldenburg, Germany	1	1924	17.0	...	...	1135	1135
Wasserkraft bauamt, Waldenburg, Germany	1	1924	17.0	...	...	580	580
Power Station, Lottin, Germany	1	1924	12.0	...	...	229	229
Power Station, Meseritz, Germany	1	1924	21.0	...	...	872	872
Electric Works, Canton Zurich, Switzerland	1	1925	24.0	...	...	376	376
Swabian Electric Works, Biberach, Munderkingen, Germany	1	1925	7.0	...	...	587	587
Siemens Schuckert Works, Berlin, Germany	1	1925	8.0	...	...	79	79
Neckar-Baudirektion, Stuttgart, Ladenburg, Germany	1	1925	27.0	...	...	3120	3120
Electric Works, Rheinbeck, Germany	1	1925	12.0	...	...	123	123
Cotton Mills, Arlen, Aach, Germany	1	1926	32.0	...	...	555	555
Power Station, Birnbaum, Meseritz, Germany	1	1926	20.0	...	...	612	612
Ruhrverband, Hengstey, Germany	3	1926	16.0	...	...	1400	4200
Siemens Schuckert, Gustrow, Germany	1	1926	7.0	...	...	80	80
Otto Sorge, Chemnitz, Germany	1	1926	12.0	...	...	103	103
Spinning Mills, Steinen, Germany	2	1926	17.0	...	...	220	440
Spinning Mills, Maulburg, Germany	1	1920	20.0	...	...	384	384
Siemens Schuckert, Niederhausen, Germany	1	1926	19.0	...	...	1760	1760
Siemens Schuckert, Niederhausen, Germany	1	1928	20.0	...	...	760	760
F. Maier Karlsruhe, Mylau, Germany	1	1926	42.0	...	...	264	264
Wolle Company, Aue, Germany	1	1927	8.0	...	...	210	210
March Electric Works, Guben, Germany	1	1927	13.0	...	...	472	472
G. Sigrist, Karlsruhe, Grotzingen	1	1927	10.0	...	...	74	74
Paper Mills, Koil, Esthonia	1	1927	11.0	...	...	360	360
Neckar A. G., Stuttgart, Horkheim, Germany	2	1927	24.0	...	...	3030	6060
Plants that have ordered Kaplan turbines, but which may not yet be in operation							
Griessen Plant, Niederlausitzer Ueberlandzential, Kottbus, Germany	1	1926	10.1	918	107	870	870
Boberullersdorf, Hirschberg, Germany	1	1926	49.3	953	214	4400	4400
Ohl-Grunscheid Plant, Dieringhausen, Rheinland, Germany	2	1927	24.5	300	300	675	1350
Lenhausen Plant, Essen a. d., Ruhr, Germany	2	1927	21.9	558	214	1160	2320
Schroedersch Paper Factory, Golzern, Germany	2	1927	10.5	1095	75	1100	2200
City Electric Works, Allenstein	2	1927	19.7	283	300	510	1020
Jastrow Plant Neberlandzentrale, Pommern, Stettin	2	1927	23.9	520	250	1190	2380
Power Plant Flederborn, Stettin, Germany	2	1927	32.1	520	300	1600	3200
Produccion y Suministro de Electricidad, Manlleu, Spain	1	1927	20.6	228	375	447	447
Neckar-A. G., Stuttgart, Anlage Cannstatt	2	1927	18.3	1000	150	1730	3460
Kraftubertragungswerke Rheinfelden, Rheinfelden, Baden	2	1927	19.6	1360	107	2550	5100
Power Plant Ryburg-Schworstadt, Baden, Germany	2	1928	37.7	11000	75	39700	79400
Hogforssen, Sweden	1	1928	33.0		214	4100	4100

[1] See *Bulletin* No. 532a, J. M. Voith, Heidenheim, Germany.

suction height must be low in order to avoid the danger of cavitation." (See *Canadian Engineer*, January 5 and 12, 1926.)

There are about 150 Kaplan turbines manufactured by European firms in actual use, having an installed capacity of over 300,000 hp. That American engineers traveling in foreign countries may know of the plants using Kaplan turbines and perhaps visit them and obtain first-hand information of their successful operation, a list of Kaplan turbine plants is given in Table 2.

This is by no means a complete list of Kaplan turbines in use in Europe, and it is only a very partial list of those made by European manufacturers for it omits most of those manufactured by Verkstaden, Kristinehamn, Sweden, and the Riva Construction Company of Milan, Italy.

A recent account of the historical development of the Kaplan menters and writers. The experience of the Escher Wyss Company is shown in Fig. 5; that of the Riva Construction Company, Milan, Italy, in Fig. 6; that of Voith in Fig. 7; and that of Kristinehamn, Verkstaden, in Fig. 8. It will be noticed that the experiences of these companies are almost identical. The flat efficiency curve of the Kaplan turbine means economy of water. Thus the ideal field for the Kaplan turbine is for low heads below 50 ft. where economy of water is essential. It will also be noticed that another advantage of the Kaplan turbine is that for the same efficiency it will carry about 50 per cent overload while the propeller type of turbine will carry only about 5 to 10 per cent and the Francis from 10 to 15 per cent. The fewer the number of units the more important is the Kaplan turbine in its field. Mr. Englessan, in "Kaplan and Propeller Turbines Com-

pared" (*Canadian Engineer*, January 5 and 12, 1926), has given an excellent example of the water savings that may be obtained by the Kaplan turbine.

On account of its slower speed there is seldom any advantage in installing a Francis turbine for low heads because of the increased cost of the generator. Therefore the Kaplan's real competitor is the propeller-type turbine. Combinations of propeller and Kaplan turbines frequently are used in Europe, because, on account of their higher partial-load efficiency, the number of units can be reduced in many cases by the use of Kaplan turbines.

The size of the Kaplan turbine is limited by the size to which

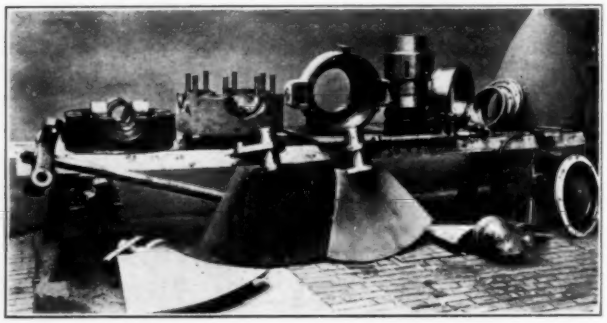


FIG. 3 KNOCKDOWN VIEW OF KAPLAN TURBINE RUNNER
(Showing small number of parts.)

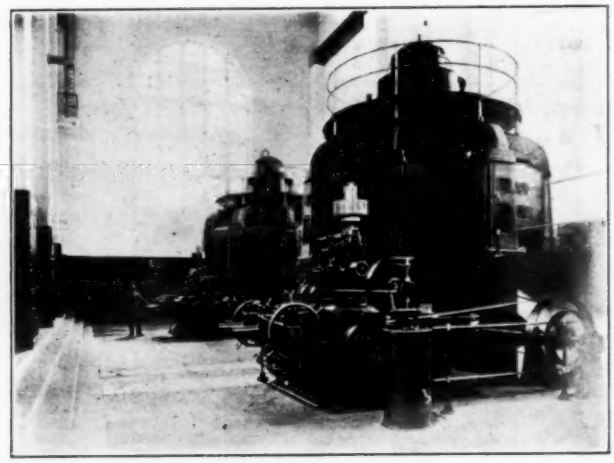


FIG. 4 POWER PLANT AT GRATWEIN, WITH TWO KAPLAN TURBINES BY J. M. VOITH
(These turbines each deliver 3540 hp. under a head of 27.9 ft. with 1380 cu. ft. per sec. of water, and operate at 167 r.p.m.)

the hub can be made, and this in turn seems to be limited by the size of the railway car for transporting it, which limits the hub to about 26 ft. diameter. As yet no Kaplan turbine has been constructed of that size.

The chief disadvantages of the Kaplan turbines are, first, their initial cost, which varies with the size of the unit from 15 to 75 per cent more than a propeller-type turbine; second, the fact that they are more complicated and therefore susceptible to more mechanical troubles; third, that they consume more oil than do propeller turbines. The author found in Europe that these objections were theoretical rather than practical and that the Kaplan turbines of today are giving satisfactory service. The savings in water and generator costs usually offset the increase in Kaplan turbine costs. The Kaplan turbine has largely passed the experimental stage; its mechanical operation seems from all reports to be as satisfactory as that of any other type of turbine. The leakage or loss of oil has not been found to be excessive or expensive. Therefore, considering these facts, it may be expected that the Kaplan turbine will soon make its initial appearance in American practice.

THE BANKI TURBINE

The second European turbine that has quite novel features is the Banki turbine (Figs. 9, 10, and 11). This turbine has not received the consideration it deserves, primarily because it is not adaptable to extremely large units. Then no doubt the fact that it is of Hungarian origin has delayed its development, because the Hungarians have not been as successful as the Germans and Americans in commercializing their ideas. There are, however, today, according to Mr. Banki, of Budapest, the son of the inventor of the turbine, more than a thousand of these turbines in actual use.

BANKI TURBINE THEORY

The theory of the Banki turbine was developed by its inventor, Prof. Donat Banki, at the Hungarian Technical University, where Professor Banki taught for many years prior to his death. (See "Neue Wasserturbine," von Donat Banki, Druck des Franklin Verein.)

The Banki turbine, as shown in Fig. 9, has a head acting on the point A of h_1, and the velocity coefficient of the nozzle is φ.

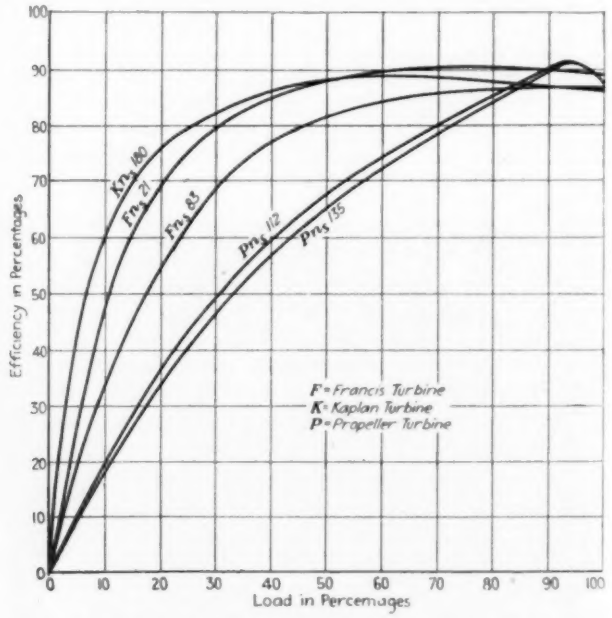


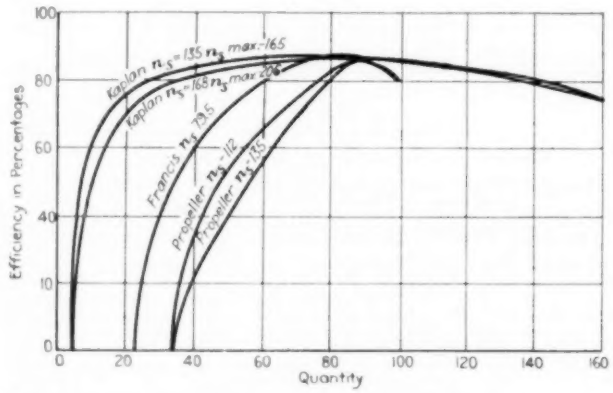
FIG. 5 COMPARATIVE EFFICIENCY CURVES OF PROPELLER, FRANCIS, AND KAPLAN TURBINES
(As obtained by Escher Wyss Company, Zurich, Switzerland.)

The spouting velocity, or the absolute velocity with which the water strikes the turbine, will be

$$c_1 = \varphi \sqrt{2gh_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

If the angle of the bucket at A is α_1 with the tangent at that point and the velocity of the runner is u_1, by the usual triangle law the relative velocity will be w_1, making an angle β with the tangent at A. If it is assumed that the water flows through the runner with undiminished velocity, it will leave B when it has reached a position b with a relative velocity w'_2, and a direction β'_2 with the tangent at B. The velocity of the runner at

this point has been reduced (because of the shorter radius) to u_2, and the absolute velocity is found by the same triangular relationship to be c'_2. The absolute path of the water through the bucket is A-b, and the water leaving point b travels in a stream line, again entering the runner at point C. The path b-C when viewed in the small glass-encased model in the Deutscher Museum at Munich, Germany, appears to be a straight line. If this is assumed, the water will follow a path b-C and enter the bucket of the runner again at C with a velocity $c'_1 = c'_2$ and in a direction



F.G. 6 Comparative Efficiency Curves of Propeller, Francis, and Kaplan Turbines

(As obtained by the Riva Construction Company, Milan, Italy.)

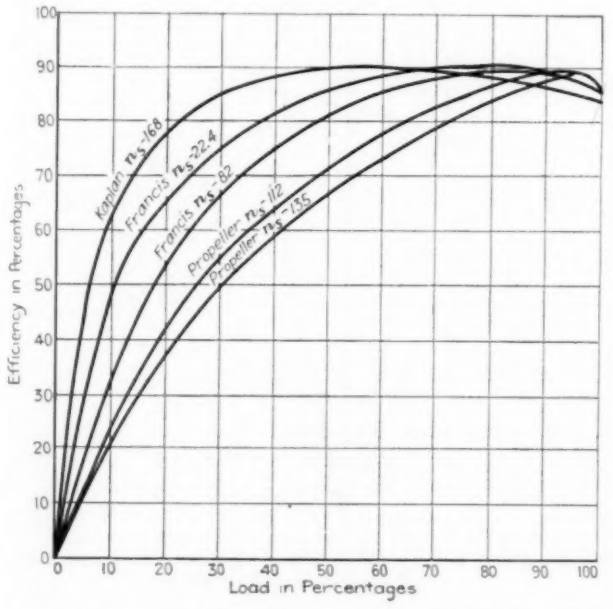


Fig. 7 Comparative Efficiency Curves of Propeller, Francis, and Kaplan Turbines

(As obtained by J. M. Voith Company, Heidenheim, Germany.)

such that $\alpha'_1 = \alpha'_2$; and since $\beta'_1 = \beta'_2$ and $\beta_2 = \beta_1$ (because of the construction of buckets), the construction of the velocity triangle gives a final absolute velocity of discharge of c_2 in such a direction as to form with the tangent at point D the angle α_2.

If this theory were correct, the stream line through the turbine would be as shown in Fig. 10-A. Actually, however, visual experiment shows that the water behaves approximately as shown in Fig. 10-B.

If L = power in foot-pounds per second
Q = quantity in cubic feet per second
γ = weight of a cubic foot of water (62.4 lb.)
g = gravitational constant (32.2)

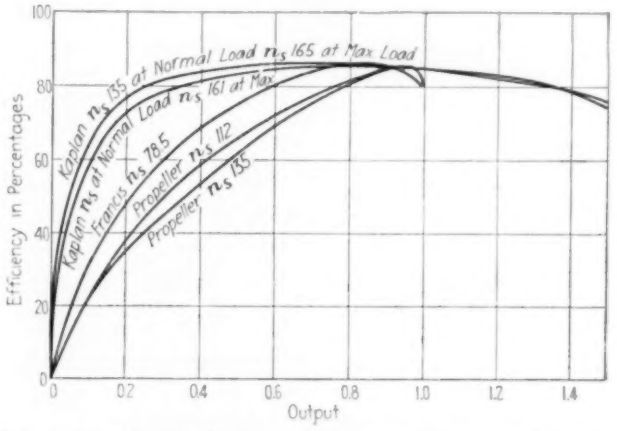


Fig. 8 Comparative Efficiency Curves of Propeller, Francis, and Kaplan Turbines

(As obtained by Kristinehamn, Verkstaden, Sweden.)

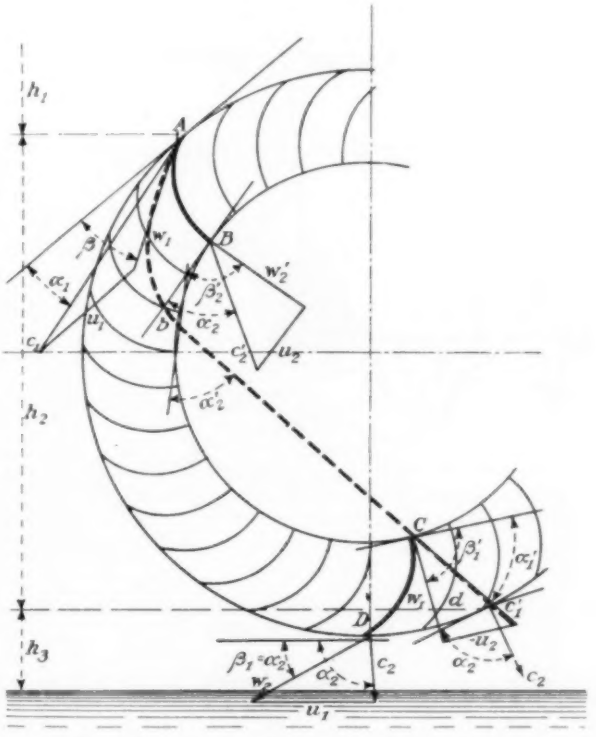


Fig. 9 Diagrammatic Sketch of a Banki Turbine

From the foregoing relationships it can be seen that

$$L = \frac{Q\gamma}{g}(c_1 \cos \alpha_1 + c_2 \cos \alpha_2)u_1 \quad \ldots\ldots\ldots\ldots [2]$$

and from the discharge triangle this same thing may be expressed in the form

$$c_2 \cos \alpha_2 = w_2 \cos \beta_2 - u_1 \quad \ldots\ldots\ldots\ldots [3]$$

and by use of the velocity coefficients ψ and φ_1, w_2 may be expressed

$$w_2 = \psi w_1 + \varphi' \sqrt{2gh_2} \quad \ldots\ldots [4]$$

and from the entrance triangle relationship

$$w_1 = \frac{c_1 \cos \alpha_1 - u_1}{\cos \beta_1} \quad \ldots\ldots [5]$$

If these are substituted in Equation [2] we have

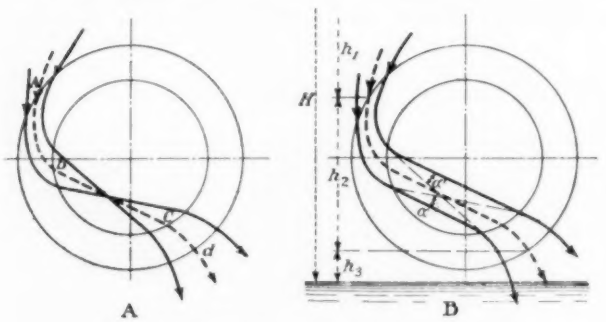


FIG. 10 THE CROSS-FLOW BANKI TURBINE
(A—Theoretical way the water travels through the turbine parts. B—Actual way the water travels.)

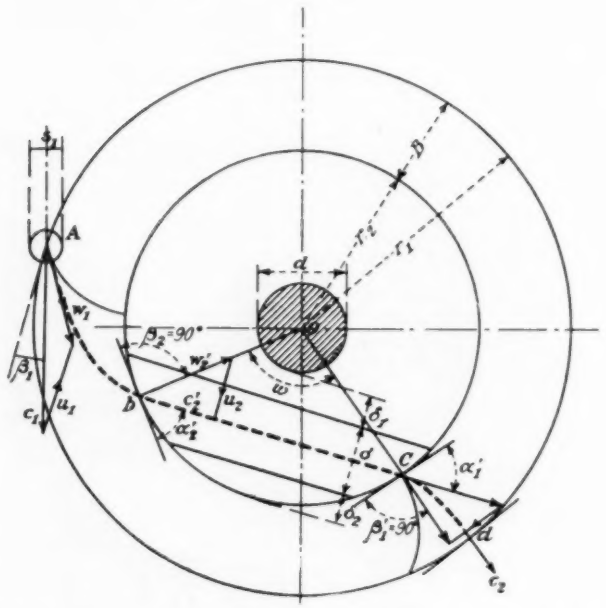


FIG. 11 BANKI TURBINE; GEOMETRICAL FIGURE FOR DETERMINING DESIGN OF RUNNER

$$L = \frac{Q\gamma}{g}\left[(c_1 \cos \alpha_1 - u_1)\left(1 + \psi \frac{\cos \beta_2}{\cos \beta_1}\right) + \varphi' \sqrt{2gh_2} \cos \beta_2\right] u_1 \quad \ldots\ldots [6]$$

The total power input is $Q\gamma h_1$, hence the efficiency would be

$$E = \frac{L}{Q\gamma h_1} = \frac{L}{Q\gamma \dfrac{c_1^2}{\varphi'^2 2g}} = 2\varphi^2 \left(1 + \psi \frac{\cos \beta_2}{\cos \beta_1}\right)\left(\cos \alpha_1 - \frac{u_1}{c_1}\right)\frac{u_1}{c_1} \quad \ldots\ldots [7]$$

For the case where $\beta_2 = \beta_1$

$$E = 2\varphi^2 (1 + \psi)\left(\cos \alpha_1 - \frac{u_1}{c_1}\right)\frac{u_1}{c_1} \quad \ldots\ldots [8]$$

To determine the point of maximum efficiency differentiate the efficiency with respect to the ratio $\frac{u_1}{c_1}$, and placing it equal to zero we obtain

$$\frac{u_1}{c_1} = \frac{\cos \alpha_1}{2} \quad \ldots\ldots [9]$$

Substituting this in Equation [8] gives a maximum efficiency

$$E \text{ (max)} = \frac{1}{2}\varphi^2 (1 + \psi) \cos^2 \alpha_1 \quad \ldots\ldots [10]$$

Professor Banki also showed that the angle should be of the order of 16 deg. and the angle β should be around 90 deg.

The radial length of the vanes B (see Fig. 12) was found to be best under the conditions

$$B = 0.17\, D_1 \quad \ldots\ldots [11]$$

where D_1 is the external diameter of the runner and

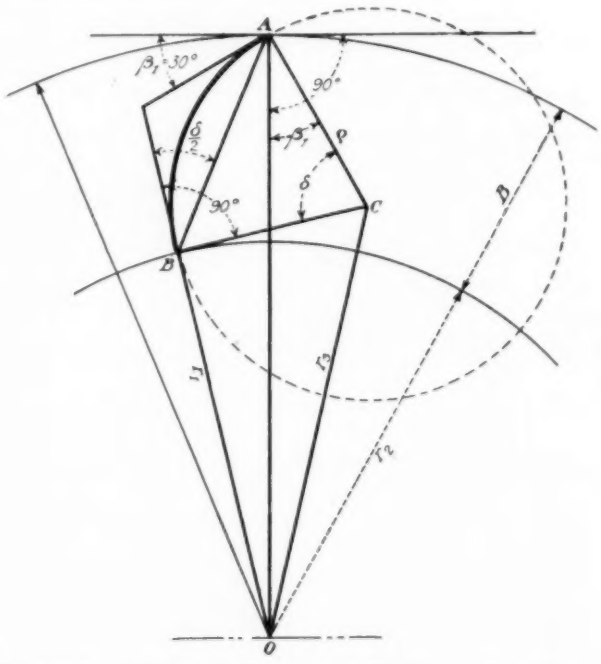


FIG. 12 BANKI TURBINE; GEOMETRICAL FIGURE FOR DETERMINING DESIGN OF RUNNER PARTS

$$D_1 = \frac{72.2\sqrt{H}}{n} \quad \ldots\ldots [12]$$

where D_1 and H_1 the head, are in feet and n is r.p.m. The design is a very simple matter because the vanes are nothing more complex than the arc of a circle (see Fig. 12) whose radius is of the order

$$r_3 = 0.326 r_2 \quad \ldots\ldots [13]$$

The number of vanes is determined by the thickness of the entering stream, which in turn determines the thickness of the water stream passing from the first set of vanes to the second (Fig. 11).

Thus σ must not be larger than

$$\sigma = 1.89\, s_1 \quad [14]$$

and δ_2 must lie between the range

$$\delta_2 = 0.0606\, D_1 \text{ and } 0.0369\, D_1 \quad [15]$$

The radial length of the Banki turbine lies between the limits

$$b = 0.263\, n\, \frac{Q}{H} \text{ and } 0.197\, n\, \frac{Q}{H} \quad [16]$$

where b and H are in feet and Q in cubic feet per second.

Thus it is seen that the design of a Banki turbine is extremely

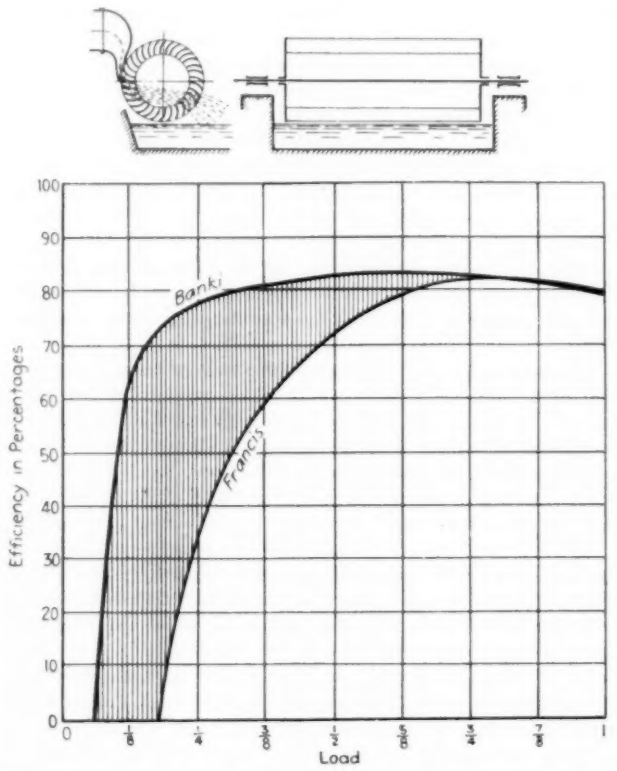


FIG. 13 COMPARATIVE EFFICIENCY CURVES OF BANKI AND FRANCIS TURBINES

simple as are also its construction and manufacture. The chief advantages claimed for this turbine are:

First, its high efficiency; for even very small units the efficiency is over 85 per cent (see Fig. 13).

Second, for heads around 17 ft., the Banki turbine will develop a speed of 127 r.p.m., whereas an overshot wheel under the same head would develop but 17 r.p.m.

Third, for a head of about 3 ft. and 21 cu. ft. per sec., a paddle wheel would not only have a low efficiency and be very large (about 21 ft. diameter), but would run at 1.36 r.p.m., while the Banki turbine under the same conditions would be 3 ft. diameter and operate at about 41 r.p.m.

Fourth, an undershot wheel operating under a head of 5 ft. and about 35 cu. ft. per sec. would run 9 r.p.m., while the Banki turbine would be smaller, would have a higher efficiency, and would run 42 r.p.m.

Fifth, in comparison with the Pelton or impulse wheel, using two nozzles, Professor Banki has shown that the Banki turbine would have a specific speed of twice the impulse wheel with two nozzles.

Sixth, the Banki turbine fills the specific speed gap between the Pelton impulse wheel and the Francis turbine; that is, it has a specific speed of from 6 to 20 English units.

Professor Banki made no comparisons with his turbine and the propeller type, because at the time his invention was made, the propeller turbine was not used and very little known. How-

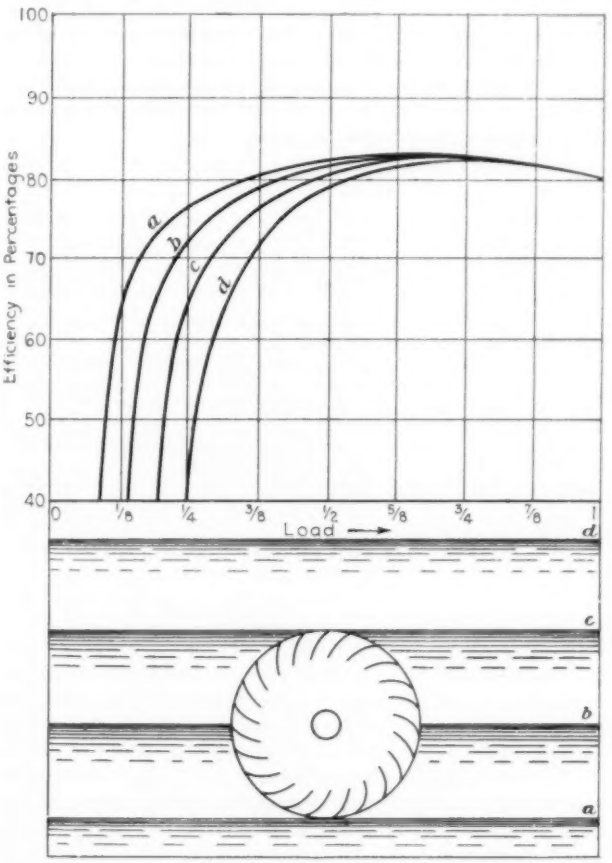


FIG. 14 SHOWING THE EFFECT OF SUBMERGENCE UPON THE BANKI TURBINE

ever, if we compare the Banki turbine mentioned with the propeller unit at Allenstein, which operates under a head of 18 ft. and a quantity of 350 cu. ft. per sec., delivering 608 hp. at 187 r.p.m., we find that its specific speed is

$$\frac{187 \times \sqrt{608}}{18\ (18)^{1/4}} = \frac{187 \times 24.5}{18 \times 2.06} = 125$$

while the Banki turbine would be lower, or about 70.

Thus it is seen that the Banki turbine does not compare so favorably with the modern propeller or Kaplan turbine from the viewpoint of speed. However, in simplicity of construction and cheapness for very small units, it is undoubtedly superior, and as shown in Fig. 13 the Banki turbine has a much higher part-load efficiency than either the Francis or the propeller type of turbine.

Fig. 14 shows that the Banki turbine is not materially affected by water submergence. Its speed regulation problems are almost the same as those of the impulse wheel, which can be and are satisfactorily solved.

The chief field for the Banki turbine seems to be where most of them are now used; namely, in small isolated farms or resort plants.

More attention could be given to this field in America although almost all American manufacturers have stock units which they sell to meet this demand. However, the cost of these units is frequently beyond the means of the farmer, and consequently there are today many gas engines in use in isolated mountain farms where small hydraulic power plants could be installed.

These European hydraulic-turbine researches are given, not because they are superior to American methods of solving our hydraulic problems, but because they are different and therefore may offer suggestions to engineers for study and research, the only road to engineering progress.

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Discussion

George A. Orrok.[2] A paper of this kind is most welcome, combining, as it does, much information on hydraulic research as well as statistics on two very interesting types of water wheels which have not been exploited to any extent in America. While the Kaplan design has been known to hydraulic engineers for some time, the Banki design is comparatively new and fits in a distinctive place where formerly the free deviation turbine was the only occupant. The Kaplan turbine is at the further end of the scheme, and with its neighbors of the propeller type has greatly increased the limits of permissible specific speed. The extensive development of low-head plants, particularly in the larger sizes, has been made practical by the use of the propeller and Kaplan designs, and heads as low as 7 to 8 ft. under proper conditions may now be utilized with good economic results.

We might be inclined to differ from Mr. Van Leer in his characterization of American contributions as the "result of a few months' thought and experimentation" when we remember that the Holyoke testing flume has been in continuous existence for more than 70 years and that its results are of an "acknowledged excellence." It is of course true that the stock wheel, built in quantity and sold by the pound, has had its greatest development in this country, but I think it is also true that our designers, with the aid of Holyoke and their own laboratories, have produced wheels of at least equal excellence when compared with the product of any European manufacturer. And I believe it is also true that our earlier hydraulic development would not have taken place were it not for the stock wheel. This, however, is ancient history.

There are many hydraulic problems still before us today which must be solved, and, as Mr. Van Leer intimates, the hydraulic laboratory and the research engineer must find the solution for us if we would keep in the vanguard of progress.

[2] Consulting Engineer, New York, N. Y. Mem. A.S.M.E.

Byron E. White.[3] It is perhaps unfortunate that Professor Van Leer's paper, taken by itself, can conceivably lead to the impression that hydraulic laboratories are a peculiarly European institution, thus leaving the inference that we know little or nothing about the theory or practice of model experiments and the conduct of hydraulic laboratories in the United States.

From personal observation it is possible to state that there is at least one technical college laboratory in the United States which rather fully measures up to the standards of the European laboratories so well described by Mr. Van Leer. The Worcester Polytechnic Institute, at Chaffins, near Worcester, Mass., has a very complete and well-equipped laboratory, operated under the direction of Prof. C. M. Allen. At this laboratory there have been carried out many experiments on a large variety of hydraulic structures and equipment which have resulted in great improvements in design and operation as well as very substantial savings in the cost of construction.

The experiments on the Conowingo spillway and the inclined apron at the foot of it were conducted there. The experience with the full-sized dam and apron have confirmed the results of those experiments. Experiments have also recently been made on models of the upper and lower Fifteen Mile Falls developments of the New England Power Company on the Connecticut River, which were studies with the view to devising proper contours of the spillway discharge so as to prevent erosion of the sand and gravel river bank below the dam on the opposite side.

This laboratory also worked out the proper design and location of a baffle pier which was used in the spillway channel around the end of the hydraulic spill dam at the Sherman development of the New England Power Company and which successfully dissipated the energy of the water so that it did not erode the river bank opposite the discharge during periods of extreme high water.

The Alabama Power Company also has a very complete hydraulic laboratory operated under the direction of Mr. I. A. Winter, hydraulic engineer, in which several of the notable structures and devices of that company were tested, revised, and proved before construction.

At the October meeting in Philadelphia of the Hydraulic Power Committee of the National Electric Light Association, Mr. Winter showed a series of lantern slides and moving pictures of model tests and of the operation of the full-sized units corresponding thereto at a time of almost maximum flood, in which the correspondence of the tests and the actual results was extremely close.

Several other power companies, as well as colleges and universities and technical schools, maintain more or less completely equipped laboratories for the study of particular or general problems. These laboratories are really linear descendants of the early experiments of Mr. James B. Francis, Lowell, Mass., and of the still later Holyoke testing flume, at Holyoke, Mass., which was conducted for so many years by that distinguished hydraulician, Mr. Clemens Herschel.

It can be safely said that anyone desiring to have experiments on models carried out can entrust them to one or more of the several excellent hydraulic laboratories in the United States with the assurance that they will be carried out under the most competent supervision and that the results may be used with the greatest confidence.

I would like also to refer to the units at the Trenton Falls, N. Y., power house. With my colleague, Mr. William Monroe White, and Mr. H. A. Hageman, who was resident engineer during the construction of the plant from 1900 to 1902, it was

[3] General Engineer, Utica Gas and Electric Co., Utica, N. Y. Mem. A.S.M.E.

my privilege in November, 1928, to examine, both internally and externally, the first of the four 1700-horsepower units, which were the first of Mr. White's design, and to note that after 26 years of operation the original runner is still in place and has pitted but little.

As this is a cylinder-gate turbine, designed in 1902, its efficiency is naturally much below that of the turbines of this date. This and the three other machines are, however, used at times of high water, sometimes several weeks each year.

It is interesting to note the comparison as to pitting between these three turbines and three other machines installed in the same station about 10 years ago. The three latter have required the replacement of runners several times on account of pitting, and in every case we have found it necessary to weld the pitted areas at least once a year. I am wondering if the practice of running these new machines at part gate for extensive periods of time does not have something to do with the prevalence of and the extent of pitting. The old machines of 1902 had their point of best efficiency at full gate and were customarily operated at that point, and it is barely possible that it is for that reason that pitting and erosion of their runners have not been more pronounced.

CLEMENS HERSCHEL.[4] The last speaker was especially clear on the benefits of models in solving hydraulic problems. Of course, that is an old story in the United States. The predominant position of turbine wheels in the United States is largely due to experiments on models made in the Holyoke testing flume, which has been there a great many years—ever since the eighties of the last century. But there is a recent development in Europe, of which I have been informed lately, which I think will be of interest, and that is the work and construction, present and future, of what I will call the National Hydraulic Observatory of Germany, rather than attempt to translate the long name given it in Germany.

The point involved is that here is an investment already of about $350,000; and remember, this is in poor Germany, and $350,000 over in Germany is certainly equivalent to six or seven hundred thousand dollars here; and they purpose to bring it up to half a million dollars. The system to be followed is to experiment in full size; though they also can test the principle of similitudes, and then try to prove what is the law to pass from models to full size. You can see from this statement and from the amount of money involved that this is no small affair.

They have formed there a company, and they call it a stock company. The German Government, the Bavarian Government, and the principal power plants of Germany are stockholders, but they do not expect any dividends in the shape of money. They help to operate the plant by an annual contribution, and for dividends they only expect information from the results of tests that are being made.

That is something that it seems to me is new and is entirely applicable to this country. There is not the slightest doubt that the results of the work done there will be of the greatest interest. The man in charge, I am pleased to say, is a hydraulic engineer, not a philosopher or, to use a much-abused word, a scientist.

S. LOGAN KERR.[5] It is very interesting to have presented to American engineers a general summary of some of the recent developments in hydraulic-turbine design in Europe. The writer had the privilege of visiting a number of the plants described in Mr. Van Leer's paper and also of visiting laboratories of four of the leading water-wheel manufacturers in Europe.

[4] Consulting Engineer, New York, N. Y.

[5] Research Engineer, I. P. Morris Corp. and De La Vergne Mach. Co., Philadelphia, Pa. Assoc-Mem. A.S.M.E.

The European testing laboratories constructed by the manufacturing companies do not differ materially from those which have been built by American turbine manufacturers. However, one additional step which has been taken in Europe has been the development of the cavitation laboratory for experimentally studying the effect of draft head and its reaction upon the performance of the turbine.

There are two types of laboratories. One of these employs the turbine with a very long draft tube installed in a specially constructed tank with provision for adjusting either the head water or tail water, or both. By this means the performance of the units under different heads can be studied, and the performance under constant head but with different draft heads can also be determined. The other type of cavitation laboratory employs two pressure chambers connected through a centrifugal pump arranged in such a manner that the supply to the turbine is drawn from one tank, discharged into another, and then pumped from the second tank back to the first. The total pressure can be controlled and the suction on the turbine can be varied by controlling the pressures in the discharge tank, and the head on the turbine can be increased by varying the pressures in the two tanks in such a manner that the desired combination is secured.

It is interesting to note that the European manufacturers have experimentally confirmed the theoretic analysis developed by Mr. L. F. Moody dealing with the permissible draft head on the turbine.

The European engineers draw a very nice distinction between the various types of so-called propeller turbines. They define a propeller turbine as any water wheel of the general form of the marine propeller operating without a runner band. The Kaplan turbine, however, is designated as a propeller-type turbine in which the blades are adjusted automatically from the governor.

There is no doubt of the commercial success of both the propeller and Kaplan turbines in Europe. In this country, however, there has not been a demand for the adjustable-blade type of runner due to the great difference between the general plan of operation of our public utilities. In small low-head plants where the adjustable-blade propeller-type turbine would find its greatest application, it is possible to operate these plants for peak-load power and to divide them into a sufficiently large number of units so that they can be operated at approximately the maximum efficiency point.

When the demand comes from the American public utilities for the adjustable-blade type of units, together with their willingness to meet the increased cost of this type of turbine, American manufacturers will be ready and willing to meet the demand. Just now, however, the fixed-blade type of unit or turbines having manually adjusted blades meet the requirements.

GEORGE A. JESSOP.[6] The author offers a very valuable contribution to the available information concerning hydraulic-turbine practice. European developments are not generally followed as closely in this country as they should be to take advantage of the research continuously going on abroad. The data concerning the high-speed Kaplan turbine are of particular interest at the present time. In Europe it is the exception to use a high-specific-speed propeller-type turbine with fixed blades, almost all installations having the automatic adjustable-blade feature. In this way full use is made of the advantages of the Kaplan turbine. A study of this paper together with the references cited will show these advantages to be three in number:

1 High efficiency under normal operation

[6] Chief Engineer, S. Morgan Smith Co., York, Pa.

2 The same high efficiency under part load (part gate) where high efficiency is desired to conserve water and to obtain the maximum output from the available limited supply of water

3 Excess capacity under flood-water conditions.

That the engineers on this side of the Atlantic are aware of the economies to be obtained by the use of the governor-controlled adjustable-blade turbine is evident from the fact that a number of such installations are now being made and will very soon be in operation in this country and Canada.

Dr. Kaplan has had ten patents issued in this country which fully cover his inventions of the fixed- and adjustable-blade axial-flow turbines.

Exclusive rights under the United States patents have been granted to the S. Morgan Smith Company, who also have the rights for Canada.

Author's Closure

Sincere thanks are extended to Messrs. George A. Orrok, Byron E. White, Clemens Herschel, S. Logan Kerr, and others who contributed much to the interest evinced in the paper by their intelligent and constructive discussions.

In view of the fact that some of the gentlemen who discussed the paper voiced the sentiment that it might lead to the conclusion that European engineers and methods were superior to those of America, the author wishes to call attention to the closing paragraph of this paper which states:

"These European hydraulic-turbine researches are given, not because they are superior to American methods of solving our hydraulic problems, but because they are different and therefore may offer suggestions to engineers for study and research, the only road to engineering progress."

Also, he wishes to again emphasize that this paper deals with "Some Interesting European Hydraulic Turbine Research." Under such a title it did not seem appropriate to laud American hydraulic engineers whose works are known to every college boy studying hydraulics. Such names as Mills, Francis, Pelton, Worthington, Gibson, Herschel, Freeman, Moody, are secure so far as the history of our hydraulic arts is concerned.

Since this question has been brought up, it would perhaps be of interest to note that the author found that Messrs. Herschel, Freeman, Moody, contemporary American engineers, were exceedingly well known in Europe; in fact, there was not a hydraulic laboratory or European manufacturer visited in Europe by the author where at least one of these names was not known, and in every instance these gentlemen were spoken of in the highest terms of praise and respect.

Perhaps the writer has been in charge of a hydraulic laboratory so long and has become so inseparably bound to the movement to secure a national hydraulic research laboratory that every paper he writes is looked upon as propaganda to that end. However, to accomplish this purpose, it is not necessary to deprecate any existing hydraulic laboratory. The excellent hydraulic laboratory at Worcester Polytechnic Institute and the one belonging to the Alabama Power Company are both described in "Hydraulic Laboratory Practice," a monumental hydraulic contribution, edited by John R. Freeman and published by the American Society of Mechanical Engineers. There are many experiments which may be carried on in these laboratories, and there are some for which they are not at present equipped. It is hoped and expected that they will continue to make valuable contributions to hydraulic engineering.

It was most gratifying to have the distinguished inventor of the venturi meter, Mr. Clemens Herschel, present at the meeting of the Hydraulic Division, and to hear him call attention in such an interesting manner to one of Germany's unique institutions. The organization to which he referred is Forschungsinstitut fur Wasserbau und Wasserkraft (the Research Institute for Hydraulic and Hydro-Electric Structures), near Munich, Germany, which is under the direction of Dr. Otto Kirschmer. This institute was described in the August, 1928, issue of *Mechanical Engineering*.

The excellent discussion by Mr. S. Logan Kerr needs no comment; it speaks for itself, and the author concurs in the points set forth.

Flow in Pipes

By MICHAEL D. AISENSTEIN,[1] OAKLAND, CALIF.

THE purpose of this paper is to present general formulas for the frictional resistance in straight circular pipes based on the experiments of different authorities and to show their application to the solution of different problems in connection with divided flow and accelerated streamline flow.

The following symbols and nomenclature are used:

t	=	time in seconds
v	=	velocity of flow in feet per second
L	=	pipe length in feet
h	=	head in feet
ν	=	kinematic viscosity in English units
μ	=	absolute viscosity in English units
T	=	viscosity in Saybolt seconds universal
w	=	weight in pounds per cubic foot
ρ	=	density in pounds per cubic foot
a	=	area in square feet
D	=	diameter in inches
d	=	diameter in feet
p	=	pressure in pounds per square inch
A.P.I.	=	American Petroleum Institute gravity
g	=	gravitational constant 32.2
vd/ν	=	Reynolds' criterion
Q	=	cubic feet per second
G	=	gallons per minute
B	=	barrels per hour
M	=	mass
$s.g.$	=	specific gravity

There are two kinds of motion of a particle of liquid, one in which the particle has a steady streamline motion and the other where the motion is unsteady accompanied by eddies.

For circular straight pipes the type of flow is determined by the Reynolds' criterion vd/ν. For values of vd/ν less than 2000 a streamline flow takes place; for vd/ν between 2000 and 3000 there is a break in the flow, and for vd/ν from 3000 up a turbulent flow takes place.

In a streamline flow the streams of fluid are symmetrical on both sides of a symmetrical obstruction (Fig. 1*a*) as the stabilizing effect of viscosity is greater than the effect of inertia forces of fluid particles. Consequently the roughness of the walls of the conduit has little effect on the flow. The velocity is very small near the walls increasing gradually toward the center. (Fig. 2.)

The ratio of the average velocity to the maximum velocity is approximately equal to 0.5. The velocity radius equation is that of a parabola

$$v = \frac{p}{4\mu}(r_0^2 - r^2) \qquad [1]$$

The turbulent flow is the most common in engineering practice. In a turbulent flow the velocity is of such a value that the inertia forces dominate the stabilizing effect of the viscosity with resulting eddies. It is evident that the roughness of the walls of the conduit increases the formation of the eddies. The average velocity is about 85 per cent of the maximum velocity.

[1] Chief Engineer, Pump Division, United Iron Works. Jun. A.S.M.E.

Contributed by the Hydraulic Division and presented at the Annual Meeting, New York, N. Y., December 3 to 7, 1928, of The American Society of Mechanical Engineers.

Friction Formulas for Streamline and Turbulent Flow

Next let us derive the general formulas for the friction drop in circular straight pipes for both types of flow.

It was found experimentally that the resistance to flow in a circular pipe is a function of some power of the diameter d in feet, length of the pipe L in feet, mean velocity of the flow v in feet per second, absolute viscosity μ in foot-poundal-second units, density ρ of the fluid in pounds per cubic foot, and depends on the surface condition of the pipe; therefore we may write,

$$p = Kd^x L^m v^n \mu^y \rho^z \qquad [2]$$

where p is the pressure in pounds per square foot at two points L feet apart along the pipe and K a dimensionless constant. The surface condition of the interior of the pipe does not enter

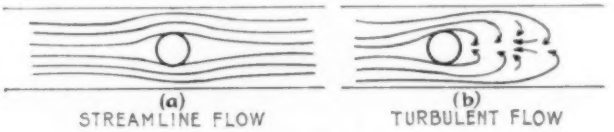


Fig. 1

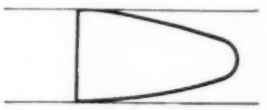


Fig. 2



Fig. 3

the equation and will be taken up later. Using the principle of dynamic similarity which is based on representation of force in terms of mass and acceleration, it is necessary that all terms in any equation having a physical significance have identical dimensions. Writing down for convenience the following table:

Quantity	Dimensions
Length	L
Time	T
Mass	M
Linear speed	LT^{-1}
Acceleration	LT^{-2}
Force	MLT^{-2}
Density	ML^{-3}
Viscosity	$ML^{-1}T^{-1}$
K	Constant, no dimensions.

Equation [2] may be rewritten

$$\frac{M}{LT^2} = L^x \left(\frac{M}{LT}\right)^y \left(\frac{M}{L^3}\right)^z \left(\frac{L}{T}\right)^n L^m \qquad [3]$$

Since the experiments indicate that the resistance to flow is, other things being equal, directly proportional to length, we can assume that m is equal to unity. Multiplying out Equation [3] and making $m = 1$

$$ML^{-1}T^{-2} = M^{z+y} L^{x-y-3z+n+1} T^{-y-n} \qquad [4]$$

According to the theory of homogeneity the dimensions on both sides of an equation in mechanics should be the same, therefore, to satisfy Equation [4],

$$x - y - 3z + n + 1 = -1$$

$$z + y = 1$$
$$-y - n = -2$$

solving these three equations for x, y, and z we have

$$x = n - 3$$
$$y = 2 - n$$
$$z = n - 1$$

substituting these values of x, y, and z in Equation [2] we have

$$p = KLd^{n-3}v^n\mu^{2-n}\rho^{n-1}$$

or because

$$p = h_f\rho g$$
$$h_f\rho g = KLd^{n-3}v^n\mu^{2-n}\rho^{n-1}$$

calling $K = \frac{K_0}{2}$ we have

$$h_f = K_0\frac{L}{d}\frac{v^2}{2g} \times \frac{v^{n-2}d^{n-2}}{\frac{\mu^{n-2}}{\rho^{n-2}}} \quad \text{.....[5]}$$

or in general

$$h_f = \frac{L}{d}\frac{v^2}{2g}\phi\left(\frac{vd}{\nu}\right) \quad \text{.....[6]}$$

for streamline flow, that is, when

$$\frac{vd}{\nu} < 2000$$

the coefficient $f = \phi\left(\frac{vd}{\nu}\right)$ was found by Poiseuille to vary with vd/ν as a rectangular hyperbola and to satisfy an equation (Fig. 4).

$$\phi\left(\frac{vd}{\nu}\right) = \frac{64}{\frac{vd}{\nu}} \quad \text{.....[7]}$$

Substituting in Equation [6] we obtain a formula for drop in head due to frictional resistance for streamline flow

$$h_f = \frac{64}{2g}\frac{L}{d^2}v\nu \quad \text{.....[I]}$$

Many experiments have been carried out to determine the friction coefficient in the turbulent region of flow. Since the roughness of the interior surface of the pipe has a great effect on the friction resistance in the turbulent type of flow, the results vary widely and many formulas have been devised to satisfy the different experimental results. From available data of different authorities the values of f and vd/ν were computed and plotted (Fig. 4).

An average curve was drawn by the author and the relation between f and vd/ν was expressed by an equation

$$f = \frac{0.167}{\left(\frac{vd}{\nu}\right)^{0.170}} \quad \text{.....[8]}$$

This equation will hold for so-called "commercially smooth" pipe. Substituting the value of f from Equation [8] in Equation [6] we obtain a formula for drop in head due to frictional resistance in the region of turbulent flow.

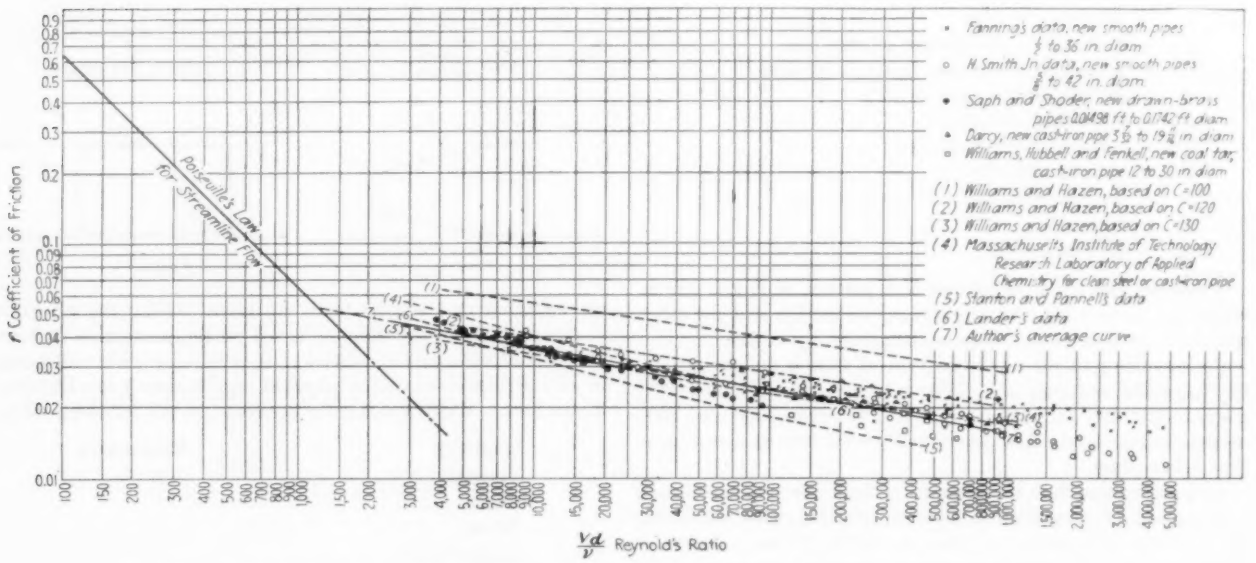


FIG. 4 RELATION BETWEEN f AND vd/ν BASED ON EXPERIMENTAL DATA OF AUTHORITIES NOTED

$$h_f = \frac{0.167}{\left(\frac{vd}{\nu}\right)^{0.170}}\frac{L}{d}\frac{v^2}{2g} \quad \text{.....[II]}$$

or

$$h_f = 0.167\,\nu^{0.170}\frac{L}{d^{1.170}}\frac{v^{1.830}}{2g} \quad \text{.....[III]}$$

in case of water having kinematic viscosity of 0.00001077 at 68 deg. fahr.

$$h_f = 0.0239\frac{L}{d^{1.170}}\frac{v^{1.830}}{2g} \quad \text{.....[IIIa]}$$

Inspecting Equations [I] and [II] we see that viscosity is a material factor in the friction drop in a region of streamline flow, and that it has a relatively small effect in a region of turbulent flow. There is an advantage in heating a viscous oil in the streamline region, which results in decrease of viscosity. In the streamline region the frictional resistance varies directly with

the viscosity and any change in viscosity will have a marked effect on frictional resistance. Since in turbulent flow the viscosity is raised to the 0.17 power, heating of the oil will not be very effective in so far as reduction of the frictional resistance is concerned.

Since there is no standard roughness and there is no way to measure it, no formula can be given to determine the coefficient of roughness. The friction drop for turbulent flow for a rough pipe is increased according to judgment. In general the effect of roughness diminishes with increase of pipe diameter. Roughness changes the slope of the f-$\frac{vd}{\nu}$ curve with a consequent increase of the exponent n in Equation [5].

Equations [I] and [II] may be rewritten for practical use as follows:

For streamline flow $vd/\nu < 2000$

$$h_f = 143200 \frac{v\nu}{D^2} \quad \text{ft. per 1000 ft. of pipe} \quad [9]$$

$$h_f = 26250000 \frac{Q\nu}{D^4} \quad \text{ft. per 1000 ft. of pipe} \quad [10]$$

$$h_f = 58600 \frac{G\nu}{D^4} \quad \text{ft. per 1000 ft. of pipe} \quad [11]$$

$$h_f = 41000 \frac{B\nu}{D^4} \quad \text{ft. per 1000 ft. of pipe} \quad [12]$$

$$P = 994 \frac{v\mu}{D^2} \quad \text{lb. per sq. in. per 1000 ft. of pipe} \quad [13]$$

$$P = 182200 \frac{Q\mu}{D^4} \quad \text{lb. per sq. in. per 1000 ft. of pipe} \quad [14]$$

$$P = 406 \frac{G\mu}{D^4} \quad \text{lb. per sq. in. per 1000 ft. of pipe} \quad [15]$$

$$P = 284.1 \frac{B\mu}{D^4} \quad \text{lb. per sq. in. per 1000 ft. of pipe} \quad [16]$$

For turbulent flow $vd/\nu > 3000$

$$h_f = 47.4\, \nu^{0.170} \frac{v^{1.830}}{D^{1.170}} \quad \text{ft. per 1000 ft. of pipe} \quad [17]$$

$$h_f = 652000\, \nu^{0.170} \frac{Q^{1.830}}{D^{4.830}} \quad \text{ft. per 1000 ft. of pipe} \quad [18]$$

$$h_f = 9.23\, \nu^{0.170} \frac{G^{1.830}}{D^{4.830}} \quad \text{ft. per 1000 ft. of pipe} \quad [19]$$

$$h_f = 4.79\, \nu^{0.170} \frac{B^{1.830}}{D^{4.830}} \quad \text{ft. per 1000 ft. of pipe} \quad [20]$$

$$P = 0.329\, \nu^{0.170} \rho \frac{v^{1.830}}{D^{1.170}} \quad \text{lb. per sq. in. per 1000 ft. of pipe} \quad [21]$$

$$P = 4530\, \nu^{0.170} \rho \frac{Q^{1.830}}{D^{4.830}} \quad \text{lb. per sq. in. per 1000 ft. of pipe} \quad [22]$$

$$P = 0.0641\, \nu^{0.170} \rho \frac{G^{1.830}}{D^{4.830}} \quad \text{lb. per sq. in. per 1000 ft. of pipe} \quad [23]$$

$$P = 0.03325\, \nu^{0.170} \rho \frac{B^{1.830}}{D^{4.830}} \quad \text{lb. per sq. in. per 1000 ft. of pipe} \quad [24]$$

Divided Flow

If a pipe line is composed of several pipes of different diameters and lengths the problem is to determine the length of a single line of a given diameter equivalent to the combination of pipes.

The streamline and turbulent flows must be treated separately.

In case of streamline flow

$$h_f = \frac{64}{2g} \nu \frac{vL}{d^2}$$

Since

$$Q = \frac{\pi d^2}{4} v$$

we have, expressing v through Q

$$h_f = \frac{64}{2g} \times \frac{4}{\pi} \nu \frac{L}{d^4} Q = 1.265\, \nu \frac{LQ}{d^4}$$

or

$$Q = \frac{h_f d^4}{1.265 \nu L} \quad [25]$$

For divided flow

$$Q = Q_1 + Q_2 + Q_3 + \ldots\ldots$$

using the same fluid, ν is constant, and substituting Q from Equation [25]

$$\frac{h_f d^4}{1.265 \nu L} = \frac{h_f}{1.265 \nu} \left[\frac{d_1^4}{L_1} + \frac{d_2^4}{L_2} + \frac{d_3^4}{L_3} + \ldots\ldots \right]$$

solving for L

$$L = \frac{d^4}{\left[\frac{d_1^4}{L_1} + \frac{d_2^4}{L_2} + \frac{d_3^4}{L_3} + \ldots \right]} \quad [A]$$

The following cases may occur in practice:

1 Determine the length of a single line of a given diameter equivalent to a single line of different diameter and equal length.

2 Determine the length of a single line of a given diameter equivalent to a number of parallel lines of the same diameter and equal length.

3 Determine the length of a single line of a given diameter equivalent to a number of parallel lines of equal length but different diameters.

4 Determine the length of a single line of a given diameter equivalent to a number of parallel lines of different length but equal diameters.

5 Determine the length of a single line of a given diameter equivalent to a number of parallel lines of different length and diameter.

Case 1.

$$L = \left[\frac{d}{d_1} \right]^4 L_1 \quad [26]$$

Case 2.

$$L_1 = L_2 = L_3 = \ldots\ldots$$
$$d_1 = d_2 = d_3 = \ldots\ldots$$

$$L = \frac{L_1}{N} \left(\frac{d}{d_1} \right)^4 \quad [27]$$

Case 3.

$$L_1 = L_2 = L_3 = \ldots\ldots$$
$$d_1 \neq d_2 \neq d_3 \neq \ldots\ldots$$

$$L = L_1 \left[\frac{d^4}{d_1^4 + d_2^4 + d_3^4 + \ldots\ldots} \right] \quad [28]$$

Case 4. $L_1 \neq L_2 \neq L_3 \neq \ldots\ldots$

$d_1 = d_2 = d_3 = \ldots\ldots\ldots$

$$L = \left[\frac{d}{d_1}\right]^4 \frac{1}{\left[\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots\right]} \quad \ldots\ldots\ldots [29]$$

Case 5. $L_1 \neq L_2 \neq L_3 \neq \ldots\ldots$

$d_1 \neq d_2 \neq d_3 \neq \ldots\ldots$

Use Equation [*A*]

In case of turbulent flow, solving Equation [III] for v

$$v = \left[\frac{2gh_f}{0.167\,\nu^{0.170}} \times \frac{d^{1.170}}{L}\right]^{\frac{1}{1.830}}$$

$$v = \left[\frac{2gh_f}{0.167\,\nu^{0.170}}\right]^{0.546} \times \frac{d^{0.638}}{L^{0.546}} \quad \ldots\ldots\ldots [30]$$

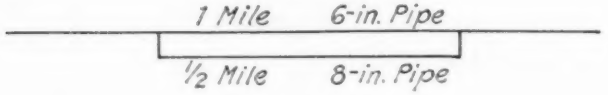


FIG. 5

since

$$Q = \frac{\pi d^2}{4} v$$

and expressing Equation [30] in terms of Q

$$Q = \frac{\pi}{4}\left(\frac{2gh_f}{0.167\,\nu^{0.170}}\right)^{0.546} \times \frac{d^{2.638}}{L^{0.546}} \quad \ldots\ldots\ldots [31]$$

Calling

$$\frac{\pi}{4}\left(\frac{2gh_f}{0.167\,\nu^{0.170}}\right)^{0.546} = K$$

$$Q = K\frac{d^{2.638}}{L^{0.546}} \quad \ldots\ldots\ldots [32]$$

If the flow is divided

$$Q = Q_1 + Q_2 + Q_3 + \ldots\ldots$$

Substituting Equation [32]

$$K\frac{d^{2.638}}{L^{0.546}} = K\left[\frac{d_1^{2.638}}{L_1^{0.546}} + \frac{d_2^{2.638}}{L_2^{0.546}} + \frac{d_3^{2.638}}{L_3^{0.546}} + \ldots\right]$$

Cancelling out K and solving for L

$$L = \left[\frac{d^{2.638}}{\frac{d_1^{2.638}}{L_1^{0.546}} + \frac{d_2^{2.638}}{L_2^{0.546}} + \frac{d_3^{2.638}}{L_3^{0.546}} + \ldots.}\right]^{1.830} \quad \ldots.[B]$$

Considering different cases in the same way as we have done for streamline flow we have:

Case 1. Equivalent length of a single pipe of equal length but different diameter

$$L = L_1\left[\frac{d}{d_1}\right]^{4.830} \quad \ldots\ldots\ldots [33]$$

Case 2. $L_1 = L_2 = L_3 = \ldots\ldots$

$d_1 = d_2 = d_3 = \ldots\ldots$

$$L = \frac{L_1}{N^{1.830}}\left[\frac{d}{d_1}\right]^{4.830} \quad \ldots\ldots\ldots [34]$$

Or

$$N = \left[\frac{L_1}{L}\right]^{0.545}\left[\frac{d}{d_1}\right]^{2.638} \quad \ldots\ldots\ldots [35]$$

where N is the number of parallel lines.

Case 3. $L_1 = L_2 = L_3 = L_4 = \ldots\ldots$

$d_1 \neq d_2 \neq d_3 \neq d_4 \neq \ldots\ldots$

$$L = L_1\left[\frac{d^{2.638}}{d_1^{2.638} + d_2^{2.638} + d_3^{2.638}}\right]^{1.930} \quad \ldots\ldots\ldots [36]$$

Case 4. $L_1 \neq L_2 \neq L_3 \neq L_4 \neq \ldots\ldots$

$d_1 = d_2 = d_3 = d_4 = \ldots\ldots$

$$L = \left[\frac{d}{d_1}\right]^{4.830} \times \frac{1}{\left[\frac{1}{L_1^{0.546}} + \frac{1}{L_2^{0.546}} + \frac{1}{L_3^{0.546}} \ldots\right]^{1.930}} \quad \ldots [37]$$

Case 5. $L_1 \neq L_2 \neq L_3 \neq L_4 \neq \ldots\ldots$

$d_1 \neq d_2 \neq d_3 \neq d_4 \neq \ldots\ldots$

Use Equation [*B*]

Illustrative examples follow.

Example 1. What is the friction drop in a 6-in. pipe line one mile long if 1000 bbl. per hr. flow through this line? Viscosity of the oil is 200 Saybolt seconds universal, gravity 30 A.P.I.

$$\nu = 0.00000237 \times 200 - \frac{0.00194}{200} = 0.0004643$$

$$s.g. = \frac{141.5}{131.5 + 30} = 0.875$$

$$d \text{ (actual)} = 6.065 \text{ in.} = 0.505 \text{ ft.}$$

$$v = \frac{1000 \times 0.286}{(6.065)^2} = 7.8 \text{ ft. per sec.}$$

$$\frac{vd}{\nu} = \frac{7.8 \times 0.505}{0.0004643} = 8500$$

This indicates a turbulent flow and Equation [20] must be used

$$h_f = 4.79 \times 0.0004643^{0.170}\,\frac{1000^{1.830}}{6.065^{4.830}} = 67 \text{ ft. per } 1000 \text{ ft.}$$

and

$h_f = 5.28 \times 67 = 353$ ft. per mile

Next the question may arise what would be the friction drop for the same conditions using a combination as indicated in Fig. 5.

$L_1 = L_2 = 0.5$ mile

$d_1 = 6.065$ in. $= 0.505$ ft.

$d_2 = 7.981$ in. $= 0.665$ ft.

Using Equation [36]

$$L = 0.5\left[\frac{6.065^{2.638}}{6.065^{2.638} + 7.981^{2.638}}\right]^{1.830} = 0.5 \times 0.127$$

$L = 0.0635$ mile

therefore the total friction drop will be

$$h_f = \frac{353}{2} + 353 \times 0.0635 = 198.9 \text{ feet}$$

Checking up whether the turbulent flow still takes place we have from Equation [III]

$$v = \left[\frac{2g \times 22.4}{0.167 \times 0.0004643^{0.170}}\right]^{0.548}\left[\frac{(0.505)^{0.638}}{(2640)^{0.548}}\right]$$

v = 3.35 ft. per sec. in $^1/_2$ mile of 6-in. pipe

and

$$\frac{vd}{\nu} = \frac{3.35 \times 0.505}{0.0004643} = 3640$$

hence turbulent flow takes place and the use of Equation [36] is justified.

Example 2. What is the friction drop in a 6-in. line one mile long if 1000 bbl. per hr. of oil are moved at the viscosity of 1500 Saybolt seconds universal and 15 A.P.I. gravity?

$$\nu = 0.00000237 \times 1500 - \frac{0.00194}{1500} = 0.00356$$

$$s.g. = \frac{141.5}{131.5 + 15} = 0.965$$

d (actual) = 6.065 in. = 0.505 ft.

v = 7.8 ft. per sec.

$$\frac{vd}{\nu} = \frac{7.8 \times 0.505}{0.00356} = 1105$$

This indicates a streamline flow and Equation [12] must be used, hence

$$h_f = 41000\,\frac{1000 \times 0.00356}{6.065^4} = 108 \text{ ft. per 1000 ft.}$$

$$h_f = 5.28 \times 108 = 572 \text{ ft. per mile.}$$

Next calculate the friction drop if two parallel lines are used. One line is 6 in. and the second line is 8 in. in diameter. Both lines are one mile long.

From Equation [28]

$$L = l_1\,\frac{6.065^4}{6.065^4 + 7.981^4}$$

$$L = 0.25 \text{ mile}$$

$$h_f = 572 \times 0.25 = 143 \text{ ft.}$$

Accelerated Streamline Flow

When a pipe line is full, a certain amount of time must elapse before a steady flow is established.

In case of a turbulent flow the time necessary for a mass of fluid to reach velocity v when starting from zero under a constant head for a series flow may be expressed approximately by the equation, assuming the pipe rigid,

$$t = \frac{Lv'}{gh} \times \frac{1}{2} \ln_e \frac{v' + v}{v' - v} \quad \text{[IV]}$$

where t is the time in seconds corresponding to velocity v in feet per second; v' the final velocity of the steady flow, L the pipe length in feet, and h the head in feet.

To the author's knowledge no expression has been derived for a streamline flow, and an equation for determining the time required to establish a steady flow for a streamline flow will be presented. This problem is of special interest in connection with oil pipe lines where a streamline flow occurs when pumping oils of high viscosities.

Imagine a pipe line filled with oil. To pump a certain quantity of oil the head produced by the pump must be equal to the sum of the static, friction, and velocity heads, thus

h = static head + friction head + velocity head

Assume a streamline flow, that is, a flow for which vd/ν is less than 2000.

Considering the forces acting on the mass of fluid, we have a force due to pressure head awh, a retarding force due to friction head $\frac{64}{2gd^2} L v \nu a w$, and a retarding force due to velocity head $aw\frac{V^2}{2g}$.

Since velocity is not steady, the system is not balanced and the resultant force is measured by the acceleration imparted to the mass of liquid

$$m\frac{dv}{dt} = aL\frac{w}{g}\frac{dv}{dt}$$

and we may write, assuming that h is constant and the pipe rigid,

$$awh - \frac{64}{2g}\frac{L}{d^2} v\nu aw - aw\frac{v^2}{2g} = aw\frac{L}{g}\frac{dv}{dt} \quad \text{[38]}$$

dividing both sides of Equation [38] by $aw\frac{L}{g}$

$$\frac{gh}{L} - \frac{64}{2}\frac{\nu}{d^2} v - \frac{v^2}{2L} = \frac{dv}{dt} \quad \text{[39]}$$

to simplify, call

$$\frac{gh}{L} = C \quad \text{[40]}$$

$$-32\frac{\nu}{d^2} = B \quad \text{[41]}$$

$$-\frac{1}{2L} = A \quad \text{[42]}$$

and substituting in Equation [39]

$$C + Bv + Av^2 = \frac{dv}{dt} \quad \text{[43]}$$

Separating the variables

$$dt = \frac{dv}{C + Bv + Av^2}$$

Integrating both sides

$$t = \int_0^v \frac{dv}{C + Bv + Av^2}$$

Solving the integral on the right side of equation

$$t = \left[\frac{1}{\sqrt{B^2 - 4AC}} \log_e \frac{2Av + B - \sqrt{B^2 - 4AC}}{2Av + B + \sqrt{B^2 - 4AC}}\right]_0^v \quad \text{[44]}$$

If $t = 0$, $v = 0$ because there is no flow, and Equation [44] becomes

$$t = \frac{1}{\sqrt{B^2 - 4AC}} \log_e \left[\frac{2Av + B - \sqrt{B^2 - 4AC}}{2Av + B + \sqrt{B^2 - 4AC}}\right] - \frac{1}{\sqrt{B^2 - 4AC}} \log_e \frac{B - \sqrt{B^2 - 4AC}}{B + \sqrt{B^2 - 4AC}}$$

or

$$t = \frac{1}{\sqrt{B^2 - 4AC}} \left\{ \log_e \frac{2Av + B - \sqrt{B^2 - 4AC}}{2Av + B + \sqrt{B^2 - 4AC}} \times \frac{B + \sqrt{B^2 - 4AC}}{B - \sqrt{B^2 - 4AC}} \right\} \quad \ldots\ldots [45]$$

If the flow becomes steady, that is, when $v = v'$, Equation [43] becomes

$$C + Bv' + Av'^2 = 0$$

and solving for v'

$$v' = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

or

$$2Av' + B = \pm \sqrt{B^2 - 4AC} \quad \ldots\ldots [46]$$

Since A and B are negative and v' is positive, the left side of Equation [46] is negative and consequently minus sign should be used for the right side of the equation

$$2Av' + B = -\sqrt{B^2 - 4AC} \quad \ldots\ldots [47]$$

Substituting in Equation [45]

$$t = \frac{1}{-2Av' - B} \log_e \frac{2Av + B + 2Av' + B}{2Av + B - 2Av' - B} \times \frac{B - 2Av' - B}{B + 2Av' + B}$$

or

$$t = \frac{1}{-2Av' - B} \log_e \frac{2A(v + v') + 2B}{2A(v - v')} \times \frac{-Av'}{B + Av'}$$

and finally

$$t = \frac{1}{-2Av' - B} \log_e \frac{A(v + v') + B}{A(v - v')} \times \frac{-Av'}{B + Av'} \quad \ldots\ldots [48]$$

When the flow is steady $v = v'$ and Equation [48] becomes

$$t = \frac{1}{-2Av' - B} \log_e \frac{2Av' + B}{A(O)} \times \frac{-Av'}{B + Av'}$$

but $\log_e \frac{K}{0} = \infty$ and it will take an infinite time for the mass of liquid to acquire the final velocity v'.

In practice the time necessary for the mass to acquire a velocity somewhat smaller than v' is of interest and a graph showing the relation between v and t must be plotted. This graph will indicate the value of t at which the flow becomes practically steady.

An example illustrative of this equation follows.

Required to pump 500 bbl. of oil per hour through a 6-in. line one mile long. The viscosity of the oil at the pumping temperature is 1000 Saybolt seconds universal, 17.5 A.P.I. gravity. Determine the time required to obtain a steady flow, assuming that the head created by the pump is constant and the pipe rigid.

First determine the ratio vd/ν.

$d = 6.065$ in. $= 0.505$ ft.

$$v = \text{bbl. per hr.} \frac{0.286}{d^2} = 500 \times \frac{0.286}{6.065^2} = 3.9 \text{ ft. per sec.}$$

$$\nu = 0.00000237T - \frac{0.00194}{T} = 0.00237$$

and

$$\frac{vd}{\nu} = \frac{3.9 \times 0.505}{0.00237} = 831$$

This indicates that a streamline flow takes place and Equation [48] should be used.

From Equations [41] and [42]

$$A = -\frac{1}{2L} = -\frac{1}{2 \times 5280} = -\frac{1}{10560}$$

$$B = -32\frac{\nu}{d^2} = -32\frac{0.00237}{0.505^2} = -0.297$$

Substituting in Equation [48]

$$t = \frac{1}{\frac{2}{10560} \times 3.9 + 0.297} \ln_e \frac{-\frac{1}{10560}(v + 3.9) - 0.297}{-\frac{1}{10560}(v - 3.9)} \times \frac{\frac{1}{10560} \times 3.9}{-0.297 - \frac{3.9}{10560}}$$

$$t = 3.36 \ln_e \frac{v + 3133.9}{3.9 - v} \times 0.001245$$

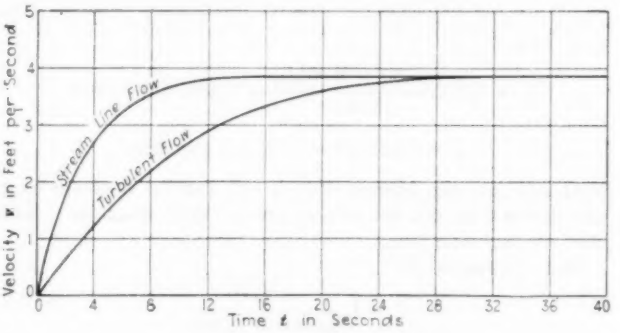


FIG. 6 TIME-VELOCITY CURVE

Substituting the different values of v and tabulating we have

v	t
0	0
1.0	1.0
2.0	2.41
3.0	4.95
3.5	7.65
3.80	12.3
3.85	14.7
3.90	$\sim$

These values are plotted (Fig. 6) and it is evident that after 13 sec. practically a steady flow is established.

If water instead of oil be pumped at the same rate (500 bbl. per hr.) through the same line,

$d = 0.505$ ft.

$v = 3.9$ ft. per sec.

$$\nu = 0.00000237 \times 30.99 - \frac{0.00194}{30.99} = 0.00001077$$

and

$$\frac{vd}{\nu} = \frac{3.9 \times 0.505}{0.00001077} = 183000$$

which indicates a turbulent flow and Equation [IV] must be used. From Equation [20]

$$h_f = 4.79 \times 0.00001077^{0.170} \frac{500^{1.830}}{6.065^{4.830}}$$

h_f = 9.8 ft. per 1000 ft.
h_f = 51.7 ft. per mile

Substituting in Equation [IV]

$$t = \frac{5280 \times 3.9}{32.2 \times 51.7} \times \frac{1}{2} \log_e \frac{3.9 + v}{3.9 - v}$$

Substituting the different values of v and tabulating the results we have,

v	t
0	0
1.0	3.25
2.0	7.02
3.0	12.65
3.5	18.1
3.80	27.0
3.85	31.3
3.90	$\sim$

These values of v and t are plotted in Fig. 6.

Discussion

J. M. Spitzglass.[2] In following the dimensional analysis of the flow formulas as given in the paper, I was in agreement with the derivation of Equations [2], [3], and [4] until I reached the substitution of h_f for p. I applied the dimensional formula to the expression $p = h_f \rho \cdot \frac{M}{LT^2}$ is not equal to $L \frac{M}{L^3}$. The correct expression should be

$$p = hg\rho \cdot \frac{M}{LT^2} = L \frac{L}{T^2} \cdot \frac{M}{L^3} = \frac{M}{LT^2}$$

This omission corrected itself later by calling $K = \frac{K_0}{2g}$; still the expression $p = h_f \rho$ as it stands is not dimensionally correct. I would suggest that the author insert the $2g$ into that expression, which would make the change from K to K_0 much clearer.

The derivation of Equation [5] requires additional explanation. It should be stated that the division of the factors is made arbitrarily to conform with the time-honored formula $h_f = 4f \frac{L}{d} \frac{V^2}{2g}$. The second factor is the dimensionless part that stands for f, the coefficient of friction. The correlation with Poiseuille's experiments is of interest. It may be stated that Equation [I] for streamline flow is more fundamental than Equation [6], even though the latter is more general.

Grant K. Palsgrove.[3] The author has presented a group of general formulas for frictional resistance offered to flow in straight, circular pipes of relatively smooth surface in a manner that makes the solution of problems of this nature comparatively simple. The writer, however, wishes to call the attention of the author to many inconsistencies, lack of proper acknowledgment, and errors in this paper.

In the first part, pertaining to the resistance offered to flow, all of the equations and derivations (with the possible exception of the form of the factor K as found from the author's Fig. 4) are to be found in the well-known book on "Hydraulics and Its Application," by A. H. Gibson (1921).

Why present Equation [1] for streamline flow and no corresponding equation for sinuous flow? We have such equations deduced by Williams, Hubbell, and Fenkell and also by Bazin and Darcy, all of which give close agreement with practice (again see Gibson).

Equation [5] is Osborne Reynolds' equation with a modified constant, Reynolds giving the equation as follows:

$$h_f = \frac{LB^n \left(\frac{\mu}{\rho}\right)^{2-n} V^n}{A\, d^{3-n}},$$

in which $\frac{B^n}{A}$ corresponds to $\frac{K_0}{2g}$ in the author's equation.

Equation [6] should be given as

$$h_f = \frac{L}{d} \frac{V^2}{2g} \phi \left(\frac{vd}{\nu}\right)^{n-2}$$

with a statement to the effect that Poiseuille found that for all surfaces with streamline flow $\phi = 64$ and $n = 1$. This is then Equation [I] given by the author. Reynolds found from various experiments with smooth pipes and sinuous or turbulent flow that $n = 1.83$ as an average, and with $\phi = 0.167$ gives the author's Equation [II].

In outlining the cases likely to be found in practice, the author has applied the law of variation to Equations [I] and [II] very well indeed.

With reference to the second part of the paper, on Accelerated Flow, the writer again wishes to call the author's attention to the work of Professor Gibson wherein the subject of accelerated flow has been covered in detail without regard to the type of flow (streamline or sinuous). Gibson derives a general equation which is rigorously true for an incompressible fluid in a rigid pipe and is of the form:

$$t = \frac{K}{2V'} \log_e \frac{v' + v}{v' - v}$$

in which $k = \frac{2L}{1 + \frac{64}{\left(\frac{vd}{\nu}\right)} \frac{l}{d}}$ for streamline flow

and $k = \frac{2L}{1 + \frac{0.167}{\left(\frac{vd}{\nu}\right)^{0.17}} \left(\frac{l}{d}\right)}$ for turbulent flow.

Applying the foregoing equation to the author's examples gives:

For oil (where streamline flow was indicated), at an assumed velocity $v' = 3$ ft. per sec.

$$t = \frac{2 \times 5280}{\left[1 + \frac{64}{831}\left(\frac{5280}{0.505}\right)\right]} \times \frac{1}{2 \times 3.9} \log_e \frac{3.9 + 3}{3.9 - 3} = 3.43 \text{ sec.}$$

and for $v' = 3.85$ ft. per sec., $t = 8.5$, etc. This shows a curve somewhat above the one indicated by the author for streamline flow in Fig. 6.

For water (where turbulent flow was indicated), at an assumed velocity $v' = 3$ ft. per sec.

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[3] Professor of Hydraulic Engineering, Rensselaer Polytechnic Institute, Troy, N. Y. Mem. A.S.M.E.

$$t = \frac{2 \times 5280}{\left[1 + \frac{0.167}{(183000)^{0.17}}\left(\frac{5280}{0.505}\right)\right]} \times \frac{1}{2 \times 3.9} \log_e \frac{3.9 + 3}{3.9 - 3}$$

$$= 12.3 \text{ sec.}$$

and for $v' = 3.85$ ft. per sec., $t = 30.5$, etc. This shows a curve which is below that for streamline flow as indicated by the author on Fig. 6, and very far different from the author's curve for turbulent flow. Upon checking Equation [IV] with the foregoing one given by Gibson, the two are identical. Further checking shows that the author not only has an error in decimal point for "t" values, but also has errors in some of the figures, which when corrected agree with the values obtained in the foregoing.

The writer would like to question the general use of Equation [48], which is deduced on the basis of Equation [47], and therefore only holds for the case of zero acceleration or $v' = v$ and for no other case. Hence the difference between the author's values of t at different velocities assumed and those of Gibson for streamline flow. Gibson's equation is based strictly on the application of the well-known Bernoulli theorem.

Author's Closure

Mr. Spitzglass correctly pointed out that to make the derivation of the Equation [5] correct the expression for the pressure should be

$$p = h_f \delta g$$

This has been changed accordingly.

In reference to Professor Palsgrove's criticism on the first part of the article, the writer wishes to state that Professor Palsgrove overlooked that due credit was given to Professors Reynolds and Poiseuille.

So far as Equation [5] is concerned, it is nothing else but the well-known exponential formula. The derivation of this equation was reviewed for the better understanding of the latter part of the article.

Having looked carefully through Professor Gibson's book on "Hydraulics" the author failed to find any attempt to solve the problem of accelerated streamline flow and did not find any equation presented in the final form as used by the author in his article. And for this reason, with all due respect to the genius of Professor Gibson, the author does not see where he could refer to Professor Gibson's "Hydraulics."

Professor Palsgrove's statement that the Equation [6] should be given in form

$$h_f = \frac{L}{d} \frac{V^2}{2_g} \phi \left(\frac{vd}{\nu}\right)^{n-2}$$

is incorrect, since ϕ is not a coefficient, but is a mathematical symbol meaning a function. Consequently, the exponents and coefficients should be omitted as long as symbol ϕ is used. (See introduction to "Differential and Integral Calculus," page 4, by Osgood.)

In reference to Professor Palsgrove's criticism on the third part of the article, the Equation [IV] was presented in its final form only for the sake of comparison with that derived by the author for the streamline flow and was in no place claimed to be original. That the final results obtained by means of this equation check up with that of Professor Gibson's equation is quite natural, and undoubtedly it will check with all similar equations presented in other treatises and text books, provided that they are correct, although they might be stated in different form.

The author does claim, as stated in the article, that to his knowledge no expression has been derived for determining the time required to establish a steady flow for a streamline flow.

To present the derivation of the Equation [48], show its application, and compare the results with those obtained for turbulent flow by means of Equation [IV] was the author's aim in the third part of the article.

Concerning the deduction of Equation [48] on the basis of Equation [47], it is correct and holds theoretically for any value of acceleration.

The substitution of one set of constants A, B, and C by another constant v' is nothing else but simple algebra and does not change the physical meaning of the equation.

The author acknowledges his appreciation for pointing out the numerical errors, which are rectified.

Experimental Study of Loss of Head in a Closed Pipe Carrying Clay Slurry[1]

By CHARLES HANOCQ,[2] LIEGE, BELGIUM

Taking as a basis for his calculations the formulas presented in a previous paper by an American investigator on the loss of head in a closed pipe carrying clear fluids, the author, a Belgian engineer, seeks to apply them to a fluid carrying a heavy proportion of solid materials. The author takes the conclusions in the original paper, which he assumes seem at first sight somewhat contradictory, and shows that no anomalies exist, and further brings out some precise deductions on the loss of head in a pipe carrying muddy water.

AN INTENSELY interesting study published in the June, 1927, number of *Mechanical Engineering* allows us, in using the method that we have proposed for the calculation of loss of head,[3] to examine in what measure these formulas for homogeneous fluids can be applied to a fluid holding in suspension a large proportion of solid materials. Having had occasion to conduct the experiments on a sufficiently large scale with a view to determine the losses of head in a pipe 4 in. in diameter supplied by a centrifugal pump, pumping a muddy water more or less charged, W. B. Gregory, professor at Tulane University in America,[4] has furnished some very complete experimental information and has brought forth certain important conclusions—although at first sight they apparently are contradictory.

In following the analysis of results by means of our method, we have been able to dispel these anomalies and to bring out some precise deductions concerning the calculation of the loss of head in the particular case of muddy waters.

The experiments have been made on a cast-iron pipe measuring in length 112.5 meters, with the amount by weight of the material in suspension varying from 0 to 35 per cent, the specific weight being respectively from

1000 at 0.0%
1130 at 18.6%
1175 at 23.4%
1225 at 29.05%
1255 at 32.5%
1285 at 35.3%

The experiments made on the new pipe with clear water have shown that the losses of head were slightly greater than those which would be calculated by means of the formula proposed by us.[5]

The unforeseen result, after this pipe had been placed in service with the muddy water, was that the losses of head were found reduced to those which are observed in a smooth pipe. It is thus that the experiments made with turbid water and of a density practically equal to unity have given values which agree exactly with those deduced from our formula

$$10^5\beta = \frac{271.8}{\sqrt[3]{\alpha}} + 3.4$$

applicable to smooth pipes.

These results can be explained by admitting that the deposit on the walls of the pipe of a coating of the materials held in suspension has had the effect of transforming the rough surface of the pipe to a practically smooth surface.

We have already been led to make an analogous observation in the case of pumping petroleum oils; the formula applicable to iron pipes must be replaced by those for smooth pipes when the fluid used is relatively very viscous and adheres perfectly to the walls.[6]

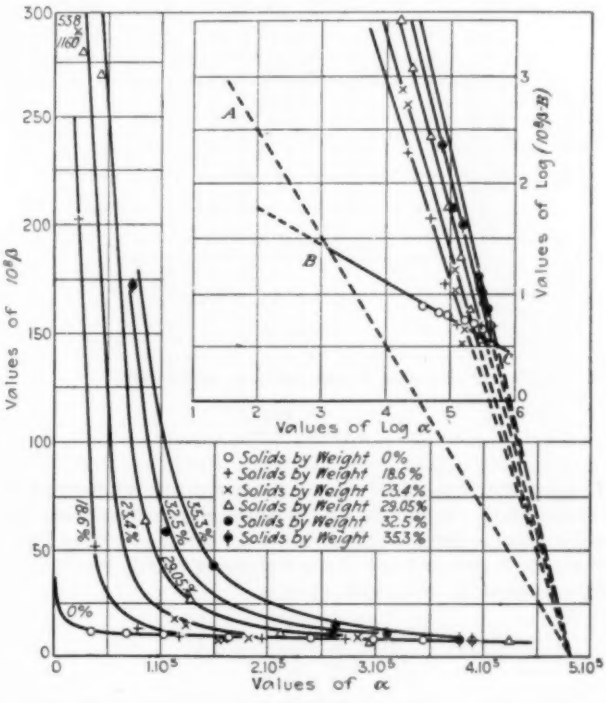


FIG. 1

Another unforeseen result is that for a certain value of the Reynolds coefficient varying with the amount of matter in suspension, the coefficient $10^5\beta$ which fixes the loss of head is found to be strictly equal to that observed in a pipe with smooth walls through which clear water is forced.

In other words, beginning with a certain critical value of α_c, turbulent flow is established and the loss of head is no longer influenced, beginning from this moment, by the presence of the suspended matter.

This conclusion does not seem likely to be general; we cannot hold it to be true, for the lack of verification of the most general conditions, of materials in suspension heavily charged and for materials in excess of 35 per cent, in round numbers.

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[1] Translation of extract from *Revue Universelle des Mines*, February 1, 1928, 7th series, vol. xvii, no. 3.

[2] Engineer A.I.Lg. and A.I.M., Professor at the University of Liege.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] See *Revue Universelle des Mines*, September 1, 1897, 7th series, vol. xv, no. 5.

[4] Tulane University, New Orleans, La.

[5] See *Revue Universelle des Mines*, February 1, 1922, 6th series, vol. xii, no. 3.

[6] See "Annales de l'Association des Ingéneurs sortis des Écoles spéciales de Gand," 5th series, vol. xvii, first number, 1927.

It is known that the loss of head in meters of fluid flowing is given by the formula

$$h = 10^4\beta \frac{w^2}{d} L$$

To use this formula we transformed the differences of pressure indicated by the author in height of meters of pure water to heights with the density of the fluid used in the experiment.

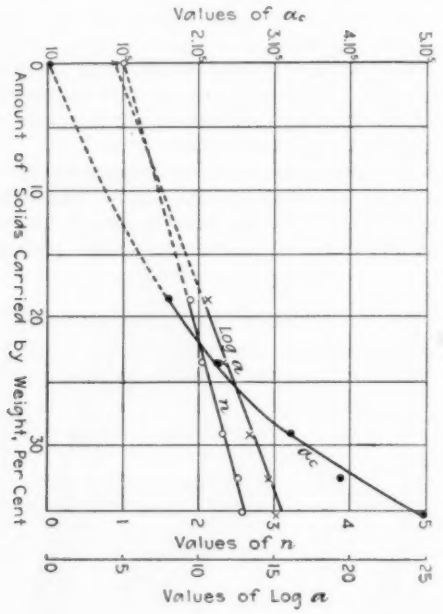


FIG. 2

In carrying as abscissas α furnished by the formula

$$\alpha = \frac{10\delta w d}{\mu}$$

(μ being the viscosity of the clear water at the temperature indicated and δ the specific weight of the mixture) we have been able to deduce from the experiments the curves of $10^4\beta$ of Fig. 1; the curve corresponding to 0 per cent, being exactly that deduced by our formulas for smooth pipes, is

$$10^4\beta = \frac{271.8}{\sqrt[3]{\alpha}} + 3.4$$

From these curves we have been able to deduce the position of the asymptotes, 3.4 to the common curve, then to calculate

$$\log (10^4\beta - 3.4)$$

In carrying these values as ordinates and the log α as abscissas we have obtained the curve BC of Fig. 1, which shows that, beginning with a certain value of α_c, variable with the amount of material carried, the law changes completely.

Admitting that at the attainment of this value α_c the appreciable formula is that of viscous flow ("régime laminaire")

$$10^4\beta = \frac{a}{\alpha^n}$$

we have calculated log $10^4\beta$ and carried the results as ordinates.

As the dotted curve AB represents the law of viscous flow for clear water, it may be admitted that as the weight of material carried diminishes, the inclination of the line representing the law of variation of log $10^4\beta$ diminished as it approaches the dotted line. We propose for lack of something better and in recognition that the results given are insufficient to obtain in a relatively sure method the direction of the curve, the law of variation of α_c as a function of the weight of materials in suspension represented by the curve of Fig. 2.

It follows from this for the lower values of α_c that the formula to apply is that of viscous flow, and must be

$$10^4\beta = \frac{a}{\alpha^n}$$

the coefficients a and n being given by the curves of Fig. 2. (a is given by log a.) It is seen that for materials carried from 18 per cent to 30 per cent the least value of n is equal to 2, then for clear water $n = 1$; $10^4\beta = \frac{a}{\alpha}$. This explains the direction of curves of the loss of head given by the author, which shows that these are independent of the velocity below a certain value of α, varying with the amount of solid material carried.

In fact, if in the formula for the loss of head $h = 10^4\beta\frac{w^2}{d}$ we introduce the value found $10^4\beta = \frac{a}{\alpha^n}$ applicable below the critical velocity, then we will have

$$h = 10^{-4} \frac{a}{\left(\frac{10\delta d}{\mu}\right)^n w^n} \frac{w^2}{d}$$

which will furnish for $n = 2$ a value of $h = a$ constant, whatever

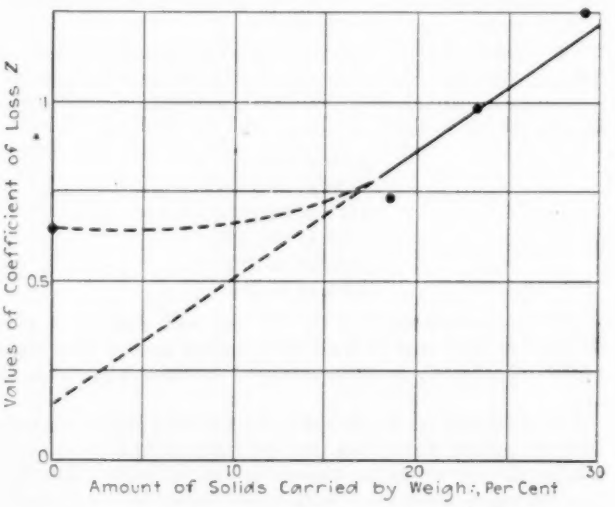


FIG. 3

is the velocity, from the moment when the critical velocity is not exceeded. This conclusion is only truly evident for the amount of material between 18 and 30 per cent, for which the average value of n stays equal to $n = 2$.

This manner of considering the total of the experimental results leads us to suppose that the presence of solid particles in suspension tends to maintain longer the viscous flow ("régime laminaire"), but that the particles in increasing the friction surfaces cause an increase in unusual proportions of the resistance to flow and consequently the loss of head. As soon as the turbulent flow is established, on the contrary the loss of head is maintained

constantly equal to that which would be observed in a smooth pipe through which runs a fluid having the viscosity of water and the same average density as the mixture.

These results deduced with our method, in terms of Reynolds' coefficient, are extremely curious because they indicate that the phenomenon of the flow of a fluid holding in suspension some particles does not depart essentially from the usual phenomenon of flow of a homogeneous fluid in the region of viscous flow at least.

This conclusion is nevertheless only true for straight pipes. For the curved pipes the results furnished by the experiments of Mr. Gregory show that the losses, if one compares them with those through which the clear water flows under the same conditions, increase rapidly with the weight of material in suspension. This difference in the assumed conclusions suggests that the deviation of the filaments has an effect to provoke a separation of the particles of the fluid themselves and to lead to a concentration of solid particles at the exterior walls. It is no longer possible from this moment to apply relatively simple laws to translate the results of the experiments.

Fig. 3 gives the value of the coefficient Z of the classic formula of supplementary losses due to an elbow

$$h = Z \frac{w^2}{2g}$$

The curve traced as a function of the per cent of solids by weight shows that the coefficient Z is proportional to the per cent of solids beginning at 18 per cent. For the observed loss with turbid water, carrying only a small quantity of solids, the author has obtained the figure 0.64 indicated on the axis of ordinates. This very high figure explains with difficulty as the one generally indicated in the formulas for a curve at 90 deg., and a corresponding value $\frac{d}{r}$ is equal to 0.15. If this last figure should be considered as the true figure for clean water in a clean pipe, we must conclude that the coefficient Z increases proportionally with the per cent solids by weight; the absence of points between 0 and 18 per cent does not permit us to venture an opinion in this regard.

This conclusion, which concerns the intersection of losses with the per cent of solids by weight, is confirmed by the figures observed for the manometric height furnished by the pump during the tests on the pipes.

In evaluating the heights observed in meters of fluid having the average density of the liquid pumped, we have been able to calculate the manometric height as a function of the discharge, the peripheral velocity remaining constant. The curve 1 (Fig. 4) refers to clear water, curve 2 to 18.6 per cent, curve 3 to 23.4 per cent, curve 4 to 29.05 per cent solids by weight.

For the curves 2, 3, and 4 the points observed group themselves relatively well on the mean curve; furthermore these curves indicate a loss of head rapidly increasing with solids carried in the passages of the impeller and of the volute.

Finally, if the law stated apropos of straight pipes remains true for curved passages, we must observe, at least in the region of the turbulent flow, a unique curve for all the per cents of solids carried up to 29 per cent.

For the per cents of solids carried below 29 per cent, the points observed are absolutely scattered, and it will be impossible to rely upon them for continuous curves; this holds true that the more and more complete separation of solid particles at the time of the passage on the blades of the wheel makes the local densities very much higher than the mean densities and that all the deductions made to define that which one expects regarding manometric heights and least discharge cease to be significant from this moment.

Conclusions

In admitting, for lack of something better, the diagram of 2 for the values of α_c, a, and n, one sees that the calculation of the loss of head for muddy water containing up to 35 per cent of material in suspension may be made in the following manner:

If α is greater than α_c, we have in meters of fluid pumped

$$h = 10^4 \beta \frac{w^2}{d}$$

with

$$10^3\beta = \frac{271.8}{\sqrt[3]{\alpha}} + 3.4$$

the coefficient α being calculated for the density equal to that of

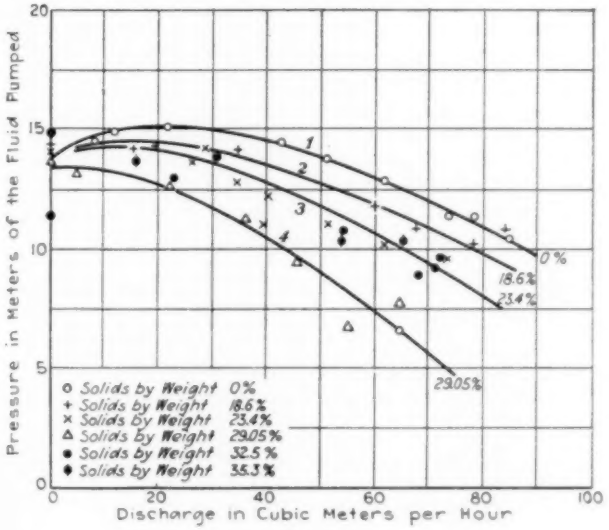


Fig. 4

the fluid pumped, and for viscosity equal to clear water at the temperature given.

If α is less than α_c, we have still

$$h = 10^4 \beta \frac{w^2}{d}$$

but with

$$10^3\beta = \frac{a}{\alpha^n}$$

the coefficient α being calculated as in the first case and the coefficients a and n being furnished by the diagrams of Fig. 2.

It is possible to use the formulas giving directly the values of $10^3\beta$ in noting that all the lines representing the viscous flow for different percentages for solids carried converge toward the same point (see Fig. 1).

This point being characterized by

$$\log \alpha = 6.88, \log 10^3\beta = -2.366$$

the relation

$$\log 10^3\beta = \log a - n \log \alpha$$

furnishes

$$\log a = 6.88\, n - 2.366$$

it follows that

$$\log 10^8\beta = n(6.88 - \log \alpha) - 2.366$$

The coefficient n being furnished as a function of the weight of solids x per cent of the materials in suspension, by the linear law,

$$n = 0.045\, x + 1$$

it will be possible to deduce a relation between $10^8\beta$ and α which will fix directly the value of β as a function of x and of α.

$$\log 10^8\beta = 0.31x + 4514 - (0.045x + 1) \log \alpha$$

Grouped under this form, the results of these very complete tests made by Mr. Gregory present a general interest which will escape none, being given in such a way that they may be considered, in my opinion, as valuable in all their applications, whatever be the nature of the solid particles entering the problem, at least if they have small enough dimensions to remain in suspension even at a low velocity.

Permissible Suction Head of High-Speed Propeller Turbines

By ARNOLD PFAU,[1] MILWAUKEE, WIS.

The paper discusses the general phenomena of pitting and cites the underlying causes. It explains the necessity for special caution in the selection of the setting of a high-speed propeller turbine. To fully realize these causes a new explanation is given of the flow conditions in a high-speed propeller, and it shows that these conditions add an item to the suction head which does not enter into the problem with reaction or action turbines. Particularly this paper emphasizes the wide variation in the characteristics of turbine runners that are termed the "propeller type" and directs attention to the difficulty that may be encountered from generalization based on the use of a single mathematical equation or expression.

IT IS known from experimental researches[2] that cavitation, and in some cases also vibration, sets in when the negative pressure becomes so low that the vapor point of the operating water is reached. At this point the water begins to "boil," and it forms air or vapor pockets which when adjacent to the surface of a blade cause the water to separate or to "cut loose." When these pockets again collapse, the water is thrown against the blade surface with a tremendous force (more than 7000 tons per square inch), so that a mechanical destruction of the material is brought about, manifested in what we term "erosion." The intense and exceedingly rapid pressure changes of an adiabatic nature also may be responsible for the possible existence of chemical processes which produce gases (oxygen, carbon dioxide, ammonia) and which attack the metal, resulting in what we term "corrosion," all of which brings about a destruction of the blade and reduces the efficiency and the life of the runners.

These negative pressures are a function of three principal factors:

1 Suction height H_s (vertical distance from runner to tail water level)
2 Regain in draft tube, $h_r = e\ (V_2^2/2g - V_3^2/2g)$, as explained farther on
3 Barometric pressure B in feet of water column (dependent on the elevation above sea level, 33.8 ft. at sea level).

The vapor point of the water is reached at a pressure equivalent to about $^1/_3$ to $^2/_3$ ft. of absolute water column at normal temperature.

Neglecting this vapor pressure, we have

$$B - H_s - e \cdot \left(\frac{V_2^2}{2g} - \frac{V_3^2}{2g}\right) \geqq 0$$

e represents the efficiency of regain (or deceleration) in the draft tube
V_2 is the absolute velocity of water below the runner (at the top of the draft tube)
V_3 is the exit velocity of water at the end of the draft tube.

Since $K_{v2} = V_2/\sqrt{2gh}$, we have $\dfrac{V_2^2}{2g} = K_{v2}^2 \cdot H$

also, $K_{v3}^2 \cdot H = \dfrac{V_3^2}{2g}$

and we can write

$$H_s \leqq B - e \cdot h(K_{v2}^2 - K_{v3}^2)$$

as the theoretical limit of suction height. It is evident that higher and higher velocities V_2 require a smaller and smaller suction height H_s. (High exit velocities V_3 and low regain or draft tube efficiencies e permit of a greater suction height H_s.) With the introduction of the modern, more efficient forms of draft tubes, the danger limit of cavitation is approached sooner than was the case with older, less efficient designs. The increase of specific speeds has increased the coefficient of discharge K_{v2}, which again brings the danger line closer. With runners of moderate specific speed K_{v2} was about 0.25 and with draft tubes of only 40 per cent efficiency and an exit velocity K_{v3} of, say, $0.1 \cdot \sqrt{2gh}$ we obtain for the setting at sea level ($B = 33.8$ ft.)

$$H_s \leqq 33.8 - 0.4 \cdot H \cdot (0.25^2 - 0.1^2) \text{ or } \leqq 33.8 - 0.021 \cdot H$$

$$\text{for } H = 100 \text{ ft.} \qquad H_s \leqq 31.7 \text{ ft.}$$

Assume now a higher specific speed with $K_{v2} = 0.45$, a draft tube efficiency of 75 per cent, and an exit velocity equivalent to $K_{v3} = 0.05$, we obtain:

$$H_s \leqq 33.8 - 0.75 \cdot H(0.45^2 - 0.05^2) \text{ or } \leqq 33.8 - 0.15 \cdot H$$

$$\text{for } H = 100 \text{ ft.} \qquad H_s \leqq 18.8 \text{ ft.}$$

The foregoing values of H_s are theoretical. A liberal margin of safety must be allowed if the water enters the draft tube, not in streamlines parallel to the axis, but discharges with a forward or backward whirl which produces centrifugal forces resulting in an additional suction effect at the inner portions of the top of the draft tube. (If the turbine is to operate at a high elevation above sea level, say, 6000 ft., B is correspondingly less, thereby reducing the allowable suction head H_s still further.)

Propeller Turbine

Literature on the subject classes the propeller turbine with the so-called "suction jet" or under-pressure turbines. This is incorrect as not every propeller turbine is an under-pressure turbine. On the other hand, it seems that under-pressure turbines can be built successfully only with runners of the propeller shape. From a purely theoretical standpoint the modern type of turbines can be placed in three typical classes:

1 Over-pressure (or reaction) turbine, in which the pressure due to the available head H is only partly transformed into velocity, so that an actual over-pressure ($p_r = a \cdot \dfrac{c_r^2}{2g} > 0$) exists between guide case and runner. This pressure, called the reaction pressure, produces the acceleration of the relative water velocity in the runner passages ($W_2^2 - W_1^2 = C_r^2 > 0$), thus $W_2 > W_1$, and a peripheral speed, which increases with the increase of p_r and which increases the specific speed n_s.

2 Action (or impulse) turbines, in which the pressure due to available head H is completely transformed into velocity so that the over-pressure p_r between guide case and runner (or

[1] Consulting Engineer, Hydraulic Department, Allis-Chalmers Mfg. Co. Mem. A.S.M.E.

[2] "Hydraulische Probleme," pp. 14–79, *V.D.I.*, Berlin, 1926.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

nozzle and buckets of an impulse wheel) is zero, i.e., atmospheric pressure. Here no acceleration of the relative water velocities takes place ($W_2^2 - W_1^2 = C_r^2 = 0$); thus ($W_2 = W_1$), except that due to centrifugal action. The absence of an over-pressure p_r is here responsible for the fact that the peripheral speed $K_u = \frac{D \cdot \pi \cdot n}{60\sqrt{2gh}}$ is fixed within much closer limits (0.42 to 0.48) than is the case with a reaction turbine where the values of K_u vary between 0.55 and 1.0 or more, depending upon the respective specific speed.

3 Under-pressure (or suction jet, or as commonly but not correctly called the propeller turbine), in which not only all of the pressure due to the available head H is transformed into velocity, but in which at a certain cross-section a velocity exceeding the spouting velocity $(\sqrt{2gh})$ is reached and where even an under-pressure exists, with subsequent retardation of the relative water velocities. ($W_2^2 - W_1^2 = C_r^2 < 0$, thus $W_2 < W_1$.)

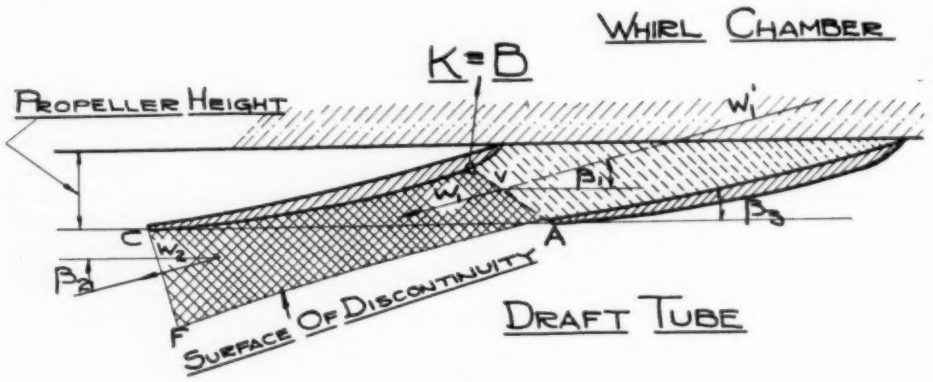


FIG. 1

In order to explain this process of the flow of water it is necessary to divide an under-pressure turbine into three stages, as illustrated in Figs. 2 and 3:

1 The guide case, where only a small portion of the pressure due to the available head H is transformed into velocity, and where the water is given an initial whirl in accordance with the arrangement of guide-vane angles or of passages between same ($V^2 = C_u^2 - C_r^2$; $C_r \gg 0$; $V \ll C_u$).

2 A whirl chamber, or free passage between the guide case and the runner (propeller). Here the water is changed from an essentially radial inward flow to an axial flow. The absolute velocity of water v is greatly increased v_1 and the whirl is somewhat modified, as will be explained in the discussion of velocity diagrams further on (α changes to α').

3 The inlet portion of the runner, extending from the entrance plane of the runner to a section in which the displacement area F by the projected runner vanes is a maximum, i.e., where the absolute water velocity is a maximum (V_{c1}).

$$V_{c1} = \frac{Q}{[(D_a^2 - D_i^2) \cdot \pi/4 - F] \cdot \sqrt{2gh}}$$

in which

D_a = runner diameter in feet at the plane in question
D_i = hub diameter in feet at the plane in question
F = projected displacement of vanes in square feet.

The corresponding maximum relative velocity W_1 is found in the cross-section A to K (Fig. 1) extending from point A (the discharge edge of the runner vane) to a point K located on the rear surface of the preceding runner vane. This is the same point as indicated and designated K, Fig. 1, *Swiss Bauzeitung*, vol. 91, no. 11, March 17, 1928, by Chief Engineer J. A. Ackeret, Zurich, and is in accord with observations made by the Aerodynamic Laboratory, Goettingen, Germany. (See Oldenbourg, Muenchen, 1927, page 132.) It is the point where the pressure p_r is a minimum, even negative.

The author's suggestion, made some nine years ago, seems therefore reasonable, i.e., to divide the under-pressure runner in two sections—the upper part ending with AK and performing similar duty to that of the rotating guide case of the so-called transformer turbine proposed about 1900 by Prof. Dr. Franz Prasil, at the Polytechnic of Zurich, Switzerland, and the lower part comprised by the triangles CKA. With this picture before us, it at once becomes evident that sections AK constitute what may be termed the "throat areas" of the runner, at which both the relative velocity W_1 and the corresponding absolute velocity V_1 are a maximum. That this absolute velocity may exceed even the value of the spouting velocity $\sqrt{2gh}$ can be explained as follows:

From the cross-sections AK, limited sidewise by the walls of the runner hub and of the stationary discharge ring (Fig. 2), the water passes downward through what I wish to term "hypothetical channels." These are formed by the rear surface CK of a blade, by the walls of runner hub and discharge ring, and by a surface of discontinuity AF which adjusts itself according to the speed and quantity of water and the angle β_3 at the throat A, which is not the outlet of the runner but the outlet of the rotating case, or accelerator.

The existence of a surface of discontinuity AF undoubtedly accounts for the great flexibility of efficiency as a function of the speed.

From the foregoing it at once becomes clear, and this is also proved by tests, that the angle β_3 can be reduced to 0, or even be negative, without seriously affecting or entirely negating the surface of discontinuity. Angles β_3 of zero value or less, if considered as relative runner discharge angles according to the old theory, would render the use of the conventional velocity diagrams impossible.

These hypothetical channels are individual diffusers in which the relative velocity W_1 at the entrance is gradually reduced to W_2 at the rear end C of the runner blade. It is at point C, therefore, where we find the actual relative discharge angle β_2 for use in our velocity diagram.

From $W_2^2 - W_1^2 = C_r \leqq 0$, the under-pressure $p_r = a \cdot C_r \leqq 0$ is at once explained, and also the fact that a "hydrodynamic suction" is produced in the under-pressure turbine, which is responsible for the high discharge capacity of this type of turbine and for the high rotating speed. To the regain in the draft tube

$$e \cdot H \cdot (K_{v2}^2 - K_{v3}^2)$$

we must therefore add the hydrodynamic regain

$$H \cdot (K_{v1}^2 - K_{v2}^2)$$

so that we obtain for the under-pressure turbine

$$H_s \leqq B - H \cdot [(K_{v1}^2 - K_{v2}^2) - e \cdot (K_{v2}^2 - K_{v3}^2)]$$

or as an approximation, by assuming that the efficiency of the hydrodynamic regain equals about that of the draft tube, we obtain

$$H_s \leqq B - e \cdot (K_{r1}^2 - K_{r3}^2);$$

for high specific speed under pressure turbines the value of K_{r1} exceeds unity. Values as high as 1.1 have been tested for $n_s > 200$. Applying this value again in the previous example, we have

$$\begin{aligned} K_{r1} &= 1.1 \\ V_3 &= 0.05 \\ e &= 0.75 \\ H_s &\leqq B - 0.75 \cdot H(1.1^2 - 0.05^2) \\ \text{or} \quad &\leqq B - 0.9 \cdot H \\ \text{for} \quad H &= 100 \text{ ft.} \\ H_s &= 33.8 - 90 = -56.2 \text{ ft.} \end{aligned}$$

From this it follows that for a 100-ft. head such an under-pressure runner would have to be submerged below the tail water 56.2 ft. in order to avoid danger of cavitation.

With the introduction of the hypothetical channel and the consequent modifications of the velocity diagrams, the process of the under-pressure turbine has been found to be identical with that and with the diagrams discussed in the literature in 1914 by Prof. Dr. Hans Baudisch of Vienna under the appropriate title "Saugstrahl Turbinen-Theorie" (Theory of the suction jet for under-pressure turbines).[3] Unfortunately the design of the runner then proposed by Professor Baudisch failed to give the desired results, because his runner was designed with four-wall channels in which it was intended to force a deceleration of the relative flow of water. The channels were too short and proper diffusion $W_2^2 - W_1^2$ could not take place. The water separated from the rear of the runner blades and did not fill the channels; $W_2^2 - W_1^2 = C_r \leqq 0$ was not fulfilled, and the typical suction effect was not obtained. In the three-walled hypothetical channels of the propeller-shaped under-pressure turbine, and thanks to the fourth hypothetical wall, the surface of discontinuity, the diffusion $W_2^2 - W_1^2 = C_r \leqq 0$ is accomplished with a liberal leeway at the disposal of nature to adjust itself and to recompense the designer with a flexible efficiency. Summing up, it may be stated:

1 Not every propeller turbine is an under-pressure turbine, but the most efficient form of an under-pressure turbine runner is that of a propeller.

[3] See U. S. Patent 1,347,578, July 27, 1920.

2 An under-pressure turbine deals with negative pressures and is therefore accordingly limited in its practical application as regards head and setting, from the point of view of cavitation, vibration, and stability. However, application to higher heads would be possible and cavitation (vibration and instability) could be avoided if the turbine is so set that the actual piezometer pressure p_r at the throat does not approach the vapor-point pressure. If a propeller of the under-pressure type could be designed physically strong enough for the power thus developed, there is no reason why it could not operate satisfactorily at a high head, provided the foregoing requirement of setting is fulfilled.

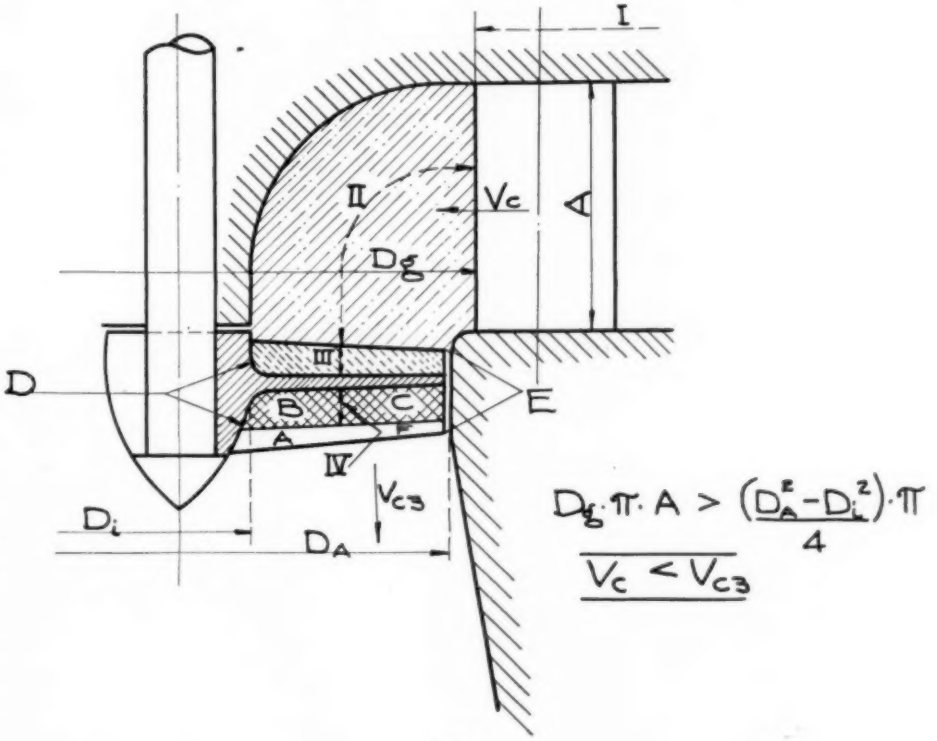


FIG. 2

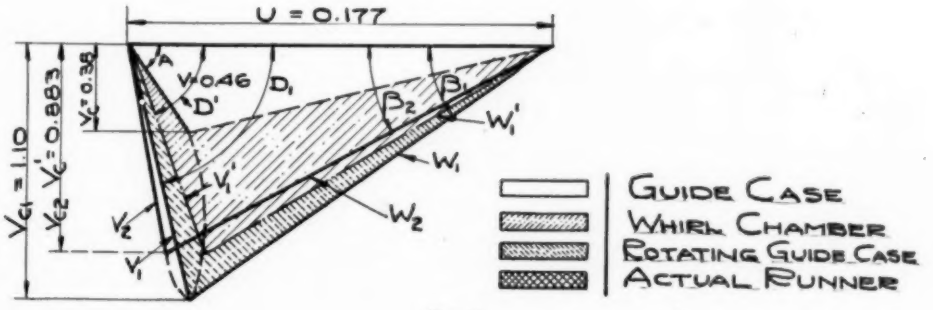


FIG. 3

3 Since the fundamental conditions involving cavitation are now sufficiently known, and since the forces producing cavitation are so destructive that any practically applicable material so far known succumbs, there is no safer way of avoiding cavitation than to keep away from the conditions which are certain to cause the cavitation; and these are high specific speed and high suction head.

Losses of Pressure Head Due to Sudden Enlargement of a Flow Cross-Section

By DR. H. C. SCHUTT,[1] NEW YORK, N. Y.

The exactness of the Borda-Carnot equation for the losses of energy in a liquid medium passing a discontinued cross-section—that is, a sudden enlargement—is discussed, the same formula is derived, based on one of the main principles in dynamics, and a report is given of experiments[2] proving the validity of this formula.

SYMBOLS

V = average velocity in section
p = intensity of pressure
A = area of cross-section
H = total head
Z = elevation head
h = lost head
h_1 = lost head due to friction
h_2 = lost head due to enlargement
g = acceleration due to gravity
γ = specific weight
η = coefficient.

THE motion of a liquid medium through a pipe, turbine, or pump is subject to losses of pressure head. There are two main causes of these losses of energy: the friction and the discontinuance of the flow cross-section. A sudden enlargement of a pipe is a typical case of the latter. This problem has been subjected to theoretical study in order to find a serviceable formula, and also subjected to experimental research to prove the correctness of the theory.

The Borda-Carnot equation

$$h_2 = \frac{(V_1 - V_2)^2}{2g}$$

gives a value for these losses, but there are divergent opinions as to its validity in all practical cases. An engineer generally applies this formula without a correcting coefficient. The reason may be that he has never encountered any appreciable deviation of his practical experience from his calculations or that the simplicity of the formula appeals to him and makes him inclined to trust those authors of treatises on hydrodynamics who advocate the unmodified validity of the Borda-Carnot equation.

But many authors recommend the correction of the formula by an experimentally determined coefficient as great as 1.1, 1.2, and even 1.3. The existing vagueness of this important and frequently applied formula led to the writing of this paper as an attempt to arrive at a more definite solution of this problem.

Borda[3] was the first to understand the mechanical process and to find a mathematical solution. His formula in the version of Carnot is still valid in modern hydrodynamics. Borda did not exclusively deal with these losses in his paper as the title already indicates; he rather determines the time that it takes to fill a submerged vessel with liquid through an orifice in the bottom. (See Fig. 1.) He correctly develops a differential equation introducing losses due to the sudden enlargement of the flow cross-section from A_1, the area of the orifice, to A_2, the area of the vessel. He assumes water as an inelastic medium and applies the mechanical law for the inelastic shock of solid bodies. As in modern physics water is still defined as incompressible and inelastic, thus justifying his assumption, and as he experimentally verified his formulas and obtained a remarkable conformity of theory and experimental results, there is reason enough to take the unrestricted validity of the formula

$$h_2 = \frac{(V_1 - V_2)^2}{2g}$$

for granted and rather examine the details of those experiments

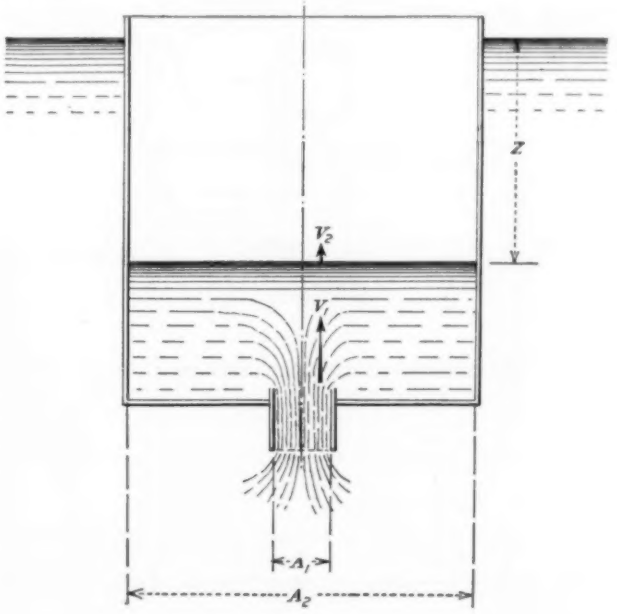


FIG. 1

apparently proving the necessity of correcting the Borda equation by a coefficient

$$h_2 = \eta \frac{(V_1 - V_2)^2}{2g}$$

Gibson[4] expresses η as dependent on the ratio of enlargement and the smaller diameter by the experimentally found relation

$$\eta = \frac{102.5 + 0.25\,m - 2D_1}{100}$$

where $m = \frac{A_2}{A_1}$ and D_1 = the smaller diameter. This equation gives a value for η in an experimental case reported below, where

$$m = 2.84;\ D_1 = 3.75;\ \text{and}\ \eta = 0.957.$$

[1] Mechanical Engineer.

[2] The experiments have been made in the laboratory of the Institute of Hydraulics in Munich, Germany. The Transactions of that Institute for 1926 contain a report of the test results.

[3] Memoire de l'Academie royale des Sciences, Paris, 1766: Memoire sur l'ecoulement des fluides par les orifices des vases.

[4] A. H. Gibson, "Hydraulic and Its Applications," p. 86.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

The validity of this auxiliary formula is wisely limited for a certain range of m and D_1, but significant is the fact that η depends on the ratio of enlargement and can even obtain a value less than 1. Generally the opinion is to give η a value greater than 1.

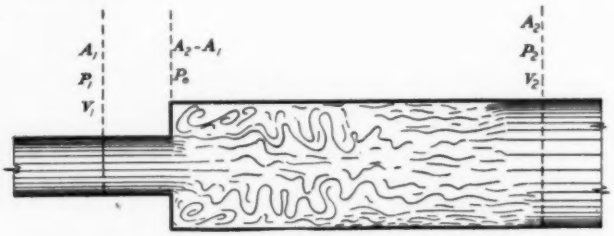


FIG. 2

Considering the mere physical occurrence of a flow of water through a suddenly enlarged pipe (see Fig. 2), there is to be included the process of the mixing of the jet leaving the smaller section with the slower moving water of the larger section. This process takes place over a greater length of the flow direction; and the actual influence of the enlargement can only be discovered by examining two cross-sections—one just before the enlargement and one far enough behind where the mechanical process has surely terminated.

Under this presumption the formula can briefly be developed in the following way. The losses are:

$$h = h_1 + h_2 = H_1 - H_2 \quad [1]$$

$$h = \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 - \left(\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2\right) \quad [2]$$

$$h = h_2 = \frac{p_1 - p_2}{\gamma} + \frac{V_1^2 - V_2^2}{2g} \quad [3]$$

omitting the friction losses h_1, as they can easily be eliminated by experiments, and making $Z_1 = Z_2$ by choosing a horizontal arrangement.

Furthermore, it is according to a law of mechanics that the sum of the forces in the direction of motion causes the change of the impulse

$$A_2 p_2 - A_1 p_1 - (A_2 - A_1)\, p_0 = m(V_1 - V_2) \quad [4]$$

where m is the mass of the liquid flowing through per second

$$m = A_2 \frac{V_2}{g} \gamma \quad [5]$$

Assuming $p_0 = p_1$, which has especially to be verified by the experiments, then is

$$A_2(p_2 - p_1) = \frac{A_2 V_2}{g} \gamma\, (V_1 - V_2) \quad [6]$$

$$\frac{p_2 - p_1}{\gamma} = \frac{V_2(V_1 - V_2)}{g} \quad [7]$$

and combining [7] and [3]

$$h_2 = \frac{V_1^2 - V_2^2}{2g} - \frac{V_2(V_1 - V_2)}{g} \quad [8]$$

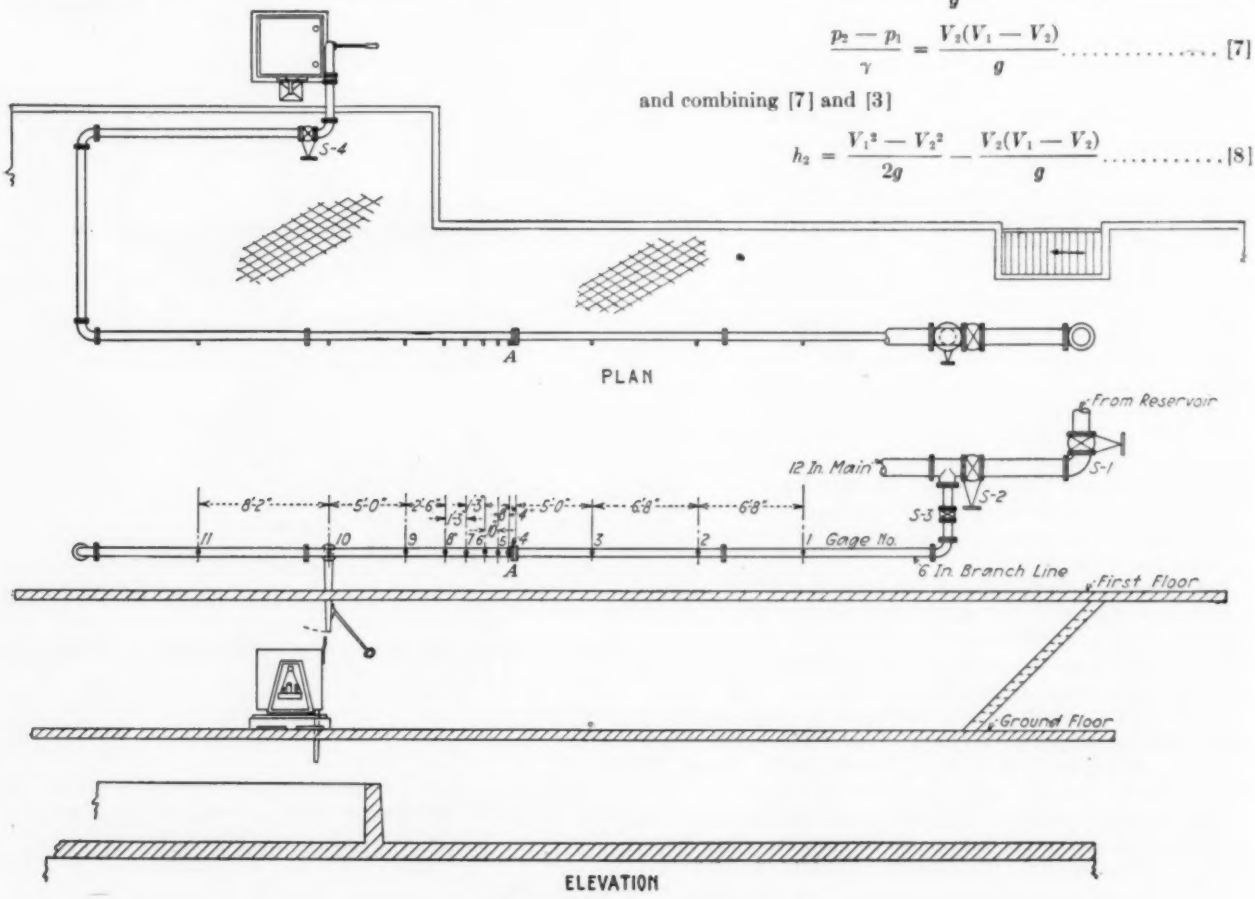


FIG. 3 EXPERIMENTAL ARRANGEMENT

$$= \frac{V_1^2 - V_2^2 + 2V_2^2 - 2V_1V_2}{2g} \quad [9]$$

$$= \frac{(V_1 - V_2)^2}{2g} \quad [10]$$

This is indeed the Borda-Carnot equation again.

Experiments have been made by the author of which a brief report is given below.

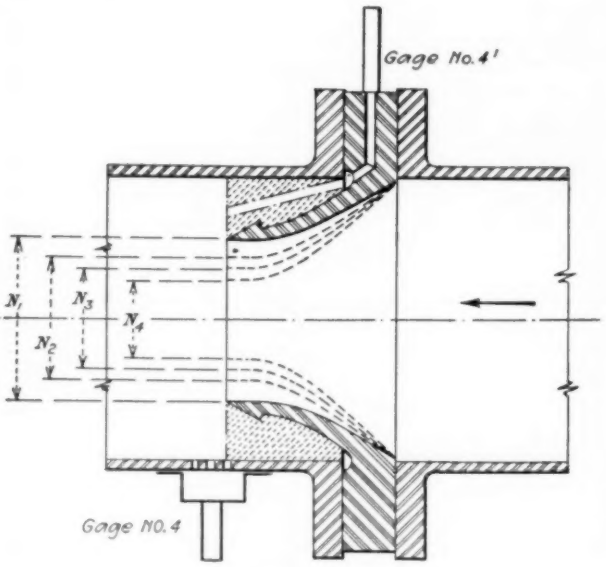


FIG. 4 SECTION AT *A*, FIG. 3

The experimental arrangement is shown in Fig. 3. The water is supplied by a high reservoir through the 12-in. water main passing the valves *S*-1 and *S*-2, and taken off by a branch line through the valve *S*-3. The reservoir has a sectional overflow, thus guaranteeing a constant head, which is throttled down by the three valves equally in order to obtain an undisturbed flow in the branch line, the actual research line with the observation spots numbered 1 to 11. At *A* a nozzle (Fig. 4) is laid in the line between two flanges, thus effecting a contraction of the flow cross-section and consequently the desired sudden enlargement. The special shape of the nozzle is to avoid any "vena contracta" at the mouth of the nozzle. The location of the observation spots is chosen for obtaining the best knowledge of the pressure gradient and consequently of the actual losses. Open water columns are used as pressure gages; the connection to the pipe is shown in Fig. 5. The quantity is measured by weight to determine the average velocity V_2.

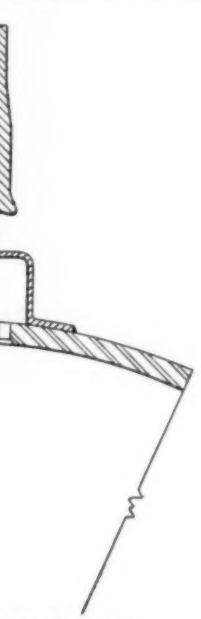


FIG. 5 TYPICAL GAGE CONNECTION

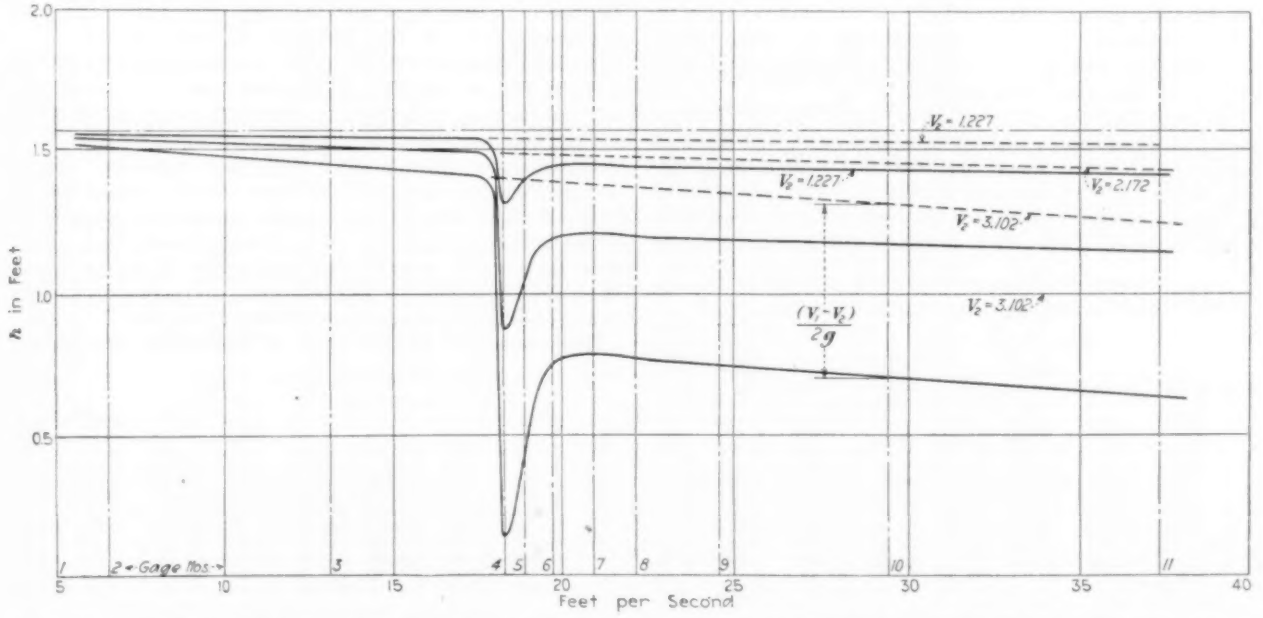


FIG. 6 OBSERVED PRESSURE-HEAD GRADIENT AT THE VELOCITIES V_2 EQUAL TO (1) 1.227, (2) 2.172, and (3) 3.102 FEET PER SECOND. THE DOTTED CURVE IS THE MERE FRICTION GRADIENT

TABLE 1

Nozzle	Pipe area, A_2 in sq. in.	Effective flow cross-section of nozzle, A_1 in sq. in.	Ratio of enlargement	Tests made within the range of V_2 in feet per second From	To
N_1.....	27.5	9.74	2.84	1.09	3.10
N_2.....	27.5	5.80	4.74	1.06	3.02
N_3.....	27.5	4.58	6.03	0.77	2.34
N_4.....	27.5	3.06	9.01	0.51	1.58

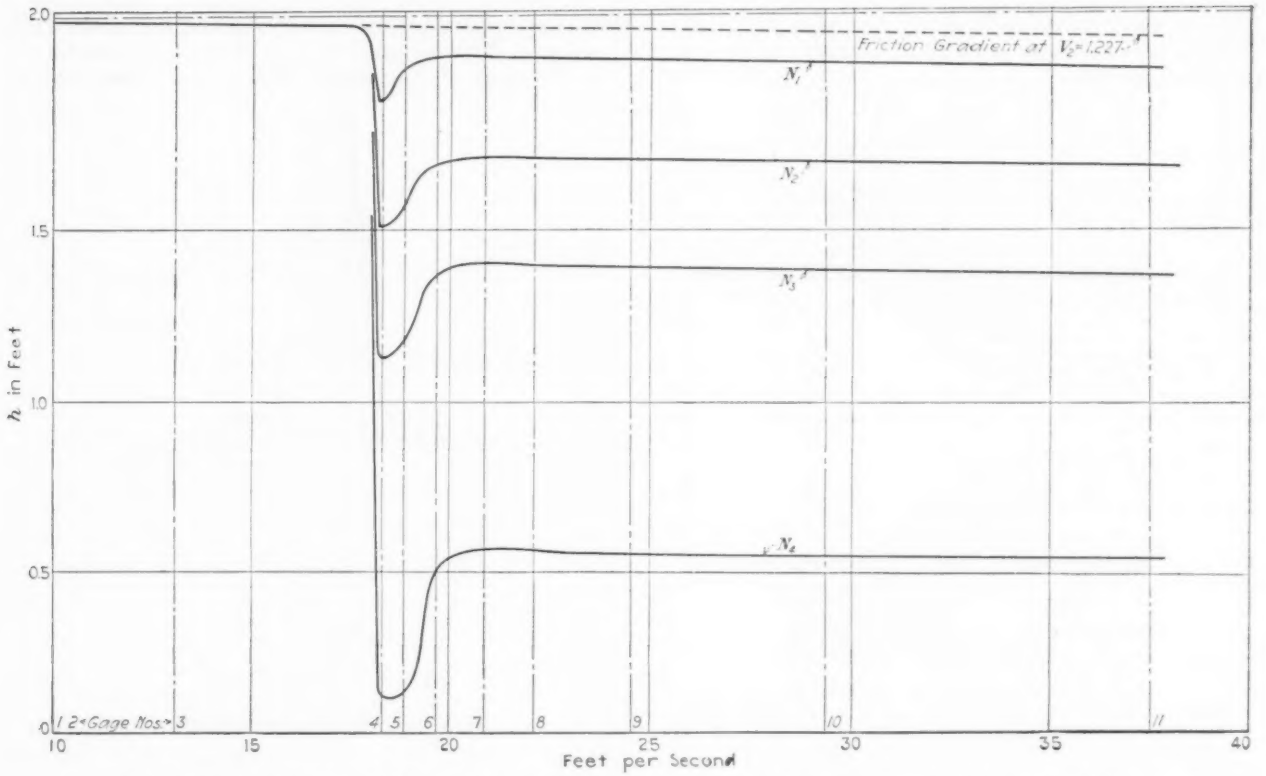


FIG. 7 PRESSURE-HEAD GRADIENT FOR THE FOUR NOZZLES AT THE SAME VELOCITY V_2 = 1.227 FEET PER SECOND

Experiments have been carried out with four nozzles. Table 1 shows some general data.

The experiments have been repeated with the space behind the nozzle filled with gypsum in order to ascertain any possible influence of this space upon the losses.

The experimentally obtained losses consist of the friction losses and those due to the sudden enlargement. In order to eliminate the friction losses, first the friction gradient of the pipe, sections 1 to 11, has been determined within the range of the velocity applied. In Fig. 6 the observed friction gradient as well as the total pressure-head gradient is shown for nozzle N_1 at three different velocities, and it is explained how the immediate test results have been valuated. Fig. 7 gives the pressure gradients for the four nozzles at the same velocity.

A summary is given in Table 2 for the two nozzles N_1 and N_2 of about 400 tests which have been made and which are graphically and numerically analyzed. Observations 1_1 to 1_{11} designate the static-pressure head at the corresponding gages 1 to 11.

In addition to Table 2, Fig. 8 shows the test results. It may be noted that the validity of the Borda-Carnot equation for the practical computation of any losses due to a sudden enlargement of the flow cross-section is proved. The deviations are not more than 1.5 per cent, as these experiments have shown, and even these deviations may be ascribed partly to mere irregularities in the observation, as they are sometimes negative.

The assumption that $p_0 = p_{11}$, as made when deriving the

TABLE 2 EXPERIMENTS WITH NOZZLE N_1

(*The space in back of the nozzle is unfilled*)

No.	V_2 in ft.	V_1 per sec.	$h_2-\frac{V_1-V_2}{2g}$ (Borda-Carnot)	1_1 to 1_{11}	1_1 to 1_{11} (friction)	h_2 experiments	Deviation $h_2$exp − h_2	Deviation in % of h_2
			in 1/100 of a foot					
1...	1.086	3.088	6.221	10.660	4.330	6.330	0.109	1.7
2...	1.227	3.490	7.965	13.460	5.420	8.040	0.075	1.0
3...	1.467	4.161	11.280	18.710	7.220	11.490	0.210	1.8
4...	1.658	4.713	14.53	23.80	9.02	14.78	0.25	1.7
5...	1.870	5.320	18.45	29.70	11.32	18.38	−0.07	−0.4
6...	2.042	5.810	22.10	35.45	13.32	22.13	0.03	0.1
7...	2.172	6.175	24.90	39.90	14.76	25.14	0.24	1.0
8...	2.328	6.610	28.52	45.95	17.22	28.73	0.21	0.7
9...	2.474	7.015	32.20	51.55	19.18	32.37	0.17	0.5
10...	2.658	7.565	37.30	59.04	21.65	37.39	0.09	0.3
11...	2.890	8.220	44.10	69.22	25.10	44.12	0.02	...
12...	3.102	8.810	50.55	78.80	28.20	50.60	0.05	0.1
			(*The space in back of the nozzle is filled*)					
1...	1.034	2.942	5.61	9.82	4.17	5.65	0.04	0.7
2...	1.270	3.642	8.75	14.43	5.48	8.95	0.2	2.0
3...	1.466	4.163	11.29	18.37	7.15	11.22	−0.07	−1.0
4...	1.595	4.533	13.40	22.32	8.37	13.95	0.55	4.0
5...	1.798	5.106	17.07	27.90	10.50	17.40	0.33	2.0
6...	1.909	5.428	19.25	32.15	11.98	20.17	0.98	5.0
7...	2.025	5.755	21.60	35.10	13.13	21.97	0.37	1.5
8...	2.156	6.122	24.46	40.00	15.09	24.91	0.45	1.8
9...	2.283	6.485	27.50	44.10	16.35	27.75	0.25	0.9
10...	2.415	6.868	30.80	49.90	18.38	31.42	0.62	2.0
11...	2.727	7.754	39.15	62.00	22.79	39.21	0.06	0.2
12...	3.082	8.670	48.40	76.10	27.68	48.42	0.02	...
						Average less than		1.5

EXPERIMENTS WITH NOZZLE N_2

(*The space in back of the nozzle is unfilled*)

1...	1.062	5.020	24.20	28.40	3.90	24.50	0.30	1.0
2...	1.231	5.838	32.80	37.80	4.90	32.90	0.1	0.3
3...	1.454	6.862	45.50	53.20	7.40	45.80	0.3	0.7
4...	1.559	7.370	52.50	60.70	8.20	52.50	...	...
5...	1.726	8.140	64.00	75.50	10.20	65.30	1.30	2.0
6...	1.877	8.890	76.10	88.60	12.00	76.60	0.50	0.7
7...	2.054	9.705	91.00	105.00	13.90	91.10	0.10	...
8...	2.258	10.660	110.00	127.50	16.75	110.75	0.75	0.7
9...	2.380	11.240	122.00	141.10	18.40	122.70	0.70	0.5
10...	2.580	12.180	143.20	165.50	21.30	144.20	1.00	0.7
11...	2.717	12.840	159.20	182.50	23.00	159.50	0.30	0.2
12...	2.972	14.050	191.20	217.60	25.60	192.00	0.80	0.4
			(*The space in back of the nozzle is filled*)					
1...	1.135	5.354	27.60	31.82	4.27	27.55	−0.05	...
2...	1.279	6.040	35.23	40.85	5.58	35.27	0.04	...
3...	1.456	6.880	45.64	53.15	7.35	45.80	0.16	0.4
4...	1.630	7.685	57.10	65.30	8.53	56.77	−0.33	0.6
5...	1.804	8.520	70.00	79.40	10.30	69.10	−0.90	1.0
6...	1.918	9.054	79.16	91.70	12.34	79.36	0.20	0.3
7	2.030	9.580	88.80	102.25	13.12	89.13	0.33	0.4
8	2.208	10.420	104.90	121.40	15.78	105.62	0.72	0.6
9...	2.440	11.525	128.40	148.00	19.20	128.80	0.40	0.3
10...	2.575	12.150	142.70	162.40	20.35	142.05	−0.65	0.4
11...	2.820	13.330	171.70	196.80	24.75	172.05	0.35	0.2
12...	2.974	14.060	191.10	217.50	26.90	190.60	−0.50	0.2
							Average less than	1.0

formula, is correct, as it has been especially observed in each case at gages 4 and 4[1]. There is no influence of the ratio of enlargement nor an effect of the dead-water space in back of the nozzle upon the losses.

As can be seen in Figs. 6 and 7, the influence of the nozzle upon the current—that is, the effect of the sudden enlargement—is still perceptible at the observation spot 8, or about ten times the pipe diameter away from the enlarged cross-section. In this fact there may be an explanation for the opinion of several authors that experiments show the necessity of the correction of the formula by a coefficient. In many experimental cases the regain of pressure head out of velocity head has not been fully observed by taking the pipe section behind the enlargement too

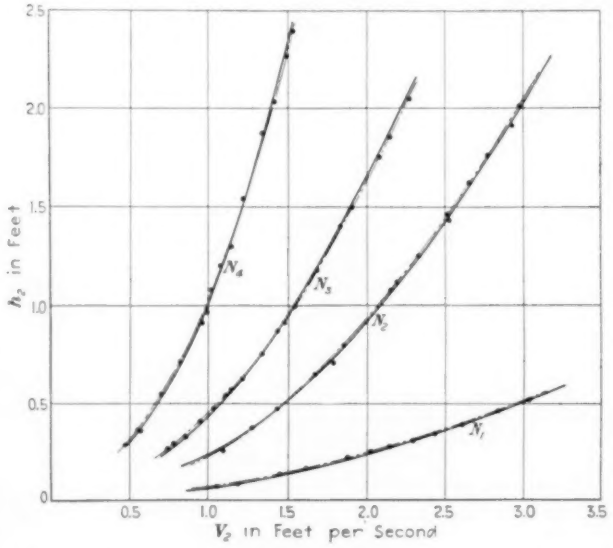


FIG. 8 FORMULA $h_2 = F(V_2)$ ACCORDING TO THE FORMULA $h_2 = \frac{(V_1 - V_2)^2}{2g}$

(The dots on the curves are placed at the points found by experiments.)

short. This was, for example, the case with the experiments of H. Baer.[5]

In hydraulics and especially in aerodynamics the most valuable knowledge can be gained only by analyzing a process qualitatively first and then making quantitative tests. This is the course generally taken in modern engineering science. Borda did not know exactly the character of the physical process taking place in case of a sudden enlargement of the flow cross-section; but his experimental arrangement seemed to prove his assumption accidentally. His explaining the process as a shock of inelastic masses may have prevented this important matter in hydraulics from really being qualitatively examined.

Though these experiments have furnished sufficient evidence as to the validity of the Borda-Carnot formula for technical calculations, the author thinks it worth while to make tests with a similar arrangement, where a greater accuracy can be obtained, and special attention given to the velocity distribution in the different cross-sections. Such experiments may at the same time complete our knowledge of the properties of flow nozzles as fluid meters.

[5] Dingler, Polytechnisches Journal, 1907, p. 177.

Discussion

BLAKE R. VAN LEER.[6] This paper is a partial translation of the article "Versuche zur Bestimmung der Energieverluste bei Plotzlicher Rohrerweiterung," pages 42–58 of vol. 1, "Mitteilungen des Hydraulischen Instituts der Technischen Hochschule Munchen," which work Dr. Schutt performed under the direction of Dr. Thoma as part of his work for his Doctor of Engineering degree.

The value of the publication of this article lies in the fact that it directs the attention of American engineers to additional experimental data upon the subject and gives this data in English units, whereas the article referred to gives it in the metric system. Also, the only other published results with which I am familiar are those published years ago by H. W. Archer in the Proceedings of the A.S.C.E., vol. 39, no. 3, March, 1913, who obtained the equation:

$$h^1 = 1.098 \frac{(V_1 - V_2)^{1.919}}{2g}$$

I think the appraising of the equation which Dr. Schutt discusses is shown in a statement by Prof. J. N. LeConte in his book "Hydraulics," McGraw-Hill Co., 1926, p. 98: "Hence, the loss due to a sudden enlargement is numerically equal to the square of the difference of the velocities divided by $2g$. The result is well borne out by experiment. . . . When a sudden enlargement exists in a pipe line, the loss can be entered in the energy equation by means of the expression $(V_1 - V_2)^2/2g$ without the use of a k-coefficient."

[6] Assistant Secretary, American Engineering Council, Washington, D. C. Mem. A.S.M.E.

Progress in Hydraulics

Contributed by the Hydraulic Division

Executive Committee: L. F. Moody, *Chairman*, B. E. White, *Secretary*, H. L. Doolittle, W. M. White, and Ely C. Hutchinson

MUCH the greater part of the developed and undeveloped water power of the United States and Canada is owned by public-utility companies. For this reason, the consolidation of electrical service and holding companies into fewer and larger business organizations is a development which vitally affects the field of hydraulic engineering, because it affects the economics of hydroelectric power. These effects are both favorable and unfavorable.

The much wider, and possibly almost universal, adoption of the so-called promotional or inducement form of rate, together with the probability of uniform rates for different classes of service throughout areas of considerable size, will accelerate the demand for electrical service, thus creating a much larger market to be supplied.

On the other hand, the much larger and more economical fuel power stations made possible by the consolidated utility groups and the greater demand, have changed the load conditions and lowered the unit costs of energy to a level with which many water powers can no longer compete.

Factors Affecting the Economics of Hydroelectric Power Development

One of the major effects of the increasingly large combinations of utility companies has been upon the economics of many water-power sites, which had hitherto been considered favorably for development. With the vastly increasing markets, the load factor demanded from many sites and plants has been profoundly changed, while at the same time the construction of vastly larger and much more efficient fuel power-generating stations has resulted in a lower competitive cost which many water-power sites cannot meet. A partial solution of this difficulty may possibly be found in a simplification of hydroelectric construction, which may result in lower construction and operating cost. The automatic and semi-automatic plants are types of the tendency in this direction. Likewise, the outdoor or semi-outdoor type of hydro generating station, consisting of one or more units remotely controlled, either from another power station or by variations in forebay water level, are variations of this tendency.

The discovery and development of an important natural-gas field in central California has made it possible to produce steam power in gas-fired installations at a lower cost than previously obtained with oil-fired boilers and many hydroelectric plants. In the opinion of many on the Pacific Coast, it is probable that new hydroelectric developments in that region will be greatly retarded, if not nearly discontinued, for a period of years by this development.

Contrasting with the opinions of some as to the economics of hydro power versus steam power, is the situation reported in the Pennsylvania area, where with all its coal mines a four-fold increase in hydro power installation has taken place within the last six years. Apparently there is still a field for the skilful hydroelectric engineer who makes his designs on the basis of dollar efficiency.

Ample water-storage capacity is becoming an ever more necessary adjunct of commercially efficient hydroelectric development, and many sites heretofore considered unavailable are constantly being found to be commercially feasible.

There is said to be an increasing tendency to purchase hydraulic equipment on the basis of price alone. It seems hardly necessary to call the attention of engineers to the fact that there is still room for advances in design and construction, which can only be brought about by continued study and experiment, at some one's cost. In this field the logical source of such experimentation is the manufacturer. Price cutting below the level of a fair profit will inevitably dry up these springs from which improvements flow, and at the same time introduce the hazards that go with poor design and construction, and details slighted or omitted to save cost, with consequent lessened efficiency and increased maintenance costs.

The outstanding tendency is to build larger and larger units, limited only by the quantities of water available from season to season, or by the limitations of mechanical design or of transportation. This has been made commercially feasible by the larger markets, and by the large aggregations of available capital resulting from the great groupings of utility organizations which have recently been brought about and are still continuing their growth.

Governmental Regulation and Supervision of Utilities and Structures

There are many who view with concern the increasing burden constantly being thrust upon utility concerns, who are by far the largest aggregate owners of water powers, and upon water-power development in particular, through the medium of legislation and administrative regulation. The margin of cost between fuel-generated and water-generated power and energy in nearly all parts of the country has been cut to such small dimensions that the taxes and license fees recently imposed have made the development of additional water power almost, if not quite, a hopeless proposition, so far as competition with steam-, oil-, or gas-generated energy is concerned.

The public authorities, as represented by the state engineers, etc., are exercising a greater degree of control and supervision over the design and construction of high dams than ever before, as a result of such failures as that of the St. Francis Dam.

Governmentally Owned Hydro Plants and Projects

During the year, the largest hydro power development thus far projected, that at Boulder Canyon, on the Colorado River, has been authorized. In connection with it there will be constructed the highest dam in the world, impounding probably the largest artificial body of water in existence. The construction of this project involves many difficult problems of engineering design and construction, as well as many problems arising out of its ownership, construction, and operation by the Government, the sale of its output to utility companies, municipalities, etc., and the transmission of the energy output to great distances.

The disposal of the Muscle Shoals power plant, or of the energy thereby produced, is still under consideration. Likewise the status of the combined St. Lawrence River power and navigation development is practically unchanged, with little to indicate that any definite steps will be taken toward its construction at an early date.

Model Testing

Turning to the more technical field, the subject of model testing, or the experimentation with small models of proposed conduits, machines, or structures, received a great impetus through the publication of John R. Freeman's book on "Hydraulic Laboratory Practice." This monumental work has brought the subject and its possibilities vividly to the attention of many engineers and executives who have hitherto not realized them. It is not surprising, therefore, that model testing, on a greatly accelerated scale, is now going on.

An increasing use of model testing is being made in the investigation and design of the water passages leading to turbine units. Many power companies are finding it well worth while to investigate the possibilities of variations from the accepted forms of intakes and water passages, or those designed by the ordinary rule-of-thumb method, as in the past. In some cases very large savings in earth or rock excavation have been made. In others, more efficient structures have been produced with the same volumes of masonry, or equal efficiency obtained with structures of decidedly less cost. These tests have also revealed the surprisingly large influence of small bends, and other variations in cross-section and alignment, upon the efficiency of the units, the elimination or alteration of which paid very handsome dividends upon the cost of the tests.

Mr. Freeman's book having been written primarily to describe the numerous European laboratories and their methods, and in advocacy of an American national hydraulic laboratory, it is perhaps natural that many received from it the impression that the laboratories in this country are decidedly inferior. It is undoubtedly true that there are but few of them which are equipped to carry on many of the investigations now being regularly made across the Atlantic. It is fair to say, however, that there are in this country laboratories which possess every requisite for the making of any type of hydraulic experiment for which any probable need may arise.

At least one of the university laboratories has been engaged for a considerable length of time in making experiments involving river hydraulics, and more particularly in developing suitable spillways, energy-dissipating devices, channels, etc. A particularly large model of a section of approximately 1½ miles of the channel of the Columbia River, at, above, and below Rock Island, has been constructed at the Alden Hydraulic Laboratory, Worcester (Mass.) Polytechnic Institute and experiments are now being made to determine the best type of dam, number of gates, etc. to effectively control the flow at high flood stages.

While the discussion of the proposed National Hydraulic Laboratory is continuing and little apparent progress is being made, various universities and governmental and private laboratories have been newly established or have been expanding their facilities. Some have expressed the belief that the parceling out of important studies among laboratories scattered throughout the country would exercise a profound influence for good upon the practice of hydraulic engineering. It is held that the much wider dissemination of the knowledge of and experience in model testing and its relation to the results with full-sized structures will stimulate research and accelerate the discarding of time-honored, rule-of-thumb methods of design.

In the field of model testing also, a method has been worked out by the engineers of the U. S. Reclamation Service, Denver, Colo., for the design of arch dams by the trial-load method. This is the first method thus far proposed in which the shape of the canyon is taken into consideration in the determination of stresses.

This is, no doubt, a direct outcome of the work of Engineering Foundation's Committee and associates at the Stevenson Creek arch dam, California, and of the corresponding model-arch tests made by Professor Beggs, of Princeton University.

Dams and Accessories

In the recent past, what had been considered minute and unnecessary details of dam construction have been receiving the attention that they deserve. It has been realized that contraction joints should be provided in at least two directions in massive unreinforced concrete masonry, in order to avoid possibly dangerous cracking. New methods of placing concrete, notably the use of articulated belt conveyors and towers, have recently been introduced, thus avoiding the horizontal flow of concrete and the segregation of material. Likewise, the water-cement ratio need no longer be influenced by the consideration that the mixture should be plastic enough to flow readily in chutes.

Recent investigations by means of telemeters imbedded in large concrete masses, such as dams, have revealed the existence of high local temperature stresses. Investigations in regard to this matter are now in progress at the Shaver Lake Dam, in southern California, and at Bull Run Dam, in Oregon.

The use of electric-arc welding has been extended to the construction of racks and gates of various types, producing structures of better design and greater strength and resistance than were heretofore obtainable with the usual bolted or riveted connections.

The increasing use of the controlled spillway instead of the formerly almost universal fixed spillway has also resulted in a marked reduction in the cost of many projects. A particular advantage is that of avoiding the occasional flowage of large areas of land, at times of flood, which are normally free of water, thus reducing the cost of flowage land and the damages from flooding.

Measurement of Water

There is also reported a markedly increasing tendency among power companies to have their units tested for efficiency under operating conditions. There is also a rapidly accelerating tendency to rate some hydraulic section adjacent to each unit for use in connection with a flow recorder, so as to be able to maintain a continuous record of water flow for comparison with electrical output, as a check against efficiency, and also to maintain a continuous record of stream flow. The latter element is often of very great importance, since the construction of many hydro plants drowns out the excellent gaging stations maintained under the supervision of the U. S. Geological Survey or state engineering departments, thus causing the loss of all reliable means of measuring the flow of the river.

Where the units are equipped with flow recorders, the forebay with a water-level recorder, and in addition a record is kept of spillway-gate or flashboard openings, it is possible to duplicate with reasonable accuracy the records formerly kept at river-gaging stations. The value of these records in checking station performance, as well as in planning future development, is hard to estimate.

In the field of pure hydraulics there should be mentioned the Parshall venturi flume developed by Ralph L. Parshall, Mem. A.S.C.E., of the U. S. Department of Agriculture, Fort Collins, Colo., for the measurement of the flow of water in open channels. This flume is extremely simple, as well as very accurate. Mr. Parshall has also conducted valuable experiments for the determination of the relation of the evaporation from large water surfaces to that from various kinds of evaporation tanks.

Adjustable-Vane Turbines

In the field of hydraulic machinery, the most notable development of the past year has been the installation, by the S. Mor-

gan Smith Company, at the Devil's River plant, in Texas, of a Kaplan-type high-speed turbine, whose blades are adjustable by means of the governor. A second unit of this kind is to be installed by the Wisconsin Power & Light Co. at their Rockton Plant, and several additional units are now being built for the Nova Scotia Power Company. High-speed runners, with hand-adjustable blades, had been previously installed at the Chippewa Falls Plant of the Northern States Power Company, and more are being installed by the Dominion Engineering Works, Ltd., Montreal, a subsidiary of I. P. Morris & De La Vergne, Inc., in the Back River project of the Montreal Power & Light Company, near Montreal, Canada, as well as at the Upper Hook Falls development of the Nova Scotia Power Commission. Two of the Back River units have been built with ejectors, which are expected to increase the output of the machines approximately 1000 hp. each during periods of high water. This type of installation permits a much more efficient use of a variable flow on a variable head than turbines with fixed vanes, and will undoubtedly have a marked effect upon the economics of many low-head, run-of-river projects.

High-Speed Propeller-Type Turbines

The successful application of propeller-type turbines to heads as high as 60 ft. has been demonstrated by the installation of a fifth 28,000-hp. I. P. Morris unit in the Great Falls plant of the Manitoba Power Company. Here the head varies from 17½ ft. to 60 ft. Control of cavitation was secured by providing sufficient blade area to give liberal values and distribution of pressure intensity.

Impulse Turbines

In the field of impulse turbines, improvements have been made in their efficiency by improving the shape of the buckets and nozzles, and by the addition of baffles to prevent water from entering the housing.

There has recently been put into operation an impulse turbine of 32,200 hp., at 143 r.p.m., under 800 ft. head, of the double overhung impulse type. The impulse wheels are each 176 in. in diameter, or 21 in. greater than that of the largest previous units. The jets impinging on the buckets are 14 in. in diameter. This unit was installed by the Allis-Chalmers Company, Milwaukee, Wis., in the San Francisquito No. 1 Plant of the Bureau of Power and Light of the City of Los Angeles, Cal.

In the Big Creek No. 2-A plant, of the Southern California Edison Company, two double overhung single-nozzle impulse units of 56,000 hp. capacity, under 2200 ft. head, were installed, one by the Pelton Water Wheel Company, San Francisco, Calif., and the other by the Allis-Chalmers Company. These machines are of record capacity for this type of turbine, and have developed a maximum of 70,000 hp. each over long periods.

The Pelton Water Wheel Company also installed two 35,000-hp., double overhung impulse turbines at the Bucks Creek plant of the Feather River Power Company, where an efficiency of 85.05 per cent was obtained. Maximum outputs of 40,000 hp. have been developed.

Active experimentation is being carried on by the Allis-Chalmers Company and the Pelton Water Wheel Company, with the object of still further increasing the efficiencies of impulse units.

Reaction Turbines

During the year, the highest-head plant in the East was placed in operation: namely, the Waterville Plant of the Carolina Power & Light Company, at Waterville, N. C., operating under 861 ft. head. The principal features of this plant are a constant-angle arch dam 180 ft. in height, a 14-ft. tunnel drilled in solid rock, approximately 6 miles in length, and the use of three S. Morgan Smith Co. reaction turbines of 49,000 hp. each.

A 44,000-hp. Pelton turbine, operating under 715 ft. head, was installed in the Southern California Edison Company's Big Creek No. 8 plant, which showed a maximum efficiency of 93 per cent when tested by the Allen salt-velocity method.

Among the interesting reaction-type units may be mentioned the 8550-hp. unit being constructed by I. P. Morris & De La Vergne, Inc., for the Turners Falls Power and Electric Co., for its Cobble Mountain, Mass., development, to operate under 420 ft. head. This unit will operate in connection with a long pipe line and, to limit pressure rises, the rate of gate closure will be limited by non-removable diaphragms in the governor piping instead of a relief valve. Leakage and waste of water through relief valves will be eliminated and the pressure rise definitely limited by a simple device.

Rubber sealing strips, set into the faces of the upper and lower distributer rings, against which the guide vanes tightly close, are a feature of two 36,000-hp. units, to be installed at the Alabama Power Company's Lower Tallassee development, under 88 ft. head. These strips reduce the gate leakage during the hours when the units are shut down, leaving them in condition for immediate starting up in case of necessity by simply opening the guide vanes, yet economizing the water supply.

Rubber-lined bearings have also proved successful in a number of large units. Bearings 22½ in. in diameter are installed at the Back River plant, Montreal, Canada.

Extensive research in the field of reaction turbines and related structures, such as conduits, draft tubes, etc., is being steadily carried forward in the excellent laboratories of turbine manufacturers, universities, and power companies. As this research proceeds, new and valuable testing equipment and methods, as well as important improvements in and data concerning turbines and accessories, are constantly being developed.

Recent advances in electric-arc welding have made it possible to produce arc-welded plate-steel scroll cases, some of which have been constructed with short, straight tangential sections, and some with bumped plates, which combine the interior smoothness and streamlines of cast-metal scroll cases with the strength and ductility of steel. The very much lighter weight obtainable with sheet-metal cases is reflected in the ease and reduced cost of transportation, and also undoubtedly eliminates the hazard of cracked scroll cases, which have been responsible for a number of serious accidents in hydro power plants.

This type of spiral case has been used by the Allis-Chalmers Company, with two 56,000-hp. vertical units, under a normal effective head of 213 ft. having 16-ft. inlet diameters, and for three 36,000-hp., 390-ft.-head turbines, the latter having been installed in Japan. The flow of water to both of these units is controlled by electric-motor-operated butterfly valves. Other companies have also built turbines with welded-steel scroll cases.

One contributor emphasizes the importance of the satisfactory operation of the Oak Grove, Oregon, 850-ft.-head reaction units as a measure of economy. The space requirements for such a unit are much less than those for impulse units of the same capacity, under the same head, thus economizing in the cost of power-house construction. These and other high-head units have now been in operation for approximately five years, with very satisfactory results, thus confirming the practicability and reliability of high-head reaction turbines, concerning which some doubt was felt at the time of installation.

During the past year the Rocky River pumped-storage hydro development has been put into service and has functioned satisfactorily.

Generators

The development of the umbrella-type generator, at the instance of A. C. Clogher, hydraulic engineer of the Electric Bond

and Share Company, has made it possible to reduce the height of power houses. This type of generator requires but two bearings, the upper a combination thrust and guide bearing, and lends itself to the semi-outdoor type of installation in which a conical hood of sheet metal protects the generator from rain entering at the top, and at the same time permits ventilation to the outdoor air.

In one installation at least, the Norwood Station of the Carolina Power & Light Co., Mt. Gilead, N. C., air washers have been installed adjacent to the generators, not only to remove the dust in the air, but to cool it to such an extent that better ratings and better generator efficiencies are obtainable in hot weather.

The recently developed Kingsbury spherical combined thrust and steady bearing seems to offer possibilities in connection with this design of unit.

Frequency and Load Control

The widespread interconnection of power systems which has occurred in recent years has made the regulation and maintenance of frequency of vastly increasing importance. There have been developed at least two systems of frequency control which have produced very satisfactory results: namely, the apparatus developed by S. Logan Kerr, of I. P. Morris & De La Vergne, Inc., in cooperation with engineers of Leeds & Northrup, and that of the Warren Telechron Company, both of which have been used sufficiently to demonstrate their value.

I. P. Morris-Leeds & Northrup have also developed a load-control apparatus which has functioned satisfactorily to regulate the load carried by two or more units in a station, in order to maintain the highest efficiency upon a given number of units for a given superimposed load. Savings of as much as 8 per cent in energy generated were disclosed by comparative tests with the use of this apparatus, and with the usual methods of hand regulation. The satisfactory operation of this apparatus at the Norwood plant of the Carolina Power & Light Company, has led to its installation at a number of other plants.

Penstocks

Improvements have also been made in the design and materials of penstocks and other accessories for conveying water to the turbines. There is a notable tendency to use welded pipe, especially in Europe where welded bell-and-spigot joints are being extensively utilized. European, and some American, engineers are using welded stiffening angles and other types of reinforcement for penstocks, wyes, branches, etc., increasingly. Anchorages of welded structural steel are also being used to a greater extent.

Improvements in welding equipment and procedure, the recently developed method of non-destructive testing of welded joints, and the improvement in the quality and strength of welded work are creating increased confidence in pipes and special fittings of welded construction. Their greater interior smoothness, the flexibility, ease and speed of design and construction, and their lower cost will undoubtedly bring about a much wider use of water conduits of this type.

Pumps

Research is continuing in the field of centrifugal and screw-type pumps, as is demonstrated by the papers presented before the Society. High-speed propeller-type pumps are growing more popular. Four Moody spiral pumps, of 70,000 g.p.m. capacity, at 400 r.p.m. under 26 ft. head, have been installed by I. P. Morris & De La Vergne, Inc., for the Robins Dry Dock and Repair Company, in New York.

Fish Screens and Fishways

Electrified fish screens have been the subject of intensive and successful experiment in the Northwest during the past year. Their efficacy in preventing fish from traveling into channels where they are unwanted has been proved beyond a doubt, as practically none were found to have passed through them. Further experiments in the way of using them to direct fish to fish ladders or fishways are under way. Likewise experiments on the various types of fish ladders or lifts and other devices for controlling the movements of fish with safety to the finny tribe, are under way, with promise of success. A reasonably satisfactory solution of this problem will remove a very large part of the objection to the construction of high dams on the Pacific Coast, and elsewhere, where anadromous or migratory fishes are present.

Hydrology

In the field of hydrology, promising advances have been made in predicting annual and even seasonal run-off by the use of cyclic methods applied to widely separated drainage areas whose hydrological conditions and topography are much unlike. Attempts to find definite relations between the volume of precipitation in a given rainfall and the succeeding flood peak and run-off, appear to hold some promise of success. A statistical method of relating annual rainfall and run-off has also been developed, whose preliminary trials on watersheds of different characteristics have given very close results. This, combined with the cyclic analysis and forecast of stream flow, holds much promise of furnishing a reasonably close approximation of past run-off where rainfall records exist but stream-flow records are absent; and also of forecasting the flow for the next season and year. With accumulating knowledge and experience, and an increasing number of investigators, further progress along these lines should be rapid.

Progress Report Committee, Hydraulic Division.
Byron E. White, *Chairman.*

An Investigation of a Rotary Pump[1]

By FRANK V. BISTROM[2] and WILLIAM W. WHITE,[2] SEATTLE, WASH.

THE pump tested was a new $1^1/_2$-in. horizontal-shaft rotary pump carrying three vanes and enclosed in a solid casing. Its rated speed was 800 r.p.m., with a capacity of 40 gal. per minute at this speed.

The pump (Fig. 1) consists of three principal parts: (1) the rotor, (2) the vane-linkage parts, and (3) the casing. The rotor, *R* in Fig. 1, is of cast iron shrunk and keyed to the steel shaft *S*. It serves to transmit the power from the shaft to the blades, and rotates about a center above that of the circular pump chamber, but is kept in close alignment by the collar *C*, set on the shaft against the housing at the opposite end of the shaft, and by the bearing *B*, which supports the rotor and its unbalanced load through the cantilever action of the shaft. The rotor is machined to a clearance of 0.002 in. between the top of the rotor and the casing in the size used in these tests, and it is essential to the efficient operation of the pump that this distance be maintained.

The vanes *V*, three rectangular blades, are propelled by the rotor through the action of small half cylinders of brass *A*. These cylinders permit the vanes to rotate about the stub shaft *F*, on the face plate. This stub shaft is centered with the chamber of the pump so that the vanes are held to a positive clearance from the casing at all times. As these vanes rotate with the drive shaft and rotor they slide back and forth through the cylinder an amount equal to twice the eccentricity of the shaft, and they also have a secondary rotative motion of an oscillatory nature. This second rotative effect is utilized to increase the volumetric displacement of the pump by locating the ports at such a position that the space between the blades will be filled with water when the secondary motion has placed them at the maximum angular distance apart.

The casing is of gray cast iron. It provides a housing for a liberal bearing surface on the forward end of the shaft and an outboard bearing at the other end. The face plate and its stub shaft are drilled and provided with alemite cups for lubrication, as is the shaft bearing immediately behind the rotor. The outboard bearing is ring oiled.

The pump was run under varied speeds and head conditions, noting its performance under the conditions of the A.S.M.E. test code, and keeping a close account of its eccentricities for further investigation. To do this a dynamometer was constructed from a General Electric $7^1/_2$-hp. three-phase induction motor by hanging it on roller bearings between two standard shafting journals. From this unit the power was belted to the pump shaft.

Water was drawn from a pump of constant level through a $2^1/_2$-in. pipe and reduced at the pump to $1^1/_2$ in. Discharge was made through the same size piping to a Fairbanks-Morse tank scales, reading the discharge directly in pounds.

Measurements of suction heads were made entirely with a Budenburg mercury gage attached between the pump and the valve regulating the suction pressure (see Fig. 2). The discharge heads, however, required two instruments for accurate measurements. Up to pressures of 7 ft. of water, a water column was attached to the discharge side of the pump, while for higher pressures a calibrated bourdon pressure gage was used.

A Fairbanks-Morse scales provided for the measurement of the dynamometer torque, and used during the first run at 900 r.p.m. proved to be insufficiently sensitive for further work, and were replaced by Fairbanks-Morse platform scales of better construction.

Motor and pump speeds were taken with a hand revolution counter and watch over a one-minute period. Several of these readings were averaged to secure a more nearly correct speed.

The tests were made with clear water at 8 deg. cent. and with the pump operating under its best mechanical condition. Data were taken for given settings of the suction or discharge valves over a ten-minute period. Each test was started by swinging the discharge pipe into the scales tank at a given time, after the valves had been adjusted to give the desired conditions with the discharge in the tank. Care was required because of the syphon effect of the long vertical discharge. During the run, record was

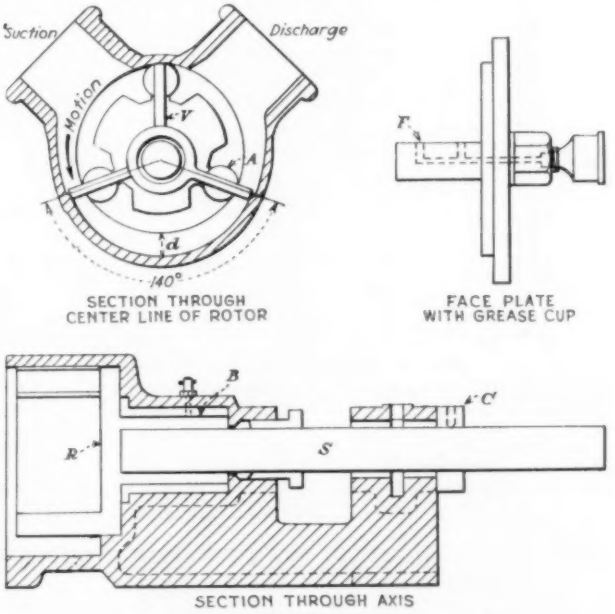


FIG. 1 SECTIONAL VIEWS OF PUMP TESTED

kept of the speed of the motor, of the speed of the pump, of the suction and discharge heads, and of the load on the dynamometer scales. Each of these readings was taken every two minutes together with notes of any unusual performance of the pump.

At the end of ten minutes the discharge was taken out of the tank, the time and the difference in weight of water read and noted. The tank was then emptied back into the sump, the valves adjusted, and the procedure followed as before.

A series of these ten-minute tests taken at the same speed but with varied heads is called a run. Each run shows the results for tests from maximum suction, zero discharge, to 250 ft. of water head on the discharge side. Four runs of 600, 700, 800, and 900 complete the first part of this investigation.

Results of the First Series of Tests

The data computed under the various items is shown in the accompanying graphs. Fig. 7 shows curves for the pump under a suction load at 900 r.p.m. It will be noticed that the point of maximum efficiency, 25 per cent, occurs at a discharge of 36 gal. per min. and under a head of 25.5 ft. of water. The sharp drop

[1] This paper received the A.S.M.E. Student Prize Award for 1929.
[2] Students at the University of Washington.

to the right of this point was accompanied by a severe chattering in the pump which increased as the head was decreased.

Fig. 11 shows the same curves for suction loads at 800 r.p.m., and also indicates the tendency of the chatter at the lower suction heads to lower the efficiency and horsepower output of the pump. It is interesting to note that the volumetric efficiencies of these two runs range between 95 and 100 per cent in the tests in which the chatter was noticed. The 800-r.p.m. efficiency

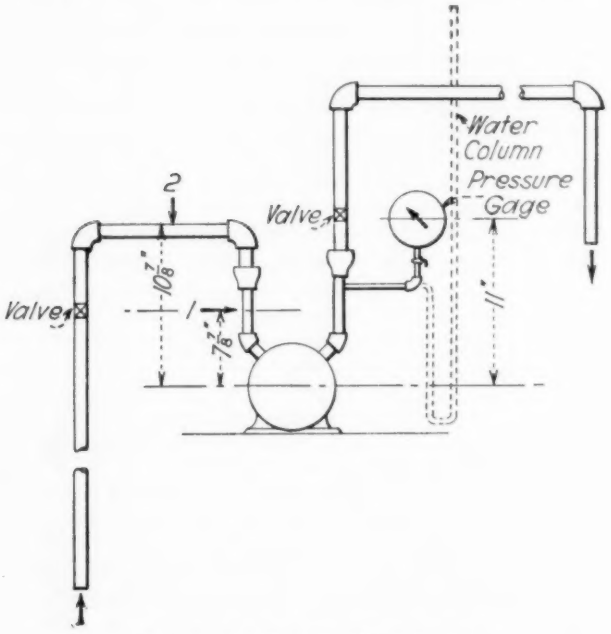


FIG. 2 SET-UP FOR FIRST TESTS

(Mercury column connection No. 1 position used for runs 1 and 2. Position No. 2 used for runs 3 and 4. Pipes 1½ in. at pump, then 2½ in.)

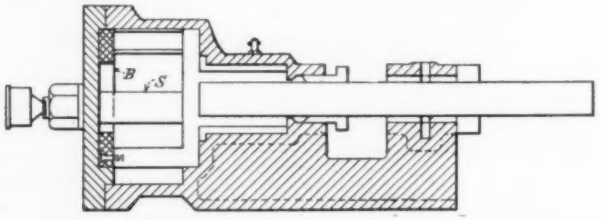


FIG. 3 PROPOSED SUPPORT FOR ROTOR

(*B* is a brass ring bolted to rotor, *S* is a stub shaft connected to the face plate.)

curve shows its maximum of 32 per cent at 32 gal. per min. and with a head of 27 ft.

The characteristic suction curves at 700 r.p.m. (Fig. 13) show a marked change in the behavior of the pump. Not only does the hammer occur at the higher heads, but it increases as the heads increase. However, the general shapes of the curves remain the same, except the head and horsepower curves, which show an ability to rise more rapidly as the quantity of water pumped is decreased. This is the equivalent of maintaining the large flow of water over a considerable increase in head and hence the rapid increase in horsepower. The maximum efficiency (29 per cent) is considerably lower at this speed than at 800 r.p.m., although it is still higher than at 900 r.p.m.

Fig. 9, 600 r.p.m., shows further the tendency to chatter at high suction heads. The chatter, however, was general over the entire run, and the head and horsepower curves rise more gradually as the quantity is decreased. The maximum efficiency at this speed is 29 per cent found at 28 gal. per min. output and under 29 ft. of water head.

Fig. 8 shows a run at 900 r.p.m. At this speed the efficiency rises rapidly to a value near 60 per cent, and then remains practically constant over the remainder of the tests. The head-capacity curve indicates no tendency to slip at high heads. There is no record of hammer in the pump during this run.

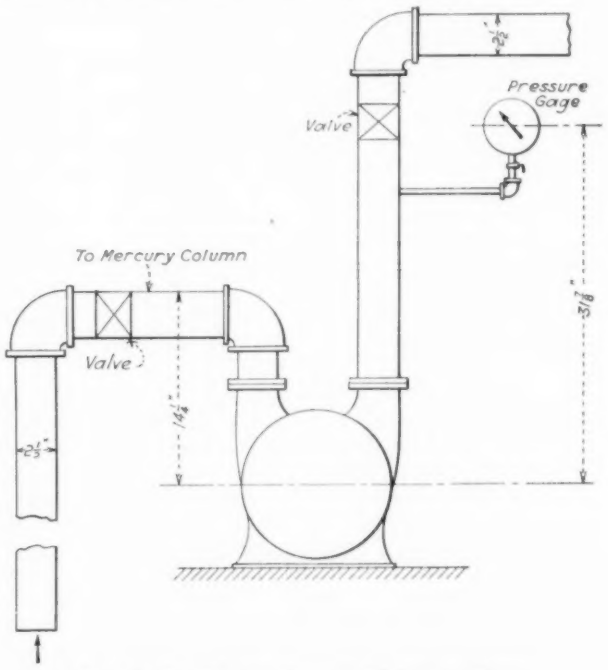
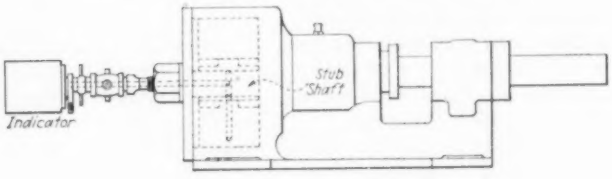


FIG. 4 SET-UP FOR SECOND SERIES OF TESTS



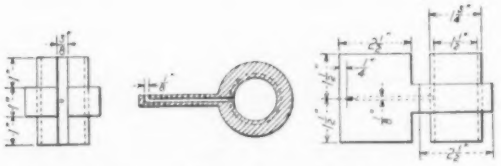


FIG. 5 INDICATOR CONNECTION DETAILS

Figs. 12 and 14 illustrating the pressure runs at 800 and 700 r.p.m., respectively, are typical curves for the pump. The efficiency rises on a curve of decreasing slope to maximums at 39 and 35 gal. per min. output. The 800-r.p.m. run again has the highest efficiency, 72 per cent at the output mentioned. These curves at the higher heads are of interest when considered together with the head-capacity curve, which is practically a straight line under these conditions. Such a consideration indicates that the mechanical losses, not the hydraulic losses, cause the major portion of the loss of efficiency. For, were the losses hydraulic in this region, the rate of decrease of the quantity of water pumped with a given increase in head would increase. Such an action would give the head curve a tendency to turn

upward. Since it does not, the losses must originate in the mechanism of the machine.

The 600-r.p.m. pressure run (Fig. 10) shows the effect of the hammering upon the pump under high heads. From 190 to 270 ft. of water head, some little blade action was noticed. This phenomenon made itself evident in the efficiency curve with a rather rapid drop to a value of 60 per cent at which it remained constant. It is also of interest to note that, during this period, there is only a very slight tendency to increase the drop in the quantity of water pumped for a given increase of head, but that the power taken by the pump shows a tendency to drop below its normal rate of increase.

To make more evident the differences in the operations of the pump at various speeds, the curves in Figs. 15 to 20 were plotted. Fig. 15 shows that with the higher suction loads the output at 800 r.p.m. is a minimum. However, as the suction decreases the output becomes practically proportional to the speed. Here, too, is illustrated the effect of cavitation. Quite evidently cavitation tends to decrease the discharge of a pump working under suction loads. Especially is the effect felt at the higher vacuums. Fig. 16 shows more readily that the capacity of the

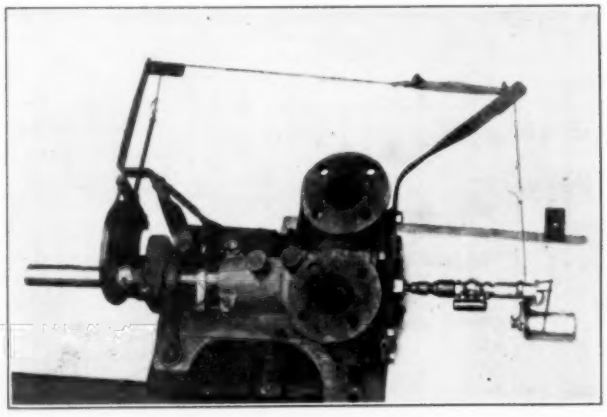


FIG. 6 INDICATOR MECHANISM USED WITH SECOND SERIES OF TESTS

pump under discharge heads is proportional to the speed of the pump, also that the cavitation affects this capacity but very little.

Figs. 19 and 20 demonstrate quite readily the effect of the "hammer" upon the output and input of the pump. In almost every case, both in suction and in discharge loaded runs, the hammer is coincident with an increase in the shaft horsepower and a decrease of the water horsepower. Figs. 17 and 18 show the same thing demonstrated upon the overall efficiency curves. Here also it is seen that the cavitation has a decidedly deterrent effect upon the volumetric efficiency. This effect is most pronounced at the very high vacuums and at the higher discharge pressures. These efficiency curves show too that the rated speed, 800 r.p.m., is unquestionably the most efficient speed. Only at very low suction pressures is the efficiency at 800 r.p.m. below that at any other speed. The volumetric efficiencies for suction runs tend to favor the higher speeds. That of 800 r.p.m. strikes a good mean between these two.

CONCLUSIONS FROM THE PERFORMANCE CURVES

From the performance curves certain conclusions may be drawn embodying not only the essential performance characteristics, but some ideas as to means of improving the performance of the pump as well.

First, regarding the performance of the pump, efficiencies ranging from 60 per cent to 70 per cent over the working range of the pump are exceptionally high. In this connection, it must be remembered that these tests were run under conditions somewhat better than those found in average practice. Es-

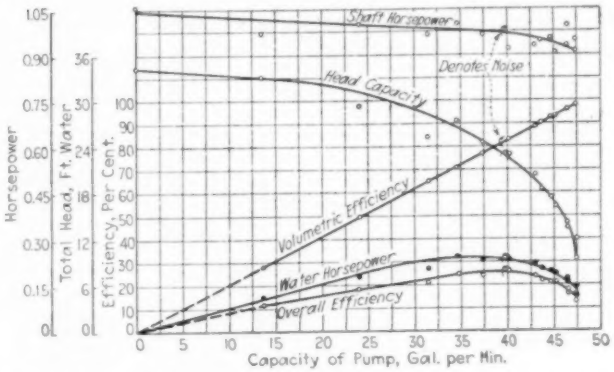


FIG. 7 CHARACTERISTIC CURVES FOR SUCTION RUN NO. 1 AT 900 R.P.M.

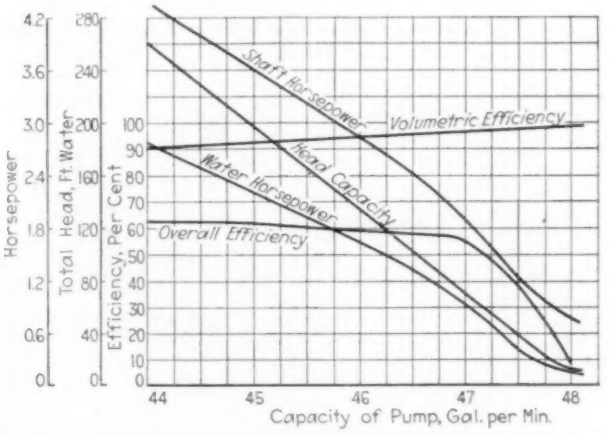


FIG. 8 CHARACTERISTIC CURVES FOR DISCHARGE RUN NO. 1 AT 900 R.P.M.

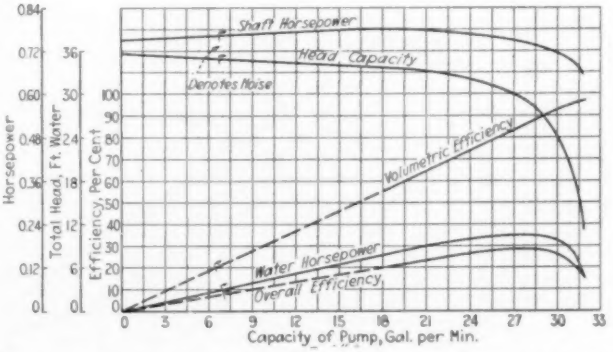


FIG. 9 CHARACTERISTIC CURVES FOR SUCTION RUN NO. 2 AT 600 R.P.M.

pecially is this significant when one considers the high suction heads carried by the pump. Since the pump tested was new and well lubricated, the test should produce results better than those to be expected after considerable wear due to dirty water and long continuous use. However, since the mechanism of the blade assembly holds these blades to a positive clearance from the casing, the wear at the blade ends will not be so severe and the deterioration which will affect most noticeably the efficiency

of the pump will occur in the cylinders and on the shaft which receives the blades.

There is an opportunity for increasing the efficiency of the pump as shown in Figs. 12 and 14. It was demonstrated that the drop in overall efficiency at the higher loads is due largely to an increase of mechanical losses. These losses may have been

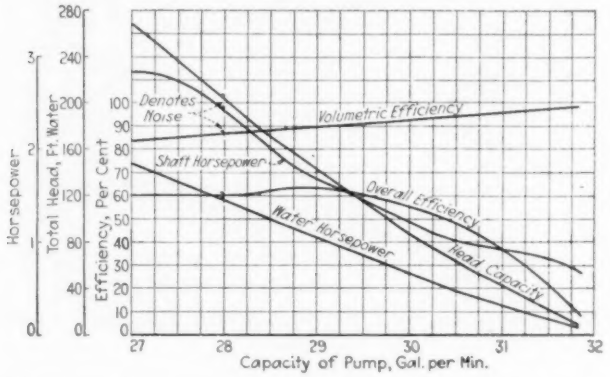


FIG. 10 CHARACTERISTIC CURVES FOR DISCHARGE RUN NO. 2 AT 600 R.P.M.

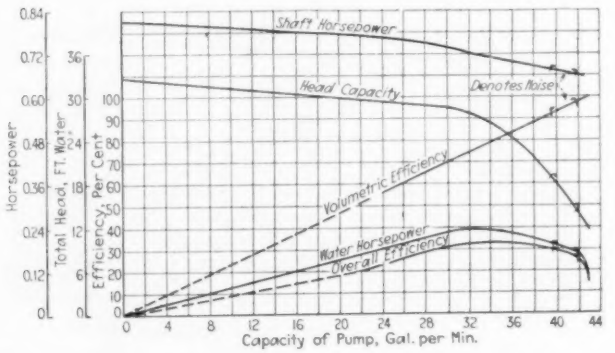


FIG. 11 CHARACTERISTIC CURVES FOR SUCTION RUN NO. 3 AT 800 R.P.M.

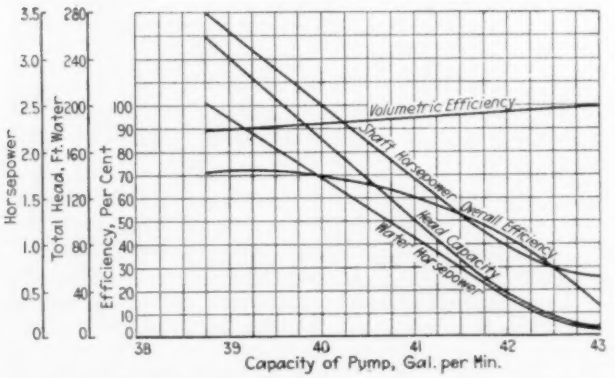


FIG. 12 CHARACTERISTIC CURVES FOR DISCHARGE RUN NO. 3 AT 800 R.P.M.

formed in three ways: First, in the bearings, as a result of the increase of bearing pressure from the belt tension at the higher loads. Second, in the increased bearing losses, a result of the eccentric loading on the rotor, increasing with the higher pressures. Third, the binding action of the rotor back against the rear wall caused by the deflection of the cantilever support of the rotor under the increased eccentric load. This latter and the second loss, to a certain extent, may be relieved by supporting the rotor at both ends. A means of accomplishing this is shown in Fig. 3.

Another means of improving the performance of this pump would be to eliminate, if possible, the hammering that is so prevalent at periods of the pump's operation. This hammering

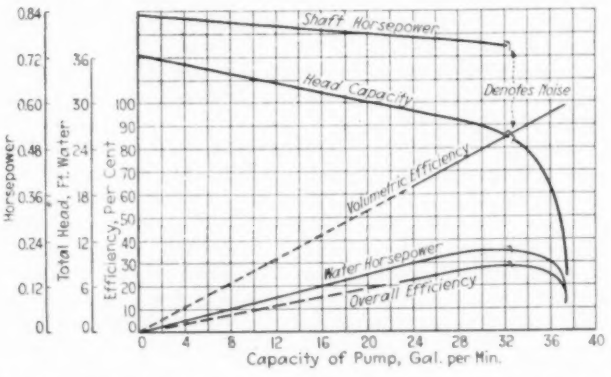


FIG. 13 CHARACTERISTIC CURVES FOR SUCTION RUN NO. 4 AT 700 R.P.M.

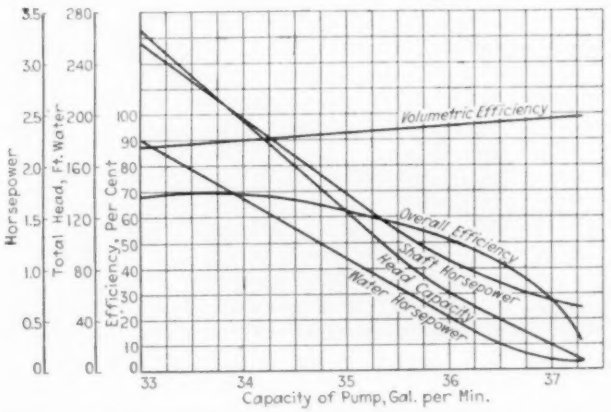


FIG. 14 CHARACTERISTIC CURVES FOR DISCHARGE RUN NO. 4 AT 700 R.P.M.

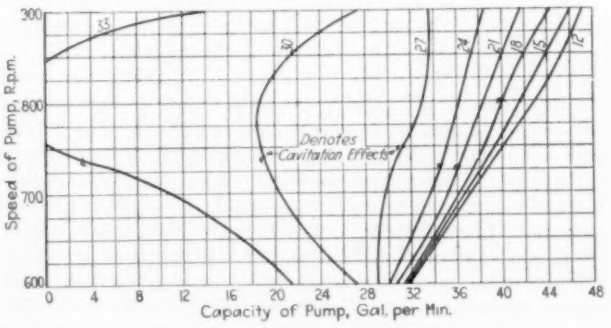


FIG. 15 CAPACITY-SPEED CURVES FOR SUCTION RUNS WITH DEFINITE HEAD CAPACITIES

decreases the capacity of the pump under certain conditions, lowers the volumetric efficiency, and decreases even further the overall efficiency.

Not only does this hammer lower the performance of the pump, but it also is objectionable in the noise that it makes, and difficulty is encountered under such conditions in maintaining tight lines and connections to the pump. Further, the loads im-

posed by such vibration may, under proper conditions, cause a failure in either the pump itself or in its connections. The determination of the nature of this "hammer," its cause, and the means of its elimination are taken up next.

Cause and Remedy for Hammering

To determine the nature, cause, and remedy for the hammering noticed in the tests, apparatus identical with that of the previous tests was used with the exception of:

1 *The pump* was considerably larger than that used in the preceding tests. It had a volumetric displacement of 66.1 cu. in. per revolution against a displacement of 12.5 cu. in. for the

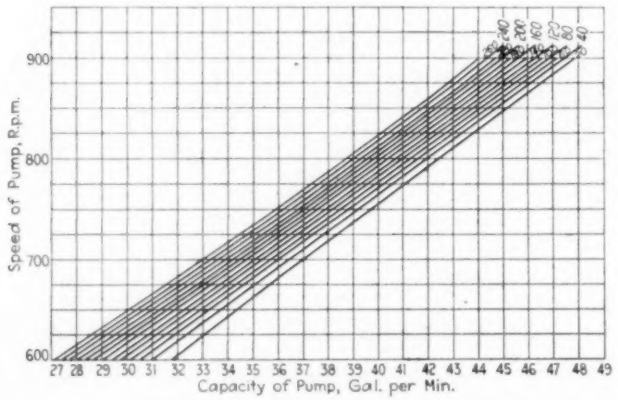


Fig. 16 Capacity-Speed Curves for Discharge Runs With Definite Head Capacities

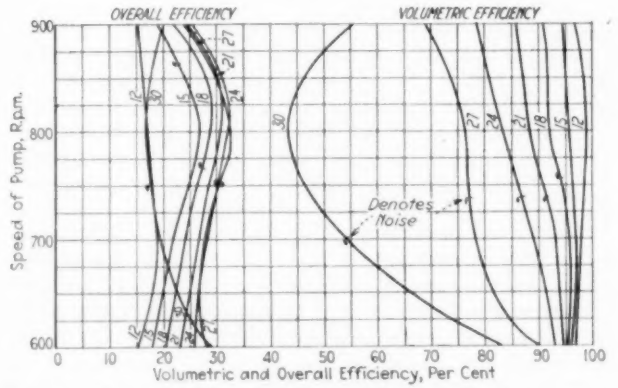


Fig. 17 Efficiency-Speed Curves for Suction Runs With Definite Head Capacities

first one used. The object of using this larger pump was to keep the blade velocities in the two tests as nearly equal as possible, and yet to have the pump running slowly enough in this second investigation to be able to attach an indicator. The casing of this pump was cast with $2^1/_2$-in. flanged connections permitting direct connection of the discharge and suction pipes. (See Fig. 4.)

2 *An indicator* was attached to the pump on the grease stud located in the center of the pump foreplate. This indicator was connected through a hole in the stub shaft to a groove in the shaft under the blade bushing, and from this groove through the blade to a position on its face as shown in Fig. 5. The cylinder movement of the indicator was taken from a cam located on the shaft at the opposite end of the pump. (See Fig. 6.)

The ordinary test was run on the pump as delivered from the factory, taking pressure readings with the indicator. Then such alterations were made in the pump as seemed advisable, and a comparative test was made to check the results of the alteration.

The first tests on the pump were run at a speed of 196 r.p.m.

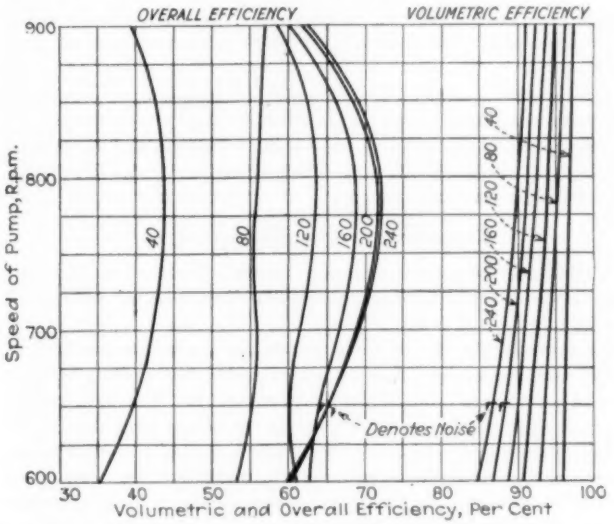


Fig. 18 Efficiency-Speed Curves for Discharge Runs With Definite Head Capacities

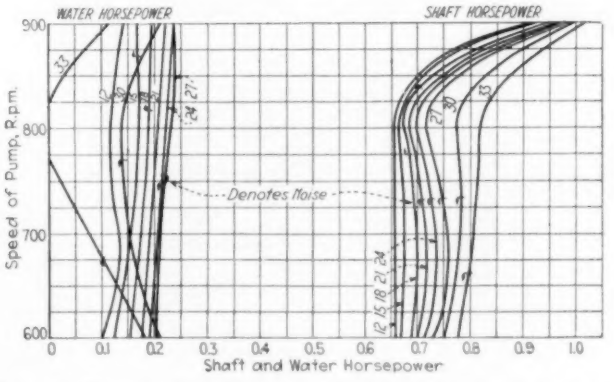


Fig. 19 Horsepower-Speed Curves for Suction Runs With Definite Head Capacities

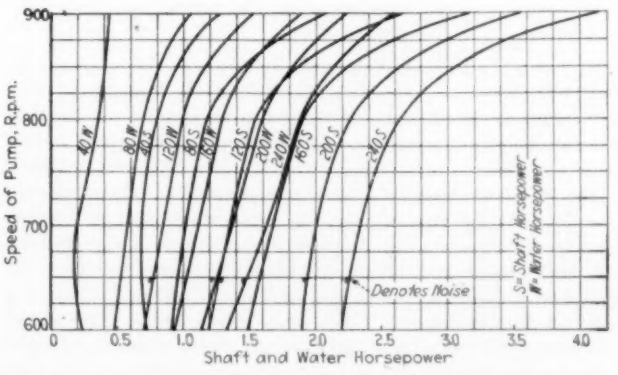


Fig. 20 Horsepower-Speed Curves for Discharge Runs With Definite Head Capacities

A five-minute interval was allowed for each test, during which time readings were taken in the same manner as in the preceding tests, with the exception of the speed determinations. In this case the count was taken over a period of one-half minute. Two

indicator cards were taken during each test as such intervals as to give a fair representation of the conditions. After the completion of the run the tapped blade was removed and reversed; that is, the pressure lead was placed on the suction side of the blade. With the blade in this position, the conditions for each test were duplicated and a "suction" card was taken for that test.

Data were taken under those conditions which would produce the worst hammer in the pump; that is, 27.3 in. of vacuum and varied discharge pressures. For this reason none of the actual values of the data taken in these tests are comparable with values from previous ones.

Results of Second Series of Tests

The data taken were plotted as curves of head capacity, shaft horsepower, water horsepower, volumetric efficiency, and overall efficiency against pump capacity.

Curves *B*, Fig. 21, with the pump set up as delivered, show that the hammering effect increases constantly as the discharge head is built up. There is a tendency also for the head capacity to fall off as this hammer increases at the higher pressures. This behavior indicates hydraulic loss, which is shown further in the drop of the efficiency curve and in the increase of the slope of the shaft-horsepower curve. Quite evidently, the hammering must be associated with loss of efficiency.

The indicator diagrams for this run, Figs. 22 and 23, are plotted to a scale in which each horizontal division represents a 30-deg. movement of the blade. This transformation from the

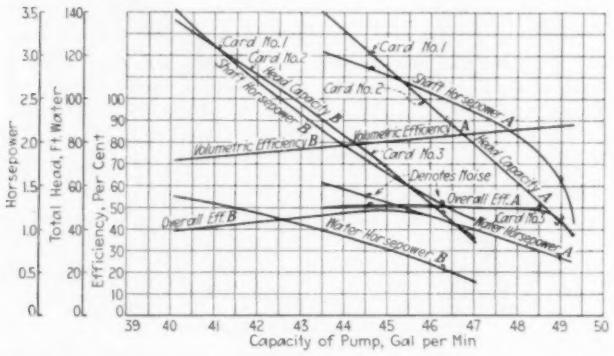


Fig. 21 Comparative Curves for Runs Nos. 5 and 6 at 196 R.P.M.

(*B* denotes run No. 5 before change, *A* denotes run No. 6 after change.)

card was made necessary through distortion of the original card by the harmonic drum motion, and further, the atmospheric line, due to leakage of the valve at the indicator, is not a reliable base for pressure measurements. However, considering the curves from a qualitative standpoint will give quite reliable indications.

Fig. 22 shows the curves from the pressure side. Starting with the left extreme representing top dead center and proceeding counter-clockwise, in the direction of rotation, it is seen that for the first 60 deg. the hole in the blade is covered by the brass cylinders. (See Fig. 5.) From 60 deg. to 140 deg. there is a marked drop in pressure. This drop occurs as the blade ahead of the tapped one moves through the region of maximum displacement increasing the volume between the blades and drawing water, as long as the suction is open, into the displacement chamber. When the tapped blade is at 110 deg., however, this increase of volume ceases. From here on the volume between blades is decreasing. The volume immediately in front of the tapped blade, however, increases until the position 180 deg. is reached. Since the decrease of volume between blades is very slight over the movements near bottom dead center, it seems reasonable to believe that the drop of pressure from 110 deg. to 150 deg. is due to the lag of the water in filling the space immediately in front of the tapped blade. Further, the decreased rate of drop tends to indicate this action, since the rate of increase of area is a harmonic wave and the counteracting effect, the decrease of volume between blades, is also. The difference of the two should give a drop as indicated. At 150 deg. the forward blade opens, dropping full discharge load upon the tapped blade.

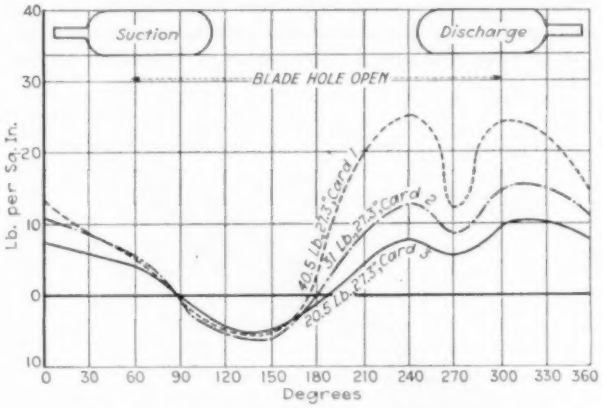


Fig. 22 Pressure Diagram for Run No. 5

The pressure builds up rapidly as the blade passes 180 deg., and the volume immediately before it starts to decrease also. By the time 210 deg. is reached, the inertia of this column is largely taken up, and the rate of increase of pressure starts to fall off. At 250 deg. the discharge part is reached. Now, the water finding an opening to the lower pressures behind the blade, rushes by it, dropping the pressure rapidly to 270 deg. At this position the following blade picks up the load and carries it upward until the hole is covered at 300 deg.

From these indications it seems that the hammer was produced as the blades picked up the discharge pressure of the foregoing blade. Such an effect, if strong enough to produce a hammer, should show on the suction card as an abrupt increase of pressure.

Fig. 23 shows the suction cards. There is only a slight indication of such an increase. At 250 deg., the point of maximum volume between blades, the suction pressure is a minimum. From there it builds up at a very moderate rate to the practically constant level of suction maintained throughout the rest of the card.

It must be remembered in analyzing this suction card that, as the pressure is dropped through to the next blade, the suction side may move away from the resulting pressure wave with sufficient speed to prevent a registration of this effect upon the indicator. If this is true, the only possibility of catching this effect upon the blade is to decrease the hammer and note the difference in the resulting curves. This is done in the curves shown in Figs. 24 and 25.

To decrease the hammering several methods were possible. The water column could be dropped more gradually through varied port shapes. Also, pressure between blades could be built up to the discharge pressure through varied discharge-port positions or through varied suction-port positions. This latter method was the one adopted.

It was reasoned that since the pump was drawing such a high vacuum with the suctions as arranged, the hammer effect, which seemed to be a cavitation at maximum displacement volume, and the losses due to slip, could be reduced considerably if the suction port was opened farther along the blade path. This would result in the maximum volume between blades being more

nearly filled, and in the pressure at the point of release being higher. Therefore, and to make some allowance for the evident effects of inertia of the water, the suction port was drained down to 120 deg. from top dead center.

With this port position the usual run was made and the results plotted as the *A* curves of Fig. 21. Many important changes are to be found under the new condition.

First, in spite of the fact that the suction port is open approximately 5 deg. after the discharge is opened, there is very little

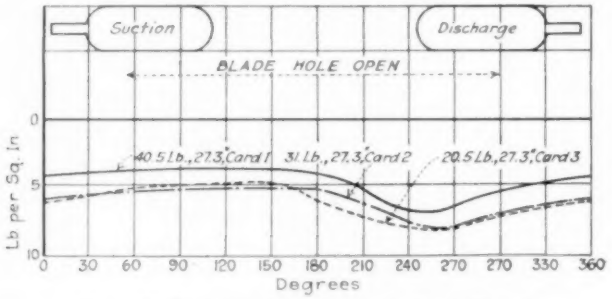


FIG. 23 SUCTION DIAGRAM FOR RUN NO. 5

tendency for the load to slip at the higher discharge pressures. This also indicates a complete occupation of the displacement volume before discharge, as one should expect with an open suction to that point.

Second, the water horsepower does not exhibit such a marked tendency to fall off at the higher loads. This is, of course, a direct result of the maintenance of the head capacity.

Third, the volumetric efficiency again shows the gain with an increase at the same heads of approximately 6 per cent.

Fourth, the overall efficiency exhibits two desirable characteristics. To start, the maximum efficiency is $2^1/_2$ per cent higher than the maximum efficiency of the previous run. It also maintains a high efficiency over a greater range. The difference lies largely in the fact that no additional losses are encountered at high pressures as were before.

Obviously, this port arrangement is superior to the original, and it seems to be because the space between the blades is fully charged with water as it is opened to the discharge. The indicator diagrams for this run (Figs. 24 and 25) check the theory.

The most noticeable feature of the first part of the pressure curve is the slight decrease of suction at low heads. The high heads seem to retain the same amount as before. Next it will be noticed that the high-pressure curves reach their maximums nearly 30 deg. before discharge. Not only does this indicate a full load of water between blades, but it shows that the pressure between the blades when relieved to discharge has been increased. It will be noted that the hammer effects are considerably less in this run, probably due to this result. Further, the drop in pressure between blade impulses is considerably decreased by this decreased discharge pressure.

If this blade displacement at discharge is more nearly full of water, and if the hammer is the cavitation between these blades, then the dip in the suction curve should be somewhat removed by the improved conditions. This we find to be the case. Also there is less tendency to draw vacuum as the blade swings by the suction port. This, while an advantage in its milder form, may become one of the limitations upon the continuation of improvement by the adjustment of the suction port.

There is another limitation to the increase of efficiencies possible by this port displacement in the slip of the water past the blades. Obviously, if the suction port is lowered too far along the periphery of the casing, the inertia of the water will no longer sustain the discharge pressure as the suction closes, but will allow some back flow into the suction line. The point at which the maximum discharge will be obtained is the point just before this back flow occurs, and will vary with the pressure and the speed of the pump. A location of this point can be made only by tests under the normal operating conditions.

CONCLUSION

In considering the last tests the conditions must be kept in mind. First, the tests were run at a speed considerably lower than those recommended for the pump. Second, conditions were maintained which were most detrimental to its operation. As a result, certain limitations must be placed upon the conclusions drawn from these discussions and the results must be taken to serve more as an indication of probable results than as an indication of the performance to be expected.

For instance, the performance of the pump when running under conditions which do not provoke hammering might not be im-

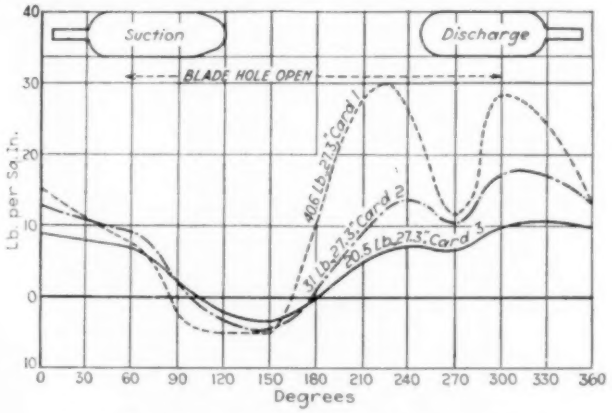


FIG. 24 PRESSURE DIAGRAM FOR RUN NO. 6

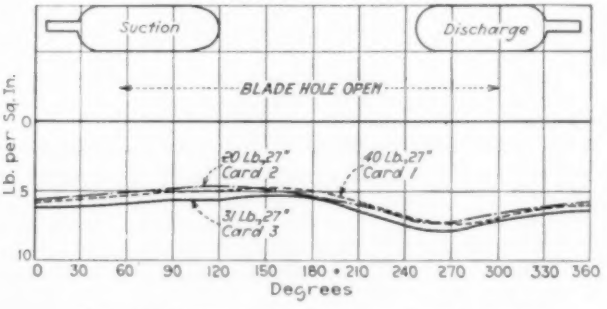


FIG. 25 SUCTION DIAGRAM FOR RUN NO. 6

proved by the port adjustment which showed so well in this test. On the other hand, if running at rated speed, 400 r.p.m., a movement of the port to even greater extremes might prove beneficial. These variations and many others are left undetermined due to the lack of time and the lack of equipment necessary for such an investigation.

Similarly, the problem of utilizing the inner pump, either as a booster or as an auxiliary, has not been touched nor has the effect of supporting the outer end of the rotor. In all these questions this paper must serve merely as a foundation for detailed work which may follow even as it has served as a foundation for the first step of the detailed solution of the hammer effect.

HYD-51-13

Mechanical Vibrations in Penstocks of Hydraulic Turbine Installations

By J. P. DEN HARTOG,[1] EAST PITTSBURGH, PA.

In this paper the author presents a theory showing that objectionable vibration phenomena are to be expected in the penstocks of Francis turbine installations when the number of buckets on the turbine runner is one less than the number of guide vanes, and that satisfactory operation should be obtained when this relation is avoided. Observations on eight different installations show good agreement with the theory.

IN THE penstocks and power houses of Francis turbine installations undesirable vibration phenomena have been observed from time to time which are known to be due to the play of the water on the buckets of the revolving runner. The subject was investigated by Mr. Roy Wilkins a number of years ago, and details of his work were published in Trans. A.I.E.E., 1923, pp. 1001, et seq. The frequency of the vibration was found by him to be the product of the revolutions per minute and the number of runner blades, and the trouble seems to have been cured by replacing the runner by another one with a different number of blades. The reason for this was unknown. The paper of Mr. Wilkins is the only one on this subject which has come to the author's notice. Regarding the choice of the number of runner buckets, there is a definite tendency among water-wheel-building concerns to avoid making this number equal to that of the stationary guide vanes, possibly on account of unpleasant experiences.

This paper is the result of certain work done in connection with a particularly troublesome installation in one of the Southern states. The vibration in that plant was reduced to satisfactory limits by replacing the existing runner by another one with a different number of buckets. A simple theory for the phenomenon was found, which is in good agreement with observations made on six other Francis installations in that neighborhood.

Theory

Fig. 1 represents schematically the scroll case and runner of a Francis turbine. The number of runner buckets n_1 is in general different from the number of guide vanes n_2. The stream of water entering the scroll case at the section AA splits up into n_2 partial streams, each of which enters the space between two guide vanes. Fixing our attention on *one* of these partial streams, it is seen that n_1 buckets pass it per revolution, so that the number of impulses per second given to it is: $n_1 \times$ (r.p.s.).

Since these impulses are not sinusoidal, higher harmonics of this frequency will also exist in the partial stream. The impulses travel away from the point of their generation with the velocity of sound in water, and finally arrive at the section AA of the penstock.

The water in the penstock at AA therefore receives impulses of the frequency $n_1 \times$ (r.p.s.) from n_2 different partial streams, which have to be added together with their proper phase relations. Under certain conditions most of these impulses will cancel each other, while in other cases they will add up to a considerable disturbance.

An expression for the phase difference between the impulses at AA of two adjacent partial streams will now be derived.

The angle between consecutive buckets is $360°/n_1$, and the angle between guide vanes is $360°/n_2$. If bucket 1 lines up with guide vane 1 (Fig. 1), the angle between bucket 2 and guide vane 2 is:

$$\left(\frac{1}{n_2} - \frac{1}{n_1}\right) \times 360° = \frac{n_1 - n_2}{n_1 n_2} \times 360°$$

The interval between the impulses on two adjacent streams is equal to the time it takes the runner to travel through this angle, namely,

$$\frac{n_1 - n_2}{n_1 n_2} \times \frac{1}{\text{(r.p.s.)}} \text{ seconds}$$

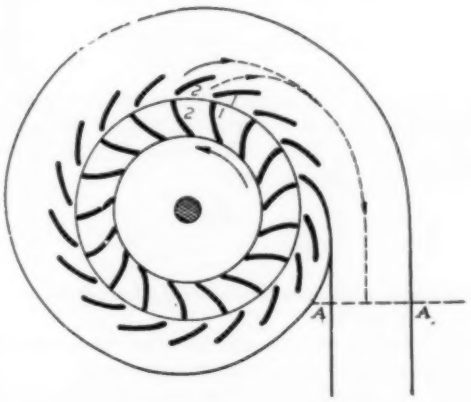


Fig. 1 Schematic Representation of Turbine Scroll Case and Runner

Let the difference in the paths of the impulses on the two streams be a feet, and the velocity of sound in water be v feet per second. It takes a/v seconds for the wave front to travel this extra distance, since the actual stream velocity of the water in the scroll case is so small as to be negligible with respect to the velocity of sound. The total time lag between two adjacent impulses at AA is therefore

$$\frac{n_1 - n_2}{n_1 n_2} \times \frac{1}{\text{(r.p.s.)}} + \frac{a}{v} \text{ seconds}$$

Since the frequency of the vibrations is $n_1 \times$ (r.p.s.), the phase difference between the impulses is found to be:

$$\frac{\varphi}{360°} = \frac{n_1 - n_2}{n_2} + \frac{n_1 a}{v} \times \text{(r.p.s.)} \quad \ldots\ldots\ldots [1]$$

In deriving this formula no definite assumption has been made regarding the path of travel of the waves from the spot of their generation to the section AA. This phenomenon is fairly complicated, consisting of many reflections against the sides, but it can be safely said that the main wave front in which we are in-

[1] Research Laboratory, Westinghouse Electric & Manufacturing Company.

Contributed by the Hydraulic Division and presented at the Annual Meeting, New York, N. Y., December 2 to 6, 1929, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

TABLE 1 DATA ON TURBINE PENSTOCK VIBRATIONS

Turbine No.	n_1	n_2	R.p.m.	Length of center line of scroll case, ft.	Wave length of vibration in water, ft.	Frequency of vibrations per sec.	φ deg.	Observed vibration amplitude at penstock, mils
1	17	18	450	41	35	127.5	3	0.55
2	16	18	450	41	37	120	18	0.25
3	18	24	514	34	29	154	72	0.03
4	21	24	514	34	25	180	25	0.03
5	19	20	154	77	92	49	3	0.83
6	16	26	200	63	85	53.3	128	0.31
7	18	20	171.4	65	88	51.5	23	0.00
8	19	20	171.4	75	83	54	2	....

terested here will move along the scroll case toward the penstock. In this case we find approximately for the distance a

$$a = \frac{2\pi R}{n_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [2]$$

where R is the radius of the center line of the scroll case, which varies but slightly throughout the scroll case.

In case the phase angle φ of Equation [1] approaches 0 deg. or 360 deg., the various impulses will be in phase and the resulting disturbance in the penstock will be considerable.

The number of guide vanes or runner blades on existing Francis turbines is in the neighborhood of 18, and the diameter is about inversely proportional to the speed. Due to this the length of the center line of the scroll case is approximately equal to one wave length of the vibration in most installations, or

$$2\pi R = \frac{v}{n_1 \times (\text{r.p.s.})} \quad \ldots\ldots\ldots\ldots\ldots\ldots [3]$$

in which the velocity of sound in the scroll case is taken to be 4500 ft. per sec. Table 1 shows the limit of accuracy within which this statement holds. Combining [1], [2], and [3], the following simple expression results:

$$\frac{\varphi}{360^\circ} = \frac{n_1 - n_2 + 1}{n_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots [4]$$

The phase angle φ becomes zero when the numerator $n_1 - n_2 + 1$ vanishes, or, in words:

Penstock vibration is to be expected when the number of runner buckets is one less than the number of guide vanes.

It can be also seen from [4] that in case the number of runner buckets is two less than or equal to the number of guide vanes, $\frac{\varphi}{360^\circ} = \frac{1}{n_2}$. Representing the various impulses by vectors will yield a diagram like Fig. 2, in which the resultant is zero, while in the previous case the vectors would all lie in a straight line and add up to one long resultant.

When there are one more or three less runner buckets than guide vanes the angle is $\frac{360^\circ}{2n_2}$; again giving a vector diagram like Fig. 2, with the difference that we go around the circle twice instead of once only.

The conclusions can be drawn, that for an equal number of runner buckets and guide vanes no particular penstock vibration trouble is to be expected. However, in such a construction the impulses occur on all runner buckets at the same time and the torque may become quite non-uniform. This may lead to non-uniformity of rotation, or in special cases to torsional vibrations between the generator and the turbine. The reactions of such an oscillating torque will set up vibration in the power house, but *not* in the penstock. Troubles experienced from this cause are probably responsible for the fact that the combination is avoided in recent constructions.

It is to be emphasized once more that the fundamental formula [1] as well as the conclusions deduced from it are only approximations and can represent only the principal traits of the phenomenon. The main inaccuracy lies probably in the assumption that the disturbances travel as units along the center line of the scroll case. The actual condition differs from this assumption in two ways. In the first place, there will be some damping of the waves in the scroll case, so that an impulse generated deep in the narrow end (Fig. 1) will arrive at AA with less intensity than one generated in the wide part. This will change the circular vector diagram of Fig. 2 into a spiral one like that shown in Fig. 3.

Then there will be diffusion. An instantaneous disturbance inside the penstock will not arrive at AA as an instantaneous occurrence. Also, the "main" effect will be instantaneous at AA, but after this there will be a train of disturbances which have traveled longer paths and have undergone one or more reflections against the walls. This tends to degenerate individual vectors of Figs. 2 or 3 into a "diffused vector" such as that represented by Fig. 4. The details of this process are too complicated to follow, but it is believed that the simple result [1] and the conclusions therefrom are sufficiently accurate, since observations made on several plants are in good agreement with it.

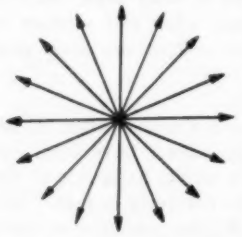


FIG. 2 VECTOR DIAGRAM OF IMPULSES WHEN NUMBER OF RUNNER BUCKETS IS TWO LESS THAN OR EQUAL TO THE NUMBER OF GUIDE VANES

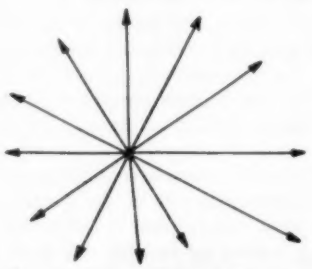


FIG. 3 SPIRAL VECTOR DIAGRAM

The vector representation can be applied only to pure sinusoidal impulses, while the actual disturbances will have higher harmonics in them. These can be considered separately and can be brought within the scope of [1] when it is considered that the second harmonics of the impulses of a runner with n_1 buckets act exactly in the same manner as the fundamental harmonics for a hypothetical runner with $2n_1$ buckets. It is found that in general these higher harmonics are of no particular importance.



FIG. 4 "DIFFUSED VECTOR" DIAGRAM

The most direct way in which to eliminate penstock vibration is of course to so design the shape of the blades as to make the individual impulses small. However, since efficiency and other considerations are of far greater importance in determining the shape of the blades, this manner of attack does not seem quite practical, especially since satisfactory operation can be secured by simply avoiding a number of buckets one less than or equal to the number of guide vanes.

EXPERIMENTAL VERIFICATION OF THEORY

The measurements of the vibration in the various hydroelectric installations tested were made by means of a simple seismic vibrograph, which was placed on the penstocks or in other suitable places near the turbine. This instrument records the vertical vibrations magnified ten times in the form of an indentation in a clear celluloid strip. A second line on the same strip carries timing marks. These records are hardly visible with the naked eye, but can be conveniently magnified. On the job we use for this purpose a hand microscope of 30 diameters magnification which is carried with the instrument and makes

it possible to immediately inspect the record, the total magnification thus being 300 times. A scale inside the microscope also shows the absolute magnitude of the amplitude of the vibration. This is more convenient than an oscillograph because the record can be immediately examined and another made in case it is not satisfactory. Later, at the office, the celluloid strips can be microscopically photographed at practically any magnification;

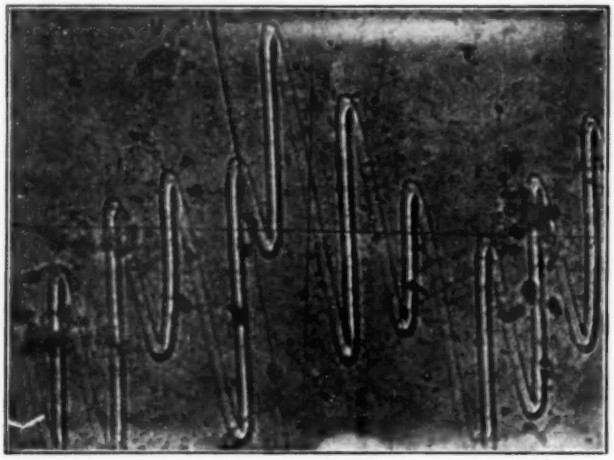


FIG. 5 VIBROGRAPH RECORD SHOWING VERTICAL VIBRATION OF PENSTOCK OF A TROUBLESOME TURBINE

the thickness of the line being the only limitation. Fig. 5 shows such a record of the vertical vibration made on the penstock of a troublesome turbine (No. 5 of the Table 1); Fig. 6 is a similar record on the penstock of a turbine with but little vibration (No. 4).



FIG. 6 VIBROGRAPH RECORD OF PENSTOCK OF A TURBINE OPERATING WITH BUT LITTLE VIBRATION

The results of the investigation on eight Francis turbines are shown in Table 1, to which the following remarks apply. Turbines Nos. 1 and 2 are the same installation with different runners. In fact, this particular station gave rise to the whole investigation, because the vibration was considered dangerously large. As a result of the analysis described in the paper, the original 17-vane runner was replaced by a 16-vane one of the same diameter and having the same clearances, which led to a decided improvement in operation. This improvement is not so apparent from the reading on the penstock itself; however, the vibration of the concrete inside the power house near the penstock entrance was reduced from 2.40 mils to 0.20 mil.

Turbines Nos. 1, 2, 3, and 4 are directly comparable because they operate under approximately the same head (600 ft.) and the penstocks are open, i.e., not buried in concrete.

Turbines Nos. 5, 6, 7, and 8 operate under a lower head (200 ft. approximately); Nos. 5 and 6 have open penstocks, No. 7 has the penstocks partly concreted in, while No. 8 has them completely concreted in. Therefore no measurement could be made on the penstocks of No. 8, but the vibration could be found on the concrete structure at the headgates and even as far as the top of the middle of the dam.

The frequency of the vibration was in all cases proportional to the product of the number of revolutions and the number of runner buckets, regardless of the difference between n_1 and n_2. This is not so clear from the records which Mr. Wilkins obtained.

In general, the observations have so far been in fair agreement with the theory. The author has had no opportunity to check an installation with the same number of buckets and vanes. It would be of considerable interest to know the nature of the trouble experienced in such cases.

ACKNOWLEDGMENT

The author wishes to express his indebtedness to the engineers of the Aluminum Co. of America, particularly Mr. J. P. Growdon, for their valuable help and cooperation.

Discussion

R. WILKINS.[2] In addition to the paper in the *Transactions* of the A.I.E.E. to which reference has been made, there was published by a subcommittee of the N.E.L.A. Hydraulic Committee of 1925–26 a report on vibration in which Fig. 7 was included.

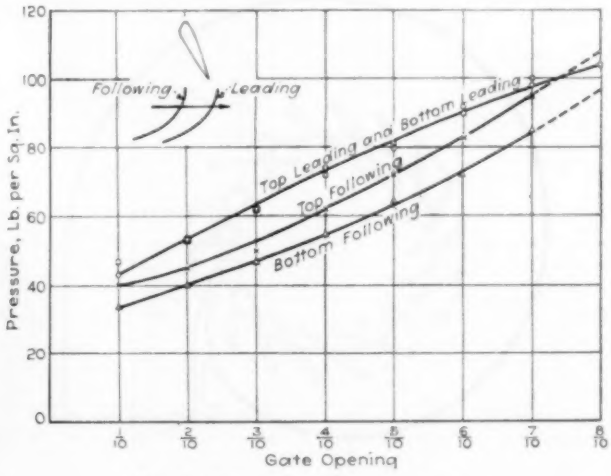


FIG. 7 PRESSURE AT RUNNER INLET
(Using explorer pipe in runner filled with air to minimize leakage and to eliminate centrifugal action.)

The values plotted are for a 40,000-hp. Francis-type turbine wherein the difference in pressure as plotted represents a considerable portion of the total head on the units. This is the fundamental reason for the resulting vibration, and in practice was partly cured by cutting off some two inches of the outer edge of the runner blades, thereby giving more clearance and a better chance for the pressure values to equalize. Contrary to expectations, this raised the efficiency of the unit. As stated in the paper, the runner was later replaced by one having a different number of vanes with still more improvement.

High-pressure differences across the face of the runner opening are due to design and are frequently encountered where the physical dimensions are reduced in order to save cost. Juggling runner and guide-vane combinations is only an expedient, not a cure, since a cure requires the removal of the cause.

A great deal of disagreeable, though not often dangerous,

[2] Consulting Engineer, Pacific Electric Mfg. Co., San Francisco, Cal.

vibration comes from draft-tube action, and this is far harder to remedy. The so-called breathing of the penstock can usually be traced to the draft tube, where the pulsations may be as slow as one per second.

One of the most difficult cases to analyze that has come to attention was caused by the balance chambers in a runner wherein, when the wearing rings wore down, the clearance was about the same as the leakage clearance. This gave an intermittent action with some 0.03 in. side motion on the turbine shaft. Replacing the wearing rings cured this trouble completely.

It is not usually sufficient to study vibration alone when hydraulic "trouble shooting;" pressure variations from various places also are desirable. Any one of several types of equipment is available, though modern oscillographs with the commercial carbon-pile telemeters form the most flexible arrangement for visual use and allow a permanent record being taken wherever desirable.

A. V. KARPOV.[3] The theory developed in the paper has a much wider application and is of much more importance than is modestly admitted by the author. The case referred to in the paper, where the disturbance that produces the vibration was due to the shape of the guiding vanes, represents only one of the many possible cases.

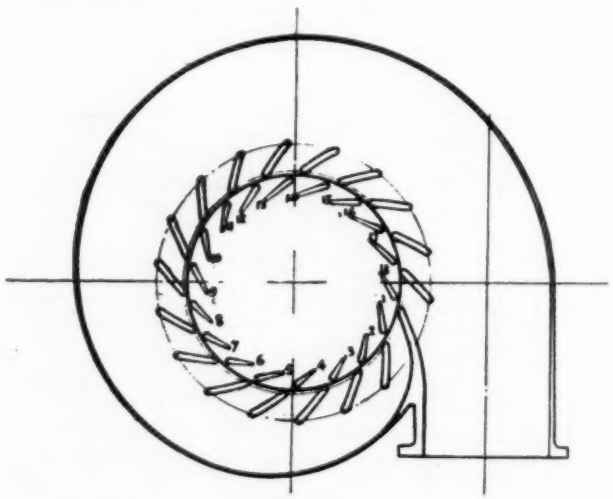


FIG. 8

Any kind of disturbances that are regularly occurring at the runner blades can accumulate and give a resultant heavy vibration either at the penstock or in the scroll case of the turbine. For instance, cavitation disturbances at the runner can produce similar vibrations in the penstock and would be subject to the same theory. It is of importance to keep that in mind, in order to avoid the possible mistake of attributing vibrations similar to that described by the author to a similar cause.

If the vibration is due to the shape of the guiding vanes, then a simple changing of the number of runner buckets will be a sufficient cure for all troubles. If the vibrations are due to cavitation disturbances, then a change in the number of runner buckets along the lines suggested by the author will eliminate the vibration, but the turbine will still be unsatisfactory due to cavitation and resulting pitting, and the fundamental cure of the troubles would be rather in eliminating the cavitation than in the simple change of the number of runner vanes.

Another interesting problem in connection with the theory developed by the author is the influence of the damping. The author mentioned the damping and the diffusion as some of the causes that would result in a difference between the figured and measured values, but does not attempt to evaluate the influence of these factors, which in some cases may be important enough to modify the conclusions.

An attempt was made to evaluate the comparative amplitudes of penstock vibrations, applying the author's theory, but taking into account the disturbing effects of damping and diffusion. The turbine mentioned in the paper under Nos. 1 and 2, with 18 guide vanes and 450 r.p.m., was investigated, and the comparative amplitudes of vibrations of the water column of the

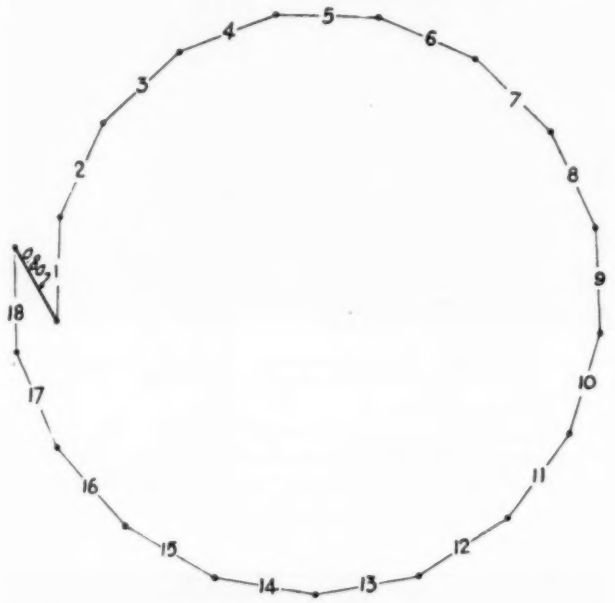


FIG. 9

scroll case and penstock were evaluated for runners with different numbers of buckets.

A graphical method was chosen, and a number of vector diagrams were drawn. In these diagrams it was assumed that a disturbance of the amplitude equal to unity was produced every time when the edge of the runner vane was passing the edge of a guiding vane.

Two sets of vector diagrams were drawn, one disregarding the damping and the other one considering the damping.

The first set of diagrams was drawn, starting with the guide vane shown as 1 on Fig. 8; the path that the disturbance has to travel until it reaches the center line of the scroll case and meets the disturbance coming from guide vane 2 was scaled and the time figured assuming the velocity of propagation of the disturbance of 4700 ft. per sec. The lag in time between the first and second impulses was figured, the path and the time necessary for the second disturbance to meet the first one was determined, and assuming sinusoidal impulses the vector of the second impulse was added to the vector of the first impulse with the proper phase angle; to this resultant vector the vector of the third impulse was added with its proper phase angle, and so on.

In these diagrams the connecting line between the starting point and any of the points of the diagram will represent the amplitude of the vibrations at the corresponding point of the scroll case, and the closing line of the diagram from starting point to the end of the vector 18 will represent the amplitude of the vibrations at the penstock. The values obtained in this

[3] Designing Engineer, Hydraulic Department, Aluminum Company of America, Pittsburgh, Pa.

way correspond to the values that were figured by the author analytically.

Figs. 9, 10, and 11 show such vector diagrams for 16-, 17-, and 19-bucket runners.

The second set of diagrams was drawn in the same way, but the damping and diffusion were taken care of, reducing by a certain amount the size of every resultant vector.

Figs. 12, 13, and 14 show such vector diagrams for the same turbine and for 16-, 17-, and 19-bucket runners in which every resultant vector was reduced by 10 per cent, which would mean that between every two impulses a 10 per cent dissipation of the vibration energy took part.

Referring to these figures, the conclusion that can be drawn from the diagrams is that, regarding or disregarding the damping and diffusion, the largest amplitude of vibrations will occur at the penstock for a 17-bucket runner.

Regarding the best possible combination with reference to the smallest vibration amplitude at the penstock, the 16-bucket runner will give the best result, if the damping is disregarded; and if the damping is considered, the best result will be obtained by using a 19-bucket runner. In contrast with the 17-bucket runner, in the 16- and 19-bucket runners the maximum amplitude of vibration will occur in the scroll case, but not in the penstock. For a 16-bucket runner, the maximum vibration that occurs in the scroll case is larger than for a 19-bucket runner. A still more interesting conclusion that can be drawn from this diagram is that the damping and diffusion decrease the vibration amplitude at the penstock for a 17- and 19-bucket runner, but considerably increase it for a 16-bucket runner.

Some time after these investigations were made, the field measurements which the author made on this turbine with 16-, 17-, and 19-bucket runners gave an exceptional opportunity to compare the theoretical and actual values.



Fig. 10

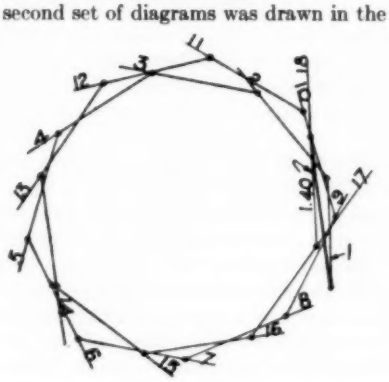


Fig. 11

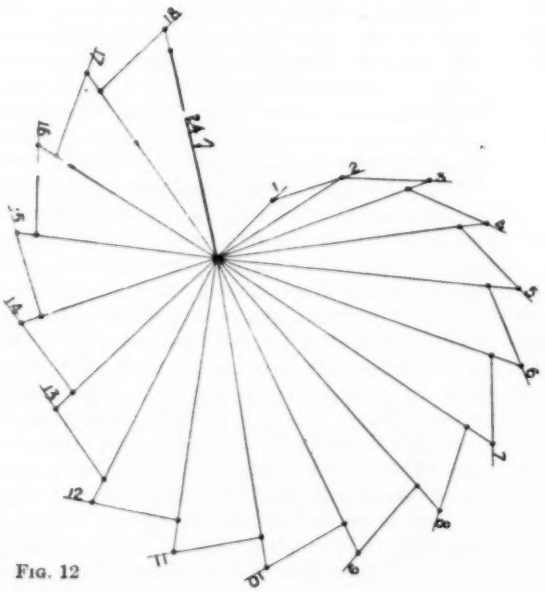


Fig. 12

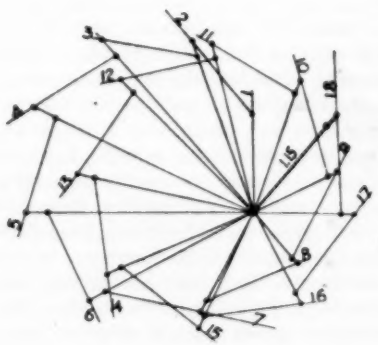


Fig. 14



Fig. 13

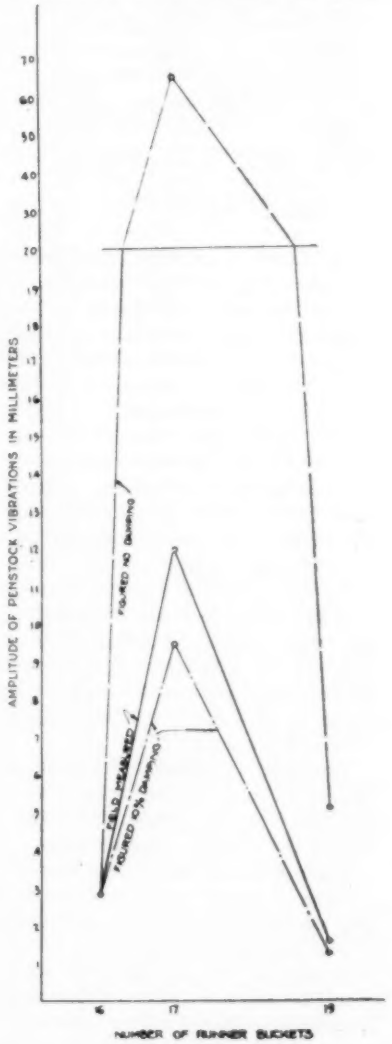


Fig. 15

In order to make such a check, the amplitudes of vibration at the penstock scaled on each set of diagrams were multiplied by a coefficient so as to have the amplitude of the vibrations of the 16-bucket runner equal to 29 mil., the value that was obtained by actual measurement.

Table 2 shows the measured and the theoretical values.

TABLE 2 AMPLITUDES OF VIBRATIONS AT THE PENSTOCK

Number of runner buckets	16	17	19
Double vibration amplitude measured at the penstock in the valve house, expressed in millimeters	0.29	1.20	0.16
Vibration amplitudes at the penstock scaled on the diagrams, disregarding damping	0.80	17.90	1.40
The same vibration amplitudes expressed in millimeters (coefficient, 0.362)	0.29	6.48	0.51
Vibration amplitudes as scaled on the diagrams, assuming 10 per cent damping between impulses	2.4	7.8	1.15
The same vibration amplitudes in millimeters (coefficient, 0.121)	0.29	0.95	0.14

The values of deflection in millimeters for the three conditions —no damping, actually measured, and 10 per cent damping between impulses—are shown on the diagram, Fig. 15. The amplitudes figured from diagrams disregarding damping and the measured amplitudes are so different for a 17-bucket runner that it was necessary to change the vertical scale of the diagram to keep it to a reasonable size.

The results figured for 10 per cent damping are amazingly close to the ones actually measured.

A further study of runners with a number of buckets larger than 19 or smaller than 16 did show that the amplitudes of vibrations at the penstock as well as the maximum amplitudes are decreasing rapidly.

For this particular kind of turbine, it can be stated that the combination of 18 guide vanes and 17 runner vanes represents the worst possible combinations so far as the vibration amplitudes in the scroll case and penstock are concerned. A 16-bucket runner represents a decisive improvement. A 19-bucket runner is still a better combination and is practically as good as a 15-bucket runner.

The general statement that was made in the paper, that a number of runner vanes equal to the number of guiding vanes less one is a bad combination, could be modified by stating that, if the number of runner vanes is to be close to the number of guiding vanes, then for the kind of turbines investigated the number of runner vanes equal to the number of guiding vanes plus one represents the best combination.

The investigation carried out for the turbines can be applied to pumps as well, but since the sense of revolution of a pump would be opposite to that of the turbines, for a pump of similar design the number of runner vanes equal to the number of guiding vanes plus one will represent the combination that ought to be avoided.

The amplitude of vibrations measured at the concrete floor of the power house does not show such a striking agreement with the theoretical values. Quite a number of additional factors involving the concrete structure enter which probably change the situation considerably.

One point that is of the utmost importance with all vibration studies is to be emphasized. The theory brought up by the author and all conclusions based on this theory are of practical importance as long as the vibration frequencies imposed on the structure are out of harmony with the natural frequencies of the whole structure or a part of it.

By changing the number of runner blades, the vibration frequencies will change, and if the new frequencies harmonize with some of the natural frequencies, then, although the original trouble is eliminated, some new disturbances may appear. Since the evaluation of the natural frequencies of vibration is a rather complicated procedure and in many cases it is impossible to obtain reliable figures, the making of a recommendation that involves a change of frequencies of vibration involves a certain gambler's chance that sometimes has to be taken.

It is impossible to refrain from admiration of the simplicity and usefulness of the theory established by the author, which cleared up the problem that used to be so very obscure.

LEWIS F. MOODY.[4] The author has pointed out some considerations affecting vibration of turbines, penstocks, and surrounding structures which are not believed to have heretofore occurred to those investigating such problems. His method of analysis is interesting and ingenious, and gives another factor to take into account in handling vibration problems.

Fortunately serious mechanical vibration, of a character which either endangers the life of structures or involves actual operating difficulties or seriously objectionable noise, is a rather rare phenomenon among the great number of hydraulic power installations extant; and consequently many of those working in the hydraulic field, including the writer, have had no occasion or opportunity to take up the problem actively. When serious vibration does occur, however, it may indeed be serious, so much so that it may be considered inadvisable to operate a unit at all until the condition is cured. It may therefore be well for those who have not been faced with the problem as a question of cure to give it due study as a matter of prevention.

It is the feeling of the writer that the most important consideration is that of eliminating or at least minimizing the initiation of vibrations at their source. This is a matter of design of guide vanes, runner vanes, their interaction, and the form of transition space between them. The author, however, seems to view this possibility with some pessimism, for he says: "since efficiency and other considerations are of far greater importance in determining the shape of the blades, this manner of attack does not seem quite practical. . . ." There are probably cases where vibration is of such serious consequence that efficiency and other considerations become relatively of minor importance instead of "far greater importance;" but the point which the writer wishes particularly to bring out is that the author seems to imply some incompatibility between smooth operation and efficiency. The writer's view is that the same design factors which lead to avoidance of vibration are also necessary for high efficiency.

If the water can be conducted through the guide vanes and then formed into an integral, uniformly moving ring of fluid before entering the runner, steady conditions of entrance will be assured. Breaking the flow up into separate jets with decided velocity variations around the circumference will be conducive to vibration, and particularly a design which permits the formation of voids or cavities in the flow between successive jets. In pronounced cases of vibration the writer should suspect the presence of such voids or cavities, since any such void, becoming filled with vapor instead of water, immediately becomes a source of pressure waves or water hammer. In cases of severe vibration it would seem likely that another undesirable accompaniment would be pitting. The writer should like to ask the author whether the severe cases of vibration which he reports showed any indications of pitting.

The author's analysis is directed to the vibration caused by pressure waves traveling through the water in the turbine casing toward the penstock. He also mentions the vibrations which travel directly into the metal structure of the turbine and then

[4] Consulting Engineer, Cramp Morris Industrials, Inc., Philadelphia, Pa. Mem. A.S.M.E.

into the foundations. His ingenious theory for the pressure-wave phenomon results in the conclusion that "vibration is to be expected" when there is one less runner vane than the number of guide vanes. The second kind of vibration just mentioned would of course be most intense when the runner and guide vanes are equal in number.

That it is possible to design runner vanes, guide vanes, and transition space so as to avoid vibration (and without sacrifice of efficiency), irrespective of the numbers of vanes used, would seem to be borne out by the records. The writer has hastily reviewed the statistics of I. P. Morris turbines, and while the great majority of the installations do not exemplify the unfavorable vane numbers, he finds that 31 units, aggregating about 860,000 hp., embody the author's condition of "expected vibration;" namely, number of runner vanes one less than number of guide vanes. He also finds one case of numbers of guide and runner vanes equal. In not one of these installations has any vibration been reported. Naturally some amount of vibration must exist in any moving machinery; but this incidental vibration is not what is meant by either the writer, or, it is believed, by the author of the paper.

As a matter of detail, it would appear to the writer from a rough calculation that the approximate formula [3] in the paper would be substantially correct for heads of around 600 ft., but for heads of 200 ft. or lower, the first member of the equation would not be much more than one-half the second member, for usual turbine proportions. In such cases, therefore, the relation derived would be modified to give a difference between n_2 and n_1 of one-half instead of one; so that the use of $n_2 - n_1 = 1$ might be sufficiently remote from the condition of exact synchronism of arrival of pressure waves at the penstock to avoid actual trouble.

It is hoped that the author may continue his studies in this field, and it is believed that he has given a useful method of attack and has pointed out some interesting considerations.

GEORGE A. JESSOP.[5] The author has offered a most interesting paper, with an admirably presented theory, explaining the cause of a certain type of vibration which has, in a few instances, caused trouble in the operation of hydraulic turbine installations and which is apparently present, to some extent, in all high-head plants. The investigations were made very carefully and thoroughly, and the gathering of data at other plants to check the theory is a particularly valuable feature.

Vibrations of hydraulic turbine installations may be classified in two groups. The first group will include the type of high-frequency vibration described by the author. The second group will include vibration caused by disturbances in the flow, the most frequently encountered instances being caused by the draft tube. In general, these vibrations are of low frequency. It is only the first group that will be discussed.

It is of interest to note that the turbines investigated by the author were all of a comparatively low-speed type, the specific speed being in each case 50 or less. Usually the types of turbine made to cover this range of specific speed are designed in such a manner that the gates are closer to the buckets than for the higher specific speed Francis runners. Since low specific speed runners are generally used under heads higher than 150 ft., the r.p.m. and the peripheral velocity are high, the number of buckets is usually greater than for low specific speed turbines, and, therefore, there are a larger number of buckets passing a given gate per second. If vibrations are set up, they are of high frequency. It is necessary to keep such vibrations at a very low amplitude in order to have entirely satisfactory operation.

[5] Chief Engineer, S. Morgan Smith Co., York, Pa.

Installation No. 1 of Table 1, where the primary investigation was made, has a turbine with a specific speed of about 26 and the normal head is 630 ft. The power house contains two units, without provision for any additional units in the future, and therefore has less mass than the usual power house with three or more units. The draft tubes are essentially of the vertical type discharging into large tail pits, each pit having openings on two sides, thus necessarily still further reducing the foundation supports. The penstock consists of pipe line and tunnel, with a large surge tank about 1000 ft. from the power house. The pipe line between the surge tank and the power house and for several hundred feet upstream from the surge tank to the first tunnel is not buried in concrete or earth, except for a short distance at and near the power house. Much of the pipe between the surge tank and the power house is supported on high bents affording little resistance to vibration. The turbines themselves are free from troublesome vibration.

Plant No. 5, which shows a vibration amplitude greater than that of plant No. 1, also has an exposed pipe line. Plant No. 8, which has been in satisfactory operation for about ten years, is a four-unit power house, with the supply pipe entirely buried. Both of these plants have turbines with the number of buckets one less than the number of gates; that is, $n_2 - n_1 = 1$, the combination found to be undesirable for plant No. 1.

For practical reasons, various bucket and gate combinations have been used by hydraulic-turbine manufacturers. There are many plants with $n_2 - n_1 = 1$, but as far as is known to the writer the only two having this bucket and gate combination which have had sufficient vibration to cause an investigation are the one discussed by Mr. Wilkins in the *Transactions* of the A.I.E.E., 1923, and referred to by the author, and the plant on which the present primary investigation was made. It is of interest to note that there has recently been installed a turbine having a 17-bucket runner and an 18-gate case on the same system where the trouble was experienced leading to Mr. Wilkins' investigation. This company was very insistent that the later installation be free from vibration, and, having been forewarned, both manufacturer and purchaser were particularly careful to be certain that this trouble should not develop. The installation has now been operating in an entirely satisfactory manner for several years and is reported to be absolutely free from vibration troubles of any kind. It is only a two-unit plant, but with elbow draft tubes which permit a rigid foundational support. The penstock is long and strongly supported where it is not entirely buried.

There are a fairly large number of installations having the same number of buckets and gates. In the writer's personal experience there is only one of these units which is reported to be operating in a manner that is not entirely satisfactory. This particular installation has been in service nearly ten years, but recent reports indicate that there may be vibration of the turbine itself. This would tend to support the author's theory that an equal number of buckets and gates have a bad effect on the turbine, but not on the pipe line.

Another installation not in the writer's personal experience, but concerning which he has received authentic reports, had the same number of buckets and gates and there was troublesome vibration both on the turbines and pipe line. A new runner was installed having the number of buckets one more than the number of gates, with very good results. Later it was desired to increase the horsepower somewhat, and to do this a runner was used having the number of buckets one less than the number of gates. Troublesome vibration in the pipe line did not occur, although the pipe is exposed.

A large number of high-head installations have been made with the number of buckets one more than the number of gates,

and, in agreement with the author's theory, without vibration trouble developing.

A very large number of plants operating under heads of 100 ft. and less are in service, with turbines having $n_2 - n_1 = 1$. These power houses are of various degrees of massiveness and rigidity, and at least some of them have exposed penstocks. Of the hundreds of installations of this kind with which the writer has been connected, there is not one where vibration of the type under discussion is known to be taking place.

The author's investigation and the experience of the art show that, where heavy power houses and buried pipe lines are used, this type of vibration has not been of sufficient amplitude to be troublesome—at least to the extent of suggesting investigation or of requiring alterations. The paper suggests to the engineer and designer the possibility of making power houses lighter and with less foundational support than has been the custom in the past. It is believed that the saving which can thus be effected is not a sufficiently large proportion of the total expense of the installation to justify the assumption of any risk. Therefore, before advocating the decreasing of the mass of the power house and using lighter foundations, with greater spacing for pipe-line supports, it would seem that the problems involved should be more fundamentally explored, as has been elsewhere recommended by the author. Hydraulic-turbine design and engineering have advanced to the extent that entirely satisfactory Francis turbines can be built without using the gate and bucket combinations that have been found undesirable according to this paper, and this can be done without adding any new limitations of head, horsepower, speed, and efficiency.

The author has made a valuable contribution to the knowledge of the performance of hydraulic turbines.

ARNOLD PFAU.[6] The author deserves high commendation for his attempt to mathematically explain these troublesome phenomena.

There is general belief in the saying that there is no rule without exception, although the writer is convinced that the apparent "exceptions" are not real exceptions to the immutable laws of nature involved, but are rather due to incomplete knowledge of some laws of nature involved in the problem. This may to some extent justify or explain the statements made by some that the rule laid down by the author does not always prove to be correct and that, in fact, many turbines are operating without vibration, although $n_2 - n_1 = 1$.

Undoubtedly other factors, not cited in the paper, also have an effect upon the results. The writer remembers particularly an experience back in 1907 when he had the pleasure of starting the 10,000-hp. horizontal-shaft, single-discharge Francis turbine at the Centerville Plant of the California Gas and Electric Corporation, now the Pacific Gas and Electric Company of San Francisco.

That turbine was a pioneer in the application of a reaction turbine to a head as high as 580 ft., and naturally offered a number of interesting problems, or "nuts to crack."

Operating one morning at fixed-gate opening (governor blocked), the writer noticed a periodic "hum" and a corresponding swing on the pressure gage located on top of the spiral casing. This swing amounted to as much as $2^1/_2$ to 3 lb. per sq. in., or 1 to $1^1/_4$ per cent of the operating penstock pressure. Since there was no fluctuation in the head or tailwater levels, it was concluded that the cause must come from an outside source and not from within the turbine. The swing was around 43 and 44 per minute, which could not be harmonized with the speed of 450 r.p.m., or the number of guide vanes (24) or runner vanes (21).

Luckily, the operator happened to make the statement that they were pulling a heavy load at the Oakland Plant. This gave a clue, and upon further inquiry, it was learned that, due to a breakdown of one of the hydro plants of the system, their gas engine units at Oakland were pulling a heavy overload. The speed of those engines, if it is remembered correctly, was 87, and they being of the two-cycle type, it at once appeared that the 43 to 44 swings per minute were identical with the cycle of the gas engines.

It is said that the engines have since been removed from the system. Evidently on account of their lack of sufficient flywheel effect, they periodically accelerated the speed of the entire system at the beginning of the cycle, and caused a dying down of it to the end of the cycle. The turbine, operating in parallel, was naturally subjected to this swing. In other words, a periodic, relative motion existed between the rotation of the runner and the guide vanes, and since the guide vanes were set for a constant opening, the discharge through these openings was affected accordingly, causing a periodic change of velocity in the penstock which resulted in the pressure variation observed on the gage. The interpretation of the phenomena proved to be correct, in that these variations immediately stopped when the gas engines were shut down.

This experience goes to show that many factors are involved in the problem under discussion. The author has made a successful beginning of theoretical analysis, and it is to be hoped that the problem can be properly analyzed as facts are added from field experience.

BLAKE VAN LEER.[7] One of the chief functions of The American Society of Mechanical Engineers is the dissemination of new and interesting information on technical subjects. This function the paper aids in an interesting, concise manner. Wherever there is trouble with mechanical installations, there is a tendency to suppress this information or to minimize it. Such a procedure may have some temporary commercial advantages to the particular manufacturer of the equipment which causes the trouble. However, it is better to face these difficulties and problems, many of which may be insurmountable or may merely be "the fortunes of war," which any one might encounter, and to freely and frankly discuss them in an open forum of technical men, than to permit a whispering campaign to go on which frequently grossly exaggerates the difficulties and their causes.

It is presumed that the author, in order to avoid any semblance of criticism of any manufacturer or group of engineers, omitted from Table 1 the name and location of the specific installations in which vibrations were studied. This, of course, is not a serious loss, because any engineer intimately interested could no doubt secure this information from the author. In the light of this, the writer wishes to commend the Hydraulic Division, the A.S.M.E., and the author for presentation of this interesting and thought-provoking paper.

D. J. MCCORMICK.[8] The author is to be commended for his fine paper which brings up a subject of timely interest. It is evident that he has obtained some valuable field data on the subject of mechanical vibrations of hydraulic-turbine installations, but since it was impossible for him to obtain data covering all combinations of number of runner vanes as compared to the number of gates used in turbine practice, the theory which he

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has applied to the data he obtained and his conclusions may be at variance from the real facts.

For example, the writer's experience has been that a runner with 16 vanes installed in a horizontal cast-iron spiral-casing turbine having 16 gates showed very serious vibration at the turbine and in the penstock. The penstock was exposed, and wires were strung both at the penstock and at the turbine to obtain the frequency of the vibrations which corresponded with the number of runner vanes, or number of gates, times the r.p.m. When a runner having 17 vanes was installed in this same casing with the 16 gates, the vibration at the turbine and in the penstock was entirely eliminated. However, this runner with 17 vanes did not show as much power as a runner with only 16 vanes. A 15-vane runner was then installed in this case with 16 gates, and there was no more vibration than with the 17-vane runner, either at the turbine or in the penstock. According to the author's theory, this 15-vane runner installed in the case with 16 gates should show very serious vibration in the penstock, but in reality it does not. Manifestly there is something wrong in his theory.

Referring to Fig. 1 of this paper, it is very doubtful whether the vibrations follow along the nice flow lines which the author has indicated. Undoubtedly, the vibrations skip across the casing and possibly right across through the runner in order to take the shortest course. This may account for the exceptions to his theory which will arise, the same as the case cited. A great deal of additional field data will be necessary in order to check up and prove out any theory which is proposed to apply for all cases of this nature.

There are a large number of turbine installations in this country and in Canada having 18 gates and 17 runner vanes. This combination has been very frequently used. However, nearly all of the large vertical installations have had the penstocks well anchored and a very substantial foundation provided for the power house so that any vibrations set up were dampened out to the extent that no serious damage would ever be expected.

In quite a few of these power houses some vibration may be found at some point in the power house, and it is the exception rather than the rule to find a power house that is entirely free of such vibration.

The question has been raised as to whether the vibration in the penstock set up by a certain combination of runner vanes and gates will cause any damage in the future due to crystallization. The writer would cite an installation made in California of two 8-ft. diameter exposed penstocks made of $^{5}/_{16}$-in. at plate the upper end where they are under a head of about 75 ft. These penstocks are vibrating sidewise from $^{1}/_{8}$ to $^{1}/_{4}$ in., depending upon the load carried. They have been operating for nine years and are still in good shape.

The high-frequency vibrations set up in penstocks due to such a combination as the author cites are not nearly of the same amplitude as the one given. They are only 0.01 of 1 in., as compared to the foregoing amplitude of $^{1}/_{8}$ to $^{1}/_{4}$ in. It is to be seen that the stresses set up in the pipe are correspondingly less, so that it is very doubtful, in the writer's opinion, whether any serious trouble should be expected in the future from such vibrations, even though they are of high frequency.

It is to be hoped that the hydraulic engineers will take an active interest in this subject and work out some method of locating penstock piers and anchorages in order to dampen out the vibrations which may be set up.

Lyle A. Whitsit.[9] The author presents valuable results of a research in hydraulic-turbine operation and an effective method of analysis that should be useful to turbine designers and manufacturers. He brings out the adaptability of methods commonly used in electrical problems, that also are serviceable in hydraulic problems.

The author indicates that, where the number of runner buckets is one less than the number of guide vanes, vibration may be expected, and in general that less trouble is to be expected if the relation of buckets to vanes is any other than this. However, among the latter there are instances where excessive vibration exists. The writer knows of one case possessing the favorable relation of 17 runner buckets to 16 guide vanes where there is vibration in the penstock. In this the superposition of impulses may be due to the dimensions and shape of the scroll case and to reflected pressure waves.

There is a distinction to be made in the effects that these impulses produce on the penstock. They may cause only a vibratory increase in pipe diameter though still maintaining the true circular outline of the pipe, or it may accentuate a deformation of the circular outline of the pipe and make the deformation vibratory—otherwise called a breathing of the penstock. At any one point in the pipe the two effects are combined into one result. Either component of the total effect may be the more evident, depending upon the position in the penstock—that is, either near the power house or near the upstream end of the penstock.

In breathing vibrations, the initial or static deformation of the pipe's cross-sectional outline is primarily due to the difference of the hydraulic pressure at the bottom and at the top of the cross-section of the pipe. During the operation of the turbine the static deformation becomes vibratory. The amplitude of motion depends on the ratio of diameter of the pipe to the pressure head on the center of cross-section. During the process of filling the pipe it flattens and reaches a maximum deformation when the level of entering water just reaches the top of the cross-section. It then decreases as the pipe continues to fill—that is, as the differential pressure from top to bottom of the cross-section becomes a less proportion of the total head on the center. The panting would, therefore, be more evident in a large penstock, especially under low head, than in a small penstock under high head.

In the first effect the expansive vibration of the circular outline may only be a few thousandths of an inch, corresponding to several thousand pounds per square inch increased stress. In the vibratory deformation the stress is caused by the bending action of the plate; that is, the main stress of tension is increased on one side of the plate and decreased on the other side. That is to be distinguished from that of the first case where uniform increased tension occurs throughout the entire perimeter of the section and uniformly across the thickness of the plate. Here again the increase in maximum stress from vibrating deformation may be only a few thousand pounds per square inch.

The writer knows of one case in which the amplitude of the breathing amounts to about $^{1}/_{2}$ in., and which seems to have a frequency bearing a definite relation to the speed of the generating unit and which went through repeated uniform cycles of amplitude.

It is conceivable that, in an installation possessing unfortunate conditions as to relative number of buckets and guide vanes and other dimensional conditions of the scroll case and speed of unit, the alternating stresses in either of these two cases might reach overstressed amounts.

The writer believes that the panting action during the operation of the unit receives its impulse from the same causes involved in the author's problem and that such means as are adopted to prevent or decrease such impulses will be beneficial also in correcting the breathing action of the penstock.

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BYRON E. WHITE.[10] The author has presented a valuable contribution to the literature of hydraulic engineering in clarifying and calling attention to the important subject of vibrations in penstocks. It is fortunate that serious difficulties arise in but few installations. Where such does occur, however, the researches described in the paper will be of great value.

Referring to Mr. Jessop's discussion, in which he calls attention to the massiveness of the usual power-house foundation, in which the turbine scroll cases are solidly encased in concrete as a means of damping out and preventing serious vibrations, the writer wishes to raise a question.

It seems to be customary to employ masses of concrete, sometimes running into many hundreds of cubic yards, in power-house foundations. There are many situations where this is entirely logical, particularly where bedrock is excavated very close to minimum clearances from the necessary water passages, etc., and but little excess concrete is needed to completely fill the voids. On the other hand, there are many situations, particularly in the case of soft ground conditions, where any reduction in mass and weight would reduce the cost of the foundation, particularly where piling is necessary.

In these latter cases, the writer would like to raise the question as to whether it would not be possible to employ very light structures, perhaps even closely analogous to the structural steel foundations frequently employed for steam turbines. The answer undoubtedly lies in the possibility of likelihood of the development of serious vibrations. It would no doubt be rather hazardous to attempt to apply this experimentally in a large installation, since the cost of remedying the conditions, should it prove bad, would be expensive both in construction cost and in the output lost during the down time of the unit, while the work was being done. If there are any cases where this method of constructing a foundation has been either successfully or unsuccessfully tried out, the power industry would be greatly benefited by having a complete recital of the facts made available.

AUTHOR'S CLOSURE

The objection has been made by Messrs. Wilkins and Moody that the method of solving the problem by properly choosing the number of vanes in the runner is not satisfactory. It is said that a more fundamental manner of attacking the problem would be to redesign the shape of the blades so as to eliminate all nonuniformity in pressure. I agree with this completely. Only it must be remembered that the design of a turbine to fit these requirements seems not possible yet with the present state of our knowledge, and a cure in the direction suggested entails hydrodynamical researches of a high order, which take a long time. The solution offered in this paper is a temporary one; as long as perfect turbines cannot be designed, the procedure proposed in the paper will obviate difficulties due to vibration in the penstock. That the perfect turbine in which no pressure variations exist will also have good efficiency is quite probable.

Mr. Moody asks whether it was found that turbines with bad penstock vibration showed at the same time large cavitation. No sufficient data to answer this satisfactorily are known to me, but it might be mentioned that the runner of turbine No. 1 (Table 1) after three months of operation had no signs of cavitation, while the runners of turbines No. 3 and 4 were regularly destroyed by cavitation in about two years.

This would indicate no relation as Mr. Moody suspects, but it is emphasized again that drawing a conclusion from two casual observations would be premature.

In the same light we have to see the case cited by Mr. McCormick in which an installation acted contrary to the theory given here. Apparently no measurements of the vibration were made, and the conclusions were drawn by comparing the actual state of vibration with one which had existed several days or weeks before. Such comparisons depend to a great extent on psychological factors. It is possible that cases will be found contrary to the theory, but I cannot accept Mr. McCormick's statement as a refutation of it. The records taken on the eight installations stated in Table 1, as well as all records published in Mr. Wilkins' article, are in agreement with the theory.

Mr. Moody's objection against the validity of Equation [3] may be true. The equation is meant to be a rough approximation, and the four last cases in Table 1, having heads of about 200 ft., are in fairly good agreement with it. In general, for any individual case it will be better to refer to the more general Equation [1] of the paper.

Mr. Karpov has given a valuable extension to the theory by making a very reasonable assumption about the damping. The influence of the damping in this case is to level down the sharp points of the theory, and thus has the same effect as damping has in other branches of engineering—for instance, in electric circuits.

Interesting remarks have been made by Messrs. White, McCormick, and Jessop on the question of the desirability of lighter concrete foundations than are now in use. An indiscriminate application of light foundations is certainly not commendable, but on the other hand it seems strange that heavy foundations should be necessary in order to insure satisfactory operation.

It seems logical that in the future the foundations of all rotating machinery, including hydraulic turbines, will be made lighter as the art of eliminating vibration advances.

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